Some Extensions of Q-Fuzzy Soft Sets

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Certificate

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A DISSERTATION SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF THE ***MASTER OF SCIENCE in MATHEMATICS***

We accept this dissertation as conforming to the required standard.

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Forwarding Sheet by Research Supervisor

The thesis entitled "Some Extensions of Q-Fuzzy Soft Sets" submitted by Mohsin Ali Khan, Registration No. 172-FBAS/MSMA/S14 in partial fulfillment of MS Degree in Mathematics has been completed under my guidance and supervision. I am satisfied with the quality of his research work and allow him to submit this thesis for further process to graduate with Master of Science Degree from the Department of Mathematics & Statistics, as per IIUI rules and regulations.

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DECLARATION

I hereby, declare that this thesis neither as a whole nor as apart thereof has been copied out from any source. It is further declared that I have prepared this thesis entirely on the basis of my personal efforts made under the sincere guidance of my kind supervisor. No portion of the work, presented in this thesis, has been submitted in the support of any application for any degree or qualification of this or any other institute of learning.

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Dedicated
To
My Family,
My
Loving parents,
My friends
And respectful teachers

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All praises to almighty **"ALLAH"** the creator of the universe, who blessed me with the knowledge and enabled me to complete the dissertation. All respects to **Holy Prophet MUHAMMAD (PBUH),** who is the last messenger, whose life is a perfect model for the whole humanity.

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Mohsin Ali Khan

Contents

Preface

Zadeh introduced the idea of fuzzy sets [18] in 1965, whose basic component is only a degree of membership for dealing with uncertainty but the problem that rose in fuzzy is that sometimes it is harder to define a membership grade for the elements of a set. To overcome this problem L. A. Zadeh, made an extension of the concept of a fuzzy set by an interval-valued fuzzy set (IVFS) [19], i.e., a fuzzy set with an interval-valued membership function. In traditional fuzzy logic, to represent, e.g., the expert's degree of certainty in different statements, numbers from the interval [0, 1] is used. It is often difficult for an expert to exactly quantify his or her certainty; therefore, instead of real number, it is more adequate to represent this degree of certainty by an interval or even by a fuzzy set. Interval-valued fuzzy sets have been actively used in real life applications, for example, in medical diagnosis, in thyroidian pathology, also in medicine etc.

The concept of Intuitionistic fuzzy sets (IFS) was introduced in 1986 by K. T. Atanassov [7], which is the generalization of fuzzy sets. Whose basic components are grade of membership and grade of non-membership, with condition that sum of both grade of membership and grade of non-membership do not exceed one.

In 1999 Molodtsov [14] firstly offered the concept of soft set. This kind of theory is a global mathematical tool to handle uncertain, fuzzy, not clearly defined objects. But this theory is different from conventional devices to deal with uncertainties, such as the idea of probability, fuzzy sets, IFS. It has been proved that this theory gives us enough capability for applications in different domains, i.e. measurement theory, Riemann integration, game theory, function smoothness, etc.

F. Adam and N. Hassan [1, 2] presented the idea of multi Q-fuzzy sets, multi Q-parameterized soft sets and defined some basic properties and operation such as complement, equality, union, intersection. F. Adam and N. Hassan also introduced Q-fuzzy soft set [3], defined some basic definitions and gave some basic operations. S. Broumi [8] established the notion of Q- intuitionistic fuzzy set (Q-IFS) by combining Q-fuzzy sets and intuitionistic fuzzy sets, Q-intuitionistic fuzzy soft set (Q-IFSS) by the combination of Q-intuitionistic fuzzy sets and soft sets. He defined some basic properties with illustrative examples. Broumi also defined some basic operation for Q-IFS and Q-IFSS [9].

The concept of neutrosophic set was introduced by Smarandache in 1999 [15]. That refers to all of the classical sets, conventional fuzzy set, IFS and IVFS. Neutrosophic set is an essential part of neutrosophy which deals with nature and scope of neutralities, along with their interactions with various notions. Neutrosophic set handle indeterminate data whereas fuzzy set theory, and IFS theory failed when the relation are indeterminate. Wang et al. [17] established another enhancement of neutrosophic set which is Single Valued Neutrosophic set (SVNS). Also Wang et al. [16] presented the idea of interval neutrosophic sets (INSs). It is characterized by an interval membership degree, interval indeterminacy degree and interval non-membership degree.

In this thesis we have introduced the concept of Q-SVNS and defined operations such as union, intersection, complement, etc. We also defined Multi Q-SVNS, and Q-SVNSS its operations, and proved some results. Also we have presented the concept of Q-INS, Multi Q-INS, Q-INSS and gave some properties and proved important results.

Structure of Thesis

Chapter 1:

This chapter is introductory. We have taken definitions from different papers and books that will help us in later chapters, It overviews soft sets, Q-fuzzy soft sets, SVNSs, and INSs.

Chapter 2:

This chapter contains the concept of Q-SVNS, multi Q-SVNS, some basic results and related properties. The idea of Q-SVNSS is defined as well. Some properties and operations of Q-SVNSS are also discussed.

Chapter 3:

This chapter contains the concept of Q-INS, multi Q-INS, some basic results and related properties.The idea of Q- INSS is defined as well. We also defined some properties and operations of Q- INSS.

Chapter 1

Preliminaries

In this chapter we recall some basic definitions and notions. These definitions will help us in later chapters. For undefined terms and notions we refer to ([1], [2], [3], [14], [16], [17]).

1.1. Soft Sets

In this section we recall soft sets, their basic operations and results.

1.1.1. Definition[14]

Let X be a universal set, E be the set of parameters and $A \subset E$. A pair (F, A) is called a soft set over X, where F is a Mapping given by

$$F: A \to P(X)$$

where $P(X)$ denotes power set of X.

1.1.2. Definition[12]

For two soft sets (F, A) and (G, B) over a common universe X, we say that (F, A) is a soft subset of (G, B) if

i. $A \subset B$

ii. $\forall \epsilon \in A,$ $F(\epsilon) and$ $G(\epsilon)$ are identical approximations.

We write, $(F, A) \subset (G, B)$.

1.1.3. Definition[12]

Two soft sets (F, A) *and* (G, B) over a common universe X are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

1.1.4. Definition[12]

Let $E = \{e_1, e_2, \ldots, e_n\}$ be a set of parameters. The NOT set of E denoted by $\rceil E$ is defined by $\rceil E = \{\neg e_1, \neg e_2, \ldots, \neg e_n\}$

where $\neg e_i =$ not e_i, $\forall\, i$

1.1.5. Definition[12]

The complement of a soft set (F, A) is denoted by $(F, A)^c = (F^c, \rceil A)$, where

F^c: $\rceil A \to P\,(X)$ is a mapping given by

$F^c\,(\alpha)$: $X - F(\rceil\, \alpha), \forall\, \alpha \in \rceil A$.

Let us denote F^c to be the soft complement function of F. clearly $(\,F^c)^c$ is the same as F and $((\,F, A)^c)^c = (F, A)$.

1.1.6. Definition[12]

A soft set (F, A) over X is said to be a Null soft set denoted by $\emptyset$, if $\forall\, \epsilon \in A$,

$F(\epsilon) = \emptyset\ (null\ set)$.

1.1.7. Definition[12]

A soft set (F, A)over X is said to be absolute soft set denoted by $\check{A}$, if $\forall\, \epsilon \in A$,

$F(\epsilon) = X$

Clearly $\check{A}^c = \emptyset$ and $\emptyset^c = \check{A}$.

1.1.8. Definition[12]

If (F, A) and (G, B) are two soft sets, then "(F, A) AND (G, B)" denoted by $(F, A) \wedge (G, B)$ is defined by $(F, A) \wedge (G, B) = (H, A \times B)$, where

$H(\propto, \beta) = F(\propto) \cap G(\beta), \forall (\propto, \beta) \in A \times B.$

1.1.9. Definition[12]

If (F, A) and (G, B) are two soft sets then "(F, A) OR (G, B) "denoted by $(F, A) \vee (G, B)$ is defined by $(F, A) \vee (G, B) = (O, A \times B)$, where

$O(\propto, \beta) = F(\propto) \cup G(\beta), \forall (\propto, \beta) \in A \times B.$

1.1.10. Definition[12]

Union of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ and $\forall e \in C$,

$$H(e) = \begin{cases} F(e) & if\ e \in A - B \\ G(e) & if\ e \in B - A \\ F(e) \cup G(e) & if\ e \in A \cap B \end{cases}$$

we write $(F, A) \cup (G, B) = (H, C)$.

1.1.11. Definition[12]

Intersection of two soft sets (F, A) and (G, B) over a common universe X is the soft set (H, C), $where$ $C = A \cap B$ and $\forall e \in C, H(e) = F(e) \cap G(e)$

we write $(F, A) \cap (G, B) = (H, C)$.

1.1.12. Proposition[12]

Let (F, A) be the soft set over X, then

i. $(F,A) \cup (F,A) = (F,A)$

ii. $(F,A) \cap (F,A) = (F,A)$

iii. $(F,A) \cup \emptyset = (F,A)$ *where* $\emptyset$ *is the null soft set*

iv. $(F,A) \cap \emptyset = \emptyset$

v. $(F,A) \cup \tilde{A} = \tilde{A}$ *where* $\tilde{A}$ *is the absolute soft set*

vi. $(F,A) \cap \tilde{A} = (F,A)$

1.1.13. Proposition[12]

If $(F, A), (G, B)$ and (H, C) are three soft sets over X, then

i. $(F,A) \cup \big((G,B) \cup (H,C)\big) = ((F,A) \cup (G,B)) \cup (H,C)$

ii. $(F,A) \cap \big((G,B) \cap (H,C)\big) = ((F,A) \cap (G,B)) \cap (H,C)$

iii. $(F,A) \cup \big((G,B) \cap (H,C)\big) = ((F,A) \cup (G,B)) \cap ((F,A) \cup (H,C))$

iv. $(F,A) \cap \big((G,B) \cup (H,C)\big) = \big((F,A) \cap (G,B)\big) \cup \big((F,A) \cap (H,C)\big)$

1.2. Q-fuzzy Soft Sets

In this section we recall multi Q-fuzzy set, Q-fuzzy soft sets and their basic operations along with results.

1.2.1. Definition[1]

Let I be a unit interval $[0,1]$, k be a positive integer, X be universal set and Q be a non-empty set. A multi Q-fuzzy set $\tilde{A}_Q$ in X and Q is a set of ordered sequences, $\tilde{A}_Q$=$\{(u,q), \mu_i(u,q): u \in X, q \in Q\}$, where $\mu_i: X \times Q \longrightarrow I^k$. The function

$\mu_1(u,q)$, $\mu_2(u,q)$,…, μ_k (u,q) is called the membership function of multi Q-fuzzy set $\tilde{A}_Q$ and $\mu_1(u,q)$+ μ_2 (u,q) +...+ μ_k $(u,q)) \leq 1$, k is called the dimension of $\tilde{A}_Q$. The set of all multi Q-fuzzy sets of dimension k in X and Q is denoted by $M^kQF(X)$.

1.2.2. Definition[2]

Let X be a universal set, E be a set of parameters, and Q be a non-empty set. Let $M^KQF(X)$ denote the power set of all multi Q-fuzzy subsets of X with dimension $k = 1$. Let $A \subset E$. A pair (F_Q, A) is called a Q-fuzzy soft set (in short $QF - S$ set) over X, where F_Q is a mapping given by

$$F_Q: A \rightarrow M^KQF(X)$$

Here a Q-fuzzy soft set can be represented by the set of ordered pairs.

$$(F_Q, A) = \{(x \in X, F_Q(x) \in M^kQF(X))\}$$

Note that the set of all Q-fuzzy soft set over X will be denoted by $QFS(X)$.

1.2.3. Example

Let $X = \{u_1, u_2, u_3, u_4, u_5\}$ be a universal set, $Q = \{p, q, r\}$ be a non-empty set, and $E = \{e_1, e_2, e_3, e_4, e_5\}$ be a set of parameters.

If

$A = \{e_1, e_2, e_3\} \subset E$,

$F_Q(e_1) = \{((u_1, p), 0.4), ((u_1, q), 0.1), ((u_1, r), 0.8)\}$

$F_Q(e_2)$

$= \{((u_1, p), 0.3), ((u_1, q), 0.1), ((u_1, r), 0.1)), ((u_4, p), 0), ((u_4, q), 0.5), ((u_4, r), 0.7)\}$

$F_Q(e_3) = \{((u_1, p), 0.9), ((u_1, q), 0.3), ((u_1, r), 0.4)\}$,

then

$(F_Q, A) = \{(e_1, \{((u_1, p), 0.4), ((u_1, q), 0.1), ((u_1, r), 0.8),$

$(e_2, \{((u_1, p), 0.3), ((u_1, q), 0.1), ((u_1, r), 0.1), ((u_4, p), 0), ((u_4, q), 0.5), ((u_4, r), 0.7)$

,

$(e_3, \{((u_1, p), 0.9), ((u_1, q), 0.3), ((u_1, r), 0.4)\})\}$

is a $QF - S$ set.

1.2.4. Definition[2]

Let $(F_Q, A) \in QFS(X)$. If $F_Q(x) = \emptyset$ for all $x \in A$,

then

(F_Q, A) is called a null $QF - S$ set denoted by $(\emptyset, A)$.

1.2.5. Definition[2]

Let $(F_Q, A) \in QFS(X)$. If $F_Q(x) = X$ for all $x \in A$,

then

(F_Q, A) is called an absolute $QF - S$ set denoted by (X, A).

1.2.6. Definition[2]

Let $(F_Q, A), (H_Q, B) \in QFS(X)$. Then we say that (F_Q, A) is a $QF - S$ subset of (H_Q, B) denoted by $(F_Q, A) \subset (H_Q, B)$, if $A \subset B$ and $F_Q(x) \subset H_Q(x)$ for all $x \in A$.

1.2.7. Proposition[2]

Let $(F_Q, A), (H_Q, B) \in QFS(X)$. Then

i. $(F_Q, A) \subset (X, E)$

ii. $(\phi, A) \subset (F_Q, A)$

iii. $(F_Q, A) \subset (H_Q, B)$ and $(H_Q, B) \subset (G_Q, C)$ then $(F_Q, A) \subset (G_Q, C)$

1.2.8. Definition[2]

Let $(F_Q, A), (H_Q, B) \in QFS(X)$. Then (F_Q, A) *and* (H_Q, B) are equal, written as

$(F_Q, A) = (H_Q, B)$ if and only if

$(F_Q, A) \subset (H_Q, B)$ *and* $(H_Q, B) \subset (F_Q, A)$.

1.2.9. Proposition[2]

Let $(F_Q, A), (H_Q, B), (G_Q, C) \in QFS(X)$. If $(F_Q, A) = (H_Q, B)$ and $(H_Q, B) =$

(G_Q, C).

Then $(F_Q, A) = (G_Q, C)$.

1.2.10. Definition[2]

Let $(F_Q, A) \in QFS(X)$ Then, the complement of $QF - S$ set denoted by $(F_Q, A)^C$ is

defined by $(F_Q, A)^C = (F_Q^C, \rceil A)$ where

$$F_Q^C : , \rceil A \longrightarrow QF(X)$$

is the mapping given by $F_Q^C\ (e) =$ Q –fuzzy complement for every $e \in \rceil A$.

1.2.11. Proposition[2]

Let $(F_Q, A) \in QFS(X)$ Then

i. $((F_Q, A)^C)^C = (F_Q, A)$

ii. $(\phi, A)^C = (U, E)$

iii. $(U, E)^C = (\phi, A)$

1.3. Single Valued Neutrosophic Sets

In this section we recall SVNSs and operations.

1.3.1. Definition[17]

Let X be a space of points (objects), with a generic element in X denoted by x. A single valued neutrosophic set (SVNS) A in X is characterized by truth-membership function T_A, indeterminacy membership function I_A and falsity-membership function F_A.

For each point x in X, $T_A(x), I_A(x), F_A(x) \in [0,1]$.

1.3.2. Example[17]

Assume that $X = \{x_1, x_2, x_3\}$, where

x_1 is capability, x_2 is trustworthiness and x_3 is price.

The values of $x_1,\ x_2, and\ x_3$ are in $[0, 1]$.

They are obtained from the questionnaire of some domain experts, their option could be a degree of "good service", a degree of indeterminacy and a degree of "poor service". A is a SVNS of X defined by

$A =< 0.3,0.4,0.5 >/x_1 +< 0.5,0.2,0.3 >/x_2 +< 0.7.,0.2,0.2 >/x_3$

B is a SVNS of X defined by

$B =< 0.6,0.1,0.1 >/x_1 +< 0.3,0.2,0.6 >/x_2 +< 0.4,0.1,0.5 >/x_3$

1.3.3. Definition[17]

The complement of a SVNS A is denoted

by $c(A)$ and is defined by

$$T_{\overline{A}}(x) = F_A(x)$$

$$I_{\overline{A}}(x) = 1 - I_A(x)$$

$$F_{\overline{A}}(x) = T_A(x)$$

for all $x \ in \ X$.

1.3.4. Example

Let A be the SVNS defined in Example 1.3.2 then,

$$\overline{A} =< 0.5,0.6,0.3 >/x_1+< 0.3,0.8,0.5 >/x_2+< 0.2.,0.8,0.7 >/x_3$$

1.3.5. Definition[17]

A SVNS A is contained in the other SVNS $B, A \subset B$, if and only if

$$T_A(x) \leq T_B(x)$$

$$I_A(x) \geq I_B(x)$$

$$F_A(x) \geq F_B(x)$$

for all $x \ in \ X$.

For example, let A and B be the SVNSs defined in Example 1.3.2.

Then, A is not contained in B and B is not contained in A.

1.3.6. Definition[17]

Two SVNSs A and B are equal, written as $A = B$, if and only if $A \subset B$ and $B \subset A$.

1.3.7. Definition[17]

The union of two SVNSs A and B is a SVNS C, written as $C = A \cup B$, whose truth-membership, indeterminacy membership and falsity-membership functions are related to those of A and B by

$$T_C(x) = max(T_A(x), T_B(x))$$

$$I_C(x) = min(I_A(x), I_B(x))$$

$$F_C(x) = min\big(F_A(x), F_B(x)\big)$$

For all $x \; in \; X$.

1.3.8. Example

Let A and B be the SVNSs defined in Example 1.3.2.

Then, $A \cup B =< 0.6, 0.4, 0.2 >/x_1 + < 0.5, 0.2, 0.3 >/x_2 \; + \; \langle 0.7, 0.2, 0.2 >/x_3$.

1.3.9. Definition[17]

The intersection of two SVNSs A and B is a SVNS C, written as $C = A \cap B$, whose truth-membership, indeterminacy-membership and falsity-membership functions are related to those of A and B by

$$T_C(x) = min(T_A(x), T_B(x))$$

$$I_C(x) = max(I_A(x), I_B(x))$$

$$F_C(x) = max\big(F_A(x), F_B(x)\big)$$

$For\ all\ x\ in\ X.$

1.3.10. Example

Let A and B be the SVNSs defined in Example 1.3.2.

Then, $A \cap B = < 0.3, 0.1, 0.5 >/x_1 + < 0.3, 0.2, 0.6 >/x_2 + < 0.4, 0.1, 0.5 >/x_3$.

1.4. Interval Neutrosophic Sets

In this section we recall some basic definitions of INSs.

1.4.1. Definition[16]

Let X be a space of points (objects), with a generic element in X denoted by x. An interval neutrosophic set (INS) A in X is characterized by truth-membership function T_A, indeterminacy membership function I_A and falsity-membership function F_A. For each point x in X, $T_A(x), I_A(x), F_A(x) \subset [0,1]$.

1.4.2. Example

Assume that $X = \{x_1, x_2, x_3\}$, x_1 is capability, x_2 is trustworthiness and x_3 is price. The values of x_1, x_2 and x_3 are in $[0,1]$. They are obtained from the questionnaire of some domain experts, their option could be degree of good, degree of indeterminacy and degree of poor. A is an INS of X defined by

$$A = < [0.2,0.4], [0.3,0.5], [0.3,0.5] > / x_1 + < [0.5,0.7], [0,0.2], [0.2,0.3] > / x_2 +$$
$$< [0.6,0.8], [0.2,0.3], [0.2,0.3] > / x_3.$$

B in an INS of X defined by

$$B = < [0.5,0.7], [0.1,0.3], [0.1,0.3] > / x_1 + < [0.2,0.3], [0.2,0.4], [0.5,0.8] > / x_2 +$$
$$< [0.4,0.6], [0,0.1], [0.3,0.4] > / x_3.$$

1.4.3. Definition[5]

An INS A is empty if and only if its

$$inf\ T_A(x) = sup\ T_A(x) = 0,$$

$$inf\ I_A(x) = sup I_A(x) = 1$$

$$and \inf F_A(x) = sup F_A(x) = 1, \quad for\ all\ x\ in\ X.$$

1.4.4. Definition[16]

An INS A is contained in the other INS B, $A \subset B$, if and only if

$$inf\ T_A(x) \le inf\ T_B(x), sup\ T_A(x) \le sup\ T_B(x),$$

$$inf\ I_A(x) \geq inf\ I_B(x)\ , supI_A(x) \geq supI_B(x),$$

$$inf\ F_A(x) \geq\ F_B(x)\ , supF_A(x) \geq\ supF_B(x),$$

$for\ all\ x\ in\ X.$

1.4.5. Definition[16]

Two INSs A and B are equal, written as $A\ =\ B$, if and only if $A\ \subset\ B$ and $B\ \subset\ A$.

1.4.6. Definition[16]

The complement of an INS A is denoted by $\overline{A}$ and is defined by

$$T_{\overline{A}}(x) = F_A(x)$$

$$Inf\ I_{\overline{A}}(x) = 1 - SupI_A(x)$$

$$SupI_{\overline{A}}(x) = 1 - InfI_A(x)$$

$$F_{\overline{A}}(x) = T_A(x)$$

for all $x\ in\ X$.

1.4.7. Example

Let A be the INS defined in Example 1.4.2. Then,

$$\overline{A} = <[0.3,0.5],[0.5,0.7],[0.2,0.4]>/\ x_1 + <[0.2,0.3],[0.8,1.0],[0.5,0.7]>/\ x_2 +$$

$$<[0.2,0.3],[0.7,0.8],[0.6,0.8]>/\ x_3.$$

1.4.8. Definition[16]

The Intersection of two INSs "A and B" is a set C where $C = A \cap B$,

Where $\overline{T_C}, \overline{I_C}, \overline{F_C}$ are realated to "A and B" by

$$Inf\, \overline{T_C}(x) = Min\left(Inf\overline{T_A}(x), Inf\overline{T_B}(x)\right)$$

$$Sup\, \overline{T_C}(x) = Min(Sup\overline{T_A}(x), Sup\overline{T_B}(x))$$

$$Inf\, \overline{I_C}(x) = Max\left(Inf\overline{I_A}(x), Inf\overline{I_B}(x)\right)$$

$$Sup\, \overline{I_C}(x) = Max(Sup\overline{I_A}(x), Sup\overline{I_B}(x))$$

$$Inf\, \overline{F_C}(x) = Max\left(Inf\overline{F_A}(x), Inf\overline{F_B}(x)\right)$$

$$Sup\overline{F_C}(x) = Max(Sup\overline{F_A}(x), Sup\overline{F_B}(x))$$

For all x in X.

1.4.9. Example

Let A and B be the INSs defined in Example 1.4.2. Then,

$$A \cap B =< [0.2,0.4], [0.3,0.5], [0.3,0.5] >/x_1+< [0.2,0.3], [0.2,0.4], [0.5,0.8]$$

$$>/ x_2 + < [0.4,0.6], [0.2,0.3], [0.3,0.4] >/ x_3.$$

1.4.10. Definition[16]

The Union of two INSs "A *and* B" is a set C, where $C = A \cup B$,

Where $\overline{T_A}, \overline{I_A}, \overline{F_A}$ are related to "A *and* B" by

$$Inf\, \overline{T_C}(x) = Max\left(Inf\overline{T_A}(x), Inf\overline{T_B}(x)\right)$$

$$Sup\, \overline{T_C}(x) = Max(Sup\overline{T_A}(x), Sup\overline{T_B}(x))$$

$$Inf\ \overline{I_C}(x) = Min\left(Inf\overline{I_A}(x), Inf\overline{I_B}(x)\right)$$

$$Sup\ \overline{I_C}(x) = Min(Sup\overline{I_A}(x), Sup\overline{I_B}(x))$$

$$Inf\ \overline{F_C}(x) = Min\left(Inf\overline{F_A}(x), Inf\overline{F_B}(x)\right)$$

$$Sup\overline{F_C}(x) = Min(Sup\overline{F_A}(x), Sup\overline{F_B}(x))$$

For all x in X.

1.4.11. Example

Let $A\ and\ B$ be the INSs defined in Example 1.4.2.

Then,

$$A \cup B = < [0.5,0.7], [0.1,0.3], [0.1,0.3] >/x_1 + < [0.5,0.7], [0,0.2], [0.2,0.3]$$
$$>/x2 + < [0.6,0.8], [0,0.1], [0.2,0.3] >/x_3$$

1.4.12. Definition[16]

The cartesian product of two INSs A defined on universe X_1 and B defined on universe X_2 is an interval neutrosophic set C, written as $C = A \times B$,

whose, truth-membership $\overline{T_C}$, indeterminacy-membership $\overline{I_C}$, and falsity-membership functions $\overline{F_C}$ are related to those of $A\ and\ B$ by

$$Inf\ \overline{T_C}(x,y) = (Inf\overline{T}_{A_Q}(x) + Inf\overline{T}_{B_Q}(y)) - Inf\overline{T}_{A_Q}(x).Inf\overline{T}_{B_Q}(y)$$

$$Sup\ \overline{T_C}((x,y) = (Sup\overline{T}_{A_Q}(x) + Sup\overline{T}_{B_Q}(y)) - Sup\overline{T}_{A_Q}(x).Sup\overline{T}_{B_Q}(y)$$

$$Inf\ \overline{I}_C(x,y) = Inf\overline{I}_{A_Q}(x), Sup\overline{I}_{B_Q}(y)$$

$$Sup\,\overline{I}_C(x,y) = (Sup\overline{I}_{A_Q}(x), Sup\overline{I}_{B_Q}(y))$$

$$Inf\,\overline{F}_C(x,y) = (Inf\overline{F}_{A_Q}(x), Inf\overline{F}_{B_Q}(y))$$

$$Sup\,\overline{F}_C(x,y) = Min(Sup\overline{F}_{A_Q}(x), Sup\overline{F}_{B_Q}(y))$$

$for\ all\ x\ in\ X_1,\ y\ in\ X_2.$

1.4.13. Example

Let *A* and *B* be the INSs defined in Example 1.4.2. Then,

$A \times B = <[0.6,0.82],[0.03,0.15],[0.03,0.15]>$

$/x_1 +\ [0.6,0.79],[0,0.08],[0.1,0.24]>/x_2 + <[0.76,0.92],[0,0.03],[0.03,0.12]>$

$/x_3$.

Chapter 2

Q-Single Valued Neutrosophic Soft Set

In this chapter we define basic definitions of Q-SVNSs, Multi Q-SVNSs, Q-SVNSSs, their operations and results.

2.1. Q-Single Valued Neutrosophic Sets

In this section we define Q-SVNSs, multi Q-SVNSs, definitions, operations and properties.

2.1.1. Definition

Let X be a universal set and $Q \neq \emptyset$. A Q −SVNS $\tilde{N}_Q$ in X and Q is an object of the form

$$\tilde{N}_Q = \{(\hat{\theta}, \hat{u}), \dot{\mu}_{\tilde{N}}(\hat{\theta}, \hat{u}), \dot{\nu}_{\tilde{N}}(\hat{\theta}, \hat{u}), \dot{\lambda}_{\tilde{N}}(\hat{\theta}, \hat{u}) : \hat{\theta} \in X, \hat{u} \in Q\},$$

Where $\dot{\mu}_{\tilde{N}_Q}: X \times Q \to [0,1]$, $\dot{\nu}_{\tilde{N}_Q}: X \times Q \to [0,1]$, $\dot{\lambda}_{\tilde{N}_Q}: X \times Q \to [0,1]$, are respectively truth-membership, indeterminacy-membership and falsity membership functions for every

$\hat{\theta} \in X \, and \, \hat{u} \in Q$ and satisfy the condition $0 \leq \dot{\mu}_{\tilde{N}_Q}(\hat{\theta},\hat{u}) + \dot{\nu}_{\tilde{N}_Q}(\hat{\theta},\hat{u}) + \dot{\lambda}_{\tilde{N}_Q}(\hat{\theta},\hat{u}) \leq 3$.

2.1.2. Example

Let $X = \{p_1, p_1, p_3\}$ and $Q = \{\hat{u}, \hat{v}\}$, then Q −SVNS $\tilde{N}_Q$ is defined below,

$$\tilde{N}_Q = \{< (p_1,\hat{u}),(0.4,0.3,0.5),(p_1,\hat{v}),(0.2,0.4,0.6),(p_2,\hat{u}),(0.6,0.1,0.3),$$

$$(p_2,\hat{v}),(0.7,0.2,0.1),(p_3,\hat{u}),(0.3,0.6,0.4),(p_3,\hat{v}),(0.5,0.4,0.6) >\}.$$

2.1.3. Definition.

Let X be a universal set, $Q \neq \emptyset$and $\tilde{N}_Q$ be a Q −SVNS. The complement of $\tilde{N}_Q$ is denoted and defined as follows

$$\tilde{N}_Q^c = \left\{(\hat{\theta},\hat{u}), \dot{\lambda}_{\tilde{N}_Q}(\hat{\theta},\hat{u}), 1 - \dot{\nu}_{\tilde{N}_Q}(\hat{\theta},\hat{u}), \dot{\mu}_{\tilde{N}_Q}(\hat{\theta},\hat{u}) : \hat{\theta} \in X, \hat{u} \in Q\right\}$$

2.1.4. Definition

Let $\tilde{A}_Q$ and $\tilde{N}_Q$be two Q −SVNS. Then the union and intersection is denoted and defined by

$$\tilde{A}_Q \cup \tilde{N}_Q = \{(\hat{\theta},\hat{u}), max(\dot{\mu}_{\tilde{A}_Q}(\hat{\theta},\hat{u}), \dot{\mu}_{\tilde{N}_Q}(\hat{\theta},\hat{u})), min(\dot{\nu}_{\tilde{A}_Q}(\hat{\theta},\hat{u}), \dot{\nu}_{\tilde{N}_Q}(\hat{\theta},\hat{u})),$$

$$min(\dot{\lambda}_{\tilde{A}_Q}(\hat{\theta},\hat{u}), \dot{\lambda}_{\tilde{N}_Q}(\hat{\theta},\hat{u})) : \hat{\theta} \in X, \hat{u} \in Q\}$$

$$\tilde{A}_Q \cap \tilde{N}_Q = \{(\hat{\theta},\hat{u}), min(\dot{\mu}_{\tilde{A}_Q}(\hat{\theta},\hat{u}), \dot{\mu}_{\tilde{N}_Q}(\hat{\theta},\hat{u})), max(\dot{\nu}_{\tilde{A}_Q}(\hat{\theta},\hat{u}), \dot{\nu}_{\tilde{N}_Q}(\hat{\theta},\hat{u})),$$

$$max(\dot{\lambda}_{\tilde{A}_Q}(\hat{\theta},\hat{u}), \dot{\lambda}_{\tilde{N}_Q}(\hat{\theta},\hat{u}))\}$$

2.1.5. Definition

Let $\tilde{A}_Q$ and $\tilde{N}_Q$ be two Q −SVNSs over two non-empty universal sets G and H respectively and Q be any non-empty set. Then the product of $\tilde{A}_Q$ and $\tilde{N}_Q$ is denoted by $\tilde{A}_Q \times \tilde{N}_Q$ and defined as

$$\tilde{A}_Q \times \tilde{N}_Q = \{< ((\hat{\theta}, b), \hat{u}), \dot{\mu}_{\tilde{A}_Q \times \tilde{N}_Q}((\hat{\theta}, b), \hat{u}), \dot{v}_{\tilde{A}_Q \times \tilde{N}_Q}((\hat{\theta}, b), \hat{u}), \dot{\lambda}_{\tilde{A}_Q \times \tilde{N}_Q}((\hat{\theta}, b), \hat{u}) >: \hat{\theta} \in G, b \in H, \hat{u} \in Q\}$$

Where

$$\dot{\mu}_{\tilde{A}_Q \times \tilde{N}_Q}((\hat{\theta}, b), \hat{u}) = \min\{\dot{\mu}_{\tilde{A}_Q}(\hat{\theta}, \hat{u}), \dot{\mu}_{\tilde{N}_Q}(b, \hat{u})\}$$

$$\dot{v}_{\tilde{A}_Q \times \tilde{N}_Q}((\hat{\theta}, b), \hat{u}) = \max\left\{\dot{v}_{\tilde{A}_Q}(\hat{\theta}, \hat{u}), \dot{v}_{\tilde{N}_Q}(b, \hat{u})\right\}$$

$$\dot{\lambda}_{\tilde{A}_Q \times \tilde{N}_Q}\left((\hat{\theta}, b), \hat{u}\right) = \max\left\{\dot{\lambda}_{\tilde{A}_Q}(\hat{\theta}, \hat{u}), \dot{\lambda}_{\tilde{N}_Q}(b, \hat{u})\right\}$$

$For\ all\ \hat{\theta}, b\ in\ G\ and\ \hat{u} \in Q$

2.1.6. Definition

Let $\tilde{A}_Q$ a Q −single valued neutrosophic subset in a set G, the strongest Q −single valued

neutrosophic relation on G, that is a Q −single valued neutrosophic relation on $\tilde{A}_Q$ is H given by

$$\dot{\mu}_H((\hat{\theta}, b), \hat{u}) = min\{\dot{\mu}_{\tilde{A}_Q}(\hat{\theta}, \hat{u}), \dot{\mu}_{\tilde{N}_Q}(b, \hat{u})\}$$

$$\dot{v}_H((\hat{\theta}, b), \hat{u}) = max\{\dot{v}_{\tilde{A}_Q}(\hat{\theta}, \hat{u}), \dot{v}_{\tilde{N}_Q}(b, \hat{u})\}$$

$$\dot{\lambda}_H\left((\hat{\theta}, b), \hat{u}\right) = max\left\{\dot{\lambda}_{\tilde{A}_Q}(\hat{\theta}, \hat{u}), \dot{\lambda}_{\tilde{N}_Q}(b, \hat{u})\right\}$$

For all $\hat{\theta}, b$ *in G and* $\hat{u} \in Q$.

2.1.7. Definition

Let X be a non-empty set and Q be any non-empty set , l be any positive integer and I be a unit interval [0,1] .A multi Q −SVNS $\tilde{A}_Q$ in X and Q is a set of ordered sequences

$$\tilde{A}_Q = \{(\hat{\theta}, \hat{u}), \dot{\mu}_j(\hat{\theta}, \hat{u}), \dot{v}_j(\hat{\theta}, \hat{u}), \dot{\lambda}_j(\hat{\theta}, \hat{u}) : \hat{\theta} \in X, \hat{u} \in Q \; for\; all\; j = 1,2,.., l\}$$

Where $\dot{\mu}_j : X \times Q \to I^K$, $\dot{v}_j : X \times Q \to I^K$, $\dot{\lambda}_j : X \times Q \to I^K$, $for\; all\; j = 1,2, \ldots, l$

and are respectively truth-membership, indeterminacy-membership and falsity membership functions for each

$\hat{\theta} \in X$ *and* $\hat{u} \in Q$ and satisfy the condition

$$0 \le \dot{\mu}_j(\hat{\theta}, \hat{u}) + \dot{v}_j(\hat{\theta}, \hat{u}) + \dot{\lambda}_j(\hat{\theta}, \hat{u}) \le 3, for\; all\; j = 1,2, \ldots, l$$

The functions $\dot{\mu}_j(\hat{\theta}, \hat{u}), \dot{v}_j(\hat{\theta}, \hat{u}), \dot{\lambda}_j(\hat{\theta}, \hat{u})$ $for\; all\; j = 1,2, \ldots, l$

are called the "truth-membership , indeterminacy-membership and falsity-membership" functions respectively of the multi Q −SVNS $\tilde{A}_Q$,

l is called the dimension of the Q −SVNS $\tilde{A}_Q$. The set of all Q −SVNS is denoted by $Z^k QSVN(X)$.

2.1.8. Example

Let $X = \{p_1, p_2, p_3\}$ be a universal set and $Q = \{\hat{u}, v\}$ be a non-empty set and $l = 2$ be a positive integer. If $\tilde{A}_Q : X \times Q \to I^2$,Then the set

$\tilde{A}_Q = \{< ((p_1,\hat{u}),(0.2,0.3,0.6),(0.6,0.2,0.3)),((p_1,\hat{v}),(0.5,0.1,0.3),(0.4,0.4,0.5)),$

$((p_2,\hat{u}),(0.4,0.3,0.5),(0.6,0.1,0.3)),((p_2,\hat{v}),(0.7,0.2,0.1),(0.2,0.4,0.8)) >\}$

is a multi Q −SVNS in X and Q.

2.1.9. Remark

Note that if $\dot{v}_j(\hat{\theta},\hat{u}) = 0$ *and* $\dot{\lambda}_j(\hat{\theta},\hat{u}) = 0$ then multi Q −SVNS reduces to multi Q −fuzzy set.

2.1.10. Definition

Let $\tilde{A}_Q$ be a Q −SVNS. The the complement of $\tilde{A}_Q$ is denoted and defined as follows

$\tilde{A}_Q^c = \{(\hat{\theta},\hat{u}),\dot{\lambda}_j(\hat{\theta},\hat{u}),1-\dot{v}_j(\hat{\theta},\hat{u}),\dot{\mu}_j(\hat{\theta},\hat{u}):\hat{\theta} \in X \, and \, \hat{u} \in Q, for \, all \, j =$

$1,2,\ldots,l\}$

2.1.11. Definition

Let $\tilde{A}_Q$ and A_Q *and* B_Q be two Q −SVNSs, and l be a positive integer such that

A=$\{(\hat{\theta},\hat{u}),\dot{\mu}_j(\hat{\theta},\hat{u}),\dot{v}_j(\hat{\theta},\hat{u}),\dot{\lambda}_j:\hat{\theta} \in X \, and \, \hat{u} \in Q \, for \, all \, j = 1,2,\ldots,l\}$ and

B=$\{(\hat{\theta},\hat{u}),\dot{\mu}_j^*(\hat{\theta},\hat{u}),\dot{v}_j^*(\hat{\theta},\hat{u}),\dot{\lambda}_j^*(\hat{\theta},\hat{u}):\hat{\theta} \in X \, and \, \hat{u} \in Q \, for \, all \, j = 1,2,\ldots,l\}$

Then we define the following basic operations for Q −SVNSs.

i. $A \subset B \, iff \, \dot{\mu}_j(\hat{\theta},\hat{u}) \leq \dot{\mu}_j^*(\hat{\theta},\hat{u}), \dot{v}_j(\hat{\theta},\hat{u}) \geq \dot{v}_j^*(\hat{\theta},\hat{u}) \, and \, \dot{\lambda}_j(\hat{\theta},\hat{u}) \geq$

   $\dot{\lambda}_j^*(\hat{\theta},\hat{u})$

   $for \, all \, j = 1,2,\ldots,l.$

ii. $A = B \, iff \; \dot{\mu}_j(\hat{\theta},\hat{u}) = \dot{\mu}_j^*(\hat{\theta},\hat{u}), \dot{v}_j(\hat{\theta},\hat{u}) = \dot{v}_j^*(\hat{\theta},\hat{u}) \, and \, \dot{\lambda}_j(\hat{\theta},\hat{u}) = \dot{\lambda}_j^*(\hat{\theta},\hat{u}) \, for \, all \, j = 1,2,\dots,l.$

iii. $A \cup B = \{(\hat{\theta},\hat{u}), \max(\dot{\mu}_j(\hat{\theta},\hat{u}), \dot{\mu}_j^*(\hat{\theta},\hat{u})), \min\left(\dot{v}_j(\hat{\theta},\hat{u}), \dot{v}_j^*(\hat{\theta},\hat{u})\right), \min(\dot{\lambda}_j(\hat{\theta},\hat{u}), \dot{\lambda}_j^*(\hat{\theta},\hat{u}))\}$

iv. $A \cap B = \{(\hat{\theta},\hat{u}), \min(\dot{\mu}_j(\hat{\theta},\hat{u}), \dot{\mu}_j^*(\hat{\theta},\hat{u})), \max\left(\dot{v}_j(\hat{\theta},\hat{u}), \dot{v}_j^*(\hat{\theta},\hat{u})\right), \max(\dot{\lambda}_j(\hat{\theta},\hat{u}), \dot{\lambda}_j^*(\hat{\theta},\hat{u}))\}$

2.2. Q−Single Valued Neutrosophic Soft Sets

In this section we introduce the concept of Q −SVNSSs by combining soft sets and Q − SVNS. We also define some basic operations and properties of Q − SVNSSs.

2.2.1 Definition

Let X be a universal set, Q be any non-empty set and E be the set of parameters. Let $Z^l QSVN(X)$ denote the set of all multi Q −single valued neutrosophic subsets of X with dimension $l = 1$ Let $K \subset E$ A pair (F_Q, K) is called Q − SVNSS over X where F_Q is a mapping given

$$F_Q : K \longrightarrow Z^l QSVN(X)$$

A Q −SVNSS can be represented by the set of ordered pairs

$$(F_Q, K) = \{\hat{\theta}, F_Q(\hat{\theta}) : \hat{\theta} \in X, F_Q(\hat{\theta}) \in Z^l QSVN(X)\}$$

2.2.2 Example

Let $X = \{p_1, p_2, p_3, p_4\}$ be a universal set, $E = \{k_1, k_2, k_3, k_4\}$ and $Q = \{\hat{u}, \hat{v}\}$ be a non-empty set. If $K = \{k_1, k_2, k_3\} \subset E$,

$F_{Q,}(k_1) = \{((p_1, \hat{u}), (0.3,0.4,0.6)), ((p_1, \hat{v}), (0.2,0.3,0.5)), ((p_2, \hat{u}), (0.6,0.2,0.4))\}$

$F_{Q,}(k_1) = \{((p_1, \hat{u}), (0.5,0.3,0.4)), ((p_1, \hat{v}), (0.4,0.1,0.7)), ((p_3, \hat{u}), (0.8,0.1,0.2))\}$

$F_{Q,}(k_3) = \{((p_1, \hat{u}), (0.9,0.1,0.1)), ((p_1, \hat{v}), (0.8,0.2,0.3)), ((p_3, \hat{v}), (0.4,0.3,0.6))\}$

Then

$(F_{Q,}K)$

$= \{(k_1, ((p_1, \hat{u}), (0.3,0.4,0.6)), ((p_1, \hat{v}), (0.2,0.3,0.5)), ((p_2, \hat{u}), (0.6,0.2,0.4)),$

$k_2, ((p_1, \hat{u}), (0.5,0.3,0.4)), ((p_1, \hat{v}), (0.4,0.1,0.7)), \ ((p_3, \hat{u}), (0.8,0.1,0.2)),$

$k_3, ((p_1, \hat{u}), (0.9,0.1,0.1)), ((p_1, \hat{v}), (0.8,0.2,0.3)), \ ((p_3, \hat{v}), (0.4,0.3,0.6))\}$

is a Q –SVNSS.

2.2.3 Definition

Let $(F_Q, K) \in QSVNSS(X)$. If $F_Q(\hat{\theta}) = \emptyset$ for all $\hat{\theta} \in K$ then (F_Q, K) is called a null Q –SVNSS denoted by $(\emptyset, K)$.

2.2.4 Example

Let X, E *and* Q be defined in Example 2.3.2 then

$(\emptyset, K) = \{(k_1, ((p_1, \hat{u}), (0,1,1)), ((p_1, \hat{v}), (0,1,1)), ((p_2, \hat{u}), (0,1,1)), k_2,$

$((p_1,\hat{u}),(0,1,1)),((p_1,\hat{v}),(0,1,1)),((p_3,\hat{u}),(0,1,1)),$

$k_3,((p_1,\hat{u}),(0,1,1)),((p_1,\hat{v}),(0,1,1)),((p_3,\hat{v}),(0,1,1))\}$

2.2.5 Definition

Let $(F_Q,K) \in QSVNSS(X)$, If $F_Q(\hat{\theta}) = X$ for all $\hat{\theta} \in K$ then (F_Q,K) is called absolute $Q - SVNSS$ denoted by (X,K).

2.2.6 Example

Let X, E and Q be defined in Example 2.3.2 then

$(X,K) = \{(k_1,((p_1,\hat{u}),(1,0,0)),((p_1,\hat{v}),(1,0,0)),((p_2,\hat{u}),(1,0,0)),$

$k_2,((p_1,\hat{u}),(1,0,0)),((p_1,\hat{v}),(1,0,0)),((p_3,\hat{u}),(1,0,0)),$

$k_3,((p_1,\hat{u}),(1,0,0)),((p_1,\hat{v}),(1,0,0)),((p_3,\hat{v}),(1,0,0))\}$

2.2.7 Definition

Let $(F_Q,K),(G_Q,L) \in QSVNS(X)$. Then (F_Q,K) is Q –SVNSS subset of (G_Q,L) , denoted by $(F_Q,K) \subset (G_Q,L)$ if $K \subset L$ and $F_Q(\hat{\theta}) \subset G_Q(\hat{\theta})$ for all $\hat{\theta} \in K$.

2.2.8 Definition

Let $(F_Q,K) \in QSVNS(X)$, Then the complement of $Q -$ SVNSS set is written as $(F_Q,K)^C$ and is defined by $(F_Q,K)^C = (F_Q^C, \neg K)$ where

$$F_Q^C : \neg K \rightarrow QSVNS(X)$$

is the mapping given by $F_Q^C(\text{e})$ Q −single valued neutrosophic complement for each $e \in K$.

2.2.9 Proposition

Let $(F_Q, K) \in QSVNS(X)$, Then

i. $((F_Q, K)^C)^C = (F_Q, K)$

ii. $(\emptyset, K)^C = (X, E)$

iii. $(X, E)^C = (\emptyset, E)$

Proof

i.

Let $k \in K$ then

$$(F_Q, K) = F_Q(k) = \{(p_1, \hat{u}), \left(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{v}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X\}$$

$$(F_Q, K)^C = (F_Q(k))^C = \left\{(p_1, \hat{u}), \left(\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), 1 - \dot{v}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{F_{Q(k)}}(p_1, \hat{u})\right)\right\}$$

$$((F_Q, K)^C)^C = \left\{(p_1, \hat{u}), \left(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), 1 - \left(1 - \dot{v}_{F_{Q(k)}}(p_1, \hat{u})\right), \dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u})\right)\right\}$$

$$((F_Q, K)^C)^C = \left\{(p_1, \hat{u}), \left(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{v}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u})\right)\right\}$$

$$((F_Q, K)^C)^C = (F_Q, K)$$

ii.

$Let\ (\emptyset, K) = (F_Q, K)$

Then for all $k \in K$

$F_Q(k) = \{(p_1, \hat{u}), \left(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{v}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X\}$

$= \{(p_1, \hat{u}), (0,1,1) : \hat{u} \in Q, p_1 \in X\}$

$(\emptyset, K)^C = (F_Q, K)^C = (F_Q(k))^C = \{(p_1, \hat{u}), (1,1-1,0) : \hat{u} \in Q, p_1 \in X\}$

$= \{(p_1, \hat{u}), (1,0,0) : \hat{u} \in Q, p_1 \in X\}$

$= (X, E)$

iii.

$Let\ (X, E) = (F_Q, E)$

Then for all $k \in K$

$F_Q(k) = \{(p_1, \hat{u}), \left(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{v}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X\}$

$= \{(p_1, \hat{u}), (1,0,0) : \hat{u} \in Q, p_1 \in X\}$

$(X, E)^C = (F_Q, E)^C = (F_Q(k))^C = \{(p_1, \hat{u}), (0,1-0,1) : \hat{u} \in Q, p_1 \in X\}$

$= \{(p_1, \hat{u}), (0,1,1) : \hat{u} \in Q, p_1 \in X\}$

$= (\emptyset, E)$

2.2.10 Definition.

Let $(F_Q, K)\ and\ (G_Q, L) \in QSVNS(X)$ .Then the union of two Q –SVNSSs $(F_Q, K)\ and\ (G_Q, L)$ is the Q –SVNSS , (M_Q, N) written as $(F_Q, K) \cup (G_Q, L) = (M_Q, N)$ where $N = K \cup L$ for all $l \in N$ and

$$(M_Q,N) = \begin{cases} F_Q(l) & if\ l\ \in K-L \\ G_Q(l) & if\ l\ \in L-K \\ F_Q(l) \cup G_Q(l) & if\ l\ \in K \cap L \end{cases}$$

2.2.11 Example.

Let $X = \{p_1, p_2, p_3\}$ be a universal set, $E = \{a_1, a_2, a_3\}$ be a set of parameters and $Q = \{\hat{u}\}$ be a non-empty set. Let $N = \{a_1, a_2\} \subset E, and\ M = \{a_2, a_3\}$

$(F_Q, N) = \{(a_1, ((p_1, \hat{u}), (0.1,0.2,0.3)), a_2, ((p_2, \hat{u}), (0.3,0.4,0.5))\}$

and

$(G_Q, M) = \{a_2, ((p_2, \hat{u}), (0.2,0.3,0.5)), a_3, ((p_3, \hat{u}), (0.7,0.8,0.9))\}$

then

$(K_Q, L) =$

$\{(a_1, ((p_1, \hat{u}), (0.1,0.2,0.3)), a_2, ((p_2, \hat{u}), (0.3,0.3,0.5)), a_3, ((p_3, \hat{u}), (0.7,0.8,0.9))\}$

2.2.12 Definition.

Let $(F_Q, K)\ and\ (G_Q, L) \in\ QSVNSS(X)$ .Then the intersection of two Q –SVNSSs, $(F_Q, K)\ and\ (G_Q, L)$ is the $Q-$ SVNSS (M_Q, N) written as $(F_Q, K) \cap (G_Q, L) = (M_Q, N)$ where $N = K \cap L$ for all $l\ \in N$ and

$$(M_Q, N) = \{e, min\left(\dot{\mu}_{F_Q}(\hat{\theta}, \hat{u}), \dot{\mu}_{G_Q}(\hat{\theta}, \hat{u})\right), max\left(\dot{v}_{F_Q}(\hat{\theta}, \hat{u}), \dot{v}_{G_Q}(\hat{\theta}, \hat{u})\right),$$

$$max\left(\dot{\lambda}_{F_Q}(\hat{\theta}, \hat{u}), \dot{\lambda}_{G_Q}(\hat{\theta}, \hat{u})\right) : \hat{\theta} \in X, \hat{u} \in Q\ and\ j = 1,2, \dots, l\}$$

2.2.13 Example

Let $X = \{p_1, p_2, p_3, p_4, p_5\}$ be a universal set, $E = \{a_1, a_2, a_3, a_4, a_5\}$ be a set of parameters and $Q = \{\hat{u}, \hat{v}, w\}$ be a non-empty set. Let $N = \{a_1, a_3, a_4\} \subset E, and\ M = \{a_1, a_2, a_3\}$

(F_Q, N)

$= \{(a_1, ((p_1, \hat{u}), (0.3,0.4,0.5)), ((p_1, \hat{v}), (0.5,0.3,0.4)), ((p_1, w), (0.6,0.1,0.2)))$

$(a_3, ((p_1, \hat{u}), (0.2,0.3,0.4)), ((p_1, \hat{v}), (0.4,0.2,0.3)), ((p_1, w), (0.6,0.2,0.4)), ((p_3, \hat{u}), (0.7,0.1,0.2)),$

$((p_3, \hat{v}), (0.8,0.2,0.2)), ((p_3, w), (0.2,0.4,0.6))\}), (a_4, \{((p_2, \hat{u}), (0.6,0.2,0.1)), ((p_2, \hat{v}), (0.4,0.2,0.5)$

$, ((p_2, w), (0.5,0.4,0.4))\},$

and

$(G_Q, M) =$

$\{(a_1, ((p_1, \hat{u}), (0.4,0.3,0.5)), ((p_1, \hat{v}), (0.3,0.3,0.4)), ((p_1, w), (0.4,0.2,0.3))),$

$(a_2, ((p_2, \hat{u}), (0.4,0.5,0.2)), ((p_2, \hat{v}), (0.7,0.1,0.1)), ((p_2, w), (0.6,0.2,0.3)),$

$(a_3, \{((p_1, \hat{u}), (0.4,0.3,0.5), (p_1, \hat{v}), (0.2,0.2,0.4), (p_1, w), (0.4,0.1,0.4))$

$, ((p_3, \hat{v}), (0.6,0.1,0.2)), ((p_3, w), (0.7,0.2,0.3))\},$

Then

(K_Q, L)

$= \{(a_1, \{((p_1, \hat{u}), (0.3,0.4,0.5)), ((p_1, \hat{v}), (0.3,0.3,0.4)), ((p_1, w), (0.4,0.2,0.3))\}),$

$a_3, ((p_1, \hat{u}), (0.2,0.3,0.5)), ((p_1, \hat{v}), (0.2,0.2,0.4)), ((p_1, w), (0.4,0.2,0.4)), (p_3, \hat{u}), (0.7,0.2,0.2),$

$(p_3, \hat{v}), (0.6,0.2,0.2), (p_3, w), (0.2,0.4,0.6))\})$

2.2.14 Proposition.

Let $(F_Q, K), (M_Q, N)$ and $(G_Q, L) \in QSVNSS(X)$. Then

i. $(F_Q, K) \cup (\emptyset, K) = (F_Q, K)$

ii. $(F_Q, K) \cup (X, K) = (X, K)$

iii. $(F_Q, K) \cup (F_{Q,}K) = (F_{Q,}K)$

iv. $(F_Q, K) \cup (G_{Q,}L) = (G_Q, L) \cup (F_{Q,}K)$

v. $(F_Q, K) \cup ((G_Q, L) \cup (M_{Q,}N)) = ((G_Q, L) \cup (F_Q, K))) \cup (M_Q, N)$

Proof

i.

We have

$(F_Q, K) = \{(p_1, \hat{u}), \left(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{v}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X$

$(\emptyset, K) = \{(p_1, \hat{u}), (0,1,1) : \hat{u} \in Q, p_1 \in X\}$

$(F_Q, K) \cup (\emptyset, K)$

$= \{(p_1, \hat{u}), max\left(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), 0\right), min\left(\dot{v}_{F_{Q(k)}}(p_1, \hat{u}), 1\right), min\left(\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), 1\right)\}$

$=\{(p_1, \hat{u}), \left(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{v}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{F_{Q(k)}}(p_1, \tilde{\hat{u}})\right) : \hat{u} \in Q, p_1 \in X$

$=(F_Q, K)$

ii.

Let

$(X, K) = (G_Q, K)$ then

$(F_Q, K) = \{(p_1, \tilde{\hat{u}}), \left(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\nu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X\}$

$(G_Q, L) = \{(p_1, \hat{u}), (1,0,0) : \hat{u} \in Q, p_1 \in X\}$

$(F_Q, K) \cup \ (G_Q, K)$

$= \{(p_1, \hat{u}), \max\left(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), 1\right), \min\left(\dot{\nu}_{F_{Q(k)}}(p_1, \hat{u}), 0\right), \min\left(\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), 0\right)\}$

$=\{(p_1, \hat{u}), (1,0,0): \hat{u} \in Q, p_1 \in X\}$

$=(G_Q, K) = (X, K)$

iii.

Let

$(F_Q, K) = \{(p_1, \hat{u}), \left(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\nu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X$

$(F_Q, K) \cup (F_Q, K) =$

$\{p_1, \hat{u}, \max\left(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{F_{Q(k)}}(p_1, \hat{u})\right), \min\left(\dot{\nu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\nu}_{F_{Q(k)}}(p_1, \hat{u})\right),$

$\min\left(\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X\} =$

$\{(p_1, \hat{u}), \left(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\nu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X\}$

$=(F_{Q,}K)$

iv.

$(F_Q, K) = \ \{(p_1, \hat{u}), \left(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\nu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X\}$

$(G_Q, L) = \ \{(p_1, \hat{u}), \left(\dot{\mu}_{G_{Q(k)}}(p_1, \hat{u}), \dot{\nu}_{G_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X\}$

$= \{(p_1, \hat{u}), max(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), (\dot{\mu}_{G_{Q(k)}}(p_1, \hat{u}),$

$\dot{\mu}_{M_{Q(k)}}(p_1, \hat{u}))min(\dot{v}_{F_{Q(k)}}(p_1, \hat{u}), (\dot{v}_{G_{Q(k)}}(p_1, \hat{u}), \dot{v}_{M_{Q(k)}}(p_1, \hat{u}))$

$(min(\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), (\dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{M_{Q(k)}}(p_1, \hat{u})), \hat{u} \in Q, p_1 \in X\}$

$=(F_Q, K) \cup ((G_Q, L) \cup (M_Q, N))$

2.2.15 Proposition

Let $(F_Q, K), (M_Q, N)$ *and* $(G_Q, L) \in QSVNSS(X)$. Then

i. $(F_Q, K) \cap (\emptyset, K) = (\emptyset, K)$

ii. $(F_Q, K) \cap (X, K) = (F_Q, K)$

iii. $(F_Q, K) \cap (F_Q, K) = (F_Q, K)$

iv. $(F_Q, K) \cap (G_Q, L) = (G_Q, L) \cap (F_Q, K)$

v. $(F_Q, K) \cap ((G_Q, L) \cap (M_Q, N)) = ((G_Q, L) \cap (F_Q, K)) \cap (M_Q, N)$

Proof

i.

We have

$(F_Q, K) = \{(p_1, \hat{u}), \left(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{v}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X$

$(\emptyset, K) = \{(p_1, \hat{u}), (0,1,1) : \hat{u} \in Q, p_1 \in X\}(F_Q, K) \cap (\emptyset, K)$

$= \{(p_1, \hat{u}), min\left(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), 0\right), max\left(\dot{v}_{F_{Q(k)}}(p_1, \hat{u}), 1\right), max\left(\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), 1\right)\}$

$=\{(x_1, \hat{u}), (0,1,1): \hat{u} \in Q, p_1 \in X\}$

$= (\emptyset, K)$

ii.

Let

$(X,K) = (G_Q, L)$ then

$$(F_Q,K) = \{(p_1,\hat{u}),\left(\dot{\mu}_{F_{Q(k)}}(p_1,\hat{u}),\dot{v}_{F_{Q(k)}}(p_1,\hat{u}),\dot{\lambda}_{F_{Q(k)}}(p_1,\hat{u})\right): \hat{u} \in Q, p_1 \in X\}$$

$$(G_Q,L) = \{(p_1,\hat{u}),(1,0,0) : \hat{u} \in Q, p_1 \in X\}$$

$$(F_Q,K) \cap (G_Q,L)$$

$$= \{(p_1,\hat{u}),\min\left(\dot{\mu}_{F_{Q(k)}}(p_1,\hat{u}),1\right),\max\left(\dot{v}_{F_{Q(k)}}(p_1,\hat{u}),0\right),\max\left(\dot{\lambda}_{F_{Q(k)}}(p_1,\hat{u}),0\right)\}$$

$$=\{(p_1,\hat{u}),\left(\dot{\mu}_{F_{Q(k)}}(p_1,\hat{u}),\dot{v}_{F_{Q(k)}}(p_1,\hat{u}),\dot{\lambda}_{F_{Q(k)}}(p_1,\hat{u})\right): \hat{u} \in Q, p_1 \in X\}$$

$$= (F_Q,K)$$

iii.

Let

$$(F_Q,K) = \{(p_1,\hat{u}),\left(\dot{\mu}_{F_{Q(k)}}(p_1,\hat{u}),\dot{v}_{F_{Q(k)}}(p_1,\hat{u}),\dot{\lambda}_{F_{Q(k)}}(p_1,\hat{u})\right): \hat{u} \in Q, p_1 \in X\}$$

$$(F_Q,K) \cap (F_Q,K) = \{(p_1,\hat{u}),(min\left(\dot{\mu}_{F_{Q(k)}}(p_1,\hat{u}),\dot{\mu}_{F_{Q(k)}}(p_1,\hat{u})\right),$$

$$min\left(\dot{v}_{F_{Q(k)}}(p_1,\hat{u}),\dot{v}_{F_{Q(k)}}(p_1,\hat{u})\right),min(\dot{\lambda}_{F_{Q(k)}}(p_1,\hat{u}),\dot{\lambda}_{F_{Q(k)}}(p_1,\hat{u})): \hat{u} \in Q, p_1 \in X\}$$

$$=\{(p_1,\hat{u}),\left(\dot{\mu}_{F_{Q(k)}}(p_1,\hat{u}),\dot{v}_{F_{Q(k)}}(p_1,\hat{u}),\dot{\lambda}_{F_{Q(k)}}(p_1,\hat{u})\right): \hat{u} \in Q, p_1 \in X\}$$

$$=(F_Q,K)$$

iv.

$$(F_Q,K) = \{(p_1,\hat{u}),\left(\dot{\mu}_{F_{Q(k)}}(p_1,\hat{u}),\dot{v}_{F_{Q(k)}}(p_1,\hat{u}),\dot{\lambda}_{F_{Q(k)}}(p_1,\hat{u})\right): \hat{u} \in Q, p_1 \in X\}$$

$(G_Q, L) = \{(p_1, \hat{u}), \left(\dot{\mu}_{G_{Q(k)}}(p_1, \hat{u}), \dot{\nu}_{G_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X\}$

$(F_Q, K) \cap (G_Q, L) = \{(p_1, \hat{u}), min(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{G_{Q(k)}}(p_1, \hat{u})), max$

$(\dot{\nu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\nu}_{G_{Q(k)}}(p_1, \hat{u})),$

$max\ (\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u})), \hat{u} \in Q, p_1 \in X\}$

$(F_Q, K) \cap (G_Q, L) = \{(p_1, \hat{u}), min(\dot{\mu}_{G_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{F_{Q(k)}}(p_1, \hat{u})), max$

$(\dot{\nu}_{G_{Q(k)}}(p_1, \hat{u}), \dot{\nu}_{F_{Q(k)}}(p_1, \hat{u})),$

$max\ (\dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u})), \hat{u} \in Q, p_1 \in X\}$

$=(G_Q, L) \cap (F_Q, K)$

v.

$(F_Q, K) = \{(p_1, \hat{u}), \left(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\nu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X\}$

$(G_Q, L) = \{(p_1, \hat{u}), \left(\dot{\mu}_{G_{Q(k)}}(p_1, \hat{u}), \dot{\nu}_{G_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X\}$

$(F_Q, K) \cap (G_Q, L) = \{(p_1, \hat{u}), min(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{G_{Q(k)}}(p_1, \hat{u})), max$

$(\dot{\nu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\nu}_{G_{Q(k)}}(p_1, \hat{u})),$

$max\ (\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u})), \hat{u} \in Q, p_1 \in X\}$

$and\ (M_Q, N) = \{(p_1, \hat{u}), \left(\dot{\mu}_{M_{Q(k)}}(p_1, \hat{u}), \dot{\nu}_{M_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{M_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X\}$

$((F_Q, K) \cup (G_Q, L)) \cup (M_Q, N) = \{(p_1, \hat{u}), min((\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{G_{Q(k)}}(p_1, \hat{u})),$

$\dot{\mu}_{M_{Q(k)}}(p_1, \hat{u})), max\ (((\dot{\nu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\nu}_{G_{Q(k)}}(p_1, \hat{u})), \dot{\nu}_{M_{Q(k)}}(p_1, \hat{u}))$

$max\,(\dot{\lambda}_{F_{Q(k)}}(p_1,\hat{u}), \dot{\lambda}_{G_{Q(k)}}(p_1,\hat{u}), \dot{\lambda}_{M_{Q(k)}}(p_1,\hat{u})), \hat{u} \in Q, p_1 \in X\}$

$= \{(p_1,\hat{u}), min(\,\dot{\mu}_{F_{Q(k)}}(p_1,\hat{u}), (\dot{\mu}_{G_{Q(k)}}(p_1,\hat{u}),$

$\dot{\mu}_{M_{Q(k)}}(p_1,\hat{u}))\; max((\dot{v}_{F_{Q(k)}}(p_1,\hat{u}), (\dot{v}_{G_{Q(k)}}(p_1,\hat{u}), \dot{v}_{M_{Q(k)}}(p_1,\hat{u}))$

$max(\dot{\lambda}_{F_{Q(k)}}(p_1,\hat{u}), (\dot{\lambda}_{G_{Q(k)}}(p_1,\hat{u}), \dot{\lambda}_{M_{Q(k)}}(p_1,\hat{u})), \hat{u} \in Q, p_1 \in X\}$

$=(F_Q,K) \cap ((G_Q,L) \cap (M_Q,N))$

2.2.16 Proposition

Let (F_Q,K) *and* $(G_Q,L) \in QSVNSS(X)$. Then

i. $((F_Q,K) \cup (G_Q,L))^C = (F_Q,K)^C \cap (G_Q,L)^C$

ii. $((F_Q,K) \cap (G_Q,L))^C = (F_Q,K)^C \cup (G_Q,L)^C$

Proof

i.

Let

$$(F_Q,K) = \{(p_1,\hat{u}), \left(\dot{\mu}_{F_{Q(k)}}(p_1,\hat{u}), \dot{v}_{F_{Q(k)}}(p_1,\hat{u}), \dot{\lambda}_{F_{Q(k)}}(p_1,\hat{u})\right) : \hat{u} \in Q, p_1 \in X\}$$

$$(G_Q,L) = \{(p_1,\hat{u}), \left(\dot{\mu}_{G_{Q(k)}}(p_1,\hat{u}), \dot{v}_{G_{Q(k)}}(p_1,\hat{u}), \dot{\lambda}_{G_{Q(k)}}(p_1,\hat{u})\right) : \hat{u} \in Q, p_1 \in X\}$$

$(F_Q,K) \cup (G_Q,L) = \{(p_1,\hat{u}), max(\,\dot{\mu}_{F_{Q(k)}}(p_1,\hat{u}), \dot{\mu}_{G_{Q(k)}}(p_1,\hat{u})),$

$min(\dot{v}_{F_{Q(k)}}(p_1,\hat{u}), \dot{v}_{G_{Q(k)}}(p_1,\hat{u})),$

$min(\dot{\lambda}_{F_{Q(k)}}(p_1,\hat{u}), \dot{\lambda}_{G_{Q(k)}}(p_1,\hat{u})), \hat{u} \in Q, p_1 \in X\}$

$((F_Q, K) \cup (G_Q, L))^C = \{(p_1, \hat{u}), min(\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u})),$

$max\,(1 - min(\dot{v}_{F_{Q(k)}}(p_1, \hat{u}), \dot{v}_{G_{Q(k)}}(p_1, \hat{u})), max(\,\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{G_{Q(k)}}(p_1, \hat{u}))\,, \hat{u} \in Q, p_1 \in$

$X\}$..........(1)

Now

$$(F_Q, K)^c = \{(p_1, \hat{u}), \left(\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), 1 - \dot{v}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{F_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X\}$$

$$(G_Q, L)^c = \{(p_1, \hat{u}), \left(\dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u}), 1 - \dot{v}_{G_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{G_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X\}$$

$(F_Q, K)^c \cap (G_Q, L)^c = \{(p_1, \hat{u}), \Big(min(\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u})), max(1 -$

$\dot{v}_{F_{Q(k)}}(p_1, \hat{u}), 1 - \dot{v}_{G_{Q(k)}}(p_1, \hat{u}))\Big), max(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{G_{Q(k)}}(p_1, \hat{u})), : \hat{u} \in Q, p_1 \in X\}$

$= \{(x_1, u), min(\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u})),$

$max\,(1 - min(\dot{v}_{F_{Q(k)}}(p_1, \hat{u}), \dot{v}_{G_{Q(k)}}(p_1, \hat{u})), max(\,\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{G_{Q(k)}}(p_1, \hat{u}))$

$, \hat{u} \in Q, p_1 \in X\}$............(2)

From Equation (1) and (2)

$((F_Q, K) \cup (G_Q, L))^C = (F_Q, K)^c \cap (G_Q, L)^c$

ii.

$$(F_Q, K) = \{(p_1, \hat{u}), \left(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{v}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X\}$$

$$(G_Q, L) = \{(p_1, \hat{u}), \left(\dot{\mu}_{G_{Q(k)}}(p_1, \hat{u}), \dot{v}_{G_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X\}$$

$(F_Q, K) \cap (G_Q, L) = \{(p_1, \hat{u}), min(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{G_{Q(k)}}(p_1, \hat{u})),$

$max(\dot{\nu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\nu}_{G_{Q(k)}}(p_1, \hat{u})),$

$max(\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u})): \hat{u} \in Q, p_1 \in X\}$

$((F_Q, K) \cap (G_Q, L))^C = \{(p_1, \hat{u}), max(\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u})),$

1- $max((\dot{\nu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\nu}_{G_{Q(k)}}(p_1, \hat{u})), min(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{G_{Q(k)}}(p_1, \hat{u})) : \hat{u} \in Q, p_1 \in X$

$\}$..........(1)

Now

$(F_Q, K)^c = \{(p_1, \hat{u}), \left(\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), 1 - \dot{\nu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{F_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X\}$

$(G_Q, L)^c = \{(p_1, \hat{u}), \left(\dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u}), 1 - \dot{\nu}_{G_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{G_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X\}$

$(F_Q, K)^c \cup (G_Q, L)^c = \{(p_1, \hat{u}), max(\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u})),$

$min(1 - \dot{\nu}_{F_{Q(k)}}(p_1, \hat{u}), 1 - \dot{\nu}_{G_{Q(k)}}(p_1, \hat{u})), min(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{G_{Q(k)}}(p_1, \hat{u})): \hat{u} \in$

$Q, p_1 \in X\}$

$= \{(p_1, \hat{u}), max(\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u})),$

1- $max((\dot{\nu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\nu}_{G_{Q(k)}}(p_1, \hat{u})), min(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{G_{Q(k)}}(p_1, \hat{u})) : \hat{u} \in Q, p_1 \in X$

$\}$..........(2)

From Equation (1) and (2)

$((F_Q, K) \cap (G_Q, L))^C$=$(F_Q, K)^c \cup (G_Q, L)^c$

2.2.17 Proposition.

Let $(F_Q, K), (M_Q, N)$ *and* $(G_Q, L) \in QSVNSS(X)$. Then

i. $(F_Q, K) \cap ((G_Q, L) \cup (M_Q, N)) = ((F_Q, K) \cap (G_Q, L)) \cup ((F_Q, K)) \cap (M_Q, N))$

ii. $(F_Q, K) \cup ((G_Q, L) \cap (M_Q, N)) = ((F_Q, K) \cup (G_Q, L)) \cap ((F_Q, K) \cup (M_Q, N)$

Proof

i.

$$(F_Q, K) = \{(p_1, \hat{u}), \left(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{v}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X\}$$

$$(G_Q, L) = \{(p_1, \hat{u}), \left(\dot{\mu}_{G_{Q(k)}}(p_1, \hat{u}), \dot{v}_{G_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X\}$$

$(G_Q, L) \cup (M_Q, N) = \{(p_1, \hat{u}), max(\dot{\mu}_{G_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{M_{Q(k)}}(p_1, \hat{u})),$

$min(\dot{v}_{G_{Q(k)}}(p_1, \hat{u}), \dot{v}_{M_{Q(k)}}(p_1, \hat{u})),$

$min(\dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{M_{Q(k)}}(p_1, \hat{u})), \hat{u} \in Q, p_1 \in X\}$

$(F_Q, K) \cap ((G_Q, L) \cup (M_Q, N)) =$

$\{(p_1, \hat{u}), min(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), max(\dot{\mu}_{G_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{M_{Q(k)}}(p_1, \hat{u}))),$

$min(\dot{v}_{F_{Q(k)}}(p_1, \hat{u}) min(\dot{v}_{G_{Q(k)}}(p_1, \hat{u}), \dot{v}_{M_{Q(k)}}(p_1, \hat{u}))),$

$min(\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), min(\dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{M_{Q(k)}}(p_1, \hat{u}))), \hat{u} \in Q, p_1 \in X\}$............(1)

ii.

$(F_Q, K) \cap (G_Q, L) = \{(p_1, \hat{u}), min(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{G_{Q(k)}}(p_1, \hat{u})), max$

$(\dot{v}_{F_{Q(k)}}(p_1, \hat{u}), \dot{v}_{G_{Q(k)}}(p_1, \hat{u})),$

$max\ (\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u})),\ \hat{u} \in Q, p_1 \in X\}$

$(F_Q, K)) \cap (M_Q, N) = \{(p_1, \hat{u}), min(\ \dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{M_{Q(k)}}(p_1, \hat{u})),$

$max\ (\dot{v}_{F_{Q(k)}}(p_1, \hat{u}), \dot{v}_{M_{Q(k)}}(p_1, \hat{u})),$

$max\ (\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{M_{Q(k)}}(p_1, \hat{u})),\ \hat{u} \in Q, p_1 \in X\}$

$(F_Q, K) \cap (G_Q, L) \cup\ (F_Q, K)) \cap (M_Q, N) =$

$\{(p_1, \hat{u}), max(min(\ \dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{G_{Q(k)}}(p_1, \hat{u}), min(\ \dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{M_{Q(k)}}(p_1, \hat{u})))),$

$min(\ max\ (\dot{v}_{F_{Q(k)}}(p_1, \hat{u}), \dot{v}_{G_{Q(k)}}(p_1, \hat{u}), max\ (\dot{v}_{F_{Q(k)}}(p_1, \hat{u}), \dot{v}_{M_{Q(k)}}(p_1, \hat{u})))),$

$min(max\ (\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u})),\ max\ (\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{M_{Q(k)}}(p_1, \hat{u}))),\ \hat{u} \in$

$Q, p_1 \in X\}$

$= \{(p_1, \hat{u}), min(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), max(\ \dot{\mu}_{G_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{M_{Q(k)}}(p_1, \hat{u}))),$

$min(\dot{v}_{F_{Q(k)}}(p_1, \hat{u}) min(\dot{v}_{G_{Q(k)}}(p_1, \hat{u}), \dot{v}_{M_{Q(k)}}(p_1, \hat{u}))),$

$min(\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), min(\dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{M_{Q(k)}}(p_1, \hat{u}))),\ \hat{u} \in Q, p_1 \in X\}$............(2)

from equation (1) and (2)

$(F_Q, K) \cap ((G_Q, L) \cup\ (M_Q, N)) = ((F_Q, K) \cap (G_Q, L)) \cup ((F_Q, K)) \cap (M_Q, N))$

$(F_Q, K) \cup ((G_Q, L) \cap\ (M_Q, N)) = ((F_Q, K) \cup (G_Q, L)) \cap ((F_Q, K) \cup (M_Q, N)$

$(F_Q, K) = \{(p_1, \hat{u}), \left(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), \dot{v}_{F_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X\}$

$(G_Q, L) = \{(p_1, \hat{u}), \left(\dot{\mu}_{G_{Q(k)}}(p_1, \hat{u}), \dot{v}_{G_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u})\right) : \hat{u} \in Q, p_1 \in X\}$

$(G_Q,L)\cap(M_Q,N)=\{(p_1,\hat{u}),min(\dot{\mu}_{G_{Q(k)}}(p_1,\hat{u}),\dot{\mu}_{M_{Q(k)}}(p_1,\hat{u})),$

$max(\dot{\nu}_{G_{Q(k)}}(p_1,\hat{u}),\dot{\nu}_{M_{Q(k)}}(p_1,\hat{u})),$

$max(\dot{\lambda}_{G_{Q(k)}}(p_1,\hat{u}),\dot{\lambda}_{M_{Q(k)}}(p_1,\hat{u})),\ \hat{u}\in Q,p_1\in X\}$

$(F_Q,K)\cup((G_Q,L)\cap(M_Q,N))=$

$\{(p_1,\hat{u}),max(\dot{\mu}_{F_{Q(k)}}(p_1,\hat{u}),min(\dot{\mu}_{G_{Q(k)}}(p_1,\hat{u}),\dot{\mu}_{M_{Q(k)}}(p_1,\hat{u}))),$

$max(\dot{\nu}_{F_{Q(k)}}(p_1,\hat{u}),max(\dot{\nu}_{G_{Q(k)}}(p_1,\hat{u}),\dot{\nu}_{M_{Q(k)}}(p_1,\hat{u}))),$

$max(\dot{\lambda}_{F_{Q(k)}}(p_1,\hat{u}),max(\dot{\lambda}_{G_{Q(k)}}(p_1,\hat{u}),\dot{\lambda}_{M_{Q(k)}}(p_1,\hat{u}))),\ \hat{u}\in Q,p_1\in X\}$............(1)

$(F_Q,K)\cup(G_Q,L)=\{(p_1,\hat{u}),max(\dot{\mu}_{F_{Q(k)}}(p_1,\hat{u}),\dot{\mu}_{G_{Q(k)}}(p_1,\hat{u})),\ min$

$(\dot{\nu}_{F_{Q(k)}}(p_1,\hat{u}),\dot{\nu}_{G_{Q(k)}}(p_1,\hat{u})),$

$min\ (\dot{\lambda}_{F_{Q(k)}}(p_1,\hat{u}),\dot{\lambda}_{G_{Q(k)}}(p_1,\hat{u})),\ \hat{u}\in Q,p_1\in X\}$

$(F_Q,K))\cap(M_Q,N)=\ \{(p_1,\hat{u}),max(\dot{\mu}_{F_{Q(k)}}(p_1,\hat{u}),\dot{\mu}_{M_{Q(k)}}(p_1,\hat{u})),$

$min\ (\dot{\nu}_{F_{Q(k)}}(p_1,\hat{u}),\dot{\nu}_{M_{Q(k)}}(p_1,\hat{u})),$

$min\ (\dot{\lambda}_{F_{Q(k)}}(p_1,\hat{u}),\dot{\lambda}_{M_{Q(k)}}(p_1,\hat{u})),\ \hat{u}\in Q,p_1\in X\}$

$(F_Q,K)\cup(G_Q,L)\cap\ (F_Q,K))\cup(M_Q,N)=$

$\{(p_1,\hat{u}),min(max(\dot{\mu}_{F_{Q(k)}}(p_1,\hat{u}),\dot{\mu}_{G_{Q(k)}}(p_1,\hat{u}),max(\dot{\mu}_{F_{Q(k)}}(p_1,\hat{u}),\dot{\mu}_{M_{Q(k)}}(p_1,\hat{u})))),$

$max(\ min\ (\dot{\nu}_{F_{Q(k)}}(p_1,\hat{u}),\dot{\nu}_{G_{Q(k)}}(p_1,\hat{u}),min\ (\dot{\nu}_{F_{Q(k)}}(p_1,\hat{u}),\dot{\nu}_{M_{Q(k)}}(p_1,\hat{u})))),$

$max(min\ (\dot{\lambda}_{F_{Q(k)}}(p_1,\hat{u}),\dot{\lambda}_{G_{Q(k)}}(p_1,\hat{u})),min\ (\dot{\lambda}_{F_{Q(k)}}(p_1,\hat{u}),\dot{\lambda}_{M_{Q(k)}}(p_1,\hat{u}))),\hat{u}\in Q,p_1$

$\in X\}$

$= \{(p_1, \hat{u}), max(\dot{\mu}_{F_{Q(k)}}(p_1, \hat{u}), min(\dot{\mu}_{G_{Q(k)}}(p_1, \hat{u}), \dot{\mu}_{M_{Q(k)}}(p_1, \hat{u})),$

$max(\dot{v}_{F_{Q(k)}}(p_1, \hat{u}), max(\dot{v}_{G_{Q(k)}}(p_1, \hat{u}), \dot{v}_{M_{Q(k)}}(p_1, \hat{u}))),$

$max(\dot{\lambda}_{F_{Q(k)}}(p_1, \hat{u}), max(\dot{\lambda}_{G_{Q(k)}}(p_1, \hat{u}), \dot{\lambda}_{M_{Q(k)}}(p_1, \hat{u}))), \hat{u} \in Q, p_1 \in X\}$............(2)

From equation (1) and (2)

$(F_Q, K) \cup ((G_Q, L) \cap (M_Q, N)) = ((F_Q, K) \cup (G_Q, L)) \cap ((F_Q, K) \cup (M_Q, N)$

2.2.18 Definition

Let $(F_Q, K), (M_Q, N)$ *and* $(G_Q, L) \in QSVNSS(X)$. Then the "AND" operation of two $Q-$ SVNSSs (F_Q, K) *and* $(G_Q, L)$is the $Q-$ SVNSS denoted by $(F_Q, K) \wedge (G_Q, L)$ and is defined by

$$(F_Q, K) \wedge (G_Q, L) = (M_Q, K \times L)$$

Where $M_Q(\gamma, \delta) = F_Q(\gamma) \cap G_Q(\delta)$ for all $\gamma \in K, \delta \in L$ is the intersection of two $Q-$ SVNSSs.

2.2.19 Definition

Let $(F_Q, K), (M_Q, N)$ *and* $(G_Q, L) \in QSVNS(X)$.Then the "OR" operation of two $Q-$ SVNSSs (F_Q, K) *and* (G_Q, L) is the $Q-$ SVNSS denoted by $(F_Q, K) \vee (G_Q, L)$ and is defined by

$$(F_Q, K) \vee (G_Q, L)) = (M_Q, K \times L)$$

Where $M_Q(\gamma, \delta) = F_Q(\gamma) \cup G_Q(\delta)$ for all $\gamma \in K$, $\delta \in L$ is the union of two $Q-$SVNSSs.

Chapter 3

Q− Interval Neutrosophic Soft Sets

In this chapter we define basic definitions of Q-INSs, Multi Q-INSs, Q-INSSs, their operations and results.

3.1. Q− Interval Neutrosophic Sets

In this section we introduce the concept of Q-Interval Neutrosophic Sets, Multi Q-Interval Neutrosophic Sets and operations.

3.1.1. Definition

Let X be a universal set and Q be any non-empty set. A Q −INS $\tilde{N}_Q$ in X and Q is an object of the form

$$\tilde{N}_Q = \{(\dot{a}, \dot{u}), \dot{\mu}_{\tilde{N}}(\dot{a}, \dot{u}), \dot{\nu}_{\tilde{N}}(\dot{a}, \dot{u}), \dot{\lambda}_{\tilde{N}}(\dot{a}, \dot{u}): \dot{a} \in X, \dot{u} \in Q\}$$

Where $\dot{\mu}_{\tilde{N}_Q}: X \times Q \rightarrow [0,1]$, $\dot{\nu}_{\tilde{N}_Q}: X \times Q \rightarrow [0,1]$, $\dot{\lambda}_{\tilde{N}_Q}: X \times Q \rightarrow [0,1]$,and are respectively truth-membership, indeterminacy-membership and falsity membership functions for each

$\dot{a} \in X, \dot{u} \in Q$ and satisfy the condition

$$0 \leq \dot{\mu}_{\tilde{N}_Q}(\dot{a},\dot{u}) + \dot{v}_{\tilde{N}_Q}(\dot{a},\dot{u}) + \dot{\lambda}_{\tilde{N}_Q}(\dot{a},\dot{u}) \leq 3.$$

3.1.2. Example

Let $X = \{q_1, q_2, q_3\}$ and $Q = \{\dot{u}, \underline{v}\}$, Then $Q -$ INS $\tilde{N}_Q$ is defined below, $\tilde{N}_Q =$

$$\{(q_1,\dot{u}),([0.1,0.2],[0.3,0.4],[0.4,0.7]),(q_1,\underline{v}),([0.2,0.3],[0.3,0.4],[0.5,0.6]),$$

$$(q_2,\dot{u}),([0.1,0.3],[0.4,0.6],[0.7,0.8])$$

$(q_2,\underline{v}),([0.1,0.4],[0.1,0.6],[0.7,0.9]),(q_3,\dot{u}),([0.3,0.7],[0.7,0.8],[0.2,0.4]),(q_3,\underline{v}),$

$([0.1,0.5],[0.2,0.6],[0.5,0.7])\}$ is Q- INS.

Now we define some basic operations for $Q -$ INS.

3.1.3. Definition

Let X be a non-empty set and Q be any non-empty set, and $\tilde{N}_Q$ be a $Q -$ INS. The complement of $\tilde{N}_Q$ is denoted and defined as follows

$$\tilde{N}_Q^C = \left\{(\dot{a},\dot{u}), \dot{\lambda}_{\tilde{N}_Q}(\dot{a},\dot{u}), P, \dot{\mu}_{\tilde{N}_Q}(\dot{a},\dot{u}) : \dot{a} \in X, \dot{u} \in Q\right\}$$

Where $P = [Inf\ I_{\tilde{N}_Q^C}(\dot{a},\dot{u}), Sup\ I_{\tilde{N}_Q^C}(\dot{a},\dot{u})]$

$$and\ Inf\ I_{\tilde{N}_Q^C}(\dot{a},\dot{u}) = 1 - Sup I_{\tilde{N}_Q}(\dot{a},\dot{u})$$

$$Sup\ I_{\tilde{N}_Q^C}(\dot{a},\dot{u}) = 1 - Inf I_{\tilde{N}_Q}(\dot{a},\dot{u})$$

For all $\dot{a} \in X, \dot{u} \in Q$.

3.1.4. Example

If we consider example 3.1.2 then

$\tilde{N}_Q^C = \{(q_1, \grave{u}), ([0.4,0.7], [0.6,0.7], [0.1,0.2]), (q_1, \underline{v}), ([0.5,0.6], [0.6,0.7], [0.2,0.3]),$

$(q_2, \grave{u}), ([0.7,0.8], [0.4,0.6], [0.1,0.3]), (q_2, \underline{v}), ([0.7,0.9], [0.4,0.9], [0.1,0.4]),$

$(q_3, \grave{u}), ([0.2,0.4], [0.7,0.8], [0.2,0.3]), (q_3, \underline{v}), ([0.1,0.5], [0.2,0.6], [0.5,0.7])\}$

3.1.5. Definition

A Q-INS A is contained in the other Q-INS B, $A \subset B$, if and only if

$$inf\ T_A(\grave{a}, \grave{u}) \leq inf\ T_B(\grave{a}, \grave{u})\ , sup\ T_A(\grave{a}, \grave{u}) \leq sup\ T_B(\grave{a}, \grave{u}),$$

$$inf\ I_A(\grave{a}, \grave{u}) \geq inf\ I_B(\grave{a}, \grave{u})\ , sup\ I_A(\grave{a}, \grave{u}) \geq sup\ I_B(\grave{a}, \grave{u}),$$

$$inf\ F_A(\grave{a}, \grave{u}) \geq inf\ F_B(\grave{a}, \grave{u})\ , sup\ F_A(\grave{a}, \grave{u}) \geq sup\ F_B(\grave{a}, \grave{u})$$

for all $\grave{a} \in X, \grave{u} \in Q$.

3.1.6. Example

Let $X = \{q_1, q_2\}$ and $Q = \{\grave{u}\}$, and

$A = \{(q_1, \grave{u}), ([0.1,0.2], [0.4,0.7], [0.7,0.8]), (q_2, \grave{u}), ([0.2,0.4], [0.4,0.6], [0.5,0.7])\}$

$B = \{(q_1, \grave{u}), ([0.2,0.4], [0.2,0.3], [0.1,0.2]), (q_2, \grave{u}), ([0.4,0.6], [0.1,0.2], [0.3,0.5])\}$.

Then $A \subset B$.

3.1.7. Definition

The Union of two Q-INSs A_Q and B_Q is a set C written as $A_Q \cup B_Q$,

where $\overline{T_C}, \overline{I_C}, \overline{F_C}$ are related to A_Q and B_Q by

$$Inf\ \overline{T_C}(\grave{a},\grave{u}) = Max\left(Inf\ \overline{T}_{A_Q}(\grave{a},\grave{u}), Inf\ \overline{T}_{B_Q}(\grave{a},\grave{u})\right)$$

$$Sup\ \overline{T_C}(\grave{a},\grave{u}) = Max(Sup\overline{T}_{A_Q}(\grave{a},\grave{u}), Sup\overline{T}_{B_Q}(\grave{a},\grave{u}))$$

$$Inf\ \overline{I}_C(\grave{a},\grave{u}) = Min\left(Inf\overline{I}_{A_Q}(\grave{a},\grave{u}), Inf\overline{I}_{B_Q}(\grave{a},\grave{u})\right)$$

$$Sup\ \overline{I}_C(\grave{a},\grave{u}) = Min(Sup\overline{I}_{A_Q}(\grave{a},\grave{u}), Sup\overline{I}_{B_Q}(\grave{a},\grave{u}))$$

$$Inf\ \overline{F}_C(\grave{a},\grave{u}) = Min\left(Inf\overline{F}_{A_Q}(\grave{a},\grave{u}), Inf\overline{F}_{B_Q}(\grave{a},\grave{u})\right)$$

$$Sup\ \overline{F}_C(\grave{a},\grave{u}) = Min(Sup\overline{F}_{A_Q}(\grave{a},\grave{u}), Sup\overline{F}_{B_Q}(\grave{a},\grave{u}))$$

For all $\grave{a} \in X, \grave{u} \in Q$.

3.1.8. Example

Let $X = \{q_1, q_2, q_3\}$ and $Q = \{\grave{u}, \underline{v}\}$, and

$A = \{(q_1,\grave{u}), ([0.1,0.2], [0.4,0.7], [0.7,0.8]), (q_2,\grave{u}), ([0.2,0.4], [0.4,0.6], [0.5,0.7])\}$

$B = \{(q_1,\grave{u}), ([0.2,0.7], [0.2,0.3], [0.1,0.2]), (q_2,\grave{u}), ([0.4,0.6], [0.1,0.2], [0.3,0.5])\}$

Then

$C = \{(q_1,\grave{u}), ([0.2,0.7], [0.2,0.3], [0.1,0.2]), (q_2,\grave{u}), ([0.4,0.6], [0.1,0.2], [0.3,0.5])\}$

3.1.9. Definition

The intersection of two Q-INSs A_Q *and* B_Q is a set C written as $A_Q \cap B_Q$,

Where $\overline{T_C}, \overline{I_C}, \overline{F_C}$ are related to A_Q *and* B_Q by

$$Inf\ \overline{T_C}(\grave{a},\grave{u}) = Min\left(Inf\overline{T}_{A_Q}(\grave{a},\grave{u}), Inf\overline{T}_{B_Q}(\grave{a},\grave{u})\right)$$

$$Sup\ \overline{T_C}(\grave{a},\grave{u}) = Min(Sup\overline{T}_{A_Q}(\grave{a},\grave{u}), Sup\overline{T}_{B_Q}(\grave{a},\grave{u}))$$

$$Inf\ \overline{I}_C(\grave{a},\grave{u}) = Max\left(Inf\overline{I}_{A_Q}(\grave{a},\grave{u}), Inf\overline{I}_{B_Q}(\grave{a},\grave{u})\right)$$

$$Sup\ \bar{I}_C(\dot{a},\dot{u}) = Max(Sup\bar{I}_{A_Q}(\dot{a},\dot{u}), Sup\bar{I}_{B_Q}(\dot{a},\dot{u}))$$

$$Inf\ \bar{F}_C(\dot{a},\dot{u}) = Max\left(Inf\bar{F}_{A_Q}(\dot{a},\dot{u}), Inf\bar{F}_{B_Q}(\dot{a},\dot{u})\right)$$

$$Sup\ \bar{F}_C(\dot{a},\dot{u}) = Max(Sup\bar{F}_{A_Q}(\dot{a},\dot{u}), Sup\bar{F}_{B_Q}(\dot{a},\dot{u}))$$

For all $\dot{a} \in X, \dot{u} \in Q$.

3.1.10. Example

Let $X = \{q_1, q_2, q_3\}$ and $Q = \{\dot{u}, \underline{v}\}$, and

$A = \{(q_1,\dot{u}), ([0.1,0.2], [0.4,0.7], [0.7,0.8]), (q_2,\dot{u}), ([0.2,0.4], [0.4,0.6], [0.5,0.7])\}$

$B = \{(q_1,\dot{u}), ([0.2,0.7], [0.2,0.3], [0.1,0.2]), (q_2,\dot{u}), ([0.4,0.6], [0.1,0.2], [0.3,0.5])\}$

Then

$C = \{(q_1,\dot{u}), ([0.1,0.2], [0.4,0.7], [0.7,0.8]), (q_2,\dot{u}), ([0.2,0.4], [0.4,0.6], [0.5,0.7])\}$

3.1.11. Definition

Let $A_Q\ and\ B_Q$ be two Q-INSs over two non empty Universal sets G & H and Q≠Ø,

Their Cartesian product is a set $C = A \times B$,whose $\overline{T_C}, \overline{I_C}, \overline{F_C}$ are related to those of

$A_Q\ and\ B_Q$ by

$$Inf\ \overline{T_C}((\dot{a},\dot{u}),y) = \left(Inf\bar{T}_{A_Q}(\dot{a},\dot{u}) + Inf\bar{T}_{B_Q}(y)\right) - Inf\bar{T}_{A_Q}(\dot{a},\dot{u}).Inf\bar{T}_{B_Q}(y)$$

$$Sup\ \overline{T_C}((\dot{a},\dot{u}),y) = \left(Sup\bar{T}_{A_Q}(\dot{a},\dot{u}) + Sup\bar{T}_{B_Q}(y)\right) - Sup\bar{T}_{A_Q}(\dot{a},\dot{u}).Sup\bar{T}_{B_Q}(y)$$

$$Inf\ \bar{I}_C((\dot{a},\dot{u}),y) = Inf\bar{I}_{A_Q}(\dot{a},\dot{u}), Sup\bar{I}_{B_Q}(y)$$

$$Sup\ \bar{I}_C((\dot{a},\dot{u}),y) = (Sup\bar{I}_{A_Q}(\dot{a},\dot{u}), Sup\bar{I}_{B_Q}(y))$$

$$Inf\ \bar{F}_C((\dot{a},\dot{u}),y) = \left(Inf\bar{F}_{A_Q}(\dot{a},\dot{u}), Inf\bar{F}_{B_Q}(y)\right)$$

$$Sup\,\overline{F}_C(\grave{a},\grave{u}) = Min(Sup\overline{F}_{A_Q}(\grave{a},\grave{u}), Sup\overline{F}_{B_Q}(y))$$

Where $y = (b,\grave{u})$.

3.1.12. Example

Let $G = \{q_1, q_2, q_3\}, H = \{p_3\}, Q = \{\grave{u}\}$

$Then\ C = A \times B = \{(q_1, q_3), (q_2, q_3)\}$

$G \times Q = \{(q_1, \grave{u}), (q_2, \grave{u})\}, H \times Q = \{(p_3, \grave{u})$

Define

$$\dot{\mu}_{G_Q}: X \times Q \to [0,1], \quad \dot{v}_{G_Q}: X \times Q \to [0,1], \dot{\lambda}_{G_Q}: X \times Q \to [0,1]$$

$$\dot{\mu}_{H_Q}: X \times Q \to [0,1], \quad \dot{v}_{H_Q}: X \times Q \to [0,1], \dot{\lambda}_{H_Q}: X \times Q \to [0,1]$$

By

$$\dot{\mu}_{G_Q}(q_1, \grave{u}) = [0.1,0.2], \dot{v}_{G_Q}(p_1, \grave{u}) = [0.3,0.4], \dot{\lambda}_{G_Q}(p_1, \grave{u}) = [0.5,0.8]$$

$$\dot{\mu}_{G_Q}(p_2, \grave{u}) = [0.3,0.5], \dot{v}_{G_Q}(p_2, \grave{u}) = [0.6,0.7], \dot{\lambda}_{G_Q}(p_2, \grave{u}) = [0.1,0.5]$$

$$\dot{\mu}_{H_Q}(p_3, \grave{u}) = [0.3,0.4], \dot{v}_{H_Q}(p_3, \grave{u}) = [0.1,0.3], \dot{\lambda}_{H_Q}(p_3, \grave{u}) = [0.1,0.4]$$

Now

$$G_Q = \{(q_1, \grave{u}), [0.1,0.2], [0.3,0.4], [0.5,0.8], (q_2, \grave{u}), [0.3,0.5], [0.6,0.7], [0.1,0.5]\}$$

$H_Q = \{(q_3, \grave{u}), [0.3,0.4], [0.1,0.3], [0.1,0.4]\}\ then$

$G_Q \times H_Q =$

$\{(q_1, q_3), \grave{u}), [0.37,0.48], [0.09,0.12], [0.05,0.32],$

$((q_2, q_3), \grave{u}), [0.51,0.7], [0.18,0.21], [0.01,0.02]\},$

is Cartesian product of $A_Q\ and\ B_Q$.

3.1.13. Definition

Let X be Universal set and $Q \neq \emptyset, l > Z^{+}$ and $I = [0,1]$, A Multi Q −Interval Neutrosophic set A_Q in X and Q is a set of ordered sequences

$$A_Q = \{(\dot{a},\dot{u}), \dot{\mu}_j(\dot{a},\dot{u}), \dot{v}_j(\dot{a},\dot{u}), \dot{\lambda}_j(\dot{a},\dot{u}): \dot{a} \in X, \dot{u} \in Q\ \forall\, j = 1,2,..,l\}$$

Where $\mu_{\tilde{N}_Q}: X \times Q \to I^l$, $v_{\tilde{N}_Q}: X \times Q \to I^l$, $\dot{\lambda}_{\tilde{N}_Q}: X \times Q \to I^l$ and are respectively "truth-membership", "indeterminacy-membership" and "falsity membership" functions for each

$\dot{a} \in X, \dot{u} \in Q$ and satisfy the condition $0 \leq \dot{\mu}_j(\dot{a},\dot{u}) + \dot{v}_j(\dot{a},\dot{u}) + \dot{\lambda}_j(\dot{a},\dot{u}) \leq 3$.

3.1.14. Example

Let $X = \{q_1, q_2, q_3\}$ be a universal set and $Q = \{\dot{u}, \underline{v}\}$ be a non-empty set and $l = 2$ be a positive integer. If $\tilde{A}_Q: X \times Q \to I^2$,Then the set

$$\tilde{A}_Q = \{< ((q_1,\dot{u}), [0.1,0.2], [0.3,0.4], [0.4,0.5], [0.5,0.6], [0.7,0.8], [0.1,0.2]),$$

$$((q_2,\dot{u}), [0.1,0.3], [0.1,0.3], [0.1,0.3], [0.3,0.6], [0.4,0.5], [0.5,0.6], [0.7,0.8]) >\}$$

is a multi $Q -$ INS in X and Q.

3.2. Q-Interval Neutrosophic Soft Sets

In this section we introduce the concept of Q-Interval Neutrosophic soft Sets, their operations and results.

3.2.1. Definition

Let X be a universal set, Q be any non-empty set and E be the set of parameters. Let $Z^{l}QINS(X)$ denote the set of all multi Q −interval neutrosophic subsets of X with dimension $l = 1$ Let $K \subset E$ A pair (F_Q, K) is called $Q -$ INSS over X where F_Q is a mapping given

$$F_Q: K \longrightarrow Z^{l}INS(X) \quad such\ that\ (F_Q(\grave{a})) = \emptyset\ \ if\ \grave{a} \notin K$$

A Q −INSS can be represented by the set of ordered pairs

$$(F_Q, K) = \{\grave{a}, F_Q(\grave{a}): \grave{a} \in X, F_Q(\grave{a}) \in Z^{l}QINS(X)\}$$

3.2.2. Example

Let $X = \{q_1, q_2, q_3\}$ be a universal set, $Q = \{\grave{u}, \underline{v}\}$ be a non-empty set,

and $E = \{\underline{k_1}, \underline{k_2}, \underline{k_3}\}, Let\ K = \{\underline{k_1}, \underline{k_2}, k_3\} \subset E$

$F_Q(\underline{k_1}) = \{(q_1, \grave{u}), [0.1,0.3], [0.4,0.5], [0.5,0.6], (q_2, \grave{u}), [0.2,0.4], [0.4,0.6], [0.6,0.8]\}$

$F_Q(\underline{k_2}) = \{(q_2, \grave{u}), [0.1,0.4], [0.4,0.5], [0.6,0.7], (q_3, \underline{v}), [0.3,0.4], [0.4,0.9], [0.2,0.7]\}$

$F_Q(\underline{k_3}) = \{(q_1, \underline{v}), [0.1,0.2], [0.3,0.5], [0.6,0.9], (q_3, \underline{v}), [0.1,0.4], [0.5,0.6], [0.3,0.8]\}$

$(F_Q, K) =$

$\{\underline{k_1}, (q_1, \grave{u}), [0.1,0.3], [0.4,0.5], [0.5,0.6], (q_2, \underline{v}), [0.2,0.4], [0.4,0.6], [0.6,0.8],$

$\underline{k_2}, (q_2, \grave{u}), [0.1,0.4], [0.4,0.5], [0.6,0.7], (q_3, \underline{v}), [0.3,0.4], [0.4,0.9], [0.2,0.7],$

$\underline{k_3}, (q_1, \underline{v}), [0.1,0.2], [0.3,0.5], [0.6,0.9], (q_3, \underline{v}), [0.1,0.4], [0.5,0.6], [0.3,0.8]\}$

is a Q-INSS.

3.2.3. Definition

Let $(F_Q, K) \in QINSS(X)$, If $F_Q(\grave{a}) = X$ for all $\grave{a} \in E$ then (F_Q, K) is called absolute $Q -$ INSS denoted by (X, K).

3.2.4. Example

Let X, E *and* Q be defined in the above example 3.3.2 then

$(X, K) = \{\underline{k}_1, (q_1, \grave{u}), [1,1], [0,0], [0,0], \underline{k}_1, (q_2, \grave{u}), [1,1], [0,0], [0,0]\}$ is absolute $Q -$ INSS.

3.2.5. Definition

Let $(F_Q, K) \in QINSS(X)$, If $F_Q(\grave{a}) = \emptyset$ for all $\grave{a} \in E$ then (F_Q, K) is called Null $Q -$ INSS denoted by (X, K).

3.2.6. Example

Let X, E *and* Q be defined in the above example 3.3.2 then

$(X, K) = \{\underline{k}_1, (q_1, \grave{u}), [0,0], [0,0], [1,1], \underline{k}_1, (q_2, \grave{u}), [0,0], [0,0], [1,1]\}$ is Null $Q -$ INSS.

3.2.7. Definition

Let $(F_Q, K), (G_Q, L) \in QINSS(X)$. Then (F_Q, K) is $Q -$interval neutrosophic soft subset of (G_Q, L) , denoted by $(F_Q, K) \subset (G_Q, L)$ if $K \subset L$ and $F_Q(\grave{a}) \subset G_Q(\grave{a})$ for all $\grave{a} \in K$.

3.2.8. Example

Let $X = \{q_1, q_2\}$ and $Q = \{\dot{u}\}$, and = $\{\underline{k}_1, \underline{k}_2\}$. Let $K = \{\underline{k}_1\}$, L=$\{\underline{k}_1, \underline{k}_2\}$,clearly

$K \subset L$

$(F_Q, K) = \{\underline{k}_1, (q_1, \dot{u}), ([0.1,0.2], [0.5,0.7], [0.7,0.8])\}$

$(G_Q, L) = \{(q_1, \dot{u}), ([0.2,0.4], [0.4,0.3], [0.1,0.2]), (q_2, \dot{u}), ([0.4,0.6], [0.1,0.2], [0.3,0.5])\}$

So

$F_Q(\grave{a}) \subset G_Q(\grave{a})$

3.2.9. Proposition

Let $(F_Q, K), (G_Q, L), (M_Q, N) \in QINSS(X)$. Then

i. $(F_Q, K) \subset (G_Q, E)$
ii. $(\emptyset, K) \subset (G_Q, L)$
iii. $(F_Q, K) \subset (G_Q, L)$ *and* $(G_Q, L) \subset (M_Q, N)$ then $(F_Q, K) \subset (M_Q, N)$.
iv. If $(F_Q, K) = (G_Q, L)$ and (G_Q, L)= (M_Q, N) *then* $(F_Q, K) = (M_Q, N)$

3.2.10. Definition

Let $(F_Q, K) \in QINSS(X)$, Then the complement of Q −interval valued neutrosophic soft set is denoted by $(F_Q, K)^C$ and is defined by $(F_Q, K)^C = (F_Q^C, \neg K)$ where

$$F_Q^C : \neg K \to QINSS(X)$$

is the mapping given by $F_Q^C(\grave{a})$ Q −single valued neutrosophic complement for each $\grave{a} \in K$.

3.2.11. Proposition

Let $(F_Q, K) \in QINSS(X)$, Then

i. $((F_Q, K)^C)^C = (F_Q, K)$

ii. $(\emptyset, K)^C = (X, E)$

iii. $(X, E)^C = (\emptyset, E)$

Proof

i.

Let $\underline{k} \in K$ then

$$(F_Q, K) = F_Q(\underline{k}) = \{(q_1, \grave{u}), \left(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u})\right) : \grave{u} \in Q, q_1 \in X\}$$

$(F_Q, K) =$

$F_Q(\underline{k})=\{(q_1, \grave{u}), \dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), [\,inf\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), suq\dot{v}_{F_{Q(k)}}], \dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}) : \grave{u} \in Q, q_1 \in X\}$

$(F_Q, K)^C = (F_Q(\underline{k}))^C$

$$= \left\{(q_1, \grave{u}), \left(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \left[1 - suq\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), 1 - inf\dot{v}_{F_{Q(k)}}(q_1, \grave{u})\right], \dot{\mu}_{F_{Q(k)}}(q_1, \grave{u})\right) : \grave{u} \in Q, q_1 \in X\right\}$$

$$((F_Q, K)^C)^C = \left\{(q_1, \grave{u}), \dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}) \left[1 - \left(1 - inf\dot{v}_{F_{Q(k)}}(q_1, \grave{u})\right), 1 - \left(1 - sup\dot{v}_{F_{Q(k)}}(q_1, \grave{u})\right)\right], \dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}) : \grave{u} \in Q, q_1 \in X\right\}$$

$=\{(x_1, \grave{u}), \dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), [\,inf\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), suq\dot{v}_{F_{Q(k)}}(q_1, \grave{u})], \dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}) : \grave{u} \in Q, q_1 \in X\}$

$$((F_Q,K)^C)^C = \left\{(q_1,\grave{u}),\left(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),\lambda_{F_{Q(k)}}(q_1,\grave{u})\right)\right\}$$

$$((F_Q,K)^C)^C = (F_Q,K)$$

ii.

$Let\ (\emptyset,K) = (F_Q,K)$

Than for all $k \in K$

$F_Q(k) = \{(q_1,\grave{u}),\left(\dot{\mu}_{F_{Q(k)}}(q_1,\underline{u}),\dot{v}_{F_{Q(k)}}(q_1,\underline{u}),\dot{\lambda}_{F_{Q(k)}}(q_1,\underline{u})\right):\underline{u} \in Q, q_1 \in X\}$

$= \{(q_1,u),[0,0],[1,1],[1,1] : \grave{u} \in Q, q_1 \in X\}$

$(\emptyset,K)^C = (F_Q,K)^C = (F_Q(k))^C = \{(q_1,\underline{u}),[1,1],[0,0],[0,0]: \grave{u} \in Q, q_1 \in X\}$

$= (X,E)$

iii.

$Let\ (X,E) = (F_Q,E)$

Then for all $k \in K$

$F_Q(k) = \{(q_1,\grave{u}),\left(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u})\right):\grave{u} \in Q, x_1 \in X\}$

$= \{(q_1,\grave{u}),[1,1],[0,0],[0,0]: \grave{u} \in Q, q_1 \in X\}$

$(X,E)^C = (F_Q,E)^C = (F_Q(k))^C = \{(q_1,\grave{u}),[0,0],[1,1],[1,1]: \grave{u} \in Q, q_1 \in X\}$

$$= (\emptyset,E)$$

3.2.12. Definition

Let (F_Q,K) *and* $(G_Q,L) \in QINSS(X)$ .Then the union of two $QINSSs$ (F_Q,K) *and* (G_Q,L) is the Q −single valued neutrosophic soft set (M_Q,N) written as $(F_Q,K) \cup (G_Q,L) = (M_Q,N)$ where $N = K \cup L$ for all e∈ N and

$$(M_Q,N) = \begin{cases} F_Q(e) & if\ e\ \in K - L \\ G_Q(e) & if\ e\ \in L - K \\ F_Q(e) \cup G_Q(e) & if\ e\ \in K \cap L \end{cases}$$

3.2.13. Example

Let $X = \{q_1, q_2, q_3\}$ be a universal set, $E = \{\underline{a}_1, \underline{a}_2, \underline{a}_3\}$ be a set of parameters and $Q = \{\grave{u}\}$ be a non-empty set. Let $N = \{\underline{a}_1, \underline{a}_2\} \subset E, and\ M = \{\underline{a}_2, \underline{a}_3\} \subset E$

$(F_Q, N) =$

$\{(\underline{a}_1, \{((q_1, \grave{u}), [0.1,0.2], [0.3,0.4], [0.4,0.5]), \underline{a}_2, (q_1, \grave{u}), [0.3,0.4], [0.5,0.6], [0.7,0.8])$

$\}$

and

$(G_Q, M) =$

$\{(\underline{a}_2, ((q_1, \grave{u}), [0.1,0.3], [0.7,0.8], [0.8,0.9]), (\underline{a}_3\ (q_1, \grave{u}), [0.1,0.2], [0.3,0.5], [0.4,0.5]$

$)\}$

Then

$(K_Q, L) = \{(\underline{a}_1, \{((q_1, \grave{u}), [0.1,0.2], [0.3,0.4], [0.4,0.5]),$

$\underline{a}_2, (q_1, \grave{u}), [0.3,0.4], [0.5,0.6], [0.7,0.8], \underline{a}_3, ((q_1, \grave{u}), [0.1,0.2], [0.3,0.5], [0.5,0.6])\}$

3.2.14. Definition

Let (F_Q,K) *and* $(G_Q,L) \in QINSS(X)$ .Then the intersection of two $QINSSs$ (F_Q,K) *and* (G_Q,L) is the Q −interval neutrosophic soft set (M_Q,N) written as $(F_Q,K) \cap (G_Q,L) = (M_Q,N)$ where $N = K \cap L$ for all e ∈ N and

$$(M_Q,N) = \{e, \min\left(\dot{\mu}_{F_Q}(\dot{a},\grave{u}), \dot{\mu}_{G_Q}(\dot{a},\grave{u})\right), \max\left(\dot{v}_{F_Q}(\dot{a},\grave{u}), \dot{v}_{G_Q}(\dot{a},\grave{u})\right),$$

$$\max\left(\dot{\lambda}_{F_Q}(\dot{a},\grave{u}), \dot{\lambda}_{G_Q}(\dot{a},\grave{u})\right) : \dot{a} \in X, \grave{u} \in Q\}$$

3.2.15. Example

Let $X = \{q_1, q_2, q_3\}$ be a universal set, $E = \{\underline{a}_1, \underline{a}_2, \underline{a}_3\}$ be a set of parameters and $Q = \{\grave{u}\}$ be a non-empty set. Let $N = \{\underline{a}_1, \underline{a}_2\} \subset E, and\ M = \{\underline{a}_2, \underline{a}_3\} \subset E$, L=N∩M

$(F_Q,N) =$

$\{(\underline{a}_1, \{((q_1,\grave{u}), [0.1,0.2], [0.3,0.4], [0.4,0.5]), \underline{a}_2, (q_2,\grave{u}), [0.3,0.4], [0.5,0.6], [0.7,0.8])$

}

and

$(G_Q,M) =$

$\{(\underline{a}_2, ((q_2,\grave{u}), [0.1,0.3], [0.7,0.8], [0.8,0.9]), (\underline{a}_3(q_3,\grave{u}), [0.1,0.2], [0.3,0.5], [0.4,0.5])$

}

Then

$(K_Q,L) = \{\ \underline{a}_2, (q_2,\grave{u}), [0.1,0.3], [0.7,0.8], [0.8,0.9]\}$

3.2.16. Proposition

Let $(F_Q,K), (M_Q,N)$ *and* $(G_Q,L) \in QINSS(X)$. Then

i. $(F_Q, K) \cup (\emptyset, K) = (F_Q, K)$

ii. $(F_Q, K) \cup (X, K) = (X, K)$

iii. $(F_Q, K) \cup (F_{Q,}K) = (F_{Q,}K)$

iv. $(F_Q, K) \cup (G_{Q,}L) = (G_Q, L) \cup (F_{Q,}K)$

v. $(F_Q, K) \cup ((G_Q, L) \cup (M_{Q,}N)) = ((F_Q, K) \cup (G_Q, L))) \cup (M_Q, N)$.

Proof

i.

We have

$$(F_Q, K) = \{(q_1, \grave{u}), \left(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u})\right) : \grave{u} \in Q, q_1 \in X$$

$$(\emptyset, K) = \{(q_1, \grave{u}), [0,0], [1,1], [1,1]) : \grave{u} \in Q, q_1 \in X\}$$

$$(F_Q, K) \cup (\emptyset, K)$$

$$= \{(q_1, \grave{u}), max\left(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), [0,0]\right), min\left(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), [1,1]\right), min\left(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), [1,1]\right)\}$$

where

$$max\left(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), [0,0]\right) = [max(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), 0), max(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), 0)]$$

$$min\left(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), [1,1]\right) = [min(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), 1), min(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), 1)]$$

$$min\left(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), [1,1]\right) = [min(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), 1), min(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), 1)]$$

$$=\{(x_1, \grave{u}), \left(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u})\right) : \grave{u} \in Q, q_1 \in X\}$$

$$=(F_Q, K)$$

ii.

Let $(X, K) = (G_Q, K)$ then

$(F_Q, K) = \{(q_1, \grave{u}), \left(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \nu_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u})\right) : \grave{u} \in Q, q_1 \in X\}$

$(G_Q, L) = \{(q_1, \grave{u}), ([1,1], [0,0], [0,0]) : \grave{u} \in Q, q_1 \in X\}$

$(F_Q, K) \cup (G_Q, K)$

$= \{(q_1, \grave{u}), max\left(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), [1,1]\right), min\left(\dot{\nu}_{F_{Q(k)}}(p_1, \grave{u}), [0,0]\right), min\left(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), [0,0]\right)\}$

where

$max\left(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), [1,1]\right) = [max(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), 1), max(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), 1)]$

$min\left(\dot{\nu}_{F_{Q(k)}}(q_1, \grave{u}), [0,0]\right) = [min(\dot{\nu}_{F_{Q(k)}}(q_1, \grave{u}), 0), min(\dot{\nu}_{F_{Q(k)}}(q_1, \grave{u}), 0)]$

$min\left(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), [0,0]\right) = [min(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), 0), min(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), 0)]$

$=\{(q_1, \underline{u}), [1,1], [1,1], [0,0] : \grave{u} \in Q, q_1 \in X\}$

$=(G_Q, K) = (X, K)$

iii.

Let

$(F_Q, K) = \{(q_1, \grave{u}), \left(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\nu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u})\right) : \grave{u} \in Q, q_1 \in X\}$

$(F_Q, K) \cup (F_Q, K) =$

$\{(x_1, \grave{u}), max\left(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \mu_{F_{Q(k)}}(q_1, \grave{u})\right),$

$min\left(\dot{\nu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\nu}_{F_{Q(k)}}(q_1, \grave{u})\right), min\left(\dot{\lambda}_{F_{Q(k)}}(q_1 \grave{u}), \dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u})\right) : \grave{u} \in Q, q_1 \in X\}$

where

$max\left(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u})\right)$

$= [\max(inf(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}), inf(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u})), \max(sup(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}), \dot{\sup}(\mu_{F_{Q(k)}}(q_1,\grave{u}))]$

$min\left(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),\dot{v}_{F_{Q(k)}}(q_1,\grave{u})\right)$

$= [min(inf\left(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}), \dot{\inf}(v_{F_{Q(k)}}(q_1,\grave{u})), min(sup(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}), \sup(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}))]$

$min\left(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u})\right)$

$= [\min(\inf(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), \inf\left(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u})\right)), min(sup(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), \sup(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}))]$

$= \{(q_1,\grave{u}),\left(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u})\right):\grave{u}\in Q, q_1\in X\}$

$=(F_Q,K)$

iv.

$(F_Q,K) = \{(q_1,\grave{u}),\left(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u})\right):\grave{u}\in Q, x_1\in X\}$

$(G_Q,L) = \{(q_1,\grave{u}),\left(\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u}),\dot{v}_{G_{Q(k)}}(q_1,\grave{u}),\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u})\right):\grave{u}\in Q, q_1\in X\}$

$(F_Q,K)\cup(G_Q,L) = \{(q_1,\grave{u}), max(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u})),$

$min(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),\dot{v}_{G_{Q(k)}}(q_1,\grave{u})),$

$min(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u})), \grave{u}\in Q, q_1\in X\}$

where

$max(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u}))=$

$\left[max(\inf(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u})), inf(\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u})), max(sup(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u})), sup(\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u})))\right]$

$min(\dot{v}_{F_{Q(k)}}(q_1,\dot{u}), \dot{v}_{G_{Q(k)}}(q_1,\dot{u}))=$

$[min(inf(\dot{v}_{F_{Q(k)}}(q_1,\dot{u}), inf(\dot{v}_{G_{Q(k)}}(q_1,\dot{u})), min(sup(\dot{v}_{F_{Q(k)}}(q_1,\dot{u})), sup(\dot{v}_{G_{Q(k)}}(q_1,\dot{u})))]$

$min(\dot{\lambda}_{F_{Q(k)}}(q_1,\dot{u}), \dot{\lambda}_{G_{Q(k)}}(q_1,\dot{u}))=$

$[min(inf(\dot{\lambda}_{F_{Q(k)}}(q_1,\dot{u}), \inf(\dot{\lambda}_{G_{Q(k)}}(q_1,\dot{u})), min(sup(\dot{\lambda}_{F_{Q(k)}}(q_1,\dot{u}), sup(\dot{\lambda}_{G_{Q(k)}}(q_1,\dot{u})))]$

$(F_Q,K) \cup (G_Q,L) = \{(q_1,\dot{u}), max(\dot{\mu}_{G_{Q(k)}}(q_1,\dot{u}), \dot{\mu}_{F_{Q(k)}}(q_1,\dot{u})),$

$min(\dot{v}_{G_{Q(k)}}(q_1,\dot{u})\dot{v}_{F_{Q(k)}}(q_1,\dot{u})),$

$min(\dot{\lambda}_{G_{Q(k)}}(q_1,\dot{u}), \dot{\lambda}_{F_{Q(k)}}(q_1,\dot{u})), \dot{u} \in Q, q_1 \in X\}$

where

$max(\dot{\mu}_{G_{Q(k)}}(q_1,\dot{u}), \dot{\mu}_{F_{Q(k)}}(q_1,\dot{u}))=$

$[max(\inf(\dot{\mu}_{G_{Q(k)}}(q_1,\dot{u})), inf(\dot{\mu}_{F_{Q(k)}}(q_1,\dot{u}))), max(sup(\dot{\mu}_{G_{Q(k)}}(q_1,\dot{u})), sup(\dot{\mu}_{F_{Q(k)}}(q_1,\dot{u})))]$

$min(\dot{v}_{G_{Q(k)}}(q_1,\dot{u})\dot{v}_{F_{Q(k)}}(q_1,\dot{u}))= min(\dot{v}_{G_{Q(k)}}(q_1,\dot{u}), \dot{v}_{F_{Q(k)}}(q_1,\dot{u}))=$

$[min(inf(\dot{v}_{G_{Q(k)}}(q_1,\dot{u})), \inf(\dot{v}_{F_{Q(k)}}(q_1,\dot{u}))), \min(\sup(\dot{v}_{G_{Q(k)}}(q_1,\dot{u})), \sup(\dot{v}_{F_{Q(k)}}(q_1,\dot{u})))]$

$min(\dot{\lambda}_{F_{G(k)}}(q_1,\dot{u}), \dot{\lambda}_{F_{Q(k)}}(q_1,\dot{u}))=$

$[min(inf(\dot{\lambda}_{G_{Q(k)}}(q_1,\dot{u})), \inf(\dot{\lambda}_{F_{Q(k)}}(q_1,\dot{u}))), \min(\sup(\dot{\lambda}_{G_{Q(k)}}(q_1,\dot{u})), \sup(\dot{\lambda}_{F_{Q(k)}}(q_1,\dot{u})))]$

$=(G_Q,L) \cup (F_Q,K)$

v.

$(F_Q,K) = \{(q_1,\dot{u}), \left(\dot{\mu}_{F_{Q(k)}}(q_1,\dot{u}), \dot{v}_{F_{Q(k)}}(q_1,\dot{u}), \dot{\lambda}_{F_{Q(k)}}(q_1,\dot{u})\right): \dot{u} \in Q, q_1 \in X\}$

$(G_Q,L) = \{(q_1,\dot{u}), \left(\dot{\mu}_{G_{Q(k)}}(q_1,\dot{u}), \dot{v}_{G_{Q(k)}}(q_1,\dot{u}), \dot{\lambda}_{G_{Q(k)}}(q_1,\dot{u})\right): \dot{u} \in Q, q_1 \in X\}$

$(F_Q, K) \cup (G_Q, L) = \{(q_1, \grave{u}), max(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})),$

$min(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{v}_{G_{Q(k)}}(q_1, \grave{u})),$

$min(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u})), \grave{u} \in Q, q_1 \in X\}$

where

$max(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1, \grave{u}))=$

$[max(inf(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u})), inf(\dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})), max(sup(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u})), sup(\dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})))]$

$min(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{v}_{G_{Q(k)}}(q_1, \grave{u}))=$

$[min(inf(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), inf(\dot{v}_{G_{Q(k)}}(q_1, \grave{u})), min(sup(\dot{v}_{F_{Q(k)}}(q_1, \grave{u})), sup(\dot{v}_{G_{Q(k)}}(q_1, \grave{u})))]$

$min(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u}))$

$[min(inf(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), inf(\dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u})), min(sup(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \sup(\dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u})))]$

and

$(M_Q, N) = \{(q_1, \grave{u}), \left(\dot{\mu}_{M_{Q(k)}}(q_1, \grave{u}), \dot{v}_{M_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{M_{Q(k)}}(q_1, \grave{u})\right) : \grave{u} \in Q, q_1 \in X\}$

$((F_Q, K) \cup (G_Q, L)) \cup (M_Q, N) = \{(q_1, \grave{u}), max((\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})),$

$\dot{\mu}_{M_{Q(k)}}(q_1, \grave{u})) min((\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{v}_{G_{Q(k)}}(q_1, \grave{u})), \dot{v}_{M_{Q(k)}}(q_1, \grave{u})),$

$min(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{M_{Q(k)}}(q_1, \grave{u})), \grave{u} \in Q, q_1 \in X\}$

where

$max((\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})),$

$\dot{\mu}_{M_{Q(k)}}(q_1, \grave{u})) = [max(inf(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), (\dot{\mu}_{G_{Q(k)}}(q_1, \grave{u}))),$

$inf(\dot{\mu}_{M_{Q(k)}}(q_1, \grave{u})), max((sup(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})), sup(\dot{\mu}_{M_{Q(k)}}(q_1, \grave{u})))]$

$min((\dot{\nu}_{F_{Q(k)}}(q_1,\grave{u}), \dot{\nu}_{G_{Q(k)}}(q_1,\grave{u})),$

$\dot{\nu}_{M_{Q(k)}}(q_1,\grave{u})) = [min(inf(\dot{\nu}_{F_{Q(k)}}(q_1,\grave{u}), \dot{\nu}_{G_{Q(k)}}(q_1,\grave{u})),$

$\inf(\dot{\nu}_{M_{Q(k)}}(q_1,\grave{u})), min(sup(\dot{\nu}_{F_{Q(k)}}(q_1,\grave{u}), \dot{\nu}_{G_{Q(k)}}(q_1,\grave{u})), sup(\dot{\nu}_{M_{Q(k)}}(q_1,\grave{u})))]$

$min((\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u})),$

$\dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u})) = [min(inf(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u})),$

$\dot{\inf}(\lambda_{M_{Q(k)}}(q_1,\grave{u})), min(sup(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u})), sup(\dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u})))]$

$= \{(q_1,\grave{u}), max(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}), (\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u}),$

$\dot{\mu}_{M_{Q(k)}}(q_1,\grave{u}))min(\dot{\nu}_{F_{Q(k)}}(q_1,\grave{u}), (\dot{\nu}_{G_{Q(k)}}(q_1,\grave{u}), \dot{\nu}_{M_{Q(k)}}(q_1,\grave{u}))$

$min(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), (\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u}), \dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u})), \grave{u} \in Q, q_1 \in X\}$

where

$max(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}), (\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u})),$

$\dot{\mu}_{M_{Q(k)}}(q_1,\grave{u})) = [max(inf(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}), \inf((\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u}),$

$(\dot{\mu}_{M_{Q(k)}}(q_1,\grave{u}))), max(sup(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}), sup((\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u}), \dot{\mu}_{M_{Q(k)}}(q_1,\grave{u})))]$

$min((\dot{\nu}_{F_{Q(k)}}(q_1,\grave{u}), ((\dot{\nu}_{G_{Q(k)}}(q_1,\grave{u}),$

$\dot{\nu}_{M_{Q(k)}}(q_1,\grave{u})) = [min(inf(\dot{\nu}_{F_{Q(k)}}(q_1,\grave{u}), inf((\dot{\nu}_{G_{Q(k)}}(q_1,\grave{u}),$

$(\dot{\nu}_{M_{Q(k)}}(q_1,\grave{u})), min(sup((\dot{\nu}_{F_{Q(k)}}(q_1,\grave{u}), \dot{\nu}_{G_{Q(k)}}(q_1,\grave{u})), sup(\dot{\nu}_{M_{Q(k)}}(q_1,\grave{u}))]$

$min((\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), ((\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u}),$

$\dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u}))) = [min(inf((\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), (\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u}))),$

$inf(\dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u}))), min(\sup((\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), (\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u})), sup(\dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u})))]$

$=(F_Q, K) \cup ((G_Q, L) \cup (M_Q, N))$

3.3.16. Proposition

Let $(F_Q,K),(M_Q,N)$ and $(G_Q,L) \in QINSS(X)$. Then

i. $(F_Q,K) \cap (\emptyset,K) = (\emptyset,K)$

ii. $(F_Q,K) \cap (X,K) = (F_Q,K)$

iii. $(F_Q,K) \cap (F_Q,K) = (F_Q,K)$

iv. $(F_Q,K) \cap (G_{Q,}L) = (G_Q,L) \cap (F_{Q,}K)$

v. $(F_Q,K) \cap ((G_Q,L) \cap (M_Q,N)) = ((G_Q,L) \cap (F_Q,K)) \cap (M_Q,N)$.

Proof

i.

We have

$$(F_Q,K) = \{(p_1,\grave{u}),\left(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u})\right):\grave{u} \in Q,q_1 \in X$$

$$(\emptyset,K) = \{(q_1,\grave{u}),([0,0],[1,1],[1,1]):\grave{u} \in Q,q_1 \in X\}$$

$(F_Q,K) \cap\ (\emptyset,K)$

$$= \{(x_1,\grave{u}),min\left(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),[0,0]\right),max\left(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),[1,1]\right),max\left(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}),[1,1]\right)\}$$

where

$$min\left(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),[0,0]\right) = [min(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),0),min(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),0)]$$

$$max\left(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),[1,1]\right) = [max(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),1),max(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),1)]$$

$$max\left(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}),[1,1]\right) = [max(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}),1),max(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}),1)]$$

$$=\{(q_1,\grave{u}),([0,0],[1,1],[1,1]):\grave{u} \in Q,q_1 \in X$$

$= (\emptyset, K)$.

ii.

Let $(X, K) = (G_Q, L)$ then

$$(F_Q, K) = \{(q_1, \dot{u}), \left(\dot{\mu}_{F_{Q(k)}}(q_1, \dot{u}), \dot{\nu}_{F_{Q(k)}}(q_1, \dot{u}), \dot{\lambda}_{F_{Q(k)}}(q_1, \dot{u})\right) : \dot{u} \in Q, q_1 \in X$$

$$(G_Q, L) = \{(q_1, \dot{u}), ([1,1], [0,0], [0,0]) : \dot{u} \in Q, q_1 \in X\}$$

$$(F_Q, K) \cap (G_Q, L)$$

$$= \{(q_1, \dot{u}), min\left(\dot{\mu}_{F_{Q(k)}}(q_1, \dot{u}), [1,1]\right), max\left(\dot{\nu}_{F_{Q(k)}}(q_1, \dot{u}), [0,0]\right), max\left(\dot{\lambda}_{F_{Q(k)}}(q_1, \dot{u}), [0,0]\right)\}$$

where

$$min\left(\dot{\mu}_{F_{Q(k)}}(q_1, \dot{u}), [1,1]\right) = [min(\dot{\mu}_{F_{Q(k)}}(q_1, \dot{u}), 1), min(\dot{\mu}_{F_{Q(k)}}(q_1, \dot{u}), 1)]$$

$$max\left(\dot{\nu}_{F_{Q(k)}}(q_1, \dot{u}), [0,0]\right) = [max(\dot{\nu}_{F_{Q(k)}}(q_1, \dot{u}), 0), max(\dot{\nu}_{F_{Q(k)}}(q_1, \dot{u}), 0)]$$

$$max\left(\dot{\lambda}_{F_{Q(k)}}(q_1, \dot{u}), [0,0]\right) = [max(\dot{\lambda}_{F_{Q(k)}}(q_1, \dot{u}), 0), max(\dot{\lambda}_{F_{Q(k)}}(q_1, \dot{u}), 0)]$$

$$=\{(q_1, \dot{u}), \left(\dot{\mu}_{F_{Q(k)}}(q_1, \dot{u}), \dot{\nu}_{F_{Q(k)}}(q_1, \dot{u}), \dot{\lambda}_{F_{Q(k)}}(q_1, \dot{u})\right) : \dot{u} \in Q, q_1 \in X\}$$

$= (F_Q, K)$.

iii.

Let

$$(F_Q, K) = \{(q_1, \dot{u}), \left(\dot{\mu}_{F_{Q(k)}}(q_1, \dot{u}), \dot{\nu}_{F_{Q(k)}}(q_1, \dot{u}), \dot{\lambda}_{F_{Q(k)}}(q_1, \dot{u})\right) : \dot{u} \in Q, q_1 \in X\}$$

$(F_Q, K) \cap (F_Q, K) =$

$\{(q_1, \grave{u}), min\left(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{F_{Q(k)}}(q_1, \grave{u})\right),$

$min\left(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{v}_{F_{Q(k)}}(q_1, \grave{u})\right), \min(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{F_{Q(k)}}(q_1, \underline{u})\, Q, x_1 \in: q_1 \in$

$X, \grave{u} \in Q\}$

$=\{(q_1, \grave{u}), \left(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u})\right): \grave{u} \in Q, q_1 \in X\}$

$=(F_Q, K).$

iv.

$(F_Q, K) = \{(q_1, \grave{u}), \left(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u})\right): \grave{u} \in Q, q_1 \in X\}$

$(G_Q, L) = \{(x_1, u), \left(\dot{\mu}_{G_{Q(k)}}(q_1, \grave{u}), \dot{v}_{G_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u})\right): \grave{u} \in Q, q_1 \in X\}$

$(F_Q, K) \cap (G_Q, L) = \{(q_1, \grave{u}), min(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})),$

$max\, (\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{v}_{G_{Q(k)}}(q_1, \grave{u})),$

$max\, (\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u})), \grave{u} \in Q, q_1 \in X\}$

where

$min(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1, \grave{u}))=$

$[min(inf(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), inf(\dot{\mu}_{G_{Q(k)}}(q_1, \grave{u}))), min(sup(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), sup(\dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})))]$

$max(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{v}_{G_{Q(k)}}(q_1, \grave{u}))=$

$[max(inf(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), inf(\dot{v}_{G_{Q(k)}}(q_1, \grave{u}))), max(sup(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), sup(\dot{v}_{G_{Q(k)}}(q_1, \grave{u})))]$

$max(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u}))=$

$[max(inf(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}),inf(\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u}))),max(sup(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}),s\dot{u}p(\lambda_{G_{Q(k)}}(q_1,\grave{u})))]$

$(F_Q,K)\cap(G_Q,L)=\{(q_1,\grave{u}),min(\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u}),\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u})),$ max

$(\dot{v}_{G_{Q(k)}}(q_1,\grave{u}),\dot{v}_{F_{Q(k)}}(q_1,\grave{u})),$

$max(\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u}),\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u})),\grave{u}\in Q,q_1\in X\}$

where

$min(\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u}),\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}))=$

$[min(inf(\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u}),inf(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}))),min(sup(\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u}),sup(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u})))]$

$max(\dot{v}_{G_{Q(k)}}(q_1,\grave{u})\dot{v}_{F_{Q(k)}}(q_1,\grave{u}))=$

$[max(inf(\dot{v}_{G_{Q(k)}}(q_1,\grave{u}),inf(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}))),max(sup(\dot{v}_{G_{Q(k)}}(q_1,\grave{u}),sup(\dot{v}_{F_{Q(k)}}(q_1,\grave{u})))]$

$max(\dot{\lambda}_{F_{G(k)}}(q_1,\grave{u}),\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}))=$

$[max(inf(\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u}),inf(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}))),max(sup(\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u}),sup(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u})))]$

$=(G_Q,L)\cap(F_Q,K)$

v.

$(F_Q,K)=\{(q_1,\grave{u}),\left(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u})\right):\grave{u}\in Q,q_1\in X\}$

$(G_Q,L)=\{(q_1,\grave{u}),\left(\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u}),\dot{v}_{G_{Q(k)}}(q_1,\grave{u}),\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u})\right):\grave{u}\in Q,q_1\in X\}$

$(F_Q,K)\cap(G_Q,L)=\{(q_1,\grave{u}),min(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u})),$ max

$(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),\dot{v}_{G_{Q(k)}}(q_1,\grave{u})),$

$max(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u})),\ \grave{u}\in Q,q_1\in X\}$

where

$min(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u}))=$

$[min(inf(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),inf(\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u}))),min(sup(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),sup(\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u})))]$

$max(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),\dot{v}_{G_{Q(k)}}(q_1,\grave{u}))=$

$[max(inf(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),inf(\dot{v}_{G_{Q(k)}}(q_1,\grave{u}))),max(sup(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),sup(\dot{v}_{G_{Q(k)}}(q_1,\grave{u})))]$

$max(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u}))=$

$[max(inf(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}),inf(\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u}))),max(sup(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}),s\dot{u}p(\lambda_{G_{Q(k)}}(q_1,\grave{u})))]$

and $(M_Q,N) = \{(q_1,\grave{u}),\left(\dot{\mu}_{M_{Q(k)}}(q_1,\grave{u}),\dot{v}_{M_{Q(k)}}(q_1,\grave{u}),\dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u})\right):\grave{u}\in Q,q_1\in X\}$

$((F_Q,K)\cap(G_Q,L))\cap(M_Q,N)=\{(q_1,\grave{u}),min((\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u})),$

$\dot{\mu}_{M_{Q(k)}}(q_1,\grave{u})),max((\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),\dot{v}_{G_{Q(k)}}(q_1,\grave{u})),\dot{v}_{M_{Q(k)}}(q_1,\grave{u})),$

$max(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u})),\dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u})),\grave{u}\in Q,q_1\in X\}$

where

$min((\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u})),$

$\dot{\mu}_{M_{Q(k)}}(q_1,\grave{u}))=[min(inf(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u})),$

$inf(\dot{\mu}_{M_{Q(k)}}(q_1,\grave{u}))),min(sup(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u})),sup(\dot{\mu}_{M_{Q(k)}}(q_1,\grave{u})))]$

$max((\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),\dot{v}_{G_{Q(k)}}(q_1,\grave{u})),$

$\dot{v}_{M_{Q(k)}}(q_1,\grave{u}))=[max(inf(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),\dot{v}_{G_{Q(k)}}(q_1,\grave{u})),$

$inf(\dot{v}_{M_{Q(k)}}(q_1,\grave{u}))),max(sup(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),\dot{v}_{G_{Q(k)}}(q_1,\grave{u})),sup(\dot{v}_{M_{Q(k)}}(q_1,\grave{u})))]$

$max((\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u})),$

$\dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u})) = [max(inf(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u})),$

$inf(\dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u}))), max(sup(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u})), sup(\dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u})))]$

$= \{(q_1,\grave{u}), min(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}), (\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u}),$

$\mu_{M_{Q(k)}}(q_1,\grave{u})), max(\dot{\nu}_{F_{Q(k)}}(q_1,\grave{u}), (\dot{\nu}_{G_{Q(k)}}(q_1,\grave{u}), \dot{\nu}_{M_{Q(k)}}(q_1,\grave{u}))$

$max(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), (\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u}), \dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u})), \grave{u} \in Q, q_1 \in X\}$

where

$min(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}), (\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u})),$

$\dot{\mu}_{M_{Q(k)}}(q_1,\grave{u})) =$

$[min(inf(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}), inf((\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u}),$

$(\dot{\mu}_{M_{Q(k)}}(q_1,\grave{u}))), min(sup(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}), sup((\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u}), \dot{\mu}_{M_{Q(k)}}(q_1,\grave{u})))]$

$max((\dot{\nu}_{F_{Q(k)}}(q_1,\grave{u}), ((\dot{\nu}_{G_{Q(k)}}(q_1,\grave{u}),$

$\dot{\nu}_{M_{Q(k)}}(q_1,\grave{u})) = [max(inf(\dot{\nu}_{F_{Q(k)}}(q_1,\grave{u}), inf((\dot{\nu}_{G_{Q(k)}}(q_1,\grave{u}),$

$\dot{\nu}_{M_{Q(k)}}(q_1,\grave{u}))), max(sup(\dot{\nu}_{F_{Q(k)}}(q_1,\grave{u}), sup((\dot{\nu}_{G_{Q(k)}}(q_1,\grave{u}), \dot{\nu}_{M_{Q(k)}}(q_1,\grave{u}))))]$

$max((\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), (\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u}),$

$\dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u}))) = [max(inf(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), inf((\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u}),$

$\dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u}))), max(sup(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), sup((\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u}), \dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u}))))]$

$=(F_Q,K) \cap ((G_Q,L) \cap (M_Q,N))$

3.3.17. Proposition

Let (F_Q, K) *and* $(G_Q, L) \in QINSS(X)$. Then

i. $((F_Q, K) \cup (G_Q, L))^C = (F_Q, K)^C \cap (G_Q, L)^C$

ii. $((F_Q, K) \cap (G_Q, L))^C = (F_Q, K)^C \cup (G_Q, L)^C$

Proof

i.

Let

$$(F_Q, K) = \{(q_1, \grave{u}), \left(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\nu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u})\right) : \grave{u} \in Q, q_1 \in X\}$$

$$(G_Q, L) = \{(q_1, \grave{u}), \left(\dot{\mu}_{G_{Q(k)}}(q_1, \grave{u}), \dot{\nu}_{G_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u})\right) : \grave{u} \in Q, q_1 \in X\}$$

$(F_Q, K) \cup (G_Q, L) = \{(q_1, \grave{u}), max(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})),$

$min(\dot{\nu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\nu}_{G_{Q(k)}}(q_1, \grave{u})),$

$min(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u})), \grave{u} \in Q, q_1 \in X\}$

where

$max(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1, \grave{u}))=$

$\left[\max(\inf(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u})), inf(\dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})), max(sup\ (\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u})), sup(\dot{\mu}_{G_{Q(k)}}(q_1, \grave{u}))\right]$

$min(\dot{\nu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\nu}_{G_{Q(k)}}(q_1, \grave{u}))=$

$[min(inf(\dot{\nu}_{F_{Q(k)}}(q_1, \grave{u}), \inf(\dot{\nu}_{G_{Q(k)}}(q_1, \grave{u})), min(sup(\dot{\nu}_{F_{Q(k)}}(q_1, \grave{u})), \sup(\dot{\nu}_{G_{Q(k)}}(q_1, \grave{u})))]$

$min(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u}))=$

$[min(inf(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \inf(\dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u})), min(sup(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \sup(\dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u})))]$

$((F_Q, K) \cup (G_Q, L))^C = \{(q_1, \grave{u}), max(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u})),$

$min\Big(([1 - sup\ \dot{v}_{F_{Q(k)}}(q_1, \grave{u}), 1 - inf\dot{v}_{F_{Q(k)}}(q_1, \grave{u})]), \Big(\Big[1 - sup\dot{v}_{G_{Q(k)}}(q_1, \grave{u}), 1 -$

$inf\dot{v}_{G_{Q(k)}}(q_1, \grave{u})\Big]\Big)\Big), min(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \mu_{G_{Q(k)}}(q_1, \grave{u}))\ , \grave{u} \in Q, q_1 \in X\}$..........(1)

Now

$(F_Q, K)^c =$

$\{(q_1, \grave{u}), (\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), (\big[1 - sup\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), 1 -$

$inf\dot{v}_{F_{Q(k)}}(q_1, \grave{u})\big]), \dot{\mu}_{F_{Q(k)}}(x_1, \grave{u})): \grave{u} \in Q, q_1 \in X\}$

$(G_Q, L)^c =$

$\{(q_1, \grave{u}), (\dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u}), (\big[1 - suq\dot{v}_{G_{Q(k)}}(q_1, \grave{u}), 1 -$

$inf\dot{v}_{G_{Q(k)}}(q_1, \grave{u})\big]), \dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})): \grave{u} \in Q, q_1 \in X\}$

$(F_Q, K)^c \cap (G_Q, L)^c = \{(q_1, \grave{u}), \Big(\Big(min(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u})), max([1 -$

$sup\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), 1 - inf\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{F_{Q(k)}}(q_1, \grave{u})]\Big), \Big(\Big[1 - sup\dot{v}_{G_{Q(k)}}(q_1, \grave{u}), 1 -$

$inf\dot{v}_{G_{Q(k)}}(q_1, \grave{u})\Big]\Big)\Big), max(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})): \grave{u} \in Q, q_1 \in X\}$........(2)

where

$min(\ \dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u}))=$

$[min(inf(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \inf(\dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u})), min(sup(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \sup(\dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u})))]$

$max(\ \dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1, \grave{u}))=$

$[max(inf(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \inf(\dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})), max(sup(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \sup(\dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})))]$

From Equation (1) and (2)

$((F_Q, K) \cup (G_Q, L))^C = (F_Q, K)^c \cap (G_Q, L)^c$

ii.

$$(F_Q, K) = \{(q_1, \dot{u}), \left(\dot{\mu}_{F_{Q(k)}}(q_1, \dot{u}), \dot{v}_{F_{Q(k)}}(q_1, \dot{u}), \dot{\lambda}_{F_{Q(k)}}(q_1, \dot{u})\right) : \dot{u} \in Q, q_1 \in X\}$$

$$(G_Q, L) = \{(q_1, \dot{u}), \left(\dot{\mu}_{G_{Q(k)}}(q_1, \dot{u}), \dot{v}_{G_{Q(k)}}(q_1, \dot{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \dot{u})\right) : \dot{u} \in Q, q_1 \in X\}$$

$(F_Q, K) \cap (G_Q, L) = \{(q_1, \dot{u}), min(\dot{\mu}_{F_{Q(k)}}(q_1, \dot{u}), \dot{\mu}_{G_{Q(k)}}(q_1, \dot{u})),$

$max(\dot{v}_{F_{Q(k)}}(q_1, \dot{u}), \dot{v}_{G_{Q(k)}}(q_1, \dot{u})),$

$max(\dot{\lambda}_{F_{Q(k)}}(q_1, \dot{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \dot{u})), \dot{u} \in Q, q_1 \in X\}$

where

$min(\dot{\mu}_{F_{Q(k)}}(q_1, \dot{u}), \dot{\mu}_{G_{Q(k)}}(q_1, \dot{u}))=$

$[min(inf(\dot{\mu}_{F_{Q(k)}}(q_1, \dot{u}), \inf(\dot{\mu}_{G_{Q(k)}}(q_1, \dot{u}))), min(sup(\dot{\mu}_{F_{Q(k)}}(q_1, \dot{u}), \sup(\dot{\mu}_{G_{Q(k)}}(q_1, \dot{u})))]$

$max(\dot{v}_{F_{Q(k)}}(q_1, \dot{u}), \dot{v}_{G_{Q(k)}}(q_1, \dot{u}))=$

$[max(inf(\dot{v}_{F_{Q(k)}}(q_1, \dot{u}), \inf(\dot{v}_{G_{Q(k)}}(q_1, \dot{u}))), max(sup(\dot{v}_{F_{Q(k)}}(q_1, \dot{u}), \sup(\dot{v}_{G_{Q(k)}}(q_1, \dot{u})))]$

$max(\dot{\lambda}_{F_{Q(k)}}(q_1, \dot{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \dot{u}))=$

$[max(inf(\dot{\lambda}_{F_{Q(k)}}(q_1, \dot{u}), \inf(\dot{\lambda}_{G_{Q(k)}}(q_1, \dot{u}))), max(sup(\dot{\lambda}_{F_{Q(k)}}(q_1, \dot{u}), \dot{\sup}(\lambda_{G_{Q(k)}}(q_1, \dot{u})))]$

$((F_Q, K) \cap (G_Q, L))^C = \{(q_1, \dot{u}), max(\dot{\lambda}_{F_{Q(k)}}(q_1, \dot{u}), \dot{\lambda}_{F_{Q(k)}}(q_1, \dot{u})),$

$max([1 - sup\, \dot{v}_{F_{Q(k)}}(q_1, \dot{u}), 1 - inf\, \dot{v}_{F_{Q(k)}}(q_1, \dot{u})]), \left(\left[1 - sup\dot{v}_{G_{Q(k)}}(q_1, \dot{u}), 1 -\right.\right.$

$\left.\left. inf\,\dot{v}_{G_{Q(k)}}(q_1, \dot{u})\right]\right), min(\dot{\mu}_{F_{Q(k)}}(q_1, \dot{u}), \mu_{F_{Q(k)}}(q_1, \dot{u})), \dot{u} \in Q, q_1 \in X\}$..........(1)

$max(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}))=$

where

$max(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u}))=$

$[max(inf(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \inf(\dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u}))), max(sup(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), sup(\dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u})))]$

$min(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1, \grave{u}))=$

$[max(inf(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \inf(\dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})), min(sup(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \sup(\dot{\mu}_{G_{Q(k)}}(q_1, \grave{u}))]$

Now

$(F_Q, K)^c =$

$\{(q_1, \grave{u}), \left(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), (\left[1 - suq\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), 1 - \right.\right.$

$\left.\left. inf v_{F_{Q(k)}}(q_1, \grave{u})\right]), \dot{\mu}_{F_{Q(k)}}(x_1, \grave{u})\right) : \grave{u} \in Q, q_1 \in X\}$

$(G_Q, L)^c =$

$\{(q_1, \grave{u}), \left(\dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u}), (\left[1 - suq\dot{v}_{G_{Q(k)}}(q_1, \grave{u}), 1 - \right.\right.$

$\left.\left. inf\boldsymbol{v}_{G_{Q(k)}}(q_1, \grave{u})\right]), \dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})\right) : \grave{u} \in Q, q_1 \in X\}$

$(F_Q, K)^c \cup (G_Q, L)^c = \{(q_1, \grave{u}), \left(max(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u})), min([1 - \right.$

$sup\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), 1 - inf\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{F_{Q(k)}}(q_1, \grave{u})]\Big), \Big(\Big[1 - sup\dot{v}_{G_{Q(k)}}(q_1, \grave{u}), 1 -$

$inf\dot{v}_{G_{Q(k)}}(q_1, \grave{u})\Big]\Big)), min(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})) : \grave{u} \in Q, q_1 \in X\}$ .... (2)

where

$min(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1, \grave{u}))=$

$[max(inf(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \inf(\dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})), min(sup(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \sup(\dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})))]$

$max(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u}))=$

$[max(inf(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \inf(\dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u}))), max(sup(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \sup(\dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u})))]$

From Equation (1) and (2)

$((F_Q, K) \cap (G_Q, L))^C=(F_Q, K)^c \cup (G_Q, L)^c$

3.3.18. Proposition

Let $(F_Q, K), (M_Q, N)$ *and* $(G_Q, L) \in QINS(X)$. Then

i. $(F_Q, K) \cup ((G_Q, L) \cap (M_Q, N)) = ((F_Q, K) \cup (G_Q, L)) \cap ((F_Q, K) \cup (M_Q, N)$

ii. $(F_Q, K) \cap ((G_Q, L) \cup (M_Q, N)) = ((F_Q, K) \cap (G_Q, L)) \cup ((F_Q, K)) \cap (M_Q, N))$

Proof

i.

$(F_Q, K) = \{(q_1, \grave{u}), \left(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u})\right) : \grave{u} \in Q, q_1 \in X\}$

$(G_Q, L) = \{(q_1, \grave{u}), \left(\dot{\mu}_{G_{Q(k)}}(q_1, \grave{u}), \dot{v}_{G_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u})\right) : \grave{u} \in Q, q_1 \in X\}$

$(M_Q, N) = \{(q_1, \grave{u}), \left(\dot{\mu}_{M_{Q(k)}}(q_1, \grave{u}), \dot{v}_{M_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{M_{Q(k)}}(q_1, \grave{u})\right) : \grave{u} \in Q, q_1 \in X\}$

$(F_Q, K) \cup (G_Q, L) =$

$\{(q_1, \grave{u}), max\left(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})\right), min\left(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{v}_{G_{Q(k)}}(q_1, \grave{u})\right),$

$min\left(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u})\right)\}$

where

$max(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1, \grave{u}))=$

$[max(inf(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u})), inf\ (\dot{\mu}_{G_{Q(k)}}(q_1, \grave{u}))), max(sup(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u})), \sup(\dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})))]$

$min(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{v}_{G_{Q(k)}}(q_1, \grave{u}))=$

$[min(inf(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \inf(\dot{v}_{G_{Q(k)}}(q_1, \grave{u})), min(sup(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), sup\ (\dot{v}_{G_{Q(k)}}(q_1, \grave{u})))]$

$min(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u}))=$

$[min(inf(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\inf}(\lambda_{G_{Q(k)}}(q_1, \grave{u})), min(sup(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u})), sup\ (\dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u})))]$

$(F_Q, K) \cup (G_Q, L) =$

$$\{(q_1, \grave{u}), \begin{pmatrix} max\left(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{M_{Q(k)}}(q_1, \grave{u})\right), min(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{v}_{M_{Q(k)}}(q_1, \grave{u})), \\ min(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{M_{Q(k)}}(q_1, \grave{u})) \end{pmatrix} : \grave{u} \in$$

$Q, q_1 \in X\}$

where

$max(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{M_{Q(k)}}(q_1, \grave{u}))=$

$[max(inf(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u})), \dot{inf}\ (\mu_{M_{Q(k)}}(q_1, \grave{u})), max(sup(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u})),$

$sup\ (\dot{\mu}_{M_{Q(k)}}(q_1, \grave{u})))]$

$min(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{v}_{M_{Q(k)}}(q_1, \grave{u}))=$

$[min(inf(\dot{v}_{F_{Q(k)}}(q_1, \grave{u})), inf\ (\dot{v}_{M_{Q(k)}}(q_1, \grave{u})), min(sup(\dot{v}_{F_{Q(k)}}(q_1, \grave{u})), sup\ (\dot{v}_{M_{Q(k)}}(q_1, \grave{u})))]$

$min(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{M_{Q(k)}}(q_1, \grave{u}))=$

$[min(inf(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{inf}\ (\lambda_{M_{Q(k)}}(q_1, \grave{u})), min(sup\left(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u})\right), sup\ (\dot{\lambda}_{M_{Q(k)}}(q_1, \grave{u})))]$

$((F_Q, K) \cup (G_Q, L)) \cap ((F_Q, K) \cup (M_Q, N)) =$

={

$(q_1,\grave{u}), min\left(max((\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1,\grave{u}), max\left(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}), \dot{\mu}_{M_{Q(k)}}(q_1,\grave{u})\right)\right),$

$max((min\left(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}), \dot{v}_{G_{Q(k)}}(q_1,\grave{u})\right), min(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}), \dot{v}_{M_{Q(k)}}(q_1,\grave{u}))),$

$max((min\left(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u})\right), min\ (\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), \dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u})))\}$.........

........(1)

$min(max(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1,\grave{u}), max(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}), (\dot{\mu}_{M_{Q(k)}}(q_1,\grave{u}))=$

$[min(inf(min(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1,\grave{u}), min(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}), (\dot{\mu}_{M_{Q(k)}}(q_1,\grave{u}))),$

$min(sup(min(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1,\grave{u}), min(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}), (\dot{\mu}_{M_{Q(k)}}(q_1,\grave{u})))]$

$max(min(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}), \dot{v}_{G_{Q(k)}}(q_1,\grave{u}), min(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}), (\dot{v}_{M_{Q(k)}}(q_1,\grave{u}))=$

$[max(inf(min(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}), \dot{v}_{G_{Q(k)}}(q_1,\grave{u}), min(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}), (\dot{v}_{M_{Q(k)}}(q_1,\grave{u}))),$

$max(sup(min(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}), \dot{v}_{G_{Q(k)}}(q_1,\grave{u}), min(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}), (\dot{v}_{M_{Q(k)}}(q_1,\grave{u})))]$

$max(min(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u}), min(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), (\dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u}))=$

$[max(inf(min(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u}), min(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), (\dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u}))),$

$max(sup(min(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u}), min(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}), (\dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u})))]$

$(G_Q, L) \cap (M_Q, N)=$

{

$(q_1,\grave{u}), min(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}), \dot{\mu}_{M_{Q(k)}}(q_1,\grave{u})), max\left(\dot{v}_{G_{Q(k)}}(q_1,\grave{u}), \dot{v}_{M_{Q(k)}}(q_1,\grave{u})\right), max(\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u}), \dot{\lambda}_{M}$

}

where

$min(\dot{v}_{G_{Q(k)}}(q_1,\grave{u}),\dot{v}_{M_{Q(k)}}(q_1,\grave{u}))$

$= [min(inf(\dot{v}_{G_{Q(k)}}(q_1,\grave{u})),inf\ (\dot{v}_{M_{Q(k)}}(q_1,\grave{u})),min(sup(\dot{v}_{G_{Q(k)}}(q_1,\grave{u})),sup\ (\dot{v}_{M_{Q(k)}}(q_1,\grave{u})))]$

$max(\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u}),\dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u})) =$

$[max(inf(\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u}),inf\ (\dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u})),max(sup(\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u})),sup\ (\dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u})))]$

$max(\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u}),\dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u}))$

$= [max(inf(\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u})),inf\ (\dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u})),max(sup(\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u})),sup\ (\dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u})))]$

$(F_Q,K) \cup ((G_Q,L) \cap (M_Q,N)) =$

$\{ (q_1,\grave{u}),max(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),min(\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u}),\dot{\mu}_{M_{Q(k)}}(q_1,\grave{u})),min(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),$

$max\left(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),\dot{v}_{M_{Q(k)}}(q_1,\grave{u})\right),$

$min(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}),max(\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u}),\dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u}))\}$

$=\{$

$(q_1,\grave{u}),min\left(max((\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u}),max\left(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\mu}_{M_{Q(k)}}(q_1,\grave{u})\right)\right),$

$max((min\left(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),\dot{v}_{G_{Q(k)}}(q_1,\grave{u})\right),min(\dot{v}_{F_{Q(k)}}(q_1,\grave{u}),\dot{v}_{M_{Q(k)}}(q_1,\grave{u}))),$

$max((min\left(\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\lambda}_{G_{Q(k)}}(q_1,\grave{u})\right),min\ (\dot{\lambda}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\lambda}_{M_{Q(k)}}(q_1,\grave{u})))\}$

.... (2)

where

$min(max(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),\dot{\mu}_{G_{Q(k)}}(q_1,\grave{u}),max(\dot{\mu}_{F_{Q(k)}}(q_1,\grave{u}),(\dot{\mu}_{M_{Q(k)}}(q_1,\grave{u}))=$

$[min(inf(min(\dot{\mu}_{F_{Q(k)}}(q_1,\dot{u}),\dot{\mu}_{G_{Q(k)}}(q_1,\dot{u}),min(\dot{\mu}_{F_{Q(k)}}(q_1,\dot{u}),(\dot{\mu}_{M_{Q(k)}}(q_1,\dot{u}))),$

$min(sup(min(\dot{\mu}_{F_{Q(k)}}(q_1,\dot{u}),\dot{\mu}_{G_{Q(k)}}(q_1,\dot{u}),min(\dot{\mu}_{F_{Q(k)}}(q_1,\dot{u}),(\dot{\mu}_{M_{Q(k)}}(q_1,\dot{u})))]$

$max(min(\dot{v}_{F_{Q(k)}}(q_1,\dot{u}),\dot{v}_{G_{Q(k)}}(q_1,\dot{u}),min(\dot{v}_{F_{Q(k)}}(q_1,\dot{u}),(\dot{v}_{M_{Q(k)}}(q_1,\dot{u}))=$

$[max(inf(min(\dot{v}_{F_{Q(k)}}(q_1,\dot{u}),\dot{v}_{G_{Q(k)}}(q_1,\dot{u}),min(\dot{v}_{F_{Q(k)}}(q_1,\dot{u}),(\dot{v}_{M_{Q(k)}}(q_1,\dot{u}))),$

$max(sup(min(\dot{v}_{F_{Q(k)}}(q_1,\dot{u}),\dot{v}_{G_{Q(k)}}(q_1,\dot{u}),min(\dot{v}_{F_{Q(k)}}(q_1,\dot{u}),(\dot{v}_{M_{Q(k)}}(q_1,\dot{u})))]$

$max(min(\dot{\lambda}_{F_{Q(k)}}(q_1,\dot{u}),\dot{\lambda}_{G_{Q(k)}}(q_1,\dot{u}),min(\dot{\lambda}_{F_{Q(k)}}(q_1,\dot{u}),(\dot{\lambda}_{M_{Q(k)}}(q_1,\dot{u}))=$

$[max(inf(min(\dot{\lambda}_{F_{Q(k)}}(q_1,\dot{u}),\dot{\lambda}_{G_{Q(k)}}(q_1,\dot{u}),min(\dot{\lambda}_{F_{Q(k)}}(q_1,\dot{u}),(\dot{\lambda}_{M_{Q(k)}}(q_1,\dot{u}))),$

$max(sup(min(\dot{\lambda}_{F_{Q(k)}}(q_1,\dot{u}),\dot{\lambda}_{G_{Q(k)}}(q_1,\dot{u}),min(\dot{\lambda}_{F_{Q(k)}}(q_1,\dot{u}),(\dot{\lambda}_{M_{Q(k)}}(q_1,\dot{u})))]$

From Equation (1) and (2)

$(F_Q,K)\cup((G_Q,L)\cap(M_Q,N))=((F_Q,K)\cup(G_Q,L))\cap((F_Q,K)\cup(M_Q,N).$

ii.

As

$(F_Q,K)=\{(q_1,\dot{u}),\left(\dot{\mu}_{F_{Q(k)}}(q_1,\dot{u}),\dot{v}_{F_{Q(k)}}(q_1,\dot{u}),\dot{\lambda}_{F_{Q(k)}}(q_1,\dot{u})\right):\dot{u}\in Q,q_1\in X\}$

$(G_Q,L)=\{(q_1,\dot{u}),\left(\dot{\mu}_{G_{Q(k)}}(q_1,\dot{u}),\dot{v}_{G_{Q(k)}}(q_1,\dot{u}),\dot{\lambda}_{G_{Q(k)}}(q_1,\dot{u})\right):\dot{u}\in Q,q_1\in X\}$

$(M_Q,N)=\{(q_1,\dot{u}),\left(\dot{\mu}_{M_{Q(k)}}(q_1,\dot{u}),\dot{v}_{M_{Q(k)}}(q_1,\dot{u}),\dot{\lambda}_{M_{Q(k)}}(q_1,\dot{u})\right):\dot{u}\in Q,q_1\in X\}$

$(G_Q,L)\cup(M_Q,N)=$

$\{(q_1,\dot{u}),max(\dot{\mu}_{G_{Q(k)}}(q_1,\dot{u}),\dot{\mu}_{M_{Q(k)}}(q_1,\dot{u})),$

$min(\dot{v}_{G_{Q(k)}}(q_1,\dot{u}),\dot{v}_{M_{Q(k)}}(q_1,\dot{u}),min(\dot{\lambda}_{G_{Q(k)}}(q_1,\dot{u}),\dot{\lambda}_{M_{Q(k)}}(q_1,\dot{u})):\dot{u}\in Q,q_1\in X\}$

where

$max(\dot{\mu}_{G_{Q(k)}}(q_1,\dot{u}),\dot{\mu}_{M_{Q(k)}}(q_1,\dot{u}))$

$= [max(inf((\dot{\mu}_{G_{Q(k)}}(q_1,\dot{u}),\dot{\mu}_{M_{Q(k)}}(q_1,\dot{u})), max(sup(\dot{\mu}_{G_{Q(k)}}(q_1,\dot{u}),\dot{\mu}_{M_{Q(k)}}(q_1,\dot{u}))))]$

$(F_Q,K) \cap ((G_Q,L) \cup (M_Q,N)) = \{(q_1,\dot{u}),$

$$\begin{pmatrix} \min((\dot{\mu}_{F_{Q(k)}}(q_1,\dot{u}), \max(\dot{\mu}_{G_{Q(k)}}(q_1,\dot{u}),\dot{\mu}_{M_{Q(k)}}(q_1,\dot{u}))), \max(\dot{v}_{G_{Q(k)}}(q_1,\dot{u}), \min(\dot{v}_{G_{Q(k)}}(q_1,\dot{u})), \\ \max(\dot{\lambda}_{F_{Q(k)}}(q_1,\dot{u}), \min(\dot{\lambda}_{G_{Q(k)}}(q_1,\dot{u}),\dot{\lambda}_{M_{Q(k)}}(q_1,\dot{u})) \end{pmatrix} : \dot{u} \in$$

$Q, q_1 \in X\}$................(1)

where

$min((\dot{\mu}_{F_{Q(k)}}(q_1,\dot{u}), max(\dot{\mu}_{G_{Q(k)}}(q_1,\dot{u}),\dot{\mu}_{M_{Q(k)}}(q_1,\dot{u}))) =$

$[min(inf((\dot{\mu}_{F_{Q(k)}}(q_1,\dot{u}), max(\dot{\mu}_{G_{Q(k)}}(q_1,\dot{u})),\dot{\mu}_{M_{Q(k)}}(q_1,\dot{u}),$

$min(sup\left(\left(\dot{\mu}_{F_{Q(k)}}(q_1,\dot{u}), max((\dot{\mu}_{G_{Q(k)}}(q_1,\dot{u}), \dot{\mu}_{M_{Q(k)}}(q_1,\dot{u})\right)\right))]$

$max((\dot{v}_{F_{Q(k)}}(q_1,\dot{u}), max(\dot{v}_{G_{Q(k)}}(q_1,\dot{u}),\dot{v}_{M_{Q(k)}}(q_1,\dot{u}))) =$

$[max(inf((\dot{v}_{F_{Q(k)}}(q_1,\dot{u}), max(\dot{v}_{G_{Q(k)}}(q_1,\dot{u})),\dot{v}_{M_{Q(k)}}(q_1,\dot{u}),$

$max(sup\left(\left(\dot{v}_{F_{Q(k)}}(q_1,\dot{u}), max((\dot{v}_{G_{Q(k)}}(q_1,\dot{u}), \dot{v}_{M_{Q(k)}}(q_1,\dot{u})\right)\right))]$

$max((\dot{\lambda}_{F_{Q(k)}}(q_1,\dot{u}), max(\dot{\lambda}_{G_{Q(k)}}(q_1,\dot{u}),\dot{\lambda}_{M_{Q(k)}}(q_1,\dot{u}))) =$

$[max(inf((\dot{\lambda}_{F_{Q(k)}}(q_1,\dot{u}), max(\dot{\lambda}_{G_{Q(k)}}(q_1,\dot{u})),\dot{\lambda}_{M_{Q(k)}}(q_1,\dot{u}),$

$max(sup\left(\left(\dot{\lambda}_{F_{Q(k)}}(q_1,\dot{u}), max((\dot{\lambda}_{G_{Q(k)}}(q_1,\dot{u}), \dot{\lambda}_{M_{Q(k)}}(q_1,\dot{u})\right)\right))]$

$(F_Q,K) \cap (G_Q,L) = \{(q_1,\dot{u}), min(\dot{\mu}_{F_{Q(k)}}(q_1,\dot{u}),\dot{\mu}_{G_{Q(k)}}(q_1,\dot{u})), max$

$(\dot{v}_{F_{Q(k)}}(q_1,\dot{u}),\dot{v}_{G_{Q(k)}}(q_1,\dot{u})),$

$max\ (\dot{\lambda}_{F_{Q(k)}}(q_1,\dot{u}),\dot{\lambda}_{G_{Q(k)}}(q_1,\dot{u})), \dot{u} \in Q, q_1 \in X\}$

where

$min(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1, \grave{u}))=$

$[min(inf(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u})), inf\ (\dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})), min(sup(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u})),$

$sup\ (\dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})))]$

$max(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{v}_{G_{Q(k)}}(q_1, \grave{u}))=$

$[min(inf(\dot{v}_{F_{Q(k)}}(q_1, \grave{u})), inf\ (\dot{v}_{G_{Q(k)}}(q_1, \grave{u})), max(sup(\dot{v}_{F_{Q(k)}}(q_1, \grave{u})), sup\ (\dot{v}_{G_{Q(k)}}(q_1, \grave{u})))]$

$max(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u}))=$

$[min(inf(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u})), inf\ (\dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u})), max(sup(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u})), sup\ (\dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u})))]$

$(F_Q, K)) \cap (M_Q, N) = \{(q_1, \grave{u}), min(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), (\dot{\mu}_{M_{Q(k)}}(q_1, \grave{u})),$

$max(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{v}_{M_{Q(k)}}(q_1, \grave{u}))$

$max(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{M_{Q(k)}}(q_1, \grave{u})), \grave{u} \in Q, q_1 \in X\}$

where

$min(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), (\dot{\mu}_{M_{Q(k)}}(q_1, \grave{u})) =$

$[min(inf(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u})), inf\ (\dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})), min(sup(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u})), sup\ (\dot{\mu}_{G_{Q(k)}}(q_1, \grave{u})))]$

$max(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), (\dot{v}_{M_{Q(k)}}(q_1, \grave{u})) =$

$[max(inf(\dot{v}_{F_{Q(k)}}(q_1, \grave{u})), inf\ (\dot{v}_{G_{Q(k)}}(q_1, \grave{u})), max(sup(\dot{v}_{F_{Q(k)}}(q_1, \grave{u})),$

$sup\ (\dot{v}_{G_{Q(k)}}(q_1, \grave{u})))]$

$max(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), (\dot{\lambda}_{M_{Q(k)}}(q_1, \grave{u})) =$

$[max(inf(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u})), inf\ \dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u})), max(sup(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), sup\ (\dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u})))]$

$((F_Q, K) \cap (G_Q, L)) \cup ((F_Q, K) \cap (M_Q, N))=$

$\{(q_1, \grave{u}), max(min(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1, \grave{u}), min(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), (\dot{\mu}_{M_{Q(k)}}(q_1, \grave{u})),$

$min(max(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{v}_{G_{Q(k)}}(q_1, \grave{u}), max(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{v}_{M_{Q(k)}}(q_1, \grave{u})))$

$min(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u}), max(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{M_{Q(k)}}(q_1, \grave{u}))), \grave{u} \in Q, q_1 \in X\}$

where

$max(min(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1, \grave{u}), min(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), (\dot{\mu}_{M_{Q(k)}}(q_1, \grave{u}))=$

$[max(inf(min(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1, \grave{u}), min(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), (\dot{\mu}_{M_{Q(k)}}(q_1, \grave{u}))),$

$max(sup(min(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{G_{Q(k)}}(q_1, \grave{u}), min(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), (\dot{\mu}_{M_{Q(k)}}(q_1, \grave{u})))]$

$min(min(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{v}_{G_{Q(k)}}(q_1, \grave{u}), min(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), (\dot{v}_{M_{Q(k)}}(q_1, \grave{u}))=$

$[min(inf(min(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{v}_{G_{Q(k)}}(q_1, \grave{u}), min(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), (\dot{v}_{M_{Q(k)}}(q_1, \grave{u}))),$

$min(sup(min(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), \dot{v}_{G_{Q(k)}}(q_1, \grave{u}), min(\dot{v}_{F_{Q(k)}}(q_1, \grave{u}), (\dot{v}_{M_{Q(k)}}(q_1, \grave{u})))]$

$min(min(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u}), min(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), (\dot{\lambda}_{M_{Q(k)}}(q_1, \grave{u}))=$

$[min(inf(min(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u}), min(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), (\dot{\lambda}_{M_{Q(k)}}(q_1, \grave{u}))),$

$min(sup(min(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u}), min(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), (\dot{\lambda}_{M_{Q(k)}}(q_1, \grave{u})))]$

$=\{(q_1, \grave{u}),$

$$\begin{pmatrix} \min(\dot{\mu}_{F_{Q(k)}}(q_1, \grave{u}), \max(\dot{\mu}_{G_{Q(k)}}(q_1, \grave{u}), \dot{\mu}_{M_{Q(k)}}(q_1, \grave{u}))), \max(\dot{v}_{G_{Q(k)}}(q_1, \grave{u}), \min(\dot{v}_{G_{Q(k)}}(q_1, \grave{u})), \\ \max(\dot{\lambda}_{F_{Q(k)}}(q_1, \grave{u}), \min(\dot{\lambda}_{G_{Q(k)}}(q_1, \grave{u}), \dot{\lambda}_{M_{Q(k)}}(q_1, \grave{u})) \end{pmatrix} : \grave{u} \in$$

$Q, q_1 \in X\}$................(2)

where

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