Free Convective Flow on Vertical Plate with Ramped Wall Velocity and Temperature

By

FARWA MINHAS

Registration no: 263-FBAS/MSMA/F15

Department of Mathematics and Statistics
Faculty of Basic and Applied Sciences
International Islamic University, Islamabad
Pakistan

2017

Fluid mechanics.

Oldroyd-B fluid

Velocity

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A Thesis
Submitted in the Partial Fulfillment of the
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MASTER OF SCIENCE
In
MATHEMATICS

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Certificate

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A DISSERTATION SUBMITTED IN THE PARTIAL FULFILLMENT OF THE

REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN MATHEMATICS

We accept this dissertation as confirming to the required standard.

1. ______________________
Dr. Khadija Maqbool
(supervisor)

2. ______________________
Dr. Tariq Javed
(Internal Examiner)

3. ______________________
Dr. Khadija Maqbool
(Chairperson)

4. ______________________
Dr. Nabeela Kousar
(External Examiner)

Department of Mathematics & Statistics
Faculty of Basic & Applied Sciences
International Islamic University, Islamabad
Pakistan

2017

DEDICATED TO

MY PARENTS

ANWAR BEG MINHAS

&

FAZAL NOOR

ACKNOWLEDGEMENT

First of all, I will like to thank **ALLAH Almighty**, for giving me the strength and health for doing this thesis. I started my thesis with ease but soon found out that it was a hard nut to crack. Without the limitless blessings of Allah, I would not have even got closer to what I wanted to achieve.

Then I offer my deepest gratitude to my thesis supervisor, **Dr. Khadija Maqbool**, who supported and guided me throughout the thesis with patience and concern. Her immense knowledge, skills and experience in the subject proved greatly helpful. Her motivation incurred an urge to work hard and put my best in the thesis. Without her kind and devoted efforts this thesis would not have been possible.

Same way I am very grateful to my university, **International Islamic University**, who actually provided us with the platform and opportunity to work on this interesting topic.

Not to forget the prayers and support of my parents **Mr. and Mrs. Anwar Beg Minhas** not only during the thesis but in every step I took to be what I am today. They were always there to encourage me at times when I felt down and helped me in any manner they could to make me move again.

Last but not the least I would also like to thank my all batch mates especially **Sambila Khalid, Hajra Ansar and Mamoona Safdar** for their effective criticism and support throughout the thesis.

Farwa Minhas

DECLARATION

I hereby declare that this thesis, neither as a whole nor a part of it, has been copied out from any source. It is further declared that I have prepared this dissertation entirely on the basis of my personal efforts made under the supervision of my supervisor **Dr. Khadija Maqbool**. No portion of the work, presented in this dissertation, has been submitted in the support of any application for any degree or qualification of this or any other learning institute.

Farwa Minhas

MS (Mathematics)

Reg. no. 263-FBAS/MSMA/F15

Department of Mathematics and Statistics

Faculty of Basic and Applied Sciences,

International Islamic University Islamabad, Pakistan.

Preface

Heat transfer plays an important role during the handling and processing of non-Newtonian fluids [1-10]. The understanding of heat transfer in flows of non-Newtonian fluid is of great importance in many engineering applications such as transpiration cooling, thermite welding, drag reduction and thermal recovery of oil etc. Many studies [11-13] have been carried out for heat and mass transfer in non-Newtonian fluids of differential type. Applications of Mass Transfer include the dispersion of contaminants, drying and humidifying, segregation and doping in materials, vaporization and condensation in a mixture, evaporation (boiling of a pure substance is not mass transfer), combustion and most other chemical processes, cooling towers, sorption at an interface (adsorption) or in a bulk (absorption), and most living-matter processes as respiration (in the lungs and at cell level), nutrition, sweating, etc. Since ramped wall velocity and ramped wall temperature conditions [14-19] are of great importance in many branches of modern science and technology. To name a few only, e.g., Ergometery or application of ramped velocity through treadmill testing (TT) is found to be very useful method for diagnosing cardiovascular diseases. Therefore, we have considered the present study to discuss the non-Newtonian fluid flow with ramped wall velocity and temperature. The present thesis is organized in a following fashion.

In the first chapter of this thesis some basic definitions are discussed.

In the second chapter, MHD convective flow of an Oldroyd-B fluid in a porous medium in which both ramped wall velocity and ramped wall temperature conditions are applied to an infinite vertical plate. The resulting equations are solved by the Laplace transform method.

In the third chapter, MHD convective flow of a second grade fluid in a porous medium in which both ramped wall velocity and ramped wall temperature conditions are applied to an infinite vertical plate. The resulting equations are solved by the Laplace transform method.

Contents

Chapter 1

Preliminaries

In this chapter some relevant definitions are provided [20, 21].

1.1 Definitions

1.1.1 Fluid Mechanics

The science that deals with the behavior of fluids at rest (fluid statics) or in motion (fluid dynamics), and the interaction of fluids with solids or other fluids at the boundaries.

1.1.2 Viscosity

Viscosity is a quantity that measures the resistance of the fluid flow.

1.1.3 Newtonian vs non-Newtonian fluid

A Newtonian fluid is that in which shear stress and deformation rate have a direct relationship between each other.

Mathematically

$$\tau_{yx} = \mu \left(\frac{du}{dy} \right),$$

where τ_{yx}, μ and $\frac{du}{dy}$ are the shear stress, constant fluid viscosity and deformation rate respectively.

In non-Newtonian fluids, shear stress is related to the deformation rate in a non-linear manner, i.e.

$$\tau_{yx} = k \left(\frac{du}{dy} \right)^n ,$$

where n is flow behavior index and k is the consistency index.

1.1.4 Steady vs Unsteady flow

A flow in which fluid properties is independent of time is called the steady flow, while in an unsteady flow the fluid properties do depend on time.

1.2 Mass Transfer

The mass transfer is defined as the transport of fluid from a region of higher concentration to lower concentration.

1.2.1 Modes of Mass Transfer

The three basic modes of mass transfer are:

- Molecular mass diffusion
- Convective mass transfer
- Mass transfer by change of phase

1.2.2 Molecular Mass Diffusion

The diffusion of molecules when the whole bulk fluid is not moving but stationary. Diffusion of molecules is due to a concentration gradient.

1.2.3 Convective Mass Transfer

The transport of net mass between a boundary surface and a moving fluid.

1.2.4 Mass Transfer by Change of Phase

This is the process in which both diffusion and convection are involved.

1.3 Heat Transfer

Heat transfer takes place when there is a temperature difference.

1.3.1 Modes of Heat Transfer

The three basic modes of heat transfer are:

- Conduction
- Convection
- Radiation

1.3.2 Conduction

The transmission of heat through a material when there is a temperature difference.

1.3.3 Convection

The transfer of heat from one place to another by the movement of fluids.

1.3.4 Radiation

Energy transferred by electromagnetic waves.

1.4 Dimensionless numbers

1.4.1 Prandtl Number

It is the non-dimensional number that gives the ratio between momentum diffusivity to thermal diffusivity.

$$P_r = \frac{c_p \mu}{k}.$$

1.4.2 Schmidt Number

Schmidt number is the non-dimensional number that defines the ratio of kinematic viscosity to the diffusivity. It can be expressed as

$$S_c = \frac{\mu}{\rho D}.$$

1.4.3 Grashof Number

It is a non-dimensional number that gives the ratio of buoyancy forces to viscous forces.

$$G_T = \frac{g\beta \left(T_s - T_\infty\right) L^3}{\nu^2}.$$

1.4.4 Mass Grashof Number

Mass Grashof number G_m appears in the free convective mass transfer by concentration difference.

$$Gm = \frac{g\beta \left(C_s - C_\infty\right) L^3}{\nu^2}.$$

1.5 Conservation Laws

1.5.1 Law of conservation of mass

This law states that the mass of the closed system remains unchanged. Mathematical expression for this law is known as the continuity equation. For incompressible fluid, law is defined as

$$\nabla.\mathbf{V} = \mathbf{0}.$$

1.5.2 Law of conservation of momentum

This law states that the total momentum of a closed system is always conserved. For incompressible fluid, the equation of motion in vector form is

$$\rho\frac{d\mathbf{V}}{dt} = \text{div}\,\mathbf{T}+\rho\mathbf{b},$$

where $\frac{d}{dt}$ is the material derivative, $\mathbf{T}$ is the Cauchy's Stress tensor and $\mathbf{b}$ is the body force.

1.5.3 Law of conservation of energy

This law states tbat total energy of a closed system is conserved i.e. energy is neither formed nor decays but changes from one form to another. For incompressible fluid, the energy equation is as follows

$$\rho c_p\frac{dT}{dt} = k\nabla^2 T,$$

where c_p, k are specific heat at constant pressure and thermal conductivity respectively.

1.5.4 Law of conservation of mass concentration

This law is also called the Fick's second law. It predicts how rate of change of concentration is caused by diffusion.

Mathematically

$$\frac{dC}{dt}=D\nabla^2 C,$$

where C, D are concentration and diffusion coefficient respectively.

1.6 Laplace Transform Method

The Laplace transform is a linear operator L that transforms the given function f of the variable τ into a new function F of the variable s.

Mathematically, it is defined as

$$L\left[f\left(\tau\right)\right] = F\left(s\right) = \int_{0}^{\infty} f\left(\tau\right) e^{-s\tau} d\tau,$$

and Inverse Laplace Transform is defined by

$$L^{-1}\left[F\left(s\right)\right] = f\left(\tau\right) = \frac{1}{2\Pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} F\left(s\right) e^{s\tau} ds.$$

Chapter 2

Flow of an Oldroyd-B Fluid with Ramped wall Temperature and Velocity

In this chapter, we have established analytic solutions for velocity, temperature and concentration profiles for convective flow of an Oldroyd-B fluid [22] past a vertical plate of vast extent with ramped wall temperature and ramped wall velocity. The results of isothermal solutions and ramped wall solutions are also compared for velocity, temperature and concentration profiles.

2.1 Mathematical Formulation

Consider flow of an Oldroyd-B fluid, which is incompressible as well as electrically conducting past on a plate parallel to y'-axis through a porous medium. x'-axis is considered in the upward direction of plate while in the direction normal to plate is y'-axis. The fluid is allowed by a constant transverse magnetic field B_0 acting along y'-axis. Along x' and z' directions the plate is of vast length and is non conductor of electricity, so all physical quantities are functions of y' and t'. Initially, fluid and plate both are stationary with constant temperature T_∞ and uniform concentration C_∞. For $t' > 0$ the velocity and temperature of the wall is non-uniformly distributed as

$$T'(0,t') = \begin{cases} T_\infty + (T_w - T_\infty)\frac{t'}{t_0} & 0 < t' \leq t_0 \\ T_w & t' > t_0 \end{cases},$$

and

$$u'(0,t') = \begin{cases} U_0\frac{t'}{t_0} & t' \leq t_0 \\ U_0 & t' > t_0 \end{cases},$$

while concentration is maintained uniformly as

$$C'(0,t') = C'_w \, .$$

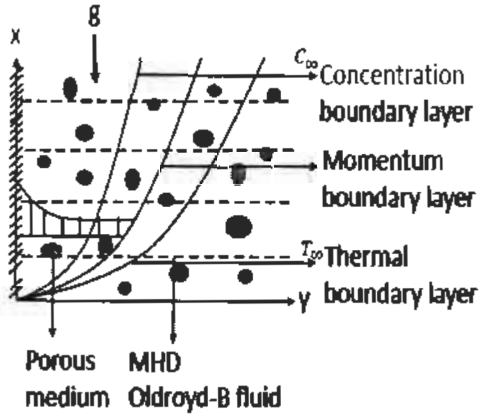


Fig. 2.1: Schematic diagram of problem

Taking assumption that induced magnetic field is imperceptible as comparing to the applied one i.e. $\mathbf{B} \equiv (0, B_0, 0)$. The polarization outcome of fluid is imperceptible as there is no outer electric field, hence we assume $\mathbf{E} \equiv 0$.

For an Oldroyd-B fluid [22], momentum equation can be written as

$$\rho\frac{d\mathbf{V}}{dt} = \operatorname{div}\mathbf{S} + (\mathbf{J}\times\mathbf{B}) + \rho\mathbf{g}\beta'(T - T_\infty) + \rho g\beta^*(C - C_\infty) + \mathbf{R},$$

where

$$\mathbf{V} = (\mathbf{u}(y,t), 0, 0),$$

and S satisfies the following relation

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\mathbf{S} = \mu\left(1+\theta\frac{\partial}{\partial t}\right)\mathbf{A}_1,$$

and

$$\mathbf{A}_1 = \nabla\mathbf{V} + (\nabla\mathbf{V})^T,$$

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\mathbf{R} = -\frac{\mu\phi}{k_p}\left(1+\theta\frac{\partial}{\partial t}\right)\mathbf{V},$$

$$\mathbf{J}\times\mathbf{B} = -\sigma B_0^2\mathbf{V}.$$

The governing equations for an Oldroyd-B fluid can be obtained as

$$\begin{aligned}\left(1+\lambda'\frac{\partial}{\partial t'}\right)\frac{\partial u'}{\partial t} &= \left(1+\lambda'\frac{\partial}{\partial t'}\right)\left(g\left(\beta^*\left(C'-C_\infty\right)+\beta'(T'-T_\infty)\right)-\frac{\sigma B_0^2 u'}{\rho}\right)\\ &+\left(1+\theta'\frac{\partial}{\partial t'}\right)\left(\nu\frac{\partial^2 u'}{\partial y'^2}-\nu\frac{\phi}{k'_p}u'\right),\end{aligned} \tag{2.1}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p}\frac{\partial q'_r}{\partial y'}, \tag{2.2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2}, \tag{2.3}$$

where u', T', g, β', β^*, ν, σ, ρ, k, k'_p, C_p, q'_r, D, λ', and θ' are fluid velocity, fluid's temperature, gravitational acceleration, volumetric coefficient of thermal expansion, volumetric coefficient of concentration expansion, coefficient of kinematic viscosity, electrical conductivity, density of the fluid, thermal conductivity, porosity parameter, specific heat at constant pressure, radiative flux vector, mass diffusion coefficient, retardation time and relaxation time respectively.

The initial conditions are

$$u'(y',0) = 0 \ , \ T'(y',0) = T_\infty, \ C'(y',0) = C_\infty \quad \text{for } y' \geq 0, \tag{2.4}$$

and the boundary conditions are

$$\begin{aligned}
u'(0,t') &= \begin{cases} U_0 \frac{t'}{t_0} & t' \leq t_0 \\ U_0 & t' > t_0 \end{cases}, \\
T'(0,t') &= \begin{cases} T_\infty + (T_w - T_\infty)\frac{t'}{t_0} & 0 < t' \leq t_0 \\ \quad T_w & t' > t_0 \end{cases}, \\
C'(0,t') &= C_w, \\
u'(\infty,t') &\longrightarrow 0, \ T'(\infty,t') \longrightarrow T_\infty, \ C'(\infty,t') \longrightarrow C_\infty \ .
\end{aligned} \tag{2.5}$$

The radiative flux vector q'_r under Rosseland approximation becomes

$$q'_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T'^4}{\partial y'}, \tag{2.6}$$

and

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \ , \tag{2.7}$$

where k^*, σ^* and T'_∞ are mean absorption coefficient, Stefan-Boltzmann constant and free stream temperature respectively.

Using Eqs. (2.6) and (2.7), in Eq. (2.2),we get the following equation

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{1}{\rho C_p}\frac{16\sigma^* T'^3_\infty}{3k^*}\frac{\partial^2 T'}{\partial y'^2}. \tag{2.8}$$

Introducing following non-dimensional quantities

$$
\begin{cases}
y = \frac{y'}{U_0 t_0},\ u = \frac{u'}{U_0},\ t = \frac{t'}{t_0}, \\
\lambda = \frac{\lambda'}{t_0},\ \theta = \frac{\theta'}{t_0}, \\
T = \frac{(T' - T_\infty)}{(T_w - T_\infty)},\ C = \frac{(C' - C_\infty)}{(C_w - C_\infty)}, \\
S_c = \frac{\nu}{D},\ M = \frac{\sigma B_0^2 \nu}{\rho U_0^2},\ k_1 = \frac{k_p U_0^2}{\phi \nu^2}, \\
G_r = \frac{g\beta\nu(T_w - T_\infty)}{U_0^3},\ G_m = \frac{g\beta^* \nu(C_w - C_\infty)}{U_0^3}, \\
P_r = \frac{\rho\nu C_p}{k}, \\
t_0 = \frac{\nu}{U_0^2}, \quad N = \frac{16\sigma^* T_\infty^3}{3kk^*}.
\end{cases} \tag{2.9}
$$

Non dimensional form of equations (2.1) , (2.3) and (2.8) are as follows

$$
\left(1 + \theta\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 u}{\partial y^2} - \frac{u}{k_1}\right)
+ \left(1 + \lambda\frac{\partial}{\partial t}\right)\left(G_r T + G_m C - Mu - \frac{\partial u}{\partial t}\right) = 0, \tag{2.10}
$$

$$
\frac{\partial T}{\partial t} = \frac{(1+N)}{P_r}\frac{\partial^2 T}{\partial y^2}, \tag{2.11}
$$

$$
\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2}, \tag{2.12}
$$

where M, k_1, G_r, G_m, P_r, S_c, λ, θ and N are magnetic parameter, porosity parameter, thermal Grashof number, mass Grashof number, Prandtl number, Schmidt number, relaxation time, retardation time and radiation parameter respectively.

2.2 Solution of the problem

2.2.1 Case 1: Ramped temperature and velocity

The non-dimensional initial and boundary conditions are

$$u\,(y,0)=0,\; T\,(y,0)=0,\; C\,(y,0)=0 \quad for \;\; y\geq 0, \tag{2.13}$$

and

$$\begin{aligned}
u\,(0,t) &= \begin{cases} t & 0<t\leq 1 \\ 1 & t>1 \end{cases}, \\
T\,(0,t) &= \begin{cases} t & 0<t\leq 1 \\ 1 & t>1 \end{cases}, \\
C\,(0,t) &= 1, \\
u\,(y,t) &\longrightarrow 0,\; T\,(y,t)\longrightarrow 0,\; C\,(y,t)\longrightarrow 0 \;as\;\; y\longrightarrow\infty.
\end{aligned} \tag{2.14}$$

Applying Laplace transform on both sides of Eqs. (2.10)-(2.12) and using Eq.(2.13), we get the following forms

$$\frac{d^2\overline{u}}{dy^2}-\left(\frac{\lambda s^2+bs+c}{1+\theta s}\right)\overline{u}+\left(\frac{1+\lambda s}{1+\theta s}\right)\left(G_r\overline{T}+G_m\overline{C}\right)=0, \tag{2.15}$$

$$\frac{d^2\overline{T}}{dy^2}-sa\overline{T}=0, \tag{2.16}$$

$$\frac{d^2\overline{C}}{dy^2}-sS_c\overline{C}=0, \tag{2.17}$$

where

$$a = \frac{P_r}{1+N}.$$

The transformed boundary conditions are

$$\begin{aligned} \overline{u}(0,s) &= \overline{T}(0,s) = \frac{(1-e^{-s})}{s^2}\ ,\ \ \overline{C}(0,s) = \frac{1}{s},\ \ for\ \ t>0, \\ \overline{u}(y,s) &= 0,\ \overline{T}(y,s) = 0\ ,\ \overline{C}(y,s) = 0 \quad as\ y \longrightarrow \infty\ \ for\ \ t>0. \end{aligned} \tag{2.18}$$

Using the boundary conditions (2.18), the solutions of Eqs. (2.15)-(2.17) becomes

$$\overline{T}(y,s) = \frac{1-e^{-s}}{s^2} e^{-y\sqrt{as}}, \tag{2.19}$$

$$\overline{C}(y,s) = \frac{e^{-y\sqrt{S_c s}}}{s}, \tag{2.20}$$

$$\begin{aligned} \overline{u}(y,s) = & \left(\frac{1-e^{-s}}{s^2}\right)\exp\left(-y\sqrt{\frac{\lambda s^2+bs+c}{1+\theta s}}\right) \\ & + \left(\frac{1-e^{-s}}{s^2}\right)\frac{G_r(1+\lambda s)}{as(1+\theta s)-(\lambda s^2+bs+c)} \\ & \left\{\exp\left(-y\sqrt{\frac{\lambda s^2+bs+c}{1+\theta s}}\right) - e^{-y\sqrt{as}}\right\} \\ & + \frac{1}{s}\frac{G_m(1+\lambda s)}{S_c s(1+\theta s)-(\lambda s^2+bs+c)} \\ & \left\{\exp\left(-y\sqrt{\frac{\lambda s^2+bs+c}{1+\theta s}}\right) - e^{-y\sqrt{S_c s}}\right\}, \end{aligned} \tag{2.21}$$

where

$$b = 1+\lambda M + \frac{\theta}{k_1},\ \ c = M + \frac{1}{k_1}.$$

Taking inverse Laplace transform of Eqs. (2.19)-(2.21), solutions for temperature profile, concentration profile and velocity profile are obtained as follows

$$T(y,t) = Q(y,t) - H(t-1)\,Q(y,t-1), \tag{2.22}$$

where

$$Q(y,t) = \left(\frac{ay^2}{2} + t\right)\operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{a}{t}}\right) - \sqrt{\frac{at}{\Pi}}\,y e^{\frac{-ay^2}{4t}},$$

$$C(y,t) = \operatorname{erf c}\left(\frac{y\sqrt{S_c}}{2\sqrt{t}}\right), \tag{2.23}$$

and the solution of velocity $u(y,t)$ can be found by Stefhest method [23]

$$u(y,t) = \frac{\ln 2}{t}\sum_{k=1}^{2n} V_k \overline{u}\left(y,k\right)\frac{\ln 2}{t}, \tag{2.24}$$

where

$$V_k = (-1)^{k+n}\sum_{j=\lfloor\frac{k+1}{2}\rfloor}^{\min(k,n)} \frac{j^n\,(2j)!}{(n-j)!j!\,(j-1)!\,(k-j)!\,(2j-k)!}.$$

The complementary error function erfc(z) is defined as

$$\operatorname{erfc}(z) = 1 - \operatorname{erf}(z),\ \operatorname{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-\eta^2}d\eta,$$

and $H(t-1)$ is the unit step function and defined as

$$H(t-1) = \begin{cases} 0 & t-1<0 \\ 1 & t-1\geq 0 \end{cases}.$$

2.2.2 Case 2: Isothermal with constant velocity

The dimensional and non-dimensional form of isothermal boundary conditions are

$$
\begin{aligned}
u'(0,t') &= U_0, \\
T'(0,t') &= T_w \qquad \text{for } t' > 0, \\
C'(0,t') &= C_w, \\
u'(\infty,t') &\longrightarrow 0,\ T'(\infty,t') \longrightarrow T_\infty,\ C'(\infty,t') \longrightarrow C_\infty\ ,
\end{aligned}
$$

and

$$
\begin{aligned}
u(0,t) &= 1, \\
T(0,t) &= 1 \qquad \text{for} \quad t > 0, \\
C(0,t) &= 1, \\
u(y,t),\ T(y,t),\ C(y,t) &\longrightarrow 0 \qquad \text{as } y \longrightarrow \infty.
\end{aligned}
$$

The transformed isothermal boundary conditions are

$$
\begin{aligned}
\overline{u}(0,s) &= \frac{1}{s}, \\
\overline{T}(0,s) &= \frac{1}{s}, \\
\overline{C}(0,s) &= \frac{1}{s}, \\
\overline{u}(y,s),\ \overline{T}(y,s),\ \overline{C}(y,s) &\longrightarrow 0 \qquad \text{as } y \longrightarrow \infty.
\end{aligned}
\tag{2.25}
$$

Using the boundary conditions (2.25) ,the solutions of Eqs. (2.15) - (2.17) are obtained as

$$
\overline{T}(y,s) = \frac{1}{s}e^{-y\sqrt{as}}, \tag{2.26}
$$

$$\overline{C}(y,s) = \frac{e^{-y\sqrt{S_c s}}}{s}, \tag{2.27}$$

$$\begin{aligned}\overline{u}(y,s) &= \frac{1}{s}\exp\left(-y\sqrt{\frac{\lambda s^2+bs+c}{1+\theta s}}\right)+\frac{1}{s}\frac{G_r(1+\lambda s)}{as(1+\theta s)-(\lambda s^2+bs+c)} \\ &\left\{\exp\left(-y\sqrt{\frac{\lambda s^2+bs+c}{1+\theta s}}\right)-e^{-y\sqrt{as}}\right\} \\ &+\frac{1}{s}\frac{G_m(1+\lambda s)}{S_c s(1+\theta s)-(\lambda s^2+bs+c)} \\ &\left\{\exp\left(-y\sqrt{\frac{\lambda s^2+bs+c}{1+\theta s}}\right)-e^{-y\sqrt{S_c s}}\right\}.\end{aligned} \tag{2.28}$$

Taking inverse Laplace transform of Eqs. (2.26) – (2.28), solutions for temperature profile, concentration profile and velocity profile are obtained as follows

$$T(y,t) = \operatorname{erf} c\left(\frac{y}{2}\sqrt{\frac{a}{t}}\right), \tag{2.29}$$

$$C(y,t) = \operatorname{erf} c\left(\frac{y}{2}\sqrt{\frac{S_c}{t}}\right), \tag{2.30}$$

and the solution of velocity $u(y,t)$ can be found by Stefhest method [23].

2.3 Skin Friction

The skin friction measures the amount of shear stress on the boundary and is given by the following expression

$$\tau_w(t) = \frac{\tau'(y,t)\,|_{y=0}}{\rho U_0^2} = \frac{\left(1+\theta\frac{\partial}{\partial t}\right)}{\left(1+\lambda\frac{\partial}{\partial t}\right)}\left.\frac{\partial u}{\partial y}\right|_{y=0}. \tag{2.31}$$

2.4 Nusselt Number

The Nusselt number measures the heat transfer rate on the boundary and is given by the following form

$$Nu = -\frac{U_0 t_0 \left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_w - T_\infty)} = -\frac{\partial T}{\partial y}\ |_{y=0},$$

For ramped temperature

$$Nu = 2\sqrt{\frac{a}{\pi}}\left(\sqrt{t} - \sqrt{t-1}H\left(t-1\right)\right), \tag{2.32}$$

For isothermal

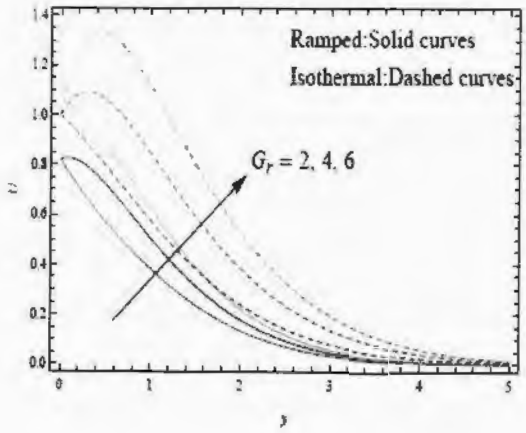
$$Nu = \sqrt{\frac{a}{\pi t}}. \tag{2.33}$$

2.5 Sherwood Number

The sherwood number measures the mass transfer rate on the boundary and is given by the following expression

$$Sh_r = -\frac{U_0 t_0 \left.\frac{\partial C}{\partial y}\right|_{y=0}}{(C_w - C_\infty)} = -\frac{\partial C}{\partial y}\ |_{y=0} = \sqrt{\frac{S_c}{\pi t}}. \tag{2.34}$$

2.6 Results and discussion



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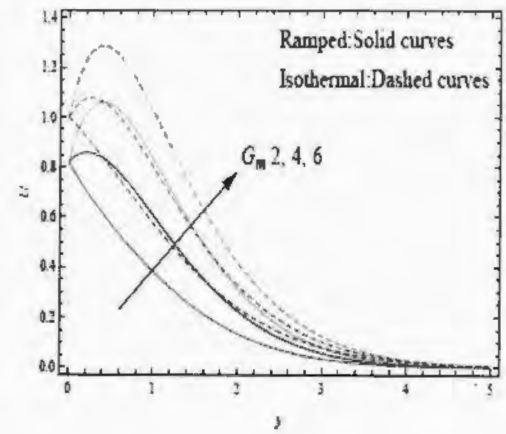

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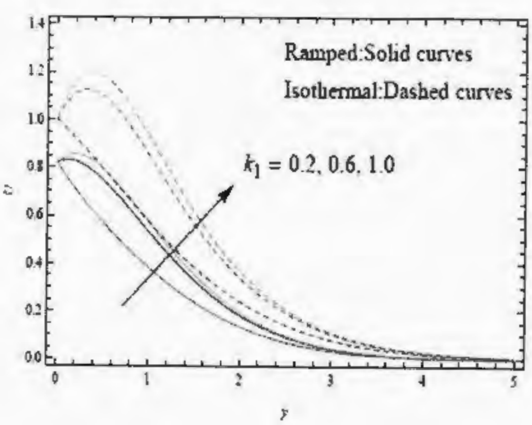
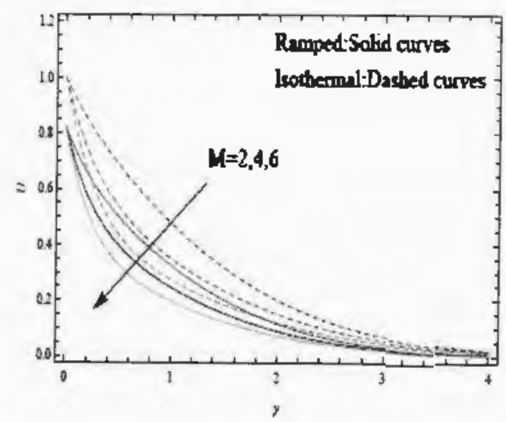


(c)



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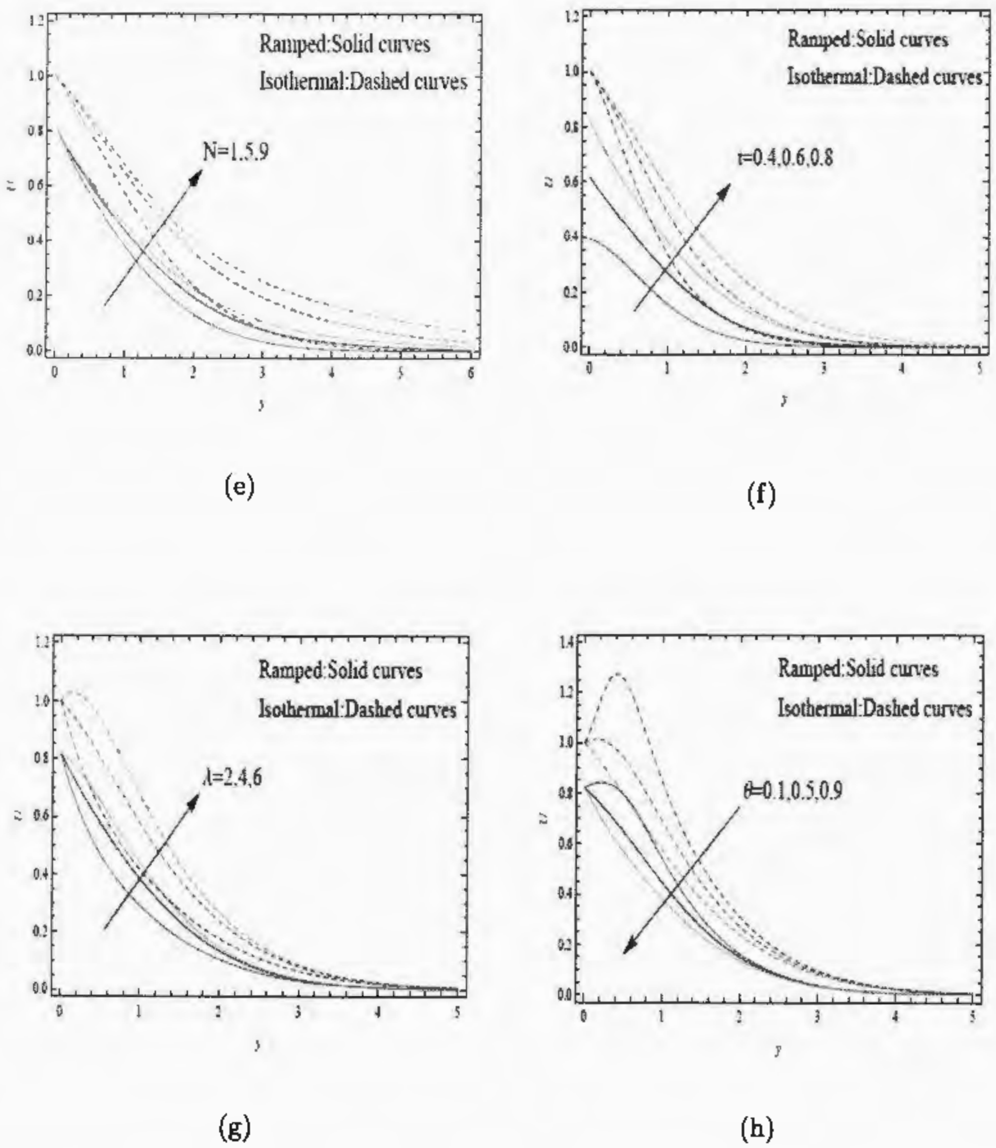
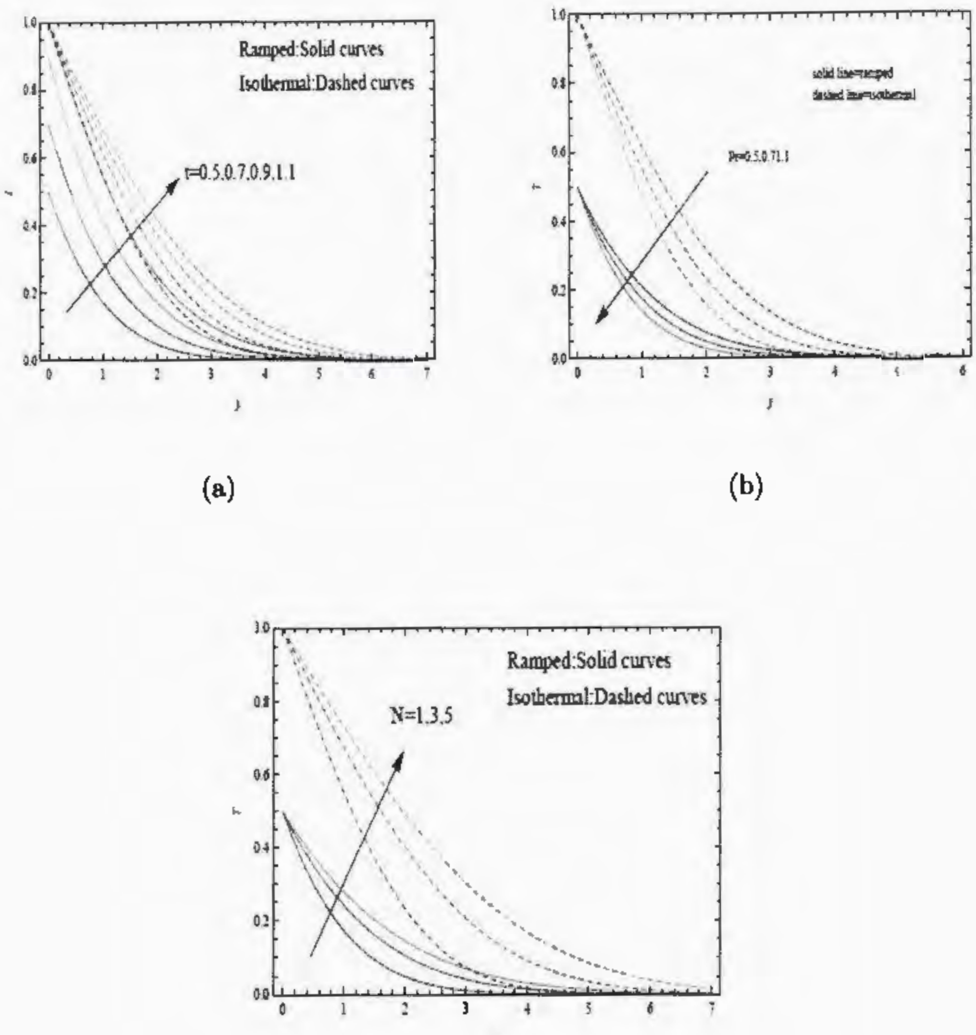


Fig. 2.2: Velocity profile for different values of (a) Grashof number G_r, (b) mass Grashof number G_m, (c) porosity parameter k_1, (d) magnetic parameter M, (e) radiation parameter N, (f) time t, (g) relaxation time λ and (h) retardation time θ with $P_r = 0.71$ and $S_c = 0.6$.



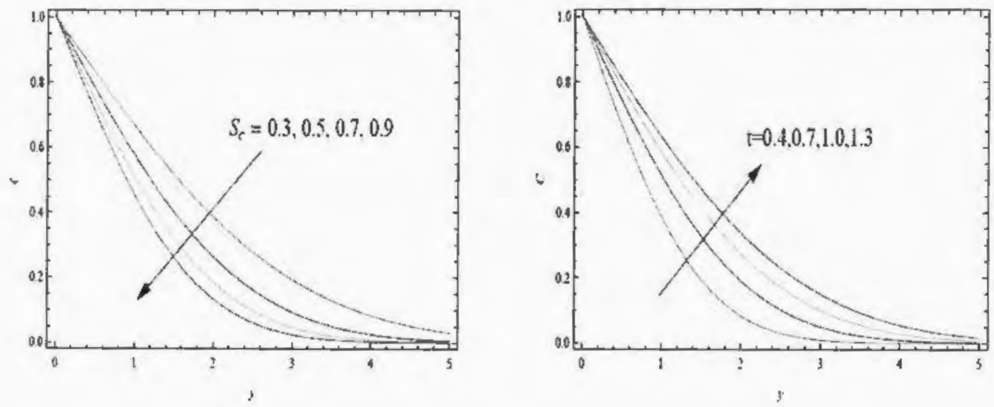
(a) (b)

(c)

Fig. 2.3: Temperature profile for different values of (a) time t, (b) Prandtl number P_r and (c) radiation parameter N.

Fig. 2.2(a) displays the effect of Grashof number G_r on fluid velocity, which shows that as G_r increases fluid velocity also increases. Hence buoyancy forces speed up the fluid flow. In Fig. 2.2(b) the effect of mass Grashof number G_m on fluid velocity is observed. In Fig. 2.2(c) the effects of porosity parameter k_1 is presented. Hence there is a proportional relation between velocity of the fluid and porosity parameter. This implies that resistance applied by porous medium reduces as k_1 increases which in result speed up fluid flow. Fig. 2.2(d) demonstrates the effect of magnetic field. So, there is an inverse relation between magnetic parameter M and velocity of the fluid. Fig. 2.2(e) describes the effect of radiation on fluid velocity. In this case we have a direct relation between fluid velocity and radiation parameter N. Fig. 2.2(f) shows the influence of time t on fluid velocity and show that velocity of the fluid speed up w.r.t time. Fig. 2.2(g) displays the effect of relaxation time λ, according to which fluid velocity u increases on increasing relaxation time λ. Fig. 2.2(h) displays the effect of retardation time θ on fluid velocity, from where we can see that as retardation time increases velocity of the fluid decreases.

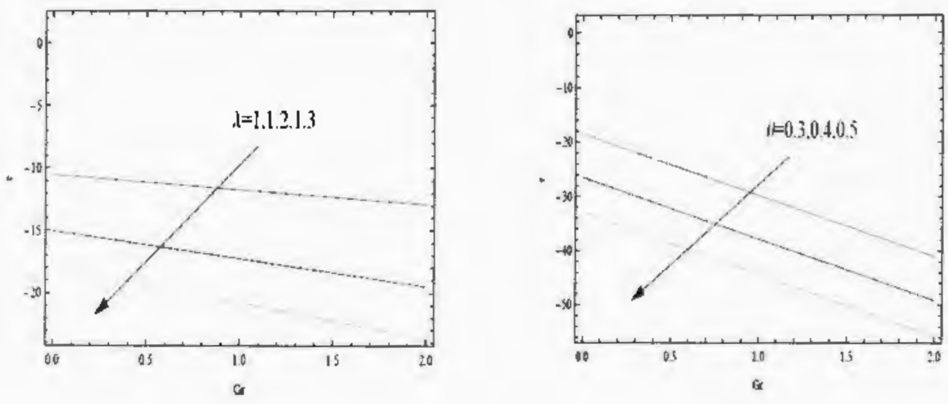
The graphs of temperature profile T are presented in Fig. 2.3. Fig. 2.3(a) shows that temperature T increases w.r.t time t. Fig. 2.3(b) describes the effect of Prandtl number P_r. Here we have an inverse relation between P_r and T , i.e. thermal diffusion tends to increase temperature of the fluid with P_r. Fig. 2.3(c) illustrates the influence of radiation on fluid temperature, where we observe that as radiation parameter N increases so does the T.



(a) (b)

Fig. 2.4: Concentration profile for different values of (a) Schmidt number S_c, (b) time t.

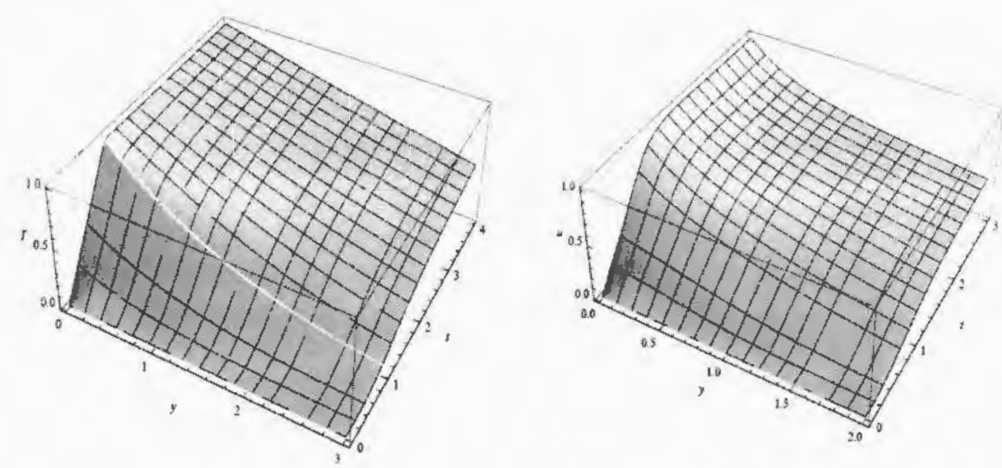
In Fig.2.4 concentration profile is presented graphically for different values of Schmidt number S_c (when $t = 0.8$) and time t (when $S_c = 0.6$). It is clear that as Schmidt number increases concentration reduces while it increases with the passage of time.



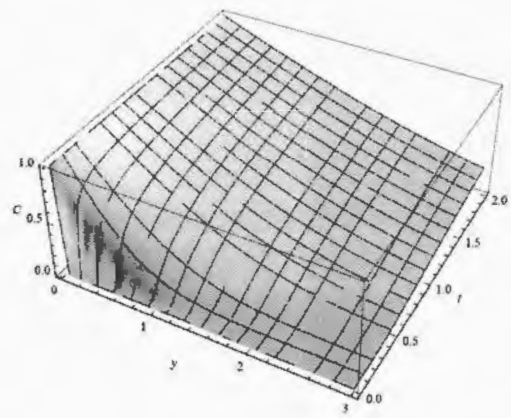
(a) (b)

Fig. 2.5 Effect of (a) relaxation time (λ) and (b) retardation time (θ) on skin friction with $P_r = 0.71$, $t = 0.8, G_m = 2$, $S_c = 0.6$, $M = 1$, $N = 1$ and $k_1 = 0.2$.

Fig. 2.5 shows the effect of relaxation time (λ) and retardation time (θ) on skin friction. It is observed that for both relaxation and retardation time magnitude of skin friction increases.



(a) (b)



(c)

Fig. 2.6(a, b and c) displays the 3D temperature profile, velocity profile (when $N = 1$) and mass concentration profile (when $S_c = 0.6$) versus y and t respectively.

Table 2.1 Nusselt Number Nu with the variation of radiation parameter N.

$N \downarrow t \longrightarrow$	0.2	0.4	0.6	0.8	1	1.2	1.4
0.5	0.3472	0.4910	0.6013	0.6944	0.7763	0.5032	0.4276
1.0	0.3007	0.4252	0.5208	0.6013	0.6723	0.4358	0.3703
5.0	0.1736	0.2455	0.3007	0.3472	0.3882	0.2516	0.2138
10.0	0.1282	0.1813	0.2221	0.2564	0.2867	0.1858	0.1579

Table 2.2 Nusselt Number Nu with the variation of Prandtl number P_r.

$P_r \downarrow t \longrightarrow$	0.4	0.6	0.8	1	1.2	1.4
0.03	0.0874039	0.107047	0.123608	0.138198	0.0895841	0.0761138
0.50	0.356825	0.437019	0.504627	0.56419	0.365725	0.310733
0.71	0.425206	0.520769	0.601332	0.672309	0.435812	0.370281
7.0	1.33512	1.63518	1.88814	2.111	1.36842	1.16266

The numerical values of non-dimensional Nusselt number Nu, calculated from the analytical expression (2.32), are shown in Table 2.1 and Table 2.2 for various values of N, P_r and t. Table

2.1 shows that Nu declines as N and t rises. So, it shows that heat transfer rate slows down as radiation rises. Table 2.2 explains that Nu and P_r have a direct relation, whereas w.r.t t it rises till $t = 1$, after which it falls down.

Table 2.3 Sherwood number Sh_r with the variation of Schmidt number S_c.

$S_c \downarrow t \longrightarrow$	0.2	0.4	0.6	0.8	1.0
0.6	0.977205	0.690988	0.56419	0.488603	0.437019
0.8	1.12838	0.797885	0.65147	0.56419	0.504627
1.0	1.26157	0.892062	0.728366	0.630783	0.56419

The numerical values of non-dimensional Sherwood number Sh_r, calculated from the analytical expression (2.34), are shown in Table 2.3. Table shows that Sh_r enhances with increase in Schmidt number S_c while declines with rise of t.

Chapter 3

Flow of a Second Grade Fluid with Ramped wall Temperature and Velocity

In this chapter, we have established analytic solutions for velocity, temperature and concentration profiles for convective flow of a second grade fluid [24] past a vertical plate of vast extent with ramped wall temperature and ramped wall velocity. The results of isothermal solutions and ramped wall solutions are also compared for velocity, temperature and concentration profiles.

3.1 Mathematical Formulation

Consider flow of a second grade fluid, which is incompressible as well as electrically conducting past on a plate parallel to y'-axis through a porous medium. x'-axis is considered in the upward direction of plate while in the direction normal to plate is y'-axis. The fluid is allowed by a constant transverse magnetic field B_0 acting along y'-axis. Along x' and z' directions the plate is of vast length and is non conductor of electricity, so all physical quantities are functions of y' and t'. Initially, fluid and plate both are stationary with constant temperature T'_∞ and uniform concentration C'_∞. For $t' > 0$ the velocity and temperature of the wall is non-uniformly distributed as

$$T'(0,t') = \begin{cases} T_\infty + (T_w - T_\infty)\frac{t'}{t_0} & 0 < t' \leq t_0 \\ T_w & t' > t_0 \end{cases},$$

and

$$u'(0,t') = \begin{cases} U_0\frac{t'}{t_0} & t' \leq t_0 \\ U_0 & t' > t_0 \end{cases},$$

while concentration is maintained uniformly as

$$C'(0,t') = C_w \, .$$

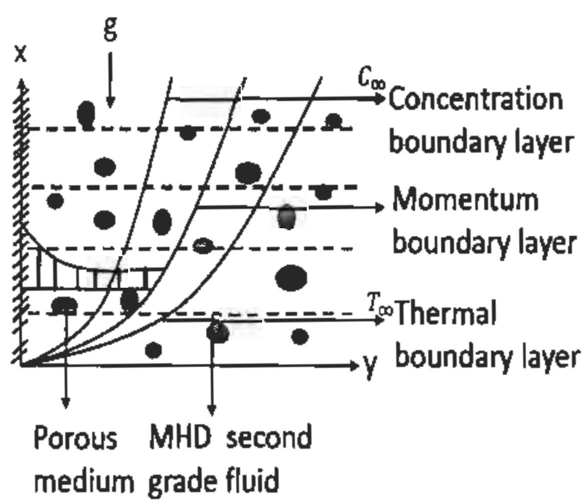


Fig. 3.1: Schematic diagram of problem

Taking assumption that induced magnetic field is imperceptible as comparing to the applied one i.e. $\mathbf{B} \equiv (0, B_0, 0)$. The polarization outcome of fluid is imperceptible as there is no outer electric field, hence we assume $\mathbf{E} \equiv 0$.

For an MHD convective flow of a second grade fluid ($\alpha_1 + \alpha_2 = 0,\ \alpha_1 > 0$) through a porous medium, momentum equation can be written as

$$\rho\frac{d\mathbf{V}}{dt} = \text{div}\,\boldsymbol{\tau} + (\mathbf{J}\times\mathbf{B}) + \rho\mathbf{g}\beta'(T - T_\infty) + \rho g\beta^*(C - C_\infty) + \mathbf{R},$$

where

$$\mathbf{V} = (\mathbf{u}(y,t), 0, 0),$$

$$\boldsymbol{\tau} = -pI + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2,$$

$$\mathbf{A}_1 = \nabla\mathbf{V} + (\nabla\mathbf{V})^T,$$

$$\mathbf{A}_2 = \frac{\partial\mathbf{A}_1}{\partial t} + (\mathbf{V}.\nabla)\mathbf{A}_1 + \mathbf{A}_1\mathbf{L} + \mathbf{L}^T\mathbf{A}_1,$$

$$\mathbf{R} = -\frac{\phi}{k_p}\left(\mu + \alpha_1\frac{\partial}{\partial t}\right)\mathbf{V},$$

$$\mathbf{J}\times\mathbf{B} = -\sigma B_0^2\mathbf{V}.$$

The governing equations for a second grade fluid can be obtained as

$$\frac{\partial u'}{\partial t'} = \left(\left(\nu + \frac{\alpha_1}{\rho}\frac{\partial}{\partial t'}\right)\left(\frac{\partial^2}{\partial y'^2} - \frac{\phi}{k'_p}\right) - \frac{\sigma B_0^2}{\rho}\right)u' + g\left(\beta'(T' - T_\infty) + \beta^*(C' - C_\infty)\right), \tag{3.1}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{1}{\rho C_p}\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T'}{\partial y'^2}, \tag{3.2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2}, \tag{3.3}$$

where α_1 is the second grade parameter and the rest are same as described in chapter 2.

The initial conditions are

$$u'(y',0) = 0\ ,\ T'(y',0) = T_\infty,\ C'(y',0) = C_\infty \quad \text{for } y' \geq 0, \tag{3.4}$$

and the boundary conditions are

$$\begin{aligned}
u\,(0,t') &= \begin{cases} U_0 \frac{t'}{t_0} & t' \leq t_0 \\ U_0 & t' > t_0 \end{cases}, \\
T'(0,t') &= \begin{cases} T'_\infty + (T'_w - T'_\infty)\frac{t'}{t_0} & 0 < t' \leq t_0 \\ \quad T_w & t' > t_0 \end{cases}, \\
C'(0,t') &= C'_w, \\
u'(\infty,t') &\longrightarrow 0,\ T'(\infty,t') \longrightarrow T_\infty,\ C'(\infty,t') \longrightarrow C_\infty\ .
\end{aligned} \tag{3.5}$$

Introducing following non-dimensional quantities

$$\left\{ \begin{aligned}
& y = \frac{y'}{U_0 t_0},\ u = \frac{u'}{U_0},\ t = \frac{t'}{t_0}, \\
& T = \frac{(T'-T_\infty)}{(T_w-T_\infty)},\ C = \frac{(C'-C_\infty)}{(C_w-C_\infty)}, \\
& S_c = \frac{\nu}{D},\ M = \frac{\sigma B_0^2 \nu}{\rho U_0^2},\ k_1 = \frac{k_p U_0^2}{\phi \nu^2}, \\
& G_r = \frac{g\beta\nu(T_w-T_\infty)}{U_0^3},\ G_m = \frac{g\beta^*\nu(C_w-C_\infty)}{U_0^3}, \\
& P_r = \frac{\rho\nu C_p}{k},\ \alpha_1^* = \frac{\alpha_1}{\mu t_0}, \\
& t_0 = \frac{\nu}{U_0^2}, \quad N = \frac{16\sigma^* T_\infty^3}{3kk^*}.
\end{aligned} \right. \tag{3.6}$$

Non dimensional form of equations (3.1) – (3.3) are as follow

$$\frac{\partial u}{\partial t} = \left(1 + \alpha_1^* \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 u}{\partial y^2} - \frac{u}{k_1}\right) - Mu + G_r T + G_m C, \tag{3.7}$$

$$\frac{\partial T}{\partial t} = \frac{(1+N)}{P_r}\frac{\partial^2 T}{\partial y^2}, \tag{3.8}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}. \tag{3.9}$$

3.2 Solution of the problem

3.2.1 Case 1: Ramped temperature and velocity

The non-dimensional initial and boundary conditions are

$$u(y,0) = 0,\ T(y,0) = 0,\ C(y,0) = 0 \quad for \ \ y \geq 0, \tag{3.10}$$

and

$$\begin{aligned} u(0,t) &= \begin{cases} t & 0 < t \leq 1 \\ 1 & t > 1 \end{cases}, \\ T(0,t) &= \begin{cases} t & 0 < t \leq 1 \\ 1 & t > 1 \end{cases}, \\ C(0,t) &= 1, \\ u(y,t) &\longrightarrow 0,\ T(y,t) \longrightarrow 0,\ C(y,t) \longrightarrow 0 \ as \ \ y \longrightarrow \infty. \end{aligned} \tag{3.11}$$

Applying Laplace transform on both sides of Eq. (3.7)-(3.9) and using Eq.(3.10), we get the following forms

$$\frac{d^2\overline{u}}{dy^2}-\left(\frac{M+\frac{1}{k_1}+\left(1+\frac{\alpha_1^*}{k_1}\right)s}{1+\alpha_1^*s}\right)\overline{u}+\frac{1}{(1+\alpha_1^*s)}\left(G_r\overline{T}+G_m\overline{C}\right)=0, \tag{3.12}$$

$$\frac{d^2\overline{T}}{dy^2}-sa\overline{T}=0, \tag{3.13}$$

$$\frac{d^2\overline{C}}{dy^2}-sS_c\overline{C}=0, \tag{3.14}$$

where

$$a=\frac{P_r}{1+N}.$$

The transformed boundary conditions are

$$\begin{aligned}\overline{u}(0,s) &= \overline{T}(0,s)=\frac{(1-e^{-s})}{s^2}\ ,\ \ \overline{C}(0,s)=1/s \qquad for\ \ t>0,\\ \overline{u}(y,s) &= 0,\ \overline{T}(y,s)=0\ ,\ \overline{C}(y,s)=0 \quad as\ y\longrightarrow\infty\ \ for\ \ t>0.\end{aligned} \tag{3.15}$$

Using boundary conditions (3.15), to solve Eqs.(3.12)-(3.14), we get the following solutious

$$\overline{T}(y,s)=\frac{1-e^{-s}}{s^2}e^{-y\sqrt{as}}, \tag{3.16}$$

$$\overline{C}(y,s)=\frac{e^{-y\sqrt{S_cs}}}{s}, \tag{3.17}$$

$$
\begin{aligned}
\overline{u}(y,s) &= \left(\frac{1-e^{-s}}{s^2}\right)\exp\left(-y\sqrt{\frac{b+cs}{1+\alpha_1^* s}}\right) \\
&+\left(\frac{1-e^{-s}}{s^2}\right)\frac{G_r}{as\left(1+\alpha_1^* s\right)-(b+cs)}\left\{\exp\left(-y\sqrt{\frac{b+cs}{1+\alpha_1^* s}}\right)-e^{-y\sqrt{as}}\right\} \\
&+\frac{1}{s}\frac{G_m}{S_c s\left(1+\alpha_1^* s\right)-(b+cs)}\left\{\exp\left(-y\sqrt{\frac{b+cs}{1+\alpha_1^* s}}\right)-e^{-y\sqrt{S_c s}}\right\}, \qquad (3.18)
\end{aligned}
$$

where

$$
b = M + \frac{1}{k_1}, \; c = 1 + \frac{\alpha_1^*}{k_1}.
$$

Taking inverse Laplace transform of Eqs. (3.16) – (3.18), solutions for temperature profile, concentration profile and velocity profile are obtained as follows

$$
T(y,t) = Q(y,t) - H(t-1)\,Q(y,t-1), \qquad (3.19)
$$

where

$$
Q(y,t) = \left(\frac{ay^2}{2}+t\right)\operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{a}{t}}\right) - \sqrt{\frac{at}{\Pi}}\, y e^{\frac{-ay^2}{4t}},
$$

$$
C(y,t) = \operatorname{erf} c\left(\frac{y\sqrt{S_c}}{2\sqrt{t}}\right), \qquad (3.20)
$$

$$
u(y,t) = \frac{\ln 2}{t}\sum_{k=1}^{2n} V_k \overline{u}\left(y,k\right)\frac{\ln 2}{t}, \qquad (3.21)
$$

where

$$
V_k = (-1)^{k+n}\sum_{j=\lfloor\frac{k+1}{2}\rfloor}^{\min(k,n)}\frac{j^n\,(2j)!}{(n-j)!j!\,(j-1)!\,(k-j)!\,(2j-k)!}.
$$

3.2.2 Case 2: Isothermal with constant velocity

The dimensional and non-dimensional form of isothermal boundary conditions are

$$
\begin{aligned}
u'(0,t') &= U_0, \\
T'(0,t') &= T_w \qquad \text{for} \quad t' > 0, \\
C'(0,t') &= C_w, \\
u'(\infty,t') &\longrightarrow 0,\ T'(\infty,t') \longrightarrow T_\infty,\ C'(\infty,t') \longrightarrow C_\infty .
\end{aligned}
$$

and

$$
\begin{aligned}
u(0,t) &= 1, \\
T(0,t) &= 1 \qquad \text{for} \quad t > 0, \\
C(0,t) &= 1, \\
u(y,t),\ T(y,t),\ C(y,t) &\longrightarrow 0 \qquad \text{as } y \longrightarrow \infty.
\end{aligned}
$$

The transformed isothermal boundary conditions are

$$
\begin{aligned}
\overline{u}(0,s) &= \frac{1}{s}, \\
\overline{T}(0,s) &= \frac{1}{s}, \\
\overline{C}(0,s) &= \frac{1}{s}, \\
\overline{u}(y,s),\ \overline{T}(y,s),\ \overline{C}(y,s) &\longrightarrow 0 \quad \text{as } y \longrightarrow \infty.
\end{aligned} \tag{3.22}
$$

Using boundary conditions (3.22) into Eqs.(3.12)-(3.14) solutions are obtained as follows

$$\overline{T}(y,s) = \frac{1}{s}e^{-y\sqrt{as}}, \tag{3.23}$$

$$\overline{C}(y,s) = \frac{e^{-y\sqrt{S_c s}}}{s}, \tag{3.24}$$

$$
\begin{aligned}
\overline{u}(y,s) &= \frac{1}{s}\exp\left(-y\sqrt{\frac{b+cs}{1+\alpha_1^* s}}\right) \\
&+\frac{1}{s}\frac{G_r}{as(1+\alpha_1^* s)-(b+cs)}\left\{\exp\left(-y\sqrt{\frac{b+cs}{1+\alpha_1^* s}}\right)-e^{-y\sqrt{as}}\right\} \\
&+\frac{1}{s}\frac{G_m}{S_c s(1+\alpha_1^* s)-(b+cs)}\left\{\exp\left(-y\sqrt{\frac{b+cs}{1+\alpha_1^* s}}\right)-e^{-y\sqrt{S_c s}}\right\}. \qquad (3.25)
\end{aligned}
$$

Taking inverse Laplace transform of Eqs. (3.23) – (3.25), solutions for temperature profile, concentration profile and velocity profile are obtained as follows

$$T(y,t) = \operatorname{erf c}\left(\frac{y}{2}\sqrt{\frac{a}{t}}\right), \qquad (3.26)$$

$$C(y,t) = \operatorname{erf c}\left(\frac{y}{2}\sqrt{\frac{S_c}{t}}\right), \qquad (3.27)$$

and the solution of velocity $u(y,t)$ can be found by Stefhest method [23].

3.3 Skin friction

The expression of skin friction for the second grade fluid is given by the following expression

$$\tau_w(t) = \frac{\tau'(y,t)\,|_{y=0}}{\rho U_0^2} = \left(1+\alpha_1^*\frac{\partial}{\partial t}\right)\frac{\partial u(y,t)}{\partial y}\bigg|_{y=0}. \qquad (3.28)$$

3.4 Nusselt Number

The Nusselt number measures the heat transfer rate on the boundary and is given by the following form

$$Nu = -\frac{U_0 t_0 \left.\frac{\partial T'}{\partial y'}\right|_{y'=0}}{(T_w' - T_\infty')} = -\frac{\partial T}{\partial y}\,|_{y=0},$$

For ramped temperature

$$Nu = 2\sqrt{\frac{a}{\pi}}\left(\sqrt{t} - \sqrt{t-1}H\left(t-1\right)\right), \tag{3.29}$$

For isothermal

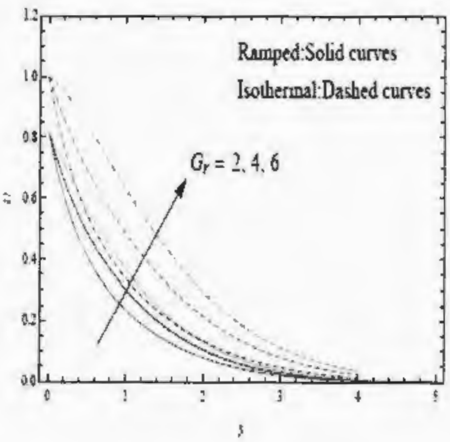
$$Nu = \sqrt{\frac{a}{\pi t}}. \tag{3.30}$$

3.5 Sherwood Number

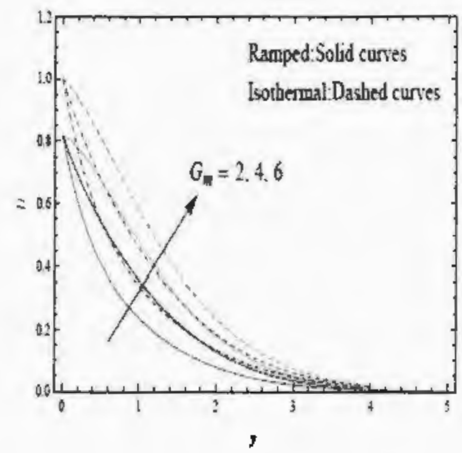
The sherwood number measures the mass transfer rate on the boundary and is given by the following expression

$$Sh_r = -\frac{U_0 t_0 \left.\frac{\partial C'}{\partial y'}\right|_{y'=0}}{(C'_w - C'_\infty)} = -\frac{\partial C}{\partial y}\,|_{y=0} = \sqrt{\frac{S_c}{\pi t}}. \tag{3.31}$$

3.6 Results and discussion



(a)



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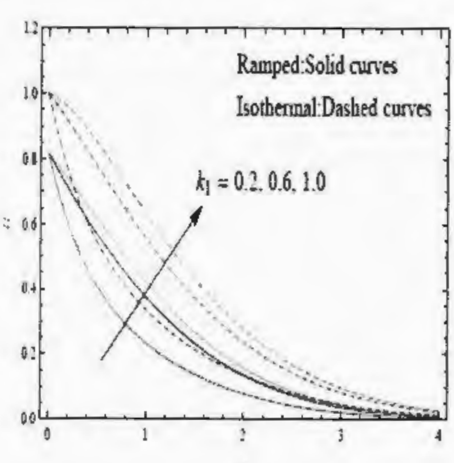

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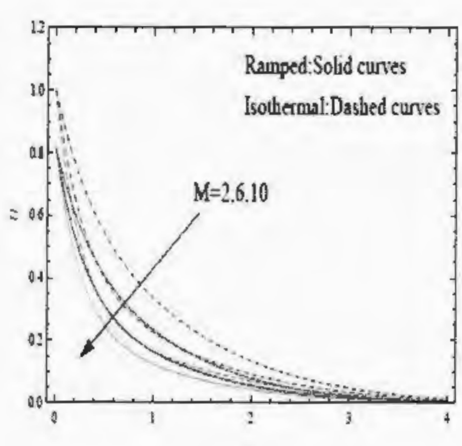

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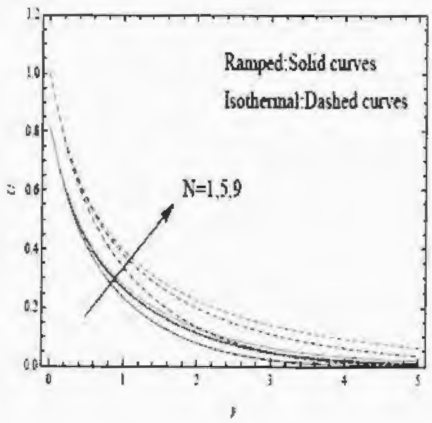

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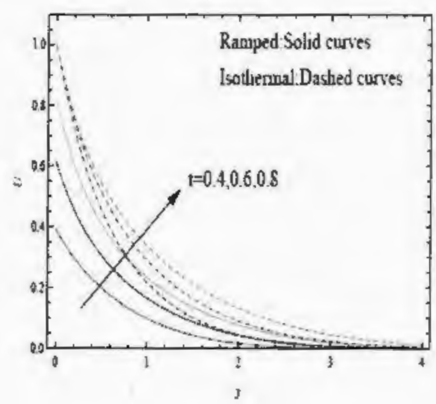

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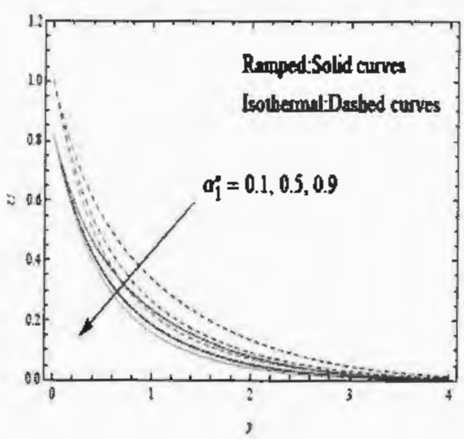
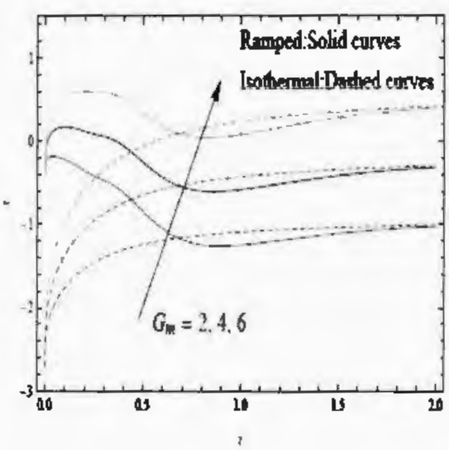


(g)

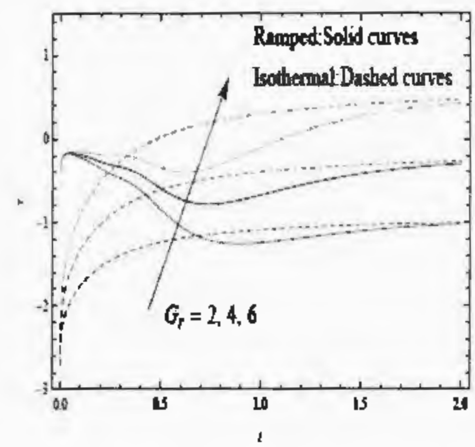
Fig. 3.2: Velocity profile for different values of (a) Grashof number G_r, (b) mass Grashof number G_m, (c) porosity parameter k_1, (d)magnetic parameter M, (e) radiation parameter N, (f) time t, (g) second grade parameter α_1^* with $P_r = 0.71$ and $S_c = 0.6$.

Fig. 3.2(a) displays the effect of Grashof number G_r on fluid velocity, which shows that as G_r increases fluid velocity also increases, hence buoyancy forces speed up the fluid flow. In Fig.

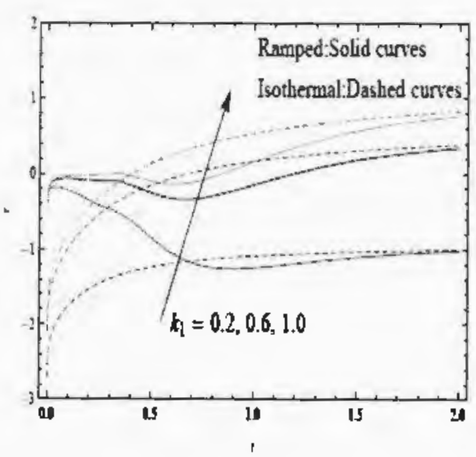
3.2(b) the effect of mass Grashof number G_m on fluid velocity is observed. In Fig. 3.2(c) the effects of porosity parameter k_1 is presented, hence there is a direct relation between velocity of the fluid and porosity parameter . This implies that resistance applied by porous medium reduces as k_1 increases which in result speed up fluid flow. Fig. 3.2(d) demonstrates the effect of magnetic field. So we can see that, there is an inverse relation between magnetic parameter M and velocity of the fluid. Fig. 3.2(e) describes the effect of radiation on fluid velocity. In this case we have a direct relation between fluid velocity and radiation parameter N. Fig. 3.2(f) shows the influence of time on fluid velocity. According to the graph velocity of the fluid speed up w.r.t time. Fig. 3.2(g) displays that speed of second grade fluid decelerated with the increase in α_1^*.


Ramped:Solid curves
Isothermal:Dashed curves
$G_m = 2, 4, 6$

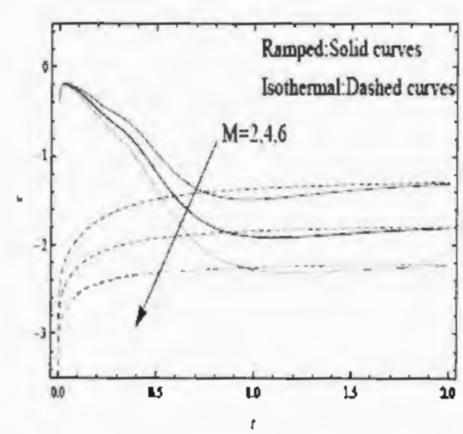
(a)


Ramped:Solid curves
Isothermal:Dashed curves
$G_r = 2, 4, 6$

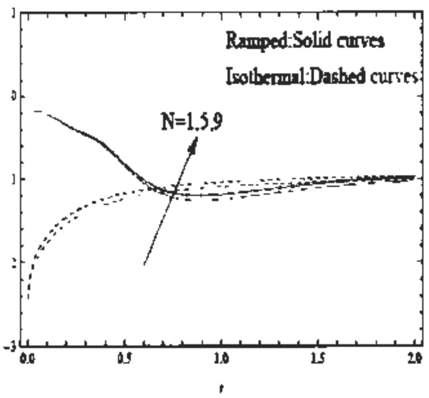
(b)


Ramped:Solid curves
Isothermal:Dashed curves
$k_1 = 0.2, 0.6, 1.0$

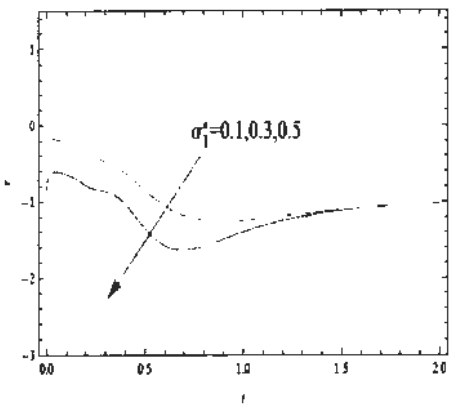
(c)


Ramped:Solid curves
Isothermal:Dashed curves
M=2,4,6

(d)



(e)



(f)

Fig. 3.3: Effect of (a) mass Grashof number G_m, (b) Grashof number G_r, (c) porosity parameter k_1, (d) magnetic parameter M, (e) radiation parameter N, (f) second grade parameter α_1^* on skin friction with $P_r = 0.71$ and $S_c = 0.6$.

In Fig. 3.3 influence of different parameters on skin friction is presented graphically. We can see that the skin friction has a non-linear behaviour before $t = 0.5$ after whicb it reaches a constant value.

The graphs of temperature and concentration profiles are same as in cbapter 2.

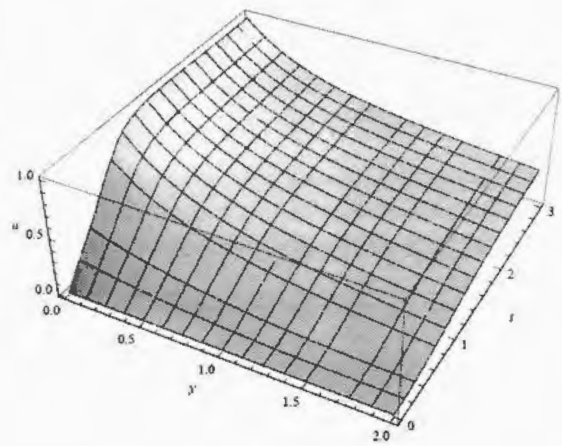


Fig. 3.4

Fig. 3.4 displays the 3D velocity profile (when $N = 1$) versus y and t respectively. The tables for Nusselt number and Sherwood number are same as in chapter 2.

3.7 Concluding Remarks

In this thesis analytical solutions for MHD Oldroyd-B and second grade fluids flow in porous medium past an infinite vertical plate subjected to ramped wall velocity and temperature has been determined. The radiative convective problem is solved by Laplace transform method. In the present study we have concluded the following observations:

- Velocity profile is the increasing function of Grashof number G_r, mass Grashof number G_m, porosity parameter k_1, radiation parameter N, time t and relaxation time λ for both isothermal and ramped wall temperature for an Oldroyd-B fluid.
- Velocity profile is the decreasing function of magnetic parameter M and retardation time θ for an Oldroyd-B fluid.
- Temperature profile is the increasing function of radiation parameter N and time t and decreasing function of Prandtl P_r.

- Concentration profile is the decreasing function of Schmidt number S_c and time t.
- Velocity is the increasing function of Grashof number G_r, mass Grashof number G_m, porosity parameter k_1, radiation parameter N and time t and decreasing function of magnetic parameter M and second grade parameter α_1^* for both isothermal and ramped wall temperature for a second grade fluid.
- The magnitude of velocity and temperature profiles are less for the ramped wall temperature as compared with the constant wall temperature.
- Skin friction reduces with increase in relaxation time λ and retardation time θ for an Oldroyd-B fluid while it reduces with increase in magnetic parameter M, and second grade parameter α_1^* while increases with increase in radiation parameter N, Grashof number G_r, mass Grashof number G_m and porosity parameter k_1 for second grade fluid.
- Both thermal diffusion and radiation decreases the heat transfer's rate at the ramped temperature plate while mass diffusion tends to reduce mass transfer's rate at the plate.

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