An Undulating Surface Model for the Gliding Motion of Bacteria

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Fluid properties

kinematic viscosity

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A THESIS SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF THE ***MASTER OF SCIENCE*** IN ***MATHEMATICS***

We accept this thesis as conforming to the required standard.

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Chapter 1

Preliminaries

The objective of this chapter is to introduce the reader with general concepts and equations related to the mechanics of fluids. Specific terms used in the subsequent chapters are also explained.

1.1 Definitions

1.1.1 Undulating surface

An undulating surface is a surface having wave like motion.

1.1.2 Bacterial gliding

Bacterial gliding is a process whereby a bacterium can move under its own power. This process does not involve the use of flagella which is more common means of motility in bacteria.

1.1.3 Motility

The ability of an organism to move by itself is called motility.

1.1.4 Slime

Slime is a viscoelastic material that is composed of macromolecular polysaccharides, glycoproteins and proteins in an aqueous medium. A thin fluid layer of slime is released by the bacterium into the extracellular environment for the transmission of forces to the substrate

1.1.5 Fluid

Fluid is a material that flows

1.1.6 Flow

A material goes under deformation when different forces act upon it. if the deformation continuously increases without limit then the phenomenon is known as flow

1.1.7 Velocity field

The velocity of a fluid is a vector field represented by

$$\mathbf{V} = \mathbf{V}(X, Y, Z, t) \tag{1.1}$$

which gives the velocity of an element of fluid at a position (X, Y, Z) at time t

In component form for two dimensional planar flow we can write it as

$$\mathbf{V} = [U(X, Y, t), V(X, Y, t), 0] \tag{1.2}$$

where U and V are the components of velocity field along X- and Y- directions respectively

1.1.8 Slip condition

Slip condition states that the velocity of the fluid in contact with the boundary is not same as that of boundary. Alternatively one can say that slip condition is applicable when the velocity of fluid is not equal to the velocity of the wall

1.1.9 Streamline

A streamline is a curve which is everywhere tangent to the velocity vector at a given instant of time

1.2 Fluid properties

1.2.1 Viscosity

The force which resists the motion of a fluid is known as viscosity. Mathematically, viscosity is the ratio of shear stress to the shear strain i.e.

$$\mu = \frac{shear\ stress}{shear\ strain}$$

where μ is called the coefficient of viscosity.

1.2.2 Density

Density of a fluid at a point is defined as the limit of mass per unit volume when the volume V shrinks to zero. Mathematically, it can be defined as

$$\rho = \lim_{V \to 0} \left(\frac{m}{V}\right)$$

where m is the mass of the fluid.

1.2.3 Pressure

Pressure is defined as the normal force F per unit area acting on a surface S. As pressure may vary on a surface, therefore it is defined using limit process at a point i.e.

$$P = \lim_{S \to 0} \left(\frac{F}{S}\right)$$

1.2.4 Kinematic viscosity

The kinematic viscosity is the ratio of dynamic viscosity μ to the density of the fluid. It is denoted by ν and given by

$$\nu = \frac{\mu}{\rho}$$

1.3 Types of Flow

1.3.1 Compressible flow

Flow in which density of fluid is not constant during the flow is called compressible flow

1.3.2 Incompressible flow

Flow in which density of fluid remains constant during the flow is called incompressible flow

1.3.3 Steady flow

A flow in which properties associated with the motion of fluid are independent of time is called a steady flow. If r represents any fluid property, then for steady flow

$$\frac{\partial r}{\partial t} = 0$$

1.3.4 Unsteady flow

All the flows in which properties associated during the motion depends on time are called unsteady flows. For unsteady flows

$$\frac{\partial r}{\partial t} \neq 0$$

1.4 Classification of fluids

1.4.1 Ideal fluid

An ideal fluid is one that is incompressible and has no viscosity. It is also known as inviscid fluid

1.4.2 Viscous fluid

All fluids for which dynamic viscosity is not zero are known as viscous fluids or real fluids. Viscous fluids are further classified as Newtonian and Non -Newtonian fluids

1.4.3 Newtonian Fluids

Fluids which obey Newton's law of viscosity are known as Newtonian fluids. According to Newtons law of viscosity shear stress is directly propotional to the rate of the deformation or shear strain. In steady uni-directional flow between parallel wall due to the motion of the wall Newtons law of viscosity becomes

$$\tau_{XY} = \mu \left(\frac{dU}{dY} \right), \tag{1.3}$$

where τ_{XY} is the shear stress and Y is transverse coordinate.

1.4.4 Non-Newtonian Fluids

Fluids in which shear stress is not directly propotional to deformation rate are known as Non-Newtonian fluids. For such fluids

$$\tau_{XY} = k \left(\frac{dU}{dY} \right)^{n}, \quad n \neq 1$$

or

$$\tau_{XY} = \eta \left(\frac{dU}{dY} \right)$$

where η is called the apparent viscosity, k is the consistency index and n is the flow behavior index.

1.5 Dimensionless number

1.5.1 Reynolds number

It is defined as the ratio of the inertial force to the viscous force. It is denoted by Re and is given as

$$Re = \frac{\rho V L}{\mu} = \frac{V L}{\nu}$$

where V is the characteristic velocity and L is the characteristic length.

1.5.2 Wave number

It is defined as ratio of mean distance to the wavelength of the wave. Mathematically it can be written as

$$\delta = \frac{2\pi h_0}{\lambda}$$

where h_0 denotes the mean distance and λ is the wavelength of the wave.

1.5.3 Amplitude ratio

It is defined as the ratio of the amplitude of the wave to the mean distance. It is denoted by ϕ. If a represents the amplitude of the wave, then

$$\phi = \frac{a}{h_0}$$

1.5.4 Deborah number

It is the ratio of material time constant to the characteristic time scale of the flow.

1.6 Governing equation

1.6.1 Law of conservation of mass

This law states that the mass of the closed system always remain constant with time. The mathematical relation expressing law of conservation of mass is known as the continuity equation.

For incompressible fluid, continuity equation becomes

$$\nabla \cdot \mathbf{V} = 0 \qquad (1.6.1)$$

1.6.2 Law of conservation of momentum

This law is defined as the total momentum of an isolated system is always conserved. For an incompressible fluid, the equation of motion in vector form is given as

$$\rho\frac{d\mathbf{V}}{dt} = \nabla \cdot \mathbf{T} + \rho\mathbf{b} \tag{1.7}$$

where $\mathbf{T}$ is the Cauchy's Stress tensor and $\mathbf{b}$ is the body force.

1.7 Method of Solution

1.7.1 Perturbation Method

This is one of the oldest technique to solve the non-linear partial differential equations of fluid mechanics in which one of the physical parameter can be assumed very small. In this method unknown variable is expanded in term of the small parameter. This assumed solution is then substituted in the original equation to get linear problems by comparing the various powers of the small parameter. The solution is usually given by few term of expansion. In the following a two term solution of an algebraic equation is presented. The expansion is carried out in term of small or large parameter which appears in the equation.

Suppose we have an equation

$$x^2 + \epsilon x - 1 = 0 \qquad\qquad \epsilon << 1 \tag{1.8}$$

We assume the expansion as

$$x = x_0 + \epsilon x_1 + \epsilon^2 x_2 + \ldots \tag{1.9}$$

Using eq (1.9) into eq (1.8) we get

$$(x_0 + \epsilon x_1 + \epsilon^2 x_2 + \ldots)^2 + \epsilon(x_0 + \epsilon x_1 + \epsilon^2 x_2 + \ldots) - 1 = 0$$

Now equate the coefficients of various power of ϵ on both sides to get

At ϵ^0 $\qquad x_0^2 - 1 = 0 \Rightarrow x_0 = \pm 1$

At ϵ^1 $\quad 2x_0x_1 + x_0 = 0 \quad x_1 = -\frac{1}{2}$

At ϵ^2 $\quad x_1^2 + 2x_0x_2 + x_1 = 0 \quad x_2 = \pm\frac{1}{8}$

Thus, when $x_0 = 1$, from eq (1.9) gives

$$x = 1 - \frac{\epsilon}{2} + \frac{\epsilon^2}{8} + O(\epsilon)^3$$

Similarly, for $x_0 = -1$, the solution is

$$x = -1 - \frac{\epsilon}{2} - \frac{\epsilon^2}{8} - O(\epsilon)^3$$

Chapter 2

Motion of gliding bacteria on a third order slime

In this chapter, motility of gliding bacteria is studied employing an undulating surface model on a layer of slime. The rheology of the slime is characterized by the constitutive equation of a third order fluid. The flow field due to the undulating glider is determined using perturbation technique. The speed of glider is determined as a function material parameters of the slime.

2.1 Mathematical Formulation

Let us consider that an organism is gliding over a flat surface due to its undulatory motion at a distance h_0 from the surface. Let X-axis be along the surface parallel to the organism and Y-axis be perpendicular to the surface. The gap between the organism and the flat surface is filled with a thin layer of slime released by the organism. The motion in the slime is generated due to the undulations of the glider surface. The undulation of the organism surface are described by the equation

$$h(X,t) = h_0 + a \sin\left[\frac{2\pi}{\lambda}(X - (c - V_g)t)\right] \tag{2.1}$$

where c is the speed of the undulation, V_g is the speed of the glider, a is the amplitude and λ is the wavelength. The flow configuration is illustrated in Fig. 2.1.

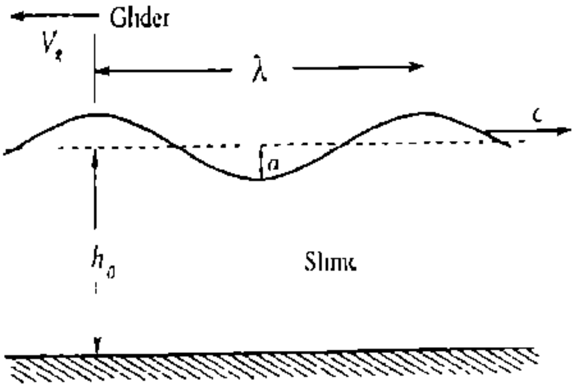

Fig. 2.1 Geometry of the undulating surface model

The equations governing the flow of slime are

$$\nabla \cdot \mathbf{V} = 0 \qquad (2.1)$$

$$\rho \frac{d\mathbf{V}}{dt} = \nabla \cdot \mathbf{T} + \rho \mathbf{b} \qquad (2.2)$$

The Cauchy stress tensor $\mathbf{T}$ for the third order fluid is given as

$$\mathbf{T} = -pI + \mathbf{S} \qquad (2.3)$$

where

$$\mathbf{S} = \eta_0 \mathbf{A}_1 + \alpha_1 \mathbf{A}_2 + \alpha_2 \mathbf{A}_1^2 + \beta_1 \mathbf{A}_3 + \beta_2 (\mathbf{A}_1 \mathbf{A}_2 + \mathbf{A}_2 \mathbf{A}_1) + \beta_3 (tr\, \mathbf{A}_1^2) \mathbf{A}_1 \qquad (2.4)$$

In the above expression α_1, α_2, β_1, β_2 and β_3 are material constants. The Rivlin-Ericksen tensors $(\mathbf{A}_n)$ are defined by the recursive relation

$$\mathbf{A}_n = \frac{d\mathbf{A}_{n-1}}{dt} + \mathbf{A}_{n-1}(\nabla \mathbf{V}) + (\nabla \mathbf{V})^T \mathbf{A}_{n-1}, \quad n > 1 \qquad (2.5)$$

The appropriate velocity field for the flow under consideration is

$$\mathbf{V} = [U(X, Y, t), V(X, Y, t), 0] \tag{2.6}$$

In view of the above definition of the velocity field, Eqs. (2.1) and (2.2) become

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2.7}$$

$$\rho\left(\frac{\partial U}{\partial t} + \left(U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}\right)U\right) + \frac{\partial P}{\partial X} = \frac{\partial S_{XX}}{\partial X} + \frac{\partial S_{XY}}{\partial Y} \tag{2.8}$$

$$\rho\left(\frac{\partial V}{\partial t} + \left(U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}\right)V\right) + \frac{\partial P}{\partial Y} = \frac{\partial S_{XY}}{\partial X} + \frac{\partial S_{YY}}{\partial Y} \tag{2.9}$$

where S_{xx}, S_{xy} and S_{yy} are the components of extra stress $\mathbf{S}$.

The boundary conditions in the fixed frame (X, Y) are

$$U = -V_g \quad at \quad Y = h \tag{2.10}$$

$$U = 0 \quad at \quad Y = 0 \tag{2.11}$$

The transformations relating coordinates, velocities and pressures between moving and fixed frame are given by

$$x = X - (c - V_g)t, \quad y = Y,$$

$$u = U - (c - V_g), \quad v = V, \quad P = p \tag{2.12}$$

where (u, v) are the velocity components in the moving frame.

Employing the transformations Eqs. (2.7)-(2.9) reduce to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.13}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) + \frac{\partial p}{\partial x} = \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} \tag{2.14}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) + \frac{\partial p}{\partial y} = \frac{\partial S_{xy}}{\partial x} + \frac{\partial S_{yy}}{\partial y} \tag{2.15}$$

The above equations can be casted in dimensionless form by defining the following variables and parameters

$$
\begin{aligned}
x^* &= \frac{2\pi x}{\lambda}, \quad y^* = \frac{y}{h_0}, \quad u^* = \frac{u}{c}, \quad v^* = \frac{v}{c}, \\
h^* &= \frac{h(x)}{h_0}, \quad p^* = \left(\frac{2\pi h_0^2}{\lambda \eta_0 c}\right) p(x), \quad \mathbf{S}^* = \left(\frac{h_0}{\eta_0 c}\right) S(x)
\end{aligned}
\tag{2.16}
$$

Making use of the above variables, Eqs (2.13)-(2.15) in dimensionless form become

$$\delta \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.17}$$

$$\mathrm{Re}\left(\delta u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \delta \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y}, \tag{2.18}$$

$$\delta\,\mathrm{Re}\left(\delta u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \delta^2 \frac{\partial S_{xy}}{\partial x} + \delta \frac{\partial S_{yy}}{\partial y} \tag{2.19}$$

where asterisks have been dropped for brevity.

Defining the stream function $\psi(x, y)$ by the relations

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\delta \frac{\partial \psi}{\partial x} \tag{2.20}$$

Eqs. (2.18) and (2.19) takes the form

$$
\begin{aligned}
\delta\,\mathrm{Re}\left[\left(\frac{\partial \psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}\right)\left(\frac{\partial \psi}{\partial y}\right)\right] + \frac{\partial p}{\partial x} &= \delta \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} && (2.21) \\
-\delta^3\,\mathrm{Re}\left[\left(\frac{\partial \psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}\right)\left(\frac{\partial \psi}{\partial x}\right)\right] + \frac{\partial p}{\partial y} &= \delta^2 \frac{\partial S_{xy}}{\partial x} + \delta \frac{\partial S_{yy}}{\partial y} && (2.22)
\end{aligned}
$$

where $\delta = \frac{2\pi h_0}{\lambda}$ and $\mathrm{Re} = \frac{\rho c h_0}{\eta_0}$ are the wave number and Reynolds number, respectively. It is pointed out that continuity equation (2.17) is satisfied identically by defining the stream function. Eliminating the pressure term from Eq. (2.21) and (2.22), the following equation emerges

$$\mathrm{Re}\,\delta\left[\left(\frac{\partial \psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}\right)(\Delta^2 \psi)\right] = \left[\left(\frac{\partial^2}{\partial y^2} - \delta^2 \frac{\partial^2}{\partial x^2}\right)(S_{xy})\right] + \delta\left[\frac{\partial^2}{\partial x \partial y}(S_{xx} - S_{yy})\right] \tag{2.23}$$

where

$$\nabla^2 = \delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

is the modified Laplacian operator

For the subsequent analysis, it is assumed that the wave number δ is very less than unity i.e.

$$\delta << 1 \tag{2.24}$$

Under the above mentioned assumption which is also known as long wavelength assumption Eqs (2.21) and (2.23) contract to

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y} S_{xy}, \tag{2.25}$$

$$\frac{\partial^2}{\partial y^2} S_{xy} = 0 \tag{2.26}$$

The tangential and normal stresses under long wavelength assumption are

$$S_{xy} = \frac{\partial^2 \psi}{\partial y^2} - 2\Gamma \left(\frac{\partial^2 \psi}{\partial y^2} \right)^3 \tag{2.27}$$

$$S_{yy} = (2\lambda_1 + \lambda_2) \left(\frac{\partial^2 \psi}{\partial y^2} \right)^2 \tag{2.28}$$

$$S_{xx} = 0 \tag{2.29}$$

where

$$\Gamma = \gamma_2 + \gamma_3,$$

$$\gamma_2 = \frac{\beta_2 c^2}{h_0^2 \eta_0} \qquad \gamma_3 = \frac{\beta_3 c^2}{h_0^2 \eta_0}$$

$$\lambda_1 = \frac{\alpha_1 c}{h_0 \eta_0} \qquad \lambda_2 = \frac{\alpha_2 c}{h_0 \eta_0}$$

The boundary conditions (2.10) and (2.11) transform to

$$\frac{\partial \psi}{\partial y} = V_b \quad at \quad y = 0 \tag{2.30}$$

$$\frac{\partial \psi}{\partial y} = 1 \quad at \quad y = h \tag{2.31}$$

where

$$h(x) = 1 + \phi \sin x$$

and

$$V_t = \frac{V_g}{c} - 1$$

Upon substituting the value of S_{xy} in (2.26), it turns out that resulting equation is a fourth order differential equation. For unique solution of this equation the boundary conditions must be four in number. But at present, we have only two of them given by (2.30) and (2.31). To development of the other two conditions, we proceed as follows.

The dimensional flow rate in fixed frame is defined as

$$Q = \int_0^h U(X, Y, t)\, dY \tag{2.32}$$

where h depends on both X and t.

The corresponding flow rate in wave frame is defined as

$$q = \int_0^h u(x, y, t)\, dy \tag{2.33}$$

where h is a function of x only.

From Eq. (2.32) and (2.33), the following relation can be derived

$$Q = q + (c - V_g)h \tag{2.34}$$

The average value of Q over one time period is

$$\bar{Q} = \frac{1}{T} \int_0^T Q\, dt \tag{2.35}$$

Insertion of Eq. (2.34) into (2.35) gives

$$\bar{Q} = q + h_0(c - V_g) \tag{2.36}$$

The normalized version of the above equation is

$$\Theta = \Gamma - V_b \tag{2.37}$$

where

$$\Theta = \frac{\bar{Q}}{h_0 c} \qquad \Gamma = \frac{q}{h_0 c}$$

Using the definition of stream function in (2.33), one can write

$$F = \int_0^h \frac{\partial \psi}{\partial y} dy = \psi(h) - \psi(0) \tag{2.38}$$

The above expressions yields the required boundary conditions on ψ i.e.

$$\begin{aligned} \psi(0) &= 0 \\ \psi(h) &= \Gamma \end{aligned} \tag{2.39}$$

In summary, we have to solve

$$\frac{\partial^2}{\partial y^2}\left[\frac{\partial^2 \psi}{\partial y^2} + 2\Gamma \left(\frac{\partial^2 \psi}{\partial y^2}\right)^3\right] = 0 \tag{2.40}$$

subject to boundary conditions

$$\begin{aligned} \psi &= 0 \quad at \quad y = 0 \\ \frac{\partial \psi}{\partial y} &= V_b \quad at \quad y = 0 \\ \psi &= \Gamma \quad at \quad y = h, \\ \frac{\partial \psi}{\partial y} &= -1 \quad at \quad y = h \end{aligned} \tag{2.41}$$

The pressure difference over one wavelength is given by

$$\Delta P_\lambda = \int_0^{2\pi} \frac{dP}{dx} dx \tag{2.42}$$

2.2 Perturbation solution

The exact solution of non-linear boundary value problem defined through Eqs. (2.40) and (2.41) is difficult to obtain. Therefore, perturbation method is employed for approximate solution. Assuming Γ to be small, the dependent variables are expanded as

$$
\begin{aligned}
u &= u_0 + \Gamma u_1 + \ldots \\
p &= p_0 + \Gamma p_1 + \ldots \\
F &= F_0 + \Gamma F_1 + \ldots
\end{aligned}
\qquad (2.43)
$$

In view of above expansions, Eqs. (2.40) and (2.25) yield the following systems

2.2.1 System of order zero

$$
\begin{aligned}
\frac{\partial^4 u_0}{\partial y^4} &= 0 \\
\frac{\partial p_0}{\partial x} &= \frac{\partial^3 u_0}{\partial y^3} \\
\frac{\partial p_0}{\partial y} &= 0
\end{aligned}
\qquad (2.44)
$$

with the boundary conditions

$$
\begin{aligned}
u_0 &= 0, \quad \frac{\partial u_0}{\partial y} = V_b \quad at \ y = 0 \\
u_0 &= F_0, \quad \frac{\partial u_0}{\partial y} = -1 \quad at \ y = h
\end{aligned}
\qquad (2.45)
$$

2.2.2 System of order one

$$
\begin{aligned}
\frac{\partial^4 u_1}{\partial y^4} &= -2\frac{\partial^2}{\partial y^2}\left[\left(\frac{\partial^2 u_0}{\partial y^2}\right)^3\right] \\
\frac{\partial p_1}{\partial x} &= \frac{\partial^3 u_1}{\partial y^3} - 2\frac{\partial}{\partial y}\left[\left(\frac{\partial^2 u_0}{\partial y^2}\right)^3\right] \\
\frac{\partial p_1}{\partial y} &= 0
\end{aligned}
\qquad (2.46)
$$

with the boundary conditions

$$\psi_1 = 0, \quad \frac{\partial \psi_1}{\partial y} = 0 \quad at\ y = 0 \qquad (2.47)$$

$$\psi_1 = F_1, \quad \frac{\partial \psi_1}{\partial y} = 0 \quad at\ y = h$$

2.2.3 Zeroth-order solution

The zeroth-order problem possess the following solution

$$\psi_0 = \frac{y^3}{6}\frac{dp_0}{dx} + \frac{y^2}{h^2}\left(3F_0 + h(1 - 2V_b)\right) + V_b y \qquad (2.48)$$

$$u_0 = \frac{y^2}{2}\frac{dp_0}{dx} + \frac{2y}{h^2}\left(3F_0 + h(1 - 2V_b)\right) + V_b \qquad (2.49)$$

$$\frac{dp_0}{dx} = -\frac{6}{h^3}\left(2F_0 + h(1 - V_b)\right) \qquad (2.50)$$

The expression of pressure drop over a wavelength (2.12) at this order gives

$$\Delta P_{\lambda_0} = \int_0^{2\pi} \frac{dp_0}{dx}\, dx = -12F_0 I_3 - 6(1 - V_b) I_2 \qquad (2.51)$$

where

$$I_2 = \frac{2\pi}{(1-\phi^2)^{\frac{3}{2}}} \qquad I_3 = \frac{\pi(2+\phi^2)}{(1-\phi^2)^{\frac{5}{2}}}$$

2.2.4 First-order solution

The stream function, longitudinal velocity and the axial pressure gradient at this order has the following forms

$$\psi_1 = \frac{1}{10}\left(\frac{dp_0}{dx}\right)^3\left[-y^5 - 3y^3h^2 - 2y^2h^3\right] + \frac{F_1}{h^3}\left[-2y^3 + 3y^2h\right] + \frac{1}{2}K(x)\left(\frac{dp_0}{dx}\right)^2\left[-y^4 + 2y^3h - y^2h^2\right] \qquad (2.52)$$

$$u_1 = \frac{1}{10}\left(\frac{dp_0}{dx}\right)^3\left[-5y^4 - 9y^2h^2 - 4yh^3\right] - \frac{F_1}{h^3}\left[-6y^2 - 6yh\right] + \frac{1}{2}K(x)\left(\frac{dp_0}{dx}\right)^2\left[-4y^3 + 6y^2h - 2yh^2\right], \qquad (2.53)$$

$$\frac{dp_1}{dx} = \frac{9}{5}h^2\left(\frac{dp_0}{dx}\right)^3 + 6K(x)h\left(\frac{dp_0}{dx}\right)^2 - 6(K(x))^2\frac{dp_0}{dx} - \frac{12F_1}{h^3} \qquad (2.54)$$

where

$$K(x) = \frac{2}{h^2}\left[3F_0 - h(1 - 2V_b)\right]$$

The pressure drop per wavelength at this order is

$$\Delta P_{\lambda_1} = -\frac{72}{5}\left[36F_0^3 I_7 - 54(1 - V_b)F_0^2 I_6 + 18(8 - 11V_b + 8V_b^2)F_0 I_5 - (7 - 11V_b + 11V_b^2 - 7V_b^3)I_4\right] - 12F_1 I_3 \qquad (2.55)$$

where

$$I_4 = \frac{\pi\left(3\phi^2 + 2\right)}{\left(1 - \phi^2\right)^{\frac{7}{2}}}$$

$$I_n = \frac{1}{1 - \phi^2}\left[\left(\frac{2n - 3}{n - 1}\right)I_{n-1} - \left(\frac{n - 2}{n - 1}\right)I_{n-2}\right] \qquad \text{for } n > 4$$

In view of the preceding analysis, the expressions of stream function, longitudinal velocity, pressure gradient and pressure drop upto first order in Γ are respectively given by

$$\begin{aligned}\psi = {} & \frac{y^3}{6}\frac{dp_0}{dx} + \frac{y^2}{h^2}\left[3F_0 - h(1 - 2V_b)\right] + V_b y \\ & + \Gamma\left\{\frac{1}{10}\left(\frac{dp_0}{dx}\right)^3\left[-y^5 - 3y^3h^2 - 2y^2h^3\right]\right. \\ & \left. + \frac{1}{2}K(x)\left(\frac{dp_0}{dx}\right)^2\left[-y^4 + 2y^3h - y^2h^2\right] - \frac{F_1}{h^3}\left[-2y^3 - 3y^2h\right]\right\}\end{aligned} \qquad (2.56)$$

$$
\begin{aligned}
u = {} & \frac{y^2}{2}\frac{dp_0}{dx} + \frac{2y}{h^2}\left[3F_0 - h\left(1 - 2V_b\right)\right] + V_b + \\
& \Gamma\left\{\frac{1}{10}\left(\frac{dp_0}{dx}\right)^3\left[-5y^4 + 9y^2h^2 - 4yh^3\right]\right. \\
& \left. + \frac{1}{2}K(x)\left(\frac{dp_0}{dx}\right)^2\left[-4y^3 + 6y^2h - 2yh^2\right] + \frac{F_1}{h^3}\left[-6y^2 + 6yh\right]\right\}
\end{aligned}
\tag{2.57}
$$

$$
\begin{aligned}
\frac{dp}{dx} = {} & \Gamma\left\{\frac{9}{5}h^2\left(\frac{dp_0}{dx}\right)^3 + 6K(x)h\left(\frac{dp_0}{dx}\right)^2 + 6(K(x))^2\frac{dp_0}{dx} - \frac{12F_1}{h^3}\right\} \\
& - \frac{6}{h^3}\left[2F_0 - h\left(1 - V_b\right)\right]
\end{aligned}
\tag{2.58}
$$

$$
\begin{aligned}
\Delta P_\lambda = {} & -12F_0I_3 - 6\left(1 - V_b\right)I_2 \\
& - \frac{72}{5}\Gamma[36F_0^3I_7 + 54\left(1 - V_b\right)F_0^2I_6 + 4\left(8 - 11V_b + 8V_b^2\right)F_0I_5 \\
& + \left(7 - 11V_b + 11V_b^2 - 7V_b^3\right)I_4] - 12\Gamma F_1I_3
\end{aligned}
\tag{2.59}
$$

Now defining F by the relation

$$
F = F_0 + \Gamma F_1 \tag{2.60}
$$

the above expression, after ignoring the terms of second and higher order in Γ, takes the form

$$
\begin{aligned}
\Delta P_\lambda = {} & -12FI_3 - 6\left(1 - V_b\right)I_2 \\
& - \frac{72}{5}\Gamma[36(F)^3I_7 + 54\left(1 - V_b\right)(F)^2I_6 \\
& + 4\left(8 - 11V_b + 8V_b^2\right)FI_5 + \left(7 - 11V_b + 11V_b^2 - 7V_b^3\right)I_4]
\end{aligned}
\tag{2.61}
$$

2.3 Forces generated by the organism

For determination of the speed of the glider, the expressions of net forces in x- and y-directions are required. To this end, we first calculate the stress vector at the organism surface by taking the dot product of normal to the organism surface $\hat{\mathbf{n}}$ with the stress tensor $\mathbf{T}$. Designating the outcome by τ, we can write

$$
\tau_x = \left(\mathbf{T}\cdot\hat{\mathbf{n}}\right)_x \tag{2.62}
$$

$$\tau_y = (\mathbf{T} \cdot \hat{\mathbf{n}})_y \qquad (2.63)$$

where

$$\hat{\mathbf{n}} = \begin{pmatrix} \frac{-h'}{\sqrt{1+(h')^2}} \\ \frac{1}{\sqrt{1+(h')^2}} \end{pmatrix}$$

and

$$\mathbf{T} = \begin{pmatrix} -p - S_{xx} & S_{xy} \\ S_{xy} & -p + S_{yy} \end{pmatrix}$$

Thus Eqs. (2.62) and (2.63) give

$$\tau_x = \frac{ph'}{\sqrt{1+(h')^2}} - \frac{S_{xx}h'}{\sqrt{1+(h')^2}} + \frac{S_{xy}}{\sqrt{1+(h')^2}} \qquad (2.64)$$

$$\tau_y = \frac{-p}{\sqrt{1+(h')^2}} - \frac{S_{xy}h'}{\sqrt{1+(h')^2}} + \frac{S_{yy}}{\sqrt{1+(h')^2}} \qquad (2.65)$$

The assumption of small wall slope i.e. $h' << 1$ leads to

$$\tau_x = ph' - S_{xx}h' + S_{xy} \qquad (2.66)$$

$$\tau_y = -p - S_{xx}h' - S_{yy} \qquad (2.67)$$

The above equations upon defining the dimensionless variables τ_x^* and τ_y^* given by

$$\tau_x^* = \frac{h_0}{\eta_0 c}\tau_x \qquad (2.68)$$

$$\tau_y^* = \frac{h_0}{\eta_0 c}\tau_y$$

take the forms

$$\tau_x^* = ph' - \delta S_{xx}h' + S_{xy} \qquad (2.69)$$

$$\tau_y^* = -\frac{1}{\delta}p - \delta S_{xx}h' - S_{yy} \qquad (2.70)$$

Under the long wavelength assumption the above expressions become

$$\tau_x = ph + S_{xy} \tag{2.71}$$

$$\tau_y = S_{yy} - \frac{1}{\delta}p \tag{2.72}$$

Now the horizontal and vertical forces per unit width per wavelength are given as

$$\begin{aligned} F_x &= \int_0^{2\pi} \tau_x|_{y=h}\, dx \\ &= \int_0^{2\pi} ph\,dx + \int_0^{2\pi} S_{xy}|_{y=h}\, dx \end{aligned} \tag{2.73}$$

$$\begin{aligned} F_y &= \int_0^{2\pi} \tau_y|_{y=h}\, dx \\ &= \int_0^{2\pi} S_{yy}|_{y=h}\, dx \end{aligned} \tag{2.74}$$

where it is assumed that for symmetrical waveform on the glider surface the integration of pressure term becomes zero in the expression of F_y.

Substituting the value of S_{xy} in Eq (2.73) we get

$$F_x = \int_0^{2\pi} \left[\left(\frac{\partial u}{\partial y}\right) + 2\Gamma \left(\frac{\partial u}{\partial y}\right)^3 - ph \right]_{y=h} dx \tag{2.75}$$

Entering the value of u and performing the integration one finally arrives at

$$\begin{aligned} F_x = \Bigg[& 6I_1 + 12FI_2 - 6I_1V_b + \Gamma\left(\frac{2592F^3I_6}{5} + \frac{3888F^2I_5}{5} + \frac{2304FI_4}{5} + \frac{504I_3}{5} - \frac{3888F^2V_bI_5}{5}\right. \\ & \left. - \frac{3168FV_bI_4}{5} - \frac{792V_bI_3}{5} + \frac{2304FV_b^2I_4}{5} + \frac{792V_b^2I_3}{5} - \frac{504V_b^3I_3}{5}\right) \\ & - 6FI_2 - 4I_1 + 2I_1V_b - \Gamma\left(\frac{1296F^3I_6}{5} + \frac{2304F^2I_5}{5} + \frac{1512FI_4}{5} + \frac{352I_3}{5} \right. \\ & \left. + \frac{1584F^2V_bI_5}{5} + \frac{1584FV_bI_4}{5} - \frac{456V_bI_3}{5} - \frac{792FV_b^2I_4}{5} - \frac{336V_b^2I_3}{5} - \frac{152V_b^3I_3}{5} \right) \Bigg] \end{aligned} \tag{2.76}$$

Similarly the expression of the lift force (F_y) reads

$$F_y = \int_0^{2\pi} \left[(2\lambda_1 + \lambda_2) \left(\frac{\partial u}{\partial y} \right)^2 \right]_{y=h} dx \qquad (2.77)$$

which on integration gives

$$\begin{aligned} F_y \;=\; & (2\lambda_1 + \lambda_2) \Big[\left\{ 36F^2 I_4 + 48FI_3 + 16I_2 - 24FV_b I_3 - 16V_b I_2 + 4V_b^2 I_2 \right\} \\ & + \Gamma \Big(-\frac{10368F^4 I_8}{5} - \frac{31104F^3 I_7}{5} - \frac{32544F^2 I_6}{5} \\ & - 2880FI_5 - \frac{2304I_4}{5} + \frac{10368F^3 V_b I_7}{5} \\ & + \frac{28224F^2 V_b I_6}{5} + \frac{21888FV_b I_5}{5} - \frac{5184V_b I_4}{5} \\ & - 288F^2 V_b^2 I_6 - \frac{6336FV_b^2 I_5}{5} - \frac{3168V_b^2 I_4}{5} \\ & - \frac{1152FV_b^3 I_5}{5} + \frac{288V_b^4 I_4}{5} \Big) \Big] \end{aligned} \qquad (2.78)$$

It is pointed out that in derivation of the expressions for F_x and F_y the relation [illegible] has been used and only term of first order in Γ are retained.

2.4 Parametric Analysis

Now in order that glider maintains a steady speed at a fixed distance from the substrate, each of the three quantities F_x, F_y and ΔP_λ must be zero. Setting these quantities equal to zero results in three non-linear algebraic equations which can be solved for three unknowns F, V_b and ω. In the present thesis, the computations are carried out using Mathematica and results are displayed in Fig. 2. It is observed that speed of the glider decreases with the increase in the material parameter Γ. Similarly, both F and ω are also found to decrease with Γ. Since the apparent viscosity of a third order fluid is greater than the viscosity of Newtonian fluid. It is therefore anticipated that gliding with a given gait is always slower in a non-Newtonian fluid.

The graphical results shown in Fig. 2 are consistent with this assertion.

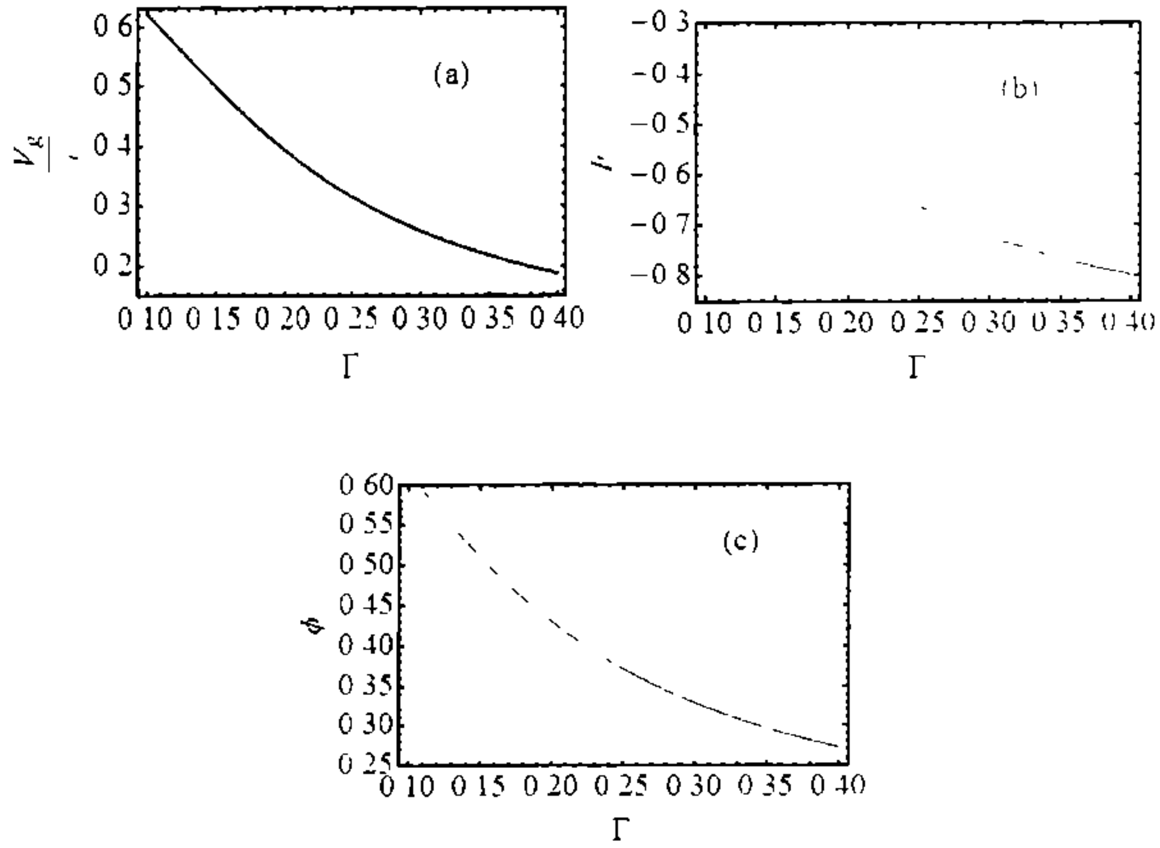

Fig. 2. Effect of Γ on (a) $\frac{V_g}{c}$ (b) F (c) ϕ when $\beta = 0$

2.5 Conclusion

A theoretical model is presented to analyze the gliding motion of a bacteria on a non-Newtonian slime. The continuity and momentum equation are utilized to obtain the governing equation of the flow between substrate and the organism. A perturbation technique is adopted to determine the flow field. The pressure rise per wavelength and forces generated by the organism are calculated and later utilized to give an estimate of glider speed as a function of material parameter of the slime. The key result of this chapter is that the speed of the glider decreases with increasing Γ a parameter which is measure of degree of increase in the viscosity of the slime

Chapter 3

Slip effect on the gliding motion of bacteria

In this chapter the results presented in chapter 2 are extended for slip flow. It is assumed that there is a certain degree of slip between fluid and bacterium and also between fluid and substrate. The flow field is determined using perturbation technique. The effects of slip on the speed of the glider are quantified. The effects of slip on distinct features of the flow field are also highlighted.

3.1 Mathematical Formulation

The geometry and the underlying assumptions of the flow are same as described in chapter 2. However, the no-slip boundary conditions imposed in chapter 2 are replaced here in favor of the slip conditions. The dimensional slip conditions in fixed frame of reference are

$$U + V_g = -\frac{l}{\eta_0} S_{XY} \qquad at \quad Y = h \tag{3.1}$$

$$U = \frac{l}{\eta_0} S_{XY} \qquad at \quad Y = 0 \tag{3.2}$$

where l is the slip length.

Employing the transformations relating fixed frame to the wave frame, the above conditions become

$$u = -c - \frac{l}{\eta_0} S_{xy} \quad at \quad y = h \tag{3.3}$$

$$u = V_g - c + \frac{l}{\eta_0} S_{xy} \quad at \quad y = 0 \tag{3.4}$$

In view of the dimensionless variables defined through (2.1), the conditions (3.3) and (3.4) after dropping the * can be written as

$$u = -1 - \beta S_{xy} \quad at \quad y = h \tag{3.5}$$

$$u = V_b - \beta S_{xy} \quad at \quad y = 0 \tag{3.6}$$

where

$$\beta = \frac{l}{h_0}$$

In term of stream function ψ, we can write (3.5) and (3.6) as

$$\frac{\partial \psi}{\partial y} = -1 - \beta \left[\frac{\partial^2 \psi}{\partial y^2} + 2\Gamma \left(\frac{\partial^2 \psi}{\partial y^2} \right)^3 \right] \quad at \ y = h \tag{3.7}$$

$$\frac{\partial \psi}{\partial y} = V_b + \beta \left[\frac{\partial^2 \psi}{\partial y^2} - 2\Gamma \left(\frac{\partial^2 \psi}{\partial y^2} \right)^3 \right] \quad at \ y = 0 \tag{3.8}$$

Therefore, in order to investigate the effects of slip on the glider speed, Eq. (2.40) must be solved together with boundary conditions (2.41), (3.7) and (3.8). Expanding the dependent variables ψ, p and F in perturbation series given by

$$\begin{aligned} \psi &= \psi_0 + \Gamma \psi_1 + \cdots \\ p &= p_0 + \Gamma p_1 + \cdots \\ F &= F_0 + \Gamma F_1 + \cdots \end{aligned} \tag{3.9}$$

and substituting this series in Eq. (2.40) and boundary conditions (2.41), (3.7) and (3.8), the following system can be readily obtained

3.1.1 System of order zero

$$
\begin{aligned}
\frac{\partial^4 \psi_0}{\partial y^4} &= 0 \\
\frac{\partial p_0}{\partial x} &= \frac{\partial^3 \psi_0}{\partial y^3} \\
\frac{\partial p_0}{\partial y} &= 0,
\end{aligned}
\tag{3.10}
$$

with the boundary conditions

$$
\begin{aligned}
\psi_0 &= 0 \quad \frac{\partial \psi_0}{\partial y} = \beta\frac{\partial^2 \psi_0}{\partial y^2} + V_b \qquad at \quad y = 0 \\
\psi_0 &= F_0 \quad \frac{\partial \psi_0}{\partial y} = -\beta\frac{\partial^2 \psi_0}{\partial y^2} - 1 \qquad at \quad y = h
\end{aligned}
\tag{3.11}
$$

3.1.2 System of order one

$$
\begin{aligned}
\frac{\partial^4 \psi_1}{\partial y^4} &= 2\frac{\partial^2}{\partial y^2}\left[\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)^3\right] \\
\frac{\partial p_1}{\partial x} &= \frac{\partial^3 \psi_1}{\partial y^3} + 2\frac{\partial}{\partial y}\left[\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)^3\right] \\
\frac{\partial p_1}{\partial y} &= 0
\end{aligned}
\tag{3.12}
$$

with the boundary conditions

$$
\begin{aligned}
\psi_1 &= 0, \ \frac{\partial \psi_1}{\partial y} = \beta\left(\frac{\partial^2 \psi_1}{\partial y^2} - \frac{16(-3h - 6\beta)F_0 + h(-h - 2(h - 3\beta)V_b))^3}{h^3(h + 2\beta)^3(h - 6\beta)^3}\right) at \ y = 0 \\
\psi_1 &= F_1 \ \frac{\partial \psi_1}{\partial y} = -\beta\left(\frac{\partial^2 \psi_1}{\partial y^2} + \frac{16(-3h - 6\beta)F_0 - h(-2(h - 3\beta) - hV_b))^3}{h^3(h - 2\beta)^3(h + 6\beta)^3}\right) at \ y = h
\end{aligned}
\tag{3.13}
$$

3.1.3 Zeroth order solution

The solution of zeroth order problem satisfying the corresponding boundary condition is

$$
\psi_0 = \frac{y((h + 2\beta)(-2y^2 + 3h(y + 2\beta))F_0 - M_1(hy - 2(h - y)\beta) - M_2V_b)}{h^2(h + 2\beta)(h + 6\beta)}
\tag{3.14}
$$

$$\frac{dp_0}{dx} = -\frac{432\beta^2 F_0}{h^5} + \frac{-6 + 6V_b}{h^2} + \frac{36\beta - 12F_0 - 36\beta V_b}{h^3} + \frac{-216\beta^2 + 72\beta F_0 + 216\beta^2 V_b}{h^4} \quad (3.15)$$

where

$$M_1 = (h^2 - hy)$$

$$M_2 = (h^2 - hy + 4h\beta - 2y\beta)$$

The pressure drop over a wavelength at this order is

$$\begin{aligned}
\Delta P_{\lambda_0} = & -432\beta^2 F_0 I_5 + I_2(-6 + 6V_b) \\
& + I_3(36\beta - 12F_0 - 36\beta V_b) \quad (3.16) \\
& + I_4(-216\beta^2 - 72\beta F_0 - 216\beta^2 V_b)
\end{aligned}$$

3.1.4 First order solution

Similarly, the stream function ψ_1 and the pressure gradient dp_1/dx are given by

$$\begin{aligned}
\psi_1 = & \frac{1}{5h^{10}} y(F_1(5h^6 h(3h - 2y)y + 6(h - 2y)(h - y)\beta) \\
& -36(h - y)(h(2F + h)(h - y)y(-18F(h - 2y) + h(h - 18y)) \\
& +2(h^3(54F^2 + 44Fh - 11h^2) - h^2(216F^2 - 76Fh - 21h^2)y \\
& -h(2F - h)(486F - 173h)y^2 - 162(2F - h)^2 y^3)\beta) \\
& +36h(h - y)\Gamma V_b(-2h(h - y)y(h(28F + 9h) \\
& -18(2F + h)y) + 4(h^3(32F + 11h) + h^2(178F - 59h)y \quad (3.17) \\
& -h(556F - 243h)y^2 + 162(2F - h)y^3)\beta \\
& -h(h(19h - 18y)(h - y)y - 2(21h^3 - 119h^2 y - 313hy^2 - 162y^3)\beta)V_b) \\
& +4(h - y)(9h(2F + h)^2(h - y)y - 3Fh - h^2 - 6Fy + 3hy)
\end{aligned}$$

$$-2(h^3(162F^3 - 198F^2h - 99Fh^2 + 19h^3)$$
$$-9h^2(2F - h)(36F^2 - Fh - 11h^2)y - 9h(2F - h)^2(81F - 23h)y^2$$
$$-243(2F - h)^3y^3)\beta + hV_b(9h(2F + h)(h - y)y(2h(7F - h)$$
$$-9(2F + h)y) - 6(2h^3(48F^2 + 33Fh + 7h^2) - 6h^2(89F^2 + 59Fh - 6h^2)y$$
$$-6h(2F - h)(139F - 52h)y^2 - 243(2F - h)^2y^3)\beta)$$
$$-hV_b(-9h(h - y)yh(19F - 7h) - 9(2F - h)y)$$
$$+6(h^3(63F + 19h) - 3h^2(119F + 47h)y - 3h(313F + 139h)y^2$$
$$-243(2F + h)y^3)\beta - h(9h(4h - 3y)(h - y)y$$
$$-2(44h^3 - 216h^2y - 522hy^2 - 243y^3)\beta)V_b)))$$

$$
\begin{aligned}
\frac{dp_1}{d\tau} = {} & \frac{622080\beta F_0^3}{5h^8} - \frac{12F_1}{h^3} \\
& + \frac{\frac{93312\beta F_0^2}{5} - \frac{2592F_0^3}{5} - \frac{93312}{5}\beta F_0^2 V_b}{h^7} \\
& + \frac{\frac{52416\beta F_0}{5} - \frac{3888F_0^2}{5} - \frac{81792}{5}\beta F_0 V_b + \frac{3888}{5}F_0^2 V_b + \frac{52416}{5}\beta F_0 V_b^2}{h^6} \\
& + \frac{-\frac{504}{5} + 72\beta F_1 + \frac{792V_b}{5} - \frac{792V_b^2}{5} + \frac{504V_b^3}{5}}{h^4} \\
& + \frac{\frac{10656\beta}{5} - \frac{2304F_0}{5} - \frac{20448\beta V_b}{5} + \frac{3168F_0V_b}{5} + \frac{20448\beta V_b^2}{5} - \frac{2304}{5}F_0V_b^2 - \frac{10656\,[illegible]}{5}}{h^5}
\end{aligned}
\qquad (3.18)
$$

The expression of pressure drop at this order is

$$
\begin{aligned}
\Delta P_{\lambda_1} = {} & \frac{12}{5}(5184\beta I_8F_0^4 - 5I_3F_1 - 216I_7F_0^3(F_0 - 36\beta(-1 + V_b)) \\
& + 6I_4(5\beta F_1 + (-1 + V_b)(7 + V_b(-4 + 7V_b))) \\
& - 24I_5(F_0(8 + V_b(-11 + 8V_b)) - 3\beta(-1 + V_b)(37 + V_b(-34 + 37V_b))) \\
& + 12I_6F_0(27F_0(-1 + V_b) - 13\beta(91 + V_b(-142 + 91V_b))))
\end{aligned}
\qquad (3.19)
$$

Combining zeroth-order and first order solutions and making use of relation

$$F = F_0 + \Gamma F_1 \qquad (3.20)$$

we can write

$$
\begin{aligned}
v \;=\; & \frac{1}{5h^{14}} y \,\big[15Fh^{12} y - 5h^{13} y - 10Fh^{11} y^2 - 5h^{12} y^2 - 5h^{14} V_b - 10h^{13} y V_b - 5h^{12} y^2 V_b \\
& + 4(h - y)\Gamma \{ -108F^3h^7 y - 72F^2h^8 y + 9Fh^9 y + 9h^{10} y - 324F^3h^6 y^2 \\
& + 396F^2h^7 y^2 + 153Fh^8 y^2 + 18h^9 y^2 - 216F^3h^5 y^3 - 324F^2h^6 y^3 - 162Fh^7 y^3 - 27h^8 y^3 \} \\
& - hV_b \{ 252F^2h^7 y - 162Fh^8 y + 18h^9 y - 576F^2h^6 y^2 - 486Fh^7 y^2 \\
& - 99h^8 y^2 + 324F^2h^5 y^3 + 324Fh^6 y^3 - 81h^7 y^3 \} - hV_b (171Fh^7 y - 63h^8 y - 333Fh^6 y^2 \\
& + 144h^7 y^2 - 162Fh^5 y^3 - 81h^6 y^3 - 36h^8 y V_b - 63h^7 y^2 V_b - 27h^6 y^3 V_b) \\
& - \frac{1}{5h^{10}} 2 \big(y (-15Fh^8 - 5h^9 + 45Fh^7 y + 20h^8 y - 30Fh^6 y^2 \\
& - 15h^7 y^2 - 648F^3h^4\Gamma - 792F^2h^5\Gamma - 396Fh^6\Gamma - 76h^7\Gamma - 1944F^3h^3 y\Gamma \\
& - 576F^2h^4 y\Gamma + 1152Fh^5 y\Gamma + 472h^6 y\Gamma + 14256F^3h^2 y^2\Gamma - 16344F^2h^3 y^2\Gamma \\
& + 5472Fh^4 y^2\Gamma - 432h^5 y^2\Gamma - 19440F^3 h y^3\Gamma - 26640F^2h^2 y^3\Gamma - 12060Fh^3 y^3\Gamma \\
& - 1800h^4 y^3\Gamma + 7776F^3 y^4\Gamma + 11664F^2 h y^4\Gamma + 58372Fh^2 y^4\Gamma + 972h^3 y^4\Gamma - 10h^4 V_b \\
& + 5256F^2h^4 y\Gamma V_b + 3156Fh^5 y\Gamma V_b + 264h^6 y\Gamma V_b - 26124F^2h^3 y^2\Gamma V_b - 21744Fh^4 y^2\Gamma V_b \\
& - 4176h^5 y^2\Gamma V_b - 25h^6 y V_b - 15h^7 y^2 V_b - 1152F^2h^5\Gamma V_b - 792Fh^6\Gamma V_b - 168h^7\Gamma V_b \\
& + 31680F^2h^2 y^3\Gamma V_b + 29160Fh^3 y^3\Gamma V_b + 29160Fh^3 y^3\Gamma V_b + 6660h^4 y^3\Gamma V_b \\
& - 11664F^2 h y^4\Gamma V_b - 11664Fh^2 y^4\Gamma V_b - 2916h^3 y^4\Gamma V_b - 756Fh^6\Gamma V_b^2 - 228h^7\Gamma V_b^2 \\
& - 3528Fh^5 y\Gamma V_b^2 - 1164h^6 y\Gamma V_b^2 + 15552Fh^4 y^2\Gamma V_b^2 - 6696h^5 y^2\Gamma V_b^2 - 17100Fh^3 y^3\Gamma V_b^2 \\
& - 7920h^4 y^3\Gamma V_b^2 + 5832Fh^2 y^4\Gamma V_b^2 + 2916h^3 y^4\Gamma V_b^2 - 176h^7\Gamma V_b^3 - 668h^6 y\Gamma V_b^3 \\
& - 2952h^5 y^2\Gamma V_b^3 + 3060h^4 y^2\Gamma V_b^3 - 972h^3 y^4\Gamma V_b^3) \big) \big]
\end{aligned}
\tag{3.21}
$$

$$
\begin{aligned}
\frac{dp}{dx} \;=\; & -\frac{12F}{h^3} - \frac{6}{h^2} - \frac{72F\beta}{h^4} + \frac{36\beta}{h^3} - \frac{432F\beta^2}{h^5} + \frac{6V_b}{h^2} - \frac{36\beta V_b}{h^3} \\
& + \Gamma \Big(-\frac{2592}{5h^7} - \frac{3888F^2}{5h^6} - \frac{2304F}{5h^5} - \frac{504}{5h^4} - \frac{62208F^3\beta}{5h^8} \\
& + \frac{93312F^2\beta}{5h^7} + \frac{52416F\beta}{5h^6} - \frac{106563}{5h^5} + \frac{432\beta^2 F_1}{5h^5}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{3888F^2\beta V_b}{5h^6} \quad \frac{3168F\beta V_b}{5h^5} \quad \frac{792\beta}{5h^4} \quad \frac{93312F^2\beta}{5h^7} \\
& -\frac{81792F\beta V_b}{5h^6} - \frac{20488\beta V_b}{5h^5} - \frac{2304F V_b^2}{5h^5} - \frac{792V_b^2}{5h^4} \\
& -\frac{52116F\beta V_b^2}{5h^6} - \frac{20148\beta V_b^2}{5h^5} - \frac{504V_b^2}{5h^4} - \frac{10656\beta V_b^3}{5} \Big)
\end{aligned}
\qquad (3.22)
$$

$$
\begin{aligned}
\Delta P_\lambda = {} & \frac{6}{5}(-5I_2 - 10FI_3 - 30\beta I_3 - 60F\beta I_4 + 5I_2V_b - 30\beta I_3V_b \\
& -12F(-5I_2 - 10FI_3 - 30\beta I_4 + 60F\beta I_4 - 5I_2V_b - 30\beta I_3V_b \\
& -36F^3(I_7 - 243I_5) - (11I_4 + 44FI_5 - 54F^2I_6)V_b \\
& -4\beta(71I_5 - 4F(71I_6 + 81FI_7))V_b \\
& -V_b^2(-11I_4 + 4(-8F - 71\beta)I_5 - 728F\beta I_6 - 7I_4 \quad 148 \; I_7 \; V_b
\end{aligned}
\qquad (3.23)
$$

3.2 Forces generated by the organism

In the presence of slip the expressions of horizontal and vertical force turn out to be

$$
\begin{aligned}
F_x = {} & \{6I_1 - 12FI_2 - 6I_1V_b - 72F\beta I_3 - 36\beta I_2 \quad 36V_b\beta I_2 \\
& -\Gamma\Big(\frac{2592F^3I_6}{5} + \frac{3888F^2I_5}{5} + \frac{2304FI_4}{5} - \frac{504I_3}{5} \\
& -\frac{62208F^3\beta I_7}{5} - \frac{93312F^2\beta I_6}{5} - \frac{52416F\beta I_5}{5} \\
& -\frac{10656\beta I_4}{5} - \frac{3888F^2V_bI_5}{5} - \frac{3168FV_bI_4}{5} \\
& \frac{792V_bI_3}{5} - \frac{93312F^2\beta V_bI_6}{5} - \frac{81792F\beta V_bI_5}{5} \\
& -\frac{20188\beta V_bI_4}{5} - \frac{2304FV_b^2I_4}{5} - \frac{792V_b^2I_3}{5} \\
& +\frac{10656\beta V_b^3I_4}{5}\Big) - 6FI_2 - I_1 - 36F\beta I_3 - 20\beta I_2 \\
& \frac{1296F^3\Gamma I_6}{5} - \frac{2304F^2\Gamma I_5}{5} - \frac{1512F\Gamma I_4}{5} \quad \frac{352\Gamma I_3}{5} \\
& -2V_bI_1 - 16\beta V_bI_2 - \frac{1584F^2\Gamma V_bI_5}{5} - \frac{1584F\Gamma V_bI_4}{5} \\
& -\frac{456\Gamma V_bI_3}{5} - \frac{792F\Gamma V_b^2I_4}{5} - \frac{336\Gamma V_b^2I_3}{5} \quad \frac{152\Gamma V_b^3I_3}{5}\Big)\Big\}
\end{aligned}
\qquad (3.24)
$$

$$
\begin{aligned}
F_y = {} & (72F^2\lambda_1 I_4 - 96F\lambda_1 I_3 + 32\lambda_1 I_2 - 864F^2\beta\lambda_1 I_5 - 1056F\beta\lambda_1 I_4 \\
& -320\beta\lambda_1 I_3 - 5760F\Gamma\lambda_1 I_5 - 48FV_b\lambda_1 I_3 \\
& -32V_b\lambda_1 I_2 - 672F\beta V_b\lambda_1 I_4 - 416\beta V_b\lambda_1 I_3 \\
& -8V_b^2\lambda_1 I_2 - 128\beta V_b^2\lambda_1 I_3 - 576F^2\Gamma V_b^2\lambda_1 I_6 \\
& -\frac{20736F^4\Gamma\lambda_1 I_8}{5} - \frac{62208F^3\Gamma\lambda_1 I_7}{5} - \frac{65088F^2\Gamma\lambda_1 I_6}{5} - \frac{1608\Gamma\lambda_1 I_4}{5} \\
& -\frac{20736F^3\Gamma V_b\lambda_1 I_7}{5} - \frac{56448F^2\Gamma V_b\lambda_1 I_6}{5} - \frac{43776F\Gamma V_b\lambda_1 I_5}{5} - \frac{10368\Gamma V_b\lambda_1 I_4}{5} \\
& -\frac{12672F\Gamma V_b^2\lambda_1 I_5}{5} - \frac{6336\Gamma V_b^2\lambda_1 I_4}{5} - \frac{2304F\Gamma V_b^3\lambda_1 I_5}{5} - \frac{576\Gamma V_b^4\lambda_1 I_4}{5} \qquad (3.25) \\
& -36F^2\lambda_2 I_4 - 48F\lambda_2 I_3 + 16\lambda_2 I_2 - 432F^2\beta\lambda_2 I_5 - 528F\beta\lambda_2 I_4 \\
& -160\beta\lambda_2 I_3 - 2880F\Gamma\lambda_2 I_5 - 24FV_b\lambda_2 I_3 - 16V_b\lambda_2 I_2 \\
& -336F\beta V_b\lambda_2 I_4 - 208\beta V_b\lambda_2 I_3 - 4V_b^2\lambda_2 I_2 - 64\beta V_b^2\lambda_2 I_3 - 288F^2\Gamma V_b^2\lambda_2 I_6 \\
& -\frac{10368F^4\Gamma\lambda_2 I_8}{5} - \frac{31104F^3\Gamma\lambda_2 I_7}{5} - \frac{32544F^2\Gamma\lambda_2 I_6}{5} - \frac{2304\Gamma\lambda_2 I_4}{5} \\
& -\frac{10368F^3\Gamma V_b\lambda_2 I_7}{5} + \frac{28224F^2\Gamma V_b\lambda_2 I_6}{5} - \frac{21888F\Gamma V_b\lambda_2 I_5}{5} \\
& -\frac{5184\Gamma V_b\lambda_2 I_4}{5} - \frac{6336F\Gamma V_b^2\lambda_2 I_5}{5} - \frac{3168\Gamma V_b^2\lambda_2 I_4}{5} \\
& -\frac{1152F\Gamma V_b^3\lambda_2 I_5}{5} - \frac{288\Gamma V_b^4\lambda_2 I_4}{5})
\end{aligned}
$$

3.3 Graphical Results

Now again to get an estimate of the glider speed we equate all the three quantities F, F_y and ΔP_λ to zero and search of possible solutions in the range $0 \le \beta < 1$, $F < 0$, $V_b < 0$. The solution of non-linear algebraic equations is found using Mathematica and results are displayed in Figs. 3.1 and 3.2. Fig. 3.1 shows that a similar trend persists with respect to Γ in the presence of slip as observed in chapter 2 i.e. all three quantities $\frac{V_b}{2}$, F and σ decrease with increase in Γ. However, as observed in Fig. 3.2 the speed of the glider decreases with increasing the slip parameter β.

The pressure rise per wavelength ΔP_λ and stream function ψ are displayed in Figs. 3.3-3.5 for $V_b = -1$ which corresponds to the case when upper wall (organism) is fixed and its wave motion entrains a net flow of slime along the x-direction. Fig. 3.3 shows that ΔP_λ in the pumping region ($\Delta P_\lambda > 0$, $\Theta > 0$) increases with increasing Γ. A similar trend is noted for free pumping flux (value of Θ for $\Delta P_\lambda = 0$). Thus greater effort is rendered by the organism to maintain the same flow rate for a third order slime as that produced for a Newtonian slime. In contrast, Fig. 3.4 shows that the pressure rise per wavelength ΔP_λ decreases with an increase in the slip parameter β. The free pumping flux also decrease with increasing β. The streamlines of the flow field generated by the wavy motion of the organism are displayed for several values of β in Fig. 3.5. It is evident that size of circulating bolus decrease and bolus disappears with increasing β.

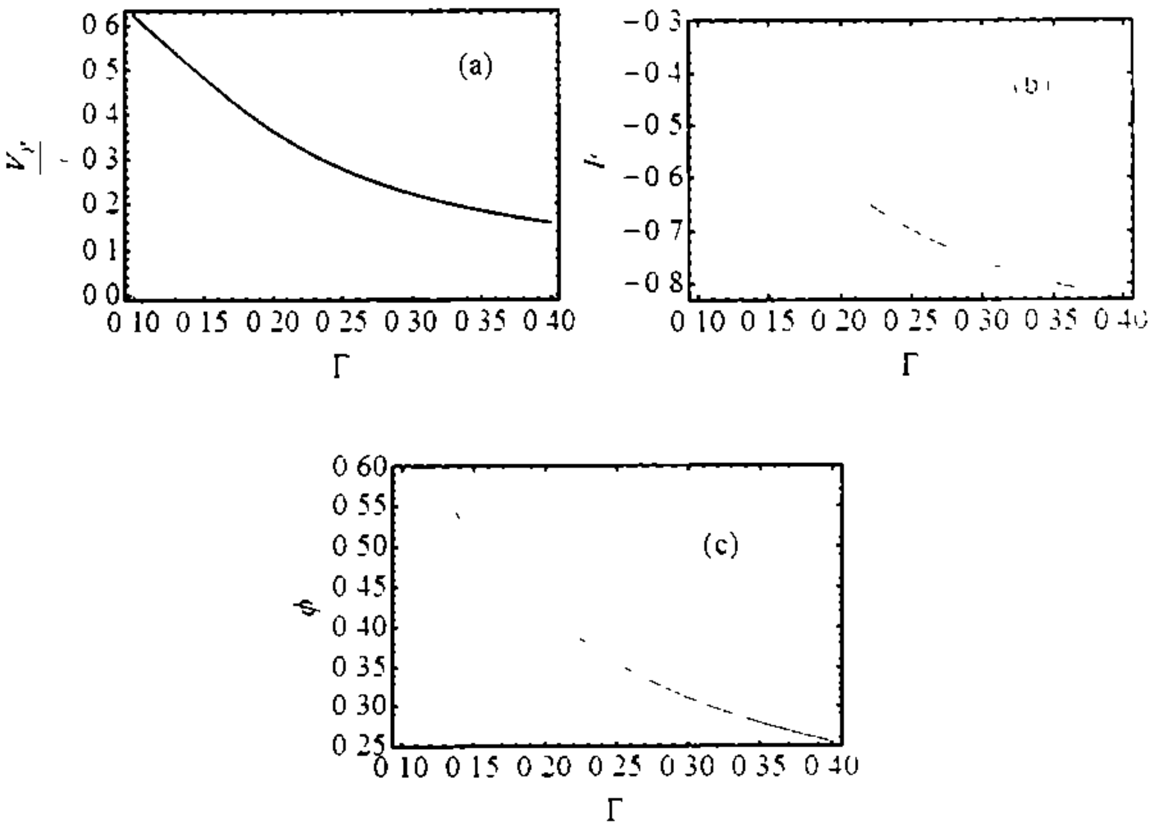

Fig. 3.1 Effect of Γ on (a) $\frac{V}{c}$ (b) F (c) ϕ
when $\beta = 0.01$

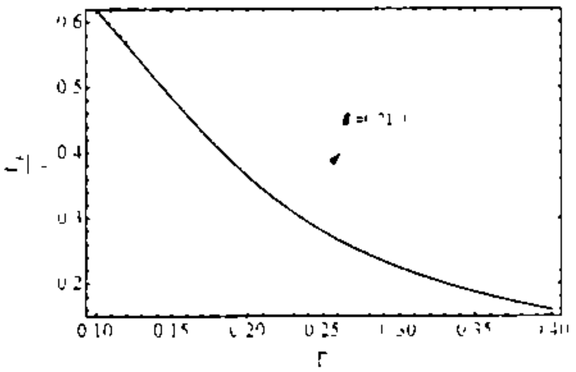

Fig 3 2 Effect of β on $\frac{t_+}{t}$

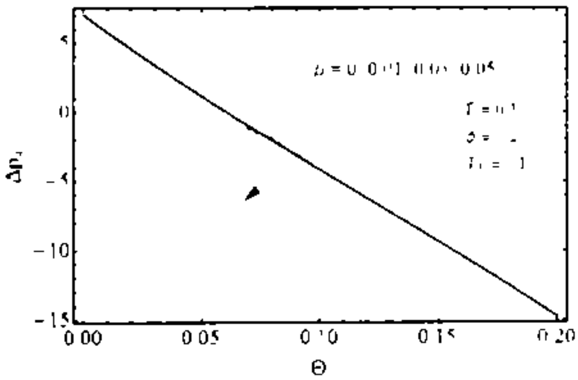

Fig 3 3 Effects of Γ on ΔP_λ

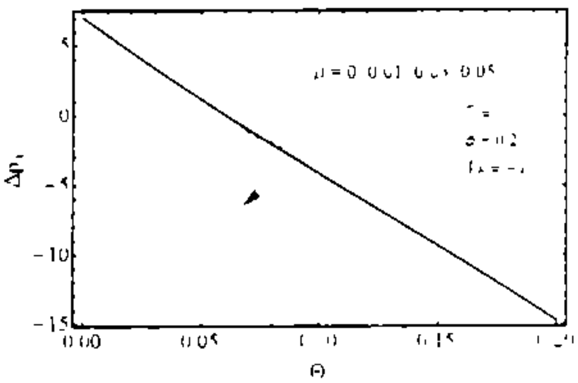

Fig 3 4 Effects of β on ΔP_λ

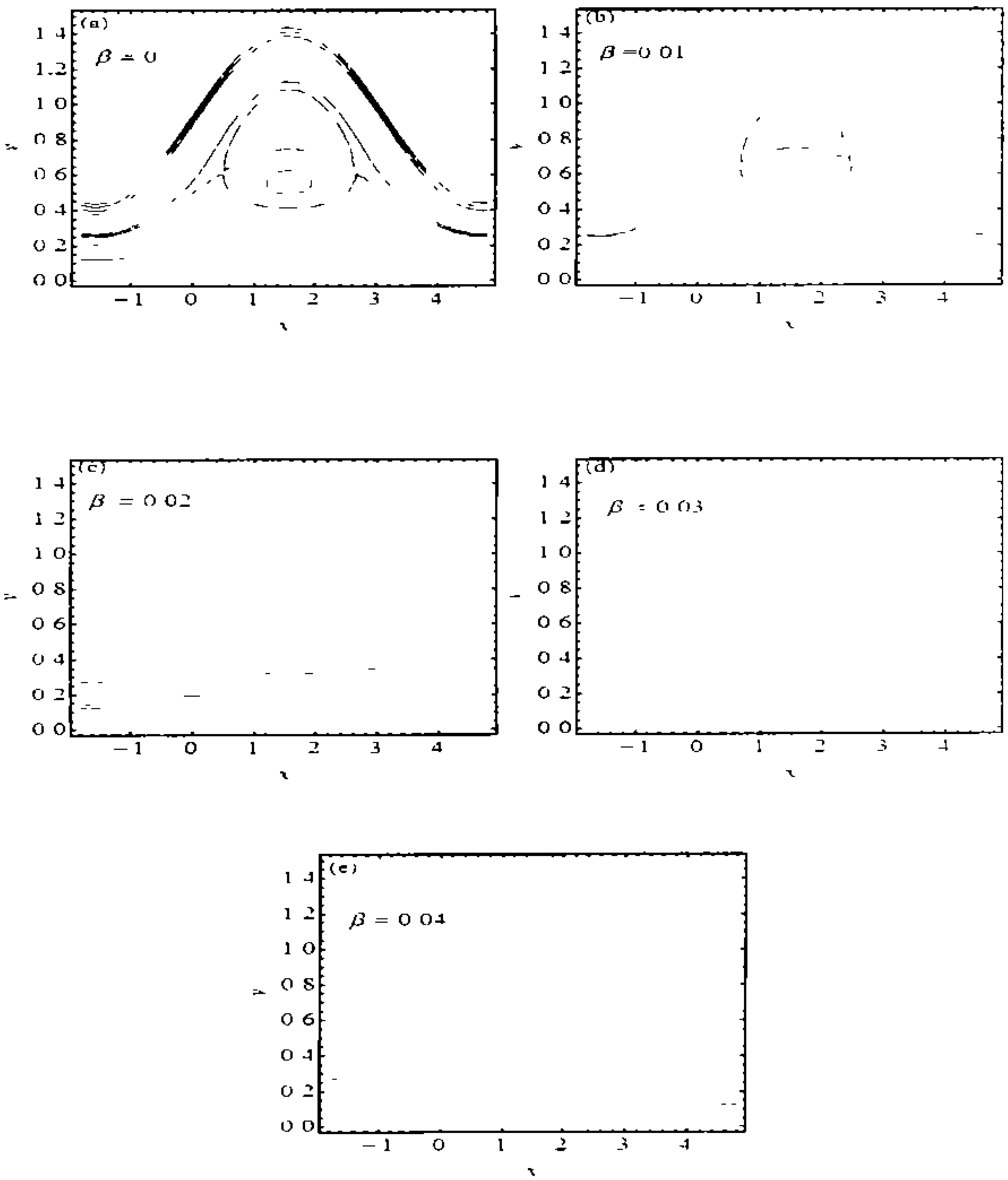

Fig. 3.5 Streamlines for different values of β

when $\phi = 0.5$, $\Gamma = 0.05$ and $Y_b = -1$

3.4 Conclusion

An analysis is carried out to investigate the effects of slip on bacterial hydrodynamics. The physical model is transformed into a fourth order non linear differential equation along with slip and prescribed flux conditions. The solution is obtained via perturbation technique for which is valid for weak non-Newtonian effects. The main findings of this analysis are

- The speed of the glider decreases with increasing slip and non-Newtonian parameters.
- The pressure rise per wavelength in the pumping region increases with increasing non-Newtonian parameter. In contrast, it decreases with increasing slip parameter.
- The bolus of fluid trapped in closed streamline decreases in size with increasing slip parameter. For larger values of slip parameter, the bolus disappears and streamlines become similar in shape to that of the boundary walls.

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