Influence of Initial Stress and Gravity on Reflection and Refraction of SV waves at Interface under Three ThermoElastic Theories

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Gravity Mathematical models.
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Maxwell equations

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A Thesis
Submitted in the Partial Fulfillment of the
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MASTER OF SCIENCE
In
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Supervised by

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Certificate

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A THESIS SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF THE **MASTER OF SCIENCE IN MATHEMATICS**

We accept this thesis as conforming to the required standard.

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2017

Dedication

"I dedicate this dissertation to my parents"

ACKNOWLEDGEMENT

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Last but not the least I would also like to thank my all friends especially **Sundas, Mariam, Ammarah, Iqra, Maheen** for their support throughout the thesis.

Ambreen Afzal

DECLARATION

I hereby declare that this thesis, neither as a whole nor a part of it, has been copied out from any source. It is further declared that I have prepared this dissertation entirely on the basis of my personal efforts made under the supervision of my supervisor **Dr. Ambreen Afsar Khan**. No portion of the work, presented in this dissertation, has been submitted in the support of any application for any degree or qualification of this or any other learning institute.

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Preface

The classical uncoupled theory of thermoelasticity predicts two phenomena not compatible with physical observations. First the equation of heat conduction does not contain any elastic term, second the heat equation is of a parabolic type, predicting infinite speed of propagation for heat wave. Biot [1] introduced the theory of coupled thermoelasticity to overcome first short coming. The governing equations of this theory are coupled eliminating the first paradox of the classical theory. However both theories share the second short coming. Since the heat equation for the coupled theory is of mixed parabolic and hyperbolic types. Different researchers have introduced different models such as Lord and Shulman [2], Green and Lindsay [3] using one and two relaxation times. The dual phase lag (DPL) model of heat conduction was proposed in refs. [4, 5] as an improved theory compared to the classical model, based on Fourier's Law and described by the heat conduction equation. Initial stresses are developed in the medium due to many reasons resulting from the difference of temperature, differential external forces, gravity, vibration etc. The earth is supposed to be under high initial stress. During the last five decades considerable attention has been directed towards this phenomenon. It was the achievement of Biot [6] to show acoustic propagation under initial stress. In classical problem of elastic waves and vibrations the effect of gravity, curvature and viscosity of the materials has not been discussed in detail. Various researchers studied the effects of reflection, refraction and initial stress which are mentioned in refs. [7-20]

Chapter one is related with the basic definitions and concepts. In chapter two, Abd-Alla et al. [8] have considered the reflection and refraction of SV wave at the solid liquid interface under the influence of magnetic field and initial stress. In chapter three, we have considered the effect of gravity and initial stress on the reflection and refraction of SV waves at the viscoelastic liquid interface.

Contents

Chapter 1

Preliminaries

In this chapter some basic definitions and concepts are discussed.

1.1 Basic Definitions

1.1.1 Waves

A wave is an oscillation accompanied by transfer of energy through a medium (space or mass). Waves involve transport of energy without the transport of matter. So waves are described as a disturbance that travels through a medium from one location to another location.

1.1.2 Wavelength

It is the distance between two consecutive points on the waves which are in the same phase, (same phase means same state of vibrations).

1.1.3 Amplitude

Amplitude is the maximum displacement of the particle from its mean position.

1.1.4 Frequency

Frequency is the number of periodic oscillations completed in one second. The unit of this measure is hertz [Hz].

1.1.5 Elasticity

The property of body due to which it tends to move back after an external force is removed is known as elasticity.

1.1.6 Viscosity

The viscosity of a fluid is a measure of its resistance to gradual deformation by stress.

1.2 Types of Waves

1.2.1 Mechanical waves

The waves which require medium for propagation are known as mechanical waves.

1.2.2 Electromagnetic waves

The waves which does not require any medium for propagation are known as electromagnetic waves.

1.2.3 Elastic waves

The waves in which the propagated disturbance is an elastic deformation of the medium. In elastic wave motion in a medium when particles are displaced a force proportional to the displacement acts on particle to restore them to their original position.

1.2.4 Surface waves

Surface waves are mechanical waves that propagates along the interface between differing media. Rayleigh waves and Love waves are examples of surface waves.

1.2.5 Body waves

Body waves are travelling through the interior of the earth emitted by an earthquake. These waves are of higher frequency than surface waves.

P-waves and S-waves are two types of body waves.

1.2.6 P waves

P waves are a type of body waves. The name P waves can stand for either pressure or primary waves as it is formed from alternating compressions and rarefactions. The motion is parallel to the particles of object. They are the first waves which are felt during an earthquake.

1.2.7 S waves

S waves are known as secondary or shear waves. It pass through the body of an object also their motion is perpendicular to the particles of object. These are the second waves to be felt during an earth quake.

S waves can be of two types.

SV waves.

SH waves.

1.2.8 SV waves

S waves polarized in vertical plane are known as SV Waves.

1.2.9 SH waves

S waves polarized in horizontal plane are known as SH Waves.

1.3 Reflection

When waves strike at interface they bounce back into the same medium. This phenomena is known as reflection of waves.

1.3.1 Laws of reflection

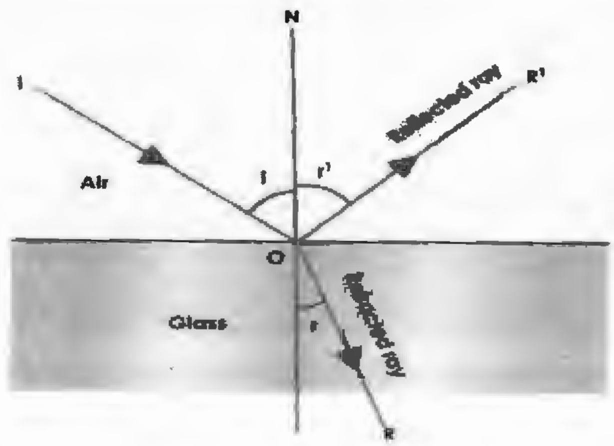
The laws of reflection are as follows:

(1) The incident ray, the reflected ray and the normal ray all lie in the same plane.

(2) The angle which the incident ray makes with the normal is equal to the angle which the reflected ray makes to the same normal.

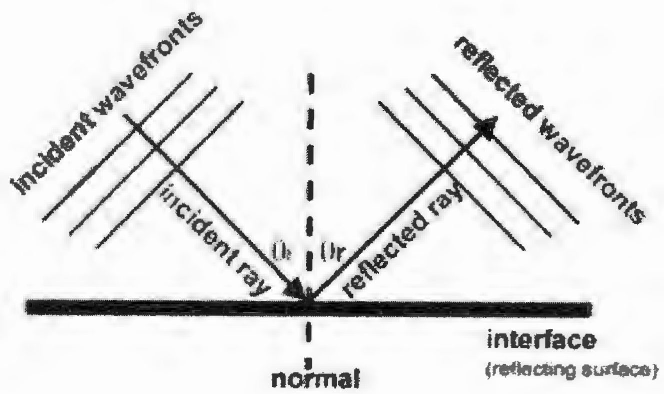
1.3.2 Refraction

Refraction is the change in direction of propagation of a wave due to change in its media. Rainbows, light bending in glass of water, glasses, camera lens are examples of refraction.



1.3.3 Interface

The point where two medium are separated.



1.4 Initial Stress

The stress applied initially on a point is known as initial stress. Its value is determined by difference of its normal component.

$$P = S_{22} - S_{11}, \tag{1.1}$$

here S_{11} and S_{22} represents normal stress component.

1.4.1 Normal initial stress

The stress acting normally on a point is known as normal initial stress.

1.4.2 Incremental stress

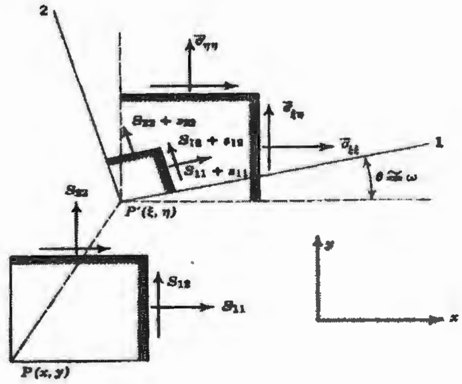
When initial stress is applied the body is deformed to a new point then the new stress components are known as incremental stress component.

1.4.3 Local rotation

When the initial stress is applied the point is displaced to a new direction by an angle θ which is known as local rotation of material.

$$\theta = \omega = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right), \tag{1.2}$$

here ω represent local rotation.



1.4.4 Strain

Strain is a measure of deformation representing the displacement between particles in the body relative to a reference length.

1.5 Elastic constants

In case of homogeneous and an isotropic medium, some elastic moduli or elastic constants are involved. The most common elastic moduli are discussed below.

1.5.1 Young's Modulus

The ratio between stress and longitudinal strain is called Young's modulus, i.e.,

$$E = \frac{F.L}{A.\Delta L}, \tag{1.3}$$

where F is applied force, A is the area, ΔL is change in length and L is the original length.

1.5.2 Bulk Modulus

The ratio between stress and volumetric strain is called bulk modulus, i.e.,

$$k = \frac{F.V}{A.\Delta V}, \tag{1.4}$$

where F is the applied force, A is the area, ΔV is change in volume and V is the original volume.

1.5.3 Poisson Ratio

It is the ratio between strains only, i.e., the ratio of relative longitudinal strain to lateral strain and is denoted by σ. When a rod of length L is elongated by ΔL and its width $W$and is contracted by ΔW, then Poisson ratio is

$$\sigma = \frac{\frac{\Delta W}{W}}{\frac{\Delta L}{L}}. \tag{1.5}$$

1.6 Equation of Motion

The equation of motion for an elastic medium is

$$\sigma_{ij,j} + f_i = \rho\left(\ddot{U}_i\right), \tag{1.6}$$

where $\sigma_{ij,j}$ is the stress force acting on the direction of x_i axis, f_i is the body force, ρ is the density of the medium and U_i is the displacement vector.

1.7 Energy Equation

The energy describe the distribution of heat in a given section over a time. The first law of thermodynamics i.e., the energy may neither be created nor be destroyed.

$$\vartheta\nabla^2 T = \rho c_e\left(\frac{\partial T}{\partial t}\right), \tag{1.7}$$

where c_e is the specific heat per unit mass, ϑ is the thermal conductivity, T is the absolute temperature of the medium ρ is density of medium.

1.8 Maxwell Equations

Maxwell equations are the sets of four laws that relate electric and magnetic field. They also relate electric and magnetic field to their sources, charge and current density. These equations are

$$\begin{aligned}
\text{curl}\,\vec{h} &= \vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}, && (1.8)\\
\text{curl}\,\vec{E} &= -\mu_e \frac{\partial \vec{h}}{\partial t}, && (1.9)\\
\text{div}\,\vec{E} &= 0, && (1.10)\\
\text{div}\,\vec{h} &= 0, && (1.11)
\end{aligned}$$

where $\vec{E}$ represents electric intensity, $\vec{h}$ is the perturbed magnetic field, $\vec{J}$ represents electric current density, μ_e and ε_0 represents magnetic and electric permeability.

Chapter 2

SV Waves between Solid Liquid Media under Initial Stress and Electromagnetic Field in reference of Three Thermoelastic Theories

2.1 Introduction

This chapter is a review work of Ref. [8]. This chapter looks at the effects of initial stress and magnetic field on SV waves under three theories. The refraction and reflection at interface of SV waves is investigated after using the boundary conditions then the results are shown by graphs.

2.2 Mathematical Formation

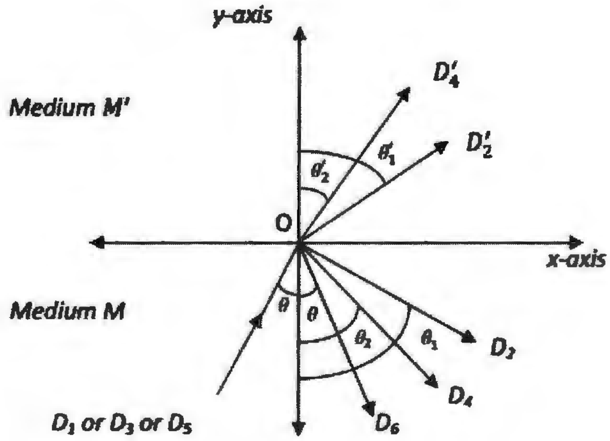


Fig. 2.1 : Geometry of the Problem

Let us assume a plane interface between solid liquid isotropic, homogeneous elastic half spaces which are at a primary temperature T_0. A plane SV wave is incident in medium M which is reflected as thermal wave, P wave, and SV wave. The rest of the waves continue to travel in the other medium M' and refracted as T waves and P waves as shown in Fig. 2.1. Where θ is the angle of the incidence for a plane wave, θ_1, θ_2, θ_1' and θ_2' are the angles of reflected waves and refracted waves The magnetic field vector **H** acts on z direction.

Consider a two dimensional unsteady problem in xy plane with O as the origin.

The equation of motions under initial stress in absence of heat source and in presence of Lorentz's force are

$$\frac{\partial \acute{s}_{11}}{\partial x} + \frac{\partial \acute{s}_{21}}{\partial y} - P\frac{\partial \omega}{\partial y} + F_1 = \rho\frac{\partial^2 u}{\partial t^2}, \tag{2.1}$$

$$\frac{\partial \acute{s}_{21}}{\partial x} + \frac{\partial \acute{s}_{22}}{\partial y} - P\frac{\partial \omega}{\partial x} + F_2 = \rho\frac{\partial^2 v}{\partial t^2}, \tag{2.2}$$

where F_1 and F_2 are the component of electromagnetic field in x and y directions respectively.

The stress strain relations are given below

$$\begin{aligned}
\acute{s}_{11} &= (\lambda + 2\mu + P)\, d_{xx} + (\lambda + P)\, d_{yy} - \gamma \left(T + \tau_1 \frac{\partial}{\partial t} T\right), &(2.3)\\
\acute{s}_{22} &= \lambda d_{xx} + (\lambda + 2\mu)\, d_{yy} - \gamma \left(T + \tau_1 \frac{\partial}{\partial t} T\right), &(2.4)\\
\acute{s}_{12} &= 2\mu d_{xy}. &(2.5)
\end{aligned}$$

The incremental strain component have the following form

$$d_{xx} = \frac{\partial u}{\partial x}, \quad d_{yy} = \frac{\partial v}{\partial y}, \quad d_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right), \tag{2.6}$$

d_{ij} is the strain component $\acute{s}_{11}$, $\acute{s}_{22}$, $\acute{s}_{12}$ are the incremental stress components, λ and μ are lame's constant.

The heat conduction equation is given as.

$$\vartheta \left(1 + \tau_\theta \frac{\partial}{\partial t}\right) \nabla^2 T = \rho c_e \left(\frac{\partial T}{\partial t} + \tau_0 \frac{\partial^2 t}{\partial t^2}\right) + T_0 \gamma \left[\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \tau_0 \delta_{ij} \frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right], \tag{2.7}$$

where c_e represents specific heat per unit mass, ϑ represents the thermal conductivity, T_0 is the natural temperature of medium, δ_{ij} is the Kronecker delta, T is the absolute temperature of the medium , τ_0 and τ_1 are the thermal relaxation times, τ_θ is phase lag of the gradient of temperature.

Lorentz's body force is given as

$$F_i = \mu_e \left(\vec{J} \times H_0\right). \tag{2.8}$$

Using Eqs. (1.8 – 1.11) in above Eq. we obtain

$$\begin{aligned}
F_1 &= \mu_e H^2 \left[\frac{\partial d}{\partial x} - \epsilon_0 \mu_e \frac{\partial^2 u}{\partial t^2}\right], &(2.9)\\
F_2 &= \mu_e H^2 \left[\frac{\partial d}{\partial y} - \epsilon_0 \mu_e \frac{\partial^2 v}{\partial t^2}\right], &(2.10)\\
F_3 &= 0. &(2.11)
\end{aligned}$$

Maxwell's stress equation is

$$\tau_{ij} = \mu_e \left[H_i h_i + H_i h_i - \left(\overrightarrow{H}_k \bullet \overrightarrow{h_k}\right)\delta_{ij}\right], \quad i,\ j = 1,\ 2,\ 3. \tag{2.12}$$

where τ_{ij} is Maxwell's stress tensor which reduces to

$$\tau_{11} = \tau_{22} = \mu_e H^2 \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right), \quad \tau_{12} = 0. \tag{2.13}$$

2.3 Solution of Problem

Substituting Eqs. (2.3 – 2.5) and (2.9 – 2.11) into Eq. (2.1, 2.2), we get

$$\begin{aligned}
&(\lambda + 2\mu + P + \mu_e H^2)\frac{\partial^2 u}{\partial x^2} + \left(\lambda + \mu + \frac{P}{2} + \mu_e H^2\right)\frac{\partial^2 v}{\partial x \partial y} + \left(\mu + \frac{P}{2}\right)\frac{\partial^2 u}{\partial y^2} \\
= \; &(\rho + \mu_e^2 H^2 \epsilon_0)\frac{\partial^2 u}{\partial t^2} + \gamma\left(\frac{\partial T}{\partial x} + \tau_1 \frac{\partial^2 T}{\partial x \partial t}\right), \qquad (2.14) \\
&\left(\mu - \frac{P}{2}\right)\frac{\partial^2 v}{\partial x^2} + \left(\lambda + \mu + \frac{P}{2} + \mu_e H^2\right)\frac{\partial^2 u}{\partial x \partial y} + (\lambda + 2\mu + \mu_e H^2)\frac{\partial^2 v}{\partial y^2} \\
= \; &(\rho + \mu_e^2 H^2 \epsilon_0)\frac{\partial^2 v}{\partial t^2} + \gamma\left(\frac{\partial T}{\partial y} + \tau_1 \frac{\partial^2 T}{\partial y \partial t}\right). \qquad (2.15)
\end{aligned}$$

Displacement scalar and vector potentials Φ and Ψ are defined as

$$u = \frac{\partial \Phi}{\partial x} - \frac{\partial \Psi}{\partial y}, \quad v = \frac{\partial \Phi}{\partial y} + \frac{\partial \Psi}{\partial x}. \tag{2.16}$$

From Eqs. (2.14) and (2.16), we get

$$\begin{aligned}
\nabla^2 \Phi &= \frac{(\rho + \mu_e^2 H_0^2 \epsilon_0)}{(\lambda + 2\mu + P + \mu_e H^2)}\left(\frac{\partial^2 \Phi}{\partial t^2}\right) + \frac{\gamma}{\lambda + 2\mu + P + \mu_e H^2}\left(T + \tau_1 \frac{\partial T}{\partial t}\right), \qquad (2.17) \\
\nabla^2 \Psi &= \frac{(\rho + \mu_e^2 H^2 \epsilon_0)}{(\mu + \frac{P}{2})}\left(\frac{\partial^2 \Psi}{\partial t^2}\right). \qquad (2.18)
\end{aligned}$$

From Eqs. (2.15) and (2.16), we get

$$\nabla^2\Phi = \frac{(\rho+\mu_e^2H^2\epsilon_0)}{(\lambda+2\mu+\mu_eH^2)}\left(\frac{\partial^2\Phi}{\partial^2 t}\right)+\frac{\gamma}{(\lambda+2\mu+\mu_eH^2)}\left(T+\tau_1\frac{\partial T}{\partial t}\right), \tag{2.19}$$

$$\nabla^2\Psi = \frac{(\rho+\mu_e^2H^2\epsilon_0)}{(\mu-\frac{P}{2})}\left(\frac{\partial^2\Psi}{\partial t^2}\right), \tag{2.20}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$.

Using Eqs. (2.16) in Eq (2.7) we have

$$\vartheta\left(1+\tau_\theta\frac{\partial}{\partial t}\right)\nabla^2 T = \rho c_e\left(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2 T}{\partial t^2}\right)+T_0\gamma\frac{\partial}{\partial t}\left(1+\tau_0\delta_{ij}\frac{\partial}{\partial t}\right)\nabla^2\Phi. \tag{2.21}$$

Eq. (2.21) under the theories have the following form

1) Classical and dynamical coupled theory

$$\delta_{ij}=0,\quad \tau_0=0,\quad \tau_\theta=0,\quad \tau_1=0.$$

2) Green and Lindsay's theory

$$\delta_{ij}=0,\quad \tau_1\geq\tau_0>0,\quad \tau_\theta=0.$$

3) Dual phase Lag theory

$$\delta_{ij}=1,\quad \tau_0>0,\quad \tau_1=0,\quad 0\leq\tau_\theta<\tau_0.$$

Eqs. (2.17) and (2.20) can be rewritten as

$$\nabla^2\Phi = \frac{\alpha}{C_1^2(1+R_H)}\frac{\partial^2\Phi}{\partial t^2}+\frac{\gamma}{\rho C_1^2(1+R_H)}\left(T+\tau_1\frac{\partial T}{\partial t}\right), \tag{2.22}$$

$$\nabla^2\Psi = \frac{\alpha}{C_2^2}\left(\frac{\partial^2\Psi}{\partial t^2}\right), \tag{2.23}$$

where

$$R_H = \frac{C_A^2}{C_1^2},\ C_1^2 = \frac{\lambda + 2\mu + P}{\rho},\ C_2^2 = \frac{\mu - \frac{P}{2}}{\rho},$$
$$C_A^2 = \frac{\mu_e H^2}{\rho},\ C^2 = \frac{1}{\mu_e \epsilon_0},\ \alpha = 1 + \frac{C_A^2}{C^2}. \tag{2.24}$$

Here R_H, C_1, C_A, C_2 represent the Alfven speed, the part of magnetic field, velocities of dilatational and rotational waves respectively in the medium M.

Eliminating T from Eqs. (2.21) and (2.22), we get a fourth order differential equation in terms of Φ as

$$T = \left(1 + \tau_1 \frac{\partial}{\partial t}\right)^{-1} \left[\frac{\rho C_1^2 (1 + R_H)}{\gamma} \nabla^2 \Phi - \frac{\rho \alpha}{\gamma} \left(\frac{\partial^2 \Phi}{\partial t^2}\right)\right], \tag{2.25}$$

$$C_3^2 (1 + R_H) \left(1 + \tau_\theta \frac{\partial}{\partial t}\right) \nabla^4 \Phi$$
$$- \left[\begin{array}{c} (1 + R_H + \varepsilon_T) \frac{\partial}{\partial t} \\ + \left((1 + R_H)\tau_0 + \varepsilon_T \tau_1 + \varepsilon_T \tau_0 \delta_{ij} + \frac{C_3^2}{C_4^2}\right) \frac{\partial^2}{\partial t^2} + \left(\frac{C_3^2}{C_4^2} \tau_\theta + \varepsilon_T \tau_0 \tau_1 \delta_{ij}\right) \frac{\partial^3}{\partial t^3} \end{array}\right] \nabla^2 \Phi$$
$$+ \frac{1}{C_4^2} \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \frac{\partial^3 \Phi}{\partial t^3} = 0, \tag{2.26}$$

where

$$C_3^2 = \frac{K}{\rho c_e},\ \varepsilon_T = \frac{T_0 \gamma^2}{\rho^2 c_e C_1^2},\ C_4^2 = \frac{C_1^2}{\alpha}. \tag{2.27}$$

ε_T is the thermoelastic constant of the solid medium M.

Let us consider the following solution

$$\Phi = q(y) \exp\left[ik(x - ct)\right], \tag{2.28}$$
$$\Psi = r(y) \exp\left[ik(x - ct)\right], \tag{2.29}$$
$$T = s(y) \exp\left[ik(x - ct)\right], \tag{2.30}$$

where $c = \frac{\omega}{k}$.

Using Eq. (2.28) into Eq. (2.26), we have

$$\frac{d^4 q}{dy^4} + A \frac{d^2 q}{dy^2} + B q(y) = 0, \tag{2.31}$$

where

$$\begin{aligned}
A &= -2k^2 + \frac{ikc}{C_3^{*2}}(1+R_H+\varepsilon_T) + \frac{k^2c^2}{C_3^{*2}}\left((1+R_H)\tau_0 + \varepsilon_T\tau_1 + \varepsilon_T\tau_0\delta_{ij} + \frac{C_3^2}{C_4^2}\right) \\
&\quad -\frac{ik^3c^3}{C_3^{*2}}\left(\frac{C_3^2}{C_4^2}\tau_\theta + \varepsilon_T\tau_0\tau_1\delta_{ij}\right), \\
B &= k^4 - \frac{k^4c^2}{C_3^{*2}}\left((1+R_H)\tau_0 + \varepsilon_T\tau_1 + \varepsilon_T\tau_0\delta_{ij} + \frac{C_3^2}{C_4^2}\right) + \frac{ik^5c^3}{C_3^{*2}}\left(\frac{C_3^2}{C_4^2}\tau_\theta + \varepsilon_T\tau_0\tau_1\delta_{ij}\right) \\
&\quad +\frac{ik^3c^3}{C_4^2C_3^{*2}}\left(1 - ikc\tau_0 - \frac{C_4^2}{c^2}(1+R_H+\varepsilon_T)\right),
\end{aligned}$$

and

$$C_3^{*2} = C_3^2(1+R_H)\left(1+\tau_\theta\frac{\partial}{\partial t}\right).$$

By solving Eq. (2.31), the Eq. 2.27 becomes

$$\Phi = \begin{bmatrix} D_1\exp(ikn_1y) + D_2\exp(-ikn_1y) \\ +D_3\exp(ikn_2y) + D_4\exp(-ikn_2y) \end{bmatrix} \exp[ik(x-ct)], \tag{2.32}$$

where

$$n_1 = \sqrt{\check{e}^2c^2 - 1}, \quad n_2 = \sqrt{p^2c^2 - 1}, \tag{2.33}$$

$$p^2 = \frac{1}{2C_4^2C_3^{*2}}\begin{bmatrix} C_4^2(\tau_o(1+R_H) + \varepsilon_T\tau_1 + \varepsilon_T\tau_o\delta_{ij}) + c_3^2 + \\ \frac{i(1+R_H+\varepsilon_T)C_4^2}{\omega} - i\omega\left(c_3^2\tau_\theta + C_4^2\varepsilon_T\tau_0\tau_1\delta_{ij}\right) + \sqrt{N} \end{bmatrix}, \tag{2.34}$$

$$\check{e}^2 = \frac{1}{2C_4^2C_3^{*2}}\begin{bmatrix} C_4^2(\tau_o(1+R_H) + \varepsilon_T\tau_1 + \varepsilon_T\tau_o\delta_{ij}) + c_3^2 + \\ \frac{i(1+R_H+\varepsilon_T)C_4^2}{\omega} - i\omega\left(c_3^2\tau_\theta + C_4^2\varepsilon_T\tau_0\tau_1\delta_{ij} - \sqrt{N}\right) \end{bmatrix}, \tag{2.35}$$

$$N = \begin{bmatrix} C_4^2(\tau_0(1+R_H) + \varepsilon_T\tau_1 + \varepsilon_T\tau_o\delta_{ij}) + c_3^2 + \\ \frac{i(1+R_H+\varepsilon_T)C_4^2}{\omega} - i\omega\left(c_3^2\tau_\theta + C_4^2\varepsilon_T\tau_0\tau_1\delta_{ij}\right) \end{bmatrix}^2 - \frac{4i(1-i\omega\tau_0)C_4^2C_3^{*2}}{\omega}. \tag{2.36}$$

Substituting Eq. (2.29) into Eq. (2.23), we have

$$\frac{d^2r}{dy^2} + k^2\left(\frac{c^2\alpha}{c_2^2} - 1\right)r = 0. \tag{2.36}$$

Eq. (2.37) suggests that the solution yields two values of $r(y)$, then Eq. (2.29) can be written as

$$\Psi = \left[D_5 \exp\left(ikn_3 y\right) + D_6 \exp\left(-ikn_3 y\right)\right] \exp\left[ik\left(x - ct\right)\right], \tag{2.38}$$

where

$$n_3 = \sqrt{\frac{\alpha c^2}{C_2^2} - 1}.$$

The constants $D_i\,(i = 1, 2, ..., 6)$ represent the amplitudes of incident SV waves and reflected (thermal waves) T, P and SV waves respectively.

Substituting from Eqs. (2.30) and (2.32) into Eq. (2.19) we get the values of $s(y)$, then the Eq. (2.30) takes the following form

$$T = \frac{\rho\alpha}{\gamma\tau}\begin{bmatrix} b_1\left(D_1 \exp\left(ikm_1 y\right) + D_2 \exp\left(ikm_1 y\right)\right) \\ +b_2\left(D_3 \exp\left(ikm_2 y\right) + D_4 \exp\left(-ikm_2 y\right)\right) \end{bmatrix} \exp\left[ik\left(x - ct\right)\right], \tag{2.39}$$

where

$$\tau = (1 - i\omega\tau_1) \quad b_1 = \omega^2\left(1 - \frac{(1 + R_H)}{\alpha}q^2 C_1^2\right) \quad b_2 = \omega^2\left(1 - \frac{(1 + R_H)}{\alpha}q^2 C_1^2\right).$$

Setting $\mu = P = 0$ in Eqs. $(2.1 - 2.7)$, we obtain the basic equations for non viscous liquid medium in the presence of body forces. We get displacement equations and temperature equation for the liquid medium M' as follows

$$\begin{aligned}
(\lambda' + \mu_e' H'^2)\frac{\partial^2 u'}{\partial x^2} + (\lambda' + \mu_e' H'^2)\frac{\partial^2 v'}{\partial x \partial y} &= \rho'\frac{\partial^2 u'}{\partial t^2} \\
&\quad + \gamma'\left(\frac{\partial T'}{\partial x} + \tau_1'\frac{\partial^2 T'}{\partial x \partial t}\right), \qquad (2.40) \\
(\lambda' + \mu_e' H'^2)\frac{\partial^2 u'}{\partial x \partial y} + (\lambda' + \mu_e' H'^2)\frac{\partial^2 v'}{\partial y^2} &= (\rho' + \mu_e'^2 \varepsilon_0' H'^2)\frac{\partial^2 v'}{\partial t^2} \\
&\quad + \gamma'\left(\frac{\partial T'}{\partial y} + \tau_1'\frac{\partial^2 T'}{\partial y \partial t}\right), \qquad (2.41) \\
K'\left(1 + \tau_\theta'\frac{\partial}{\partial t}\right)\nabla^2 T' &= \rho' c_e'\left(\frac{\partial T'}{\partial t} + \tau_0'\frac{\partial^2 T'}{\partial t^2}\right) \\
&\quad + T_0'\gamma'\frac{\partial}{\partial t}\left(1 + \tau_0'\delta_{ij}\frac{\partial}{\partial t}\right)(\nabla^2\Phi). \qquad (2.42)
\end{aligned}$$

The primes show quantities in the liquid medium M' which are defined in case of solid medium M.

Taking

$$u' = \frac{\partial \Phi'}{\partial x}, \quad v' = \frac{\partial \Phi'}{\partial y}, \tag{2.43}$$

we obtain

$$\nabla^2 \Phi' = \frac{a'}{C_1'^2 \left(1 + R_H'\right)} \frac{\partial^2 \Phi'}{\partial t^2} + \frac{\gamma^2}{\rho' C_1'^2 \left(1 + R_H'\right)} \left(1 + \tau_1' \frac{\partial}{\partial t}\right) T', \tag{2.44}$$

$$K' \left(1 + \tau_\theta' \frac{\partial}{\partial t}\right) \nabla^2 T' = \rho' c_e' \left(\frac{\partial T'}{\partial t} + \tau_0' \frac{\partial^2 T'}{\partial t^2}\right) + T_0' \gamma' \frac{\partial}{\partial t} \left(1 + \tau_0' \delta_{ij} \frac{\partial}{\partial t}\right) \left(\nabla^2 \Phi'\right), \tag{2.45}$$

where $C_1'^2 = \frac{\lambda'}{\rho'}$.

Solving Eqs. (2.44) and (2.45) and proceeding exactly in a same way as in solid medium M, we get the appropriate solution for Φ' and T' as

$$\Phi' = \left[D_2' \exp\left(ikm_1'\gamma\right) + D_4' \exp\left(ikm_2'\gamma\right)\right] \exp\left[ik\left(x - ct\right)\right], \tag{2.46}$$

$$T' = \frac{\rho' a'}{\gamma' \tau'} \left[D_2' \exp\left(ikm_1'\gamma\right) + D_4' \exp\left(ikm_2'\gamma\right)\right] \exp\left[ik\left(x - ct\right)\right], \tag{2.47}$$

where

$$\tau' = \left(1 - i\omega t_1'\right), \quad b_1' = \omega^2 \left(1 - \frac{\left(1 + R_H'\right)}{a'} q'^2 C_1'^2\right), \quad b_2' = \omega^2 \left(1 - \frac{\left(1 + R_H'\right)}{a'} p'^2 C_1'^2\right).$$

2.4 Boundary Conditions

(1) At the interface, normal displacement is continuous, i.e., $v = v'$, and this leads to

$$\frac{\partial \Phi}{\partial y} + \frac{\partial \Psi}{\partial x} = \frac{\partial \Phi'}{\partial y}. \tag{2.48}$$

Using Eqs. (2.32), (2.38), and (2.46) in the above continuity relation, we get

$$\begin{aligned}
&\left\{\begin{array}{c} D_1\,(n_1)\exp\left(ikn_1y\right)+D_2\,(-n_1)\exp\left(-ikn_1y\right)+ \\ D_3\,(n_2)\exp\left(ikn_2y\right)+D_4\,(-n_2)\exp\left(-ikn_2\right)y \end{array}\right\} \\
&+\left\{D_5\exp\left(ikn_3y\right)+D_6\exp\left(-ikn_3y\right)\right\} \\
=\;&\left\{D_2'\left(n_1'\right)\exp\left(ikn_1'y\right)+D_4'(n_2')\exp(ikn_2'y)\right\},
\end{aligned}$$

at $y=0$

$$n_1D_1-n_1D_2+n_2D_3-n_2D_4+D_5+D_6-n_1'D_2'-n_2'D_4'=0. \tag{2.49}$$

(2) At the interface, tangential displacement must be zero, i.e., $u=0$.

This leads to

$$\frac{\partial\Phi}{\partial x}-\frac{\partial\Psi}{\partial y}=0. \tag{2.50}$$

Using Eqs. (2.32) and (2.38), we get

$$\begin{aligned}
&\left\{\begin{array}{c} D_1\exp(ikn_1y)+D_2\exp(-ikn_1y) \\ +D_3\exp(ikn_2y)+D_4\exp(-ikn_2y) \end{array}\right\} \\
&-\left\{D_5(n_3)\exp(ikn_3y)+D_6(-n_3)\exp(-ikn_3y)\right\}=0,
\end{aligned}$$

at $y=0$

$$D_1+D_2+D_3+D_4-n_3D_5+n_3D_6=0. \tag{2.51}$$

(3) At the interface, normal force per unit initial area must be continuous, i.e., $\nabla fy=\nabla f'y$.

This leads to

$$s_{22}+\tau_{22}=s_{22}'+\tau_{22}',$$

where

$$\tau_{ij}=\mu_e\left[H_ih_j+H_jh_i-(\mathbf{H}.\mathbf{h})\delta_{ij}\right],\; i,j=1,2,3.$$

Using Eqs. $(2.3-2.5)$, (2.7), (2.12),(2.13),(2.16) and (2.43), we obtain

$$(\lambda+\mu_eH^2)\left(\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}\right)+2\mu\left(\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial\Psi}{\partial x\partial y}\right)-\gamma\left(T+\tau_1\frac{\partial T}{\partial t}\right)$$

$$= \left(\gamma' + \mu_e' H'^2\right)\left(\frac{\partial^2 \Phi'}{\partial x^2} + \frac{\partial^2 \Phi'}{\partial y^2}\right) - \gamma^2 \left(T' + \tau_1 \frac{\partial T'}{\partial t}\right). \tag{2.52}$$

Substituting Eqs. (2.32), (2.38), (2.39), (2.46) and (2.47) into the Eq. (2.52), we get

$$\left(-(2+\beta) + c^2\left(\frac{1}{c_2^2} - \beta q^2\right)\right) D_1 \exp(ikn_1 y) + D_2 \exp(-ikn_1 y)$$

$$+ \left(-(2+\beta) + c^2\left(\frac{1}{c_2^2} - \beta p^2\right)\right) (D_3 \exp(ikn_1 y) + D_4 \exp(-ikn_1 y))$$

$$+ (2+\beta)\, n_3 (D_5 \exp(ikn_3 y) - D_6 \exp(-ikn_3 y))$$

$$-\rho^* \left(1 + n_3^2\right)\left(D_2' \exp\left(ikn_1' y\right) + D_4' \exp\left(ikn_2' y\right)\right) = 0,$$

at $y = 0$

$$\left[-(2+\beta) + c^2\left(\frac{1}{c_2^2} - \beta q^2\right)\right](D_1 + D_2) + \left[-(2+\beta) + c^2\left(\frac{1}{c_2^2} - \beta p^2\right)\right]$$

$$(D_3 + D_4) + (2+\beta)\, n_3 (D_5 - D_6) - \rho^* \left(1 + n_3^2\right)\left(D_2' + D_4'\right) = 0, \tag{2.53}$$

where $\rho^* = \frac{\rho'}{\rho}$ and $\beta = \frac{P}{\rho C_2^2}$.

(4) At interface, tangential force per unit initial area vanish, i.e., $\nabla f x = 0$.

$$s_{12} + Pe_{xy} + \tau_{12} = 0.$$

Using Eqs. (2.3 – 2.5), (2.7), (2.12), and (2.13), (2.16), (2.32), and (2.38), we obtain

$$D_1 m_1 \exp(ikn_1 y) + D_2 n_1 y) = n_2 (D_3 \exp(ikn_2 y) - D_4 \exp(-ikn_2 y))$$

$$-\frac{1}{2}\left(\begin{array}{c} D_5 n_3^2 \exp(ikn_3 y) + D_6 n_3^2 \exp(-ikn_3 y) \\ +\frac{1}{2}(D_5 \exp(ikn_3 y) + D_6 \exp(-ikn_3 y)) \end{array}\right) = 0,$$

at $y = 0$

$$n_1 (D_1 - D_2) + n_2 (D_3 - D_4) - \frac{1}{2}\left(n_3^2 - 1\right)(D_5 + D_6) = 0. \tag{2.54}$$

(5) At the interface, temperature is continuous, i.e, $T = T'$.

Using Eqs. (2.39) and (2.47) with making some algebraic simplifying, we get

$$\begin{aligned}
& \left(1-\frac{(1+R_H)}{\alpha}q^2C_1^2\right)\left(D_1\exp\left(ikn_1y\right)+D_2\exp\left(-ikn_1y\right)\right) \\
+ \quad & \left(1-\frac{(1+R_H)}{\alpha'}p^2C_1^2\right)\times\left(D_3\exp\left(ikn_2y\right)+D_4\exp\left(-ikn_2y\right)\right) \\
= \quad & \frac{\rho*\alpha*}{\gamma*\tau*}\left[\begin{array}{c}\left(1-\frac{(1+R'_H)}{\alpha'}q'^2C_1'^2\right)D'_2\exp\left(ikn'_1y\right)+ \\ \left(1-\frac{(1+R'_H)}{\alpha'}p'^2C_1'^2\right)D'_4\exp\left(ikn'_2y\right)\end{array}\right],
\end{aligned}$$

at $y=0$

$$\begin{aligned}
& \left(1-\frac{(1+R_H)}{\alpha}q^2C_1^2\right)(D_1+D_2)+\left(1-\frac{(1+R_H)}{\alpha'}p^2C_1^2\right)\times(D_3+D_4) \\
& -\frac{\rho*\alpha*}{\gamma*\tau*}\left[\begin{array}{c}\left(1-\frac{(1+R'_H)}{\alpha'}q'^2C_1'^2\right)D'_2+ \\ \left(1-\frac{(1+R'_H)}{\alpha'}p'^2C_1'^2\right)D'_4\end{array}\right]=0, \qquad (2.55)
\end{aligned}$$

where $\gamma^*=\frac{\gamma'}{\gamma}$ and $\tau^*=\frac{\tau'}{\tau}$.

2.5 Equations for the Refraction and Reflection Cofficients

Consider the refraction and reflection of SV-wave incident at the solid-liquid interface at $y=0$ making an angle θ with the $y-axis$.

For SV-waves incident, put $c=c_2\text{cosec}\theta$ and $D_1=D_3=0$.

Generalizing, a system of five nonhomogeneous equations has been obtained

$$\sum_{i=1}^{5}a_{ij}Z_j=y_j,\ \ (j=1,2,..,5)\,, \qquad (2.56)$$

where

$$\begin{aligned}
a_{11} &= -n_1,\ a_{12} = -n_2,\ a_{13} = 1,\ a_{14} = -n_1,\ a_{15} = -n_2,\ a_{21} = a_{22} = 1,\ a_{23} = n_3, \\
a_{24} &= a_{25} = 0,\ a_{31} = \left[-(2+\beta) + c^2\left(\frac{\alpha}{C_2^2} - \beta q^2\right)\right],\ a_{32} = \left[-(2+\beta) + c^2\left(\frac{\alpha}{C_2^2} - \beta p^2\right)\right], \\
a_{33} &= -(2+\beta)\,n_3,\ a_{34} = a_{35} = -\rho * \left(1 + n_3^2\right),\ a_{41} = -n_1,\ a_{42} = -n_2,\ a_{43} = -0.5\left(n_3^2 - 1\right), \\
a_{44} &= 0,\ a_{45} = 0,\ a_{51} = \left(1 - \frac{(1+R_H)}{\alpha} q^2 C_1^2\right),\ a_{52} = \left(1 - \frac{(1+R_H)}{\alpha} p^2 C_1^2\right), \\
a_{53} &= 0,\ a_{54} = -\frac{\rho * \alpha *}{\gamma * \tau *}\left(1 - \frac{(1+R'_H)}{\alpha'} q'^2 C_1'^2\right),\ a_{55} = -\frac{\rho * \alpha *}{\gamma * \tau *}\left(1 - \frac{(1+R'_H)}{\alpha'} p'^2 C_1'^2\right).
\end{aligned}$$

where $(j = 1, 2, .., 5)$ are the ratios of amplitudes of reflected thermal, P, SV-waves and refracted thermal, P-waves for the incident wave, respectively

$$y_1 = -a_{13},\ y_2 = a_{23},\ y_3 = a_{33},\ y_4 = -a_{43},\ y_5 = a_{53},$$

$$Z_1 = \frac{D_2}{D_5},\ Z_2 = \frac{D_4}{D_5},\ Z_3 = \frac{D_6}{D_5},\ Z_4 = \frac{D'_2}{D_5},\ Z_5 = \frac{D'_4}{D_5}. \quad (2.57)$$

2.6 Numerical results and discussion

For numerical analysis we have used the data given in Refs. [17].

For solid medium(M crust "Granite")

$$\begin{aligned}
\lambda &= \mu = 3 \times 10^{10} Nm^{-2},\quad \alpha_t = 1.0667 \times 10^{-5} K^{-1},\quad \omega = 7.5 \times 10^{13} S^{-1}, \\
C_e &= 1100 JKg^{-1}K^{-1},\quad \rho = 2900 Kgm^{-3},\quad K = 3 Wm^{-1}K^{-1},\quad T_0 = 300K.
\end{aligned}$$

For liquid medium(M' water)

$$\begin{aligned}
\lambda' &= \mu' = 20.4 \times 10^9 Nm^{-2},\quad \alpha'_t = 69 \times 10^{-6} K^{-1},\quad K' = 0.6 Wm^{-1}K^{-1}, \\
C'_e &= 4187 JKg^{-1}K^{-1},\quad \rho' = 1000 Kgm^{-3},\quad T'_0 = 300K
\end{aligned}$$

Considering

$$\tau_0 = \tau'_0 = 0.9, \tau_1 = \tau'_1 = 0.9, \tau_\theta = \tau'_\theta = 0.8.$$

Fig. 2.2 shows the variation of angle of incidence of SV waves for three models in thermoelasticity the Couple dynamic, Green-Lindsay theories as well as the DPL theory. $|Z_1|$, $|Z_2|$, $|Z_4|$ and $|Z_5|$ starts from maximum value and interrupted to zero at $\theta = 90°$ but $|Z_3|$ arrives to unity. It is shown that DPL theory in $|Z_1|$ and $|Z_4|$ take the smallest values comparing with the existing two relaxation times (G-L), while if there aren't relaxation times (CD) $|Z_1|$ and $|Z_4|$ take the largest value. It is shown that Green-Lindsay theory for $|Z_2|$ takes the smallest values comparing with (DPL)theory, while if there are no relaxation times (CD) theory $|Z_2|$ takes the largest value. It is shown that that Green-Lindsay theory for $|Z_5|$ takes the smallest value comparing with DPL theory, but for the CD thoery $|Z_5|$ takes the largest value.

Fig. 2.3 explains the variation of amplitudes with the angle of incidence of SV wave for variation of initial stress under the effect of thermal relaxation times to DPL theory. $|Z_1|$, $|Z_2|$, $|Z_4|$ and $|Z_5|$ starts from maximum value and interrupted to zero at $\theta = 90°$ but $|Z_3|$ starts from maximum value and interrupted to one at $\theta = 45°$ and after this their direction is reversed. The amplitude ratios $|Z_1|$, $|Z_2|$, $|Z_4|$ and $|Z_5|$ decreases with an increasing initial stress.

Fig. 2.4 depicts the variation of the amplitudes with the angle of incidence of SV wave for variation of magnetic field under effect of thermal relaxation times to DPL theory. $|Z_1|$, $|Z_2|$, $|Z_4|$ and $|Z_5|$ start from maximum value and interrupted to zero at $\theta = 90°$ but $|Z_3|$ arrives to unity at $\theta = 90°$ and have same trend for all the values of magnetic field. The amplitude ratios $|Z_2|$, $|Z_4|$ decreases with an increasing magnetic field, but the amplitude ratios $|Z_1|$and $|Z_5|$ increases with an increasing magnetic field.

Fig. 2.5 depicts the variation of the amplitudes with the angle of incidence of SV wave for variation of electric field under effect of thermal relaxation times to DPL theory. $|Z_1|$, $|Z_2|$, $|Z_4|$ and $|Z_5|$ start from maximum value and interrupted to zero at $\theta = 90°$ but $|Z_3|$ arrives to unity at $\theta = 90°$ and have same trend for all the values of electric field. The amplitude ratios $|Z_5|$ increases with an increasing electric field.

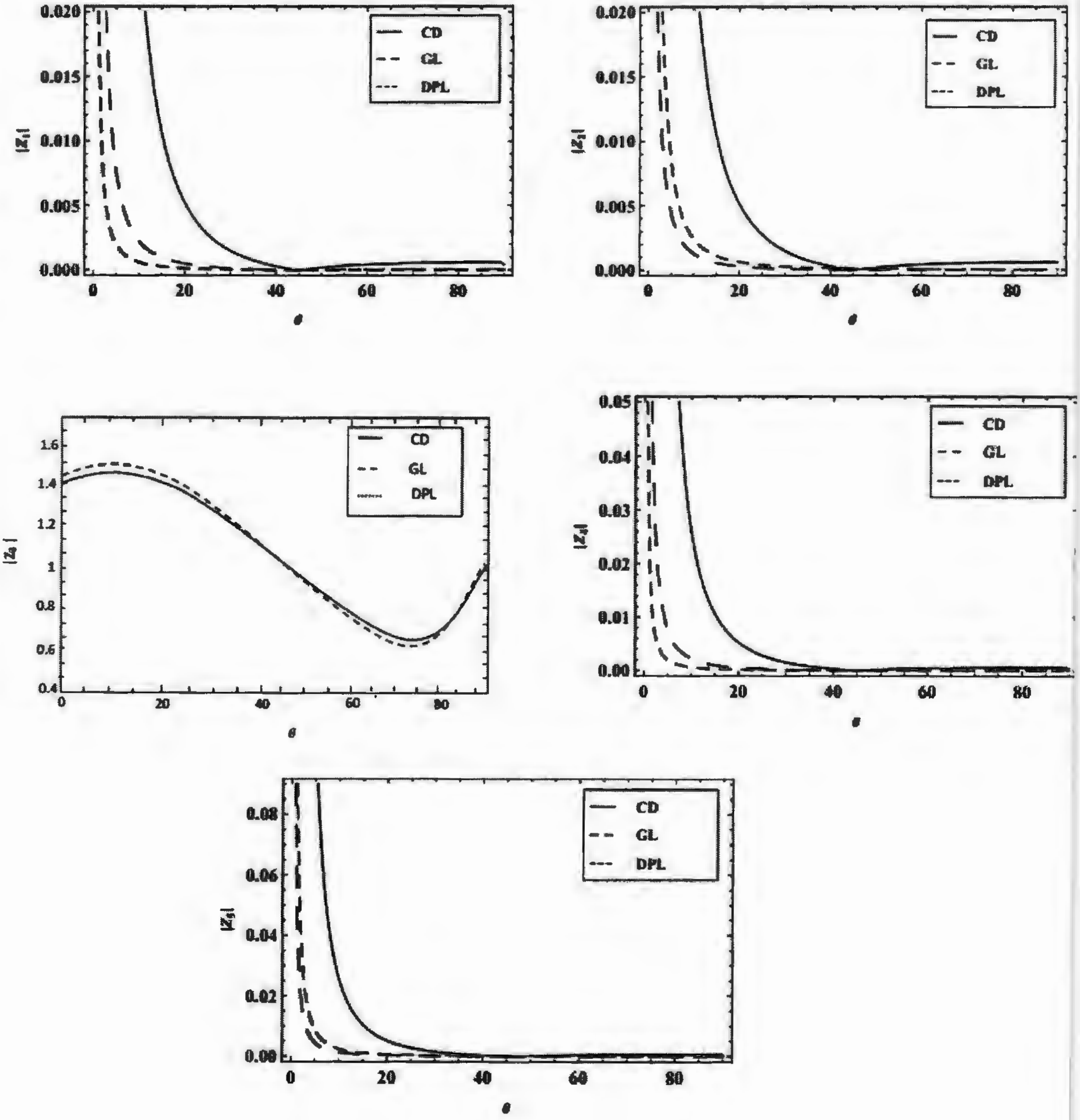


Fig. 2.2: Variation of the amplitudes of waves for three models.

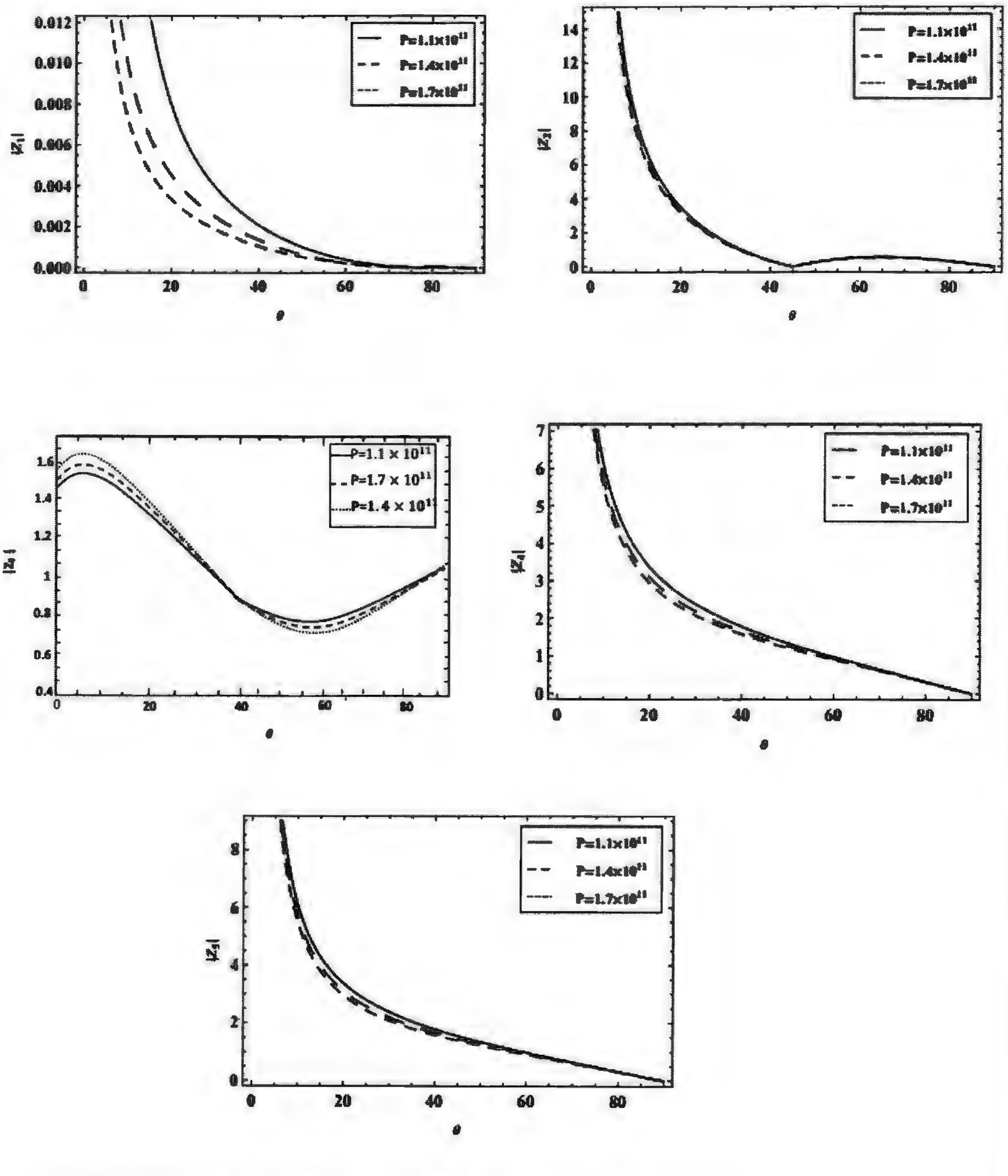


Fig. 2.3: Variation of the amplitudes of waves for change of initial stress.

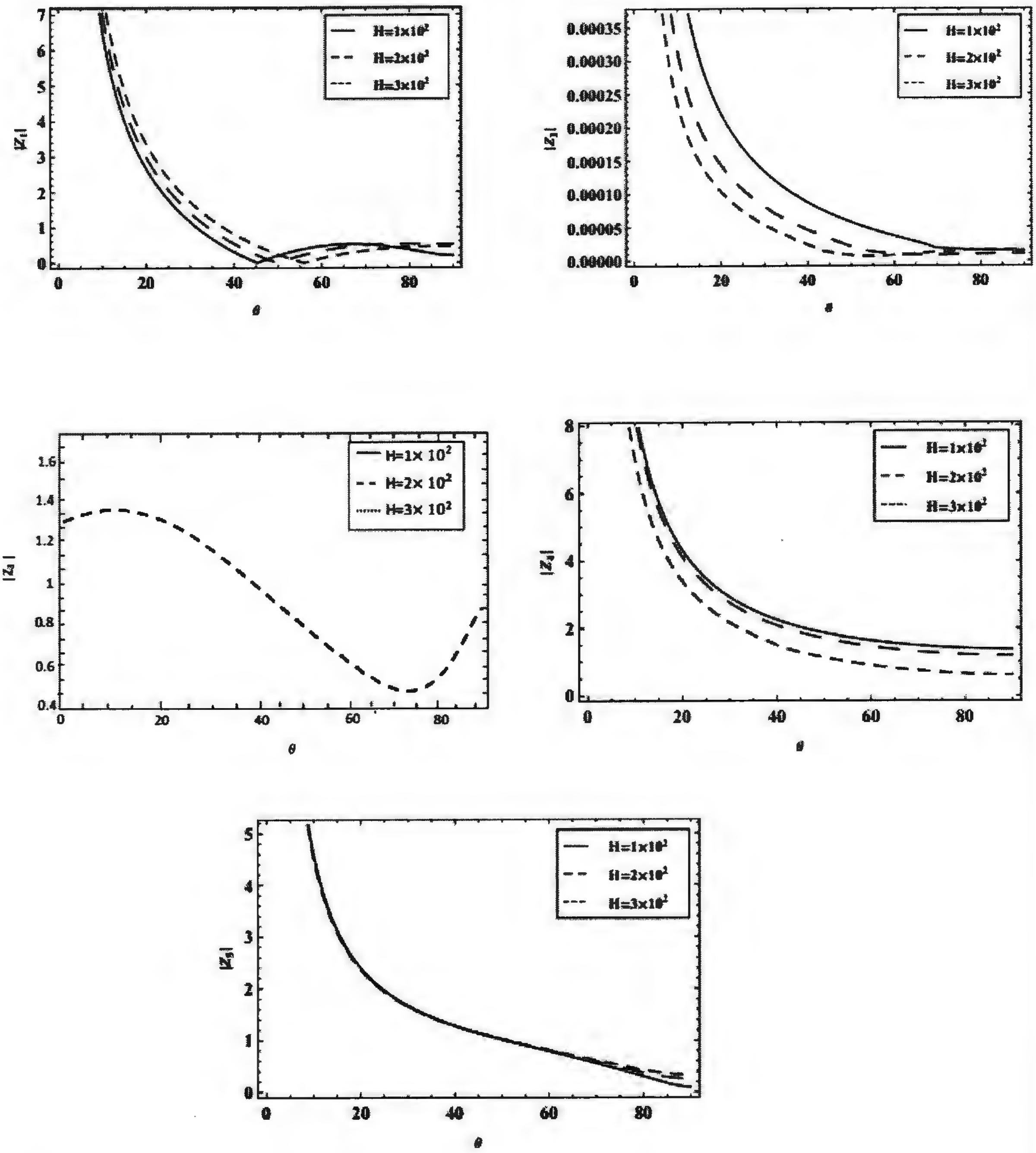


Fig. 2.4: Variation of the amplitudes of waves for change of magnetic field.

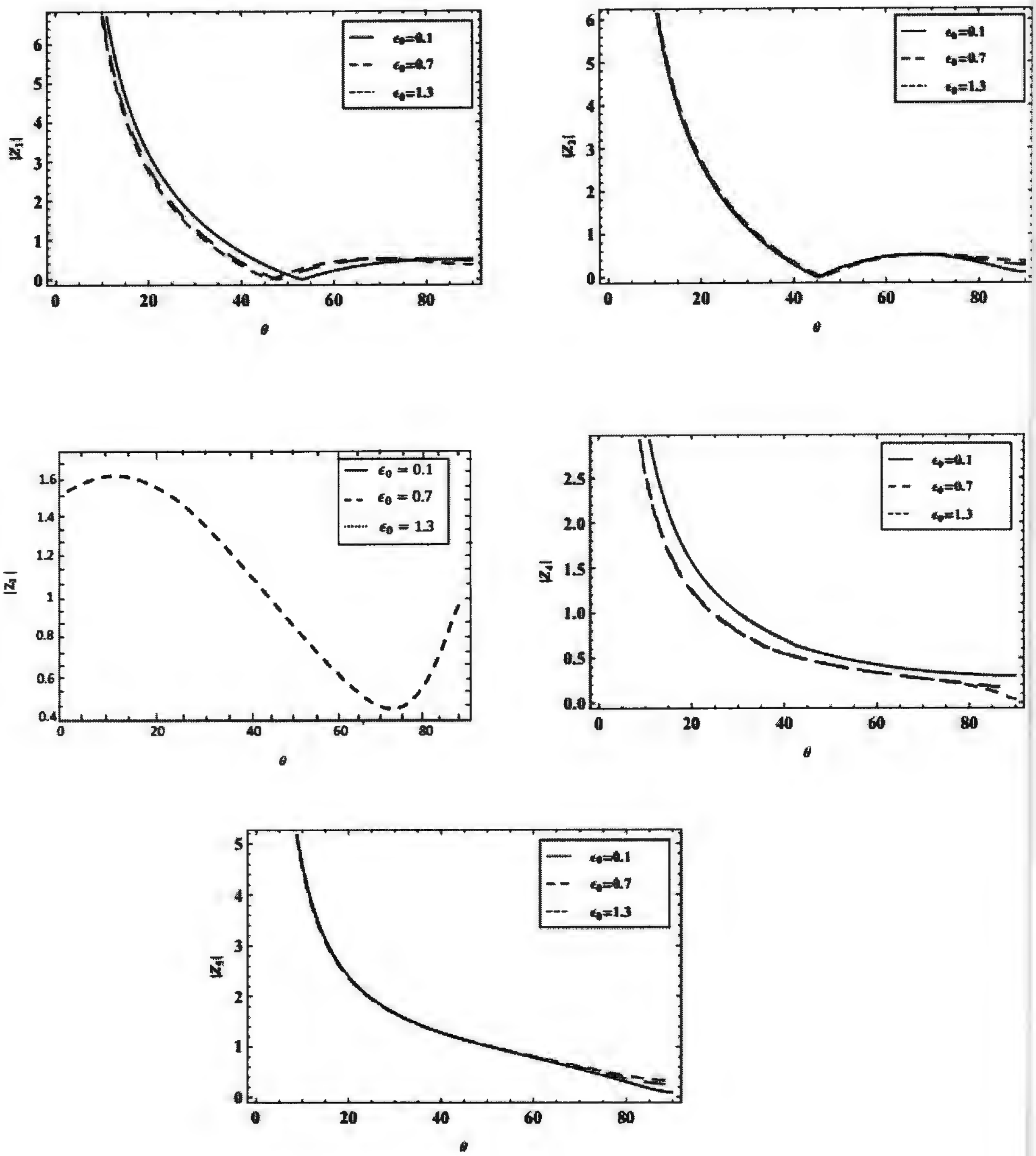


Fig. 2.5: Variation of the amplitudes of waves for change of magnetic field.

Chapter 3

Influence of Initial Stress and Gravity on Reflection and Refraction of SV waves at Interface under Three Thermoelastic Theories

This chapter looks at the influence of viscosity, magnetic field, initial stress, and gravity on SV waves while traveling through the interface of two visco thermoelastic liquid layers. The basic equations in context of three theories have been solved to drive equations for refracted P and thermal waves and reflected thermal, SV and P waves. After using the boundary conditions the amplitude ratios have been computed in matrix form.

3.1 Formation of the Problem

Let us assume a plane interface which is differentiating two isotropic, homogeneous viscoelastic liquid half spaces under the effects of initial stress, gravity and magnetic field. Let T_0 be the primary temperature of both medium, a SV wave in medium N strikes at the interface of two visco thermo elastic liquid layers, which is reflected as well as transmitted. A SV wave is reflected as SV, P and thermal waves in the same medium N and transmitted in the form of

P and T waves in the medium N'. We assume a rectangular coordinate system with origin O and interface is held at $y = 0$. A constant magnetic field $\mathbf{H_0} = (0, 0, H)$ acts along z direction and both medium are placed under initial stress. Since we have two dimensional problem, we restrict our analysis in the xy plane. All variables in medium N are without primes where as the variables in the second medium N' are shown by primes.

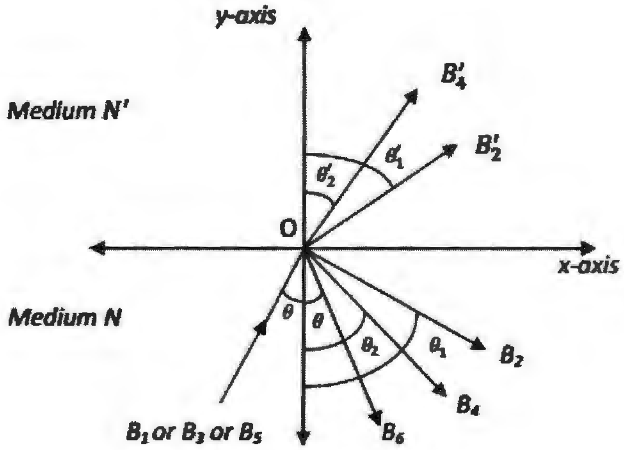


Fig 3.1 : Geometry of problem

Biot [1] proposed the equation of motion in presence of Lorentz force and gravity, when no heat source is present formulated as follows,

$$\frac{\partial \acute{s}_{11}}{\partial x} + \frac{\partial \acute{s}_{22}}{\partial y} - P\frac{\partial \omega}{\partial y} + F_1 - \rho g\frac{\partial v}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{3.1}$$

$$\frac{\partial \acute{s}_{21}}{\partial x} + \frac{\partial \acute{s}_{22}}{\partial y} - P\frac{\partial \omega}{\partial x} + F_2 + \rho g\frac{\partial u}{\partial x} = \rho\frac{\partial^2 v}{\partial t^2}. \tag{3.2}$$

3.2 Solution of the Problem

To obtain the equations of motion in visco elastic liquid medium N the lame's parameter λ is replaced by $\left(K - \frac{2}{3}\mu\right)$, the effect of fluid viscosity is accounted for replacing μ by the operator

$(\eta\frac{\partial}{\partial t})$ in Eqs. (2.3 − 2.5), where η is the fluid viscosity we have

$$\acute{s}_{11} = \left(K + P + \frac{4}{3}\eta\frac{\partial}{\partial t}\right) d_{xx} + \left(K + P - \frac{2}{3}\eta\frac{\partial}{\partial t}\right) d_{yy} - \gamma\left(T + \tau_1\frac{\partial T}{\partial t}\right), \tag{3.3}$$

$$\acute{s}_{22} = \left(K - \frac{2}{3}\eta\frac{\partial}{\partial t}\right) d_{xx} + \left(K + \frac{4}{3}\eta\frac{\partial}{\partial t}\right) d_{yy} - \gamma\left(T + \tau_1\frac{\partial T}{\partial t}\right), \tag{3.4}$$

$$\acute{s}_{21} = 2\eta\frac{\partial}{\partial t}d_{xy}. \tag{3.5}$$

Making use of Eqs. (2.9 − 2.11) and (3.3 − 3.5) in Eqs. (3.1) and (3.2), we obtain

$$\begin{aligned} &\left(K + P + \mu_e H^2 + \frac{4}{3}\eta\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial x^2} + \left(K + \frac{P}{2} + \mu_e H^2 + \frac{1}{3}\eta\frac{\partial}{\partial t}\right) \\ &\frac{\partial^2 v}{\partial x \partial y} + \left(\eta\frac{\partial}{\partial t} + \frac{P}{2}\right)\frac{\partial^2 u}{\partial y^2} - \rho g\frac{\partial v}{\partial x} \\ &= \left(\rho + \epsilon_0\mu_e^2 H^2\right)\frac{\partial^2 u}{\partial t^2} + \gamma\left(\frac{\partial T}{\partial x} + \tau_1\frac{\partial^2 T}{\partial t \partial x}\right), \end{aligned} \tag{3.6}$$

$$\begin{aligned} &\left(K + P + \mu_e H^2 + \frac{4}{3}\eta\frac{\partial}{\partial t}\right)\frac{\partial^2 v}{\partial y^2} + \left(K + \frac{P}{2} + \mu_e H^2 + \frac{1}{3}\eta\frac{\partial}{\partial t}\right) \\ &\frac{\partial^2 u}{\partial x \partial y} + \left(\eta\frac{\partial}{\partial t} - \frac{P}{2}\right)\frac{\partial^2 v}{\partial x^2} + \rho g\frac{\partial u}{\partial x} \\ &= \left(\rho + \epsilon_0\mu_e^2 H^2\right)\frac{\partial^2 v}{\partial t^2} + \gamma\left(\frac{\partial T}{\partial y} + \tau_1\frac{\partial^2 T}{\partial t \partial y}\right), \end{aligned} \tag{3.7}$$

using displacement potential Φ and Ψ defined as

$$u = \frac{\partial \Phi}{\partial x} - \frac{\partial \Psi}{\partial y}, \quad v = \frac{\partial \Phi}{\partial y} + \frac{\partial \Psi}{\partial x}. \tag{3.8}$$

Using Eq. (3.8) into Eqs. (3.6) and (3.7), we get

$$\left(K + P + \mu_e H^2 + \frac{4}{3}\eta\frac{\partial}{\partial t}\right)\nabla^2\Phi - \rho g\left(\frac{\partial\Phi}{\partial y}\right) = \left(\rho + \epsilon_0\mu_e^2 H^2\right)\frac{\partial^2\Phi}{\partial t^2} + \gamma\left(T + \tau_1\frac{\partial T}{\partial t}\right), \tag{3.9}$$

$$\left(\eta\frac{\partial}{\partial t} + \frac{P}{2}\right)\nabla^2\Psi + y\rho g\left(\frac{\partial^2\Psi}{\partial x^2}\right) = \left(\rho + \epsilon_0\mu_e^2 H^2\right)\frac{\partial^2\Psi}{\partial t^2}, \tag{3.10}$$

$$\left(K + \mu_e H^2 + \frac{4}{3}\eta\frac{\partial}{\partial t}\right)\nabla^2\Phi + \rho g y\left(\frac{\partial^2\Phi}{\partial x^2}\right) = \left(\rho + \epsilon_0\mu_e^2 H^2\right)\frac{\partial^2\Phi}{\partial t^2} + \gamma\left(T + \tau_1\frac{\partial T}{\partial t}\right), \tag{3.11}$$

$$\left(\eta\frac{\partial}{\partial t} - \frac{P}{2}\right)\nabla^2\Psi - \rho g\left(\frac{\partial\Psi}{\partial y}\right) = \left(\rho + \epsilon_0\mu_e^2 H^2\right)\frac{\partial^2\Psi}{\partial t^2}. \tag{3.12}$$

Eqs. (3.9) and (3.12), have scalar and vector potential Φ and Ψ, represent the dilatational and rotational components of wave equation moving along x-axis which are obtained by partially integrating with respect to x. The other two Eqs. (3.10) and (3.11) involving vector potential Φ and Ψ represent dilatational and rotational wave propagating along y axis which are obtained by partially integrating with respect to y. We consider the propagation of wave along x axis, so we consider the dilatational and rotational wave Eqs. (3.9) and (3.12) only.

Making use of Eq. (3.8) in Eq. (2.7), we get

$$\vartheta\left(1 + \tau_\theta\frac{\partial}{\partial t}\right)\nabla^2 T = \rho c_e\left(\frac{\partial T}{\partial t} + \tau_0\frac{\partial^2 T}{\partial t^2}\right) + T_0\gamma\left[\frac{\partial}{\partial t}\left(\nabla^2\Phi\right) + \tau_0\delta_{ij}\frac{\partial^2}{\partial t^2}\left(\nabla^2\Phi\right)\right]. \tag{3.13}$$

Rewriting Eqs. (3.9) and (3.12) as

$$\nabla^2\Phi = \frac{\alpha}{c_1^2\left(1 + R_H + \zeta + \frac{4\eta}{3K}\frac{\partial}{\partial t}\right)}\frac{\partial^2\Phi}{\partial t^2} + \frac{\gamma}{\rho c_1^2\left(1 + R_H + \zeta + \frac{4\eta}{3K}\frac{\partial}{\partial t}\right)}\left[T + \tau_1\frac{\partial T}{\partial t}\right] + \frac{g}{c_1^2\left(1 + R_H + \zeta + \frac{4\eta}{3K}\frac{\partial}{\partial t}\right)}\frac{\partial\Phi}{\partial y}, \tag{3.14}$$

and

$$
\begin{aligned}
\nabla^2\Psi &= \frac{\alpha}{c_2^2}\frac{\partial^2\Psi}{\partial t^2} + \frac{g}{c_2^2}\frac{\partial\Psi}{\partial y}, && (3.15)\\
R_H &= \frac{\mu_e}{\rho c_1^2}, \quad \zeta = \frac{P}{K}, \quad c_2^2 = \frac{\eta\left(\frac{\partial}{\partial t}\right) - \frac{P}{2}}{\rho}, \quad c_1^2 = \frac{K}{\rho}, \quad \alpha = 1 + \frac{\epsilon_0\mu_e^2 H^2}{\rho}.
\end{aligned}
$$

Here R_H, c_1, c_2, α, ζ show magnetic pressure number, isothermal dilatational elastic wave velocity, rotational wave velocity and initial stress parameter respectively.

Eliminating T from Eqs. (3.13) and (3.14), we get 4^{th} order ordinary differential equation of scalar potential Φ.

$$
T = \left(1 + \tau_1\frac{\partial}{\partial t}\right)^{-1}\begin{bmatrix} \frac{\rho c_1^2\left(1+R_H+\zeta+\frac{4\eta}{3K}\frac{\partial}{\partial t}\right)}{\gamma}\nabla^2\Phi \\ -\frac{g\rho}{\gamma}\left(\frac{\partial\Phi}{\partial y}\right) - \frac{\rho\alpha}{\gamma}\left(\frac{\partial^2\Phi}{\partial t^2}\right) \end{bmatrix}, \qquad (3.16)
$$

$$
\begin{aligned}
&\rho c_1^2\left(1 + R_H + \zeta + \frac{4\eta}{3K}\frac{\partial}{\partial t}\right)c_3^2\nabla^4\Phi \\
&- \begin{bmatrix} \left(1 + R_H + \zeta + \frac{4\eta}{3K}\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t} + \\ \left\{ \begin{matrix} \tau_0\left(1 + R_H + \zeta + \frac{4\eta}{3K}\frac{\partial}{\partial t}\right) + \varepsilon_T\tau_1 \\ + \frac{c_3^2\alpha}{c_1^2} + \varepsilon_T\tau_0\delta_{ij} \end{matrix} \right\} \\ \frac{\partial^2}{\partial t^2} + \left(\frac{c_3^2\alpha\tau_\theta}{c_1^2} + \varepsilon_T\tau_0\delta_{ij}\right)\frac{\partial^3}{\partial t^3} + \frac{c_3^2 g}{c_1^2}\frac{\partial}{\partial y} + \frac{c_3^2 g\tau_\theta}{c_1^2}\frac{\partial^2}{\partial y\partial t} \end{bmatrix}\nabla^2\Phi \\
&+ \frac{\alpha}{c_1^2}\left(1 + \tau_0\frac{\partial}{\partial t}\right)\frac{\partial^3\Phi}{\partial t^3} + \frac{g}{c_1^2}\left(1 + \tau_0\frac{\partial}{\partial t}\right)\frac{\partial^2\Phi}{\partial y\partial t} = 0, \qquad (3.17)
\end{aligned}
$$

where $c_3^2 = \frac{K}{\rho c_e}$ and $\varepsilon_T = \frac{T_0\gamma^2}{\rho^2 c_e c_1^2}$ represents thermoelastic constant of the liquid medium.

We consider the solution of the form

$$
\begin{aligned}
\Phi &= f(y)\exp\left[ik\left(x - ct\right)\right], && (3.18)\\
\Psi &= e(y)\exp\left[ik\left(x - ct\right)\right], && (3.19)\\
T &= h(y)\exp\left[ik\left(x - ct\right)\right], && (3.20)
\end{aligned}
$$

where $c = \frac{\omega}{K}$ is the phase velocity, k is the wave number and ω is the circular frequency.

Substituting Eq. (3.18) into Eq. (3.17), we get

$$\frac{d^4f}{dy^4}+U\frac{d^3f}{dy^3}+V\frac{d^2f}{dy^2}+W\frac{df}{dy}+Xf\left(y\right)=0, \tag{3.21}$$

where

$$\begin{aligned}
U &= -\frac{c_3^2}{c_3^{*2}c_1^2}g, \\
V &= -2k^2+\frac{1}{c_3^{*2}}\left[\begin{array}{c}\left(1+R_H+\zeta+\varepsilon_T-\frac{i4\eta\omega}{3}+\varepsilon_T\tau_1+\frac{c_3^2}{c_4^2}+\varepsilon_T\tau_0\delta_{ij}\right)\\ +\left(\frac{c_3^2}{c_4^2}\tau_\theta+\varepsilon_T\tau_0\tau_1\delta_{ij}\right)i\omega\end{array}\right], \\
W &= -\frac{1}{c_3^{*2}}\left[\frac{c_3^2k^2g}{c_1^2}+\frac{gi\omega}{c_1^2}+\frac{g\omega\tau_0}{c_1^2}\right], \\
X &= k^4-\frac{1}{c_3^{*2}c^2}\left[\left(1+R_H+\zeta+\varepsilon_T-\frac{i4\eta\omega}{3}\right)i\omega^3+\left(\frac{c_3^2}{c_4^2}\tau_\theta+\varepsilon_T\tau_0\tau_1\delta_{ij}\right)i\omega^3\right. \\
&\quad \left.+\left(\left(+R_H+\zeta+\varepsilon_T-\frac{i4\eta\omega}{3}\right)\tau_0+\varepsilon_T\tau_1+\frac{c_3^2}{c_4^2}+\varepsilon_T\tau_0\delta_{ij}\right)k^4c^2-\frac{1}{c_4^2}i\omega^3-\frac{\tau_0}{c_4^2}\omega^4\right].
\end{aligned}$$

Eq. (3.21) is 4^{th} order ordinary differential equation, it has four solutions for $f\left(y\right)$ and the general solution of the scalar potential Φ is given as

$$\Phi=\left[B_1\exp\left(n_1y\right)+B_2\exp\left(n_2y\right)+B_3\exp\left(n_3y\right)+B_4\exp\left(n_4y\right)\right]\exp\left[ik\left(x-ct\right)\right], \tag{3.22}$$

where

$$\begin{aligned}
n_1 &= -\frac{U}{4}-\frac{1}{2}\sqrt{p}-\frac{1}{2}\sqrt{q-z}, \\
n_2 &= -\frac{U}{4}-\frac{1}{2}\sqrt{p}+\frac{1}{2}\sqrt{q-z}, \\
n_3 &= -\frac{U}{4}+\frac{1}{2}\sqrt{p}-\frac{1}{2}\sqrt{q-z}, \\
n_4 &= -\frac{U}{4}+\frac{1}{2}\sqrt{p}+\frac{1}{2}\sqrt{q-z}, \\
n_5 &= \frac{1}{2c_2^2}\left(g-\sqrt{\left(g\right)^2-4k^2c^2\alpha c_2^2+4k^2c_2^4}\right), \\
n_6 &= \frac{1}{2c_2^2}\left(g+\sqrt{\left(g\right)^2-4k^2c^2\alpha c_2^2+4k^2c_2^4}\right),
\end{aligned}$$

$$n_7=\frac{2}{3}k\eta\omega,$$

$$\begin{aligned}
p &= \frac{U^2}{4} - \frac{2V}{3} + \frac{2^{\frac{1}{3}}\left(V^2 + 12X - 3UW\right)}{3h} + \frac{1}{3 * 2^{\frac{1}{3}}}h, \\
q &= \frac{U^2}{2} - \frac{4V}{3} - \frac{2^{\frac{1}{3}}\left(V^2 + 12X - 3UW\right)}{3h} - \frac{1}{3 * 2^{\frac{1}{3}}}h, \\
h &= \left(O + \sqrt{\left(-4\left(V^2 + 12X - 3UW\right)^3 + (O)^2\right)}\right), \\
O &= 2V^3 + 27U^2X - 72VX - 9UVW + 27W^2,
\end{aligned}$$

$$\begin{aligned}
l_1 &= K\left(-k^2 + n_2^2\right) - \frac{2}{3}\eta k^2\omega - \frac{4}{3}\eta n_2^2 i\omega - \gamma d_2 - \tau_1 i\omega d_2, \\
l_2 &= K\left(-k^2 + n_4^2\right) - \frac{2}{3}\eta k^2\omega - \frac{4}{3}\eta n_4^2 i\omega - \gamma d_4 - \tau_1 i\omega d_4,
\end{aligned}$$

$$l_3 = -\frac{2}{3}k\eta\omega,$$

$$\begin{aligned}
d_1 &= \frac{\rho}{\gamma\tau}\left(-c_1^2k^2\left(1 + R_H + \zeta - \frac{i4\eta\omega}{3}\right) + n_1^2c_1^2\left(1 + R_H + \zeta - \frac{i4\eta\omega}{3} - gn_1\right)\right), \\
d_2 &= \frac{\rho}{\gamma\tau}\left(-c_1^2k^2\left(1 + R_H + \zeta - \frac{i4\eta\omega}{3}\right) + n_2^2c_1^2\left(1 + R_H + \zeta - \frac{i4\eta\omega}{3} - gn_2\right)\right).
\end{aligned}$$

Using Eq. (3.19) into (3.15), we get

$$\frac{d^2e}{dy^2} - \frac{g}{c_2^2}\frac{de}{dy} - \left(\frac{\alpha}{c_2^2}c^2 - 1\right)k^2 e\,(y) = 0, \tag{3.23}$$

after solving, we get

$$\Psi = \left[B_5 \exp\left(n_5 y\right) + B_6 \exp\left(n_6 y\right)\right] \exp\left[ik\left(x - ct\right)\right]. \tag{3.24}$$

The amplitudes are represented as B_i where $(i = 1, .., 6)$ B_1, B_3, B_5 are the amplitudes of incident waves where as B_2, B_4, B_6 corresponds to amplitudes of reflected waves.

Using Eqs. (3.18) and (3.22) in to Eq. (3.24), we get the value of T as

$$T = \left[d_1B_1 \exp\left(n_1y\right) + d_2B_2 \exp\left(n_2y\right) + d_3B_3 \exp\left(n_3y\right) + d_4B_4 \exp\left(n_4y\right)\right] \exp\left[ik\left(x - ct\right)\right]. \tag{3.25}$$

Considering primes to all quantities of the basic equations in the visco thermo elastic liquid medium N', we obtain the basic equations for visco thermoelastic liquid medium N', the quantities c_1, c_2, c_3, R_H, ε_T, and ζ in medium N change in medium N' as given below

$$\begin{aligned} R'_H &= \frac{\mu'_e}{\rho' c_1'^2}, \quad \zeta' = \frac{P'}{K'}, \quad c_2'^2 = \frac{\eta'\left(\frac{\partial}{\partial t}\right) - \frac{P'}{2}}{\rho'}, \\ c_1'^2 &= \frac{K'}{\rho'}, \quad c_3'^2 = \frac{K'}{\rho' c'_e}, \quad \varepsilon'_T = \frac{T'_0 \gamma'^2}{\rho'^2 c'_e c_1'^2}. \end{aligned}$$

and we get solution in the form

$$\Phi' = \left[B'_2 \exp\left(n'_2 y\right) + B'_4 \exp\left(n'_4 y\right)\right] \exp\left[ik\left(x - ct\right)\right], \tag{3.26}$$

$$T' = \left[d'_2 B'_2 \exp\left(n'_2 y\right) + d'_4 B'_4 \exp\left(n'_4 y\right)\right] \exp\left[ik\left(x - ct\right)\right]. \tag{3.27}$$

The amplitudes of refracted thermal and P-waves are represented by B'_2 and B'_4.

3.3 Boundary Conditions

The boundary conditions for incident SV-wave and letting $B_1 = B_3 = 0$ at $y = 0$ are as follows

(1) At the interface, the continuity of normal displacement i.e., $v = v'$

$$\frac{\partial \Phi}{\partial y} + \frac{\partial \Psi}{\partial x} = \frac{\partial \Phi'}{\partial y'}. \tag{3.28}$$

Using Eqs. (3.22), (3.24) and (3.26) in Eq. (3.28), we get

$$n_2 B_2 + n_4 B_4 + ikB_5 + ikB_6 - n'_2 B'_2 - ikB'_4 = 0. \tag{3.29}$$

(2) At the interface, tangential displacement must disappear i.e., $u = 0$

$$\frac{\partial \Phi}{\partial x} - \frac{\partial \Psi}{\partial y} = 0. \tag{3.30}$$

Using Eqs. (3.22), (3.24), in Eq. (3.30), we get

$$ikB_2 + ikB_4 - n_5 B_5 - n_6 B_6 = 0. \tag{3.31}$$

(3) At the interface, normal force per unit area is continuous i.e., $\sigma_{22} = \sigma'_{22}$.

This leads to

$$l_1 B_2 + l_2 B_4 - n_7 B_5 + l_3 B_6 + l'_1 B_4 + l'_2 B'_4 = 0. \tag{3.32}$$

(4) At the interface, tangential force per unit primary area must disappear i.e., $\sigma_{12} = 0$.

$$2\frac{\partial^2 \Phi}{\partial x \partial y} + \frac{\partial^2 \Psi}{\partial x^2} - \frac{\partial^2 \Psi}{\partial y^2} = 0, \tag{3.33}$$

$$2ikn_2 B_2 + 2ikn_4 B_4 + (ik)^2 B_5 + (ik)^2 B_6 - n_5^2 B_5 - n_6^2 B_6 = 0. \tag{3.34}$$

(5) At the interface, continuity of temperature i.e., $T = T'$.

$$d_2 B_2 + d_4 B_4 - d'_2 B'_2 - d'_4 B_4 = 0. \tag{3.35}$$

3.4 Equations for the Reflection and Refraction Coefficient

Consider the refraction and reflection of SV wave incident at viscoelastic interface at y=0 making an angle θ with y axis.

For incident SV wave we put $c = c_2 \cos ec\theta$ and take $B_1 = B_3 = 0$.

We get system of five non homogeneous equations,

$$\sum_{i=1}^{5} c_{ij} Z_j = y_j \qquad (j = 1, ..., 5),$$

where

$$\begin{aligned}
c_{11} &= n_2,\ c_{12} = n_4,\ c_{13} = ik,\ c_{14} = -n'_2,\ c_{15} = -n'_4, \\
c_{21} &= ik,\ c_{22} = ik,\ c_{23} = -n_6,\ c_{24} = 0,\ c_{25} = 0, \\
c_{31} &= l_1,\ c_{32} = l_2,\ c_{33} = l_3,\ c_{34} = l'_1,\ c_{35} = l'_2, \\
c_{41} &= 2ikn_2,\ c_{42} = 2ikn_4,..c_{43} = -\left(k^2 + n_6^2\right),\ c_{44} = 0,\ c_{45} = 0, \\
c_{51} &= d_2,\ c_{52} = d_4,\ c_{53} = 0,\ c_{54} = -d'_2,\ c_{55} = -d'_4,
\end{aligned}$$

$$y_1 = -ik, \; y_2 = n_5, \; y_3 = n_7, \; y_4 = \left(k^2 + n_5^2\right), \; y_5 = 0,$$
$$Z_1 = \frac{B_2}{B_5}, \; Z_2 = \frac{B_4}{B_5}, \; Z_3 = \frac{B_6}{B_5}, \; Z_4 = \frac{B_2'}{B_5}, \; Z_5 = \frac{B_4'}{B_5},$$

where $j = 1, ..., 5$ are the ratios of amplitude of reflected T, Pressure and SV-waves and transmitted thermal and pressure waves respectively.

3.5 Numerical result and discussion

For numerical study we used data for two visco elastic liquid layer following Dziewonski and Anderson [20].

For visco elastic liquid medium N:

$$k = 748.400 \times 10^9 Nm^{-2}, \; \alpha = 15.991 \times 10^{-6} k^{-1}, \; c_e = 811 Jkg^{-1}K^{-1}$$
$$\rho = 10327.26 kgm^{-3}, \; \vartheta = 28.5 Wk^{-1}m^{-1}, \; g = 10 ms^{-2}.$$

For visco elastic liquid medium N':

$$k' = 674.300 \times 10^9 Nm^{-2}, \; \alpha' = 17.386 \times 10^{-6} k^{-1}, \; c_e' = 814 Jkg^{-1}K^{-1}$$
$$\rho' = 10029.4 kgm^{-3}, \; \vartheta' = 28.4 Wk^{-1}m^{-1}.$$

Fig. 3.2 shows variation of amplitude ratio with incidence angle of SV-wave by taking three model of thermoelasticity in presence of gravity. The couple dynamic, Green and Lindsay and DPL theories. $|Z_1|$, $|Z_2|$, $|Z_4|$ and $|Z_5|$ begins with the maximum values and reaches to zero at $\theta = 90°$ but $|Z_3|$ comes to unity at $\theta = 90°$. It can be seen that the DPL theory in $|Z_1|$ obtains minimum values in comparison of GL and CD theories. We see that CD theory in $|Z_4|$ and $|Z_5|$ obtain minimum values in comparison of GL and DPL theories. We observe that GL theory in $|Z_1|$ obtains minimum values in comparison of DPL and CD theories. In Fig. 3.2, we consider the effect of gravity where as in Fig. 3.3 we neglect the effect of gravity. In Fig. 3.2 we see that the amplitudes of all wave reduces. In $|Z_5|$ CD and DPL theories have the same effect in comparison of G-L theory. It can be seen that CD theory in $|Z_1|$ and $|Z_2|$ reaches to zero after $\theta = 45°$ and then it start increasing.

Fig. 3.4 shows the effect of viscosity on the amplitude ratios with incidence angle of SV waves under the influence of DPL theory. The amplitude ratio $|Z_1|$, $|Z_4|$ and $|Z_5|$ increases with increasing viscosity but the amplitude ratio $|Z_2|$ decreases with increasing viscosity. The amplitude ratio $|Z_3|$ reaches to unity at $\theta = 50°$, before $\theta = 50°$ the amplitude ratio increases after $\theta = 50°$ it decreases.

Fig. 3.5 gives the influence of magnetic field on the amplitude ratio with θ under the influence of DPL theory. $|Z_1|$, $|Z_4|$ and $|Z_5|$ increases by increasing magnetic field, the amplitude ratios $|Z_2|$ and $|Z_3|$ decreases with increasing H. In $|Z_2|$ all curves coincide and moves to zero as θ approaches to 90° but $|Z_3|$ after $\theta = 45°$ it has opposite effect.

Fig. 3.6 depicts the variation of initial stress on the amplitude ratio under the influence of DPL theory. We observe that $|Z_1|$, $|Z_2|$ increases by increasing initial stress. The amplitude ratios $|Z_3|$, $|Z_4|$ and $|Z_5|$ decreases as initial stress increases but in $|Z_3|$ initially it decreases by increasing initial stress and reaches to unity at $\theta = 50°$ after $\theta = 50°$ it has opposite behaviour.

Fig. 3.7 illustrates the behavior of electric field on amplitude ratios with incidence angle θ under the influence of DPL theory. The amplitude ratios $|Z_1|$ and $|Z_4|$ increases as electric field increases, where as $|Z_2|$ and $|Z_5|$ decreases by increasing electric field while $|Z_3|$ reaches to unity at $\theta = 50°$, before $\theta = 50°$ it increases as electric field increases after $\theta = 50°$ it starts decreasing.

3.6 Conclusion

The influence of gravity, magnetic field, electric field and initial stress on the reflection and refraction of SV waves under three theories is observed. The amplitude ratios of all waves reduces in the absence of gravity. The change in electric field produces maximum amplitude of reflected and refracted thermal waves. The change in initial stress produces maximum amplitude ratio of reflected and refracted P waves. The change in initial stress and electric field produce maximum amplitude ratio of reflected SV wave.

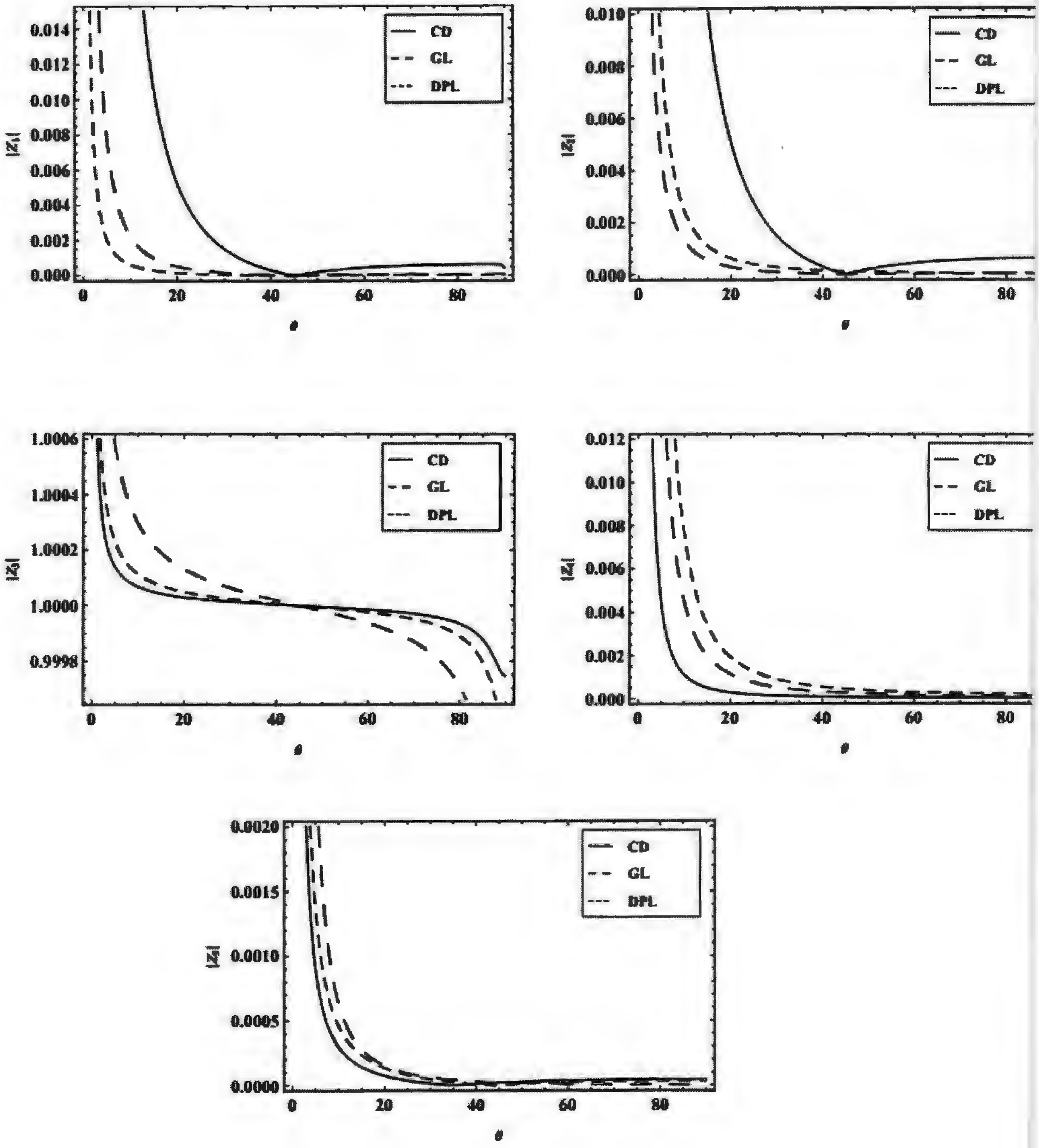


Fig. 3.2: Variation of amplitude with $|Z_i|\,(1,...,5)$ with the angle of incidence of SV-wave in presence of gravity.

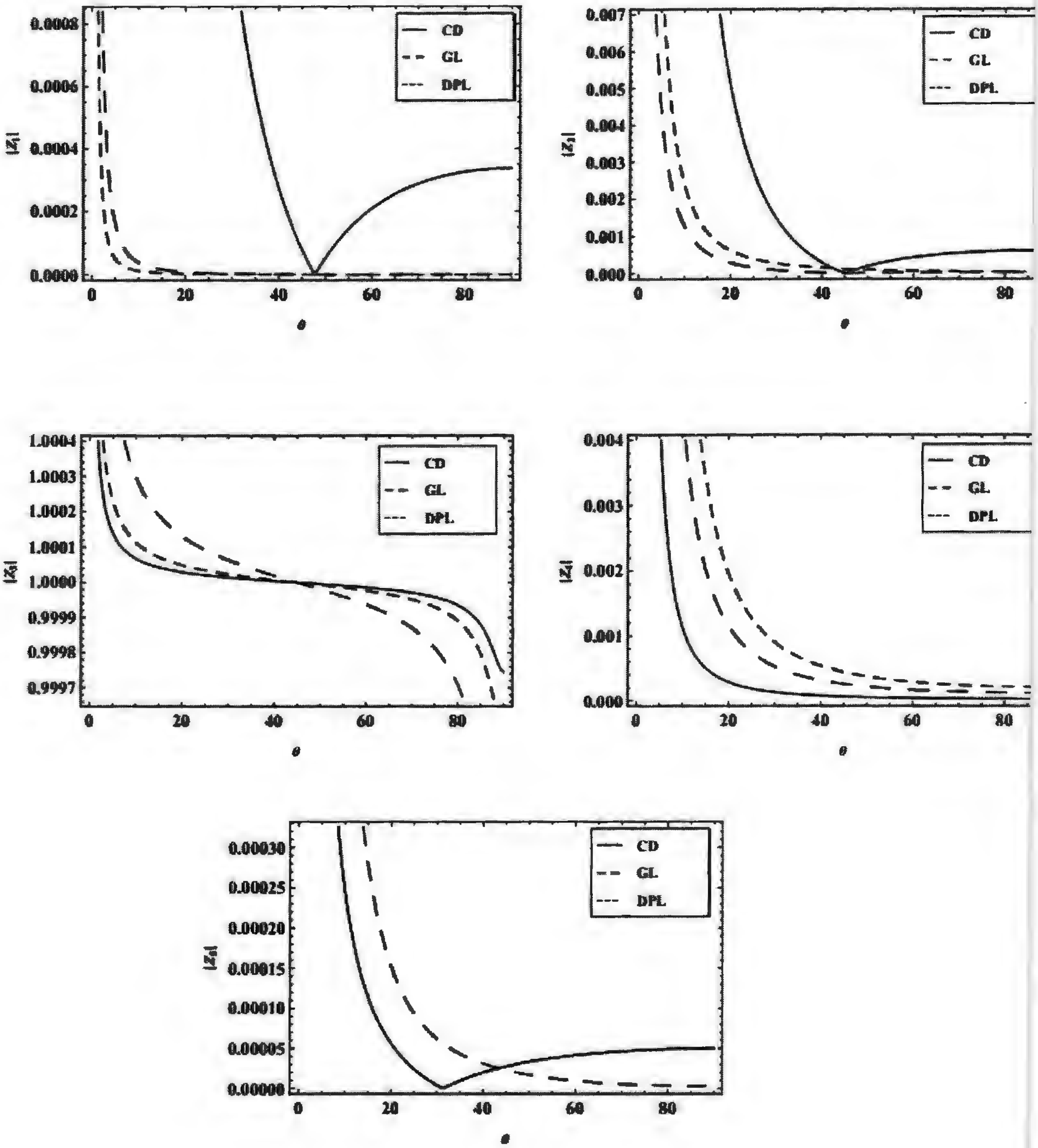


Fig. 3.3: Variation of amplitude with $|Z_i|\,(1,...,5)$ with the angle of incidence of SV-wave in absence of gravity

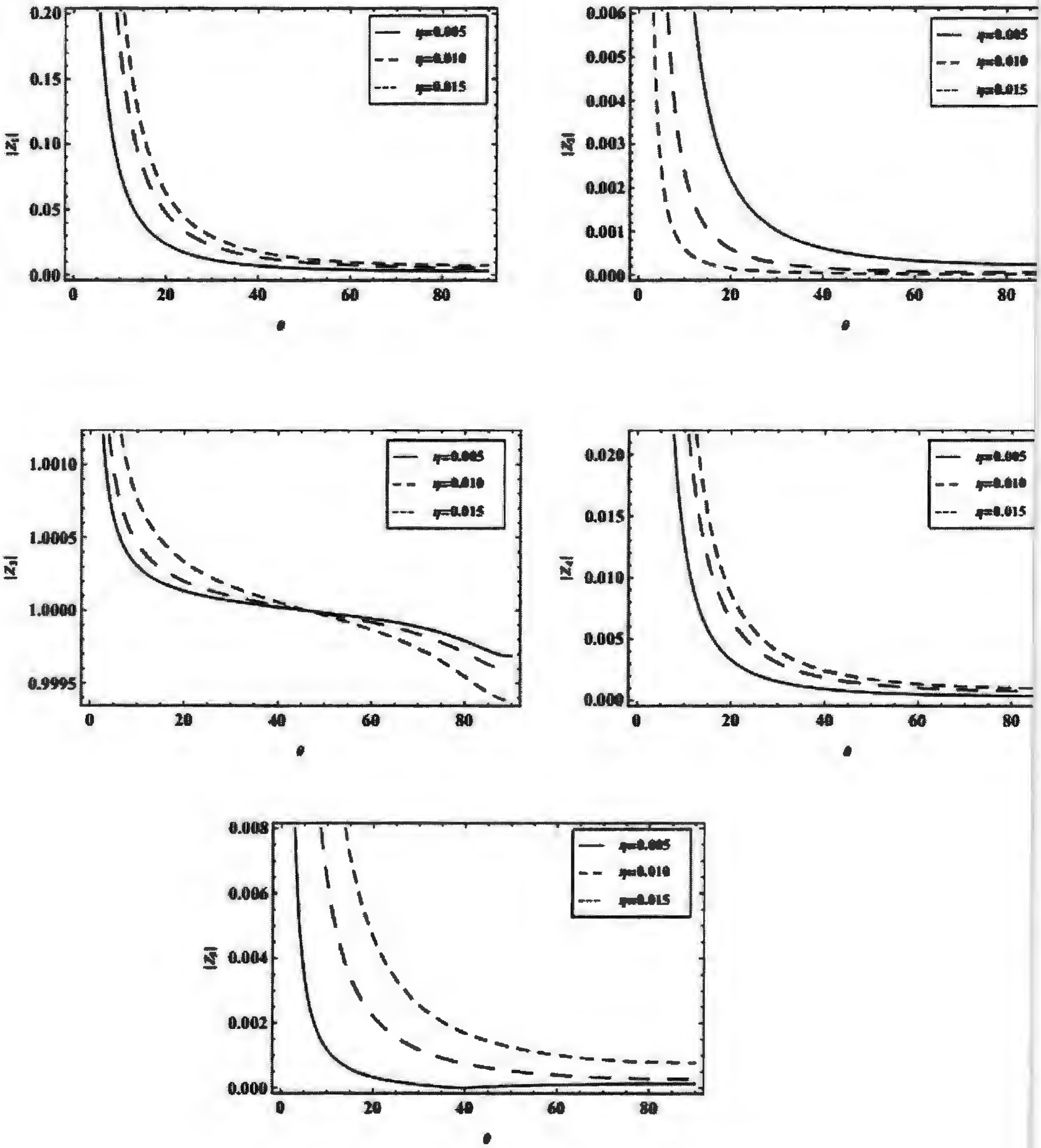


Fig. 3.4: Variation of amplitude with $|Z_i|$ $(1, 2, ..., 5)$ with the angle of incidence of SV-wave for variation of viscosity.

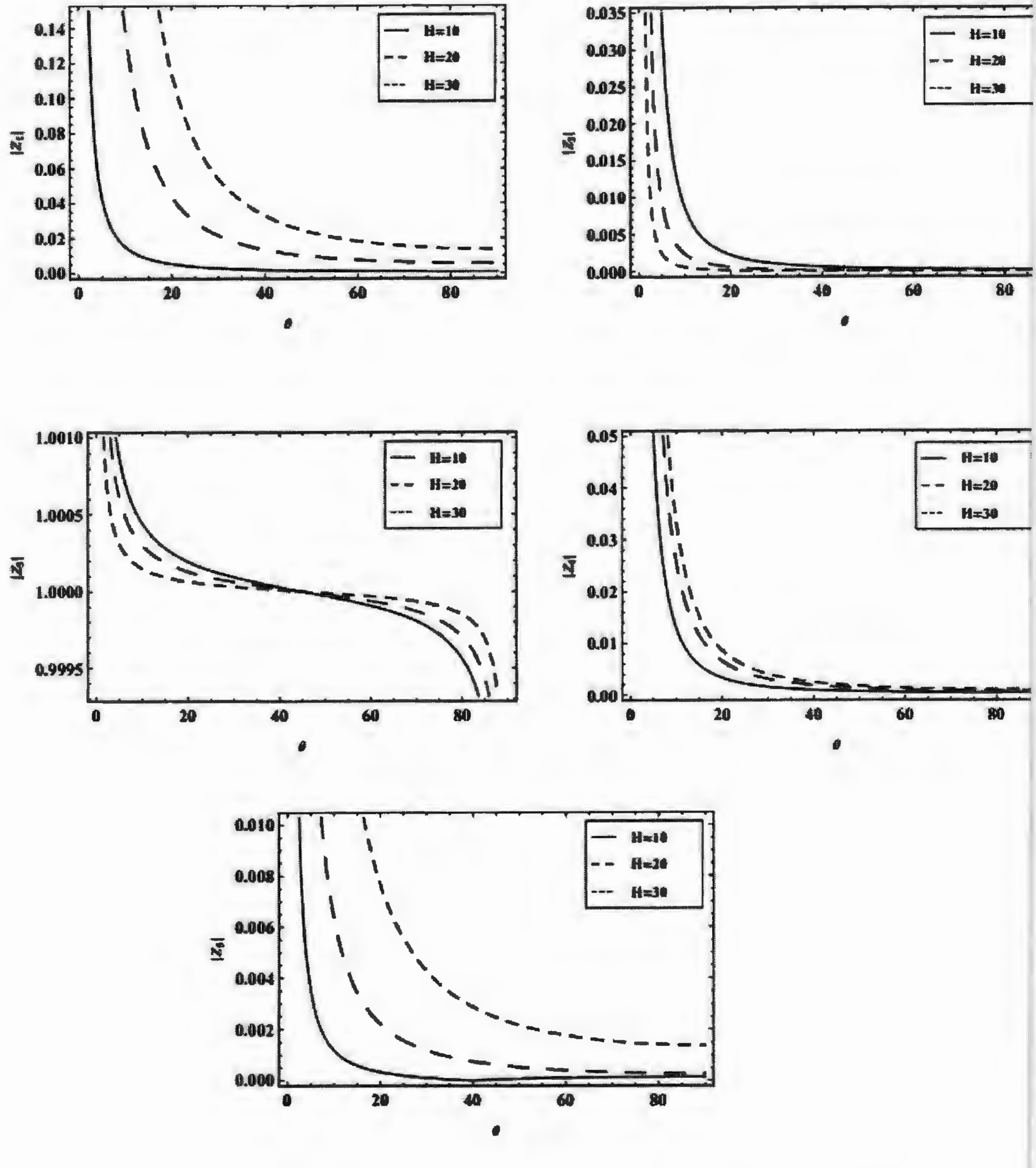


Fig. 3.5: Variation of amplitude with $|Z_i|\,(1,.....5)$ with the angle of incidence of SV-wave for variation of magnetic field.

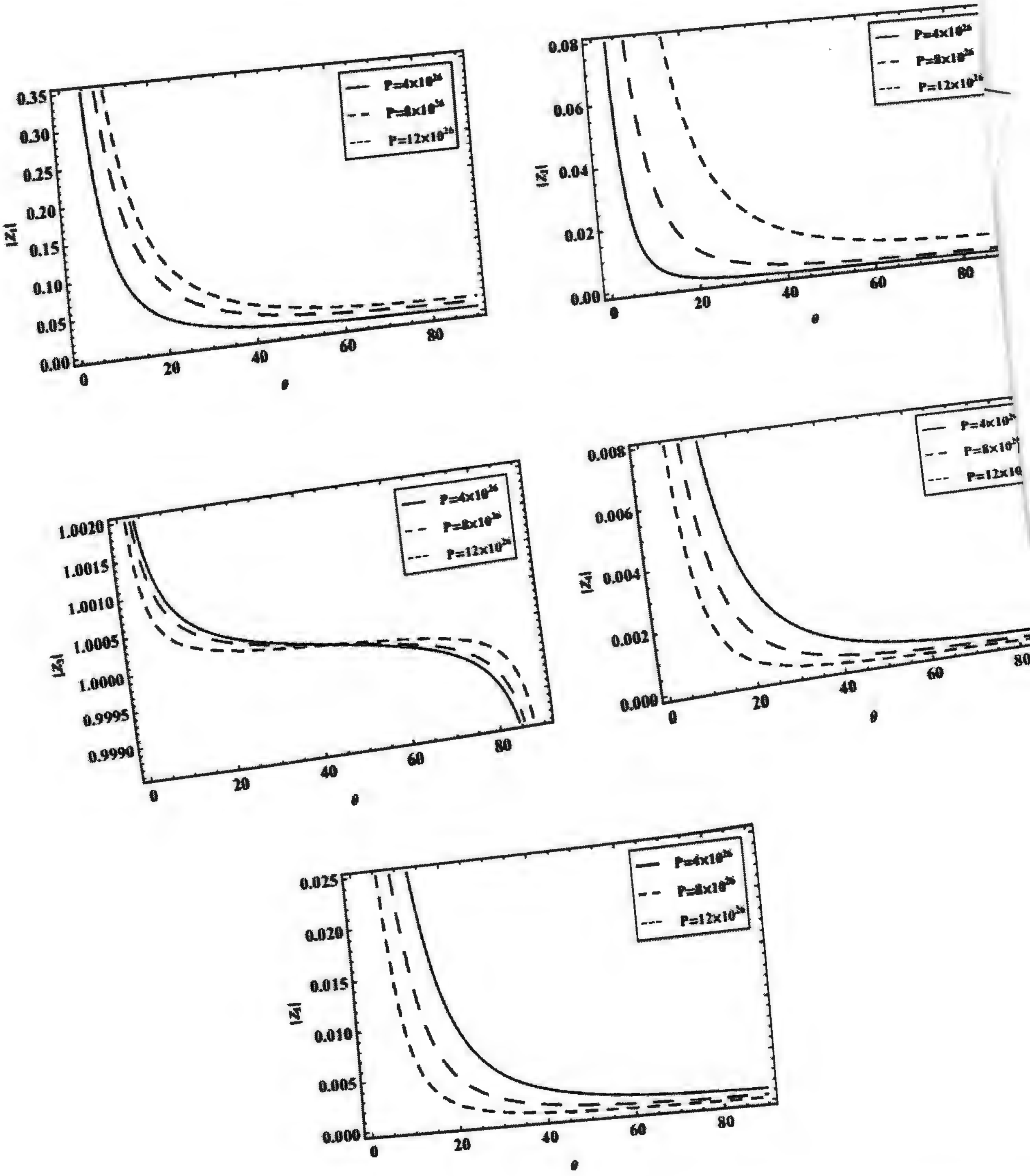


Fig. 3.6: Variation of amplitude with $|Z_i|\,(1,......5)$ with the angle of incidence of SV-wave for variation of initial stress.

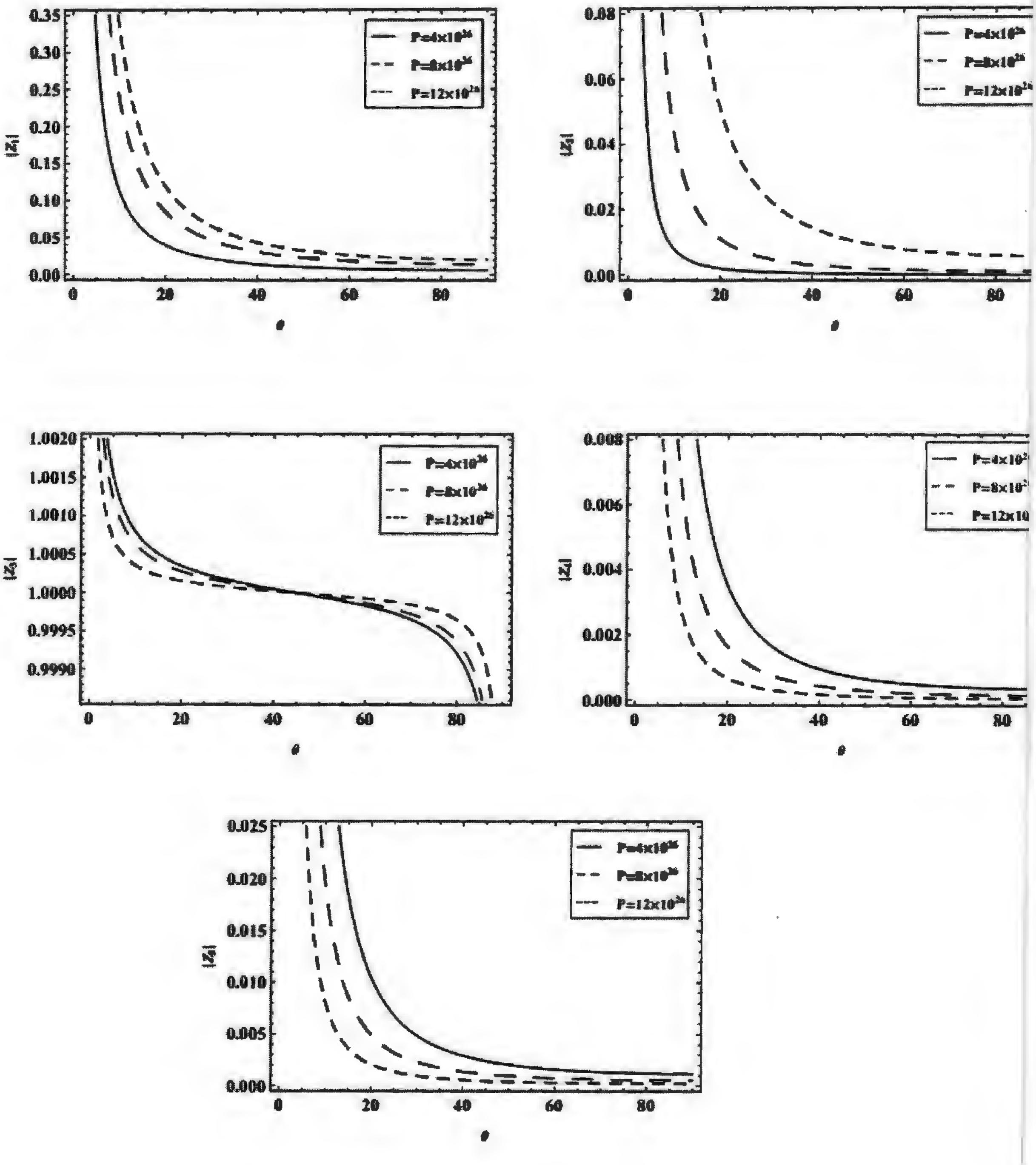


Fig. 3.6: Variation of amplitude with $|Z_i|\,(1,......5)$ with the angle of incidence of SV-wave for variation of initial stress.

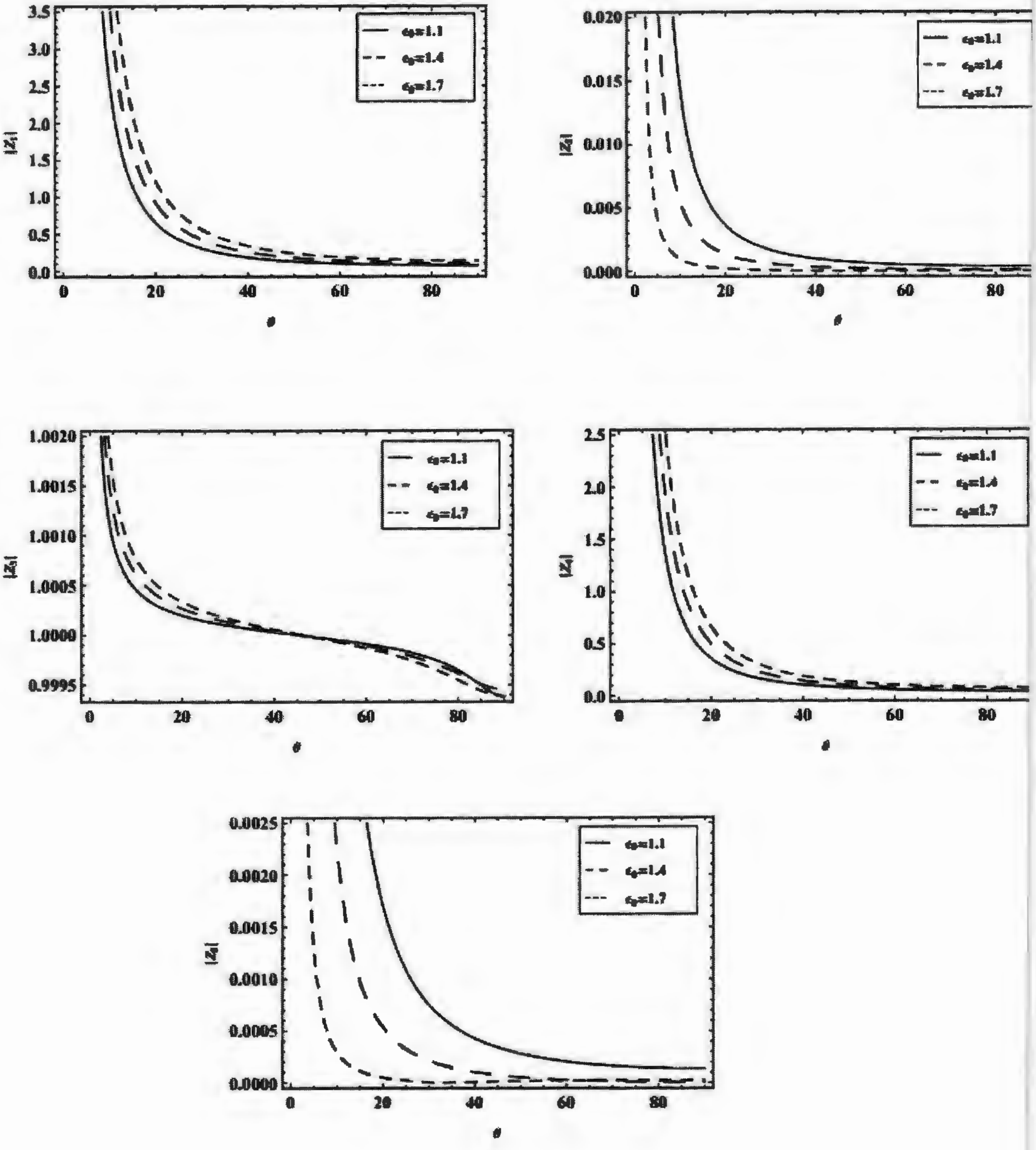


Fig. 3.7: Variation of amplitude with $|Z_i|\,(1, ...5)$ with the angle of incidence of SV-wave for variation of magnetic field.

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