Fractional Optimization driven Accelerated Prediction Strategy for Effective Matrix Factorization in Personalized Recommender Systems

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DISSERTATION

A dissertation submitted to the Department of Electrical Engineering, International Islamic University Islamabad as a partial fulfillment of the requirements for the award of the degree.

Department of Electrical Engineering

Faculty of Engineering and Technology

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DEDICATION

I dedicate this thesis to my beloved parents, respected teachers and all those who prayed for our success.

"Alhamdulillah for everything, we can never thank Allah enough for the countless bounties He blessed us with"

CERTIFICATE OF APPROVAL

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I certify that research work titled "Fractional Optimization driven Accelerated Prediction Strategy for Effective Matrix Factorization in Personalized Recommender Systems" is my own work and has not been presented elsewhere for assessment. Moreover, the material taken from other sources has also been acknowledged properly.

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ABSTRACT

To tackle rise in customer's request due to the rise in size of items and users, the e-commerce industry needs more efficient, reliable and accurate recommender systems. The reputation of a recommender system depends on a few inevitable factors like sparseness, scalability, predictive accuracy, recommendations speed and computational complexity. It has been observed that for learning useful latent features, stochastic gradient descent driven approaches suggested in the literature for efficiently solving RS problem through matrix factorization, merely utilize integer order gradients in the weight learning mechanisms. Moreover, the integer-order-based SGD models are deficient of capturing the useful information encapsulated in the historical users' ratings for various items. The deficiency of lacking historical feedback influences the performance of RSs regarding recommendations predictive accuracy and speed.

Motivated by the strong underlying mathematical concepts of fractional calculus, we aim to develop an improved SGD model termed as accelerated fractional SGD (AF-SGD) by exploiting enhanced fractional order gradient concepts for providing fast and precise recommendations. The suggested method utilizes the historical feedback of the users for certain items efficiently for providing effective and precise recommendations. The suggested method outperforms baseline and state-of-the-art models. The fitness of the model is valuated through various evaluation metrics for two benchmark datasets.

Key words: Accelerated Fractional SGD, Stochastic Gradient Decent, Recommender System.

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CHAPTER 1
INTRODUCTION

1.1 Introduction

This chapter describes the need, importance, and valuable applications of recommendation systems. It also discusses the standard method of recommender system as well as their benefits and drawbacks, and also acknowledged the important role of collaborative filtering methods for designing adaptive, innovative, robust, convergent and reliable algorithms for rating and ranking prediction in recommender systems. Finally, an introduction of suggested matrix factorization model for evaluation estimate based on collaborative filtering is presented.

1.2 Inspiration and Background

To enhance online sales, the emphasis of an e-commerce is to meet customers' demands by recommending them the set of most useful and relevant items efficiently. To fulfill diversified customers' demands, it is hard for the e-commerce industry to develop a strategy for approximating the users' interest out of billions of products. Recommender systems (RSs) are intelligent programs used by the e-commerce industry for estimating the relevancy of the useful products to users [1, 2].

RSs recommend related recommendations to users in three-steps. (1) A rating matrix is build using the users' responses for several products. By utilizing the feedback given by users, a RS estimates users' interest for the set of products not rated by users[3] . RSs are playing a vital role both for clients and sellers. RSs are also used the vendors to exploit and recognize the attractive, appropriate, and amazing news-content for the interested clients[4].

Moreover, auxiliary information such as demographic characteristics were utilized by RSs to enhance the contractors' sales for developing the customers' trust. RSs are extensively applicable for offering useful recommendations including web-page recommendations, house-rent recommendations, e-books recommendations, travel-service recommendations, identical products recommendations, songs recommendations, friends' recommendations, and similar-movie recommendations[5] .

The strategy used by a RS for providing relevant recommendations plays a significant role for recognizing the type of RS. Different types of RSs used for obtaining the trust of clients such as knowledge-based RS, collaborative-filtering (CF) RS, hybrid-RS, content-based (CB) RS and demographic-RS[6, 7] . CB[8] [9] and CF[10, 11] are the most frequently used algorithms in RSs. CB utilizes the identical historical feedback given by the users for the recommendations of new items to the same users[12] . However, CF incorporates the similarity in liking of users (correlation) with similar interests to recommend a new product to the candidate user[13]. CF proved to be an effective strategy in RSs for providing accurate and useful recommendations to the candidate users. Moreover, model-based CF[14] and memory-based CF[15] are the two sub-categories of CF for RSs. CB utilizes neighborhood knowledge such as item-item and user-user for estimating relevant and missing feedback. Recommendations to the users using model-based (latent factor-based) CF are given through a users' feedback (rating) model also termed as user's rating-model. CF centered rating models are constructed by utilizing data-mining and artificial intelligence algorithms including Genetic

Algorithms[16] , Bayesian Classifiers[17] , Matrix Factorization (MF)[18] , and Deep Neural Networks (DNN)[19]. Fig. 1 given below, graphically demonstrates the idea behind CB and CF.

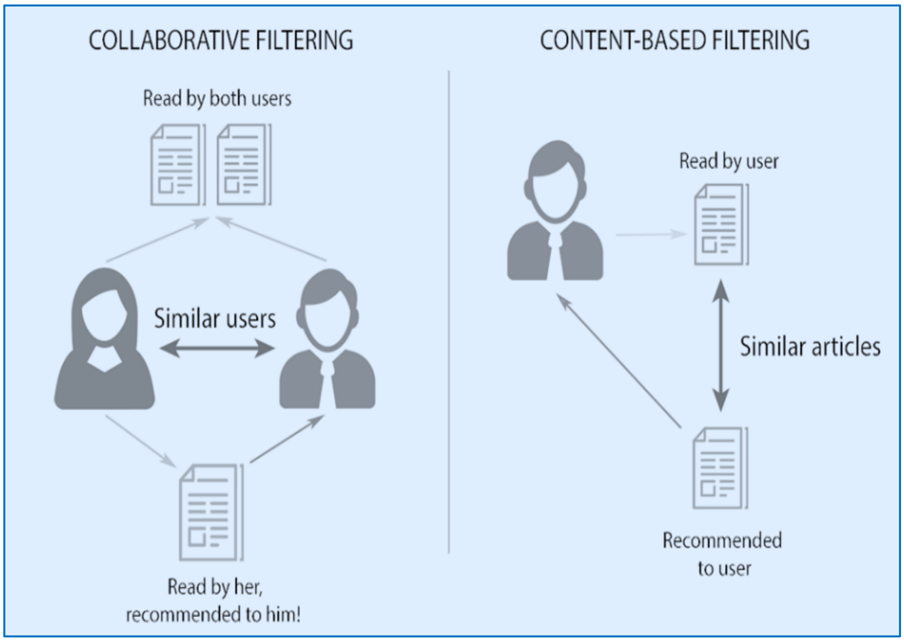


Figure 1: Popular types of RS [12]

The missing user-item ratings in a sparse rating matrix are efficiently estimated through MF for solving RS problem. The hidden characteristics of the users and items decomposed matrices are effectively learned by[20]. Enhancement of fractional gradient-based least mean square (EFDLMS) fractional stochastic gradient descent is suggested in order to factorize a huge user-item rating matrix is a challenging job. The fast and improved MF for RSs is achieved through the refined variants[21] of stochastic gradient descent based optimizer, for solving the data sparsity and large-level factorization problems. Fig. 2 describes MF mechanism for factorizing the users' input rating matrix built through user-item interactions.

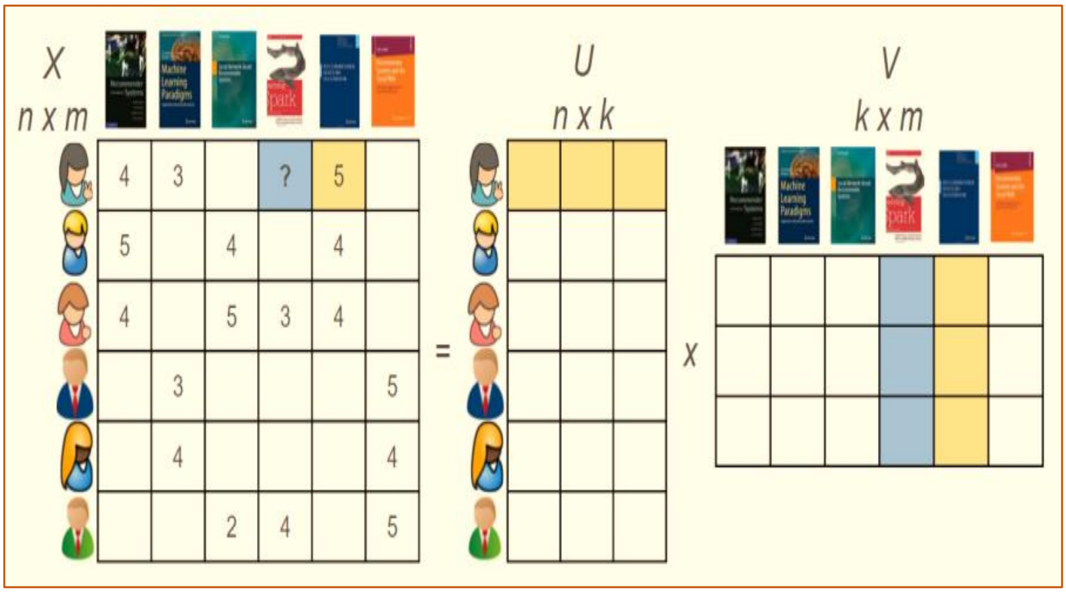


Figure 2: RSs using matric factorization [12]

Fractional calculus has gained the attention through deep theoretical concepts and mathematical interpretations. Recently, fractional gradient-based SGD optimizers[22-25] are introduced by the researchers to increase the recommendations speed and quality of recommendation in terms of accuracy.

In [26], an enhanced fractional gradient-based least mean square method (EFDLMS) is suggested. Another researcher tested EFDLMS for the precise and fast power signal parameters approximation in 2022[27] . The concepts of enhanced fractional gradient-based SGD optimizer (EFDLMS) have not been explored yet for enhancing the recommendations speed and precision through effective MF.

Using the concepts of EFDLMS, an accelerated fractional stochastic gradient descent AF-SGD optimizer is proposed in this study to provide quick and correct recommendations through superior MF procedure for solving RS problem. The hierarchical representation of the research problem is given in Fig. 3.

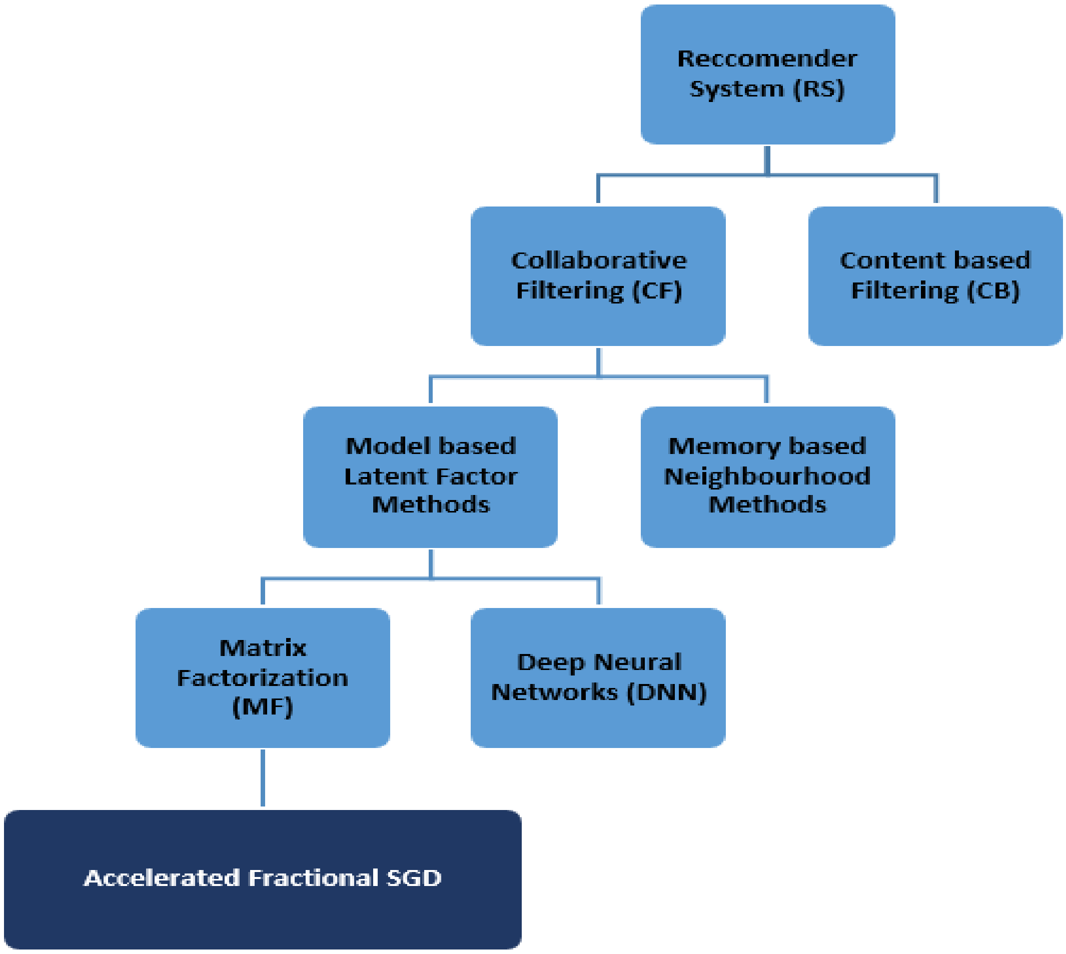


Figure 3: Hierarchical representation of the research problem

1.3 Problem Statement

To tackle rise in customers, demand due to increase in size of items and users, the e-commerce industry needs more efficient and reliable and accurate approaches. From the literature it is noticed that some inevitable factors including sparseness, less scalability, low predictive accuracy, slow convergence speed and high computational complexity are affecting the performance of RSs. It has been observed that SGD-based approaches [21] introduced in the literature for solving

RS problem efficiently through MF, merely utilize integer order gradients in the weights learning mechanisms for learning useful latent factors/features. Moreover, the integer-order-based SGD models are deficient of capturing the useful information encapsulated in the historical users' ratings for various items. The deficiency of lacking historical feedback influences the performance of RSs regarding recommendations predictive accuracy and speed.

In [26], For the purpose of making prompt and precise recommendations, an enhanced fractional gradient-based least mean square (EFDLMS) Faa-di-Bruno fractional order derivative for the effective power signals parameters approximation [27]. The EFDLMS proposed in [26] utilized the estimate of the squared errors in the latent features learning mechanism for the improvement of convergence speed.

It is evident from the literature that EFDLMS has not been used for learning user/item latent features in RSs. Motivated by the strong underlying mathematical concepts of EDFLMS, we aim to design an improved SGD model termed as accelerated fractional SGD (AF-SGD) by exploiting enhanced fractional concepts for providing fast and accurate recommendations.

1.4 Goals and Objective

The present study's primary goals are:

- To make MF procedure efficient by providing fast recommendations.
- Enhancing recommendations accuracy by manipulating users' rating history.
- Exploiting and improving user/item hidden characteristics to boost recommendation accuracy.

1.4 Contributions

The suggested research work can be considered as a healthy contribution to the SGD-based optimization techniques such as [43][44] for the development of an accurate, efficient, robust, and effective rating prediction-based procedure in the field of RS.

The practical significance of the proposed AF-SGD will be assessed through fast and correct predictions provided by the proposed model to the clients (to explore new, exciting, and relevant products) and service providers such as (to gain shoppers' trust, increase transactions and to gather knowledge about customers). The applicability domain of the suggested AF-SGD is graphically demonstrated in Fig 4.

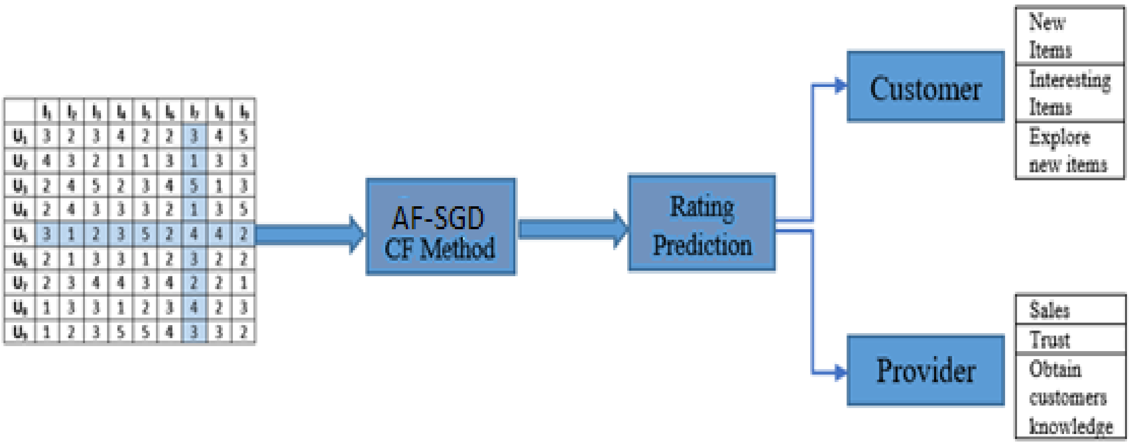


Figure 4 Applicability domain of the suggested AF-SGD

1.6 Organization of the Thesis

The following is the composition of the work presented in this research project.

Chapter1: Provides a conceptual overview of the entire thesis, consisting of research gaps, statements, and definitions that clearly define research goals and hypotheses, as well as background and motivations for the determination of problematic issues and research problem definitions.

Chapter 2: provides detail of the work done so far by discussing the advantages and disadvantages of already suggested methods in the literature.

Chapter 3: describes the research methodology with MF model by elaborating the proposed AF-SGD technique. It also contains the pseudocode of the proposed hybrid model.

Chapter 4: includes hyper-parameters selection details. Moreover, it provides simulation results in the form of tables and learning curves for a thorough comparison between the standard SGD and the suggested AF-SGD using two benchmark datasets, namely Film Trust and ML-100K, which might be an expansion of an existing study.

Chapter 5: holds the conclusions drawn from the research work along with future research directions for the likely extension of a present learning.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter includes the basic concepts of matrices factorization specific to RS, points out key differences and limitations from previous work matrix factorization methods for recommender systems, and presents a system model for matrix factorization. Additionally, this chapter also includes our contributions and a summary at the end.

2.2 Stochastic Gradient Descent (SGD) Variants for RS

Matrix factorization is one of the superior CF methods used by RS for proposing quick, useful and related recommendations[28]. To deal with the growing number of users and items, probabilistic MF [14], alternating least squares (ALS)[29] , probabilistic latent semantic analysis[30] , and maximum margin matrices factorization[31]. Stochastic gradient descent (SGD) and ALS have proved to be comparatively improved strategies for MF. Coordinate descent algorithms employing ALS[32] and momentum-based SGD [21] have accelerated the speed of recommendations though MF.

The newly proposed MF methods need to be more scalable to handle the growth in extent of items and customer. SGD-based scalable algorithms [21] were proposed to improve predictive-accuracy and reduce the training-time. In[33] , asymmetric factor methods and biased MF models were presented. The convergence rate of RSs was further improved through adaptive learning rates in

[34]. SGD-based algorithms for handling large dimensions MF were suggested in [35].

To speed up and parallelize the standard-SGD, a distributed-SGD (D-SGD) was presented in[36] , SGD, a distributed SGD (DSGD), was introduced. Locking and memory discontinuity are two of DSGD's drawbacks., an SGD alternate (FPSG) with shared memory was introduced in[37] . Moreover, a minibatch SGD strategy (KMSGD) was given in[38]. Researchers tried to establish a stable behavior between efficacy of KMSGD and its robustness, but KMSGD lacks to solve the best learning rate selection problem. The exactness and speed of recommendations through latent factor (LF) based SGD were improved further by the eight extended LF-based SGD methods presented in[39]. The extended LF methods were more expensive in terms of computational cost as compared to SGD.

The convergence speed of the standard SGD was well tackled by the over-parameterized methods given in[40] but the training time of the over-parameterized methods was more than the standard SGD. In [41], an incremental model was introduced to improve the fitness of the technique regarding speed and accuracy of recommendations. Incremental approach fails to address the convergence issue among the batch variations and predictive abilities through MF. The concept of pre-initializing the latent factors for achieving the speed, precision and scalability of RSs was provided in [42]. The pre-initialized factor models merely depend on pre-initialized factors for the achievement of improved performance.

SGD-based approaches [21] introduced in the literature for efficiently solving RS problem through MF, merely utilize integer order gradients in the weights learning mechanisms for learning useful latent factors/features. Moreover, the integer-order-based SGD models are deficient of capturing the useful information encapsulated in the historical users' ratings for various items. The deficiency of lacking historical feedback influences the performance of RSs regarding recommendations predictive accuracy and speed. Researchers addressed the inherent drawbacks of integer-order-gradient-based SGD approaches for solving RS problem by developing the fractional-gradient-based SGD approaches [22 - 25] for effective MF in RSs.

In [26], By creating fractional gradient-based SGD models, the enhanced fractional order gradient based least-mean (EFDLMS) Faa-di-Bruno fractional order derivative was developed to address the model weakness of the integer order gradient-based SGD methods for solving the RS problem and provide quick and precise recommendations for the effective power signals parameters approximation [27]. The EFDLMS proposed in [26] utilized the estimate of the squared errors in the latent features learning mechanism for the improvement of convergence speed.

It is evident from the literature that EFDLMS has not been used for learning user/item latent features in RSs. Motivated by the strong underlying mathematical concepts of EDFLMS, we aim to design an improved SGD model termed as accelerated fractional SGD (AF-SGD) by exploiting enhanced fractional concepts for providing fast and accurate recommendations. The summary of the related work done for solving RS problem and provide quick and precise recommendations. The synopsis of the in Table 1.

Table 1: Summary of the SGD based variants for RS

Category of Improved SGD methods for RS	Methods Representation	Advantages	Limitations	Metrics	Datasets
To speed up and parallelize the standard SGD [36][37]	DSGD, FPSG	Improved SGD speed through parallelism. FPSG introduces randomized strategy and locking-free planning mechanism to avoid memory-discontinuity and locking problems observed in DSGD.	DSGD has locking and memory discontinuity issues. FPSG is a viable solution to shared memory systems only.	RMSE	Hugewiki, Netflix, Yahoo!Music, MovieLens
Minibatch SGD strategy [38]	KMSGD	Established a stable behavior between efficacy of KMSGD and its robustness. Lowering the variations in latent	Lacks to solve the best learning rate selection problem.	RMSA	ML -1M, ML -100K

		factors revisions which results in convergence stability.			
Extended LF-based SGD methods [39]	Stochastic Proximal Gradient Descent-Based LF, Nesterov Accelerated SGD-Based LF, Momentum-Incorporated SGD-Based LF, Adaptive Moment	The exactness and speed of recommendations through latent factor (LF) based SGD were improved further by the eight extended LF-based SGD methods for tackling High-Dimensional-Sparse-Matrices (HIDS).	Appropriate methods for dealing mostly with High-Dimensional-Sparse-Matrices (HIDS). The extended LF methods were more expensive in terms of computational cost as compared to SGD.	RMSE	Netflix, ML -20M

	Estimation SGD-Based LF Adaptive SGD-Based LF, RMSprop SGD-Based LF, Ada-Delta SGD-Based LF, Accelerated Stochastic Proximal Gradient Descent-Based LF,				

Increasing convergence speed of MF through over-parameterization [40]	OP_RMSprop, OP_Adadelta, OP_AdaGrad, OP_SGD, OP_Adam.	Convergence speed of the standard SGD was well tackled by the over-parameterized methods	Over-parameterized methods consume more training time as compared to the standard SGD	MSE	Film Trust, Yahoo! Movies, Epinions, ML -1M, AMI
Accelerates MF speed by utilizing positive-only-feed-back [41]	ISGD	ISGD enhances the model's capability regarding speed and accuracy of recommendations.	Incremental approach fails to address the convergence issue among the batch variations and predictive abilities through MF.	Recall@10 Recall@5 Recall@1	Music-listen, Lastfm-600k, Music-playlist, ML -1M
Introduces concept of pre-initializing latent factors [42]	Pre-initialized models	The pre-initialized latent user/item factor matrices improve speed, precision and scalability of RSs.	The pre-initialized factor models merely depend on pre-initialized factors for the	RMSE	ML-100K,

		Also, pre-initialization reduces sparsity in user-item rating matrix.	achievement of improved performance		
Fractional stochastic gradient descent for recommender systems [23]	FSGD	FSGD is more precise with regard to RMSE as compared to standard SGD for fractional order values less than 1.	Incapable of incorporating the memory effect by capturing users historical feedback efficiently. slow convergence speed	RMSE	ML-100K
Momentum Fractional Stochastic Gradient Descent for Recommender System [25]	mF-SGD	mF-SGD method has achieved improved estimation-accuracy and convergence rate.	Utilizes previous ratings history by using only past gradients information.	RMSE	ML-100K, ML-1M
Normalized fractional SGD computing paradigm	NF-SGD	NF-SGD adaptively tunes the learning rate	less scalability.	RMSE	ML-100K, ML-1M

for recommender systems [24]		for speedy and accurate recommendations.	Unable to recognize and choose clusters of highly interrelated latent factors. low predictive accuracy		
Generalized fractional strategy for recommender systems with chaotic ratings behavior [22]	GFSGD	GFSGD guarantees speedy convergence for fractional order values greater than 1.	Not capable of discovering and selection collections of highly associated latent characteristics	RMSE, MAE, sMAPE, NSE	ML-100K, ML-1M, Film Trust
Rating pattern aware sliding window strategy for RSs [43]	RP-SWSGD	Achieved speedy convergence and precision window length > 1.	Unable to recognize and choose clusters of highly interrelated latent factors. Lacks Generalization capability.	RMSE, MAE	ML-100K, Film Trust,

			Slow convergence speed		
Moving Information-based SGD for RSs[44]	MISGD	Enhanced the correctness of the RSs via historical feedback by exploiting the fuzziness among user-item interactions.	Incapable of finding and picking groups of highly related latent features.	MAE, RMSE	ML-100K, ML-1M

	Adaptive SGD-Based LF, RMSprop SGD-Based LF, Ada-Delta SGD-Based LF, Adaptive Moment Estimation SGD-Based LF				
Accelerating MF by overparameterization [35]	OP_SGD, OP_AdaGrad, OP_Adadelta, OP_RMSprop, OP_Adam.	The overparameterized model converges to a small loss with fewer epochs.	Takes much more computational time than baselines.	MSE	Epinions, MovieLens-1M, Film Trust, Yahoo! Movies, AMI

Fast Incremental MF with positive only feed-back [41]	ISGD	ISGD has attained useful cumulative scores and appears to achieve valuable outcome by utilizing a progressive learning process.	Suggested for streaming data environment for positive-only feedback. It is required to observe how precisely it progresses over time by assessing via ranking based evaluation measures. Moreover, it did not find a convergent solution between the incremental prediction function and the batch extension of the MF.	Recall@10 Recall@5 Recall@1	Music-listen, Lastfm-600k, Music-playlist, MovieLens-1M
Improving precision of estimation in CF by initialization of factor matrices [42]	Initialized method	The pre-initialization of hidden factors in MF has two substantial benefits: (1) lessens sparsity and increases scalability (2) provides fast convergence as well as matrix factorization.	Depicts quality-performance only by Pre-initialization of factor matrices.	RMSE	MovieLens-100k

2.3 Our Work

SGD base optimization technique consist of Adam, Momentum, Adamax and RMS prose are established to use incomplete data exponentially weight moving average of preceding and present values. the pattern-rating and histories-rating of client shows great promising result for update hidden-feature-vectors of matrix factorization to resolve the problem of recommendation system by using matrix- factorization technique. For this purpose, we use efficient stochastic gradient decent gradients by exploiting user-rating pattern for useful recommendation and accelerated fractional perceptions to increase the speed of recommendation. The accelerated fractional technique is named as Accelerated fractional stochastic gradient decent technique for RS.

2.4 Mathematical Model of Recommender System

The cost function for solving RS through MF is represented as:

$$\aleph(\mathbf{o},\mathbf{p}) = \min_{\substack{\mathbf{O}\in\Re^{m\times x}\\ \mathbf{P}\in\Re^{m\times y}}} \sum_{(u,v)\in\lambda} (\mathbf{Q}_{uv} - \mathbf{o}_u^{\mathrm{T}}\mathbf{p}_v)^2 = \min_{\substack{\mathbf{O}\in\Re^{m\times x}\\ \mathbf{P}\in\Re^{m\times y}}} \sum_{(u,v)\in\lambda} \Im_{uv}^2 , \tag{1}$$

$$\Im_{uv}^2 = (\mathbf{Q}_{uv} - \mathbf{o}_u^{\mathrm{T}}\mathbf{p}_v)^2 = (\mathbf{Q}_{uv} - \hat{\mathbf{Q}}_{uv}) , \tag{2}$$

Where $\mathbf{Q}\in\Re^{x\times y}$ is sparse matrix with y items and x users representing explicit user-item interactions. The error between observed and estimated ratings is denoted by $\Im_{uv}^2$.

While $\aleph(\mathbf{o},\mathbf{p})$ represents the cost function for the RS. Whereas $\mathbf{O}\in\Re^{m\times x}$ and $\mathbf{P}\in\Re^{m\times y}$ are factorized matrices of users and items. The u^{th} and v^{th} feature vectors for the users and items decomposed matrices are represented by $\mathbf{o}$ and $\mathbf{p}$

respectively. The latent features for the decomposed user/item matrices are denoted by m. The indices for the actual feedback given by the u^{th} user for v^{th} item is shown by λ.

2.5 Summary

The general overview of literature has been discussed along with limitation of these existing methods. Moreover, Mathematical model of our recommender system is provided. The next Chapter describes the detailed oversight of the suggested methodology and standard-SGD.

CHAPTER 3

PROPOSED METHODOLOGY

This chapter presents the standard SGD and the suggested AF-SGD, together with the suggested approach. Additionally, the derivations of the learning rules for the standard SGD and the proposed AF-SGD are included in the Chapter 3 subsections.

3.1 Standard-Stochastic-Gradient-Descent (SGD)

Mathematical update expressions of stand. SGD for the customer and product feature-vectors are estimated independently by calculating the integer-order-gradient of the objective-function $\aleph(\mathbf{o},\mathbf{p})$. The update relations for the nth iterative step are given as:

$$\mathbf{o}_u(n+1) = \mathbf{o}_u(n) - \frac{\mu}{2}\Delta_{\mathbf{o}_u}\aleph(\mathbf{o},\mathbf{p}) \tag{3}$$

$$\mathbf{p}_v(n+1) = \mathbf{p}_v(n) - \frac{\mu}{2}\Delta_{\mathbf{p}_v}\aleph(\mathbf{o},\mathbf{p}) \tag{4}$$

Here, μ indicates the step-size for estimating user/item latent features. The integer derivative of objective-function $\aleph(\mathbf{o},\mathbf{p})$ for user latent factor vector $\mathbf{o}_u$ is calculated as:

$$\Delta_{\mathbf{o}_u}\aleph(\mathbf{o},\mathbf{p}) = -2\Im_{uv}\mathbf{p}_v \tag{5}$$

Similarly, the integer-derivative of objective-function $\aleph(\mathbf{o},\mathbf{p})$ for item latent factor vector $\mathbf{p}_v$ is computed as:

$$\Delta_{\mathbf{p}_v} \aleph(\mathbf{o},\mathbf{p}) = -2\Im_{uv}\mathbf{o}_u \tag{6}$$

The learning expressions for the users/items factor vectors using standard SGD are finally computed by putting the gradients given in the equations (5) and (6) to the equations (3) and (4) respectively. The step size for user/item latent factors is indicated by the update relations for the nth iterative step. In equations (7) and (8), the user/item factor vectors and the integer derivative of the objective function are provided, respect.

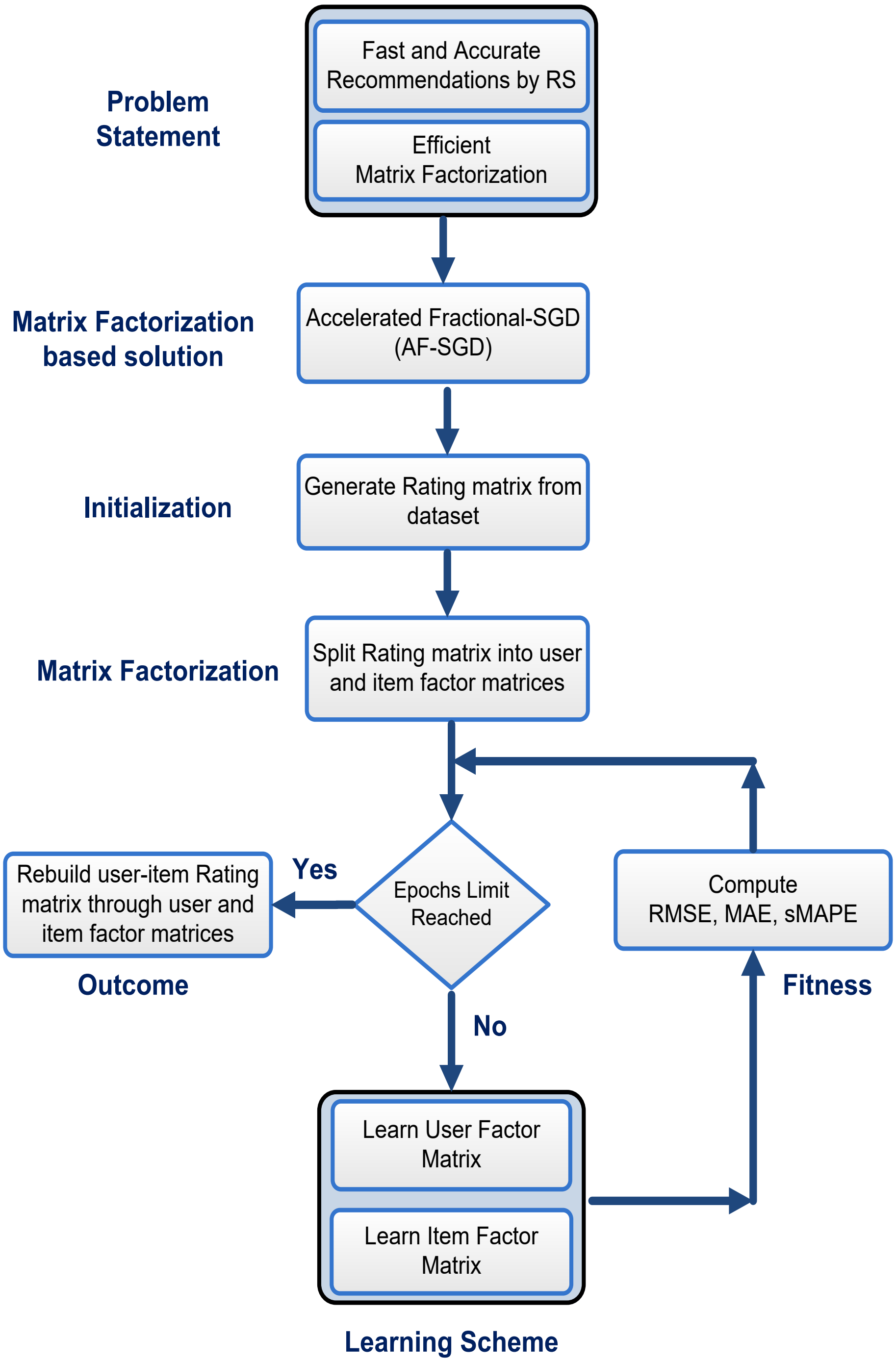


Figure 5: Graphical-Flowchart of the proposed AF-SGD for RS

$$\mathbf{o}_u(n+1) = \mathbf{o}_u(n) - \mu \Im_{uv} \mathbf{p}_v(n) \tag{7}$$

$$\mathbf{p}_v(n+1) = \mathbf{p}_v(n) - \mu \Im_{uv} \mathbf{o}_u(n) \tag{8}$$

The weight-apprise relations of the stand.SGD demonstrate memory less nature of SGD Because SGD only uses the input from the current iteration to learn the user/item latent factors for the next iteration, the weight update relations of the conventional SGD illustrate the memoryless nature of SGD.

3.2. Enhanced-Fractional-SGD (EF-SGD)

Least-means-square model on accelerated fractional sechostics gradient decent (AF-SGD) presented now in [6] .We see that it is also discover it uses in parameter-estimation in power-signals by reaserchers in [27]. The AF-SGD is novel algorithm for problem resoving in RS .In this study we will suggest a noval Accelerated Fractional Stochastic Gradient Descent AF-SGD variation of EFDLMS for RS using matrices fectorization

The basic mathematical ground work of fractional calculus are apply for updating of weight relation of stand. SGD for customer/product feature vectors, putting the fractional order derivatives for integer order derivatives of objective function it allows the effective use of historical user feedback for many sets of products.

By using a fractional order gradient to the objective function in place of an integer order gradient, the expressions for learning user/item latent factor vectors for the proposed AF-SGD are created. The relations are expressed as follows:

$$\mathbf{o}_u(n+1) = \mathbf{o}_u(n) - \frac{\mu}{2}\, {}_{\alpha}\Delta^{\omega}_{\mathbf{o}_u} \aleph(\mathbf{o},\mathbf{p}) \tag{9}$$

$$\mathbf{p}_v(n+1) = \mathbf{p}_v(n) - \frac{\mu}{2}\, {}_{\alpha}\Delta^{\omega}_{\mathbf{p}_v} \aleph(\mathbf{o},\mathbf{p}) \tag{10}$$

The learning expression for the user’s latent factor vector after applying fractional order gradient to the objective function with regard to the user's latent factor vector, by taking advantage of the simplified Faa di Bruno weight update expression in [40] is demonstrated as:

$$ {}_{\alpha}\Delta_{\mathbf{o}_u}^{\omega}\aleph(\mathbf{o},\mathbf{p})=\frac{[\mathbf{o}_u-\alpha]^{-\omega}}{\Gamma(1-\omega)}\Im_{uv}^{2}+\frac{\Gamma(1+\omega)}{\Gamma(\omega)\Gamma(2)}1!\times\frac{[\mathbf{o}_u-\alpha]^{1-\omega}}{\Gamma(2-\omega)}\,{}_{\alpha}\Delta_{\mathbf{Q}_{uv}}^{1}\Im_{uv}^{2}\frac{1}{1!}\left[\frac{{}_{\alpha}\Delta_{\mathbf{o}_u}^{1}\mathbf{Q}_{uv}}{1!}\right]^{1}. \tag{11} $$

weight update expression for the item's latent factor vector, and then taking the fractional order gradient of the objective function $\aleph(\mathbf{o},\mathbf{p})$ regarding item’s latent factor vector $\mathbf{q}_o$, by employing simplified Faa di Bruno formula is presented as:

$$ {}_{\alpha}\Delta_{\mathbf{p}_v}^{\omega}\aleph(\mathbf{o},\mathbf{p})=\frac{[\mathbf{p}_v-\alpha]^{-\omega}}{\Gamma(1-\omega)}\Im_{uv}^{2}+\frac{\Gamma(1+\omega)}{\Gamma(\omega)\Gamma(2)}1!\times\frac{[\mathbf{p}_v-\alpha]^{1-\omega}}{\Gamma(2-\omega)}\,{}_{\alpha}\Delta_{\mathbf{Q}_{uv}}^{1}\Im_{uv}^{2}\frac{1}{1!}\left[\frac{{}_{\alpha}\Delta_{\mathbf{p}_v}^{1}\mathbf{Q}_{uv}}{1!}\right]^{1} \tag{12} $$

.

The equations (11) to (12) represent fractional order gradients ${}_{\alpha}\Delta_{\mathbf{o}_u}^{\omega}\aleph(\mathbf{o},\mathbf{p})$ and ${}_{\alpha}\Delta_{\mathbf{p}_v}^{\omega}\aleph(\mathbf{o},\mathbf{p})$ with α indicating the lower bound and ω representing the fractional order Nevertheless, the modified learning procedures for learning user/item latent factor vectors are (10) due to the gamma function involved in fractional order gradient for user/item latent factor vectors expressions (13) and (14). by compounding the first order gradient chain rule with gamma function features. The updated user/item latent factor vectors are respectively represented as:

$$ {}_{\alpha}\Delta_{\mathbf{o}_u}^{\omega}\Im_{uv}^2 = \frac{\left[\mathbf{o}_u - \alpha\right]^{-\omega}}{\Gamma(1-\omega)}\Im_{uv}^2 + \frac{\omega\left[\mathbf{o}_u - \alpha\right]^{1-\omega}}{\Gamma(2-\omega)}\,{}_{\alpha}\Delta_{\mathbf{o}_u}^{1}\Im_{uv}^2 . \tag{13} $$

$$ {}_{\alpha}\Delta_{\mathbf{p}_v}^{\omega}\Im_{uv}^2 = \frac{\left[\mathbf{p}_v - \alpha\right]^{-\omega}}{\Gamma(1-\omega)}\Im_{uv}^2 + \frac{\omega\left[\mathbf{p}_v - \alpha\right]^{1-\omega}}{\Gamma(2-\omega)}\,{}_{\alpha}\Delta_{\mathbf{p}_v}^{1}\Im_{uv}^2 . \tag{14} $$

The information in [26] also depicted that the destruction of the gradient's information can occur through weights approximation. Using this notion to the weight update expressions (13) and (14), the amended learning rules for learning customer/products latent-feature-vectors are stated as follows:

$$ {}_{\alpha}\Delta_{\mathbf{o}_u}^{\omega}\Im_{uv}^2 = \frac{1}{\Gamma(1-\omega)}\Im_{uv}^2 + \frac{\omega}{\Gamma(2-\omega)}\,{}_{\alpha}\Delta_{\mathbf{o}_u}^{1}\Im_{uv}^2 . \tag{15} $$

$$ {}_{\alpha}\Delta_{\mathbf{p}_v}^{\omega}\Im_{uv}^2 = \frac{1}{\Gamma(1-\omega)}\Im_{uv}^2 + \frac{\omega}{\Gamma(2-\omega)}\,{}_{\alpha}\Delta_{\mathbf{p}_v}^{1}\Im_{uv}^2 . \tag{16} $$

The properties of sign function have been exploited in user/item weight update expressions (15) and (16) for avoiding fractional extreme points and supporting the enhancement in rate of convergence by utilizing squared-error-approximation [26]. The updated user/item learning rules are specified as:

$$ {}_{\alpha}\Delta_{\mathbf{o}_u}^{\omega}\Im_{uv}^2 = \frac{\Im_{uv}^2 \operatorname{sgn}\left[{}_{\alpha}\Delta_{\mathbf{o}_u}^{1}\Im_{uv}^2\right]}{\Gamma(1-\omega)} + \frac{\omega\left[{}_{\alpha}\Delta_{\mathbf{o}_u}^{1}\Im_{uv}^2\right]}{\Gamma(2-\omega)} . \tag{17} $$

$$ {}_{\alpha}\Delta_{\mathbf{p}_v}^{\omega}\Im_{uv}^2 = \frac{\Im_{uv}^2 \operatorname{sgn}\left[{}_{\alpha}\Delta_{\mathbf{p}_v}^{1}\Im_{uv}^2\right]}{\Gamma(1-\omega)} + \frac{\omega\left[{}_{\alpha}\Delta_{\mathbf{p}_v}^{1}\Im_{uv}^2\right]}{\Gamma(2-\omega)} . \tag{18} $$

The accelerated matrix factorization of RS through proposed AF-SGD is accomplished by putting equations (17) and (18) to equations (9) and (10). The reviewed terms for customer/products latent factor vectors are denoted in equations (19) and (20) as:

$$\mathbf{o}_u(n+1) = \mathbf{o}_u(n) + \mu\left[\frac{\Im_{uv}^2 \operatorname{sgn}\left[\Im_{uv}\mathbf{p}_v(n)\right]}{\Gamma(1-\omega)} + \frac{\omega\left[\Im_{uv}\mathbf{p}_v(n)\right]}{\Gamma(2-\omega)}\right] \tag{19}$$

$$\mathbf{p}_v(n+1) = \mathbf{p}_v(n) + \mu\left[\frac{\Im_{uv}^2 \operatorname{sgn}\left[\Im_{uv}\mathbf{o}_u(n)\right]}{\Gamma(1-\omega)} + \frac{\omega\left[\Im_{uv}\mathbf{o}_u(n)\right]}{\Gamma(2-\omega)}\right] \tag{20}$$

It is inferred from expressions (19) and (20) that for the fractional-order ω rate equal to one in (19) and (20), the proposed AF-SGD becomes standard SGD.

The next chapter shows the simulation and analysis results of the suggested algorithm compared to the standard SGD method for various datasets.

CHAPTER 4

SIMULATIONS AND ANALYSES

4.1 Introduction

The Simulation and Analysis Chapter provides the simulation details in terms of simulation parameters setting and tuning. Furthermore, case studies to explain the impact and improvement in results.

4.2 Simulations and results

This part is included that explanation of Hyper-parameters. datasets and estimation (evaluation matrices) using foe authentication of AF-SGD and also concluded the outcomes and examination of outcomes by using some parameters.

4.2.1 Simulation Parameters

This part shows simulations setting regarding many hyper parameters to show the validation of AF-SGD. this portion also explain the evaluation matrix and data sets that utilize to describe accurateness and scalability of suggested algorithm. And also explain the criteria of selecting optimal hyper parameters. Table-2 shows the description of data sets that we are using in this project,

Table 2: Datasets Particulars

Dataset	Film Trust	ML-100K
Ratings (F)	35497	100K
Users (C)	1508	943
Items (O)	2071	1682
Density = F / (O*C) * 100	1.136	6.30

The accuracy of suggested algorithm AF-SGD is tested on different evaluation matrix such as sMAPE, RMSE, NSE and MAE .and the equations these evaluation matrices are.

Root-Mean-Square-Error

$$RMSE_{test} = \sqrt{mean\left[\sum_{(u,v)\in \alpha_{\text{test}}} \Im_{uv}^{2}\right]}, \quad \textbf{(21)}$$

Mean Absolute Error

$$MAE_{test} = mean\left[\sum_{(u,v)\in \alpha_{\text{test}}} \left|\Im_{uv}^{2}\right|\right], \quad \textbf{(22)}$$

Symmetric mean absolute percentage error

$$sMAPE_{test} = mean\sum_{(u,v)\in \alpha_{\text{test}}}\left[\frac{\left|\Im_{uv}^{2}\right|}{\left|\mathbf{Q}_{uv}\right| + \left|\mathbf{o}_{u}^{\mathrm{T}}\mathbf{p}_{v}\right|}\right]\times 100, \quad \textbf{(23)}$$

Nash–Sutcliffe Efficiency coefficient

$$NSE_{test} = 1 - \sum_{(u,v)\in \alpha_{\text{test}}} \left[\frac{\mathfrak{I}_{uv}^{2}}{\left(\mathbf{Q}_{uv} - \hat{\mathbf{Q}}_{uv}\right)^{2}} \right], \quad \textbf{(24)}$$

Where $0.1 \leq NSE \leq 1$ and $NSE = 0.9$ represent the ideal fit

This table shows the set of hyper parameters use in the assessment along with their ideal values.

Table 3: Hyper-Parameters with best values

Hyper-Parameters	**Latent-Features**	**Fractional Orders**	**Learning Rate**
Notation	m	ω	μ
Tunned values	5, 15, 30	0.6, 0.75, 0.9	0.001

Simulation Setting

Simulation Environment Using the Windows-11 platform, Python-based simulations are run on a laptop equipped with a Core i5 12th Gen 4005U CPU running at 3.40 GHz and 8 GB of DDR4 RAM.

The main Objective to design suggested Accelerated fractional SGD use to approximate and recommended the sparse-ratings that have not rated yet by the customer for the products by manipulating the data which is not given by the customers-rating-pattern by using the customer-rating -history via AF-SGD.

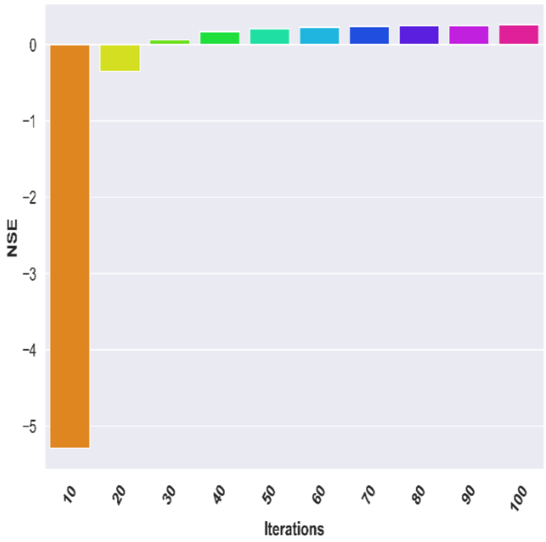
Case study 1: ML-100k dataset performance analysis

Case-study1 consist of the outcomes of film ML-100k set best rate of learning μ

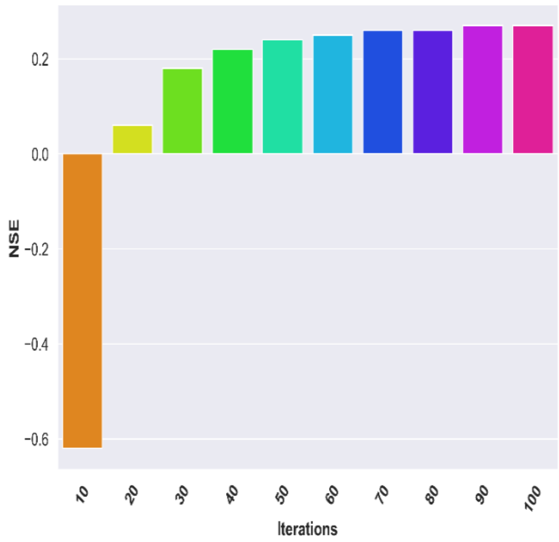
=0.001 latent factors of m=[5,15,30]and4fractional-order-values ω=[0.6,0.75.0.90, and 1.0] .This case-study consist of outcomes explanation w.r.t Some parameters-performances consisting convergence -speed and precision .

Predictability vs Fractional order

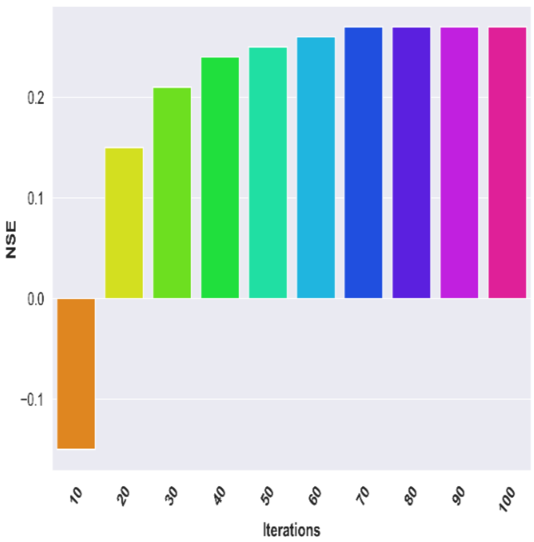
The predictability of the proposed AF-SGD as compared to the standard SGD ($\omega = 1$) for various latent features (m) i.e., [5, 15, 30] and fractional order (ω) values i.e., [0.6, 0.75, 0.9] is tested by means of NSE for ML-100K dataset. Figures 5 – 7 include the vertical bar graphs representing NSE outcomes with four fractional orders against number of iterations. Table 5 contains the NSE values regarding three latent features and four fractional orders for 100 iterations. It is observed from the Figure 5(a) – (d) that for $m = 5$ NSE increases gradually by increasing the fractional order values from $\omega = 1$ to $\omega = 0.6$ for all iterations. Similarly, the same performance trend has been noticed for $m = 15$ and $m = 30$ presented in Figures 6 and 7 respectively. It is also noticed that the proposed AF-SGD consumes smaller iterations to attain the NSE achieved by the standard SGD ($\omega = 1$) after 100 iterations. Moreover, it is noticed that NSE also improves by increasing the latent features and the best NSE value is achieved by the AF-SGD for $\omega = 0.9$ and $m = 30$. It is also seen that for large m values, the predictability of the proposed AF-SGD improves for higher iterations with large values of fractional orders.



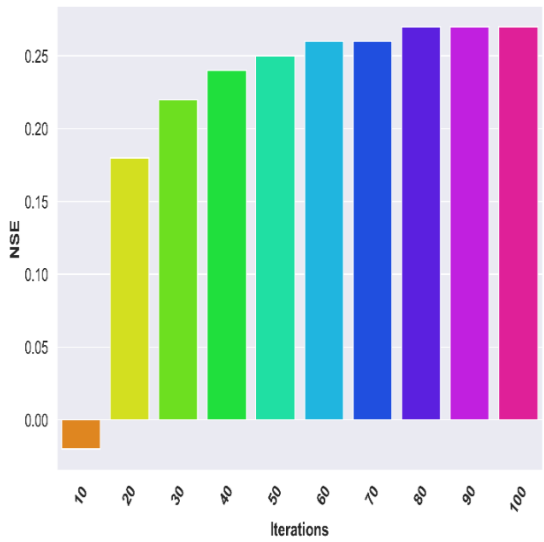
(a) $\omega = 1$



(b) $\omega = 0.9$

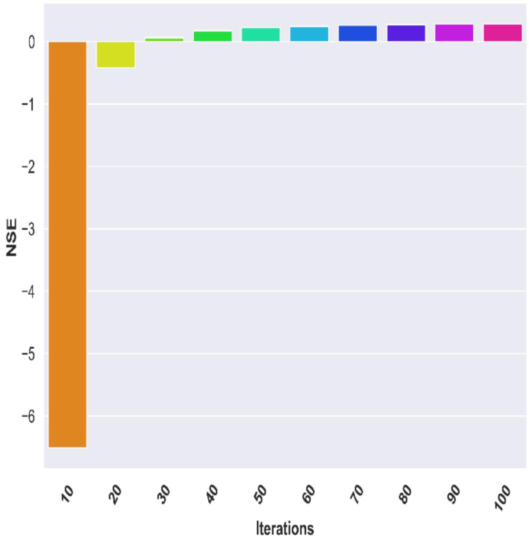


(c) $\omega = 0.75$

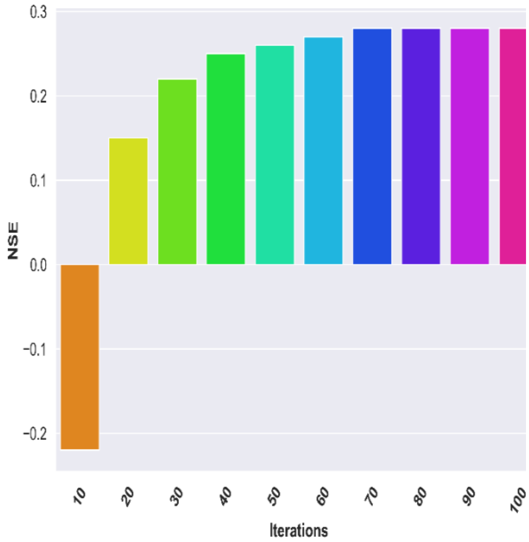


(d) $\omega = 0.6$

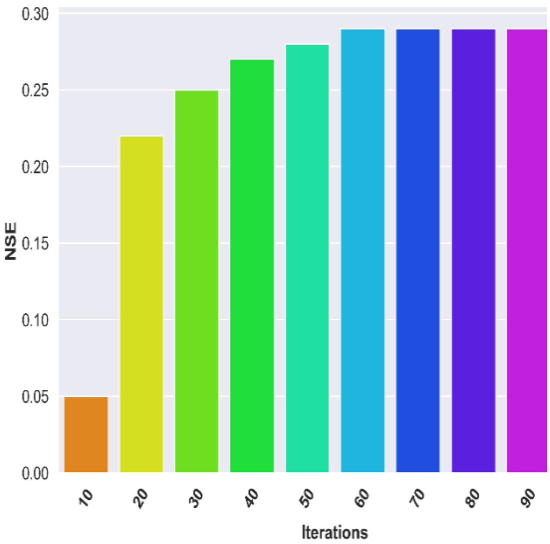
Figure 6: NSE-based Investigation of AF-SGD for four fractional orders variations using latent dimension of m = 5 with ML-100k dataset.



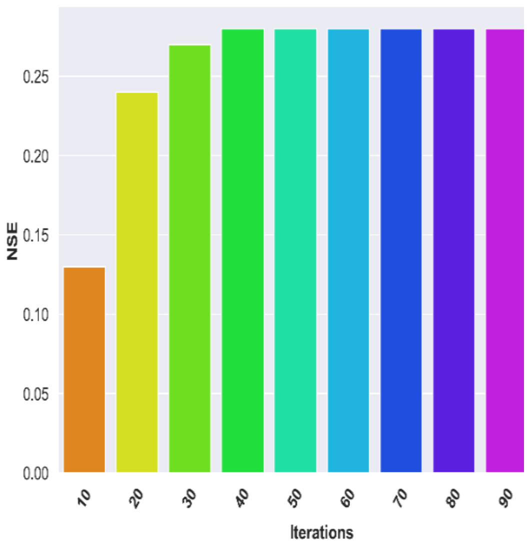
(a) $\omega = 1$



(b) $\omega = 0.9$

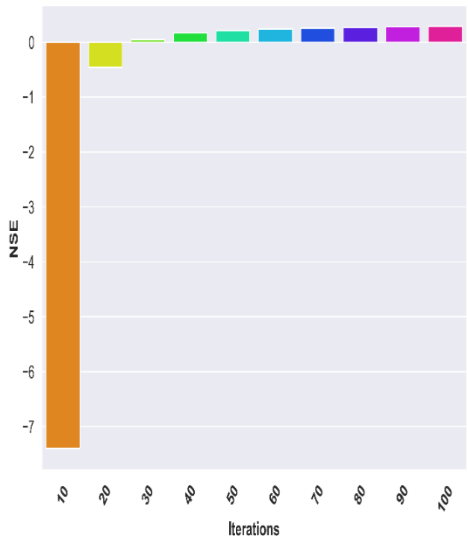


(c) $\omega = 0.75$

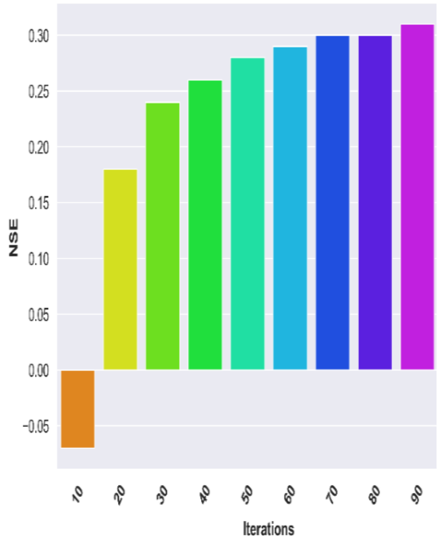


(d) $\omega = 0.6$

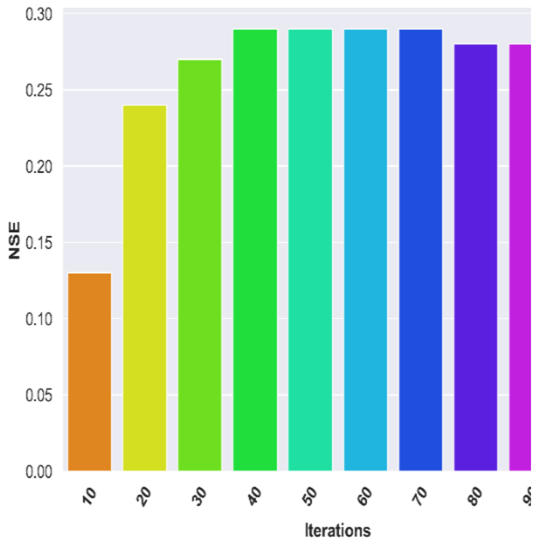
Figure 7: NSE-based Investigation of SGD for four fractional orders variations using latent dimension of m = 15 with ML-100k dataset.



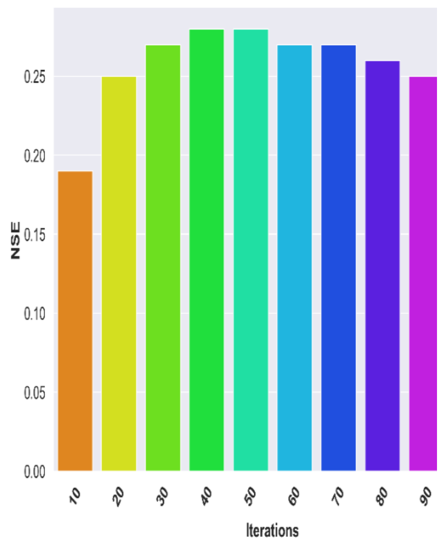
(a) $\omega = 1$



(b) $\omega = 0.9$



(c) $\omega = 0.75$



(d) $\omega = 0.6$

Figure 8: NSE-based Investigation of AF-SGD for four fractional orders variations using latent dimension of m = 30 with ML-100k dataset.

Table 4 : NSE Comparison of the proposed method with standard SGD (AF-SGD for ω=1) using ML-100K dataset.

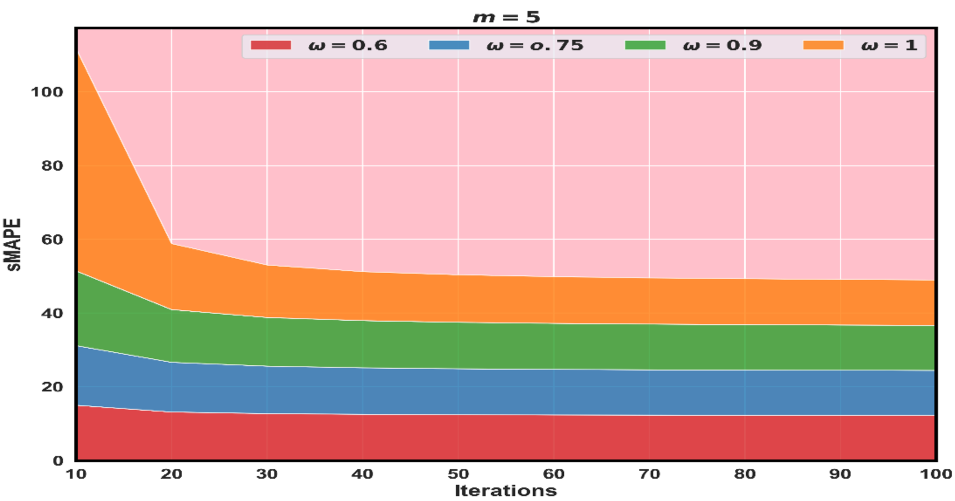
Dataset	Metric	Method	m	Iterations	ω 0.6	0.75	0.9	1
ML-100K	**NSE**	**AF-SGD**	**5**	20	0.18	0.15	0.06	-0.35
				40	0.24	0.24	0.22	0.17
				60	0.26	0.26	0.25	0.23
				80	0.27	0.27	0.26	0.25
				100	0.27	0.27	0.27	0.26
			15	20	0.24	0.22	0.15	-0.42
				40	0.28	0.27	0.25	0.17
				60	0.28	0.29	0.27	0.24
				80	0.28	0.29	0.28	0.27
				100	0.28	0.29	0.28	0.28
			30	20	0.25	0.24	0.18	-0.46
				40	0.28	0.29	0.26	0.17
				60	0.27	0.29	0.29	0.24
				80	0.26	0.28	0.3	0.27
				100	0.23	0.27	0.31	0.29

Percentage Performance for Number of Iterations

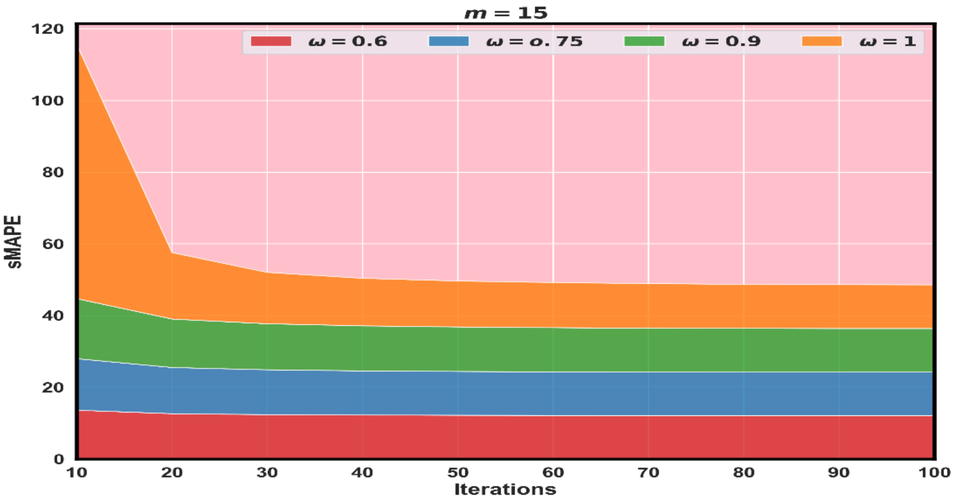
The ability of the proposed AF-SGD compared the standard SGD ($\omega = 1$) is also monitored through sMAPE as evaluation measure for dataset ML-100K. sMAPE learning curves for three latent features (m) i.e., [5, 15, 30] and 4 fractional-order (ω) variants for example , [0.6, 0.75, 0.9, 1] are demonstrated through vertical stack plots presented in Figure 8. Additionally, the sMAPE values for 100 iterations in tabular form are given in Table 5. The width of the vertical stacks at different iterations exhibits the percentage error attained by AF-SGD over the standard SGD ($\omega = 1$) for three m and ω variants. It is witnessed from the Figures 8(a) – (c) that the AF-SGD takes smaller iterations to accomplish sMAPE which is achieved by the standard SGD ($\omega = 1$) after 100 iterations. For instance, for $\omega = 30$, standard SGD ($\omega = 1$) attains sMAPE = 12.15 after 100 iterations. Whereas AF-SGD achieves the same sMAPE with $\omega = 0.9$ and $\omega = 0.75$ in around 75 and 55 iterations respectively for $\omega = 30$. Moreover, AF-SGD gains better percentage error in smaller number of iterations with smaller ω variations.

Table 5: sMAPE assessment of the suggested model with standard SGD (AF-SGD for ω=1) using ML-100K dataset

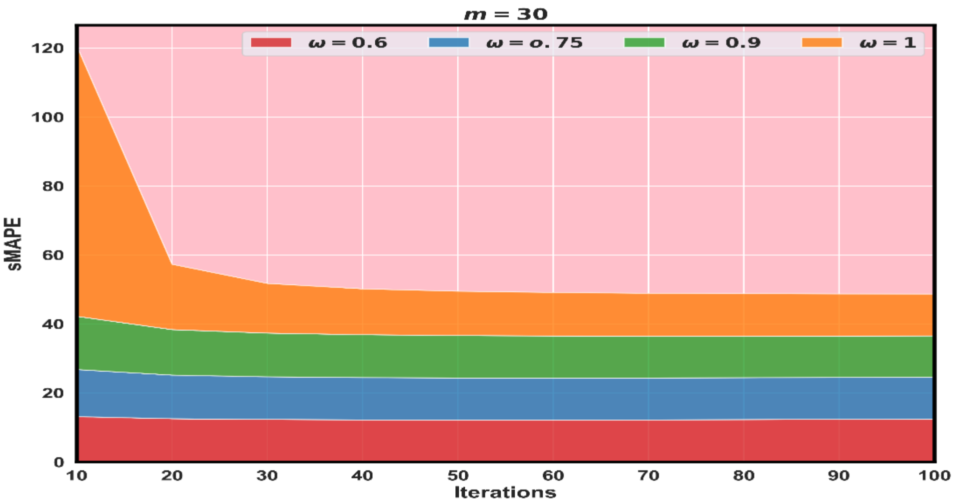
Dataset	**Metric**	**Method**	m	**Iterations**	ω			
					0.6	**0.75**	**0.9**	**1**
ML-100K	**sMAPE**	**AF-SGD**	**5**	20	13.18	13.48	14.29	17.89
				40	12.56	12.61	12.8	13.27
				60	12.38	12.37	12.46	12.67
				80	12.31	12.26	12.31	12.45
				100	12.26	12.19	12.23	12.33
			15	20	12.65	12.88	13.52	18.51
				40	12.27	12.32	12.56	13.25
				60	12.17	12.16	12.31	12.58
				80	12.14	12.1	12.18	12.3
				100	12.16	12.09	12.14	12.18
			30	20	12.57	12.65	13.19	18.94
				40	12.27	12.23	12.43	13.29
				60	12.24	12.14	12.18	12.62
				80	12.32	12.14	12.03	12.33
				100	12.44	12.19	11.96	12.15



(a)



(b)



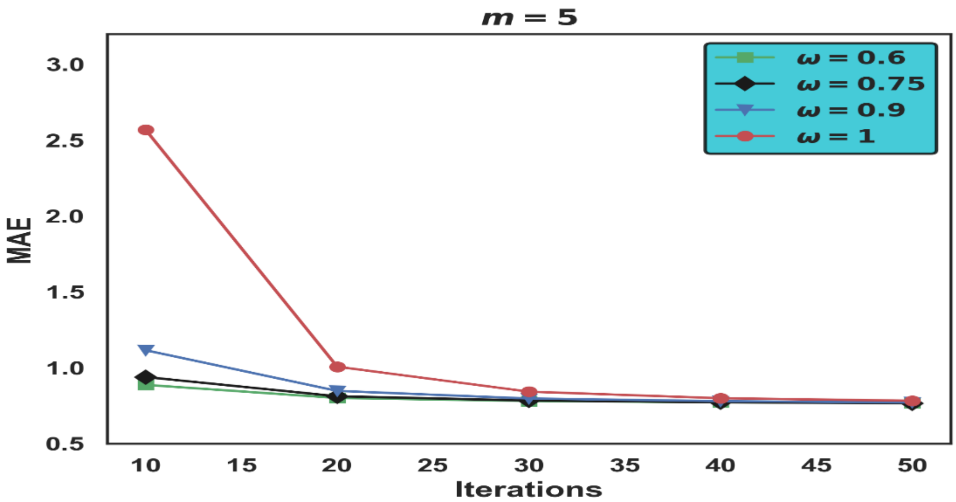
(c)

Figure 9: sMAPE-based Investigation of AF-SGD for fractional orders variations using latent dimension with ML-100k dataset.

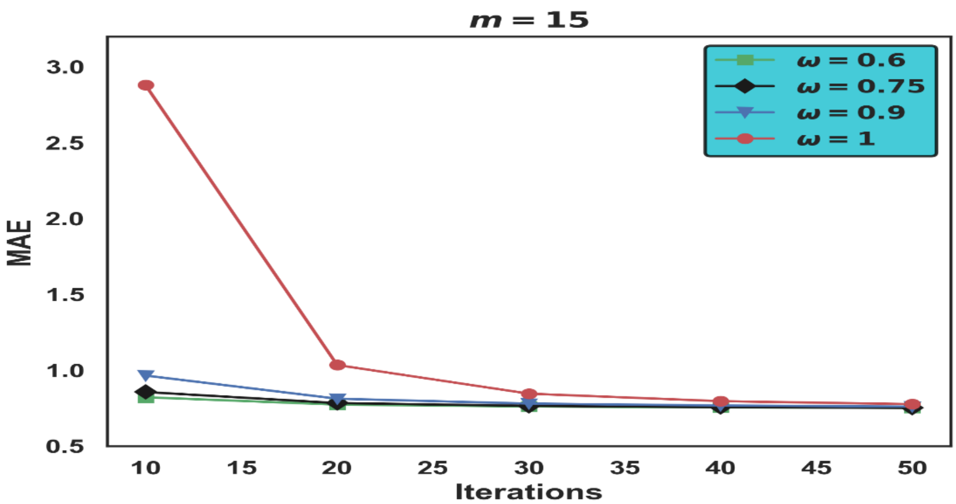
Performance Analysis for Convergence Speed and Accuracy

The rate of convergence and correctness of suggested AF-SGD over the standard SGD ($\omega = 1$) are validated through RMSE and MAE for different fractional order (ω) variations i.e., [0.6, 0.75, 0.9, 1] using dataset of ML-100K. MAE plots for three latent features and four fractional order values are presented in Figure 11. While the RMSE-based curves with three latent features (m) i.e., [5, 15, 30] are respectively shown in Figures 12, 13 and 14. Furthermore, MAE and

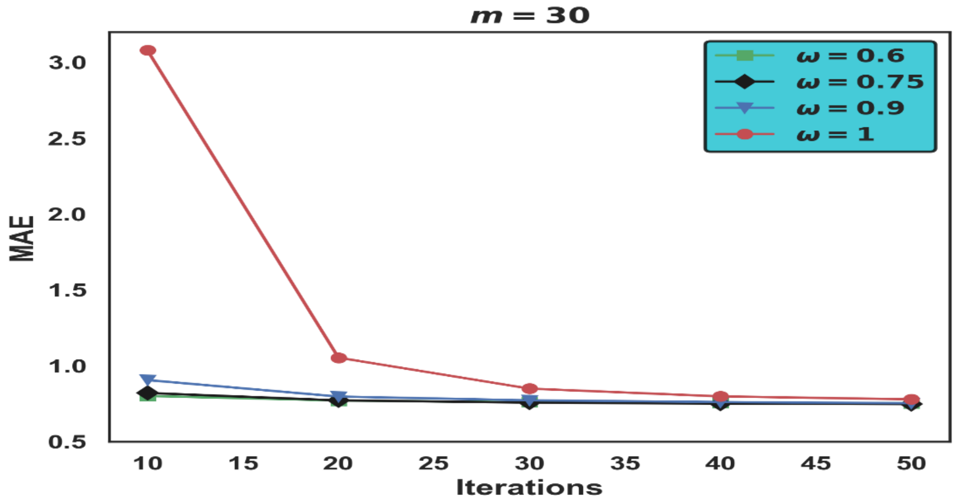
RMSE values attained for multiple repetitions are shown in Table 6 and Table 7 respectively. It is seen from the Figures 12-14 that AF-SGD converges faster than the standard SGD ($\omega = 1$) for three fractional-order (ω) values and latent-factors (m). Moreover, AF-SGD also achieved superior accuracy after 100 iterations for ω and m variations. AF-SGD attains the minimum accuracy with respect to RMSE (0.939) and MAE (0.734) after 100 iterations for $\omega = 0.9$ and $m = 30$.



(a)



(b)



(c)

Figure 10: Performance comparison with standard fractional-SGD and momentum-SGD for ML-100k.

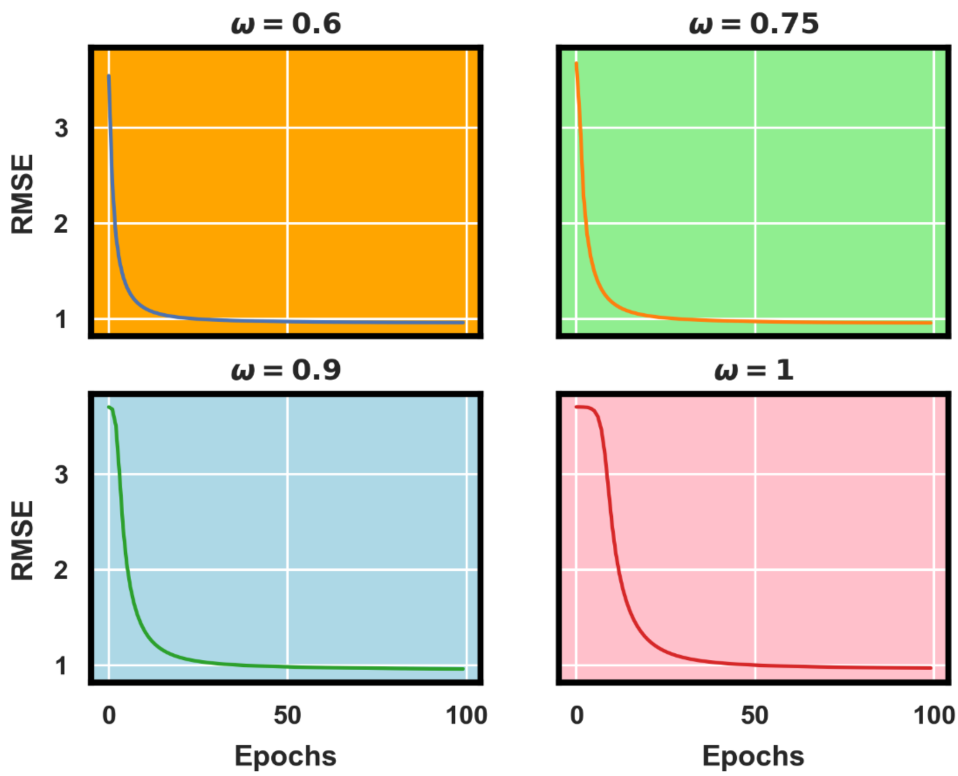


Figure 11**:** RMSE-based Investigation of AF-SGD for four fractional orders variations using latent dimension of m = 5 with ML-100k dataset**.**

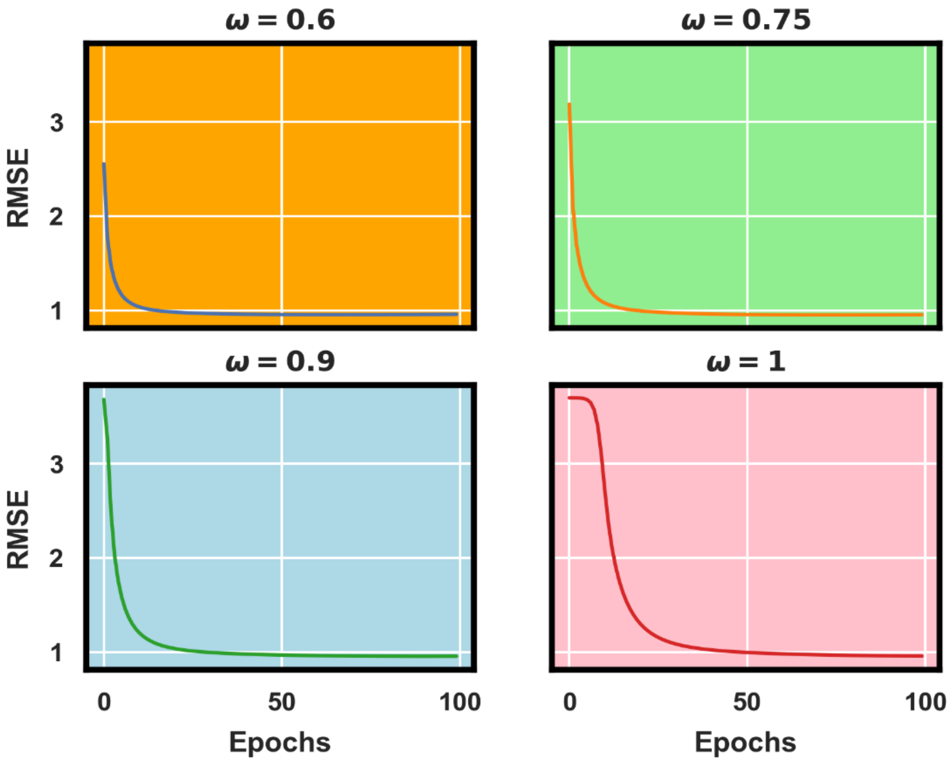


Figure 12: RMSE-based Investigation of AF-SGD for four fractional orders variations using latent dimension of m = 15 with ML-100k dataset.

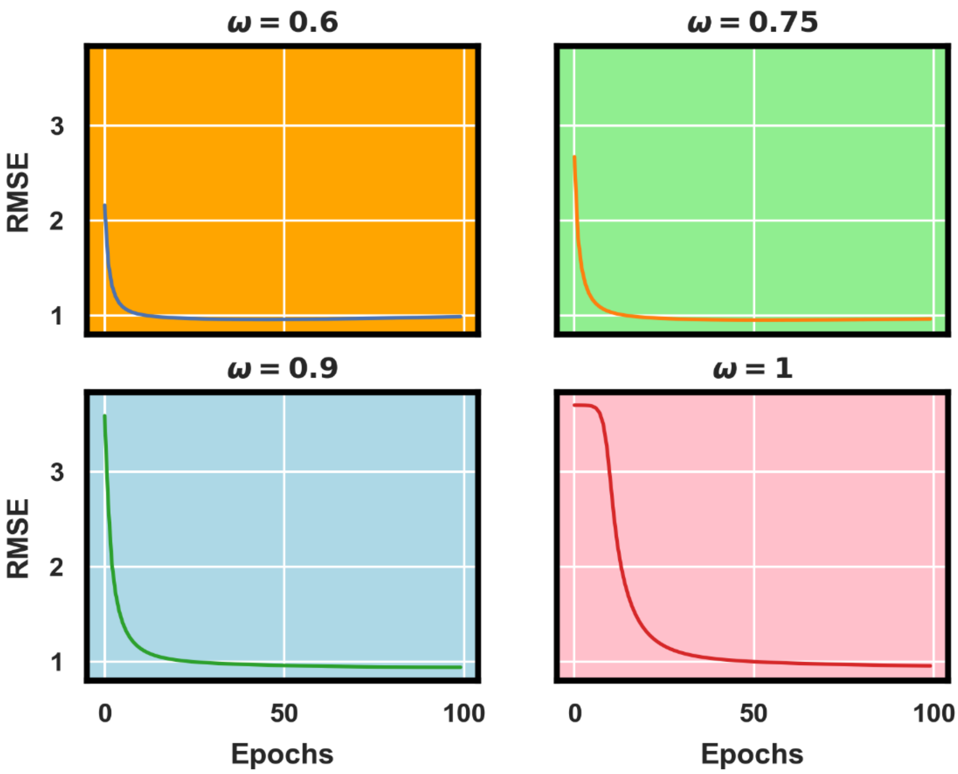


Figure 13: NSE-based Investigation of AF-SGD for four fractional orders variations using latent dimension of m = 30 with ML-100k dataset.

Table 6: MAE Comparison of the suggested model with stand. SGD (AF-SGD for ω=1) using ML-100K dataset

Dataset	Metric	Method	m	Iterations	ω			
					0.6	**0.75**	**0.9**	**1**
ML-100K	**MAE**	**AF-SGD**	**5**	20	0.80	0.81	0.85	1.01
				40	0.77	0.77	0.78	0.80
				60	0.76	0.76	0.76	0.77
				80	0.76	0.76	0.76	0.76
				100	0.76	0.75	0.75	0.76
			15	20	0.77	0.78	0.81	1.03
				40	0.75	0.76	0.77	0.80
				60	0.75	0.75	0.75	0.77
				80	0.75	0.74	0.75	0.75
				100	0.75	0.74	0.75	0.75
			30	20	0.77	0.77	0.80	1.05
				40	0.75	0.75	0.76	0.80
				60	0.75	0.75	0.75	0.77
				80	0.76	0.75	0.74	0.75
				100	0.77	0.75	0.73	0.74

Table 7: RMSE Comparison of the proposed method with standard SGD (AF-SGD for ω=1) using ML-100K dataset

Dataset	Metric	Method	m	Iterations	ω			
					0.6	**0.75**	**0.9**	**1**
ML-100K	**RMSE**	**AF-SGD**	**5**	20	1.02	1.04	1.09	1.31
				40	0.98	0.99	1.00	1.03
				60	0.97	0.97	0.98	0.99
				80	0.97	0.96	0.97	0.98
				100	0.96	0.96	0.96	0.97
			15	20	0.98	1.00	1.04	1.34
				40	0.96	0.96	0.98	1.03
				60	0.96	0.95	0.96	0.98
				80	0.96	0.95	0.96	0.96
				100	0.96	0.95	0.95	0.96
			30	20	0.98	0.98	1.02	1.37
				40	0.96	0.95	0.97	1.03
				60	0.96	0.95	0.95	0.98
				80	0.97	0.96	0.94	0.96
				100	0.99	0.96	0.94	0.95

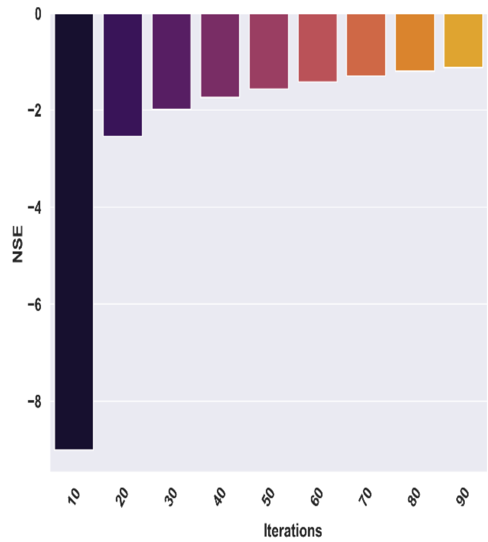
Case-Study II: performance analysis for film trust dataset

Case-study 2 also consist of learning-rate-values μ =0.0009 latent factors of m= [5,15,30] and 4 fractional-order-values ω=[0.6,0.75.0.90, and 1.0] by applying data set of film trust. This case-study consist of outcomes explanation w.r.t Some parameters-performances consisting convergence -speed and precision.

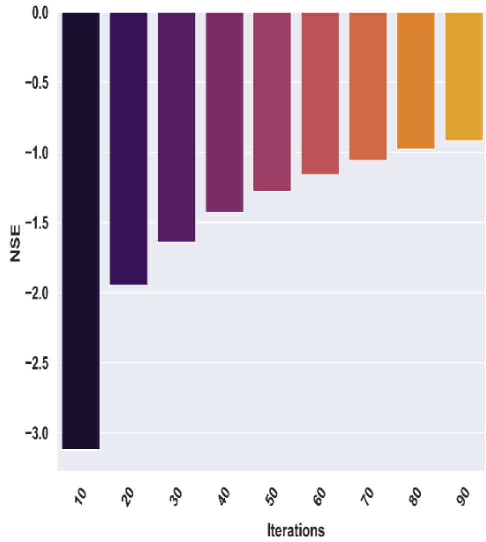
In this case study, we have executed the proposed AF-SGD approach for another benchmark database; Film Trust for proving the flexibility and salableness of the designed algorithm. Similar to case study-I, the proposed fractionally accelerated matrix factorization model is executed for multiple latent features (m) = [5,15,30] with specified variations of fractional order [0.1, 0.7, 0.85, 1] along optimal learning rate of μ= [0.0009].

Initially, the performance behavior is observed through the Nash-Sutcliffe Efficiency (NSE). **Table-8** provides the insights about the achieved NSE for each latent feature and fractional order variations. From **Table-8** it is observable that the proposed method with fractional orders have attained promising feedback in comparison with the stand. SGD ($\omega = 1$) in terms of a reaching NSE closer to 1. For m=5, it is observed that the best NSE of -0.84 is reached with AF-SGD ($\omega = 0.75$). Whereas, for m= 15 and m=30, the best NSE of -0.73 and -0.62 are attained with AF-SGD ($\omega = 0.9$). **Fig.15-17** displays the bar chart visulaization of the computed NSE for all latent feature variations. From these bar charts, it is noteworthy to observe the convergence difference between the fractional order SGD and standard SGD. For all latent features, it is depicted that the proposed method of AF-SGD with fractional orders like $\omega = 0.9$, achieves the same NSE in partial amount of repetitions in comparision with the standard SGD which shows the convergence efficiency achived in the recommendations with the integration of fractional order derivatives instead of integer-orders in SGD. The superior convergence behavior is more clear from the **Fig-18**, showing the Symmetric Mean Absolute Percentage Error (sMAPE) vs Iterations plots. Similar to NSE, the best sMAPE are achieved with fractional order of $\omega = 0.75 \; and \; \; \omega = 0.9$ for latent features of (m) = [5,15,30] as shown in **Table-9**. Seeing the accuracy and computational efficiency in terms of achieved NSE and sMAPE, it is noticeable that the incorporation of fractional orders has significantly accelerates the recommendation process.

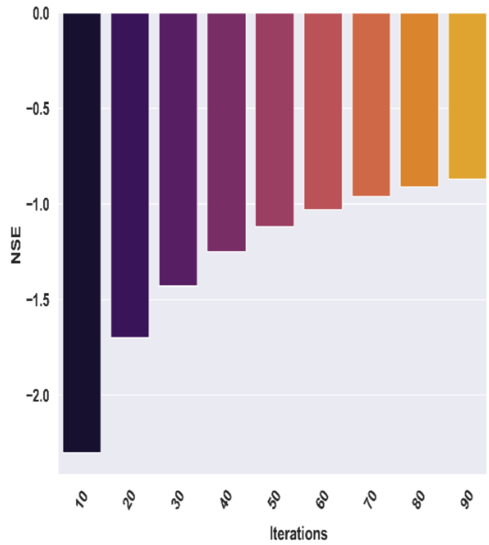
Afterwards, the recommendation performance of the suggested model is further analyzed through the popular evaluation metrics of Root-Mean-Square-Error (RMSE) and Mean-Abs- Error (MAE). **Table-10** demonstrates a performance assessment of the suggested AF-SGD approach with standard SGD regarding MAE. For each variation of latent features, it is evident that the fractional order specially $\omega = 0.9$ attains lowest possible MAE among its counterparts. Additionally, the speedy convergence of the AF-SGD with distinct fractional order variations is graphically presented through the error reduction curves given in **Fig-17.** Furthermore, the evaluation with respect to RMSE is presented in **Table-11**. The proposed AF-SGD approach with latent feature m=30 and fractional order $\omega = 0.9$ resulted in the best RMSE of 1.14 for the task at hand. **Fig.18-20** presents the iteration performance plots with respect to RMSE, which explicitly provides the insights about the acceleration in terms of convergent gains achieved with the integrations of fractional orders for all specified latent feature variations. From the above detailed discussions, we have concluded that the proposed approach of matrix factorization method with AF-SGD is accurate, scalable, adaptable and highly efficient for the reliable and speedy recommendation tasks.



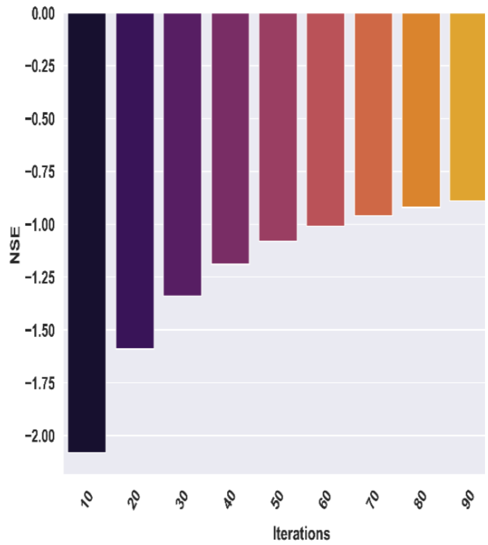
(a) $\omega = 1$



(b) $\omega = 0.9$

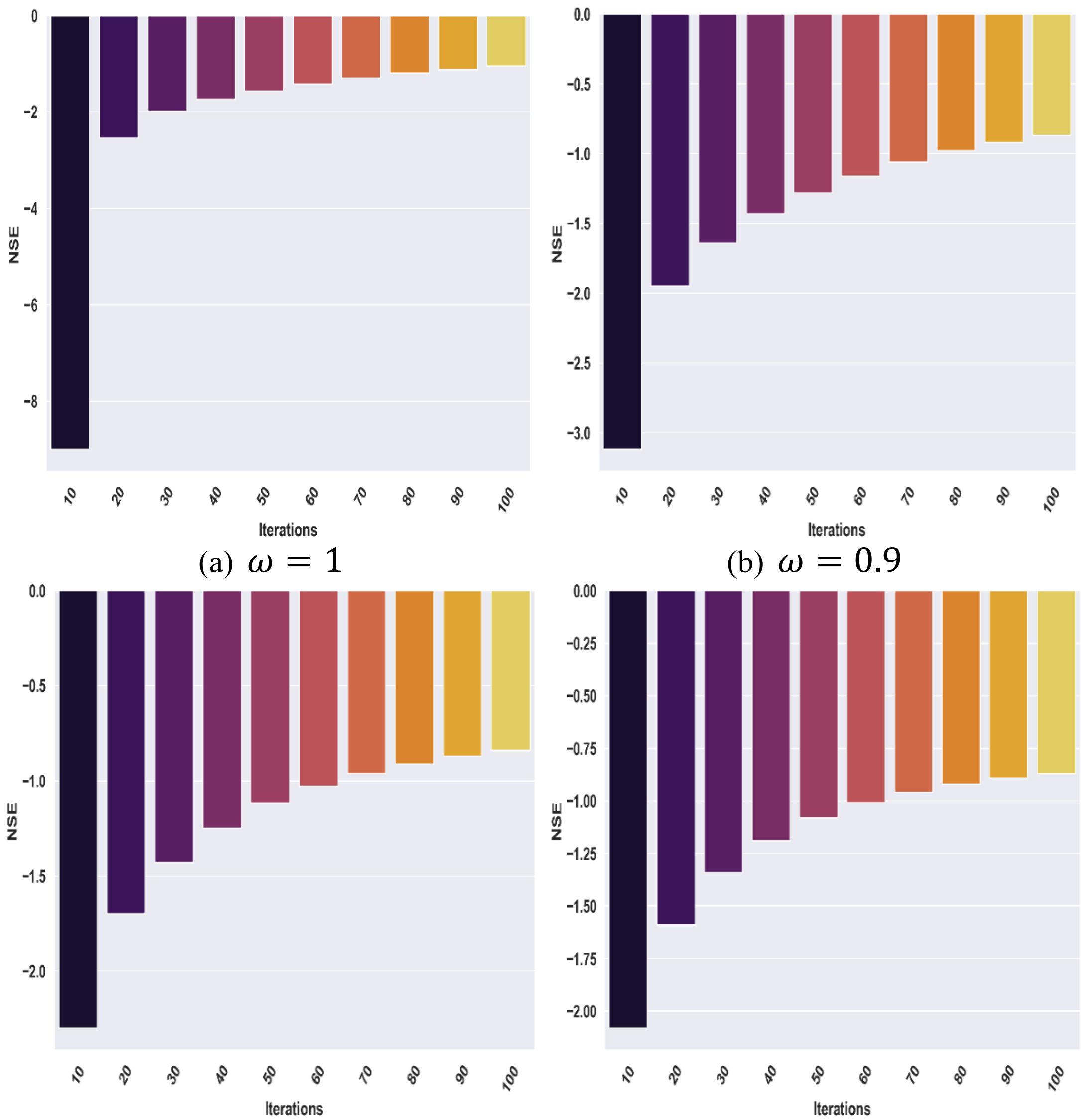


(c) $\omega 0 = .75$



(d) $\omega = 0.6$

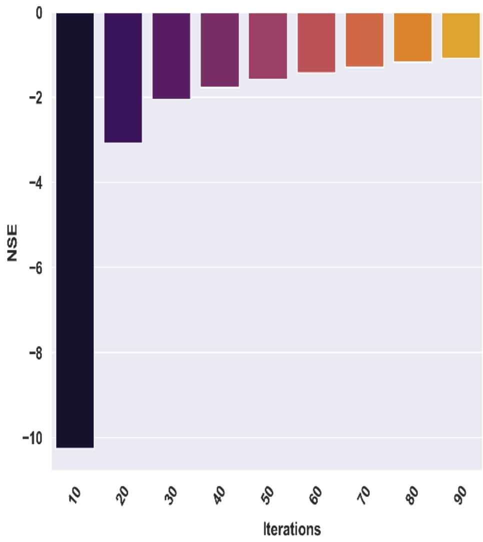
Figure 14: NSE-based Investigation of AF-SGD for four fractional orders variations using latent dimension of m = 5 with Film Trust dataset.



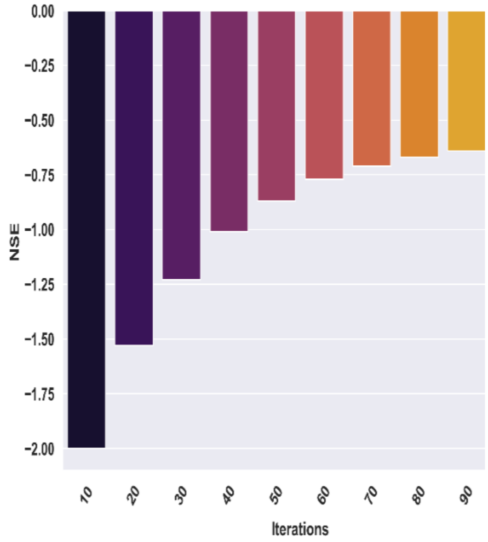
(a) $\omega = 1$ (b) $\omega = 0.9$

(c) $\omega = 0.75$ (d) $\omega = 0.6$

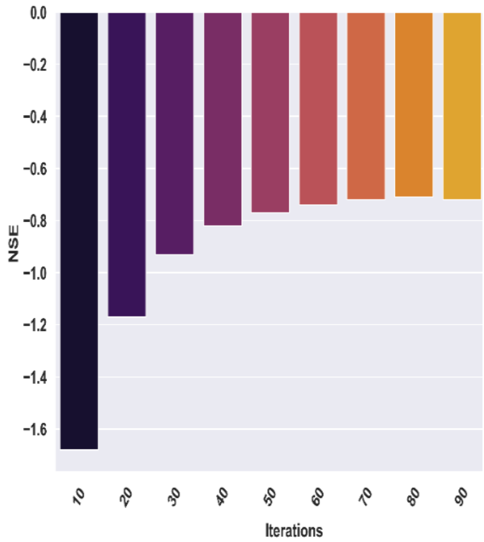
Figure 15: NSE-based Investigation of AF-SGD for four fractional orders variations using latent dimension of m = 15 with Film Trust dataset.



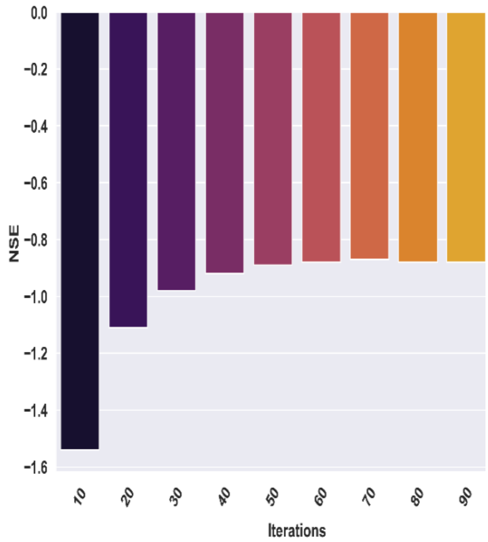
(a) $\omega = 1$



(b) $\omega = 0.9$



(c) $\omega = 0.75$

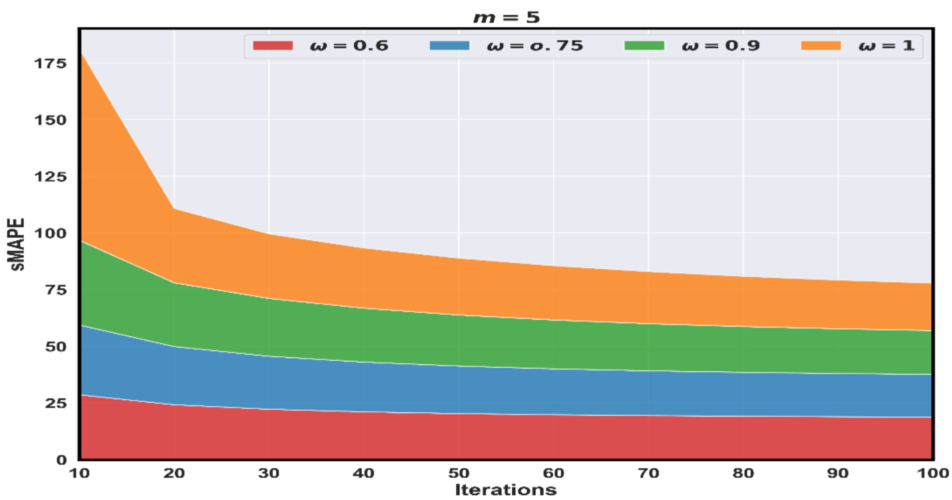


(d) $\omega = 0.6$

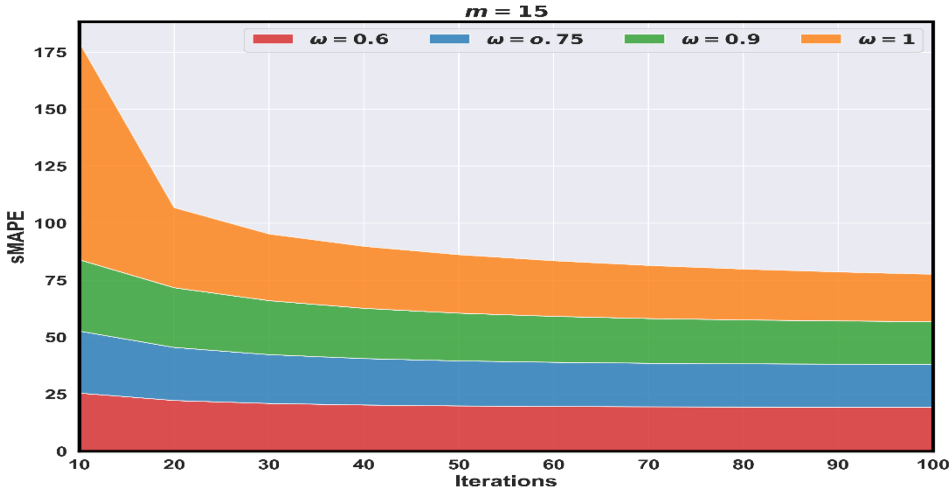
Figure 16: NSE-based Investigation of AFSGD for four fractional orders variations using latent dimension of m = 30 with Film Trust dataset.

Table 8: Performance comparison of the suggested AF-SGD with regard to NSE against SGD (AF-SGD for ω=1) by utilizing Film Trust dataset.

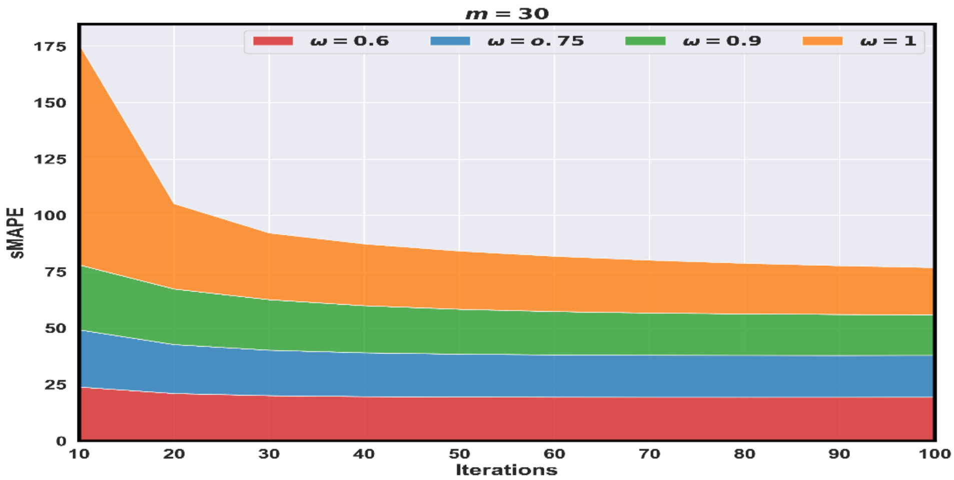
Dataset	Metric	Method	m	Iterations	ω 0.6	0.75	0.9	1
Film Trust	**NSE**	**AF-SGD**	**5**	20	-1.59	-1.7	-1.95	-2.54
				40	-1.19	-1.25	-1.43	-1.74
				60	-1.01	-1.03	-1.16	-1.42
				80	-0.92	-0.91	-0.98	-1.2
				100	-0.87	-0.84	-0.87	-1.05
			15	20	-1.29	-1.37	-1.67	-2.76
				40	-1.01	-0.97	-1.17	-1.75
				60	-0.92	-0.83	-0.92	-1.41
				80	-0.89	-0.77	-0.8	-1.17
				100	-0.88	-0.76	-0.73	-0.97
			30	20	-1.11	-1.17	-1.53	-3.08
				40	-0.92	-0.82	-1.01	-1.77
				60	-0.88	-0.74	-0.77	-1.43
				80	-0.88	-0.71	-0.67	-1.18
				100	-0.89	-0.72	-0.62	-0.98



(a)



(b)



(c)

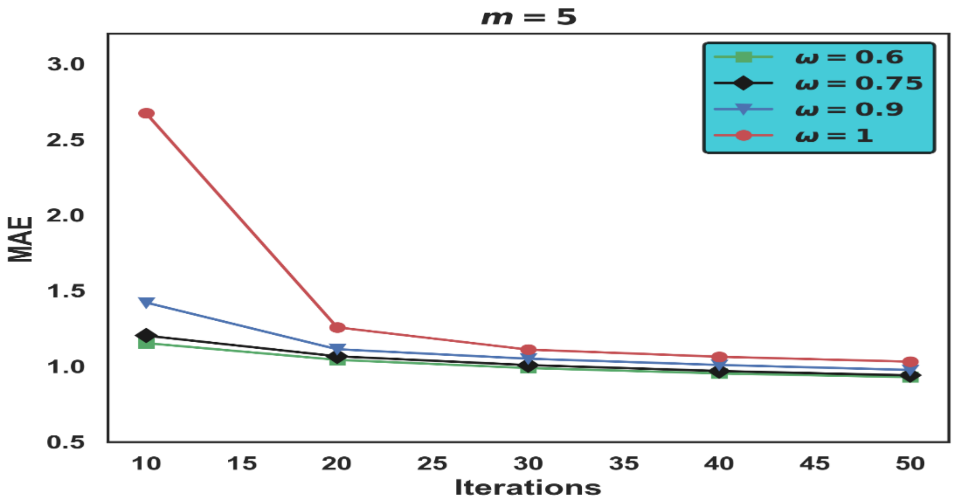
Figure 17: Performance comparison with standard fractional-SGD and momentum-SGD for Film Trust

Table 9: Performance comparison of the suggested AF-SGD with regard to sMAPE against SGD (AF-SGD for ω=1) by utilizing Film Trust dataset.

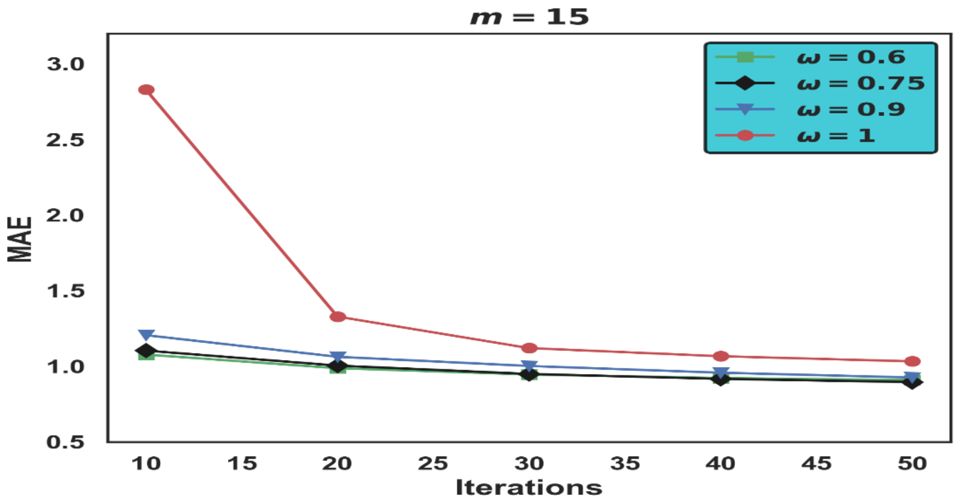
Dataset	Metric	Method	*m*	Iterations	*ω* 0.6	0.75	0.9	1
Film Trust	**sMAPE**	**AF-SGD**	**5**	20	24.31	25.66	28.12	32.82
				40	21.19	21.94	23.78	26.56
				60	19.91	20.26	21.54	23.94
				80	19.26	19.36	20.19	22.17
				100	18.9	18.82	19.34	20.91
			15	20	22.32	23.25	26.19	35.13
				40	20.32	20.38	22.03	27.19
				60	19.69	19.36	20.17	24.39
				80	19.46	18.94	19.23	22.32
				100	19.39	18.77	18.74	20.78
			30	20	21.06	21.7	24.65	37.83
				40	19.69	19.42	20.9	27.39
				60	19.41	18.79	19.21	24.53
				80	19.36	18.58	18.42	22.46
				100	19.4	18.55	18.05	20.9

Table 10: Performance comparison of the suggested AF-SGD regarding MAE with conventional SGD (AF-SGD for ω=1) for Film Trust dataset

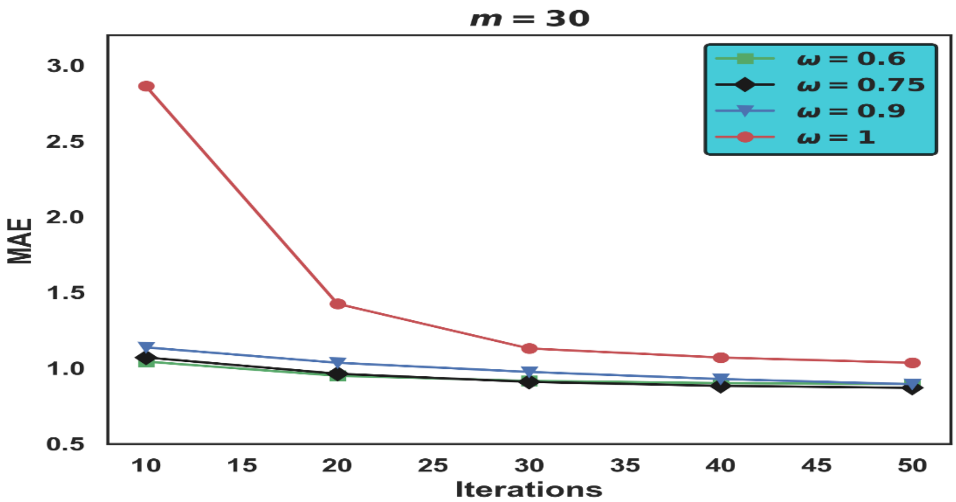
Dataset	Metric	Method	*m*	Iterations	*ω* 0.6	0.75	0.9	1
Film Trust.	**MAE**	**AF-SGD**	**5**	20	1.04	1.06	1.11	1.26
				40	0.95	0.97	1.01	1.06
				60	0.91	0.92	0.95	1.00
				80	0.89	0.89	0.91	0.96
				100	0.88	0.87	0.88	0.92
			15	20	0.99	1.00	1.06	1.33
				40	0.92	0.92	0.96	1.07
				60	0.90	0.88	0.90	1.00
				80	0.89	0.87	0.87	0.95
				100	0.89	0.86	0.86	0.91
			30	20	0.95	0.96	1.03	1.42
				40	0.90	0.88	0.93	1.07
				60	0.89	0.86	0.87	1.01
				80	0.89	0.86	0.84	0.96
				100	0.89	0.86	0.83	0.91



(a)



(b)



(c)

Figure 18: Performance comparison with AF-SGD and momentum-SGD for Film Trust

Table 11: Performance comparison of the suggested AF-SGD regarding RMSE with conventional SGD (AF-SGD for ω=1) for Film Trust dataset.

Dataset	Metric	Method	*m*	Iterations	*ω* 0.6	0.75	0.9	1
Film Trust.	RMSE	AF-SGD	5	20	1.44	1.47	1.54	1.68
				40	1.32	1.34	1.39	1.48
				60	1.27	1.27	1.31	1.39
				80	1.24	1.23	1.26	1.33
				100	1.22	1.21	1.22	1.28
			15	20	1.35	1.38	1.46	1.73
				40	1.27	1.25	1.32	1.48
				60	1.24	1.21	1.24	1.39
				80	1.23	1.19	1.20	1.32
				100	1.23	1.18	1.18	1.25
			30	20	1.30	1.32	1.42	1.81
				40	1.24	1.21	1.27	1.49
				60	1.22	1.18	1.19	1.39
				80	1.22	1.17	1.15	1.32
				100	1.23	1.17	1.14	1.26

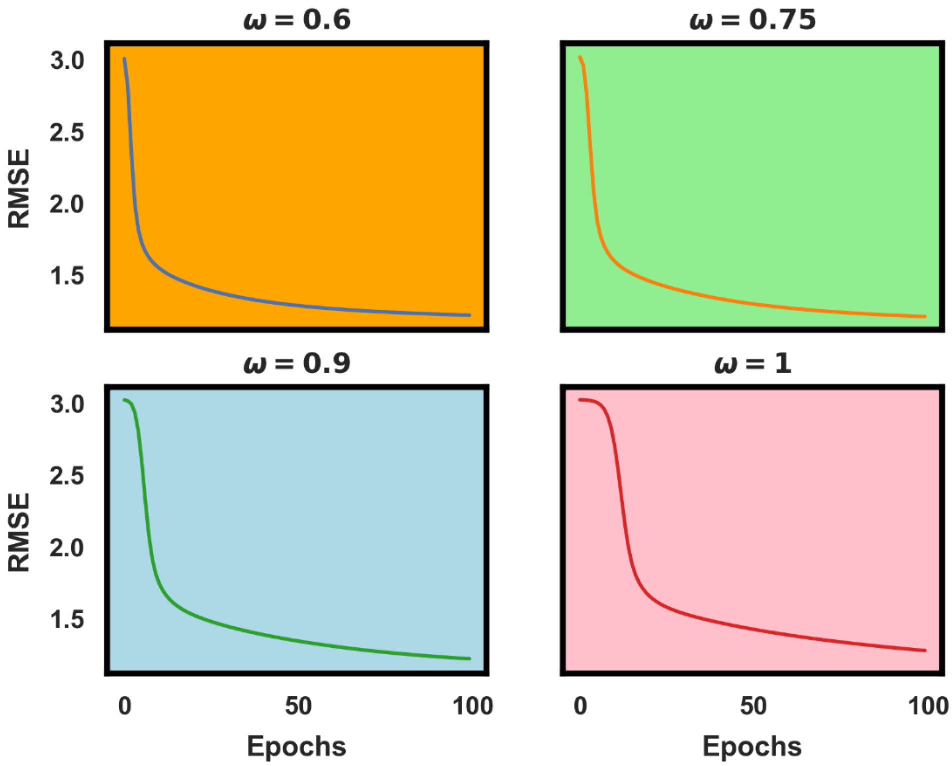


Figure 19: RMSE-based Investigation of AF-SGD for four fractional orders variations using latent dimension of m = 05 with Film Trust dataset.

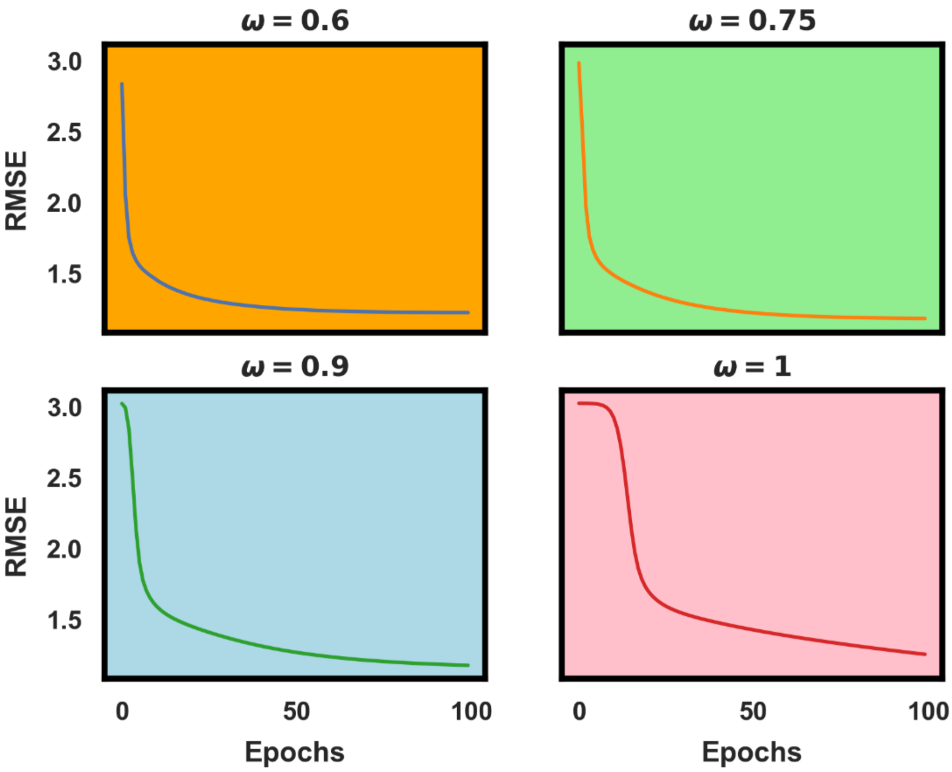


Figure 20: RMSE-based Investigation of AF-SGD for four fractional orders variations using latent dimension of m = 15 with Film Trust dataset

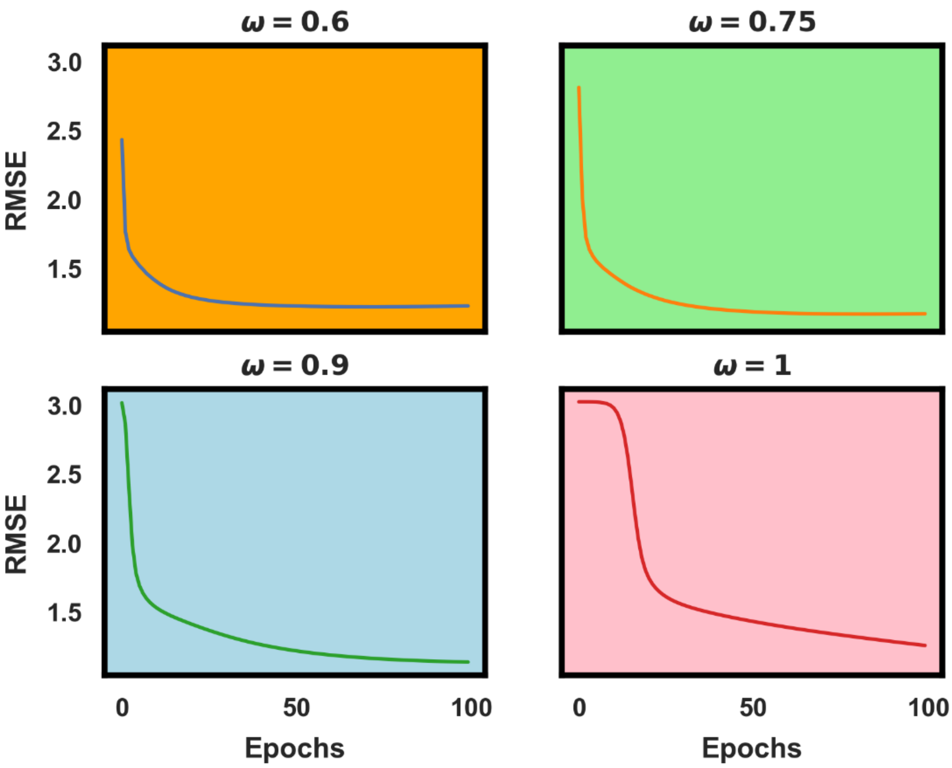


Figure 21: RMSE-based Investigation of AF-SGD for four fractional orders variations using latent dimension of m = 30 with Film Trust dataset.

4.4 Summary

The simulation environment tunning and case studies are discussed briefly. In next chapter the conclusion and future works will be explained.

CHAPTER 5

CONCLUSIONS AND FUTURE WORK

5.1 Introduction

This chapter presents conclusions deduced from the suggested matrix factorization and the standard SGD method for recommender system discussed in the previous chapters. Apart from conclusions, this chapter also includes guidelines for scholars interested in doing future research by applying proposed methods or different variations of suggested methods in different fields.

5.2 Conclusions

The conclusion of results for this learning are as under.

- A latent factor based enhanced MF model (AF-SGD) is designed for effectively dealing the disorder present in the chaotic users' feedback by providing fast and accurate recommendations.
- The predictability and the percentage performance of the proposed AF-SGD as compared to the standard SGD ($\omega = 1$) in terms of a reaching NSE closer to 1 and sMAPE respectively improves for all fractional order values.
- AF-SGD converges faster than the standard SGD ($\omega = 1$) for three fractional order (ω) values and latent features (m). Moreover, AF-SGD also achieved higher accuracy after 100 iterations for ω and m variations.
- The superior performance of AF-SGD over standard SGD for two benchmark datasets, various fractional order values and multiple latent features confirmed that the proposed approach of applying matrix factorization with AF-SGD is scalable, adaptable, accurate, and highly efficient for the reliable and speedy recommendation tasks.

5.3 Future Work

- The dynamic way of this work is to merge fractional base idea for MF with deep machine learning method like denoising auto encoder etc..
- This concept (suggested method) can also redesign using Accelerated fractional base concept (AF-SGD) for precise estimation and fast matrices factorization by recommender system.
- One of the future directions of our suggested work is the application of suggested model Accelerated fractional for implicit as well as explicit response for positioning and assessment estimation using RS.
- Additional potential research way for suggested model will be explore the Accelerated fractional SGD with neighborhood (memory-based) technique to resolve the problem of recommender system.

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