Extreme Risk and Value at Risk in the Pakistan Stock Market

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A Dissertation

Submitted in the Partial Fulfillment of the Requirements

for the degree of

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Certificate

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A DISSERTATION SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF THE MASTER OF SCIENCE IN STATISTICS

We accept this dissertation as conforming to the required standard.

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2017

Dedication

I most humbly dedicate all my efforts to my loving family

And

to my devoted teachers, for all their love, guidance, care and moral support.

Forwarding Sheet by Research Supervisor

The thesis entitled "**Extreme Risk and Value at Risk in the Pakistan Stock Market**" submitted by Zishan Ali Syed (Registration # 28-FBAS/MSST/S13) in partial fulfillment of M.S degree in Statistics has been completed under my guidance and supervision. I am satisfied with the quality of his research work and allow him to submit this thesis for further process to graduate with Master of Science degree from Department of Mathematics and Statistics, as per IIU Islamabad rules and regulations.

Dated:____________________

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I pay all homage and shower my gratitude to the Creature of Universe, the Omnipotent, the Omnipresent and the Omniscient who bestowed knowledge to Man.

I offer my humble thanks to Holy Prophet **Hazrat Muhammad** (Peace Be Upon Him), the Mentor, the Guide and source of inspiration for all the scholars of the world.

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This arduous task would not been possible without the aid and support of all my family members. I offer thanks and apologies to those that I could not mention.May Allah Almighty shower His Blessings to all who have helped and assisted me in attaining my goal.

Zishan Ali Syed

DECLARATION

I hereby declare that this thesis, neither as a whole nor a part of it, has been copied out from any source. It is further declared that I have prepared this dissertation entirely on the basis of my personal efforts made under the supervision of my supervisor **Dr. Ishfaq Ahmad**. No portion of the work, presented in this dissertation, has been submitted in the support of any application for any degree or qualification of this or any other learning institute.

Signature: ______________________

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Abstract

The Extreme Value Theory (EVT) has been used to model and measure the distribution of extreme minima of Byco petroleum in Pakistan stock market over the period from 2005 to 2012. This thesis covers the investigation of distributions including generalized extreme value theory, generalized logistics and generalized Pareto distribution mostly used in finance. L-moments ratio diagram is being used to find the appropriate distributions among the distributions. L-moment diagram depicts that generalized extreme value and generalized Logistics distributions are suitable to represent the extremes of Byco Petroleum Pakistan limited. Thereafter Probability Weighted Moment (PWM) method has been used in order to estimate the parameters of probability distributions. Further Anderson Darling goodness of fit test is employed to test the goodness of fit among generalized extreme value and generalized Logistics distribution, and it is clear from the results that generalized Logistics distribution is more reliable and applicable for extreme minima of Byco Petroleum Company in the Pakistan stock exchange market. EVT and traditional methods are used for Value at Risk analysis. The analysis indicates that EVT methods are more suitable for risk measurement in comparison to traditional methods.

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CHAPTER# 01

1.1 Introduction

The insufficiency of risk management was very much obvious after the collapse of stock market happened during the period of 1970s. These crashes affected all the stock markets around the world. Specially, "the oil price shock" and fixed exchange rate system. Oil price shock negatively influenced the United States economy. The changes of these prices led to competitive positions change in many industries, like automobile industry. These events left no room for doubt that effective management of financial risk is inevitable. The crisis did not stop there but these catastrophes continued in the form of worldwide crisis in 1987, the Mexican crisis in 1995, the financial crisis of Asia in 1997 which increased the threat of worldwide economic collapse. Orange County, Bank Barings and Long Term Capital Management were also some of the notable scenarios of crisis. By this time, financial crisis, their risks and the poor financial management was evident. All these issues forced the regulators to address these problems at the earliest. In 1996 the bank for international settlements (BIS) put forward the amendment to capital accord to incorporate market risk which introduced the amendment to capital accord to save from drastic price risk. This amendment also forced qualitative standard to improve risk management techniques for all the financial markets. It presented the model of value at risk (VAR) for estimating the capital adequacy requirements (CAR). The basic purpose of CAR was to cover the possible lose that results from market position and help to fight financial crisis.

VaR helps to examine the lose with c% confidence covering specific time period. It can be termed as the lower quantile of distribution that do not cross the certain limit.

There are three traditional methods that are used to calculate VaR; the Variance-covariance (VC), the historical simulation (HS) and the Monte Carlo Simulation (MCS).The VC method depends on the thought that financial returns have normal distribution. But other researches set forth that they have fatter tails than normal distribution. The presumption is that probability of lose is more than suggested by normal distribution. Conversely, HS method does not need to infer returns of distributions in order to find VaR. It only delineate that historical returns can be good to predicate future returns. But this method lags behind because of the insufficiency of data which can lead to unreliable results. MCS can be used to calculate VaR but it needs more computing skills to bring about results.

Even though, VaR is being widely used still there is no concordance among professionals and scholars for which method can be best in calculating VaR. The common flaw of traditional method is that they do not rightly manipulate lower tails of distribution. They are unable to give right results when we need them the most.

On the contrary, Extreme Value Theory that is the branch of the statistics which deals with the extremes to asses given random variable to find the probability in events which are more extreme than the last observation. EVT based models are specially designed for extreme events. It provides the models with which we can represent the tails of distribution. The theory has been applied in many areas, for instance, engineering, hydrology and after that areas of finance and insurance showed increased interest in it. The application of EVT is not an old development but flourished instantaneously. The pioneer to use EVT in finance was Parkinson (1980). He used extreme returns data that were available to compare the variance of estimated returns. He postulated that the extreme value method can render much better evaluation of variance than the old methods. Another leading name in putting forward the Extreme

Value Theory(EVT) in finance was no doubt was Longin, Longin examined the distribution of extremes in stock market of USA(Longin, 1996). He studied the extreme movement of US Stock Market containing the daily observations of over hundred years (1885-1990). These extreme variables depend on the returns distributed and selected time period. Empirically, Longin posed that minima and maxima could accurately be characterized by the Frechet distribution. Frechet distribution is important occurrence in GEV distribution. Jondeau and Rockinger (2003) gave the international extreme value theory application. They used the data of stock markets for about 20 countries. They evaluate to congruence in right and left tail of returns, all across countries. They used extreme value theory (EVT) to show that the left tail returns are same as right tail returns and concluded that this phenomenon is not the result of clustering of extremes. They also established the point that Generalized Pareto (GP) gave good fit to the experimental data set. Withal, they rationalized that extreme value theory (EVT) for estimation of VAR but it is not as good as it could not figure out the S&P 500 index large negative daily returns of -22.83% in October.

The methods that are being used in this paper were also implied to European stock markets. Lux (1996) manipulated the data sample of DAX index covering the period from 1988 to 1994. He posed the thought that the tails of distribution are thinner using Pareto distribution. He evaluated the German stock market data by fitting the GEV distribution and concluded that Frechet distribution is good to fit in maximum occurrences. He also found the same results on DAX index returns implying intra-day data.

Another important step was the application of EVT in estimating VaR. This has been tested and put forward by Pownall and Koedijk (1999). They collected the data set of Asian Stock Markets and made the comparison of V-a-R computation by normal distributions. They also used the Risk Metrics model of Morgan (1996) with evaluation originated by EVT. They concluded that V-a-R estimated apparently better than other models and techniques. Currently, Danielson (2002) collected the data from foreign exchanges, major assets and commodities. The dataset covered the time span of 15 years. He used risk models to estimate the value at risk (V-a-R). These models were VC, HS, EVT, and GARCH. He also considered extremes and the risk level was 99%. He computed data with each model. He concluded that empirically and theoretically all these models lack in efficiency to foretell risks. One does need great care to implement these models for measuring risk otherwise the results can be misleading.

In EVT, GEV and GP are thought to be the only techniques which can fit extremes (minima and maxima). In recent studies Gettinby et al (2004) delineated the distribution of extremes. For that, he used the data of shares of an index of UK consisting the time period from 1975 to 2000. He examined the number of distributions: GEV, Frechet, GP, Weibull and GL. He examined and compared each distribution to find the best fit. He evaluated the extremes of UK index with each of distribution separately. He concluded that empirically GL fits the minima and maxima. GL gives better outcomes as compare to GEV. Da Silva and Mendes (2003) evaluated the returns of 10 Asian Stock Markets. They used probability weighted Moment (PWM) to estimate the parameters of limiting distributions of extremes. Their main emphasis was on GEV, which proved to be good fit to that data. Their study suggests that EVT based VaR is more authentic in comparison to normal distribution based results. Another work that seconds Gettinby's study is recently put forward by Tolikas and Brown (2006). They also presented that, Generalized Logistics (which is not given as attention as GEV and GP) when it comes to the EVT application. Despite, it gives more stable results for Athens Stock Exchange. They gathered the data of ASE of the period from 1986 to 2001 and found the minimum daily returns applying EVT method. Hence empirically, they concluded GL distribution to be far better than GEV and GP and GL has fatter tails than GEV. Bystrom (2004) presented his work on the management of risks using Conditional Extreme Value Theory. As the financial risk management mainly focuses on the low probability occurring in the tails of distributions. For that, there should be models that focus only on the behaviour of tails. In his paper, he applied both conditional and unconditional EVT models. He chose the data of negative tails distributions of AFF Sweden and Dow Jones industrial Average, United States. He combined block maxima method with time series modeling. In the result it was clear that conditional EVT model gives the accurate estimation either in tranquil or volatile time span. Year by year back testing showed that traditional EVT models are less improved.

In this dissertation, we mainly focused on the measurement of extreme risk of Byco Petroleum Company in the Pakistan Stock Market. We collected the data set of Byco Petroleum Pakistan covering the time span of 8 years from 2005 to 2012. We found the daily, weekly and monthly logarithmic returns of chosen sample of that dataset. After finding logarithmic returns, two distributions were fitted to these returns. L-moments ratio diagram was the technique to identify the distribution of the extreme minimum daily returns. These distributions were Generalized Extreme Value (GEV) and Generalized Logistic (GL). Anderson Darling goodness of fit test was used to test the goodness of fit. Findings of this thesis indicate that Generalized Logistics is better fit for these returns.

1.2 Objectives of the study

- To employ extreme value theory to the daily stock exchange behaviour in Pakistan.
- To find estimates of different distributions.
- To find most appropriate distribution is appropriate for fitting the tail of the loss function(minima) through Anderson darling test and L-moment ratio diagram.
- To perform value at risk analysis for risk assessment in this study.

CHAPTER#02

Literature Review

The foremost purpose of this chapter is to review all the literature that has been put to use for the research.

Extreme Value Theory (EVT) which is mainly concerned with the extreme events of finance industry. EVT aims to estimate the probability from the given sample. Parkinson(1980) was the pioneer to practically use Extreme Value Theory(EVT) in finance. He collected the data of extreme returns and computed the variance using different models. In the end, he was able to conclude that EVT based models showed better results in calculating variance and forecasting risk.

Hosking (1990) first time developed L-moment estimation technique. In his study he derived a method of estimation for population parameters of any continuous probability distribution if its mean exists. This technique is parallel to the conventional moment estimation method. It is based on linear combination of order statistics. This technique has some advantages over conventional moment estimation. Because L-moments less affected from the extreme values. In his study he developed L-moment estimators, L-skewness and L-kurtosis for some distributions. For example, name of distribution are as 1) Uniform 2) Exponential 3) Gumbel 4) Logistics 5) Normal 6) Generalized Pareto 7) Generalized Extreme value 8) Generalized Logistics 9) Log-Normal 10) Gamma

Embrechts, Resnick and Somorodnistky (1998) used the extreme value theory in risk management for insurance data..The financial business as well as banking and insurance, is undergoing major changes. Insurance business more and more exposed to harmful losses. The securitization of risk and different risk transfer highlight the convergence of finance and insurance at the extreme level. Extreme value theory plays a vital role at interval risk management for insurance and finance. .

In the favor of Extreme Value Theory (EVT) in risk management, Alexander J. McNeil (1999) also put his work forward. He emphasized more on the modern models like Peak over Threshold (POT) than the old models as Block Maxima (BM). He argued that EVT correctly predicts even the rare risk event. It is concluded in this paper that Extreme Value Theory will play an important role in risk management and this is becoming true by now.

Extreme Value Theory is considered as a tool to measure the risk with extreme events. It is useful while in the situation of extreme stress and helps to find even the minor consequences. But when it comes to application of EVT, many problems arises such as data's inefficiency to provide extremes.Younes Bessalah(2000) applied the EVT techniques to the Canadian exchange rate over a period of 5 years(1995-2000). He compared and contrasted EVT method with V-a-R method and pointed out two pitfalls of V-a-R methods. Firstly, Value at Risk is not able to measure high quantile risks. Secondly, in V-a-R all series is needed for estimation. EVT concentrates on the behavior of extremes only. But there are some cases when EVT also has some limitations. Such as all the results go right with univariate case, not with multi variate case.

Aparicio, F. M. and Estrada, J. (2001) used the extreme value theory methods to find the distribution of the lower tail daily returns for the Athens stock market. Generalized Logistic distribution showed adequate description of the extreme minima of the Athens stock exchange.

Foreign exchange faces many fluctuations daily. These fluctuations can lead to unpredictable serious losses overnight. To combat this issue, certain limit and prediction should be set. In usual cases, Gaussian model is used to set the limits. Peter and Dacoronga(2002) in their study illustrated how this model can sometimes lead to unexpected result. They proposed the more accurate model to predict the extreme risk based on Extreme Value Theory. They collected the daily log returns of foreign exchange FX. In the empirical result it was found that risk was higher than the predicted risk by Gaussian model. So this model is not able to predict fluctuations accurately. On the contrary, they posed that simple method based on EVT can rightly study the behavior of tails and it can set realistic limits for FX foreign exchange.

(Jondeau & Rockinger, 2003) used 20 countries database of daily stock market to check whether they are similar or not. They used the Monte Carlo, Maximum likelihood and the likelihood ratio test approach to check the behaviors of extremes. They found that the dispersion among extremes varies and they are at different levels across countries. They verified that the left tailed extremes are heavier than the right tailed but not due to clustering of extreme.

Hans N.E Bystrom(2004) detailed the method of conditional EVT for managing risks. He gathered the data of "Affiavarlden Generalindex"AFF and U.S Dow Jones. But in collecting data, he solely focused on the negative tails of distribution. After computing, in the conclusion it was evident that the conditional EVT shows better result in estimating extremes and VaR.

It is evident that Extreme Value Theory is widely applied in risk management. But very little emphasize was put on the use of EVT to understand the tail behaviour in tranquil and volatile period. Blake LeBaron and Ritirupa (2004) applied the EVT to understand the level of fatness of tails in markets. They applied Generalized Extreme Value(GEV) for this purpose. They applied the 'Hill Index" to evaluate the distribution of tails. They developed the Bootstrap and Monte Carlo test and found that negative tails are fatter than positive tails.

(Fernandez, 2005) in his work presented the application of EVT for risk management. He used the sample data of stock indices of Europe, Asia and Latin America. He computed VaR and financial market dependence using the Extreme Value Theory. Heposed that EVT is powerful tool for predicting risk both in tranquil and volatile scenerio. He concluded that EVT estimates the most accurately.

With the growing interest in the management of credit risk, there are numbers of model developed for computation of default probabilities. These models can work well for non-commercial firms but not very effective for banking system.Hans Bystrom(2006) postulated his thoughts on this phenomenon. His paper is based on the Hall and Miles. This paper works as an extension to their work. Bystrom applied the Extreme Value Theory to the Swedish banking sector covering the period of Bank crisis of 1990s. he used EVT based Hall and Miles model. Which was very effective to predict the crisis and it gave the probablities that were more stable.

Financial risk management maily focuses on the probability events. Son one should rely on the methodologies that assess the behavior of tails. Many studies have been conducted on these methodologies. (Ané, 2006) conducted the study on the use of Traditional Extreme Value Theory(TEVT) distributions. She detailed the use of TCEV distributions. She assessed the data of MSCI indices of Pacific region with this distribution. After the goodness of fit test , the results of paper demostrated that TCEV added to the improvement of accuracy in forecasting extreme events.

Enmah , li and Liv(2006) trumpted the thought that sometimes in externe settings data is insufficient for indicating the risk. But Extreme Value Theory is able to fight this problem. They used multivariate Extreme value Theory to the airline performance measures. They used this technique to develop a threshold system.They developed this threshold to assign it to different levels. This techniques was very much able to assess the performance of airline system.

Tolikas and Brown(2006) detailed the method of using Extreme Value Theory to assess the asymptotic distribution. For the research on their case, they presented the sample based on the dataset of Athens Stock Exchange. It covered the period from 1986 to 2001. They applied the Generalized Logistic distribution to examine the behaviour of sample. It was found that Generalized Logistic provided accurate explication of the behavior of ASE minia. They also used the moving window techniques on the parameter. It was found that the capacity of tail to be more or less can vary and the models and methods which are able to assess the time changing behaviour can result in more accuracy. This is, therefore, important for risk management.

The empirical research conducted by (Ahmed Ghorbel, 2007) mainly emphasized on the methodologies named Block Maxima(BM) and Peak over Threshold(POT).VaR models were also comparatively studied. They applied both conditional and unconditional EVT models to Tunisian Stock Exchange(BVMT) and CAC 40 indexes. Through this study, they aimed to assess the performance of different estimation techniques. The empirical result of this tudy showed the conditional POT-EVT gives the more accurate estimation of extreme events and VaR and unconditional Block Maxima lacks in accuracy.

(Saf, 2009) analyzed the flood flow data of 47 sites of West Mediterranean region of Turkey. She fitted generalized logistic, General extreme value, Generalized normal, Pearson type III, generalized Pareto, four parameter Kappa, and Wakeby distributions. For the goodness of fit author uses the L-moments diagrams, and decide the best distribution for different sights.

(Tolikas & Gettinby, 2009) aim to apply the EVT distributions to assess the probability distributions. For this purpose, they cumulate the data of daily returns of Singapore Stock Exchange covering the period from1973 to 2005. They fitted Generalized Pareto, Generalized Extreme Value(GEV) and Generalized Logistics(GL) to be able to see the suitability of these distributions. The result of report exhibit that Generalized Logistic fitted best to empirical data. In comparison, GEV and GP which are more in use and more popular than GL can lead to inaccurate estimation of risk. Therefore, these cannot give protection against devastating crisis.

(Qayyum & Faisal, 2011) threw spotlight on the use of Extreme Value Theory(EVT) on Pakistani financial dataset of Karachi Stock Exchange KSE. The sample consisted on the negative index. The data covered the period from 1993 to 2009. They applied the Generalized Extreme Value (GEV) distribution to assess the behavior of tails. After applying the distribution, they implemented the Peak over Threshold method on the sample. The attained daily returns were assessed by Block Maxima method. VaR and POT methods were compared. The findings of this paper covers that POT method is more precise and proper in giving results. It also shows how efficient is EVT in risk management and it can contribute a big part for predicting rare events.

U. A. Nadiah, Ani Shabri and A. Z. Zakaria (2011) derived TL-moments (1, 0) for the Generalized Pareto distribution. They compared (1, 0) TL-moments with L-moments on simulated data and stream flow data. They made conclusion that (1, 0) TL-moments is better than L-moments.

(Afreen & Faqir, 2012) studied the "Flood frequency analysis of various dams and barrages in Pakistan". They fitted Gamma distribution and Generalized Logistics Distribution on the flood flow data of different dams and barrages in Pakistan. They estimated there parameters and check there goodness of fit through different Statistical tests. For parameters estimation they used Probability Weighted Moments and Maximum Likelihood estimation methods. They fitted both distributions on all sits data. They checked the closeness of fitted and observed parametric values and make recommendations which distribution is best for any specific site on the basis of goodness of fit test. They also estimated the quintiles for the different return period for all sits.

N. Vivekanandan (2014) conducted study on rainfall data for different return periods. In his study, he fitted the six distributions to the data and estimated there parameters through L-moments. Through different goodness of fit tests he suggested appropriate distribution for different regions.

(Deka, 2014) (Aparicio & Estrada, 2001) fitted the three extreme value distribution named as Generalized Extreme Value distribution, Generalized Logistic distribution, Generalized Pareto distribution on maximum daily rainfall data of nine distantly located stations of North East India. He estimated there parameters through Trimmed L-moments. On the bases of goodness of fit test he proposed Generalized Extreme Value distribution is most appropriate.

As the expanding scope of EVT, it has become an authentic application for risk management. Hussain and Li (2015) trumpeted their thoughts on the implication of extreme returns for risk management. They presented the sample based on the dataset of daily returns of Sbanghai Stock Exchange (SSE). They implemented three distributions: Generalized Extreme Value, Generalized Logistic and Generalized Pareto, consisted on the time period from 1991 to 2013. They computed the parameters of distributions by Power Weighted Method. The result stemmed from the research showed that the Generalized logistic was better implication to minima series and GEV is good fit maxima series.

CHAPTER#03

MATERIAL AND METHODS

3.1Extreme value theory

Extreme value theory is a branch of statistics which deals with the extreme observation. The main role of Extreme Value Theory is to empirically estimate and determine the extreme behaviour of random variables. According to Longin (1996) extreme observations are either minimum or maximum of returns of certain intervals such as (daily, weekly and monthly). For more clear understanding, let us assume variables Y_1, Y_2, ..., Yn as the time series of daily index and denote the extreme minima with X_1= min (Y_1,.......,Ym), X_2= min (Y_{m+1}, ..., Y_{2m}),......, $X_{n/m}$= min(Y_{n-m}, ..., Y_n). Now the point here is to acquire the adequate probability distribution which will assess and explain the behaviour of minimum X_1, X_2, ..., $X_{n/m}$.

According to pre supposition, returns are independent and identically distributed (iid). Gnedenko (1943) based his study on this thought; he explained that Generalized Extreme Value distribution is a limiting distribution of extremes. GEV holds huge support theoretically but there is also great backing to the thought that financial returns show serial correlation and heteroscedasticity. Kearns and Pagan (1997) also underlie their study on iid assumption. They used Monte Carlo simulation (MCS) technique to present that violating iid assumption exaggerate the shape parameter estimates. Leadbetter et al (1983) examined the extreme value theory and findings of his study showed that it can deal with weak data structures. Jondeau and Rockinger (2003) gathered the data of DAX index daily returns covering the period of 1969 to 1998. They analyzed the extremes of German stock market. They also examined time that high values of shape parameter depict large negative returns or heightened probability of crash.

L-moments are used to identify the suitable distribution of all the three chosen distributions. L-moments estimation technique was developed by Hosking (1990). This technique is used widely because one can identify the best fitted distribution and get more accurate parametric estimates as compared to conventional moments estimators. Let X_1, X_2, X_3, …….., X_n be a random sample and their corresponding order statistics is $X_{1:n} \le X_{2:n} \le X_{3:n} \le \cdots \le X_{n:n}$.

L-moments defined by Hosking (1990) is:

$$\lambda_r = \frac{1}{r}\sum_{k=0}^{r-1} \binom{r-1}{k}(-1)^k E\,(X_{r-k:n}) \qquad \text{where } r=1,2,3,4,\ldots\ldots..$$

λ_1 and λ_2 denote the location and scale parameters respectively. Skewness and kurtosis are denoted by τ_3 and τ_4. L-skewness and L-kurtosis are plotted on L-moments diagram to find the suitable distribution for data. L-moments have robust properties against extreme values. This technique provides efficient estimates as compared to the conventional moments.

In the next step, probability weighted moment method is used to estimate the parameter of distributions. In this approach sample moments of GL and GEV are equated to estimate parameters. Hosking (1986) gave an easy description of PWM. Let X be the random variable having finite mean and F be the distribution function, then

$$\alpha_r = E[\,X\{1-F(X)\}^r\,], \qquad r = 1, 2, \ldots\ldots..$$

PWM is useful while dealing with small samples of data because extremes are not high even in large samples. Hosking established the view that robust L-moment and PWM have linear relationship, which is showed in four equations of L-moments.

$$\lambda_1 = E(X_{1:1}) = \alpha_0$$

$$\lambda_2 = \frac{1}{2}\left(E(X_{2:2}) - E(X_{1:2})\right) = \alpha_0 - 2\alpha_1$$

$$\lambda_3 = \frac{1}{3}\left(E(X_{3:3}) - 2\,E(X_{2:3}) + E(X_{1:3})\right) = \alpha_0 - 6\,\alpha_1 + 6\,\alpha_2$$

$$\lambda_4 = \frac{1}{4}\left(E(X_{4:4}) - 3\,E(X_{3:4}) + 3E(X_{2:4}) - E(X_{1:4})\right) = \alpha_0 - 12\,\alpha_1 + 30\alpha_2 - 20\,\alpha_3$$

The next step after estimating parameters is to know which distribution is the best fit. Anderson darling test is used for this purpose. Anderson darling test statistics is defined by

$$\int_{-\infty}^{\infty} [F_n(x) - F(x)]^2 \emptyset(x)\,dF(x)$$

Where F(x) is the distribution function of X and n is the number of observations.

At last, after properly modeling the distributions of extremes, lower quantiles have been put to use in order to calculate VaR estimates. As the length of minima is specified of certain time period, so confidence level is adjusted to get the desired frequency of EVT based VaR. Probability of extreme returns is denoted by $P_{ext.}$ It is presumed that collected daily returns will not exceed VaR. The probability of extreme returns is $p_{ext} = 1 - (1 - p)^T$.

CHAPTER#03

MATERIAL AND METHODS

3.1Extreme value theory

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3.2 Methodology

Following steps are employed to estimate VaR using EVT. First of all, length of minima is selected for a certain time span. Secondly, distributions are chosen in order to find the right distribution for extreme minima returns. Thirdly, the goodness of fit test is employed to these distributions to accumulate the distribution which is best fit for data. At the end VaR estimates are derived by different methods.

The length of extreme intervals is defined by m. Parameter of distribution become less efficient as we increase the size of intervals of returns. Logarithmic returns are divided into daily, weekly and monthly intervals. Daily extremes are divided into 10 and 30 sub-periods, weekly into 2, 4 and 10 sub-periods and monthly into 2 sub-periods.

After dividing the extremes into sub-periods of daily, weekly and monthly further job is to look for the suitable distributions for the data. Those distributions will be considered for analysis which can represent fat tail. GEV, GL and GP were selected because of the influence of flood frequency analysis. These distributions deal with the three parameters. These three parameters are: location (α), scale (β) and shape (γ). The first parameter location represents mean; the higher value of mean signifies the larger extremes. The second parameter scale corresponds to standard deviation. High value of standard deviation shows that distribution of extremes is largely spread. The third parameter shape is most important parameter of all because higher value of shape parameter is analogous to fatter tails. In the frame reference of finance, it means

PWM is useful while dealing with small samples of data because extremes are not high even in large samples. Hosking established the view that robust L-moment and PWM have linear relationship, which is showed in four equations of L-moments.

$$\lambda_1 = E(X_{1:1}) = \alpha_0$$

$$\lambda_2 = \frac{1}{2}\left(E(X_{2:2}) - E(X_{1:2})\right) = \alpha_0 - 2\alpha_1$$

$$\lambda_3 = \frac{1}{3}\left(E(X_{3:3}) - 2E(X_{2:3}) + E(X_{1:3})\right) = \alpha_0 - 6\alpha_1 + 6\alpha_2$$

$$\lambda_4 = \frac{1}{4}\left(E(X_{4:4}) - 3E(X_{3:4}) + 3E(X_{2:4}) - E(X_{1:4})\right) = \alpha_0 - 12\alpha_1 + 30\alpha_2 - 20\alpha_3$$

The next step after estimating parameters is to know which distribution is the best fit. Anderson darling test is used for this purpose. Anderson darling test statistics is defined by

$$\int_{-\infty}^{\infty} [F_n(x) - F(x)]^2 \emptyset(x)\, dF(x)$$

Where F(x) is the distribution function of X and n is the number of observations.

At last, after properly modeling the distributions of extremes, lower quantiles have been put to use in order to calculate VaR estimates. As the length of minima is specified of certain time period, so confidence level is adjusted to get the desired frequency of EVT based VaR. Probability of extreme returns is denoted by $P_{ext.}$ It is presumed that collected daily returns will not exceed VaR. The probability of extreme returns is $p_{ext} = 1 - (1 - p)^T$.

VaR is thought to be effective only when they provide accurate results and to know their efficiency these models are needed to be validated. Back testing can be used for validation of VaR models. VaR is rightly specified when the confidence level is equal to the violations of returns. For VaR validation, we employed Christoffersen (1998) test.

3.3 Data description:

The dataset consist of 1920 daily logarithmic returns collected from data stream and covers the 8 years period from 3 January 2005 to 21 December 2012. The table of descriptive statistics contains the descriptive statistics of Byco Petroleum Pakistan Lmited index for the two sub-periods and for whole period.

Table 3.1 includes descriptive statistics of byco-petrolium company index daily, weekly and monthly returns over the period from 2005 to 2012 and the two sub periods from 2005 to 2008.5 and from 2008.5 to 2012. n denotes the number of observations, m denotes the length of extremes selection interval, St.Dev denotes the standard deviation of returns, Min denotes the minimum return and max denotes the maximum return.

Table 3.1 **Descriptive statistics for Byco petroleum company daily returns**

	n	m	mean	St.Dev	Min	Max	Skewness	Kurtosis	SW
2005-2012									
Daily	1920		0.02	0.04	-0.19	0.32	-1.07	11.22	0.8675
Minima									
Weekly	384	5	0.03	0.08	-0.77	0.48	-1.41	20.31	0.8349
Monthly	97	20	0.01	0.15	-0.9	0.035	-2.01	10.23	0.8570
2005-2008.5									
Daily	959		0.01	0.04	-0.19	0.28	0.49	8.31	0.8936
Weekly	191	5	-0.02	0.01	0.77	0.49	-1.57	18.99	0.8049
Monthly	48	20	0.03	0.19	0.9	0.35	-1.83	6.72	0.8587
2008.5-2012									
Daily	960		0.01	0.03	-0.13	0.32	2.03	15.64	0.8314
Weekly	192	5	0.04	0.06	-0.25	0.22	-0.09	2.8	0.9319
Monthly	48	20	0.01	0.1	0.21	0.25	0	0.4	0.9759

In Byco Petroleum daily returns, negative skewness of -1.07 and a kurtosis 11.22 can be seen. It clearly demonstrates that large number of daily returns deviate from normality. This deviation from normality was also assessed by Shapiro-Wilk test. It is clear from table 3.1 that both sub periods deviated from normality but second sub-period was more volatile in comparison to first sub-period.

Table3.2 **Frequency of Byco Petroleum Company index large negative daily returns**

Threshold	<μ-2σ	<μ-3σ	<μ-4σ
2005-2012			
Total	102	45	15
Expected on the assumption of normality	70	15	0
In cluster	55	20	8
2005-2008.5			
Total	52	25	7
Expected on the assumption of normality	34	8	0
In cluster	25	10	5
2008.5-2012			
Total	50	20	8
Expected on the assumption of normality	36	7	0
In cluster	30	10	3

In this table (μ) is denoted as mean which contains the values of large negative daily returns exceeding four defined threshold. σ is the standard deviation. Total values are divided into two sub-periods. It also contains the values expected on the normality assumption. In Cluster row includes the values of daily returns preceding another daily return within two trading weeks.

Daily returns of Byco petroleum are examined as shown in table 3.2 in which the main focus is on left tail distribution. In the table μ is denoted as overall mean and σ is denoted as overall standard deviation. Frequency of the negative daily returns exceeded all three thresholds: $\mu - 2\sigma$, $\mu - 3\sigma$, $\mu - 4\sigma$. The result of this table showed that all the daily returns which were above the limit of threshold were also higher than assumed normality.

After finding values above threshold, the next job was to look for the tendency to cluster of these returns with the passage of time. For this purpose, returns were computed with their lower and higher values of certain time period of two weeks. It can be noticed that clustering heightened under the first threshold μ - 2σ. This table showed that the investors who will take this normal distribution as a base while investing in Pakistan Stock Market, they will not be able to accurately foresee the risk.

On the whole, descriptive statistics demonstrate that distribution of daily returns is different in two sub-periods examined.

CHAPTER#04

Results and Discussion

4.1 Analysis of the Extremes in Pakistan Stock market:

Byco Petroleum daily index covering the period of 8 years is collected to find the daily, weekly and monthly minima. Table 3.1 showed the descriptive statistics of minima. It clearly illustrated that mean minima increased as we increased interval and all sub-periods deviated from normality. Subsection 4.1.1 deals with examination of distribution to identify the accurate distribution. Subsection 4.1.2 gives the parameter estimation and explains how the goodness of fit test is fitted to the distributions to find the best fit.

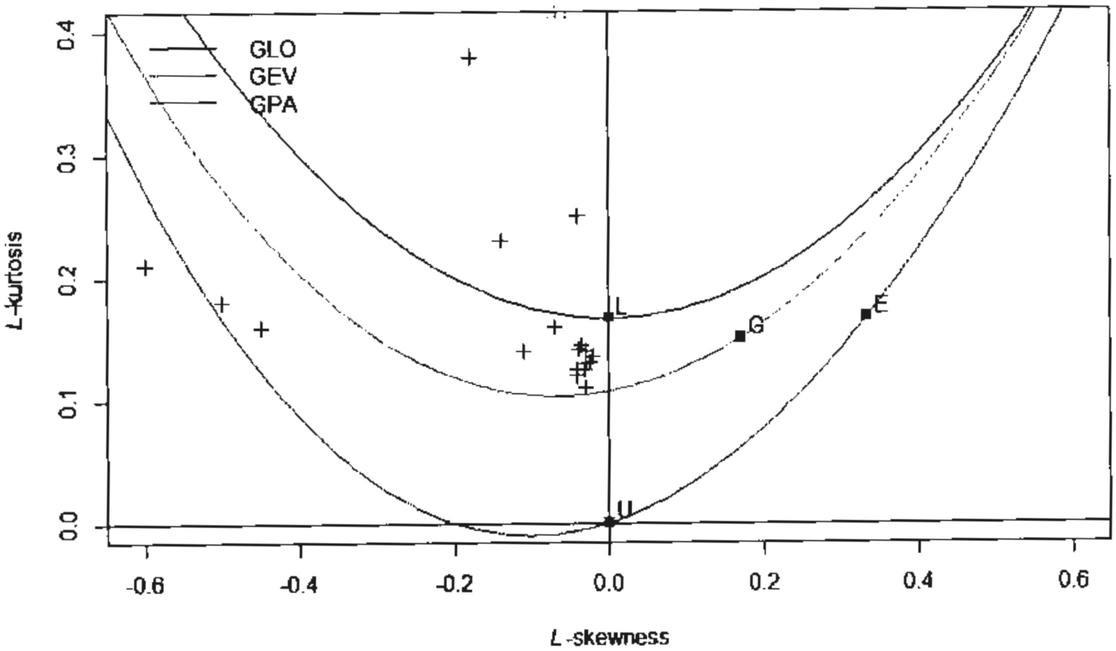
4.1.1 Identification of the distribution of the extreme minimum daily returns using L-moment diagram

L-moments diagram is used to determine the suitable distribution for extreme daily returns. Using the data of daily minima, L- Skewness and L-Kurtosis were calculated for 30 sub-periods of daily returns. Then these calculations of L-Skewness and L-Kurtosis were plotted on L-moment diagram. In the diagram, it can be seen that τ_3 and τ_4 were divided into the series of 30 sub-periods for daily minima. One can clearly infer in the first glance, that GL and GEV are only suitable distributions excluding other distributions, because all the points of τ_3 and τ_4 lie around the curves of GL and GEV. Despite all, L-moment does not clearly determine which distribution is to be focused among GL and GEV. For that purpose, these two distributions need to be analyzed by goodness of fit test.

4.1.2 Parameter estimates and goodness of fit test

GL and GEV distributions were used to fit the whole sample and sub-periods of daily, weekly and monthly minima of Byco Petroleum Pakistan limited. After that, using probability weighted moment (PWM) parameters of Generalized logistics and Generalized Extreme value distribution were calculated for all the sample and sub-periods of daily, weekly and monthly minima. Then p-values of AD goodness of fit test were found as shown in following tables. GL and GEV distributions were now fitted to the whole sample. In which p-value indicated that both of the distributions do not give adequate description. But when the distributions were fitted to the sub-periods as compare to whole sample, GL seem to be fitting adequately.

Figure 4.1: **L-Moments Ratio diagram**



L-moments ratio diagram for the daily returns of Byco Petrolium Pakistan Limited indicates that the L-skewness and L-kurtosis are mainly concntrated around the theoretical curves of the Generalized Logistics and Generalized extreme value distribution indicating that these two distributions are likely to fit adequately the empirical data.

In GEV distribution, the shape parameter sometimes takes positive values which signify Weibull distribution whereas negative values go for Frechet. In previous studies (Longin, 1996 in the USA and Gettinby et al., 2004 in UK) showed no change of sign in shape parameter. In figure 4.2 it can be noticed that shape parameter fitted with GL and GEV was stable in first half and greatly variable in the second half. The variability of shape parameter in second half can affect VaR estimates significantly. Higher values of shape parameter can lead to higher estimation of Value at Risk.

Table 4.1: Daily minima GEV and GL PWM parameter estimates and AD p-value

Generalized extreme value estimates

Sub periods (s)	N	alpha	beta	gamma	p-value
S=1					
	1920	-0.01217	0.0288	0.1951	0.000
S=10					
	192	-0.0133	0.02631	0.1411	0.701
	192	-0.0112	0.03562	0.2507	0.742
	192	-0.0110	0.0212	0.1923	0.556
	192	-0.0085	0.027	0.2444	0.678
	192	-0.0234	0.0515	0.2973	0.221
	192	-0.0129	0.0292	0.0961	0.123
	192	-0.0155	0.0305	0.0487	0.334
	192	-0.0077	0.0179	0.1923	0.702
	192	-0.0074	0.0207	0.3197	0.536
	192	-0.0091	0.0235	0.3303	0.159
S=30					
	64	-0.0148	0.0280	0.2070	0.325
	64	-0.0155	0.0267	0.1347	0.576
	64	-0.0094	0.0243	0.0989	0.062
	64	-0.0074	0.0332	0.1679	0.771
	64	-0.0096	0.0365	0.2540	0.774
	64	-0.0167	0.0366	0.3184	0.738
	64	-0.0091	0.0187	0.2865	0.070

Sub periods (s)	N	alpha	beta	gamma	p-value
	64	-0.0116	0.0199	0.0979	0.701
	64	-0.0125	0.0251	0.2088	0.117
	64	-0.0658	0.0292	0.2381	0.758
	64	-0.0112	0.0261	0.2106	0.391
	64	-0.0077	0.0262	0.2988	0.682
	64	-0.0096	0.0182	0.3900	0.439
	64	-0.0258	0.0480	0.2559	0.201
	64	-0.0349	0.0797	0.2471	0.767
	64	-0.0224	0.0457	0.0549	0.229
	64	-0.0076	0.0166	0.1127	0.248
	64	-0.0086	0.0212	0.0657	0.002
	64	-0.0138	0.0250	-0.1336	0.771
	64	-0.0155	0.0292	0.2120	0.001
	64	-0.0165	0.0370	0.0971	0.274
	64	-0.0046	0.0176	0.2146	0.178
	64	-0.0093	0.0180	0.2571	0.247
	64	-0.0090	0.0184	0.1384	0.750
	64	-0.0095	0.0240	0.4561	0.310
	64	-0.0078	0.0197	0.3811	0.200
	64	-0.0168	0.0347	0.0911	0.034
	64	-0.0057	0.0133	0.1996	0.160
	64	-0.0050	0.0181	- 0.0122	0.567
	64	-0.0046	0.0179	0.1142	0.302

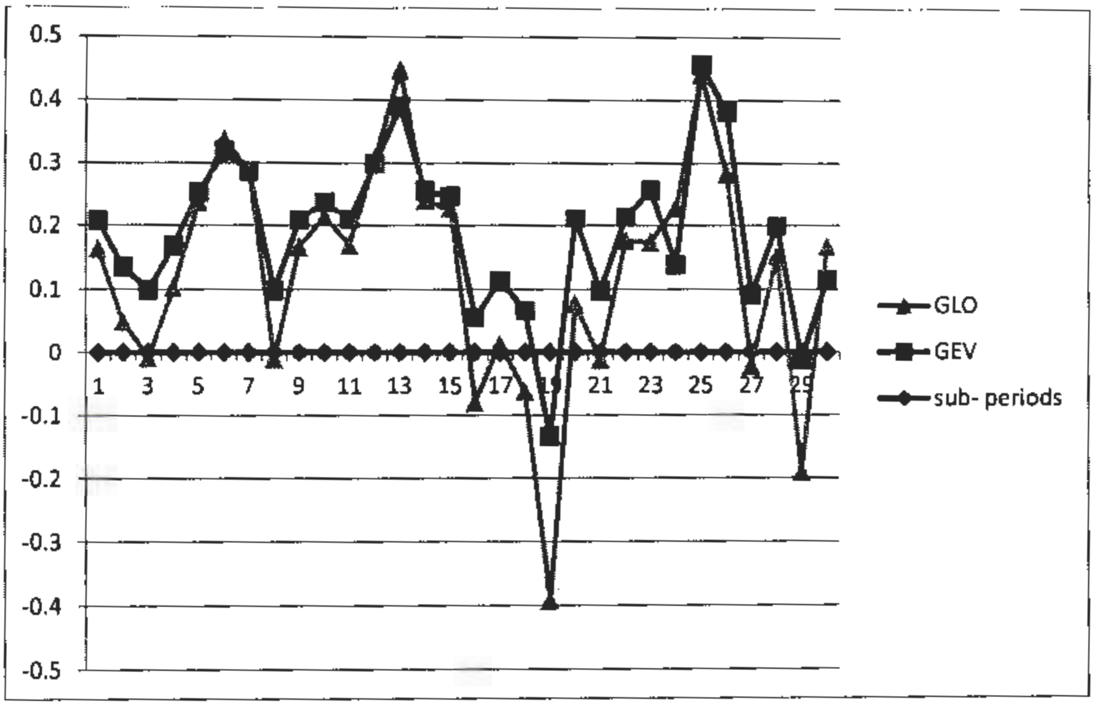
Generalized Logistic distribution estimates

Sub periods (s)	N	alpha	beta	gamma	p-value	Better fit
S=1						
	1920	-0.0016	0.0171	-0.0505	0.022	GL
S=10						
	192	-0.0036	0.0160	-0.0823	0.923	GL
	192	-0.0014	0.0205	-0.0185	0.760	GL
	192	-0.0033	0.0126	-0.0521	0.810	GL
	192	0.0011	0.0156	-0.0221	0.761	GL
	192	-0.0054	0.0289	0.0075	0.050	GEV
	192	-0.0019	0.0183	-0.1096	0.261	GL
	192	-0.0038	0.0196	-0.1389	0.034	GL
	192	-0.0012	0.0106	-0.0520	0.380	GEV
	192	-0.0001	0.0115	0.0198	0.231	GL
	192	-0.0004	0.0453	-0.0944	0.615	GEV
S=30						
	64	-0.0047	0.0165	-0.0435	0.910	GL
	64	-0.0056	0.0163	-0.0861	0.443	GEV
	64	-0.0002	0.0152	-0.1078	0.552	GL
	64	0.0047	0.0200	-0.0664	0.916	GL
	64	0.0033	0.0210	-0.0166	0.940	GL
	64	-0.0040	0.0204	0.0191	0.820	GL
	64	-0.0025	0.0106	0.0015	0.412	GL
	64	-0.0041	0.0124	- 0.1084	0.547	GEV
	64	-0.0034	0.0148	-0.0425	0.326	GL

Sub periods (s)	N	alpha	beta	gamma	p-value	better fit
	64	0.0038	0.0169	-0.0256	0.867	GL
	64	-0.0017	0.0153	-0.0415	0.800	GL
	64	0.0014	0.0147	0.0083	0.665	GEV
	64	-0.0034	0.0098	0.0576	0.510	GL
	64	-0.0087	0.0276	-0.0155	0.113	GEV
	64	-0.0064	0.0460	-0.0205	0.887	GL
	64	-0.0050	0.0293	-0.1351	0.204	GEV
	64	-0.0014	0.0103	-0.0994	0.270	GL
	64	-0.0006	0.0135	-0.1283	0.030	GL
	64	-0.0038	0.0178	-0.2587	0.911	GL
	64	-0.0050	0.0293	-0.1351	0.002	GL
	64	-0.0026	0.0232	-0.1089	0.630	GL
	64	0.0016	0.0103	-0.0391	0.821	GEV
	64	-0.0029	0.0103	-0.0149	0.467	GL
	64	-0.0021	0.0112	-0.0840	0.308	GL
	64	-0.0016	0.0124	0.0918	0.096	GL
	64	-0.0020	0.0111	-0.0985	0.400	GL
	64	-0.0037	0.0218	-0.1127	0.048	GL
	64	-0.0008	0.0078	-0.0478	0.905	GEV
	64	-0.0019	0.0120	-0.1784	0.825	GEV
	64	-0.0011	0.0106	0.0528	0.309	GL

The probability weighted moment estimates and Anderson Darling P-value of generalized extreme value theory and generalized logistics distributions for the daily minima returns of Byco Petroleum Pakistan Limited were calculated in the above table. N denotes the number of extreme observations in each period. Alpha denotes the location parameter, beta denotes the scale parameter and gamma denotes the shape parameter.

Figure 4.2: **Shape parameter estimates for GEV and GLO**



This figure illustrates the behavior of shape parameter estimates of GL and GEV distribution for the daily minima divided over 30 sub-periods.

In figure 4.2 it can be noticed that shape parameter fitted with GL and GEV was stable in first half and greatly variable in the second half. The variability of shape parameter in second half can affect VaR estimates significantly. Higher values of shape parameter can lead to higher estimation of Value at Risk.

The whole series of extremes and the series of sub-periods were fitted to GL and GEV in order to evaluate the variability of extremes. Table 4.2 illustrates the findings of weekly minima of whole period. It showed that GEV gave the adequate fit with the p-value of 0.136. On the other hand, GL couldn't prove to be good fit for series of weekly minima of whole period. But when the sub-periods of weekly minima were fitted to GL and GEV, GL proved to be a better fit.

Table 4.2: Weekly minima GEV and GL PWM parameter estimates and AD p-value

Generalized extreme value distribution estimates

Sub periods (s)	N	alpha	beta	gamma	p-value
S=1					
	384	-0.0249	0.0732	0.3339	0.136
S=2					
	192	-0.0323	0.0864	0.3617	0.338
	192	-0.0183	0.0582	0.2578	0.035
S=4					
	96	-0.0319	0.0705	0.1735	0.696
	96	-0.0314	0.1003	0.5300	0.029
	96	-0.0174	0.0633	0.2624	0.455
	96	-0.0197	0.0536	0.2432	0.160
S=10					
	38	-0.0286	0.0710	0.3456	0.610
	38	-0.0356	0.0824	0.0425	0.640
	38	-0.0246	0.0587	0.4581	0.589
	38	-0.0034	0.0615	0.5681	0.009
	38	-0.0661	0.1445	0.5422	0.330
	38	-0.0140	0.0592	0.2419	0.210
	38	-0.0226	0.0855	0.2170	0.035
	38	0.0217	0.0393	0.2585	0.164
	38	-0.0186	0.0572	0.4396	0.052
	38	-0.0130	0.5508	0.2001	0.620

Generalized Logistic distribution estimates

Sub periods (s)	N	alpha	beta	gamma	p-value	better fit
S=1						
	384	-0.0003	0.0403	0.02757	0.039	GEV
S=2						
	192	-0.0028	0.0470	0.0425	0.185	GEV
	192	0.0023	0.0334	-0.1456	0.149	GL
S=4						
	96	-0.0061	0.0423	-0.0631	0.335	GEV
	96	0.0007	0.0562	0.1288	0.069	GL
	96	0.0050	0.0362	-0.0119	0.520	GL
	96	-0.0006	0.0308	-0.0171	0.160	GL
S=10						
	38	-0.0041	0.0389	0.0338	0.435	GEV
	38	-0.0042	0.0532	-0.1428	0.460	GEV
	38	-0.0053	0.0304	0.0928	0.367	GEV
	38	0.0160	0.0302	0.1473	0.165	GL
	38	-0.0198	0.0724	0.1347	0.178	GEV
	38	0.0071	0.0342	-0.0235	0.289	GL
	38	0.0081	0.0502	0.0378	0.066	GL
	38	-0.0077	0.0225	-0.0141	0.106	GEV
	38	0.0003	0.0299	0.0834	0.063	GL
	38	0.0069	0.0326	-0.0475	0.936	GL

The probability weighted moment estimates and Anderson Darling P-value of generalized extreme value theory and generalized logistics distributions for the weekly minima returns of Byco Petroleum Pakistan were calculated in the above table. N denotes the number of extreme observations in each period. Alpha denotes the location parameter, beta denotes the scale parameter and gamma denotes the shape parameter.

We examined the behaviour of stock exchange extremes by fitting the whole data and sub-periods of Byco Petroleum Pakistan Limited. The results of both the distributions generalized extreme value and generalized logistics distributions were presented in the above tables with the Anderson darling P-value. When fitted to the daily extremes the generalized logistics distribution shows the better fit, while for the sub-periods it showed the mixed behaviour sometimes it is GEV which showed the better fit and sometimes it is GL. But overall the GL showed the better fit for the daily returns as well as for the weekly returns.

Table 4.3: **Monthly minima GEV and GL PWM parameter estimates and AD p-Value**

Generalized extreme value distribution estimates

Sub periods (s)	N	alpha	beta	gamma	p value
s=1					
	96	-0.0368	0.1488	-0.5397	0.310
S=2					
	48	-0.0584	0.1915	0.6218	0.270
	48	-0.0197	0.0984	0.2913	0.721

Generalized Logistic distribution estimates

Sub periods (s)	N	alpha	beta	gamma	p value	better fit
s=1						
	96	0.0106	0.0742	0.1335	0.340	GL
S=2						
	48	0.0006	0.0919	0.1728	0.261	GEV
	48	0.0148	0.0555	0.0041	0.800	GL

The probability weighted moment estimates and Anderson Darling P-value of generalized extreme value theory and generalized logistics distributions for the monthly minima returns of Byco Petroleum Pakistan Limited were calculated in the above table. N denotes the number of extreme observations in each period. Alpha denotes the location parameter, beta denotes the scale parameter and gamma denotes the shape parameter.

In a nutshell, all the examination of extremes proved that GL is a better fit for the extremes of Byco petroleum daily returns. GL performed better than GEV in more sub-periods. But in few cases GEV go beyond GL in performance. It is also important to note that behavior of extremes can change as the nature of shape parameter changed with sub-periods.

4.2 Estimating and comparing Value-at-Risk

EVT methods are evaluated in order to determine the efficiency of these methods to calculate VaR. Lower quantiles of extremes were used to produce VaR estimates of daily returns. To that end, parameter estimates of both GL and GEV distributions for daily extremes were put to use. Moving window method is employed on the grounds that, distribution of extremes have time varying behaviour. Traditional methods including HS, MCS and VC were used to generate VaR estimates. For that purpose 250 past daily returns were examined but for HS VaR estimates, 1000 historical returns were used because 250 returns are fewer to calculate VaR at higher confidence levels. The Christofferson p-value is presented for all methods in parenthesis.

The results mentioned in table 4.4 throw spot light on different facts. At first it can be noticed that VC method over estimated the risk at lower confidence levels but at 99.75% and 99.90% it predicted the VaR rightly where the number of violations are 20 and 16. Like VC method, HS method also performed well at high confidence levels. However with 1000 past daily returns, HS method seemed to be forcasting accurately even at lower confidence levels. MCS, on the other hand, worked unsatisfyingly at all confidence levels. All traditional methods did not accurately measured the risk of extremes daily negative returns.

Further, it can be noticed from the results of table that EVT methods based on static approach performed badly in forcasting the risk. Only GL with static approach worked well at 99.90% confidence level. The reason behined this poor performance is the time varying behaviour of distribution of extremes. So we can see moving window approach eliminated this problem upto some extent. EVT methods with moving window produced accurate results at higher confidence levels. For instance expected violations are 9, 4 and 2 at 99.50%, 99.75% and 99.90% confidence levels, respectively. However, Generalized Logistics distribution with static approach resulted in 29,20 and 2 voilations and outcome of GL with MW approach are 13, 6 and 1 voilations respectively. The findings are summerized, which showed that Generalized Logistics and Generalized Extreme Value performed better than HS, VC and MCS. GL even surpass GEV in some cases because GL have fatter tail and can evaluate larger extremes efficiently.

Table 4.4 Daily VaR backtesting results at various confidence levels

Confidence level	**90.00%**	**95.00%**	**97.50%**	**99.00%**
Number of returns: 1920				
Expected violations	192	95	47	18
VC250	124(0.000)	78(0.063)	44(0.092)	32(0.003)
HS250	176(0.000)	92(0.089)	52(0.184)	18(0.264)
HS1000	218(0.001)	116(0.001)	50(0.173)	23(0.003)
MCS250	140(0.000)	83(0.059)	41(0.052)	29(0.003)
GL static	229(0.000)	144(0.001)	83(0.007)	44(0.000)
GEV static	230(0.002)	136(0.000)	78(0.004)	43(0.000)
GL MW	223(0.000)	116(0.002)	53(0.187)	23(0.001)
GEV MW	221(0.000)	115(0.001)	52(0.183)	23(0.001)

Confidence level	**99.50%**	**99.75%**	**99.90%**
Number of returns: 1920			
Expected violations	9	4	2
VC250	24(0.001)	20(0.000)	16(0.001)
HS250	7(0.860)	92([illegible]	-
HS1000	16(0.002)	10(0.017)	4(0.412)
MCS250	23(0.001)	20(0.000)	16(0.002)
GL static	29(0.001)	20(0.000)	2(0.829)
GEV static	33(0.001)	24(0.000)	12(0.001)
GL MW	13(0.091)	6(0.884)	1(0.974)
GEV MW	13(0.091)	7(0.610)	4(0.418)

CHAPTER # 05

CONCLUSIONS

In this dissertation, daily returns of Byco petroleum Pakistan Limited were investigated to determine which distribution is better able to model the risk. The whole sample and the different sub-periods of the daily minima were fitted by the Generalized Logistics (GL) and Generalized Extreme Value (GEV) distributions, Probability Weighted Moments method were implied to estimate the parameters of these distributions. Anderson Darling goodness of fit test p-value and parameter estimates of both the distributions are presented in the tables. The findings of this thesis clearly indicate that Generalized Logistics can adequately explain the extreme minima of daily returns. This result is important in changing the perspective regarding the poor application of GL. In EVT applications, GP and GEV are thought to be only applicable distributions but, it's evident in the conclusion that GL distribution is good fit for lower tail data. It can also be seen that division of extremes into sub-periods changes the nature of extremes with the passage of time.

EVT based VaR analysis indicate that EVT methods were adequate in risk measurement of extreme returns with lower probabilities. Among traditional methods HS was the only method that gave accurate estimates at higher confidence levels but HS method needs more data points for accurate estimation as compare to other methods employed in the study. It can be noticed that GL and GEV were not able to estimate VaR at lower confidence levels because main focus of both distributions is only on lower quantiles. After that when MW approach was employed it improved the ability of GL and GEV to estimate VaR. Finally results make it clear that applying the moving window technique helped in making the VaR estimates more accurate. It is recommended to expand the study by examining the efficiency of macroeconomic factors to explain the behaviour of extreme distribution parameters.

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Appendix A

Root mean square error and BIAS

Table A.1 DAILY RETURNS

A.1.1 Generalized Logistic distribution

Sample size	RMSE	BIAS
20	0.04863	0.009296
40	0.04319	-0.006740
60	0.04758	0.0026671

A.1.2 Generalized extreme value distribution

Sample size	RMSE	BIAS
20	0.06388	-0.01500
40	0.04736	-0.0000348
60	0.03453	0.007039

Table A.2 WEEKLY RETURNS

A.2.1 Generalized Logistic distribution

Sample size	RMSE	BIAS
20	0.01529	-0.00388
40	0.01942	0.00059
60	0.02078	0.001189

A.2.2 Generalized extreme value distribution

Sample size	RMSE	BIAS
20	0.014229	-0.003924
40	0.017229	0.0011963
60	0.016118	-0.000140

Table A.3 MONTHLY RETURNS

A.3.1 Generalized Logistic distribution

Sample size	RMSE	BIAS
20	0.009765	-0.001026
40	0.009989	-0.0008639
60	0.008086	0.0005483

A.3.2 Generalized extreme value distribution

Sample size	RMSE	BIAS
20	0.01317038	0.004388
40	0.0078468	0.0006075
60	0.0085080	0.0006156

Appendix B

The GEV, GL and GP are three parameter distributions which have the following PDF's, CDF's and quantile function X(F). The parameter α and β are called scale and location respectively while the parameter κ is called the shape parameter and it determines the type of distributions.

Generalized Extreme Value

PDF:

$$f(x) = \alpha^{-1} - (1-k)ye^{-1(1-k)y}\ e^{-e^{-y}}$$

Where

$$y = \begin{cases} -k^{-1}log\{1-k(x-\beta)/\alpha\}, & k \neq 0 \\ (x-\beta)/\alpha, & k = 0 \end{cases}$$

Subject to the restrictions

$$\begin{cases} \beta + \alpha/k \leq x < \infty, k < 0 \\ -\infty < x < \infty \quad ,k = 0 \\ -\infty < x \leq \beta + \alpha/k, k > 0 \end{cases}$$

CDF:

$$F(x) = e^{-e^{-y}}$$

X(F):

$$X(F) = \begin{cases} \beta + \alpha\{1 - (-logF)^k\}/k, k \neq 0 \\ \beta - \alpha log(-logF), \quad k = 0 \end{cases}$$

The Weibull distribution is the special case of the GEV when $k > 0$, the Gumbel distribution is the special case for $k = 0$ and the Frechet distribution is the special case for $k < 0$

Generalized logistic

PDF;

$$f(x) = \frac{\alpha^{-1} e^{-(1-k)y}}{(1 + e^{-y})^2}$$

Where

$$y = \begin{cases} -k^{-1}\log\{1 - k(x - \beta)/\alpha\}, k \neq 0 \\ (x - \beta)/\alpha, \quad k = 0 \end{cases}$$

Subject to the restrictions

$$\begin{cases} \beta + \alpha/k \le x < \infty, k < 0 \\ -\infty < x < \infty, \quad k = 0 \\ -\infty < x \le \beta + \frac{\alpha}{k}, k > 0 \end{cases}$$

CDF:

$$F(x) = 1/(1 + e^{-y})$$

X(F):

$$X(F) = \begin{cases} \beta + \alpha[1 - \{(1-F)/F\}^{k}]/k, k \neq 0 \\ \beta - \alpha \log\{(1-F)/F\}, \quad k = 0 \end{cases}$$

The logistic distribution is the special case of GL when $\kappa = 0$

Generalized Pareto

PDF:

$$f(x) = \alpha^{-1} - (1-k)ye^{-1(1-k)y}$$

Where

$$\begin{cases} -k^{-1}log\{1-k\,(x-\beta)/a\}, k \neq 0 \\ (x-\beta)/a, \quad s \qquad\qquad k=0 \end{cases}$$

Subject to the restrictions

$$\begin{cases} 0 < x < \infty \quad ,k \leq 0 \\ 0 < x \leq a/k, k > 0 \end{cases}$$

CDF:

$$F(x) = 1 - e^{-y}$$

X(F):

$$X(F) = \begin{cases} \beta + a\{1(1-F)^k\}/k, k \neq 0 \\ \beta - alog(1-F), \qquad k=0 \end{cases}$$

The exponential and the uniform distributions are the special case of the GP when $k = 0$ and $k = 1$, respectively, on the interval $0 \leq x \leq a$.