Some Generalizations of Best Worst Method for Multi-Criteria Decision-Making

By

Aleena Ansar

Department of Mathematics and Statistics
Faculty of Sciences
International Islamic University, Islamabad
Pakistan
2024

Some Generalizations of Best Worst Method for Multi-Criteria Decision-Making

By

Aleena Ansar

Reg. No. 834-FBAS/MSMA/F22

Supervised By

Dr. Sajida Kousar

Department of Mathematics and Statistics
Faculty of Sciences
International Islamic University, Islamabad
Pakistan

2024

Some Generalizations of Best Worst Method for Multi-Criteria Decision-Making

By

Aleena Ansar

Reg. No. 834-FBAS/MSMA/F22

A dissertation submitted in the partial fulfillment of requirements for the award of degree of ***MASTER OF SCIENCE*** *in* ***MATHEMATICS***

Supervised By

Dr. Sajida Kousar

Department of Mathematics and Statistics
Faculty of Sciences
International Islamic University, Islamabad
Pakistan

2024

<u>Certificate of Approval</u>

Some Generalizations of Best Worst Method for Multi-Criteria Decision-Making

By

Aleena Ansar

Reg. No. 834-FBAS/MSMA/F22

A DISSERTATION SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN MATHEMATICS

We accept this dissertation as confirming to the required standard.

Prof. Dr. Madad Khan (*External Examiner*)	*Dr. Amna Kalsoom* (*Internal Examiner*)
Dr. Sajida Kousar (*Supervisor*)	*Prof. Dr. Nasir Ali* (*Chairperson*)

Department of Mathematics and Statistics
Faculty of Sciences
International Islamic University, Islamabad
Pakistan
2024

Author's Declaration

I hereby declare that this thesis, in whole and no portion of it, is completely free from any form of copying. I hereby declare that I have carefully compiled this dissertation exclusively with my own efforts, with the support and guidance of my mentor, Dr. Sajida Kousar. All utilized sources have been appropriately referenced and cited in accordance with academic standards. The information provided in this dissertation has not been submitted for a degree or qualification application from any other academic institution before.

Name of Student: Aleena Ansar

Reg. No. 834-FBAS/MSMA/F22

Dated: August 21, 2024

Forwarding Sheet by Research Supervisor

The Thesis entitled **Some Generalizations of Best Worst Method for Multi-Criteria Decision-Making** submitted by **Aleena Ansar**, Reg. No 834-FBAS/MSMA/F22 in partial fulfillment of MS Degree in Mathematics has been completed under my guidance and supervision. I am satisfied with the quality of her research work and allow her to submit this thesis for further process to graduate with Master of Science degree from the Department of Mathematics and Statistics, as per IIUI rules and regulations.

Dr. Sajida Kousar
Assistant Professor
Department of Mathematics and Statistics,
International Islamic University,
Islamabad.

Dedicated to

my parents and brother

Preface

Decision Making (DM) is a broad and extensive topic of research that encompasses various subject fields. It can be defined as the act of choosing a logical alternative from a range of possible choices. Regarding people, the process of making decisions or reasoning is intricate and influenced by numerous elements, both internal and external. The way people think has a direct impact on human reasoning. Nevertheless, replicating this behavior accurately with a computer is exceedingly challenging. Computers rely exclusively on logical reasoning for their decision-making process, in contrast to humans. DM is a methodical process of choosing from multiple options by recognizing the need, assembling relevant information, appraising, and comparing potential options. Multi-Criteria Decision-Making (MCDM) helps select the best option in real-life situations by developing alternatives, assigning weights to criteria, and assessing performance. MCDM is a structured method for DM that compares alternatives using multiple criteria and addresses potential contradictions in strategy development practices and procedures. MCDM methods help identify the most effective solution for selecting the best alternative in various applications. Since the time of its establishment in the $20th$ century, numerous researchers have devoted their time to creating new MCDM algorithms and strategies. Despite the exponential growth of the MCDM method due to research, there is still room for improvement, as little effort is being made to establish its mathematical foundation.

Chapter 1 establishes the framework to explore the foundations of DM. This study presents a comprehensive analysis of the DM process by emphasizing its complex elements and the importance of different criteria in DM situations. Moreover, MCDM methodologies and their crucial role in managing the complexities inherent in DM processes has been discussed. The chapter concludes with an examination of the weight of criteria which is a crucial component of MCDM approaches.

In Chapter 2, the Best-Worst-Entropy-TOPSIS (BWET) MCDM technique is explained. By using a rigorous mathematical framework, this chapter provides a comprehensive explanation of the Best-Worst Approach, the Entropy Method, and the Technique for Order of Preference by Similarity to the Ideal Solution (TOPSIS). The chapter begins with an instructive example which demonstrates the use of the BWET approach followed by a thorough investigation of its application, specifically in the context of solar panel installation.

Chapter 3 is based on the discussion which introduces the Best-Worst-MEREC (BW-MEREC) MCDM technique. As in previous study, $Chapter 2$, this chapter introduces a mathematical model that specifically examines the method of the removal effects of criteria (MEREC) approach within the BW-MEREC framework. This study offers a practical demonstration of applying the BW-MEREC method to effectively address the pressing issue of smog in Pakistan. The conclusions are derived from the BW-MEREC approach by implementing a thoughtful examination.

Acknowledgement

First and foremost, I extend my heartfelt gratitude to Almighty Allah, the source of all blessings, for granting me the strength, guidance, and opportunities to embark on this academic journey.

I am very thankful to my supervisor, **Dr. Sajida Kousar**, for her constant support, valuable advice, and excellent guidance during my research and thesis work. Your dedication to teaching, providing feedback, and sharing knowledge guided my research effectively. Your encouragement, patience, and belief in my abilities have given me confidence and a clear sense of purpose. I am deeply grateful for your unwavering support and for motivating me to do my best in everything I do.

I want to express my heartfelt thanks to my dear parents, **Mr. and Mrs. Ansar**, for their endless love, constant support, and many sacrifices. Your encouragement, guidance, and strong belief in my abilities have been the main reasons for my success. I am truly grateful for the things you have given up, the wise advice you have offered, and your unwavering faith in what I can achieve. Your care and backing have been the best things in my life, and I will always appreciate your constant love and unwavering support.

I also want to thank my brother, **Daniyal Ansar**, from the bottom of my heart for always being there to support me, encourage me, and understand me during this journey. His presence in my life has always given me strength and inspiration. His belief in what I can do, along with his dependable support, has been like a guiding light throughout my time in school. I really appreciate having him in my life, as he has been a source of strength and encouragement to me.

I would like to extend my deepest appreciation to seniors **Ms. Gul Freen**, **Ms. Urooj Shafqat**, **Dr.Sumbal Khalil**, **Ms. Asma Alvi**, and **Ms. Tania Hussain Satti** for their invaluable guidance, friendship, and support throughout this journey. Your camaraderie and encouragement have enriched this experience, making it both enjoyable and rewarding. I am sincerely grateful for your steadfast support and the strong bonds we have formed.

Contents

List of Tables

List of Figures

Chapter 1

Preliminaries

1.1 Decision-making (DM)

Decision-making (DM) is a methodical process for choosing from various options by recognizing the need, assembling relevant information, appraising, and comparing potential choices.
DM is a cognitively arduous process that includes identifying desired outcomes based on various physiological, cultural, and social factors. Authority and risk levels can influence DM, whether it is logical or irrational [1]. Contemporary DM challenges are resolved using mathematical equations, statistics, principles, economic theories, and computerized tools for automated computation, evaluation and estimation of results.

1.1.1 Multi-criteria decision-making (MCDM)

Multi-criteria decision-making (MCDM) is a DM technique used in real-life situations to choose the best, engage in developing alternatives, assign weights to the criteria, and assess performance.
MCDM is an area that integrates different operations research disciplines, encompassing Multi-Attribute Decision-Making (MADM) and Multi-Objective Decision-Making (MODM). MODM investigates continuous decision areas with

endless options or continuous DM problems. The answer is a viable region, but there is no clear-cut solution. Criteria serve as goals, while traits remain unstated. Limits are clear, and DMs communicate frequently. MADM, also known as discrete problems, is an evaluation problem with clear goals, attributes, and choices that lacks contact between decision makers (DMs) and has limited boundaries [2].

1.1.2 Components of DM

Criteria

Criteria are essential aspects of DM, representing qualitative and quantitative factors like sustainability, safety, and social impact. They help assess and contrast options, ensuring accurate evaluation and well-informed judgments by determining their relative significance.

Alternatives

In a DM process, one can consider alternatives, which range from concrete options like products or technology to abstract plans or policies, and evaluate and contrast them based on their ability to meet the chosen goals.

Examples

Example 1: Using six criteria—price, quality, safety, style, fuel consumption, and color—we choose the most suitable car out of four options, that are, BMW, Honda, Audi, and Toyota.

Example 2: Six laptop brands are HP, Dell, Lenovo, Apple, Toshiba, and Acer, each ranked based on price, storage capacity, RAM, and weight.

Example 3: There are eight alternatives for selecting a leader. Daniyal, Hamza, Imran, Faisal, Ayaan, Junaid, Bilal, and Haroon are based on three criteria: age, education, and experience.

1.1.3 MCDM methods

Multi-criteria decision-making methods provide logical, comprehensible, and viable options in real-life scenarios, identifying options, determining their importance, assessing efficiency, and ranking alternatives based on overall performance. Several MCDM strategies were proposed, including some of the most popular MCDM methods as shown in Table 1.1, including the Simple Additive Weighting (SAW), Analytic Hierarchy Process (AHP), Weighted Product Model (WPM), VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR), Data Envelopment Analysis (DEA), Multi-attribute utility analysis (MAUT), ELimination Et Choix Traduisant la REalité (ELECTRE), Grey Relational Analysis (GRA), and Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE).

Table 1.1: MCDM methods

Methods	Year	Description	Reference
AHP	1970	The AHP compares hierarchical criteria pairwise, considering various facts, organizing problems hierarchically, and emphasizing the importance of each component in DM for adaptability and ease of use.	Saaty [3]
DEA	1957	DEA evaluate efficiency through standardized modules with diverse inputs and outputs, allowing peer comparisons and managing multiple inputs and outputs without relationships.	Charnes *et al.* [4]
GRA	1980s	GRA is a crucial method in MCDM for assessing the link between alternatives and criteria, especially useful in limited data-rich situations for efficient ranking and selection.	Mahmoudi and Javed [5]
WPM	1922	WPM are versatile DM tools that aid in weighting criteria, evaluating alternatives, prioritizing issues, promoting impartiality, and sensitivity analysis across various academic disciplines.	Aruldoss *et al.* [6]
ELECTRE	1968	A ranking algorithm called ELECTRE has been used to compare pairs of data and surpass the correlation between alternatives.	Yu *et al.* [7]

VIKOR	1990	The VIKOR method is a widely used technique in MCDM, involving evaluating alternatives, balancing trade-offs, and considering specific criteria for effective results.	Gavade [8]
MAUT	1957	MAUT focus on adding ambiguity and risk preference factors to multi-criteria decision-support techniques.	Taufik *et al.* [9]
SAW	1954	The process involves adding scores representing goal achievements, considering all criteria, and multiplying by the criteria weights.	Wang [10]
PROMETHEE	1982	The application employs an outranking methodology to evaluate alternatives and compare them to the choice requirements to solve a decision problem.	Kabir *et al.* [11]

1.1.4 Weight of criteria

Several techniques were created to establish criteria weights Ayan *et al.* [12]. Three categories exist for weighting methods: subjective, objective, and integrated weighting methods. Subjective weighting methods, like direct ranking, depend on the preferences of the people making the decisions. As the number of variables increases, these methods become less beneficial. Objective weighting methods, on the other hand, employ particular techniques to compute results, and the subjective and objective methods are combined in the integrated weighing method.

MCDM weighting methods are shown in Table 1.2, including Integrated Determination of Objective CRIteria Weights (IDOCRIW), CRiteria Importance Through Intercriteria Correlation (CRITIC), Criterion Impact Loss (CILOS), and MEthod based on the Removal Effects of Criteria (MEREC), which are objective weighting methods. Level Based Weight Assessment (LBWA), Full Consistency Method (FUCOM), and Simple Multi-Attribute Rating Technique (SMART) are subjective weighting methods. Figure 1.1 illustrates the classification of weighting methods for the MCDM problem.

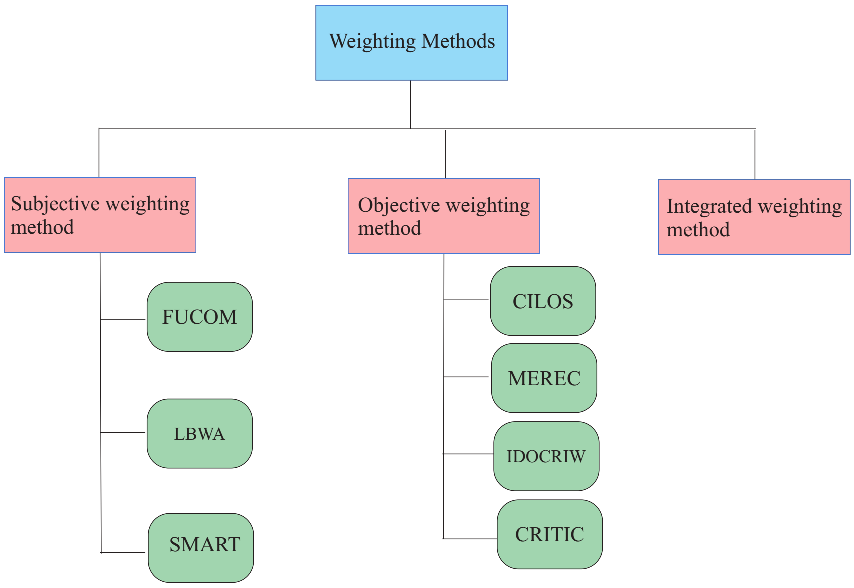

Figure 1.1: Classification of weighting methods in MCDM

Table 1.2: MCDM weighting methods

Methods	Year	Description	Reference
CILOS	1996	When one reaches the ideal, the CILOS approach, an objective technique, weighs the impact of other criteria, increasing the weight as the proportionate loss of effect decreases.	Zavadskas and Podvezko [13]
IDOCRIW	2018	The IDOCRIW technique is a hybrid approach combining ENTROPY and CILOS weights, demonstrating variability in criteria values, with significance decreasing as losses increase.	Zavadskas and Podvezko [13]
CRITIC	1995	CRITIC is a method for objective weights in MCDM issues, utilizing assessment matrix analysis to quantify intrinsic information, considering standard deviation and correlation.	Peng *et al.*[14]
FUCOM	1983	FUCOM is a comparison-based MCDM method that ensures consistent criteria weight coefficients and correct results by requiring fewer comparisons and checking the difference from maximum consistency.	Pamucar *et al.* [15]
LBWA	2015	The LBWA method is a subjective scoring method that simplifies BWM and AHP methods, prevents inconsistent expert opinions, and allows sensitivity analysis of MCDM models.	Žižović and Pamucar [16]
SMART	1971	The process involves assigning weights to criteria based on their significance and translating these weights into numerical values.	Patel *et al.* [17]

Chapter 2

BestWorst-Entropy-TOPSIS Multi-criteria Decision Making Method

This chapter introduces a MCDM methodology that combines the Best-Worst method, Entropy, and TOPSIS and provides and explaination of each component of the BWET technique for criteria weighting, uncertainty management, and ranking alternatives. To illustrate the systematic application of the BWET approach, a concrete example is discussed in this study. Additionally, the BWET MCDM methodology is applied to a real-world solar panel installation issue, demonstrating its effectiveness in managing practical DM obstacles. The application demonstrates the efficiency of the BWET technique in managing uncertainty in multi-criteria, and it defines a helpful instrument for DMs in the renewable energy sector. In this chapter there are three sections: $Section 2.1$ introduces the mathematical pattern for MCDM, which encomprises the Best-Worst approach, Entropy, and TOPSIS. $Section 2.2$ includes the BWET MCDM methodology to address the issue of solar panel installation. $Section 2.3$ concludes the chapter by reviewing the main discoveries and emphasizes the contribution of the BWET method to MCDM.

2.1 Mathematical model for MCDM

2.1.1 Best worst method (BWM)

The BWM is a widely used and effective method of problem-solving using pairwise comparison, was popularized by Rezaei [18]. BWM uses mathematical frameworks to provide a pair-wise comparison with a rational framework, helping decision-makers (DMs) determine the most suitable and the most detrimental criteria. It determines priorities and the importance of linguistic perceptions of comparisons using a calculated value from 1 to 9. However, in DM problems with different levels, BWM cannot achieve the final ranking of alternatives, as it only determines the weights of criteria and alternatives, hindering the overall aggregated weights and ranking.

2.1.2 Entropy method

Shannon and Weaver developed the entropy weights method in 1947. Probability theory quantifies efficiency dispersion in DM to calculate entropy, which is a measure of data scarcity. It utilizes the concept of entropy, an objective weighting technique, to assess the significance of each pertinent criterion. The entropy methodology determines the weights of criteria by assessing the significance of each feature, regardless of the DM's preference. The entropy weight measurement process then includes the decision structure, criteria, attribute-matched chance, variance, criterion, feature entropy value, and finally the entropy weight. A higher score is considered superior to a lower one, Chodha *et al.*[19].

2.1.3 TOPSIS method

This study employs an intuitive MCDM strategy that depends on the TOPSIS method. It not only offers simplicity and ease of use but also provides solutions for a variety of choices and standards. Hwang et al. initially suggested this approach Gavade[8]. The TOPSIS method aims to select an option with the least mathematical dispersion from the optimal negative outcome and the biggest mathematical dispersion towards the optimal positive outcome Chodha *et al.*[19]. This method determines each attribute weight, normalization, mathematical distance, and ideal solution. Standardization in MCDM is necessary due to the typically inconsistent measurement of requirements or criteria. The TOPSIS method features restorative procedures that allow for quality compromises. The best result associated with different qualities balances out each quality's negative impact Opricovic and Tzeng-Selamzade *et al.* [20]-[21].

This section outlines the specific procedures for implementing the BW criteria for a DM problem, utilizing the concepts of entropy weights and TOPSIS methods. The DM methodology is shown in Figure 2.1

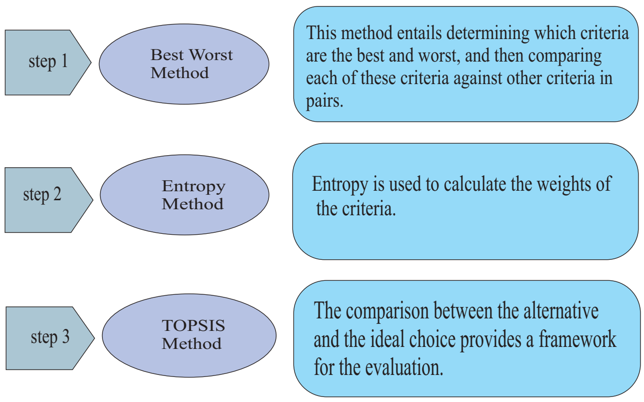

Figure 2.1: DM methodology

We have carried out the whole process in the following way:

Step 1: DMs must have defined criteria $c1$, $c2$,...,cn and then determine the best and worst criteria.

Step 2: Determine which criteria you consider to be the most important over all other criteria. Create a pairwise comparison vector for the best criteria over all other criteria by placing each criterion on a scale of 1 to 9.

Step 3: Determine which criteria you consider the least important of all the others. Create a pairwise comparison vector for all criteria over the worst criteria by placing each criterion on a scale of 1 to 9.

Step 4: The entropy approach finds the data weights of the best and worst criteria by requiring the determination of the normalized data value.

$$h_{ij} = \frac{u_{ij}}{\sum_{j=1}^{n} u_{ij}} \tag{2.1.1}$$

Step 5: Calculate entropy from the given Equation 2.1.2.

$$\begin{aligned} e_{bw} &= -L \sum_{j=1}^{n} h_{ij} ln h_{ij} \\ where - L &= \frac{1}{lnv} \end{aligned} \tag{2.1.2}$$

The number of alternatives can be represented as v.

Step 6: Determining the optimal weights for the chosen criteria.

$$w_b = \frac{1 - e_b}{(1 - e_b) + (1 - e_w)} \tag{2.1.3}$$

Step 7: Determining the weights for the worst criteria.

$$w_w = \frac{1 - e_w}{(1 - e_w) + (1 - e_b)} \tag{2.1.4}$$

Step 8: The TOPSIS technique has been utilized to normalize the decision matrix, focusing on the best and worst criteria from the m alternatives and n criteria.

$$\overline{c_{ij}} = \frac{u_{ij}}{\sqrt{\sum_{j=1}^{n} (u_{ij})^2}} \tag{2.1.5}$$

Step 9: Weighted normalized decision data for Equation 2.1.6 was created by multiplying columns of c_{ij} by allocated weights (w_b) and (w_w).

$$\begin{aligned} f_{ij} &= w_b \times \overline{c_{ij}} \\ f_{ij} &= w_w \times \overline{c_{ij}} \end{aligned} \tag{2.1.6}$$

Step 10: The ideal best and worst values for the best and worst criteria have been calculated using Equation 2.1.7.

$$\begin{aligned} B^+ &= [f_1^+, f_2^+, \ldots\ldots.., f_j^+, \ldots., f_n^+] \\ B^- &= [f_1^-, f_2^-, \ldots\ldots.., f_j^-, \ldots., f_n^-] \end{aligned} \tag{2.1.7}$$

Evaluate the j criteria using f_j^+ and f_j^-, respectively.

Step 11: Euclidean distances referred in Equations 2.1.8 and 2.1.9 are used to determining the ideal best G_i^+ and the ideal worst G_i^-.

$$G_{i^-} = \sqrt{\sum (f_{ij - f_{j^-}})^2} \tag{2.1.8}$$

$$G_{i^+} = \sqrt{\sum (f_{ij - f_{j^+}})^2} \tag{2.1.9}$$

Step 12: Calculate the Multiple Composite Score (MCS) M_i values for each alternative as in Equation 2.1.10.

$$M_i = \frac{G_{i^-}}{G_{i^-} + G_{i^+}} \tag{2.1.10}$$

When ranking alternatives, the MCS obtained are subsequently arranged in descending order.

2.1.4 Example

Table 2.1 presents a selection of a mobile phone with five alternatives: IPhone, Nokia, Redme, Samsung, Vivo and five criteria: price, space, camera, and looks. Tables 2.2 and 2.3 presents a pair-wise comparison of the best and worst criteria.

Table 2.1: List of criteria used in a phone selection

Criteria	Price	Space	Camera	Looks
IPhone	250	16	12	5
Nokia	200	16	8	3
Redme	300	32	16	4
Samsung	275	32	8	4
Vivo	225	16	16	2

Table 2.2: A pairwise comparison vector with an optimal criterion

Criteria	Price	Space	Camera	Looks
Best criteria: Price	1	3	4	7

Table 2.3: Worst-criterion pairwise comparison vector

Criteria	Worst criteria:Looks
price	7
space	5
camera	5
looks	1

The normalization process is facilitated using Equation 2.1.1 in Tables 2.4 and 2.5.

Table 2.4: Normalized best criteria (entropy weight method)

Criteria	Price	Space	Camera	Looks
Best criteria: Price	0.0667	0.2	0.2667	0.4667

Table 2.5: Normalized worst criteria (entropy weight method)

Criteria	Worst criteria :Looks
price	0.3889
space	0.2778
camera	0.2778
looks	0.0556

Calculate the entropy for the best and worst using Equation 2.1.2. The best criteria (price) weight 51.88%, while the worst criteria (looks) weight 48.12%, are calculated using Equations 2.1.3 and 2.1.4, as shown in Table 2.6.

Table 2.6: The entropy weights calculations of the best and worst criteria

Criteria	e_b	e_w	w_b	w_w
Price	0.112	0.228		
Space	0.200	0.221		
Camera	0.219	0.221		
Looks	0.221	0.100		
Total	0.752	0.770	0.519	0.481

Equation 2.1.5 is utilized to normalize the best and worst criteria with the use of the TOPSIS method out of five alternatives, as illustrated in Table 2.7. Table 2.8 displays the weighted normalized decision data obtained using Equation 2.1.6.

Table 2.7: Normalized decision data using TOPSIS method

Criteria	B.C Price	W.C Looks
IPhone	0.4428	0.5979
Nokia	0.3542	0.3586
Redme	0.5314	0.4781
Samsung	0.4871	0.4781
Vivo	0.3981	0.2390

Table 2.8: Weighted normalized decision data using TOPSIS method

Criteria	B.C Price	W.C Looks
IPhone	0.2297	0.2876
Nokia	0.1838	0.1726
Redme	0.2757	0.2301
Samsung	0.2527	0.2301
Vivo	0.2065	0.1150
f_{j^+}	0.1838	0.2876
f_{j^-}	0.2757	0.1150

Step 10 identifies the optimal values for the best and worst criteria, considering their beneficial or non-beneficial aspects. For beneficial criteria, a higher value is the ideal best, and a lower value is the ideal worst. For non-beneficial criteria (cost), the ideal best is the minimum value, while the ideal worst is the maximum value. Calculating the Euclidean distance using Equations 2.1.8 and 2.1.9, as shown in Table 2.9. The MCS was generated using Equation 2.1.10. Table 2.10 shows a rank comparison of various mobiles utilizing the TOPSIS method, with mobile 1 (the iPhone) attaining the top rank because of its MCS value of 0.7955.

Table 2.9: Ideal best and ideal worst euclidean distance

Criteria	G_{i^+}	G_{i^-}	$G_{i^+} + G_{i^-}$
IPhone	0.0459	0.1786	0.2245
Nokia	0.115	0.1085	0.2235
Redme	0.1084	0.1151	0.2235
Samsung	0.0897	0.1174	0.2071
Vivo	0.1741	0.0692	0.2433

Table 2.10: MCS and rank of each alternatives

Alternatives	M_i	Rank
IPhone	0.7955	1
Nokia	0.4855	4
Redme	0.5149	3
Samsung	0.5669	2
Vivo	0.2844	5

2.2 Application

Warming temperatures, ice melting, sea level rise, and unexpected events like floods and droughts are contributing to global environmental change. Fossil fuels cause greenhouse gas emissions, impact ecosystems, and increase temperatures. Manufacturing operations require energy, and countries are investing in energy efficiency and renewable energy sources to address financial issues and mitigate climate change. Renewable energy reduces carbon emissions, purifies the atmosphere, and promotes equitable growth by reducing energy supply dependence, guaranteeing an adequate supply, and protecting the environment. Both technological and financial innovations are contributing to the increasing importance of solar power in environmentally friendly power solutions. Nevertheless, achieving efficiency, reliability, and economic sustainability poses a complicated problem. Solar-generating installations need thorough assessments that consider technological feasibility, environmental sustainability, and changing factors, which can render typical assessment techniques insufficient.

Active and passive techniques harness solar energy to generate heat and power. Utilizing advanced technologies such as flat plate, concentrated photovoltaic (PV), and concentrated solar power (CSP) systems, these techniques transform solar radiation into electrical energy directly or through thermal processes. Recent technological advancements have significantly enhanced the efficiency of solar energy systems, facilitating their widespread use in both standalone and grid-connected applications. The fundamental challenge with solar energy is its significant upfront expenses, despite its eco-friendly and cost-efficient qualities. Therefore, the installation of sophisticated PV and CSP solar power systems requires thoughtful evaluation of ideal locations and conditions. Investors are crucial in the DM process, assessing several solar energy projects to find a balance between social, ecological, technological, and economic variables, ultimately selecting the most ideal development.

Daily sunshine exposure and the frequency of sunny days determine the location for constructing solar energy systems. Sunny countries are implementing regulations to oversee and direct investments in solar energy. Investors must select suitable solar projects by considering both natural and man-made variables. Investors find it challenging to determine the optimal location for solar facilities due to uncertainties and variable circumstances. The interplay of these elements complicates and adds risk to the DM process for investors. Investors selecting solar energy installations prioritize solutions with minimal risk and high return potential. Due to advanced technology, photovoltaic systems offer reliable power, low maintenance, and low waste, but they require costly investment and ample storage space.

Current study introducing a generalized method for evaluating solar panel installations that utilizes SWOT assessment, the Best Worst Method (BWM), the entropy method for objective weighting, and the Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS). Decision-makers (DMs) are evaluating five alternatives. To evaluate these possibilities, we establish twenty-one criteria across five alternatives of the SWOT analysis through discussions with DMs. An objective weighting technique should determine the most and least important criteria. Entropy determines the weight calculation of solar panel installation and ranks the alternatives using the TOSIS technique. The findings indicated that the MCDM strategies for solar panel installation are highly beneficial.

This study utilizes data from surveys intended for solar panel installation to inform a thorough DM technique. The SWOT analysis consists of four aspects (Strength, Weakness, Opportunity, Threat), each defining 21 criteria, as shown in Figure 2.2.

criteria as $Q1$ (Dependability and stainability), $Q2$ (Environmental advantages), $Q3$ (Technological advancement), $Q4$ (Energy independence), $Q5$ (Cost Reduction), $Q6$ (Initial cost obstacles), $Q7$ (Fluctuation in solar energy), $Q8$ (Social and cultural barriers), $Q9$ (Maintenance and repair cost), $Q10$ (Employment generation), $Q11$ (Innovative technologies), $Q12$ (Energy storage), $Q13$ (Government incentives), $Q14$ (Initial cost impact), $Q15$ (Policy changes), $Q16$

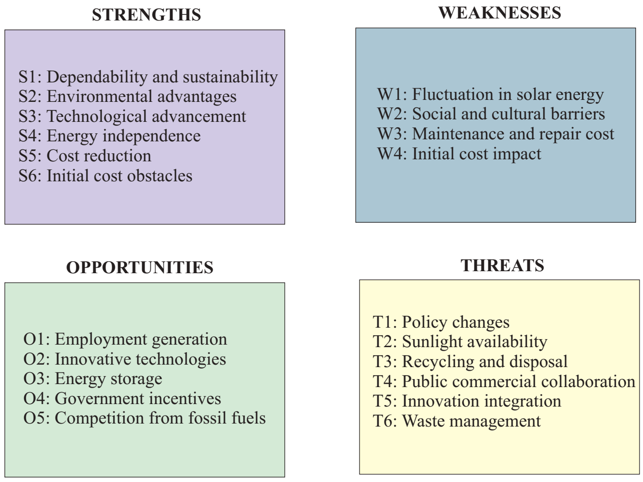

Figure 2.2: The SWOT analysis

(Sunlight availability), $Q17$ (Recycling and disposal), $Q18$ (Public commercial collaboration), $Q19$ (Innovation integration), $Q20$ (Competition from fossil fuels), $Q21$ (Waste management). Five choices are strongly agree (SA), agree (A), neutral (N), disagree (D), and strongly disagree (SD).

The researchers assigned numerical values to the linguistic terms in the questionnaire responses: SA (5), A (4), N (3), D (2), and SD (1). We analyzed the questionnaire results using the COUNTIF Excel solver algorithm for each of the 21 criteria. The values of 5 and 4 were combined by adding the counts of responses falling under them. Responses categorized as 3, 2, and 1 were combined by adding their respective counts. Dependability and sustainability (solar panels are commonly acknowledged as a dependable and ecologically sustainable means of producing electrical energy) received the highest value when we added all the answers, leading us to select them as the best criteria. In the same way, fluctuation in solar energy (the fluctuation in solar energy, which depends on the level of solar irradiance, is a major problem in the field of energy generation) selects the worst criteria.

Next, apply steps 2 and 3 for normalization, utilizing Equation 2.1.1 from step 4 in Table 2.11. To calculate the entropy of the best and worst criteria, apply Equation 2.1.2 in step 5. Equations 2.1.3 and 2.1.4 calculate the weights for the best and worst criteria. The best criteria weigh 50.4%, and the worst criteria weigh 49.6%, as depicted in Table 2.12.

Table 2.11: Best worst method using entropy weights

Questions	SWOT	BC	WC	h_i	h_j
Q1	S1	1	9	0.012	0.114
Q2	S2	2	6	0.024	0.076
Q3	S3	1	7	0.012	0.089
Q4	S4	1	7	0.012	0.089
Q5	S5	1	7	0.012	0.089
Q6	S6	2	5	0.024	0.063
Q7	W1	9	1	0.106	0.013
Q8	W2	7	1	0.082	0.013
Q9	W3	2	5	0.024	0.063
Q14	W4	5	2	0.059	0.025
Q10	O1	4	4	0.047	0.051
Q11	O2	5	2	0.059	0.025
Q12	O3	5	2	0.059	0.025
Q13	O4	5	2	0.059	0.025
Q20	O5	5	2	0.059	0.025
Q15	T1	4	4	0.047	0.051
Q16	T2	7	1	0.082	0.013
Q17	T3	4	4	0.047	0.051
Q18	T4	4	4	0.047	0.051
Q19	T5	5	2	0.059	0.025
Q21	T6	6	2	0.071	0.025
	Total	85	79		

Table 2.12: Best worst method using entropy weights

Questions	SWOT	e_b(best)	e_w(worst)	w_b(best)	w_w(worst)
Q1	S1	0.038	0.179		
Q2	S2	0.064	0.141		
Q3	S3	0.038	0.155		
Q4	S4	0.038	0.155		
Q5	S5	0.038	0.155		
Q6	S6	0.064	0.126		
Q7	W1	0.172	0.040		
Q8	W2	0.148	0.040		
Q9	W3	0.064	0.126		
Q14	W4	0.120	0.067		
Q10	O1	0.104	0.109		
Q11	O2	0.120	0.067		
Q12	O3	0.120	0.067		
Q13	O4	0.120	0.067		
Q20	O5	0.120	0.067		
Q15	T1	0.104	0.109		
Q16	T2	0.148	0.040		
Q17	T3	0.104	0.109		
Q18	T4	0.104	0.109		
Q19	T5	0.120	0.067		
Q21	T6	0.135	0.067		
	Total	2.081	2.062	0.504	0.496

The TOPSIS method is utilized to normalize the best and worst criteria using Equation 2.1.5 out of 5 alternatives, which is depicted in Table 2.13. Table 2.14 uses Equation 2.1.6 to calculate weighted normalized decision data. Step 10 determines the ideal best and worst values for the best and worst criteria, taking into account whether they are beneficial or not. Beneficial criteria demand higher values, with the maximum value representing the ideal best and the minimum value representing the ideal worst. Non-beneficial criteria should have lower values, with the ideal being lower for the best and higher for the worst. Dependability and sustainability are beneficial criteria, whereas fluctuations in solar energy are not that are present in Table 2.15. Equation 2.1.10 generates MCS for all alternatives. Table 2.16 ranks all the alternatives to solar panel installation according to MCS. The SA rank is 1 according to an MCS of 0.637, followed by an A of 0.567 and an SD of 0.445.

Table 2.13: Normalized decision data using the TOPSIS method

Data	$Q1$	$Q1 * 2$	$Q7$	$Q7 * 2$	BC	WC
SA	76	5776	21	441	0.471	0.144
A	140	19600	104	10816	0.868	0.715
N	21	441	96	9216	0.130	0.660
D	6	36	24	576	0.037	0.165
SD	12	144	10	100	0.074	0.069
Total		25997		21149		

Table 2.14: Weighted normalized

Data	$wb * c_{ij}$	$ww * c_{ij}$
SA	0.238	0.072
A	0.438	0.354
N	0.066	0.327
D	0.019	0.082
SD	0.038	0.034
f_{j^+}	0.438	0.034
f_{j^-}	0.019	0.354

Table 2.15: Ideal best and ideal worst euclidean distance

Data	G_{i^-}	G_{i^+}	$G_{i^+} + G_{i^-}$
SA	0.358	0.204	0.5614
A	0.419	0.320	0.7396
N	0.054	0.474	0.5281
D	0.273	0.422	0.6946
SD	0.321	0.400	0.7213

Table 2.16: MCS and rank of each alternative

Data	M_i	Rank
SA	0.637	1
A	0.567	2
N	0.103	5
D	0.392	4
SD	0.445	3

2.3 Conclusion

This study utilizes data from surveys intended for solar panel installation. The questionnaire uses a SWOT analysis framework to gather information on solar panel installation. It has twenty-one criteria and considers five alternatives: strongly agree, agree, neutral, disagree, and strongly disagree. We used MS Excel to solve the given decision data. After combining the results, Solar panels are commonly acknowledged as a dependable and ecologically sustainable means of producing electrical energy chose the best criteria, and the fluctuation in solar energy, which depends on the level of solar irradiance, is a major problem in the field of energy generation selected the worst. The objective weighting technique, the entropy weight method, calculates the BW criteria weights. 51.88% is the best, and 48.12% is the worst weight for the criteria. We use the TOPSIS method to find the best alternative for solar panel installation. Ranking all the alternatives to solar panel installation is based on a multiple composite score (MCS). The SA rank is 1, according to an MCS of 0.637. If SA is not available, then choose the next option.

Chapter 3

Best-Worst-MEREC Multi-criteria Decision Making Method

The chapter includes the Best-Worst-MEREC (BW-MEREC) MCDM method, which is a new methodology that integrates the Best-Worst method for determining the importance of criteria and the MEREC method for evaluating and rating different options. This approach is specifically developed for intricate DM difficulties marked by the presence of uncertainty in multiple conflicting criteria. This study presents the mathematical model for the BW-MEREC technique and provides a comprehensive example that is applied to control the problems regarding pollution in Pakistan. The BW-MEREC approach is effective to evaluate options by combining a variety of criteria and professional opinions which makes it suitable for complex DM problems in the range of different fields. This chapter comprises of three section: $Section$3.1 introduces the mathematical model for MCDM which interprets the Best-Worst technique and MEREC. $Section$3.2 involves the BW-MEREC MCDM methodology to address the Pakistan smog issue. $Section$3.3 concludes the chap which highlights the contributions of the BW-MEREC technique to the area of MCDM.

3.1 Mathematical model for MCDM

3.1.1 MEREC method

Keshavarz-Ghorabaee *et al.* [22] developed the MEREC technique which a more precise and reliable method for MCDM. This approach leverages the impact of removing each requirement on the estimation of alternatives to get the weights of the criteria. The method of evaluating a choice by eliminating the criterion of considering deviations is a novel approach to determining the weights of criteria. A criterion carries significant importance when its elimination results in a greater influence on the overall performance of alternatives. This particular viewpoint not only establishes the impartial significance of each criterion but also facilitates the DM process by enabling DMs to exclude specific criteria. The MEREC method employs an exclusion viewpoint and removal effects to determine objective criteria weights, distinct from other approaches that use an inclusion perspective [23].

The DM methodology is shown in Figure 3.1. The procedure for addressing a DM problem can be followed in detail.

Step 1: The decision matrix, a n $\times$ m matrix with m criteria and n alternatives, is constructed by assigning performance values to each row and column, denoted by x_{ij}.

Step 2: The decision matrix has been standardized. We used the following

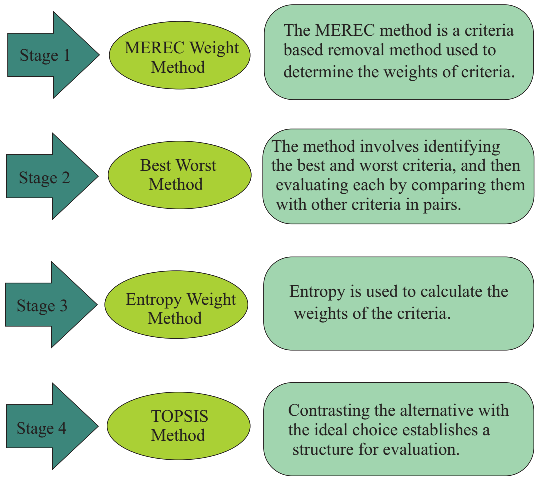

Figure 3.1: DM methodology

formula to standardize the decision matrix:

$$z_{ij}^x = \begin{cases} \frac{\min\limits_k x_{kj}}{x_{ij}} & \text{if } j \in M \\ \frac{x_{ij}}{\max\limits_k x_{kj}} & \text{if } j \in D \end{cases} \tag{3.1.1}$$

The DM process is influenced by a set of beneficial criteria (M) and non-beneficial criteria (D) refers to the factors that influence the DM process.

Step 3: Evaluate the alternative's overall performance (A_i).

$$A_i = ln(1 + (\frac{1}{m}\sum_j |ln(z_{ij^x})|)) \tag{3.1.2}$$

Step 4: The performance of the alternatives is ascertained by removing each criterion ($A_{ij'}$).

$$A_{ij'} = ln(1 + (\frac{1}{m}\sum_{k,k\neq j} |ln(z_{ik^x})|)) \tag{3.1.3}$$

Step 5: Determine the sum of the absolute deviation (H_j).

$$H_j = \sum_i |A_{ij'} - A_i| \tag{3.1.4}$$

Step 6: You can determine the weights of the criteria using the following formula:

$$W_j = \frac{H_j}{\sum_k H_k} \tag{3.1.5}$$

Step 7: To find the best and worst criteria, we used MEREC weights that were calculated in step 6.

Step 8: The pairwise comparison vector compares the best and worst criteria on a scale of 1 to 9.

Step 9: We have used the entropy technique to normalize the best and worst criteria.

$$u_{ij} = \frac{v_{ij}}{\sum_{j=1}^{n} v_{ij}} \tag{3.1.6}$$

Step 10: We compute entropy using the following formula:

$$\begin{aligned} e_{bw} &= -R \sum_{j=1}^{n} f_{ij} ln f_{ij} \\ where - R &= \frac{1}{lna} \end{aligned} \tag{3.1.7}$$

a indicates the number of alternatives.

Step 11: Determining the best criteria weights.

$$w_b = \frac{1 - e_b}{(1 - e_b) + (1 - e_w)} \tag{3.1.8}$$

Step 12: Calculating the weights for the worst criteria.

$$w_w = \frac{1 - e_w}{(1 - e_w) + (1 - e_b)} \tag{3.1.9}$$

Step 13: The BWM proposes a consistency ratio to evaluate the validity of the comparisons.

$$CR = \frac{\epsilon}{CI} \tag{3.1.10a}$$

$$\epsilon = |\frac{w_b - e_b}{e_b}| \tag{3.1.10b}$$

$$\epsilon = |\frac{w_w - e_w}{e_w}| \tag{3.1.10c}$$

$$CI = a_{bw} \tag{3.1.10d}$$

Step 14: We have used the TOPSIS technique to normalize the decision matrix. Out of the 7 alternatives and 5 criteria, we only have to consider the best and worst criteria.

$$\overline{b_{ij}} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n}(x_{ij})^2}} \tag{3.1.11}$$

Step 15: Equation 3.1.12 is used to create weighted normalized decision data by multiplying the columns of b_{ij} by the allocated weights.

$$\begin{aligned} g_{ij} &= w_b \times \overline{b_{ij}} \\ g_{ij} &= w_w \times \overline{b_{ij}} \end{aligned} \tag{3.1.12}$$

Step 16: The ideal best and worst values for the best and worst criteria should be determined using Equations 3.1.13 and 3.1.14.

$$C^+ = [g_1^+, g_2^+, \ldots\ldots., g_j^+, \ldots., g_n^+] \tag{3.1.13}$$

$$C^- = [g_1^-, g_2^-, \ldots\ldots., g_j^-, \ldots., g_n^-] \tag{3.1.14}$$

g_j^+ and g_j^- are the best and worst values of the j criteria. where j=1, 2, ..., n.

Step 17: Calculate the ideal best K_i^+ and ideal worst K_i^- Euclidean distance.

$$K_{i^-} = \sqrt{\sum (g_{ij - g_{j^-}})^2} \tag{3.1.15}$$

$$K_{i^+} = \sqrt{\sum (g_{ij - g_{j^+}})^2} \tag{3.1.16}$$

Step 18: Calculate the Multiple Composite Score (MCS) N_i values for each alternative.

$$N_i = \frac{K_{i^-}}{K_{i^-} + K_{i^+}} \tag{3.1.17}$$

After ranking the alternatives, we arrange the multiple composite scores obtained in descending order.

3.1.2 Example

Table 3.1 represents decision data for the selection of a car with seven alternatives: Toyota, Audi, Tesla, Kia, BMW, Wagon R, Cultus and five criteria: price, fuel (km/lit), safety, comfort, and brand reputation (BR). Safety, comfort, and BR are beneficial criteria, as denoted by M. D denotes price and fuel as non-beneficial criteria. Safety, comfort, and BR are linguistic terms. To convert them into numerical values, we have used the rating scale shown in Table 3.2. Table 3.3 represents decision data, and each value in each cell is called a performance value.

Table 3.1: Decision data

Alternative	Price(D)	Fuel(Km/lit)(D)	Safety(M)	Comfort(M)	BR(M)
Toyota	3	15	A	A	G
Audi	7	16	V.G	O	O
Tesla	4.5	12	E	V.G	V.G
Kia	5	10	A	G	A
BMW	2.5	11	G	A	A
Wagon R	10	17	O	E	O
Cultus	7.5	14	E	V.G	E

Table 3.2: Rating scale

Average	Good	V.Good	Excellent	Outstanding
1	2	3	4	5

Table 3.3: Decision data

Alternative	Price(D)	Fuel(Km/lit)(D)	Safety(M)	Comfort(M)	BR(M)
Toyota	3	15	1	1	2
Audi	7	16	3	5	5
Tesla	4.5	12	4	3	3
Kia	5	10	1	2	1
BMW	2.5	11	2	1	1
Wagon R	10	17	5	4	5
Cultus	7.5	14	4	3	4

Equation 3.1.1 is the normalized decision data that is shown in Table 3.4.

Table 3.4: Normalized (z_{ij})

Alternative	Price(D)	Fuel(D)	Safety(M)	Comfort(M)	BR(M)
Toyota	0.30	0.88	1.00	1.00	0.50
Audi	0.70	0.94	0.33	0.20	0.20
Tesla	0.45	0.71	0.25	0.33	0.33
Kia	0.50	0.59	1.00	0.50	1.00
BMW	0.25	0.65	0.50	1.00	1.00
Wagon R	1.00	1.00	0.20	0.25	0.20
Cultus	0.75	0.82	0.25	0.33	0.25

In Table 3.5, DMs should assess the overall performance of the alternatives according to Equation 3.1.2.

Table 3.5: Overall performance of the alternatives

Alternative	Price(D)	Fuel(D)	Safety(M)	Comfort(M)	BR(M)	A_i
Toyota	0.30	0.88	1.00	1.00	0.50	0.340
Audi	0.70	0.94	0.33	0.20	0.20	0.666
Tesla	0.45	0.71	0.25	0.33	0.33	0.666
Kia	0.50	0.59	1.00	0.50	1.00	0.325
BMW	0.25	0.65	0.50	1.00	1.00	0.407
Wagon R	1.00	1.00	0.20	0.25	0.20	0.653
Cultus	0.75	0.82	0.25	0.33	0.25	0.626

DMs use Equation 3.1.3 to calculate the overall performance of alternatives by eliminating each criterion as illustrated in Table 3.6.

Table 3.6: The values of A_{ij}

Alternative	Price(D)	Fuel(D)	Safety(M)	Comfort(M)	BR(M)
Toyota	0.152	0.322	0.340	0.340	0.236
Audi	0.629	0.660	0.547	0.486	0.486
Tesla	0.580	0.629	0.512	0.546	0.546
Kia	0.219	0.245	0.325	0.219	0.325
BMW	0.204	0.348	0.311	0.407	0.407
Wagon R	0.653	0.653	0.469	0.497	0.469
Cultus	0.595	0.605	0.466	0.501	0.466

Using Equations 3.1.4 and 3.1.5 to calculate the absolute deviation and weight calculation are shown in Table 3.7 and 3.8.

Table 3.7: Absolute deviation (H_j)

Alternative	Price(D)	Fuel(D)	Safety(M)	Comfort(M)	BR(M)
Toyota	0.30	0.88	1.00	1.00	0.50
Audi	0.70	0.94	0.33	0.20	0.20
Tesla	0.45	0.71	0.25	0.33	0.33
Kia	0.50	0.59	1.00	0.50	1.00
BMW	0.25	0.65	0.50	1.00	1.00
Wagon R	1.00	1.00	0.20	0.25	0.20
Cultus	0.75	0.82	0.25	0.33	0.25
H_j	1.0588	0.9167	1.3354	1.5018	1.5631

Table 3.8: Weight calculation using MEREC

Alternative	Price(D)	Fuel(D)	Safety(M)	Comfort(M)	BR(M)	Total
H_j	1.0588	0.9167	1.3354	1.5018	1.5631	6.3758
W_j	0.1661	0.1438	0.2094	0.2355	0.2452	

The MEREC method selects brand reputation as the most significant criterion and fuel (km/liter) as the least significant criterion for car selection. In Table 3.9 best criteria and Table 3.10 worst criteria, pair-wise comparisons have been made using a number between 1 to 9.

Table 3.9: Best for others

Best criteria	Price(D)	Fuel(D)	Safety(M)	Comfort(M)	BR(M)	Total
BR	4	9	5	2	1	21

Table 3.10: Others for worst

Worst criteria	Fuel
Price	6
Fuel	1
Safety	4
comfort	3
BR	9
Total	23

In Table 3.11, use Equation 3.1.6 for normalization.

Table 3.11: Normalized best and worst criteria

Criteria	u_i	u_j
Price	0.1905	0.2609
Fuel	0.4286	0.0435
Safety	0.2381	0.1739
comfort	0.0952	0.1304
BR	0.0476	0.3913

To calculate entropy we have to use Equation 3.1.7 and determine weights for the best and worst criteria using Equations 3.1.8 and 3.1.9. The best criteria weight is 51.59% and the worst criteria weight is 48.41%. The consistency ratio of the best and worst criteria is 0.1 and 0.1 using Equations 3.1.10 is present in Tables 3.12 and 3.14. To calculate the consistency index, see Table 3.13.

Table 3.12: The entropy weight calculations of the best and worst criteria

Criteria	e_b	e_w	w_b	w_w
Price	0.1623	0.1801		
Fuel	0.1866	0.0701		
Safety	0.1756	0.1563		
Comfort	0.1151	0.1365		
BR	0.0745	0.1887		
Total	0.7141	0.7317	0.5159	0.4841

Table 3.13: Consistency index

abw	1	2	3	4	5	6	7	8	9
CI	0.00	0.44	1.00	1.63	2.30	3.00	3.73	4.47	5.23

Table 3.14: Consistency ratio

	Best	Worst
CI	0.3	0.3
CR	0.1	0.1

Table 3.15 TOPSIS technique has been used to normalized data and in Table 3.16 weighted normalized the decision data using Equations 3.1.11, 3.1.12.

Step 15 determines the ideal best and worst values for the best and worst criteria.

Table 3.15: Normalized (b_{ij})

Alternative	BC(BR)	WC(Fuel)
Toyota	0.2222	0.4112
Audi	0.5556	0.4386
Tesla	0.3333	0.3289
Kia	0.1111	0.2741
BMW	0.1111	0.3015
Wagon R	0.5556	0.4660
Cultus	0.4444	0.3837

Table 3.16: Weighted normalized (g_{ij})

Alternative	$w_b * normalized$	$w_w * normalized$
Toyota	0.1146	0.1990
Audi	0.2866	0.2123
Tesla	0.1720	0.1592
Kia	0.0573	0.1327
BMW	0.0573	0.1460
Wagon R	0.2866	0.2256
Cultus	0.2293	0.1858
g_{j^+}	0.2866	0.1327
g_{j^-}	0.0573	0.2256

Determining the ideal best and the ideal worst Euclidean distance have been using Equations 3.1.15, 3.1.16 as depicted in Table 3.17.

Table 3.17: Euclidean distance

Alternative	K_{i^-}	K_{i^+}	$K_{i^-} + K_{i^+}$
Toyota	0.0632	0.1843	0.2475
Audi	0.2297	0.0796	0.3093
Tesla	0.1325	0.1177	0.2501
Kia	0.0929	0.2293	0.3222
BMW	0.0796	0.2297	0.3093
Wagon R	0.2293	0.0929	0.3222
Cultus	0.1765	0.0781	0.2546

MCS is calculated using Equations 3.1.17, as shown in Table 3.18. After calculating the MCS, rank the alternatives, with Audi being the first because its MCS is 0.7426, the highest value of all the alternatives.

Table 3.18: Multiple composite score (MCS)

Alternative	N_i	Rank
Toyota	0.2552	7
Audi	0.7426	1
Tesla	0.5295	4
Kia	0.2883	5
BMW	0.2574	6
Wagon R	0.7117	2
Cultus	0.6932	3

3.2 Application

Pakistan is a nation situated in southern Asia, sharing borders with several significant countries, including India, China, Iran, and Afghanistan, all of which face substantial pollution challenges. Both China and India, two major economies, have several cities that are among the most polluted in the world. Pakistan is also not exempt from this issue. Pakistan constitutes a historically significant territory that has witnessed the emergence and collapse of various civilizations and dynasties. Currently, it is established as an Islamic republic, boasting a substantial population of over 231.4 million individuals, ranking it as the nation with the fifth largest population globally.

In 2022, Pakistan came in with a PM2.5 reading of $70.9\mu g/m^3$. the country has been ranked third out of the most polluted countries globally, according to an IQAIR 2022 [24] rating. In 2023, Pakistan came in with a PM2.5 reading of 73.7 $\mu g/m^3$ (more than 14.7 times higher than the WHO PM2.5 annual guideline), not only putting it into the 'unhealthy' rating category but also into the second place position out of the most polluted countries in the world [25]. Smog is becoming more prevalent in Punjab, where two-thirds of the country population lives. The problem gets worse in the winter. According to IQAIR 2022 [26], Lahore, the world's most polluted city in 2022, experienced an average annual PM2.5 concentration of $97.4\mu g/m^3$, and in 2023 IQAIR [25] report PM2.5 reading of $99.5\mu g/m^3$, exceeds by over 10 times.

The objective of this current study is to employ MCDM methodologies to examine points of view on smog-related issues in Pakistan. The analysis will specifically concentrate on the health, economic, social, and ecological consequences of smog, as well as the solutions employed to mitigate its effects. By employing various approaches, such as the MEREC objective weighting technique to find the weights of each criterion and choose the essential criteria, in the best-worst technique, pair-wise comparisons are performed on the best and worst criteria that are selected through MEREC. We have created a designated consistency ratio for the BWM to assess the reliability of comparisons. Entropy is an objective weighting technique used to calculate the weights of the best as well as worst criteria, and we employ the TOPSIS technique to determine the suitable alternative from a collection of choices by evaluating their resemblance to an ideal answer.

In this section, we have applied MCDM techniques and utilized data from the questionnaire intended for Exploring Stakeholder Perceptions of Smog Factors in Pakistan: Implications for Health, Economy, and Society. In this survey, we have 36 questions, and each question has nine different options. Criteria as **Causes of Smog in Pakistan** $Q1$ (vehicle emissions), $Q2$ (agricultural practices), $Q3$ (electricity generation), $Q4$ (deforestation), $Q5$ (brick kilns), and $Q6$ (incineration of waste). **Health impact of smog in Pakistan** $Q7$ (respiratory issues), $Q8$ (cardiovascular health), $Q9$ (skin irritation), $Q10$ (eye irritations), $Q11$ (aggravation of existing conditions), and $Q12$ (increased mortality rates). **Economic impact of smog in Pakistan** $Q13$ (increased health care costs), $Q14$ (effect on work efficiency), $Q15$ (business disruptions), $Q16$ (agricultural losses), $Q17$ (increased energy costs), and $Q18$ (tourism decline). **Social impact of smog in Pakistan** $Q19$ (impact on outdoor activities), $Q20$

(impact on social events), $Q21$ (disruption of daily activities), $Q22$ (economic inequality), $Q23$ (psychological stress), and $Q24$ (education-related problems). **Environmental impact of smog in Pakistan** $Q25$ (ecosystem damage), $Q26$ (deterioration of air quality), $Q27$ (climate change), $Q28$ (impact on agricultural productivity), $Q29$ (depletion of ozone), and $Q30$ (formation of acid rain). **Mitigation strategies for smog in Pakistan** $Q31$ (promotion of public transportation), $Q32$ (adoption of renewable energy sources), $Q33$ (agricultural practices and crop residue management), $Q34$ (afforestation and ecological areas), $Q35$ (renovation of brick kilns), and $Q36$ (waste management and recycling).

The linguistic terms in the questionnaire responses were assigned numerical values: strongly agree (9), agree (8), somewhat agree (7), slightly agree (6), neutral (5), slightly disagree (4), somewhat disagree (3), disagree (2), strongly disagree (1). We analysed the questionnaire results using the Excel solver algorithm COUNTIF for each of the thirty-six criteria. A questionnaire with six sections, each containing six questions, generates 36 criteria. Combining each section's questions results in six criteria. C1: Causes of Smog in Pakistan, C2: Health Impact of Smog in Pakistan, C3: Economic Impact of Smog in Pakistan, C4: Social Impact of Smog in Pakistan, C5: Environmental Impact of Smog in Pakistan, C6: Mitigation Strategies for Smog in Pakistan. We have selected nine options as alternatives for each question. A1: strongly agree, A2: agree, A3: somewhat agree, A4: slightly agree, A5: neutral, A6: slightly disagree, A7: somewhat disagree, A8: disagree, and A9: strongly disagree. Objective weighting technique MEREC is used to select the best and worst criteria. In the MEREC methodology, we get the weight of each criterion and, considering these weights, select the most suitable and less beneficial criteria. In Table 3.19, decision data is represented. We select C1, C2, C3, C4, and C5 as non-beneficial criteria, denoted as D, and C6 as beneficila criteria, denoted by M. Normalization and overall performance of the alternatives are depicted in Tables 3.20 and 3.21 using Equations 3.1.1 and 3.1.2. Overall performance of alternatives by eliminating each criterion as illustrated in Table 3.22 from the Equation 3.1.3. The absolute deviation is calculated using Equation 3.1.4 depicted in Table 3.23, and weights are calculated for each criterion using Equation 3.1.5, as shown in Table 3.24. After applying the MEREC technique, we calculate weights $C6 > C2 > C5 > C3 > C1 > C4$ and assign the highest weight to C6. Therefore, we selected C6 as the best criterion and C4 as the worst.

Table 3.19: Smog in Pakistan: responses

Alternatives	C1(D)	C2(D)	C3(D)	C4(D)	C5(D)	C6(M)
A1	66	64	39	42	72	72
A2	86	93	95	87	91	86
A3	24	18	19	28	19	16
A4	16	19	25	18	16	18
A5	20	21	32	30	17	26
A6	6	1	5	7	3	3
A7	3	9	3	5	1	5
A8	6	7	13	12	10	7
A9	7	2	3	5	5	1

Table 3.20: Normalized (z_{ij})

Alternatives	C1	C2	C3	C4	C5	C6
A1	0.767	0.688	0.411	0.483	0.791	0.014
A2	1.000	1.000	1.000	1.000	1.000	0.012
A3	0.279	0.194	0.200	0.322	0.209	0.063
A4	0.186	0.204	0.263	0.207	0.176	0.056
A5	0.233	0.226	0.337	0.345	0.187	0.038
A6	0.070	0.011	0.053	0.080	0.033	0.333
A7	0.035	0.097	0.032	0.057	0.011	0.200
A8	0.070	0.075	0.137	0.138	0.110	0.143
A9	0.081	0.022	0.032	0.057	0.055	1.000

Table 3.21: Overall performance of the alternatives

Alternatives	C1	C2	C3	C4	C5	C6	A_i
A1	0.767	0.688	0.411	0.483	0.791	0.014	0.755
A2	1.000	1.000	1.000	1.000	1.000	0.012	0.555
A3	0.279	0.194	0.200	0.322	0.209	0.063	0.981
A4	0.186	0.204	0.263	0.207	0.176	0.056	1.030
A5	0.233	0.226	0.337	0.345	0.187	0.038	0.983
A6	0.070	0.011	0.053	0.080	0.033	0.333	1.351
A7	0.035	0.097	0.032	0.057	0.011	0.200	1.391
A8	0.070	0.075	0.137	0.138	0.110	0.143	1.172
A9	0.081	0.022	0.032	0.057	0.055	1.000	1.279

Table 3.22: The values of A_{ij}

Alternatives	C1	C2	C3	C4	C5	C6
A1	0.734	0.725	0.683	0.696	0.737	0.347
A2	0.555	0.555	0.555	0.555	0.555	0.000
A3	0.898	0.873	0.875	0.907	0.878	0.791
A4	0.925	0.931	0.947	0.932	0.921	0.841
A5	0.888	0.886	0.913	0.914	0.873	0.756
A6	1.229	1.133	1.215	1.236	1.192	1.303
A7	1.242	1.290	1.237	1.265	1.184	1.322
A8	1.024	1.029	1.064	1.064	1.051	1.066
A9	1.155	1.083	1.104	1.137	1.135	1.279

Table 3.23: Absolute deviation (H_j)

Alternatives	C1	C2	C3	C4	C5	C6
A1	0.767	0.688	0.411	0.483	0.791	0.014
A2	1.000	1.000	1.000	1.000	1.000	0.012
A3	0.279	0.194	0.200	0.322	0.209	0.063
A4	0.186	0.204	0.263	0.207	0.176	0.056
A5	0.233	0.226	0.337	0.345	0.187	0.038
A6	0.070	0.011	0.053	0.080	0.033	0.333
A7	0.035	0.097	0.032	0.057	0.011	0.200
A8	0.070	0.075	0.137	0.138	0.110	0.143
A9	0.081	0.022	0.032	0.057	0.055	1.000
H_j	0.8482	0.9936	0.9048	0.7905	0.9731	1.7927

Table 3.24: Weight calculation using MEREC

	C1	C2	C3	C4	C5	C6	TOTAL
H_j	0.8482	0.9936	0.9048	0.7905	0.9731	1.7927	6.3029
W_j	0.1346	0.1576	0.1436	0.1254	0.1544	0.2844	

In Table 3.25 and 3.26, apply Step 8; the entropy method is used for normalization; refer to Equation 3.1.6 as shown in Table 3.27. Calculate entropy employing Equation 3.1.7 and weight determination of both best and worst criteria utilizes Equations 3.1.8 and 3.1.9 as depicted in Table 3.28. The best criteria weight is 0.483 and the worst criteria weight is 0.517; to calculate consistency, use Equation 3.1.10. The consistency ratio of the best and worst criteria is 0.08 and 0.07, respectively, as shown in Table 3.29. This suggests a high level of consistency. Next, we employ the TOPSIS technique to determine the ideal alternative from a collection of choices by evaluating their resemblance to an ideal answer. In Table 3.30, normalized the best and worst criteria out of 9 alternatives using Equation 3.1.11, for weighted normalization utilizing Equation 3.1.12 as shown in Table 3.31. Step 16 determines the ideal best and worst values for the best and worst criteria. Using Equations 3.1.15 and 3.1.16 to find the ideal best and ideal worst Euclidean distance as shown in Table 3.32, we can then use Equation 3.1.17 to find the MCS for each option and use Table 3.33 to rank each option based on its MCS. The A1 alternative rank is 1, according to the MCS of 0.6487.

Table 3.25: Best for others

Best	C1	C2	C3	C4	C5	C6	Total
C6	7	4	6	8	4	1	30

Table 3.26: Others for worst

Worst	C4
C1	6
C2	5
C3	7
C4	2
C6	8
Total	29

Table 3.27: Normalized

Criteria	u_i	u_j
C1	0.233	0.207
C2	0.133	0.172
C3	0.200	0.241
C4	0.267	0.034
C5	0.133	0.069
C6	0.033	0.276

Table 3.28: Entropy weight calculation of the best and worst

Criteria	e_b	e_w	w_b	w_w
C1	0.155	0.148		
C2	0.122	0.138		
C3	0.146	0.156		
C4	0.160	0.053		
C5	0.122	0.084		
C6	0.052	0.162		
Total	0.758	0.741	0.483	0.517

Table 3.29: Consistency ratio

	Best	Worst
CI	4.47	4.47
CR	0.08	0.07

Table 3.30: Normalization (b_{ij}) using the TOPSIS technique

Alternatives	BC(C6)	WC(C4)	b_i	b_j
A1	72	42	0.0052	0.0036
A2	86	87	0.0062	0.0075
A3	16	28	0.0011	0.0024
A4	18	18	0.0013	0.0016
A5	26	30	0.0019	0.0026
A6	3	7	0.0002	0.0006
A7	5	5	0.0004	0.0004
A8	7	12	0.0005	0.0010
A9	1	5	0.0001	0.0004

Table 3.31: Weighted normalized (g_{ij})

Alternatives	w_b*normalized	w_w*normalized
A1	0.0025	0.0019
A2	0.0030	0.0039
A3	0.0006	0.0013
A4	0.0006	0.0008
A5	0.0009	0.0013
A6	0.0001	0.0003
A7	0.0002	0.0002
A8	0.0002	0.0005
A9	0.0000	0.0002
g_{j^+}	0.0030	0.0002
g_{j^-}	0.0000	0.0039

Table 3.32: Euclidean distance

Alternatives	K_{i^-}	K_{i^+}	$K_{i^+} + K_{i^-}$
A1	0.0032	0.0017	0.0049
A2	0.0030	0.0037	0.0066
A3	0.0027	0.0026	0.0053
A4	0.0031	0.0024	0.0056
A5	0.0027	0.0024	0.0051
A6	0.0036	0.0029	0.0065
A7	0.0037	0.0028	0.0065
A8	0.0034	0.0028	0.0061
A9	0.0037	0.0030	0.0066

Table 3.33: Multiple composite scores

Alternatives	N_i	Rank
A1	0.6487	1
A2	0.4462	9
A3	0.5045	8
A4	0.5635	3
A5	0.5323	7
A6	0.5535	5
A7	0.5659	2
A8	0.5487	6
A9	0.5538	4

3.3 Conclusion

Pakistan faces a persistent threat from smog due to industrialization, fossil fuel reliance, and automobile emissions. This issue poses risks to humans and plants contributing to global health issues and negatively impacting sectors like health, ecology, transportation, and education. This research uses MCDM techniques and data from a questionnaire to explore stakeholder perceptions of smog factors in Pakistan. The questionnaire has 36 questions with nine options, including causes of smog, health impact, economic impact, social impact, environmental impact, and mitigation strategies. The linguistic terms in the responses were assigned numerical values. The results were analyzed using the Excel solver algorithm COUNTIF for each of the thirty-six criteria. Six criteria were generated, with nine options selected as alternatives for each question. The objective weighting technique MEREC was used to select the best and worst criteria. The current study ranked C6 as the best criterion and C4 as the worst, normalized the overall performance of alternatives, and calculated their absolute deviation. The study uses the entropy method for normalization, calculating entropy, and weight determination of the best and worst criteria. The best criteria weight is 0.483 and the worst criteria weight is 0.517. The consistency ratios are 0.08 and 0.07, indicating high consistency. The TOPSIS technique is used to determine the ideal alternative from a collection of choices. The ideal best and worst values are determined using Euclidean distance and to rank all the alternatives to smog in Pakistan is based on multiple composite score (MCS). The A1 rank is 1 according to an MCS of 0.6487.

Bibliography

[1] H. Taherdoost and M. Madanchian, *Multi-criteria decision making (MCDM) methods and concepts*, Encyclopedia **3** (2023), no. 1, 77-87.

[2] F. Eltarabishi, O. H. Omar, I. Alsyouf, and M. Bettayeb, *Multi-criteria decision making methods and their applications-A literature review*, 10th Annual International IEOM Conference, IEOM 2020, 2020, pp. 2654-2663.

[3] T. L. Saaty, *Decision making—the analytic hierarchy and network processes (AHP/ANP)*, Journal of Systems Science and Systems Engineering **13** (2004), 1-35.

[4] A. Charnes, W. W. Cooper, and E. Rhodes, *Measuring the efficiency of decision making units*, European Journal of Operational Research **2** (1978), no. 6, 429-444.

[5] A. Mahmoudi and S. A. Javed, *Grey Multiple Criteria Decision Making Methods: A*.

[6] M. Aruldoss, T. M. Lakshmi, and V. P. Venkatesan, *A survey on multi criteria decision making methods and its applications*, American Journal of Information Systems **1** (2013), no. 1, 31-43.

[7] X. Yu, S. Zhang, X. Liao, and X. Qi, *ELECTRE methods in prioritized MCDM environment*, Information Sciences **424** (2018), 301-316.

[8] R. K. Gavade, *Multi-Criteria Decision Making: An overview of different selection problems and methods*, International Journal of Computer Science and Information Technologies **5** (2014), no. 4, 5643-5646.

[9] I. Taufik, C. N. Alam, Z. Mustofa, A. Rusdiana, and W. Uriawan, *Implementation of Multi-Attribute Utility Theory (MAUT) method for selecting diplomats*, IOP Conference Series: Materials Science and Engineering **1098** (2021), no. 3, 032055.

[10] Y. J. Wang, *A fuzzy multi-criteria decision-making model based on simple additive weighting method and relative preference relation*, Applied Soft Computing **30** (2015), 412-420.

[11] G. Kabir, R. Sadiq, and S. Tesfamariam, *A review of multi-criteria decision-making methods for infrastructure management*, Structure and Infrastructure Engineering **10** (2014), no. 9, 1176-1210.

[12] B. Ayan, S. Abacıoğlu, and M. P. Basilio, *A comprehensive review of the novel weighting methods for multi-criteria decision-making*, Information **14** (2023), no. 5, 285.

[13] E. K. Zavadskas and V. Podvezko, *Integrated determination of objective criteria weights in MCDM*, International Journal of Information Technology & Decision Making **15** (2016), no. 02, 267-283.

[14] X. Peng, X. Zhang, and Z. Luo, *Pythagorean fuzzy MCDM method based on CoCoSo and CRITIC with score function for 5G industry evaluation*, Artificial Intelligence Review **53** (2020), no. 5, 3813-3847.

[15] D. Pamučar, Ž. Stević, and S. Sremac, *A new model for determining weight coefficients of criteria in MCDM models: Full consistency method (FUCom)*, Symmetry **10** (2018), no. 9, 393.

[16] M. Žižović and D. Pamucar, *New model for determining criteria weights: Level Based Weight Assessment (LBWA) model*, Decision Making: Applications in Management and Engineering **2** (2019), no. 2, 126-137.

[17] M. R. Patel, M. P. Vashi, and B. V. Bhatt, *SMART-Multi-criteria decision-making technique for use in planning activities*, New Horizons in Civil Engineering (NHCE 2017) (2017), 1-6.

[18] J. Rezaei, *Best-worst multi-criteria decision-making method*, Omega **53** (2015), 49-57.

[19] V. Chodha, R. Dubey, R. Kumar, S. Singh, and S. Kaur, *Selection of industrial arc welding robot with TOPSIS and Entropy MCDM techniques*, Materials Today: Proceedings **50** (2022), 709-715.

[20] S. Opricovic and G. H. Tzeng, *Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS*, European Journal of Operational Research **156** (2004), no. 2, 445-455.

[21] F. Selamzade, Y. Ersoy, Y. Ozdemir, and M. Y. Celik, *Health efficiency measurement of OECD countries against the COVID-19 pandemic by using DEA and MCDM methods*, Arabian Journal for Science and Engineering **48** (2023), no. 11, 15695-15712.

[22] M. Keshavarz-Ghorabaee, M. Amiri, E. K. Zavadskas, Z. Turskis, and J. Antucheviciene, *Determination of objective weights using a new method based on the removal effects of criteria (MEREC)*, Symmetry **13** (2021), no. 4, 525.

[23] N. Keleş, *Measuring performances through multiplicative functions by modifying the MEREC method: MEREC-G and MEREC-H*, International Journal of Industrial Engineering and Operations Management (2023), ahead-of-print.

[24] *World's Most Polluted Countries*, 2022. https://www.iqair.com/world-most-polluted-countries.

[25] *World Air Quality Report*, 2023. Accessed: 2024-04-04.

[26] *World's Most Polluted Cities*, 2022. https://www.iqair.com/world-most-polluted-cities.