On Lattice Ordered Intuitionistic Fuzzy Soft Sets

By

Muhammad Aamir Malik

Reg#276-FBAS/MSMA/F16

Department of Mathematics & Statistics
Faculty of Basic & Applied Sciences
International Islamic University Islamabad
Pakistan

2017

Set theory
Fuzzy sets

On Lattice Ordered Intuitionistic Fuzzy Soft Sets

By

Muhammad Aamir Malik

Reg#276-FBAS/MSMA/F16

Supervised By

Dr. Tahir Mahmood

Department of Mathematics & Statistics
Faculty of Basic & Applied Sciences
International Islamic University Islamabad
Pakistan

2017

On Lattice Ordered Intuitionistic Fuzzy Soft Sets

By

Muhammad Aamir Malik

A thesis Submitted in partial fulfillment of the requirements for the degree of ***Master of Science (MS) in Mathematics*** at the Department of Mathematics & Statistics, Faculty of Basic & Applied Sciences, International Islamic University Islamabad, Pakistan.

Supervised By

Dr. Tahir Mahmood

Department of Mathematics & Statistics
Faculty of Basic & Applied Sciences
International Islamic University Islamabad
Pakistan.

2017

Certificate

On Lattice Ordered Intuitionistic Fuzzy Soft Sets

By

Muhammad Aamir Malik

A DISSERTATION SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF THE MASTER OF SCIENCE IN STATISTICS

We accept this dissertation as conforming to the required standard.

1. ______________________
Dr. Saqib Hussain
External Examiner

2. ______________________
Prof. Dr. M. Arshad Zia
Internal Examiner

3. ______________________
Dr. Tahir Mahmood
Supervisor

4. ______________________
Prof. Dr. Muhammad Sajid, T.I
Chairman

Department of Mathematics & Statistics
Faculty of Basic and Applied Sciences
International Islamic University, Islamabad
Pakistan

2017

DEDICATION

This work is dedicated

To

My beloved Father Malik Muhammad Yamin Awan, Valued Teacher Dr. Tahir Mahmood for Supporting and encouraging me.

DECLARATION

I hereby declare that this thesis neither as a whole nor as a part thereof has been copied out from any source. It is further declare that I have prepared this thesis entirely on the basis of my personal efforts made under the sincere guidness of my supervisor.

No portion of the work presented in this thesis has been submitted in support of an application for any degree or qualification of this or other institute of learning.

Muhammad Aamir Malik

Reg#276-FBAS/MSMA/F16

MS-Mathematics

Department of Mathematics & Statistics

Faculty of Basic & Applied Sciences

International Islamic University Islamabad, Pakistan.

ACKNOWLEDGEMENTS

I would like to express my deep gratitude to my supervisor, Dr. Tahir Mahmood. I have learned many things since I became Dr. Tahir Mahmood's student. He spends vary much time instructing me how to write a paper, how to search literature and how to collect data. I am also grateful to Dr. Muhammad Irfan Ali for spending time to read this thesis and providing useful suggestions about this thesis. They are all hard-working professors and I believe their academic achievements will continue to increase. Special thanks are given to Department of Mathematic & Statistics.

During the period of two years, many friends are helpful to color my life. I have to acknowledge all my colleagues in research room. Nevertheless, it is lucky for me to meet some friends who inspirit my effort to overcome my difficulties. These friends are Shahzaib Ashraf and Waseem Ahmed.

I owe more than thanks to my family members which includes my parents, wife, sisters and brothers, for their support and encouragement throughout my life. Without their support, it is impossible for me to finish my college and graduate education seamlessly.

Muhammad Aamir Malik

Reg#276-FBAS/MSMA/F16

MS-Mathematics

Department of Mathematics & Statistics

Faculty of Basic & Applied Sciences

International Islamic University Islamabad, Pakistan.

Introduction

In our real life, lots of models and traditional methods, are being used to handle uncertainty and imprecision. Many traditional areas such as medical, social and physical sciences, which utilized the imprecision and uncertainty for recognizing the data, these methods and tools are used extensively. To handle such situations, different theories and ideas are being presented these days. In 1965 L. A. Zadeh [28] introduced the concept of fuzzy sets that is applicable in many situations. In addition to this several theories are also introduced including Rough sets [22], Intuitionistic fuzzy sets [8, 9].

Molodtsov [21], in 1999, presented the concept of soft set theory to deal data involving uncertainties. Since then soft sets [21] attain keen attention for researchers and are extensively spotlight both in the theory of logic and in the theory of algebraic structures. Maji [19] introduced the concept of operations in soft sets and further studied soft set theory and used this theory to solve some decision making problems [16]. Aktas and Cagman [1] initiated the study of soft sets in algebraic structures and defined soft groups. The concept of soft topology is discussed in [13, 25]. The concept of soft fuzzy sets is developed and discussed in [17], of intuitionistic fuzzy soft sets in [2, 18, 26] and the concept of fuzzy parameterized interval valued fuzzy soft sets is developed by Alkhazaleh [5]. Jiang [14] extended the applications of soft sets in description logic. Roy et al. [24] presented a fuzzy soft sets theoretic approach towards decision making problems. In [4] the concept of lattice ordered soft sets is discussed and in [7] the concept of lattice ordered fuzzy soft sets is discussed. The applications of soft sets are also used in [3, 6, 11, 12, 15, 23, 27].

In this thesis, we define extended union, extended intersection, basic union, basic intersection, $\vee$-Product and $\wedge$-Product on intuitionistic fuzzy soft sets. Further we define lattice ordered intuitionistic fuzzy soft set. Some operations like restricted union, restricted intersection, extended union, basic union, basic intersection, complement, $\vee$-Product and $\wedge$-Product on lattice ordered intuitionistic fuzzy soft sets are also defined. In the last section a Multi attribute decision making method based on lattice ordered intuitionistic fuzzy soft sets is used to solve a daily life problem.

CONTENTS

Chapter 1

Preliminaries

Chapter 2

Intuitionistic Fuzzy Soft Set Theory

Chapter 3

Lattice Ordered Intuitionistic Fuzzy Soft Set

CHAPTER I

PRELIMINARIES

This chapter is of introductory nature including some basic definitions and results, which will help us in the subsequent chapters. In this chapter we discuss lattices, fuzzy sets, intuitionistic fuzzy sets, soft sets, soft fuzzy sets, lattice ordered soft sets and lattice ordered fuzzy soft sets.

1.1 Lattices

In this section we discuss partial order, upper and lower bound, supremum, infimum and lattice. For undefined term and notion we refer to [10].

1.1.1 Definition

$L_\varkappa \neq \emptyset$ is said to be partially ordered (PO), if for binary relation $\leq$ defined on $L_\varkappa$ the following conditions hold.

(1) For all $l \in L_\varkappa$, $l \leq l$,

(2) for $l_1, l_2 \in L_\varkappa$, if $l_1 \leq l_2$ and $l_2 \leq l_1$, then $l_1 = l_2$,

(3) for $l_1, l_2, l_3 \in L_\varkappa$, if $l_1 \leq l_2$ and $l_2 \leq l_3$, then $l_1 \leq l_3$.

1.1.2 Example

Let $P(L_\varkappa)$ be the collection of all possible subsets of $L_\varkappa$. Then "$\subseteq$" is a PO on $P(L_\varkappa)$.

1.1.3 Definition

Let $L_\varkappa$ be a PO and $L_{\varkappa_1} \subseteq L_\varkappa$. Then an element $j \in L_\varkappa$ is said to be

(1) lower bound of $L_{\varkappa_1}$ iff $\forall\, m \in L_{\varkappa_1}$, $j \leq m$,

(2) upper bound of $L_{\varkappa_1}$ iff $\forall\, m \in L_{\varkappa_1}$, $m \leq j$.

1.1.4 Definition

Let $L_\varkappa$ be a PO and $L_{\varkappa_1} \subseteq L_\varkappa$. Then

(1) A lower bound α of $L_{\varkappa_1}$ is said to be greatest lower bound or infimum of $L_{\varkappa_1}$ iff $\forall$ lower bounds j of $L_{\varkappa_1}$, $j \leq \alpha$. Then we write $\alpha = glb(L_{\varkappa_1})$ or $\alpha = \inf(L_{\varkappa_1})$.

(2) An upper bound β of $L_{\varkappa_1}$ is said to be least upper bound or supremum of $L_{\varkappa_1}$ iff for every other upper bounds j of $L_{\varkappa_1}$, $\beta \leq j$. Then we write $\beta = lub(L_{\varkappa_1})$ or $\beta = \sup(L_{\varkappa_1})$.

1.1.5 Definition[10]

Let $\leq$ be a PO on $L_\varkappa$. The pair $L_\varkappa = (L_\varkappa, \leq)$ is a lattice if $\forall\ l_1, l_2 \in L_\varkappa$ the set $\{l_1, l_2\}$ has supremum and an infimum in $L_\varkappa$, if $\exists\ 0$ and $1 \in L_\varkappa$ s.t $0 \leq j$ and $j \leq 1$, $\forall\ j \in L_\varkappa$. Then $L_\varkappa$ is called a bounded lattice.

1.1.6 Theorem

Let $L_\varkappa \neq \emptyset$ and $\wedge$ and $\vee$ be two binary operations on $L_\varkappa$, Then $L_\varkappa = (L_\varkappa, \wedge, \vee)$ is a lattice iff for each l_1, l_2, $l_3 \in L_\varkappa$ the following hold:

(1) $l_1 \wedge l_2 = l_2 \wedge l_1$ and $l_1 \vee l_2 = l_2 \vee l_1$

(2) $(l_1 \wedge l_2) \wedge l_3 = l_1 \wedge (l_2 \wedge l_3)$ and $(l_1 \vee l_2) \vee l_3 = l_1 \vee (l_2 \vee l_3)$

(3) $l_1 \wedge l_1 = l_1$ and $l_1 \vee l_1 = l_1$

(4) $l_1 \wedge (l_1 \vee l_2) = l_1$ and $l_1 \vee (l_1 \wedge l_2) = l_1$.

1.1.7 Example

(1) Suppose $L_\varkappa$ is a set of positive integer and $\vee$ denotes the least common multiple and $\wedge$ denotes greatest common divisor. Then $L_\varkappa$ is lattice.

(2) For any non-empty set W, $(P(W), \cap, \cup)$ is a bounded lattice.

1.2 Intuitionistic Fuzzy Set

1.2.1 Definition[28]

Suppose $X_\varkappa \neq \emptyset$. Then $A_\varkappa = \{(\varkappa, f(\varkappa)) | \varkappa \in X_\varkappa \wedge f : X_\varkappa \to [0,1]\}$.

is known to be a fuzzy set on $X_\varkappa$, when f is known to be membership function. According to some authors in this case f is called fuzzy subset of $X_\varkappa$. $FP(X_\varkappa)$ represents the collection of all fuzzy subsets of $X_\varkappa$.

1.2.2 Definition

Suppose f and g be any two fuzzy subsets of a non empty set $X_\varkappa$. Then

$f \subseteq g$, if $f(\varkappa) \leq g(\varkappa)$, for each $\varkappa \in X_\varkappa$.

1.2.3 Definition[8]

An intuitionistic fuzzy set F_ζ in $X_\varkappa$ is defined as:

$$F_\zeta = \{(T_{F_\zeta}(\varkappa), K_{F_\zeta}(\varkappa)) : \varkappa \in X_\varkappa\}$$

where $T_{F_\zeta} : X_\varkappa \rightarrow [0,1]$ and $K_{F_\zeta} : X_\varkappa \rightarrow [0,1]$ define the degree of membership and degree of non membership of the element $\varkappa \in X_\varkappa$ and $0 \leq T_{F_\zeta}(\varkappa) + K_{F_\zeta}(\varkappa) \leq 1$. For notation, collection of all intuitionistic fuzzy sets over $X_\varkappa$ is represented by $IFS(X_\varkappa)$.

1.2.4 Theorem

Consider F_ζ, $G_\zeta \in IFS(X_\varkappa)$. Then

- $F_\zeta \subseteq G_\zeta \Leftrightarrow T_{F_\zeta}(\varkappa) \leq T_{G_\zeta}(\varkappa)$, $K_{F_\zeta}(\varkappa) \geq K_{G_\zeta}(\varkappa) : \varkappa \in X_\varkappa$
- $F_\zeta \cap G_\zeta = \{\varkappa, \min\{T_{F_\zeta}(\varkappa), T_{G_\zeta}(\varkappa)\}, \max\{K_{F_\zeta}(\varkappa), K_{G_\zeta}(\varkappa)\} : \varkappa \in X_\varkappa\}$
- $F_\zeta \cap G_\zeta = \{\varkappa, \max\{T_{F_\zeta}(\varkappa), T_{G_\zeta}(\varkappa)\}, \min\{K_{F_\zeta}(\varkappa), K_{G_\zeta}(\varkappa)\} : \varkappa \in X_\varkappa\}$
- $F_\zeta^c = \{K_{F_\zeta}(\varkappa), T_{F_\zeta}(\varkappa) : \varkappa \in X_\varkappa\}$

1.3 Soft Set

In this section we define soft sets(SSs), soft subsets, soft equal, relative null SSs, relative whole SSs, restricted union of two SSs, restricted intersection of two SSs, extended union of two SSs, extended intersection of two SSs, basic union of two SSs, basic intersection of two SSs and complement of a SSs.

1.3.1 Definition[21]

Suppose $X_\varkappa$ is an objects collection represented as $\varkappa$ and $L_\varkappa \subseteq E_\varkappa$ is a set of parameters. Then $(F_\zeta, L_\varkappa)$ is known to be a soft set over $X_\varkappa$, where mapping F_ζ defined as $F_\zeta : L_\varkappa \rightarrow P(X_\varkappa)$. In further notation for soft set over $X_\varkappa$, denotes as $SS(X_\varkappa)$.

1.3.2 Definition[18]

For two $SS(X_\varkappa)$, $(F_\zeta, L_\varkappa) \subseteq (G_\zeta, O_\varkappa)$, if it gratify the conditions.

(1) $L_\varkappa \subseteq O_\varkappa$

(2) $F_\zeta(\varrho) = G_\zeta(\varrho) \; \forall \; \varrho \in L_\varkappa$.

$(F_\zeta, L_\varkappa)$ is said to be a soft super set of $(G_\zeta, O_\varkappa)$, if $(G_\zeta, O_\varkappa)$ is soft subset of $(F_\zeta, L_\varkappa)$. We denote it by $(F_\zeta, L_\varkappa) \supseteq (G_\zeta, O_\varkappa)$.

1.3.3 Definition[3]

Suppose universe set is $X_\varkappa$ and $L_\varkappa \subseteq E_\varkappa$ is set of parameters. Then

(1) $(F_\zeta, L_\varkappa)$ is known to be relative null soft set. if $F_\zeta(l) = \emptyset \;\; \forall \; l \in L_\varkappa$, For notation, relative null soft set represented by $\emptyset_{L_\varkappa}$.

(2) $(F_\zeta, L_\varkappa)$ is known to be relative whole soft set, if $F_\zeta(l) = X_\varkappa \; \forall \; l \in L_\varkappa$. For notation, relative whole soft set represented by $U_{L_\varkappa}$.

1.3.4 Definition[3]

Extended union of two soft sets $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$ over a common universe $X_\varkappa$ is a soft set $(H_\zeta, P_\varkappa)$, where $P_\varkappa = L_\varkappa \cup O_\varkappa$ and for all $\varrho \in P_\varkappa$.

$$H_\zeta(\varrho) = \begin{cases} F_\zeta(\varrho) & \text{if } \varrho \in L_\varkappa - O_\varkappa \\ G_\zeta(\varrho) & \text{if } \varrho \in L_\varkappa - O_\varkappa \\ F_\zeta(\varrho) \cup G_\zeta(\varrho) & \text{if } \varrho \in L_\varkappa \cap O_\varkappa \end{cases}$$

We write $(F_\zeta, L_\varkappa) U_{EXT} (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$.

1.3.5 Definition[3]

Extended intersection of two soft sets $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$ over a common universe $X_\varkappa$ is a soft set $(H_\zeta, P_\varkappa)$, where $P_\varkappa = L_\varkappa \cup O_\varkappa$ and for all $\varrho \in P_\varkappa$.

$$H_\zeta(\varrho) = \begin{cases} F_\zeta(\varrho) & \text{if } \varrho \in L_\varkappa - O_\varkappa \\ G_\zeta(\varrho) & \text{if } \varrho \in L_\varkappa - O_\varkappa \\ F_\zeta(\varrho) \cap G_\zeta(\varrho) & \text{if } \varrho \in L_\varkappa \cap O_\varkappa \end{cases}$$

We write $(F_\zeta, L_\varkappa) \cap_{EXT} (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$.

1.3.6 Definition[3]

Let $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$ be two soft sets over a same universe $X_\varkappa$, such that $L_\varkappa \cap O_\varkappa \neq \emptyset$. Then restricted union of $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$ is denoted by $(F_\zeta, L_\varkappa)U_{RES}(G_\zeta, O_\varkappa)$ and is defined by $(F_\zeta, L_\varkappa)U_{RES}(G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$, where $P_\varkappa = L_\varkappa \cap O_\varkappa$ and for all $\varrho \in P_\varkappa$, $H_\zeta(\varrho) = F_\zeta(\varrho) \cup G_\zeta(\varrho)$

if $L_\varkappa \cap O_\varkappa = \emptyset$, then $(F_\zeta, L_\varkappa) \cup_{RES} (G_\zeta, O_\varkappa) = \emptyset$.

1.3.7 Definition[3]

Let $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$ be two soft sets over a same universe $X_\varkappa$, such that $L_\varkappa \cap O_\varkappa \neq \emptyset$. Then restricted intersection of $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$ is denoted by $(F_\zeta, L_\varkappa) \cap_{RES} (G_\zeta, O_\varkappa)$ and is defined by $(F_\zeta, L_\varkappa) \cap_{RES} (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$, where $P_\varkappa = L_\varkappa \cap O_\varkappa$ and for all $\varrho \in P_\varkappa$, $H_\zeta(\varrho) = F_\zeta(\varrho) \cap G_\zeta(\varrho)$

if $L_\varkappa \cap O_\varkappa = \emptyset$, then $(F_\zeta, L_\varkappa) \cap_{RES} (G_\zeta, O_\varkappa) = \emptyset$.

1.3.8 Definition[23]

Let $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$ be two soft sets over a common universe $X_\varkappa$. Then the basic union of $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$ is denoted by $(F_\zeta, L_\varkappa) \vee (G_\zeta, O_\varkappa)$ and is defined as $(H_\zeta, P_\varkappa) = (F_\zeta, L_\varkappa) \vee (G_\zeta, O_\varkappa)$, where $P_\varkappa = L_\varkappa \times O_\varkappa$ and $H_\zeta(l, o) = F_\zeta(l) \cup G_\zeta(o)$ for all $(l, o) \in L_\varkappa \times O_\varkappa$.

1.3.9 Definition[23]

Let $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$ be two soft sets over the common universe $X_\varkappa$. Then the basic intersection of $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$ is denoted by $(F_\zeta, L_\varkappa) \wedge (G_\zeta, O_\varkappa)$ and is defined as $(H_\zeta, P_\varkappa) = (F_\zeta, L_\varkappa) \wedge (G_\zeta, O_\varkappa)$, where $P_\varkappa = L_\varkappa \times O_\varkappa$ and $H_\zeta(l, o) = F_\zeta(l) \cap G_\zeta(o)$ for all $(l, o) \in L_\varkappa \times O_\varkappa$.

1.3.10 Definition[20]

Suppose parameter set is $L_\varkappa$, $O_\varkappa \subseteq E_\varkappa$.Then conjunction and disjunction parameters are denoted and defined as.

$L_\varkappa \otimes O_\varkappa = \{(l \wedge o) : (l, o) \in L_\varkappa \times O_\varkappa\}$

$L_\varkappa \oplus O_\varkappa = \{(l \vee o) : (l, o) \in L_\varkappa \times O_\varkappa\}$.

1.3.11 Definition[20]

Let $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$ be two soft sets over a common universe $X_\varkappa$. Then

(1) $(F_\zeta, L_\varkappa) \cap_\wedge (G_\zeta, O_\varkappa) = (H_\zeta, L_\varkappa \otimes O_\varkappa)$ is the soft set defined as $H_\zeta(l \wedge o) = F_\zeta(l) \cap G_\zeta(o)$ for all $l \wedge o \in L_\varkappa \otimes O_\varkappa$.

(2) $(F_\zeta, L_\varkappa) \cup_\vee (G_\zeta, O_\varkappa) = (H_\zeta, L_\varkappa \oplus O_\varkappa)$ is the soft set defined as $H_\zeta(l \vee o) = F_\zeta(l) \cup G_\zeta(o)$ for all $l \vee o \in L_\varkappa \oplus O_\varkappa$.

1.4 Lattice Ordered Fuzzy Soft Set

In this section we define lattice (anti lattice) ordered soft set, fuzzy soft set, some results on lattice (anti lattice) ordered fuzzy soft set also investigated.

1.4.1 Definition[4]

$(F_\zeta, L_\varkappa) \in (SS(X_\varkappa))$ is called a lattice (anti lattice) ordered soft set over $X_\varkappa$ $(LOSS(X_\varkappa))$ if for mapping $F_\zeta : L_\varkappa \to P(X_\varkappa)$,

if $\varrho_1 \leq \varrho_2$, then $F_\zeta(\varrho_1) \subseteq F_\zeta(\varrho_2)$ $\quad (F_\zeta(\varrho_2) \subseteq F_\zeta(\varrho_1))$ $\forall\, \varrho_1, \varrho_2 \in L_\varkappa$.

1.4.2 Example

Suppose $P = \{p_1, p_2, p_3, p_4\}$ is a set of four stores and $L_\varkappa = \{\varrho_1, \varrho_2, \varrho_3, \varrho_4\} \subseteq E_\varkappa$,where

ϱ_1 = Large store

ϱ_2 = very large store

ϱ_3 = Huge store

ϱ_4 = Very Huge store.

Ordered can be describe as $\varrho_1 \leq \varrho_2 \leq \varrho_3 \leq \varrho_4$. $(F_\zeta, L_\varkappa) \in (SS(X_\varkappa))$ may be represented by $\{F_\zeta(\varrho_1) = \{p_2\},\ F_\zeta(\varrho_2) = \{p_1, p_2\},\ G_\zeta(\varrho_3) = \{p_1, p_2, p_3\},\ F_\zeta(\varrho_4) = \{p_1, p_2, p_3, p_4\}\}$. Soft set $(F_\zeta, L_\varkappa)$ in tabular form is shown in Table 1.1.

$(F_\zeta, L_\varkappa)$	ϱ_1	ϱ_2	ϱ_3	ϱ_4
p_1	0	1	1	1
p_2	1	1	1	1
p_3	0	0	1	1
p_4	0	0	0	1

Table 1.1

Clearly $F_\zeta(\varrho_1) \subseteq F_\zeta(\varrho_2) \subseteq F_\zeta(\varrho_3) \subseteq F_\zeta(\varrho_4)$. Thus $(F_\zeta, L_\varkappa)$ is an $LOSS(X_\varkappa)$.

1.4.3 Definition[27]

A pair $(\lambda_\zeta, L_\varkappa)$ is known to be fuzzy soft set over $X_\varkappa$, where λ_ζ is a mapping given by $\lambda_\zeta : L_\varkappa \to FP(X_\varkappa)$. For notation, collection of all fuzzy soft sets over $X_\varkappa$ is represented by $FSS(X_\varkappa)$.

1.4.4 Definition[7]

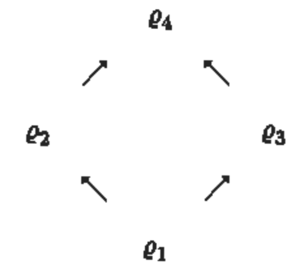
Let fuzzy soft set be $(\lambda_\zeta, L_\varkappa)$. Then it is known to be lattice (anti lattice) ordered fuzzy soft set over $X_\varkappa$, where λ_ζ is a mapping defined by $\lambda_\zeta : L_\varkappa \to FP(X_\varkappa)$. If

$\varrho_1 \leq \varrho_2$, then $\lambda_\zeta(\varrho_1) \subseteq \lambda_\zeta(\varrho_2)$ $(\lambda_\zeta(\varrho_2) \subseteq \lambda_\zeta(\varrho_1))$ $\forall$ $\varrho_1, \varrho_2 \in L_\varkappa$.

For notation, lattice ordered fuzzy soft sets over $X_\varkappa$ is represented by $LOFSS(X_\varkappa)$.

1.4.5 Example

Mr. Usama wants to enroll his grandson in a computer programme. He visited a few institutions for this purpose. Suppose $X_\varkappa = \{\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4\}$ is the set of institutions and $L_\varkappa = \{\varrho_1$(cheaper), ϱ_2(english medium), ϱ_3(highly qualified staff), ϱ_4(good environment)$\} \subseteq E_\varkappa$ is the parameters set and order is defined as.



Then a $LOFSS(X_\varkappa)$ given below.

$(\lambda_\zeta, L_\varkappa)$	ϱ_1	ϱ_2	ϱ_3	ϱ_4
$\varkappa_1$	0.1	0.1	0.2	0.5
$\varkappa_2$	0.3	0.4	0.4	0.6
$\varkappa_3$	0.4	0.5	0.6	0.7
$\varkappa_4$	0.5	0.6	0.7	0.8

Table 1.2

Here for $\varrho_1 \leq \varrho_2 \leq \varrho_4$ and $\varrho_1 \leq \varrho_3 \leq \varrho_4$ we have $\lambda_\zeta(\varrho_1) \subseteq \lambda_\zeta(\varrho_2) \subseteq \lambda_\zeta(\varrho_4)$ and $\lambda_\zeta(\varrho_1) \subseteq \lambda_\zeta(\varrho_3) \subseteq \lambda_\zeta(\varrho_4)$ respectively.

1.4.6 Definition[7]

Let $(\lambda_\zeta, L_\varkappa)$, $(\mu_\zeta, O_\varkappa) \in LOFSS(X_\varkappa)$. Then $(\lambda_\zeta, L_\varkappa) \cup_{EXT} (\mu_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$, where $P_\varkappa = L_\varkappa \cup O_\varkappa$ and for all $\varrho \in P_\varkappa$

$$H_\zeta(\varrho) = \begin{cases} \lambda_\zeta(\varrho) & \text{if } \varrho \in L_\varkappa - O_\varkappa \\ \mu_\zeta(\varrho) & \text{if } \varrho \in L_\varkappa - O_\varkappa \\ \lambda_\zeta(\varrho) \cup \mu_\zeta(\varrho) & \text{if } \varrho \in L_\varkappa \cap O_\varkappa \end{cases}$$

1.4.7 Definition[7]

Let $(\lambda_\zeta, L_\varkappa)$, $(\mu_\zeta, O_\varkappa) \in LOFSS(X_\varkappa)$. Then $(\lambda_\zeta, L_\varkappa) \cap_{EXT} (\mu_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$, where $P_\varkappa = L_\varkappa \cup O_\varkappa$ and for all $\varrho \in P_\varkappa$

$$H_\zeta(\varrho) = \begin{cases} \lambda_\zeta(\varrho) & \text{if } \varrho \in L_\varkappa - O_\varkappa \\ \mu_\zeta(\varrho) & \text{if } \varrho \in L_\varkappa - O_\varkappa \\ \lambda_\zeta(\varrho) \cap \mu_\zeta(\varrho) & \text{if } \varrho \in L_\varkappa \cap O_\varkappa \end{cases}$$

1.4.8 Definition[7]

Let $(\lambda_\zeta, L_\varkappa)$, $(\mu_\zeta, O_\varkappa) \in LOFSS(X_\varkappa)$. Then $(\lambda_\zeta, L_\varkappa) U_{RES} (\mu_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$, where $P_\varkappa = L_\varkappa \cap O_\varkappa$ and for all $\varrho \in P_\varkappa$, $H_\zeta(\varrho) = \lambda_\zeta(\varrho) \cup \mu_\zeta(\varrho)$

if $L_\varkappa \cap O_\varkappa = \emptyset$, then $(\lambda_\zeta, L_\varkappa) \cup_{RES} (\mu_\zeta, O_\varkappa) = \emptyset$.

1.4.9 Definition[7]

Let $(\lambda_\zeta, L_\varkappa)$, $(\mu_\zeta, O_\varkappa) \in LOFSS(X_\varkappa)$. Then $(\lambda_\zeta, L_\varkappa) \cap_{RES} (\mu_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$, where $P_\varkappa = L_\varkappa \cap O_\varkappa$ and for all $\varrho \in P_\varkappa$, $H_\zeta(\varrho) = \lambda_\zeta(\varrho) \cap \mu_\zeta(\varrho)$

if $L_\varkappa \cap O_\varkappa = \emptyset$, then $(\lambda_\zeta, L_\varkappa) \cap_{RES} (\mu_\zeta, O_\varkappa) = \emptyset$.

1.4.10 Definition[7]

Let $(\lambda_\zeta, L_\varkappa)$, $(\mu_\zeta, O_\varkappa) \in LOFSS(X_\varkappa)$. Then the basic union of $(\lambda_\zeta, L_\varkappa)$ and $(\mu_\zeta, O_\varkappa)$ is denoted by $(\lambda_\zeta, L_\varkappa) \vee (\mu_\zeta, O_\varkappa)$ and is defined as $(H_\zeta, P_\varkappa) = (\lambda_\zeta, L_\varkappa) \vee (\mu_\zeta, O_\varkappa)$, where $P_\varkappa = L_\varkappa \times O_\varkappa$, and $H_\zeta(l, o) = \lambda_\zeta(l) \cup \mu_\zeta(o)$ for all $(l, o) \in L_\varkappa \times O_\varkappa$.

1.4.11 Definition[7]

Let $(\lambda_\zeta, L_\varkappa)$, $(\mu_\zeta, O_\varkappa) \in LOFSS(X_\varkappa)$. Then the basic intersection of $(\lambda_\zeta, L_\varkappa)$ and $(\mu_\zeta, O_\varkappa)$ is denoted by $(\lambda_\zeta, L_\varkappa) \wedge (\mu_\zeta, O_\varkappa)$ and is defined as $(H_\zeta, P_\varkappa) = (\lambda_\zeta, L_\varkappa) \wedge (\mu_\zeta, O_\varkappa)$, where $P_\varkappa = L_\varkappa \times O_\varkappa$, and $H_\zeta(l, o) = \lambda_\zeta(l) \cap \mu_\zeta(o)$ for all $(l, o) \in L_\varkappa \times O_\varkappa$.

1.4.12 Definition[7]

Let $(\lambda_\zeta, L_\varkappa)$, $(\mu_\zeta, O_\varkappa) \in LOFSS(X_\varkappa)$. Then

(1) $(\lambda_\zeta, L_\varkappa) \cap_\wedge (\mu_\zeta, O_\varkappa) = (H_\zeta, L_\varkappa \otimes O_\varkappa)$ is the $LOFSS(X_\varkappa)$ defined as $H_\zeta(l \wedge o) = \lambda_\zeta(l) \cap \mu_\zeta(o)$ for all $l \wedge o \in L_\varkappa \otimes O_\varkappa$.

(2) $(\lambda_\zeta, L_\varkappa) \cup_\vee (\mu_\zeta, O_\varkappa) = (H_\zeta, L_\varkappa \oplus O_\varkappa)$ is the $LOFSS(X_\varkappa)$ defined as $H_\zeta(l \vee o) = \lambda_\zeta(l) \cup \mu_\zeta(o)$ for all $l \vee o \in L_\varkappa \oplus O_\varkappa$.

1.4.13 Proposition[7]

Suppose $(\lambda_\zeta, L_\varkappa)$, $(\mu_\zeta, O_\varkappa) \in LOFSS(X_\varkappa)$. Then $(\lambda_\zeta, L_\varkappa) \cup_{RES} (\mu_\zeta, O_\varkappa) \in LOFSS(X_\varkappa)$.

1.4.14 Proposition[7]

Suppose $(\lambda_\zeta, L_\varkappa)$, $(\mu_\zeta, O_\varkappa) \in LOFSS(X_\varkappa)$. Then $(\lambda_\zeta, L_\varkappa) \cap_{RES} (\mu_\zeta, O_\varkappa) \in LOFSS(X_\varkappa)$.

1.4.15 Proposition[7]

Suppose $(\lambda_\zeta, L_\varkappa)$, $(\mu_\zeta, O_\varkappa) \in LOFSS(X_\varkappa)$. Then $(\lambda_\zeta, L_\varkappa) \cup_{EXT} (\mu_\zeta, O_\varkappa) \in LOFSS(X_\varkappa)$, if $(\lambda_\zeta, L_\varkappa) \subseteq (\mu_\zeta, O_\varkappa)$ or $(\mu_\zeta, O_\varkappa) \subseteq (\lambda_\zeta, L_\varkappa)$.

1.4.16 Proposition[7]

Suppose $(\lambda_\zeta, L_\varkappa)$, $(\mu_\zeta, O_\varkappa) \in LOFSS(X_\varkappa)$. Then $(\lambda_\zeta, L_\varkappa) \cap_{EXT} (\mu_\zeta, O_\varkappa) \notin LOFSS(X_\varkappa)$.

1.4.17 Proposition[7]

Suppose $(\lambda_\zeta, L_\varkappa)$, $(\mu_\zeta, O_\varkappa) \in LOFSS(X_\varkappa)$. Then $(\lambda_\zeta, L_\varkappa) \vee (\mu_\zeta, O_\varkappa) \in LOFSS(X_\varkappa)$.

1.4.18 Proposition[7]

Suppose $(\lambda_\zeta, L_\varkappa)$, $(\mu_\zeta, O_\varkappa) \in LOFSS(X_\varkappa)$. Then $(\lambda_\zeta, L_\varkappa) \wedge (\mu_\zeta, O_\varkappa) \in LOFSS(X_\varkappa)$.

CHAPTER II

INTUITIONISTIC FUZZY SOFT SET THEORY

In this chapter the research paper [2] is reviewed, which contains some core material providing a base work for our work. The work in this chapter is about intuitionistic fuzzy soft set containing some useful results and basic operations.

2.1 Intuitionistic Fuzzy Soft Set

2.1.1 Definition[12]

Suppose $X_{\varkappa}$ is an initiative universe, $IF(X_{\varkappa})$ is the set of all $(IFS(X_{\varkappa}))$, $E_{\varkappa}$ is a parameters set and $L_{\varkappa} \subseteq E_{\varkappa}$. Then, an intuitionistic fuzzy soft set $(F_{\zeta},\ L_{\varkappa})$ over $X_{\varkappa}$ is function from $E_{\varkappa}$ in to $IF(X_{\varkappa})$.

$$(F_{\zeta}, L_{\varkappa}) = \{(\varkappa,\ T_{F_{\zeta}(\varrho_i)}(\varkappa),\ K_{F_{\zeta}(\varrho_i)}(\varkappa)) : \varrho_i \in L_{\varkappa} \wedge \varkappa \in X_{\varkappa}\}$$

where $T_{F_{\zeta}(\varrho_i)}(\varkappa)$ and $K_{F_{\zeta}(\varrho_i)}(\varkappa)$ are degree of membership and degree of non membership of $\varkappa$ to the parameter ϱ_i respectively. For notation, intuitionistic fuzzy soft set represented by $IFSS(X_{\varkappa})$.

2.1.2 Example

Let $X_{\varkappa}$ be the set of houses and $E_{\varkappa}$ is the set of of parameters. Consider $X_{\varkappa} = \{\varkappa_1,\ \varkappa_2,\ \varkappa_3,\ \varkappa_4,\ \varkappa_5\}$ and $L_{\varkappa} \subseteq E_{\varkappa}$,

$L_{\varkappa} = \{\varrho_1\,(\text{beautiful}),\ \varrho_2\,(\text{wooden}),\ \varrho_3\,(\text{costly}),\ \varrho_4\,(\text{moderate})\}$. Suppose that,

$$F_{\zeta}(\varrho_1) = \left\{ \begin{array}{c} \langle \varkappa_1, 0.1, 0.2\rangle,\ \langle \varkappa_2, 0.7, 0.1\rangle,\ \langle \varkappa_3, 0.4, 0.2\rangle, \\ \langle \varkappa_4, 0.2, 0.6\rangle,\ \langle \varkappa_5, 0.2, 0.4\rangle \end{array} \right\},$$

$$F_{\zeta}(\varrho_2) = \left\{ \begin{array}{c} \langle \varkappa_1, 0.1, 0.5\rangle,\ \langle \varkappa_2, 0.2, 0.7\rangle,\ \langle \varkappa_3, 0.1, 0.8\rangle, \\ \langle \varkappa_4, 0.4, 0.4\rangle,\ \langle \varkappa_5,\ 0.5,\ 0.5\rangle \end{array} \right\},$$

$$F_\zeta(\varrho_3) = \left\{ \begin{array}{c} \langle \varkappa_1, 0.2, 0.5\rangle,\ \langle \varkappa_2,\ 0.4, 0.1\rangle,\ \langle \varkappa_3, 0.3, 0.5\rangle, \\ \langle \varkappa_4, 0.3, 0.5\rangle,\ \langle \varkappa_5, 0.4, 0.1\rangle \end{array} \right\},$$

$$F_\zeta(\varrho_4) = \left\{ \begin{array}{c} \langle \varkappa_1, 0.3, 0.3\rangle,\ \langle \varkappa_2,\ 0.3,\ 0.6\rangle,\ \langle \varkappa_3, 0.4, 0.3\rangle, \\ \langle \varkappa_4, 0.5, 0.2\rangle,\ \langle \varkappa_5, 0.3, 0.7\rangle \end{array} \right\}.$$

The $IFSS(X_\varkappa)$ $(F_\zeta,\ L_\varkappa)$ is a parametrized family.

Thus we can view the $IFSS(X_\varkappa)$ $(F_\zeta, L_\varkappa)$ as a collection of approximation below,

$$(F_\zeta, L_\varkappa) = \left\{ \begin{array}{l} \varrho_1 = \left\{ \begin{array}{c} \langle \varkappa_1, 0.1,\ 0.2\rangle,\ \langle \varkappa_2, 0.7, 0.1\rangle,\ \langle \varkappa_3, 0.4, 0.2\rangle, \\ \langle \varkappa_4, 0.2, 0.6\rangle,\ \langle \varkappa_5, 0.2, 0.4\rangle \end{array} \right\}, \\ \varrho_2 = \left\{ \begin{array}{c} \langle \varkappa_1, 0.1, 0.5\rangle,\ \langle \varkappa_2, 0.2, 0.7\rangle,\ \langle \varkappa_3, 0.1, 0.8\rangle, \\ \langle \varkappa_4, 0.4, 0.4\rangle,\ \langle \varkappa_5,\ 0.5,\ 0.5\rangle \end{array} \right\}, \\ \varrho_3 = \left\{ \begin{array}{c} \langle \varkappa_1, 0.2, 0.5\rangle,\ \langle \varkappa_2,\ 0.4, 0.1\rangle,\ \langle \varkappa_3, 0.3, 0.5\rangle, \\ \langle \varkappa_4, 0.3, 0.5\rangle,\ \langle \varkappa_5, 0.4, 0.1\rangle \end{array} \right\}, \\ \varrho_4 = \left\{ \begin{array}{c} \langle \varkappa_1, 0.3, 0.3\rangle,\ \langle \varkappa_2,\ 0.3,\ 0.6\rangle,\ \langle \varkappa_3, 0.4, 0.3\rangle, \\ \langle \varkappa_4, 0.5, 0.2\rangle,\ \langle \varkappa_5, 0.3, 0.7\rangle \end{array} \right\} \end{array} \right\},$$

where two parts of each approximation : (1) a predicate and (2) an approximate value-set.

For example, $\varrho_1 = \{\langle \varkappa_1, 0.1, 0.2\rangle,\ \langle \varkappa_2, 0.7, 0.1\rangle,\ \langle \varkappa_3, 0.4, 0.2\rangle, \langle \varkappa_4, 0.2, 0.6\rangle,\ \langle \varkappa_5, 0.2, 0.4\rangle\}$, we have (1) the predicate name "beautiful houses" and (2) the approximate value-set is $\{\langle \varkappa_1, 0.1, 0.2\rangle,\ \langle \varkappa_2, 0.7, 0.1\rangle,\ \langle \varkappa_3, 0.4, 0.2\rangle, \langle \varkappa_4, 0.2, 0.6\rangle,\ \langle \varkappa_5, 0.2, 0.4\rangle\}$.

For the purpose $IFSS(X_\varkappa)$ store in a computer, we present form of a table. In this table, the entries are c_{ij} corresponding to the house $\varkappa_i$ and the parameter ϱ, where

c_{ij} = (true-membershipvalue of $\varkappa_i$, falsity-membership value of $\varkappa_i$).

$(F_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$	$\varkappa_4$
ϱ_1	$(0.1, 0.2)$	$(0.7, 0.1)$	$(0.4, 0.2)$	$(0.2, 0.6)$	$(0.2, 0.4)$
ϱ_2	$(0.1, 0.5)$	$(0.2, 0.7)$	$(0.1, 0.8)$	$(0.4, 0.4)$	$(0.5, 0.5)$
ϱ_3	$(0.2, 0.5)$	$(0.4, 0.1)$	$(0.3, 0.5)$	$(0.3, 0.5)$	$(0.4, 0.1)$
ϱ_4	$(0.3, 0.3)$	$(0.3, 0.6)$	$(0.4, 0.3)$	$(0.5, 0.2)$	$(0.3, 0.7)$

Table 2.1

2.1.3 Example

Suppose $X_{\varkappa} = \{\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4, \varkappa_5, \varkappa_6\}$ is a set of men and $L_{\varkappa}$ =$\{\varrho_1$(educated), ϱ_2(government employee), ϱ_3(businessman), ϱ_4(smart)$\}\subseteq E_{\varkappa}$. Then $IFSS(X_{\varkappa})$ is given below.

$(F_{\zeta}, L_{\varkappa})$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$	$\varkappa_5$	$\varkappa_6$
ϱ_1	$(0.6, 0.4)$	$(0.9, 0.1)$	$(0.5, 0.5)$	$(0.3, 0.5)$	$(0.7, 0.3)$	$(0.8, 0.2)$
ϱ_2	$(0.7, 0.3)$	$(0, 1)$	$(0.7, 0.3)$	$(0.9, 0.1)$	$(0.6, 0.4)$	$(0.5, 0.5)$
ϱ_3	$(0, 1)$	$(0.7, 0.3)$	$(0.9, 0.1)$	$(0.6, 0.4)$	$(0.5, 0.5)$	$(0.8, 0.2)$
ϱ_4	$(0.3, 0.7)$	$(0.1, 0.9)$	$(0.4, 0.6)$	$(0.2, 0.8)$	$(0.5, 0.5)$	$(0.3, 0.5)$

Table 2.2

2.2 Basic Operations on Intuitionistic Fuzzy Soft sets

2.2.1 Definition

Suppose $L_{\varkappa}$, $O_{\varkappa} \subseteq E_{\varkappa}$, $(F_{\zeta}, L_{\varkappa})$, $(G_{\zeta}, O_{\varkappa}) \in IFSS(X_{\varkappa})$, Then $(F_{\zeta}, L_{\varkappa})$ is known to be intuitionistic fuzzy soft subset ($IFSSub(X_{\varkappa})$ of $(G_{\zeta}, O_{\varkappa})$ if

(1) $L_{\varkappa} \subseteq O_{\varkappa}$

(2) $F_{\zeta} \subseteq G_{\zeta} \Leftrightarrow T_{F_{\zeta}(\varrho)}(\varkappa) \leq T_{G_{\zeta}(\varrho)}(\varkappa)$ and $K_{G_{\zeta}(\varrho)}(\varkappa) \geq K_{F_{\zeta}(\varrho)}(\varkappa)\ \forall\ \varkappa \in X_{\varkappa}$.

This relationship is denoted by $(F_{\zeta}, L_{\varkappa}) \subseteq (G_{\zeta}, O_{\varkappa})$.

2.2.2 Definition

Suppose $(F_{\zeta}, L_{\varkappa})$, $(G_{\zeta}, O_{\varkappa}) \in IFSS(X_{\varkappa})$. Then $(F_{\zeta}, L_{\varkappa})$ is an intuitionistic fuzzy soft equal to $(G_{\zeta}, O_{\varkappa})$, if and only if $(F_{\zeta}, L_{\varkappa}) \subseteq (G_{\zeta}, O_{\varkappa})$ and $(G_{\zeta}, O_{\varkappa}) \subseteq (F_{\zeta}, L_{\varkappa})$ and written by $(F_{\zeta}, L_{\varkappa}) = (G_{\zeta}, O_{\varkappa})$.

2.2.3 Definition[15]

Suppose $(F_{\zeta}, L_{\varkappa}) \in IFSS(X_{\varkappa})$, if $(F_{\zeta}, L_{\varkappa}) = \{(\varkappa,\ 0,\ 1) : \forall \varkappa \in X_{\varkappa}\}$. Then $(F_{\zeta}, L_{\varkappa})$ is called null $IFSS(X_{\varkappa})$ and denoted by $(F_{\zeta}, \emptyset_{L_{\varkappa}}.)$.

2.2.4 Definition[12]

Suppose $(F_\zeta, L_\varkappa) \in IFSS(X_\varkappa)$, if $(F_\zeta, L_\varkappa) = \{(\varkappa,\ 1,\ 0) : \forall \varkappa \in X_\varkappa\}$. Then $(F_\zeta, L_\varkappa)$ is called universal $IFSS(X_\varkappa)$ and denoted by $(F_\zeta, U_{L_\varkappa}.)$.

2.2.5 Definition[12]

Suppose $(F_\zeta, L_\varkappa) \in IFSS(X_\varkappa)$. Then complement of $(F_\zeta, L_\varkappa)$ denoted by $(F_\zeta, L_\varkappa)^c$ and is defined by,

$$(F_\zeta, L_\varkappa)^c = \{(\varkappa,\ K_{F_\zeta(\varrho_i)}(\varkappa),\ T_{F_\zeta(\varrho_i)}(\varkappa)) : \varrho_i \in E_\varkappa \wedge \varkappa \in X_\varkappa\}.$$

2.2.6 Example

Suppose $X_\varkappa = \{\varkappa_1,\ \varkappa_2,\ \varkappa_3,\ \varkappa_4,\ \varkappa_5\}$ is a set of shops to buy toys and $E_\varkappa$ is the parameters set and $L_\varkappa = \{\varrho_1(\text{cheap}),\ \varrho_2(\text{beautiful}),\ \varrho_3(\text{quality}),\ \varrho_4(\text{discount})\} \subseteq E_\varkappa$.

Then $IFS(X_\varkappa)$ and its compliment is shown in Table 2.3 and Table 2.4 respectively.

$(F_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$	$\varkappa_5$
ϱ_1	(0.1, 0.9)	(0.2, 0.7)	(0.3, 0.7)	(0.5, 0.4)	(0.1, 0.4)
ϱ_2	(0.2, 0.7)	(0.3, 0.6)	(0.4, 0.6)	(0.3, 0.7)	(0.5, 0.3)
ϱ_3	(0.6, 0.3)	(0.6, 0.4)	(0.2, 0.8)	(0.6, 0.2)	(0.3, 0.2)
ϱ_4	(0.4, 0.5)	(0.9, 0.1)	(0.9, 0.1)	(0.8, 0.1)	(0.7, 0.1)

Table 2.3

$(F_\zeta, L_\varkappa)^c$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$	$\varkappa_5$
ϱ_1	(0.9, 0.1)	(0.7, 0.2)	(0.7, 0.3)	(0.4, 0.5)	(0.4, 0.1)
ϱ_2	(0.7, 0.2)	(0.6, 0.3)	(0.6, 0.4)	(0.7, 0.3)	(0.3, 0.5)
ϱ_3	(0.3, 0.6)	(0.4, 0.6)	(0.8, 0.2)	(0.2, 0.6)	(0.2, 0.3)
ϱ_4	(0.5, 0.4)	(0.1, 0.9)	(0.1, 0.9)	(0.1, 0.8)	(0.1, 0.7)

Table 2.4

2.2.7 Definition

Suppose $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in IFSS(X_\varkappa)$. Then restricted union is denoted and defined by $(F_\zeta, L_\varkappa) \cup_{RES} (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$, where $P_\varkappa = L_\varkappa \cap O_\varkappa$ and $\forall\ p \in P_\varkappa$, $\varkappa \in X_\varkappa$ we have $H_\zeta(p) = F_\zeta(p) \cup G_\zeta(p)$.

$$T_{H_\zeta(p)}(\varkappa) = Max\{T_{F_\zeta(p)}(\varkappa), T_{G_\zeta(p)}(\varkappa)\}$$

$$K_{H_\zeta(p)}(\varkappa) = Min\{K_{F_\zeta(p)}(\varkappa), K_{G_\zeta(p)}(\varkappa)\}.$$

2.2.8 Example

Suppose set of cars is $X_\varkappa = \{\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4, \varkappa_5\}$ and parameters set is $E_\varkappa = \{\varrho_1$(color), ϱ_2(price), ϱ_3(Tax), ϱ_4(speed)$\}$ and $L_\varkappa$, $O_\varkappa \subseteq E_\varkappa$, $L_\varkappa = \{\varrho_1, \varrho_2, \varrho_3\}$, $O_\varkappa = \{\varrho_2, \varrho_3, \varrho_4\}$.

$(F_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$	$\varkappa_5$
ϱ_1	(0.6, 0.3)	(0.4, 0.2)	(0.5, 0.4)	(0.6, 0.1)	(0.1, 0.6)
ϱ_2	(0.4, 0.1)	(0.1, 0.9)	(0.3, 0.7)	(0.2, 0.5)	(0.7, 0.2)
ϱ_3	(0.6, 0.1)	(1, 0)	(0.7, 0.1)	(0.9, 0.1)	(0.8, 0.1)

Table 2.5

$(G_\zeta, O_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$	$\varkappa_5$
ϱ_2	(0.1, 0.6)	(0.4, 0.6)	(0.6, 0.1)	(0.1, 0.6)	(0.1, 0.6)
ϱ_3	(0.4, 0.2)	(0.1, 0.2)	(0.3, 0.6)	(0.5, 0.3)	(0.2, 0.2)
ϱ_4	(0.8, 0.1)	(0.6, 0.2)	(0.7, 0.3)	(0.3, 0.1)	(0.8, 0.1)

Table 2.6

Then clearly $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in IFSS(X_\varkappa)$ and $(F_\zeta, L_\varkappa) \cup_{RES} (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$ is the restricted union of two $IFSS(X_\varkappa)$.

$(H_\zeta, P_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$	$\varkappa_5$
ϱ_2	$(0.4, 0.1)$	$(0.4, 0.6)$	$(0.6, 0.1)$	$(0.2, 0.5)$	$(0.7, 0.2)$
ϱ_3	$(0.6, 0.1)$	$(1, 0)$	$(0.7, 0.1)$	$(0.9, 0.1)$	$(0.8, 0.1)$

Table 2.7

2.2.9 Definition

Suppose $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in IFSS(X_\varkappa)$. Then restricted Intersection of $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$ is denoted and defined by $(F_\zeta, L_\varkappa) \cap_{RES} (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$, where $P_\varkappa = L_\varkappa \cap O_\varkappa$ and $\forall\ p \in P_\varkappa$, $\varkappa \in X_\varkappa$ we have $H_\zeta(p) = F_\zeta(p) \cap G_\zeta(p)$.

$$T_{H_\zeta(p)}(\varkappa) = Min\{T_{F_\zeta(p)}(\varkappa), T_{G_\zeta(p)}(\varkappa)\}$$

$$K_{H_\zeta(p)}(\varkappa) = Max\{K_{F_\zeta(p)}(\varkappa), K_{G_\zeta(p)}(\varkappa)\}$$

2.2.10 Example

Suppose $X_\varkappa = \{\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4, \varkappa_5\}$ is a set of shoes and $E_\varkappa = \{\varrho_1(\text{price}), \varrho_2(\text{color}), \varrho_3(\text{quality}), \varrho_4(\text{comfort})\}$ is the parameters set and $L_\varkappa$, $O_\varkappa \subseteq E_\varkappa$, $L_\varkappa = \{\varrho_1, \varrho_2, \varrho_3\}$, $O_\varkappa = \{\varrho_2, \varrho_3, \varrho_4\}$.

$(F_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$	$\varkappa_5$
ϱ_1	$(0.9, 0.1)$	$(0.4, 0.2)$	$(0.5, 0.4)$	$(0.1, 0.5)$	$(0.8, 0.1)$
ϱ_2	$(0.4, 0.2)$	$(0.2, 0.1)$	$(0.3, 0.7)$	$(0.6, 0.2)$	$(0.3, 0.5)$
ϱ_3	$(0.6, 0.1)$	$(1, 0)$	$(0.7, 0.1)$	$(0.3, 0.5)$	$(0.7, 0.2)$

Table 2.8

$(G_\zeta, O_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$	$\varkappa_5$
ϱ_2	$(0.3, 0.6)$	$(0.4, 0.6)$	$(0.5, 0.5)$	$(0.1, 0.3)$	$(0.5, 0.4)$
ϱ_3	$(0.4, 0.2)$	$(0.1, 0.9)$	$(0.2, 0.1)$	$(0.3, 0.5)$	$(0.3, 0.2)$
ϱ_4	$(0.2, 0.9)$	$(0.6, 0.2)$	$(0.7, 0.3)$	$(0.7, 0.1)$	$(0.5, 0.2)$

Table 2.9

Then clearly $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in IFSS(X_\varkappa)$ and $(F_\zeta, L_\varkappa) \cap_{RES} (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$ is the restricted intersection of two $IFSS(X_\varkappa)$.

$(H_\zeta, P_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$	$\varkappa_5$
ϱ_2	$(0.3, 0.6)$	$(0.2, 0.6)$	$(0.3, 0.7)$	$(0.1, 0.3)$	$(0.3, 0.5)$
ϱ_3	$(0.4, 0.2)$	$(0.1, 0.9)$	$(0.2, 0.1)$	$(0.3, 0.5)$	$(0.3, 0.5)$

Table 2.10

CHAPTER III

LATTICE ORDERED INTUITIONISTIC FUZZY SOFT SETS

In this chapter, we give some discussion on extended union, extended intersection, $\vee$-product, $\wedge$-product, basic union, basic intersection on $IFSS(X_\varkappa)$, then we define lattice ordered intuitionistic fuzzy soft set over $X_\varkappa$ ($LOIFSS(X_\varkappa)$), anti $LOIFSS(X_\varkappa)$, restricted union, restricted intersection, extended union, extended intersection, $\vee$-product, $\wedge$-product, basic union, basic intersection, complement of $LOIFSS(X_\varkappa)$. Further we use this concept to find score function. In this chapter the set of parameter $E_\varkappa$ will be lattice, if otherwise stated.

3.1 Some New Operations on Intuitionistic Fuzzy Soft Set

3.1.1 Definition

Suppose $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in IFSS(X_\varkappa)$. Then extended Union of $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$ is denoted and defined by $(F_\zeta, L_\varkappa) \cup_{EXT} (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$, where $P_\varkappa = L_\varkappa \cup O_\varkappa$.

$$(H_\zeta, P_\varkappa) = \begin{cases} < T_{F_\zeta(\varrho)}(\varkappa), K_{F_\zeta(\varrho)}(\varkappa) > \text{ if } \varrho \in L_\varkappa - O_\varkappa \\ < T_{G_\zeta(\varrho)}(\varkappa), K_{G_\zeta(\varrho)}(\varkappa) > \text{ if } \varrho \in O_\varkappa - L_\varkappa \\ < Max\{T_{F_\zeta(\varrho)}(\varkappa), T_{G_\zeta(\varrho)}(\varkappa)\}, Min\{K_{F_\zeta(\varrho)}(\varkappa), K_{G_\zeta(\varrho)}(\varkappa) > \text{if } \varrho \in L_\varkappa \cap O_\varkappa \end{cases}$$

3.1.2 Example

Suppose $X_\varkappa = \{\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4, \varkappa_5\}$ is a set of cars and $E_\varkappa = \{\varrho_1(\text{color}), \varrho_2(\text{price}), \varrho_3(\text{Tax}), \varrho_4(\text{speed}), \varrho_5(\text{comfort})\}$ is the parameters set and $L_\varkappa, O_\varkappa \subseteq E_\varkappa$, $L_\varkappa = \{\varrho_1, \varrho_2, \varrho_3\}$, $O_\varkappa = \{\varrho_2, \varrho_3, \varrho_4, \varrho_5\}$.

$(F_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$	$\varkappa_5$
ϱ_1	(0.2, 0.6)	(0.4, 0.2)	(0.5, 0.4)	(0.1, 0.3)	(0.6, 0.1)
ϱ_2	(0.8, 0.1)	(0.2, 0.5)	(0.3, 0.5)	(0.5, 0.2)	(0.3, 0.2)
ϱ_3	(0.6, 0.4)	(1, 0)	(0.7, 0.1)	(0.5, 0.2)	(0.3, 0.2)

Table 3.1

$(G_\zeta, O_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$	$\varkappa_5$
ϱ_2	(0.3, 0.6)	(0.4, 0.6)	(0.5, 0.5)	(0.5, 0.2)	(0.5, 0.3)
ϱ_3	(0.1, 0.9)	(0.2, 0.8)	(0.2, 0.4)	(0.3, 0.3)	(0.6, 0.1)
ϱ_4	(0.8, 0.1)	(0.6, 0.2)	(0.7, 0.3)	(0.6, 0.1)	(0.1, 0.2)
ϱ_5	(0.3, 0.2)	(0.1, 0.2)	(0.3, 0.3)	(0.1, 0.2)	(0.5, 0.2)

Table 3.2

$(F_\zeta, L_\varkappa)U_{EXT}(G_\zeta, O_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$	$\varkappa_5$
ϱ_1	(0.2, 0.6)	(0.4, 0.2)	(0.5, 0.4)	(0.1, 0.3)	(0.6, 0.1)
ϱ_2	(0.8, 0.1)	(0.4, 0.5)	(0.5, 0.5)	(0.5, 0.2)	(0.5, 0.2)
ϱ_3	(0.6, 0.4)	(1, 0)	(0.7, 0.1)	(0.5, 0.2)	(0.6, 0.1)
ϱ_4	(0.8, 0.1)	(0.6, 0.2)	(0.7, 0.3)	(0.6, 0.1)	(0.1, 0.2)
ϱ_5	(0.3, 0.2)	(0.1, 0.2)	(0.3, 0.3)	(0.1, 0.2)	(0.5, 0.2)

Table 3.3

3.1.3 Definition

Suppose $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in IFSS(X_\varkappa)$. Then extended Intersection of $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$ is denoted and defined by $(F_\zeta, L_\varkappa) \cap_{EXT} (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$ where $P_\varkappa = L_\varkappa \cup O_\varkappa$.

$$(H_\zeta, P_\varkappa) = \begin{cases} < T_{F_\zeta(\varrho)}(\varkappa), K_{F_\zeta(\varrho)}(\varkappa) > \text{ if } \varrho \in L_\varkappa - O_\varkappa \\ < T_{G_\zeta(\varrho)}(\varkappa), K_{G_\zeta(\varrho)}(\varkappa) > \text{ if } \varrho \in O_\varkappa - L_\varkappa \\ < Min\{T_{F_\zeta(\varrho)}(\varkappa), T_{G_\zeta(\varrho)}(\varkappa)\}, Max\{K_{F_\zeta(\varrho)}(\varkappa), K_{G_\zeta(\varrho)}(\varkappa) > \text{if } \varrho \in L_\varkappa \cap O_\varkappa \end{cases}$$

3.1.4 Example

Suppose $X_\varkappa = \{\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4, \varkappa_5\}$ is a set of men under consideration and $E_\varkappa = \{\varrho_1$(educated), ϱ_2(businessman), ϱ_3(smart), ϱ_4(government employee), ϱ_5(bank balance)$\}$ is the set of parameters and $L_\varkappa$, $O_\varkappa \subseteq E_\varkappa$, and $L_\varkappa \subseteq O_\varkappa$, $L_\varkappa = \{\varrho_2, \varrho_3, \varrho_4\}$, $O_\varkappa = \{\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5\}$.

$(F_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$	$\varkappa_5$
ϱ_2	(0.1, 0.9)	(0.2, 0.4)	(0.2, 0.7)	(0.2, 0.6)	(0.3, 0.4)
ϱ_3	(0.7, 0.1)	(0.6, 0.2)	(0.5, 0.2)	(0.8, 0.1)	(0.5, 0.4)
ϱ_4	(0.6, 0.3)	(0.5, 0.4)	(0.55, 0.4)	(0.5, 0.4)	(0.2, 0.3)

Table 3.4

$(G_\zeta, O_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$	$\varkappa_5$
ϱ_1	(0.2, 0.8)	(0.1, 0.9)	(0.3, 0.7)	(0.5, 0.4)	(0.2, 0.3)
ϱ_2	(0.5, 0.2)	(0.3, 0.4)	(0.5, 0.5)	(0.1, 0.9)	(0.5, 0.2)
ϱ_3	(0.5, 0.4)	(0.1, 0.8)	(0.6, 0.3)	(0.7, 0.2)	(0.6, 0.1)
ϱ_4	(0.2, 0.2)	(0.7, 0.3)	(0.1, 0.8)	(0.5, 0.4)	(0.5, 0.2)
ϱ_5	(0.8, 0.1)	(0.9, 0.1)	(0.9, 0.1)	(0.5, 0.4)	(0.1, 0.6)

Table 3.5

$\cap_{EXT}$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$	$\varkappa_5$
ϱ_1	(0.2, 0.8)	(0.1, 0.9)	(0.3, 0.7)	(0.5, 0.4)	(0.2, 0.3)
ϱ_2	(0.1, 0.9)	(0.2, 0.4)	(0.2, 0.7)	(0.1, 0.9)	(0.3, 0.4)
ϱ_3	(0.5, 0.4)	(0.1, 0.8)	(0.5, 0.3)	(0.7, 0.2)	(0.5, 0.4)
ϱ_4	(0.2, 0.3)	(0.5, 0.4)	(0.1, 0.8)	(0.5, 0.4)	(0.2, 0.3)
ϱ_5	(0.8, 0.1)	(0.9, 0.1)	(0.9, 0.1)	(0.5, 0.4)	(0.1, 0.6)

Table 3.6

3.1.5 Definition

Suppose $(F_\zeta, L_\varkappa), (G_\zeta, O_\varkappa) \in IFSS(X_\varkappa)$. Then $(F_\zeta, L_\varkappa) \vee (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$ is known to be basic union of two $IFSS(X_\varkappa)$, where $P_\varkappa = L_\varkappa \times O_\varkappa$ define $H_\zeta(l, o) = F_\zeta(l) \cup_{RES} G_\zeta(o)$ and

$$T_{H_\zeta(l,o)}(\varkappa) = Max\{T_{H_\zeta(l)}(\varkappa),\ T_{G_\zeta(o)}(\varkappa)\}$$
$$K_{H_\zeta(l,o)}(\varkappa) = Min\{T_{H_\zeta(l)}(\varkappa),\ T_{G_\zeta(o)}(\varkappa)\}\ \forall\ (l, o) \in P_\varkappa, \varkappa \in X_\varkappa.$$

3.1.6 Example

Suppose universe set is $X_\varkappa = \{\varkappa_1,\ \varkappa_2,\ \varkappa_3,\ \varkappa_4\}$ and parameters set is $L_\varkappa$, $O_\varkappa \subseteq E_\varkappa = \{\varrho_1,\ \varrho_2,\ \varrho_3,\ \varrho_4\}$ and $L_\varkappa = \{\varrho_1,\ \varrho_2,\ \varrho_3\}$, $O_\varkappa = \{\varrho_2,\ \varrho_3,\ \varrho_4\}$.

$(F_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
ϱ_1	(0.1, 0.9)	(0.2, 0.7)	(0.3, 0.7)	(0.5, 0.4)
ϱ_2	(0.2, 0.7)	(0.3, 0.6)	(0.4, 0.6)	(0.56, 0.3)
ϱ_3	(0.3, 0.5)	(0.6, 0.4)	(0.55, 0.4)	(0.6, 0.2)

Table 3.7

$(G_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
ϱ_2	(0.2, 0.8)	(0.4, 0.3)	(0.1, 0.6)	(0.5, 0.3)
ϱ_3	(0.4, 0.4)	(0.7, 0.2)	(0.4, 0.4)	(0.7, 0.2)
ϱ_4	(0.6, 0.2)	(0.9, 0.1)	(0.8, 0.2)	(0.9, 0.1)

Table 3.8

Then $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in IFSS(X_\varkappa)$ and $(F_\zeta, L_\varkappa) \vee (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$ is known to be basic union of two $IFSS(X_\varkappa)$.

$(H_\zeta, P_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
(ϱ_1, ϱ_2)	$(0.2, 0.8)$	$(0.4, 0.3)$	$(0.3, 0.6)$	$(0.5, 0.3)$
(ϱ_1, ϱ_3)	$(0.4, 0.4)$	$(0.7, 0.2)$	$(0.4, 0.4)$	$(0.7, 0.2)$
(ϱ_1, ϱ_4)	$(0.6, 0.2)$	$(0.9, 0.1)$	$(0.8, 0.2)$	$(0.9, 0.1)$
(ϱ_2, ϱ_2)	$(0.2, 0.7)$	$(0.4, 0.3)$	$(0.4, 0.4)$	$(0.7, 0.2)$
(ϱ_2, ϱ_3)	$(0.4, 0.4)$	$(0.7, 0.2)$	$(0.4, 0.4)$	$(0.7, 0.2)$
(ϱ_2, ϱ_4)	$(0.6, 0.2)$	$(0.9, 0.1)$	$(0.8, 0.2)$	$(0.9, 0.1)$
(ϱ_3, ϱ_2)	$(0.3, 0.5)$	$(0.6, 0.3)$	$(0.55, 0.4)$	$(0.6, 0.2)$
(ϱ_3, ϱ_3)	$(0.4, 0.4)$	$(0.7, 0.2)$	$(0.55, 0.4)$	$(0.7, 0.2)$
(ϱ_3, ϱ_4)	$(0.6, 0.2)$	$(0.9, 0.1)$	$(0.8, 0.2)$	$(0.9, 0.1)$

Table 3.9

3.1.7 Definition

Suppose $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in IFSS(X_\varkappa)$. Then $(F_\zeta, L_\varkappa) \wedge (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$ is known to be basic intersection of two $IFSS(X_\varkappa)$, where $P_\varkappa = L_\varkappa \times O_\varkappa$ define $H_\zeta(l, o) = F_\zeta(l) \cap_{RES}$ $G_\zeta(o)$ and

$T_{H_\zeta(l,o)}(\varkappa) = Min\{T_{H_\zeta(l)}(\varkappa), T_{G_\zeta(o)}(\varkappa)\}$

$K_{H_\zeta(l,o)}(\varkappa) = Max\{T_{H_\zeta(l)}(\varkappa), T_{G_\zeta(o)}(\varkappa)\}\ \forall\ (l, o) \in P_\varkappa,\ \varkappa \in X_\varkappa$.

3.1.8 Example

Suppose $X_\varkappa = \{\varkappa_1,\ \varkappa_2,\ \varkappa_3,\ \varkappa_4\}$ and $E_\varkappa = \{\varrho_1,\ \varrho_2,\ \varrho_3,\ \varrho_4\}$ is the parameters set and $L_\varkappa$, $O_\varkappa \subseteq E_\varkappa$, $L_\varkappa = \{\varrho_1,\ \varrho_2,\ \varrho_3\}$, $O_\varkappa = \{\varrho_2,\ \varrho_3,\ \varrho_4\}$.

$(F_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
ϱ_1	$(0.1, 0.9)$	$(0.2, 0.7)$	$(0.3, 0.7)$	$(0.5, 0.4)$
ϱ_2	$(0.2, 0.7)$	$(0.3, 0.6)$	$(0.4, 0.6)$	$(0.56, 0.3)$
ϱ_3	$(0.3, 0.5)$	$(0.6, 0.4)$	$(0.55, 0.4)$	$(0.6, 0.2)$

Table 3.10

$(G_\zeta, O_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
ϱ_2	$(0.2, 0.8)$	$(0.4, 0.3)$	$(0.1, 0.6)$	$(0.5, 0.3)$
ϱ_3	$(0.4, 0.4)$	$(0.7, 0.2)$	$(0.4, 0.4)$	$(0.7, 0.2)$
ϱ_4	$(0.6, 0.2)$	$(0.9, 0.1)$	$(0.8, 0.2)$	$(0.9, 0.1)$

Table 3.11

Then $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in IFSS(X_\varkappa)$ and $(F_\zeta, L_\varkappa) \wedge (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$ is known to be basic intersection of two $IFSS(X_\varkappa)$.

$(H_\zeta, P_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
(ϱ_1, ϱ_2)	$(0.1, 0.9)$	$(0.2, 0.7)$	$(0.1, 0.7)$	$(0.5, 0.4)$
(ϱ_1, ϱ_3)	$(0.1, 0.9)$	$(0.2, 0.7)$	$(0.3, 0.7)$	$(0.5, 0.4)$
(ϱ_1, ϱ_4)	$(0.1, 0.9)$	$(0.2, 0.7)$	$(0.3, 0.7)$	$(0.5, 0.4)$
(ϱ_2, ϱ_2)	$(0.2, 0.8)$	$(0.3, 0.6)$	$(0.1, 0.6)$	$(0.5, 0.3)$
(ϱ_2, ϱ_3)	$(0.2, 0.7)$	$(0.3, 0.6)$	$(0.4, 0.6)$	$(0.56, 0.3)$
(ϱ_2, ϱ_4)	$(0.2, 0.7)$	$(0.3, 0.6)$	$(0.4, 0.6)$	$(0.56, 0.3)$
(ϱ_3, ϱ_2)	$(0.2, 0.8)$	$(0.4, 0.4)$	$(0.1, 0.6)$	$(0.5, 0.3)$
(ϱ_3, ϱ_3)	$(0.3, 0.5)$	$(0.6, 0.4)$	$(0.4, 0.4)$	$(0.6, 0.2)$
(ϱ_3, ϱ_4)	$(0.3, 0.5)$	$(0.6, 0.4)$	$(0.55, 0.4)$	$(0.6, 0.2)$

Table 3.12

3.2 Lattice Ordered Intuitionistic Fuzzy Soft Set

3.2.1 Definition

An $IFSS(X_\varkappa)$

$(F_\zeta, L_\varkappa) = \{\varkappa, T_{F_\zeta(\varrho_i)}(\varkappa), K_{F_\zeta(\varrho_i)}(\varkappa) : \varrho_i \in L_\varkappa \wedge \varkappa \in X_\varkappa\}$ is known to be $LOIFSS(X_\varkappa)$.

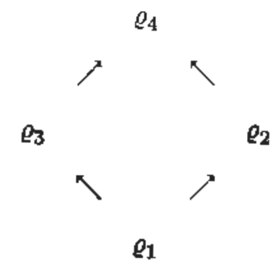
if for $\varrho_1, \varrho_2 \in L_\varkappa$, such that $\varrho_1 \leq \varrho_2$.

$\Rightarrow F_\zeta(\varrho_1) \subseteq F_\zeta(\varrho_2)$, i.e $T_{F_\zeta(\varrho_1)}(\varkappa) \leq T_{F_\zeta(\varrho_2)}(\varkappa)$ and $K_{F_\zeta(\varrho_1)}(\varkappa) \geq K_{F_\zeta(\varrho_2)}(\varkappa)\ \forall\ \varkappa \in X_\varkappa$.

3.2.2 Example

Suppose $X_\varkappa = \{\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4\}$ is a set of shops to buy toys and

$L_\varkappa = \{\varrho_1(\text{cheap}), \varrho_2(\text{beautiful}), \varrho_3(\text{quality}), \varrho_4(\text{discount})\} \subseteq E_\varkappa$.Then Ordered among the elements of $L_\varkappa$ is given.



then $LOIFSS(X_\varkappa)$ is given below.

$(F_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
ϱ_1	$(0.2, 0.4)$	$(0.1, 0.8)$	$(0.3, 0.4)$	$(0.5, 0.5)$
ϱ_2	$(0.4, 0.3)$	$(0.3, 0.6)$	$(0.4, 0.3)$	$(0.6, 0.3)$
ϱ_3	$(0.6, 0.2)$	$(0.6, 0.4)$	$(0.6, 0.2)$	$(0.8, 0.2)$
ϱ_4	$(0.7, 0.1)$	$(0.9, 0.1)$	$(0.9, 0.1)$	$(1, 0)$

Table 3.13

Clearly $F_\zeta(\varrho_1) \subseteq F_\zeta(\varrho_2) \subseteq F_\zeta(\varrho_4)$ and $F_\zeta(\varrho_1) \subseteq F_\zeta(\varrho_3) \subseteq F_\zeta(\varrho_4)$, so it is $LOIFSS(X_\varkappa)$.

3.3 Operations and Results on Lattice Ordered Intuitionistic Fuzzy Soft Sets

3.3.1 Definition

Suppose $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. Then $(F_\zeta, L_\varkappa)$ is known to be Lattice Ordered intuitionistic fuzzy soft subset $(LOIFSSub(X_\varkappa))$ of $(G_\zeta, O_\varkappa)$,

if $L_\varkappa \subseteq O_\varkappa$, then $T_{F_\zeta(\varrho)}(\varkappa) \leq T_{G_\zeta(\varrho)}(\varkappa)$ and $K_{F_\zeta(\varrho)}(\varkappa) \geq K_{G_\zeta(\varrho)}(\varkappa)$ $\forall$ $\varrho \in E_\varkappa$, $\varkappa \in X_\varkappa$. We denote$(F_\zeta, L_\varkappa) \subseteq (G_\zeta, O_\varkappa)$.

3.3.2 Definition

Suppose $(F_\zeta, L_\varkappa), (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. Then restricted union of $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$ is denoted and defined by $(F_\zeta, L_\varkappa) \cup_{RES} (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$, where $P_\varkappa = L_\varkappa \cap O_\varkappa$ and $\forall$ $\varrho \in P_\varkappa$, $\varkappa \in X_\varkappa$,we have $H_\zeta(\varrho) = F_\zeta(\varrho) \cup G_\zeta(\varrho)$.

$$T_{H_\zeta(\varrho)}(\varkappa) = Max\{T_{F_\zeta(\varrho)}(\varkappa), T_{G_\zeta(\varrho)}(\varkappa)\}$$

$$K_{H_\zeta(\varrho)}(\varkappa) = Min\{K_{F_\zeta(\varrho)}(\varkappa), K_{G_\zeta(\varrho)}(\varkappa)\}.$$

3.3.3 Theorem

Suppose $(F_\zeta, L_\varkappa), (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. Then $(F_\zeta, L_\varkappa) \cup_{RES} (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$.

Proof. Since $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$ contain in the set $LOIFSS(X_\varkappa)$. Then by Definition 3.3.2

$F_\zeta(\varrho) \cup G_\zeta(\varrho) = H_\zeta(\varrho)$ where $P_\varkappa = L_\varkappa \cap O_\varkappa$,

if $L_\varkappa \cap O_\varkappa = \emptyset$, then trivially hold the require result.

Now for $L_\varkappa \cap O_\varkappa \neq \emptyset$, since $L_\varkappa$, $O_\varkappa \subseteq E_\varkappa$, so $L_\varkappa$ and $O_\varkappa$ inherit the partial order from $E_\varkappa$ therefore for any

$l_1 \leq_{L_\varkappa} l_2$ we have $F_\zeta(l_1) \subseteq F_\zeta(l_2)$ $\forall$ l_1, $l_2 \in L_\varkappa$,

also for any $o_1 \leq_{O_\varkappa} o_2$ we have $G_\zeta(o_1) \subseteq G_\zeta(o_2)$ $\forall$ o_1, $o_2 \in O_\varkappa$,

Now for any p_1, $p_2 \in P_\varkappa$ and $p_1 \leq_{P_\varkappa} p_2$

$\Rightarrow p_1, p_2 \in L_\varkappa \cap O_\varkappa$

$\Rightarrow p_1, p_2 \in L_\varkappa$ and $p_1, p_2 \in O_\varkappa$

$\Rightarrow F_\zeta(p_1) \subseteq F_\zeta(p_2)$ and $G_\zeta(p_1) \subseteq G_\zeta(p_2)$ whenever $p_1 \leq_{L_\varkappa} p_2$, $p_1 \leq_{O_\varkappa} p_2$

$\Rightarrow$

$$T_{F_\zeta(p_1)}(\varkappa) \leq T_{F_\zeta(p_2)}(\varkappa), \qquad T_{G_\zeta(p_1)}(\varkappa) \leq T_{G_\zeta(p_2)}(\varkappa)$$

$$K_{F_\zeta(p_2)}(\varkappa) \leq K_{F_\zeta(p_1)}(\varkappa), \qquad K_{G_\zeta(p_2)}(\varkappa) \leq K_{G_\zeta(p_1)}(\varkappa)$$

$\Rightarrow$

$$Max\{T_{F_\zeta(p_1)}(\varkappa), T_{G_\zeta(p_1)}(\varkappa)\} \leq Max\{T_{F_\zeta(p_2)}(\varkappa), T_{G_\zeta(p_2)}(\varkappa)\}$$

$$Min\{K_{F_{\zeta(p_2)}}(\varkappa),\ K_{G_{\zeta(p_1)}}(\varkappa)\} \leq Min\{K_{F_{\zeta(p_1)}}(\varkappa), K_{G_{\zeta(p_1)}}(\varkappa)\}$$

$\Rightarrow$

$$T_{(F_\zeta(p_1)\cup G_\zeta(p_1))}(\varkappa) \leq T_{(F_\zeta(p_2)\cup G_\zeta(p_2))}(\varkappa)$$

$$K_{(F_\zeta(p_2)\cup G_\zeta(p_2))}(\varkappa) \leq K_{(F_\zeta(p_1)\cup G_\zeta(p_1))}(\varkappa)$$

$\Rightarrow$

$$T_{(F_\zeta\cup G_\zeta)(p_1)}(\varkappa) \leq T_{(F_\zeta\cup G_\zeta)(p_2)}(\varkappa)$$

$$K_{(F_\zeta\cup G_\zeta)(p_2)}(\varkappa) \leq K_{(F_\zeta\cup G_\zeta)(p_1)}(\varkappa)$$

$\Rightarrow$

$$T_{H_\zeta(p_1)}(\varkappa) \leq T_{H_\zeta(p_2)}(\varkappa)$$

$$K_{H_\zeta(p_2)}(\varkappa) \leq K_{H_\zeta(p_1)}(\varkappa)$$

$\Rightarrow$

$$H_\zeta(p_1) \subseteq H_\zeta(p_2) \text{ for } p_1 \leq_{P_\varkappa} p_2$$

$\Rightarrow (F_\zeta, L_\varkappa) \cup_{RES} (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. ■

3.3.4 Example

Suppose $X_\varkappa = \{\varkappa_1,\ \varkappa_2,\ \varkappa_3\}$ is a set of cars and $E_\varkappa = \{\varrho_1(\text{color}),\ \varrho_2(\text{price}),\ \varrho_3(\text{Tax}),\ \varrho_4(\text{speed})\}$ is the parameters set and $L_\varkappa,\ O_\varkappa \subseteq E_\varkappa$, $L_\varkappa = \{\varrho_1,\ \varrho_2,\ \varrho_3\}$, $O_\varkappa = \{\varrho_2,\ \varrho_3,\ \varrho_4\}$.

$(F_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$
ϱ_1	$(0.2, 0.3)$	$(0.4, 0.2)$	$(0.5, 0.4)$
ϱ_2	$(0.4, 0.2)$	$(0.8, 0.1)$	$(0.6, 0.2)$
ϱ_3	$(0.6, 0.1)$	$(1, 0)$	$(0.7, 0.1)$

Table 3.14

$(G_\zeta, O_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$
ϱ_2	$(0.3, 0.6)$	$(0.4, 0.6)$	$(0.5, 0.5)$
ϱ_3	$(0.4, 0.2)$	$(0.5, 0.3)$	$(0.6, 0.4)$
ϱ_4	$(0.8, 0.1)$	$(0.6, 0.2)$	$(0.7, 0.3)$

Table 3.15

Then $(F_\zeta, L_\varkappa), (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$ and $(F_\zeta, L_\varkappa) \cup_{RES} (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$ is the restricted union of two $LOIFSS(X_\varkappa)$.

$\cup_{RES}$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$
ϱ_2	$(0.4, 0.2)$	$(0.8, 0.1)$	$(0.6, 0.2)$
ϱ_3	$(0.6, 0.1)$	$(1, 0)$	$(0.7, 0.1)$

Table 3.16

$\Rightarrow H_\zeta(\varrho_2) \subseteq H_\zeta(\varrho_3)$, So it is an $LOIFSS(X_\varkappa)$.

3.3.5 Example

let $X_\varkappa = \{\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4\}$ be the set of bikes and

$E_\varkappa = \{\varrho_1(\text{cheap}), \varrho_2(\text{good looking}), \varrho_3(\text{good condition}), \varrho_4(\text{speed})\}$ be a set of parameter and let$L_\varkappa$, $O_\varkappa \subseteq E_\varkappa$, $L_\varkappa = \{\varrho_1, \varrho_2, \varrho_3\}$, $O_\varkappa = \{\varrho_2, \varrho_3, \varrho_4\}$.

$(F_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
ϱ_1	$(0.1, 0.9)$	$(0.2, 0.7)$	$(0.3, 0.5)$	$(0.5, 0.4)$
ϱ_2	$(0.2, 0.7)$	$(0.3, 0.6)$	$(0.4, 0.6)$	$(0.56, 0.3)$
ϱ_3	$(0.3, 0.5)$	$(0.6, 0.4)$	$(0.55, 0.4)$	$(0.6, 0.2)$

Table 3.17

$(G_\zeta, O_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
ϱ_2	$(0.2, 0.8)$	$(0.4, 0.3)$	$(0.1, 0.6)$	$(0.5, 0.1)$
ϱ_3	$(0.4, 0.4)$	$(0.7, 0.2)$	$(0.4, 0.4)$	$(0.5, 0.1)$
ϱ_4	$(0.6, 0.2)$	$(0.9, 0.1)$	$(0.8, 0.2)$	$(0.6, 0.3)$

Table 3.18

Then $(F_\zeta, L_\varkappa), (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$ and $(F_\zeta, L_\varkappa) \cup_{RES} (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$ is the restricted union of two $LOIFSS(X_\varkappa)$.

$(H_\zeta, P_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
ϱ_2	$(0.2, 0.7)$	$(0.4, 0.3)$	$(0.4, 0.6)$	$(0.56, 0.1)$
ϱ_3	$(0.4, 0.4)$	$(0.7, 0.2)$	$(0.55, 0.4)$	$(0.6, 0.1)$

Table 3.19

$\Rightarrow H_\zeta(\varrho_2) \subseteq H_\zeta(\varrho_3)$, So it is an $LOIFSS(X_\varkappa)$.

3.3.6 Definition

Suppose $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. Then restricted Intersection of $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$ is denoted and defined by $(F_\zeta, L_\varkappa) \cap_{RES} (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$, where $P_\varkappa = L_\varkappa \cap O_\varkappa$ and $\forall\ \varrho \in P_\varkappa$, $\varkappa \in X_\varkappa$ we have $H_\zeta(\varrho) = F_\zeta(\varrho) \cap G_\zeta(\varrho)$.

$$T_{H_{\zeta(\varrho)}}(\varkappa) = Min\{T_{F_{\zeta(\varrho)}}(\varkappa), T_{G_{\zeta(\varrho)}}(\varkappa)\}$$

$$K_{H_\zeta(\varrho)}(\varkappa) = Max\{K_{F_\zeta(\varrho)}(\varkappa), K_{G_\zeta(\varrho)}(\varkappa)\}$$

3.3.7 Theorem

Suppose $(F_\zeta, L_\varkappa), (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. Then $(F_\zeta, L_\varkappa) \cap_{RES} (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$.

Proof. Since $(F_\zeta, L_\varkappa), (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. Then by Definition 3.3.6,$F_\zeta(p) \cap G_\zeta(p) = H_\zeta(p)$ where $P_\varkappa = L_\varkappa \cap O_\varkappa$,

if $L_\varkappa \cap O_\varkappa = \emptyset$, then the required result hold trivially.

Now for $L_\varkappa \cap O_\varkappa \neq \emptyset$, since $L_\varkappa$, $O_\varkappa \subseteq E_\varkappa$ so $L_\varkappa$ and $O_\varkappa$ inherit the partial order from $E_\varkappa$ therefore for any $l_1 \leq_{L_\varkappa} l_2$ we have $F_\zeta(l_1) \subseteq F_\zeta(l_2)$ $\forall$ l_1, $l_2 \in L_\varkappa$ also for any $o_1 \leq_{O_\varkappa} o_2$ we have $G_\zeta(o_1) \subseteq G_\zeta(o_2)$ $\forall$ o_1, $o_2 \in O_\varkappa$,

therefore for any p_1, $p_2 \in P_\varkappa$ and $p_1 \leq_{P_\varkappa} p_2 \Rightarrow p_1$, $p_2 \in L_\varkappa \cap O_\varkappa$

$\Rightarrow p_1$, $p_2 \in L_\varkappa$ and p_1, $p_2 \in O_\varkappa$

$\Rightarrow F_\zeta(p_1) \subseteq F_\zeta(p_2)$ and $G_\zeta(p_1) \subseteq G_\zeta(p_2)$ whenever $p_1 \leq_{L_\varkappa} p_2$, $p_1 \leq_{O_\varkappa} p_2$

$\Rightarrow$

$$\begin{aligned} T_{F_\zeta(p_1)}(\varkappa) &\leq T_{F_\zeta(p_2)}(\varkappa)\ , T_{G_\zeta(p_1)}(\varkappa) \leq T_{G_\zeta(p_2)}(\varkappa) \\ K_{F_\zeta(p_2)}(\varkappa) &\leq K_{F_\zeta(p_1)}(\varkappa)\ , K_{G_\zeta(p_2)}(\varkappa) \leq K_{G_\zeta(p_1)}(\varkappa) \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} Min\{T_{F_\zeta(p_1)}(\varkappa), T_{G_\zeta(p_1)}(\varkappa)\} &\leq Min\{T_{F_\zeta(p_1)}(\varkappa), T_{G_\zeta(p_1)}(\varkappa)\} \\ Max\{K_{F_\zeta(p_2)}(\varkappa),\ K_{G_\zeta(p_2)}(\varkappa)\} &\leq Max\{K_{F_\zeta(p_1)}(\varkappa), K_{G_\zeta(p_1)}(\varkappa)\} \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} T_{(F_\zeta(p_1) \cap G_\zeta(p_1))}(\varkappa) &\leq T_{(F_\zeta(p_2) \cap G_\zeta(p_2))}(\varkappa) \\ K_{(F_\zeta(p_2) \cap G_\zeta(p_2))}(\varkappa) &\leq K_{(F_\zeta(p_1) \cap G_\zeta(p_1))}(\varkappa) \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} T_{(F_\zeta \cap G_\zeta)(p_1)}(\varkappa) &\leq T_{(F_\zeta \cap G_\zeta)(p_2)}(\varkappa) \\ K_{(F_\zeta \cap G_\zeta)(p_2)}(\varkappa) &\leq K_{(F_\zeta \cap G_\zeta)(p_1)}(\varkappa) \end{aligned}$$

$\Rightarrow$

$$T_{H_\zeta(p_1)}(\varkappa) \leq T_{H_\zeta(p_2)}(\varkappa)$$
$$K_{H_\zeta(p_2)}(\varkappa) \leq K_{H_\zeta(p_1)}(\varkappa)$$

$\Rightarrow$

$$H_\zeta(p_1) \subseteq H_\zeta(p_2) \text{ for } p_1 \leq_{P_\varkappa} p_2$$

$\Rightarrow (F_\zeta, L_\varkappa) \cap_{RES} (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. ■

3.3.8 Example

Suppose $X_\varkappa = \{\varkappa_1, \varkappa_2, \varkappa_3\}$ is a set of shoes and

$E_\varkappa = \{\varrho_1(\text{price}), \varrho_2(\text{color}), \varrho_3(\text{quality}), \varrho_4(\text{comfort})\}$ is the parameters set and $L_\varkappa$, $O_\varkappa \subseteq E_\varkappa$, $L_\varkappa = \{\varrho_1, \varrho_2, \varrho_3\}$, $O_\varkappa = \{\varrho_2, \varrho_3, \varrho_4\}$.

$(F_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$
ϱ_1	$(0.2, 0.3)$	$(0.4, 0.2)$	$(0.5, 0.4)$
ϱ_2	$(0.4, 0.2)$	$(0.8, 0.1)$	$(0.6, 0.2)$
ϱ_3	$(0.6, 0.1)$	$(1, 0)$	$(0.7, 0.1)$

Table 3.20

$(G_\zeta, O_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$
ϱ_2	$(0.3, 0.6)$	$(0.4, 0.6)$	$(0.5, 0.5)$
ϱ_3	$(0.4, 0.2)$	$(0.5, 0.3)$	$(0.6, 0.4)$
ϱ_4	$(0.8, 0.1)$	$(0.6, 0.2)$	$(0.7, 0.3)$

Table 3.21

Then $(F_\zeta, L_\varkappa), (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$ and $(F_\zeta, L_\varkappa) \cap_{RES} (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$ is the restricted intersection of two $LOIFSS(X_\varkappa)$.

$\cap_{RES}$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$
ϱ_2	$(0.3, 0.6)$	$(0.4, 0.6)$	$(0.5, 0.5)$
ϱ_3	$(0.4, 0.2)$	$(0.5, 0.3)$	$(0.6, 0.4)$

Table 3.22

$\Rightarrow H_\zeta(\varrho_2) \subseteq H_\zeta(\varrho_3)$, So it is an $LOIFSS(X_\varkappa)$.

3.3.9 Example

Let $E_\varkappa = \{\varrho_1, \varrho_2, \varrho_3, \varrho_4\}$ be a set of parameter and $L_\varkappa$, $O_\varkappa \subseteq E_\varkappa$, $L_\varkappa = \{\varrho_1, \varrho_2, \varrho_3\}$, $O_\varkappa = \{\varrho_2, \varrho_3, \varrho_4\}$ and $X_\varkappa = \{\varkappa_1, \varkappa_2, \varkappa_3, , \varkappa_4\}$.

$(F_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
ϱ_1	$(0.1, 0.9)$	$(0.2, 0.7)$	$(0.3, 0.5)$	$(0.5, 0.4)$
ϱ_2	$(0.2, 0.7)$	$(0.3, 0.6)$	$(0.4, 0.6)$	$(0.56, 0.3)$
ϱ_3	$(0.3, 0.5)$	$(0.6, 0.4)$	$(0.55, 0.4)$	$(0.6, 0.2)$

Table 3.23

$(G_\zeta, O_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
ϱ_2	$(0.2, 0.8)$	$(0.4, 0.3)$	$(0.1, 0.6)$	$(0.1, 0.9)$
ϱ_3	$(0.4, 0.4)$	$(0.7, 0.2)$	$(0.4, 0.4)$	$(0.7, 0.1)$
ϱ_4	$(0.6, 0.2)$	$(0.9, 0.1)$	$(0.8, 0.2)$	$(0.9, 0.1)$

Table 3.24

Then $(F_\zeta, L_\varkappa), (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$ and $(F_\zeta, L_\varkappa) \cap_{RES} (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$ is the restricted intersection of two $LOIFSS(X_\varkappa)$.

$(H_\zeta, P_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
ϱ_2	$(0.2, 0.8)$	$(0.3, 0.6)$	$(0.1, 0.6)$	$(0.1, 0.9)$
ϱ_3	$(0.3, 0.5)$	$(0.6, 0.4)$	$(0.4, 0.4)$	$(0.6, 0.2)$

Table 3.25

$\Rightarrow H_\zeta(\varrho_2) \subseteq H_\zeta(\varrho_3)$, So it is an $LOIFSS(X_\varkappa)$.

3.3.10 Definition

Suppose $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. Then extended Union of $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$ is denoted and defined by $(F_\zeta, L_\varkappa) \cup_{EXT} (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$, where $P_\varkappa = L_\varkappa \cup O_\varkappa$and $\forall$ $\varrho \in P_\varkappa, \varkappa \in X_\varkappa$.

$$(H_\zeta, P_\varkappa) = \begin{cases} < T_{F_\zeta(\varrho)}(\varkappa), K_{F_\zeta(\varrho)}(\varkappa) > \text{ if } \varrho \in L_\varkappa - O_\varkappa \\ < T_{G_\zeta(\varrho)}(\varkappa), K_{G_\zeta(\varrho)}(\varkappa) > \text{ if } \varrho \in O_\varkappa - L_\varkappa \\ < Max\{T_{F_\zeta(\varrho)}(\varkappa), T_{G_\zeta(\varrho)}(\varkappa)\}, Min\{K_{F_\zeta(\varrho)}(\varkappa), K_{G_\zeta(\varrho)}(\varkappa) > \text{if } \varrho \in L_\varkappa \cap O_\varkappa \end{cases}$$

3.3.11 Proposition

Suppose $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. Then $(F_\zeta, L_\varkappa) \cup_{EXT} (G_\zeta, O_\varkappa)$ is $LOIFSS(X_\varkappa)$, if one of them is a $LOIFSSub(X_\varkappa)$ of other.

Proof. Since $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. Then $(F_\zeta, L_\varkappa) \cup_{EXT} (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$, $F_\zeta(p) \cup G_\zeta(p) = H_\zeta(p)$ where $P_\varkappa = L_\varkappa \cup O_\varkappa$, $\forall \varrho \in P_\varkappa, \varkappa \in X_\varkappa$.

$$(H_\zeta, P_\varkappa) = \begin{cases} < T_{F_\zeta(\varrho)}(\varkappa), K_{F_\zeta(\varrho)}(\varkappa) > \text{ if } \varrho \in L_\varkappa - O_\varkappa \\ < T_{G_\zeta(\varrho)}(\varkappa), K_{G_\zeta(\varrho)}(\varkappa) > \text{ if } \varrho \in O_\varkappa - L_\varkappa \\ < Max\{T_{F_\zeta(\varrho)}(\varkappa), T_{G_\zeta(\varrho)}(\varkappa)\}, Min\{K_{F_\zeta(\varrho)}(\varkappa), K_{G_\zeta(\varrho)}(\varkappa) > \text{if } \varrho \in L_\varkappa \cap O_\varkappa \end{cases}$$

Suppose $(F_\zeta, L_\varkappa)$ is a $LOIFSSub(X_\varkappa)$ of $(G_\zeta, O_\varkappa)$. Then $L_\varkappa \subseteq O_\varkappa$ and $T_{F_{\zeta(\varrho)}}(\varkappa) \leq T_{G_{\zeta(\varrho)}}(\varkappa)$ and $K_{F_{\zeta(\varrho)}}(\varkappa) \geq K_{G_{\zeta(\varrho)}}(\varkappa)$ $\forall$ $\varrho \in L_\varkappa$, $\varkappa \in X_\varkappa$, since $L_\varkappa$, $O_\varkappa \subseteq E_\varkappa$ so $L_\varkappa$ and $O_\varkappa$ inherit the partial order from $E_\varkappa$, therefore for any $l_1 \leq_{L_\varkappa} l_2$ we have $F_\zeta(l_1) \subseteq F_\zeta(l_2)$

$\forall\ l_1,\ l_2 \in L_{\varkappa}$, also for any $o_1 \leq_{O_\varkappa} o_2$ we have $G_\zeta(o_1) \subseteq G_\zeta(o_2)\ \forall\ o_1,\ o_2 \in O_\varkappa$. Therefore for any $p_1,\ p_2 \in P_\varkappa$ and $p_1 \leq_{P_\varkappa} p_2$ implies $p_1,\ p_2 \in L_\varkappa \cup O_\varkappa$ implies $p_1,\ p_2 \in L_\varkappa \cap O_\varkappa$ or p_1, $p_2 \in O_\varkappa$ and $p_1,\ p_2 \notin L_\varkappa$ because $L_\varkappa \subseteq O_\varkappa$.

Now take $p_1,\ p_2 \in L_\varkappa \cap O_\varkappa$

$\Rightarrow p_1,\ p_2 \in L_\varkappa$ and $p_1,\ p_2 \in O_\varkappa$.

$\Rightarrow F_\zeta(p_1) \subseteq F_\zeta(p_2)$ and $G_\zeta(p_1) \subseteq G_\zeta(p_2)$ whenever $p_1 \leq_{L_\varkappa} p_2$ and $p_1 \leq_{O_\varkappa} p_2$

$\Rightarrow$

$$\begin{aligned} T_{F_\zeta(p_1)}(\varkappa) &\leq T_{F_\zeta(p_2)}(\varkappa),\ T_{G_\zeta(p_1)}(\varkappa) \leq T_{G_\zeta(p_2)}(\varkappa) \\ K_{F_\zeta(p_2)}(\varkappa) &\leq K_{F_\zeta(p_1)}(\varkappa),\ K_{G_\zeta(p_2)}(\varkappa) \leq K_{G_\zeta(p_1)}(\varkappa) \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} Max\left\{T_{F_\zeta(p_1)}(\varkappa), T_{G_\zeta(p_1)}(\varkappa)\right\} &\leq Max\left\{T_{F_\zeta(p_2)}(\varkappa), T_{G_\zeta(p_2)}(\varkappa)\right\} \\ Min\left\{K_{F_\zeta(p_2)}(\varkappa), K_{G_\zeta(p_2)}(\varkappa)\right\} &\leq Min\left\{K_{F_\zeta(p_1)}(\varkappa), K_{G_\zeta(p_1)}(\varkappa)\right\} \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} T_{F_\zeta(p_1)\cup G_\zeta(p_1)}(\varkappa) &\leq T_{F_\zeta(p_2)\cup G_\zeta(p_2)}(\varkappa) \\ K_{F_\zeta(p_2)\cup G_\zeta(p_2)}(\varkappa) &\leq K_{F_\zeta(p_1)\cup G_\zeta(p_1)}(\varkappa) \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} T_{(F_\zeta\cup G_\zeta)(p_1)}(\varkappa) &\leq T_{(F_\zeta\cup G_\zeta)(p_2)}(\varkappa) \\ K_{(F_\zeta\cup G_\zeta)(p_2)}(\varkappa) &\leq K_{(F_\zeta\cup G_\zeta)(p_1)}(\varkappa) \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} T_{H_\zeta(p_1)}(\varkappa) &\leq T_{H_\zeta(p_2)}(\varkappa) \\ K_{H_\zeta(p_2)}(\varkappa) &\leq K_{H_\zeta(p_1)}(\varkappa) \end{aligned}$$

$\Rightarrow H_\zeta(p_1) \subseteq H_\zeta(p_2)$ for $p_1 \leq_{P_\varkappa} p_1$.

Thus $(F_\zeta, L_\varkappa) \cup_{EXT} (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$ if $p_1,\ p_2 \in L_\varkappa \cap O_\varkappa$.

Now suppose for any $p_1,\ p_2 \in O_\varkappa$ and $p_1,\ p_2 \notin L_\varkappa$ and $p_1 \leq_{O_\varkappa} p_2$,

$\Rightarrow G_\zeta(p_1) \subseteq G_\zeta(p_2)$ whenever $p_1 \leq_{O_\varkappa} p_2$.

implies this is also a $LOIFSS(X_\varkappa)$.

Hence $(H_\zeta, L_\varkappa) \cup_{EXT} (G_\zeta, O_\varkappa)$ is a $LOIFSS(X_\varkappa)$ for both cases.

Similarly we can prove for $(G_\zeta, O_\varkappa)$ is a $LOIFSS(X_\varkappa)$ of $(H_\zeta, L_\varkappa)$.

Thus extended union of two $LOIFSS(X_\varkappa)$ is a $LOIFSS(X_\varkappa)$ if one of them is a Lattice ordered intuitionistic fuzzy soft subset of other. ■

3.3.12 Example

Suppose $X_\varkappa = \{\varkappa_1, \varkappa_2, \varkappa_3\}$ is a set of men under consideration and

$E_\varkappa = \{\varrho_1$(educated), ϱ_2(businessman), ϱ_3(smart), ϱ_4(government employee), ϱ_5(bank balance)$\}$ is the parameters set and $L_\varkappa$, $O_\varkappa \subseteq E_\varkappa$ and $L_\varkappa \subseteq O_\varkappa$, $L_\varkappa = \{\varrho_2, \varrho_3, \varrho_4\}$, $O_\varkappa = \{\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5\}$.

$(F_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$
ϱ_2	$(0.1, 0.9)$	$(0.1, 0.7)$	$(0.3, 0.7)$
ϱ_3	$(0.2, 0.7)$	$(0.3, 0.6)$	$(0.4, 0.6)$
ϱ_4	$(0.6, 0.3)$	$(0.6, 0.4)$	$(0.55, 0.4)$

Table 3.26

$(G_\zeta, O_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$
ϱ_1	$(0.1, 0.9)$	$(0.1, 0.9)$	$(0.3, 0.7)$
ϱ_2	$(0.2, 0.6)$	$(0.3, 0.4)$	$(0.5, 0.5)$
ϱ_3	$(0.5, 0.4)$	$(0.5, 0.4)$	$(0.6, 0.3)$
ϱ_4	$(0.7, 0.2)$	$(0.7, 0.3)$	$(0.6, 0.2)$
ϱ_5	$(0.8, 0.1)$	$(0.9, 0.1)$	$(0.9, 0.1)$

Table 3.27

Then clearly $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$ and $(F_\zeta, L_\varkappa) \subseteq (G_\zeta, O_\varkappa)$ then extended union is defined as follows.

$\cup_{EXT}$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$
ϱ_1	$(0.1, 0.9)$	$(0.1, 0.9)$	$(0.3, 0.7)$
ϱ_2	$(0.2, 0.6)$	$(0.3, 0.4)$	$(0.5, 0.5)$
ϱ_3	$(0.5, 0.4)$	$(0.5, 0.4)$	$(0.6, 0.3)$
ϱ_4	$(0.7, 0.2)$	$(0.7, 0.3)$	$(0.6, 0.2)$
ϱ_5	$(0.8, 0.1)$	$(0.9, 0.1)$	$(0.9, 0.1)$

Table 3.28

which is $LOIFSS(X_\varkappa)$.

3.3.13 Example

Let $E_\varkappa = \{\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5\}$ be a set of parameter and $L_\varkappa$, $O_\varkappa \subseteq E_\varkappa$ and $L_\varkappa \subseteq O_\varkappa$, $L_\varkappa = \{\varrho_2, \varrho_3, \varrho_4\}$, $O_\varkappa = \{\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5\}$ and $X_\varkappa = \{\varkappa_1, \varkappa_2, \varkappa_3, , \varkappa_3\}$.

$(F_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
ϱ_2	$(0.1, 0.9)$	$(0.3, 0.5)$	$(0.3, 0.7)$	$(0.1, 0.8)$
ϱ_3	$(0.2, 0.7)$	$(0.4, 0.4)$	$(0.4, 0.5)$	$(0.3, 0.5)$
ϱ_4	$(0.9, 0.1)$	$(0.5, 0.3)$	$(0.6, 0.4)$	$(0.6, 0.3)$

Table 3.29

$(G_\zeta, O_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
ϱ_1	$(0.2, 0.2)$	$(0.4, 0.4)$	$(0.3, 0.6)$	$(0.2, 0.5)$
ϱ_2	$(0.3, 0.4)$	$(0.4, 0.3)$	$(0.4, 0.5)$	$(0.4, 0.4)$
ϱ_3	$(0.4, 0.2)$	$(0.5, 0.3)$	$(0.6, 0.4)$	$(0.6, 0.3)$
ϱ_4	$(0.9, 0.1)$	$(0.7, 0.2)$	$(0.6, 0.3)$	$(0.8, 0.2)$
ϱ_5	$(0.1, 0.1)$	$(0.9, 0.1)$	$(0.66, 0.1)$	$(0.9, 0.1)$

Table 3.30

Then clearly $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$ and $(F_\zeta, L_\varkappa) \subseteq (G_\zeta, O_\varkappa)$ then extended union is defined as follows.

$\cup_{EXT}$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
ϱ_1	(0.2, 0.2)	(0.4, 0.4)	(0.3, 0.6)	(0.2, 0.5)
ϱ_2	(0.3, 0.4)	(0.4, 0.3)	(0.4, 0.5)	(0.4, 0.4)
ϱ_3	(0.4, 0.2)	(0.5, 0.3)	(0.6, 0.4)	(0.6, 0.3)
ϱ_4	(0.9, 0.1)	(0.7, 0.2)	(0.6, 0.3)	(0.8, 0.2)
ϱ_5	(0.1, 0.1)	(0.9, 0.1)	(0.66, 0.1)	(0.9, 0.1)

Table 3.31

which is $LOIFSS(X_\varkappa)$.

3.3.14 Definition

Suppose $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. Then extended Intersection of $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$ is denoted and defined by

$(F_\zeta, L_\varkappa) \cap_{EXT} (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$, where $P_\varkappa = L_\varkappa \cup O_\varkappa$ and $\forall\ \varrho \in P_\varkappa, \varkappa \in X_\varkappa$.

$$(H_\zeta, P_\varkappa) = \begin{cases} < T_{F_\zeta(\varrho)}(\varkappa), K_{F_\zeta(\varrho)}(\varkappa) > \text{ if } \varrho \in L_\varkappa - O_\varkappa \\ < T_{G_\zeta(\varrho)}(\varkappa), K_{G_\zeta(\varrho)}(\varkappa) > \text{ if } \varrho \in O_\varkappa - L_\varkappa \\ < Min\{T_{F_\zeta(\varrho)}(\varkappa), T_{G_\zeta(\varrho)}(\varkappa)\}, Max\{K_{F_\zeta(\varrho)}(\varkappa), K_{G_\zeta(\varrho)}(\varkappa) > \text{ if } \varrho \in L_\varkappa \cap O_\varkappa \end{cases}$$

3.3.15 Example

Suppose $X_\varkappa = \{\varkappa_1, \varkappa_2, \varkappa_3\}$ is a set of men under consideration and

$E_\varkappa = \{\varrho_1$(educated), ϱ_2(businessman), ϱ_3(smart), ϱ_4(government employee), ϱ_5(bank balance)$\}$ is the parameters set and $L_\varkappa$, $O_\varkappa \subseteq E_\varkappa$, and $L_\varkappa \subseteq O_\varkappa$, $L_\varkappa = \{\varrho_2, \varrho_3, \varrho_4\}$, $O_\varkappa = \{\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5\}$.

$(F_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$
ϱ_2	$(0.1, 0.9)$	$(0.2, 0.7)$	$(0.3, 0.7)$
ϱ_3	$(0.2, 0.7)$	$(0.3, 0.6)$	$(0.4, 0.6)$
ϱ_4	$(0.6, 0.3)$	$(0.6, 0.4)$	$(0.55, 0.4)$

Table 3.32

$(G_\zeta, O_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$
ϱ_1	$(0.2, 0.8)$	$(0.1, 0.9)$	$(0.3, 0.7)$
ϱ_2	$(0.3, 0.6)$	$(0.3, 0.4)$	$(0.5, 0.5)$
ϱ_3	$(0.5, 0.4)$	$(0.5, 0.4)$	$(0.6, 0.3)$
ϱ_4	$(0.7, 0.2)$	$(0.7, 0.3)$	$(0.6, 0.2)$
ϱ_5	$(0.8, 0.1)$	$(0.9, 0.1)$	$(0.9, 0.1)$

Table 3.33

Then clearly $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$ and $(F_\zeta, L_\varkappa) \subseteq (G_\zeta, O_\varkappa)$ then extended intersection is defined as follows.

$\cap_{EXT}$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$
ϱ_1	$(0.2, 0.8)$	$(0.1, 0.9)$	$(0.3, 0.7)$
ϱ_2	$(0.1, 0.9)$	$(0.2, 0.7)$	$(0.3, 0.7)$
ϱ_3	$(0.2, 0.7)$	$(0.3, 0.6)$	$(0.4, 0.6)$
ϱ_4	$(0.6, 0.3)$	$(0.6, 0.4)$	$(0.55, 0.4)$
ϱ_5	$(0.8, 0.1)$	$(0.9, 0.1)$	$(0.9, 0.1)$

Table 3.34

which is not a $LOIFSS(X_\varkappa)$.

3.3.16 Definition

Suppose $(F_\zeta, L_\varkappa) \in LOIFSS(X_\varkappa)$. Then it is known to be null $LOIFSS(X_\varkappa)$ with respect to parameter $L_\varkappa$, if $T_{F_\zeta(\varrho)} = 0$ and $K_{F_\zeta(\varrho)} = 1 \ \forall \ \varkappa \in X_\varkappa$ and $\varrho \in L_\varkappa$. It is denoted by $\emptyset_{L_\varkappa}$.

3.3.17 Definition

Suppose $(F_\zeta, L_\varkappa) \in LOIFSS(X_\varkappa)$. Then it is known to be relative whole $LOIFSS(X_\varkappa)$ witb respect to parameter $L_\varkappa$, if $T_{F_\zeta(\varrho)} = 1$ and $K_{F_\zeta(\varrho)} = 0 \ \forall \ \varkappa \in X_\varkappa$ and $\varrho \in L_\varkappa$. It is denoted by $U_{L_\varkappa}$.

3.3.18 Proposition

Suppose $(F_\zeta, L_\varkappa) \in LOIFSS(X_\varkappa)$. Then

1. $(F_\zeta, L_\varkappa) \cap_{RES} (F_\zeta, L_\varkappa) = (F_\zeta, L_\varkappa)$

2. $(F_\zeta, L_\varkappa) \cup_{RES} (F_\zeta, L_\varkappa) = (F_\zeta, L_\varkappa)$

3. $(F_\zeta, L_\varkappa) \cap_{RES} \emptyset_{L_\varkappa} = \emptyset_{L_\varkappa}$

4. $(F_\zeta, L_\varkappa) \cup_{RES} \emptyset_{L_\varkappa} = (F_\zeta, L_\varkappa)$

Proof. 01: Suppose $(F_\zeta, L_\varkappa) \cap_{RES} (F_\zeta, L_\varkappa) = (H_\zeta, P_\varkappa)$ where $P_\varkappa = L_\varkappa \cap L_\varkappa$.

Then by Definition 3.3.2 for $\varrho \in P_\varkappa$ and $\forall \ \varkappa \in X_\varkappa$

$T_{H_\zeta(\varrho)}(\varkappa) = Min\{T_{F_\zeta(\varrho)}(\varkappa), T_{F_\zeta(\varrho)}(\varkappa)\} = T_{F_\zeta(\varrho)}(\varkappa)$

$K_{H_\zeta(\varrho)}(\varkappa) = Max\{K_{F_\zeta(\varrho)}(\varkappa), K_{F_\zeta(\varrho)}(\varkappa)\} = K_{F_\zeta(\varrho)}(\varkappa)$

$\Rightarrow (H_\zeta, P_\varkappa) = (F_\zeta, L_\varkappa)$

$\Rightarrow (F_\zeta, L_\varkappa) \cap_{RES} (F_\zeta, L_\varkappa) = (F_\zeta, L_\varkappa)$

02: Suppose $(F_\zeta, L_\varkappa) \cup_{RES} (F_\zeta, L_\varkappa) = (H_\zeta, P_\varkappa)$ where $P_\varkappa = L_\varkappa \cap L_\varkappa$.

Then by Definition 3.3.6 for $\varrho \in P_\varkappa$ and $\forall \ \varkappa \in X_\varkappa$

$T_{H_\zeta(\varrho)}(\varkappa) = Max\{T_{F_\zeta(\varrho)}(\varkappa), T_{F_\zeta(\varrho)}(\varkappa)\} = T_{F_\zeta(\varrho)}(\varkappa)$

$K_{H_\zeta(\varrho)}(\varkappa) = Min\{K_{F_\zeta(\varrho)}(\varkappa), K_{F_\zeta(\varrho)}(\varkappa)\} = K_{F_\zeta(\varrho)}(\varkappa)$

$\Rightarrow (H_\zeta, P_\varkappa) = (F_\zeta, L_\varkappa)$

$\Rightarrow (F_\zeta, L_\varkappa) \cup_{RES} (F_\zeta, L_\varkappa) = (F_\zeta, L_\varkappa)$

03: Suppose $(F_\zeta, L_ϰ) \cap_{RES} (\emptyset, L_ϰ) = (H_\zeta, P_ϰ)$ where $P_ϰ = L_ϰ \cap L_ϰ$.

Then by Definition 3.3.6 for $\varrho \in P_ϰ$ and $\forall\, ϰ \in X_ϰ$

$T_{H_\zeta(\varrho)}(ϰ) = Min\{T_{F_\zeta(\varrho)}(ϰ), T_{\emptyset(\varrho)}(ϰ)\} = Min\{T_{F_\zeta(\varrho)}(ϰ), 0\} = 0 = T_{\emptyset(\varrho)}(ϰ)$

$K_{H_\zeta(\varrho)}(ϰ) = Max\{K_{F_\zeta(\varrho)}(ϰ), K_{\emptyset(\varrho)}(ϰ)\} = Max\{T_{F_\zeta(\varrho)}(ϰ), 1\} = 1 = K_{\emptyset(\varrho)}(ϰ)$

$\Rightarrow (H_\zeta, P_ϰ) = (\emptyset, L_ϰ)$

$\Rightarrow (F_\zeta, L_ϰ) \cap_{RES} \emptyset_{L_ϰ} = \emptyset_{L_ϰ}$

04: Suppose $(F_\zeta, L_ϰ) \cup_{RES} (\emptyset, L_ϰ) = (H_\zeta, P_ϰ)$ where $P_ϰ = L_ϰ \cap L_ϰ$.

Then by Definition of 3.3.2 for $\varrho \in P_ϰ$ and $\forall\, ϰ \in X_ϰ$

$T_{H_\zeta(\varrho)}(ϰ) = Max\{T_{F_\zeta(\varrho)}(ϰ), T_{\emptyset(\varrho)}(ϰ)\} = Max\{T_{F_\zeta(\varrho)}(ϰ), 0\} = T_{F_\zeta(\varrho)}(ϰ)$

$K_{H_\zeta(\varrho)}(ϰ) = Min\{K_{F_\zeta(\varrho)}(ϰ), K_{\emptyset(\varrho)}(ϰ)\} = Min\{T_{F_\zeta(\varrho)}(ϰ), 1\} = K_{F_\zeta(\varrho)}(ϰ)$

$\Rightarrow (H_\zeta, P_ϰ) = (\emptyset, L_ϰ)$

$\Rightarrow (F_\zeta, L_ϰ) \cup_{RES} \emptyset_{L_ϰ} = (F_\zeta, L_ϰ)$. ■

3.3.19 Definition

Suppose $(F_\zeta, L_ϰ) \in LOIFSS(X_ϰ)$. Then complement of $(F_\zeta, L_ϰ)$ denoted by $(F_\zeta, L_ϰ)^C$, and is defined by

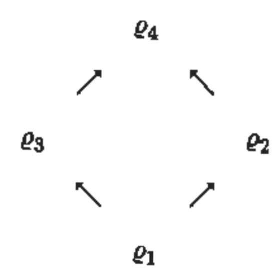
$(F_\zeta, L_ϰ)^C = \{ϰ, K_{F_\zeta(\varrho_i)}(ϰ), T_{F_\zeta(\varrho_i)}(ϰ)) : \varrho_i \in L_ϰ,\ ϰ \in X_ϰ\}$

tben $(F_\zeta, L_ϰ)^C$ is anti $LOIFSS(X_ϰ)$.

3.3.20 Example

Suppose $X_ϰ = \{ϰ_1,\ ϰ_2,\ ϰ_3,\ ϰ_4\}$ is a set of shops to buy toys and

$L_ϰ = \{\varrho_1(\text{cheap}),\ \varrho_2(\text{beautiful}),\ \varrho_3(\text{quality}),\ \varrho_4(\text{discount})\}$ is the parameters set. The order among the elements of $L_ϰ$ is given below



Then $LOIFSS(X_ϰ)$ and its compliment is shown in Table 3.35 and Table 3.36 respectively.

$\Rightarrow$

$$T_{F_\zeta^c(\varrho_2)}(\varkappa) \leq T_{F_\zeta^c(\varrho_1)}(\varkappa)$$
$$K_{F_\zeta^c(\varrho_1)}(\varkappa) \leq K_{F_\zeta^c(\varrho_2)}(\varkappa)$$

$\Rightarrow$

$$F_\zeta^c(\varrho_2) \subseteq F_\zeta^c(\varrho_1)$$

$\Rightarrow (F_\zeta, L_\varkappa)^c$ is anti $LOIFSS(X_\varkappa)$. ■

3.3.22 Proposition

Suppose $(F_\zeta, L_\varkappa) \in LOIFSS(X_\varkappa)$. Then$((F_\zeta, L_\varkappa)^c)^c = (F_\zeta, L_\varkappa)$.

Proof. Since $(F_\zeta, L_\varkappa) \in LOIFSS(X_\varkappa)$. Then complement of $(F_\zeta, L_\varkappa)$ is

$T_{F_\zeta^c(\varrho)}(\varkappa) = K_{F_\zeta(\varrho)}(\varkappa)$

$K_{F_\zeta^c(\varrho)}(\varkappa) = T_{F_\zeta(\varrho)}(\varkappa)$

Now the complement of $(F_\zeta, L_\varkappa)^C$ is

$T_{(F_\zeta^c)^c(\varrho)}(\varkappa) = K_{F_\zeta^c(\varrho)}(\varkappa) = T_{F_\zeta(\varrho)}(\varkappa)$

$K_{(F_\zeta^c(\varrho))^c}(\varkappa) = T_{F_\zeta^c(\varrho)}(\varkappa) = K_{F_\zeta(\varrho)}(\varkappa)$

$\Rightarrow ((F_\zeta, L_\varkappa)^c)^c = (F_\zeta, L_\varkappa)$. ■

3.3.23 Proposition

Suppose $(F_\zeta, L_\varkappa), (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. Then $(F_\zeta, L_\varkappa) \cap_{RES} (G_\zeta, O_\varkappa)$ is largest $LOIFSS(X_\varkappa)$w is contain in $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$.

Proof. As $(F_\zeta, L_\varkappa) \cap_{RES} (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$ where $P_\varkappa = L_\varkappa \cap O_\varkappa \neq \emptyset$

and for $p \in P_\varkappa$

we have $T_{H_\zeta(p)}(\varkappa) = Min\{T_{F_\zeta(p)}(\varkappa), T_{G_\zeta(p)}(\varkappa)\}$

$K_{H_\zeta(p)}(\varkappa) = Max\{K_{F_\zeta(p)}(\varkappa), K_{G_\zeta(p)}(\varkappa)\}$

then $T_{H_\zeta(p)}(\varkappa) \leq T_{F_\zeta(p)}(\varkappa)$ and $T_{H_\zeta(p)}(\varkappa) \leq T_{G_\zeta(p)}(\varkappa)$

also $K_{F_\zeta(p)}(\varkappa) \leq K_{H_\zeta(p)}(\varkappa)$ and $K_{G_\zeta(p)}(\varkappa) \leq K_{H_\zeta(p)}(\varkappa)$

$\Rightarrow H_\zeta(p) \subseteq F_\zeta(p)$ and $H_\zeta(p) \subseteq G_\zeta(p)$

then $H_\zeta(p) \subseteq F_\zeta(p) \cap_{RES} G_\zeta(p)$

$H_\zeta(p) \subseteq (F_\zeta \cap_{RES} G_\zeta)(p)$

thus $(H_\zeta, P_\varkappa) \subseteq (F_\zeta, L_\varkappa) \cap_{RES} (G_\zeta, O_\varkappa)$

Suppose $(Y_\zeta, D_\varkappa)$ is another $LOIFSS(X_\varkappa)$ contained in both $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$. Then

$Y_\zeta(p) \subseteq F_\zeta(p) \cap_{RES} G_\zeta(p)$

$Y_\zeta(p) \subseteq (F_\zeta \cap_{RES} G_\zeta)(p)$

$Y_\zeta(p) \subseteq (H_\zeta, P_\varkappa)\ \forall\ p \in D_\varkappa$, where $D_\varkappa = L_\varkappa \cap O_\varkappa \neq \emptyset$

$\Rightarrow (Y_\zeta, D_\varkappa) \subseteq (H_\zeta, P_\varkappa)\ \forall\ p \in P_\varkappa,$

$\Rightarrow (H_\zeta, P_\varkappa)$ is largest $LOIFSS(X_\varkappa)$ that contain in $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$. ■

3.3.24 Definition

Suppose $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in IFSS(X_\varkappa)$. Then $(F_\zeta, L_\varkappa) \cap_\wedge (G_\zeta, O_\varkappa) = (H_\zeta, L_\varkappa \otimes O_\varkappa)$, defined as $H_\zeta(l \wedge o) = F_\zeta(l) \cap_{RES} G_\zeta(o)\ \forall\ l \wedge o \in L_\varkappa \otimes O_\varkappa$.

3.3.25 Definition

Suppose $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in IFSS(X_\varkappa)$. Then $(F_\zeta, L_\varkappa) \cup_{\underline{\vee}} (G_\zeta, O_\varkappa) = (H_\zeta, L_\varkappa \oplus O_\varkappa)$, defined as $H_\zeta(l \vee o) = F_\zeta(l) \cup_{RES} G_\zeta(o)\ \forall\ l \wedge o \in L_\varkappa \oplus O_\varkappa$.

3.3.26 Proposition

Suppose $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. Then $(F_\zeta, L_\varkappa) \cap_\wedge (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$.

Proof. Since $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$ (i.e $\forall\ l_1,\ l_2 \in L_\varkappa$), $F_\zeta(l_1) \subseteq F_\zeta(l_2)$, whenever $l_1 \leq_{L_\varkappa} l_2$ and $\forall\ o_1,\ o_2 \in O_\varkappa$, $G_\zeta(o_1) \subseteq G_\zeta(o_2)$, whenever $o_1 \leq_{O_\varkappa} o_2$. Then

we have to prove that $(K, L_\varkappa) \cap_\wedge (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$.

As $(F_\zeta, L_\varkappa) \cap_\wedge (G_\zeta, O_\varkappa) = (H_\zeta, L_\varkappa \otimes O_\varkappa)$

defined as $H_\zeta(l_1 \wedge o_1) = F_\zeta(l_1) \cap G_\zeta(o_1)\ \forall\ (l_1 \wedge o_2) \in L_\varkappa \otimes O_\varkappa$

and $H_\zeta(l_2 \wedge o_2) = F_\zeta(l_2) \cap G_\zeta(o_2)$

$F_\zeta(l_1) \subseteq F_\zeta(l_2)$ and $G_\zeta(o_1) \subseteq G_\zeta(o_2)$

03: Suppose $(F_\zeta, L_\varkappa) \cap_{RES} (\emptyset, L_\varkappa) = (H_\zeta, P_\varkappa)$ where $P_\varkappa = L_\varkappa \cap L_\varkappa$.

Then by Definition 3.3.6 for $\varrho \in P_\varkappa$ and $\forall\ \varkappa \in X_\varkappa$

$T_{H_\zeta(\varrho)}(\varkappa) = Min\{T_{F_\zeta(\varrho)}(\varkappa), T_{\emptyset(\varrho)}(\varkappa)\} = Min\{T_{F_\zeta(\varrho)}(\varkappa), 0\} = 0 = T_{\emptyset(\varrho)}(\varkappa)$

$K_{H_\zeta(\varrho)}(\varkappa) = Max\{K_{F_\zeta(\varrho)}(\varkappa), K_{\emptyset(\varrho)}(\varkappa)\} = Max\{T_{F_\zeta(\varrho)}(\varkappa), 1\} = 1 = K_{\emptyset(\varrho)}(\varkappa)$

$\Rightarrow (H_\zeta, P_\varkappa) = (\emptyset, L_\varkappa)$

$\Rightarrow (F_\zeta, L_\varkappa) \cap_{RES} \emptyset_{L_\varkappa} = \emptyset_{L_\varkappa}$

04: Suppose $(F_\zeta, L_\varkappa) \cup_{RES} (\emptyset, L_\varkappa) = (H_\zeta, P_\varkappa)$ where $P_\varkappa = L_\varkappa \cap L_\varkappa$.

Then by Definition of 3.3.2 for $\varrho \in P_\varkappa$ and $\forall\ \varkappa \in X_\varkappa$

$T_{H_\zeta(\varrho)}(\varkappa) = Max\{T_{F_\zeta(\varrho)}(\varkappa), T_{\emptyset(\varrho)}(\varkappa)\} = Max\{T_{F_\zeta(\varrho)}(\varkappa), 0\} = T_{F_\zeta(\varrho)}(\varkappa)$

$K_{H_\zeta(\varrho)}(\varkappa) = Min\{K_{F_\zeta(\varrho)}(\varkappa), K_{\emptyset(\varrho)}(\varkappa)\} = Min\{T_{F_\zeta(\varrho)}(\varkappa), 1\} = K_{F_\zeta(\varrho)}(\varkappa)$

$\Rightarrow (H_\zeta, P_\varkappa) = (\emptyset, L_\varkappa)$

$\Rightarrow (F_\zeta, L_\varkappa) \cup_{RES} \emptyset_{L_\varkappa} = (F_\zeta, L_\varkappa)$. ■

3.3.19 Definition

Suppose $(F_\zeta, L_\varkappa) \in LOIFSS(X_\varkappa)$. Then complement of $(F_\zeta, L_\varkappa)$ denoted by $(F_\zeta, L_\varkappa)^C$, and is defined by

$(F_\zeta, L_\varkappa)^C = \{\varkappa, K_{F_\zeta(\varrho_i)}(\varkappa), T_{F_\zeta(\varrho_i)}(\varkappa)) : \varrho_i \in L_\varkappa,\ \varkappa \in X_\varkappa\}$

then $(F_\zeta, L_\varkappa)^C$ is anti $LOIFSS(X_\varkappa)$.

3.3.20 Example

Suppose $X_\varkappa = \{\varkappa_1,\ \varkappa_2,\ \varkappa_3,\ \varkappa_4\}$ is a set of shops to buy toys and

$L_\varkappa = \{\varrho_1$(cheap), ϱ_2(heautiful), ϱ_3(quality), ϱ_4(discount)$\}$ is the parameters set. The order among the elements of $L_\varkappa$ is given below

$$\begin{array}{ccc} & \varrho_4 & \\ \nearrow & & \nwarrow \\ \varrho_3 & & \varrho_2 \\ \nwarrow & & \nearrow \\ & \varrho_1 & \end{array}$$

Then $LOIFSS(X_\varkappa)$ and its compliment is shown in Table 3.35 and Tahle 3.36 respectively.

$(F_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
ϱ_1	$(0.1, 0.9)$	$(0.2, 0.7)$	$(0.3, 0.7)$	$(0.5, 0.4)$
ϱ_2	$(0.2, 0.7)$	$(0.3, 0.6)$	$(0.4, 0.6)$	$(0.56, 0.3)$
ϱ_3	$(0.3, 0.5)$	$(0.6, 0.4)$	$(0.55, 0.4)$	$(0.6, 0.2)$
ϱ_4	$(0.4, 0.45)$	$(0.9, 0.1)$	$(0.9, 0.1)$	$(0.8, 0.1)$

Table 3.35

$(F_\zeta, L_\varkappa)^C$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
ϱ_1	$(0.9, 0.1)$	$(0.7, 0.2)$	$(0.7, 0.3)$	$(0.4, 0.5)$
ϱ_2	$(0.7, 0.2)$	$(0.6, 0.3)$	$(0.6, 0.4)$	$(0.3, 0.56)$
ϱ_3	$(0.5, 0.3)$	$(0.4, 0.6)$	$(0.4, 0.55)$	$(0.2, 0.6)$
ϱ_4	$(0.45, 0.4)$	$(0.1, 0.9)$	$(0.1, 0.9)$	$(0.1, 0.8)$

Table 3.36

which is clearly an anti $LOIFSS(X_\varkappa)$.

3.3.21 Proposition

Suppose $(F_\zeta, L_\varkappa) \in LOIFSS(X_\varkappa)$. Then complement of $(F_\zeta, L_\varkappa)$ is an Anti $LOIFSS(X_\varkappa)$.

Proof. Since $(F_\zeta, L_\varkappa) \in LOIFSS(X_\varkappa)$. Then for $\varrho_1 \leq_{L_\varkappa} \varrho_2$

$\Rightarrow$

$$F_\zeta(\varrho_1) \subseteq F_\zeta(\varrho_2)$$

$\Rightarrow$

$$T_{F_\zeta(\varrho_1)}(\varkappa) \leq T_{F_\zeta(\varrho_2)}(\varkappa)$$
$$K_{F_\zeta(\varrho_2)}(\varkappa) \leq K_{F_\zeta(\varrho_1)}(\varkappa)$$

$\Rightarrow$

$$T_{F_\zeta^c(\varrho_2)}(\varkappa) \leq T_{F_\zeta^c(\varrho_1)}(\varkappa)$$
$$K_{F_\zeta^c(\varrho_1)}(\varkappa) \leq K_{F_\zeta^c(\varrho_2)}(\varkappa)$$

$\Rightarrow$

$$F_\zeta^c(\varrho_2) \subseteq F_\zeta^c(\varrho_1)$$

$\Rightarrow (F_\zeta, L_\varkappa)^c$ is anti $LOIFSS(X_\varkappa)$. ∎

3.3.22 Proposition

Suppose $(F_\zeta, L_\varkappa) \in LOIFSS(X_\varkappa)$. Then$((F_\zeta, L_\varkappa)^c)^c = (F_\zeta, L_\varkappa)$.

Proof. Since $(F_\zeta, L_\varkappa) \in LOIFSS(X_\varkappa)$. Then complement of $(F_\zeta, L_\varkappa)$ is

$T_{F_\zeta^c(\varrho)}(\varkappa) = K_{F_\zeta(\varrho)}(\varkappa)$

$K_{F_\zeta^c(\varrho)}(\varkappa) = T_{F_\zeta(\varrho)}(\varkappa)$

Now the complement of $(F_\zeta, L_\varkappa)^C$ is

$T_{(F_\zeta^c)^c(\varrho)}(\varkappa) = K_{F_\zeta^c(\varrho)}(\varkappa) = T_{F_\zeta(\varrho)}(\varkappa)$

$K_{(F_\zeta^c(\varrho))^c}(\varkappa) = T_{F_\zeta^c(\varrho)}(\varkappa) = K_{F_\zeta(\varrho)}(\varkappa)$

$\Rightarrow ((F_\zeta, L_\varkappa)^c)^c = (F_\zeta, L_\varkappa)$. ∎

3.3.23 Proposition

Suppose $(F_\zeta, L_\varkappa), (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. Then $(F_\zeta, L_\varkappa) \cap_{RES} (G_\zeta, O_\varkappa)$ is largest $LOIFSS(X_\varkappa)$w is contain in $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$.

Proof. As $(F_\zeta, L_\varkappa) \cap_{RES} (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$ where $P_\varkappa = L_\varkappa \cap O_\varkappa \neq \emptyset$

and for $p \in P_\varkappa$

we have $T_{H_\zeta(p)}(\varkappa) = Min\{T_{F_\zeta(p)}(\varkappa), T_{G_\zeta(p)}(\varkappa)\}$

$K_{H_\zeta(p)}(\varkappa) = Max\{K_{F_\zeta(p)}(\varkappa), K_{G_\zeta(p)}(\varkappa)\}$

then $T_{H_\zeta(p)}(\varkappa) \leq T_{F_\zeta(p)}(\varkappa)$ and $T_{H_\zeta(p)}(\varkappa) \leq T_{G_\zeta(p)}(\varkappa)$

also $K_{F_\zeta(p)}(\varkappa) \leq K_{H_\zeta(p)}(\varkappa)$ and $K_{G_\zeta(p)}(\varkappa) \leq K_{H_\zeta(p)}(\varkappa)$

$\Rightarrow H_\zeta(p) \subseteq F_\zeta(p)$ and $H_\zeta(p) \subseteq G_\zeta(p)$

then $H_\zeta(p) \subseteq F_\zeta(p) \cap_{RES} G_\zeta(p)$

$H_\zeta(p) \subseteq (F_\zeta \cap_{RES} G_\zeta)(p)$

thus $(H_\zeta, P_\varkappa) \subseteq (F_\zeta, L_\varkappa) \cap_{RES} (G_\zeta, O_\varkappa)$

Suppose $(Y_\zeta, D_\varkappa)$ is another $LOIFSS(X_\varkappa)$ contained in both $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$. Then

$Y_\zeta(p) \subseteq F_\zeta(p) \cap_{RES} G_\zeta(p)$

$Y_\zeta(p) \subseteq (F_\zeta \cap_{RES} G_\zeta)(p)$

$Y_\zeta(p) \subseteq (H_\zeta, P_\varkappa)\ \forall\ p \in D_\varkappa$, where $D_\varkappa = L_\varkappa \cap O_\varkappa \neq \emptyset$

$\Rightarrow (Y_\zeta, D_\varkappa) \subseteq (H_\zeta, P_\varkappa)\ \forall\ p \in P_\varkappa$,

$\Rightarrow (H_\zeta, P_\varkappa)$ is largest $LOIFSS(X_\varkappa)$ that contain in $(F_\zeta, L_\varkappa)$ and $(G_\zeta, O_\varkappa)$. ■

3.3.24 Definition

Suppose $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in IFSS(X_\varkappa)$. Then $(F_\zeta, L_\varkappa) \cap_\wedge (G_\zeta, O_\varkappa) = (H_\zeta, L_\varkappa \otimes O_\varkappa)$, defined as $H_\zeta(l \wedge o) = F_\zeta(l) \cap_{RES} G_\zeta(o)\ \forall\ l \wedge o \in L_\varkappa \otimes O_\varkappa$.

3.3.25 Definition

Suppose $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in IFSS(X_\varkappa)$. Then $(F_\zeta, L_\varkappa) \cup_{\underline{\vee}} (G_\zeta, O_\varkappa) = (H_\zeta, L_\varkappa \oplus O_\varkappa)$, defined as $H_\zeta(l \vee o) = F_\zeta(l) \cup_{RES} G_\zeta(o)\ \forall\ l \wedge o \in L_\varkappa \oplus O_\varkappa$.

3.3.26 Proposition

Suppose $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. Then $(F_\zeta, L_\varkappa) \cap_\wedge (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$.

Proof. Since $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$ (i.e $\forall\ l_1,\ l_2 \in L_\varkappa$), $F_\zeta(l_1) \subseteq F_\zeta(l_2)$, whenever $l_1 \leq_{L_\varkappa} l_2$ and $\forall\ o_1,\ o_2 \in O_\varkappa$, $G_\zeta(o_1) \subseteq G_\zeta(o_2)$, whenever $o_1 \leq_{O_\varkappa} o_2$. Then

we have to prove that $(K, L_\varkappa) \cap_\wedge (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$.

As $(F_\zeta, L_\varkappa) \cap_\wedge (G_\zeta, O_\varkappa) = (H_\zeta, L_\varkappa \otimes O_\varkappa)$

defined as $H_\zeta(l_1 \wedge o_1) = F_\zeta(l_1) \cap G_\zeta(o_1)\ \forall\ (l_1 \wedge o_2) \in L_\varkappa \otimes O_\varkappa$

and $H_\zeta(l_2 \wedge o_2) = F_\zeta(l_2) \cap G_\zeta(o_2)$

$F_\zeta(l_1) \subseteq F_\zeta(l_2)$ and $G_\zeta(o_1) \subseteq G_\zeta(o_2)$

$\Rightarrow$

$$\begin{aligned} T_{F_\zeta(l_1)}(\varkappa) &\leq T_{F_\zeta(l_2)}(\varkappa) & T_{G_\zeta(o_1)}(\varkappa) \leq T_{G_\zeta(o_2)}(\varkappa)\ \forall\ \varkappa \in X_\varkappa \\ K_{F_\zeta(l_2)}(\varkappa) &\leq K_{F_\zeta(l_1)}(\varkappa) & K_{G_\zeta(o_2)}(\varkappa) \leq K_{G_\zeta(o_1)}(\varkappa)\ \forall\ \varkappa \in X_\varkappa \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} T_{F_\zeta(l_1)}(\varkappa) \cap T_{G_\zeta(o_1)}(\varkappa) &\leq T_{F_\zeta(l_2)}(\varkappa) \cap T_{G_\zeta(o_2)}(\varkappa) \\ K_{F_\zeta(l_2)}(\varkappa) \cap K_{G_\zeta(o_2)}(\varkappa) &\leq K_{F_\zeta(l_1)}(\varkappa) \cap K_{G_\zeta(o_1)}(\varkappa) \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} (T_{F_\zeta(l_1)} \cap T_{G_\zeta(o_1)})(\varkappa) &\leq (T_{F_\zeta(l_2)} \cap T_{G_\zeta(o_2)})(\varkappa) \\ (K_{F_\zeta(l_2)} \cap K_{G_\zeta(o_2)})(\varkappa) &\leq (K_{F_\zeta(l_1)} \cap K_{G_\zeta(o_1)})(\varkappa) \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} (T_{F_\zeta(l_1) \cap G_\zeta(o_1)})(\varkappa) &\leq (T_{F_\zeta(l_2) \cap G_\zeta(o_2)})(\varkappa) \\ (K_{F_\zeta(l_2) \cap G_\zeta(o_2)})(\varkappa) &\leq (K_{F_\zeta(l_1) \cap G_\zeta(o_1)})(\varkappa) \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} T_{H_\zeta(l_1 \cap o_1)}(\varkappa) &\leq T_{H_\zeta(l_2 \cap o_2)}(\varkappa) \\ K_{H_\zeta(l_2 \cap o_2)}(\varkappa) &\leq K_{H_\zeta(l_1 \cap o_1)}(\varkappa) \end{aligned}$$

$\Rightarrow H_\zeta(l_1 \wedge o_1) \subseteq H_\zeta(l_2 \wedge o_2)\ \forall\ l_1 \leq_{L_\varkappa} l_2,\ o_1 \leq_{O_\varkappa} o_2,\ l_1 o_1 \leq l_2 \wedge o_2$ for $l_1 \wedge o_1,\ l_2 \wedge o_2 \in L_\varkappa \otimes O_\varkappa$.

$\Rightarrow (F_\zeta, L_\varkappa) \cap_\wedge (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. ∎

3.3.27 Proposition

If $(F_\zeta, L_\varkappa), (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. Then $(F_\zeta, L_\varkappa) \cup_{\underline{\vee}} (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$.

Proof. If $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$ (i.e $\forall\ l_1,\ l_2 \in L_\varkappa$) $F_\zeta(l_1) \subseteq F_\zeta(l_2)$, whenever $l_1 \leq_{L_\varkappa} l_2$ and $\forall\ o_1, o_2 \in O_\varkappa$, $G_\zeta(o_1) \subseteq G_\zeta(o_2)$, whenever $o_1 \leq_{O_\varkappa} o_2$. Then we have to prove that $(F_\zeta, L_\varkappa) \cup_\vee (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$.

As $(F_\zeta, L_\varkappa) \cup_\vee (G_\zeta, O_\varkappa) = (H_\zeta, L_\varkappa \oplus O_\varkappa)$

defined as $H_\zeta(l_1 \vee o_1) = F_\zeta(l_1) \cup G_\zeta(o_1) \;\forall\; (l_1 \vee o_2) \in L_\varkappa \oplus O_\varkappa$

and $H_\zeta(l_2 \vee o_2) = F_\zeta(l_2) \cup G_\zeta(o_2)$

$F_\zeta(l_1) \subseteq F_\zeta(l_2)$ and $G_\zeta(o_1) \subseteq G_\zeta(o_2)$

$\Rightarrow$

$$\begin{aligned} T_{F_\zeta(l_1)}(\varkappa) &\leq T_{F_\zeta(l_2)}(\varkappa) & T_{G_\zeta(o_1)}(\varkappa) \leq T_{G_\zeta(o_2)}(\varkappa) \;\forall\; \varkappa \in X_\varkappa \\ K_{F_\zeta(l_2)}(\varkappa) &\leq K_{F_\zeta(l_1)}(\varkappa) & K_{G_\zeta(o_2)}(\varkappa) \leq K_{G_\zeta(o_1)}(\varkappa) \;\forall\; \varkappa \in X_\varkappa \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} T_{F_\zeta(l_1)}(\varkappa) \cup T_{G_\zeta(o_1)}(\varkappa) &\leq T_{F_\zeta(l_2)}(\varkappa) \cup T_{G_\zeta(o_2)}(\varkappa) \\ K_{F_\zeta(l_2)}(\varkappa) \cup K_{G_\zeta(o_2)}(\varkappa) &\leq K_{F_\zeta(l_1)}(\varkappa) \cup K_{G_\zeta(o_1)}(\varkappa) \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} (T_{F_\zeta(l_1)} \cup T_{G_\zeta(o_1)})(\varkappa) &\leq (T_{F_\zeta(l_2)} \cup T_{G_\zeta(o_2)})(\varkappa) \\ (K_{F_\zeta(l_2)} \cup K_{G_\zeta(o_2)})(\varkappa) &\leq (K_{F_\zeta(l_1)} \cup K_{G_\zeta(o_1)})(\varkappa) \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} (T_{F_\zeta(l_1) \cup G_\zeta(o_1)})(\varkappa) &\leq (T_{F_\zeta(l_2) \cup G_\zeta(o_2)})(\varkappa) \\ (K_{F_\zeta(l_2) \cup G_\zeta(o_2)})(\varkappa) &\leq (K_{F_\zeta(l_1) \cup G_\zeta(o_1)})(\varkappa) \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} T_{H_\zeta(l_1 \cup o_1)}(\varkappa) &\leq T_{H_\zeta(l_2 \cup o_2)}(\varkappa) \\ K_{H_\zeta(l_2 \cup o_2)}(\varkappa) &\leq K_{H_\zeta(l_1 \cup o_1)}(\varkappa) \end{aligned}$$

$\Rightarrow H_\zeta(l_1, o_1) \subseteq H_\zeta(l_2, o_2) \;\forall\; l_1 \leq l_2,\; o_1 \leq o_2,\; l_1 \vee o_1 \leq l_2 \vee o_2$ for $l_1 \vee o_1,\; l_2 \vee o_2 \in L_\varkappa \oplus O_\varkappa$.

$\Rightarrow (F_\zeta \vee L_\varkappa) \cup_{\underline{\vee}} (G_\zeta \vee O_\varkappa) \in LOIFSS(X_\varkappa)$. ■

3.3.28 Definition

Suppose $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. Then $(F_\zeta, L_\varkappa) \vee (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$ is known to be basic union of two $LOIFSS(X_\varkappa)$, where $P_\varkappa = L_\varkappa \times O_\varkappa$ and $H_\zeta(l, o) = F_\zeta(l) \cup G_\zeta(o)$.

$T_{H_\zeta(l,o)}(\varkappa) = Max\{T_{H_\zeta(l)}(\varkappa), T_{G_\zeta(o)}(\varkappa)\}$

$K_{H_\zeta(l,o)}(\varkappa) = Min\{T_{H_\zeta(l)}(\varkappa), T_{G_\zeta(o)}(\varkappa)\}\ \forall\ (l,o) \in P_\varkappa,\ \varkappa \in X_\varkappa.$

3.3.29 Theorem

Suppose $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. Then $(F_\zeta, L_\varkappa) \vee (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$.

Proof. Suppose $(F_\zeta, L_\varkappa), (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. Then $(F_\zeta, L_\varkappa) \vee (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$,where $P_\varkappa = L_\varkappa \times O_\varkappa$

Since $L_\varkappa, O_\varkappa \subseteq E_\varkappa$ so both $L_\varkappa$ and $O_\varkappa$ inherit partial order from $E_\varkappa$ also $H_\zeta(l,o) = F_\zeta(l) \vee G_\zeta(o) = F_\zeta(l) \cup_{RES} G_\zeta(o)$

Now $l_1 \leq_{L_\varkappa} l_2$ we have $F_\zeta(l_1) \subseteq F_\zeta(l_2)\ \forall\ l_1, l_2 \in L_\varkappa$

also for $o_1 \leq_{O_\varkappa} o_2$,we have $G_\zeta(o_1) \subseteq G_\zeta(o_2)\ \forall\ o_1, o_2 \in O_\varkappa$.

Now for any $(l_1, o_1), (l_2, o_2) \in P_\varkappa$ and $\leq$ is partial order on $P_\varkappa$ which is induced by partial orders on $L_\varkappa$ and $O_\varkappa$.

The order on $L_\varkappa \times O_\varkappa$ is $(l_1, o_1) \leq (l_2, o_2)$,whenever $l_1 \leq_{L_\varkappa} l_2$ and $o_1 \leq_{O_\varkappa} o_2$.

$\Rightarrow$

$$F_\zeta(l_1) \subseteq F_\zeta(l_2) \qquad \text{and} \qquad G_\zeta(o_1) \subseteq G_\zeta(o_2)$$

$\Rightarrow$

$$\begin{aligned} T_{F_\zeta(l_1)}(\varkappa) &\leq T_{F_\zeta(l_2)}(\varkappa), \qquad & T_{G_\zeta(o_1)}(\varkappa) \leq T_{G_\zeta(o_2)}(\varkappa) \\ K_{F_\zeta(l_2)}(\varkappa) &\leq K_{F_\zeta(l_1)}(\varkappa), \qquad & K_{G_\zeta(o_2)}(\varkappa) \leq K_{G_\zeta(o_1)}(\varkappa) \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} T_{F_\zeta(l_1)}(\varkappa) \vee T_{G_\zeta(o_1)}(\varkappa) &\leq T_{F_\zeta(l_2)}(\varkappa) \vee T_{G_\zeta(o_2)}(\varkappa) \\ K_{F_\zeta(l_2)}(\varkappa) \vee K_{G_\zeta(o_2)}(\varkappa) &\leq K_{F_\zeta(l_1)}(\varkappa) \vee K_{G_\zeta(o_1)}(\varkappa) \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} T_{(F_\zeta(l_1) \vee G_\zeta(o_1))}(\varkappa) &\leq T_{(F_\zeta(l_2) \vee G_\zeta(o_2))}(\varkappa) \\ K_{(F_\zeta(l_2) \vee G_\zeta(o_2))}(\varkappa) &\leq K_{(F_\zeta(l_1) \vee G_\zeta(o_1))}(\varkappa) \end{aligned}$$

⇒

$$T_{H_\zeta(l_1,o_1)}(\varkappa) \leq T_{H_\zeta(l_2,o_2)}(\varkappa)$$
$$K_{H_\zeta(l_2,o_2)}(\varkappa) \leq K_{H_\zeta(l_1,o_1)}(\varkappa)$$

⇒

$H_\zeta(l_1,o_1) \subseteq H_\zeta(l_2,o_2)\ \forall\ (l_1,o_1) \leq (l_2,o_2)$

thus $(F_\zeta, L_\varkappa) \vee (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. ■

3.3.30 Example

Suppose $X_\varkappa = \{\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4\}$ and $E_\varkappa = \{\varrho_1, \varrho_2, \varrho_3, \varrho_4\}$ is the parameters set and $L_\varkappa, O_\varkappa \subseteq E_\varkappa, L_\varkappa = \{\varrho_1, \varrho_2, \varrho_3\}, O_\varkappa = \{\varrho_2, \varrho_3, \varrho_4\}$.

$(F_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
ϱ_1	(0.1, 0.9)	(0.2, 0.7)	(0.3, 0.7)	(0.5, 0.4)
ϱ_2	(0.2, 0.7)	(0.3, 0.6)	(0.4, 0.6)	(0.56, 0.3)
ϱ_3	(0.3, 0.5)	(0.6, 0.4)	(0.55, 0.4)	(0.6, 0.2)

Table 3.37

$(G_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
ϱ_2	(0.2, 0.8)	(0.4, 0.3)	(0.1, 0.6)	(0.5, 0.3)
ϱ_3	(0.4, 0.4)	(0.7, 0.2)	(0.4, 0.4)	(0.7, 0.2)
ϱ_4	(0.6, 0.2)	(0.9, 0.1)	(0.8, 0.2)	(0.9, 0.1)

Table 3.38

Then $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$ and $(F_\zeta, L_\varkappa) \vee (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$ is known to be basic union of two $LOIFSS(X_\varkappa)$.

$(H_\zeta, P_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
(ϱ_1, ϱ_2)	$(0.2, 0.8)$	$(0.4, 0.3)$	$(0.3, 0.6)$	$(0.5, 0.3)$
(ϱ_1, ϱ_3)	$(0.4, 0.4)$	$(0.7, 0.2)$	$(0.4, 0.4)$	$(0.7, 0.2)$
(ϱ_1, ϱ_4)	$(0.6, 0.2)$	$(0.9, 0.1)$	$(0.8, 0.2)$	$(0.9, 0.1)$
(ϱ_2, ϱ_2)	$(0.2, 0.7)$	$(0.4, 0.3)$	$(0.4, 0.4)$	$(0.7, 0.2)$
(ϱ_2, ϱ_3)	$(0.4, 0.4)$	$(0.7, 0.2)$	$(0.4, 0.4)$	$(0.7, 0.2)$
(ϱ_2, ϱ_4)	$(0.6, 0.2)$	$(0.9, 0.1)$	$(0.8, 0.2)$	$(0.9, 0.1)$
(ϱ_3, ϱ_2)	$(0.3, 0.5)$	$(0.6, 0.3)$	$(0.55, 0.4)$	$(0.6, 0.2)$
(ϱ_3, ϱ_3)	$(0.4, 0.4)$	$(0.7, 0.2)$	$(0.55, 0.4)$	$(0.7, 0.2)$
(ϱ_3, ϱ_4)	$(0.6, 0.2)$	$(0.9, 0.1)$	$(0.8, 0.2)$	$(0.9, 0.1)$

Table 3.39

We observe that $(H_\zeta, P_\varkappa) \in LOIFSS(X_\varkappa)$.

3.3.31 Definition

Suppose $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. Then $(F_\zeta, L_\varkappa) \wedge (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$ is known to be basic intersection of two $LOIFSS(X_\varkappa)$, where $P_\varkappa = L_\varkappa \times O_\varkappa$ and $H_\zeta(l, o) = F_\zeta(l) \cap G_\zeta(o)$.

$T_{H_\zeta(l,o)}(\varkappa) = Min\{T_{H_\zeta(l)}(\varkappa), T_{G_\zeta(o)}(\varkappa)\}$

$K_{H_\zeta(l,o)}(\varkappa) = Max\{T_{H_\zeta(l)}(\varkappa), T_{G_\zeta(o)}(\varkappa)\}\ \forall\ (l, o) \in P_\varkappa, \varkappa \in X_\varkappa$.

3.3.32 Theorem

Suppose $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. Then $(F_\zeta, L_\varkappa) \wedge (G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$..

Proof. Suppose $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$. Then $(F_\zeta, L_\varkappa) \wedge (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$,where $P_\varkappa = L_\varkappa \times O_\varkappa$

Since $L_\varkappa, O_\varkappa \subseteq E_\varkappa$ so hoth $L_\varkappa$ and $O_\varkappa$ inherit partial order from $E_\varkappa$ also $H_\zeta(l, o) = F_\zeta(l) \vee G_\zeta(o) = F_\zeta(l) \cup_{RES} G_\zeta(o)$

Now $l_1 \leq_{L_\varkappa} l_2$ we have $F_\zeta(l_1) \subseteq F_\zeta(l_2)\ \forall\ l_1, l_2 \in L_\varkappa$

also for $o_1 \leq_{O_\varkappa} o_2$ we have $G_\zeta(o_1) \subseteq G_\zeta(o_2)\ \forall\ o_1, o_2 \in O_\varkappa$

Now for any$(l_1, o_1), (l_2, o_2) \in P_{\varkappa}$ and $\leq$ is partial order on $P_{\varkappa}$ which is induced by partial orders on $L_{\varkappa}$ and $O_{\varkappa}$.

The order on $L_{\varkappa} \times O_{\varkappa}$ is $(l_1, o_1) \leq (l_2, o_2)$,whenever $l_1 \leq_{L_{\varkappa}} l_2$ and $o_1 \leq_{O_{\varkappa}} o_2$.

$\Rightarrow$

$$F_{\zeta}(l_1) \subseteq F_{\zeta}(l_2) \qquad \text{and} \qquad G_{\zeta}(o_1) \subseteq G_{\zeta}(o_2)$$

$\Rightarrow$

$$\begin{aligned} T_{F_{\zeta}(l_1)}(\varkappa) &\leq T_{F_{\zeta}(l_2)}(\varkappa) \qquad & T_{G_{\zeta}(o_1)}(\varkappa) \leq T_{G_{\zeta}(o_2)}(\varkappa) \\ K_{F_{\zeta}(l_2)}(\varkappa) &\leq K_{F_{\zeta}(l_1)}(\varkappa) \qquad & K_{G_{\zeta}(o_2)}(\varkappa) \leq K_{G_{\zeta}(o_1)}(\varkappa) \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} T_{F_{\zeta}(l_1)}(\varkappa) \wedge T_{G_{\zeta}(o_1)}(\varkappa) &\leq T_{F_{\zeta}(l_2)}(\varkappa) \wedge T_{G_{\zeta}(o_2)}(\varkappa) \\ K_{F_{\zeta}(l_2)}(\varkappa) \wedge K_{G_{\zeta}(o_2)}(\varkappa) &\leq K_{F_{\zeta}(l_1)}(\varkappa) \wedge K_{G_{\zeta}(o_1)}(\varkappa) \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} T_{(F_{\zeta}(l_1) \wedge G_{\zeta}(o_1))}(\varkappa) &\leq T_{(F_{\zeta}(l_2) \wedge G_{\zeta}(o_2))}(\varkappa) \\ K_{(F_{\zeta}(l_2) \wedge G_{\zeta}(o_2))}(\varkappa) &\leq K_{(F_{\zeta}(l_1) \wedge G_{\zeta}(o_1))}(\varkappa) \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} T_{H_{\zeta}(l_1, o_1)}(\varkappa) &\leq T_{H_{\zeta}(l_2, o_2)}(\varkappa) \\ K_{H_{\zeta}(l_2, o_2)}(\varkappa) &\leq K_{H_{\zeta}(l_1, o_1)}(\varkappa) \end{aligned}$$

$\Rightarrow$

$H_{\zeta}(l_1, o_1) \subseteq H_{\zeta}(l_2, o_2) \ \forall \ (l_1, o_1) \leq (l_2, o_2)$

$\Rightarrow (F_{\zeta}, L_{\varkappa}) \wedge (G_{\zeta}, O_{\varkappa}) \in LOIFSS(X_{\varkappa})$. ■

3.3.33 Example

Suppose $X_{\varkappa} = \{\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4\}$ and $E_{\varkappa} = \{\varrho_1, \varrho_2, \varrho_3, \varrho_4\}$ is the parameters set and $L_{\varkappa}, O_{\varkappa} \subseteq E_{\varkappa}$, $L_{\varkappa} = \{\varrho_1, \varrho_2, \varrho_3\}, O_{\varkappa} = \{\varrho_2, \varrho_3, \varrho_4\}$.

$(F_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
ϱ_1	$(0.1, 0.9)$	$(0.2, 0.7)$	$(0.3, 0.7)$	$(0.5, 0.4)$
ϱ_2	$(0.2, 0.7)$	$(0.3, 0.6)$	$(0.4, 0.6)$	$(0.56, 0.3)$
ϱ_3	$(0.3, 0.5)$	$(0.6, 0.4)$	$(0.55, 0.4)$	$(0.6, 0.2)$

Table 3.40

$(G_\zeta, O_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
ϱ_2	$(0.2, 0.8)$	$(0.4, 0.3)$	$(0.1, 0.6)$	$(0.5, 0.3)$
ϱ_3	$(0.4, 0.4)$	$(0.7, 0.2)$	$(0.4, 0.4)$	$(0.7, 0.2)$
ϱ_4	$(0.6, 0.2)$	$(0.9, 0.1)$	$(0.8, 0.2)$	$(0.9, 0.1)$

Table 3.41

Then $(F_\zeta, L_\varkappa)$, $(G_\zeta, O_\varkappa) \in LOIFSS(X_\varkappa)$ and $(F_\zeta, L_\varkappa) \wedge (G_\zeta, O_\varkappa) = (H_\zeta, P_\varkappa)$ is known to be basic intersection of two $LOIFSS(X_\varkappa)$.

$(H_\zeta, P_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
(ϱ_1, ϱ_2)	$(0.1, 0.9)$	$(0.2, 0.7)$	$(0.1, 0.7)$	$(0.5, 0.4)$
(ϱ_1, ϱ_3)	$(0.1, 0.9)$	$(0.2, 0.7)$	$(0.3, 0.7)$	$(0.5, 0.4)$
(ϱ_1, ϱ_4)	$(0.1, 0.9)$	$(0.2, 0.7)$	$(0.3, 0.7)$	$(0.5, 0.4)$
(ϱ_2, ϱ_2)	$(0.2, 0.8)$	$(0.3, 0.6)$	$(0.1, 0.6)$	$(O.5, 0.3)$
(ϱ_2, ϱ_3)	$(0.2, 0.7)$	$(0.3, 0.6)$	$(0.4, 0.6)$	$(0.56, 0.3)$
(ϱ_2, ϱ_4)	$(0.2, 0.7)$	$(0.3, 0.6)$	$(0.4, 0.6)$	$(0.56, 0.3)$
(ϱ_3, ϱ_2)	$(0.2, 0.8)$	$(0.4, 0.4)$	$(0.1, 0.6)$	$(0.5, 0.3)$
(ϱ_3, ϱ_3)	$(0.3, 0.5)$	$(0.6, 0.4)$	$(0.4, 0.4)$	$(0.6, 0.2)$
(ϱ_3, ϱ_4)	$(0.3, 0.5)$	$(0.6, 0.4)$	$(0.55, 0.4)$	$(0.6, 0.2)$

Table 3.42

we observe that $(H_\zeta, P_\varkappa) \in LOIFSS(X_\varkappa)$.

3.4 An Application of LOIFSS($X_\varkappa$) in a Decision Making Problem

3.4.1 Definition:Comparison matrix

The comparison matrix is a matrix whose columns are name of object of the universe such as $\varkappa_1, \varkappa_2, ..., \varkappa_n$ and the rows are labelled by the parameters $\varrho_1, \varrho_2, ..., \varrho_m$. The entries are p_{ij},where p_{ij}, is the number of parameters for which the value of $\varkappa_i$ exceeds or equal to the value $\varkappa_j$. We calculate that entries by $p_{ij} = l - o$,where "l" is the integer calculated as "how many times $T\varkappa_i(\varrho_j)$ exceeds or equal to $T_{\varkappa_j}(\varrho_j)$", for $\varkappa_i \neq \varkappa_j$, $\forall\ \varkappa_k \in X_\varkappa$,"$o$" is the integer calculated as "how many times $K\varkappa_i(\varrho_j)$ exceeds or equal to $K_{\varkappa j}(\varrho_j)$", for $\varkappa_i \neq \varkappa_j$ $\forall\ \varkappa_k \in X_\varkappa$.

3.4.2 Definition:Score of an object

The score of an object $\varkappa_i$ is S_i and is calculated as :

$$S_i = \sum_j p_{ij}$$

Now the algorithm for most appropriate selection of an object will be as follows.

1. Consider the $LOIFSS(X_\varkappa)$ $(F_\zeta, L_\varkappa)$ and written in tabular form.
2. Compute the comparison matrix of $LOIFSS(X_\varkappa)$ $(F_\zeta, L_\varkappa)$.
3. Compute the score S_i of $\varkappa_i, \forall\ i$.
4. Find $S_k = \max S_i$.

   To understand the basic idea of the algorithm,now we apply it to the $LOIFSS(X_\varkappa)$ based decision making problem.

3.4.3 Example

Mr. Hamza wants to buy a land and he have four options for land $\{\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4\}$ and $L_\varkappa = \{\varrho_1(\text{Rate}), \varrho_2(\text{Near to road}), \varrho_3(\text{Sewage system}), \varrho_4(\text{Security System}), \varrho_5(\text{Environment})\} \subseteq E_\varkappa$ be the parameters set and we have to take decision which one is better for this there is an order between them given below.

$$\begin{array}{ccccc} & & \varrho_5 & & \\ & \nearrow & & \nwarrow & \\ \varrho_3 & & & & \varrho_4 \\ & \nwarrow & & \nearrow & \\ & & \varrho_2 & & \\ & & \downarrow & & \\ & & \varrho_1 & & \end{array}$$

$(F_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
ϱ_1	$(0.1, 0.9)$	$(0.2, 0.7)$	$(0.3, 0.7)$	$(0.5, 0.4)$
ϱ_2	$(0.2, 0.7)$	$(0.3, 0.6)$	$(0.4, 0.6)$	$(0.56, 0.3)$
ϱ_3	$(0.3, 0.5)$	$(0.6, 0.4)$	$(0.55, 0.4)$	$(0.6, 0.2)$
ϱ_4	$(0.5, 0.4)$	$(0.8, 0.2)$	$(0.6, 0.2)$	$(0.9, 0.1)$
ϱ_5	$(0.7, 0.2)$	$(0.9, 0.1)$	$(0.8, 0.1)$	$(1, 0)$

Table 3.43: Tabular form of $LOIFSS(X_\varkappa)(F_\zeta, L_\varkappa)$

$(F_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
ϱ_1	-3	-1	0	3
ϱ_2	-3	-1	0	3
ϱ_3	-3	1	-1	3
ϱ_4	-3	0	-1	3
ϱ_5	-3	0	-1	3

Table 3.44: Comparison matrix of $LOIFSS(X_\varkappa)$ $(F_\zeta, L_\varkappa)$

$(F_\zeta, L_\varkappa)$	$\varkappa_1$	$\varkappa_2$	$\varkappa_3$	$\varkappa_4$
Score(S_i)	-15	-1	-3	15

Table 3.45: Score

The selection of the land will be judged by maximum score in table 3.45. Thus the ranking of land is given by table 3.46.

Land	Score(S_i)	Rank
$\varkappa_1$	-15	4th
$\varkappa_2$	-1	2nd
$\varkappa_3$	-3	3rd
$\varkappa_4$	15	1st

Table 3.46: Ranking

Hence best choice for Mr. X is to be $\varkappa_4$,followed by $\varkappa_2$.

REFERENCES

[1] H. Aktas and N. Cagman, Soft sets and soft groups, inform. Sci. 177 (2007) 2726-2735.

[2] K. Alhazaymeh, S. A. Halim, A.R. Salleh and N. Hassan, Soft Intuiotionistic Fuzzy Sets, App. Mathematical Sciences, 6 (2012) 2669-2680.

[3] M. I. Ali, F. Feng, X. Y. Liu, W. K. Min and M. Shabir, On some new operations in soft set theory, Comput. Math. Appl. (2008) 2621:2628

[4] M. I. Ali, T. Mehmood, M. Mati ur rehman and M. F. Aslam, On lattice ordered soft set, Applied Soft Computing, 36 (2015) 499-505.

[5] S. Alkhazaleh. A. R. Salleh and N. Hassan, Fuzzy parametrized interval-valued fuzzy soft set, Applied Mathematical Science, 67 (2011) 3335-3346.

[6] S. Alkhazaleh. A. R. Salleh and N. Hassan, Soft multisets theory, Applied Mathematical Science, 72 (2011) 3561-3573.

[7] M. F. Aslam, Study of fuzzy soft sets with some order on set of parameters, MS Thesis IIUI, 2014.

[8] K. Atanassov, Intuitionistic fuzzy sets and systems, 20 (1986) 87-96.

[9] K. Atanassov, Operators over interval valued intuitionistic fuzzy sets, Fuzzy Sets and systems, 64 (1994) 159-174.

[10] G. Birkhoff, Lattice Theory, American Mathematical Society (1967).

[11] N. Cagman and S. Enginoglu, Soft set theory and uni-int decision making, 207 (2010) 848-855.

[12] N. Cagman and S. Karatas, Intuitionistic fuzzy soft set theory and its applications in decision making, Journal of Intelligent and fuzzy systems, (2012).

[13] N. Cagman, S. Karatas and S. Enginoglu, Soft topology, Comput. Math. appl. 62 (2011) 351-358.

[14] Y. Jiang, Y. Tang, Q. Chen, J. Wang and S. Tang, Extending soft sets with description logics, Comput. Math. Appl. 59 (2010) 2087-2096.

[15] F. Karaaslan, N. Cagman and S. Yilmaz, Intuitionistic fuzzy soft set theory and its applications, (2013).

[16] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, J. Fuzzy Math. 9 (2001) 589-602.

[17] P. K. Maji, R. Biswas and A. R. Roy, Intuitionistic fuzzy soft sets, J. Fuzzy Math. 9 (2001) 677-692.

[18] P. K. Maji, R. Biswas and A. R. Roy, Soft set thoery, Comput. Math. Appl. 45 (2003) 555-562.

[19] P. K. Maji and A. R. Roy, An application of soft sets in a decision making problems, Comput. Math. Appl. 44 (2002) 1077-1083.

[20] W. K. Min, Similarity in soft set theory, App. Math. Supposet. 25 (2012) 310-314.

[21] D. Molodtsov, Soft set theory-first result, Comut. Math. Appl. 37 (1999) 19-31.

[22] Z. Pawlak, Rough sets, International Journal of Computers Science, 11 (1982)341−356.

[23] D. Pei, D. Miao, From soft sets to information systems, granular computing, IEEE Int.Conf. 2. (2005) 617-621.

[24] A. R. Roy and P. K. Maji, A fuzzy soft set theoretic approach to decision making problems, J. Comput. Appl. Math. 203 (2007) 412-418.

[25] M. Shabir and M. Naz, On soft topological spaces, Comp. & Maths. with Appl., 61 (7) (2011) 1786-1799.

[26] Y. Xu, Y. Sun and D. Li, Intuitionistic fuzzy soft set, Science and Research Department Dalian Naval Academy Dalian China, (2010).

[27] B. Yao, L. Jin-liang and Y. Rui-xia, Fuzzy soft set and soft fuzzy set, Fourth International Conference on Natural Computation, (2008).

[28] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338-353.