

**Simultaneous Adaptive Attitude Control and Power  
Tracking Using Variable Speed Control Moment  
Gyroscopes**



**Faisal Mehmood**

**REG No 200-FET/MSEE/F08**

**Supervisor:**

**Dr. Sajjad Asghar**

**Scientist, NESCOM**

**Islamabad**

**Department of Electronic Engineering  
Faculty of Engineering and Technology  
International Islamic University, (IIU), Islamabad**

**2011**

Created with



**nitro PDF**

**professional**

download the free trial online at [nitropdf.com/professional](http://nitropdf.com/professional)

download the free trial online at [nitropdf.com/professional](http://nitropdf.com/professional)

# Simultaneous Adaptive Attitude Control and Power Tracking Using Variable Speed Control Moment Gyroscopes



**FAISAL MEHMOOD**  
**200-FET/MSEE/F08**

This dissertation is submitted to Faculty of Engineering and Technology, International Islamic University Islamabad Pakistan for partial fulfillment of the degree of **MS Electronic Engineering** With specialization in **Control System Engineering** at the Department of Electronic Engineering

**Faculty of Engineering and Technology, (FET)**

**International Islamic University, (IIU), Islamabad**

Supervisor:

Dr. Sajjad Asghar

22 July 2011

Created with



بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

*In the name of Allah (SWT) the most beneficent and the most merciful.*

Created with



**nitro** PDF<sup>®</sup>

professional

download the free trial online at [nitropdf.com/professional](http://nitropdf.com/professional)

download the free trial online at [nitropdf.com/professional](http://nitropdf.com/professional)

*Dedicated to my Parents, my son Hassan and  
Family*

Created with



**nitro** PDF<sup>®</sup>

professional

download the free trial online at [nitropdf.com/professional](http://nitropdf.com/professional)

download the free trial online at [nitropdf.com/professional](http://nitropdf.com/professional)

# Certificate of Approval

**Title of Thesis:** Simultaneous Adaptive Attitude control and Power Tracking Using  
Variable Speed Control Moment Gyroscopes

**Name of Student:** Faisal Mehmood

**Registration No:** 200-FET/MSEE/F08

Accepted by the Faculty of Engineering and Technology, International Islamic University, Islamabad, in partial fulfillment of the requirement for thy MS Degree in Electronic Engineering.

## Viva voce committee

### External Examiner

Dr.Nisar Ahmed  
Associate Professor,  
GIKI, Swabi.

.....

### Internal Examiner

Dr.M Ijaz  
Associate Professor,  
IIU, Islamabad.

.....

### Supervisor

Dr.Sajjad Asghar  
Scientist,  
NESCOM, Islamabad.

.....

22 July 2011

Created with

## **Declaration**

I certify that except where due acknowledgments has been made, the work has not been submitted previously, in whole, to qualify for any other academic award, the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program.

.....  
**Faisal Mehmood**

**200-FET/MSEE/F08**

Created with



## Abstract

Control laws for Simultaneous Adaptive Attitude and Power Tracking (SAAPT) using variable speed single gimbal control moment gyroscopes (VSCMGs) is termed as SAAPT-VSCMGs system, which is established for satellite. The variable speed single gimbal control moment gyroscopes have extra advantage of variable wheel spin rates whereas the wheel spin rates of conventional control moment gyroscopes are constant. Therefore, VSCMGs have extra degrees of freedom and can be applied to accomplish further goals, for example simultaneous attitude control with energy storages. We use VSCMGs in combination with an SAAPT system. The gimbal rates of the VSCMGs are applied to provide the reference-tracking torques, whereas the wheel accelerations are used for both attitude and power reference tracking. The latter goal has been attained by storing or releasing the kinetic energy in the wheels. These control algorithms perform both the attitude and power tracking goals simultaneously in this nonlinear MIMO system. An adaptive control law with uncertain inertia properties has been developed for satellite. Moreover, the control law for equalization of the wheel speeds is also proposed. Wheel speed equalization distributes evenly the kinetic energy among the wheels, minimizing the possibility of wheel speed saturation and the occurrence of zero-speed singularities. Finally, a numerical example for a satellite in a low Earth, near-polar orbit has been provided to test the proposed SAAPT-VSCMGs algorithm.

Created with

## **Acknowledgements**

I would like at this juncture to express my deepest appreciation and gratitude to my kind supervisor Professor Dr. Sajjad Asghar for his support, throughout my research period at International Islamic University. It has been a great pleasure to work with him and to benefit from his knowledge. His door is open to me whenever I try to seek help. Thanks to all my friends who have given me intellectual and emotional support during my MS education and research.

In the end, I would like to thank my family members for their support and encouragement .Their love and care is the deriving force for my studies and life.

**Faisal Mehmood**

Created with





# Table of Contents

<b>Abstract .....</b>	<b>vi</b>
<b>Acknowledgements.....</b>	<b>vii</b>
<b>List of Figures.....</b>	<b>xi</b>
<b>List of Table.....</b>	<b>xii</b>
<b>Chapter 1.....</b>	<b>1</b>
<b>Introduction.....</b>	<b>1</b>
Research Question Background.....	1
Simultaneous Attitude and Power Tracking Using Variable Speed Control Moment Gyroscopes for satellite (SAPT-VSCMG System).....	2
Adaptive Attitude and Power Tracking Controller Using Variable Speed Control Moment Gyroscopes for Satellite.....	4
Problem Statement.....	4
Aim and Objectives of Thesis.....	5
Scope of This Thesis.....	6
<b>Chapter 2.....</b>	<b>7</b>
<b>Literature Review.....</b>	<b>7</b>
Power and Attitude Tracking in satellite using fly wheels technology.....	7
First phase of fly wheel technology.....	8

Created with



**nitro** PDF<sup>®</sup>  
Created with

**professional**  
PDF

download the free trial online at [nitropdf.com/professional](http://nitropdf.com/professional)

download the free trial online at [nitropdf.com/professional](http://nitropdf.com/professional)

Second phase of fly wheel technology.....	8
Third phase of fly wheel technology.....	10
Control Moment Gyroscopes.....	10
Control Moment Gyroscopes on Orbit Applications.....	12
Variable Speed Control Moment Gyroscopes.....	14
Adaptive Attitude Tracking Control of Satellite.....	15
<b>Chapter 3.....</b>	<b>16</b>
<b>Simultaneous Adaptive Attitude Control and Power Tracking Using Variable Speed CMG systems for Satellite.....</b>	<b>16</b>
Equations of Attitude Motion for Satellite Equipped with VSCMGs.....	16
Kinematics of Satellite Maneuvering.....	23
Satellite Attitude Tracking Using Lyapunov Stability Theory.....	24
Velocity Based Steering Law for Attitude Tracking Using Variable Control Moment Gyroscopes.....	33
Power Tracking.....	34
SAAPT – VSCMG (Simultaneous Adaptive Attitude and Power Tracking with VSCMG) Controller.....	35
Singularity Avoidance with Weighting Matrix.....	35
Variable Control Moment Gyroscopes Wheel Speed Equalization.....	37
<b>Chapter 4.....</b>	<b>40</b>
<b>Simulation Results and Analysis.....</b>	<b>40</b>
Simultaneous Adaptive Attitude and Power Tracking with Wheel Speed Equalization in a Low Earth Orbit.....	40

Simulation Results Analysis.....	55
<b>Chapter 5</b> .....	59
<b>Conclusions and Future Recommendations</b> .....	59
Conclusions.....	59
Future Recommendations.....	60
<b>References</b> .....	63

Created with



## List of Figures

Figure 1: Axis System associated with VSCMG.....	17
Figure 2: Angular velocities of satellite.....	43
Figure 3: Attitude Trajectory of satellite.....	45
Figure 4: Power Profile.....	47
Figure 5: VSCMGs Gimbal angles without wheel speed equalization.....	48
Figure 6: VSCMGs Gimbal angles with wheel speed equalization.....	48
Figure 7: VSCMGs Gimbal rates.....	49
Figure 8: VSCMGs wheel speeds without wheel speed equalization.....	50
Figure 9: VSCMGs wheel speeds with wheel speed equalization.....	50
Figure 10: VSCMGs wheel speeds rates without wheel speed equalization.....	51
Figure 11: VSCMGs wheel speeds rates with wheel speed equalization.....	51
Figure 12: Condition number of matrix C.....	52
Figure 13: Estimated Moment of Inertia ( $J_{xx}$ ).....	53
Figure 14: Estimated Moment of Inertia ( $J_{yy}$ ).....	53
Figure 15: Estimated Moment of Inertia ( $J_{zz}$ ).....	54
Figure 16: Estimated Moment of Inertia ( $J_{xx}, J_{yy}, J_{zz}$ ).....	54
Figure 17: Estimated Product of Inertia ( $J_{xy}, J_{xz}, J_{yz}$ ).....	55

# List of Tables

Table 1: Simulation Parameters .....42

# CHAPTER 1

## INTRODUCTION

### 1.1 RESEARCH QUESTION BACKGROUND

In order to make lighter, high quality and low cost satellites, the researchers have endeavored hard to decrease payload capacity, fabrications, launching cost and bus mass of satellites in their spacecraft research programs. The designing parameter keeps on changing in order to achieve the higher operational performance in satellites. In huge size satellites, high quality attitude tracking controllers are required for large maneuverings. Normally gas jet thrusters have been used as actuator for amplification of torque in three axes attitude tracking controller in these big satellites. The use of gas jet thrusters as actuator in satellite increases the total mass of satellite. On the other hand, the purposes of flywheels technologies as actuators for attitude tracking have solved the higher mass problems in satellites. The applications of flywheels technologies have an extra advantage that we can design a controller model for both attitude tracking and power tracking of satellite simultaneously. The American AFR laboratories have successfully used flywheels as mechanical batteries for energy storage requirements in satellites. [17]

## **1.2 SIMULTANEOUS ATTITUDE AND POWER TRACKING USING VARIABLE SPEED CONTROL MOMENT GYROSCOPES FOR SATELLITE (SAPT-VSCMG System)**

The nature of satellite system has become more complex in this modern age. The applications of flywheels as actuator have been used for both attitude tracking and energy storage requirements. So fly wheels have been used as mechanical batteries, which is replaced for conventional chemical batteries in satellites. This idea is introduced as SAPT-VSCMG system (simultaneous attitude and power tracking using VSCMGs). In sunset period the solar panel of satellite convert the solar energy into electrical energy which is stored in chemical batteries and during eclipse period the electrical energy is acquired for satellite operations from these chemical batteries. The use of chemical batteries has certain disadvantages for example extra mass system is required for controlling charging and discharging of batteries, short batteries lifetime (less than five years) and hard temperature conditions for satellite (less than  $20^{\circ}\text{C}$  in a LEO).

In SAPT-VSCMG systems flywheels have been used as a substitute of chemical batteries to store energies in their wheels in form of kinetic energies and are known as mechanical batteries. These mechanical batteries have higher efficiencies in hard temperature conditions. In shapes of flywheels, a single device is used both for attitude tracking and energy storage simultaneously in satellites. The use of flywheels as actuator increases efficiency of attitude controller and decreases mass of whole satellite remarkably. The idea of combined attitude tracking and energy storage was presented during 1960's. Recently, the idea of SAPT-VSCMG system has become

Created with

into reality in practical form in NASA GR laboratories where flywheels operates continuously at 60,000 rates per minute supported by magnetic bearing.

## COMPARISON BETWEEN MECHANICAL BATTERIES AND ELRCTROCHEMICAL BATTERIES

<b>MECHANICAL BATTERIES</b>	<b>ELECTROCHEMICAL BATTERIES</b>
<ul style="list-style-type: none"> <li>• No Facility grows fainter more than life</li> <li>• Determined status of charge</li> <li>• 85-95% round trip efficiency</li> <li>• Overcharging is handled without difficulty</li> <li>• Much higher rates are feasible</li> <li>• Energy storage and power capacity are not coupled</li> </ul>	<ul style="list-style-type: none"> <li>• Facility grows fainter more than life</li> <li>• Complexity in measuring the charge status</li> <li>• 75% round trip efficiency</li> <li>• Overcharging is handled with difficulty</li> <li>• Charging / Discharging rate restrictions</li> <li>• Coupled energy storage and power capacities</li> </ul>



### **1.3 ADAPTIVE ATTITUDE TRACKING CONTROLLER USING VARIABLE SPEED CONTROL MOMENT GYROSCOPES FOR SATELLITE**

The key objective of variable control moment gyroscope is to provide required torque for attitude tracking in satellites. Several scientists have used control moment gyroscopes as actuator for attitude tracking in satellites [11, 61, 4, 5]. In this thesis, model based techniques have been used to design attitude tracking controller using variable speed control moment gyroscopes as actuator for satellites maneuverings. The purpose of this thesis is to derive adaptive attitude control law using VSCMGs for simultaneous adaptive attitude and power tracking (SAAPT). An Adaptive attitude tracking controller has been designed for satellite, because in some satellite operations the exact elements of inertia matrix are not exactly known. For example utilization of fuel, docking, removing a payload and receiving a spacecraft at international space station. So an adaptive attitude control law has been derived for exact attitude and power tracking of satellite.

### **1.4 PROBLEM STATEMENT**

The simultaneous attitude and power tracking of satellite using Variable Speed Control Moment Gyroscopes (VSCMGs) have been studied in this MS research Thesis. The power requirement of the satellite has to be met b

spinning of VSCMGs, such that the generated torques does not disturb the attitude tracking of satellite. The focus of this study is to develop the simultaneous adaptive attitude and power tracking controller using VSCMGs with wheel speed equalization for satellite maneuverings.

## **1.5 AIM AND OBJECTIVES OF THESIS**

The aim of this MS research thesis is to develop an efficient simultaneous adaptive attitude and power tracking with wheel speed equalization controller of satellite using Variable Speed Control Moment Gyroscopes (VSCMGs). In order to accomplish this aim the following objectives are set:

- Development of an attitude controller for nonlinear Satellite system.
- Development of an adaptive attitude controller for nonlinear MIMO system.
- Designing of simultaneous adaptive attitude and power tracking controller using VSCMGs for nonlinear Satellite system.
- Designing of an adaptive attitude and power tracking controller with wheel speed equalization using VSCMGs for nonlinear Satellite system.
- Implementation of developed control strategy in MATLAB simulation.
- Simulation analysis of the developed technique.

## 1.6 SCOPE OF THIS THESIS

In chapter 2, the literature review has been presented. Specific concentrations have been given to that studies in which fixed speed control moment gyroscopes or variable speed control gyroscopes have been used as actuators for energy storage and attitude tracking of satellites. Adaptive attitude control strategies also presented in this literature review. In this chapter, an overview of SAAPT- VSCMGs system concept has been discussed in detail.

Chapter 3 represents the kinematics and dynamical equations of motion of satellite using variable speed control moment gyroscopes as actuator. In this chapter, we used Modified Rodriguez Parameters (MRPs) for attitude representation of satellite. Simultaneous adaptive attitude and power tracking controllers using variable speed control moment gyroscopes in pyramid configuration have been derived with the help of Lyapunov stability control theory. In these controllers, simultaneous adaptive attitude and power tracking have been achieved with wheel speed equalization of VSCMGs. These control laws have been introduced as SAAPT- VSCMGs systems and used for satellites operations. This chapter is designed for SAAPT-VSCMGs control theory for nonlinear adaptive MIMO system.

Chapter 4 represents the simulations results of simultaneous adaptive attitude and power tracking controllers of satellites. In this chapter, analyses of these simulations results using SAAPT –VSCMG systems have also been discussed.

In Chapter 5, we have presented the concluding remarks and future recommendations.

## CHAPTER 02

### LITERATURE REVIEW

In literature review, the subject related to attitude and power tracking for satellite has been discussed well. The flywheel technology has been studied for attitude control and energy storage since in early sixty decade [18]. In this chapter, the important literature regarding with both attitude and power tracking problems using fixed speed control moment gyroscopes are presented.

Literature review is divided into two portions. The first portion is linked with power tracking using fly wheels[18] and second portion of literature relates to attitude tracking with single gimbal CMG's [21].

#### 2.1 POWER AND ATTITUDE TRACKING IN SATELLITE USING FLYWHEEL TECHNOLOGY

In 1960's the idea of utilizing fly wheels for energy storing purposes was introduced. This idea emerged during the studies of high kinetic energy in flywheels. In 1961, a composite flywheel 17-w.hr/kg, spinning at ten to twenty thousand rate per minute on a magnetic bearing was presented by Hall and Rose. These flywheels have been presented as mechanical batteries for energy storage. In these mechanical batteries,

Created with

two flywheels are revolving in anti-clockwise direction and two flywheels are moving in clock wise direction. But these flywheels have not been used for attitude tracking of satellite [27].In Reference [25,26], the author has given the applications of flywheel technologies in various parts of industries.

This idea became more promising during the energy shortage issues in the late seventies [24]. Now fly wheels technologies are being utilized in various parts of industries. For example, energy saving scheme in automobile, UPS and in production plants.[25,26,18].In the next phase, we have discussed fly wheel technology of satellite in 1970's and onward.

## **2.2 FIRST PHASE OF FLY WHEEL TECHNOLOGY (1960 to 1979)**

First in reference [27] the idea of combined attitude tracking and power tracking using fly wheels was presented by Anderson. Anderson replaced the joint attitude tracking and power tracking term with Integrated Power and Attitude Control System (IPACS) in mid seventies in Ref [28, 30, 32].The idea of IPACS further studied in Ref [28,29,30,31,32] by NASA.

## **2.3 SECOND PHASE OF FLY WHEEL TECHNOLOGY (1980 to 1989)**

The second phase of fly wheel technology period was from 1981 to1989.In this period, the following references [33,34,35,36,37,38,39,40,41,42] h

In these references, the NASA scientists in space station have studied the CARES and IPACS system. In References [34, 35, 17], Downor and Eiienshavre utilized CARES system for the solution of fly wheels actual technical problems concerned with build up Momentum Exchange Device ( MED ) on magnetic bearings. The flywheels technology problems for example parameter calculation of components, hardware properties and designing parameters have been discussed in Department of CARES at NASA [35].In CARES system, the attitude tracking is around three axis i.e. x axis, y axis and z axis, and gimbal of CMG's have been presented by magnetic bearings in Ref [35].In Downor and Eiienshavre research paper, satellite attitude tracking along x-axis has been achieved by wheel as a Reaction Wheel (RW) and satellite other two attitude tracking along y-axis and z-axis have been achieved by wheels as CMG's.

In Ref [41], the technical problems related with integrated power and attitude control system have been comprehensively discussed. In IPACS discussion, the technical problems of flywheels, for example electrical and mechanical energy conversion issues, material composition issues, elasticity and rigidity of material issues and high speed motor technical issues have been considered. In Ref [38] energy conversation related issues in IPACS have also been discussed. In a huge spacecraft, extremely large amount of power is needed for spacecraft maneuvering. The IPACS model has given the solution of these huge power requirements for satellite maneuvering in Ref [38]. In Ref [39], fly wheels revolve in anti clock-wise for power tracking purposes.

In Ref [36] during simultaneous attitude and energy storage, Flatley used four fly wheels in tetrahedron shape. He described the voltage pa

these four wheels tetrahedron shape. In 1970, flywheels technology was in initial phase. In Ref [42] flywheels technology has been used for ACE system. Rodriguez and Studer highlighted the actual technical problems in ACE system. During the 1980's, many scientists highlighted the practical implementation problems of fly wheels for IPACS and ACE system.

## **2.4 THIRD PHASE OF FLY WHEEL TECHNOLOGY (1990 to Onwards)**

The third phase of flywheel technology belongs to 1990 to onwards. In this period, the high tech technologies for example, magnetic bearing technology have been applied to solve flywheel technical implementation issues in Ref [43, 18, 19]. In Ref [19], Tsiosstros, Shen and C D Hall used the flywheel as momentum wheel for complete derivations of nonlinear equation of motion. Ref [44] described the modern techniques for flywheel implementations. American Air Force Research Laborites and NASA are working on flywheel technologies to produce low cost, high quality and lighter weight and more efficient control satellite.

## **2.5 CONTROL MOMENT GYROSCOPES**

The basic goal of fixed speed control moment gyroscopes or variable speed control moment gyroscopes is to provide required torque for attitude tracking in satellite. A control moment gyroscope (CMG) is a device used as an actuator for the attitude control of a satellite. It generates torques through angular momentum transfer to and from the main satellite body. This is achieved by changing the direction of the angular momentum vector of a gimballed flywheel.

Created with



nitro PDF

professional

download the free trial online at [nitropdf.com/professional](http://nitropdf.com/professional)

download the free trial online at [nitropdf.com/professional](http://nitropdf.com/professional)

In last three decades, the CMGs are extensively discussed for attitude tracking only.

## **TYPES OF CONTROL MOMENT GYROSCOPES**

The types of control moment gyroscopes are:

- Single Gimbal Control Moment Gyroscopes
- Double Gimbal Control Moment Gyroscopes
- Variable Speed Control Moment Gyroscopes

The literature related to control moment gyroscopes is divided into three portions:

### **2.5.1 INITIAL DEVELOPMENT OF CONTROL MOMENT GYROSCOPES**

The space discoveries have become more prominent in the period of 1960's. The ideas of attitude tracking using CMG's have been introduced in this period in Ref [45]. Liska and Jacot described first time the ideas about CMG's advantages, configuration and designing parameters. In their research paper, they derived the linear equation of motion using control moment gyroscopes and after that most of researchers used that linear equation in their research papers [45]. The observation, construction and applications of CMG's have been verified in laboratories and initially CMG's were produced very little torque for satellites. The use of control moment gyroscopes in satellites is also used for the following application:

- (i) Huge satellite large angle maneuvering in small time
- (ii) Satellite maneuvering around its three axes
- (iii) Single attitude and power tracking device

Created with



**nitro** PDF<sup>®</sup>

**professional**

download the free trial online at [nitropdf.com/professional](http://nitropdf.com/professional)

download the free trial online at [nitropdf.com/professional](http://nitropdf.com/professional)



## 2.6 CONTROL MOMENT GYROSCOPES ON ORBIT APPLICATIONS

Control moment gyroscopes have been used as actuator in spacecraft for attitude tracking since 1960 to onwards. In Ref [23] Be. Wie described that in NASA Skylab Space Station, three double gimbal control moment gyroscopes in orthogonal configuration and six single gimbal control moment gyroscopes have been implemented successfully. Four double gimbal control moment gyroscopes have also been used effectively in MIR Station (Russian space station). The control moment gyroscopes have been applied for attitude tracking in huge satellite, but CMGs have never been applied in commercial satellites, due to their complex construction nature and implementations issues.

### 2.6.1 CONTROL MOMENT GYROSCOPES EQUATION OF MOTION

Margulies and Aubrun developed the first actual practical mathematical model of control moment gyroscope in his research paper in 1973 [46]. His work described the designing parameters of control moment gyroscope and discussed the singularities issues [46]. In Ref [47], complete descriptions of nonlinear control moment gyroscope have been given by Valadli and Oh. Dumnor. The singularities avoidance techniques have been given for IPACS in Ref [48, 49, 50, 51]. Ref [63] represents a design of a novel free-flying maintenance robot (known as a Maintenance Bot.) The Maintenance Bot uses Control Moment Gyros (CMGs) for manipulator arm and attitude control. This architecture provides high authority control in a compact

Created with

low power package. In Ref [64], a real-time optimization method is presented that includes null motion in a closed-loop end-effectors tracking problem.

Ref [65] discussed the potential benefits of optimizing these gimbal axis configurations and compares these results to existing configurations such as the box, rooftop, and pyramid. A static optimization is performed to find the correct gimbal axis configuration in terms of Euler angles for an attitude control system (ACS) consisting of four CMGs. A four CMG configuration is chosen for minimal redundancy in avoiding singularities. Ref [66] based on singular value decomposition (SVD) theory, the mechanism of Escaping / avoiding singularity using generalized singularity-robust (GSR) steering law and weighted singularity-robust (WSR) steering law is analyzed for a spacecraft, which uses single gimbal control moment gyros (SGCMGs) as the actuators for attitude control system.

Ref [67] proposed a generalized framework for steering laws that are explicitly linear with respect to the gimbal rates. This formulation is followed by a discussion of characteristics of constraints used in this law, and general principles for designing a singularity-free constraint. Ref [68] described the development of a hybrid steering logic that maintains attitude tracking precision while avoiding hyperbolic internal singularities or escaping elliptic singularities inherent to single-gimbal control moment gyroscopes.

## 2.7 VARIABLE SPEED CONTROL MOMENT GYROSCOPES

Hall and Ford in Ref [21, 52] had presented first practical research of CMG's with Momentum wheel or Reaction wheel. In Ref [53], Schaub described that Hall and Ford did not give any explanation about simultaneous attitude control using CMG's and momentum wheels. To solve this problem Valdli, Schaub and Junkins had replaced CMG's with VSCMGs to escape from singularities issues in Ref [53,54,55]. In Ref [53, 54, 55] ,the authors also derived the real and practically applicable non linear equation of motion for attitude tracking using variable control moment gyroscopes in satellite. More singularities avoidance techniques have also been given in references [53, 54, 55]. First time, the VSCMGs as actuators were used in satellites for simultaneous attitude tracking with energy storage in 1985, by O'Dea in Ref [37]. But the author failed to derive the complete nonlinear equation for simultaneous attitude tracking control and energy storage using VSCMGs as actuator. This deficiency invites researchers to derive full nonlinear equation of for simultaneous attitude tracking control and energy storage using VSCMGs. There is acute shortage of literature related with VSCMGs for simultaneous attitude control and energy storage. In Ref [63], Yoon and Tsiosstros derived the nonlinear equation of motion for combined attitude and power tracking using VSCMGs in their research paper.

## 2.8 ADAPTIVE ATTITUDE TRACKING CONTROL OF SATELLITE

Many adaptive control schemes have been presented in Ref [1,2, 3,10, 12,13, 16]. In these references, Reaction Wheel, Mechanical Wheel, Gas Jet Thruster and Fixed speed control moment gyroscopes have been as actuator for attitude tracking. Rodriguez parameters (RPs) and Euler angles have been used for adaptive attitude control in Ref [13]. The applications of RPs and Euler angle for attitude tracking are applicable in short range only due to kinematics singularities issues. Fixed speed control moment gyroscopes have been used for attitude control with condition that angular velocities must be calculate-able in Ref [12]. If angular velocities are not being calculate-able, then approximation techniques will be applied for attitude tracking. In Ref [2], adaptive attitude tracking is only possible, if we know prior values of higher and lower elements of principle inertia matrix. Proportional Integral Derivative techniques have been used for attitude tracking with inertia matrix error in Ref [10]. In Ref [3,12], author presented the analysis of attitude tracking using fixed speed control moment gyroscopes. Double gimbal CMGs has been used as actuator with unknown elements of inertia matrix to derive adaptive control law in Ref [6]. Adaptive attitude tracking and energy storage of satellite have been achieved by applying complete nonlinear equation with uncertain moment of inertia matrix using four variable speed control moment gyroscopes as actuators

## CHAPTER 3

# SIMULTANEOUS ADAPTIVE ATTITUDE CONTROL AND POWER TRACKING USING VARIABLE SPEED CMG SYSTEMS FOR SATELLITE

In this chapter, I derived the kinematics and dynamical nonlinear equations of motion of satellite using variable speed control moment gyroscopes as actuator. I used Modified Rodriguez Parameters (MRPs) for attitude representation of satellite. Simultaneous adaptive attitude and power tracking controllers using variable speed control moment gyroscopes in pyramid configuration have been derived with the help of Lyapunov stability control theory for nonlinear MIMO system. In these controllers, simultaneous adaptive attitude control and power tracking for satellite have been achieved with wheel speed equalization of VSCMGs. These control laws have been introduced as SAAPT- VSCMGs systems and used for satellites operations. This chapter is designed for SAAPT- VSCMGs control theory for nonlinear MIMO system.

### 3.1 EQUATIONS OF ATTITUDE MOTION FOR SATELLITE EQUIPED WITH VSCMGs

The wheel speed of a conventional CMG is kept constant, where as the wheel speed of a VSCMG is allowed to vary smoothly. As a result, VSCMGs have extra degrees of freedom and can be used for additional objectives such as energy storage, singularity avoidance, as well as attitude control

Created with

### 3.1 .1 DYNAMICAL MODEL

Single Gimbal Variable Speed Control Moment Gyroscopes as actuator have been used to produce required attitude tracking torque in satellites which can be shown as in figure 1.

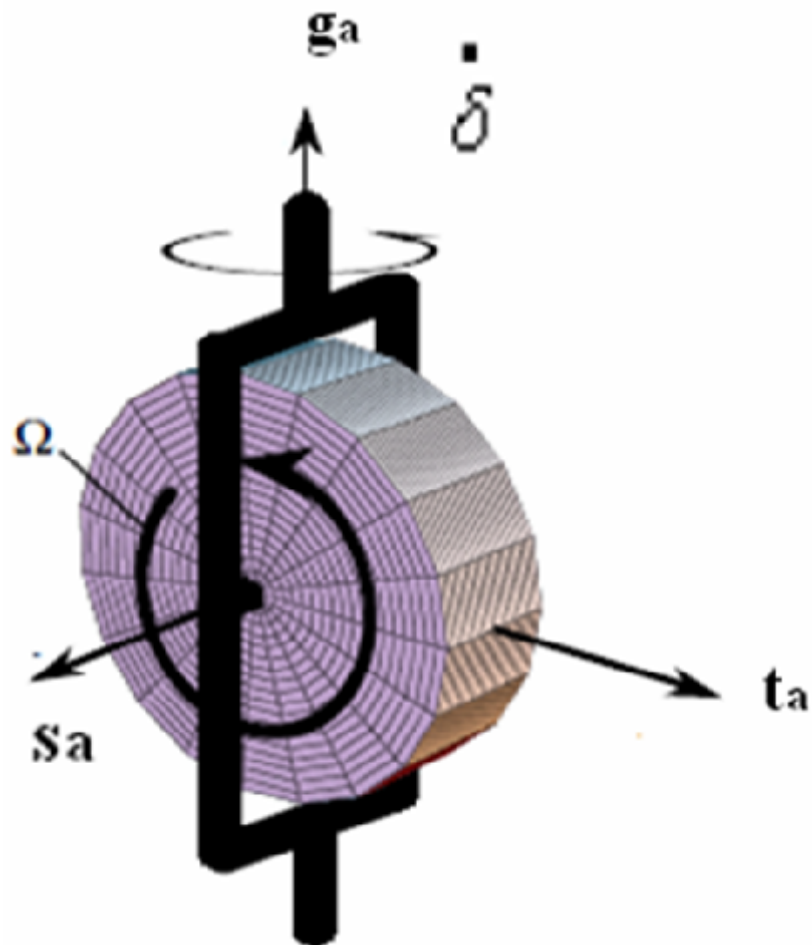


Figure 1: Axis System associated with VSCMG.

In above figure1, gimbal axis vector is indicated by  $\mathbf{g}_a$  and spin axis vector is denoted by  $\mathbf{s}_a$  and transverse (torque) axis vector is represented as  $\mathbf{t}_a$

$$\mathbf{t}_a = \mathbf{g}_a \times \mathbf{s}_a$$

The whole angular momentum of a satellite using N number of variable speed control moment gyroscopes cluster in satellite frame of reference is given as [62]

$$\mathbf{h} = \mathbf{I} \boldsymbol{\omega} + \mathbf{h}_c$$

Where

$$\mathbf{h}_c = \mathbf{A}_{ga} \mathbf{I}_{gac} \dot{\boldsymbol{\delta}} + \mathbf{A}_{sa} \mathbf{I}_{saw} \boldsymbol{\Omega}$$

Then

$$\mathbf{h} = \mathbf{I} \boldsymbol{\omega} + \mathbf{A}_{ga} \mathbf{I}_{gac} \dot{\boldsymbol{\delta}} + \mathbf{A}_{sa} \mathbf{I}_{saw} \boldsymbol{\Omega} \dots\dots\dots (1)$$

$\boldsymbol{\omega}$  is the angular velocity of satellite, The gimbal angles and wheel speeds of variable speed control moment gyroscopes are given respectively as

$$\boldsymbol{\delta} = (\delta_1, \dots, \delta_N)^T \in \mathbb{R}^N$$

$$\boldsymbol{\Omega} = (\Omega_1, \dots, \Omega_N)^T \in \mathbb{R}^N$$

The total inertia matrix of satellite with actuator (VSCMGs) is represented as follows

$$\mathbf{I} = \mathbf{I}_B + \mathbf{A}_{sa} \mathbf{I}_{sac} \mathbf{A}_{sa}^T + \mathbf{A}_{ta} \mathbf{I}_{sc} \mathbf{A}_{ta}^T + \mathbf{A}_{ga} \mathbf{I}_{gac} \mathbf{A}_{ga}^T \dots\dots\dots (2)$$

Inertia matrix of satellite without VSCMGs is a constant matrix and is represented as  $\mathbf{I}_B$ .  $\mathbf{I}_r$  represents diagonal gimbals and wheels composition inertia matrix of VSCMGs. Where ' $\mathbf{r}$ ' represents  $\mathbf{g}_a$ ,  $\mathbf{s}_a$  and  $\mathbf{t}_a$ .  $\mathbf{I}_{rw}$  represents wheels composition inertia matrix of VSCMGs.

$$\mathbf{I}_{rc} = \mathbf{I}_{rg} + \mathbf{I}_{rw}$$

$$\mathbf{I}_{rg} = \text{diag} [ I_{rc1}, \dots, I_{rc \times N} ]$$

$$\mathbf{I}_{rw} = \text{diag} [ I_{rw1}, \dots, I_{rwN} ]$$

The matrix  $\mathbf{A}_r \in \mathbb{R}^{3 \times N}$  represents columns of spin, torque and gimbal unit vectors respectively and are given as

$$\mathbf{A}_s = [ \mathbf{s}_{a1} \dots \mathbf{s}_{aN} ]$$

$$\mathbf{A}_t = [ \mathbf{t}_{a1} \dots \mathbf{t}_{aN} ]$$

$$\mathbf{A}_g = [ \mathbf{g}_{a1} \dots \mathbf{g}_{aN} ]$$

Where  $\mathbf{A}_s(\delta)$  and  $\mathbf{A}_g(\delta)$  both depend on gimbal angle as follows

$$\mathbf{A}_s = \mathbf{A}_{sa}(\delta) \quad \text{and} \quad \mathbf{A}_g = \mathbf{A}_{ga}(\delta)$$



Satellite total inertia matrix ( I ) is also dependent on gimbal angles i.e.  $I = I(\delta)$ . The equation of motion of a rigid satellite equipped with cluster of N VSCMGs is given as

$$\dot{h} + [\omega^x](I\omega + A_{ga} I_{gac} \dot{\delta} + A_{sa} I_{saw} \Omega) = T_{ext}$$

Where  $T_{ext}$  is external torque and is considered to be zero for simplicity.

$$\dot{h} + [\omega^x](I\omega + A_{ga} I_{gac} \dot{\delta} + A_{sa} I_{saw} \Omega) = 0$$

By taking derivative of equation (1), we can get

$$\dot{h} = \dot{I}\omega + I\dot{\omega} + \dot{h}_c$$

$$\dot{I}\omega + I\dot{\omega} + \dot{h}_c + [\omega^x](I\omega + A_{ga} I_{gac} \dot{\delta} + A_{sa} I_{saw} \Omega) = 0 \dots \dots (3)$$

Consider angular velocity vector as  $\omega = [\omega_1, \omega_2, \omega_3]^T \in \mathbb{R}^3$

$$[\omega^x] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

Where  $[\omega^x]$  represents skew symmetric matrix

At initial value i.e.  $t = 0$ .

$$A_{ga} = A_{gao}$$

$$A_{sa} = A_{sao} [\cos(\delta)]^d + A_{tao} [\sin(\delta)]^d$$

$$A_{ta} = A_{tao} [\cos(\delta)]^d - A_{sao} [\sin(\delta)]^d$$

Where

$\cos(\delta) = [\cos(\delta_1), \dots, \cos(\delta_N)]^T \in \mathbb{R}^N$  is column vector.

and

$\sin(\delta) = [\sin(\delta_1), \dots, \sin(\delta_N)]^T \in \mathbb{R}^N$  is also a column vector.

It shows

$$\dot{A}_{sa} = A_{ta} \text{diag} [\dot{\delta}]$$

$$\dot{A}_{ta} = -A_{sa} \text{diag} [\dot{\delta}]$$

The time derivative of  $I$  and  $h_c$  are calculated as

$$\dot{h}_c = A_{ga} I_{gac} \ddot{\delta} + A_{ta} I_{saw} \text{diag} [\Omega] \dot{\delta} + A_{sa} I_{saw} \dot{\Omega}$$

and

$$\dot{I} = A_{ta} \text{diag} [\dot{\delta}] (I_{sac} - I_{cta}) A_{sa}^T + A_{sa} \text{diag} [\dot{\delta}] (I_{csa} - I_{cta}) A_{ta}^T$$

Created with

Now by applying this law as

$$\text{diag} [\dot{\delta}] \Omega = \text{diag} [\Omega] \dot{\delta}$$

Now put  $\dot{h}_c$  and  $\dot{I}$  values in equation (3)

$$\begin{aligned} & \{A_{ta} \text{diag} [\dot{\delta}] (I_{sac} - I_{ca}) A_{sa}^T + A_{sa} \text{diag} [\dot{\delta}] (I_{csa} - I_{cta}) A_{ta}^T\} \omega + I \dot{\omega} + \\ & A_{ga} I_{gac} \ddot{\delta} + A_{ta} I_{saw} \text{diag} [\Omega] \dot{\delta} + A_{sa} I_{saw} \dot{\Omega} + [\omega^x] (I \omega + A_{ga} I_{gac} \dot{\delta} + A_{sa} I_{saw} \Omega) = 0 \end{aligned} \quad \dots(4)$$

The equation (4) is the **final dynamical nonlinear equation of motion** using variable speed control moment gyroscopes as actuator for satellite.

### 3.1 .3 Dynamical equation of motion for Reaction / Momentum wheel system

If we put gimbal angles i.e  $\delta = \text{constant}$  in equation (4) instead of gimbal rates then the above equation becomes the dynamical equation for reaction /momentum wheel.

### 3.1 .4 Dynamical equation of motion for fixed speed control moment gyroscopes

If we replace variable wheel speed  $\dot{\Omega}$  with constant wheel speed  $\Omega$  in equation (4) then equation (3) becomes the dynamical equation for fixed speed CMGs.

## 3.2 KINEMATICS OF SATELLITE MANEUVERING

In research papers, variety of methods has been used for attitude tracking representations of satellite. In this thesis, Modified Rodriguez Parameters (MRPs) techniques have been applied to design SAAPT-VSCMG controller system for adaptive attitude and power tracking of satellite.

### 3.2.1 Modified Rodriguez Parameters (MRPs) as Attitude Representations

In this thesis, MRPs have been used to measure the attitude kinematics error of the of satellite and defined as

$$\sigma = [\sigma_1, \sigma_2, \sigma_3]^T = \hat{\eta} \tan(\phi/4)$$

By using MRPs, the range of satellite maneuvering is possible without redundancies

i.e.  $\phi \in [0, 2\pi]$

$$[q_1, q_2, q_3]^T = \hat{\eta} \sin(\phi/2)$$

$$q_4 = \cos(\phi/2)$$

$$\sigma_i = \frac{q_i}{1+q_4} \quad \text{where } i = 1, 2, 3, 4.$$

Created with

Differential form MRPs have described the kinematics of satellite as follows

$$\dot{\sigma} = L(\sigma)\omega \quad \dots\dots\dots (5)$$

Where

$$L(\sigma) = \frac{1}{2} \{ I_{id} + [\sigma^\times] + \sigma\sigma^T - [\frac{1}{2}(1 + \sigma^T\sigma)] I_{id} \}$$

Where  $I_{id}$  is  $3 \times 3$  identity matrix

## 3.2 SATELLITE ATTITUDE TRACKING USING LYAPUNOV STABILTY THEORY

In this thesis, attitude tracking controller has been designed by applying Lyapunov stability analysis. First it is considered that the elements of inertia matrix of satellite and variable speed control moment gyroscopes as actuator are precisely identified.

### 3.3.1 ATTITUDE TRACKING CONTROLLER

In kinematics and dynamics equation of motion of satellite, we have used few identified parameters. These parameters have been represented as follows  $\sigma(t), \sigma_d(t)$  and  $\omega(t), \omega_d(t)$  for  $t \geq 0$ , to design attitude tracking controller in a desired frame of reference. Modified Rodriguez Parameters vector is denoted by  $\sigma(t)$  for attitude tracking and  $\omega$  represents the angular velocities vector.  $\sigma_d(t)$  and  $\omega_d(t)$  represents

desired attitude and desired angular velocities vector of satellite respectively.  $\sigma_e(t)$  represents the attitude tracking error and is given as

$$\sigma_e = \sigma - \sigma_d$$

and  $\omega_e$  represents angular velocity tracking error and is given as

$$\omega_e = \omega - \omega_d$$

The following Lyapunov function is considered to achieve angular velocity tracking error ( $\omega_e$ ) and attitude tracking error ( $\sigma_e$ ) approaches to zero as given in Refs [11,14, 15]

$$V = \frac{1}{2} \omega_e^T I \omega_e + 2 K_s \ln(1 + \sigma_e^T \sigma_e)$$

Where  $K_s > 0$  and this function is positive definite.

The time derivative of V is

$$\dot{V} = \frac{1}{2} (\omega - \omega_d)^T \dot{I} (\omega - \omega_d) + (\omega - \omega_d)^T I (\dot{\omega} - \dot{\omega}_d) + \frac{2k_s \sigma_e^T \dot{\sigma}_e}{1 + \sigma_e^T \sigma_e}$$

$$\dot{V} = -(\omega - \omega_d)^T \left\{ -\frac{1}{2} \dot{I} (\omega - \omega_d) - I (\dot{\omega} - \dot{\omega}_d) - K_s \sigma_e \right\}$$

Created with

We choose the following function to ensure stability

$$-\frac{1}{2} \dot{I} (\omega - \omega_d) - I (\dot{\omega} - \dot{\omega}_d)^T - K_s \sigma_e = C (\omega - \omega_d)$$

Where C is a positive definite 3×3 gain matrix. The resulting system is Lyapunov stable because  $\dot{V}$  is non negative definite. It is obvious that  $\dot{V}$  is zero if and only if

$$\omega = \omega_d \quad \text{and} \quad \sigma = \sigma_d$$

It can be shown that the system trajectories are stable about the desired attitude tracking and the resulting system is globally asymptotically stable.

Calculation shows that

$$\dot{h}_c + \frac{1}{2} \dot{I} (\omega + \omega_d) + [\omega^x] A_{ga} I_{gac} \dot{\delta} = C (\omega - \omega_d) + K_s \sigma_e - I \dot{\omega}_d -$$

$$[\omega^x] (I \omega + A_s I_{saw} \Omega)$$

The left hand side of above equation consists of the controllers gimbal input  $\dot{\delta}$  and wheel speed input  $\dot{\Omega}$  and is given as follows

$$\dot{h}_c + \frac{1}{2} \dot{I} (\omega + \omega_d) + [\omega^x] A_{ga} I_{gac} \dot{\delta} = B \ddot{\delta} + D \dot{\delta} + E \dot{\Omega}$$

Created with

$$B = A_{ga} I_{gac}$$

$$D = A_{ta} I_{saw} \text{diag} [\Omega]$$

And

$$E = A_{sa} I_{saw}$$

By combining the above equations, we get the required attitude tracking torque as

$$T_{rm} = C (\omega - \omega_d) + K_s \sigma_e - I \dot{\omega}_d - [\omega^x] (I \omega + A_{sa} I_{saw} \Omega)$$

$$T_{rm} = C (\omega - \omega_d) + K_s \sigma_e - I \dot{\omega}_d - [\omega^x] (I \omega) - [\omega^x] (A_{sa} I_{saw} \Omega) \dots \dots (6)$$

### 3.3.2 ADAPTIVE ATTITUDE TRACKING CONTROLLER

As discussed earlier, the satellite inertia matrix is depended on gimbal angle i.e  $I = I(\delta)$ . So satellite total inertia matrix ( $I$ ) is variable by using VSCMGs, but time rate change of satellite inertia matrix ( $I$ ) is known. The time rate change of inertia matrix ( $I$ ) is obtained by taking derivative of gimbal angles. In order to derive the adaptive control law for attitude tracking of satellites, we have used estimated inertia matrix of satellite.



$$\frac{1}{2} \dot{I} \omega + I \dot{\omega} + [\omega^x] (I \omega + A_{sa} I_{saw} \Omega) + B \ddot{\delta} + D \dot{\delta} + E \dot{\Omega} = 0$$

B, C and D have already been determined. The matrix B has very small values, so it can be neglected, then above equation becomes as follows

$$\frac{1}{2} \dot{I} \omega + I \dot{\omega} + [\omega^x] (I \omega + A_{sa} I_{saw} \Omega) + D \dot{\delta} + E \dot{\Omega} = 0 \dots\dots\dots (7)$$

By differentiating equation (5), we get

$$\omega = L^{-1}(\sigma) \dot{\sigma} \quad \text{and} \quad \ddot{\sigma} = L(\sigma) \dot{\omega} + \dot{L}(\sigma, \dot{\sigma}) \omega$$

Now the equation (7) becomes as

$$\frac{1}{2} \dot{I} \omega + I L^{-1}(\sigma) \ddot{\sigma} - I L^{-1}(\sigma) \dot{L}^{-1}(\sigma, \dot{\sigma}) \omega + [\omega^x] (I \omega + A_{sa} I_{saw} \Omega) + D \dot{\delta} + E \dot{\Omega} = 0$$

If we suppose

$$h_1 = I \omega \quad \text{and} \quad h_2 = A_{sa} I_{saw} \Omega$$

Then the above equation can also be written in standard form of system equation as in reference [62]

$$H_1(\sigma)\ddot{\sigma} + D_1(\sigma, \dot{\sigma})\dot{\sigma} = F \dots \dots \dots (8)$$

Where

$$H_1(\sigma) = L^{-T}(\sigma) I L^{-1}(\sigma)$$

$$D_1 = -L^{-T}(\sigma) I \dot{L}^{-1}(\sigma) L^{-1}(\sigma) - L^{-T}(\sigma) [\dot{h}_1^x] L^{-1}(\sigma)$$

$$F = L^{-T}(\sigma) [\dot{h}_2^x] \omega - L^{-T}(\sigma) (D \dot{\delta} + E \dot{\Omega}) - \frac{1}{2} L^{-T} \dot{I} \omega$$

The LHS of equation (8) is linear in term of the elements of inertia matrix ( I ), which are the unknown parameters to be estimated.

And

$$\dot{L}(\sigma, \dot{\sigma}) = \frac{1}{2} ([\dot{\sigma}^x] + \dot{\sigma} \sigma^T + \sigma \dot{\sigma}^T - \dot{\sigma}^T \sigma I)$$

Consider the inertia parameter vector is as follows

$$a = (I_{11} \ I_{12} \ I_{13} \ I_{22} \ I_{23} \ I_{33})^T \in \mathbb{R}^6$$

and suppose estimated inertia parameter vector is given as follows

$$\hat{a} = (\hat{I}_{11} \ \hat{I}_{12} \ \hat{I}_{13} \ \hat{I}_{22} \ \hat{I}_{23} \ \hat{I}_{33})^T \in \mathbb{R}^6$$

$a_e = (\hat{a} - a)$  is the parameter estimation error and  $\hat{\sigma} = (\sigma - \sigma_d)$  is the attitude tracking error. To design adaptive control law, we consider the following Lyapunov function as given in reference [62]

$$V_a = \frac{1}{2} s^T H_1(\sigma) s + \frac{1}{2} a_e^T \Gamma^{-1} a_e$$

Where “s” is the measure of attitude tracking error and defines as

$$s = \dot{\hat{\sigma}} + \lambda a_e = \dot{\sigma} - \dot{\sigma}_r \quad (\lambda > 0)$$

$\dot{\sigma}_r = \dot{\sigma}_d - \lambda \hat{\sigma}$  is the reference velocity tracking vector and  $\Gamma$  is positive constant matrix .

By taking the derivative of Lyapunov function, it becomes as

$$\dot{V}_a = s^T (F - H_1(\sigma) \ddot{\sigma}_r - D_1(\sigma, \dot{\sigma}) \dot{\sigma}_r + \frac{1}{2} L^T(\sigma) \dot{I} L^{-1}(\sigma) s) + a_e F^{-1} \dot{a}_e$$

Now assume a control law as follows

$$F = H_2(\sigma) \ddot{\sigma}_r + D_2(\sigma, \dot{\sigma}) \dot{\sigma}_r - \frac{1}{2} L^T(\sigma) \dot{I} L^{-1}(\sigma) s$$

Where

$$H_2(\sigma) = L^T(\sigma) \hat{I} L^{-1}(\sigma)$$

$$D_2 = L^{-T}(\sigma) \hat{I} L^{-1}(\sigma) \dot{L}(\sigma, \dot{\sigma}) L^{-1}(\sigma) - L^{-T}(\sigma) [h_1^*] L^{-1}(\sigma)$$

Then it follows as

$$\dot{V}_a = s^T (F - H_1(\sigma)) \ddot{\sigma}_r - D_1(\sigma, \dot{\sigma}) \dot{\sigma}_r + a_e \Gamma^{-1} (\dot{a}_e - \dot{a})$$

$$H(\sigma) = H_2(\sigma) - H_1(\sigma)$$

$$D(\sigma) = D_2(\sigma, \dot{\sigma}) - D_1(\sigma, \dot{\sigma})$$

A known matrix  $Y^*(\sigma, \dot{\sigma}, \dot{\sigma}_r, \ddot{\sigma}_r)$  is defined with the help of Linearization of the dynamics as follows

$$Y^*(\sigma, \dot{\sigma}, \dot{\sigma}_r, \ddot{\sigma}_r) \hat{a} = H(\sigma) \dot{\sigma}_r + D(\sigma, \dot{\sigma}) \ddot{\sigma}_r$$

And choose adaptation law as

$$\dot{\hat{a}} = \Gamma (Y^*) s + \dot{a} \dots \dots \dots (9)$$

Normally satellite inertia matrix and point masses of variable speed control moment gyroscopes are represented by combined inertia matrix ( I ). The most part of the total inertia matrix ( I ) is occupied by satellite inertia matrix ( I<sub>B</sub> ). So very small part of satellite total inertia matrix ( I ) depends on gimbal angles of VSCMG's, which is

negligible. Therefore we supposed that the total inertia matrix of satellite is a constant matrix and as a result the term  $\dot{a}$  is neglected in equation (9). Now with this supposition, the equation of motion and F becomes as follows

$$\frac{1}{2} \dot{I} \omega + I \dot{\omega} + [\omega^x] (I \omega + A_{sa} I_{saw} \Omega) + D \dot{\delta} + E \dot{\Omega} = 0$$

$$F = L^{-T}(\sigma) [h_2^x] \omega - L^{-T}(\sigma) (D \dot{\delta} + E \dot{\Omega})$$

And adaptation law becomes as follows

$$\dot{\hat{a}} = \Gamma (\dot{Y})^* s \dots \dots \dots (10)$$

Now control law as given in equation (6) with estimated inertia matrix of satellite is taken form as follows

$$T_{rm} = C (\omega - \omega_d) + K_s \sigma_e - \hat{I} \dot{\omega}_d - [\omega^x] (\hat{I} \omega + A_{sa} I_{saw} \Omega)$$

### 3.4 VELOCITY BASED STEERING LAW FOR ATTITUDE TRACKING USING VARIABLE CONTROL MOMENT GYROSCOPES

In this attitude tracking controller, the term  $B\ddot{\delta}$  is represented the gimbals acceleration. But this gimbals acceleration term is neglected because B matrix is very small matrix as compared to D matrix and E matrix. So the required attitude tracking torque equation becomes as follows

$$T_{rm} = D\dot{\delta} + E\dot{\Omega}$$

Gimbal rates ( $\dot{\delta}$ ) and wheel spin rates ( $\dot{\Omega}$ ) of VSCMG's are control inputs .The above equation can also be written as

$$[D \ E] \begin{bmatrix} \dot{\delta} \\ \dot{\Omega} \end{bmatrix} = T_{rm} \dots \dots \dots (11)$$

The equation (11) is known as velocity based steering law for attitude tracking control in satellite using variable speed control moment gyroscopes.

### 3.5 POWER TRACKING

In this section, power tracking has been described for the case of rigid satellite using variable speed control moment gyroscopes as actuator. The total kinetic energy “E” stored in VSCMGs is given as

$$E = \frac{1}{2} \Omega^T I_{saw} \Omega$$

and rate of change of the kinetic energy is known as power, which is given as follows

$$P = \frac{dE}{dt}$$

$$P = [0 \quad \Omega^T I_{saw}] \begin{bmatrix} \dot{\delta} \\ \dot{\Omega} \end{bmatrix} \dots\dots\dots(12)$$

$$P = \Omega^T I_{saw} \dot{\Omega}$$

### 3.6 SAAPT – VSCMG (SIMULTANEOUS ADAPTIVE ATTITUDE AND POWER TRACKING USING VSCMG) CONTROLLER

The existing control inputs are obtained from velocity-based steering law in equation (11) and  $U$  is assumed as collective controller input and presented as

$$U = \begin{bmatrix} \dot{\delta} \\ \dot{\Omega} \end{bmatrix} \quad \text{Where } U \in \mathbb{R}^N$$

The equation (11) is expanded to simultaneous adaptive attitude and power tracking equation using variable speed control moment gyroscopes is described as follows

$$T_{rp} = \begin{bmatrix} Trm \\ P \end{bmatrix}$$

$$\begin{bmatrix} D & E \\ 0 & v \end{bmatrix} [U] = T_{rp}$$

$$Q_{4 \times 2N} = \begin{bmatrix} D & E \\ 0 & v \end{bmatrix}$$

Where

$$v = \Omega^T I_{saw} \quad , \quad D = A_{ta} I_{saw} \text{diag} [\Omega]$$

and

$$E = A_{sa} I_{saw}$$

Created with



The solution of velocity based steering law is calculated as

$$Q U = T_{rp}$$

$$Q_{inv} = Q^T (Q Q^T)^{-1}$$

Then

$$U = Q_{inv} T_{rp} \dots \dots \dots (13)$$

### 3.7 SINGULARITY AVOIDANCE WITH WEIGHTING MATRIX

The weighting matrix W is defined for singularities avoidance, as

$$W_s = w_1 \exp^{-w_2 \sigma^c} I_N$$

$$W = \begin{bmatrix} W_s & 0_N \\ 0_N & I_N \end{bmatrix}$$

$W_1$  and  $W_2$  are positive gain.  $\sigma_c$  represents the condition number of  $C$  and  $0_N$  represents the  $N \times N$  zeros matrix and  $I_N$  represents the  $N \times N$  identity matrix. Now the solution of velocity based steering law with weighted matrix is calculated as

$$Q_{inv} = W Q^T (Q W Q^T)^{-1}$$

$$U = Q_{inv} T_{rp} \dots \dots \dots (14)$$

### **3.8 VARIABLE CONTROL MOMENT GYROSCOPES WHEEL SPEED EQUALIZATION**

Sometimes during the process of simultaneous attitude and power tracking, the following two situations may occur [62]. In first situation, wheel spin rates become too large and as a result the wheel speed rates go into saturation and in second situation wheel spin rates become too low and as a result the required tracking torque may not be enough generated for the change of gimbal angles. To avoid these two situations, the wheel speed equalization techniques have been applied for required attitude tracking of satellites.

To achieve the wheel speed equalization of VSCMGs, the following control law has been used. In this control law, an additional factor (constraint) has been added. This new factor is used to convert high wheel speeds of VSCMGs to the average wheel speed of variable speed control moment gyroscopes.

This new factor (constraint) is given as

$$S_w(\Omega_1, \dots, \Omega_N) = \frac{1}{2} \Omega_e^T I \Omega_e$$

Where

$$\Omega_e = [I_N - (\frac{1}{N}) 1_{N \times N}] \Omega$$

Where  $1_{N \times N}$  is  $N \times N$  matrix whose all elements are 1's.

$$\frac{d}{dt}(S_w) = \nabla S_w \dot{\Omega} = -K_2 S_w$$

And

$$\nabla S_w = \Omega_e^T$$

Now control law becomes as follow

$$\begin{bmatrix} D & E \\ 0_N & v \\ 0_N & \nabla S_w \end{bmatrix} \begin{bmatrix} \dot{\delta} \\ \dot{\Omega} \end{bmatrix} = \begin{bmatrix} T_{rm} \\ P \\ -K_2 S_w \end{bmatrix}$$

Where

$$Q_1 = \begin{bmatrix} D & E \\ 0_N & v \\ 0_N & \nabla S_w \end{bmatrix}$$

$$Q_{invs} = W Q_1^T (Q_1 W Q_1^T)^{-1}$$

$$U = Q_{invs} \begin{bmatrix} T_{rm} \\ P \\ -K_2 S_w \end{bmatrix} \dots\dots\dots(15)$$

The above equation (15) is represented as the SAAPT-VSCMG controller system equation with wheel speed equalization for simultaneous adaptive attitude and power tracking of satellite using variable speed control moment gyroscopes.

## CHAPTER 4

### SIMULATION RESULTS AND ANALYSIS

#### 4.1 SIMULTANEOUS ADAPTIVE ATTITUDE AND POWER TRACKING WITH WHEEL SPEED EQUALIZATION OF A LOW EARTH ORBIT SATELLITE

In order to perform simultaneous adaptive attitude and power tracking with wheel speed equalization using variable control moment gyroscopes algorithm, the following example has been considered. In this example, the following physical model is considered, i.e Lower Earth Orbit rigid satellite with four variable speed control moment gyroscopes in pyramid shapes [62].

In order to achieve higher efficiency of the panel, it is assumed that the satellite keeps tracking the ground station and the sun even when these are not directly visible due to the location of the Earth. The 680 watts is usual power needs for this rigid spacecraft and during eclipse and an extra need of 4kw is also required up to five minutes. The time period is considered 98 minutes for low earth satellite. The eclipse time period is considered 33 minutes with power need of 4kw up to five minutes. Now the time period for Sunlight becomes 65 minutes. In sunlight the energy storing is started in wheels of variable control moment gyroscopes and goes up zero power

Created with

level to 1.5kw. The controller simulation parameters for simultaneous adaptive attitude and power tracking with wheel speed equalization using variable control moment gyroscopes of satellite have been given in following table.

Created with



**Table 1: Simulation Parameters**

Symbol	Value	Unit
N	4	unitless
$\theta$	54.75	deg
$\omega(t)$	$[0,0,0]^T$	rad /sec
$\omega_d(t)$	$(0.02) \times [\sin(2 \pi t/800), \sin(2\pi t /600), \sin(2 \pi t /400)]^T$	rad /sec
$\dot{\omega}(t)$	$[0,0,0]^T$	rad /sec <sup>2</sup>
$\sigma(t)$	$[0.33,0.33,0.33]^T$	unit less
$\sigma_d(t)$	$[1,0,0]^T$	unit less
$\delta(t)$	$[\pi/4, -\pi/4, -\pi/4, \pi/4]^T$	rad
$\dot{\delta}(t)$	$[0,0,0,0]^T$	rad/sec <sup>2</sup>
$\Omega(t)$	$[50000,60000,55000,65000]^T$	RPM
$\dot{\Omega}(t)$	$[0,0,0,0]^T$	RPM
$I_B$	$\begin{bmatrix} 15053 & 3000 & -1000 \\ 3000 & 6500 & 2000 \\ 1000 & 2000 & 11122 \end{bmatrix}$	Kg m <sup>2</sup>
$I_{wsa}$	diag [0.7,0.7,0.7,0.7]	Kg m <sup>2</sup> /sec <sup>2</sup>
k	0.01	unit less
c	0.1	unit less
$k_s$	$1 \times 10^{-3}$	unit less
$\Gamma$	$1 \times 10^{-7} \times I_{6 \times 6}$	unit less

Created with

This chapter represents the simulation results analysis of SAAPT (Simultaneous Adaptive Attitude with Power Tracking) –VSCMG controller algorithm of satellite. Large numbers of simulation tests have been executed. These simulation results have been shown in following figures.

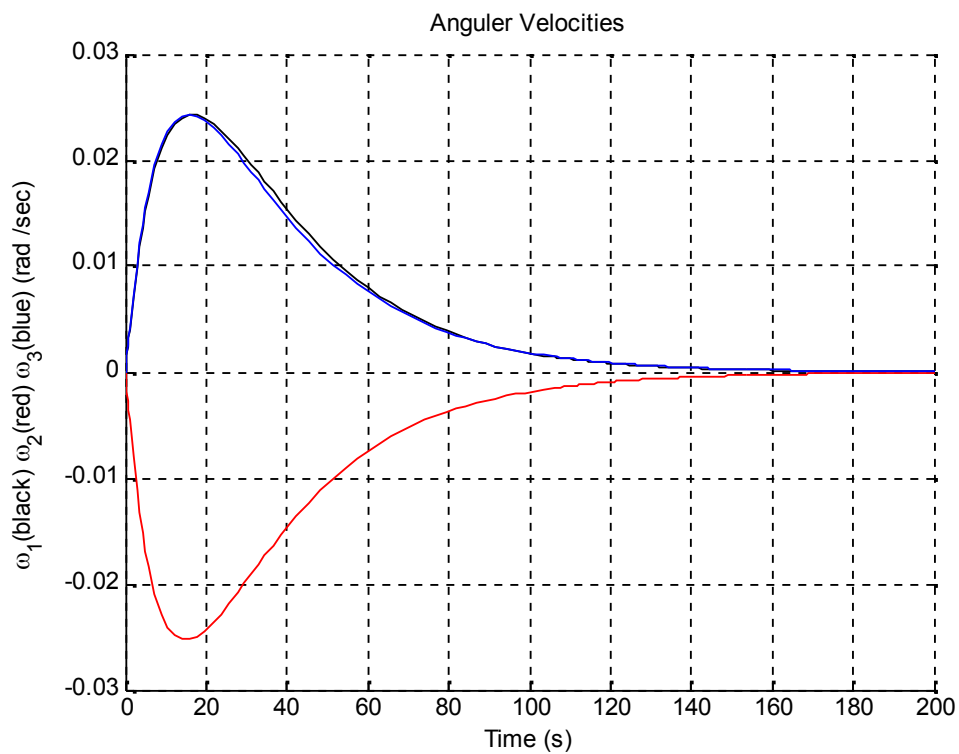
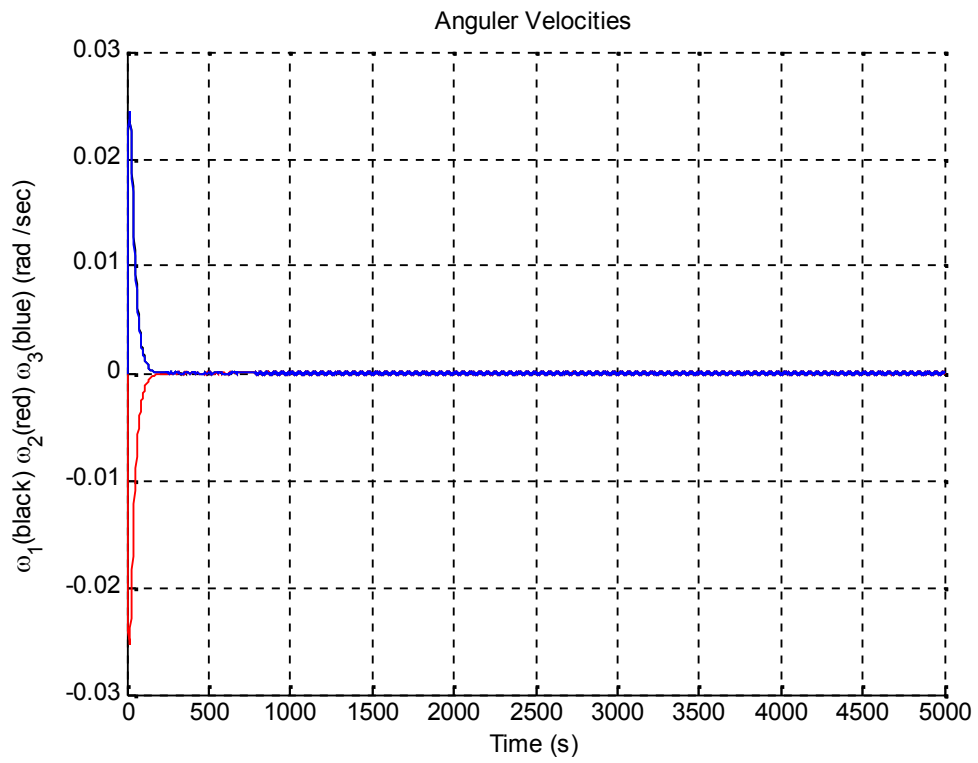


Figure 2: Angular velocities of Satellite





**Figure2: Angular velocities of satellite**

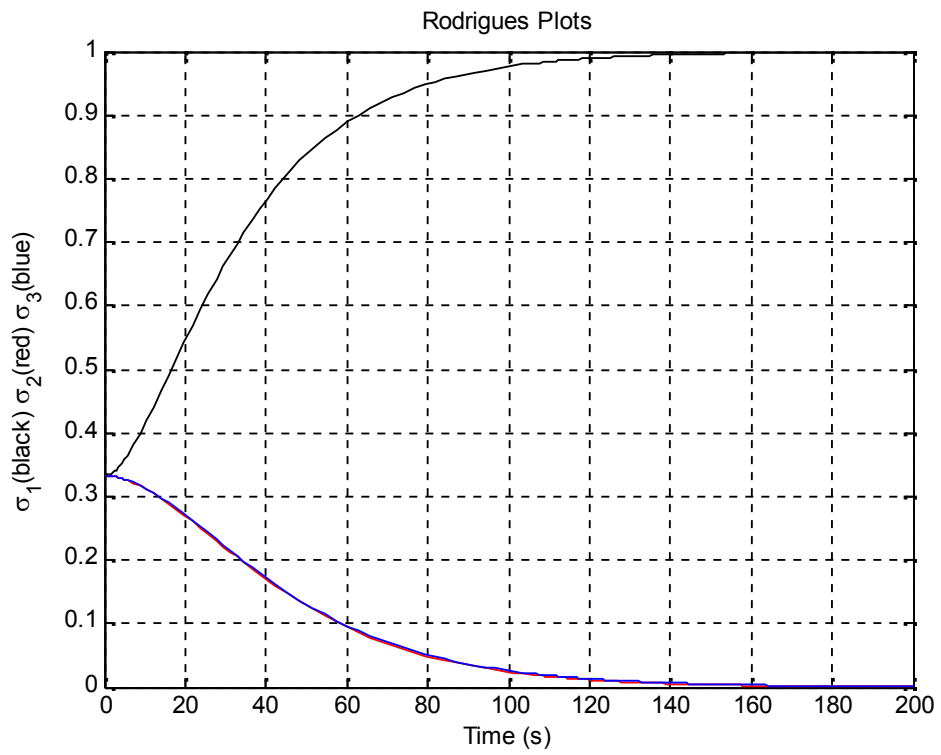
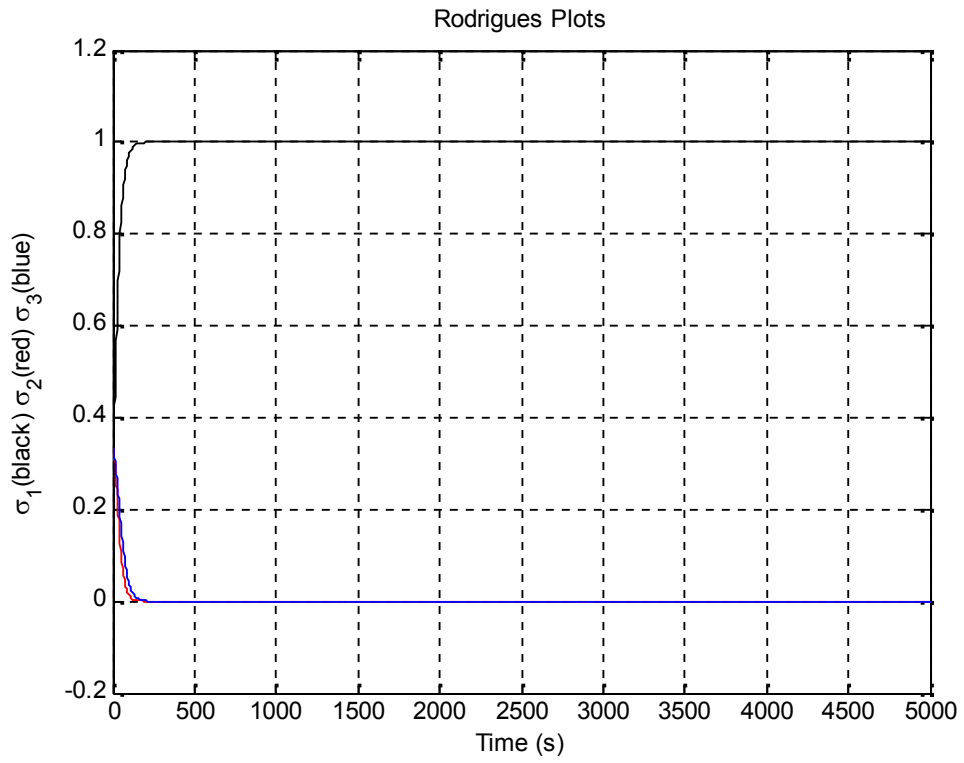


Figure 3: Attitude Trajectory of Satellite



**Figure 3: Attitude Trajectory of satellite**

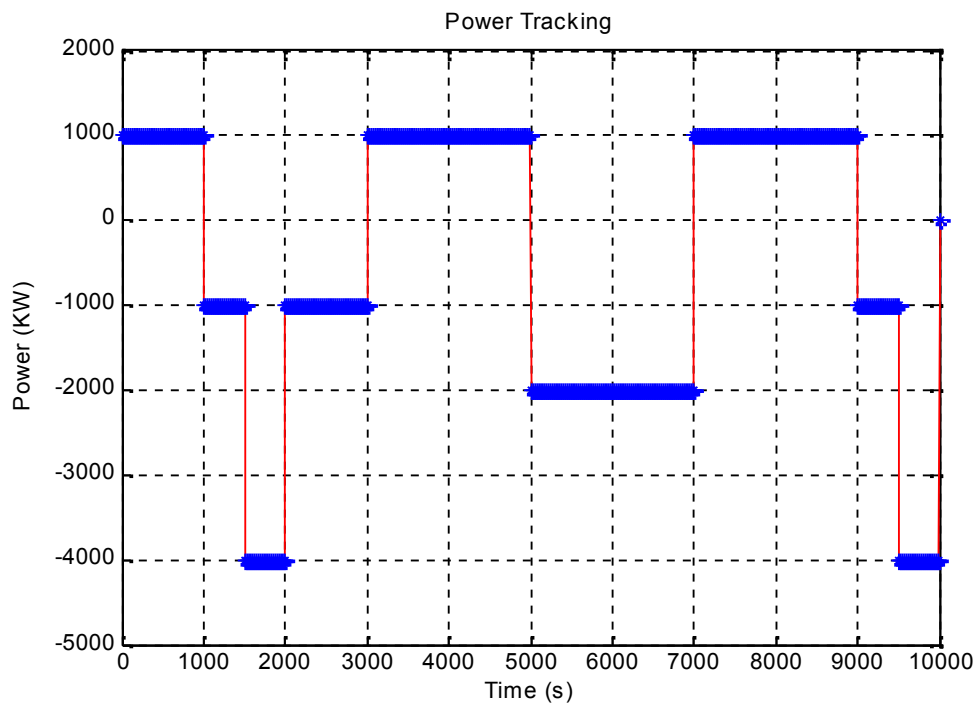
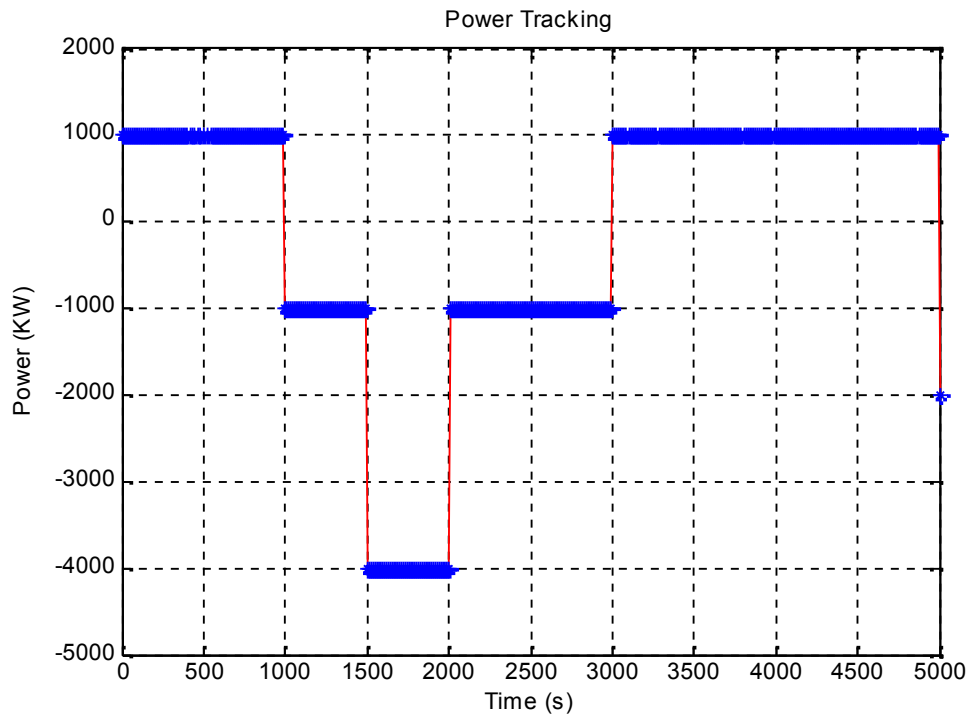


Figure 4: Desired (\*' blue) and Actual ('-red) Power Profile

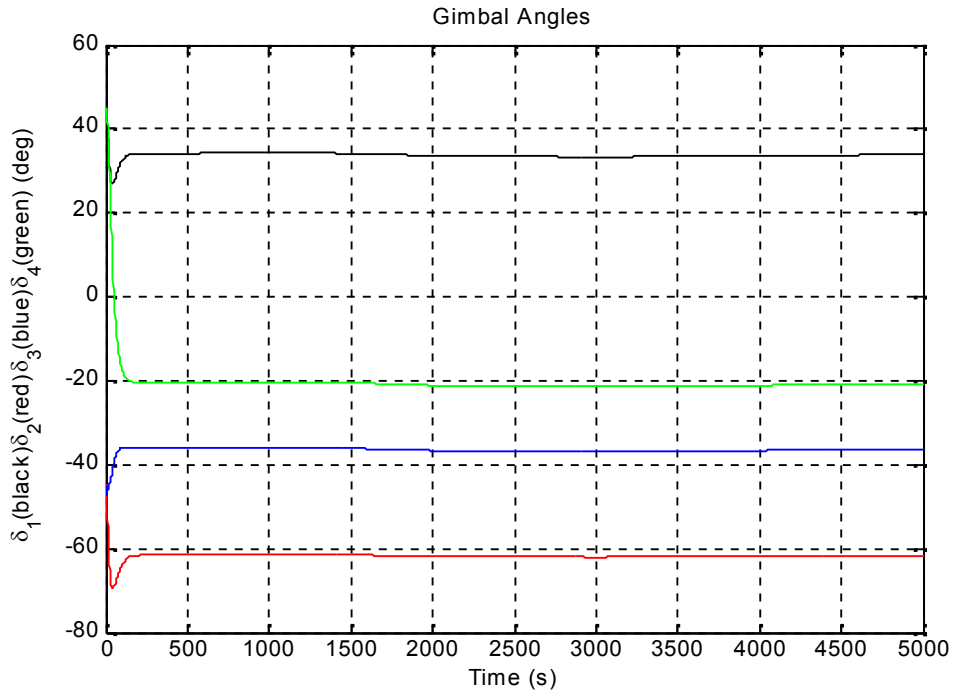


Figure 5: VSCMGs Gimbal angles without wheel speed equalization

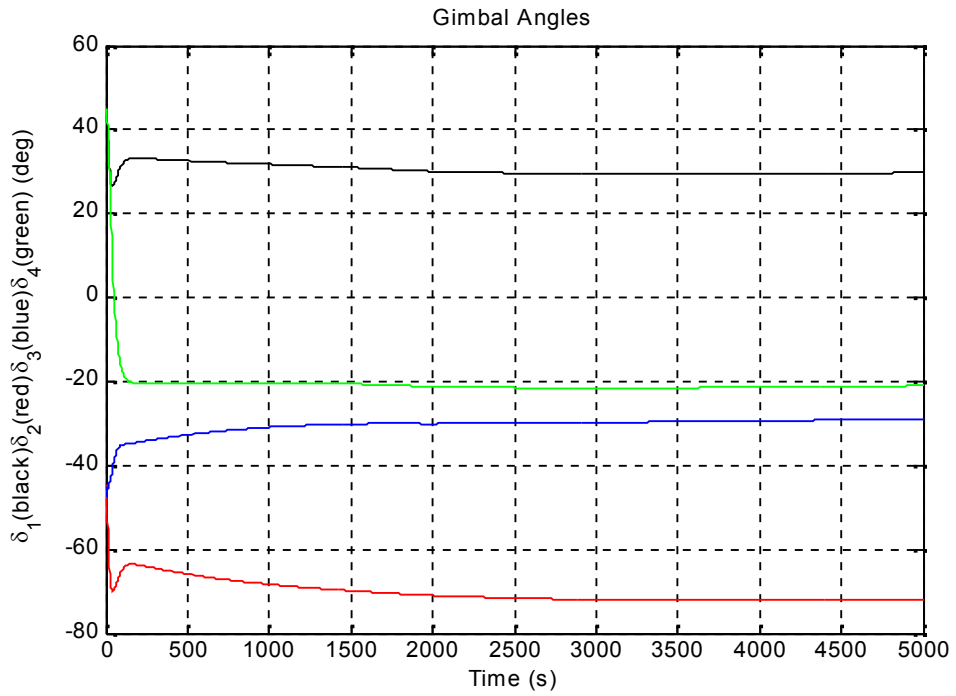


Figure 6: VSCMGs Gimbal angles with wheel speed equalization

Created with

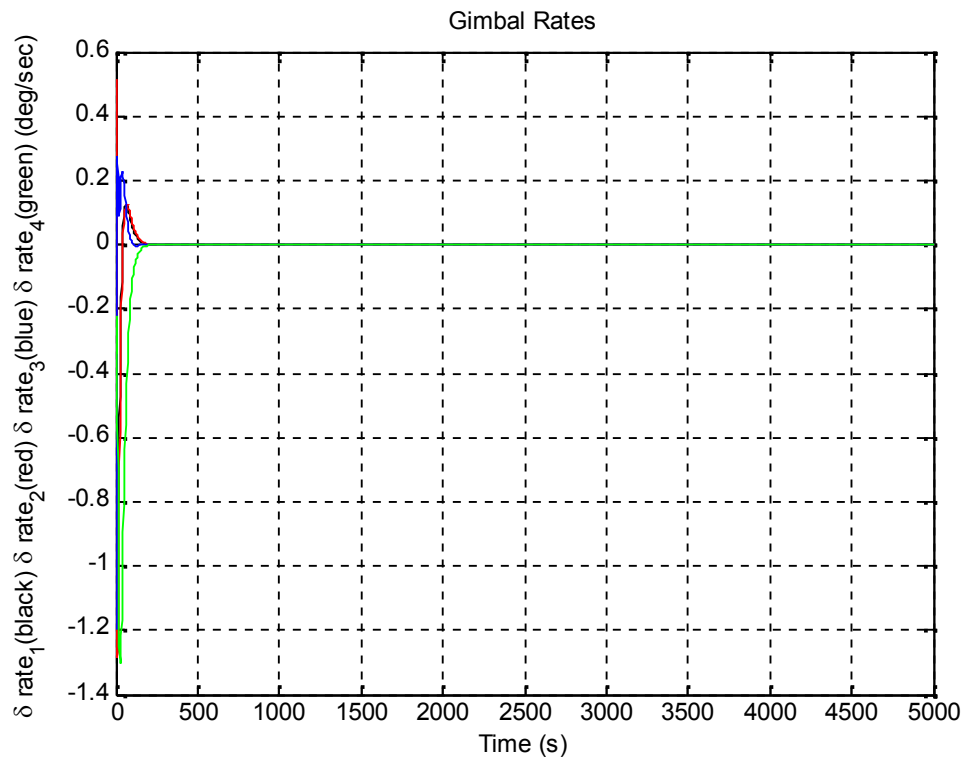
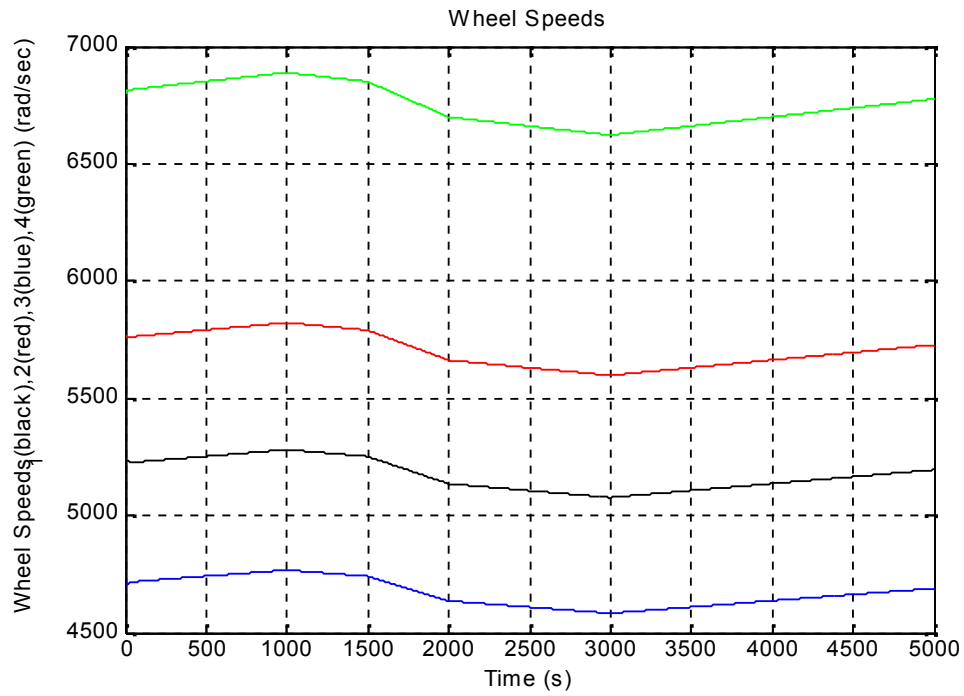
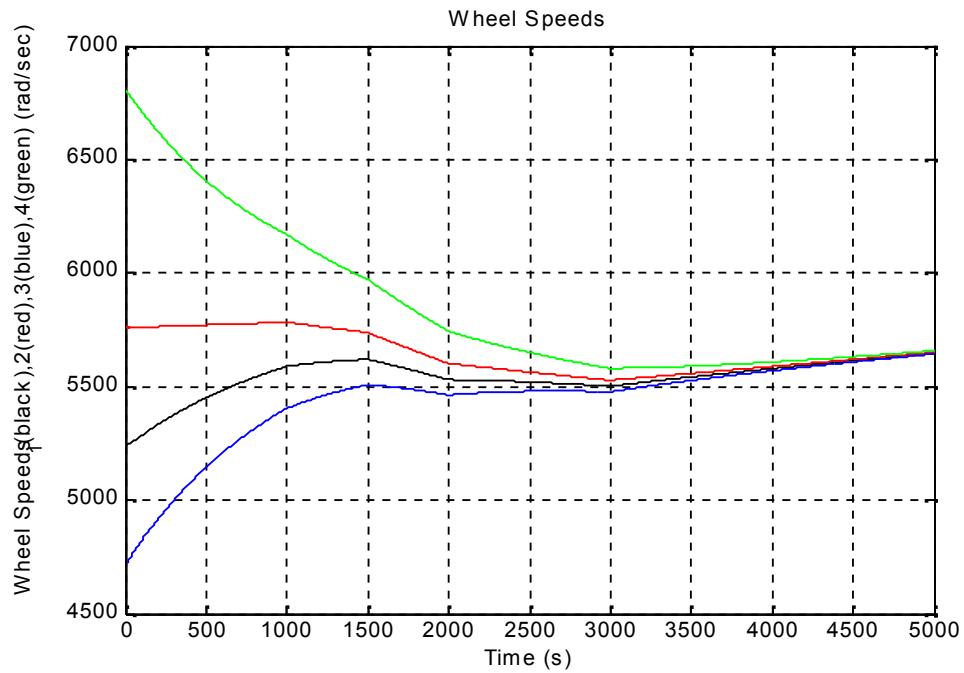


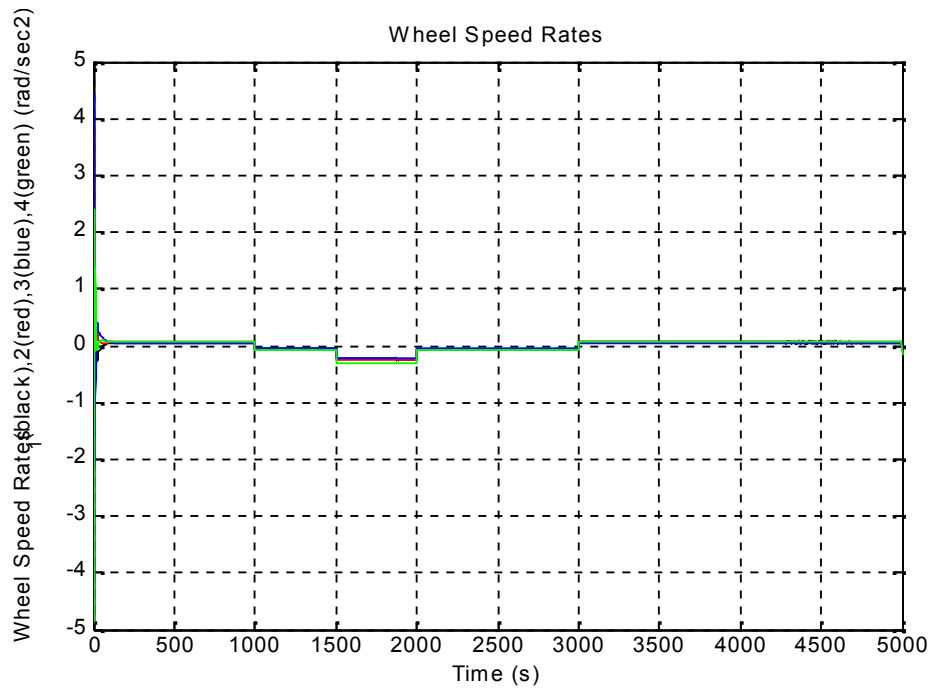
Figure 7: VSCMGs Gimbal rates



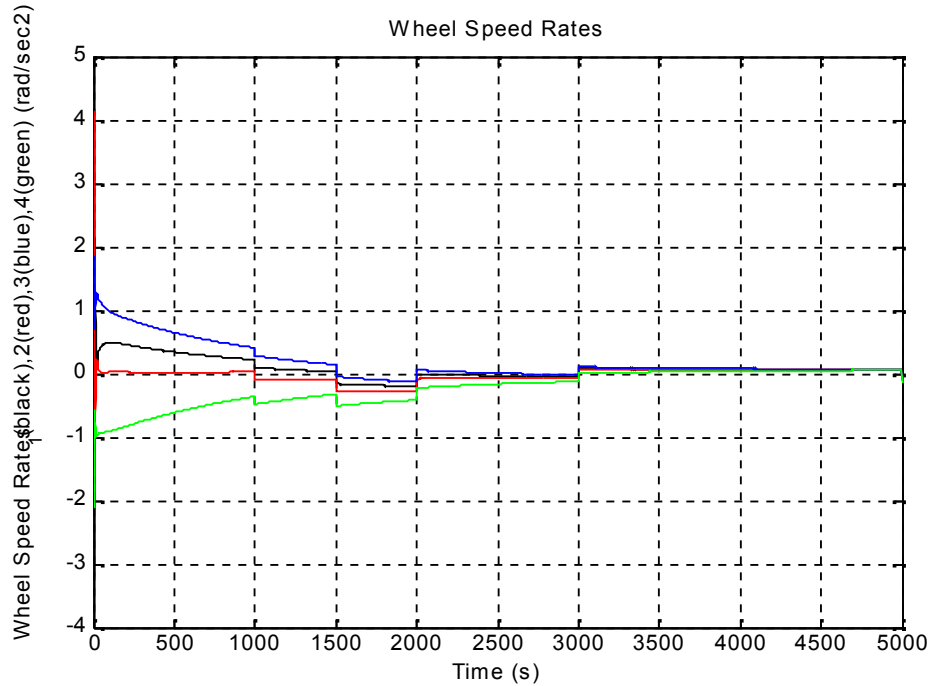
**Figure 8: VSCMGs wheel speeds without wheel speed equalization**



**Figure 9 : VSCMGs wheel speeds with wheel speed equalization**

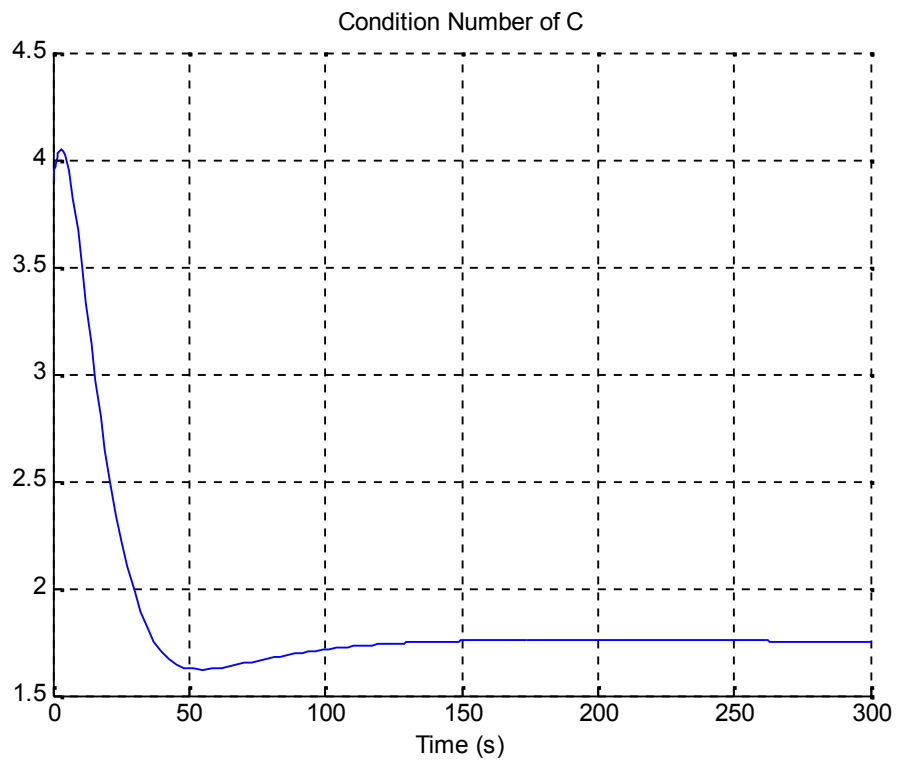


**Figure 10: VSCMGs wheel speeds rates without wheel speed equalization**



**Figure 11: VSCMGs wheel speeds rates with wheel speed equalization**





**Figure12: Condition Number of Matrix C**

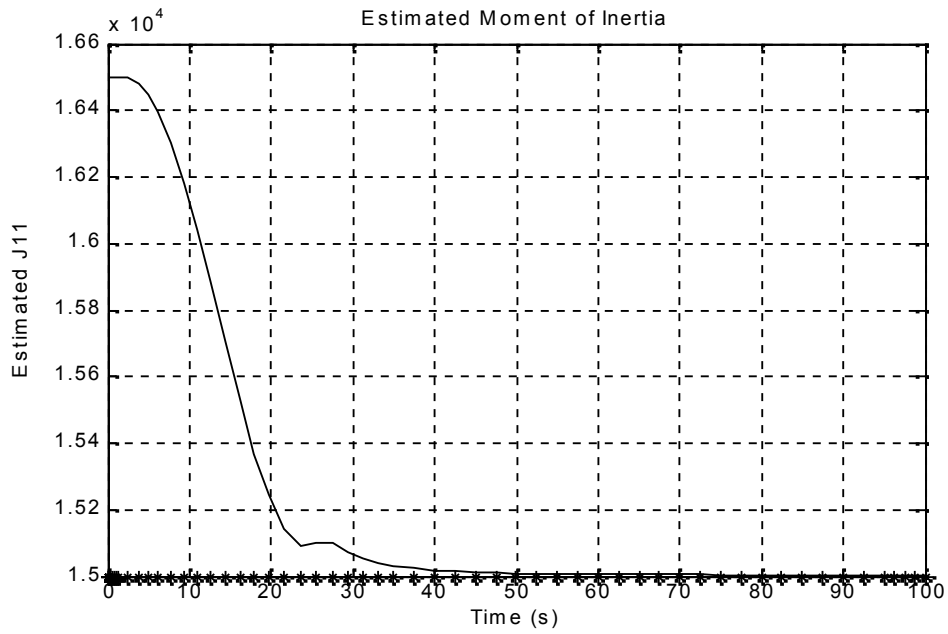


Figure13: Desired (\*) and Actual (-) Estimated Moment of Inertia ( $J_{xx}$ ,  $\text{Kgm}^2$ )

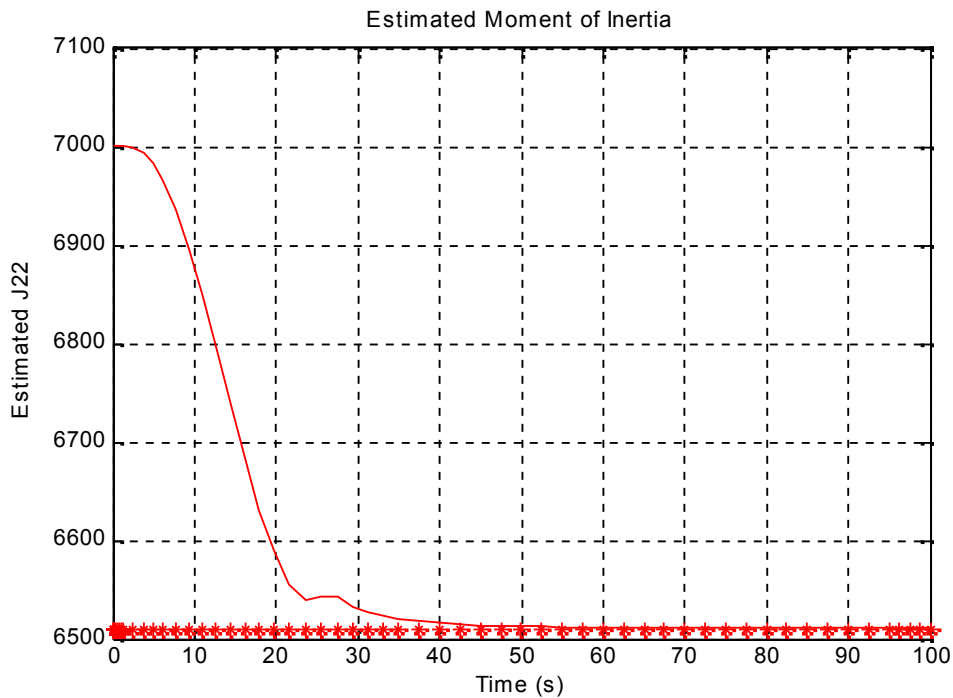


Figure 14: Desired (\*) and Actual (-) Estimated Moment of Inertia ( $J_{yy}$ ,  $\text{Kgm}^2$ )

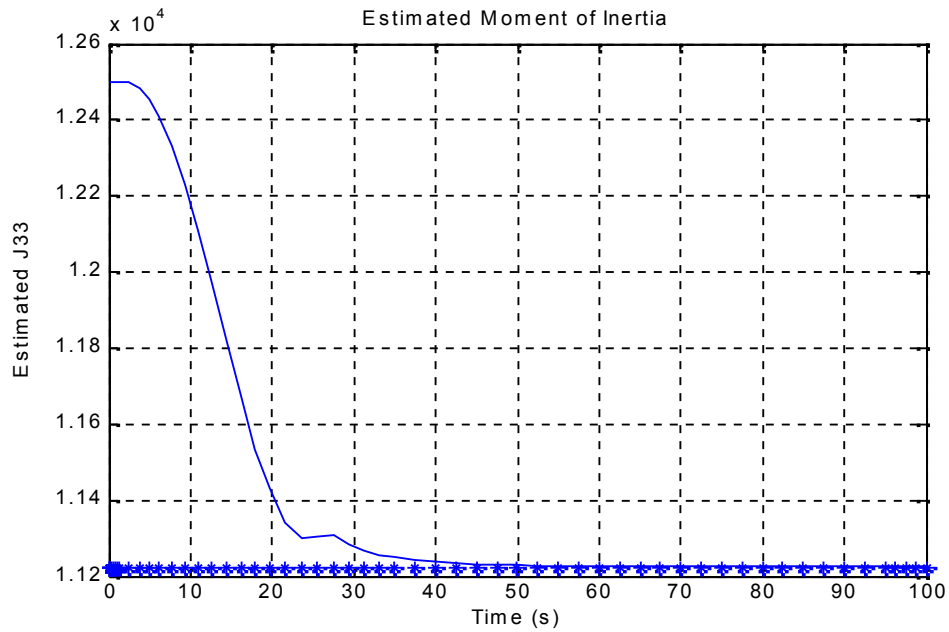


Figure15: Desired (\*) and Actual (-) Estimated Moment of Inertia ( $J_{zz}$ ,  $\text{Kgm}^2$ )

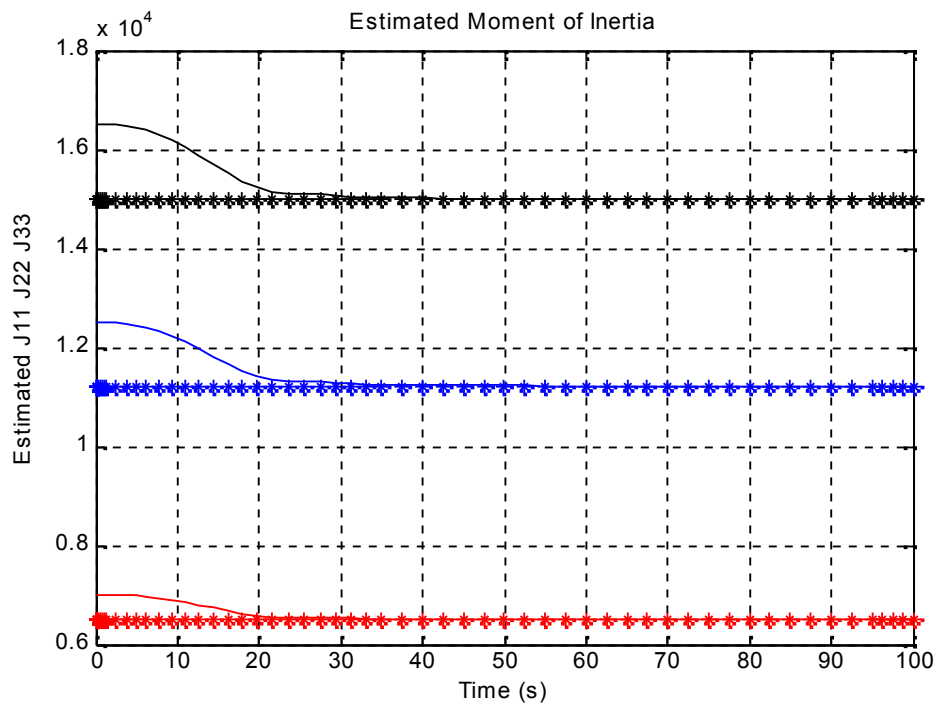


Figure 16 : Desired (\*) and Actual (-) Estimated Moment of Inertia

( $J_{xx}$ ,  $J_{yy}$ ,  $J_{zz}$ ,  $\text{Kgm}^2$ )

Created with

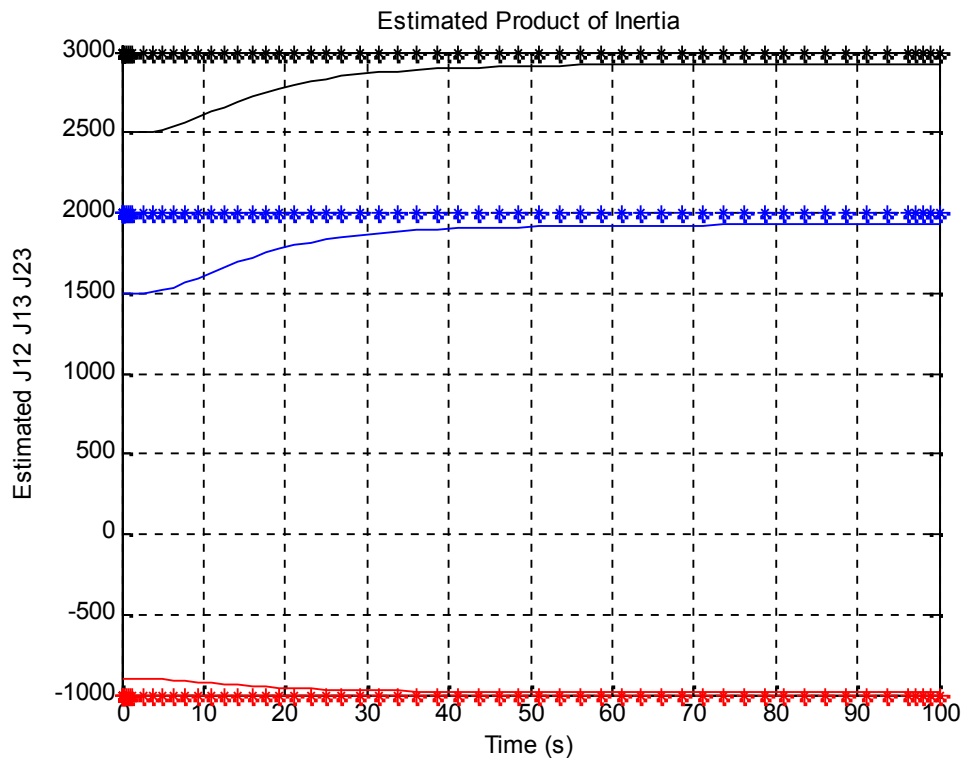


Figure 17: Desired (\*) and Actual (-) Estimated Product of Inertia

$$(J_{xy}, J_{xz}, J_{yz}, \text{Kgm}^2)$$

## 4.2 SIMULATION RESULTS ANALYSIS

In Fig 2, the black line represents the angular velocity ( $\omega_1$ ) of satellite in its x – axis and red and blue represents the angular velocities ( $\omega_2$  and  $\omega_3$ ) in its y and z axes of body references frame respectively. These plots shows that during the process of simultaneous adaptive attitude and power tracking, the desired angular velocities have

been achieved in 160 seconds, so this SAAPT-VSCMG controller algorithm ensures stability of attitude maneuver of satellite. Fig 3 shows the attitude tracking trajectory with Modified Rodriguez Parameters (MRPs). In Fig2, the desired satellite attitude tracking have been achieved with help of modified Rodriguez parameters. The black line in Fig.3 represents the first MRP parameter ( $\sigma_1$ ) and this first parameter is exactly tracked the desired attitude tracking trajectory i.e ( $\sigma_{d1}=1$ ) in 160 seconds. Similarly in Fig 3, red and blue lines represent the second and third modified Rodriguez parameters ( $\sigma_2$  and  $\sigma_3$ ) respectively .These parameters also tracked precisely their desired MRPs values i.e. ( $\sigma_{d2}=0$  and  $\sigma_{d3}=0$ ) in 160 seconds. These results show SAAPT-VSCMG algorithm guarantees perfect adaptive attitude tracking with power tracking. Power tracking plot has been shown in Fig 4. In these plots, symbol (\*) represents actual power and (--) represents desired power tracking. In this plot, actual power is successfully tracked the desired power profile during the simultaneous adaptive attitude with power tracking using VSCMG'

Four variable speed control moment gyroscopes have been used in pyramid configuration in this SAAPT-VSCMG controller. In this thesis, there are two scenarios related to gimbal angles. First scenario represents gimbal angles of VSCMGs without wheel speed equalization in Fig 5. In Fig.5 the gimbal angle of first VSCMG changes from 45 degree to 35 degree. The gimbal angle of second VSCMG changes from - 45 degree to -58 degree and gimbal angles of third and fourth VSCMGs have been changed from -45 degree to -36 degree and 45 degree to 20 degree respectively .These gimbal angles result reflects that this SAAPT-VSCMG controller algorithm effectively avoid the singularities problems. Second scenario represents gimbal angles of VSCMGs with wheel speed equalizati

the gimbal angle of first VSCMG changes from 45 degree to 30 degree instead of 35 as in Fig 5. The gimbal angle of second VSCMG changes from - 45 degree to -70 degree instead of -58 as in Fig 5 and gimbal angles of third and fourth VSCMGs have been changed from -45 degree to -30 degree instead of -36 in Fig 5 and 45 degree to 20 degree .These gimbal angles results reflect that this SAAPT-VSCMG controller algorithm effectively avoids the singularities problem .Singularities issues successfully avoid in both cases.

Gimbal rates of four VSCMGs are shown in Fig.7. This figure shows first all gimbal rates of VSCMGs change to a certain degrees and then all gimbal rates stabilize to their initial values i.e. zero degree. Fig 8 represents the four wheel speeds of VSCMGs without wheel speed equalization techniques. In this SAAPT-VSCMG algorithm, we consider initially very high variable wheel speed values for simultaneous attitude and power tracking. In this figure the variable speed nature of four flywheels have been observed in which energy is stored in the form of kinetic energies.

Higher and lower parts of curves in these four plots represent the high wheel speed with higher kinetic energy and low wheel speed with lower kinetic energy. Fig 9 represents the wheel speeds of VSCMGs with wheel speed equalization techniques because sometimes during the process of simultaneous attitude and power tracking, the following two problems may occur. In first problem, the wheel spin rates become too large as a result wheel speed rates go into saturation and in second problem wheel spin rates become too low then required tracking torque may not be generated by the change of gimbal angle. In Fig 9, this SAAPT-VSCMG controller

algorithm has successfully applied with wheel speed equalization techniques in these wheel speed simulation plots. So by using the wheel speed equalization techniques all wheel speeds of VSCMGs have converged into a common (single) value 57000 RPM irrespective of their different initial values. Four wheel speed rates without and with wheel equalization have been plotted in Fig.10 and Fig11 respectively. These four wheel speeds rates have changed up to certain ranges of wheel speed and then come back quickly to zero wheel speed value. These four wheel speed rates vary according to power requirement of satellites. In Fig12 condition number of matrix C has been plotted which reflects there is no singularities problem in this controller.

Adaptive attitude tracking with uncertain moment of inertia plots has been given in Fig.13, Fig14 and Fig.15. These figures show that estimated moment of inertias values have been converged successfully into its actual moment of inertia values. In Fig13 estimated moment of inertia value  $16000 \text{ Kg m}^2$  is converted into its original moment of inertia values in 60 seconds. Similarly in Fig.14 and Fig.15, the estimated moment of inertia values are  $7000 \text{ Kg m}^2$  and  $12000 \text{ Kg m}^2$  have also been converted into its original moment of inertia values in 60seconds. Fig 17 show that estimated product of inertia values have been converged successfully into its actual product of inertias values with slight difference.

## CHAPTER 5

### CONCLUSIONS AND FUTURE RECOMMENDATIONS

#### 5.1 CONCLUSIONS

This thesis has introduced SAAPT (Simultaneous Adaptive Attitude and Power Tracking) –VSCMG system controller for satellites operations. We used variable speed control moment gyroscopes as actuator in pyramid configuration in SAAPT-VSCMG algorithm.

In third chapter theoretical development of SAAPT-VSCMG controller system has been achieved. The gimbal rates and wheel speed rates have been used as control inputs. In SAAPT-VSCMG algorithm the required attitude tracking torque has been achieved with the help of gimbal rates of VSCMGs and power tracking has been accomplished with application of wheel speed rates of VSCMGs. The variable nature of wheels speed produced variable kinetic energies in their wheels. These variable kinetic energies in wheels have been used for energy storage or power tracking purposes as mechanical batteries in satellites. Adaptive attitude control laws with uncertain element of inertia matrix of satellite have been derived. Wheel speed equalization techniques have been developed for adaptive attitude tracking to achieve average (common) wheels speed of VSCMGs. By applying wheel



techniques, wheels of VSCMGs have been achieved average kinetic energies during the simultaneous adaptive attitude and power tracking of satellite and as a result, we can successfully avoid singularities problems in adaptive attitude tracking and saturation problems in wheels speeds of VSCMGs.

The fourth chapter is represented simulation results and their analysis. The SAAPT-VSCMG controller algorithm has been applied successfully to achieve precise simulation results for simultaneous adaptive attitude and power tracking with wheel speed equalization. In this control algorithm simulation results largely depend on initial values of gimbal angles of actuator and initial wheels speed values of variable speed control moment gyroscopes.

## 5.2 FUTURE RECOMMENDATIONS

In this section, we will present some recommendations for future research. These recommendations are mainly divided into following areas.

Firstly, in this thesis simulation results of control laws have been achieved by applying complete equation of motion of rigid satellites. We design SAAPT-VSCMG control algorithm for large satellites with slow maneuvering by using nonlinear equation of motion for VSCMGs system. In future, the researchers can contribute in the development of control algorithm for large satellites with fast maneuvering by using nonlinear equation of motion for VSCMGs. Several scientists have been used fixed speed control moment gyroscopes and variable speed control moment gyroscopes for small satellite maneuverings [7,8,9] . In order to achieve fast

maneuvering with large satellites, we must consider satellites dynamics. In the process of controller designing of large satellite with fast maneuvering capabilities, the application of nonlinear equation of motion will become more difficult. As a consequence, the singularities avoidance techniques turn out to be further complex.

Secondly, we used Matlab m files codes for simulations results in this thesis. In future these Matlab m files codes can be converted into C language code. By the conversions of Matlab m files controller codes to C language controller codes then controller simulations results will become more fast and precise and will be used for real time application with DSP tools.

Thirdly, we used four VSCMGs in pyramid configuration for the implementations of SAAPT-VSCMG controller's algorithm. In future one should explore that how minimum number of variable speed control moment gyroscopes will be sufficient for the implementations of SAAPT-VSCMG controller algorithm with most suitable VSCMGs configurations.

Fourthly, we used adaptive nonlinear controller laws for attitude and power tracking of satellites. Some other nonlinear controller techniques for example Robust controller and Neural Network controller can also be used in future for designing of SAAPT-VSCMG controller algorithm. By applying above-mentioned nonlinear controller techniques then we get more valuable insight information for the best performance of controller system and as a outcome, we will achieve better simulation results.

Fifthly, we have designed SAAPT-VSCMG controller algorithm for rigid satellite with unknown element of inertia matrix of satellite. This controller algorithm can be used for non-rigid (flexible) satellite with slight variations in future research. Controller algorithm with flexible satellites dynamics give more practically applicable and highly accurate simulations results for simultaneous adaptive attitude tracking and power tracking of satellites.

Created with



## RERFERNCES

- [1] Ahmed, J. and Bernstein, D., “Adaptive control of a dual-axis CMG with an unbalanced rotor,” in Proceedings of the 37th IEEE Conference on Decision and Control, (Tampa, FL), pp. 4531–4536, Dec.1992.
- [2] Ahmed, J., Coppola, V., and Bernstein, D., “Adaptive asymptotic tracking of spacecraft attitude motion with inertia matrix identification,” Journal of Guidance, Control, and Dynamics, vol. 21, no. 5, pp. 684–691, 1998.
- [3] Bishop, R., Paynter, S., and Sunkel, J., “Adaptive control of space station with control moment gyros,” in IEEE Control Systems Magazine, vol. 12, pp. 23–28, Oct.1992.
- [4] Ford, K. A. and Hall, C. D., “Flexible spacecraft reorientations using gimballed momentum wheels,” in Advances in the Astronautical Sciences, Astrodynamics(Hoots, F., Kaufman, B., Cefola, P. J., and Spencer, D. B., eds.), vol. 97, Univelt, San Diego), pp. 1895–1914, 1997.
- [5] Ford, K. A. and Hall, C. D., “Singular direction avoidance steering for control moment gyros,” Journal of Guidance, Control, and Dynamics, vol. 23, no. 4, pp. 648–656, 2000.

- [6] Junkins, J. L. and Kim, Y., Introduction to Dynamics and Control of Flexible Structures, pp. 48–49. New York: AIAA, 1993.
- [7] Kailath, T., Linear Systems, pp. 135–139. Englewood Cliffs, New Jersey: Prentice-Hall, 1980.
- [8] Lappas, V. J., Steyn, W. H., and Underwood, C. I., “Practical results on the development of a control moment gyro based attitude control system for agile small satellites,” in Sixteenth Annual AIAA/USU Conference on Small Satellites, (Logan, UT), Utah State University, Aug., 2002.
- [9] Lappas, V. J. and Underwood, C. I., “Experimental testing of a CMG cluster for agile micro satellites,” in 54th International Astronautical Congress of the International Astronautical Federation (IAF), (Bremen, Germany), Sep. - Oct., 2003.
- [10] Schaub, H., Akella, M. R., and Junkins, J. L., “Adaptive realization of linear closed-loop tracking dynamics in the presence of large system model errors,” Journal of the Astronautical Sciences, vol. 48, pp. 537–551, Oct.-Dec. 2000
- [11] Schaub, H., Vadali, S. R., and Junkins, J. L., “Feedback control law for variable speed control moment gyroscopes,” Journal of the Astronautical Sciences, vol. 46, no. 3, pp. 307–328, 1998.
- [12] Sheen, J. and Bishop, R., “Adaptive nonlinear control of spacecraft,” in American Control Conference, (Baltimore, Maryland), pp. 2867–2871, 1994.

- [13] Slotine, J. and Li, W., *Applied Nonlinear Control*. New Jersey: Prentice Hall, 1991.
- [14] Tsiotras, P., “Stabilization and optimality results for the attitude control problem,” *Journal of Guidance, Control, and Dynamics*, vol. 19, no. 4, pp. 772–779, 1996.
- [15] Tsiotras, P., Shen, H., and Hall, C., “Satellite attitude control and power tracking with energy/momentum wheels,” *Journal of Guidance, Control, and Dynamics*, vol. 24, no. 1, pp. 23–34, 2001.
- [16] Zaremba, A., “An adaptive scheme with parameter identification for spacecraft attitude control,” in *Proceedings of American Control Conference*, pp. 552–556, 1997.
- [17] Richie, D. J., Tsiotras, P., and Fausz, J. L., “Simultaneous attitude control and energy storage using VSCMGs: Theory and simulation,” in *Proceedings of American Control Conference*, pp. 3973–3979, 2001.
- [18] Junkins, J. L. and Turner, J., *Optimal Spacecraft Rotational Maneuvers*. New York: Elsevier, 1986.
- [19] Tsiotras, P., Shen, H., and Hall, C., “Satellite attitude control and power tracking with energy/momentum wheels,” *Journal of Guidance, Control, and Dynamics*, vol. 24, no. 1, pp. 23–34, 2001.

- [20] Shen, H., and Hall, C., "Satellite attitude control and power tracking with momentum wheels," *Journal of Guidance, Control, and Dynamics*, 1999.
- [21] Ford, K. A. and Hall, C. D., "Flexible spacecraft reorientations using gimballed momentum wheels," in *Advances in the Astronautical Sciences, Astrodynamics* vol. 97, 1997.
- [22] M. Kaplan. *Modern Spacecraft Dynamics and Control*. John Wiley and Sons, New York, 1976.
- [23] Wie, B., *Space Vehicle Dynamics and Control*. Reston, VA: American Institute of Aeronautics and Astronautics, Inc., 1998.
- [24] D. Davis and A. Csomor The new age of high performance kinetic energy storage systems. In proceeding of the 15<sup>th</sup> Intersociety Energy Conversion Engineering Conference, volume2, 1980.
- [25] J.A.Kirk. Flywheel energy storage part I: Basic concept *International Journal of Mechanical Science* 1977.
- [26] J.A.Kirk and P. A .Studer. Flywheel energy storage part ii: Magnetically suspended super fly wheel. *International Journal of Mechanical Science*, 1977.
- [27] Roes, J. B., "An electro-mechanical energy storage system for space application," in *Progress in Astronautics and Rocketry*, vol. 3, pp. 613–622, Academic Press, 1964.

- [28] Anderson, W. and Keckler, C., “An integrated power/attitude control system(IPACS) for space application,” in Proceedings of the 5th IFAC Symposium on Automatic Control in Space, (New York), pp. 81–82, Pergamon, 1973.
- [29] Cormack III, A., “Three axis flywheel energy and control systems,” tech. rep., NASA Technical Report TN-73-G&C-8, North American Rockwell Corp., 1973.
- [30] C.R. Keckler and K.L. Jacobs. A spacecraft integrated power/attitude control system. In 9<sup>th</sup> Intersociety Energy conversion Engineering Conference, 1974.
- [31] Notti, J., Cormack III, A., Schmill, W., and Klein, W., “Integrated power/attitude control system (IPACS) study : Volume ii - conceptual designs,” tech. rep., NASA Technical Report CR-2384, Rockwell International Space Division, Downey, CA, 1974.
- [32] R.W. Will, C .r. Keckler, and K. L. Jacobs. Description and simulation of an integrated and power and attitude control system concept for space vehicle application. Technical Report NASA, 1974.
- [33] D. Anand, J. A .Kirk and D.A. Frommer. Design considerations for magnetically suspended flywheel systems. In Proceedings of the 20<sup>th</sup> Intersociety Energy Conversion Engineering Conference, volume2,1985.



- [34] J. Dower, D. Eisenhaure, R. Hockney, B. Johnson, and S. O'Dea. Magnetic suspension design options for satellite attitude control and energy storage. In Proceedings of the 20th Intersociety Engineering Conference, 1985.
- [35] D.B. Eisenhaure, J.R. Dower, T.E. Bliamptis and S.D. Hendrie. A low authority control law for under actuated rigid spacecraft. In 22<sup>nd</sup> Aerospace Sciences Meeting, 1984.
- [36] T. Flately. Tetrahedron array of reaction wheels for attitude control and energy storage. In Proceedings of the 20<sup>th</sup> Intersociety Energy Conversion Engineering Conference, volume 2, 1985.
- [37] S. O'Dea, P. Burdick, J. Downer, D. Eisenhaure, and L. Larkin, Design and development of high efficiency effectors for the control of attitude and power in space systems. Proceedings of the 20<sup>th</sup> Inter society Energy Conversion Engineering Conference, volume 3, 1986.
- [38] R. E Oglevie and D. B Eisenhaure. Integrated power and attitude control system (IPACS) technology. In Proceedings of the 21<sup>st</sup> Intersociety Energy Conversion Engineering Conference, volume 3, 1986.
- [39] D.R. Olmsted. Feasibility of flywheel energy storage in spacecraft application, In Proceedings of the 20<sup>th</sup> Intersociety Energy Conversion Engineering Conference, volume 2, 1985.

- [40] M. Olszewski and D.U, O'kain. Advances in flywheel technology for space power applications. In Proceedings of the 20<sup>th</sup> Intersociety Energy Conversion Engineering Conference, volume 3, 1986.
- [41] W.E Simon and K.E. Van Tassel. Inertial energy storage for advanced space station applications. In Proceedings of the 20<sup>th</sup> Intersociety Energy Conversion Engineering Conference, volume 2, 1985.
- [42] P.A Studer and G.E Rodriguez. High speed reaction wheel for satellite attitude control and energy storage. In Proceedings of the 20<sup>th</sup> Intersociety Energy Conversion Engineering Conference, volume 2, 1985.
- [43] M. Ahrens,L. Kucera and R. Larssonneur. Performance of magnetically suspended flywheel energy storage device. IEEE Transactions on control system teachnology, 1996.
- [44] D.T. Radzykewycz, J.L Fausz and W.R James. Energy storage technology development at the air force research laboratory space vehicle directorate. In Proceedings of the 1999 space technology conference and exposition, 1999.
- [45] A. D. Jacot and D. Liska. Control moment gyros in attitude control. Journal of spacecraft and rockets, 1966.
- [46] G. Margulies and J.N Aubrun. Geometric theory of single gimbal control moment gyro systems. Journal of Astronautically Sciences, 1978.

- [47] Oh, H. and Vadali, S., "Feedback control and steering laws for spacecraft using single gimbal control moment gyro," The Journal of the Astronautical Sciences, vol. 39, no. 2, pp. 183–203, 1991.
- [48] Bedrossian, N. S., Paradiso, J., Bergmann, E. V., and Rowell, D., "Redundant single gimbal control moment gyroscope singularity analysis," Journal of Guidance, Control, and Dynamics, vol. 13, no. 6, pp. 1096–1101, 1990.
- [49] B. R. Hoelscher and S. R. Vadali. Optimal open loop and feedback control using single gimbal control moment gyroscopes. Journal of Astronautical Sciences, 1994.
- [50] S.R. vadali and S. Krishnan. Suboptimal command generation control moment gyroscopes and feedback control of spacecraft. Journal of Guidance, control, and Dynamics. 1995
- [51] Vadali, S. R., Oh, H. S., and Walker, S. R., "Preferred gimbal angles for single gimbal control moment gyros," Journal of Guidance Control, and Dynamics, vol. 13 ,no. 6, pp. 1090–1095, 1990.
- [52] K. A. Ford and C.D Hall. Singular direction avoidance steering for control moment gyros. AIAA Journal of Guidance, control and dynamics, 2000

- [53] Schaub, H., Novel Coordinates for Nonlinear Multibody Motion with Application to Spacecraft Dynamics and Control. PhD thesis, Aerospace Engineering Department, Texas A&M University, May 1998
- [54] H. Schaub and J. L Junkins. Singularity avoidance using null motion and variable speed control moment gyros .AIAA Journal of Guidance control and Dynamics.2000.
- [55] H. Schaub ,S.R Vadali and J.L Junkins, Feedback control law for variable speed control moment gyros. Journal of the Astronautical sciences, 1998.
- [56] D. Richie, P. Tsiostras and J Fausz. Simultaneous attitude control and energy storage using VSCMGs theory and simulation. In Proc. American control conference, 2001.
- [57] T. R Kane and D.A Levinson. Dynamics theory and applications. McGraw Hill, 1985.
- [58] R.A Horn and C.R Johnson., Matrix analysis. Cambridge university press, Cambridge, United /kingdom, 1985.
- [59] A. Brain and M. Breiner .MATLAB 5 for engineers, Addison Wesley.1999.
- [60] C. Hall, P. Tsiostras and H. Shen. Tracking rigid body motion using thrusters and reaction wheels, In AIAA / AAS Astrodynamics Spacecraft Conference, Boston, 1998.

- [61] Schaub, H. and Junkins, J. L., “Singularity avoidance using null motion and variable-speed control moment gyros,” *Journal of Guidance Control, and Dynamics*, vol. 23, no. 1, pp. 11–16, 2000.
- [62] Hyunjoo Yoon and Panagiotis Tsiotras, *Spacecraft Adaptive Attitude and Power Tracking with Variable Speed Control Moment Gyroscopes* *Journal of Guidance, Control, and dynamics* Vol. 25, No. 6, November–December 2002
- [63] Mason A. Peck, *Control-Moment Gyroscopes for Joint Actuation: A New Paradigm in Space Robotics*, 1st Space Exploration Conference: Continuing the Voyage of Discovery 30 January - 1 February 2005, Orlando, Florida
- [64] Michele D. Carpenter, *Power-Optimal Steering of a Space Robotic System Driven by Control-Moment Gyroscopes*, AIAA Guidance, Navigation and Control Conference and Exhibit 18 - 21 August 2008, Honolulu, Hawaii
- [65] Frederick A. Leve, *Optimization in Choosing Gimbal Axis Orientations of a CMG Attitude Control System*, AIAA InfoTech @ Aerospace Conference, 6 - 9 April 2009, Seattle, Washington.
- [66] Jin JIN, Jingrui ZHANG and Zaozhen LIU, *Output-torque Error Analysis and Steering Law Design of SGCMGs Based on SVD Theory*, AIAA Guidance, Navigation, and Control Conference 10 - 13 August 2009, Chicago, Illinois.

- [67] Laura L. Jones and Mason, A. Peck A Generalized Framework for Linearly-Constrained Singularity-Free Control Moment Gyro Steering Laws, AIAA Guidance, Navigation, and Control Conference 10 - 13 August 2009, Chicago, Illinois.
- [68] Frederick A. Leve and Norman G. Fitz-Coy, Hybrid Steering Logic for Single-Gimbal Control Moment Gyroscopes, Journal of Guidance, Control, and Dynamics Vol. 33, No. 4, July–August 2010.