ADAPTIVE BEAMFORMING BASED ON GENERALIZED SIDELOBE CANCELLATION

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Adaptive Beamforming Based on Generalized Sidelobe Cancellation

Muhammad Siddiq

Thesis submitted in partial fulfillment of requirements for the
MS degree in Electronic Engineering

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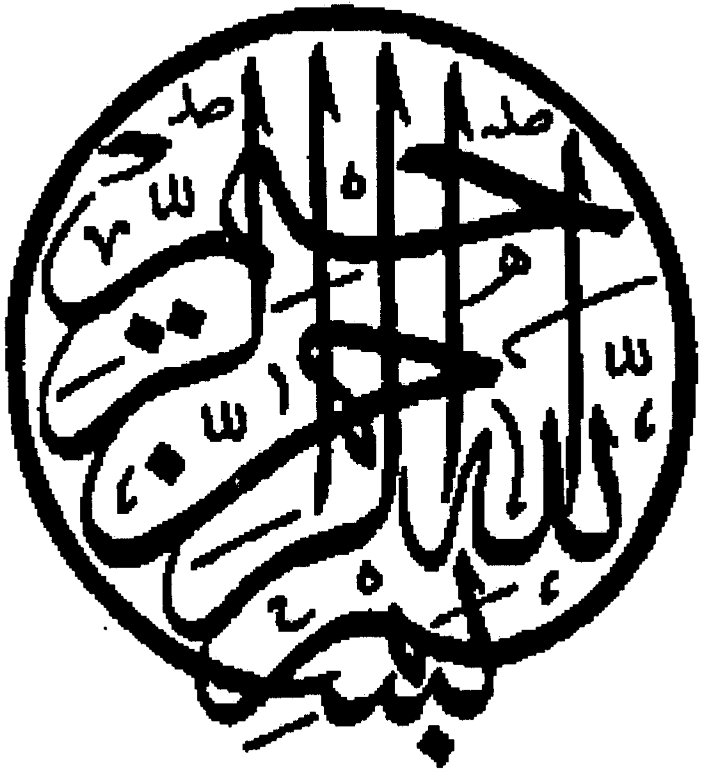
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Dedicated to my respected teachers

ABSTRACT

A beamformer is an array of sensors controlled by some computational machine, which can perform spatial filtering. The aim of this research work is to develop an efficient beamforming array, which can be used in the real time environments where the decision time is limited, especially in military applications such as radar. A linear array with total ten antenna elements have been simulated in this work using MATLAB. Simulations are carried out to find quiescent weights, which are called optimum weights, for multiplying with the outputs of array sensors so as to produce a far field pattern that optimizes the reception of a target signal along the direction of interest. At the same time algorithm produces nulls for the blocking of interfering/jamming signals arriving from the unwanted directions.

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ACRONYMS

NULA	Non Uniform Linear Array
VLA	Very Large Array
SONAR	Sound Navigation and Ranging
RADAR	Radio Angle Detection and Ranging
CDMA	Code Division Multiple Access
GSM	Global System for Mobile Communication
DOA	Direction of Arrival
MVDR	Minimum Variance Distortionless Response
URA	Uniform Rectangular Array
UCA	Uniform Circular Array
AOA	Angle of Arrival
MUSIC	Multiple Signal Classification
PAWS	Phased Array War System
AEW	Airborne Early Warning
STAP	Space Time Adaptive Processing
ML	Maximum Likelihood
MMSE	Minimum Mean Square Error
MSINR	Maximum Signal to Interference and Noise Ratio
MSNR	Maximum Signal to Noise Ratio

CHAPTER 1

INTRODUCTION

1.1 Introduction

Adaptive beam forming is an active area of research which finds its applications in numerous fields. Communication, radar, sonar, seismic and oil exploration are quite evident. That is why, a lot of research and study have been carried out in this area. In our country limited use has been experienced and also it is in the assembled and modular form, which are normally provided by the system manufacturers.

The motivation behind the research work is being covered in the following section followed by problem statement. The thesis contributions are then summarized and then outline of the thesis has been given at the end of the chapter.

1.2 Motivation

In recent decades adaptive beamforming has been widely used in many areas, in a variety of applications such as radar, sonar, communication systems, geo-physical explorations and other systems that require to reject interferences. For all above mentioned systems the general goal is to reconstruct a desired signal in the presence of interferences and noise. Antenna arrays and digital signal processing capabilities are the important part of adaptive beamforming. There are different algorithms which are used to select weights for getting maximum required information and generalized sidelobe cancellation at the same time. Mechanical beam steering due to bulkiness and slow in response has clearly been left behind by the electronic scanning radar systems (ESRS)[1].

1.3 Problem Statement

To achieve the required information in the presence of noise and interference is the key problem of this research work. It will be achieved by using signal processing techniques applied on the electromagnetic radiation received from the antenna array. The signal from the source of interest is picked up by an array of omni-directional antennas in the presence of noises and interference sources in surrounding of the source. Some times such interfering sources are intentionally generated to undermine the signal of interest. Usually in case of radars, the enemy targets (fighter planes) when moving in the opponent area generate some interference signals to deceive the radar. The antenna array will pickup these composite signals. These signals are then processed by some algorithm to filter out the interfering data from the actual information. The antenna arrays pickup these composite signals with different phases at each antenna element. Then the output of these individual antenna elements is multiplied with some multiplication factor called weight. These weights are optimized by using specialized algorithm. With these optimized weights, all outputs of the antenna elements are summed up to eliminate the interferences and generate strong signal of interest. At the same time by using the digital beamforming it is possible to monitor many sources in the space as per our requirement and utilization. We will be able to steer the receiving/transmitting beam with minimum power and in a narrow band. While steering the receiving beam we will be able to create some nulls in required directions where we feel any interference/ jammer source[2].

1.5 Contribution of Thesis

Generalized sidelobe cancellation has been implemented using beamforming technique. Total of two interferences / Jammers have been tested at different angles from the source of interest. The source of interest has been tried / tested at different locations within the 180°. The interferences / jammers have been tried /tested on either side of the source of interest. The algorithm has been implemented in MATLAB and result has been given in the form of plots. The retrieved signal power has been plotted at different angles of arrivals to determine the performance of the algorithm. The performance in the presence of jamming signal has also been evaluated at different angle of arrivals for signal of interest.

1.6 Outline of Thesis

Chapter 2 introduces the concept, different structures and schemes of antenna arrays. It also includes the practical uses of antenna arrays for the elimination of different interfering sources. Chapter 3 covers beamforming, its concept and classifications of beamforming and spatial filtering, analog and digital beamforming. Chapter 4 is the key part of the subject under consideration; it covers the concept and method adapted in the topic. An effort ha been carried out to explain the theory under discussion and mathematical base has also been discussed in detail. Finally generalized sidelobe cancellation has been demonstrated by the calculation of quiescent weights of the beamformer. Chapter 5 explains the simulation and results. A MATLAB program has been developed for the simulation of sidelobe cancellation. The result of the beamformer based on generalized sidelobe canceller with source of interest at different locations/angles has been demonstrated. Secondly the result with source of interest and some interference source / jammer has been demonstrated in this chapter. Chapter 6 covers conclusion and future work.

We will denote throughout this thesis, vectors with lower case boldface letters while a matrix with uppercase bold face letter. Superscript H will denote hermitian. Assuming vectors to be column one.

1.7 Summary

This chapter covered the introduction, problem statement, the motivation behind the research work, the structure and organization of the thesis report. A brief historical perspective has also been presented.

CHAPTER 2

PRINCIPLE AND CONCEPT OF ANTENNA ARRAYS

2.1 Introduction

This chapter will explain the basic concept and different structures of an antenna. Antenna elements and sensors constitute antenna arrays. Antenna array processing and beamforming technique is being used now a day for commercial as well as military uses. Beamforming using arrays of sensors is widely used in radars, sonars, communication networks and mobile communication. There are a large number of other areas in modern and advance world i.e weapon systems, navigation, monitoring, exploration and guidance where beamforming based on sensor arrays are really used.

2.2 Antenna

Antenna generally is a combination of electrical conductors, pipes and some wires. These elements create magnetic and electric fields in the surrounding space. When these fields are changing, information propagates outward through the space. These electromagnetic waves travel at the speed of light. Speed of light is a constant quantity and taken as $C = 3 \times 10^8$ meters/sec in vacuum. It is true for any antenna that it transmits some information and at the same time it can also receive the information. Some antennas are used for transmitting purposes while some are receiving antennas. In the process of receiving, currents are produced in the antenna conductors/ sensors when they are excited with electromagnetic waves. When impinging electromagnetic waves pass through antenna some energy is captured by it and is converted into the electrical signal. In order to design an antenna, its dimensions are carefully calculated according to the wavelength of signal to transmit or receive.

A circular surface radiating electromagnetic waves uniformly is considered as a satellite dish antenna. This type of antenna has narrow central “beam” with high gain. In order to narrow down the central beam, the dish diameter in wavelengths is increased. On either side of the central beam there are sidelobes. Nulls are the directions where the signal strength is zero [3].

2.3 Antenna Arrays

An array of sensors is used to enhance the signal strength of the distant source of interest. These sensors are distributed over a horizontal plane surface to receive a propagating electromagnetic wave field with the following objectives:

i To detect the location of the source.

ii To find the direction of arrival of the propagating source.

iii To receive the message from a distant source.

We shall study the basic structure of a sensor array system and in the sequel learn how the above objectives are achieved.

The most commonly used array geometries are

Uniform linear array(**ULA**).

Uniform circular array(**UCA**).

Uniform planar array(**UPA**).

In uniform linear array, the array sensors are placed on a single line with equal distance between each other according to transmitting or receiving wave length as shown in figure 2.2.

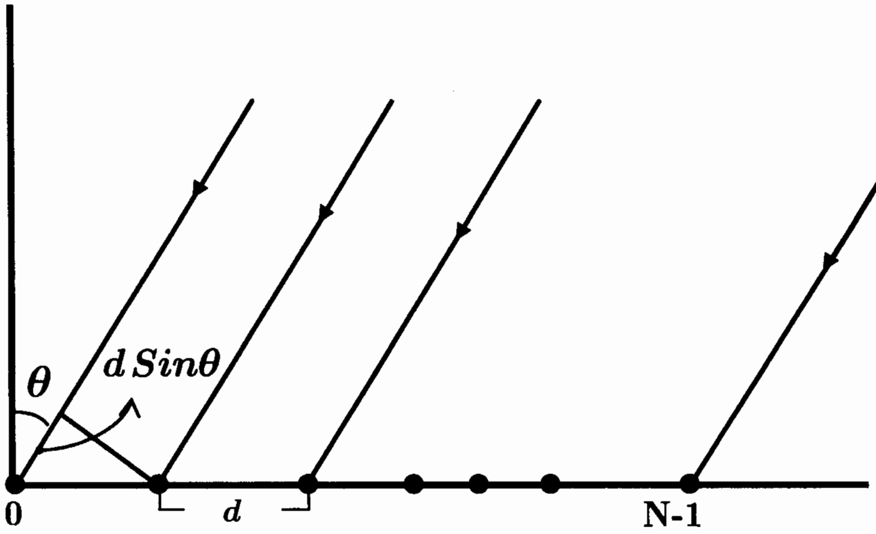

Figure 2.2 , A uniformly spaced linear array of N sensors.

Borrowed from [4] Section 2.1.1 page 16.

The central beam becomes narrower with the increase in the width of the array. To make the sidelobes smaller, the number of elements is increased. In the uniform circular array sensors are placed in an equispaced circle. In a uniform planer array the sensors are placed on an equispaced rectangular grid which is more common in large military phased array systems.

2.4 Antenna Array Principle

When an information wavefront propagates across the array of sensors as shown in figure 2.2. This information impinges on the 0th element/sensor. An electrical signal is picked up by this sensor. The same wave front interfaces the 1st sensor/element, it is sooner by some time with respect to 0th sensor because it has to cover some less distance.

This time difference is represented by some equivalent phase difference. All the sensors are at equidistant from each other therefore the time delay (the phase difference) will be same for all consecutive sensors/elements. In this way the wave front is picked up by all sensors with $(N-1)$ time delays, where n is the number of antenna elements/ sensors. Thus, we have all the elements or sensors with their individual outputs. These individual outputs are multiplied by some constants called weights. After the multiplication by respective weights all the individual outputs are summed up to form the output of the antenna array. The difference of the this output with the desired output is the error, which has to be minimized by adjusting the weights. Some recursive algorithm may be used to achieve the final output with optimized weights, which is the basic antenna array concept. In the worst case, individual sensor outputs are strongly corrupted with noise and other interference, leaving a very little resemblance among them. Array processing now involves combining all sensor outputs in some optimal manner so that the coherent signal emitted by the source is received and all other inputs are maximally discarded. The aperture of an array, that is, the spatial extent of the sensor distribution, is a limiting factor on resolution. However, the aperture can be synthetically increased by moving a source or sensor. The synthetic aperture concepts are extensively used in mapping radars and sonar [4].

CHAPTER 3

BEAMFORMING

3.1 Introduction

In this chapter, basic technologies and concept employed in beamforming have been introduced. In the first section of this chapter we will discuss the operation of the beamforming and spatial filtering. Then we introduce second order statistics and explain the difference between narrowband and broadband beamforming. In the final section various types of beamformers and the algorithms have been explained. A brief summary has been presented at the end of the chapter.

3.2 Concept and Terminology of Beamforming

The term beamforming seems to apply only on the radiation of energy however it also refers to the energy reception. In beamforming an array of elements is adjusted in such a manner that the reception of a signal arriving from a desired direction is maximized whereas the noise and interference is cancelled. This is done by approximating the signal arriving from the desired direction.

The beamforming objectives are obtained by choosing suitable weights for each antenna element. The spatial beamformers separate the desired signal from the noise by exploiting the fact that the radiations emitted from different sources even on same frequency arrive at an antenna from different directions. However in adaptive beamforming complex algorithms repeatedly adjust the optimum weights by monitoring different signal parameters. The basic operation in beamforming is to adjust the phase feed of each antenna element such that an in-phase signal is obtained from all the antenna elements in a particular direction. The amplitudes and the phases are selected so that the reception of the desired signal is maximized [4].

In beamforming the most common spatial processing technique is used by antenna array. In a cellular system, the desired and the interfering signals originate from two different spatial locations. This spatial separation is exploited by a beamformer. It is regarded as a spatial filter, which separate desired signal from the interference. The signals from the different antenna elements are weighted and summed to optimize the quality of the signal. It is possible to point the beam towards the direction of the desired user and / or place nulls in the direction of the interferers. If we have an antenna array consisting of N elements and *k* signals with distinct angle of arrival (AOA) impinging on it, the received signal vector on each element can be written as

$$\underline{x}(t) = \sum_{i=1}^{k} s_i(t)\,\underline{a}(\theta_i) + \underline{n}(t) \qquad (3.1)$$

where $s_i(t)$ is the *ith* signal with an AOA of θ_i, $\underline{a}(\theta_i)$ is the Nx1 antenna response vector for the AOA of θ_i and $\underline{n}(t)$ is the thermal noise. The output of the antenna array at some time (*t*) is given by

$$y(t) = \underline{w}^H \underline{x}(t) \qquad (3.2)$$

$$\underline{w} = [w_1 \;\; w_2 \; w_3 \; .. \; w_N]^T$$

Here $\underline{w}$ is Nx1 weight vector and H denotes the hermitian operator. The weight vector is chosen to optimize beamforming criterion. The well known adaptive beamforming techniques include minimum mean square error (MMSE), Maximum signal to interference and noise ratio (MSINR), Maximum signal to noise ratio (MSNR) and Maximum likelihood (ML).

3.3 Array Ambiguity

Let us consider the array response vector of a uniform linear array (ULA), given by equation (3.2) obviously the information having incident angles θ_1 and θ_2, related as $\theta_2 = \pi - \theta_1$, will have the same array response vector i.e. $a\ (\theta_1) = a\ (\theta_2)$. As a result, it is impossible for a uniform linear array to distinguish between the desired signal coming from the direction θ_1 and a interfering signal coming from the direction θ_2. This ambiguity makes it impossible to place a null in the direction of the interferer without nulling the desired signal itself. Any linear array suffers from this ambiguity although the relation between the ambiguous AOAs depends on the inter-element spacing. This ambiguity can be avoided if sectorization is employed and/or the individual elements are in isotropic [4].

3.4 Beamforming and Spatial Filtering

An adaptive array configuration for processing narrow band signals is shown in figure 3.1a. The individual antenna element is connected to a variable weight. The weighted signals are summed as shown below. This type of beamformer samples the propagating wave field (electromagnetic radiations) in space. It is typically used for processing narrowband signals. The out put of beamformer at time k is denoted by y(k) and it is given by a linear combination of the data at *l* sensors at time k

$$y(k) = \sum_{l=1}^{J} \mathbf{w}_l^* \, \mathbf{x}_l(k) \tag{3.3}$$

where * represents complex conjugate.

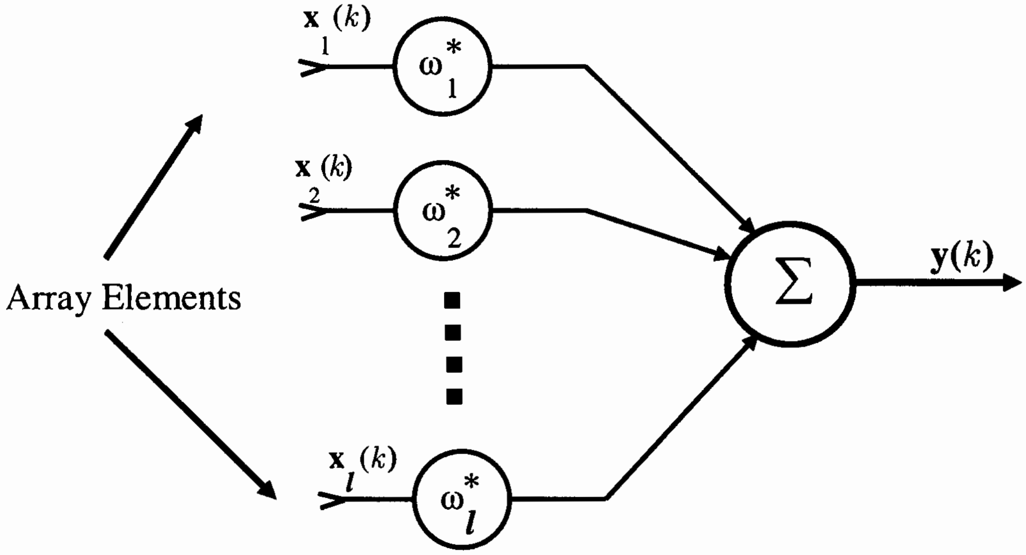

Figure 3.1a Narrow band spatial filter

Each sensor is assumed to have the necessary receiver electronics and AD converter if beamforming is performed digitally. Figure. 3.1a shows a way to combine the antenna element signals in an adjustable linear structure when the received signals and noises are narrow band.

To receive signals over a range of frequencies, each of the phase shifters in figure. 3.1a is replaced by an adaptive transversal filter as shown in figure. 3.1b.This taped delay time permits adjustment of gain and phase as desired at a number of frequencies over the band of interest. If the weight spacing is sufficiently small, this network approximates this ideal filter which could allow complete control of the gain and phase at each frequency in the pass band.

A common broadband beamformer is illustrated here. The beamformer samples the propagating wave field in both space and time and is often used when signals of significant frequency (broadband) are of interest.

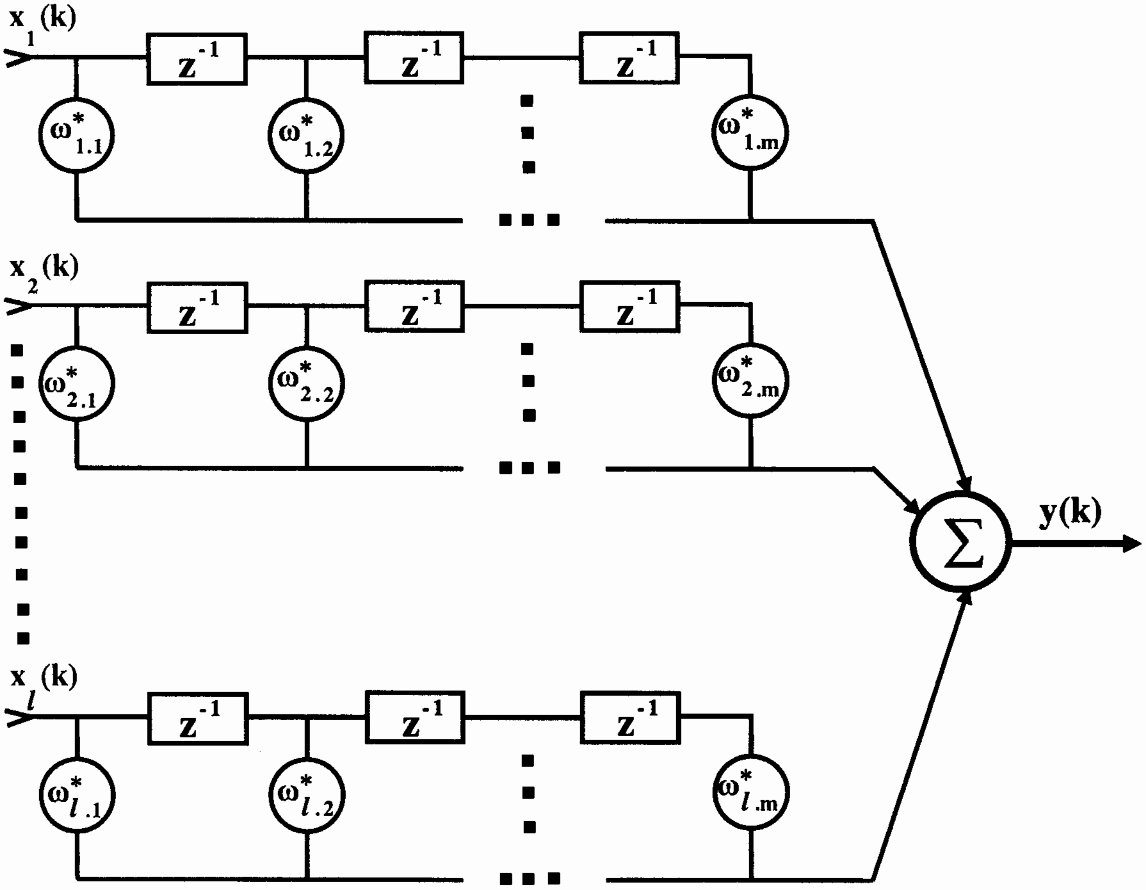

Figure 3.1b Broadband spatial filter

Borrowed figure 3.1 [7] Section 61.1 page 1303.G

Problem under consideration (Adaptive beamforming using GSC) also belongs to narrow band spatial filtering or narrow band beamformer. It is because of the reason that we want to receive signals from the source of interest from some specified direction/

location while at the same time we want to reject or suppress signals from the source of interference at some locations/ direction or we want to create nulls at these locations.

According to figure 3.1a the output of the narrow band beamformer is given by the output of the beamformer or equation (3.2) in vector form can also be written as

$$y(k) = \mathbf{w}^H \mathbf{x}(k) \qquad (3.4)$$

w is a column vector such as

$$\mathbf{w} = [w_1\, w_2\, w_3\, ..w_l]^T \qquad (3.5)$$

By defining a data vector $\mathbf{x}(k)$ and the weight vector $\mathbf{w}$ while $\mathbf{x}(k)$ and $\mathbf{w}$ are N dimensional. This shows that N=l when referring to Equation 3.3. We do not show the time index and it is understood during the adaptive algorithms. The frequency response of an FIR filter with tap weights $\mathbf{w}_p^*$, $1 \leq p \leq J$. Equation 3.4 can be written as

$$y = \mathbf{w}^H \mathbf{x} \qquad (3.6)$$

$$r(w) = \sum_{p=1}^{J} \mathbf{w}_p^* e^{-j\omega T(p-1)} \qquad (3.7)$$

or

$$r(\omega) = \mathbf{w}^H \boldsymbol{d}(\omega) \qquad (3.8)$$

where

$$\mathbf{w}^H = \left[\mathbf{w}_1^*\, \mathbf{w}_2^* .. \mathbf{w}_l^*\right]^T \qquad (3.9)$$

and

$$d(\omega) = \left[1, e^{j\omega T}, e^{j2\omega T}, .. e^{j(J-1)\omega T}\right] \qquad (3.10)$$

$r(\omega)$ shows filter response with input of a complex sinusoid of frequency ω while d(ω) is the phase of the complex sinusoid at each tap in the filter relative to the tap associated with w_1 [7].

3.5 Analog Beamforming

The term beamforming relates to the function performed by a device or apparatus in which energy radiated by an aperture antenna is directed along a specific direction in space. The objective is either to preferentially receive a signal from that direction or to transmit a signal in that direction. For example in a parabolic antenna system, the dish is a beamforming network in that it takes the energy that lies within the aperture formed by the perimeter of the dish and focuses it on to the antenna feed. The dish and the feed operate as a spatial integrator. Energy from the far field source, which is assumed to be aligned with the antenna preferred direction, arrives at the feed temporarily aligned and is thereby summed coherently. In general, sources in other directions arrive at the feed unaligned and add incoherently. For this reason beamforming is often referred to as spatial filtering.

3.6 Digital Beamforming

Foundations of the digital beamforming was first developed by workers of sonar and radar systems. Antenna technology and digital technology were married and gave rise to the digital beamforming. Device that converts spatiotemporal signals into strictly temporal signals is called an antenna. It makes signals available to a wide variety of signal processing techniques. In this way all of the desired information that is being carried by these signals can be extracted. An optimum antenna is a device that carries out the conversion of the signals that arrive at its face without introducing any distortion to the signals. It is why a digital beamforming antenna should be considered to be an optimum antenna. From a conceptual point of view, its sampled outputs represent all of the data arriving at the antenna aperture. No information is destroyed, at least not until the processing begins and any computations that are made in the processing [4].

In antenna applications point to point communication is of prime interest, highly directive antenna beam is used to get the advantage. The directional beam can be achieved by forming an array consisting of a number of elements. With the increase of directivity of antenna the gain is also increased. At the receiver end the increase in directivity means that the antenna receives less interference from its surrounding signal. For the signal level at a receiving antenna, if we increase the gain by a factor of 10 we can reduce the transmitted power by a factor of 10. In digital beamforming when the data signals from the array elements are directly multiplied by a set of weights to form a beam at the desired angle, it is called element space beam forming. Multiplying the data signals by different set of weights we can produce a set of beams with pointing angles directed anywhere in the field defined by the elements used in the array. Each beamformer creates an independent beam by applying independent weight to the array signals.

3.7 Classification of Beamformers

Beamformers can be divided into statistically optimum and data independent beamformers based on the method of selection of weights. The statistically optimum beamformer optimizes the array response by selecting the weights depending upon the statistical analysis of the array data. In this method nulls are generated in the interfering direction. As a result the signal to noise ratio is enhanced at the output. The data independent beamformer performs a specific response for every signal as well as interference; this response is generated by selecting weights which are independent of array data. The methods designed for data independent beamforming are mostly used in statistically optimum beamforming. These techniques include constraint design in linearly constrained minimum variance beamforming. The array data changes with time thus adaptive algorithms are used to calculate the weights. Such algorithms are employed which converge the beamformer output to an optimum solution. The data independent beamformer creates an output that is fairly close to the desired response by selecting weights in a specific manner. Hence it satisfies the objective of classical FIR filter using a different approach. Although both the types use different approaches but reach a common objective, which is to optimize the output with minimum response of noise and signal radiating from a source other than the source of interest.

For example multiple sidelobe canceller and maximization of signal to noise ratio belong to the category of statistically optimum beamformers.

3.8 Data Independent Beamforming

In this technique the beamformers output is made close to the desired response. It is obtained by selecting weights, which do not depend upon the array data. This approximation of output is similar to that of obtained by classical FIR filter. They use a classical approach to generate a beam. This way of beamforming produce output equal to unity for the signals coming from the desired direction and it is zero output for the signals coming from all other directions.

3.9 Classical Beamforming

In order to separate some complex frequency component from the mixed frequency components, we can use taped filter method. In this method, the frequency response of the beamformer will be unity against the frequency of interest ω_0 and its response will be zero for all other frequencies. The objective can be achieved by selecting a weight vector $\mathbf{w}$ such that it minimizes the mean square error, which is the difference between the actual response and desired response. The actual response represents the main lobe and is accompanied by many sidelobes. The sidelobes are removed by tapering(gradual decrease) of the amplitudes of vector $\mathbf{w}$ to make the output shape similar to the desired response. The tapering is carried out by considering real valued taper weights. However if we want to receive a signal from a specific location, we have to assume the signal to be narrow band. Since the output of each sensor is phase shifted prior to summation therefore this beamformer and the resultant array is termed as the phased array.

3.10 Statistically Optimum Beamforming

Weights of the statistical beamformer are selected on the bases of received array data. In wide sense it is required to achieve the optimized output of the beamformer with minimum response/ effect due to noise and interference as input.

3.11 Multiple Sidelobe Canceller

Multiple sidelobe canceller is one of the first among the statistically optimum beamformers. Main channel and the several auxiliary channels constitute the (MSC) as shown in figure (3.2) below The main channel consist of a data independent beamformer or an antenna with very high gain. Its response is highly directional and desired response can be obtained in a specific direction. The sidelobes of the main channel are considered to be interference signals.

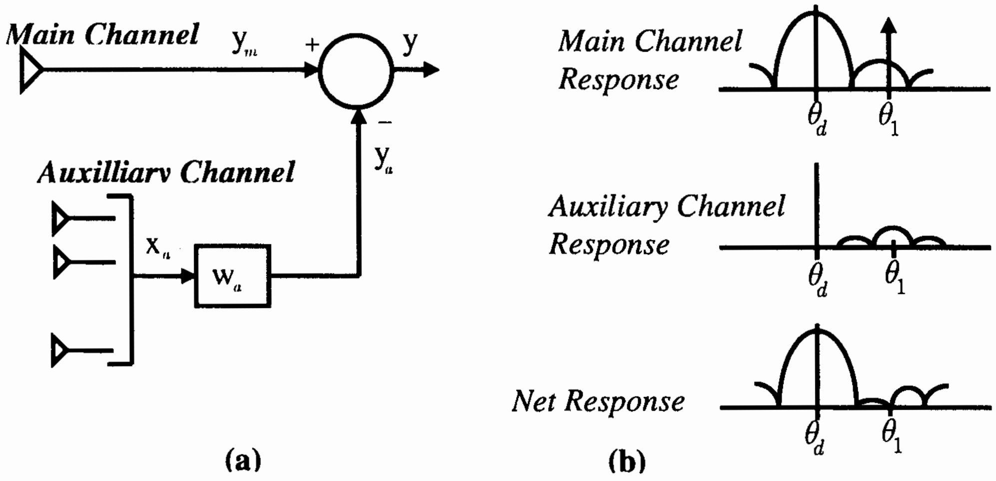

Figure 3.2: Multiple Sidelobe Canceller

Borrowed Figure. [7] section 61.4.1 page 1313.

The weights of the auxiliary channel are selected so as to eliminate the interference which has entered as sidelobes of the main channel [7]. Interference signals also enter via auxiliary channels. The weights of the auxiliary channels are selected in such a way that they cancel the interference component of the main channel. For this reason the effect of the interference on the main channel as well as the auxiliary channel is kept equal. It is important that the system should show a zero overall response/output for the interference only. However this is an ideal condition and it cannot be completely achieved in practice. Even if it is achieved, it will result in very high white noise gain. The expected value of total output power is minimized by the selection of weights. Since the total output power is minimized, the desired signal being a part of total output is also reduced in magnitude. As the amplitude of the desired signal increases its contribution to total output power also increases causing a greater percentage of the desired signal to be cancelled. This property of (MSC) is un-wanted; however it is effectively used in applications where the signal of interest is low in amplitude as compared to the interference. This is because the weights do not take the desired signal into consideration since they are very weak. The (MSC) is particularly effective in situations where the desired signal completely disappear during a short time interval of the transmission.

3.12 Use of Reference Signal

In the discussions of beamformer we have considered that the desired signal is known and by keeping the desired signal as the reference the weights are adjusted to get the output. It is not possible to have a desired signal in practice. Secondly if we really have the desired signal then the beamformer is no more required. However in certain situations a signal closely resembling the desired signal is generated from the information known prior to the reception of the signal. This produced resembling signal is called the reference signal. The mean square error which is the difference between the reference signal and output of the beamformer is minimized by selecting optimum weights. Cross covariance between the reference signal and the unknown desired signal decides the weight vector of the beamformer. If the covariance of the unknown desired signal is calculated with itself, an acceptable performance is achieved. For example the reference signal of an amplitude modulated transmission is obtained by considering carrier of the AM signal as reference signal. The satisfactory performance is obtained by the use of carrier as reference signal. However it is also considered that the interference is uncorrelated with the reference signal. In this technique the reference signal is independent of direction and so this type of beamforming is often referred to as blind beamforming.

CHAPTER 4

LCMV BEAMFORMING BASED ON GENERALIZED SIDELOBE CANCELLATION.

4.1 Introduction

This chapter consists of the basic steps involved in beamforming. It starts with the brief introduction of Wiener filter, transversal filter and their correlation with linear array of sensors. Then a beamformer is constructed using linear array of sensors. Finally generalized sidelobe canceller is discussed in detail.

4.2 LCMV

In case of LCMV a Wiener filter is used to minimizes the mean-square value of the estimation error, which is the difference between desired response and output of the filter. It is shown in the figure below.

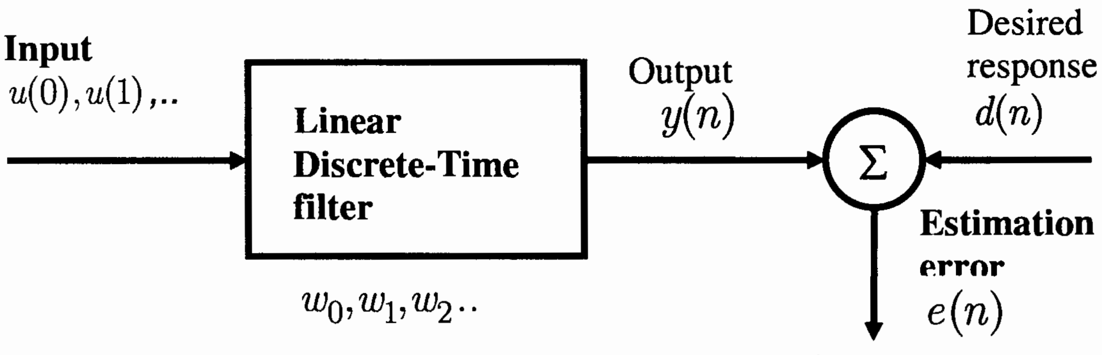

Figure 4.1 block diagram of linear optimum filter

Borrowed from [9] page 195 3rd edition.

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$$e(n) = d(n) - y(n) \tag{4.1}$$

Where $e(n)$ is estimated error, $d(n)$ is desired response and $y(n)$ is final output of the filter.

In the solution of this optimization no constraints has been imposed. The solution leads only to the minimization of the mean-square error. But in some filtering applications /requirements it is necessary during the designing of the filter that minimizes the mean-square error subject to a specific constraint. The requirement of the subject constraint may be that the average power of the filter is minimized while the response of the filter measured at some specific frequency of interest is required to remain constant (that constant may be some fixed constant value, say g). The basic idea behind linearly constrained minimum variance (LCMV) beamforming is to constrain the response of the beamformer for the signals from the direction of interest to passed with specified gain and phase. The weights are chosen to minimize output variance or power subject to response the constraint. This has the effect of preserving the desired signal while minimizing contributions to the output due to interfering signals and noise arriving from directions other than the direction of interest [7].

The base of the **LCMV** beamforming is linearly constrained minimum variance filter. Consider a linear transversal filter as shown in Figure. (4.2)

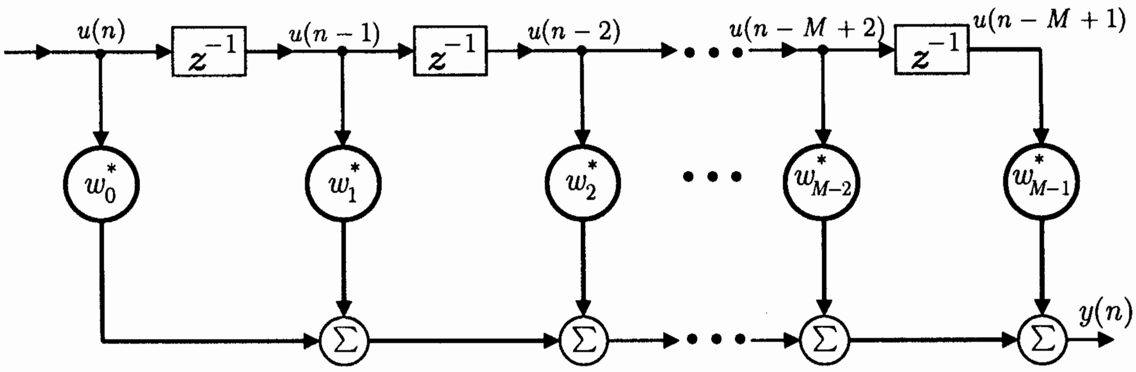

Figure 4.2, A transversal filter of order M

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In above presented transversal filter, the tap inputs are,

be $u(n), u(n-1), u(n-2), .. u(n-M+1)$ cause the order of the filter is M.

The output of the filter in response to the tap inputs are

$u(n), u(n-1), u(n-2), .. u(n-M+1)$ is

$$y(n) = \sum_{k=0}^{M-1} w_k^* u(n-k) \tag{4.2}$$

In above expression $u(n), u(n-1), u(n-2), .. u(n-M+1)$ are tap inputs which are temporal in nature.

If the input of the filter is sinusoidal, it is called a special case of sinusoidal excitation. Such as

$$u(n) = e^{j\omega n} \tag{4.3}$$

Then the output of the filter using equation (4.3) in equation (4.2) is given by

$$y(n) = \sum_{k=0}^{M-1} w_k^* e^{j\omega n} e^{-j\omega k} \tag{4.4}$$

$$y(n) = e^{j\omega n} \sum_{k=0}^{M-1} w_k^* e^{-j\omega k} \tag{4.5}$$

Where ω is the angular frequency of the excitation, which is normalized with respect to the sampling rate. The summation part of the equation (4.5) is frequency response of the given filter. For the solution of the constrained optimization problem, we will find the optimum set of filter weights i.e. $w_{o0}, w_{o1}, w_{o2}, .. w_{o(M-1)}$ which minimizes the mean-square value of output of the filter $y(n)$, subject to the condition of the linear constraint

$$\sum_{k=0}^{M-1} w_k^* e^{-j\omega_o k} = \text{g} \tag{4.6}$$

Where ω_o is a prescribed value of the normalized angular frequency ω, lying within the range $-\pi < \omega \leq \pi$, and **g** is a complex valued gain.

The constrained optimization filtering problem represented by equations described above from equation (4.2) to equation (4.6) is temporal in nature. We want to formulate a spatial version of this constrained optimization problem. So we consider a beamformer which is composed of a linear array of uniformly spaced antenna elements.[9]

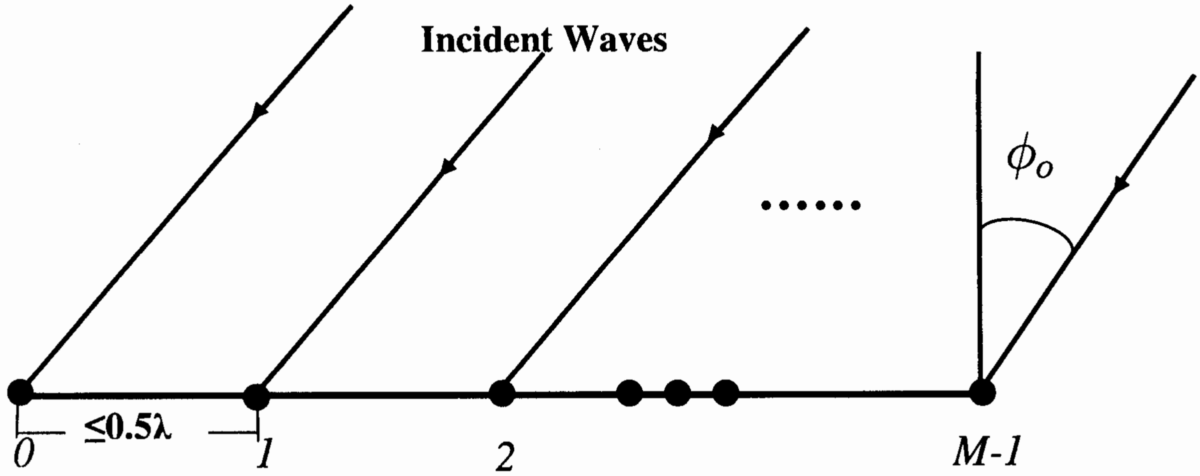

Figure 4.3, Linear array of M sensors

Borrowed from [9] page 223, 3rd edition.

The array of elements is illuminated by an isotropic source which is present in the far field (at a long distance), we suppose that at time n, a plane wave impinges on this linear array along the direction making an angle ϕ_o with the perpendicular to the array. It is also confirmed that the spacing between all sensors of the array is equal or less than λ / 2 , where λ is the wavelength of the transmitted signal of the distant source, same is being received. The out put of the beamformer relating to this antenna is

$$y(n) = u_0(n) \sum_{k=0}^{M-1} w_k^* e^{-jk\theta_o} \qquad (4.7)$$

The direction of arrival is defined by the electrical angle θ_0 and its relation leads to the actual angle of incidence ϕ_0. In equation (4.7), $u_0(n)$ is the electrical signal picked up by the antenna element which is labeled 0 and we mark it as point of reference. The symbol w_k denotes the element weight of the beamformer. The spatial version of the constrained optimization problem is solved by finding the optimum set of elemental weights which are

$$w_{o0}, w_{o1}, w_{o2}, .. w_{o(M-1)}$$

These are the weights when optimized, minimizes the mean square value of the beamformer output subject to the linear constraint.

$$\sum_{k=0}^{M-1} w_k^* e^{-jk\theta_o} = \mathrm{g} \qquad (4.8)$$

$$y(n) = u_0(n) \sum_{k=0}^{M-1} w_k^* e^{-jk\theta_o}$$

Where θ_0 is a prescribed (special value) of the electrical angle, which lies in the range $-\pi < \theta_o \leq \pi$ and **g** is a complex –valued gain.

The beamformer is a narrowband. It means that its output response is concentrated at a single frequency. On comparison between transversal filter and beamformer described above we see that though they address entirely different physical situations, yet they have same mathematical formulations. Both cases have exactly the same constrained optimization problem. To solve this type of constrained optimization problem, we use the method of Lagrange Multipliers. For its solution we define a real valued cost function J that combines the two parts of the constrained optimization problem. The cost function for this problem is

$$J = E\left[\, y(n)\, y^*(n) \right] + \mathrm{Re}\left[\lambda^* \left(\sum_{k=0}^{M-1} w_k^* e^{-jk\theta_o} - \mathrm{g} \right) \right] \qquad (4.9)$$

Which can be written as

$$J = \underbrace{\sum_{k=0}^{M-1} \sum_{i=0}^{M-1} w_k^{*} w_i r(i-k)}_{\text{Output power}} + \underbrace{\mathrm{Re}\left[\lambda^{*} \left(\sum_{k=0}^{M-1} w_k^{*} e^{-jk\theta_0} - g \right) \right]}_{\text{Linear constraint}} \qquad (4.10)$$

In above equation λ is a complex valued Lagrange multiplier. In the definition of the cost function denoted by J, there is no desired response, bur it has a linear constraint that has to be satisfied for the prescribed or special value of electrical angle θ_0 in the beamforming. On the other hand the angular frequency w_0 in the context of transversal filtering. In both cases the linear constraint maintains the required signal of interest, and minimizes the value of cost function J. It also attenuates interference or noise that can be troublesome if left unchecked/untreated.

We have to solve equation (4.10) for the optimum values of the weights of the beamformer. The optimum weights minimize J as defined above. From the solution of the cost function J, we will determine the gradient vector ∇J, and then set it equal to zero for minimum value. Differentiating equation (4.10) with respect to w_k^{*} as described in appendix B [9] and find the kth element of the gradient vector ∇J that is

$$\nabla J = 2 \frac{\partial J}{\partial \mathbf{w}_k^{*}}$$

$$\nabla_k J = 2 \sum_{i=o}^{M-1} w_i r(i-k) + \lambda^{*} e^{-jk\theta_0} \qquad (4.11)$$

Suppose $\mathbf{w}_{oi}$ be the ith element of the optimum weight vector $\mathbf{w}_o$ .Then beamformer with weight vector w_{oi} is represented as

$$\sum_{i=0}^{M-1} \mathbf{w}_{oi}\, r(i-k) = -\frac{\lambda^*}{2} e^{-jk\theta_o} \quad ,k = 0,1,..M-1 \qquad (4.12)$$

The above beamformer represent M optimum values of the weight vector by simultaneous equations. These equations are somewhat similar to that of the Wiener-Hopf equations.

At this point in the analysis, we find it convenient to switch to matrix notation. In particular, we may rewrite the system of M simultaneous equations given in (4.12) simplifies as

$$\mathbf{R}\,\mathbf{w}_o = -\frac{\lambda^*}{2}\,\mathbf{S}\,(\theta_o) \qquad (4.13)$$

Where $\mathbf{R}$ is the MxM correlation matrix and $\mathbf{w}_o$ is Mx1 optimum weight vector of the constrained beamformer. The Mx1 *Steering vector* is defined by

$$\mathbf{S}(\theta_o) = \left[1 e^{-j\theta_o}\; e^{-j2\theta_o}\; e^{-j3\theta_o}\; ..\; e^{-j(M-1)\theta_o}\right]^T$$

Solving equation (4.13) for $\mathbf{w}_o$ we thus have

$$\mathbf{w}_o = -\frac{\lambda^*}{2}\,\mathbf{R}^{-1}\,\mathbf{S}(\theta_o) \qquad (4.14)$$

here $\mathbf{R}^{-1}$ is the inverse of the correlation matrix $\mathbf{R}$, we assume that $\mathbf{R}$ is non singular(its determinant is not zero) . This assumption is perfectly justified in practice by virtue of the fact that, in the context a beamformer, the received signal at the output of each antenna element of the systems may include a white (thermal) noise component representing sensor noise.

The solution for the optimum weight vector $\mathbf{w}_o$ given in equation 4.14 is not complete as it involves the unknown Lagrange multiplier λ (or its complex conjugate, to be precise). To eliminate λ from this expression, we fist use the linear constraint of equation (4.8) to write.

$$\mathbf{w}_o^H \mathbf{S}(\theta_o) = g \qquad (4.15)$$

Hence taking the hermitian transpose of both sides of equation. 4.15, post multiplying by $\mathbf{S}(\theta_o)$ and then using the linear constraint of equation (4.15) we get

$$\lambda = -\frac{2g}{\mathbf{S}^H(\theta_o)\mathbf{R}^{-1}\mathbf{S}(\theta_o)} \qquad (4.16)$$

Where we have used the fact that $\mathbf{R}^{\mathbf{H}} = \mathbf{R}^{-1}$. The quadratic form $\mathbf{S}^{\mathbf{H}}(\theta_o)\mathbf{R}^{-1}\mathbf{S}(\theta_o)$ is real valued. Hence substituting Equation 4.15 into Equation 4.16 we get the desired formula for the optimum weight vector:

$$\mathbf{w}_o = \frac{g^*\mathbf{R}^{-1}\mathbf{S}(\theta_o)}{\mathbf{S}^H(\theta_o)\mathbf{R}^{-1}\mathbf{S}(\theta_o)} \qquad (4.17)$$

By minimizing the output power subject to the linear constraint defined in Equation 4.15, signals/information incident on the array along direction different from the prescribed value θ_o will be attenuated.

For obvious reasons, a beamformer characterized by the weight vector $\mathbf{w}_o$ is referred to as a *linearly constrained minimum variance* (**LCMV**) beamformer. For a zero mean input and therefore a zero mean output, "minimum variance" and "minimum mean square value" are indeed synonymous.

4.3 Minimum Variance Distortionless Response Beamformer

From the previous results we come to know that the complex constant g defines the output response of an **LCMV** beamformer for the input signal impinges at the electrical angle θ_o. For a special case when $\mathbf{g} = 1$, the optimum solution given in the Equation 4.17 becomes

$$\mathbf{w_o} = \frac{\mathbf{R}^{-1}\mathbf{s}(\theta_o)}{\mathbf{s}^H(\theta_o)\mathbf{R}^{-1}\mathbf{s}(\theta_o)} \qquad (4.18)$$

The output of the beamformer described by equation (4.18) is constrained equal to unity, where signal of interest is incident at an electrical angle θ_0. In other words, we say that this beamformer will give out a distortionless response along the look direction corresponding to θ_0.

The output of the beamformer having minimum mean-square value (average power) of the optimum beamformer is expressed as

$$J_{\min} = \mathbf{w}_o^H \mathbf{R} \mathbf{w}_o \qquad (4.19)$$

Substituting value of $\mathbf{w}_o^H$ and $\mathbf{w}_o$ from equation (4.18) into equation (4.19) it will be

$$J_{\min} = \frac{1}{\mathbf{S}^H(\theta_o)\mathbf{R}^{-1}\mathbf{S}(\theta_o)} \qquad (4.20)$$

Now the beamformer has become optimum and it is constrained to pass the target signal (signal of interest) with unit response, while at the same time the output variance is minimized. In this process of variance minimization it also attenuates interference and noise which is not originating/impinging at the electrical angle θ_0. As a result we can say that $J_{\min}$ describes an estimate of the variance of the signal impinging on the array along the direction corresponding to θ_0. We can also generalize this result by obtaining an estimate of variance as a function of direction. $J_{\min}$ will be formulated as a function of θ.

In this way we will obtain the **MVDR** spatial power spectrum as

$$\mathrm{S}_{MVDR}(\theta) = \frac{1}{\mathbf{S}^H(\theta_o)\mathbf{R}^{-1}\mathbf{S}(\theta_o)} \qquad (4.21)$$

Where

$$\mathbf{S}(\theta) = \begin{bmatrix} 1 & e^{-j\theta} & e^{-j2\theta} & e^{-j3\theta} .. e^{-j(M-1)\theta} \end{bmatrix}^T \qquad (4.22)$$

$\mathbf{S}(\theta)$ which is Mx1 vector, it is called the spatial scanning vector in the context of the beamforming environment. In the transversal filter a *frequency scanning* vector with ω in place of θ as represented in figure (4.2). By definition $\mathrm{S}_{MVDR}(\theta)$ and $\mathrm{S}_{MVDR}(\omega)$

give out the power. In case of beamformer the input is electrical angle θ while the input of the transversal filter is angular frequency ω and both give out a power spectrum estimate. Indeed, it is commonly referred to as the minimum variance distortion less response (**MVDR**) spectrum.

Knowledge of desired signal strength and also the reference signal is mostly required in all beamforming techniques. We can overcome these limitations through the application of linear constraints to the weight vector. LCMV spatial filters are beamformers that choose their weights so as to minimize the filter's output variance or power subject to constraints. This criterion together with other constraints ensure signal preservation at the location of interest while minimizing the variance effects of signals originating from other locations.

4.4 Generalized Sidelobe Canceller

Just like other algorithms null steering algorithms do not look for the signal presence and then enhance it, instead they examine where nulls are located or the desired signal is not present and minimize the output signal power. One technique based on this approach is to minimize the mean squared value of the array output. LCMV narrowband beamformer defined by the linear constraint is

$$\sum_{k=0}^{M-1} w_k^* e^{-j\theta_0 k} = \mathrm{g} \qquad (4.23)$$

this constraint represents the inner product (the vector form of equation (4.23) is

$$\mathbf{w}^H \mathbf{S}(\theta_\circ) = \mathbf{g} \qquad (4.24)$$

Here $\mathbf{w}$ is the weight vector and $\mathbf{S}(\theta_\circ)$ is the steering vector pointing along the electrical angle $\theta_\circ$. The steering vector is Mx1 vector, where M is the total number of antenna elements in the beamformer. We consider multiple linear constraints instead of a single linear constraint to generalize the idea. So we have to construct a constraint matrix as given below

$$\mathbf{C}^H \mathbf{w} = \mathbf{g} \tag{4.25}$$

Where matrix $\mathbf{C}$ is called the constraint matrix, and the vector $\mathbf{g}$, named as the gain vector. It has constant elements or numerical values. Assuming that there are L linear constraints, $\mathbf{C}$ is MxL matrix and $\mathbf{g}$ is Lx1 vector; each column of the matrix $\mathbf{C}$ represents a single linear constraint. It is also assumed that the constraints matrix $\mathbf{C}$ has linearly independent columns.

The example below has only two steering vectors $\mathbf{S}(\theta_\circ)$ and $\mathbf{S}(\theta_1)$ where $\mathbf{S}(\theta_\circ)$ is the steering vector along the electrical angle $\theta_\circ$ and $\mathbf{S}(\theta_1)$ is the steering vector along the electrical angle θ_1. The constraint against the $\mathbf{S}(\theta_\circ)$ is 1 (one) while the constraint against the $\mathbf{S}(\theta_1)$ is 0. Mathematically it is

$$\left[\mathbf{S}(\theta_o), \mathbf{S}(\theta_1)\right]^H \mathbf{w} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{4.26}$$

The **LCMV** narrowband beamformer preserves the signal of interest impinging on the array along the electrical angle θ_o equal to the constraint defined above, and at the same time suppresses an interference known to originate along the electrical angle θ_1, whose constraint is zero at that time.

Suppose that the columns of Mx(M - L) matrix $\mathbf{C}_a$ be defined as a basis for the orthogonal complement of the space spanned by the columns of the matrix $\mathbf{C}$. Using the definition of an orthogonal complement we may thus write

$$\mathbf{C}^H \mathbf{C}_a = \mathbf{0} \qquad (4.27)$$

It is also true for

$$\mathbf{C}_a^H \mathbf{C} = \mathbf{0} \qquad (4.28)$$

The order of null matrix $\mathbf{0}$ in equation (4.27) is Lx (M - L) where as in equation (4.28) it is (M - L)xL ; we know that the constraints are less than the total antenna elements, so M > L. We now want to define a MxM partitioned matrix which is

$$\mathbf{U} = \left[\mathbf{C} \vdots \mathbf{C}_a\right] \qquad (4.29)$$

Whose columns (equal to number of antenna elements) span the entire M-dimensional signal space. The inverse of this matrix which is $\mathbf{U}^{-1}$ exists because of the fact that the determinant of matrix $\mathbf{U}$ is non zero. Next, the order of the weight vector of the beamformer is Mx1 and we will write it in terms of the matrix $\mathbf{U}$ as

$$\mathbf{w}_{\mathrm{Mx1}} = \mathbf{U}_{\mathrm{Mx1}} \mathbf{q}_{\mathrm{Mx1}} \qquad (4.30)$$

Equivalently, the Mx1 vector $\mathbf{q}$ is defined by

$$\mathbf{q}_{\mathrm{Mx1}} = \mathbf{U}^{-1}{}_{\mathrm{MxM}}\, \mathbf{w}_{\mathrm{Mx1}} \qquad (4.31)$$

Let $\mathbf{q}_{\mathrm{Mx1}}$ be partitioned in a manner compatible with that in Equation (4.29), as given by

$$\mathbf{q}_{\mathrm{Mx1}} = \begin{bmatrix} \mathbf{V}_{\mathrm{Lx1}} \\ \cdots\cdots\cdots \\ \mathbf{w}_{a(\mathrm{M-L})\mathrm{x1}} \end{bmatrix} \qquad (4.32)$$

Where $\mathbf{V}$ vector is of the order of $L\mathrm{x}1$ and that of the $\mathbf{w}_a$ vector is $(M-L)\mathrm{x}1$, i.e the portion of weight vector $\mathbf{w}_{MX1}$ which is not affected by the constraints. Using the Equation (4.29) and (4.32) in Equation (4.30). So new form of equation (4.30) is (by putting values of $\mathbf{U}$ and $\mathbf{q}$)

$$\mathbf{w}_{M\mathrm{x}1} = \left[\mathbf{C}_{M\mathrm{x}L} : \mathbf{C}_{L\mathrm{x}(L-M)}\right] \begin{bmatrix} \mathbf{V}_{L\mathrm{x}1} \\ \cdots\cdots\cdots \\ -\mathbf{w}_{a(M-L)\mathrm{x}1} \end{bmatrix} \tag{4.33}$$

$$\mathbf{w} = \mathbf{C}\,\mathbf{v} - \mathbf{C}_a\,\mathbf{w}_a \tag{4.34}$$

Now we can apply the multiple linear constraints. Multiplying Equation (4.34) by $\mathbf{C}^H$ we get

$$\begin{aligned} \mathbf{C}^H\mathbf{w} = \mathbf{g} &= \mathbf{C}^H\,\mathbf{C}\,\mathbf{v} - \mathbf{C}^H\mathbf{C}_a\mathbf{w}_a \\ \mathbf{g} &= \mathbf{C}^H\mathbf{C}\,\mathbf{v} - \mathbf{C}^H\mathbf{C}_a\mathbf{w}_a \\ \mathbf{g} &= \mathbf{C}^H\mathbf{C}\,\mathbf{v} \end{aligned} \tag{4.35}$$

But, from equation (4.27) we know that $\mathbf{C}^H\mathbf{C}_a$ is $\mathbf{0}$ (null matrix). Putting in Equation (4.35) it becomes

$$\mathbf{C}^H\mathbf{C}\,\mathbf{v} = \mathbf{g} \tag{4.36}$$

Solving (4.36) for the vector $\mathbf{v}$ we will get

$$\mathrm{v} = (\mathrm{C}^H\mathrm{C})^{-1}\,\mathbf{g} \tag{4.37}$$

From Equation (4.37) it is clear that the multiple linear constraints do not effect $\mathbf{w}_a$.

Let us define a fixed beamformer component represented by $\mathbf{w}_q = \mathbf{C}\,\mathbf{v}$, It is given by left multiplying equation (4.37) by $\mathbf{C}$, we will get

$$\mathbf{w}_q = \mathbf{C}\,\mathbf{v} = \mathbf{C}\,(\mathbf{C}^H\mathbf{C})^{-1} = \mathbf{g} \qquad (4.38)$$

This $\mathbf{w}_q$ is orthogonal to the columns of matrix $\mathbf{C}_a$ and it is in accordance with the property described in the equation (4.28); the fundamental reason for using the subscript **q** in $\mathbf{w}_q$ will be explained later. From equation (4.38), putting value of $\mathbf{w}_q = \mathbf{C}\mathbf{v}$ in equation (4.34). The new form of equation (4.34) will give out the overall weight vector of the beamformer.

$$\mathbf{w} = \mathbf{w}_q - \mathbf{C}_a\mathbf{w}_a \qquad (4.39)$$

Substituting Equation (4.39) into Equation(4.25) which is $\mathbf{C}^H\,\mathbf{w} = \mathbf{g}$ will give out

$$\mathbf{C}^H\mathbf{w}_q - \mathbf{C}^H\mathbf{C}_a\mathbf{w}_a = \mathbf{g} \qquad (4.40)$$

By use of Equation (4.27), Equation (4.40) becomes

$$\mathbf{C}^H\mathbf{w}_q = \mathbf{g} \qquad (4.41)$$

Equation (4.41) expresses the weight vector $\mathbf{w}_q$ which is the part of the weight vector **w** satisfying the constraints. On the other hand the vector $\mathbf{w}_a$ is unaffected by the constraints and does not take part in constraint calculation. It therefore provides the degrees of freedom built into the design of the beamformer. So according to the equation (4.40), the beamformer may be represented by the block diagram shown in Figure 4.4.

The beamformer described in Figure 4.4 is referred to as generalized side lob canceller (GSC).[9]

According to equation (4.38) we can perform an unconstrained minimization of the mean square value of the beamformer output y(n) with respect to the adjustable weight vector $\mathbf{w}_a$. According to Equation (4.7) beamformer output can be written as

$$y(n) = \mathbf{w}^H \mathbf{u}(n) \tag{4.42}$$

Where

$$\mathbf{u}(n) = \mathbf{u}_{0(n)}\left[1 \; e^{-j\theta_o} \; e^{-j2\theta_o} \; ..e^{-j(M-1)\theta_o}\right]^T \tag{4.43}$$

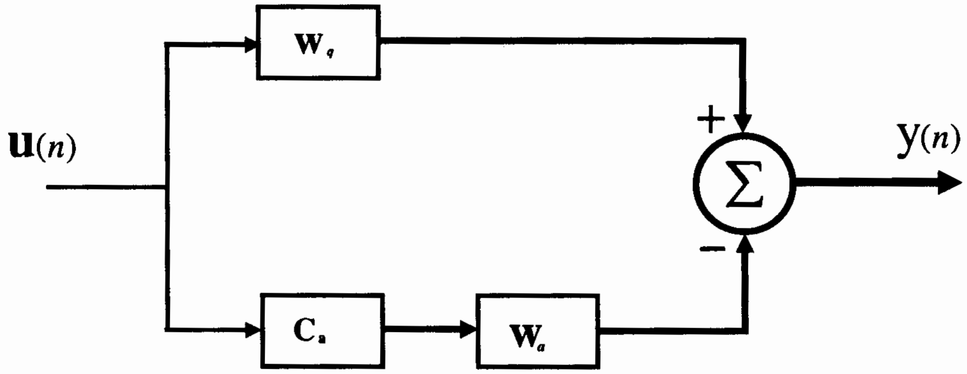

Figure 4.4 Block diagram of generalized sidelobe canceller

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u(n) is the input signal vector impinging on the linear array, in which the electrical angle θ_o is defined by the direction of arrival of the incoming plane wave and $u_0(n)$ is the electrical signal picked up by the antenna element 0 (zero) of the linear array in figure 4.3 at time n. Hence substituting equation (4.39) into equation(4.42) gives

$$y(n) = \mathbf{w}_q^H \mathbf{u}(n) - \mathbf{w}_a^H \mathbf{C}_a^H \mathbf{u}(n) \tag{4.44}$$

for simplicity we can write

$$\mathbf{w}_q^H \, \mathbf{u}(n) = d(n) \tag{4.45}$$

And

$$\mathbf{C}_a^H \, \mathbf{u}(n) = \mathbf{x}(n) \tag{4.46}$$

The Equation (4.44) becomes in a form that resembles the standard Wiener filter exactly, as shown by

$$y(n) = d(n) - \mathbf{w}_a^H \, \mathbf{x}(n) \tag{4.47}$$

While $d(n)$ play the role of desired response for the GSC and $\mathbf{x}(n)$ plays the role of input vector. We thus see that the combined use of vectors $\mathbf{w}_q$ and matrix $\mathbf{C}_a$ has converted the linearly constrained optimization problem into a standard optimum filtering problem. In particular we now have an unconstrained optimization problem involving the adjustable portion $\mathbf{w}_a$ of the weight vector, which may be formally written as

$$\min_{\mathbf{w}_a} E\left[\left|y(n)\right|^2\right] = \min_{\mathbf{w}_a}(\sigma_d^2 - \mathbf{w}_a^H \mathbf{p}_x - \mathbf{p}_x^H \mathbf{w}_a + \mathbf{w}_a^H \mathbf{R}_x \mathbf{w}_a) \tag{4.48}$$

Where the (M - L)x1 cross-correlation vector

$$\mathbf{p}_x = E\left[\mathbf{x}(n)\; d^*(n)\right] \tag{4.49}$$

and (M - L)x(M - L) correlation matrix.

$$\mathbf{R}_{\mathrm{x}} = E\left[\mathbf{x}(n)\, \mathbf{x}^{\mathrm{H}}(n)\right] \tag{4.50}$$

The cost function of the Equation (4.48) is quadratic in the unknown vector $\mathbf{w}_a$ which provides as stated before available degrees of freedom in the GSC. Most importantly this cost function has exactly the same mathematical form as that of the standard Weiner filter. We may use our previous results to obtain the value of $\mathbf{w}_a$ as

$$\mathbf{w}_{ao} = \mathbf{R}_x^{-1}\mathbf{p}_x \tag{4.51}$$

Using equation (4.45) and equation (4.46) in equation (4.49), we will get the vector $\mathbf{P}_x$ as

$$\begin{aligned}\mathbf{p}_x &= E\left[\mathbf{C}_a^H\,\mathbf{u}(n)\mathbf{u}^H(n)\,\mathbf{w}_q\right]\\ &= \mathbf{C}_a^H\left[\mathbf{u}(n)\mathbf{u}^H(n)\right]\mathbf{w}_q \qquad (4.52)\\ \mathbf{p}_x &= \mathbf{C}_a^H\mathbf{R}\mathbf{w}_q\end{aligned}$$

Where $\mathbf{R}$ is the correlation matrix of the incoming data vector $\mathbf{u}$(n). Similarly using equation (4.46) in equation (4.50) we can express the matrix $\mathbf{R}_x$ as

$$\begin{aligned}\mathbf{R}_x &= E\left[\mathbf{C}_a^H\mathbf{u}(n)\mathbf{u}^H(n)\mathbf{C}_a\right]\\ &= \mathbf{C}_a^H\left[\mathbf{u}(n)\mathbf{u}^H(n)\right]\mathbf{C}_a \qquad (4.53)\\ \mathbf{R}_x &= \mathbf{C}_a^H\mathbf{R}\mathbf{C}_a\end{aligned}$$

The matrix $\mathbf{C}_a$ has full rank, and the correlation matrix $\mathbf{R}$ is a positive definite, since the incoming data always contain some form of additive sensor noise, with the result that

$\mathbf{R}_x$ is non singular. Accordingly we may rewrite the optimum solution from equation (4.51) as

$$\mathbf{w}_{ao} = (\mathbf{C}_a^H \mathbf{R} \mathbf{C}_a)^{-1} \mathbf{C} \mathbf{R} \, \mathbf{w}_q \tag{4.54}$$

We suppose that P_o denote the minimum output power of the GSC attained by using the optimum solution $\mathbf{w}_{ao}$. Then adapting the previous result derived for the standard Wiener filter and proceeding in the same manner, we will express P_o as

$$\begin{aligned} p_o &= \sigma_d^{2} - \mathbf{P}_x^{H} \mathbf{R}_x^{-1} \mathbf{P}_x \\ &= \mathbf{w}_q^H \mathbf{R} \, \mathbf{w}_q - \mathbf{w}_q^H \mathbf{R} \; \mathbf{C}_a (\mathbf{C}_a^H \mathbf{R} \mathbf{C}_a)^{-1} \mathbf{C}_a^H \mathbf{R} \, \mathbf{w}_q \end{aligned} \tag{4.55}$$

Special case:

Consider that there is no signal of interest but only noise is present. It is called quiet environment, for which the received signal consists of white noise acting alone. Let the corresponding value of the correlation matrix **R** will then be

$$\mathbf{R} = \sigma^2 \mathbf{I} \tag{4.56}$$

Where $\mathbf{I}$ is MxM identity matrix and σ^2 is the noise variance. Under this condition we readily find using Equation (4.54) that

$$\mathbf{w}_{ao} = (\mathbf{C}_a^H \mathbf{C}_a)^{-1} \mathbf{C}_a^H \, \mathbf{w}_q \tag{4.57}$$

By defining the weight vector $\mathbf{w}_q$ orthogonal to the columns of matrix $\mathbf{C}_a$. It follows therefore that the optimum weight vector $\mathbf{w}_{ao}$ is identically zero for the quiet environment described by Equation (4.56). Thus with $\mathbf{w}_{ao}$ equal to zero, equation (4.39) that $\mathbf{w} = \mathbf{w}_q$. It is for this reason that $\mathbf{w}_q$ is often referred to as the quiescent weight vector hence the use of subscript q.

4.5 Practical Example of Null Steering (GSC)

Every One's ear act as acoustic sensor and receive sound signal from surrounding.

Because of the separation between the ears, each ear receives the signal with a different time delay.

The human brain, a specialized signal processor, does a large number of calculations to correlate information and compute the location of the received sound. To better provide explanation of the smart antenna system and its working, let us consider two persons having conversation inside an isolated room. There is a listener among them who is capable of determining the location of the speaker as he moves about the room because the voice of the speaker arrives at each acoustic sensor, the ear, at a different time. The brain (human) which is the signal processor, estimates the direction of the speaker with the help of time differences or delays which it has received from the two ears. Afterward, the brain increases the strength of the signals coming from each ear to focus on the sound of the estimated direction with the help of a similar process, the human brain is capable of distinguishing between multiple signals that have different directions of arrival (DOA). Thus, if additional speakers join the conversation, the brain is able to enhance the received signal from the speaker of interest and tune out unwanted interferers. Therefore, the listener has the ability to distinguish one person's voice, among many people talking simultaneously, and concentrate on one conversation at a time. In this way, any unwanted interference is attenuated. On the other hand, the listener may attend back to the same direction of the speaker of interest by orienting his transmitter, his/her mouth, toward the speaker.

Electrical smart antenna systems also work in the same manner with two antennas just as two ears. It also has a signal processor in place of the brain. Thus, based on the time delays due to the impinging signals onto the antenna elements, the digital signal processor computes the direction-of-arrival (DOA) of the signal-of-interest (SOI), and then it varies the excitations (gains and phases of the signals) to generate an excitation pattern which is then focused on the SOI and minimizing the interferers or signals-not-of-interest (SNOI).

Transferring the same idea to mobile communication systems, the base station plays the role of the listener, and the active cellular telephones simulate the role of the several sounds heard by human ears. In mobile communication system digital signal processor located at the base station works in conjunction with the antenna array and is responsible for adjusting various system parameters to filter out any interferers or signals-not-of-interest (SNOI) while enhancing desired communication or signals-of-interest (SOI). Thus, the system forms the radiation pattern in an adaptive manner, responding dynamically to the signal environment and its alterations. The principle of beam forming is essentially to weight the transmit signals in such a way that obtains a constructive superposition of different signal parts. Note that some knowledge of the transmission channel at the transmitter is necessary to make transmission feasible through beamforming, such that maximum radiated power is produced in the directions of desired mobile users and deep nulls are generated in the directions of undesired signals representing co-channel interference from mobile users in adjacent cells. Prior to adaptive beamforming, the directions of users and interferes must be obtained using a direction of arrival algorithm [7].

CHAPTER 5

SIMULATION AND RESULTS

5.1 Introduction

This chapter provides information about the simulation setup used for the simulation results. Results of the setup have been acquired using MATLAB software and finally plotted. The results are achieved according to the different angles of the source of interest and interference sources.

5.2 Simulation Setup

The beamformer has been simulated in MATLAB for following conditions.

1. For quiet environment (noise only, no signal of interest and interference is present)

2. Only single source of interest is present, with no interference.

3. One source of interest and one interference.

4. One source of interest and two interference sources.

The Beamformer used have 10 sensors (antenna elements) arranged in a linear array fashion with the separation between the sensors (elements) d=0.5 m. For the case of adaptive Beamformer every sensor branch has its tap weight. The space around or environment around it consist of two interference sources and one signal of interest source. Figure 5.1 shows the convention followed for the angles.

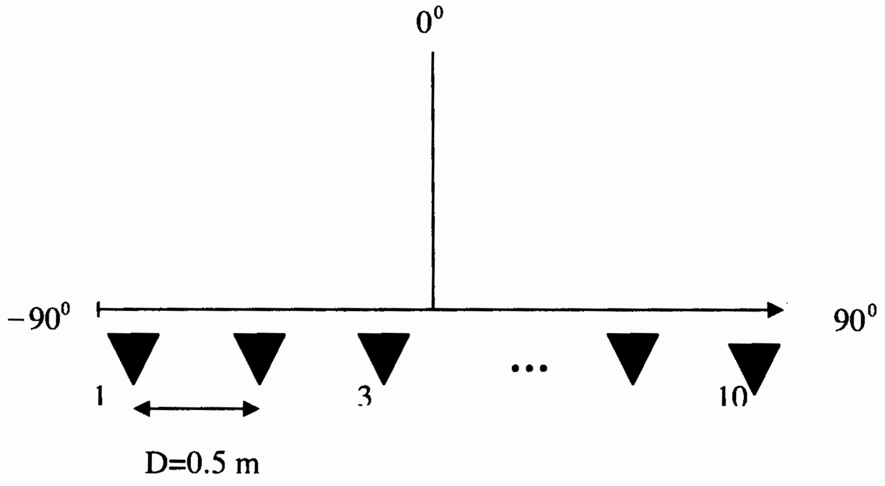

Figure 5.1 Linear array of 10 sensors.

Borrowed from [20] Course project report ENEE 624, fall 2001, page 14).

The simulation has been done for the following 2 environments.

1. The signal from the source of interest only with no interference signal

2. The signal from the source of interest as well as interference source

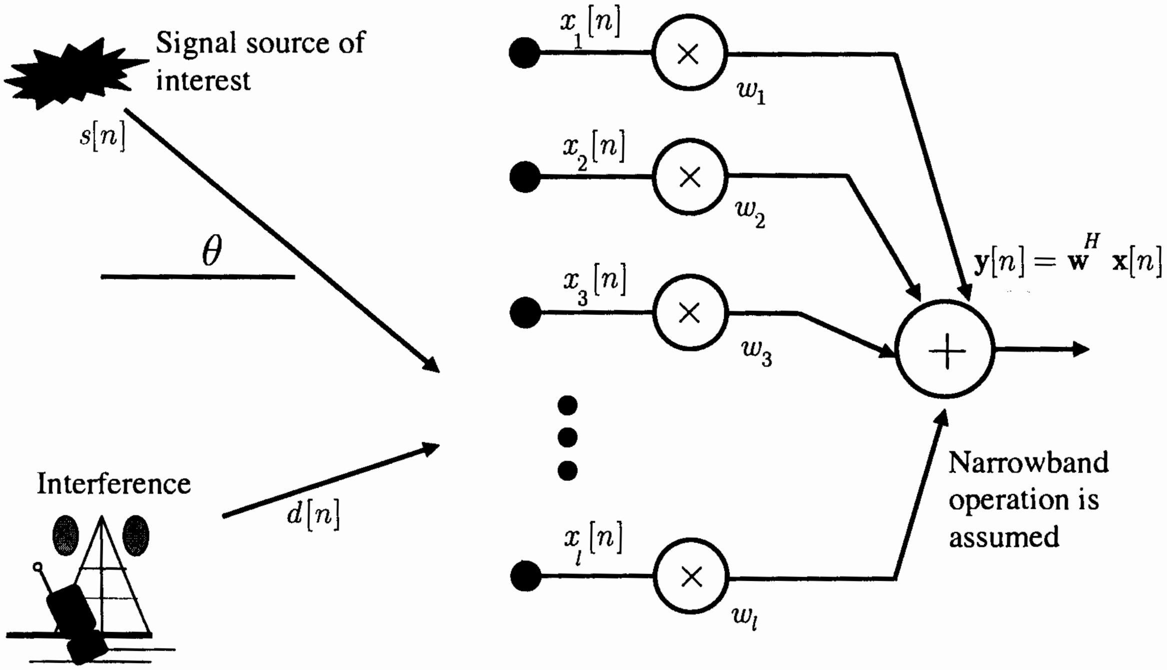

Figure 5.2 Signal and beamforming model,

Borrowed from, Interference Cancellation with a Focal Plane Array Brigham Young University page 18,

NRAO, Green Bank, Nov. 5, 2007

In the simulation all 10 antenna elements have been taking part in the signal construction. In reference to figure 5.1, the location of the source of interest is measured from reference line which is normal to the antenna array and it is marked as 0°(zero degree), The angle of incidence is taken between л and – л. So the source of interest may lie in all four quadrants, similarly the source of interference may also be present in any of the four quadrants.

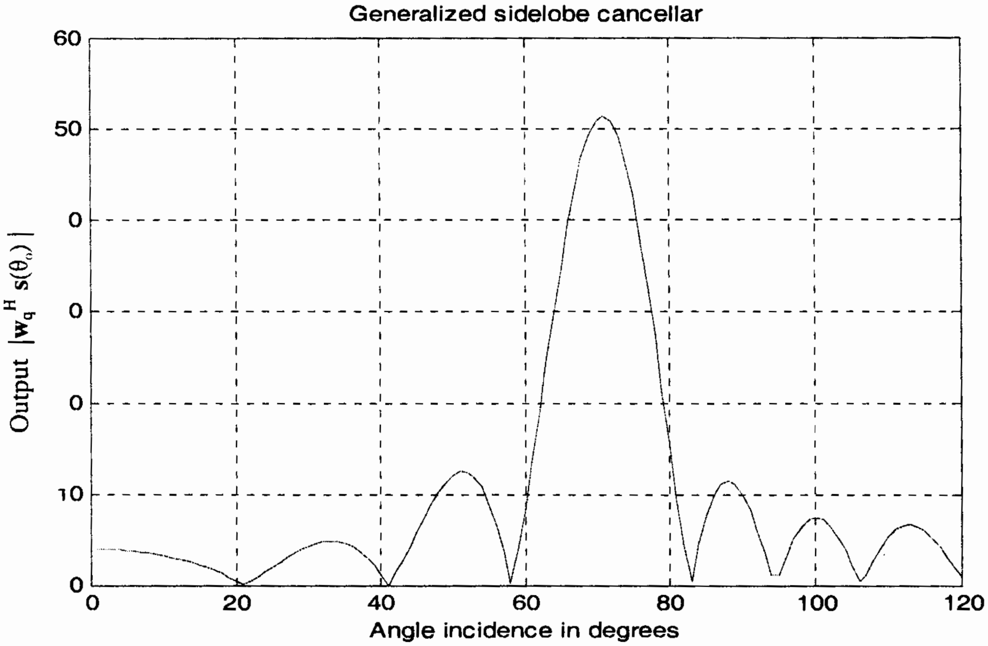

Figure 5.3 Beamformer output with source at 70° and no interference.

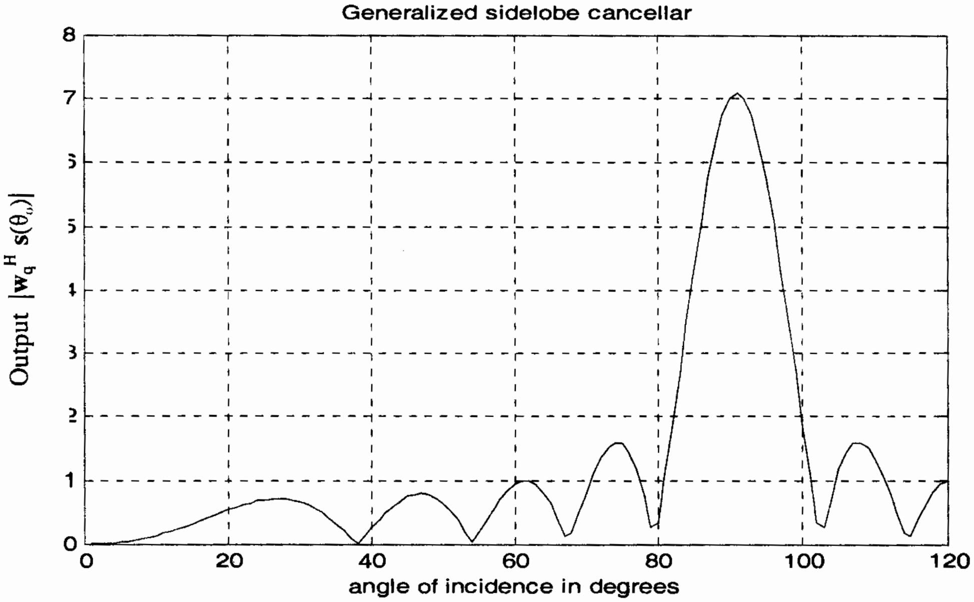

Figure 5.4 Beamformer output with source at 90° and no interference.

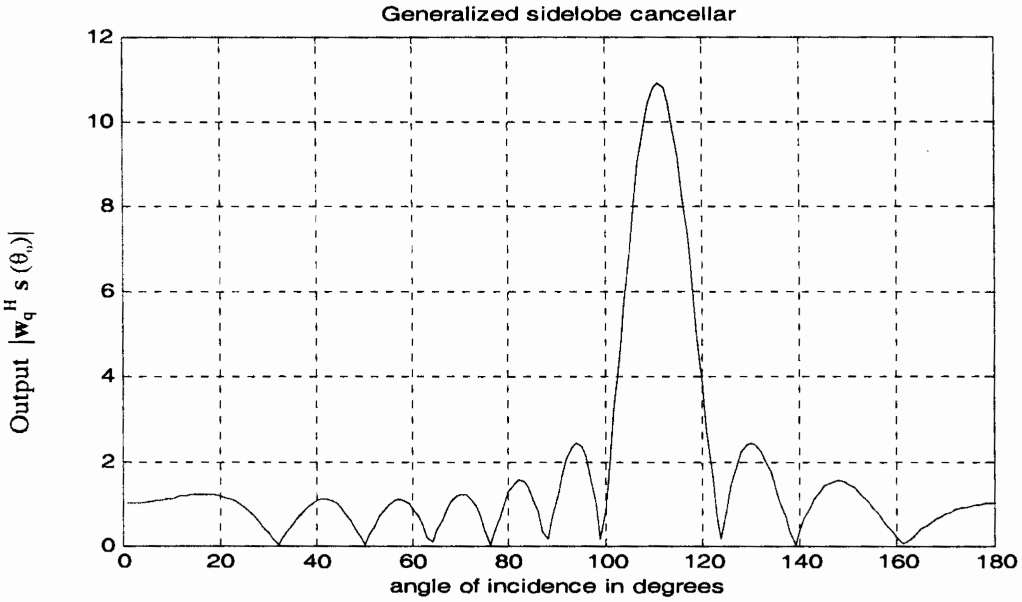

Figure 5.5 Beamformer output with source at 110° and no interference.

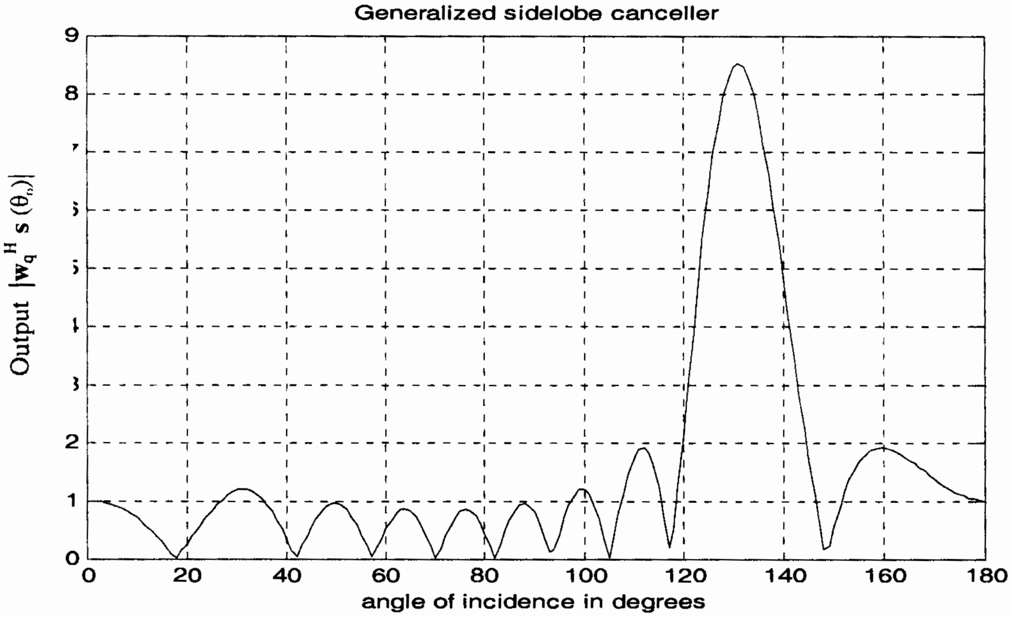

Figure 5.6 Beamformer output with source at 130° and no interference.

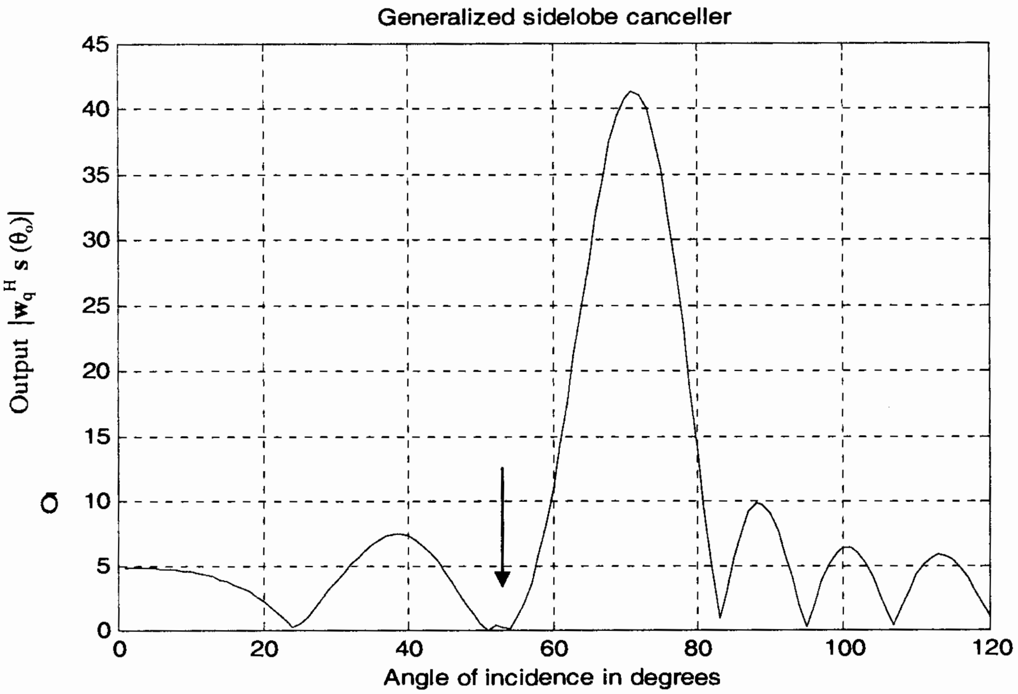

Figure 5.7 Beamformer output with source at 70° and interference at 50°.

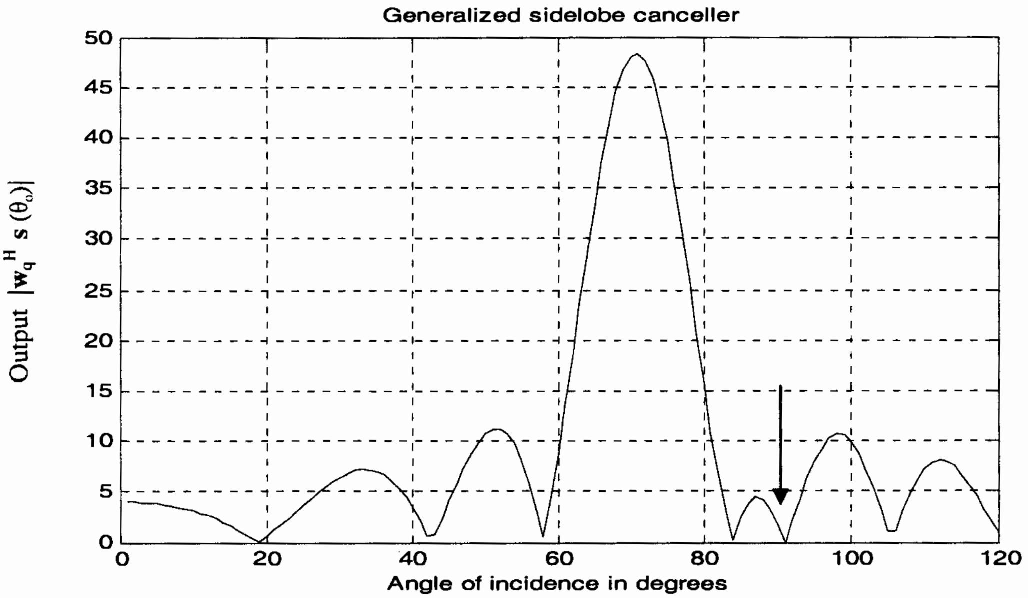

Figure 5.8 Beamformer output with source at 70° and interference at 90°.

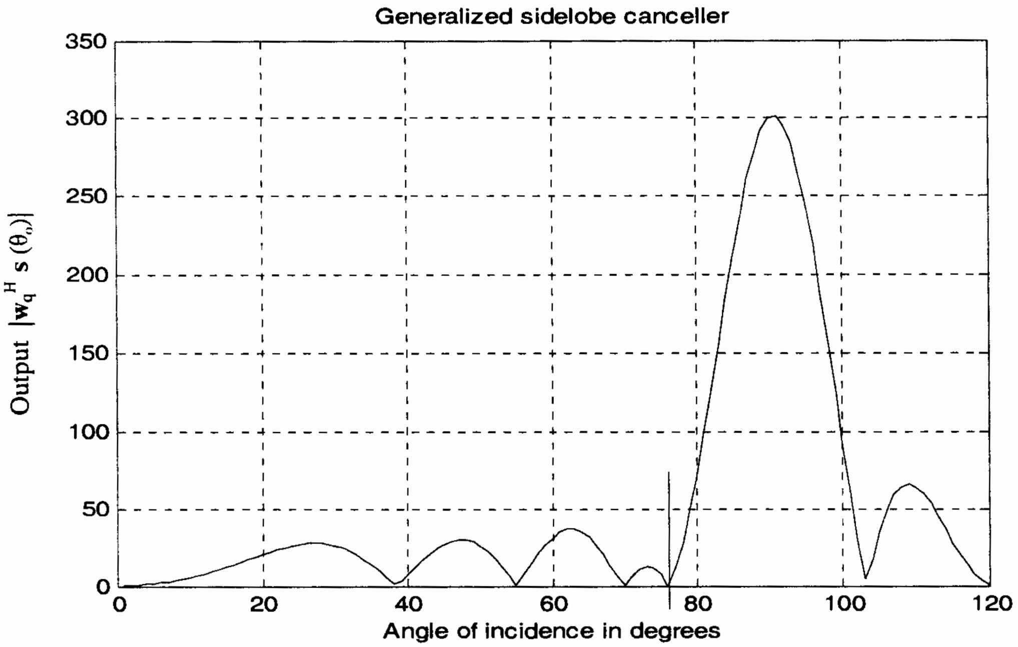

Figure 5.9 Beamformer output with source at 90° and interference at 75°.

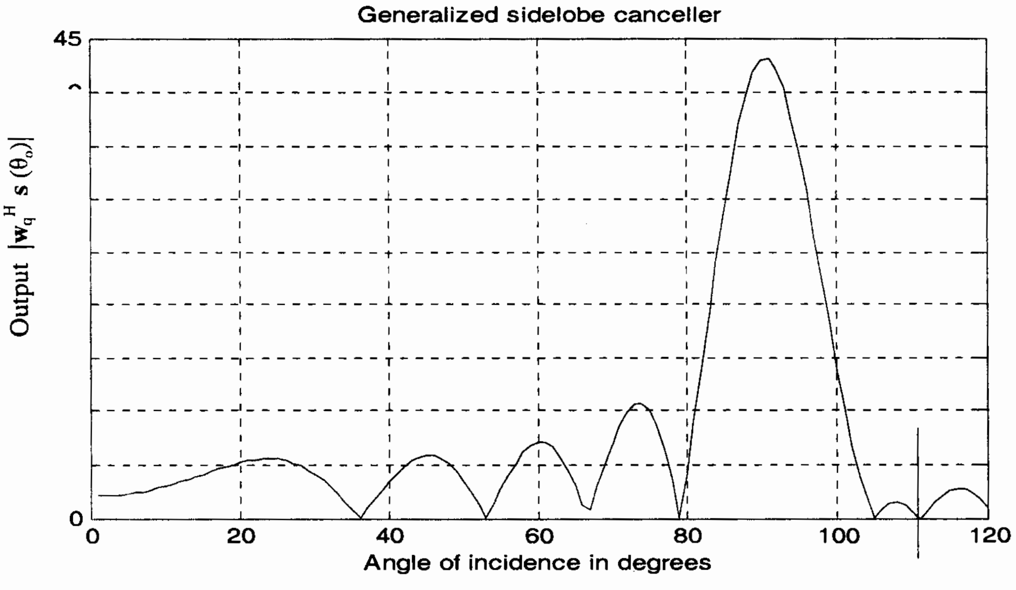

Figure 5.10 Beamformer output with source at 90° and interference at 110°.

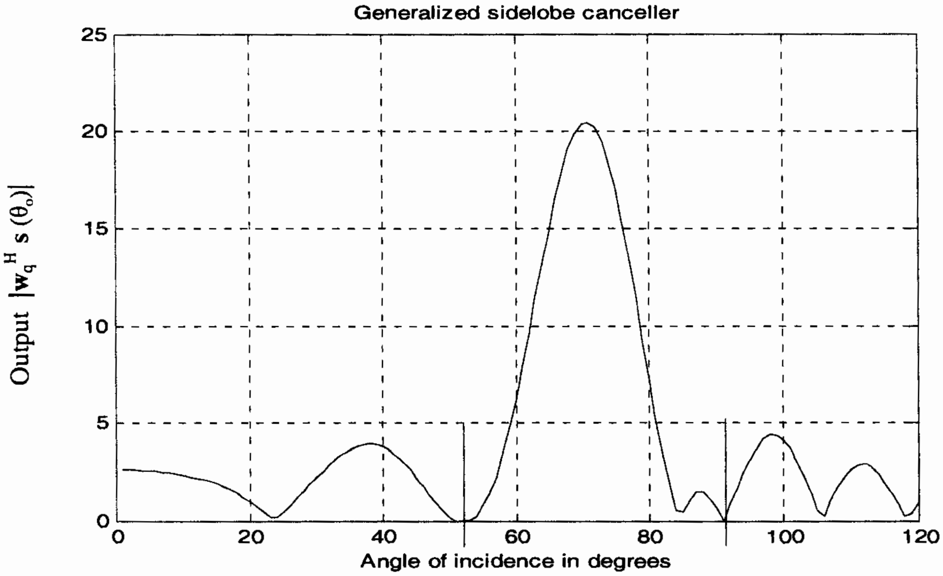

Figure 5.11 Beamformer output with source at 70° while interference at 50° and 90°

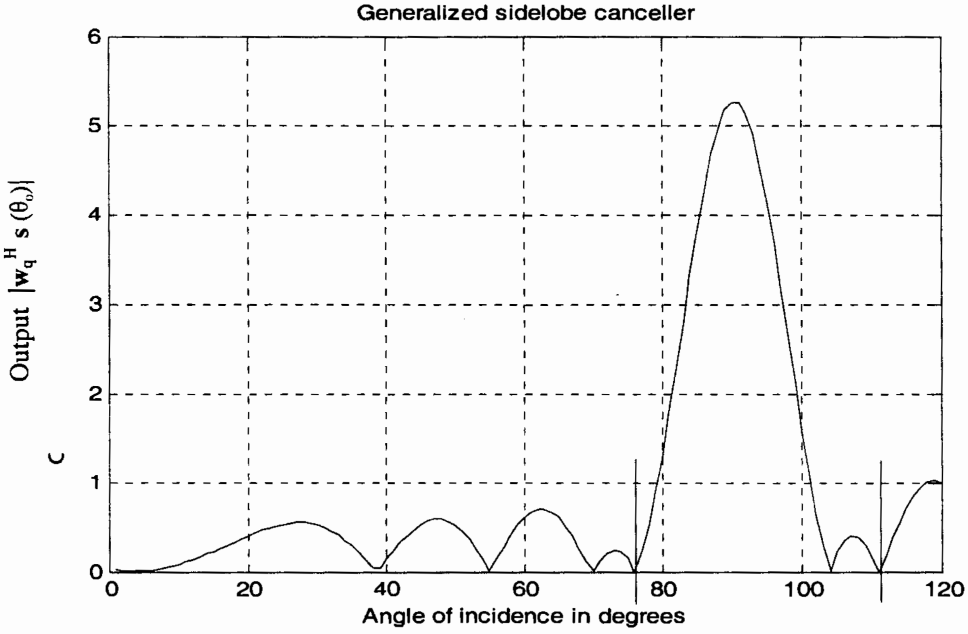

Figure 5.12 Beamformer output with source at 90° while interference at 75° and 110°

Chapter 6

Conclusion and Future Work

Conclusion

In my research work information was extracted from the received signal using adaptive beamforming and null steering technique. Nulls were placed in the direction of interfering sources for reducing the interference signal to minimum. The basic feature of this method is minimum loss of required information during interference cancellation.

The results have enabled us to effectively monitor and acquire the information from some desired direction or some specific location. It can also improve the quality of information being received with minimum computational effort, time delay and working cost.

Widespread application of this technique has revolutionized the telecom sector. It has also been successfully implemented in military and medical fields. Sonar and radar have been improved and much effective by using this technique. Noise cancellers and echo suppressor are using this technique to provide the best results. This technique has also made communication immune to Jammers.

Future Work

I have used uniform linear array as antenna in my study. The future prospect of this technique includes the use of circular array antenna instead of the linear array. High performance of this experiment may be achieved by using planar array antenna strips.

In present work we have used single multiplier / weight for each antenna element to optimize the system performance. In future however we may use a tap filter at each antenna input instead of single weight. This may improve the performance at the cost of hardware and computational complexity.

Another area of utilization may be sonars which may be considered with its constraints by proposing some basic changes in the idea.

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