

# **Modeling and Forecasting Exchange Rate Volatility and it's Impact on Pakistan's Economy**



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2016**





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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

**Modeling and Forecasting Exchange Rate Volatility and  
it's Impact on Pakistan's Economy**



BY  
**MUHAMMAD FAIZAN**

*A dissertation  
Submitted in the partial fulfillment of the  
Requirements for the degree of  
MASTER OF SCIENCE  
IN  
STATISTICS*

SUPERVISED BY:

*Dr. Zahid Iqbal*

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Faculty of Basic and Applied Sciences  
International Islamic University, Islamabad  
Pakistan  
2016**

# Certificate


## **Modeling and Forecasting Exchange Rate Volatility and its Impact on Pakistan's Economy**

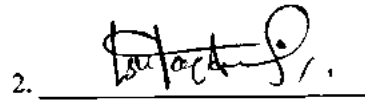
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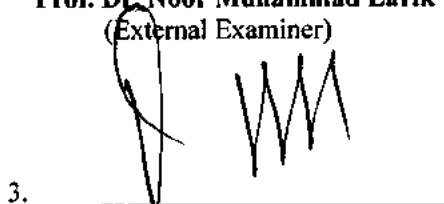
***Muhammad Faizan***


A DISSERTATION SUBMITTED IN THE PARTIAL FULFILLMENT OF THE  
REQUIREMENTS FOR THE DEGREE OF THE MS IN STATISTICS

We accept this dissertation as conforming to the required standard.

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**2016**

***For My Worthy Parents***

***Thank you for allowing me to follow my heart and  
encouraging me to pursue my dreams.***



## Forwarding Sheet by Research Supervisor

The thesis entitled “Modeling and Forecasting Exchange Rate Volatility and it’s Impact on Pakistan’s Economy” submitted by Muhammad Faizan, Registration No: 25-FBAS/MSST/S-13, in partial fulfillment of MS degree in Statistics has been completed under my guidance and supervision. I am satisfied with the quality of his research work and allow him to submit this thesis for further process to graduate with Master of Science Degree from Department of Mathematics and Statistics, as per IIU Islamabad rules and regulations.

Dated \_\_\_\_\_



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**MUHAMMAD FAIZAN**

## **DECLARATION**

I, **Muhammad Faizan**, hereby declare that this dissertation is original and has never been presented in any other institution. I, moreover, declare that any secondary information used in this dissertation has been duly acknowledged.

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## **LIST OF ABBREVIATIONS**

**ACF- Auto Correlation Function**

**AIC - Akaike Information Criteria**

**SIC – Schwartz information Criteria**

**ADF - Augmented Dickey-Fuller**

**AR - Auto Regressive**

**ARCH - Auto Regressive Conditional Heteroscedasticity**

**ARIMA - Auto Regressive Integrated Moving Average**

**EGARCH - Exponential Generalized Auto Regressive Conditional Heteroscedasticity**

**GARCH - Generalized Auto Regressive Conditional Heteroscedasticity**

**GED - Generalized Error Distribution**

**GNP - Gross National Product**

**MA - Moving Average**

**MAE - Mean Absolute Error**

**MAPE - Mean Absolute Percentage Error**

**PAC - Partial Auto Correlation**

**PACF - Partial Auto Correlation Function**

**RMSE - Root Mean Square Error**

**USD - United States Dollar**

**GDP – Gross Domestic Product**

**FDI – Foreign Direct Investment**

**REM – Remittances**

**INF - Inflation Rate**

**VAR – Vector autoregressive**

## ABSTRACT

This dissertation describe an empirical study of modeling and forecasting time series data of Exchange rate and it's impact on Pakistan's economy. The modeling and forecasting of exchange rate and its volatility has important implications for macro-economic decisions making. This research is divided into two parts, at first part the study aims to capture volatility patterns using different time series models. Monthly and yearly exchange rate of Pakistan to USD for the period ranging 1981 to 2014 obtained from State Bank of Pakistan. We have ran AR (Autoregressive), MA (Moving Average) and different combination of both ARIMA (Autoregressive Integrated Moving Average) models. The diagnostic checking has shown that ARIMA (3,1,2) and ARIMA (1,1,2) [on the basis of Akaike Information Criteria (AIC) and Schwartz Information Criteria (SIC)] are appropriate models for monthly and yearly Exchange rate returns, respectively. ARIMA (3,1,2) and ARIMA (1,1,2) selected, amongst different sets of AR (Autoregressive), MA (Moving Averages) and ARIMA, as Mean models for further applying ARCH family of models. For diagnostic checking we also performed forecast from 1981 to 2014 as well as 2010 to 2014 with the help of these selected models. The performance of forecast was evaluated by RMSE (Root Mean Square Error), MAE (Mean Absolute Error), MAPE (Mean Absolute Percentage Error) and Theil's Inequality coefficient. The ARCH-LM test and residual of ARIMA (3,1,2) and ARIMA (1,1,2) have justified to run ARCH family of models.

Similarly, for capturing the volatility of Exchange Rate returns, used ARCH family of model i.e. Autoregressive conditional heteroscedasticity (ARCH), Generalized Autoregressive conditional heteroscedasticity (GARCH), Integrated Generalized

Autoregressive conditional heteroscedasticity (IGARCH) and Exponential Generalized Autoregressive conditional heteroscedasticity (EGARCH). After running the different sets of ARCH, GARCH, EGARCH, IGARCH. The diagnostic checking has shown that for monthly exchange rate data the ARIMA(3,1,2)-EGARCH (1,1) and for yearly exchange rate data the ARIMA(1,1,2)-EGARCH(1,1) are the best fitted models for modeling and forecasting Exchange Rate volatility. We have made forecast from 1981 to 2020 with these models. The performance of forecast was evaluated by RMSE (Root Mean Square Error), MAE (Mean Absolute Error), MAPE (Mean Absolute Percentage Error) and Theil's Inequality coefficient. These forecasts would be helpful for the policy makers to foresee the future requirements. Monthly and Yearly Exchange Rate data (PKR/Us-Dollar) is obtained from the State Bank of Pakistan (SBP).

The second part of this study was designed to investigate the effect of Exchange Rate's volatility on the Real Gross Domestic Product (GDP) of Pakistan. The study has used six variables, the data for the variables i.e. Inflation Rate (INF), Foreign Direct Investment (FDI), Exports, Real GDP and Foreign Remittances obtained from The State Bank of Pakistan. The empirical relationship between exchange rate volatility and Real Gross Domestic Product (GDP) have been found while employing an econometric study based on Vector Autoregressive (VAR) methodology for yearly data collected from 1981 to 2014. We have also used the Vector Autoregressive (VAR) model to estimate the Impulse Response functions and Variance Decompositions for Gross Domestic Product in order to determine how Exchange Rate shock effects Real GDP. in short run the Real GDP account for 67.32% variation of the fluctuation in Real GDP (own shock) and in long run i.e. 10 years is 32.32% variation of the fluctuation in Real GDP (own shocks). Shock to Exchange rate can cause 2.63% in short run but contribution in long run is 7.76% to the Real GDP in Pakistan. Impulse response shows due to exchange rate shocks, GDP



Changes negatively in the short run but after that it steadily change in positive direction for the effects of one standard deviation in Exchange Rate. EViews software is used for the data analysis.

**Key words:** Exchange Rate, ARIMA, ARCH, GARCH, IGARCH, EGARCH, AIC, VAR.

## CHAPTER 1

### INTRODUCTION

After the conversion of exchange rate from fixed to floating system, exchange rate volatility became an important issue, Exchange rate determines rate of change of currencies, it is a conversion factor (Khan & Azim 2013, Javed & Farooq 2009). Exchange rate volatility is the amount of uncertainty or risk associated with size of changes in the value of currency. A higher volatility means that the value of currency spread out over large range. This means that the price of the currency suddenly change over short period of time in either direction (Mukhtar & Malik, 2010).

Exchange rate is the most important variable for an economy because all countries have its own separate currency and all countries trade internationally, so it's necessary to convert one country's currency in terms of another. Exchange rate is a factor for conversion, it is the value of one country's currency in terms of another country's currency. Exchange rate is the comparison of value of goods and services produced in different countries in an efficient way.

Foreign Exchange rate is necessary for international transaction. It is very important for all countries, it's equally important for emerging as well as for developing countries.

In fact, some professionals have said that exchange rate policies followed by some developing countries after 1970s were inappropriate, and this caused severe overvaluation of their currencies and finally contributed to their debt crises. Such overvaluation may cut exports, harm agriculture and create weakening capital outflows in the developing countries. Problems like energy crisis which became predominant in the 1970s, stimulated a new concern in matters of adjustment and behavior of exchange rate to external shocks since oil was being imported on a large scale by developing as well as emerging.

Before World War II, in the 1930s there was flexible, high volatile exchange rate and competitiveness in exchange rate policies. On 27th December, 1945, the Bretton

Woods conference was held, participants of different countries was agreed to begin a period of fixed, but adjustable exchange rate system. It was supposed that a more stable exchange rate system would uphold the development of international trade. It was decided that the parity value of each associate should be expressed in terms of gold as a common denominator or in terms of the US dollar. Moreover, the maximum as well as the minimum rates should not differ from the parity by more than 1%.

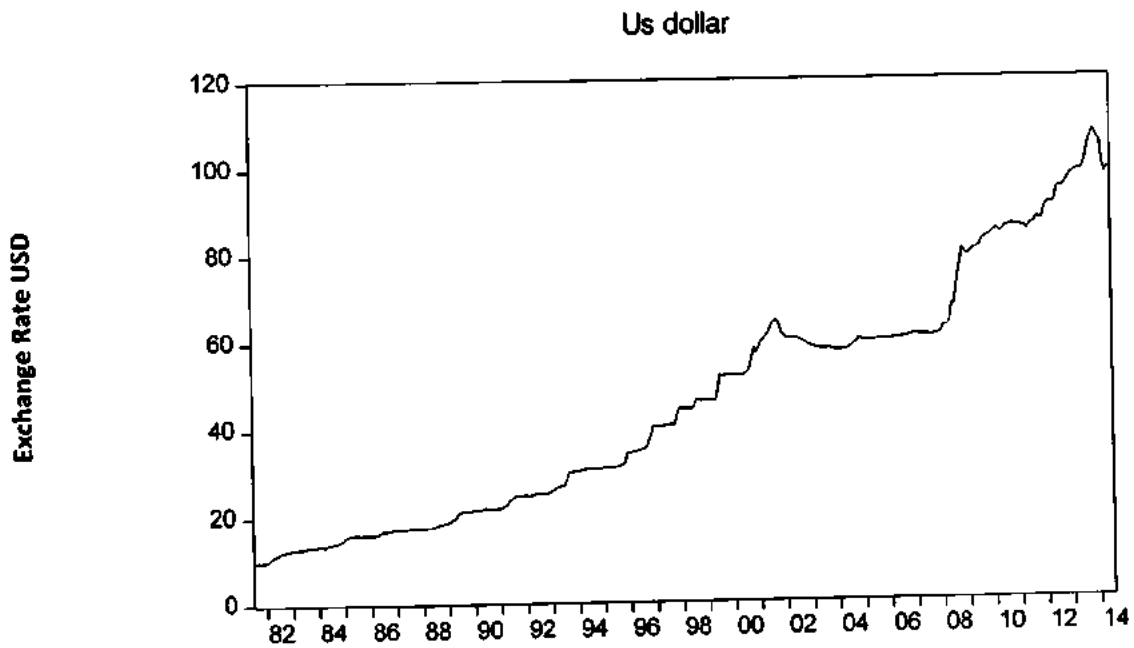
Exchange rate plays an important role in international trade and finance. Exchange rate fluctuations may have significant impact on prices, wages, interest rates, unemployment and level of output (Ramzan, Ramzan & Zahid, 2013). It also influences on decision of policy makers and play an important role to determine the volume of imports and exports. Exchange rate volatility directly influences the prices of exports and imports, manufacturing and their growth rates. Moreover, its effect on allocation of manufacturing goods, reserve money and balance of payments (Javed & Farooq, 2009). It helps to maintain balance of capital in a free market system. The dynamics in exchange rate play an important role to determine trade balance.

The high degree of uncertainty and volatility of exchange fluctuations observed in Pakistan is of great concern to policy-makers and researchers, to investigate the nature and extent of the impact of such fluctuation on Pakistan's volume of trade (Mustafa & Nishat, 2004). In many countries, it is experienced that volatility in exchange rate has reduced the trade by creating about future profit from exports (Mustafa & Nishat, 2004).

Historically, Pakistan's foreign exchange system has remained dynamic under fixed and flexible exchange rate regimes. In the beginning, it was pegged to the Pound Sterling, which continued up to the 1970s in order to keep the exchange rate fixed under the Bretton Woods policy consensus. However, during the early 1970s, the peg-currency (Pound Sterling) was replaced by the US dollar, at PKR 4.76 per USD as an initial exchange rate. The first shock occurred in 1972, when the Rupee fell by 56.7 percent in terms of gold. A flexible currency band accommodating fluctuations up to 4.5 percent was introduced to resolve this issue (Khan & Azim 2013).

Pakistan has tried to incorporate some reforms within the financial sector over time, such as making stock markets accessible to a large number of investors and following a flexible exchange rate policy. Since 1970, the Rupee was depreciated against the US dollar on average from PKR 4.76 to PKR 10.34 in 1970s, to PKR 17.55 in 1980s, to PKR 46.82 during 1990s, and up to PKR 87.16 per US dollar during the 2000s to PKR 98.2 during 2010s (*State Bank of Pakistan*).

**Figure: 1.1 US Dollar Exchange Rate July 1981 to Jun 2014**

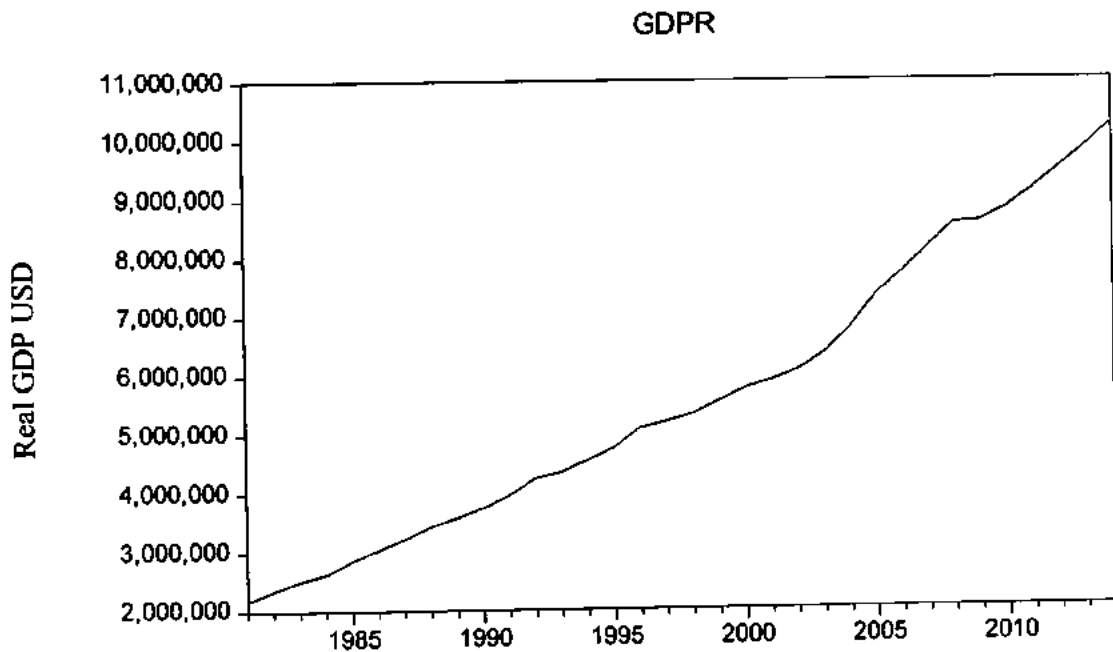


*Source: State Bank of Pakistan*

## 1.1 Real Gross Domestic Product

Gross domestic product (GDP) is the monetary value of all the finished goods and services produced within a country's borders in a specific time period. Gross Domestic Product (GDP) observed during the year 2014 is 243.63 billion USD. GDP in Pakistan averaged 59.54 billion USD. Pakistan's GDP reaching an all-time in the year 2014 and record low of 3.71 billion USD in 1960. (World Bank Group).

Figure: 1.2 Real GDP, in US dollar 1981 to 2014

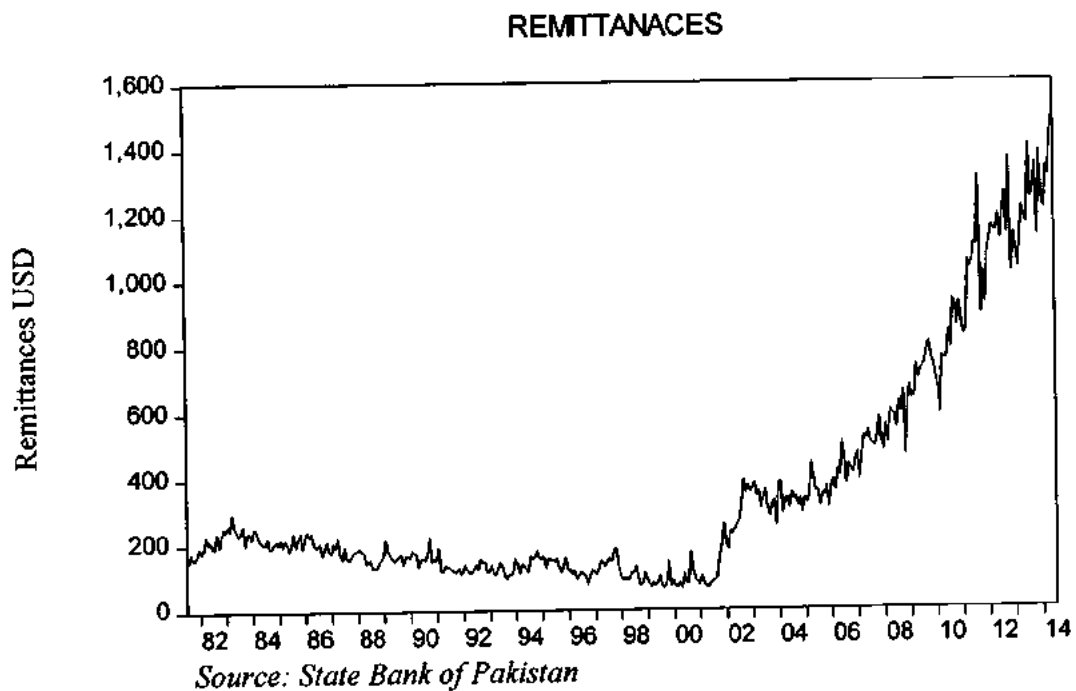


Source: State Bank of Pakistan

## 1.2 Foreign Remittances

Worker's remittances are different from FDI, aids and loans it means the transfer of money from migrants to their family in home country. For developing countries like Pakistan it is the largest source of earnings. The foreign exchange in the year 2013-14 the total worker's remittances amount is 15833.67 US million dollars. (*State Bank of Pakistan*).

Figure: 1.3 Foreign Remittance in million USD, July 1981 to June 2014



### 1.3 Inflation Rate

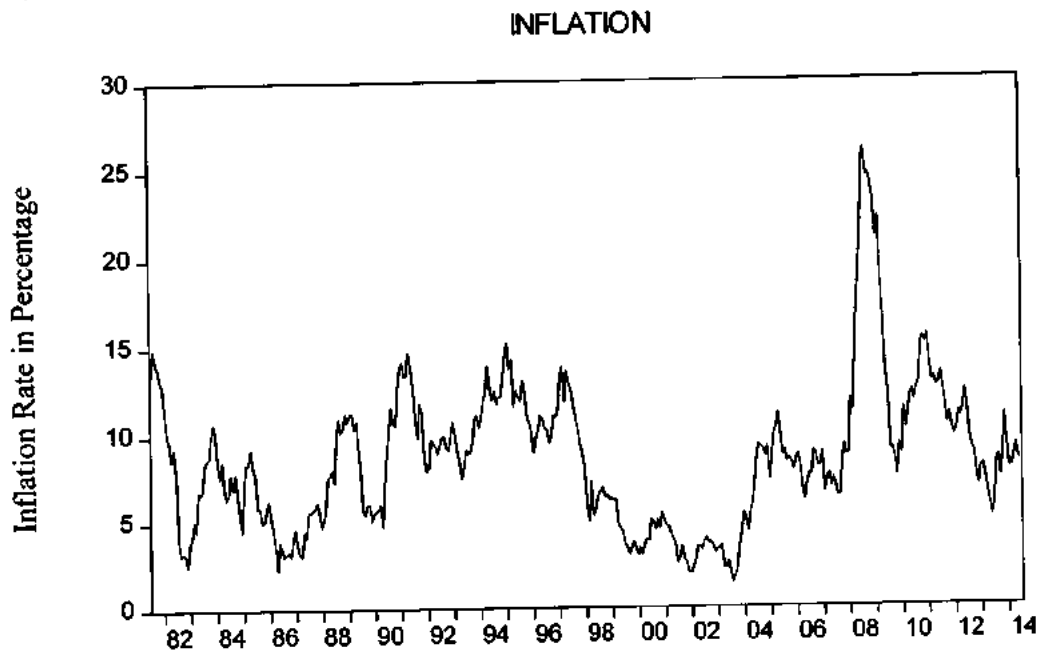
(Fischer, 1993) describes that the relationship between GDP growth Rate and inflation has recognized much attention particularly in the past era. Most empirical results have proven an inverse relationship between GDP growth rate and inflation.

The Relentless rise in prices of goods and services over time, delays efficient Allocation of resource by obscuring the motioning role of relative price variations which is a significant guide for effective decision making.

Inflation makes exports quite expensive, reducing international competitiveness of any country. It makes the cost of living is very high, it has negative effects on balance of payments. It's infect inflation is the public enemy. But in positive sense inflation redistributes real income, encourages employment, increases government revenue and encourages investment.

The inflation rate is recorded during fiscal year 2013-14 is 8.62 from 1981 to 2014 reaching high of 21.03 percent and a recorded low of 3.11 in the year 2002-03. (*State Bank of Pakistan*).

**Figure: 1.4** Inflation Rate July 1981 to June 2014



Source: *State Bank of Pakistan*

#### **1.4 Foreign Direct Investment (FDI)**

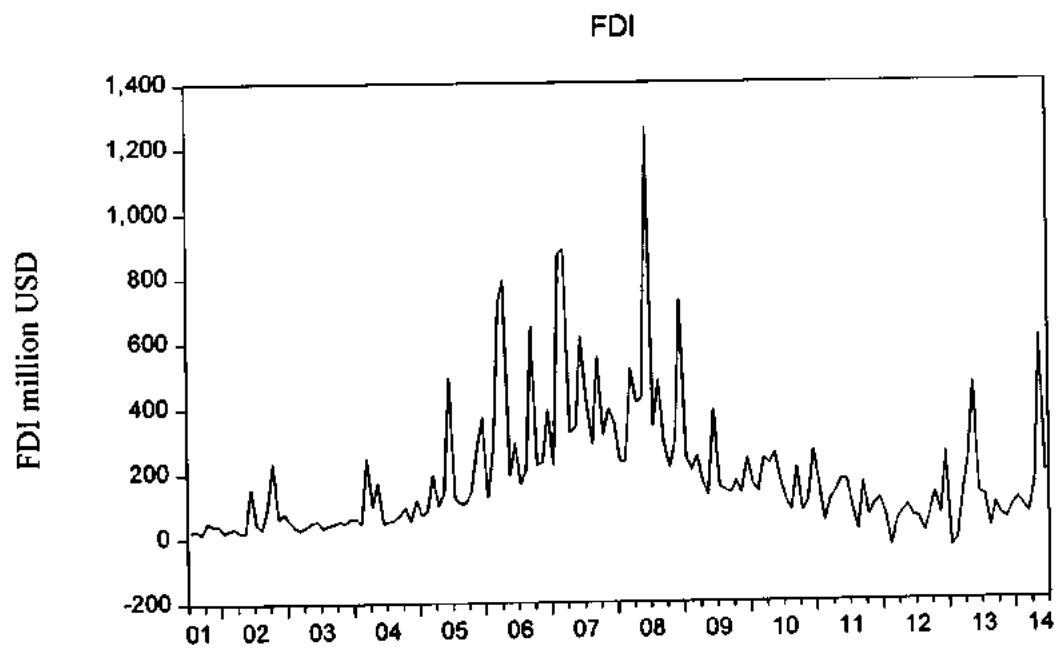
The role of foreign Direct Investment (FDI) for the progress of developing countries is vital. The foreign stakeholders are encouraged to invest in the host country, if the predictions of earning long term revenues by contributing to production sector of that country are very much obvious. FDI not only contributes to capital formation in developing countries, but this investment is also a source of transfer of new technologies and innovative skills from advanced to emerging countries.

In the light of its importance for the progress of developing countries, developing countries offer some incentives to the overseas investors in order to attract Foreign Direct Investment.

Furthermore, these countries take some steps to make certain developments in their environment i.e. law and order situation, security peace in order to make it conducive for investors. Pakistan is facing some serious problems due to law and order situation and political instability. Inflow of Foreign Direct Investment during the fiscal year 2006-07 is 5409.84 million USD which is highest but during the year 2013-14 reached to 1631.26 million USD. Pakistan has been ranked among top ten reforming countries in the world. In terms of ease of doing business Pakistan is at 128th position according to the recent World Bank report. (*World Bank Group, Pakistan Country Partnership Strategy 2015-19*).



**Figure 1.5 Foreign Direct Investment**

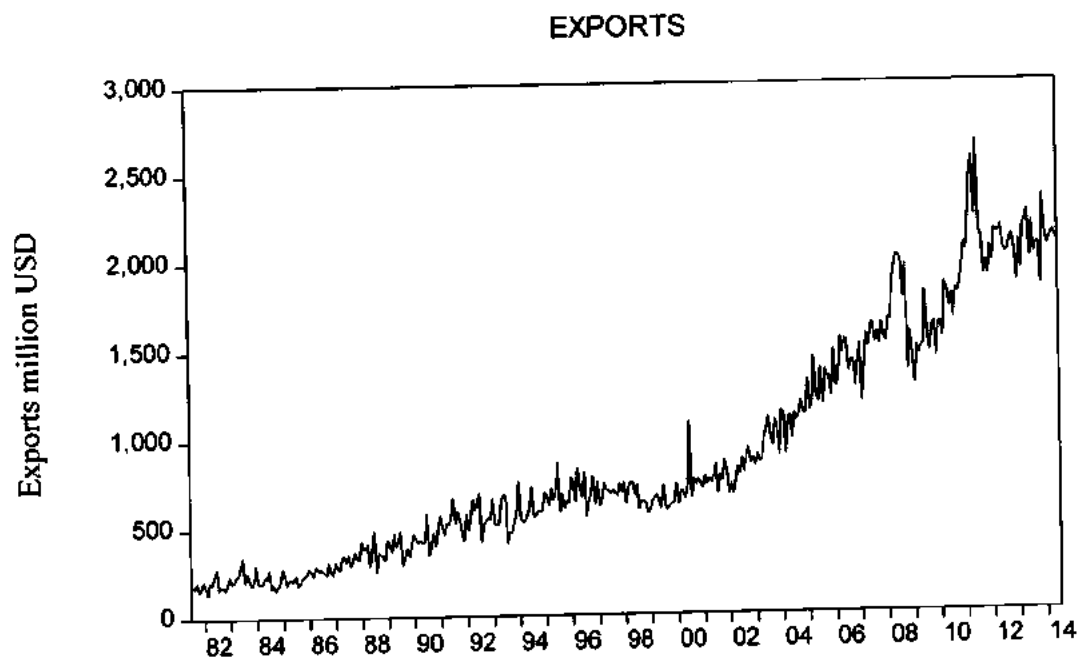


*Source: State Bank of Pakistan*

## 1.5 Pakistan's Exports

Exports play a vital role to improve growth of any country. Exports are the source of earnings of a country because it reflected in foreign money. Exports depend on some factors i.e. price of product, economic conditions of importing country and quality of product. Exports have strong effect on exchange rate. During the fiscal year 2010-11 total exports value is 25354.44 million USD which is highest and in last year i.e. 2013-14 is 25162.08 million USD. (*State Bank of Pakistan*).

**Figure : 1.6** Pakistan's Exports in million USD, July 1981 to June 2014



Source: *State Bank of Pakistan*

## **1.6 OBJECTIVES OF THE STUDY**

Followings are the objectives of the study

1. To search the best time series model among ARIMA, ARCH/GARCH family models to capture the exchange rate volatility.
2. To check the significance of ARCH family of model, for modeling and forecasting on Pakistan exchange rate's data.
3. The high degree of volatility and uncertainty of exchange rate fluctuations observed in Pakistan is of great concern to policy makers and researchers to investigate the nature and extent of the impact for such fluctuation on Pakistan's Economy.

## Chapter 2

### REVIEW OF LITERATURE

According to Engle, (1982) & Bollerslev,(1986) in financial time series ARCH family of models are more reliable for clustering volatility because these models are specifically designed for volatility modeling.

Copelman and Werner (1996), used VAR model with five variables for Mexico's exchange rate, depreciation of nominal exchange rate, interest rate, and a measure for money balances, revealed that dropped of output are observed after a devaluation.

Kamin (1996) indicated that the level of the exchange rate was a main determinant of the inflation rare in Mexico during the period of 1980 and 1990.

McKenzie, (1999) investigated that the modeling and forecasting of exchange rate and their volatility has important implications on economics and finance. He compares the ability of ARCH, AR and Mean models to forecast the magnitude of change in 19 Australian bilateral exchange rate series. He has suggested that ARCH model generate superior forecasting performance in situation of squared return of an exchange rate series.

Kamin and Roger (2000) observed the influence of depreciation on output and inflation rate of Mexico by applying VAR model with four variables; exchange rate, price index, output, and interest rate of US using quarterly data for the period of 1981-1995.

**AHMAD et al, (2002).** They used GARCH models with modifications for capturing the volatility of the exchange rates. The Q-statistic and LM tests suggest that long memory GARCH models should be used instead of the short-term memory and high order ARCH model. The GARCH models in out-of-sample and one-step-ahead forecasting. When using random walk model as the simple benchmark, all GARCH models outperform this model in forecasting the volatility of the exchange rates.

**Mukherjee et al (2003),** VAR structure is a dominant tool for analyzing impulse responses to systemic shocks and variance decomposing in variables due to these shocks.

**Sandoval (2006)** used the time series models i.e. ARMA, GARCH and EGARCH models for exchange rate modeling and capturing significant characteristic of data. He took four years daily exchange rate data from 2000 to 2004 from Asian and Latin American countries.

**Suliman Zakaria Suliman Abdalla, (2012)** used the generalized autoregressive conditional heteroscedastic approach to model the exchange rate volatility of nineteen Arab countries using daily returns over the period of 1st January 2000 to 19th November 2011. He applies both symmetric and asymmetric models that capture most common stylized facts about exchange rate returns such as volatility clustering and leverage effect. Based on the GARCH (1,1) model. Moreover, the asymmetrical EGARCH (1,1) results provide evidence of leverage effect for majority of currencies, indicating that negative shocks imply a higher next period volatility than positive shocks. Finally, the paper concludes that the exchange rates volatility can be adequately modeled by the class of GARCH models.

**Abdullah, (2012)** applied time series modeling for estimation and forecasting of gold bullion prices. He has concluded that ARIMA (2, 1, 2) is best model to forecast the price of gold bullion.

**Asad, (2012)** predicted the electricity demand in Australia for first seven days of May, 2011. He has used one year data of fiscal year 2010 to 2011. He has selected the ARIMA model for forecasting electricity demand and used Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE) to measure the accuracy of forecast.

**Ramzan et al. (2012)** explained that ARCH families of model are best for prediction of exchange rate. They also studied the working of GARCH model, and concluded that GARCH (1,2) is best model for removing persistence in volatility. Moreover, EGARCH (1,1) successfully overcome the leverage effect in the exchange rate returns. They also investigated the performance of GARCH family models in forecasting the volatility behavior of Pakistan foreign exchange market, and found that the EGARCH based evaluation of foreign exchange rates showed asymmetric behavior of volatility, where TARARCH model showed insignificant.

**Muhsin et al (2012).** Examined the performance of GARCH family models including EGARCH model in forecasting the volatility behavior of Pakistan's Exchange market. Daily Exchange rates data, from January, 2001 to December, 2009 was used for volatility capturing.

**Corliss, (2013)** shows that ARIMA is the most suitable for modeling and forecasting financial data. He reviewed the basic statistical concepts of ARIMA model and its application in statistical research.

**Devi et al, (2013)** they have selected top 4 companies from the Nifty Midcap50. Time series data of 5 years has selected and applied ARIMA model for forecasting. To check the accuracy of model they applied Akaike info Criterion and Bayesian info Criterion.

**Obeng et al. (2013)**, they have proved in their study that the relationship between exchange rate and GDP growth rate is positive, to check the correlation between both variable they used Pearson's Product Moment Correlation Coefficient (PPMC).

**Steve c, (2014)** analyzed exchange rate data of Naira/Dollar through ARIMA model. He has used 29 years data from 1982 to 2011 and concluded that the ARIMA model is most suitable for modeling and forecasting of exchange rate of Naira/Dollar.

## CHAPTER 3

### RESEARCH METHODOLOGY

#### 3.1. Source of Data

For volatility modeling and forecasting, the data used in present study consist average foreign exchange rates of Pakistan (Pak rupees per US \$). Monthly and yearly exchange rate data from 1981 to 2014, obtained from State Bank of Pakistan (SBP). Different time series model are used to model this data like ARMA, ARIMA ARCH and GARCH family of models. To investigate the effect of Exchange Rate's volatility on the Real Gross Domestic Product (GDP) of Pakistan. The study has used six variables, the data for the variables i.e. Inflation Rate (INF), Foreign Direct Investment (FDI), Exports, Real GDP and Foreign Remittances obtained from The State Bank of Pakistan. employing The empirical relationship between exchange rate volatility and Real Gross Domestic Product (GDP) have been found while employing an econometric study based on Vector Autoregressive (VAR) methodology for yearly data collected from 1981 to 2014..

#### 3.2. Time Series Models

##### 3.2-1 Time Series

CHATFIELD (2001) defines that "A time series is a set of observations measured sequentially through time".

##### 3.2-2 Univariate Time Series

The term univariate means a time-series that consist only one variable recorded chronologically through time in this study i.e. Exchange rate.

##### 3.2-3 Time Series Model

Time series model based on mathematical representations and theoretical foundations. An econometric model has to fulfil the following qualities, FROHN (1995)

- theoretical plausibility



- reliable parameter estimation
- good adjustment (i.e. the process that generates the data should be captured)
- good forecast
- simplicity (i.e. a model with less variables or easier functional form should be preferred)

#### **3.2-4 ACF (Autocorrelation Function)**

The important properties of any random variable are the first, two moments, i.e. its mean and the variance. The time series is the stochastic process. The relationship between different time periods is very important.

This is known as autocorrelation or we can say that auto-covariance. The auto-covariance function ( $X_k$ ) of a time-series is defined as:

$$Y_k = E \{ [X_t - E(X_t)][X_{t-k} - E(X_{t-k})] \},$$

Where,  $Y_t$  stands for the time-series. The autocorrelation function ( $\rho_k$ ) is defined as:

$$\rho_k = \frac{X_k}{X_0}$$

and the graph of this function is called correlogram. The correlogram is very important for the analysis, because it contained observed series which is time dependent. Since  $X_k$  and  $\rho_k$  only differ in the constant factor  $X_0$ , i.e. the auto variance of the time-series, it is sufficient to plot just one of these two functions.

Correlogram is used for inspection the randomness in the data. If these autocorrelations are close to zero for any or all time lags then the data is random. It also used to identify the order of an AR and MA process.

On the vertical axis the autocorrelation coefficients ( $X_k$ ) and on the horizontal axis the time lag.

### 3.2-5 PACF (Partial Autocorrelation Function)

The PACF ( $\pi_k$ ), where  $k \geq 2$ , is defined as the partial correlation between  $Y_t$  and  $Y_{t-k}$  under firm the random variables between  $Y_u$ . It looks to be clear, that the PACF is defined for lags equal to two or greater. The partial correlogram is also used for model identification in Box & Jenkins models. On the y-axis they display the partial autocorrelations coefficients and on the x-axis the time lag  $k$ .

### 3.2-6 Augmented Dickey-Fuller Test

In case of time series analysis stationary of the data is very important otherwise classical linear regression model (CLRM) assumption will be not valid and estimates of parameters will not be Best Linear Unbiased (BLUE) estimators and results may be spurious. To check the stationary of time series data the Augmented Dickey-Fuller unit root test is best. Stationary can be confirmed through first order difference or higher orders difference of the series. Augmented Dickey-Fuller (ADF) has following structure.

$$EX_t = \alpha + \beta_t + \gamma EX_{t-1} + \sum_{k=1}^p \delta_k \Delta EX_{t-k} + e_t$$

Where EX (Exchange Rate) is the variable under consideration,  $\Delta$  is first difference operator,  $t$  captures any time trend,  $e_t$  is a random error  $p$  is the maximum lag length while  $\alpha, \beta, \gamma$  and  $\delta$  are the parameters be estimated.

### 3.2-7 ARMA (Autoregressive and Moving Average) Models

The ARMA model is the combination of an autoregressive (AR) and Moving Average (MA) Models. In an ARMA (p,q) process there are two integers (p,q) where p is autoregressive and q is moving average.

$$EX_t = \varphi_0 + \varepsilon_t + \sum_{i=1}^p \varphi_i EX_{t-i} + \sum_{i=1}^q \theta_i \varepsilon_{t-i}$$

Where

$\phi_1, \phi_2, \dots, \phi_p$  Are the coefficients of auto-regressive.

$\theta_1, \theta_2, \dots, \theta_p$  Are the coefficients of moving averages.

For the stationary process it must have constant Mean  $\mu$  for all time periods. It is assumed that the mean of  $y_t$  and  $\varepsilon_t$  are uncorrelated random variables with

$$E[\varepsilon_t] = 0 \text{ and } \text{var}[\varepsilon_t] = \sigma_t^2.$$

### 3.2-8 ARIMA (Autoregressive Integrated Moving Average) Models

The model is developed by Box & Jenkins (1976), we shall use this model for exchange rate volatility forecasting. Model is a combination of autoregressive (AR) and moving average (MA). An ARIMA (p, d, and q) is

$$\phi(L)(1-L)^d EX_t = \theta_q(L)\varepsilon_t$$

$$(1 - \phi_1 L - \phi_2 L^2 - \dots - \phi_p L^p) EX_t = (1 - \theta_1 L - \theta_2 L^2 - \dots - \theta_q L^q) \varepsilon_t$$

Where,  $EX_t$ , represent the exchange rate, L is the lag operator.

$\phi_1, \phi_2, \dots, \phi_p$  are autoregressive parameters

$\theta_1, \theta_2, \dots, \theta_p$ , are moving average parameters

$\varepsilon_t$  is error term,

p, d and q denote the autoregressive, differenced order and moving average parameters.

### 3.2-9 ARCH (Autoregressive Conditional Heteroscedasticity) Model

It is a nonlinear model which is specifically built for volatility capturing. It has wide application in finance. It is an extension of AR (Autoregressive) model and its various extension from ARMA(p,q) model, we assume that error terms is white noise, stochastic process i.e.  $\mu_t \sim N(0, \sigma^2)$ . However, it is unlikely in the context of same time series,

especially financial time series that the variance of the error will be constant over time i.e. Heteroscedasticity. Another important feature is that volatility clustering exist in financial time series which leads us to construct such a model that would be able to capture the phenomena of volatility clustering of the time series by allowing to vary the variance of residual in the model i.e.

If the model is AR (1)

$$EX_t = \varphi_0 + \varphi_1 EX_{t-1} + \epsilon_t$$

And ARCH (1) model is

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2$$

$\sigma_t^2$  Depends on lag square term  $\epsilon_{t-1}^2$ .

It is assume that in ARCH models, variance of the current error term is related to the size of the previous time periods, error term, often the variance is related to square of the previous innovations. Engle, (1982) introduced ARCH model, these models apply on modelling financial time series that exhibit time varying clustering.

Let  $\epsilon_t$  denotes error terms, and  $\epsilon_t$  are split into two pieces  $z_t$  stochastic and  $\sigma_t$  time dependent standard deviation, so that

$$\epsilon_t = \sigma_t z_t$$

Where  $z_t \sim iid N(0,1)$  and the series  $\sigma_t^2$  are modeled by

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2$$

Where  $\alpha_0 > 0$  and  $\alpha_i \geq 0, i > 0$

### 3.2-10 GARCH (Generalized Autoregressive Integrated Moving Average) Model

There is some limitations of ARCH (q) model which first the decision of the value of "q" is difficult task and second is the value of "q" to capture all of the dependence in the conditional variance might be large, resultantly, a large conditional variance of the

model. Due to the above limitations ARCH (q) model has been extended to GARCH model which overcomes some of the problems of ARCH (q) model. The GARCH models were developed by Engle, (1982) & Baillie, (1986), GARCH family model have proved successful in forecasting volatility in many situation. It allows the conditional variance to be dependent upon previous own lags along with lag residual terms i.e. The GARCH (p,q) model is formulated as

$$\begin{aligned}\sigma_t^2 &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \dots + \alpha_q \varepsilon_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \dots + \beta_p \sigma_{t-p}^2 \\ &= \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^p \beta_i \sigma_{t-i}^2\end{aligned}$$

Where q is the order of the ARCH terms, and p is the order of the GARCH terms.

$\sigma_t^2$  may be interpreted as a weighted function of a long term average value that gives us long term average value that gives us information about volatility during the previous period ( $\alpha_1 \varepsilon_{t-1}^2$ ) and the fitted variance from the model during the previous period ( $\beta_1 \sigma_{t-1}^2$ ).

### 3.2-11 IGARCH (Integrated Generalized Autoregressive Integrated Moving Average) Model

GARCH models apply both autoregressive and moving average structure to the variance,  $\sigma_t^2$ . The IGARCH is specified as

$$\varepsilon_t = \sigma_t z_t; \sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \ln \sigma_{t-j}^2$$

Integrated Generalized Autoregressive Conditional Heteroscedasticity (IGARCH) is a restricted version of the GARCH model, where the sum of persistence parameters sum up to 1, and therefore there is a unit root in the GARCH process. The condition for this is:

$$\sum_{i=1}^p \beta_i + \sum_{i=1}^q \alpha_i = 1$$

However if sum of  $\alpha$ 's and  $\beta$ 's is one or more, then there will be problem of unit root in the autoregressive representation for squared returns. To account for this problem integrated GARCH is used.

### 3.2-12 EGARCH (Exponential Generalized Autoregressive Integrated Moving Average) Model

The Exponential General Autoregressive Conditional Heteroscedasticity model is a popular GARCH model which is developed by Nelson (1991). EGARCH model is successfully capture the leverage effect and ensures that the variance is always positive even if the parameters may be negative.

The EGARCH is generally specified as

$$\varepsilon_t = \sigma_t z_t; \ln \sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \ln \sigma_{t-j}^2$$

The following model also used in the financial data, (Dhamija and Bhalla)

$$\ln \sigma_t^2 = \alpha + \beta_i \varepsilon_{t-1}^2 + \sum_{j=1}^p \gamma_j \ln(\sigma_{t-j}^2) + \sum_{i=1}^p \omega_i \left( \frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} \sqrt{\frac{2}{\pi}} \right)$$

### 3.3 Comparison Methods

For the purpose of comparison of forecasting performance the Mean Absolute Error, Mean Absolute Percentage and Root Mean Square Error and Theil's inequality coefficient U are commonly used. These measures use for evaluation of forecasting performance.

### 3.3-1 Mean Absolute Error

The Mean absolute error (MAE) is the common measure of forecast error in time series analysis. It is dependent on the scale of the dependent variable. It is less sensitive than the usual loss to large deviation.

$$MAE = \frac{1}{n} \sum_{t=1}^n \left| \hat{EX}_t - EX_t \right|$$

### 3.3-2 Mean Absolute Percentage Error

The Mean absolute percentage error (MAPE) is scale independent accuracy measure. It is also referred as Mean absolute percentage deviation (MAPD). There are some important problems MAPE is affected are: (1) Absolute percentage error is greater if equal errors above the actual value; (2) When the value of the original series is small then percentage errors will be large; (3) while in empirical studies outliers may distort the comparison

$$MAPE = \left[ \frac{1}{n} \sum_{t=1}^n \left| \frac{\hat{EX}_t - EX_t}{EX_t} \right| \right] \times 100$$

### 3.3-3 Root Mean Square Error

The Root mean square error (RMSE) is frequently used accuracy measure. It is depend on the scale of the dependent variable. It is relative measure to compare forecasts for the same series across different models. Problem with RMSE forecast error variance vary across time because of variation in exogenous variables and nonlinearities in the model.

$$RMSE = \sqrt{\frac{1}{n} \sum_{t=1}^n \left( \hat{EX}_t - EX_t \right)^2}$$



### 3.3-4 Theil's Inequality Coefficient

The Theil inequality coefficient is scale invariant and it always lies between 0 and 1, where 0 indicates perfect fit.

$$U = \frac{\sqrt{\frac{1}{n} \sum_{t=1}^n (\overline{EX}_t - EX_t)^2}}{\sqrt{\frac{1}{n} \sum_{t=1}^n \overline{EX}_t^2} + \sqrt{\frac{1}{n} \sum_{t=1}^n EX_t^2}}$$

### 3.4 Vector Autoregressive (VAR) MODEL:

The general VAR model for this study is specified as

VAR structural Model

$$BY_t = r_1 Y_{t-1} + \dots + r_p Y_{t-p} + \varepsilon_t$$

VAR Model reduced form is

$$Y_t = \gamma_1 Y_{t-1} + \dots + \gamma_p Y_{t-p} + e_t$$

Where  $Y_t = (6 \times 1)$  vector of system endogenous variables at time  $t$  such as Exchange Rate, Gross Domestic Production (GDP), Exports, Foreign Direct Investment (FDI), Inflation Rate and Foreign Remittances.

$Y_{t-1}$  are lagged values of the system endogenous variables,  $t = 1, 2, \dots, p$

$\gamma_i = (6 \times 6)$  matrices of determined variables coefficient to be estimated,  $i = 1, 2, \dots, p$

$e_t = (6 \times 1)$  vector of innovations at time  $t$ .

### 3.4-1 Impulse Response Function (IRF)

**Objective:** the reaction of the system to a shock

$$Y_t = \gamma_1 Y_{t-1} + \dots + \gamma_p Y_{t-p} + e_t$$

The term  $e_t$  represents innovations in the system.

If the system is stable,

$$Y_t = \mu + \phi(L)e_t = \mu + e_t + \phi_1 e_{t-1} + \phi_2 e_{t-2} + \dots$$

$$\phi(L) = [\gamma(L)]^{-1}$$

Redating at time  $t+s$

$$Y_{t+s} = \mu + e_{t+s} + \phi_1 e_{t+s-1} + \phi_2 e_{t+s-2} + \dots + \phi_s e_t + \phi_{s+1} e_{t-1} + \dots$$

$$\frac{\partial Y_{t+s}}{\partial e_t} = \phi_s = [\phi_{ij}^{(s)}] \text{ S is a multiplier}$$

$$\frac{\partial Y_{i,t+s}}{\partial e_{jt}} = [\phi_{ij}^{(s)}] \text{ Reaction of the i variables to a unit change in innovation j}$$

The interpretation of the  $\phi$  matrices is that they symbolizes marginal effects, or the model's reaction to a unit innovation at time S in every of the variable

### 3.4-2 Impulse Response Function

Response of  $Y_{i,t+s}$  to one-time impulse in  $Y_{jt}$  with all other variables time  $t$  or earlier is constant. Normally an impulse response function traces the influence of a one-time shock to one of the innovations on present and future values of the endogenous variables

$$\frac{\partial Y_{i,t+s}}{\partial e_{jt}} = \phi_{ij}^s$$

## Chapter 4

### RESULTS AND DISCUSSIONS

#### 4.1 The Augmented Dickey–Fuller (ADF) test

The Augmented Dickey–Fuller (ADF) test is presented in table 4.1. The result shows that all the variables are not stationary at level and stationary at first difference. It is essential to check the stationarity of the data before using the time series models like Autoregressive Moving Average (ARMA), Autoregressive Integrated Moving Average (ARIMA), Autoregressive Conditional Heteroscedasticity (ARCH), Generalized Autoregressive Conditional Heteroscedasticity (GARCH), Integrated Generalized Autoregressive Conditional Heteroscedasticity (IGARCH) and Exponential General Autoregressive Conditional Heteroskedastic (EGARCH), which are being used in this study.

Table: 4.1 Unit Root test

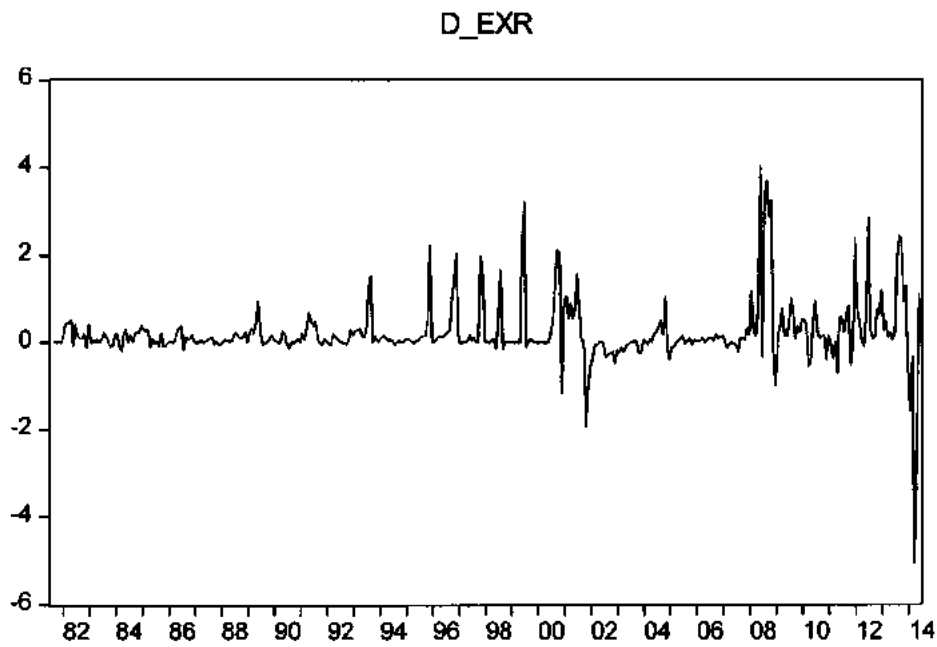
| Variable              | Level    |                    | First Difference |                    |
|-----------------------|----------|--------------------|------------------|--------------------|
|                       | Constant | Constant and Trend | Constant         | Constant and Trend |
| GDP                   | -0.57    | -1.91              | -5.10**          | -5.03**            |
| Export                | -1.85    | -1.52              | -6.28***         | -6.58**            |
| Yearly Exchange rate  | -0.15    | -1.77              | -4.45***         | -3.87**            |
| FDI                   | -2.30    | 2.56               | -4.90**          | -4.92***           |
| Inflation rate        | -2.59    | -2.55              | -6.59**          | -6.50***           |
| Remittances           | -0.93    | -1.33              | -5.09**          | -5.07**            |
| Monthly Exchange Rate | 0.7954   | -1.9566            | -12.4775***      | -12.5518***        |

\*\*\* Significant at 1% level

\*\* Significant at 5% level

The above table 4.1 shows the stationary level of exchange rate. Results indicate that exchange rate is integrated of level one. It is stationary at first difference. The correlogram and graphical representation of Exchange Rate shows in Figure 4.1 and Table 4.2 no trend in data hence, suggesting that the exchange rate data is stationary. Figure: 4.1. Representation of Exchange Rate stationary at first difference

Graph: 4.1 Graphical representation of Exchange Rate at first difference



**Table 4.2. Auto Correlation and Partial Auto Correlation of Exchange Rate**

| No | AC     | PAC    | Q-Stat | Prob  |
|----|--------|--------|--------|-------|
| 1  | 0.431  | 0.431  | 73.943 | 0.000 |
| 2  | 0.204  | 0.022  | 90.564 | 0.000 |
| 3  | 0.163  | 0.083  | 101.19 | 0.000 |
| 4  | 0.010  | -0.114 | 101.23 | 0.000 |
| 5  | 0.018  | 0.049  | 101.37 | 0.000 |
| 6  | -0.006 | -0.036 | 101.38 | 0.000 |
| 7  | -0.101 | -0.094 | 105.49 | 0.000 |
| 8  | -0.039 | 0.041  | 106.12 | 0.000 |
| 9  | 0.044  | 0.084  | 106.92 | 0.000 |
| 10 | 0.053  | 0.031  | 108.07 | 0.000 |

After taking the first difference of the exchange rate data and transformed sample Autocorrelation function (ACF) and Partial Autocorrelation Function (PACF) shows that the exchange rate series is now become stationary.

#### **4.2 Model Fitting**

Model fitting is well-known as model identification, generally starts with autocorrelation analysis. We can draw the conclusion about a suitable model through the time series plot and correlogram. Now for checking the autoregressive pattern the below table 4.4 shows the Autoregressive process of difference Exchange rate returns.

Table 4.3 Autoregressive model Variable Difference of Exchange Rate

| Model  | Coefficients |        |        |        |        |        |        |        |        |         |
|--------|--------------|--------|--------|--------|--------|--------|--------|--------|--------|---------|
|        | ar (1)       | ar (2) | ar (3) | ar (4) | ar (5) | ar (6) | ar (7) | ar (8) | ar (9) | ar (10) |
| AR(1)  | .43***       | -      | -      | -      | -      | -      | -      | -      | -      | -       |
| AR(2)  | .42***       | .02    | -      | -      | -      | -      | -      | -      | -      | -       |
| AR(3)  | .42*         | -.02   | .09*   | -      | -      | -      | -      | -      | -      | -       |
| AR(4)  | .43***       | -.01   | .14**  | -.13** | -      | -      | -      | -      | -      | -       |
| AR(5)  | .43***       | -.02   | .13**  | -.15** | .05    | -      | -      | -      | -      | -       |
| AR(6)  | .44***       | -.02   | .14**  | .15**  | .07    | -.04   | -      | -      | -      | -       |
| AR(7)  | .43***       | -.02   | .12**  | -.14** | .07    | .02    | -.13** | -      | -      | -       |
| AR(8)  | .44***       | -.02   | .12**  | -.13** | .06    | .02    | -.15** | .06    | -      | -       |
| AR(9)  | .43***       | -.00   | .12**  | -.14** | .07    | .01    | -.15** | .01    | .11*   | -       |
| AR(10) | .43***       | -.00   | .13**  | -.14** | .07    | .01    | -.15** | .01    | .09    | .04     |

\*\*\* Significant at 1% level of Significance

\*\* Significant at 5% level of Significance

Above table 4.3 shows models of Autoregressive process in rows with capital letter while their coefficients are shown in columns with small alphabets. It is clear that the coefficients are significant up to AR (9). We can conclude that Exchange Rate is an AR (9) Process. Smallest Akaike info criterion (AIC) value 1.9697 shows AR (1) is better fitted model amongst Autoregressive AR models.

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**Table: 4.4. Moving Average model Variable Difference of Exchange Rate**

| Model | Coefficients |            |            |           |           |           |           |           |           |            |
|-------|--------------|------------|------------|-----------|-----------|-----------|-----------|-----------|-----------|------------|
|       | ma<br>(1)    | ma<br>(2)  | ma<br>(3)  | ma<br>(4) | ma<br>(5) | ma<br>(6) | ma<br>(7) | ma<br>(8) | ma<br>(9) | ma<br>(10) |
| MA(1) | .41<br>***   | -          | -          | -         | -         | -         | -         | -         | -         | -          |
| MA(2) | .42<br>***   | .08        | -          | -         | -         | -         | -         | -         | -         | -          |
| MA(3) | .24<br>***   | .17**<br>* | .18**<br>* | -         | -         | -         | -         | -         | -         | -          |
| MA(4) | .43<br>***   | .17**<br>* | .17**<br>* | -.01      | -         | -         | -         | -         | -         | -          |
| MA(5) | .43<br>***   | .17**<br>* | .18**<br>* | -.002     | .02       | -         | -         | -         | -         | -          |
| MA(6) | .44<br>***   | .19**<br>* | .20**<br>* | .02       | .06       | .09       | -         | -         | -         | -          |

\*\*\* Significant at 1% level of Significance

\*\* Significant at 5% level of Significance

Above table 4.4 shows models of Moving Average (MA) in rows with capital letter while their coefficients are shown in columns with small alphabets. It is clear that the coefficients are significant up to MA (3). We can conclude that Moving Average is an MA (3) Process. Smallest Akaike info criterion (AIC) value 1.9583 shows MA (3) is better fitted model amongst Autoregressive MA models.

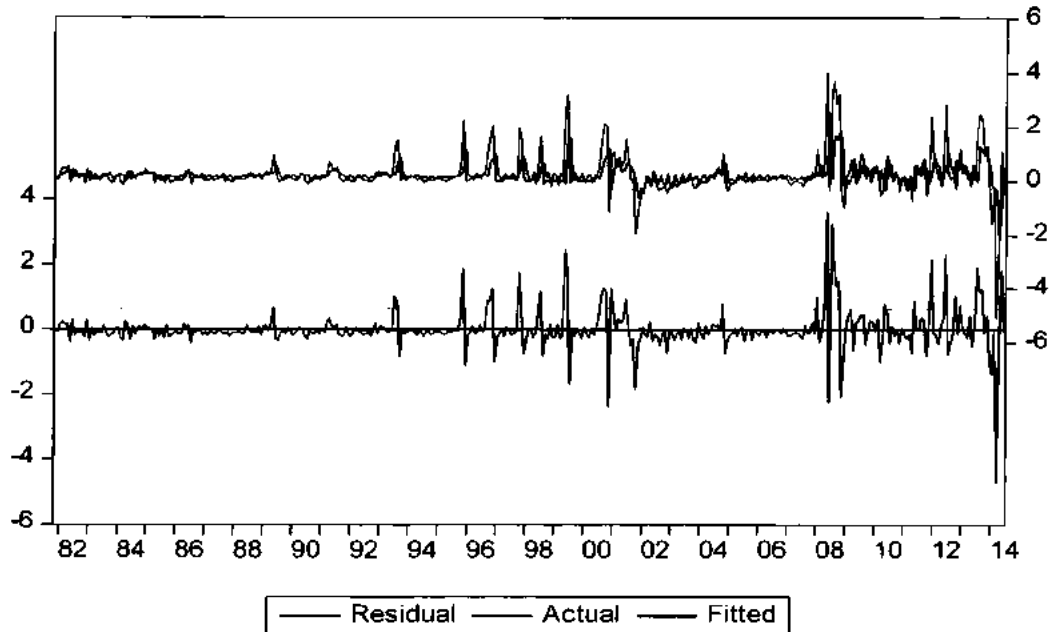
**Table: 4.5. Autoregressive Integrated Moving Average (ARIMA)**

| <b>ARIMA</b>   | <b>SIC</b>    | <b>AIC</b>    | <b>LM</b>                   |
|--|---------------|---------------|-----------------------------|
| 1 1 1  | 2.004         | 1.9740        | Reject H <sub>0</sub>       |
| 1 1 2  | 2.0160        | 1.9756        | Accept H <sub>0</sub>       |
| 1 1 3  | 2.0163        | 1.9658        | Reject H <sub>0</sub>       |
| 2 1 1  | 2.0167        | 1.9762        | Accept H <sub>0</sub>       |
| 2 1 2  | 2.0239        | 1.9733        | Accept H <sub>0</sub>       |
| 2 1 3  | 2.0142        | 1.9536        | Accept H <sub>0</sub>       |
| 3 1 1  | 2.0226        | 1.9720        | Accept H <sub>0</sub>       |
| <b>3 1 2</b>   | <b>2.0124</b> | <b>1.9516</b> | <b>Accept H<sub>0</sub></b> |
| 3 1 3  | 2.0264        | 1.9554        | Accept H <sub>0</sub>       |
| 4 1 1  | 2.0350        | 1.9741        | Reject H <sub>0</sub>       |
| 4 1 2  | 2.0292        | 1.9582        | Accept H <sub>0</sub>       |
| 4 1 3  | 2.0376        | 1.9564        | Accept H <sub>0</sub>       |
| 5 1 1  | 2.0524        | 1.9812        | Accept H <sub>0</sub>       |
| 5 1 2  | 2.0441        | 1.9627        | Accept H <sub>0</sub>       |
| 5 1 3  | 2.0160        | 1.9244        | Reject H <sub>0</sub>       |
| 6 1 1  | 2.0670        | 1.9855        | Reject H <sub>0</sub>       |
| 6 1 2  | 2.0349        | 1.9432        | Accept H <sub>0</sub>       |
| 6 1 3  | 2.0635        | 1.9616        | Accept H <sub>0</sub>       |
| 7 1 1  | 2.0736        | 1.9817        | Accept H <sub>0</sub>       |
| 7 1 2  | 2.0852        | 1.9831        | Accept H <sub>0</sub>       |
| 7 1 3  | 2.0624        | 1.9501        | Accept H <sub>0</sub>       |
| 8 1 1  | 2.0821        | 1.9798        | Accept H <sub>0</sub>       |
| 8 1 2  | 2.0913        | 1.9788        | Accept H <sub>0</sub>       |
| 8 1 3  | 2.0962        | 1.9734        | Accept H <sub>0</sub>       |
| Accept H <sub>0</sub> means there is no serial correlation |               |               |                             |

Table 4.5 shows the different combination of ARIMA models for checking the serial correlation we performed LM test. For selection of best model amongst ARIMA models we used AIC and SIC criteria and we conclude that the ARIMA (3,1,2) is best fitted model amongst ARIMA models.



**Graph: 4.2 Actual, Fitted, Residual Graph**



Residual of low volatility tends to be followed by periods of low volatility for a prolonged periods and periods of high volatility is followed by periods of high volatility for a prolonged periods when this thing is happen for residual then we have the justification to run ARCH family of model

**Table: 4.6 ARIMA (3,1,2) model, Monthly Exchange Rate**

| Variable | Coefficient | Standard Error | t-statistic | p-value |
|----------|-------------|----------------|-------------|---------|
| Constant | 0.227279    | 0.060337       | 3.766837**  | 0.0002  |
| AR(1)    | -0.639882   | 0.051818       | -12.34872** | 0.0000  |
| AR(2)    | -0.454253   | 0.057560       | -7.891750** | 0.0000  |
| AR(3)    | 0.482224    | 0.046453       | 10.38089**  | 0.0000  |
| MA(1)    | 1.077240    | 0.033861       | 31.81365**  | 0.0000  |
| MA(2)    | 0.944986    | 0.033872       | 27.89856**  | 0.0000  |

Note: \*\* show significant at 1%

$$EX_t = 0.2272 + 0.63988 EX_{t-1} + 0.4525 EX_{t-2} + 0.4822 EX_{t-3} + 1.0772 \epsilon_{t-1} + 0.9449 \epsilon_{t-2}$$

Table 4.6 shows the variable of ARIMA (3, 1, 2) model on the basis of correspondence p-values we can conclude that all the variable are highly significant.

**Table: 4.7 The LM test on ARIMA(3,1,2)**

| Adj R-squared | DW-Stat  | F-stat   | LM-test F-stat | LM-test P-value |
|---------------|----------|----------|----------------|-----------------|
| 0.210254      | 1.985371 | 21.81916 | 0.6884         | 0.6833          |

ARCH effect even at 1% level of significance.

#### 4.5 ARCH Test on ARIMA (3,1,2)

The Autoregressive Conditional Heteroscedasticity (ARCH) family of models necessitates the existence of 'ARCH effect' in the residuals. For testing the presence of ARCH effect, use the Lagrange Multiplier (LM) test for differential exchange rate returns series. The results of LM test are presented in Table 4.7. The p-value shows that there is indication of ARCH effect. Hence, we reject the null hypothesis of nonappearance of ARCH effect.

**Table: 4.8 ARCH-LM test on ARIMA (3,1,2)**

| Lag(P) | Obs-R <sup>2</sup> | Df | Prob.> Chi <sup>2</sup> |
|--------|--------------------|----|-------------------------|
| 1      | 17.58337           | 1  | 0.0000                  |

H<sub>0</sub>: There is no ARCH effect and alternate is H<sub>1</sub>: there is ARCH (p) disturbance

The above Table 4.8 shows that the ARIMA (3,1,2) model has ARCH effect, hence we have the evidence to perform the ARCH family of model for capturing the volatility of exchange rate returns.

**Table: 4.9 ARIMA (1,1,2) Model, Yearly Exchange Rate**

| Variable | Coefficient | Standard Error | t-statistic | p-value |
|----------|-------------|----------------|-------------|---------|
| C        | 0.614708    | 0.677443       | 0.907395    | 0.0319  |
| AR(1)    | 0.755466    | 0.153229       | 4.930294    | 0.0000  |
| MA(1)    | -0.790940   | 0.262502       | -3.013078   | 0.0054  |
| MA(2)    | -0.933684   | 0.264170       | -3.534402   | 0.0014  |

Note: \*\* show significant at 1%

$$EX_t = 0.614708 + 0.755466EX_{t-1} - 0.790940 \epsilon_{t-1} - 0.933684\epsilon_{t-2}$$

Table 4.9 shows the variable of ARIMA (1, 1, 2) model on the basis of correspondence p-values we can conclude that all the variable are highly significant.

**Table: 4.10 The LM test on ARIMA(1,1,2)**

| Adj R-squared | DW-Stat  | F-stat  | LM-test E-stat | LM-test P-value |
|---------------|----------|---------|----------------|-----------------|
| 0.189405      | 1.969754 | 5.14016 | 0.117334       | 0.8897          |

ARCH effect even at 1% level of significance.

#### 4.5-1 ARCH Test on ARIMA (1,1,2)

The Autoregressive Conditional Heteroscedasticity (ARCH) family of models necessitates the existence of 'ARCH effect' in the residuals. For testing the presence of ARCH effect, use the Lagrange Multiplier (LM) test for differential exchange rate returns series. The results of LM test are presented in Table 4.10. The  $p$ -value shows that there is indication of ARCH effect. Hence, we reject the null hypothesis of nonappearance of ARCH effect.

**Table: 4.11 ARCH-LM test on ARIMA (1,1,2)**

| Lag(p) | Obs-R <sup>2</sup> | DF | Prob > Chi <sup>2</sup> |
|--------|--------------------|----|-------------------------|
| 1      | 0.254045           | 1  | 0.0000                  |

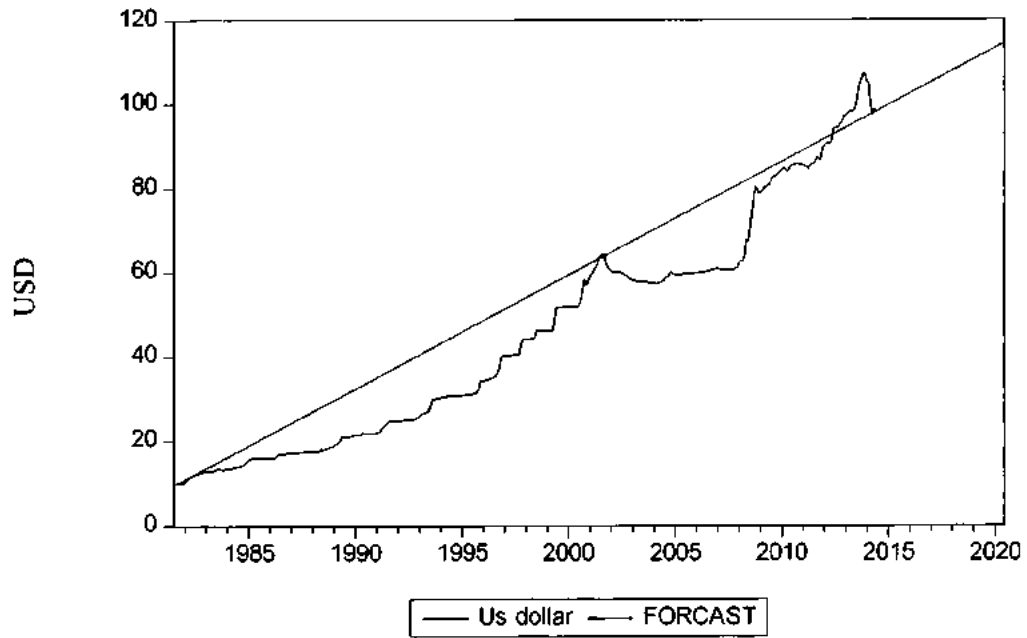
H<sub>0</sub>: There is no ARCH effect and alternate is H<sub>1</sub>: there is ARCH (p) disturbance

The above Table 4.11 shows that the ARIMA (1,1,2) model has ARCH effect, hence we have the evidence to perform the ARCH family of model for capturing the volatility of exchange rate returns.

#### 4.6 Forecasting Analysis

Forecasting performance of the fitted ARIMA(3,1, 2) model of exchange rate returns is examined through Mean Absolute Error (MAE), Root Mean Square Error (RMSE), Mean Absolute Percent Error (MAPE), and Theil inequality coefficient. The results are shown in Table. 4.12

Graph: 4.3 Forecast comparisons of Monthly Exchange Rate with actual Monthly Exchange rate data.

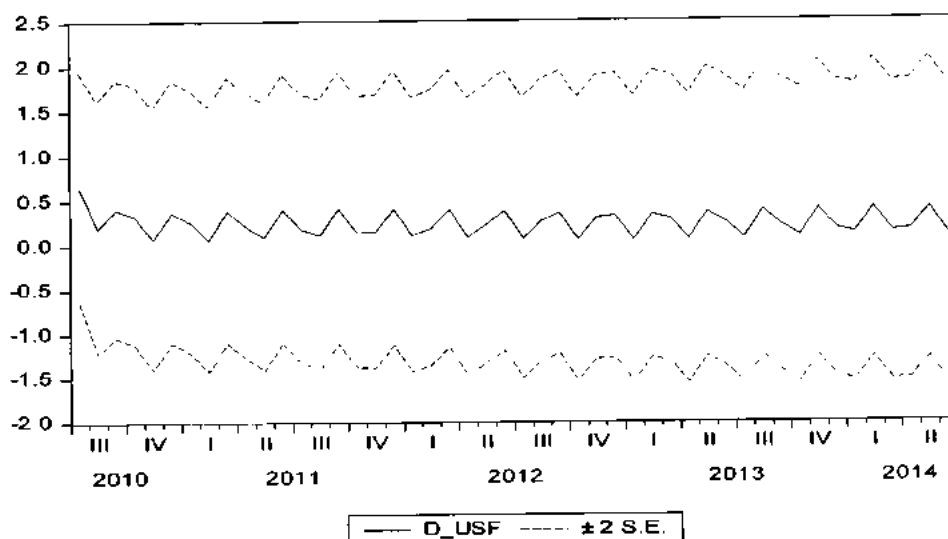


**Table 4.12** Forecasting Performance of Monthly and yearly Exchange Rate.

|                              | ARIMA(3,1,2) MONTHLY EXCHANGE RATE DATA | ARIMA (1,1,2) YEARLY EXCHANGE RATE DATA |
|------------------------------|---|---|
| RMSE                         | 1.190930                                | 1.632320                                |
| MAE                          | 0.745808                                | 0.454367                                |
| MAPE                         | 172.6314                                | 182.8976                                |
| Theil Inequality Coefficient | 0.0780373                               | 0.076897                                |
| Bias Proportion              | 0.000856                                | 0.000745                                |
| Variance Proportion          | 0.082410                                | 0.467589                                |
| Covariance Proportion        | 0.775037                                | 0.237845                                |

The value of Theil inequality is 0.076897 indicating the model is a better fit. The bias proportion and variance proportion are close to zero. We can say that this model is good for forecasting purpose along with capturing the volatility

**Graph: 4.4.** Forecasting Graph of Monthly Exchange rate from 2010 to 2014



For checking forecasting performance of ARIMA (3,1,2) model. We forecast for the period of July, 2010 to June, 2014 the forecasting of Exchange Rate returns is between plus and minus 2 Standard deviation.

This research motivations on building a model for the exchange rate of Pakistan using time series methodology, Autoregressive (AR), Moving Average (MA) and Autoregressive Integrated Moving Average (ARIMA) for the selection of best fitted model we have choose the Akaike info criterion (AIC). The small value of AIC shows the better estimation. We have selected three best fitted models amongst AR, MA and ARIMA. AR (1) is 1.9697, MA (3) is 1.9583 and ARIMA (3,1,2) is 1.9516. Hence, here the best fitted model is ARIMA (3,1,2). The performance of forecast is measured by RMSE, MAE, MAPE and Thiel Inequality.

#### 4.7 Autoregressive Conditional Heteroscedasticity Model

Table 4.13 ARCH model on ARIMA (3,1,2)

| Model | AIC      | SIC      | Serial correlation test |
|-------|----------|----------|-------------------------|
| 1     | 1.201014 | 1.282960 | Accept H <sub>0</sub>   |
| 2     | 0.942117 | 1.033294 | Reject H <sub>0</sub>   |
| 3     | 1.267981 | 1.369288 | Reject H <sub>0</sub>   |
| 4     | 1.099763 | 1.211201 | Reject H <sub>0</sub>   |
| 5     | 1.268648 | 1.390217 | Reject H <sub>0</sub>   |
| 6     | 1.245182 | 1.376882 | Reject H <sub>0</sub>   |
| 7     | 1.321264 | 1.463095 | Reject H <sub>0</sub>   |
| 8     | 1.296858 | 1.448820 | Reject H <sub>0</sub>   |
| 9     | 1.325635 | 1.487728 | Reject H <sub>0</sub>   |

Accept H<sub>0</sub> means there is no serial correlation

Table 4.13 shows the different ARCH models for checking the serial correlation we performed LM test. For selection of best model we used AIC and SIC criterions.

#### 4.8 Generalized Autoregressive Conditional Heteroscedasticity Model

Table 4.14 GARCH model on ARIMA (3,1,2)

| Model | AIC      | SIC      | Serial Correlation Test |
|-------|----------|----------|-------------------------|
| 1,1   | 1.272734 | 1.363911 | Accept H0               |
| 1,2   | 1.116379 | 1.217686 | Accept H0               |
| 1,3   | 1.041951 | 1.153390 | Accept H0               |
| 1,4   | 1.254303 | 1.375872 | Accept H0               |
| 1,5   | 1.346488 | 1.478188 | Reject H0               |
| 1,6   | 1.107776 | 1.249606 | Reject H0               |
| 1,7   | 1.047357 | 1.199318 | Accept H0               |
| 1,8   | 1.034944 | 1.197036 | Accept H0               |
| 1,9   | 1.276233 | 1.448456 | Accept H0               |
| 2,1   | 0.961197 | 1.062504 | Accept H0               |
| 2,2   | 0.875452 | 0.986891 | Accept H0               |
| 2,3   | 0.999345 | 1.120914 | Accept H0               |
| 2,4   | 1.052511 | 1.184211 | Reject H0               |
| 2,5   | 1.037497 | 1.179328 | Reject H0               |
| 2,6   | 1.034056 | 1.186018 | Reject H0               |
| 2,7   | 0.927956 | 1.090048 | Reject H0               |
| 2,8   | 0.880974 | 1.053198 | Reject H0               |
| 2,9   | 0.946956 | 1.129310 | Reject H0               |
| 3,1   | 0.980607 | 1.092046 | Reject H0               |
| 3,2   | 0.998848 | 1.120417 | Accept H0               |
| 3,3   | 0.954946 | 1.086646 | Reject H0               |
| 3,4   | 0.996172 | 1.138002 | Reject H0               |
| 3,5   | 1.027689 | 1.179650 | Accept H0               |
| 3,6   | 1.054789 | 1.216882 | Accept H0               |
| 3,7   | 0.970194 | 1.142417 | Reject H0               |
| 3,8   | 0.988467 | 1.170821 | Reject H0               |
| 3,9   | 0.918412 | 1.110897 | Reject H0               |
| 4,1   | 1.270797 | 1.392366 | Reject H0               |
| 4,2   | 1.290103 | 1.421803 | Reject H0               |
| 4,3   | 1.292217 | 1.434047 | Reject H0               |

Accept  $H_0$  means there is no serial correlation

Table 4.14 shows the different combination of GARCH models for checking the serial correlation we performed LM test. For selection of best model we used AIC and SIC criterions.



#### 4.9 Integrated Generalized Autoregressive Conditional Heteroscedasticity Model

**Table 4.15** IGARCH model on ARIMA (3,1,2)

| Model | AIC      | SIC      | Serial Correlation Test |
|-------|----------|----------|-------------------------|
| 1,1   | 1.409964 | 1.480879 | Reject H0               |
| 1,2   | 1.400527 | 1.481573 | Reject H0               |
| 1,3   | 1.343087 | 1.434264 | Reject H0               |
| 1,6   | 1.204152 | 1.325722 | Reject H0               |
| 1,7   | 1.328376 | 1.460076 | Reject H0               |
| 1,8   | 1.272177 | 1.414008 | Accept H0               |
| 1,9   | 1.145223 | 1.297184 | Reject H0               |
| 2,1   | 1.049914 | 1.30260  | Accept H0               |
| 2,2   | 1.558736 | 1.649913 | Reject H0               |
| 2,3   | 1.147756 | 1.249064 | Reject H0               |
| 2,4   | 1.362550 | 1.473989 | Reject H0               |
| 2,5   | 1.231142 | 1.352711 | Reject H0               |
| 2,6   | 1.177189 | 1.308889 | Reject H0               |
| 2,7   | 1.257310 | 1.399140 | Reject H0               |
| 2,8   | 1.467310 | 1.297310 | Reject H0               |
| 2,9   | 1.662550 | 1.862560 | Reject H0               |
| 3,1   | 1.084078 | 1.175254 | Accept H0               |
| 3,2   | 1.085565 | 1.186872 | Accept H0               |
| 3,3   | 1.078014 | 1.189453 | Accept H0               |
| 3,4   | 1.214523 | 1.336092 | Accept H0               |
| 3,5   | 1.215488 | 1.347188 | Reject H0               |
| 3,6   | 1.130056 | 1.271887 | Reject H0               |
| 3,7   | 1.274305 | 1.426267 | Reject H0               |
| 3,8   | 1.330552 | 1.492644 | Reject H0               |
| 3,9   | 1.178999 | 1.351222 | Reject H0               |
| 4,1   | 1.087995 | 1.189303 | Accept H0               |
| 4,2   | 1.110229 | 1.221667 | Accept H0               |
| 4,3   | 1.126037 | 1.247606 | Accept H0               |
| 4,4   | 1.144590 | 1.276290 | Accept H0               |
| 4,5   | 1.377709 | 1.519540 | Reject H0               |
| 4,6   | 1.171164 | 1.323126 | Reject H0               |
| 4,7   | 1.242775 | 1.404867 | Reject H0               |

Accept H<sub>0</sub> means there is no serial correlation

Table 4.15 shows the different combination of IGARCH models for checking the serial correlation we performed LM test. For selection of best model we used AIC and SIC criterions.

#### 4.10 Exponential Generalized Autoregressive Conditional Heteroscedasticity Model

Table 4.16 EGARCH model on ARIMA (3,1,2)

| Model | AIC      | SIC      | Serial Correlation Test |
|-------|----------|----------|-------------------------|
| 1,1   | 0.758059 | 0.862612 | Accept H0               |
| 1,2   | 0.967384 | 1.078822 | Reject H0               |
| 1,3   | 0.947586 | 1.069156 | Reject H0               |
| 1,4   | 1.014194 | 1.145894 | Reject H0               |
| 1,5   | 0.862117 | 1.003948 | Reject H0               |
| 1,6   | 0.954958 | 1.106920 | Reject H0               |
| 1,7   | 0.962414 | 1.113994 | Reject H0               |
| 1,8   | 0.896211 | 1.343943 | Reject H0               |
| 1,9   | 0.762227 | 1.983998 | Reject H0               |
| 2,1   | 1.252803 | 1.364242 | Reject H0               |
| 2,2   | 0.774738 | 0.896307 | Accept H0               |
| 2,3   | 0.922264 | 1.053964 | Reject H0               |
| 2,4   | 0.906392 | 1.048223 | Reject H0               |
| 2,5   | 0.893471 | 1.045432 | Reject H0               |
| 2,6   | 0.967922 | 1.004829 | Reject H0               |
| 2,7   | 0.926294 | 1.068541 | Reject H0               |
| 2,8   | 0.816639 | 0.948445 | Reject H0               |
| 2,9   | 0.866981 | 0.108431 | Reject H0               |
| 3,1   | 0.804055 | 0.925624 | Accept H0               |
| 3,2   | 0.806731 | 0.938431 | Reject H0               |
| 3,3   | 0.820609 | 0.962440 | Reject H0               |

Accept  $H_0$  means there is no serial correlation

Table 4.16 shows the different combination of EGARCH models for checking the serial correlation we performed LM test. For selection of best model we used AIC and SIC criterions. We have selected EGARCH (1,1) model for capturing the volatility of Exchange Rate data monthly as well as yearly.

#### 4.11 Best Fitted Model

**Table: 4.17** ARIMA (3,1,2) EGARCH(1,1),Mean Equation, Monthly Exchange Rate

| Variable | Coefficient | Std. Error | z-Statistic | Prob.  |
|----------|-------------|------------|-------------|--------|
| C        | 0.069188    | 0.031986   | 2.163087    | 0.0305 |
| AR(1)    | 0.552573    | 0.060463   | 9.139087    | 0.0000 |
| AR(2)    | 0.900300    | 0.009851   | 91.38798    | 0.0000 |
| AR(3)    | -0.472570   | 0.057128   | -8.272202   | 0.0000 |
| MA(1)    | 0.002813    | 0.003173   | 0.886554    | 0.3753 |
| MA(2)    | -0.976045   | 0.002756   | -354.1375   | 0.0000 |

**Table: 4.18** ARIMA (3,1,2) EGARCH(1,1),Variance Equation, Monthly Exchange Rate

| Variable | Coefficient | Std. Error | z-Statistic | Prob.  |
|----------|-------------|------------|-------------|--------|
| C(7)     | -0.461995   | 0.034727   | -13.30381   | 0.0000 |
| C(8)     | 0.535711    | 0.053967   | 9.926641    | 0.0000 |
| C(9)     | 0.302832    | 0.044388   | 6.822409    | 0.0000 |
| C(10)    | 0.953808    | 0.009531   | 100.0706    | 0.0000 |

$$LN(GARCH) = -0.4619 + 0.5357ABS(RESID(-1)/@SQRT(GARCH(-1))) + 0.3028$$

$$RESID(-1)/@SQRT(GARCH(-1)) + 0.9538LN(GARCH(-1))$$

EGARCH parameters, displayed in Table 4.17 and Table 4.18 shows mean equation variables of EGARCH (1,1) model, the calculated coefficients and the p' values of the EGARCH model on monthly exchange rate returns also presented. The results shows that, there is a first order autoregressive behavior in the exchange rate as in the mean equation, as all the Autoregressive and Moving averages lag are significant at 1% in monthly exchange rate returns. The constant C is also significant at 5% in monthly exchange rate returns. In the variance equation, all the terms are significance at 1% level for the monthly exchange rate returns. The EGARCH model proves to be the best model to explain the behavior of exchange rate monthly data as the most of the coefficients of mean and variance equation are significant.

**Table: 4.19** ARIMA (1,1,2) EGARCH(1,1)Mean Equation yearly Exchange Rate

| Variable | Coefficient | Std. Error | z-Statistic | Prob.  |
|----------|-------------|------------|-------------|--------|
| C        | 3.525186    | 1.587157   | 2.221069    | 0.0263 |
| AR(1)    | 0.911973    | 0.104118   | 8.759007    | 0.0000 |
| MA(1)    | -0.610614   | 0.186334   | -3.276985   | 0.0010 |
| MA(2)    | -0.315140   | 0.198853   | -2.584793   | 0.0130 |

**Table: 4.20** ARIMA (1,1,2) EGARCH(1,1)Variance Equation yearly Exchange Rate

| Variable | Coefficient | Std. Error | z-Statistic | Prob.  |
|----------|-------------|------------|-------------|--------|
| C(5)     | 0.331116    | 0.517818   | 2.639445    | 0.0425 |
| C(6)     | -0.272655   | 0.769568   | -1.354297   | 0.0231 |
| C(7)     | 0.106108    | 0.206847   | 3.512978    | 0.0080 |
| C(8)     | 1.022685    | 0.154416   | 6.622931    | 0.0000 |

$$LN(GARCH) = 0.3311 - 0.2726ABS(RESID(-1)/@SQRT(GARCH(-1))) + 0.1061$$

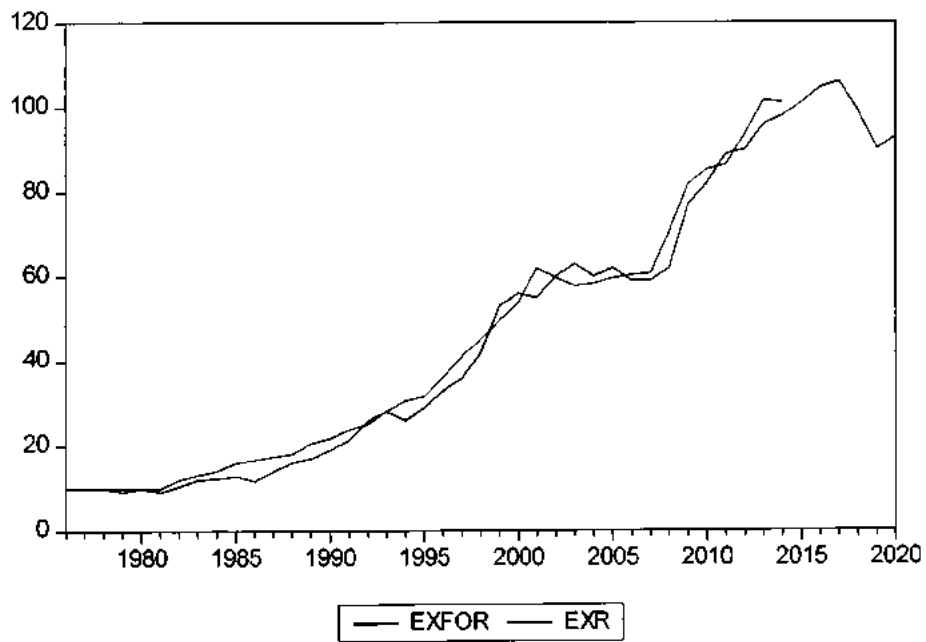
$$RESID(-1)/@SQRT(GARCH(-1)) + 1.0226LN(GARCH(-1))$$

EGARCH parameters, displayed in Table 4.19 and Table 4.20 shows mean equation variables of EGARCH (1,1) model, the calculated coefficients and the p' values of the EGARCH model on yearly exchange rate returns also presented. The results shows that, there is a first order autoregressive behavior in the exchange rate as in the mean equation, as all the Autoregressive and Moving averages lag are significant at 1% in yearly exchange rate returns. The constant C is also significant at 5% in yearly exchange rate returns. In the variance equation, C(7) and C(8) terms are significance at 1% level and C(5) and C(6) are significant at 5% level for yearly exchange rate returns. The EGARCH model proves to be the best model to explain the behavior of exchange rate yearly data as the most of the coefficients of mean and variance equation are significant.

#### 4.12 Forecasting Analysis

Forecasting performance of the fitted EGARCH (1,1) model of exchange rate returns is examined through Mean Absolute Error (MAE), Root Mean Square Error (RMSE), Mean Absolute Percent Error (MAPE), and Theil inequality coefficient. The results are shown in Table 4.14.

Graph: 4.5 Yearly Exchange Rate



Graph: 4.6 Monthly Exchange Rate

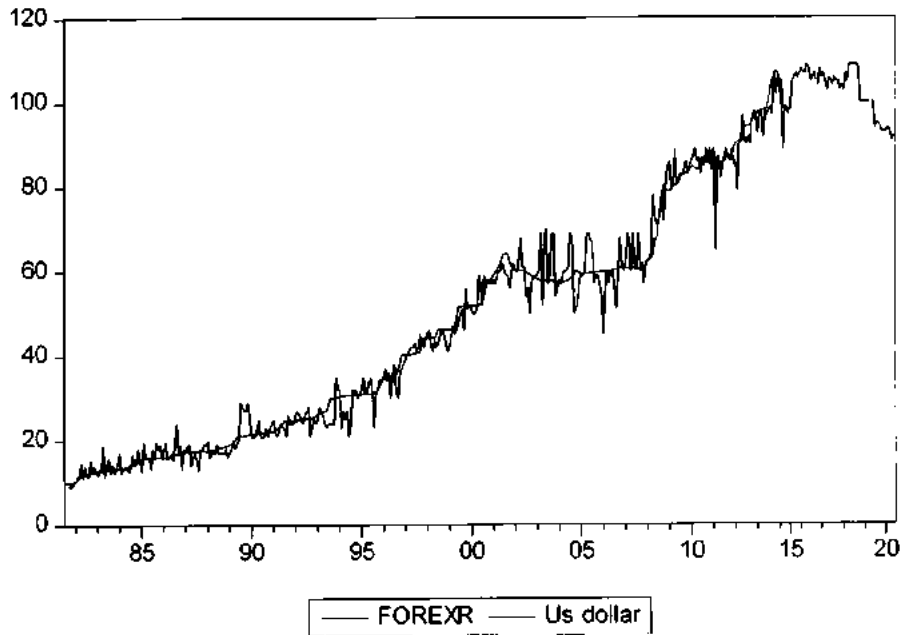


Table: 4.21 Forecast Performance

|                              | EGARCH(1,1) MONTHLY EXCHANGE RATE DATA | EGARCH (1,1) YEARLY EXCHANGE RATE DATA |
|------------------------------|--|--|
| RMSE                         | 0.818931                               | 0.732360                               |
| MAE                          | 0.765453                               | 0.352010                               |
| MAPE                         | 198.6314                               | 174.7645                               |
| Theil Inequality Coefficient |  |  |
| Bias Proportion              | 0.000467                               | 0.003324                               |
| Variance Proportion          | 0.072470                               | 0.851989                               |
| Covariance Proportion        | 0.805037                               | 0.884714                               |

The value of Theil inequality is 0.060137 indicating the model is a better fit. The bias proportion and variance proportion are close to zero. We can say that this model is good for forecasting purpose along with capturing the volatility

#### 4.13 VECTOR AUTOREGRASSIVE MODEL RESULTS

All the variables are stationary at first difference Augmented Dickey Fuller test has been used for the unit root test. The said test verified at level all variables are non-stationary after taking the first difference became stationary at 1% level of significance the earlier table shows that studied variables are stationary I(1).

**Table 4.22 VAR Lag Selection Criteria**

| Lag | LogL      | LR        | FPE      | AIC       | SC        | HQ        |
|-----|-----------|-----------|----------|-----------|-----------|-----------|
| 0   | -78.86196 | NA        | 9.62e-06 | 5.474965  | 5.752511  | 5.565438  |
| 1   | 133.5104  | 328.8346* | 1.15e10* | -5.903895 | 3.961074* | 5.270584* |
| 2   | 171.2754  | 43.85616  | 1.37e-10 | -6.017768 | -2.409671 | -4.841618 |
| 3   | 220.5658  | 38.16028  | 1.39e-10 | 6.875211* | -1.601839 | -5.156223 |

\* indicates lag order selected by the criterion

The basic step before using appropriate Vector Autoregressive model is the suitable lag selection for the variables. For the selection of suitable lag length there are many criterions for lag length LR sequential modified LR test statistic (each test at 5% level), FPE Final prediction error, AIC Akaike information criterion, SC Schwarz information criterion and HQ Hannan-Quinn information criterion. The study used one optimal lag because from all six criterions five are favor one lag.

**Table:4.23** Vector Auto regression Estimates

| DEPENDENT<br>INDEPENDENT | LGDP                                  | LEXR                                  | LNEXPP                                | LFDI                                  | INF                                   | LREM                                  |
|--------------------------|---------------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|
| LGDP(-1)                 | 0.826657<br>(0.08438)<br>[ 9.79650]   | 0.726911<br>(0.26247)<br>[ 2.76949]   | 0.107789<br>(0.66732)<br>[ 0.16153]   | 1.679118<br>(2.28145)<br>[ 0.73599]   | 24.99964<br>(14.8064)<br>[ 1.68844]   | -2.864176<br>(1.25409)<br>[ -2.2838]  |
| LEXR(-1)                 | 0.034582<br>(0.03774)<br>[ 0.91629]   | 0.699185<br>(0.11739)<br>[ 5.95598]   | 0.088944<br>(0.29846)<br>[ 0.29801]   | -1.459993<br>(1.02040)<br>[ -1.43081] | -19.24141<br>(6.62225)<br>[ -2.90557] | 1.454900<br>(0.56090)<br>[ 2.59387]   |
| LNEXPP(-1)               | 0.049496<br>(0.02211)<br>[ 2.23894]   | -0.202985<br>(0.06876)<br>[ -2.95196] | 0.717645<br>(0.17482)<br>[ 4.10494]   | 1.578861<br>(0.59770)<br>[ 2.64156]   | 5.267510<br>(3.87900)<br>[ 1.35795]   | 0.238525<br>(0.32855)<br>[ 0.72600]   |
| LFDI(-1)                 | 0.009595<br>(0.00612)<br>[ 1.56664]   | -0.010789<br>(0.01905)<br>[ -0.56632] | 0.032671<br>(0.04843)<br>[ 0.67455]   | 0.529452<br>(0.16559)<br>[ 3.19738]   | 0.149590<br>(1.07466)<br>[ 0.13920]   | 0.151905<br>(0.09102)<br>[ 1.66887]   |
| INF(-1)                  | -0.002112<br>(0.00081)<br>[ -2.61936] | 0.005327<br>(0.00251)<br>[ 2.12460]   | -0.007167<br>(0.00638)<br>[ -1.12423] | -0.048275<br>(0.02180)<br>[ -2.21496] | 0.205955<br>(0.14145)<br>[ 1.45604]   | -9.82E-05<br>(0.01198)<br>[ -0.00820] |
| LREM(-1)                 | 0.017347<br>(0.00533)<br>[ 3.25701]   | -0.038153<br>(0.01657)<br>[ -2.30297] | 0.000480<br>(0.04212)<br>[ 0.01140]   | -0.109544<br>(0.14400)<br>[ -0.76071] | -0.563315<br>(0.93455)<br>[ -0.60277] | 1.108143<br>(0.07916)<br>[ 13.9995]   |
| C                        | 1.759352<br>(0.96603)<br>[ 1.82121]   | -7.039680<br>(3.00482)<br>[ -2.34279] | 1.681186<br>(7.63956)<br>[ 0.22006]   | -37.38749<br>(26.1185)<br>[ -1.43145] | -376.5015<br>(169.506)<br>[ -2.22117] | 34.05446<br>(14.3570)<br>[ 2.37197]   |
| R-squared                | 0.999215                              | 0.996727                              | 0.969974                              | 0.936160                              | 0.683560                              | 0.948378                              |
| Adj. R-squared           | 0.999034                              | 0.995971                              | 0.963045                              | 0.921428                              | 0.610535                              | 0.936466                              |
| F-statistic &<br>Prob    | 5516.221<br>(0.0000)                  | 1319.498<br>(0.0000)                  | 139.9873<br>(0.0000)                  | 63.54498<br>(0.0000)                  | 9.360658<br>(0.0000)                  | 79.6109<br>(0.0000)                   |

The results of the unrestricted VAR model are showed in Table 4.23, Considered Real Gross Domestic Product (GDP) equation (1<sup>st</sup> Column of Table 4.23 ). We have six dependent variable, Inflation Rate (INF), foreign direct investment (FDI), Exports, Real GDP and foreign remittances and Exchange Rate of Pakistan. We have Chosen one lag with the guidance of lag selection criterions, so, there are one lag for each model and constant for this model. We have six dependent variable and every variable have seven independent variable, so; total 42 coefficients have in this VAR model. Now for



checking the significance we need P-value. The guideline is if P-value is less than 5% than the variable has significant impact on endogenous variable.

**Table: 4.24** Coefficients of VAR model

|       | Coefficient | Std. Error | t-Statistic | Prob.  |
|-------|-------------|------------|-------------|--------|
| C(1)  | 0.826657    | 0.084383   | 9.796498    | 0.0000 |
| C(2)  | 0.034582    | 0.037741   | 0.916292    | 0.3609 |
| C(3)  | 0.049496    | 0.022107   | 2.238942    | 0.0266 |
| C(4)  | 0.009595    | 0.006125   | 1.566644    | 0.1192 |
| C(5)  | -0.002112   | 0.000806   | -2.619363   | 0.0097 |
| C(6)  | 0.017347    | 0.005326   | 3.257009    | 0.0014 |
| C(7)  | 1.759352    | 0.966033   | 1.821214    | 0.0705 |
| C(8)  | 0.726911    | 0.262471   | 2.769488    | 0.0063 |
| C(9)  | 0.699185    | 0.117392   | 5.955977    | 0.0000 |
| C(10) | -0.202985   | 0.068763   | -2.951956   | 0.0036 |
| C(11) | -0.010789   | 0.019050   | -0.566322   | 0.5720 |
| C(12) | 0.005327    | 0.002507   | 2.124603    | 0.0352 |
| C(13) | -0.038153   | 0.016567   | -2.302970   | 0.0226 |
| C(14) | -7.039680   | 3.004823   | -2.342794   | 0.0204 |
| C(15) | 0.107789    | 0.667316   | 0.161526    | 0.8719 |
| C(16) | 0.088944    | 0.298462   | 0.298007    | 0.7661 |
| C(17) | 0.717645    | 0.174825   | 4.104935    | 0.0001 |
| C(18) | 0.032671    | 0.048434   | 0.674548    | 0.5010 |
| C(19) | -0.007167   | 0.006375   | -1.124230   | 0.2626 |
| C(20) | 0.000480    | 0.042120   | 0.011403    | 0.9909 |
| C(21) | 1.681186    | 7.639563   | 0.220063    | 0.8261 |
| C(22) | 1.679118    | 2.281452   | 0.735986    | 0.4628 |
| C(23) | -1.459993   | 1.020396   | -1.430810   | 0.1545 |
| C(24) | 1.578861    | 0.597700   | 2.641561    | 0.0091 |
| C(25) | 0.529452    | 0.165590   | 3.197376    | 0.0017 |
| C(26) | -0.048275   | 0.021795   | -2.214958   | 0.0282 |
| C(27) | -0.109544   | 0.144001   | -0.760714   | 0.4480 |
| C(28) | -37.38749   | 26.11853   | -1.431455   | 0.1543 |
| C(29) | 24.99964    | 14.80635   | 1.688440    | 0.0933 |
| C(30) | -19.24141   | 6.622249   | -2.905570   | 0.0042 |
| C(31) | 5.267510    | 3.879003   | 1.357955    | 0.1764 |
| C(32) | 0.149590    | 1.074656   | 0.139198    | 0.8895 |
| C(33) | 0.205955    | 0.141448   | 1.456045    | 0.1474 |
| C(34) | -0.563315   | 0.934550   | -0.602767   | 0.5475 |
| C(35) | -376.5015   | 169.5061   | -2.221168   | 0.0278 |
| C(36) | -2.864176   | 1.254086   | -2.283875   | 0.0237 |
| C(37) | 1.454900    | 0.560899   | 2.593870    | 0.0104 |
| C(38) | 0.238525    | 0.328548   | 0.725996    | 0.4689 |
| C(39) | 0.151905    | 0.091023   | 1.668875    | 0.0971 |
| C(40) | -9.82E-05   | 0.011981   | -0.008200   | 0.9935 |
| C(41) | 1.108143    | 0.079156   | 13.99955    | 0.0000 |
| C(42) | 34.05446    | 14.35703   | 2.371970    | 0.0189 |

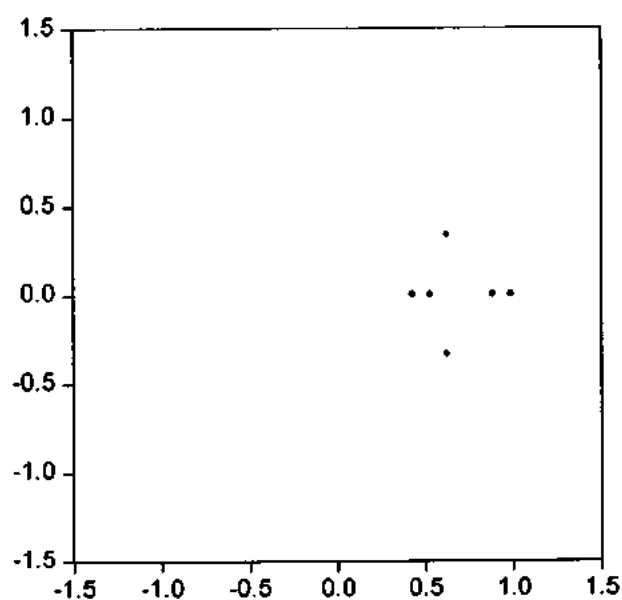
|                    |          |                    |          |
|--------------------|----------|--------------------|----------|
| R-squared          | 0.999215 | Mean dependent var | 15.47185 |
| Adjusted R-squared | 0.999034 | S.D. dependent var | 0.434494 |
| S.E. of regression | 0.013505 | Sum squared resid  | 0.004742 |

$$\begin{aligned}
 LGDPR = & 1.759352 + 0.826657 LGDPR(-1) + 0.034582LEXR(-1) + 0.049496LNEXPP(-1) + \\
 & 0.009595LFDI(-1) + -0.002112INF(-1) + 0.017347LREM(-1)
 \end{aligned}$$

Exchange rate has no significant impact on variability of Real Gross Domestic Product (GDP) in the Pakistan. Lag of GDP has significant impact on Real GDP similarly inflation rate and foreign remittances have significantly affect Real GDP of Pakistan.

**Graph: 4.7 AR Roots Graph**

Inverse Roots of AR Characteristic Polynomial



AR Roots Graph shows the inverse roots of the characteristic AR polynomial. The above graph shows that estimated VAR is stable (stationary) because all roots have modulus lie inside the unit circle so result are valid.

#### 4.14 VARIANCE DECOMPOSITION ANALYSIS:

There are two methods variance decomposition analysis and impulse response function to expand investigation and study of the effects of shocks to Gross Domestic Product (GDP). Variance Decomposition functions as a tool for assessing dynamic relations and strengths of causal relations among variables in the system. Below table 4.26 depict the results of the variance Decomposition, they are presenting that there are significant role played by the shocks in Exchange rate in accounting for the variability in the Goss Domestic Product (GDP) in Pakistan.

**Table No. 4.25** Variance decomposition of Real GDP.

| Period | S.E.     | LGDP     | LEXR     | LNEXPP   | LFDI     | INF      | LREM     |
|--------|----------|----------|----------|----------|----------|----------|----------|
| 1      | 0.013505 | 100.0000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 2      | 0.020011 | 83.59865 | 2.427145 | 4.215583 | 2.400988 | 5.135455 | 2.222182 |
| 3      | 0.025494 | 67.32039 | 2.639610 | 9.425471 | 6.012614 | 8.454782 | 6.147137 |
| 4      | 0.029951 | 55.89060 | 1.957701 | 12.87170 | 9.559972 | 9.327901 | 10.39212 |
| 5      | 0.033546 | 48.17350 | 1.846611 | 14.30930 | 12.52045 | 8.924453 | 14.22569 |
| 6      | 0.036523 | 42.82195 | 2.635304 | 14.36258 | 14.73310 | 8.098626 | 17.34844 |
| 7      | 0.039076 | 38.98446 | 3.987754 | 13.73342 | 16.26775 | 7.264912 | 19.76170 |
| 8      | 0.041325 | 36.15880 | 5.454020 | 12.89705 | 17.30483 | 6.557559 | 21.62774 |
| 9      | 0.043348 | 34.01886 | 6.746561 | 12.08347 | 18.02825 | 5.984748 | 23.13811 |
| 10     | 0.045205 | 32.32902 | 7.766846 | 11.37049 | 18.57318 | 5.519000 | 24.44147 |

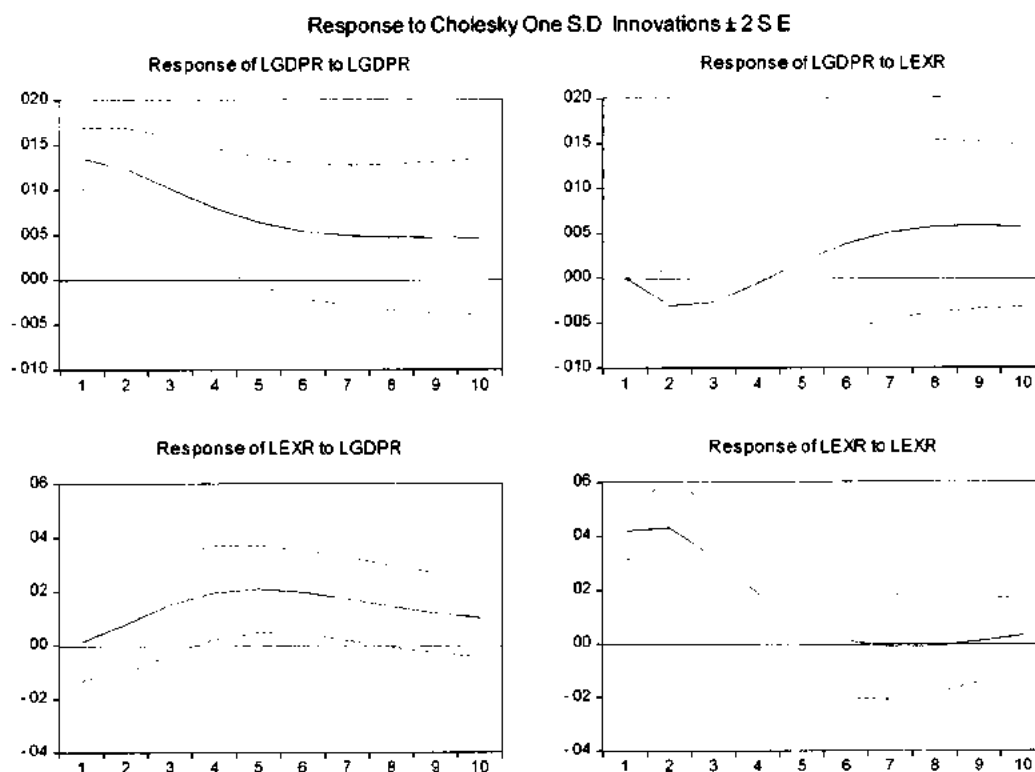
The result shows that in short run impulse of innovation of shock to Real GDP account for 67.32% variation of the fluctuation in Real GDP (own shock) and in long run i.e. 10 years is 32.32% variation of the fluctuation in Real GDP (own shocks). Shock to Exchange rate can cause 2.63% in short run but contribution in long run is 7.76% to the Real GDP in Pakistan.

#### 4.15 Impulse Response Function:

The Impulse response function is a shock to a VAR system. The Impulse responses identify the responsiveness of the dependent variable (Endogenous variable) in the VAR when a shock is put to the error term.

The impulse response graphs indicate that response of Real Gross Domestic Product to one standard deviation shock in Exchange Rate of Pakistan initially during the first four years decline and negative effect rising subsequently in the direction of the equilibrium level in the fifth year of the forecast period. Real GDP is increasing steadily through the last five year in response of shock to Exchange rate.

Graph: 4.8 Impulse response function



## Chapter 5

### Conclusion

This research motivation on building a model for the exchange rate volatility of Pakistan using time series methodology. As the monetary time series like exchange rate may have volatility, an effort is made to model this volatility using ARCH family of models. Increasing role of exchange rate in corporate decision making is becoming famous in the emerging economies. Exchange rate volatility occupied a significant position all over the world in Investment decision. Exchange rate volatility is equally helpful in many micro as well as macro-economic decision-making. Monthly as well as yearly foreign exchange rates of Pakistan for the period ranging from July 1981 to May 2014 are used for this purpose. The stationarity of the exchange rate returns is inspected through graphical analysis, Augmented Dicky Fuller test and correlogram which showed the series is nonstationary at level but after taking first difference it became stationary. For selection of mean model we have used Autoregressive (AR), Moving Average (MA) and Autoregressive Integrated Moving Average (ARIMA). ARIMA (3,1,2) and ARIMA (1,1,2) selected appropriate models for monthly and yearly Exchange rate data, respectively. We have performed forecast from 1981 to 2014 as well as 2010 to 2014 with the help of these selected models. The performance of forecast was evaluated by RMSE (Root Mean Square Error), MAE (Mean Absolute Error), MAPE (Mean Absolute Percentage Error) and Theil's Inequality coefficient. The ARCH-LM test and residual of ARIMA (3,1,2) and ARIMA (1,1,2) have justified to run ARCH family of models. We have used the different sets of ARCH, GARCH, EGARCH, and IGARCH.

The diagnostic checking has shown that for monthly exchange rate data the ARIMA(3,1,2)-EGARCH (1,1) and for yearly exchange rate data the ARIMA(1,1,2)-EGARCH(1,1) are the best fitted models for modeling and forecasting Exchange Rate volatility. We have made forecast from 1981 to 2020 with these models. The performance of forecast was evaluated by RMSE (Root Mean Square Error), MAE (Mean Absolute Error), MAPE (Mean Absolute Percentage Error) and Theil's Inequality coefficient. These forecasts would be helpful for the policy makers to foresee the future requirements.

On second part the study examined the impact of exchange rate volatility on Pakistan's economy using yearly data from 1981 to 2014. The finding of this study showed significant impact of exchange rate on Real Gross Domestic Product of Pakistan. The study has used six variables i.e. the data for the variables i.e. Exchange Rate, inflation Rate (INF), foreign direct investment (FDI), Exports, Real GDP and foreign remittances obtained from The State Bank of Pakistan. The empirical relationship between exchange rate volatility and Real Gross Domestic Product (GDP) have been found while employing an econometric study based on Vector Autoregressive (VAR) methodology. We have chosen one lag by the guidance of different lag selection criterions. The result of VAR model shows Exchange rate volatility has significant impact on variability of Real Gross Domestic Product (GDP) in the Pakistan and lag of Real GDP has significant impact on Real GDP. Similarly inflation rate and foreign remittances have significant effect on Real GDP of Pakistan. AR Roots Graph showed estimated VAR is stable (stationary) because all roots have modulus lie inside the unit circle so result are valid. Study shows that in short run impulse of innovation of shock to Real GDP account for 67.32% variation of the fluctuation in Real GDP (own shock) and in long run i.e. 10

years is 32.32% variation of the fluctuation in Real GDP (own shocks). Shock to Exchange rate can cause 2.63% in short run but contribution in long run is 7.76% to the Real GDP in Pakistan. The impulse response graphs indicate that response of Real Gross Domestic Product to one standard deviation shock in Exchange Rate of Pakistan initially during the first four years decline and negative effect rising subsequently in the direction of the equilibrium level in the fifth year of the forecast period. Real GDP is increasing steadily through the last five year in response of shock to Exchange rate.

## Chapter 6

### REFERENCES

As'ad. M, (2012). Finding the Best ARIMA Model to Forecast Daily Peak Electricity Demand, *University of Wallongong Research online*, (ASEARC).

Abdullah. L, (2012). ARIMA for Gold Bullion Coin Selling Prices Forecasting. *Institute of Advanced Engin & Sci*.

Attah-Obeng, P. Osei-Gyimah,F,E,P. Opoku,C,D,K. (2013). An Econometric Analysis of the Relationship between GDP growth rate and Exchange rate in Ghana. *J.Eco.sustainable.Dev. vol (4). No. 9*.

Abdalla. S.Z. (2012). "Modeling Exchange Rate Volatility using GARCH Models: Empirical Evidence from Arab Countries", *International Journal of Economics and Finance Vol. 4, No. 3; March 2012*

Box and Jenkins (1976). *Time Series Analysis, Forecasting and Control*; Prentice Hall.

Boollerslev, T. Chou, R. and Kroner, K.F. (1992). ARCH Modeling in Finance: A review of the Theory and empirical evidence. *Econometrica*, vol (52), 5-59.

Chatfield, C. (2001), *Time Series Forecasting*, Chapman and Hall.

CHONG . C.W, Chun. L.S, Ahmed. M.I. (2002). "Modeling the Volatility of Currency Exchange Rate Using GARCH Model" *Pertanika J. Soc. Sci. & Hum. 10(2): 85-95 (2002)*



C Steve. N, (2014). Autoregressive Integrated Moving Average (ARIMA) Model for Exchange Rate (Naira to Dollar). *Academic.J. Interdisciplinary Studies MC SER, Rome, Italy*; vol. 3-4: pp 428-430.

Corliss. D.J, (2013). Non-Temporal ARIMA Models in Statistical Research. *University of Toledo Physics & Astronomy*; pp 457-459.

Devi. U, Sundar.D and Alli.P, (2013). An Effective Time Series Analysis for Stock Trend Prediction Using ARIMA Model for Nifty Midcap50, *int, J Data Mining & Knowledge Management Process (IJKP)*, vol. 3.

Engle, R.F. (1982). Autoregressive Conditional Heteroscedasticity with estimates of the variance United Kingdom inflation. *Econometrica*, vol ( 50), 987-1007.

Javed, Z.H. and Farooq, M. (2009). Economic Growth and Exchange Rate Volatility in Case of Pakistan. *Pak.j.life soc . sci*, 7(12), 112-118.

Khan, A.J. and Azim, P. (2013). One-step-Ahead forecastability analysis of USD- and PKR- Based exchange rate volatilities. *The Lahore Journal of Economics*,18(1), 1-38.

Kamin, Steven B. (1996). "Exchange Rates and Inflation in Exchange Rate based Stabilization: an Empirical Examination", *International Finance Discussion Paper*, 555, Federal Reserve System, Washington, D.C.

Kamin, Steven B., and Rogers, John H. (2000). "Output and the Real Exchange Rate in Developing Countries: An Application to Mexico," *Journal of Development Economics*: 80-110.

Kamal. Y, Ghani. H.U, Muhsin. M. (2012). "Modeling the exchange rate volatility, using generalized autoregressive conditionally heteroscedastic (GARCH) type models:

Evidence from Pakistan” *African Journal of Business Management* Vol. 6(8), pp. 2830-2838, 29 February, 2012

Mukhtar, T. and Malik, S.J. (2010). Exchange Rate Volatility and Export Growth: Evidenced From selected South Asian countries. >>SPOUDI>> vol.60, 3-4, *university of Piraeus*, 58-68.

Mckenzie, M.D. (1999). Forecasting Australian Exchange Rate Volatility: A comparative study Of alternate Modeling techniques and the impact of power of transformation. *Applied Financial Economics*, 7,1-31.

Mustafa, M. and Nishat, M. (2004). Volatility of Exchange Rate and Export Growth in Pakistan The structure and Independence in Regional Markets. *The Pakistan Development Review*, 43(4), 813-828.

Mahmood, I. Ehsanullah,M. and Ahmed,H. (2011). Exchange rate volatility and macroeconomic variables in Pakistan. *BM Dynamics*, vol (1) No. 2, 11-22.

Mukherjee, Chandan, (2003). *Econometrics and Data Analysis for Developing Countries*, London: Routledge.

Nelson, D.B. (1991). Conditional Heteroscedasticity in asset returns: A new approach. *Econometrica*, 59(2), 347-370.

Ramzan, S. Ramzan, S. and Zahid, F.M. (2012). Modeling and Forecasting Exchange Rate Dynamics in Pakistan using ARCH family of Model. *Electronic.J.App.Stat.anal*, vol(5), Issue 1, 15-29.

Woschnagg, E. and Cipan, J. (2004). Evaluation Forecast Accuracy. *Okonometrische prognose university of Viena department of Economics*.