SPECTRAL DENSITY ESTIMATORS

AND

NG-PERRON TEST

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DECLARATION

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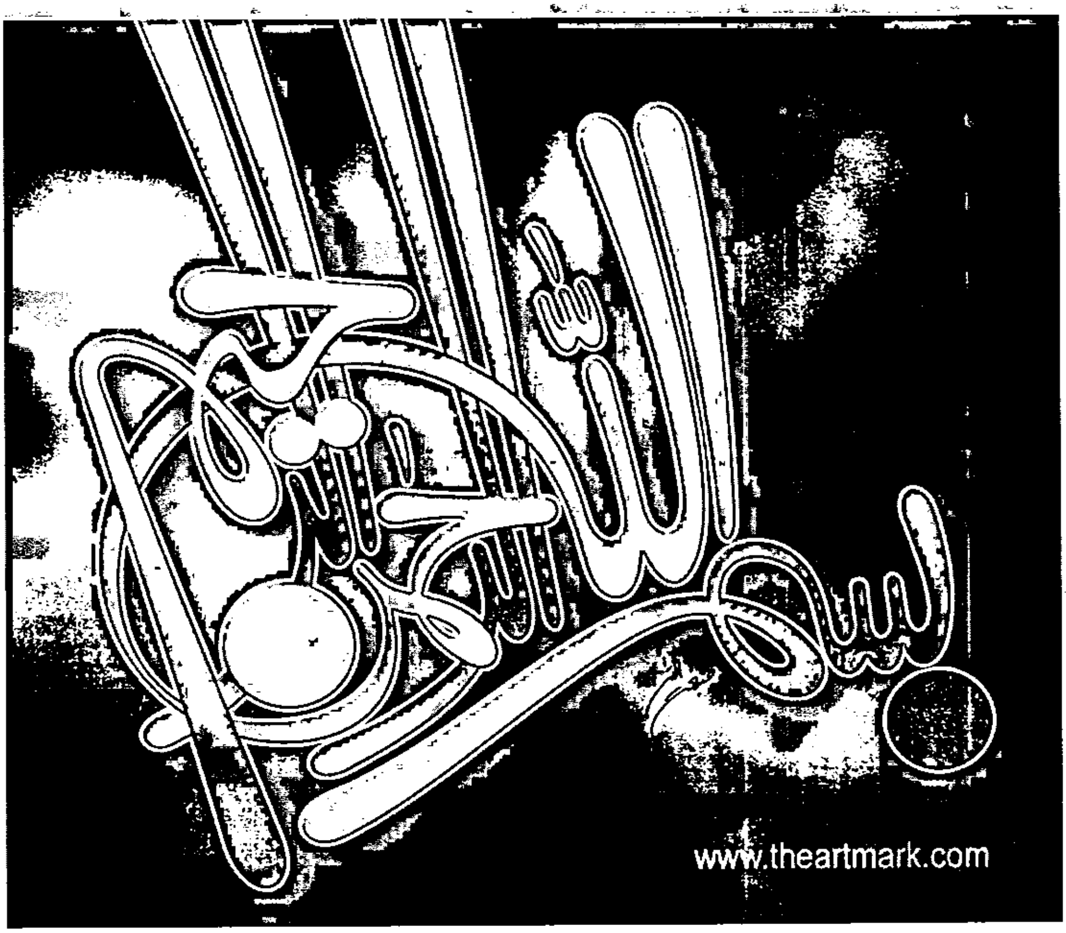
Muhammad Irfan Malik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By

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Accepted by the International Institute of Islamic Economics, International Islamic University, Islamabad, as partial fulfillment of the requirements for the award of degree of **Master of Philosophy in Econometrics**

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ABSTRACT

Unit root testing is most debated issue in econometrics for last three decade after the seminal paper of Nelson and Plosser (1982). Latest and significant development in unit root literature is Ng-Perron test introduced by Ng and Perron (2001). In this test, spectral density estimator is used to estimate the long run variance. The output of Ng-Perron test is heavily dependent on choice of these spectral density estimators however no guidance for the selection of most appropriate estimator of spectral density was available. In this thesis we have investigated the properties Ng-Perron test for various spectral density estimators. We found that appropriate choice of spectral density estimator depends on type of moving average in the data generating process. If there is positive moving average or no moving average kernel based estimator is better choice and if there is negative moving average then autoregressive estimator is better. We have also developed a procedure to find the nature of moving average in the given time series. The Monte Carlo results show that this procedure perform very well in term of successfully detecting the sign of the moving average.

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All praises are to Allah Almighty, Peace and Blessing of Allah be on his last Prophet Muhammad صلى الله تعالى عليه وآله وسلم who taught the human beings humanity.

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Muhammad Irfan Malik

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CHAPTER 1

Introduction to Unit Root

Unit root testing is well known and most debated issue in econometrics. There are lots of economic and econometric implications of existence of unit root in time series data including the incidence of spurious regression [see for example Libanio (2005) and Rehman (2011)]. Due to its importance, many tests and testing procedures were developed for this purpose. However the size and power properties of unit root tests have always been subject to debate.

It is well known that classical unit root tests perform poorly in the presence of negative moving average in the time series data [Schwert (1989), Perron and Ng (1996)]. Ng and Perron (2001) proposed a new test for unit root testing which was designed to overcome the problem of size distortion in the presence of negative moving average. This test uses an estimator of the spectral density. There are number spectral density estimators which are substantially different in finite samples and give rise to quite different output of Ng-Perron test. However, it is not clear which estimate of spectral density should be used to get reliable results.

This thesis aims at analyzing effect of choice of estimators of spectral density on the size and power properties of Ng-Perron test. This study will help the researchers in the selection of most appropriate estimator of spectral density for Ng-Perron test to get dependable results about unit root.

1.1 Ng-Perron Test (2001)

Ng-Perron test was introduced by Serena Ng and Pierre Perron (2001). It consists of a suite of four tests, namely MZa, MZt, MSB and MPT. Dufour & King (1991) and Elliott et al. (1996) found that local GLS detrending of the data yields significant power gains. Ng & Perron applied the idea of GLS detrending to modify some of the existing techniques like the PP test (1988) test, Bhargava test (1984) and Elliot test (1996). They showed that significant size and power gains can be made when GLS detrending is used in conjunction with an autoregressive spectral density estimator at zero frequency provided that the truncation lag is appropriately selected.

1.2 Variation in Output of Ng-Perron Test

As discussed above in the introduction, results of Ng-Perron test are sensitive to the choice of estimator of spectral density. The results may change with estimator of spectral density. Some practical examples are given below.

Ng-Perron test has been applied to the GDP of Pakistan and UK. The results are given below.

1.2.1 Result for Pakistan GDP

The results of Ng-Perron test for Pakistan's GDP using annual data from 1957 to 2007 with different estimators of spectral density are given below in table 1.1.

Table # 1.1: Results of Ng-Perron test for Pakistan GDP for different estimator of spectral density

Spectral Density Estimator	Ng-Perron tests			
	MZa	MZt	MSB	MPT
Autoregressive Estimator	-1359**	-26.02**	0.0191**	0.123**
Kernel Based estimator with Bartlett Kernel	3.12	1.506	0.483	78.02
Kernel Based estimator with Parzen Kernel	3.60	1.972	0.548	100.5
Kernel Based estimator with Quadratic Spectral Kernel	3.816	2.24	0.587	115.41
Kernel Based estimator with Truncated Kernel	2.16	0.866	0.401	53.907
Kernel Based estimator with Tukey-Hanning Kernel	4.08	2.649	0.649	141.17
Critical Values				
1%	-23.8	-3.42	0.143	4.03
5%	-17.3	-2.91	0.168	5.48

Critical values were provided by Ng and Perron (2001)
*significant at 5% level **significant at 1% level

Form above table 1.1we observed that Ng-Perron test with autoregressive (AR) estimator of spectral density, calculated values of all four tests are less than critical values both at 5% and 1% level of significance for Pakistan GDP. This implies the output favor stationarity in series. Ng-Perron test with kernel based (KB) estimator of spectral density, results of all tests with KB estimator suggest the Pakistan GDP series is unit root.

1.2.2 Result for UK GDP

The tests results for UK GDP annual data from 1948 to 2007. The results are given below for different estimate of spectral density.

Table # 1.2: Results of Ng-Perron test for UK GDP for different estimator of spectral density

Spectral Density Estimator	Ng-Perron tests			
	MZa	MZt	MSB	MPT
Autoregressive Estimator	-26.93**	-3.52**	0.131**	4.29*
Kernel Based estimator with Bartlett Kernel	1.149	0.726	0.632	100.6
Kernel Based estimator with Parzen Kernel	1.467	1.073	0.731	134.78
Kernel Based estimator with Quadratic Spectral Kernel	1.282	0.857	0.668	112.51
Kernel Based estimator with Truncated Kernel	0.367	0.182	0.496	61.895
Kernel Based estimator with Tukey-Hanning Kernel	1.541	1.175	0.762	146.36
Critical Values				
1%	-23.8	-3.42	0.143	4.03
5%	-17.3	-2.91	0.168	5.48

Critical values were provided by Ng and Perron (2001)
*significant at 5% level **significant at 1% level

Table 1.2 shows results for Ng-Perron test when applied to UK GDP. UK GDP is stationary at 5% and 1% level of significance when autoregressive estimator of spectral density is used. The results of the test with kernel based estimator of spectral density support the unit root in UK series.

Choice of spectral density estimators leads to different conclusion. It is necessary to investigate which of these results is more reliable.

1.3 Objectives of the Study

This study conducted to acquire the following objectives:

1. To evaluate the performance of Ng-Perron unit root test under different estimators of spectral density.
2. To find the kernel which gives good size and power properties of Ng-Perron test when kernel based estimator of spectral density is used.

1.4 Significance of Study

Unit root testing is the starting point in the analysis of time series. Ng-Perron test getting popularity because, it accumulates the intellectual heritage of many previous unit root tests. However the properties of Ng-Perron are ambiguous. Properties of the test depend upon the choice of spectral density estimator and nature of moving average. Our research makes it clear to the practitioners, which estimator of spectral density gives good size and power among autoregressive and kernel based estimator. In kernel based estimator of spectral density, which kernel give good size and power of Ng-Perron test.

CHAPTER 2

Literature Review

In modern day analysis of econometric models, presence or absence of unit root in variable has attracted considerable attention in last three decade. There was consensus among the econometricians that the economic time series behave like stationary oscillation around some deterministic trend. This consensus was challenged by Nelson & Plosser (1982). Nelson and Plosser applied Dickey Fuller (1979) unit root test to a number of American macroeconomic time series and found that they are unable to reject unit root for the majority of these series. Because non-stationarity has significant consequences from the point of view of economics & statistics, research about unit root was accelerated. There is vast literature on development of statistical theory for unit root (integrated) processes and the lists of empirical applications are even more extensive. It is now a familiar practice to test for unit root on each variable involved in the model. Various tests have been developed for this purpose. Most common tests are AD test [Dickey and Fuller (1979)], ADF [Said and Dickey (1984)], PP test [Phillips and Perron (1988)], KPSS test [Kwiatkowski, Phillips, Schmidt and Shin (1992)], ERS point optimal test (Elliot, Rothenberg and Stock 1996) and Ng-Perron test [Ng and Perron (2001)].

2.1 Ng-Perron Unit Root Test (2001)

Elliott et al. (1996) and Dufour & King (1991) showed that local GLS detrending of the data yields considerable power gains. Ng & Perron (2001) apply the idea of GLS detrending to some modified tests and show that considerable size and power gains can be made, when used in combination with an autoregressive spectral density estimator at frequency zero, provided the truncation lag is appropriately selected. This is a frequently used test in modern day analysis of data to investigate the presence of unit root. This test consists of suite of four tests, which are the modified versions of Phillips and Perron (1988) test, Bhargava (1984) test and Elliot (1996) test. Computational details are given in chapter 3 on methodology.

2.2 Spectral Density Estimation at Zero Frequency

In many economic time series models, errors may have heterogeneity and temporal dependence of unknown form. This is the main source of size and power distortion of unit root tests. To draw more accurate inference from estimates of parameters it has now become essential to construct test based on long run variance (LRV) which takes into account serial correlation and heterogeneity.

The key aspect to construct LRV is to estimate the spectral density at zero frequency. Unit root tests, like Phillips Perron (1988) test, KPSS (1992) test and Ng-Perron (2001) test are based on the estimates of spectral density at zero frequency. Spectral density estimator can be divided into two categories

(i) Parametric Estimator (autoregressive estimator)

(ii) Non parametric Estimator (kernel based estimator)

2.2.1 Parametric Estimator (Autoregressive Estimator)

Parametric estimator of spectral density was proposed by Stock (1990) (see also Stock 1994 and Perron and Ng 1998) known as autoregressive estimator of spectral density[1]

[1] *Mathematical and computational details are in chapter 3 section I.*

This estimator based on the estimation of parametric model is identical to the equation of ADF test equation. In order to capture the heteroskedasticity and autocorrelation in error, the choice of the lag length is very important [Ng and Perron 2001]. In real world applications, lag length is decided by the practitioners by using information criteria or himself. In literature many information criteria are available. These information criteria are e.g. Akaike Information Criterion (AIC), Schwarz Information Criterion(SIC), Hannan-Quinn Criterion(HQC) and their modified versions MAIC, MSIC, MHQC etc used to select the lag length in order to estimate autoregressive estimator of spectral density.

2.2.2 Non Parametric Estimator (Kernel Based Estimator)

Non parametric estimator[2] of spectral density was proposed by Phillips (1987) and then restructured by Phillips and Perron (1988). Kernel based estimator of spectral density is the weighted sum of auto covariance, in which weights are being decided by the kernel and bandwidth parameter. In literature a variety of kernels are available. Kernels which we considered in our study are

a. Bartlett Kernel
b. Parzen Kernel
c. Quadratic Spectral Kernel
d. Tukey-Hanning Kernel
e. Truncated Kernel

Properties of these kernels were investigated by Andrew (1991). Bandwidth parameter acts as lag truncation. In practices bandwidth parameter is decided by the practitioner or

[2] *Mathematical and computational details are in chapter 3 section I*

by bandwidth selection methods. Most popular are the two Andrew Bandwidth selection method was proposed by Andrew (1991) and Newey-West Bandwidth selection method was proposed by Newey and West (1994). Properties of above estimators of spectral density were analyzed by Ng and Perron (1996).

2.3 Ng-Perron Test and Spectral Density Estimator

The performance of Ng-Perron test is very sensitive to the estimates of spectral density. From section 1.4 [table 1.1 and table 1.2] the same has been observed when Ng-Perron test was applied to the GDP of Pakistan and UK in order to test the presence of unit root. We get different results of Ng-Perron test for both estimator of spectral density. The GDP is stationary when autoregressive estimator of spectral density and difference stationary with kernel based estimator used in the implementation of Ng-Perron test.

This test has been widely acknowledged by applied researchers in the situations of heterogeneity and temporal dependence in error of unknown form. Recent studies of Wickremasinghe (2004), Shehbaz et al. (2010) and Awan et al. (2010) and many others used Ng-Perron test. According to Ng and Perron (2001), this test gives good size and power in the presence of negative moving average in error when Modified Akaike Information Criterion (MAIC) is used to select the lag length with autoregressive spectral density estimator and GLS detrending. Perron and Qu (2007), in their study, showed that simple modification in MAIC improve the small sample properties of this test.

No study is available investigating power/ size of Ng-Perron test under different spectral density estimators. We in this study investigate the characteristics of Ng-Perron unit root test by manipulating the estimators of spectral density at different lag length/ lag truncations. We also try to explore the behavior of this test in the presence of positive moving average in error as well. This study will help the researcher indicating which spectral density estimator give good size and power for Ng-Perron test in the presence of positive and negative moving average.

CHAPTER 3

Methodology

The methodology consists of two sections. First section describes the procedures and test that we will be using in our study, second section describe the Monte Carlo design we have formulated.

SECTION: 1

3.1 GLS Detrending

Let $y_1, y_2, y_3 \dots . y_{(T-1)}, y_T$ is given time series. The following steps are involved in GLS detrending.

- We find the Quasi difference

$$\nabla y_t = \begin{cases} y_t & if\ t = 1 \\ y_t - a y_{(t-1)} & if\ t > 1 \end{cases}$$

Where

$$a = \left(1 + \frac{\bar{c}}{T}\right)$$

$$\bar{c} = \begin{cases} -7 & if \quad d_t^0 \\ -13.5, & if \quad d_t^1 \end{cases}$$

d_t^i $[here\ i = 0\ and\ 1]$ is deterministic part in the data generating process (DGP).

$d_t^0 = \{1\}$ DGP with constant only.

$d_t^1 = \{1, t\}$ DGP with constant and time trend.

In the same way we find the quasi difference of d_t^i

- Now we consider an OLS regression of the quasi- difference data, ∇y_t on quasi-difference ∇d_t^i

$$\nabla y_t = \nabla d_t^i \hat{\pi} + \varepsilon_t \tag{3.1}$$

- In this step we solve for GLS detrending

$$\tilde{y}_t = y_t - d_t^i \hat{\pi} \tag{3.2}$$

$\hat{\pi}$ is the coefficient of equation (3.1)

3.2 Ng-Perron Test (2001)

Let $\tilde{y}_1, \tilde{y}_2, \tilde{y}_3, \tilde{y}_4, \tilde{y}_5 \dots \tilde{y}_{(T-1)}, \tilde{y}_T$ be a GLS detrended series resulting from equation (3.2). The four test statistics proposed by Ng-Perron are.

$$MZ_a = \frac{\left((T^{-1}\,\tilde{y}_t)^2 - \hat{f}(0)\right)}{2k}$$

$$MZ_t = MZ_a * MSB$$

$$MSB = \left(\frac{k}{\hat{f}(0)}\right)^{\frac{1}{2}}$$

$$MPT = \begin{cases} \dfrac{\bar{c}^2 k - \bar{c}T^{-1}(\,\tilde{y}_t)^2}{\hat{f}(0)} & when\ d_t^0 \\ \dfrac{\bar{c}^2 k + (1-\bar{c})T^{-1}(\,\tilde{y}_t)^2}{\hat{f}(0)} & when\ d_t^1 \end{cases}$$

Where $k = \sum_{t=1}^{T} \frac{\left(\tilde{y}_{(t-1)}\right)^2}{T^2}$ and $\hat{f}(0)$ is an estimator of spectral density at zero.

3.3 Estimators of Spectral Density

I. **Autoregressive estimator of spectral density.**

Given the GLS detrending series estimate the regression equation (3.3) given below.

$$\Delta\tilde{y}_t = \rho\tilde{y}_{(t-1)} + \sum_{l=1}^{T}\Delta\tilde{y}_{(t-l)}\hat{\beta}_l + \varepsilon_t \tag{3.3}$$

Autoregressive estimator of spectral density is

$$\hat{f}(0) = \frac{\hat{\sigma}_\varepsilon^2}{\left(1-\hat{\beta}(1)\right)^2} \tag{3.4}$$

where

$\hat{\beta}(1) = \sum_{l=1}^{T}\hat{\beta}_l$ And $\hat{\sigma}_\varepsilon^2 = \frac{\sum_{t=1}^{(T-l)}\hat{\varepsilon}_t^2}{(T-l)}$

$\hat{\beta}(1)$ is the sum of coefficients of lags of $\Delta\tilde{y}_t$. Here $\Delta\tilde{y}_t = \tilde{y}_t - \tilde{y}_{(t-1)}$ and $\hat{\sigma}_\varepsilon^2$ is variance of residuals ($\hat{\varepsilon}_t$) from the equation (3.3)

II. **Kernel Based estimator of spectral density.**

Estimate the equation (3.5) using GLS detrended series

$$\Delta\tilde{y}_t = \rho\tilde{y}_{(t-1)} + \varepsilon_t \tag{3.5}$$

The kernel based estimator given as

$$\hat{f}(0) = \sum_{j=-(T-l)}^{(T-l)}\hat{\gamma}(j).K(j/l) \tag{3.6}$$

$$\hat{\gamma}(j) = \frac{\sum_{t=1}^{T-j}\hat{\varepsilon}_t * \hat{\varepsilon}_{t-j}}{T-j}$$

Where l is bandwidth parameter (which act as a truncation lag in the covariance weighting), K is kernel function, $\hat{\gamma}(j)$ is j^{Th} order auto covariance of residual from equation (3.5).

In the estimation of kernel estimator of spectral density we used the following kernels in this study.

1. Bartlett Kernel

$$K_{BT}(x) = \begin{cases} 1 - |x| & for\ |x| \leq 1 \\ 0 & otherwise, \end{cases}$$

2. Parzen Kernel

$$K_{PR}(x) = \begin{cases} 1 - 6x^2 + 6|x|^3 & for\ 0 \leq |x| \leq 1/2 \\ 2(1 - |x|)^3 & for\ 1/2 \leq |x| \leq 1 \\ 0 & otherwise \end{cases}$$

3. Quadratic Spectral Kernel

$$K_{QS}(x) = \frac{25}{12\pi^2 x^2}\left(\frac{\sin\left(\frac{6\pi x}{5}\right)}{\frac{6\pi x}{5}} - \cos\left(\frac{6\pi x}{5}\right)\right)$$

4. Tukey-Hanning Kernel

$$K_{TH}(x) = \begin{cases} \frac{1+\cos(\pi x)}{2} & for\ |x| \leq 1 \\ 0 & otherwise \end{cases}$$

5. Truncated Kernel

$$K_{TR}(x) = \begin{cases} 1 & for\ |x| \leq 1 \\ 0 & otherwise, \end{cases}$$

Here $(x = j/l)$ for all kernels. Asymptotically all these kernels are equivalent[3]

SECTION: 2

3.4 Monte Carlo Experiment

The main objective of Monte Carlo experiment is to compare the performance different estimators of spectral density with respect to size and power of Ng-Perron test. The series are generated with known data generating process (DGP) and Ng-Perron test with different spectral density were applied. Details are given below.

3.4.1 Methodology.

(i) Generate the series with pre-specified DGP

(ii) Apply Ng-Perron unit root test with different estimates of spectral density.

(iii) Evaluate the output of Ng-Perron test

Repeat the above steps to calculate the size and power of Ng-Perron test for pre-specified DGP under different combinations of sample size, value of autoregressive coefficient ρ and moving average coefficient.

Sample Size

The Monte Carlo experiment is repeated for sample size 40, 80, 150, 250 and 400

Data Generating Process

We consider the following DGP's in this study

DGP-I $\quad y_t = \alpha + u_t \qquad u_t = \rho u_{(t-1)} + \delta e_{(t-1)} + e_t$

[3] For detail see Andrew (1991)

Errors with ARMA (1,1) process with positive moving average with no time trend

DGP-II $y_t = \alpha + u_t$ $u_t = \rho u_{(t-1)} + (-\delta)e_{(t-1)} + e_t$

Errors with ARMA (1,1) process with negative moving average with no time trend

DGP-III $y_t = \alpha + \beta t + u_t$ $u_t = \rho u_{(t-1)} + \delta e_{(t-1)} + e_t$

Errors with ARMA (1,1) process with positive moving average with time trend

DGP-IV $y_t = \alpha + \beta t + u_t$ $u_t = \rho u_{(t-1)} + (-\delta)e_{(t-1)} + e_t$

Errors with ARMA (1,1) process with negative moving average with time trend

Autoregressive Coefficient

To calculate the size of the Ng-Perron test, the autoregressive coefficient (ρ) is "1" for unit root and for power coefficients (ρ) are 0.99, 0.98, 0.95, 0.90, 0.85, 0.80 and 0.70

Moving Average Coefficient

Coefficient of moving average: -0.80, -0.60, -0.40, -0.20, 0, 0.2, 0.4, 0.6 and 0.8

Here t= 1, 2.............T

In this study we calculate the size and power of the Ng-Perron test at 5% level of significance. Monte Carlo experiment is simulated 25000 times

3.4.2 Testing the series for unit root

a. When we have drift as deterministic part in the DGP then the null and alternative hypothesis are

$$H_0: |\rho| = 1 \; with\ no\ constant$$

$$H_1: |\rho| < 1 \quad with\ constant$$

b. DGP with drift and trend as deterministic part then null and alternative hypothesis are

$$H_0: |\rho| = 1\ with\ drift\ and\ no\ trend$$

$$H_1: |\rho| < 1 \quad with\ drift\ and\ trend$$

The results of size and power are in percentage.

CHAPTER 4

Equivalence of four tests MZa, MZt, MSB & MPT and Kernels in Kernel Based Estimator of Spectral Density

The main objectives of this study to find out the spectral density estimator that gives the optimal performance of Ng-Perron unit root test. This test consists of a set of four statistics namely MZa, MZt, MSB and MPT for testing unit root. This test uses estimator of spectral density at frequency zero in its estimation procedure. The most commonly used estimators are autoregressive (AR) estimator and kernel based (KB) estimator. We consider five different kernels for kernel based estimator of spectral density. If we analyze all possible combinations of tests with spectral density estimators, this will make the study cumbersome. So we need to simplify our analysis.

Fortunately the four test statistics proposed by Ng-Perron give nearly same size and power for any sample size, lag length/truncation and estimator of spectral density at zero frequency (autoregressive estimator or kernel based estimator) for given data generating process. In section 4.1, we provide the evidence of equivalence of four tests under different conditions. In section 4.2, conformation of kernel based estimator based on Bartlett, Parzen, Quadratic Spectral and Tukey-Hanning are nearly identical in size and power except truncated kernel which behaves poorly so we discard it from our analysis. Out of these four kernels we select Parzen kernel as representative. Comparison of spectral density estimators is provided in chapter 5.

4.1 Equivalence of MZa, MZt, MSB & MPT

In this section, we shall provide the evidence that four tests proposed by Ng-Perron are equivalent with respect to their size and power properties. To prove this we have calculated size and power of four tests for all DGP's given in section 3.4.1

We observed that suite of four tests are alike in size and power. In subsection, 4.1.1, equivalence of MZa, MZt, MSB and MPT under AR spectral density estimator has been established subsection, 4.1.2 establishes the equivalence of four tests for different kernels in kernel based estimator of spectral density.

4.1.1 Equivalence of Tests with Autoregressive Estimator of Spectral Density

It has been observed that with autoregressive estimator of spectral density under different data generating process, the size as well as power of MZa, MZt, MSB and MPT are nearly equal for given sample size, lag length and the value autoregressvie root ρ . The following figures provides the comparison.

Figure # 4.1: The size and power of MZa, MZt, MSB, MPT when sample size is 80, lag length=2 and $\delta = 0.4$ with DGP-I

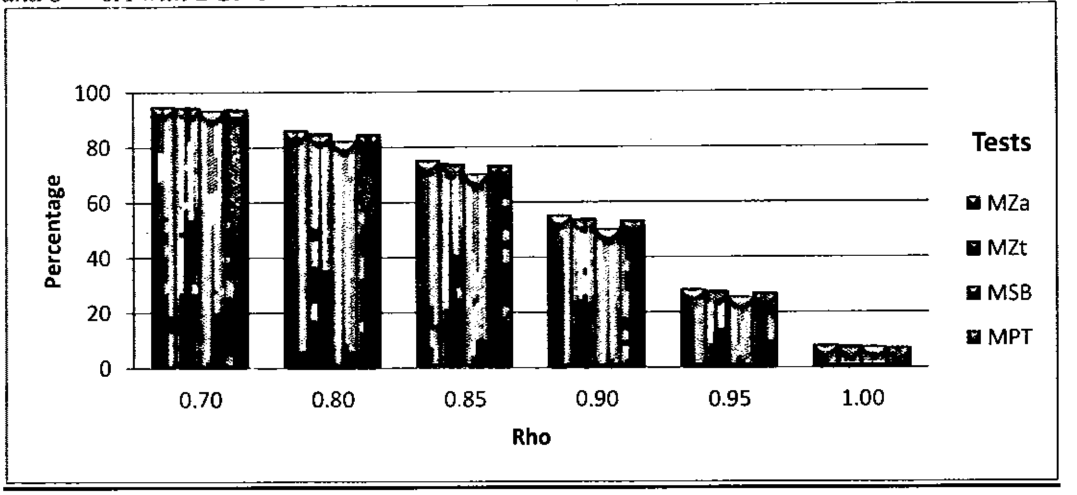


Figure # 4.2: The size and power of MZa, MZt, MSB, MPT when sample size is 150,lag length is 4 and $\delta = -0.4$ with DGP-II

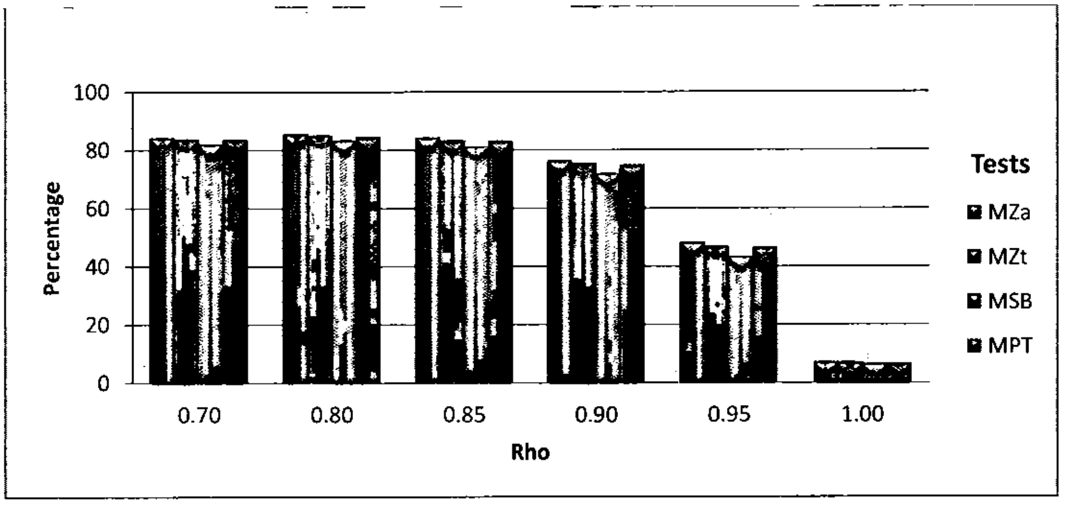


Figure # 4.3: *The size and power of MZa, MZt, MSB, MPT when sample size is 80, lag length=4 and* $\delta = 0.4$ *with DGP-III*

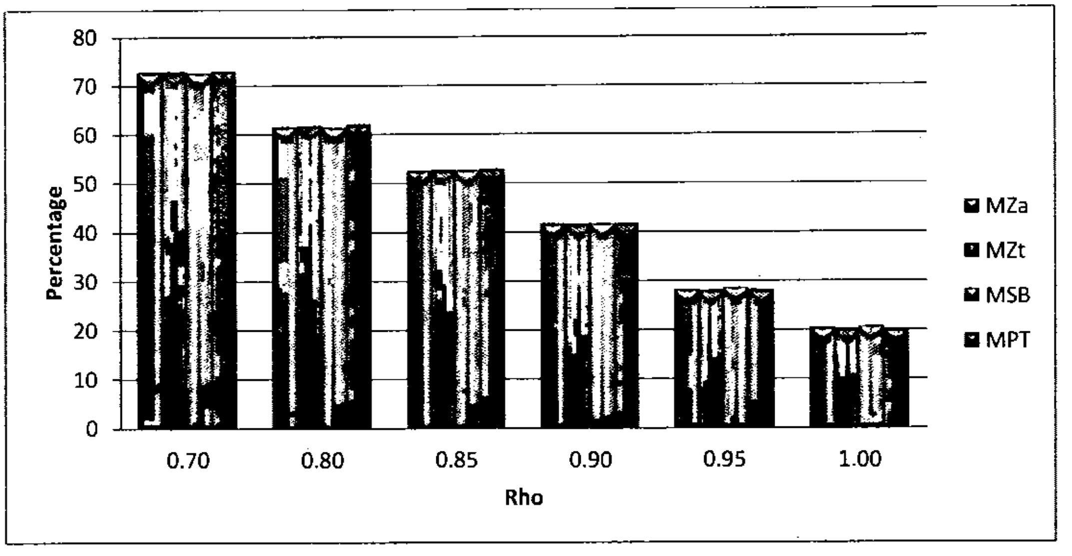
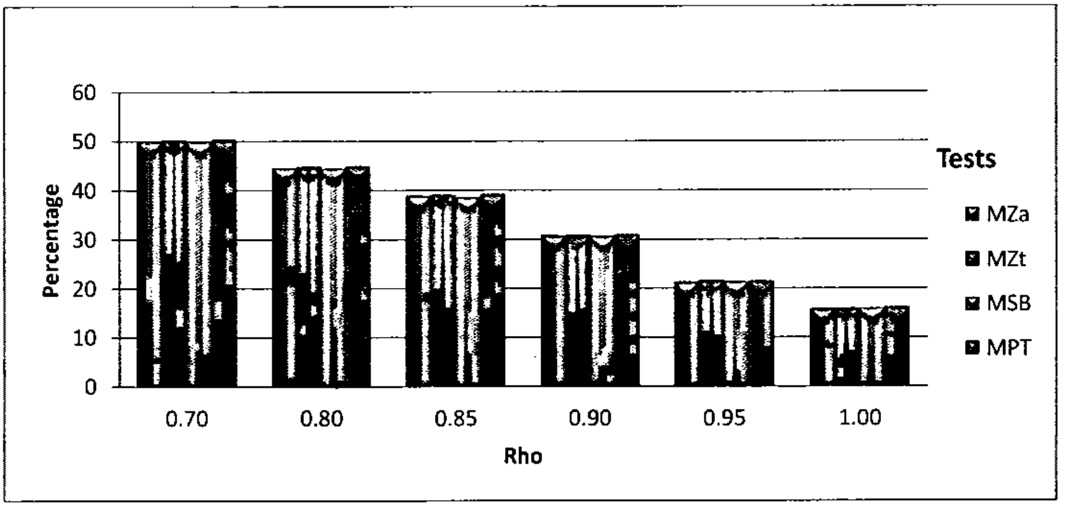


Figure # 4.4: *The size and power of MZa, MZt, MSB, MPT when sample size is 80, lag length is 5 and* $\delta = -0.6$ *with DGP-IV*



From the above figures (4.1, 4.2, 4.3 and 4.4), it can be concluded that all four tests in the suite of Ng-Perron test are nearly equivalent in their size and power for given sample size and lag length for DGP's.

4.1.2 Equivalence of MZa, MZt, MSB AND MPT for Different Kernels in Kernel Based Estimator of Spectral Density

Our simulations results show that when kernels in kernel based estimator of spectral densnity were changed, all the four tests MZa, MZt, MSB and MPT are nearly equivalent in size and power for given sample size, lag truncation and DGP.

Equivalence of MZa, MZt, MSB AND MPT for Bartlett kernel

The four tests are identical in size and power when Bartlett kernel was uesd in the estimation of kernel based estimator of spectral density under differnet DGP's

Figure # 4.5: *The size and power of MZa, MZt, MSB, MPT when, truncation lag= 3, sample size is 80 with* δ = 0.8 *with DGP-1*

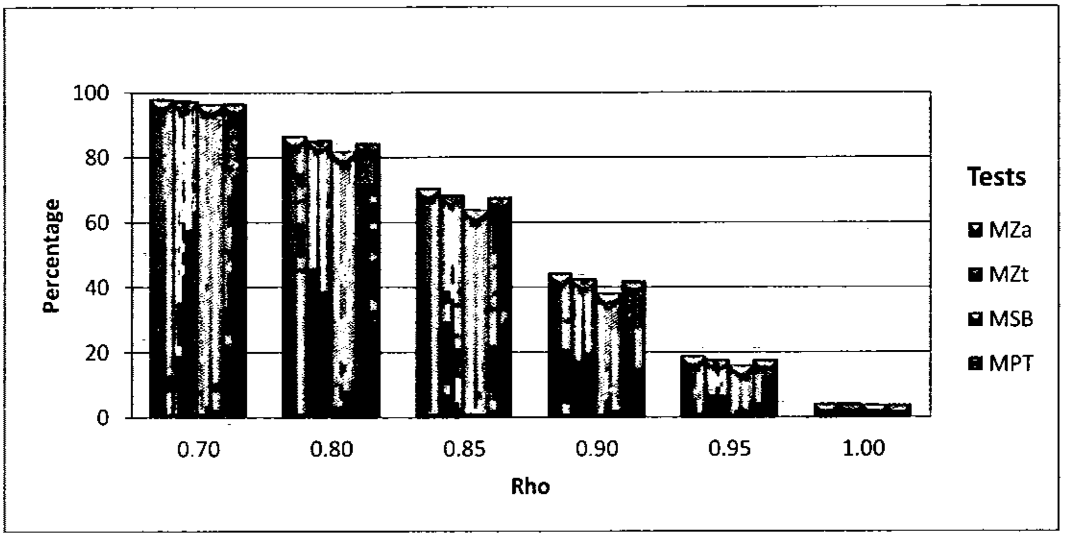


Figure # 4.6: The size and power of MZa, MZt, MSB, MPT when, truncation lag= 4, sample size is 80 with $\delta = -0.4$ with DGP-II

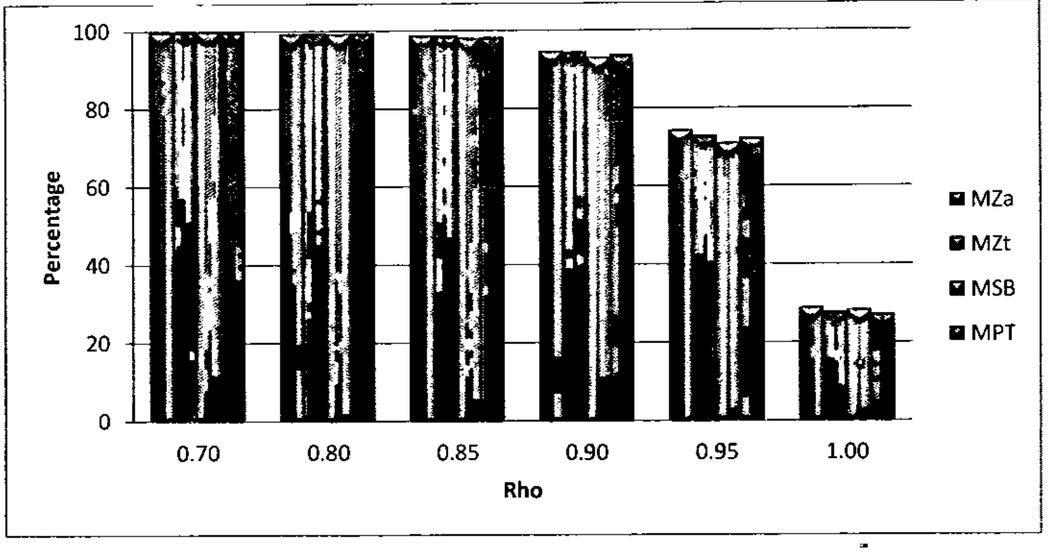


Figure 4.5 & 4.6 shows that four tests give same size and power when bartlett kernel kernel is used in kernel based estimator of spectral density for DGP I & II, also they are identical for other DGP's for any sample size and lag truncation when bartlett kernel in kernel based estimator of specrtal density is used.

It has also been observed that the size and power properties of MZa, MZt, MSB and MPT are nearly similar for Parzen Kernel, Quadratic Spectral kernel, Tuncated karnel and Tuckey-Hanning kernel for given sample size, lag truncation and DGP's.

4.1.3 Conclusions

From the above discussion it can be concluded that suite of four tests in Ng-Perron test are neraly equivalent with repect to size and power for any combination of sample size, lag length/lag truncation, moving average coefficient, spectral density estimator and deterministic part.

Since all test four tests in the suit of Ng-Perron test are nearly equivilant in size and power, so we take MZa test as a representative of this suit in further analysis.

4.2 Equivalence of Kernels in Kernel Based Estimator of Spectral Density

In section 4.1 it has been concluded that the four tests (MZa, MZt, MPT and MSB) give approximately same size and power for given estimator of spectral density and DGP. From the suite of these tests, we select MZa as representative. Now we compare the performance of kernels used in KB estimator of spectral density as given in section 3.3.

4.2.1 Equivalence at Zero Lag Truncation.

It can be prove analytically, for zero lag length/ lag truncation all kernels in KB estimator of spectral density at zero frequency are equivalent. From equation 3.6 the kernel based estimator of spectral density at zero frequency is

$$\hat{f}(0) = \sum_{j=-(T-l)}^{(T-l)} \hat{\gamma}(j).K(j/l)^{4} \tag{4.1}$$

$\hat{\gamma}(j)$ is an auto-covariance and K is kernel, since auto-covariance are symmetric i.e.

$$\hat{\gamma}(j) = \hat{\gamma}(-j) \tag{4.2}$$

We can write the equation (4.1) as

$$\hat{f}(0) = \hat{\gamma}(0) + 2\sum_{j=1}^{l} \hat{\gamma}(j).K(j/l) \tag{4.3}$$

l is truncation lag and $j = 0,1,2 \ldots\ldots l$. In kernel based estimator of spectral density all kernels assign zero weights to all auto-covariance when$j > l$, i.e. $K\left(\frac{j}{l}\right) = 0$. For $l = 0$ equation 4.3 reduce to

$$\hat{f}(0) = \hat{\gamma}(0) \tag{4.4}$$

[4] For detail see subsection 3.3

From equation (4.4) for any sample size, moving average coefficient and DGP, all the kernel based estimators of spectral density are identical at zero lag truncation.

4.2.2 Equivalence at Non Zero Lag Truncation

When lag truncation is zero all five kernels are mathematically identical. In this subsection, equivalence of kernels has been investigated for non zero lag truncation under different DGP's. Our simulation results reveals that the size and power of Ng-Perron test is nearly same for all kernels except truncated kernel, which is different in size and power of MZa from all other kernels.

Figure # 4.7: *The size and power of MZa when sample size is 150 and* $\delta = 0.4$ *with DGP-I*

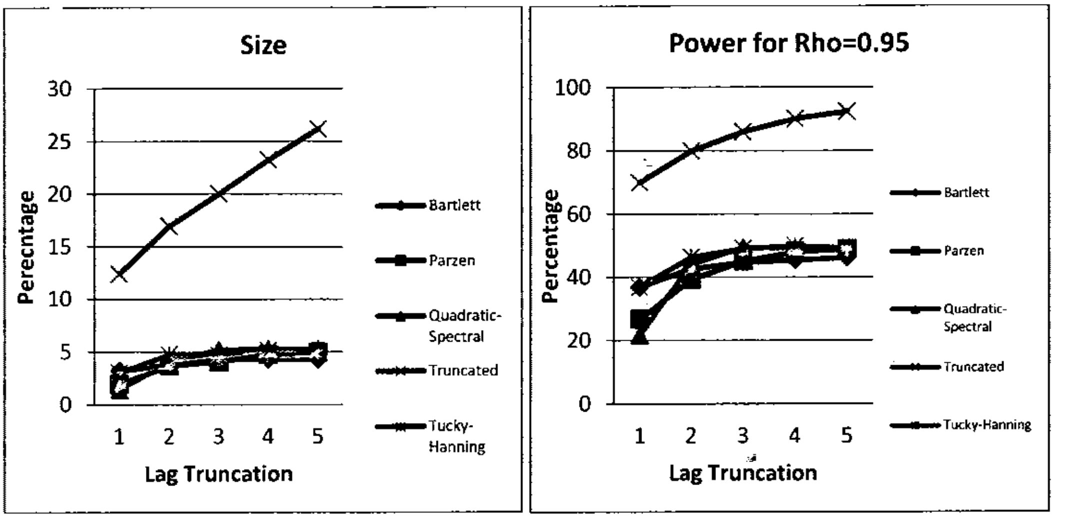
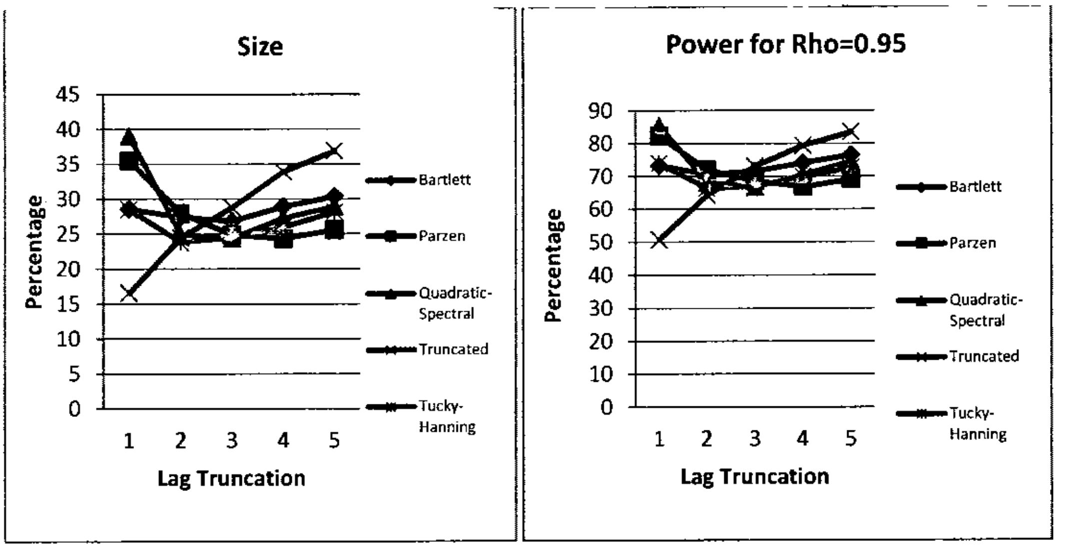


Figure # 4.8: The size and power of MZa when sample size is 80, $\delta = -0.4$ with DGP-II



From the figures 4.7 and 4.9 above, that all four kernels are identical with no significant difference in size and power of MZa except truncted kernel for DGP I and II. It has been also observed that these kernels are nearly equivient to each others for other DGP's aswell given in section 3.4.1for different sample size and moving average coeffieients.

4.2.3 CONCLUSIONS

From the above analysis we can conclude that all the kernels in KB estimator spectral density are equivalent in size and power of MZa except truncated kernel which is different from all other kernels. The size and power of turncated kernel is not stable so we exclude it from our further disucssion. In order to keep our analysis simple, we take one kernel as representative of these kernels. In our later dicussion and analysis, we use Parzen kernel in the estimation of kernel based estimator of spectral density.

CHAPTER 5

Comparison of Autoregressive (AR) and Kernel Based Estimators of Spectral Density

In previous chapter, it has been concluded that statistics in the suite of Ng-Perron test are nearly identical with respect to size and power. So any one can be taken as representative. Similarly kernels in kernel based estimator of spectral density are also identical. In futher analysis We take MZa as representative of this suite and Parzen kernel as repesentative these kernels. Now we compare AR estimator with KB estimator on the basis of size distortion and effective power of Ng-Perron test.

5.1 Performance at Zero Lag Length/ Lag Truncation

When the lag length/ truncation lag is zero analytically both estimators of spectral density are identical.

Autoregressive estimator of spectral density

From equation 3.4

$$\hat{f}(0) = \frac{\hat{\sigma}_\varepsilon^2}{\left(1-\hat{\beta}(1)\right)^2}$$

$$\hat{\beta}(1) = \sum_{l=1}^{T}\hat{\beta}_l \; and \; \hat{\sigma}_\varepsilon^2 = \frac{\sum_{t=1}^{(T-l)}\hat{\varepsilon}_t^2}{(T-l)}$$

To estimate the autoregressive spectral density estimator we use equation

$$\Delta\tilde{y}_t = \rho\tilde{y}_{(t-1)} + \sum_{l=1}^{T}\Delta\tilde{y}_{(t-l)}\hat{\beta}_l + \varepsilon_t$$

For zero lag length, our equation reduce to

$\Delta\tilde{y}_t = \rho\tilde{y}_{(t-1)} + \varepsilon_t$

Then

$\hat{\beta}(1) = \sum_{l=1}^{T}\hat{\beta}_l = 0$ and $\hat{\sigma}_\varepsilon^2 = \frac{\sum_{t=1}^{(T}\hat{\varepsilon}_t^2}{T}$

So autoregressive estimator of spectral density at zero lag length is

$$\hat{f}(0) = \hat{\sigma}_\varepsilon^2 \tag{5.1}$$

Kernel based estimator of spectral density

In section 4.1 we have already showed mathematically that for zero lag truncation, kernel based estimator of spectral density is

$\hat{f}(0) = \hat{\gamma}(0)$

$where\ \hat{\gamma}(0) = \frac{\sum_{t=1}^{T}\hat{\varepsilon}_t * \hat{\varepsilon}_t}{T} = \hat{\sigma}_\varepsilon^2$

$$\hat{f}(0) = \hat{\sigma}_\varepsilon^2 \tag{5.2}$$

Equation 5.1 and 5.2 shows that AR estimator and KB estimator are identical mathematically when lag length/ truncation lag is zero for any DGP.

5.2 Performance at Non Zero Lag Length/ Lag Truncation

In previous section, it has been proved mathematically that both estimators are identical at zero lag length/ lag truncation for any type of DGP given section 3.4.1.

When the size of the test is not stable, power comparison has no sense. For useful power comparison of both estimators of spectral density we use effective power. Effective power of the test is the difference between the observed power minus observed size of the test for given sample size, lag length/ lag truncation, estimator of spectral density and DGP. In this section we examine the performance of AR estimator and KB estimator on the basis of their distortion in size and effective power at different lag length/ lag truncation for given data generating processes. Size distortion is the difference between observed minus nominal size of the test

5.2.1 Performance with DGP I

From figure 5.1 & 5.2 given below, as lag length/ lag truncation increases effective power of test increase with both estimators of spectral density. With AR estimator, size distortion increase with lag length in small and large samples. This distortion in size with large moving average coefficient is small at even lag length than same is the case with power. The size distortion of test with KB estimator is always close to zero for any value of positive moving average coefficient. In small samples the effective power of test with AR estimator is more than the KB estimator but with large size distortion.

Figure # 5.1: The size distortion and effective power Ng-Perron test with autoregressive estimator and kernel based estimator when MA=0.2 for DGP-I

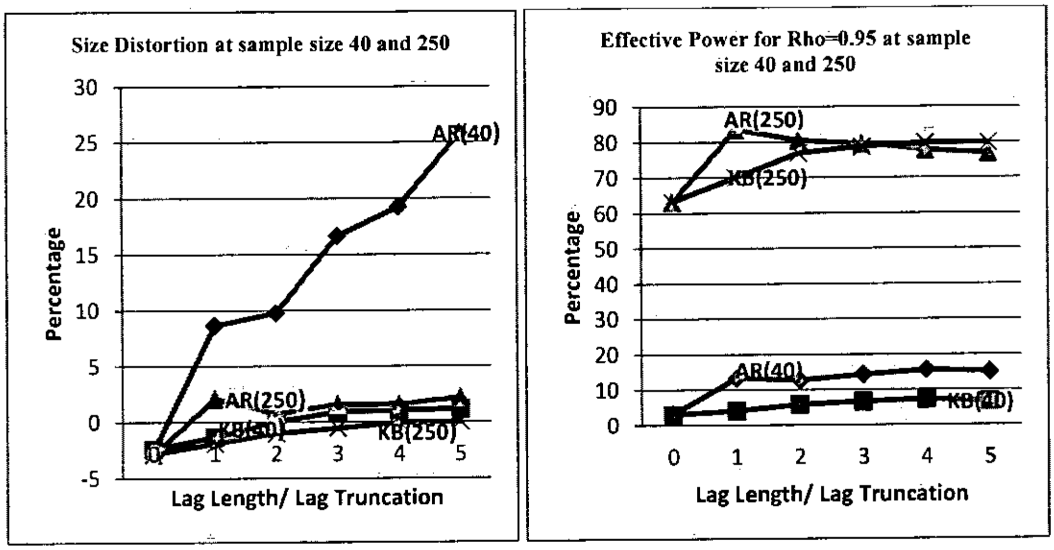
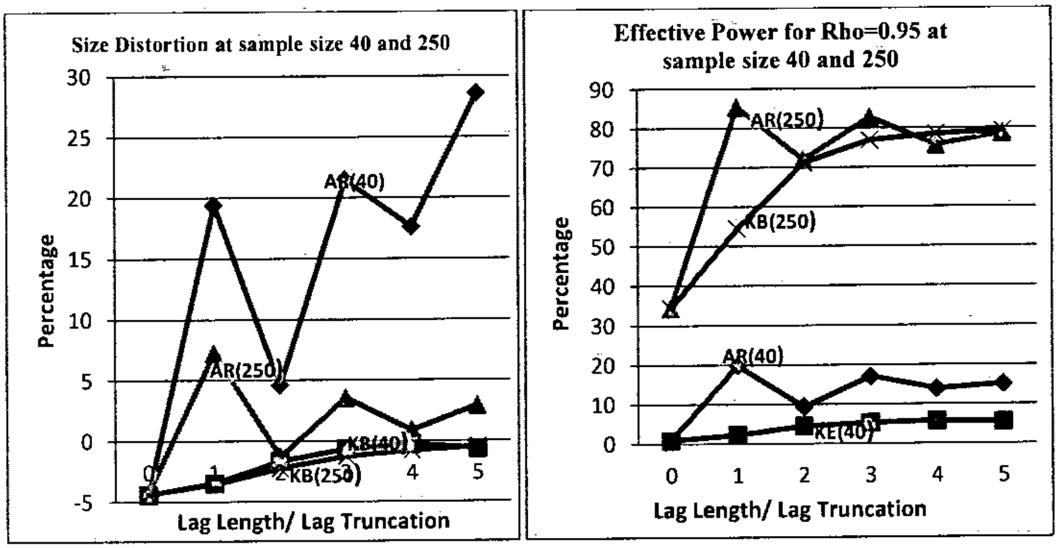


Figure # 5.2: The size distortion and effective power Ng-Perron test with autoregressive estimator and kernel based estimator when MA=0.6 for DGP-I



5.2.2 Performance with DGP- II

Simulation results reveals that for both AR and KB estimator, in small samples the size distortion of Ng-Perron test decrease and then increase after some lags with small negative moving average coefficient. In large samples with same negative moving coefficient, size distortion decrease. DGP with large negative moving average coefficient size distortion decrease for all samples as lag length/ lag truncation increases with both estimators of spectral density at zero frequency. The size distortion converges toward zero with AR estimator. The effective power of test with KB estimator is less than its size distortion in small samples when negative moving average is large. These findings are given in figures 5.3 and 5.4 below.

Figure # 5.3: *The size distortion and effective power Ng-Perron test with autoregressive estimator and kernel based estimator when MA=-0.20 for DGP-II*

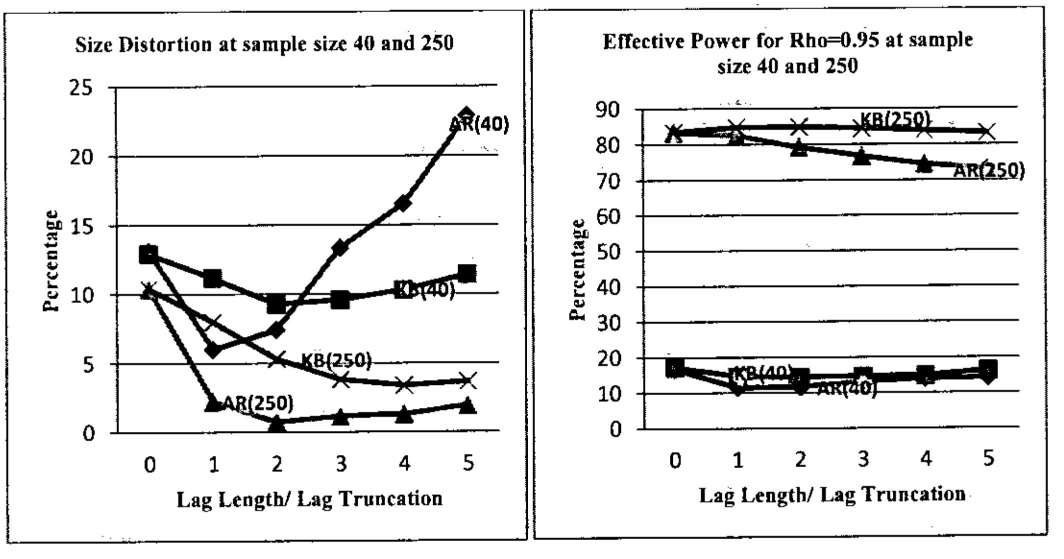
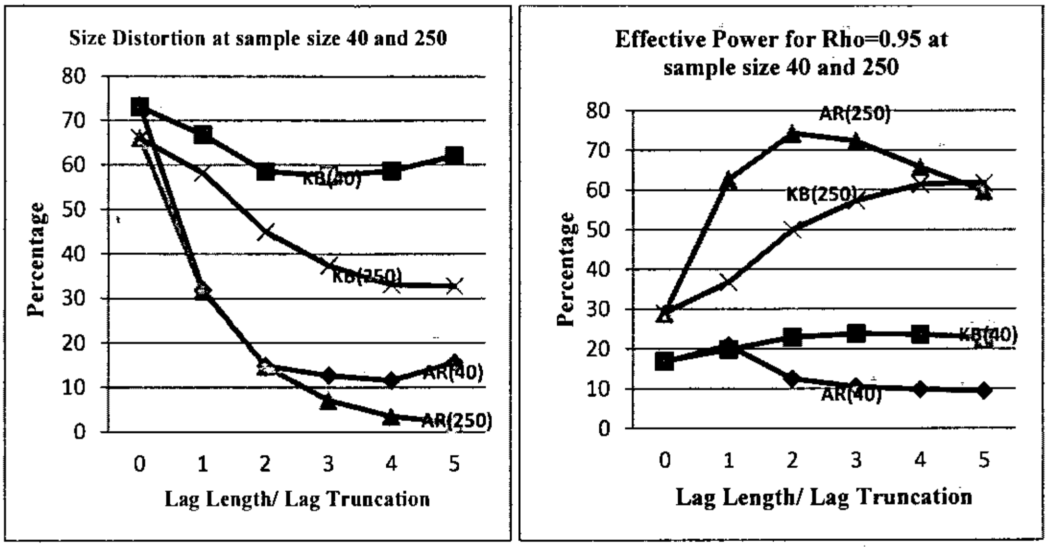


Figure # 5.4: The size distortion and effective power Ng-Perron test with autoregressive estimator and kernel based estimator when MA=-0.60 for DGP-II



5.2.3 Performance with DGP-III

Figure 5.5 and 5.6 given below shows that, there is big size distortion in Ng-Perron test with AR estimator of spectral density in small samples for any value of positive moving average coefficient in DGP-III with lag length/ lag truncation increase from zero to five and effective power is very small. The size distortion of test at even lags is less than that of odd lags with large moving average coefficient. With KB estimator of spectral density the size distortion remain close to zero for any value of moving average coefficient and sample size. The effective power of test in small sample is close to zero and increases with increase in sample size.

Figure # 5.5: *The size distortion and effective power Ng-Perron test with autoregressive estimator and kernel based estimator when MA=0.2 for DGP-III*

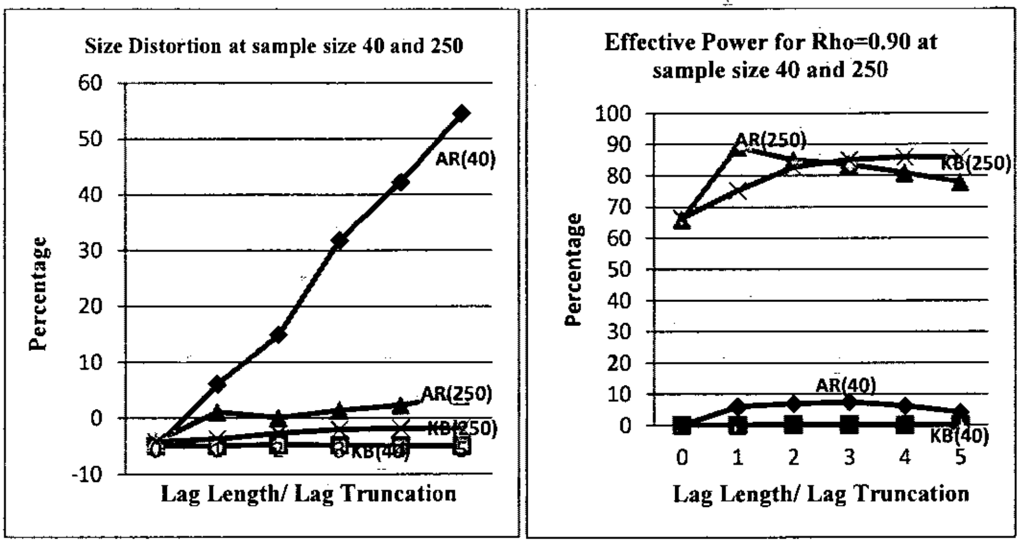
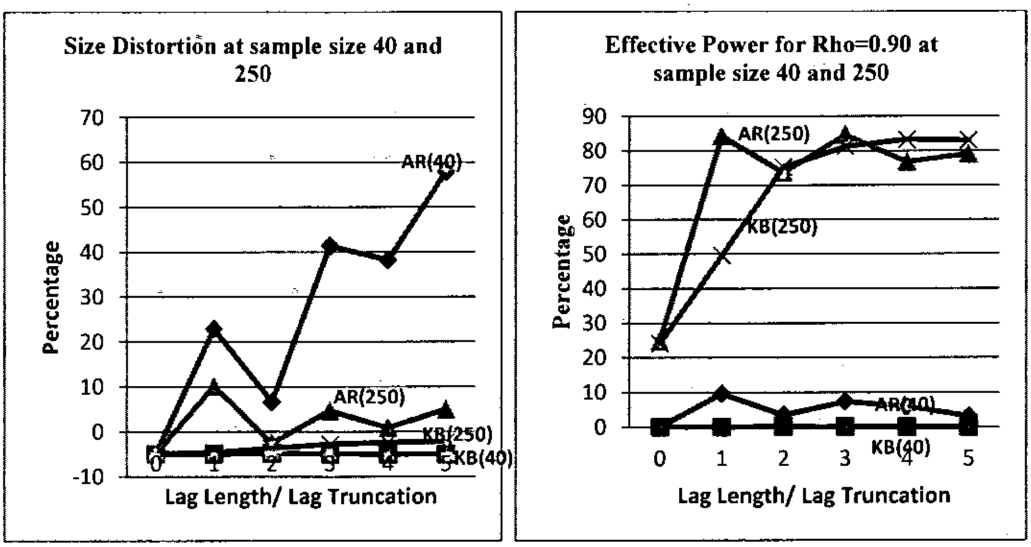


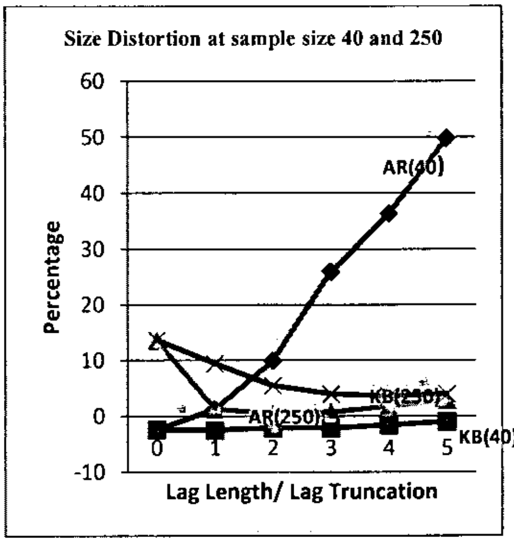
Figure # 5.6: *The size distortion and effective power Ng-Perron test with autoregressive estimator and kernel based estimator when MA=0.6 for DGP-III*



5.2.4 Performance with DGP-IV

It has been observed that with DGP-IV, with KB estimator the size distortion increases and effective power of test decreases with the increase in negative moving average coefficient for given sample size. For large negative coefficient of moving average the effective power become zero with high size distortion. Size distortion in Ng-Perron test with AR estimator increase in small samples for small coefficient of negative moving average and vice versa when lag length increases. In large samples distortion in size converges toward zero with increase lag length. At zero lag length/ lag truncation size distortion increases significantly for given coefficient of negative moving average when sample size gets large. The graphs are given below.

Figure # 5.7: *The size distortion and effective power Ng-Perron test with autoregressive estimator and kernel based estimator when MA=-0.2 for DGP-IV*



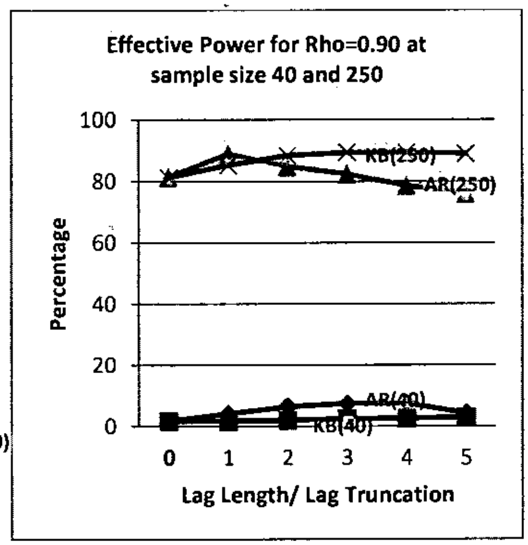
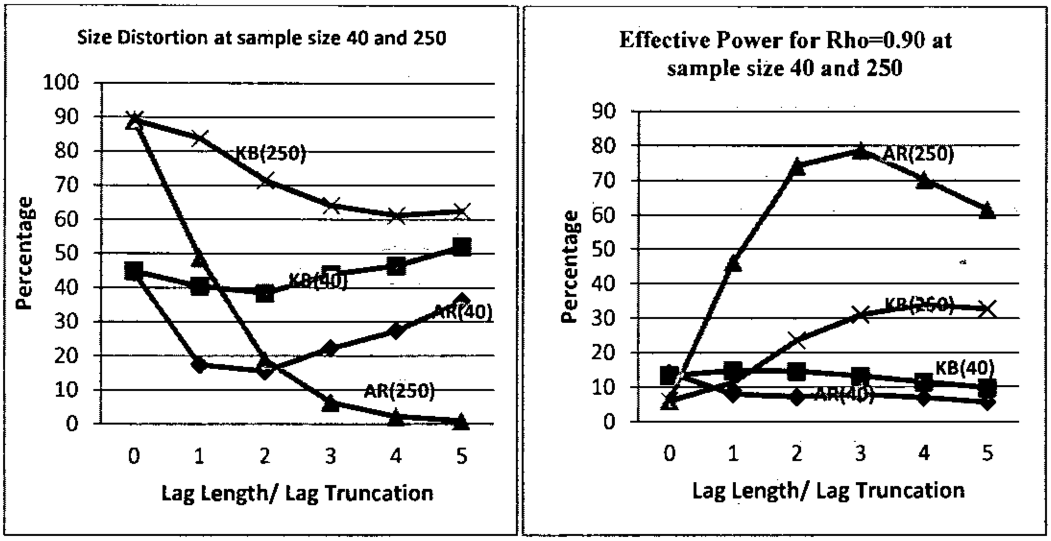


Figure # 5.8: *The size distortion and effective power Ng-Perron test with autoregressive estimator and kernel based estimator when MA=-0.6 for DGP-IV*



5.3 Summary

We summarize the above discussion with respect to

Effect of DGP

Effect of Moving Average

Effect of Sample Size

Effect of Lag Length/ Lag Truncation

Effect of DGP

When there is no moving average, DGP-I & II[5] are identical to each other, similarly DGP-III & IV[6] are also identical to each other. Now the only difference between DGP-I & II and DGP-III & IV is of deterministic part. Drift as deterministic part in DGP-I & II and DGP-III & IV with drift and time trend as deterministic part. The effective power and size distortion in Ng-Perron test at zero lag length/ lag truncation for DGP's with drift is always greater than the DGP's with drift and trend as deterministic part. There is no size

[5] For detail see section 3.4.1
[6] For detail see section 3.4.1

distortion in Ng-Perron test for any DGP. The effective power of Ng-Perron test is almost zero in small samples for DGP-III & IV. Effective power of Ng-Perron test improves in large samples for all DGP's. When lag length/ lag truncation is zero both AR estimator and KB estimator of spectral density are mathematically equivalent[7]. The table 5.1 of effective power of Ng-Perron test for Rho= 0.95 and size distortion when Rho=1 with AR estimator of spectral density are given below.

Table # 5.1: *Effect of DGP on effective power and size distortion with no moving average at zero lag length/ lag truncation*

DGP	Effective Power for Rho=0.95		Size Distortion	
	Sample Size=40	Sample Size=250	Sample Size=40	Sample Size=250
I & II	7.36	82.40	1.53	0.50
III & IV	0.03	33.58	-4.71	-0.97

Effect of Moving Average

When lag length/ lag truncation is zero both estimator of spectral density (AR and KB) are equivalent (section 5.1). Performance of Ng-Perron test is different in the presence of positive and negative moving average. The results presented here for AR estimator only.

Positive Moving Average

Effective power of Ng-Perron test decreases with the increase in positive moving average in the errors of DGP-I &III and there is no distortion in size. Effective power and size distortion in Ng-Perron test at sample size 40 and 250 for DGP-I are given in tables 5.2

[7] See section 5.1

Table # 5.2: *Effect of positive moving average on effective power and size distortion with DGP-I at zero lag length/ lag truncation*

Moving average	Effective Power for Rho=0.95		Size Distortion	
	Sample Size=40	Sample Size=250	Sample Size=40	Sample Size=250
0.2	3.11	63.11	-2.57	-2.89
0.4	1.46	44.56	-4.02	-4.03
0.6	0.90	34.40	-4.42	-4.30
0.8	0.67	29.96	-4.53	-4.46

Negative Moving Average

Distortion in the size of Ng-Perron test increases and effective power decreases with the increase of negative moving average in errors of DGP-II & IV for any sample size. The distortion in size and effective power of Ng-Perron test is given below in table 5.3 at sample size 40 and 250 for DGP-II.

Table # 5.3: *Effect of negative moving average on effective power and size distortion with DGP-II at zero lag length/ lag truncation*

Moving average	Effective Power for Rho=0.95		Size Distortion	
	Sample Size=40	Sample Size=250	Sample Size=40	Sample Size=250
-0.20	16.62	83.31	13.13	10.36
-0.40	25.69	62.94	37.18	32.05
-0.60	16.74	28.90	73.35	66.10
-0.80	0.91	1.36	93.99	93.64

Effect of sample size

The effective power of Ng-Perron test increases in large samples for all DGP's (section 3.4.1) and size distortion nearly remains same. Both AR estimator and KB estimator are identical mathematically (section 3.4.1). The tables of effective power and size distortion of Ng-Perron with AR estimator are given below in table 5.4 for data generating process I & II

Table # 5.4: *Effect of sample size on effective power and size distortion at zero lag length/lag truncation at different sample size*

Sample Size	Effective Power for Rho=0.95				Size Distortion			
	DGP-I		DGP-II		DGP-I		DGP-II	
	MA=0.2	MA=0.6	MA=-0.2	MA=-0.6	MA=0.2	MA=0.6	MA=-0.2	MA=-0.6
40	2.96	0.78	17.16	16.88	-2.41	-4.41	12.92	73.08
80	9.58	3.12	38.79	21.58	-2.66	-4.44	12.74	72.98
150	27.23	10.89	68.71	26.25	-2.74	-4.36	11.62	68.72
250	63.14	34.34	83.39	28.96	-2.85	-4.38	10.38	66.03
400	93.88	79.98	85.47	30.46	-2.80	-4.43	9.51	64.53

Effect of sample lag length/ lag truncation

The effect of lag length/ lag truncation has been discussed in detail in section 5.2. Findings of this section are summarized as

- DGP with positive moving average, as lag length increase, in small samples there is distortion in size of Ng-Perron test with AR estimator of spectral density with increase in the effective power. There is no size distortion due to increase in lag length/ lag truncation with KB estimator of spectral density. In large samples there is no size distortion due to increase in lag length/ lag truncation in Ng-Perron test with both estimators of spectral density and effective power of test increases. Both AR estimator and KB estimator are nearly identical in large samples. See figures 5.1, 5.2, 5.5 &5.6
- DGP with negative moving average, size distortion in Ng-Perron test decreases with the increase in lag length/ lag truncation with both estimators of spectral

density. With suitable lag length/ lag truncation size distortion converge to zero with AR estimator of spectral density. Size distortion in Ng-Perron test is always high with KB estimator of spectral density when coefficient of negative moving average is large for any sample size with small effective power. See figures 5.3, 5.4, 5.7 & 5.8

It is very important for the implementation of Ng-Perron test to detect the sign of moving average in the given time series. Now the question arise how to detect the nature of moving average in the real life data when there is no information about the true DGP. This issue has been discussed in detail in chapter 6.

CHAPTER 6

Detection of Nature of Moving Average

In chapter 5, we conclude that the results of Ng-Perron unit root test depend heavily on the nature of moving average and spectral density estimator. In real life analysis it is important to make correct decision about the estimator of spectral density. Any wrong selection may leads to incorrect conclusion about the unit root. The selection of estimator is based on the nature of moving average in the time series data. Now the question arise, how to detect the nature of moving average. In this study we developed a procedure to detect the sign of moving average. This procedure is very simple. Our simulation result shows that, this procedure work well in small sample size even with small moving average. This technique is applied to any type of DGP with or without unit root.

6.1 Procedure for Detection of Sign of Moving Average

- For the given series estimate any equation from the set of equations (6.1) and find residuals e_t.

$$\left\{\begin{array}{c} y_t = \rho y_{(t-1)} + e_t \\ y_t = \alpha + \rho y_{(t-1)} + e_t \\ y_t = \alpha + \beta t + \rho y_{(t-1)} + e_t \end{array}\right\} \tag{6.1}$$

- Calculate the autocorrelation coefficient using residuals and note the sign of correlation coefficient.

Now apply the Ng-Perron test and use the estimator of spectral density according to the sign of autocorrelation. If sign of autocorrelation coefficient is positive, then use Kernel based estimator of spectral density otherwise Autoregressive estimator. As we mentioned above that this procedure work well in any kind of DGP. To check the reliability of the results of this detection method we conduct the Monte Carlo simulation. The details of Monte Carlo design are given below.

6.2 Monte Carlo Design

Our Monte Carlo design consists of following steps.

1. Generate the data using pre specified DGP's
2. Estimate autoregressive model and find the residuals.
3. Estimate the autocorrelation coefficient and note its sign.

We perform our Monte Carlo experiment 10,000 times for different values of moving average coefficient and sample size. Also we estimate three different AR models for each DGP. Reason to estimate three models for each DGP is to investigate the effect of mismatch with true DGP.

The DGP's are

DGP-1

$$y_t = u_t \qquad u_t = \rho u_{(t-1)} + \delta e_{(t-1)} + e_t \qquad (6.2)$$

DGP- 2

$$y_t = \alpha + u_t \qquad u_t = \rho u_{(t-1)} + \delta e_{(t-1)} + e_t \qquad (6.3)$$

DGP- 3

$$y_t = \alpha + \beta t + u_t \qquad u_t = \rho u_{(t-1)} + \delta e_{(t-1)} + e_t \qquad (6.3)$$

The sample size is 40, 80, 150 and 250

The autoregressive coefficient Rho i.e. ρ: 1, 0.95 and 0.90

Moving average coefficient: 0.60, 0.40, 0.20, -0.20, -0.40 and -0.60

The models are

Model - I

$$y_t = \rho y_{(t-1)} + v_t \tag{6.5}$$

Model - II

$$y_t = \alpha + \rho y_{(t-1)} + v_t \tag{6.6}$$

Model - III

$$y_t = \alpha + \beta t + \rho y_{(t-1)} + v_t \tag{6.7}$$

The tables given below give the percentage of detection of sign identical to the sign of moving average coefficient in the DGP.

Table # 6.1: *The percentage of detection of sign of moving average for DGP-1 with model I*

Sample Size	Moving Average	Autoregressive Coefficient (ρ)		
		1	0.95	0.9
40	0.6	99.88	99.86	99.84
	0.4	98.33	98.12	98.32
	0.2	85.46	86.55	85.46
	-0.2	87.03	84.96	83.73
	-0.4	97.44	95.90	94.28
	-0.6	98.05	95.75	92.62

Table # 6.2: The percentage of detection of sign of moving average for DGP-3 with model I

Sample Size	Moving Average	Autoregressive Coefficient (ρ)		
		1	0.95	0.9
40	0.6	99.81	99.89	99.82
	0.4	98.28	98.25	98.07
	0.2	86.17	86.06	84.98
	-0.2	87.69	85.24	83.63
	-0.4	97.82	96.20	93.60
	-0.6	98.16	96.38	92.89

6.3 Summary

The simulation results reveals that, the percentage of detection of sign of moving average coefficient matching with the sign of moving average in original DGP is very good. From the table 6.1 & 6.2 percentage of detection of nature of moving average matching with the sign that was in DGP is nearly 85% when sample size was 40 and moving average coefficient is just ±0.2. The result also confirms that there is no effect on the performance of this method when disparity with DGP and model to estimate the residuals, which we used in the estimation of autocorrelation. We have same results for all other combinations of DGP and autoregressive models.

CHAPTER 7

Conclusions

The Ng-Perron test is recent and significant development in unit root testing and getting popularity. However its application is unnecessarily complicated. The Ng-Perron consists of a suite four tests and it was not clear that which of four would be optimal. Similarly there are various methods of estimating spectral density and output of the test changes with the choice of spectral density estimator. So practitioners don't know which test should be used and at the same time which estimator of spectral density should be used. In the process of comparing different estimators of spectral density, we were able to introduce several simplifications to the application of this test.

The main findings of this study are summarized below:

7.1 Equivalence of Four Tests Statistics Proposed by Ng-Perron

Ng and Perron proposed four tests statistics for testing unit root namely MZa, MZt, MSB and MPT. The evidence in section 4.1 shows that all four tests are identical in size and power for given sample size, estimator of spectral density, moving average coefficient, lag length/ lag truncation and ρ. One of them can be used to test for unit root.

7.2 Equivalence of Kernels in Kernel Based Estimator of Spectral Density

In the estimation of kernel based estimator of spectral density estimator at frequency zero we used Bartlett Kernel, Parzen Kernel, Quadratic Spectral Kernel, and Tukey-Hanning Kernel. All these kernels are equivalent asymptotically (Andrew 1991). The result in section 4.2 shows that these kernels are equivalent in small samples as well. The choice of kernel has no effect on the performance of Ng-Perron test. In the estimation of kernel based estimator of spectral density any kernel can be used.

7.3 Comparison of AR Estimator and KB Estimator

7.3.1 DGP with Positive Moving Average.

In the presence of positive moving average in the DGP there is almost zero size distortion when lag truncation/ lag length is zero. For non zero lag truncation/ lag length the size distortion increases with AR estimator of spectral density in small samples. There is no size distortion with kernel based estimator for any non zero lag length/ lag truncation. In large samples kernel based estimator give good effective power with zero distortion in size for any value of moving average coefficient.

7.3.2 DGP with Negative Moving Average.

There is size distortion in the presence of negative moving average in the DGP for both estimators of spectral density. This size distortion increases as coefficient of negative moving average coefficient increases even in large samples. Distortion in size of Ng-Perron test with AR estimator of spectral density decrease with non zero lags. However, it has been seen that as negative moving average coefficient the chances of spurious reduced. Where as positive moving increases the chances of spurious regression. Therefore we need to protect against positive moving average.

7.4 Detection of Nature of Moving Average

From sections 7.3, the nature of moving average plays pivotal role in the selection of suitable spectral density estimator and power of Ng-Perron test. The method introduced in chapter 6, detect the exact sign of moving average with high probability in the DGP. This method is very simple and detects the nature of moving average even in small sample sizes.

7.5 Recommendations

In the light of above discussion a practitioner who wants to apply Ng-Perron test have to chose an estimator of spectral density. The choice of estimator is very important and has effects on the Ng-Perron test result about unit root. Prior to the application of Ng-Perron test it is necessary to detect the sign of moving average in the data using the procedure introduced in chapter 6. If the sign of moving average is positive it is better to use kernel based estimator of spectral density with non zero lag truncation. Data with negative moving average then autoregressive estimator of spectral density is always a better choice. Ng-Perron test is not a batter choice in small samples because its effective power is very low.

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