HEATING OF PLASMA WITH HIGH HARMONIC FAST WAVES IN SPHERICAL TOKAMAK

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Final Approval

It is certified that the work presented in this thesis entitled **"Heating of Plasma with High Harmonic Fast Waves in Spherical Tokamak"** by **Shaukat Mehmood, Registration No: 65-FBAS/MSPHY/F11** is of sufficient standard in scope and quality for the award of degree of Master of Science(MS) Physics from International Islamic University Islamabad

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Shaukat Mehmood

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This work is Dedicated to my inspiring

Dearest Brother

for

His care and endless support who have always stood

by me and dealt with all of my absence

from many family occasions

with a smile

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First of all thanks to Allah Almighty 'Who taught (the writing) by the pen (and) taught human being that which he knew not Then His Prophet (peace be upon him) for even the knowledge contained in 'the Pen and the Board is a fragment of the knowledge he (PBUH) is the master of I pray to Allah to turn my seek for the knowledge towards the the right path in the supervision of His beloved ones (Ameen)

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Shaukat Mehmood

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Abstract

The purpose of this present work is to analyze the effective heating of plasma with high harmonic fast waves (HHFW) in spherical tokamak Heating with HHFW is a promising scheme in addition to ohmic heating in spherical tokamak (ST) The idea of HHFW heating was coined by Ono in [9] Small aspect ratio high beta regime is used with low value of magnetic field to achieve the fusion reaction in spherical tokamak In this work heating scheme in ST with high harmonic fast waves is investigated with the help of dielectric tensor refered by Thomas H Stix in his book Waves in Plasmas We have used Maxwell s equations to achieve required dielectric tensor The confined electrons in the plasma in spherical tokamak by the magnetic field are gyrating with proportional frequency to strength of magnetic field When electromagnetic waves of resonance frequency (electron cyclotron frequency) are injected in confined plasma the energy of electromagnetic waves is transferred and temperature of electrons is raised In other words energy is absorbed by plasma

Chapter 1

Introduction

In this chapter, we have discussed about the nature and important properties of plasma its different features and requirements followed by different plasma confinement techniques

1.1 What is plasma

The electromagnetic force is the main building block to create structure e g atoms and molecules etc The binding energy of crystaline solids are larger than their thermal energy On heating they change their phase e g decompose melt or disassociate as shown in fig 1 1 When atoms got the energy greater than ionization energy atoms decompose into positive and negitive ions These ions are not totally free from electromagnetic force and are strongly affected by the force which is produced by other ions Although charges are no longer bound, they are capable of collective motion of great vigor on their assemblage Such an assemblage is said to be a plasma Of course bound systems can show extreme complexity of structure e g a protein molecule Plasma complexity is somewhat different since thermal excitation breaks interatomic bonds and ionized most terrestrial plasmas seen as gases In fact plasma is sometimes defined as an ionized gas that exhibit plasma-like behaviour Plasma like behaviour ensures after a small fraction of gas has ionized Therefore, partially ionized gases show most of the exotic phenomenon of fully ionized gases Plasmas produced by ionization of neutral gases have approximately equal number of positive and negative ions In this case the oppositely charged fluids will be strongly coupled and forced to electrically neutralize each other on macroscopic length-scales. Such plasmas are called quasi-neutral ('quasi' means the small variation from exact neutrality which have important dynamic consequences for certain types of plasma modes). Non-neutral plasmas contain charges of only one sign occuring primarily in the laboratory experiments. Their equilibrium depends opon the intensity of magnetic fields in which the charged fluid rotates. It is approximated as 95% (or 99% which depends on whom we are trying to impress) of the universe consists of plasma. In earlier approaches everything of the universe was plasma. In the present approach stars, nebulae and even interstellar space are filled with plasma. The solar system also consists of plasma of solar wind form. The plasma surrounding the earth is trapped by its magnetic field. We can also find terrestrial plasmas in lightning, fluorescent lamps, in laboratory experiments and in industrial processes. In fact, the glow discharge has recently become the mainstay of the micro-circuit fabrication industry [2]. A relationship between temperature and number density of plasma is shown in fig 1.1 in different types of plasma.

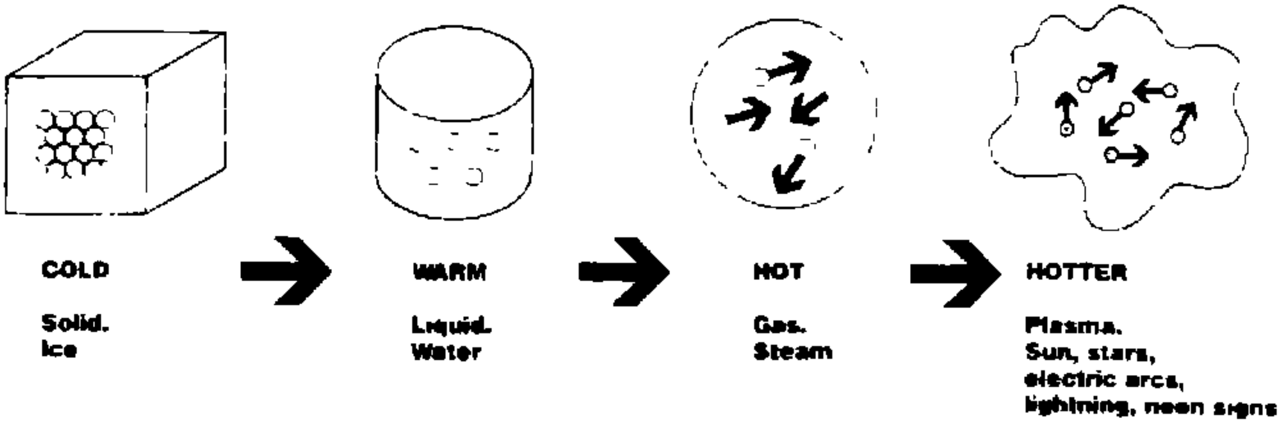


Figure 1.1: States of matter[1]

1.2 A brief history of plasma physics

When the constituent parts of blood are separated, a transparent liquid called plasma is left behind. The word plasma is taken from a Greek word $\pi\lambda\alpha\sigma\mu\alpha$, which means a "moldable substance" or a "jelly". This name is given by Czech scientist Johannes

Purkinje (1787 – 1869) He coined the term ionized gas in 1927 Langmuir said that blood plasma consists of red and white corpuscles by the way as electrons and ions in an electrified fluid Langmuir and his colleague Lewi Tonks were experimenting the physics and chemistry of tungsten filament light-bulbs He discovered certain regions of plasma discharge tube exhibiting periodic variations in electron density which are now called as Langmuir waves Langmuir s research provided theoretical basis of most plasma processing techniques in fabrication of intigrated circuits Langmuir s research leads to following five important developments

1 Radio broadcasting which led towards the earth s ionosphere

2 This helped the astrophysicists in recognizing that the most of the universe consists of plasma The pioneer of this field is Hannes Alfven who developed magnetic reconnection and dynamo theory

3 Creation of hydrogen bomb opened the door for controlled thermonuclear fusion which is possible energy source for the future

4 In 1958, James A Van Allen discovered Van Allen radiation belts that surrounds the earth

5 This helped in developing high powered lasers and opened the new field of laser plasma physics in 1960 2,

As shown in fig 1 2 the plasma medium is described macroscopically (on a large scale) by its temperature and density and changes in the plasma are calculated by using conservation equations such as conservation of energy momentum and mass

1.3 Basic parameters for plasma

All phenomena of plasma are due to the motion of gas molecules and macroscopic quantities Plasma system can be explained with the help of following parameters

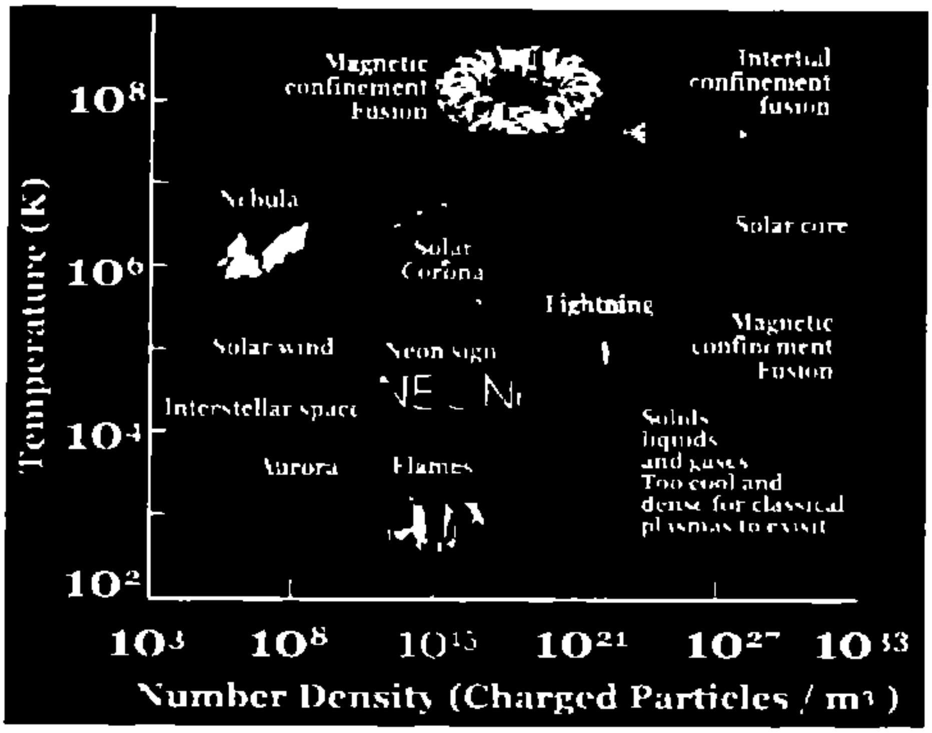


Figure 1 2 Number Density vs Temperature[1]

1 Debye shielding

2 Plasma frequency

3 Quasi-neutrality

1.3.1 Debye Shielding

When a charge carrier is placed in its solution then the meaurement of its net electrostatic effect and at which distance those electrostatic effects continue is called Debye shielding The volume of a sphere of radius equal to the Debye length is called Debye sphere There is no any electrical effect outside the sphere Debye length is the characterstic of plasma and is given by

$$\lambda_D = \left(\frac{k_B T}{4\pi n_e e^2}\right)^{1/2} \quad (cgs\ units) \qquad (1\ 1)$$

Above relation shows that Debye length "λ_D" is directly proportional to temperature and inversely proportional to the number density of electrons (n_e) This is obvious from the above formula that "λ_D" increases with increase in kinetic energy of electrons and decreases due to increase in concentration of electrons [3]

1.3.2 Plasma frequency

Collective behaviour of plasma can be explained by using time scale parameter. Plasma frequency or langmuir frequency "ω_p" suggests the reaction time. It is given by

$$\omega_{pe} = \left(\frac{4\pi n_0 e^2}{m_e}\right)^{1/2} (cgs\ units) \qquad (1.2)$$

where e, n_0 and m are the charge, number density and mass of electron respectively. This equation shows that plasma frequency is different for different species. Positive ions have greater mass and so have low frequency as compared to that of electrons [2].

1.3.3 Quasi-neutrality

The word "quasi" is a Latin word meaning small deviations from neutrality. As on macroscopic level, there exist equal no of positive and negative charges. Due to the cancellation of fields of positive and negative charges, plasma becomes neutral. Now we consider the Debye length of microscopic scale length at which plasma is not exactly neutral. So electric field is created due to deviation from perfect neutrality.

Deviation from perfect neutrality is achieved at length less than λ_D. But, on length scale above than λ_D, plasma must be quasi-neutral. The deviation from quasi-neutrality is obtained due to insufficient internal energy to become considerably non-neutral for larger distances than the Debye length.

One of the fundamental requirements of plasma is being quasi neutral, given as

$$n_i \simeq n_e \equiv n$$

where n_i = number density of ions, n_e = number density of electrons and "n" is called common density which is also called plasma density [4].

1.4 Plasma criteria

To achieve the perfect plasma state an ionized gas must fullfil the following three conditions.

1 $L \gg \lambda_D$

2 $N_D \gg 1$

3 $\omega_p \tau > 1$

1.4.1 $L \gg \lambda_D$

For conversion of an ionized gas into the plasma, the first condition is that "λ_D" must be much smaller then "L" where "L" is the scale length

1.4.2 $N_D \gg 1$

Debye shielding occures only when there are huge number of particles in the cloud For small number of particles Debye shielding becomes statistically invalid To find the number of particles in Debye sphere we can use the following equation

$$N_D = \frac{4}{3} n_0 \pi \lambda_D^3 = 1.38 \times 10^6 \sqrt{T^3 / n_0} \quad (T \text{ in } K)$$

1.4.3 $\omega_p \tau > 1$

The condition for plasma is that $\omega_p \tau$ must be greater than 1 where ω_p is the plasma frequency and "τ" is mean time between collisions with neutral atoms i.e

$$\omega_p \tau > 1 \tag{1.3}$$

Charged particles collide so frequently with neutral atoms that their motion is controlled by ordinary hydrodynamic forces rather than by electromagnetic forces If ω is the frequency of typical plasma oscillations and τ is the mean time between collisions with neutral atoms we require $\omega_p \tau > 1$ for the gas to behave like a plasma rather than a neutral gas

To shield the charge particles which are confined in the plasma system, there should be large number of electrons in the Debye sphere Oscillations especially of electron must be greater than the collision frequency f_{ne} of neutrals and electrons i e

$$f_p = \frac{\omega_p}{2\pi} > f_{ne} \qquad (1\,4)$$

If this condition is not achieved by the plasma system then the motion of neutral dominates and no collective action will occur in the system This target can be fullfilled for plasma system by increasing the degree of ionization [3]

1.5 Confinement

Here we explain some techniques to confine plasma We also discuss the advantages and disadvantages of these techniques Basically plasma confinement is accumulation of plasma in certain place Plasma heating and confinement are two major problems in the fusion reactor There are three methods from which plasma could be confined but two of them is used on earth for controlled thermonuclear fusion Plasma at high temperature will create a pressure and it will expand abruptly to control this expansion we need an opposite force against this thermal pressure This force could be gravitational e g in stars, magnetic force in magnetic confinement devices or may be plasma inertia that keeps the plasma together [5]

1.6 Gravitational confinement

One of force capable of confining the plasma fuel well enough to satisfy the Lawson criterion is gravity The required mass is so high for gravitational confinement so could survive only in stars for example in red dwarfs and brown dwarfs They fuse deuterium and lithium for which they have enough heavy mass In Figure 1 3 fusion is shown in the sun

Figure 1 3 Gravitational force of the sun makes the thermonuclear Fusion [6]

1.7 Inertial confinement

Inertial confinement [12]

In this kind of confinement large part of pellet surface is taken as a fusion fuel at very high temperature and pressure Fuel should be hot enough and dense enough that will proceed to fusion reaction Inertia of this fusion fuel will confine it only for nano seconds For ignition source lasers ions/electrons or z-pinch is used Inertial confinement is used in Hydrogen bomb and in controlled nuclear fusion as well Four stages of fusion by using inertial confinement are shown in Figure 1 4

1.8 Magnetic confinemet

This technique is used for plasma confinement as charge particles ions and electrons traces the lines of magnetic field, these magnetic force lines are used to confine the plasma In this fusion fuel is trapped by using strong magnetic fields

There are number of magnetic geometries used for magnetic confinement for example

- Magnetic Mirror
- Stellarators
- Tokamaks

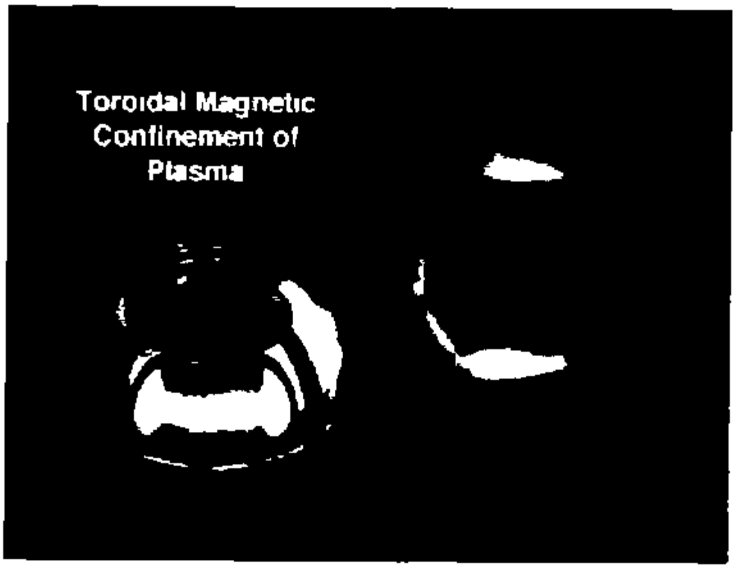


Figure 1 4 Magnetic confinement geomatery[12]

1.8.1 Magnetic Mirrors

In this type of magnetic confinement charged particles are reflected back from the densor magnetic field The invarience of magnetic moment 'μ' is the basic parameter of plasma confinement

$$\mu = \frac{1}{2} m v_{\perp}^{2} / B$$

As seen from above relation by increasing magnetic field "B" perpendicular velocity '$v_{\perp}$' will also increase to keep magnetic moment 'μ' constant To conserve total energy

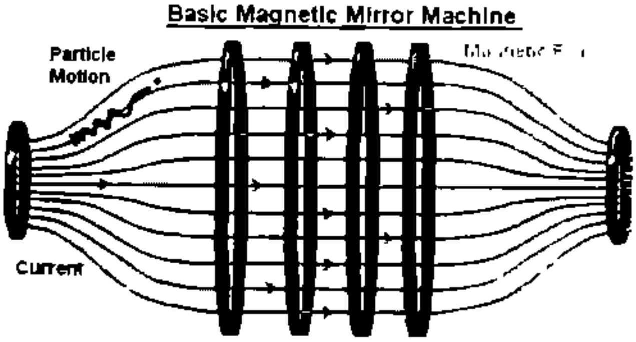


Figure 1 5 Mirror machine

parallel velocity component "$v_{\|}$" must decrease at throat There is high magnetic field so parallel velocity component "$v_{\|}$" will become zero and perpendicular velocity "$v_{\perp}$" is very high as a result particle have to reflect back In this geometry particles proceed to and fro motion between two ends of magnetic mirror as shown in fig1 5 [3]

1.8.2 Stellerators

It is the device to confine plasma fuel along the magnetic field lines to produce controlled thermo nuclear fusion In this machine closed flux surfaces are produced by externally used magenets while in "Tokamak" it is not like this Generally a stellarator consist of helical conductors in the form of pairs which are arrange antiparallel to each other to produce magnetic field by using current helical structure can be understood by fig1 6 Lyma spitzer first invented it in 1950 but after the invention of Tokamak which have better results than stellators are fallen from favor in 1970s [6]

1.8.3 Tokamaks

Tokamak is the confinement system which confines the plasma toroidaly Toroidal field is responsible for confinement, but in order to achieve equilibrium (to balance the plasma pressure) poloidal magnetic field is also applied Poloidal magnetic field in the tokamak mainly produced by plasma itself The direction of current due to poloidal field is toroidal Toroidal field "B_o" and Poloidal field "B_θ" both give rise the plasma in the helical trajectory in the torus Toroidal field "B_o" is produced by passing the current arround the

Figure 1 6 Stallerator [6]

torus, as shown in Figure 1 8 The product of particle density and temperature is called the plasma pressure The parameters that are particle density and temperature increased the fusion reactivity Strength of the magnetic field increases the plasma pressure, which is confined Strength of toridal field is limited due to technological factors The toroidal magnetic field is inversely proportional to the major radius the resulting field at the center of plasma would be around 6 ~ 8 tesla the toroidal fields in large scale Tokamak is lower than 6 ~ 8 tesla value For toroidal magnetic field the confined plasma pressure increases the plasma current up to limiting value The resulting poloidal field is lower than the toroidal field In large scale Tokamak current of several mega amperes is produced, for example in JET Tokamak 7 MA is produced In figures 1 7 1 8 and 1 8 schematic diagram of tokamak cutout model of tokamak is shown to elaborate the geomatery of tokamak

Since plasma current is driven by a toroidal electric field produced by the flux changing action of transforming The energy confinement time with the size is found experimentally which is equal to $\frac{1}{2}r_p^2$ where "r_p is the mean minor radius of the plasma In JET Tokamak confinement time is more than one seconds It is found experimentally that energy confinement time increases with plasma current and decreases with plasma pressure Tokamak plasmas are heated upto few KeV by Ohmic heating of plasma current the required temperature of $\geqslant 10\ KeV$ are then achieved by additional heating by Neutral Beam Injection (NBI) or Electromagnetic (EM) wave heating

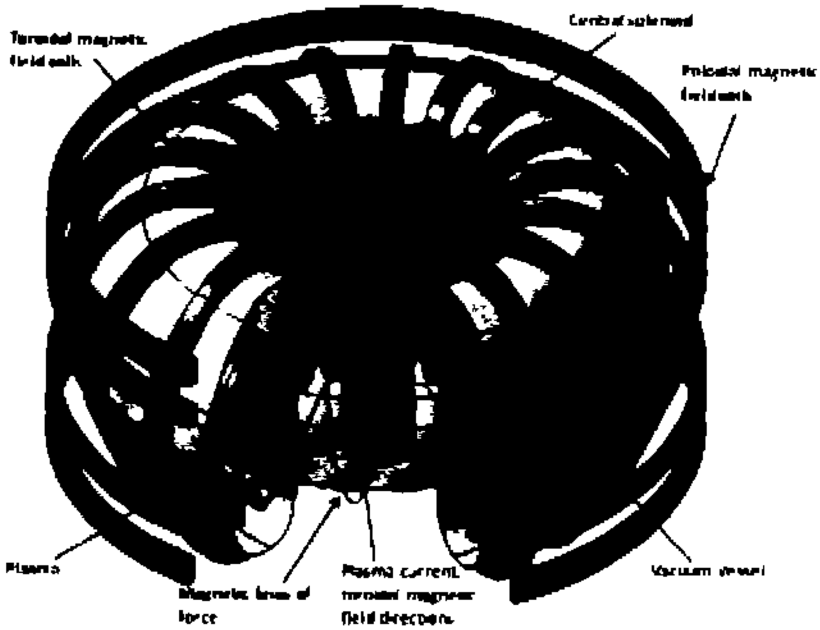


Figure 1.7 Schematic diagram of a tokamak [10]

Present days Tokamaks are operated in particle densities of the order of $10^{19} \sim 10^{20} m^{-3}$ which is lower than the atmospheric pressure of the order of 10^{6} [7]

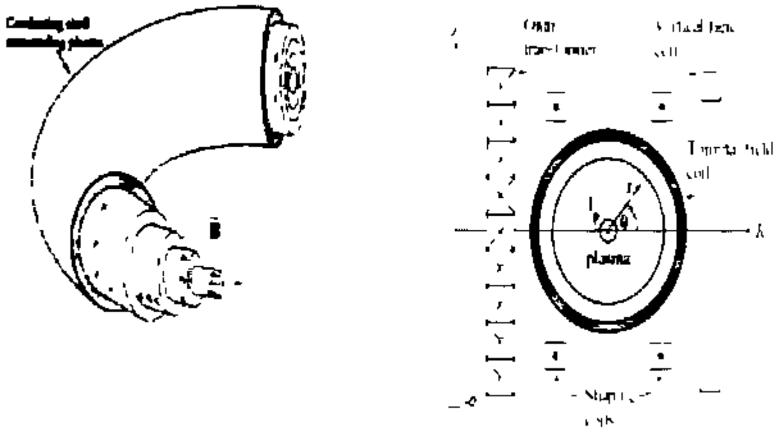


Figure 1.8 Torus [7]

1.9 Spherical Tokamaks

In 1986 Peng and Stricker developed new configuration with ultratight aspect ratio $\left(\frac{R}{a}\right)$ which is in the range of $1.2 \sim 1.4$. Number of spherical tokamak experiments are being

Figure 1 9 MAST [11]

done from some decades Here in fig 1 10 diffrence between conventional and spherical tokamak is shown

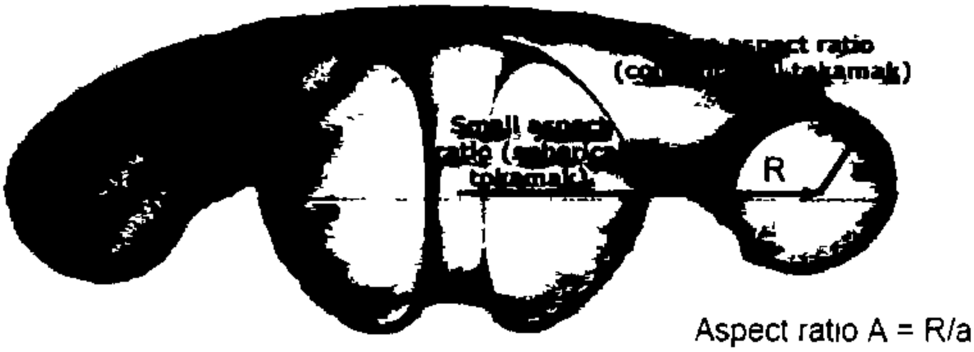


Figure 1 10 Diffrence between conventional and spherical tokamak[11]

(MAST) Mega Ampere Spherical Tokamak at Culham Laboratory in United Kingdom

(NSTX) National Spherical torus Experiment which is at Prinston Plasma physics Lab United States of America

The main advantage of spherical tokamak is that we can get high-beta due to low value of B (Magnetic field) Due to ultratight configuration and high beta condition additional heating is also required For this additional heating neutral beam injection is presently used in Small Tight Aspect Ratio Tokamak (START) and Mega Ampere

Spherical Tokamak (MAST) [8] [9] A schematic diagram for power genaration by controlled plasma fusion reaction is shown in fig 1 11

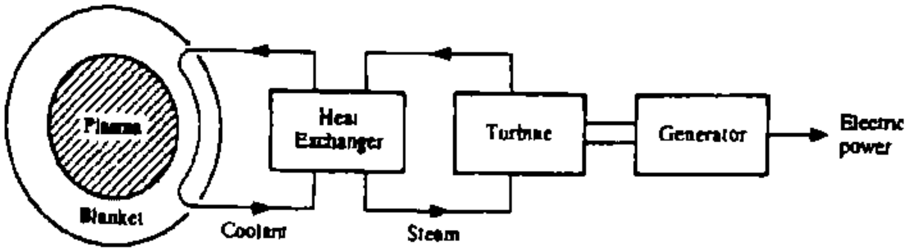


Figure 1 11 Schematic of Energy from Spherical tokamak [7]

Chapter 2

Plasma Heating

In this chapter different heating methods for the fusion reaction have been discussed. As high energy is required to fuse the light nuclei and different methods can be used to achieve the target. In the graph below relation between reaction rate and temperature is shown.

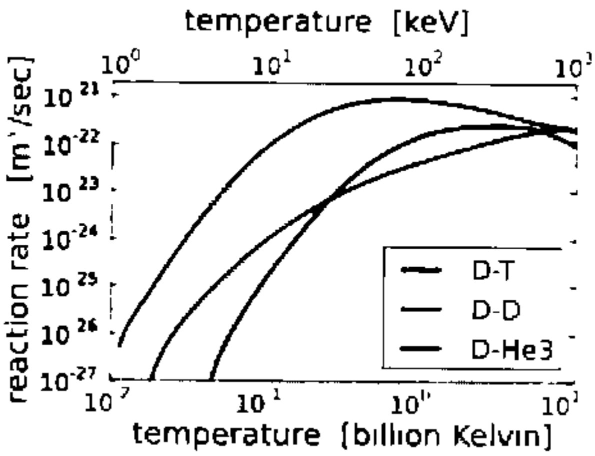


Figure 2.1 Temperature vs reaction rate

To start controlled thermo-nuclear fusion reaction we need temperature of million C°. Following are few known techniques to provide necessary energy to the fuel

- Ohmic Heating
- Magnetic Compression Heating

- Nuetral Beam Injection Heating
- Radio Frequency Heating

2.1 Ohmic heating

When high electric current passes through the plasma heat is produced due to power dissipation This type of heating is called "Ohmic Heating" which depends on the resistance of plasma through which current is passed It is just like electric heater used to heat the water The main problem in this type of heating is that when temperature of plasma increases its resistance decreases due to which heating is not so much effective

When temprature of plasma rises collision frequency decreases due to which resistance of plasma also decreases At high temperature plasma becomes collisionless resulting low resistance Because of this we can not rise the temperature of plasma upto certain limit by Ohmic heating Consequently other techniques also required to heat the plasma

2.2 Magnetic compression heating

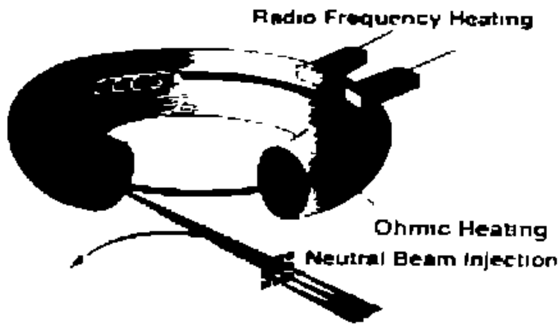
The second technique for plasma heating is "Magnetic Compression" Plasma is confined due to pressure exerted by a magnetic field Plasma confinement is strongly affected by the change in applied magnetic field As magnetic field strength is increased, plasma ions come closer and closer hence compressed This produces an extra heat So the use of magnetic field to heat the plasma has an extra advantage to compress the plasma which is requirement of nuclear fusion The probability of fusion increases by increasing plasma density The effectiveness of this regime depends upon the technique used to ignite the plasma

2.3 Neutral beam injection

To achieve the target of heating the plasma another technique is to insert the accelerated neutral particles into the plasma vessel Electric field cannot accelerate the neutral atoms so there is no direct way to inject the accelerated neutral beam into plasma so we have to use indirect way to achieve this target For this purpose positively charged deuterium ions are accelerated to enough energy before entering into the plasma vessel, accelerated deuterium ions pass through the neutral deuterium gas When accelerated duetrium ions pass through neutral deuterium gas they exchange their charge and become neutral Then these energetic deuterium atom will enter the plasma vessel When neutral atoms strikes the plasma deuterium atoms will ionized again and will trapped in the magnetic field During the collision process energetic deuterium atoms transfer some part of energy to plasma and consequently plasma is heated Neutral beam injection is additional energy which can be used with ohmic heating to ignite the plasma [1]

2.4 Radio frequency heating

The fourth way to heat the plasma is by radiating the radio frequency waves into the plasma It is just like microwaves which are used to heat the food as the microwaves agitate the polar molecules in the food and they are subject to vibration and consequently food is heated



Radio Frequency Heating

Radio frequency waves also heat the plasma as the plasma is composed of ions The main difference between radio frequency heating and microwave heating is quantity of radiation used to heat the plasma Charged particles of plasma absorbs the energy from the radio frequency waves and collision between them is increased which will increase the temperature of the plasma This heating is used in addition with ohmic heating because of limitations of ohmic heating [8] In fig 2 2 Ohmic heating NBI heating and radio frequency heating is shown in the torus

Chapter 3

High Harmonics Fast Waves Heating and Current Drive

In this chapter we have discussed the wave properties of waves interaction of waves with plasma and using the Maxwell s equations for derivation of dielectric tensor in order to heat the plasma

Let us begin with the basic definition and explanations

3.1 Waves

Any periodic motion which transfer energy is called waves Waves transfer energy from one place to other without transferring matter

3.2 Electromagnetic waves

The electromagnetic waves can be defined as disturbance of electric and magnetic fields The direction of these waves is perpendicular to both electric and magnetic fields These waves will continue propagating until absorbed by any matter

mathematically

$$E = \bar{A}e^{-i(\omega t - \bar{k}\cdot\bar{r})} \tag{3.1}$$

where

$$k = \frac{\omega}{v_p} \tag{3.2}$$

and v_p is velocity of phase propagation

3.3 Harmonics

The integral multiple of fundamental frequency is called harmonic. As the harmonic number is increased the frequency also increased, for example second harmonic has the frequency $2f$ and n^{th} harmonic has frequency nf and wavelength is $\frac{\lambda}{n}$

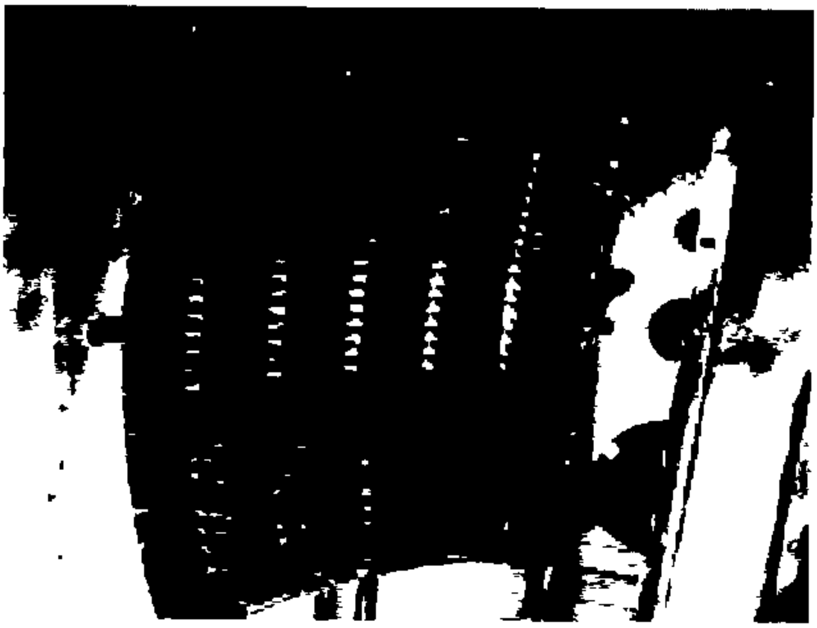


Figure 3.1: Combline antenna used in TST–2 spherical tokamak [13]

To produce high harmonic fast wave frequencies combline antenna is used in spherical tokamak. Working of this antenna is just like a band pass filter in which current is driven through first strap and other straps works due to mutual induction.

3.4 Cyclotron Resonance

For achieving the temperature upto required level radio frequency heating is usefull technique. For resonance or energy transfer our wave frequency must be equal to electron

S/No	Frequency	Source	Remarks
1	$< 100MHz$	High Power vacum tubes	
2	$\sim 1 - 10GHz$	Klystrons	Microwaves
3	$\sim 10 - 300GHz$	Gyrotrons	Submillimeter waves

Table 3 1 Table showing Frequency and Resource

cyclotron frequency to heat the electrons For example

$$\omega_{ce} = \frac{eB}{m_e} \qquad \text{at } B = 1T \tag{3 3}$$

$$\omega_{ce} = \frac{1\,6 \times 10^{-19} \times 1}{9\,1 \times 10^{-31}}$$

$$\omega_{ce} = 0\,17 \times 10^{12} rad/\sec$$

$$f = \frac{0\,17 \times 10^{12}}{2\pi} = 0\,02799 \times 10^{12} Hz$$

$$f = 28GHz$$

It means we required a wave of 28 GHz frequency to heat the electrons

And similarly for ions

$$\omega_{ci} = \frac{eB}{m_i} \qquad \text{for } B = 1T \tag{3 4}$$

$$\omega_{ci} = \frac{1\,6 \times 10^{-19} \times 1}{1\,6 \times 10^{-27}}$$

$$\omega_{ci} = 1 \times 10^{14} rad/\sec$$

$$f = \frac{\omega}{2\pi} = 0\,159 \times 10^{8} Hz$$

$$f = 16\ MHz$$

Which shows we need 16 MHz frequency wave to heat the ions

In High Harmonic Fast Waves (HHFW) $\omega_{pi} \leqslant \omega \leqslant \omega_{pe}$ is our promising Fast wave heating regime The sources of these frequencies are as follows

High power vacum tubes could be used for ion cyclotron heating (ICH) Gyrotron for electron cyclotron heating (ECH) and for lower hybrid current drive (LHCD) Klystron could be used [8]

3.5 Plasma Oscillations

Plasma is composed of electrons and ions Ions are obviously heavier than electrons when electrons are displaced from their mean position electric field is built to restore electrons back to their original position but due to inertia they will overshoot and continue to oscillate around the mean position which is called plasma frequency Although ions are not stationary but with respect to electrons they are supposed to be fixed due to their heavy mass These oscillations also called plasma waves The plasma frequency is given by [3]

$$\omega_p = \left(\frac{n_o e^2}{\varepsilon_o m}\right)^{\frac{1}{2}} rad/\sec$$

3.6 Electron Plasma Waves

Due to thermal effect plasma oscillations propagate this effect can be treated by including the term "$-\nabla P$" in the equation of motion The dispersion relation after including the thermal effect will becomes [3]

$$\omega = \omega_p + \frac{3}{2}k^2 V_{th}^2 \quad (3.5)$$

where $V_{th}^2 \equiv \frac{2kT_e}{m}$

3.7 Ion Waves

These are pressure waves propagate layer by layer In plasma there are no neutral atoms ions transmit their vibrations between each other due to their charge These vibrations are of low frequency The dispersion relation of an ion acoustic waves is given as [3]

$$\frac{\omega}{k} = \left(\frac{KT_e + \gamma_i KT_i}{M}\right)^{\frac{1}{2}} \equiv v_s \qquad (3.6)$$

Where v_s is speed of sound in plasma for ion waves the group velocity is equivalent to phase velocity

3.8 Upper Hybrid Frequency

For longitudnal waves in which $k \parallel E$ k and E along x-axis B_0 along z-axis so equations of motion will become

$$-i\omega m v_x = -eE - e v_y B_0$$

$$-i\omega m v_y = e v_x B_0$$

Solving for v_x and using $\omega_c = \frac{eB_0}{m}$ we get

$$v_x = \frac{eE/i\omega m}{1 - \frac{\omega_c^2}{\omega^2}}$$

Above equation shows that v_x becomes infinite at $\omega_c = \omega$

Now using equation of continuty in linearized form

$$\frac{\partial n_{e1}}{\partial t} + n_0 \nabla v_{e1} = 0$$

Or it can be written as

$$n_1 = \frac{k}{\omega} n_0 v_x$$

From linearized Poisson equation we get

$$\varepsilon_0 \nabla E = -e n_{e1}$$

$$\text{or } \varepsilon_0 ikE = -e n_1$$

Putting the value of n_1 and v_x we get

$$E\left(1-\frac{\omega_c^2}{\omega^2}\right)=\frac{-e^2 E n_0}{i m\omega^2\varepsilon_0}$$

Since $\omega_p^2=\frac{-e^2 n}{m\varepsilon_0}$ the above relation becomes on simplifying

$$\frac{\omega^2}{\omega_p^2}-\frac{\omega_c^2}{\omega_p^2}=1$$

The dispersion relation get the form

$$\omega^2=\omega_p^2+\omega_c^2\equiv\omega_h^2 \tag{3.7}$$

ω_h is called upper hybrid frequency [3]

3.9 Lower hybrid frequency

When angle (θ) between k and B_0 is exactly 90° the electrons goes along the lines of force and neutrality between electron and ions destroyed. Now they will not obey Boltzmann's relation rather obey the full equation of motion. For finite mass of electron equation of motion will be non trivial.

Equation of motion for ions using Boltzmann's relation is given as

$$v_{ix}=\frac{ek}{M\omega}\phi_1\left(1-\frac{\Omega_c^2}{\omega^2}\right)^{-1}$$

and for electrons

$e=-e$

$M=m$

$\Omega_c^2=-\omega_c$

so above relation becomes

$$v_{ex}=\frac{ek}{m\omega}\phi_1\left(1-\frac{\omega_c^2}{\omega^2}\right)^{-1}$$

and equations of continuity for ions and electrons becomes

$$n_{i1} = n_0 \frac{k}{\omega} v_{i1}$$

$$n_{e1} = n_0 \frac{k}{\omega} v_{e1}$$

As quazi-neutrality of plasma demands

$n_i \simeq n_e$

so above equations can be written as

$$M\left(1 - \frac{\Omega_c^2}{\omega^2}\right) = -m\left(1 - \frac{\omega_c^2}{\omega^2}\right)$$

$$\omega^2 = \frac{e^2 B^2}{Mm} = \Omega_c \omega_c$$

The dispersion relation takes the form

$$\omega^2 = (\Omega_c \omega_c)^{\frac{1}{2}} \equiv \omega_L \qquad (3.8)$$

Where "ω_l" is the lower hybrid frequency

by using the Poisson's equation $\epsilon_0 \nabla E1 = -e n_{i1}$ above relation takes the form

$$\frac{1}{\omega_l^2} = \frac{1}{\Omega_c \omega_c} - \frac{1}{\Omega_p^2}$$

In low density plasmas $\frac{1}{\Omega_p^2}$ term dominates. The plasma approximation is not valid at such high frequencies, so lower hybrid frequencies can be observed only if $\theta \simeq \frac{\pi}{2}$ [3]

3.10 Ordinary waves

In this case we take electric field is parallal to applied magnetic field i-e $E1 \parallel B_c$. These wave are called ordinary waves because they are not effected by magnetic field. The direction of these fields are as follows

$$B_o = B_o\hat{z}$$

$$E_1 = E_1\hat{z}$$

$$k = k\hat{x}$$

By solving the equation of motion for ordinary wave equation will become

$$\omega^2 = \omega_p^2 + c^2k^2$$

3.11 Extra-Ordinary waves

When electric field is transverse to direction of magnetic field B_o, electron motion will be changed (reflected) and also dispersion relation will be changed. In this case wave is elliptically polarized instead of plane polarized and wave is partially longitudnal and partially transverse [3]

The non trivial equations (x and y components) are

$$v_x = \frac{-ie}{m\omega}(E_x + v_y B_0)$$

$$v_y = \frac{-ie}{m\omega}(E_y + v_x B_0)$$

The dispersion relation for extra-ordinary waves will be given as

$$\frac{c^2k^2}{\omega^2} = \frac{c^2}{v_\phi^2} = 1 - \frac{\omega_p^2}{\omega^2}\frac{\omega^2 - \omega_p^2}{\omega^2 - \omega_h^2} \qquad (3.9)$$

3.12 Cutoffs and Rasonances

When an electromagnetic wave is injected into the vessel either it is absorbed in the plasma or reflected back. Reflection is reffered as cutoff and absorption is reffered as resonance.

3.12.1 Condition for Cutoff

A cutoff occur in the plasma when refractive index goes to zero i.e.

$$n = \frac{ck}{\omega} = 0 \tag{3.10}$$

$$\Longrightarrow \lambda \longrightarrow \infty$$

As dispersion relation of extra-ordinary wave by putting "$k = 0$" we get

$$1 - \frac{\omega_p^2}{\omega^2}\frac{\omega^2 - \omega_p^2}{\omega^2 - \omega_h^2} = 0 \tag{3.11}$$

we get a quadratic equation

$$\omega^2 \mp \omega\omega_c - \omega_p^2 = 0 \tag{3.12}$$

The solution of this equation give two roots for "ω" i-e

$$\omega_R = \frac{1}{2}[\omega_c + (\omega_c^2 + 4\omega_p^2)^{\frac{1}{2}}] \tag{3.13}$$

$$\omega_L = \frac{1}{2}[-\omega_c + (\omega_c^2 + 4\omega_p^2)^{\frac{1}{2}}] \tag{3.14}$$

ω_R and ω_L are called right hand and left hand cutoffs [3]

3.12.2 Condition for Rasonance

Resonance occurs in the plasma when refractive index goes to infinity

$$n = \frac{ck}{\omega} = \infty \qquad (3.15)$$
$$\Longrightarrow \lambda \longrightarrow 0$$

When a waves of given frequency "ω" approaches the resonance point both its phase velocity and group velocity approaches to zero and wave energy is converted into upper hybrid oscillations. Wave particle resonance dispersion relation is given by

$$\omega = k_{\|} v_{\|} + l\omega_c \qquad (3.16)$$

Where $l = 0, 1, 2, 3$

When Doppler shift frequency harmonics is exactly equal to cyclotron frequency wave is absorbed. This kind of absorption produces the high harmonic heating and current drive.

For "$l = 0$" resonance is called "Landau damping", for "$l = 1$" known as heating at the fundamental frequency and for "$l = 2$" resonance is second harmonic heating and so on.

It is an important phenomenon for electron cyclotron heating (ECH) and as well as for ion cyclotron heating (ICH) [8].

3.13 The dielectric tenser

3.13.1 Maxwell's equations

Considering electromagnetic waves propagating in any arbitrary medium, for example in a plasma. Generally, the behavior of a plasma is not too simple to be modeled by a simple permittivity ε or permeability μ as for dielectrics or magnetic materials. Instead we assume that plasma particles are immersed in vacuum characterized by ε_0 and μ_0

Then interaction of the charged particles and the waves propagation characteristics are examined explicitly by calculating the charge density σ and current density J by means of a suitable plasma model. The results of plasma behavior are then conveniently examined by means of the resulting "dielectric tensor".

Now we consider the Maxwell's equations

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{3.17}$$

$$\nabla \times B = \mu_\circ J + \frac{1}{c^2}\frac{\partial E}{\partial t} \tag{3.18}$$

$$\nabla \cdot E = \frac{\sigma}{\epsilon_\circ} \tag{3.19}$$

$$\nabla \cdot B = 0 \tag{3.20}$$

Using the small-amplitude allows one to expand all quantities as

$$Q_{(r,t)} = Q_\circ + \widetilde{Q}_{1(r,t)} \tag{3.21}$$

Where $Q_1 \ll Q_\circ$

Assuming the homogeneity we consider that Fourier analysis can be used in space and time as

$$\widetilde{Q}_{1(r,t)} = Q_1 exp(-i\omega t + ik \cdot r) \tag{3.22}$$

By taking $\frac{\partial}{\partial t} = i\omega$, $\nabla = ik$ in Equation 3.17 – 3.20 we get

$$ik \times E_1 = +i\omega B_1 \tag{3.23}$$

$$ik \times B = \mu_\circ J_1 - \frac{i\omega}{c^2}E_1 \tag{3.24}$$

$$ik \cdot E = \frac{\sigma_1}{\epsilon_\circ} \tag{3.25}$$

$$ik \cdot B = 0 \tag{3.26}$$

From Equation 3.23

$$B_1 = \frac{ik \times E}{i\omega} \qquad (3.27)$$

$$\Longrightarrow B_1 = \frac{k \times E}{\omega} \qquad (3.28)$$

Putting values of B_1 in Equation 3.24, we get

$$ik \times \frac{k \times E}{\omega} = \mu_c J_1 - \frac{i\omega}{c^2} E_1 \qquad (3.29)$$

$$-k \times \frac{k \times E}{i\omega} = \mu_c J_1 - \frac{i\omega}{c^2} E_1 \qquad (3.30)$$

$$-k \times k \times E = i\omega\mu_c J_1 + \frac{\omega^2}{c^2} E_1 \qquad (3.31)$$

$$\text{or} \quad k \times k \times E = -\frac{\omega^2}{c^2} E_1 - i\omega\mu_c J_1 \qquad (3.32)$$

This relationship is not isotropic, implying tensor relationship

$$J_1 = \overrightarrow{\sigma} E_1 \qquad (3.33)$$

Where $\overline{\sigma}$ is defined as the conducting tensor so Equation 3.32 can be written as

$$k \times k \times E_1 = -\frac{\omega^2}{c^2} E_1 - i\omega\mu_c \overrightarrow{\sigma} E_1 \qquad (3.34)$$

$n\omega = ck \Longrightarrow n = \frac{c}{\omega} k$

$$\Longrightarrow k = n\frac{\omega}{c} \qquad (3.35)$$

The dielectric tensor is given by

$$\overline{K} = \overrightarrow{1} + \frac{i}{\epsilon_c \omega} \overrightarrow{\sigma} \qquad (3.36)$$

Then Equation 3.34 becomes

$$\left(\frac{\omega}{c}\right)^2 [n \times n \times E_1] = -\frac{\omega^2}{c^2}\left[E_1 + \frac{i}{\omega} c^2 \mu_c \overrightarrow{\sigma} E_1\right] \qquad (3.37)$$

$$\Longrightarrow -E_1\left[1+\frac{i}{\omega}\overline{\sigma}c^2\mu_c\right] \quad (3.38)$$

Since

$$\epsilon_c = \frac{1}{c^2\mu_c} \quad (3.39)$$

$$n \times n \times E_1 + \overrightarrow{K} E_1 = 0 \quad (3.40)$$

$$D(\omega\ k) = o - or - D(\omega\ n) = 0 \quad (3.41)$$

Equation 3 37 can be written in three separate components as follows

$$\left[\overrightarrow{n} \times \overrightarrow{n} \times \overrightarrow{\ } + \overrightarrow{K}\right] E_1 = 0 \quad (3.42)$$

$$E_1 \neq 0$$

Hence

$$\overrightarrow{n} \times \overrightarrow{n} \times \overrightarrow{\ } + \overrightarrow{K} = 0 \quad (3.43)$$

By using vector identity

$$A \times B \times C = B(A\ C) - C(A\ B) \quad (3.44)$$

Hence

$$\overrightarrow{n}(\overrightarrow{n}\ \overrightarrow{\ }) - \overrightarrow{\ }(\overrightarrow{n}\ \overrightarrow{n}) \quad (3.45)$$

$$n = n_x i + n_y j \quad (3.46)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3.47)$$

$$\overrightarrow{K} = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{xy} & k_{yy} & k_{yz} \\ k_{xz} & k_{yz} & k_{zz} \end{bmatrix} \quad (3.48)$$

$$\vec{n}(\vec{n}\cdot\overrightarrow{\ }) = (n_x i + n_y j + n_z k)\left[(n_x i + n_y j + n_z k)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right] \qquad (3.49)$$

$$= (n_x i + n_y j + n_z k)\left[(n_x i + n_y j + n_z k)\cdot(i + j + k)\right] \qquad (3.50)$$

$$= (n_x i + n_y j + n_z k)(n_x + n_y + n_z) \qquad (3.51)$$

$$= \left(n_x^2 + n_x n_y + n_x n_z\right) i + \left(n_x n_y + n_y^2 + n_y n_z\right) j + \left(n_x n_z + n_y n_z + n_z^2\right) k \qquad (3.52)$$

This relation can be written as in matrix form

$$= \begin{bmatrix} n_x^2 & n_x n_y & n_x n_z \\ n_x n_y & n_y^2 & n_y n_z \\ n_x n_z & n_y n_z & n_z^2 \end{bmatrix} \qquad (3.53)$$

And

$$\overrightarrow{\ }(\vec{n}\cdot\vec{n}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}(n_x^2 + n_y^2 + n_z^2) \qquad (3.54)$$

$$= \begin{bmatrix} n_x^2 + n_y^2 + n_z^2 & 0 & 0 \\ 0 & n_x^2 + n_y^2 + n_z^2 & 0 \\ 0 & 0 & n_x^2 + n_y^2 + n_z^2 \end{bmatrix} \qquad (3.55)$$

Subtracting Equation 3.55 from Equation 3.52 we get

$$\vec{n} \times \vec{n} \times \overrightarrow{\ } = \vec{n}(\vec{n}\cdot\overrightarrow{\ }) - \overrightarrow{\ }(\vec{n}\cdot\vec{n}) \qquad (3.56)$$

$$\vec{n} \times \vec{n} \times \overrightarrow{\ } = \begin{bmatrix} n_x^2 & n_x n_y & n_x n_z \\ n_x n_y & n_y^2 & n_y n_z \\ n_x n_z & n_y n_z & n_z^2 \end{bmatrix} - \begin{bmatrix} n_x^2 + n_y^2 + n_z^2 & 0 & 0 \\ 0 & n_x^2 + n_y^2 + n_z^2 & 0 \\ 0 & 0 & n_x^2 + n_y^2 + n_z^2 \end{bmatrix} \qquad (3.57)$$

$$\overrightarrow{n} \times \overrightarrow{n} \times \overrightarrow{\;} = \begin{bmatrix} -\left(n_y^2 + n_z^2\right) & n_x n_y & n_x n_z \\ n_x n_y & -\left(n_x^2 + n_z^2\right) & n_y n_z \\ n_x n_z & n_y n_z & -\left(n_x^2 + n_y^2\right) \end{bmatrix} \tag{3.58}$$

Now

$$\overrightarrow{n} \times \overrightarrow{n} \times \overrightarrow{\;} + \overline{K} = \begin{bmatrix} -\left(n_y^2 + n_z^2\right) & n_x n_y & n_x n_z \\ n_x n_y & -\left(n_x^2 + n_z^2\right) & n_y n_z \\ n_x n_z & n_y n_z & -\left(n_x^2 + n_y^2\right) \end{bmatrix} + \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{xy} & k_{yy} & k_{yz} \\ k_{xz} & k_{yz} & k_{zz} \end{bmatrix} \tag{3.59}$$

$$\overrightarrow{n} \times \overrightarrow{n} \times \overrightarrow{\;} + \overline{K} = \begin{bmatrix} k_{xx} - \left(n_y^2 + n_z^2\right) & k_{xy} + n_x n_y & k_{xz} + n_x n_z \\ k_{xy} + n_x n_y & k_{yy} - \left(n_x^2 + n_z^2\right) & k_{yz} + n_y n_z \\ k_{xz} + n_x n_z & k_{yz} + n_y n_z & k_{zz} - \left(n_x^2 + n_y^2\right) \end{bmatrix} \tag{3.60}$$

Or by reversing signs we have

$$\Longrightarrow \begin{bmatrix} n_y^2 + n_z^2 - k_{xx} & -n_x n_y - k_{xy} & n_x n_z \\ n_x n_y & n_x^2 + n_z^2 - k_{yy} & n_y n_z \\ n_x n_z & n_y n_z & n_x^2 + n_y^2 - k_{zz} \end{bmatrix} = 0 \tag{3.61}$$

When solving this disperssion relation there will be multiple roots given for ω, n_y, n_z may lead to multiple solutions for n_x. Each root corresponds to independent wave with different propagation characteristics. These roots are independent when trying to understand RF heating & current drive in a plasma [8]

Dielectric tensor can be written in another form i-e

$$\text{The dielectric tensor} = \begin{bmatrix} S & -iD & 0 \\ -iD & S & 0 \\ 0 & 0 & P \end{bmatrix} \tag{3.62}$$

Where S is for sum, D is for difference, P is for plasma

$$S = 1/2\,(R + L) \tag{3.63}$$

$$D = 1/2(R - L) \tag{3 64}$$

$$R = 1 + \sum \chi^- = 1 - \sum \frac{\omega_{ps}^2}{\omega(\omega + \Omega\ )} \tag{3 65}$$

$$L = 1 + \sum \chi^+ = 1 - \sum \frac{\omega_{ps}^2}{\omega(\omega - \Omega\ )} \tag{3 66}$$

$$P = 1 - \sum \frac{\omega_{ps}^2}{\omega^2} \tag{3 67}$$

From Maxwell s equation

$$\nabla \times B = \frac{4\pi J}{c} + \frac{1}{c}\frac{\partial E}{\partial t} = \frac{1}{c}\frac{\partial D}{\partial t} \tag{3 68}$$

$$\nabla \times E = -\frac{1}{c}\frac{\partial B}{\partial t} \tag{3 69}$$

It is one of the form which is priviously derived

$$\text{i e } \; n \times (n \times E) + \epsilon E = 0 \tag{3 70}$$

$$\Longrightarrow [n \times (n \times \ ) + \epsilon]\, E = 0 \tag{3 71}$$

$$n \times n \times \ = \begin{bmatrix} -\left(n_y^2 + n_z^2\right) & n_x n_y & n_x n_z \\ n_x n_y & -\left(n_x^2 + n_z^2\right) & n_y n_z \\ n_x n_z & n_y n_z & -\left(n_x^2 + n_y^2\right) \end{bmatrix} \tag{3 72}$$

$$[n \times (n \times \ ) - \epsilon] = \begin{bmatrix} -\left(n_y^2 + n_z^2\right) & n_x n_y & n_x n_z \\ n_x n_y & -\left(n_x^2 + n_z^2\right) & n_y n_z \\ n_x n_z & n_y n_z & -\left(n_x^2 + n_y^2\right) \end{bmatrix} - \begin{bmatrix} S & -iD & 0 \\ -iD & S & 0 \\ 0 & 0 & P \end{bmatrix} \tag{3 73}$$

$$= \begin{bmatrix} S - \left(n_y^2 + n_z^2\right) & -iD + n_x n_y & n_x n_z \\ -iD + n_x n_y & S - \left(n_x^2 + n_z^2\right) & n_y n_z \\ n_x n_z & n_y n_z & P - \left(n_x^2 + n_y^2\right) \end{bmatrix} \tag{3 74}$$

Since n is in $(x\ z)$ plane

$$n_y = 0 \tag{3 75}$$

$$n_y^2 + n_z^2 = n^2 \cos^2\theta \tag{3.76}$$

$$n_x n_y = 0 \tag{3.77}$$

$$n_x n_z = n^2 \cos\theta \sin\theta \tag{3.78}$$

$$n_x^2 + n_z^2 = n^2 \tag{3.79}$$

$$n_y n_z = 0 \tag{3.80}$$

$$n_x^2 + n_y^2 = n^2 \sin^2\theta \tag{3.81}$$

Hence

$$[n \times (n \times \quad) + \epsilon] = \begin{bmatrix} S - n^2\cos^2\theta & -iD & n^2\cos\theta\sin\theta \\ -iD & S - n^2 & 0 \\ n^2\cos\theta\sin\theta & 0 & P - n^2\sin^2\theta \end{bmatrix} \tag{3.82}$$

The identities S D and P means "Sum" "Diffrence" and "Parallel" It is clear that the P element corresponds to the cold-plasma limit of the parallel dielectric i e $P = 1 + \chi_i + \chi_e$ where $\chi_\sigma = \frac{-\omega_{p\sigma}^2}{\omega^2}$ This is just the cold limit of the unmagnetized dielectric because behavior involving parallel motions in a magnetized plasma is identical to that in an unmagnetized plasma In the limit of no plasma $\overrightarrow{K}$ becomes the unit tensor and describes the effect of the vacuum displacement current only [14]

3.13.2 High Harmonic Fast Wave Dispersion Relation

For high beta scenario dielectric constant " $\in$ " is large since beta can be defined as ratio between plasma pressure to the magnetic pressure

mathematically ion beta

$$\beta_i = \frac{8\pi n_i k T_i}{B^2} \tag{3.83}$$

And similarly electron beta

$$\beta_e = \frac{8\pi n_e k T_e}{B^2}$$

Since electron plasma frequency

$$\omega_{pe} = \left(\frac{4\pi n_e e^2}{m_e}\right)^{\frac{1}{2}} \tag{3.84}$$

And

$$v_{Te} = \left(\frac{2kTe}{m_e}\right)^{\frac{1}{2}} \tag{3.85}$$

$$\Omega_e = \frac{eB}{m_e c} \tag{3.86}$$

Hence

$$\beta_e = \left(\frac{\omega_{pe}}{\Omega_e}\right)^2 \left(\frac{v_{Te}}{c}\right)^2 \tag{3.87}$$

$$= \in \left(\frac{v_{Te}}{c}\right)^2 \tag{3.88}$$

We get

$$\in = \beta_e \left(\frac{c}{v_{Te}}\right)^2 \tag{3.89}$$

For deuterium plasma in NSTX

$n_e \approx 5 \times 10^{13} cm^{-3}$

$T_e = 1keV = 11605 \times 10^3 K$

$B_T \approx 2.5 KG = 2500G$

Solving for electron beta "β_e"

$$\beta_e = \frac{8 \times 3.14 \times 5 \times 10^{13} \times 1.38 \times 10^{-16} \times 11605 \times 10^3}{2500 \times 2500}$$

$$\beta_e = 32.2\%$$

Now solving for thermal velocity

$m_e = 9.1 \times 10^{-28} g$

$$v_{Te} = \left(\frac{2 \times 1.38 \times 10^{-16} \times 11605 \times 10^3}{9.1 \times 10^{-28}}\right)^{\frac{1}{2}}$$

$$v_{Te} = 187.6 \times 10^7 cm/s$$

And dielectric constant ϵ for spherical tokamak

$$\epsilon = 0.322\left(\frac{3\times 10^{10}}{187.6\times 10^{7}}\right)^{2}$$

$$\epsilon = 82.34$$

These values are used in typically ST experiment

If we take the values of

$n_e \approx 5\times 10^{13} cm^{-3}$

$T_e = 200eV = 2\times 11605\times 10^{2}K$

$B_T \approx 1KG = 1000G$

Solving for electron beta"β_e"

$$\beta_e = \frac{8\times 3.14\times 5\times 10^{13}\times 1.38\times 10^{-16}\times 2\times 11605\times 10^{2}}{1000\times 1000}$$

$$\beta_e = 8\%$$

Now solving for thermal velocity

$m_e = 9.1\times 10^{-28}g$

$$v_{Te} = \left(\frac{2\times 1.38\times 10^{-16}\times 2\times 11605\times 10^{2}}{9.1\times 10^{-28}}\right)^{\frac{1}{2}}$$

$$v_{Te} = 84\times 10^{7} cm/s$$

And dielectric constant ϵ for spherical tokamak

$$\epsilon = 0.80\left(\frac{3\times 10^{10}}{84\times 10^{7}}\right)^{2}$$

$$\epsilon = 103.68$$

Now high harmonic fast wave dispersion relation can be written as

$$\det \begin{bmatrix} k_{xx} - n_\parallel^2 & -ik_{xy} & k_{xz} + n_\perp n_\parallel \\ ik_{xy} & k_{yy} - n^2 & ik_{yz} \\ k_{xz} + n_\perp n_\parallel & -ik_{yz} & k_{zz} - n_\perp^2 \end{bmatrix} = 0 \tag{3.90}$$

Where

$$k_{xx} = 1 + \sum_\sigma \frac{\omega_{p\sigma}^2}{\omega} \sum_{n=-\infty}^{\infty} \frac{n^2 I_n(\lambda) e^{-\lambda}}{\lambda} \frac{1}{k_\parallel v_{T\sigma}} Z_0(y_n) \tag{3.91}$$

$$k_{xy} = \sum_\sigma \frac{\omega_{p\sigma}^2}{\omega} \sum_{n=-\infty}^{\infty} n\left(I_n - I_n'\right) e^{-\lambda} \frac{1}{k_\parallel v_{T\sigma}} Z_0(y_n) \tag{3.92}$$

$$k_{yy} = 1 + \sum_\sigma \frac{\omega_{p\sigma}^2}{\omega} \sum_{n=-\infty}^{\infty} \left(\frac{n^2}{\lambda} I_n + 2\lambda I_n - 2\lambda I_n'\right) \times \frac{e^{-\lambda}}{k_\parallel v_{T\sigma}} Z_0(y_n) \tag{3.93}$$

$$k_{xz} = -\sum_\sigma \frac{\omega_{p\sigma}^2}{\omega} \sum_{n=-\infty}^{\infty} e^{-\lambda} \frac{k_\perp}{\Omega} \frac{nI_n}{\lambda} \frac{1}{2k_\parallel} \frac{dZ_0(y_n)}{dy_n} \tag{3.94}$$

$$k_{yz} = -\sum_\sigma \frac{\omega_{p\sigma}^2}{\omega} \sum_{n=-\infty}^{\infty} e^{-\lambda} \frac{k_\perp}{\Omega} \left(I_n - I_n'\right) \frac{1}{2k_\parallel} \frac{dZ_0(y_n)}{dy_n} \tag{3.95}$$

$$k_{zz} = -\sum_\sigma \frac{\omega_{p\sigma}^2}{\omega} \sum_{n=-\infty}^{\infty} e^{-\lambda} \frac{2(\omega - n\Omega)}{k_\parallel v_{T\sigma}^2} I_n \frac{1}{2k_\parallel} \frac{dZ_0(y_n)}{dy_n} \tag{3.96}$$

Where $y_n \equiv \frac{(\omega - n\Omega)}{k_\parallel v_{T\sigma}}$ and $\lambda \equiv \frac{(k_\perp^2 kT_\sigma)}{(\Omega_\sigma^2 m_\sigma)} = \frac{(k_\perp \rho_\sigma)^2}{2}$

Here I_n and I_n' are the nth-order modified Bessel function and its derivative, and Z_0 is the plasma dispersion function. The subcript σ is over all species.

3.14 Electron Absorption Processes

For the present analysis, the wave frequency is high compared to the ion cyclotron frequency ($\omega \approx 21\Omega_D$) but well below the electron cyclotron frequency ($\omega \approx 0.006\Omega_e$). We can also neglect the finite Larmor radius (FLR) effects on electron $\left[\lambda_e \approx \left(\frac{m_e}{m_i}\right)\lambda_i \ll 1\right]$. The terms that contribute to the electron damping are the n=0 terms in the k_{yy}, k_{yz}, k_{zy} and k_{zz} elements of the dielectric tensor. For the present case, it is sufficient to keep the lowest-order terms in λ_e. If we neglect the ion FLR terms, the dielectric elements are simplified to

$$K_{xxc} = 1 + \frac{\omega_{pe}^2}{\Omega_e^2} - \sum_i \frac{\omega_{pi}^2}{\omega^2 - \Omega_i^2} \tag{3.97}$$

$$K_{xyc} = \frac{\omega_{pe}^2}{\omega\Omega_e^2} + \sum_i \frac{\omega_{pi}^2 \Omega_i}{\omega(\omega^2 - \Omega_i^2)} \tag{3.98}$$

$$K_{yyc} = K_{xxc} - n_\perp^2 \frac{\omega_{pe}^2}{\Omega_e^2} \frac{v_{Te}^2}{c^2} \frac{\omega}{k_\parallel v_{Te}} Z_0(y_0) = K_{xxc} - n_\perp^2 \delta_m \tag{3.99}$$

$$K_{xzc} = -n_\perp n_\parallel \sum_i \frac{\omega_{pi}^2 \omega^2 v_{Ti}^2}{c^2 (\omega^2 - \Omega_i^2)^2} = n_\perp n_\parallel \delta \tag{3.100}$$

$$K_{yz} \cong -n_\perp \frac{v_{Te}^2 k_\parallel}{2c\Omega_e} K_{zze} = -n_\perp \delta_x K_{zzc} \tag{3.101}$$

and

$$K_{zz} \cong 1 - \sum_i \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pe}^2}{k_\parallel^2 v_{Te}^2} \frac{dZ_0(y_0)}{dy_0} \cong -\frac{\omega_{pe}^2}{k_\parallel^2 v_{Te}^2} \frac{dZ_0(y_0)}{dy_0} = K_{zze} \tag{3.102}$$

where δ_m is proportional to the electron beta is the magnetic pumping (MP) term and K_{zze} gives the electron Landau damping (ELD) Here δ_x represents the cross ($\times$) term in K_{yz} For cold ions δ in K_{xz} though being kept here is actually quite small (≤ 0.01) In the high beta regime usually δ_x is small but δ_m is of order of unity Since δ_m is of the order 1, the usual perturbation approximation by assuming small δ_m is no longer accurate One can then rewrite the matrix in Eq (1) in the following simple form

$$\det \begin{bmatrix} K_{xxc} - n_\parallel^2 & -iK_{xyc} & n_\perp n_\parallel (1+\delta) \\ iK_{xyc} & K_{xxc} - n_\parallel^2 - n_\perp^2 (1-\delta_m) & -in_\perp \delta_x K_{zz} \\ n_\perp n_\parallel (1+\delta) & in_\perp \delta_x K_{zz} & K_{zz} - n_\perp^2 \end{bmatrix} = 0 \tag{3.103}$$

The above determinant is convenient for solving $n_\perp$ for a given ω and $n_\parallel$ since all K's and δ's are independent of $n_\perp$ It is worthwhile to note that one can readily solve for $n_\perp$ exactly by solving the detrminant of the 3×3 matrix

$$K_{xxc} - n_\parallel^2 [(K_{xxc} - n_\parallel^2 - n_\perp^2 (1-\delta_m))(K_{zz} - n_\perp^2) - (-in_\perp \delta_x K_{zz})(in_\perp \delta_x K_{zz})] + iK_{xyc}[(iK_{xyc})(K_{zz} - n_\perp^2) -$$

$$(-in_\perp\delta_x K_{zz})(n_\perp n_\parallel(1+\delta))] + n_\perp n_\parallel(1+\delta)[(iK_{xyc})(in_\perp\delta_x K_{zz})$$

$$-(K_{xxc} - n_\parallel^2 - n_\perp^2(1+\delta_m))(n_\perp n_\parallel(1+\delta))] = 0 \qquad (3.104)$$

Lets take

$C_0 = K_{xxc}$ $C_1 = n_\parallel$ $C_2 = \delta$ $C_3 = K_{xyc}$ $C_4 = K_{xxc}$ $C_5 = \delta_m$ $C_6 = \delta_x$ $C_7 = K_{zz}$

Then above equation takes the form

$$C_0 - C_1^2[(C_0 - C_1^2 - n_\perp^2(1+C_5))(C_7 - n_\perp^2) - (-in_\perp C_6 C_7)(in_\perp C_6 C_7)] + iC_3[(iC_3)(C_7 - n_\perp^2) -$$

$$(-in_\perp C_6 C_7)(n_\perp C_1(1+C_2))] + n_\perp C_1(1+C_2)[(iC_3)(in_\perp C_6 C_7)$$

$$-(C_0 - C_1^2 - n_\perp^2(1-C_5))(n_\perp C_1(1+C_2))] = 0 \qquad (3.105)$$

$$C_0 - C_1^2[(C_0 - C_1^2)(C_7 - n_\perp^2) - n_\perp^2(1+C_5)(C_7 - n_\perp^2) - n_\perp^2 C_6^2 C_7^2] - C_3^2(C_7 - n_\perp^2)$$

$$-n_\perp^2 C_1 C_3 C_6 C_7(1-C_2) - n_\perp^2 C_1 C_3 C_6 C_7(1+C_2)$$

$$-n_\perp^2 C_1^2(1+C_2)^2[C_0 - C_1^2 - n_\perp^2(1-C_5)] = 0 \qquad (3.106)$$

$$C_0 - C_1^2[(C_7)(C_0 - C_1^2) - n_\perp^2(C_0 - C_1^2) - n_\perp^2 C_7(1+C_5) + n_\perp^4(1+C_5) - n_\perp^2 C_6^2 C_7^2]$$

$$-C_3^2C_7 + n_\perp^2 C_3^2 - 2n_\perp^2 C_1 C_3 C_6 C_7 (1 + C_2) - n_\perp^2 C_1^2 (1 + C_2)^2 (C_0 - C_1^2)$$

$$+n_\perp^4 C_1^2 (1 + C_2)^2 (1 - C_5) = 0 \quad (3\ 107)$$

$$C_7(C_0 - C_1^2)^2 - n_\perp^2 (C_0 - C_1^2)^2 - n_\perp^2 C_7 (1 + C_5)(C_0 - C_1^2) + n_\perp^4 (1 + C_5)(C_0 - C_1^2)$$

$$-n_\perp^2 C_6^2 C_7^2 (C_0 - C_1^2) - C_3^2 C_7 + n_\perp^2 C_3^2 - 2n_\perp^2 C_1 C_3 C_6 C_7 (1 + C_2)$$

$$-n_\perp^2 C_1^2 (1 + C_2)^2 (C_0 - C_1^2) + n_\perp^4 C_1^2 (1 + C_2)^2 (1 - C_5) = 0 \quad (3\ 108)$$

Rearranging the terms for $n_\perp^4$ and $n_\perp^2$

$$[(1 + C_5)(C_0 - C_1^2) + C_1^2 (1 + C_2)^2 (1 - C_5)]n_\perp^4 + [C_3^2 - (C_0 - C_1^2)^2 - C_7 (1 + C_5)(C_0 - C_1^2)$$

$$-C_6^2 C_7^2 (C_0 - C_1^2) - 2C_1 C_3 C_6 C_7 (1 + C_2) - C_1^2 (1 + C_2)^2 (C_0 - C_1^2)]n_\perp^2$$

$$+C_7 (C_0 - C_1^2)^2 - C_3^2 C_7 = 0 \quad (3\ 109)$$

Let put back the values $C_0 = K_{xxc}$ $C_1 = n_\parallel$ $C_2 = \delta$ $C_3 = K_{xyc}$ $C_5 = \delta_m$, $C_6 = \delta_\tau$ $C_7 = K_{zz}$ we get

$$[(1 + \delta_m)(K_{xxc} - n_\parallel^2) + n_\parallel^2 (1 + \delta)^2 (1 - \delta_m)]\mathbf{n}_\perp^4 + [K_{xyc}^2 - (K_{xxc} - n_\parallel^2)^2 - K_{zz} (1 + \delta_m)(K_{xxc} - n_\parallel^2)$$

$$-\delta_x^2 K_{zz}^2 (K_{xxc} - n_\parallel^2) - 2n_\parallel K_{xyc}\delta_x K_{zz}(1+\delta) - n_\parallel^2 (1+\delta)^2 (K_{xxc} - n_\parallel^2)]\mathbf{n}_\perp^2$$

$$+K_{zz}(K_{xxc} - n_\parallel^2)^2 - K_{xyc}^2 K_{zz} = 0 \quad (3.110)$$

This is the quadratic form in $n_\perp^2$ as

$$an_\perp^4 + bn_\perp^2 + c = 0 \quad (3.111)$$

Where

$a = (1+\delta_m)(K_{xxc} - n_\parallel^2) + n_\parallel^2 (1+\delta)^2 (1-\delta_m)$

$b = K_{xyc}^2 - (K_{xxc} - n_\parallel^2)^2 - K_{zz}(1+\delta_m)(K_{xxc} - n_\parallel^2) - \delta_x^2 K_{zz}^2 (K_{xxc} - n_\parallel^2)$

$-2n_\parallel K_{xyc}\delta_x K_{zz}(1+\delta) - n_\parallel^2 (1+\delta)^2 (K_{xxc} - n_\parallel^2)$

and

$c = K_{zz}(K_{xxc} - n_\parallel^2)^2 - K_{xyc}^2 K_{zz}$

"Further analysis of the above relation can gives the dependence of the magnetic pumping (MP) term and the electron Landau damping (ELD) on electron density. High value of MP will lead to large electron absorption. However this analysis is beyond the scope of this thesis work."

Chapter 4

Summary/ Conclusion

In this work our objective is to study the heating of plasma with high harmonics fast waves in spherical tokamak In this regard we have discussed in earlier chapters about the plasma its requirements conditions and its confinement procedures later on we came on in our objective heating of plasma is explained, in which different heating techniques are discussed In our discussion and letrature review it is obvious that Ohmic heating is not enough for controlled thermo-nuclear fusion reaction we need an extra source of heating to achieve the required temperature In this regard we have discussed different methods for heating As we have discussed high harmonic wave is a pominant source for heating

In this work some basic properties of electromagnetic waves (EM) have been discussed in the context of RF heating and current drive in the high beta plasmas It is concluded that the waves transmitted from outside into the center of plasma which is related by the dispersion relation $D(\omega k)$ Our target was to achieve accessibility of waves into center of plasma and then wave particle resonance in order to transmit waves energy into plasma particles Our major task has been to apply basic electromagnetic wave principles just described to learn how to heat the plasma and current drive efficiently For introductory RF heating and current drive, the more appropriate to focus on the analytical solution of simple models

The simple model is of the form

$$D(\omega k) = D_r(\omega k) + D_i(\omega k)$$

The functions D_r and D_i could be real if ω and k are real The function D_r associated with the wave propagation characteristics which describes

- Propagation
- Cutoffs
- Resonance

The second function D_i Which is always $D_i << D_r$ discirbes the dissipation of the wave due to resonance of wave and particle

We have derived the dielectric tensor by using Maxwell s equations later on discussed the dispersion relation for high harmonics fast wave At the end we explained the electron absorption process which leads us to dispersion relation in determinant form and further analysis of this determinant gives the result in qudratic form of $n_{\perp}^2$ Which have the very complex form of coefficients further analysis of this quadratic equation is beyond the scope of this work

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