A Numerical Solution for the Flow of Magnetic Fluid through a Channel

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A Dissertation
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Certificate

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A DISSERTATION SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF THE MASTER OF SCIENCE IN MATHEMATICS

We accept this dissertation as conforming to the required standard.

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2016

Dedicated To

My Mother
A strong and gentle soul who taught me to trust in Allah, Belive in hard work and that so much could be done with little.

My Father
For earning an honest living for us and for supporting and encouraging me to believe in my self

Declaration

I hereby declare that this project, neither as a whole nor as a part has been copied out from any source. It is further declare that I have developed this project and accompanied report entirely on the basis of my personal effort made under the sincere guidance of my supervisor Dr. Ahmed Zeeshan. No portion of the work presented in this report has been submitted in the support of any other degree or qualification of this or any other university or institute of learning, If found guilty I stand responsible.

Signature: ______________

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Preface

Magnetically responsive fluids called ferrofluids composed of ferromagnetic particles of approximately domain size homogeneously dispersed in a liquid carrier have recently been synthesized in the laboratory [l]. One particular class of ferrofluids consisting of colloidal dispersions of ferrite particles in organic based liquids are made stable against particle agglomeration by the addition of a surfactant. The mechanism preventing agglo- meration is here due to the short range repulsive force arising from the compression of an absorbed layer of surfactant on the particle surface balancing the attractive London and magnetic forces [2].

In this thesis, a numerical scheme namely Generalized Differential Quadrature Matheod (GDQ) is employed to solve problem generated due to flow of magnetic fluid. The thesis contain three chapters. First chapter contain the introduction of the tools and basic definations which envolves in the next chapter.

Two problems are discussed in this thesis. In Chapter 2, flow of magnetic fluid with two dipoles 'd' distance appart over flate plate is resolved using GDQ method and the result of Necringer [15] are replicated.

Chapter three, discussed the flow of magnetic fluid due to point dipole in a channel with wall is stretched proportional to length. Due to symmetry half channel is considered and problem is solved using GDQ method. The graphical results are displayed and discussed. Main findings of this research are as follows:

- The velocity component to the channel wall decreases monotonically as the magnetic field force increases.
- In the action of a adequate strong magnetic field, the temperature decreases with an increase in the prandtl number.

- It is interesting to notice that, when at velocity decreases with the variation of which is close to the boundary layer wall and opposite trend is occur for .
- It is also observe that velocity profile asymptotically approaches to one that is satisfy the boundary condition.
- The skin friction coefficient increases by increasing ferrohydrodynamic interaction parameter as well as Prandtl number Pr.
- By boosting Prandtl number Pr the Nusselt number boost up but the reverse behavior is seen in case of ferrohydrodynamic interaction parameter.

Contents

Chapter 1

Preliminaries

This chapter includes some basic definitions and concept relevant to the material presented in the subsequent chapters.

1.1 Fluid mechanics

Fluid mechanics is that branch of mechanics of applied mechanics that deals with the behavior of fluids (liquids or gases) at rest or in motion.

Fluid mechanics may be divided into three categories: fluid statics, Fluid kinematics, fluid dynamics. Fluid statics deal with the study of fluids at rest, while fluid kinematics is the study of fluid in motion without considering the forces which cause or accompany the motion. On the other hand fluid dynamics is the study of fluids in motion considering the forces acting on the fluid.

1.2 Fluid

A fluid is a substance that deforms continuously when subjected to a shear stress, no matter how small that shear stress may be. In simple words, a fluid is a substance which is capable of flowing and which conforms to the shape of containing vessel.

1.3 Some physical properties of the fluid

1.3.1 Viscosity

The viscosity of a fluid is a measure of its resistance to deformation i.e. resistance to a shearing force when the fluid is in motion . For liquids, it corresponds to the informal notion of "thickness". For example, honey has a higher viscosity than water. Mathematically, it is define as the ratio of shear stress to the range of shear strain i.e.

$$Viscosity = \mu = \frac{shear\ stress}{Rate\ of\ shear\ strain}, \tag{1.1}$$

Where μ is called dynamic viscosity. Its unit are pa.s.

1.3.2 Density

The density of a fluid denoted by ρ is defined as the mass per unit volume. Thus if m is the mass enclosed in a volume V, then

$$Density = \frac{mass\ of\ fluid}{volume\ of\ fluid}, \tag{1.2}$$

or

$$\rho = \frac{m}{\upsilon}$$

The unit of density are kgm^{-3}.

1.3.3 Kinematic viscosity

The kinematic viscosity (also called "momentum diffusivity") is the ratio of the absolute viscosity μ to the density of the fluid ρ. It is usually denoted by the Greek letter nu (υ).

$$\upsilon = \frac{\mu}{\rho}, \qquad (1.3)$$

Its unit is $m^2 s^{-1}$.

1.3.4 Temperature

Temperature of a body is defined as a measure of the intensity of heat. Heat always flows from a region of higher temperature to one of lower temperature. Physical state of a substance change with temperature. For example, water at low temperature is ice, at higher temperature is ice, at higher temperature is water and at still a higher temperature is steam. Temperature can be measure by different scales. Three common used temperature scales are the Celsius (or centigrade), Kelvin (or absolute) and the Fahrenheit scale.

1.3.5 Energy

Energy is the capacity of a physical system to perform work. Energy exists in several forms such as heat, kinetic or mechanical energy, light, potential energy, electrical, or other than form.

1.3.6 Stress

The stress or stress vector is defined as the force per unit area of the force on which it acts. If the stress is uniformly distributed over the plan area A, the stress called the average stress is defined as $\frac{\vec{F}}{A}$. The stress at any point P in the fluid is defined as

$$\text{Stress any point P} = \underset{\Delta s \to 0}{Lt} \frac{\Delta \vec{F}}{\Delta S}$$

Where $\Delta \vec{F}$ is the force acting on an element of surface area ΔS enclosing the point P.

1.4 Types of fluids

1.4.1 Compressible and incompressible fluids

It is usual to divide fluids into two groups: liquids and gases. All known liquids are slightly compressible and their density varies little with temperature and pressure. For most practical purposes, liquids are considered to be incompressible. But for situation involving either sudden or great changes in pressure or temperature, their compressibility becomes important.

1.4.2 Inviscid fluids

An inviscid fluids is that fluid having zero viscosity. With zero viscosity the fluid offers no internal resistance to a change in shape. Thus an inviscid fluid, whether at rest or in motion, can exert only a normal stress(i.e. pressure) or any surface with which it is in constant it is in contact. Consequently the shear stress in this case are zero.

1.4.3 Ideal fluids

A fluid which is both inviscid and incompressible is called an ideal fluid.

1.4.4 Real fluid

A real fluid is one which has finite viscosity and thus can exert a tangential stress on a surface with which it is in contact. The flow of real fluid is called a viscous flow. Real fluids can further be subdivided into Newtonian fluids and non-Newtonian fluids.

1.4.5 Newtonian fluids

A fluid which obeys the Newtonian's law of viscosity is called Newtonian's fluid. In this case, the shear stress is linearly proportional to the velocity gradient. The viscosity μ is constant for each Newtonian fluid at a given temperature and pressure. Water and air are the example of Newtonian fluids.

1.4.6 Non-Newtonian fluid

A fluid which does not obey the Newtonian's law of viscosity is known as non-Newtonian fluid. In this case, the shear stress is non-linearly proportional to the velocity gradient.

1.5 Types of flows

1.5.1 Incompressible and compressible flows

The flow of an incompressible fluid (i.e. for which the density remains constant throughout the fluid) is said to be incompressible flow. On the other hand, the flow of a compressible fluid i.e. for which the density is not constant is called a compressible flow.

1.5.2 Ideal and real flows

The flow of an ideal (i.e. in viscid and incompressible) fluid is said to be ideal flow, while the flow of a real (i.e. viscous) fluid is called a real flow.

1.5.3 Uniform and non uniform flow

A flow is said to be uniform when the velocity vector as well as other fluid properties do not change from point in the fluid. Thus

$$\frac{\partial \vec{V}}{\partial s} = 0, \frac{\partial \rho}{\partial s} = \frac{\partial p}{\partial s} = \ldots = 0$$

i.e. the partial derivative w.r.t 'distance' of any quantity vanishes.

A flow is said to be non-uniform when velocity, density, pressure, etc. change from point to point in the fluid flow i.e.

$$\frac{\partial \vec{V}}{\partial s} \neq \vec{0}$$

For example, a liquid floe through a long straight pipe of constant diameter is s uniform flow. On the other hand, a liquid flow through a pipe of reducing section or through a curved pipe is a non-uniform flow.

1.5.4 Steady and unsteady flow

A flow is said to be steady (i.e. stationary) when the velocity vector and other fluid properties at every point in a fluid do not change with time so that flow pattern remains unchanged i.e.

$$\frac{\partial \vec{V}}{\partial t} = 0, \frac{\partial \rho}{\partial t} = \frac{\partial p}{\partial t} = \ldots = 0$$

i.e. the partial derivative w.r.t time of any quantity vanishes. Thus the velocity is constant w.r.t time but it may vary from point to point. Hence the steady flow may be uniform or non-uniform.

A flow is said to be unsteady when fluid properties and conditions at any point in a fluid change with time i.e. $\frac{\partial \vec{V}}{\partial t} \neq \vec{0}$ etc.

1.5.5 Laminar and turbulent flow

A flow is said to be laminar if the fluid particles move along straight parallel path in layers or laminar. Thus in this flow, the curves traced out by any two different fluid particles do not intersect. On the other hand, a flow is said to be turbulent if the particle of the fluid move in irregular fashion in all directions.

1.5.6 Irrotational and rotational flow

A flow is said to be irrotational if the fluid particles do not rotates about their own axes during the flow. On the other hand, a flow is said to be rotational if the fluid particles go on rotating about their own axis during the flow.

An external flow is a flow over the outside surface of an object. Common examples are the flow around a vehicle, a truck, and an aircraft as they speed along. Depending on the geometry, external flows can be very simple or quite complicated.

1.5.7 Internal and external flow

Internal flows are those where fluid flow through confined spaces such as pipes, open channel, and fluid mechanics. The internal flow of liquids in which the channel does not flow full is called an open channel flow. For example, flow in rivers and irrigation canals.

External flows occur over bodies immersed in an unbounded fluid, such as atmosphere through which airplanes, missiles, and space vehicle travel, or the ocean water through which submarines and torpedoes.

1.6 Prandtl number

The Prandtl number (Pr) is a dimensionless number, named after the German physicist Ludwig Prandtl, defined as the ratio of momentum diffusivity to thermal diffusivity. That is, the Prandtl number is given as

$$\mathrm{Pr} = \frac{\upsilon}{\alpha} = \frac{viscous\ diffusion\ rate}{thermal\ diffusion\ rate} = \frac{\mu / \rho}{k / c_p \rho} = \frac{c_p \mu}{k}$$

1.7 Reynold's number

In fluid mechanics, the Reynolds number (Re) is a dimensionless quantity that is used to help predict similar flow patterns in different fluid flow situations. The concept was introduced by George Gabriel Stokes in 1851, but the Reynolds number is named after Osborne Reynold (1842–1912), who popularized its use in 1883.

1.8 Ferro hydrodynamics

The field of study concerned with the flow and other mechanical properties of ferrofluids.

1.9 Introduction of Ferro Fluid

Ferrofluids were first developed and classified in the 1960s by Stephen Pappell at National Aeronautics and Space Administration (NASA) as a method for controlling fluids in space. The scientists at NASA found that they could make to flow this amazing ferrofluid by varying the external magnetic field. Ferrofluid is a liquid which becomes strongly magnetized in the presence of magnetic field. There are at least three components required to prepare ferrofluid i.e. magnetic particles of colloidal size, carrier liquid and stabilizer (surfactant). They are stable suspensions of colloidal single domain ferromagnetic particles of the order of 10nm in suitable non-magnetic carrier liquid. A typical ferrofluid contain 5% magnetic solids, 10% surfactant, and 85% carrier liquids 11[3]. If the size of permanently magnetized nano-particles will be less than 1-2 nm, the magnetic properties will disappear and colloidal motion increases with decreasing the size of the particle. The colloidal particles, typically made from magnetite (Fe3O4), are coated with surfactants to avoid their agglomeration under Vander Waals attraction forces and dipole-dipole interaction among them. The presence of surfactant helps to maintain proper spacing between the particles to provide colloidal stability [4,5].

Ferrofluid is one of such smart materials, which are not available Free State in nature, but are to be synthesized. These fluids have variety of applications in the field of sciences and engineering like instrumentation, electrical and electronics engineering. etc., which are being commercialized. Ferrofluids are widely used in sealing of computer hard disk drives, rotating X-ray tubes, rotating shafts rods and sink-float systems for separation of materials. These are used as lubricants in bearing and dumpers. Also Ferrofluids are used as heat controller in electric motors and hi-fi speaker systems without the need of change in their geometrical shape Hathway [6]. Ferrofluids are being greatly used in many magnetic fluid based scientific devices like sensor, densimeters, accelerometer, pressure transducers etc., also in actuating mechanics like electromechanical converters and energy converter Raj and Moskowitz [7]. In field of biomedicine also, they have been found very useful. These can be used to deliver certain drugs to a certain area of human blood. There is also an idea to use ferrofluids for cancer treating by heating the tumor soaked in ferrofluids by means of an alternating magnetic fields is the prospect of influencing flow by the magnetic field and vice-versa [8,9]. One special application of ferrofluids is their use as magnetic ink for high-speed, inexpensive and silent printers Maruno et al. [10]

1.10 Differential Quadrature Method.

The method of DQ is based on the idea that the partial derivative of a function with respect to a space variable at a given discrete point can be expressed as a weighted linear sum of the function values at all discrete points in the domain of that variable.

Let us take the first derivative of a one-dimensional $(1-D)$ function $u(x,\ t)$ as an example, the higher-order partial derivatives will have essentially same formation. A DQ approximation of the first derivative of the function u(x, t) at the i^{th} discrete point on a grid is given by

$$u_x(x_i,t)=\sum_{j=1}^{N} c^{(1)}{}_{ij} u(x_j,t) \quad \text{for } i=1,2,\ldots,N, \tag{1.4}$$

Where $u_x(x_i,t))$ is the first derivative of $u(x,\ t)$ with respect to x at x_i, N is the number of discrete grids $c_{ij}^{(1)}$ are the weighting coefficients for the first derivative approximation. As known, the most important part of the DQ method is to determine the weighting coefficients $c_{ij}^{(1)}$. Two approaches have been used in the method of DQ.

1.10.1 Approach 1

The first one is to let equation (1.4) be exact for all polynomials of degree less than or equal to

$$(N-1), g(x) = x^k, k = 0,1,...,N, \tag{1.5}$$

Substituting this set of polynomials into equation (1.4), we have

$$\sum_{j=1}^{N} c_{ij} x_j^{\,k} = k x_i^{\,k-1} \text{ for } k = 0,1,...,N-1 \text{ and } i = 1,2,...,N, \tag{1.6}$$

Once the grid (i.e x_i) are given, this relationship leads to a set of $N \times N$ linear algebraic equations. This set of equations has a unique solution since its matrix is of vandermonde form. Unfortunately, it has been found that this set of equations become ill-conditioned and it is difficult to be solved when N is large.

In order to quantify this singularity, weighting coefficients have been calculated for equally space grids based on Equation (1.6) for various number of grid points. From the computation, it is found that the maximum number of grid point is 22. Once the grid number is greater then 22, the set of linear algebraic equations become singular and cannot be solved. The computed weighted coefficients are also compared with the accurate coefficients obtained from the generalized DQ method to be introduced later, it is found that the results from Equation (1.6) have some errors when grid number is more than 20. Therefore, the maximum number of grid points is practically 20 for equally spaced grids if this method is used for determining the weighting coefficients.

In addition, we have to solve a set of $N \times N$ linear equations for every each order of derivatives in the governing equations.

1.10.2 Approach 2

The other approach to determine the weighting coefficients is similar to the first one with an exception that a different set of test functions $g(x)$ is chosen for satisfying Equation (1.4) exactly as

$$g(x) = \frac{L_N(x)}{(x - x_j)L_N{}'(x_j)} \quad \text{for j=1,2,...,N}, \tag{1.7}$$

where N is the number of the grid points. $L(x)$ is the N^{th} order Legendre polynomial and $L'(x)$ the first derivative of $L'(x)$.

By choosing x_i to be the roots of the shifted Legendre polynomial and substituting equation (1.7) into Equation (1.4), Bellman et al. [11] obtained a direct simple algebraic expression for the weighting coefficients $c^{(1)}{}_{ij}$,

$$c^{(1)}{}_{ij} = \frac{L'_N(x_i)}{(x_i - x_j)L'_N(x_j)} \quad \text{for } i \neq j, \tag{1.8}$$

$$c^{(1)}{}_{ii} = \frac{1 - 2x_i}{2x_i(x_i - 1)} \quad \text{for } i = j, \tag{1.9}$$

for $i, j = 1, 2, \ldots N$.

It is obvious that once the number of grids N is specified, the roots of the shifted Legendre polynomial are given, thus the distribution of the grid points is fixed no matter what physical problems are considered. This imposes a major restriction on the applications of this method to problems in structural analysis, since all sorts of boundary conditions could appear and different mesh grids may be needed for different boundary conditions and different structure geometry in practice.

From the above discussion, some deficiencies are found in the originally proposed DQ. These deficiencies impose some restrictions on this method to structural analysis problems. This is probably one of the main reasons that the method of DQ is not widely used in structural analysis. In order to overcome such deficiencies, a GDQ will be introduced and applied to solve some problems in structural analysis. The ease of use and the accuracy of the GDQ will be demon- strated through the numerical examples.

1.11 Generalized Differential Quadrature

As discussed in section 1.10, two approaches have been adopted for determining the weighting coefficients by Belman el al. [11]. Both the method needs some attention. The first one restricts small number of the grids to be meshed besides the need to solve sets of linear equations. The second limits the distribution of the grid points which is critical to structural analysis. To remedy these deficiencies Shu and Richard. [13] gave a good method to determine the weighting coefficients so that the method has no limitation on the choice of grid meshes and still gives an algebraic expression for partial differential equation. In order to find a simple algebraic expression for calculating the weighting coefficients without restricting the choice of grid meshes, let us choose the Lagrange interpolated polynomial as the set of test functions $g(x)$ instead of using the power polynomials or the Legendre polynomials.

$$g_i(x) = \frac{M(x)}{(x - x_i)M^{(1)}(x_i)} \text{ for } i = 1, 2, \ldots, N, \tag{1.10}$$

where

$$M(x) = \prod_{j=1}^{N} (x - x_j), \tag{1.11}$$

and $M^{(1)}(x)$ is the first derivative of $M(x)$ defined as,

$$M^{(1)}(x_i) = \prod_{j=1, j \neq i}^{N} (x_i - x_j), \tag{1.12}$$

and N is the number of grid points.

For simplicity, we set

$$M(x) = N(x, x_i)(x - x_i) \; i = 1, 2, \ldots N, \tag{1.13}$$

with

$$N(x_i, x_j) = M^{(1)}(x_i)\delta_{ij}$$

Where δ_{ij} is the Kronecker operator.

Thus we have

$$M^{(k)}(x) = N^{(k)}(x, x_j)(x - x_j) + kN^{(k-1)}(x, x_j) \text{ for } k = 1, 2, \ldots, N - 1, \tag{1.14}$$

where $M^{(k)}(x)$ and $N^{(k)}(x, x_j)$ indicate the k^{th} order derivative of $M(x)$ and $N(x, x_j)$. Substituting Equation (1.10) into (1.4) yields

$$c^{(1)}_{ij} = \frac{M^{(1)}(x_i)}{(x_i - x_j)M^{(1)}(x_j)} \text{ for } i \neq j, \tag{1.15}$$

$$c^{(1)}_{ii} = \frac{M^{(2)}(x_i)}{2M^{(1)}(x_i)} \text{ for } i = j, \tag{1.16}$$

for $i, j = 1, 2, ..., N$.

Equation (1.15), (1.16) is a simple expression for computing $c^{(1)}_{ij}$ without any restriction on the choice of the co-ordinates of grid points x_i. It is obvious that once the grids (x_i) are given, $M^{(1)}(x)$ is very easy to be obtained from Equation (1.12). Hence, $c^{(1)}_{ij}$ can be easily calculated for $i \neq j$. The calculation of $c^{(1)}_{ii}$ is based on the calculation of the second derivative of $M(x)$ which is more difficult to obtain. Instead of using Equation (1.15), a more convenient relationship can be obtained and used for calculating $c^{(1)}_{ii}$. It can be shown by using Taylor series expansion that the following relationship exists for $c^{(1)}_{ij}$

$$\sum_{j=1}^{N} c^{(1)}_{ij} = 0 \text{ for } i = 1, 2, ..., N, \tag{1.17}$$

Thus, from Equation (1.17), the coefficient $c^{(1)}_{ii}$ can be calculated from $c^{(1)}_{ij}$ $i \neq j$. That is

$$c^{(1)}_{ii} = -\sum_{j=1, j\neq i}^{N} c^{(1)}_{ij} \text{ for } i = 1, 2, ..., N, \tag{1.18}$$

The weighting coefficients for the second and the higher-order derivatives can be similarly obtained. Let us consider the discretization of m^{th} order derivative of $u(x,t)$, the following DQ approximation is assumed

$$u^{(m)}{}_x(x_i,t)=\sum_{j=1}^{N}c^{(m)}{}_{ij}u(x_j,t) \text{ for } i=1,2,...,N, \tag{1.19}$$

Again, using Lagrange interpolated polynomials as test functions, an amazing recurrence relationship will be found for the m^{th} order weighting coefficients $c^{(m)}{}_{ij}$.

$$c^{(m)}{}_{ij}=m\left(c^{(m-1)}{}_{ii}c_{ij}-\frac{c^{(m-1)}{}_{ij}}{x_i-x_j}\right) \text{ for } i\neq j,\ m=2,3,...,N-1,\ i,j=1,2,...,N \tag{1.20}$$

,

Where $c^{(m)}{}_{ij}$ is the weighting coefficients for the m^{th} order derivative.

The calculation of $c^{(m)}{}_{ii}$ can be obtained from the relationship similar to Equation (1.18)

$$c^{(m)}{}_{ii}=-\sum_{j=1,j\neq i}^{N}c^{(m)}{}_{ij} \text{ for } i=1,2,...N, \tag{1.21}$$

Therefore, Equations (1.20) and (1.21) together with equations (1.15) and (1.18) give a convenient and general form for determining the weighting coefficients for the first through $(N-1)^{th}$ order derivatives. There are no restrictions on the co-ordinates of the chosen grid points. There is no need to solve for the weighting coefficients from a set of algebraic equations which could be ill-conditioned when the number of grids is large. Furthermore, this set of expressions for the determination of the weighting coefficients is so compact and simple and is very easy to be implemented in formulating and programming because of the recurrence feature. All these features give a great convenience to this GDQ for solving practical problems in structural analysis. Thus, it is of great potential to be used in structural analysis.

Extension of the method to two-dimensional $(2-D)$ problems is straightforward. Each dimension can be simply treated individually as a $(1-D)$ case. Assuming that there are N, grid points in the x -direction $x_1,...,x_{N_x}$, and N, grid points in the y -direction $y_1,...,y_{N_y}$. The n^{th} order partial derivative of $u(x,y)$ with respect to x and the m^{th} order partial derivative of $u(x,y)$ with respect to y at x_i, y_j can be discretized as

$$u^{(n)}{}_x(x_i,y_j)=\sum_{k=1}^{N_x} c^{(n)}{}_{ik}u(x_k,y_j),\ n=1,...,N_x-1, \tag{1.22}$$

$$u^{(m)}{}_y(x_i,y_j)=\sum_{k=1}^{N_y} c^{(m)}{}_{jk}u(x_i,y_k),\ m=1,...,N_y-1, \tag{1.23}$$

As usual, this GDQ method can be used in structural analysis for solving both ordinary differential equations and partial differential equations. The application of this method for static problems will lead to a set of algebraic equations with the function values at the grid points as unknowns. While the application to time-dependent dynamic problems will result in a set of ordinary differential equations with the time-dependent function values at the grid points as unknowns. The time-dependent ordinary differential equations can then be solved by existing integration scheme. Finally, once the function values at all grids are obtained, it is very easy to determine the function values in the overall domain in terms of polynomial approximation, that is

$$u(x,y)=\sum_{i=1}^{N_x}\sum_{j=1}^{N_y} u(x_i,y_j)r_i(x)s_j(y), \tag{1.24}$$

where $r_i(x)$ and $s_j(y)$ are the Lagrange interpolated polynomials along the x - and y - direction, respectively.

Chapter 2

Numerical Solution of Ferro hydrodynamic flow Equation

In this chapter, numerical solution of the flow problem of a saturated ferrofluid along a wall is examined. The Generalized Differential Quadrature Method (GDQ) is employed for numerical simulations. The temperature distribution drops linearly versus distance from leading edge under the influence of transverse magnetic field. The effect of sundry parameter on the velocity, temperature, skin friction coefficient and Nusselt number are depicted through graphs, tables and discussed in detail.

2.1 Mathematical Formulation

The flow problem is modeled using continuity, momentum and energy equation as Joseph L. Necringer [17]. Consider a 2-D flat plate flow problem. The pair of line currents perpendicular to and directed out of the flow plane generates the field. The wall temperature is $T_w = T_\theta(1 - x/l)$ where l is plate length. The selection of a linear variation of the wall temperature will be visible with distance from the front edge will be visible.

The magnetic scalar potential $\phi(H = -\nabla\phi)$ at any point (x, y) and the corresponding field components are

$$\phi = -\frac{I_0}{2\pi}\left[\tan^{-1}\left(\frac{y+d}{x}\right) + \tan^{-1}\left(\frac{y-d}{x}\right)\right],$$

$$H_x = -\frac{\partial \phi}{\partial x} = -\frac{I_0}{2\pi}\left[\frac{y+d}{x^2+(y+d)^2} + \frac{y-d}{x^2+(y-d)^2}\right],$$

$$H_y = -\frac{\partial \phi}{\partial y} = \frac{I_0}{2\pi}\left[\frac{x}{x^2+(y-d)^2} + \frac{x}{x^2+(y+d)^2}\right].$$

Also $$\frac{\partial^2 \phi}{\partial x^2} = -\frac{\partial^2 \phi}{\partial y^2} = -\frac{I_0}{2\pi}\left\{\frac{2x(y+d)}{\left[x^2+(y+d)^2\right]} + \frac{2x(y-d)}{\left[x^2+(y-d)^2\right]^2}\right\},$$

and

$$\frac{\partial^2 \phi}{\partial x \partial y} = -\frac{I_0}{2\pi}\left\{\frac{(y-d)^2 - x^2}{\left[x^2+(y-d)^2\right]^2} + \frac{(y+d)^2 - x^2}{\left[x^2+(y+d)^2\right]^2}\right\}.$$

For boundary layer region, and for distance from the large front edge equating to the distance of the line source from the plate, i.e. $x \gg d$,

$$(\nabla H_x) = -\frac{I_0}{\pi}\frac{1}{x^2}$$

Using above stated relation for the horizontal"magnetic"force, the solution will be suitable only for $x \gg d$.

The effect of the magnetic field on the flow is limited to a thin region close to the wall where the fluid temperature T is practical different from its free stream Curie temperature $T < T_\theta$, the magneto-thermo mechanical interaction can be narrated using boundary layer theory.

From boundary layer theory, the continuity, momentum and energy equations become,

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}, \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=-\frac{I_0\mu_0 K}{\pi\rho}(T_\theta-T)\frac{1}{x^2}+\nu\frac{\partial^2 u}{\partial y^2}, \tag{2.2}$$

$$\frac{\partial^2 T}{\partial y^2}=\frac{c}{k}\left[u\frac{\partial u}{\partial x}+\upsilon\frac{\partial T}{\partial y}\right], \tag{2.3}$$

where in the energy equation ignoring the source terms in parallel to the conduction and convection terms. For obtaining a solution of the equations (2.1), (2.2), and (2.3) with corresponding boundary conditions

$$\begin{aligned}&\text{at } y=0,\quad u=\upsilon=0;\quad T=T_\theta(1-x/l),\\ &\text{at } y=\infty,\quad u=u_0;\quad T=T_\theta,\end{aligned} \tag{2.4}$$

For obtaining the similar solution, Firstly introducing the new independent variables x and η, defined by

$$x=x,\eta=\left(\frac{u_0}{\nu x}\right)^{\frac{1}{2}}y, \tag{2.5}$$

and then introducing the non dimensional stream function and temperature functions $f(\eta)$ and $g(\eta)$ defined through the non dimensional stream function and temperature functions ψ and T defined by

$$\psi=(\nu u_0 x)^{\frac{1}{2}}f(\eta)\ ,\ T=T_\theta\left[1-\frac{x}{l}g(\eta)\right], \tag{2.6}$$

Using the transformation equations (2.5) and (2.6) in (2.2) and (2.3) and boundary conditions equation (2.4) and remembering that

$$u = \frac{\partial \psi}{\partial y} = u_0 f'(\eta); \upsilon = -\frac{\partial \psi}{\partial x} = \left(\frac{\nu u_0}{x}\right)^{\frac{1}{2}} \left[\frac{\eta f'(\eta)}{2} - \frac{f(\eta)}{2}\right], \tag{2.7}$$

Equation (2.2) and (2.3) and boundary conditions equation (2.4) becomes

$$f''' + \frac{ff''}{2} - \gamma g = 0, \tag{2.8}$$

$$g'' + \Pr\left\{\frac{fg}{2} - f'g\right\} = 0, \tag{2.9}$$

And the corresponding B.C's are

$$\begin{aligned} &at\ \eta = 0, f = f' = 0; g = 1, \\ &at\ \eta = \infty, f' = 1; g = 0. \end{aligned} \tag{2.10}$$

where the Ferrohydrodynamic interaction parameter

$$\gamma = \frac{I_0 \mu_0 K T_\theta}{\pi \rho l u_0^{\ 2}},$$

2.2 Solution of the problem

Assuming that the number of grid points is N, and applying the Generalized Differential Quadrature approximation to equations (2.8) and (2.9) at each isolated point on the grid, We get

$$\sum_{j=1}^{N} c^{(3)}{}_{ij} f_j + \frac{f_i \sum_{j=1}^{N} c^{(2)}{}_{ij} f_j}{2} - \gamma g_i = 0 \quad \text{for } i = 1,2,...N \tag{2.11}$$

$$\sum_{j=1}^{N} c^{(2)}{}_{ij} g_j + \Pr\left\{ \frac{f_i g_i}{2} - \left(\sum_{j=1}^{N} c^{(1)}{}_{ij} f_j \right) g_i \right\} = 0 \quad \text{for } i = 1,2,...,N \tag{2.12}$$

and boundary conditions at each isolated point on the grid are

$$\text{at } \eta\text{=}0, f_1 = \sum_{j=1}^{N} c^{(1)}{}_{1j} f_j = 0, g_1 = 1 \tag{2.13}$$

$$\text{at } \eta\text{=}\infty, \sum_{j=1}^{N} c^{(1)}{}_{Nj} f_j = 1, g_N = 0 \tag{2.14}$$

The set of equations $(2.11)-(2.14)$ is superfluous because there are five boundary condition in (2.13) and (2.14) plus N equations in (2.11) and (2.12). In order to remove this difficulty, we just drop the equations for $i = 1, 2, (N-1)$ and N in equation (2.11) and for $i = 1, (N-1)$ and N in (2.12). Thus

$$\sum_{j=1}^{N} c^{(3)}{}_{ij} f_j + \frac{f_i \sum_{j=1}^{N} c^{(2)}{}_{ij} f_j}{2} - \gamma g_i = 0 \quad \text{for } i = 3,4,...,N\text{-}1 \tag{2.15}$$

$$\sum_{j=1}^{N} c^{(2)}{}_{ij} g_j + \Pr\left\{ \frac{f_i g_i}{2} - \left(\sum_{j=1}^{N} c^{(1)}{}_{ij} f_j \right) g_i \right\} = 0 \quad \text{for } i = 2,3,...,N-1 \tag{2.16}$$

By expanding Equations (2.15) and (2.16) for $i = 3,4,...,N\text{-}1$ and for $i = 2,3,...,N-1$ respectively. For $i = 3,4,5$, Equation (2.15) takes the form

$$c^{(3)}{}_{31} f_1 + c^{(3)}{}_{32} f_2 + c^{(3)}{}_{33} f_3 + c^{(3)}{}_{34} f_4 + c^{(3)}{}_{35} f_5 +$$
$$f_3 \left(\frac{c^{(2)}{}_{31} f_1 + c^{(2)}{}_{32} f_2 + c^{(2)}{}_{33} f_3 + c^{(2)}{}_{34} f_4 + c^{(2)}{}_{35} f_5}{2} \right) - \gamma g_3 = 0$$

(2.17)

$$c^{(3)}{}_{41}f_1+c^{(3)}{}_{42}f_2+c^{(3)}{}_{43}f_3+c^{(3)}{}_{44}f_4+c^{(3)}{}_{45}f_5+$$
$$f_4\left(\frac{c^{(2)}{}_{41}f_1+c^{(2)}{}_{42}f_2+c^{(2)}{}_{43}f_3+c^{(2)}{}_{44}f_4+c^{(2)}{}_{45}f_5+}{2}\right)-\gamma g_4=0$$

(2.18)

$$c^{(3)}{}_{51}f_1+c^{(3)}{}_{52}f_2+c^{(3)}{}_{53}f_3+c^{(3)}{}_{54}f_4+c^{(3)}{}_{55}f_5+$$
$$f_5\left(\frac{c^{(2)}{}_{51}f_1+c^{(3)}{}_{52}f_2+c^{(3)}{}_{53}f_3+c^{(3)}{}_{54}f_4+c^{(3)}{}_{55}f_5}{2}\right)-\gamma g_5=0$$

(2.19)

for i=2,3,4, Equation (2.16) takes the form,

$$c^{(2)}{}_{21}g_1+c^{(2)}{}_{22}g_2+c^{(2)}{}_{23}g_3+c^{(2)}{}_{24}g_4+c^{(2)}{}_{25}g_5+$$
$$\Pr\left\{\frac{f_2g_2}{2}-\left(\frac{c^{(1)}{}_{21}g_1+c^{(1)}{}_{22}g_2+c^{(1)}{}_{23}g_3+c^{(1)}{}_{24}g_4+c^{(1)}{}_{25}g_5}{2}\right)g_2\right\}=0$$

(2.20)

$$c^{(2)}{}_{31}g_1+c^{(2)}{}_{32}g_2+c^{(2)}{}_{33}g_3+c^{(2)}{}_{34}g_4+c^{(2)}{}_{35}g_5+$$
$$\Pr\left\{\frac{f_3g_3}{2}-\left(\frac{c^{(1)}{}_{31}g_1+c^{(1)}{}_{32}g_2+c^{(1)}{}_{33}g_3+c^{(1)}{}_{34}g_4+c^{(1)}{}_{35}g_5}{2}\right)g_3\right\}=0$$

(2.21)

$$c^{(2)}{}_{41}g_1+c^{(2)}{}_{42}g_2+c^{(2)}{}_{43}g_3+c^{(2)}{}_{44}g_4+c^{(2)}{}_{45}g_5+$$
$$\Pr\left\{\frac{f_4g_4}{2}-\left(\frac{c^{(1)}{}_{41}g_1+c^{(1)}{}_{42}g_2+c^{(1)}{}_{43}g_3+c^{(1)}{}_{44}g_4+c^{(1)}{}_{45}g_5}{2}\right)g_4\right\}=0$$

(2.22)

From boundary conditions (2.13) and (2.14) it is clear that $f_1=g_N=0$. From equation (2.14) expanding boundary condition implies that

$$c^{(1)}{}_{51}f_1+c^{(1)}{}_{52}f_2+c^{(1)}{}_{53}f_3+c^{(1)}{}_{54}f_4+c^{(1)}{}_{55}f_5=1 \qquad (2.23)$$

After eliminating f_1 and g_5 from above equations the remaining set of linear equations are

$$\left(c^{(3)}{}_{32}+f_3c^{(2)}{}_{32}\right)f_2+\left(c^{(3)}{}_{33}+f_3c^{(2)}{}_{33}\right)f_3+\left(c^{(3)}{}_{34}+f_3c^{(2)}{}_{34}\right)f_4+$$
$$\left(c^{(3)}{}_{35}+f_3c^{(2)}{}_{35}\right)f_5=-\gamma g_3 \qquad (2.24)$$

$$\left(c^{(3)}{}_{42}+f_4c^{(2)}{}_{42}\right)f_2+\left(c^{(3)}{}_{43}+f_4c^{(2)}{}_{43}\right)f_3+\left(c^{(3)}{}_{44}+f_4c^{(2)}{}_{44}\right)f_4+$$
$$\left(c^{(3)}{}_{45}+f_4c^{(2)}{}_{45}\right)f_5=-\gamma g_4 \tag{2.25}$$

$$\left(c^{(3)}{}_{52}+f_5c^{(2)}{}_{52}\right)f_2+\left(c^{(3)}{}_{53}+f_5c^{(2)}{}_{53}\right)f_3+\left(c^{(3)}{}_{54}+f_5c^{(2)}{}_{54}\right)f_4+$$
$$\left(c^{(3)}{}_{55}+f_5c^{(2)}{}_{55}\right)f_5=-\gamma g_5 \tag{2.26}$$

$$c^{(1)}{}_{52}f_2+c^{(1)}{}_{53}f_3+c^{(1)}{}_{54}f_4+c^{(1)}{}_{55}f_5=1 \tag{2.27}$$

$$\left\{c^{(3)}{}_{11}+\Pr\left(\frac{f_1g_1}{2}-\frac{c^{(2)}{}_{11}}{2}\right)\right\}g_1+\left\{c^{(3)}{}_{12}+\Pr\left(\frac{f_1g_1}{2}-\frac{c^{(2)}{}_{12}}{2}\right)\right\}g_2+$$
$$\left\{c^{(3)}{}_{13}+\Pr\left(\frac{f_1g_1}{2}-\frac{c^{(2)}{}_{13}}{2}\right)\right\}g_3+\left\{c^{(3)}{}_{14}+\Pr\left(\frac{f_1g_1}{2}-\frac{c^{(2)}{}_{14}}{2}\right)\right\}g_4=0 \tag{2.28}$$

$$\left\{c^{(3)}{}_{21}+\Pr\left(\frac{f_2g_2}{2}-\frac{c^{(2)}{}_{21}}{2}\right)\right\}g_1+\left\{c^{(3)}{}_{22}+\Pr\left(\frac{f_2g_2}{2}-\frac{c^{(2)}{}_{22}}{2}\right)\right\}g_2+$$
$$\left\{c^{(3)}{}_{23}+\Pr\left(\frac{f_2g_2}{2}-\frac{c^{(2)}{}_{23}}{2}\right)\right\}g_3+\left\{c^{(3)}{}_{24}+\Pr\left(\frac{f_2g_2}{2}-\frac{c^{(2)}{}_{24}}{2}\right)\right\}g_4=0 \tag{2.29}$$

$$\left\{c^{(3)}{}_{31}+\Pr\left(\frac{f_3g_3}{2}-\frac{c^{(2)}{}_{31}}{2}\right)\right\}g_1+\left\{c^{(3)}{}_{32}+\Pr\left(\frac{f_3g_3}{2}-\frac{c^{(2)}{}_{32}}{2}\right)\right\}g_2+$$
$$\left\{c^{(3)}{}_{33}+\Pr\left(\frac{f_3g_3}{2}-\frac{c^{(2)}{}_{33}}{2}\right)\right\}g_3+\left\{c^{(3)}{}_{34}+\Pr\left(\frac{f_3g_3}{2}-\frac{c^{(2)}{}_{34}}{2}\right)\right\}g_4=0 \tag{2.30}$$

$$\left\{c^{(3)}{}_{41}+\Pr\left(\frac{f_4g_4}{2}-\frac{c^{(2)}{}_{41}}{2}\right)\right\}g_1+\left\{c^{(3)}{}_{42}+\Pr\left(\frac{f_4g_4}{2}-\frac{c^{(2)}{}_{42}}{2}\right)\right\}g_2+$$
$$\left\{c^{(3)}{}_{43}+\Pr\left(\frac{f_4g_4}{2}-\frac{c^{(2)}{}_{43}}{2}\right)\right\}g_3+\left\{c^{(3)}{}_{44}+\Pr\left(\frac{f_4g_4}{2}-\frac{c^{(2)}{}_{44}}{2}\right)\right\}g_4=0 \tag{2.31}$$

with Pr=10. The combination of (2.24) and (2.31) with the help of boundary conditions (2.13) and (2.14) gives N equations with N unknown function values $f_1, f_2, \ldots, f_N$ and $g_1, g_2, \ldots, g_N$ for the interaction parameter $\gamma = 0, 0.4$. The stream function $f(\eta)$ and heat transfer $g(\eta)$ are obtained by solving these set of algebraic equations. It is important to point out that the handling of boundary conditions here is diverse from that applied in structural problem using the original Differential

Quadrature method. The values of velocity and heat transfer rate are listed below in Table. 1 for $\gamma = 0, 0.4$ and Pr=10.

Table. 1 Values of velocity and heat transfer for $\gamma = 0, 0.4$ and $\Pr = 10$.

$\gamma = 0$		$\gamma = 0.4$	
f_i'	g_i	f_i'	g_i
$f'(0.05) = -1.206 \times 10^{-12}$	$g(0.05) = 1$	$f'(0.05) = -2.190 \times 10^{-12}$	$g(0.05) = 1$
$f'(0.10) = 0.053$	$g(0.10) = 0.909$	$f'(0.10) = 0.048$	$g(0.10) = 0.91$
$f'(0.15) = 0.107$	$g(0.15) = 0.819$	$f'(0.15) = 0.098$	$g(0.15) = 0.82$
$f'(0.20) = 0.161$	$g(0.20) = 0.732$	$f'(0.20) = 0.148$	$g(0.20) = 0.73$
$f'(0.25) = 0.214$	$g(0.25) = 0.649$	$f'(0.25) = 0.200$	$g(0.25) = 0.65$
$f'(0.30) = 0.268$	$g(0.30) = 0.570$	$f'(0.30) = 0.252$	$g(0.30) = 0.57$
$f'(0.35) = 0.322$	$g(0.35) = 0.496$	$f'(0.35) = 0.305$	$g(0.35) = 0.50$
$f'(0.40) = 0.375$	$g(0.40) = 0.427$	$f'(0.40) = 0.358$	$g(0.40) = 0.43$
$f'(0.45) = 0.429$	$g(0.45) = 0.364$	$f'(0.45) = 0.411$	$g(0.45) = 0.37$
$f'(0.50) = 0.482$	$g(0.50) = 0.306$	$f'(0.50) = 0.465$	$g(0.50) = 0.31$
$f'(0.55) = 0.535$	$g(0.55) = 0.255$	$f'(0.55) = 0.519$	$g(0.55) = 0.26$
$f'(0.60) = 0.588$	$g(0.60) = 0.209$	$f'(0.60) = 0.573$	$g(0.60) = 0.21$
$f'(0.65) = 0.641$	$g(0.65) = 0.168$	$f'(0.65) = 0.628$	$g(0.65) = 0.17$
$f'(0.70) = 0.693$	$g(0.70) = 0.132$	$f'(0.70) = 0.682$	$g(0.70) = 0.13$
$f'(0.75) = 0.746$	$g(0.75) = 0.101$	$f'(0.75) = 0.736$	$g(0.75) = 0.10$
$f'(0.80) = 0.797$	$g(0.80) = 0.074$	$f'(0.80) = 0.789$	$g(0.80) = 0.07$
$f'(0.85) = 0.849$	$g(0.85) = 0.051$	$f'(0.85) = 0.843$	$g(0.85) = 0.05$
$f'(0.90) = 0.900$	$g(0.90) = 0.031$	$f'(0.90) = 0.895$	$g(0.90) = 0.03$
$f'(0.95) = 0.950$	$g(0.95) = 0.014$	$f'(0.95) = 0.948$	$g(0.95) = 0.01$
$f'(1) = 1$	$g(1) = 0$	$f'(1) = 1$	$g(1) = 0$

2.3 Results and discussion

The numerical results are obtained using the generalized differential quadrature method (GDQ) by taking N=20 grid points. The set of algebraic equations and B.C's are solved with the help of Mathematica. In Table. 2 dimensionless wall skin friction coefficients, $f''(0)$ are listed as function of γ. Also listed as function of γ are the ratios of shear stress, τ / τ_0 with and without the magnetic interaction. These are also plotted in Fig 2.3. It is observed that the increase of these ratios with increase in γ is lightly non-linear with the decrease in heat transfer, again covering noticeably the decrease in skin friction.

Table. 2 Dimensionless wall skin friction $f''(0)$ for several values of γ.

γ	$f''(0)$ Pr=10	$f''(0)$ Pr=5	$f''(0)$ Pr=0	$\frac{\tau}{\tau_0}$ Pr=10	$\frac{\tau}{\tau_0}$ Pr=5	$\frac{\tau}{\tau_0}$ Pr=0
0	1.02116	1.02116	1.02116	1.00000	1.00000	1.00000
0.05	1.00796	1.00657	1.00439	0.98707	0.98571	0.98357
0.10	0.99474	0.99197	0.98761	0.97412	0.97141	0.96714
0.15	0.9815	0.97734	0.97084	0.96116	0.95708	0.95072
0.20	0.96822	0.96269	0.95406	0.94815	0.94274	0.93429
0.25	0.95492	0.94802	0.93729	0.93513	0.92837	0.91786
0.30	0.9416	0.93333	0.92051	0.92208	0.91398	0.90143
0.35	0.92824	0.91861	0.90374	0.90900	0.89957	0.88501
0.40	0.91486	0.9038	0.88696	0.89590	0.88507	0.86858

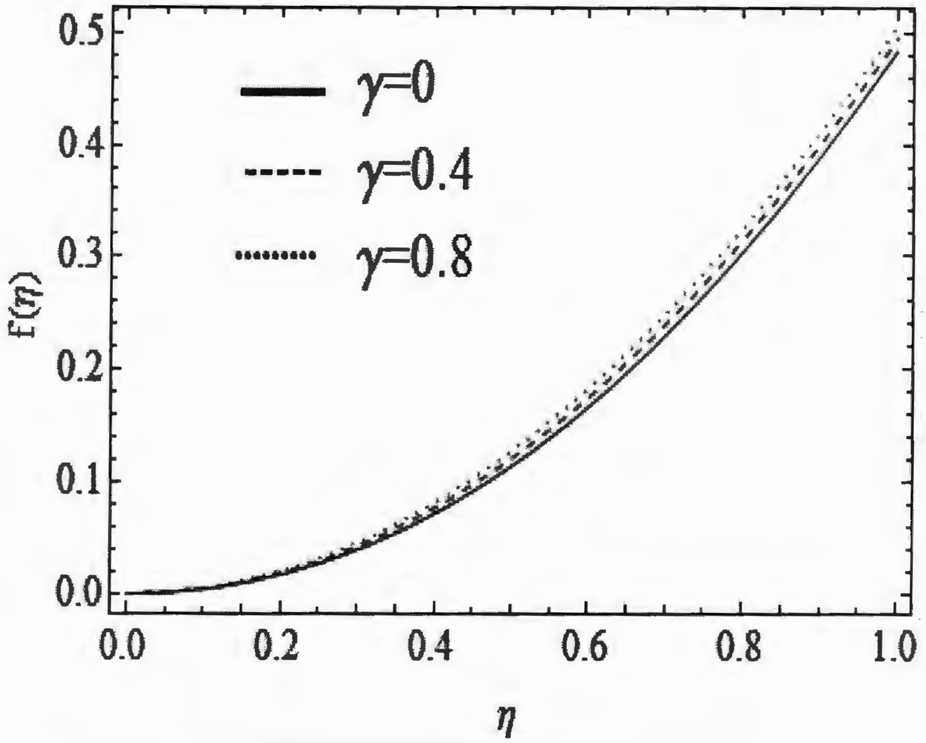

Figure 2.1 Impact of γ on stream function $f(\eta)$.

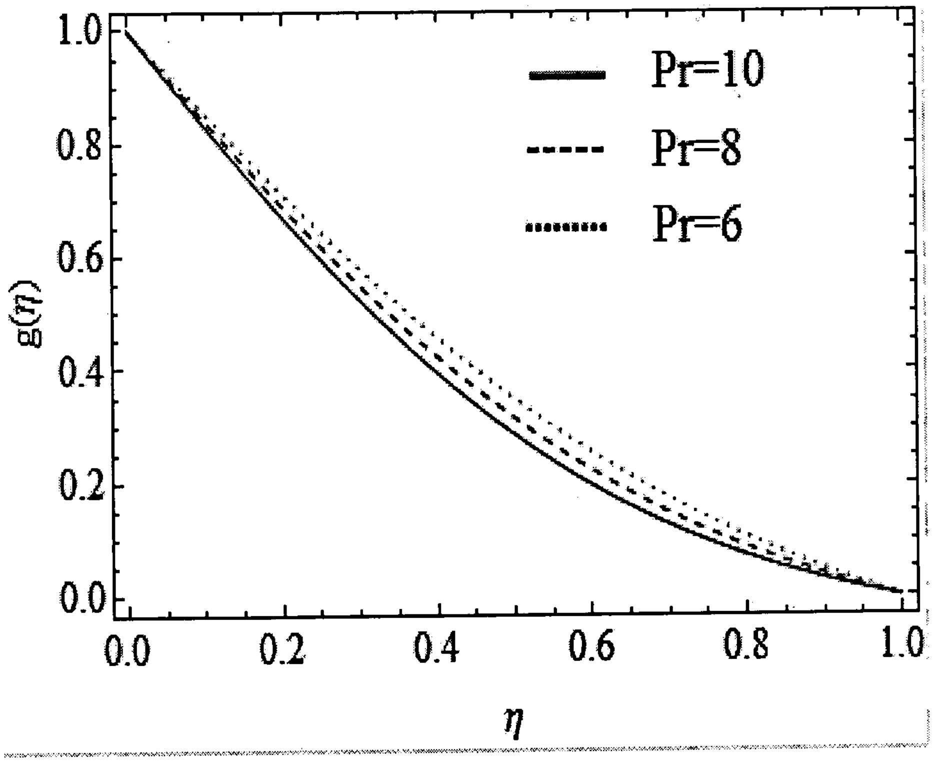

Figure 2.2 Impact of Pr on Temperature distribution $g(\eta)$.

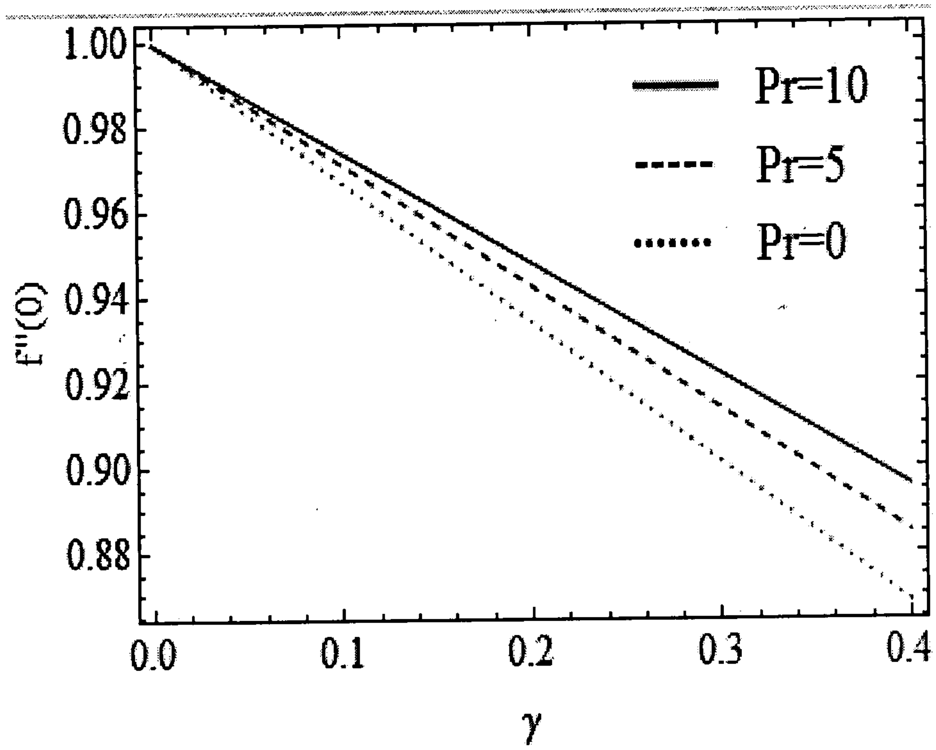

Figure 2.3 Impact of Pr on Skin friction coefficient for various valued of γ.

Chapter 3

Numerical solution of magnetic fluid flow and heat transfer in a channel with a stretching wall

In this chapter, we analyze the flow and heat transfer characteristics of a magnetic fluid in a parallel plate channel with a stretching wall. The flow equation for viscous Newtonian magnetic fluid are modeled with two point magnetic dipole. The partial differential equations are reduced to ordinary differential equation using similarity transform. Generalized Differential Quadrature method (GDQ) is used to obtain numerical result of the governing nonlinear differential equation. The obtain results are presented through graph for several set of the parameters, and the salient features of the solution are analyzed.

3.1 Mathematical Formulation

Let us consider the steady laminar flow of an incompressible fluid in a parallel plate channel bounded by the planes $y = \pm a$. The steady (2-D) boundary layer equation for flow and energy equation in usual notation are

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \gamma\frac{\partial^2 u}{\partial y^2}, \tag{3.1}$$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0, \tag{3.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=K\frac{\partial^2 T}{\partial y^2}, \tag{3.3}$$

Where (u,v) is the fluid velocity component along x and y-directions. $T(x,y)$ is the temperature at any point and K is the thermal diffusivity of the fluid. By considering the flow is to be symmetric about the line $y=0$ of the channel. Our main attention for the flow in the region $0 \le y \le a$ only.

The corresponding B.C's are

$$u=bx, v=0 \text{ at } y=a, \tag{3.4}$$

$$\frac{\partial u}{\partial y}=0, v=0 \text{ at } y=0, \tag{3.5}$$

$$\frac{\partial T}{\partial y}=0 \text{ at } \eta=0, \tag{3.6}$$

$$T=T_w \text{ at } \eta=\text{a}, \tag{3.7}$$

Where T_w is a constant.

Equations (3.1) and (3.2) declare a self similar solution of the form

$$u=bxf'(\eta), v=-abf(\eta), \eta=\frac{y}{a}, \tag{3.8}$$

by introducing non dimensional variables as

$$x^*=\frac{x}{a}, \eta=\frac{y}{a}, u^*=\frac{u}{ab}, v^*=\frac{v}{ab}, \theta=\frac{T}{T_w} \tag{3.9}$$

with these non dimensional variables, Equation (3.8) can be written as

$$u^* = x^* f'(\eta), v^* = -f'(\eta), \tag{3.10}$$

It is Clear that u and v satisfy the continuity Equation (3.2). Substituting these new variable in Equation (3.1), (3.2) and (3.3), we have

$$f''' + ff'' - (f') - \gamma\theta = 0, \tag{3.11}$$

$$\theta'' + \Pr f\theta = 0, \tag{3.12}$$

With the use of transformation (3.8) − (3.10) the boundary condition (3.4) − (3.7) becomes

$$f(\eta) = 0 \text{ at } \eta = 0, \tag{3.13}$$

$$f'(\eta) = 1, f(\eta) = 0 \text{ at } \eta = 1, \tag{3.14}$$

$$\theta'(\eta) = 0 \text{ at } \eta = 0, \tag{3.15}$$

$$\theta(\eta) = 1 \text{ at } \eta = 1, \tag{3.16}$$

where $\Pr = \frac{k}{\mu}$ is a Prandtl number.

3.2 Solution of the problem

Assuming that the number of grid points is N, and applying the Generalized differential Quadrature (GDQ) approximation to equations (3.11) and (3.12) at each discrete point on the grid, We have

$$\sum_{j=1}^{N} c^{(3)}{}_{ij} f_j + f_i \sum_{i=1}^{N} c^{(2)}{}_{ij} f_j - \left(\sum_{i=1}^{N} c^{(1)}{}_{ij} f_j \right)^2 - \gamma g_i = 0 \quad \text{for } i = 1, 2, \ldots N, \tag{3.17}$$

$$\sum_{j=1}^{N} c^{(2)}{}_{ij} \theta_j + \Pr(f_i \theta_i) = 0 \quad \text{for } i = 1, 2, \ldots, N, \tag{3.18}$$

And boundary conditions

$$at\ \eta = 0, f_1 = 0,$$
$$at\ \eta = 1, \sum_{i=1}^{N} c^{(1)}{}_{ij} f_j = 1, f_N = 0\ , \tag{3.19}$$

and

$$at\ \eta = 0, \sum_{i=1}^{N} c^{(1)}{}_{ij} \theta_j = 0\ ,$$
$$at\ \eta = 1, \theta_N = 1, \tag{3.20}$$

The set of Equations $(3.17)-(3.20)$ is superfluous because there are five boundary condition in (3.19) and (3.20) plus N Equations in (3.17) and (3.18). In order to remove this difficulty, we simply drop the Equations for $i = 1, 2,$ and N in Equation (3.17) and for $i = 1,$ and N in (3.18). Thus

$$\sum_{j=1}^{N} c^{(3)}{}_{ij} f_j + f_i \sum_{i=1}^{N} c^{(2)}{}_{ij} f_j - \left(\sum_{i=1}^{N} c^{(1)}{}_{ij} f_j \right)^2 - \gamma\theta_i = 0 \quad \text{for } i = 3, 4, \ldots, (N-1) \tag{3.21}$$

$$\sum_{j=1}^{N} c^{(2)}{}_{ij} \theta_j + \Pr(f_i \theta_i) = 0 \quad \text{for } i = 2, 3, \ldots, N-1 \tag{3.22}$$

With

$\Pr = 0.02$ and $\gamma = 2, 3, 4$, The combination of (3.21) and (3.22) with the help of boundary conditions (3.19) and (3.20) gives N equations with N unknown function values $f_1, f_2, \ldots, f_N$ and $\theta_1, \theta_2, \ldots, \theta_N$. $f(\eta)$ and $\theta(\eta)$ are obtained by solving these algebraic equations given in (3.21) and (3.22). The values of velocity and heat transfer rate are listed below in Table. 3 for $\gamma = 2, 3$ and Pr=0.

Table. 3 Values of velocity and heat transfer for $\gamma = 2,3$ and Pr=0.

$\gamma = 2$		$\gamma = 3$	
f_i'	g_i	f_i'	g_i
$f'(0.05)$=−0.601	$g(0.05)$ = 0.99820	$f'(0.05)$=−0.445	$g(0.05)$ = 0.9982
$f'(0.10)$=−0.575	$g(0.10)$ = 0.99829	$f'(0.10)$=−0.450	$g(0.10)$ = 0.99823
$f'(0.15)$=−0.543	$g(0.15)$ = 0.99838	$f'(0.15)$=−0.447	$g(0.15)$ = 0.99827
$f'(0.20)$=−0.505	$g(0.20)$ = 0.99848	$f'(0.20)$=−0.434	$g(0.20)$ = 0.99831
$f'(0.25)$=−0.460	$g(0.25)$ = 0.99857	$f'(0.25)$=−0.413	$g(0.25)$ = 0.99836
$f'(0.30)$=−0.408	$g(0.30)$ = 0.99867	$f'(0.30)$=−0.383	$g(0.30)$ = 0.99841
$f'(0.35)$=−0.350	$g(0.35)$ = 0.99876	$f'(0.35)$=−0.344	$g(0.35)$ = 0.99846
$f'(0.40)$=−0.286	$g(0.40)$ = 0.99886	$f'(0.40)$=−0.296	$g(0.40)$ = 0.99853
$f'(0.45)$=−0.216	$g(0.40)$ = 0.99895	$f'(0.45)$=−0.239	$g(0.40)$ = 0.99860
$f'(0.50)$=−0.139	$g(0.50)$ = 0.99905	$f'(0.50)$=−0.173	$g(0.50)$ = 0.99868
$f'(0.55)$=−0.055	$g(0.55)$ = 0.99914	$f'(0.55)$=−0.098	$g(0.55)$ = 0.99877
$f'(0.60)$=0.034	$g(0.60)$ = 0.99924	$f'(0.60)$=−0.014	$g(0.60)$ = 0.99887
$f'(0.65)$=0.130	$g(0.65)$ = 0.99933	$f'(0.65)$=0.078	$g(0.65)$ = 0.99898
$f'(0.70)$=0.233	$g(0.70)$ = 0.99943	$f'(0.70)$=0.180	$g(0.70)$ = 0.99910
$f'(0.75)$=0.343	$g(0.75)$ = 0.99952	$f'(0.75)$=0.292	$g(0.75)$ = 0.99923
$f'(0.80)$=0.459	$g(0.80)$ = 0.99962	$f'(0.80)$=0.414	$g(0.80)$ = 0.99937
$f'(0.85)$=0.583	$g(0.85)$ = 0.99971	$f'(0.85)$=0.545	$g(0.85)$ = 0.99952
$f'(0.90)$=0.714	$g(0.90)$ = 0.99981	$f'(0.90)$=0.686	$g(0.90)$ = 0.99967
$f'(0.95)$=0.853	$g(0.95)$ = 0.99990	$f'(0.95)$=0.837	$g(0.95)$ = 0.99983
$f'(1)$=1	$g(1) = 1$	$f'(1)$=1	$g(1) = 1$

3.3 Results and discussion

The numerical results are obtained using the generalized differential quadrature method (GDQ) by taking N=20 grid points. By expanding Equations (3.21) and (3.22) w.r.t boundary conditions given in (3.19) and (3.20), 18 algebraic equations with 18 unknowns i.e. $f_2, f_3, f_4, \ldots f_{19}$ are obtained. The set of algebraic equations and boundary conditions are solved with the help of Mathematica. Table. 4 and Table. 5

lists the non dimensional wall skin friction $f''(1)$ and heat transfer coefficients $g'(0)$ as function of γ. Also listed as function of γ are the ratios of shear stress and heat transfer, τ/τ_0 and Q/Q_0 with and without the magnetic interaction. These ratios are also plotted in Fig 3.4 and Fig 3.5. It is observed that the increase of these ratios with increase in γ is lightly non-linear with the reduction in heat transfer, again covering noticeably the decrease in skin friction.

Table. 4 dimensionless wall skin friction $f''(1)$ for different values of γ.

γ	$f''(1)$ Pr=0	$f''(1)$ Pr=5	$f''(1)$ Pr=10	$\frac{\tau}{\tau_1}$ Pr=0	$\frac{\tau}{\tau_1}$ Pr=5	$\frac{\tau}{\tau_1}$ Pr=10
0	2.22352	2.22352	2.22352	1.00000	1.00000	1.00000
0.05	2.23943	2.23842	2.23757	1.00715	1.00670	1.00631
0.10	2.25535	2.25332	2.25164	1.01431	1.01340	1.01264
0.15	2.27127	2.26824	2.26573	1.02147	1.02011	1.01898
0.20	2.28719	2.28317	2.27982	1.02863	1.02682	1.02532
0.25	2.30311	2.29810	2.29393	1.03579	1.03354	1.03166
0.30	2.31904	2.31305	2.30804	1.04295	1.04026	1.03801
0.35	2.33498	2.32800	2.32218	1.05012	1.04698	1.04437
0.40	2.35091	2.34297	2.33632	1.05729	1.05372	1.05073

Table. 5 dimensionless heat transfer $g'(0)$ for different values of Pr.

Pr	$g'(0)$ $\gamma=0$	$g'(0)$ $\gamma=0.4$	$g'(0)$ $\gamma=0.8$	$\frac{Q}{Q_0}$ $\gamma=0$	$\frac{Q}{Q_0}$ $\gamma=0.4$	$\frac{Q}{Q_0}$ $\gamma=0.8$
0	−0.11953	−0.11953	−0.11953	1.00000	1.00000	1.00000
0.05	−0.12346	−0.12330	−0.12315	1.03287	1.03154	1.03028
0.10	−0.12738	−0.12707	−0.12676	1.06567	1.06308	1.06048
0.15	−0.13129	−0.13083	−0.13037	1.09838	1.09453	1.09068
0.20	−0.13520	−0.13459	−0.13397	1.13109	1.12599	1.12080
0.25	−0.13910	−0.13833	−0.13757	1.16372	1.15728	1.15092
0.30	−0.14299	−0.14207	−0.14116	1.19626	1.18857	1.18095
0.35	−0.14687	−0.14581	−0.14474	1.22872	1.21986	1.21090
0.40	−0.15075	−0.14954	−0.14832	1.26118	1.25106	1.24086

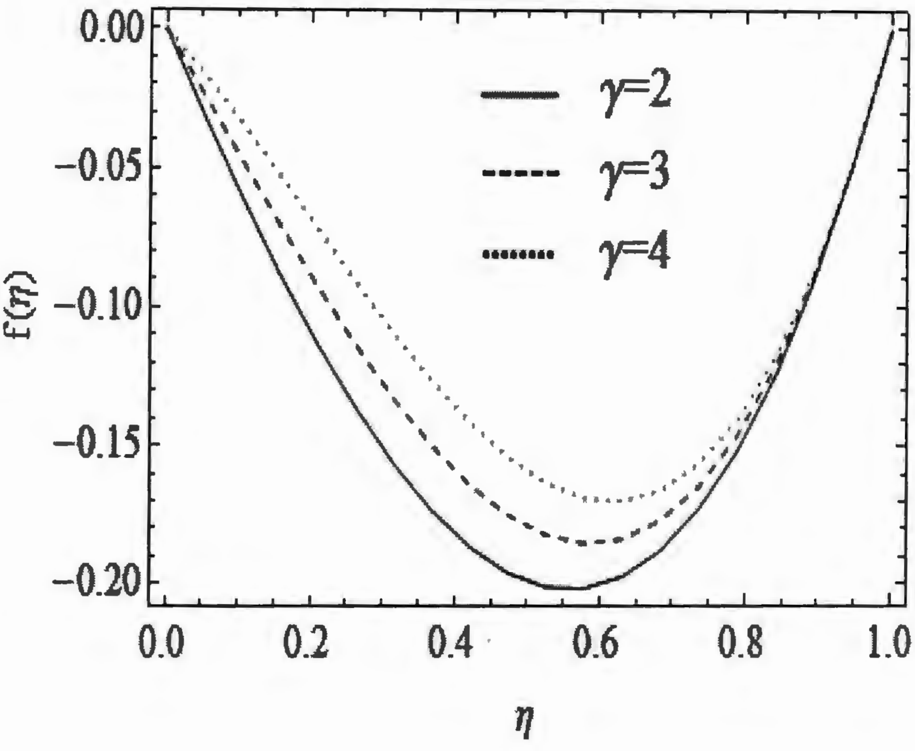

Figure 3.1 Impact of γ on stream function $f(\eta)$.

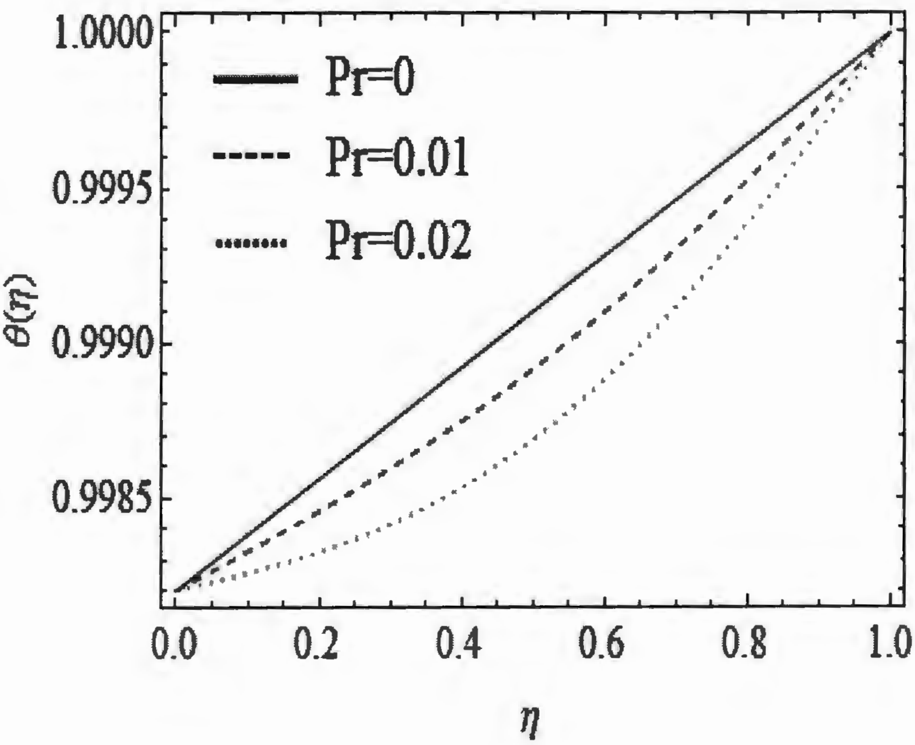

Figure 3.2 Impact of Pr on temperature distribution $\theta(\eta)$.

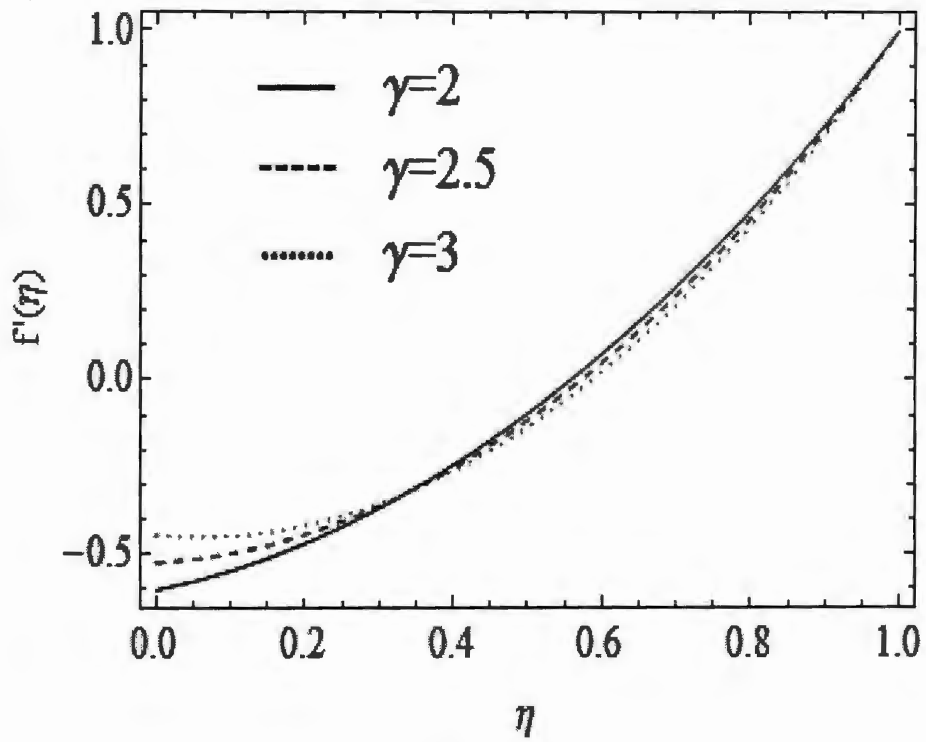

Figure 3.3 Impact of γ on velocity distribution $f'(\eta)$.

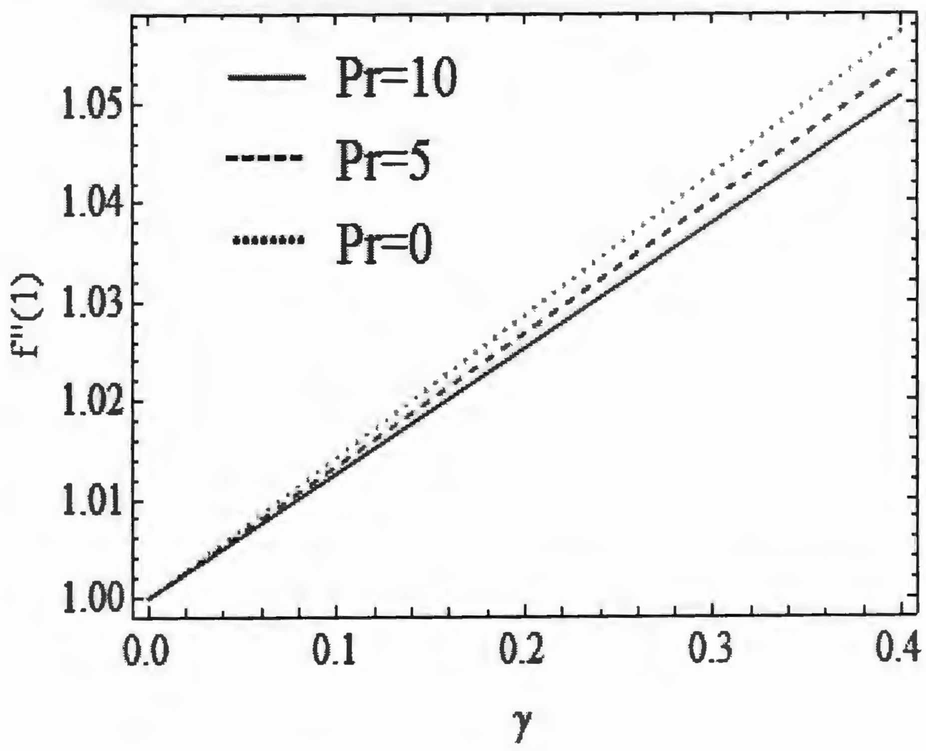

Figure 3.4 Impact of Pr on Skin friction coefficient for various values of γ.

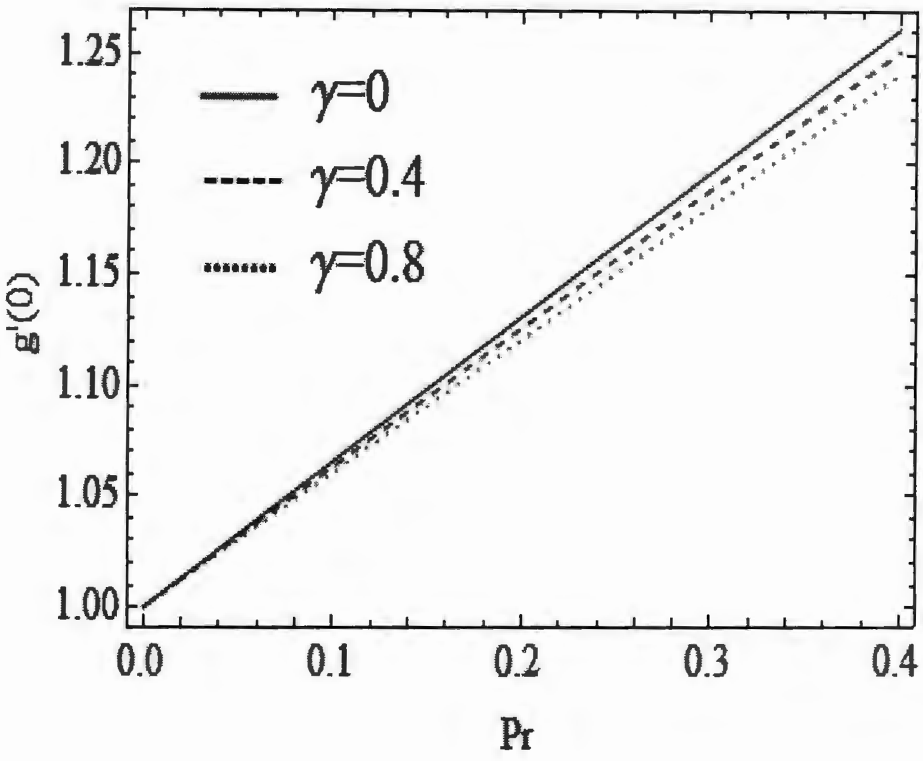

Figure 3.5 Impact of Pr on Nusselt number for various values of γ.

Chapter 4

Conclusion

In this thesis, a numerical scheme namely Generalized Differential Quadrature Matheod (GDQ) is employed to solve problem generated due to flow of magnetic fluid. Two problems are discussed. In Chapter 2, flow of magnetic fluid with two dipoles 'd' distance appart over flate plate is resolved using GDQ method and the result of Necringer [15] are replicated. Chapter three, discussed the flow of magnetic fluid due to point dipole in a channel with wall is strechted propotional to length x. Due to symmetry half channel is considered and problem is solved using GDQ method. The graphical results are displayed and discussed. Main findings of this research are as follows:

- The velocity to the channel wall decreases monotonically as the magnetic field force increases.
- In the action of a sufficient strong megantic field, the temperature decreases with an increase in the prandtl number.
- It is remarkable to notice that, when at $\eta > 0.3$ velocity decreases with the variation of γ which is close to the bounday layer wall and opposite trend is occur for $\eta > 0.3$.
- It is aslo observe that velocity profile asymptotically approches to one that is satisfy the boundary conditon.
- The skin friction coefficient increases by increasing ferrohydrodynamic interaction parameter γ as well as Prandtl number Pr.
- By boosting Prandtl number Pr the Nusselt number boost up but the reverse behaviour is seen in case of ferrohydrodynamic interaction parameter γ.

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