Effect of Chemical Reaction and Heat Source on MHD Couette Flow of Non-Newtonian Fluid

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1. Fluid dynamics

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A Thesis
Submitted in the Partial Fulfillment of the
Requirement for the Degree of
MASTER OF SCIENCE
In
MATHEMATICS

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We accept this thesis as conforming to the required standard.

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2016

This thesis is dedicated to my parents Rashid Ahmad and Naseem Rashid

For their endless love, support and encouragement

DECLARATION

I hereby declare that this thesis has been prepared by me under the able guidance and supervision of **Dr. Khadija Maqbool**. It is further declared that I have not copied from any other students' work or from any other sources and that it has not been submitted anywhere for any award.

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Thank you.

Preface

There are many fluids in nature those exhibit non-Newtonian behavior. In such fluids the shear stress is not directly proportional to deformation rate. Non-Newtonian fluids are classified into three types namely differential, rate and integral type fluids. Maxwell fluid is the simplest model of the rate type fluids. This fluid model has become the most popular as it can predict stress relaxation and also excludes the complicating effects of shear-dependent viscosity. The flow between two parallel plates is a classical problem that has many applications in accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, purification of crude oil, fluid droplets and sprays. Such flow models are of great interest not only for its theoretical significance, but also for its wide application to geophysics and engineering. Heat and mass transfer with chemical reaction has immense important in chemical and hydrometallurgical industries. Many researcher [1-10] have investigated the heat and mass transfer flow of viscous fluid with flow geometry. Many researchers [11-20] have discussed the Maxwell fluid flow in different regimes. Recently Joseph [21] have discussed the heat and mass transfer of Couette flow with viscous dissipation and slip condition. Garg [22] have studied oscillatory convective flow of second grade fluid in a vertical rotating channel with heat radiation and slip condition. MHD Couette flow of a non-Newtonian fluid in a rotating system with heat and mass transfer have studied by Manjusa [23].

No study have been reported with heat source and chemical reaction of Maxwell fluid flow in a rotating channel with slip effect.

Keeping in view all above applications this dissertation comprises on following chapters. Chapter one consists of preliminaries of the proposed work.

In chapter two we have reviewed the work of Bhattacharya [6] he considered the free convicted MHD Couette flow of a viscous fluid with heat source and chemical reaction.

Chapter three is extended for the Maxwell fluid model in rotating frame with heat source and chemical reaction and results are deduce for the Newtonian fluid.

Contents

Chapter 1

Preliminaries

This chapter comprises the basic definitions of fluid mechanics.

1.1 Fluid Mechanics

Fluid mechanics is the branch of engineering that examines the properties and nature of the fluid at rest or in motion.

1.2 Classification of the fluids

1.2.1 Inviscid fluid

Fluids with negligible viscosity is known as the inviscid or ideal fluid e.g. water.

1.2.2 Real fluid

Fluids which possesses non-zero viscosity is known as real fluid e.g. petrol and castor oil.

1.2.3 Newtonian fluid

Fluids in which shear stress is directly proportional to the deformation rate .are Newtonian fluids. Mathematically defined as

$$\tau_{yx} = \mu \frac{du}{dy}.$$

1.2.4 Non-Newtonian fluid

A non-Newtonian fluid is a fluid whose viscosity is variable based on applied stress or force. The most common everyday example of a non-Newtonian fluid is cornstarch dissolved in water. Mathematically defined as

$$\tau_{yx} = \eta \frac{du}{dy},$$

η is called the apparent viscosity and given by the following expression

$$\eta = k \left(\frac{du}{dy} \right)^{n-1}.$$

1.3 Types of flows

1.3.1 Steady vs unsteady flow

Steady flow refers to the flow where the fluid properties at a point in the system do not change with time, otherwise flow is called unsteady.

1.3.2 Rotational vs irrotational flow

Rotational flow in which the fluid particles also rotate about their own axis while flowing, otherwise flow is irrotational.

1.3.3 Incompressible vs compressible flow

If density of a fluid is constant with respect to space and time then flow is incompressible otherwise it is compressible.

1.3.4 Couette flow vs Poiseuille flow

When two parallel plane walls are in relative motion a simple shearing motion with linear profile is obtained and this flow is termed as Couette flow. Whereas Poiseuille flow is defined as the flow between two parallel stationary walls and the flow between these two plates is due to pressure.

1.4 Heat Transfer

Heat is a form of energy which flows from a body of higher temperature to another body at a lower temperature by virtue of the temperature difference between the bodies. There are three different ways of heat transfer, conduction, convection and radiation.

1.4.1 Conduction

Conduction is transfer of heat through molecules of medium example of conduction is the heat through a stationary wall or pipe.

1.4.2 Convection

Convected heat transfer can be defined as transport of heat from one point to another in a fluid as a result of macroscopic motions of the fluid. Convective heat transfer may be categorized into two forms according to the nature of flow.

- Free Convection
- Forced Convection

Free Convection

If fluid motion is caused by buoyancy forces that are induced by density differences due to variation in fluid temperature or due to variation in densities of two different fluids adjacent to each other or solid fluid interfaces are at different temperatures.

Forced Convection

Fluid is forced over the surface by external means such as a fan, pump, mixer or wind.

1.4.3 Radiation

Energy is emitted by matter in the form of electromagnetic waves as a result of changes in electronic configurations of atoms or molecules.

1.5 Mass Transfer

Movement of components from one phase to another due to concentration difference between the phases is called mass transfer.

Mass is certainly transferred whenever there is a bulk fluid motion, this is not what we have in mind. For example, we do not use the term mass transfer to describe the motion of air that is induced by a fan or the motion of water being forced through a pipe. In both cases, there is gross or bulk fluid motion due to mechanical work. We do, however, use the term to describe the relative motion of species in a mixture due to the presence of concentration gradients. One example is the dispersion of oxides of sulfur released from a power plant smoke stack into the environment. Another example is the transfer of water vapor into dry air, as in a home humidifier.

1.6 Chemical Reaction

A chemical reaction is the change of a substance into a new one that has a different chemical identity. A chemical reaction is usually accompanied by easily observed physical effects, such as the emission of heat and light, the formation of a precipitate, the evolution of gas or a color change.

1.7 Dimensionless Number

1.7.1 Reynold Number

The Reynold numbers Re is a dimensionless number that gives a measure of the ratio of inertial forces to the viscous forces. Mathematically defined as

$$\mathrm{Re} = \frac{Vl\rho}{\mu},$$

where V is the velocity of the fluid, l is the length, ρ is the density and μ is the dynamic viscosity.

1.7.2 Prandtl Number

Prandtl number Pr is a dimensionless number, defines as the ratio of momentum diffusivity to thermal diffusivity. Mathematically defined as

$$\mathrm{Pr} = \frac{\nu}{\alpha},$$

where ν is kinematic viscosity and α is the thermal diffusivity.

1.7.3 Eckert Number

The Eckert number is a dimensionless number which express the relationship between flow's kinetic energy and enthalpy. Mathematically defined as

$$Ec = \frac{V^2}{c_p(T_s - T_\infty)},$$

where V is the velocity of the fluid and c_p is the specific heat.

1.7.4 Schmidt Number

Schmidt number is a dimensionless number, defined as a ratio of momentum diffusivity to mass diffusivity. Mathematically defined as

$$Sc = \frac{\nu}{D}.$$

1.8 Conservation Laws

1.8.1 Law of conservation of mass

The mass conservation equation is also called the equation of continuity. It is derived from the law of conservation of mass. The law of conservation of mass assumes that mass can neither be created nor destroyed and that on a steady flow process, the stored mass in a control volume does not change. A steady flow process is one where the flow rate does not change over time. This implies that inflow into the control volume equals outflow. For steady Incompressible flow the continuity equation is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0.$$

1.8.2 Law of conservation of momentum

The law of the conservation of momentum states that the rate of change of momentum in the control volume is equal to the sum of the net momentum flux into the control volume and any external forces acting on the control volume. This implies that the total momentum of a closed system is constant. The general momentum equation is

$$\rho\frac{d\mathbf{V}}{dt}=\nabla.\mathbf{T}+\mathbf{F},$$

where $\mathbf{T}$ is the Cauchy stress tensor, ρ is the density and $\mathbf{F}$ is a body force.

1.8.3 Law of conservation of energy

To obtain the energy equation, we have to apply the law of conservation of energy which states that energy cannot be created or destroyed, but only changed from one form to another.

For incompressible flow the energy equation is

$$\rho\frac{de}{dt}=\tau.\mathbf{L}-\nabla.\mathbf{q}+\rho r, \tag{1.1}$$

in which e is the internal energy per unit mass, $\mathbf{q}$ is heat flux vector and τ is the Cauchy stress tensor.

By using the following Fourier law of heat conduction

$$\mathbf{q}=-k\nabla T,$$

Eq. (1.1) becomes

$$\rho C_p\frac{dT}{dt}=\tau.\mathbf{L}+k\nabla^2 T+\rho r.$$

where T is the temperature, r is the radial heating, C_p is the specific heat and k is the thermal conductivity.

1.8.4 Law of Concentration field

The mass conservation or concentration equation in the presences of a chemical reaction has the following form in three dimensions

$$\frac{\partial C}{\partial t} + (\mathbf{V}.\nabla)C = D\nabla^2 C - R(C - C_\infty),$$

where D is mass diffusivity, R is the chemical reaction parameter.

1.9 Method of Solution

In the field of physical science there are both linear and non-linear problems, according to the phenomena. Specially most of the problem encountered in fluid mechanics are non-linear. To find the solution for the non-linear problem is often hard. Thus many analytic techniques are developed to solve these type of problems. Perturbation method and Homotopy perturbation method are among these analytic techniques. we have used these techniques in the subsequent chapters to get the analytic solution.

1.9.1 Perturbation method

A set of mathematical methods often used to obtain approximate solution to the equations for which no exact solution is possible, or known. According to this technique, the solution is given by few term of expansion. This methods rely on there being a dimensionless parameter in the problem that may be small or large. The solution is given by few terms of expansion

$$A = A_0 + \epsilon A_1 + \epsilon^2 A_2 + ...$$

There are two types of perturbation

- Regular Perturbation
- Singular Perturbation

Regular Perturbation

In regular perturbation the power series in ϵ having non-zero radius of convergence and exact solution for $\epsilon \to 0$ approaches to zeroth order solution.

Singular perturbation

In singular perturbation power series may has a vanishing radius of convergence also exact solution for $\epsilon \to 0$ may not approach to the zeroth order solution.

1.9.2 Homotopy Perturbation Method

Consider the non-linear differential equation

$$A(u) - f(r) = 0, \quad r \in K, \tag{1.2}$$

with the boundary condition as

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad r \in \Gamma, \tag{1.3}$$

where A is general differential operator which can be divided into two parts linear L and non-linear N, and B is the boundary operator and $f(r)$ is the source term, K is the domain and Γ is its boundary. Now Eq.(1.2) can be rewrite as

$$L(u) + N(u) - f(r) = 0, \quad r \in K, \tag{1.4}$$

As Liao propose the homotopy technique, we first construct the homotopy of the Eq.(1.2) as

$$H(r,p) : v \times [0,1] \to \mathbb{R},$$

which satisfy

$$\begin{aligned} H(v,p) &= (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0, \\ p &\in [0,1]\ ;\ r \in K. \end{aligned} \tag{1.5}$$

where u_0 is the initial approximation which satisfies the boundary conditions and $p \in [0,1]$ is the embedding parameter. From Eq.(1.5) it is observed that as p varies from 0 to 1 the solution varies from initial value to the final value.

The linear term can be replaced by infinite series

$$v = \sum_{n=0}^{\infty} v_n p^n, \tag{1.6}$$

and for non-linear terms He's polynomial is defined as

$$N(v) = \sum_{n=0}^{\infty} A_n p^n,$$

$$A_n = \frac{1}{n!}\frac{d^n}{dp^n}\left[N\left(\sum_{i=0}^{n} v_i p^i\right)\right]_{p=0}.$$

The solution of Eq.(1.2) can easily be obtained as

$$u = \lim_{p\to 1}\sum_{n=0}^{\infty} p^n v_n. \tag{1.7}$$

Chapter 2

Couette flow with heat source and chemical reaction

In this chapter we review the work of Bhattacharyya [6]. He discussed the free convected MHD couette flow of a viscous fluid in the presence of heat source and chemical reaction.

2.1 Formulation of problem

Consider the flow of an incompressible viscous electrically conducting fluid through porous medium bounded by two infinite vertical porous plates separated by a distance h in presence of transverse magnetic field, chemical reaction and heat source by making the following assumptions:

- All fluid properties except the density in the buoyancy force are constant.
- The Eckert number Ec is small i.e. Ec<< 1.
- The magnetic Reynolds number is so small that the induced magnetic field is negligible.

X-axis is taken along the wall of channel and Y-axis is normal to it. A uniform magnetic field of strength B_0 has been applied perpendicular to the wall of the channel. On the Boussinesq's approximation the steady flow, is governed by the following equations

$$\frac{dv'}{dy'} = 0, \text{ implies } v = -v_0 = \text{constant.}$$

$$-v_0\frac{du'}{dy'} = \nu\frac{d^2u'}{dy'^2} + g\beta(T' - T_s') + g\beta_c(C' - C_s') - \frac{\nu u'}{K'} - \frac{\sigma B_0^2}{\rho}u', \tag{2.1}$$

$$-v_0\frac{dT'}{dy'} = \frac{k}{\rho C_p}\frac{d^2T'}{dy'^2} + \frac{\nu}{C_p}\left(\frac{du'}{dy'}\right)^2 - Q'(T' - T_s'), \tag{2.2}$$

$$-v_0\frac{dC'}{dy'} = D\frac{d^2C'}{dy'^2} - R'(C' - C_s'). \tag{2.3}$$

The corresponding boundary conditions are

$$\left.\begin{aligned} u' = 0;\ T' = T_0';\ C' = C_0' \text{ at } y' = 0,\\ u' = U';\ T' = T_1';\ C' = C_1' \text{ at } y' = h. \end{aligned}\right\} \tag{2.4}$$

Let us introduce the following non-dimensional quantities

$$\left.\begin{aligned} y = \tfrac{y'}{h},\ \theta = \tfrac{T'-T_s'}{T_0'-T_s'},\ \phi = \tfrac{C'-C_s'}{C_0'-C_s'},\ Sc = \tfrac{\nu}{D},\ \Pr = \tfrac{\mu C_p}{k},\ \text{Re} = \tfrac{v_0 h}{\nu},\\ Gr = \tfrac{hg\beta(T_0'-T_s')}{v_0^2}, Gm = \tfrac{hg\beta_c(C_0'-C_s')}{v_0^2}, Ec = \tfrac{v_0^2}{C_p(T_0'-T_s')}, K = \tfrac{K'}{h^2},\\ M = \tfrac{\sigma B_0^2 h^2}{\rho\nu}, m = \tfrac{T_1'-T_s'}{T_0'-T_s'}, n = \tfrac{C_1'-C_s'}{C_0'-C_s'},\ R = \tfrac{hR'}{v_0},\ Q = \tfrac{hQ'}{v_0}, U = \tfrac{U'}{v_0}. \end{aligned}\right\} \tag{2.5}$$

The equations (2.1) – (2.4) in non dimensional form are

$$-\frac{du}{dy} = \frac{1}{\text{Re}}\frac{d^2u}{dy^2} + Gr\theta + Gm\phi - \frac{u}{\text{Re}\,K} - \frac{Mu}{\text{Re}}, \tag{2.6}$$

$$-\frac{d\theta}{dy} = \frac{1}{\Pr\text{Re}}\frac{d^2\theta}{dy^2} + \frac{Ec}{\text{Re}}\left(\frac{du}{dy}\right)^2 - Q\theta, \tag{2.7}$$

$$-\frac{d\phi}{dy} = \frac{1}{Sc\text{Re}}\frac{d^2\phi}{dy^2} - R\phi. \tag{2.8}$$

Subject to the boundary condition

$$\left.\begin{aligned} u = 0;\ \theta = 1;\ \phi = 1 \text{ at } y = 0,\\ u = U;\ \theta = m;\ \phi = n \text{ at } y = 1. \end{aligned}\right\} \tag{2.9}$$

2.2 Method of Solution:

The solution of the equation (2.8) subject to the conditions (2.9) is

$$\phi = A_4 e^{A_1 y} + A_3 e^{A_2 y}, \tag{2.10}$$

where $A_1 = \dfrac{-Sc\,\mathrm{Re} + \sqrt{Sc^2\,\mathrm{Re}^2 + 4R\,\mathrm{Re}\,Sc}}{2}$, $A_2 = \dfrac{-Sc\,\mathrm{Re} - \sqrt{Sc^2\,\mathrm{Re}^2 + 4R\,\mathrm{Re}\,Sc}}{2}$,

$$A_3 = \frac{n - e^{A_1}}{e^{A_2} - e^{A_1}}, \quad A_4 = 1 - A_3.$$

In order to solve the equations (2.6) and (2.7) subject to the boundary condition (2.9), we use perturbation method with perturbation parameter Ec << 1.

The solution of Eq. (2.6) and Eq. (2.7) can be written in the form of series.

$$\left.\begin{aligned} u &= u_0(y) + (Ec)\,u_1(y) + (Ec)^2\,u_2(y) + ..., \\ \theta &= \theta_0(y) + (Ec)\,\theta_1(y) + (Ec)^2\,\theta_2(y) + ..., \end{aligned}\right\} \tag{2.11}$$

Using Eq. (2.11) into equation (2.6) – (2.7) we get the following system of equation

Zeroth order system

$$u_0'' - A_6 u_0' + A_7 u_0 = A_8 \theta_0 + A_{10} \phi, \tag{2.12}$$

$$\theta_0'' + \Pr \mathrm{Re}\,\theta_0' - \Pr \mathrm{Re}\,Q \theta_0 = 0. \tag{2.13}$$

$$\left.\begin{aligned} u_0(0) = 0;\ \theta_0(0) = 1, \\ u_0(1) = U;\ \theta_0(1) = m. \end{aligned}\right\} \tag{2.14}$$

First order system

$$u_1'' - A_6 u_1' + A_7 u_1 = A_8 \theta_1. \tag{2.15}$$

$$\theta_1'' + \Pr \mathrm{Re}\,\theta_1'' - \Pr \mathrm{Re}\,Q \theta_1 = -\Pr \left(u_0'\right)^2. \tag{2.16}$$

$$\left.\begin{aligned} u_1(0) = 0;\ \theta_1(0) = 0, \\ u_1(1) = 0;\ \theta_1(1) = 0. \end{aligned}\right\} \tag{2.17}$$

$$a_5 = \frac{-\Pr A_1^2 a_3^2}{4A_1^2 + 2\Pr \mathrm{Re}\, A_1 - Q \Pr \mathrm{Re}}; \quad a_6 = \frac{-\Pr A_2^2 a_4^2}{4A_2^2 + 2\Pr \mathrm{Re}\, A_2 - Q \Pr \mathrm{Re}},$$

$$a_7 = \frac{-\Pr B_1^2 a_1^2}{4B_1^2 + 2\Pr \mathrm{Re}\, B_1 - Q \Pr \mathrm{Re}}; \quad a_8 = \frac{-\Pr B_2^2 a_2^2}{4B_2^2 + 2\Pr \mathrm{Re}\, B_2 - Q \Pr \mathrm{Re}},$$

$$a_9 = \frac{-\Pr B_5^2 B_7^2}{4B_5^2 + 2\Pr \mathrm{Re}\, B_5 - Q \Pr \mathrm{Re}}; \quad a_{10} = \frac{-\Pr B_6^2 B_8^2}{4B_6^2 + 2\Pr \mathrm{Re}\, B_6 - Q \Pr \mathrm{Re}},$$

$$a_{11} = \frac{-2\Pr A_1 A_2 a_3 a_4}{(A_1 + A_2)^2 + \Pr \mathrm{Re}\,(A_1 + A_2) - Q \Pr \mathrm{Re}}; \quad a_{12} = \frac{-2\Pr A_1 B_1 a_1 a_3}{(A_1 + B_1)^2 + \Pr \mathrm{Re}\,(A_1 + B_1) - Q \Pr \mathrm{Re}},$$

$$a_{13} = \frac{-2\Pr A_2 B_1 a_1 a_4}{(A_2 + B_1)^2 + \Pr \mathrm{Re}\,(A_2 + B_1) - Q \Pr \mathrm{Re}}; \quad a_{14} = \frac{-2\Pr A_1 B_2 a_2 a_3}{(A_1 + B_2)^2 + \Pr \mathrm{Re}\,(A_1 + B_2) - Q \Pr \mathrm{Re}},$$

$$a_{15} = \frac{-2\Pr A_2 B_2 a_2 a_4}{(A_2 + B_2)^2 + \Pr \mathrm{Re}\,(A_2 + B_2) - Q \Pr \mathrm{Re}}; \quad a_{16} = \frac{-2\Pr B_1 B_2 a_2}{(B_1 + B_2)^2 + \Pr \mathrm{Re}\,(B_1 + B_2) - Q \Pr \mathrm{Re}},$$

$$a_{17} = \frac{-2\Pr A_1 B_5 B_7 a_3}{(A_1 + B_5)^2 + \Pr \mathrm{Re}\,(A_1 + B_5) - Q \Pr \mathrm{Re}}; \quad a_{18} = \frac{-2\Pr B_5 B_7 a_2 a_4}{(A_2 + B_5)^2 + \Pr \mathrm{Re}\,(A_2 + B_5) - Q \Pr \mathrm{Re}},$$

$$a_{19} = \frac{-2\Pr B_1 B_5 B_7 a_1}{(B_1 + B_5)^2 + \Pr \mathrm{Re}\,(B_1 + B_5) - Q \Pr \mathrm{Re}}; \quad a_{20} = \frac{-2\Pr B_2 B_5 B_7 a_2}{(B_2 + B_5)^2 + \Pr \mathrm{Re}\,(B_2 + B_5) - Q \Pr \mathrm{Re}},$$

$$a_{21} = \frac{-2\Pr A_1 B_6 B_8 a_3}{(A_1 + B_6)^2 + \Pr \mathrm{Re}\,(A_1 + B_6) - Q \Pr \mathrm{Re}}; \quad a_{22} = \frac{-2\Pr A_2 B_6 B_8 a_4}{(A_2 + B_6)^2 + \Pr \mathrm{Re}\,(A_2 + B_6) - Q \Pr \mathrm{Re}},$$

$$a_{23} = \frac{-2\Pr B_1 B_6 B_8 a_1}{(B_1 + B_6)^2 + \Pr \mathrm{Re}\,(B_1 + B_6) - Q \Pr \mathrm{Re}}; \quad a_{24} = \frac{-2\Pr B_2 B_6 B_8 a_2}{(B_2 + B_6)^2 + \Pr \mathrm{Re}\,(B_2 + B_6) - Q \Pr \mathrm{Re}},$$

$$a_{25} = \frac{-2\Pr B_5 B_6 B_7 B_8}{(B_5 + B_6)^2 + \Pr \mathrm{Re}\,(B_5 + B_6) - Q \Pr \mathrm{Re}}; \quad a_{26} = \frac{A_8 a_5}{4A_1^2 - 2A_6 A_1 + A_7},$$

$$a_{27} = \frac{A_8 a_6}{4A_2^2 - 2A_6 A_2 + A_7}; \quad a_{28} = \frac{A_8 a_{11}}{(A_1 + A_2)^2 - A_6(A_1 + A_2) + A_7},$$

$$a_{29} = \frac{A_8 B_9}{B_1^2 - A_6 B_1 + A_7}; \quad a_{30} = \frac{A_8 a_7}{4B_1^2 - 2A_6 B_1 + A_7},$$

$$a_{31} = \frac{A_8 a_{12}}{(A_1 + B_1)^2 - A_6(A_1 + B_1) + A_7}; \quad a_{32} = \frac{A_8 a_{13}}{(A_2 + B_1)^2 - A_6(A_2 + B_1) + A_7},$$

$$a_{33} = \frac{A_8 B_{10}}{B_2^2 - A_6 B_2 + A_7}; \quad a_{34} = \frac{A_8 a_8}{4B_2^2 - 2A_6 B_2 + A_7},$$

$$a_{35} = \frac{A_8 a_{14}}{(A_1 + B_2)^2 - A_6(A_1 + B_2) + A_7}; \quad a_{36} = \frac{A_8 a_{15}}{(A_2 + B_2)^2 - A_6(A_2 + B_2) + A_7},$$

$$a_{37} = \frac{A_8 a_{16}}{(B_1+B_2)^2 - A_6(B_1+B_2) + A_7}; \quad a_{38} = \frac{A_8 a_9}{4B_5^2 - 2A_6 B_5 + A_7},$$

$$a_{39} = \frac{A_8 a_{17}}{(A_1+B_5)^2 - A_6(A_1+B_5) + A_7}; \quad a_{40} = \frac{A_8 a_{18}}{(A_2+B_5)^2 - A_6(A_2+B_5) + A_7},$$

$$a_{41} = \frac{A_8 a_{19}}{(B_1+B_5)^2 - A_6(B_1+B_5) + A_7}; \quad a_{42} = \frac{A_8 a_{20}}{(B_2+B_5)^2 - A_6(B_2+B_5) + A_7},$$

$$a_{43} = \frac{A_8 a_{10}}{4B_6^2 - 2A_6 B_6 + A_7}; \quad a_{44} = \frac{A_8 a_{21}}{(A_1+B_6)^2 - A_6(A_1+B_6) + A_7},$$

$$a_{45} = \frac{A_8 a_{22}}{(A_2+B_6)^2 - A_6(A_2+B_6) + A_7}; \quad a_{46} = \frac{A_8 a_{23}}{(B_1+B_6)^2 - A_6(B_1+B_6) + A_7},$$

$$a_{47} = \frac{A_8 a_{24}}{(B_2+B_6)^2 - A_6(B_2+B_6) + A_7}; \quad a_{48} = \frac{A_8 a_{25}}{(B_5+B_6)^2 - A_6(B_5+B_6) + A_7},$$

$$G_2 = \sum_{i=5}^{25} a_i, \quad G_5 = \sum_{i=26}^{48} a_i,$$

$$\begin{aligned} G_3 \;=\; & a_5 e^{2A_1} + a_6 e^{2A_2} + a_{11} e^{(A_1+A_2)} + a_7 e^{2B_1} + a_{12} e^{(A_1+B_1)} + a_{13} e^{(A_2+B_1)} + a_8 e^{2B_2} + a_{14} e^{(A_1+B_2)} \\ & + a_{15} e^{(A_2+B_2)} + a_{16} e^{(B_1+B_2)} + a_9 e^{2B_5} + a_{17} e^{(A_1+B_5)} + a_{18} e^{(A_2+B_5)} + a_{19} e^{(B_1+B_5)} + a_{20} e^{(B_2+B_5)} \\ & + a_{10} e^{2B_6} + a_{21} e^{(A_1+B_6)} + a_{22} e^{(A_2+B_6)} + a_{23} e^{(B_1+B_6)} + a_{24} e^{(B_2+B_6)} + a_{25} e^{(B_5+B_6)}. \end{aligned}$$

$$\begin{aligned} G_6 \;=\; & a_{26} e^{2A_1} + a_{27} e^{2A_2} + a_{28} e^{(A_1+A_2)} + a_{29} e^{B_1} + a_{30} e^{2B_1} + a_{31} e^{(A_1+B_1)} + a_{32} e^{(A_2+B_1)} + a_{33} e^{B_2} + a_{34} e^{2B_2} \\ & + a_{35} e^{(A_1+B_2)} + a_{36} e^{(A_2+B_2)} + a_{37} e^{(B_1+B_2)} + a_{38} e^{2B_5} + a_{39} e^{(A_1+B_5)} + a_{40} e^{(A_2+B_5)} + a_{41} e^{(B_1+B_5)} \\ & + a_{42} e^{(B_2+B_5)} + a_{43} e^{2B_6} + a_{44} e^{(A_1+B_6)} + a_{45} e^{(A_2+B_6)} + a_{46} e^{(B_1+B_6)} + a_{47} e^{(B_2+B_6)} + a_{48} e^{(B_5+B_6)}. \end{aligned}$$

$$B_7 = -B_8 - \sum_{i=1}^{4} a_i,$$

$$B_8 = \frac{\left(a_1 e^{B_1} + a_2 e^{B_2} + a_3 e^{A_1} + a_4 e^{A_2}\right) - (a_1 + a_2 + a_3 + a_4) e^{B_5} - U}{e^{B_5} - e^{B_6}},$$

$$B_{10} = \frac{\left(G_3 - G_2 e^{B_1}\right)}{\left(e^{B_1} - e^{B_2}\right)}; \; B_9 = -\left(G_2 + B_{10}\right), \; B_{12} = \frac{G_6 - G_5 e^{B_5}}{e^{B_5} - e^{B_6}}, \; B_{11} = -\left[B_{12} + G_5\right].$$

2.3 Result and discussion

In order to get clear insight of the physical problem, numerical computations for the representative velocity field, temperature field and concentration field at the plate have been carried out for different values of the parameters.

Figs.(2.1) – (2.2) exhibit the behavior of the velocity field u versus y due to the variations of reaction parameter R and heat sink parameter Q for $Sc = .6, \mathrm{Pr} = .71, M = 2, Gr = 2, Gm = 2, \mathrm{Re} = 2, K = 1, U = 1, R = 2, m = 1, n = 1, Ec = .001, Q = 1$. From these figures we have observed that the velocity of the fluid is retarded due to imposition of chemical reaction and heat sink. The effect of the parameter Q on the temperature field for $Sc = .6, \mathrm{Re} = 1, M = 1, K = 1, Gr = 1, Gm = 1, U = 1, m = 1, R = 1, \mathrm{Pr} = .71, n = 2, Ec = .001$ is depicted in the figure (2.3), it is observed that the temperature profile decreases with the increase of heat sink parameter. The variation of the concentration distribution C versus y under the influence of Schmidt number Sc and chemical reaction parameter R for $Sc = 0.66, \mathrm{Re} = 1, n = 2$ $R = 1$,is presented in the figure (2.4) and (2.5). The concentration decreases as increasing the chemical reaction and Schmidt parameter.

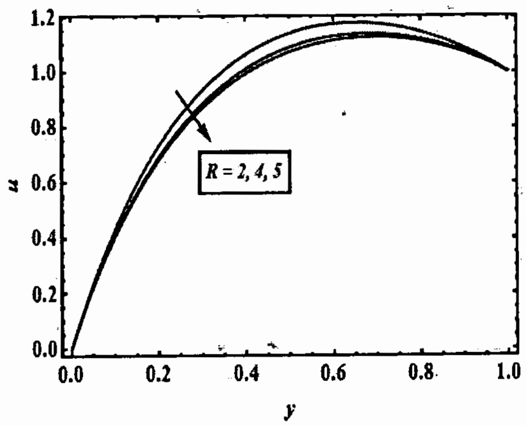

Fig. 2.1: Effect of R on $u(y)$.

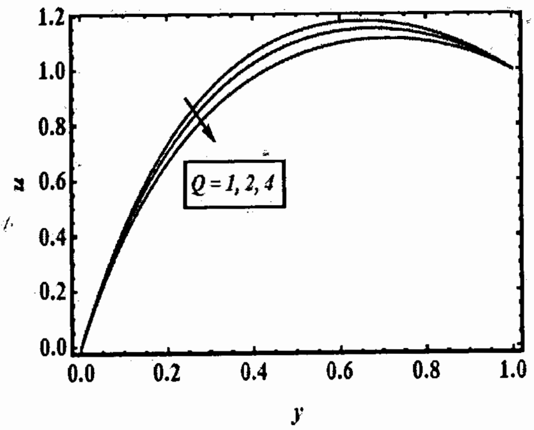

Fig. 2.2: Effect of Q on $u(y)$.

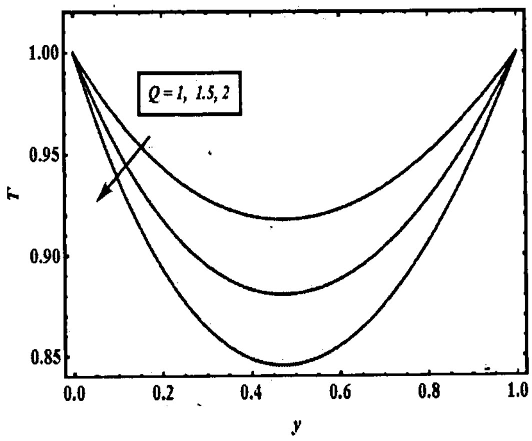

Fig. 2.3: Effect of Q on $T(u)$.

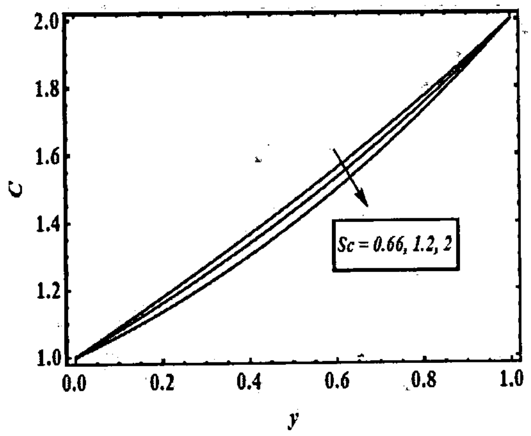

Fig. 2.4: Effect of Sc on $C(y)$.

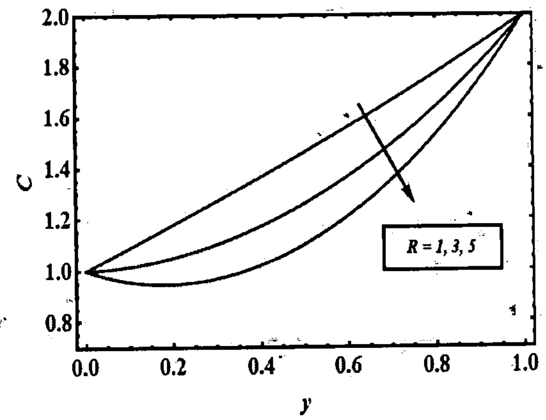

Fig. 2.5: Effect of R on $C(y)$.

Chapter 3

Rotating flow of Newtonian and Maxwell fluid with heat source and chemical reaction

In this chapter we have discussed the rotating flow of Newtonian and Maxwell fluid with heat source or sink and chemical reaction in the presence of viscous dissipation. The coupled non-linear system of equations are solved by the analytical technique Homotopy Perturbation Method (HPM). The comparison of viscous and Maxwell fluid have been made by the help of graphs.

3.1 Problem 1

We consider the steady flow of an incompressible viscous electrically conducting fluid through a porous medium between two porous plates separated by a distance h in presence of magnetic field, heat source or sink and chemical reaction. The plates are rotating along z-axis, with uniform angular velocity Ω'. Both the plates and fluid are rotating along z-axis. The magnetic field B_0 is applied perpendicular to the plates. The induced magnetic field due to the small magnetic Reynolds number is negligible and Eckert number is small.

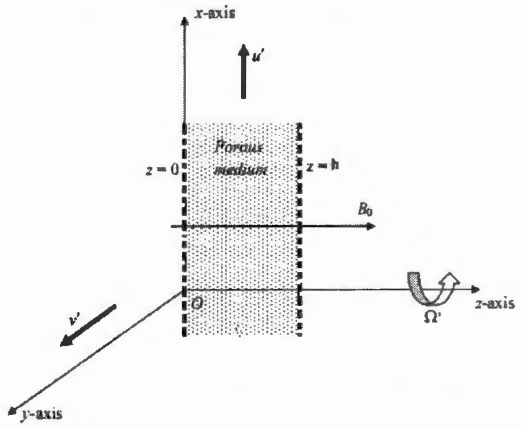

Figure 3.1: Geometery of problem 1

The governing equations for the rotating viscous fluid flow under the Boussinesq's approximation are

$$-w_0\frac{du'}{dz'} - 2\Omega' v' = \nu\frac{d^2u'}{dz'^2} - \frac{\nu u'}{K'} - \frac{\sigma B_0^2}{\rho}u' + g\beta(T' - T_s') + g\beta_c(C' - C_s'). \tag{3.1}$$

$$-w_0\frac{dv'}{dz'} + 2\Omega' u' = \nu\frac{d^2v'}{dz'^2} - \frac{\nu v'}{K'} - \frac{\sigma B_0^2}{\rho}v'. \tag{3.2}$$

The energy and concentration equation in the presence of viscous dissipation, heat source or sink and chemical reaction are in the following form

$$-w_0\frac{dT'}{dz'} = \frac{k}{\rho C_p}\frac{d^2T'}{dz'^2} + \frac{\nu}{C_p}\left[\left(\frac{du'}{dz'}\right)^2 + \left(\frac{dv'}{dz'}\right)^2\right] - Q'(T' - T_s'). \tag{3.3}$$

$$-w_0\frac{dC'}{dz'} = D\frac{d^2C'}{dz'^2} - R'(C' - C_s'). \tag{3.4}$$

The corresponding boundary conditions are

$$\left.\begin{array}{l} u' = 0;\ v' = 0;\ T' = T_0';\ C' = C_0' \text{ at } z' = 0, \\ u' = U_0';\ v' = V_0';\ T' = T_1';\ C' = C_1' \text{ at } z' = h. \end{array}\right\} \tag{3.5}$$

To non-dimensionalize the problem we use the following parameters.

$$\left.\begin{array}{c} z = \frac{z'}{h},\ u = \frac{u'}{w_0}, v = \frac{v'}{w_0}\ \theta = \frac{T'-T_s'}{T_0'-T_s'},\ \phi = \frac{C'-C_s'}{C_0'-C_s'},\ Sc = \frac{\nu}{D},\ \Pr = \frac{\mu C_p}{k},\ \mathrm{Re} = \frac{w_0 h}{\nu}, \\ Gr = \frac{hg\beta(T_0'-T_s')}{w_0^2}, Gm = \frac{hg\beta_c(C_0'-C_s')}{w_0^2}, Ec = \frac{w_0^2}{C_p(T_0'-T_s')}, K = \frac{K'}{h^2},\ M = \frac{\sigma B_0^2 h^2}{\rho\nu}, \\ \Omega = \frac{\Omega' h}{v_0},\ m = \frac{T_1'-T_s'}{T_0'-T_s'}, n = \frac{C_1'-C_s'}{C_0'-C_s'},\ R = \frac{hR'}{w_0},\ Q = \frac{hQ'}{w_0},\ U_0 = \frac{U_0'}{w_0}, V_0 = \frac{V_0'}{w_0}. \end{array}\right\} \tag{3.6}$$

where u' and $v'$are the velocity components in the x' and y' direction respectively, β and β_c are the thermal and concentration volume expansion coefficient respectively, k is the thermal conductivity, C' is the species concentration, T' is the fluid temperature, C_s' and T_s' are the concentration and temperature in static condition respectively, C_p is the specific heat at constant pressure, C_0' and C_1' are the species concentration at the lower plate and upper plate respectively, D is the chemical molecular diffusivity, g is the acceleration due to gravity, h is the distance between two plates, R' is the rate of chemical reaction, Q' is the constant heat sink / source (It may be noted that $Q' < 0$ for heat source and $Q' > 0$ for heat sink), ρ is the fluid density, ν is the kinematic viscosity.

Eqs. (3.1) – (3.5) in non dimensional form are

$$\frac{d^2u}{dz^2} + \mathrm{Re}\frac{du}{dz} + 2\,\mathrm{Re}\,\Omega v - A_5 u + Gr\,\mathrm{Re}\,\theta + Gm\,\mathrm{Re}\,\phi = 0, \tag{3.7}$$

$$\frac{d^2v}{dz^2} + \mathrm{Re}\frac{dv}{dz} - 2\,\mathrm{Re}\,\Omega u - A_5 v = 0, \tag{3.8}$$

$$-\frac{d\theta}{dz} = \frac{1}{\Pr\mathrm{Re}}\frac{d^2\theta}{dz^2} + \frac{Ec}{\mathrm{Re}}\left[\left(\frac{du}{dz}\right)^2 + \left(\frac{dv}{dz}\right)^2\right] - Q\theta, \tag{3.9}$$

$$-\frac{d\phi}{dz} = \frac{1}{Sc\,\mathrm{Re}}\frac{d^2\phi}{dz^2} - R\phi. \tag{3.10}$$

subject to the boundary condition

$$\left.\begin{array}{c} u=0;\ v=0;\ \theta=1;\ \phi=1 \text{ at } z=0, \\ u=U_0;\ v=V_0;\ \theta=m;\ \phi=n \text{ at } z=1. \end{array}\right\} \tag{3.11}$$

3.2 Method of Solution

The solution of equation (3.10) subject to the boundary conditions (3.11) is

$$\phi = A_4 e^{A_1 z} + A_3 e^{A_2 z}, \tag{3.12}$$

where $A_1 = \dfrac{-Sc\,\mathrm{Re} + \sqrt{Sc^2\,\mathrm{Re}^2 + 4R\,\mathrm{Re}\,Sc}}{2}$, $A_2 = \dfrac{-Sc\,\mathrm{Re} - \sqrt{Sc^2\,\mathrm{Re}^2 + 4R\,\mathrm{Re}\,Sc}}{2}$,

$$A_3 = \frac{n - e^{A_1}}{e^{A_2} - e^{A_1}}, A_4 = 1 - A_3.$$

Solving equations (3.7)-(3.9) by homotopy perturbation method, the following homotopies are defined as

$$H_1(\widetilde{u}, p) = (1-p)\left(\frac{d^2\widetilde{u}}{dz^2} - \frac{d^2 u_0}{dz^2}\right) + p\left[\begin{array}{c} \frac{d^2\widetilde{u}}{dz^2} + \mathrm{Re}\,\frac{d\widetilde{u}}{dz} + 2\,\mathrm{Re}\,\Omega\widetilde{v} - A_5\widetilde{u} \\ +Gr\,\mathrm{Re}\,\widetilde{\theta} + Gm\,\mathrm{Re}\,\phi \end{array}\right] = 0, \tag{3.13}$$

$$H_2(\widetilde{v}, p) = (1-p)\left(\frac{d^2\widetilde{v}}{dz^2} - \frac{d^2 v_0}{dz^2}\right) + p\left[\frac{d^2\widetilde{v}}{dz^2} + \mathrm{Re}\,\frac{d\widetilde{v}}{dz} - 2\,\mathrm{Re}\,\Omega\widetilde{u} - A_5\widetilde{v}\right] = 0, \tag{3.14}$$

$$H_3\left(\widetilde{\theta}, p\right) = (1-p)\left(\frac{d^2\widetilde{\theta}}{dz^2} - \frac{d^2\theta_0}{dz^2}\right) + p\left[\begin{array}{c} \frac{d^2\widetilde{\theta}}{dz^2} + \mathrm{Pr}\,\mathrm{Re}\,\frac{d\widetilde{\theta}}{dz} - Q\,\mathrm{Pr}\,\mathrm{Re}\,\widetilde{\theta} \\ +Ec\,\mathrm{Pr}\left[\left(\frac{d\widetilde{u}}{dz}\right)^2 + \left(\frac{d\widetilde{v}}{dz}\right)^2\right] \end{array}\right] = 0, \tag{3.15}$$

where $p \in [0, 1]$ is the embedding parameter.

Assume the solution in the following form

$$\widetilde{u} = \sum_{n=0}^{\infty} \widetilde{u}_n p^n,\ \widetilde{v} = \sum_{n=0}^{\infty} \widetilde{v}_n p^n,\ \widetilde{\theta} = \sum_{n=0}^{\infty} \widetilde{\theta}_n p^n, \tag{3.16}$$

where

$$N(\widetilde{u}, y) = \left[\left(\frac{d\widetilde{u}}{dz}\right)^2 + \left(\frac{d\widetilde{v}}{dz}\right)^2\right] = \sum_{n=0}^{\infty} B_n p^n,$$

$$B_n = \frac{1}{n!}\frac{d^n}{dp^n}\left[N\left(\sum_{i=0}^{n}\widetilde{u}_i p^i, \sum_{i=0}^{n}\widetilde{v}_i p^i\right)\right]_{p=0}, \tag{3.17}$$

using equations (3.16) in (3.13) – (3.15), and equating the coefficient of like powers of p we get

p^0 :

$$\left.\begin{aligned} &\widetilde{u}_0''(z) = u_0''(z), \quad \widetilde{u}_0(0) = 0, \ \widetilde{u}_0(1) = U_0, \\ &\widetilde{v}_0''(z) = v_0''(z), \quad \widetilde{v}_0(0) = 0, \ \widetilde{v}_0(1) = V_0, \\ &\widetilde{\theta}_0''(z) = \theta_0''(z), \ \widetilde{\theta}_0(0) = 1, \ \widetilde{\theta}_0(1) = m. \end{aligned}\right\} \tag{3.18}$$

p^1 :

$$\left.\begin{aligned} &\widetilde{u}_1''(z) + \mathrm{Re}\,\widetilde{u}_0'(z) + 2\,\mathrm{Re}\,\Omega\widetilde{v}_0(z) - A_5\widetilde{u}_0(z) \\ &\quad + \mathrm{Re}\left(Gr\widetilde{\theta}_0(z) + Gm\phi(z)\right) = 0, \\ &\widetilde{v}_1''(z) + \mathrm{Re}\,\widetilde{v}_0'(z) - 2\,\mathrm{Re}\,\Omega\widetilde{u}_0(z) - A_5\widetilde{v}_0(z) = 0, \\ &\widetilde{\theta}_0''(z) + \mathrm{Pr}\,\mathrm{Re}\,\widetilde{\theta}_0'(z) - Q\,\mathrm{Pr}\,\mathrm{Re}\,\widetilde{\theta}_0(z) + Ec\,\mathrm{Pr}\,B_1 = 0, \\ &\widetilde{u}_1(0) = 0, \ \widetilde{u}_1(1) = 0, \\ &\widetilde{v}_1(0) = 0, \ \widetilde{v}_1(1) = 0, \\ &\widetilde{\theta}_1(0) = 1, \ \widetilde{\theta}_1(1) = 0. \end{aligned}\right\} \tag{3.19}$$

p^2 :

$$\left.\begin{aligned} &\widetilde{u}_2''(z) + \mathrm{Re}\,\widetilde{u}_1'(z) + 2\,\mathrm{Re}\,\Omega\widetilde{v}_1(z) - A_5\widetilde{u}_1(z) + Gr\,\mathrm{Re}\,\widetilde{\theta}_1(z) = 0, \\ &\widetilde{v}_2''(z) + \mathrm{Re}\,\widetilde{v}_1'(z) - 2\,\mathrm{Re}\,\Omega\widetilde{u}_1(z) - A_5\widetilde{v}_1(z) = 0, \\ &\widetilde{\theta}_2''(z) + \mathrm{Pr}\,\mathrm{Re}\,\widetilde{\theta}_1'(z) - Q\,\mathrm{Pr}\,\mathrm{Re}\,\widetilde{\theta}_1(z) + Ec\,\mathrm{Pr}\,B_2 = 0, \\ &\widetilde{u}_2(0) = 0, \ \widetilde{u}_2(1) = 0, \\ &\widetilde{v}_2(0) = 0, \ \widetilde{v}_2(1) = 0, \\ &\widetilde{\theta}_2(0) = 1, \ \widetilde{\theta}_2(1) = 0. \end{aligned}\right\} \tag{3.20}$$

Solving equations (3.18) – (3.20), we obtain the series solution for velocity profile and temperature distribution as follows

$$\left.\begin{aligned} u &= \lim_{p\to 1}\left(\widetilde{u}_0 + p\widetilde{u}_1 + p^2\widetilde{u}_2 + ...\right), \\ v &= \lim_{p\to 1}\left(\widetilde{v}_0 + p\widetilde{v}_1 + p^2\widetilde{v}_2 + ...\right), \\ \theta &= \lim_{p\to 1}\left(\widetilde{\theta}_0 + p\widetilde{\theta}_1 + p^2\widetilde{\theta}_2 + ...\right), \end{aligned}\right\}$$

where

$$\begin{aligned} \widetilde{u}_0 &= U_0 z, \\ \widetilde{v}_0 &= V_0 z, \\ \widetilde{\theta}_0 &= 1 + (m-1)z. \end{aligned}$$

$$\begin{aligned} \widetilde{u}_1 &= a_1 + a_2 z + a_3 z^2 + a_4 z^3 - a_5 e^{A_1 z} - a_6 e^{A_2 z}, \\ \widetilde{v}_1 &= a_7 z - a_8 z^2 + a_9 z^3, \\ \widetilde{\theta}_1 &= a_{10} z + a_{11} z^2 + a_{12} z^3. \end{aligned}$$

$$\begin{aligned} \widetilde{u}_2 &= a_{13} + a_{14} z + a_{15} z^2 + a_{16} z^3 + a_{17} z^4 + a_{18} z^5 - a_{19} e^{A_1 z} - a_{20} e^{A_2 z}, \\ \widetilde{v}_2 &= a_{21} + a_{22} z - a_{23} z^2 + a_{24} z^3 + a_{25} z^4 + a_{26} z^5 - \frac{(2A_2 a_5 \Pr \mathrm{Re}\,\Omega)}{A_1} e^{A_1 z} \\ &\quad - \frac{(2A_1 a_5 \Pr \mathrm{Re}\,\Omega)}{A_2} e^{A_2 z}, \\ \widetilde{\theta}_2 &= -\frac{(2(A_2 a_5 + A_1 a_6) Ec \Pr U_0)}{A_1 A_2} + a_{27} z + a_{28} z^2 + a_{29} z^3 + a_{30} z^4 \\ &\quad + \frac{a_{12} \Pr Q \,\mathrm{Re}}{20} z^5 + \frac{(2a_5 Ec \Pr U_0)}{A_1} e^{A_1 z} + \frac{(2a_6 Ec \Pr U_0)}{A_2} e^{A_2 z}. \end{aligned}$$

where

$$a_1 = \left(\frac{A_3}{A_2^2} + \frac{A_4}{A_1^2}\right) Gm \,\mathrm{Re},$$

$$a_2 = \frac{1}{6A_1^2 A_2^2}\left(\begin{array}{c} -A_1^2 A_2^2 (A_5 - 3\,\mathrm{Re}) U_0 + \mathrm{Re}(6A_2^2 A_4(-1 + e^{A_1}) Gm \\ +A_1^2(6A_3(-1 + e^{A_2}) Gm + A_2^2 Gr(2+m)) + 2A_1^2 A_2^2 \Omega V_0 \end{array}\right),$$

$$a_3 = -\frac{\mathrm{Re}(Gr+U_0)}{2}, a_4 = \frac{A_5U_0+\mathrm{Re}(Gr-Grm-2\Omega V_0)}{6}, a_5 = \frac{A_4 Gm\,\mathrm{Re}}{A_1^2},$$

$$a_6 = \frac{A_3 Gm\,\mathrm{Re}}{A_2^2}, a_7 = -\frac{(2\,\mathrm{Re}\,\Omega U_0+(A5-3\,\mathrm{Re})V_0)}{6}, a_8 = -\frac{\mathrm{Re}\,V_0}{2},$$

$$a_9 = \frac{(2\,\mathrm{Re}\,\Omega U_0+A_5V_0)}{6}, a_{10} = -\frac{Pr((3+m(-3+Q)+2Q)\,\mathrm{Re}-3EcV_0^2)}{6},$$

$$a_{11} = -\frac{Pr((-1+m-Q)\,\mathrm{Re}+Ec(V_0^2+U_0^2))}{2}, a_{12} = \frac{(-1+m)\,\mathrm{Pr}\,Q\,\mathrm{Re}}{6},$$

$$a_{13} = \frac{(a_5A_5)}{A_1^2} - \frac{(a_5\,\mathrm{Re})}{A_1} + \frac{a_6(A_5-A_2\,\mathrm{Re})}{A_2^2},$$

$$a_{14} = \frac{1}{60A_1^2A_2^2}\begin{pmatrix} 60A_2^2a_5A_5(-1+e^{A_1}) - 60A_1A_2^2a_5(-1+e^{A_1})\,\mathrm{Re}+A_1^2(60A_5a_6(-1+e^{A2}) \\ -10a_2A_2^2(A_5-3\,\mathrm{Re}) - 60A_2a_6(-1+e^{A_2})\,\mathrm{Re}+A_2^2(-3a_4(A_5-5\,\mathrm{Re}) \\ -5a_3(A_5-4\,\mathrm{Re}) + (10a_{10}+5a_{11}+3a_{12})Gr\,\mathrm{Re}) + (10a_8\,\mathrm{Re}+6a_9\,\mathrm{Re})\,\Omega \\ -30a_1A_1^2A_2^2A_5. \end{pmatrix},$$

$$a_{15} = \frac{(a_1A_5-a_2\,\mathrm{Re})}{2},\; a_{16} = \frac{(a_2A_5-(2a_3+2a_7\Omega+a_{10}Gr)\,\mathrm{Re})}{6},$$

$$a_{17} = \frac{a_3A_5-\mathrm{Re}(3a_4+a_{11}Gr-2a_8\Omega)}{12},\; a_{18} = \frac{a_4A_5-\mathrm{Re}(a_{12}Gr+2a_9\Omega)}{12},$$

$$a_{19} = \frac{a_5(-A_5+A_1\,\mathrm{Re})}{A_1^2},\; a_{20} = \frac{a_6(-A_5+A_2\,\mathrm{Re})}{A_2^2},\; a_{21} = 2\left(\frac{a_5}{A_1^2}+\frac{a_6}{A_2^2}\right)\mathrm{Re}\,\Omega,$$

$$a_{22} = -\frac{1}{60A_1^2A_2^2}\begin{pmatrix} -120A_2^2a_5(-1+e^{A_1})\,\mathrm{Re}\,\Omega + A_1^2(-120a_6(-1+e^{A_2})\,\mathrm{Re}\,\Omega \\ +A_2^2(A_5(10a_7-5a_8+3a_9)+\mathrm{Re}\,(-30a_7+20a_8-15a_9+60a_1\Omega \\ +20a_2\Omega+10a_3\Omega+6a_4\Omega) \end{pmatrix},$$

$$a_{23} = -\frac{\mathrm{Re}(a_7-2a_1\Omega)}{2},\; a_{24} = \frac{A_5a_7+2\,\mathrm{Re}(a_8+a_2\Omega)}{6},$$

$$a_{25} = -\frac{(A5a8+3a_9\,\mathrm{Re}-2a_3\,\mathrm{Re}\,\Omega)}{12},\; a_{26} = \frac{A_5a_9+2a_4\,\mathrm{Re}\,\Omega}{12},$$

$$a_{27} = \frac{-1}{60A_1A_2}\begin{pmatrix} \mathrm{Pr}(A_1A_2(3a_{12}(-5+Q)+5a_{11}(-4+Q)+10a_{10}(-3+Q))\,\mathrm{Re} \\ -10(-12A_2a_5(-1+e^{A_1})+A_1\,(6a_2A_2+4A_2a_3+3A_2a_4 \\ +12a_6-12a_6e^{A2})\;EcU_0 \end{pmatrix},$$

$$a_{28} = -\frac{\mathrm{Pr}(a_{10}Re+2a_2EcU_0+2a_7EcV_0)}{2}, a_{29} = -\frac{\mathrm{Pr}(2a_{11}\,\mathrm{Re}-a_{10}Q\,\mathrm{Re}+4a_3EcU_0-4a_8EcV_0)}{6},$$

$$a_{30} = -\frac{\Pr(3a_{12}\,\mathrm{Re} - a_{11} Q\,\mathrm{Re} + 6a_4 EcU_0 + 6a_9 EcV_0)}{12},$$

3.3 Problem 2

We consider the steady MHD Maxwell incompressible electrically conducting fluid between two porous plates separated by a distance h in presence of magnetic field and chemical reaction. The plates are rotating along z-axis, with uniform angular velocity Ω'. Both the plates and fluid are rotating along z-axis. The magnetic field B_0 is applied perpendicular to the plates. The induced magnetic field due to the small magnetic Reynolds number is negligible and Eckert number is small.

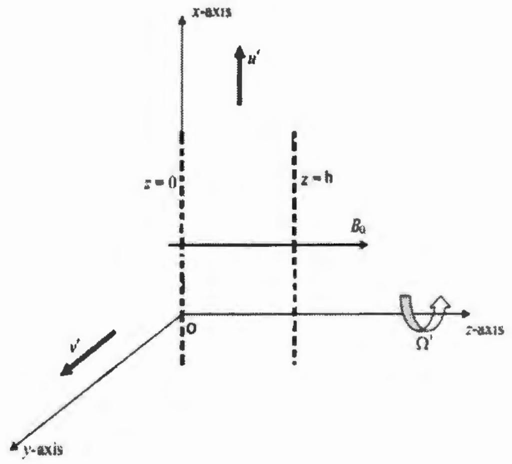

Figure 3.2 : Geometery of problem 2.

The continuity and momentum equation for the maxwell fluid are as follows

$$\frac{du'}{dx'} + \frac{dv'}{dy'} + \frac{dw'}{dz'} = 0. \tag{3.21}$$

$$\rho\left[\frac{d\mathbf{V}}{dt} + 2\mathbf{\Omega}' \times \mathbf{v} + \mathbf{\Omega}' \times (\mathbf{\Omega}' \times \mathbf{r})\right] = \operatorname{div}\mathbf{T} + \mathbf{J} \times \mathbf{B} + \rho g\beta(T' - T_s') + \rho g\beta_c(C' - C_s'). \tag{3.22}$$

where Cauchy stress T is defined as

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S}, \tag{3.23}$$

and extra stress tensor for the maxwell fluid is defined by the following expression

$$(1+\lambda\frac{D}{Dt})\mathbf{S}=\mu\mathbf{A}_1, \tag{3.24}$$

$$\mathbf{A}_1=\nabla\mathbf{v}+(\nabla\mathbf{v})^T, \tag{3.25}$$

$$\frac{D\mathbf{S}}{Dt}=(\mathbf{v}.\nabla)\mathbf{S}-(\nabla\mathbf{v})\mathbf{S}-\mathbf{S}(\nabla\mathbf{v})^T.$$

Stress tensor and velocity components in 3-dimension are defined as

$$\mathbf{S}(z)=\begin{pmatrix} S'_{xx} & S'_{xy} & S'_{xz} \\ S'_{yx} & S'_{yy} & S'_{yz} \\ S'_{zx} & S'_{zy} & S'_{zz} \end{pmatrix}, \qquad \mathbf{V}=\begin{pmatrix} u' \\ v' \\ -w'_0 \end{pmatrix}. \tag{3.26}$$

Using Eqs. (3.24) – (3.26), we obtain the following equations

$$\left(1-\lambda w'_0\frac{d}{dz'}\right)S'_{xx}-2u'_zS'_{zx}=0, \tag{3.27}$$

$$\left(1-\lambda w'_0\frac{d}{dz'}\right)S'_{xy}-\left(u'_zS'_{zy}+v'_zS'_{zx}\right)=0, \tag{3.28}$$

$$\left(1-\lambda w'_0\frac{d}{dz'}\right)S'_{xz}-u'_zS'_{zz}=\mu u'_z, \tag{3.29}$$

$$\left(1-\lambda w'_0\frac{d}{dz'}\right)S'_{yy}-2v'_zS'_{zy}=0, \tag{3.30}$$

$$\left(1-\lambda w'_0\frac{d}{dz'}\right)S'_{yz}-v'_zS'_{zz}=\mu v'_z, \tag{3.31}$$

$$\left(1-\lambda w'_0\frac{d}{dz'}\right)S'_{zz}=0. \tag{3.32}$$

From Eq. (3.32) $S'_{zz}=0$ use this in Eqs. (3.29) and (3.31) we get

$$\left.\begin{aligned} S'_{xz}-\lambda w'_0\tfrac{dS'_{xz}}{dz'}&=\mu u'_z, \\ S'_{yz}-\lambda w'_0\tfrac{dS'_{yz}}{dz'}&=\mu v'_z. \end{aligned}\right\} \tag{3.33}$$

so equation (3.22) in component form is given by the following expressions

$$-w_0\frac{du'}{dz'} - 2\Omega' v' = \frac{1}{\rho}\frac{dS'_{xz}}{dz'} - \frac{\sigma B_0^2}{\rho}u' + g\beta(T' - T'_s) + g\beta_c(C' - C'_s), \tag{3.34}$$

$$-w_0\frac{dv'}{dz'} + 2\Omega' u' = \frac{1}{\rho}\frac{dS'_{yz}}{dz'} - \frac{\sigma B_0^2}{\rho}v'. \tag{3.35}$$

From Eqs. (3.34) – (3.35) we find the values of $\frac{dS'_{xz}}{dz'}$ and $\frac{dS'_{yz}}{dz'}$ and use this in Eq (3.33) we get the value of S'_{xz} and S'_{yz}

$$S'_{xz} = \rho\lambda' w_0\left[-w_0\frac{du'}{dz'} - 2\Omega' v' + \frac{\sigma B_0^2}{\rho}u' - g\beta(T' - T'_s) - g\beta_c(C' - C'_s)\right] + \mu\frac{du'}{dz'}, \tag{3.36}$$

$$S'_{yz} = \rho\lambda' w_0\left[-w_0\frac{dv'}{dz'} + 2\Omega' u' + \frac{\sigma B_0^2}{\rho}u'\right] + \mu\frac{dv'}{dz'}. \tag{3.37}$$

The energy and concentration equation with heat source and chemical reactions are as follows

$$-w_0\frac{dT'}{dz'} = \frac{k}{\rho C_p}\frac{d^2T'}{dz'^2} + \frac{1}{\rho C_p}\left[\frac{du'}{dz'}S'_{xz} + \frac{dv'}{dz'}S'_{yz}\right] - Q'(T' - T'_s), \tag{3.38}$$

$$-w_0\frac{dC'}{dz'} = D\frac{d^2C'}{dz'^2} - R'(C' - C'_s). \tag{3.39}$$

The corresponding boundary conditions are

$$\left.\begin{array}{l} u' = 0;\ v' = 0;\ T' = T'_0;\ C' = C'_0 \text{ at } z' = 0, \\ u' = U'_0;\ v' = V'_0;\ T' = T'_1;\ C' = C'_1 \text{ at } z' = h. \end{array}\right\} \tag{3.40}$$

We introduce the following non-dimensional quantities

$$\left.\begin{array}{c} z = \frac{z'}{h},\ u = \frac{u'}{w_0},\ v = \frac{v'}{w_0},\ S_{xz} = \frac{hS'_{xz}}{\mu w_0},\ S_{yz} = \frac{hS'_{yz}}{\mu w_0},\ \theta = \frac{T' - T'_s}{T'_0 - T'_s},\ \phi = \frac{C' - C'_s}{C'_0 - C'_s}, \\ Sc = \frac{\nu}{D}, \mathrm{Pr} = \frac{\mu C_p}{k},\ \mathrm{Re} = \frac{w_0 h}{\nu},\ Gr = \frac{hg\beta(T'_0 - T'_s)}{w_0^2},\ Gm = \frac{hg\beta_c(C'_0 - C'_s)}{w_0^2}, \\ Ec = \frac{w_0^2}{C_p(T'_0 - T'_s)}, M = \frac{\sigma B_0^2 h^2}{\rho\nu},\ m = \frac{T'_1 - T'_s}{T'_0 - T'_s},\ n = \frac{C'_1 - C'_s}{C'_0 - C'_s},\ R = \frac{hR'}{w_0},\ Q = \frac{hQ'}{w_0}, \\ \lambda = \frac{\lambda' w_0}{h},\ \Omega = \frac{\Omega' h}{w_0}, U_0 = \frac{U'_0}{w_0},\ V_0 = \frac{V'_0}{w_0}. \end{array}\right\} \tag{3.41}$$

Where u' and v' are the velocity components in the x' and y' direction respectively, β and β_c are the thermal and concentration volume expansion coefficient respectively, k is the thermal conductivity, C' is the species concentration, T' is the fluid temperature, C'_s and T'_s are the concentration and temperature in static condition respectively, C_p is the specific heat at constant pressure, C'_0 and C'_1 are the species concentration at the lower plate and upper plate respectively, D is the chemical molecular diffusivity, g is the acceleration due to gravity, h is the distance between two plates, R' is the rate of chemical reaction, Q' is the constant heat sink / source (It may be noted that $Q' < 0$ for heat source and $Q' > 0$ for heat sink), ρ is the fluid density, ν is the kinematic viscosity.

3.4 Method of Solution

The non dimensional form of Eqs. (3.34) – (3.40) is given by the following equations

$$-\frac{d\phi}{dz} = \frac{1}{Sc\,\mathrm{Re}}\frac{d^2\phi}{dz^2} - R\phi, \tag{3.42}$$

where

$$\phi = A_4 e^{A_1 z} + A_3 e^{A_2 z}. \tag{3.43}$$

$$A_1 = \frac{-Sc\,\mathrm{Re} + \sqrt{Sc^2\,\mathrm{Re}^2 + 4R\,\mathrm{Re}\,Sc}}{2}, \quad A_2 = \frac{-Sc\,\mathrm{Re} - \sqrt{Sc^2\,\mathrm{Re}^2 + 4R\,\mathrm{Re}\,Sc}}{2},$$

$$A_3 = \frac{n - e^{A_1}}{e^{A_2} - e^{A_1}}, \quad A_4 = 1 - A_3.$$

$$\frac{d^2u}{dz^2} - A_6\frac{du}{dz} + A_7 u - A_8 v + A_9\frac{dv}{dz} = A_{10}\theta - A_{11}\frac{d\theta}{dz} + A_{12}\phi - A_{13}\frac{d\phi}{dz}, \tag{3.44}$$

$$\frac{d^2v}{dz^2} - A_6\frac{dv}{dz} + A_7 v + A_8 u - A_9\frac{du}{dz} = 0, \tag{3.45}$$

$$-\frac{d\theta}{dz} = \frac{1}{\mathrm{Pr}\,\mathrm{Re}}\frac{d^2\theta}{dz^2} + \frac{Ec}{\mathrm{Re}}\left[\frac{du}{dz}\mathbf{S}_{xz} + \frac{dv}{dz}\mathbf{S}_{yz}\right] - Q\theta, \tag{3.46}$$

Subject to the boundary condition

$$\left.\begin{aligned} &u = 0, v = 0,\ \theta = 1,\ \phi = 1 \text{ at } z = 0, \\ &u = U_0, v = V_0,\ \theta = m,\ \phi = n \text{ at } z = 1. \end{aligned}\right\} \tag{3.47}$$

where

$$A_6 = \frac{\mathrm{Re}+\lambda M}{\mathrm{Re}\,\lambda-1},\ A_7=\frac{M}{\mathrm{Re}\,\lambda-1},\ A_8=\frac{2\Omega\,\mathrm{Re}}{\mathrm{Re}\,\lambda-1},\ A_9=\lambda A_8,\ A_{10}=\frac{\mathrm{Re}\,Gr}{\mathrm{Re}\,\lambda-1},$$
$$A_{11} = \lambda A_{10},\ A_{12}=\frac{Gm\,\mathrm{Re}}{\mathrm{Re}\,\lambda-1},\ A_{13}=\lambda A_{12}.$$

$$S_{xz}=(1-\lambda\,\mathrm{Re})\frac{du}{dz}+\lambda Mu-2\,\mathrm{Re}\,\lambda\Omega v-\lambda Gr\,\mathrm{Re}\,\theta-\lambda Gm\,\mathrm{Re}\,\phi,$$

$$S_{yz}=(1-\lambda\,\mathrm{Re})\frac{dv}{dz}+\lambda Mv+2\,\mathrm{Re}\,\lambda\Omega u.$$

Solving equations (3.43) to(3.45) by Homotopy Perturbation Method, the following homotopies are defined as

$$H_1(\widetilde{u},p)=(1-p)\left(\frac{d^2\widetilde{u}}{dz^2}-\frac{d^2u_0}{dz^2}\right)+p\begin{bmatrix}\frac{d^2\widetilde{u}}{dz^2}-A_6\frac{d\widetilde{u}}{dz}+A_7\widetilde{u}-A_8y+A_9\frac{dy}{dz}\\ -A_{10}\widetilde{\theta}+A_{11}\frac{d\theta}{dz}-A_{12}\phi+A_{13}\frac{d\phi}{dz}\end{bmatrix}=0, \tag{3.48}$$

$$H_2(\widetilde{v},p)=(1-p)\left(\frac{d^2\widetilde{v}}{dz^2}-\frac{d^2v_0}{dz^2}\right)+p\left[\frac{d^2\widetilde{v}}{dz^2}-A_6\frac{d\widetilde{v}}{dz}+A_7\widetilde{v}-A_8\widetilde{u}+A_9\frac{d\widetilde{u}}{dz}\right]=0, \tag{3.49}$$

$$H_3\left(\widetilde{\theta},p\right)=(1-p)\left(\frac{d^2\widetilde{\theta}}{dz^2}-\frac{d^2\theta_0}{dz^2}\right)+p\begin{bmatrix}\frac{d^2\widetilde{\theta}}{dz^2}+\Pr\mathrm{Re}\frac{d\widetilde{\theta}}{dz}-Q\Pr\mathrm{Re}\,\widetilde{\theta}\\ +Ec\Pr\left(N(\widetilde{u},\widetilde{v},\widetilde{\theta})\right)\end{bmatrix}=0. \tag{3.50}$$

where

$$N(\widetilde{u},\widetilde{v},\widetilde{\theta}) = (1-\lambda\,\mathrm{Re})\left(\left(\frac{d\widetilde{u}}{dz}\right)^2+\left(\frac{d\widetilde{v}}{dz}\right)^2\right)+\lambda M\left(\widetilde{u}\frac{d\widetilde{u}}{dz}+\widetilde{v}\frac{d\widetilde{v}}{dz}\right)$$
$$+2\,\mathrm{Re}\,\lambda\Omega\left(\widetilde{u}\frac{d\widetilde{v}}{dz}-\widetilde{v}\frac{d\widetilde{u}}{dz}\right)-\lambda Gr\,\mathrm{Re}\,\widetilde{\theta}\frac{d\widetilde{u}}{dz}-\lambda Gm\,\mathrm{Re}\,\phi\frac{d\widetilde{u}}{dz}.$$

where $p \in [0,1]$ is the embedding parameter.

Assume the solution in the form of

$$\widetilde{u}=\sum_{n=0}^{\infty}\widetilde{u}_np^n,\ \widetilde{v}=\sum_{n=0}^{\infty}\widetilde{v}_np^n,\ \widetilde{\theta}=\sum_{n=0}^{\infty}\widetilde{\theta}_np^n,\ N(\widetilde{u},\widetilde{v},\widetilde{\theta})=\sum_{n=0}^{\infty}B_np^n, \tag{3.51}$$

where

$$B_n=\frac{1}{n!}\frac{d^n}{dp^n}\left[N\left(\sum_{i=0}^{n}\widetilde{u}_ip^i,\sum_{i=0}^{n}\widetilde{v}_ip^i,\sum_{i=0}^{n}\widetilde{\theta}_ip^i\right)\right]_{p=0}.$$

using equation (3.51) in (3.48) - (3.50), and equating the coefficient of like powers of p we get:

p^0 :

$$\left.\begin{array}{l}\widetilde{u}_0''(z) = u_0''(z), \quad \widetilde{u}_0(0) = 0, \ \widetilde{u}_0(1) = U_0, \\ \widetilde{v}_0''(z) = v_0''(z), \quad \widetilde{v}_0(0) = 0, \ \widetilde{v}_0(1) = V_0, \\ \widetilde{\theta}_0''(z) = \theta_0''(z), \ \widetilde{\theta}_0(0) = 1, \ \widetilde{\theta}_0(1) = m.\end{array}\right\} \tag{3.52}$$

p^1 :

$$\left.\begin{array}{c}\widetilde{u}_1''(z) - A_6\frac{d\widetilde{u}_0}{dz} + A_7\widetilde{u}_0 - A_8\widetilde{v}_0 + A_9\frac{d\widetilde{v}_0}{dz} \\ -A_{10}\widetilde{\theta}_0 + A_{11}\frac{d\widetilde{\theta}_0}{dz} - A_{12}\phi + A_{13}\frac{d\phi}{dz} = 0, \\ \widetilde{v}_1''(z) - A_6\frac{d\widetilde{v}_0}{dz} + A_7\widetilde{v}_0 - A_8\widetilde{u}_0 + A_9\frac{d\widetilde{u}_0}{dz} = 0, \\ \widetilde{\theta}_0''(z) + \Pr\mathrm{Re}\,\widetilde{\theta}_0'(z) - Q\Pr\mathrm{Re}\,\widetilde{\theta}_0(z) + Ec\Pr B_1 = 0, \\ \widetilde{u}_1(0) = 0, \ \widetilde{u}_1(1) = 0, \\ \widetilde{v}_1(0) = 0, \ \widetilde{v}_1(1) = 0, \\ \widetilde{\theta}_1(0) = 1, \ \widetilde{\theta}_1(1) = 0.\end{array}\right\} \tag{3.53}$$

p^2 :

$$\left.\begin{array}{c}\widetilde{u}_2''(z) - A_6\frac{d\widetilde{u}_1}{dz} + A_7\widetilde{u}_1 \\ -A_8\widetilde{v}_1 + A_9\frac{d\widetilde{v}_1}{dz} - A_{10}\widetilde{\theta}_1 + A_{11}\frac{d\widetilde{\theta}_1}{dz} = 0, \\ \widetilde{v}_2''(z) - A_6\frac{d\widetilde{v}_1}{dz} + A_7\widetilde{v}_1 - A_8\widetilde{u}_1 + A_9\frac{d\widetilde{u}_1}{dz} = 0, \\ \widetilde{\theta}_2''(z) + \Pr\mathrm{Re}\,\widetilde{\theta}_1'(z) - Q\Pr\mathrm{Re}\,\widetilde{\theta}_1(z) + Ec\Pr B_2 = 0, \\ \widetilde{u}_2(0) = 0, \ \widetilde{u}_2(1) = 0, \\ \widetilde{v}_2(0) = 0, \ \widetilde{v}_2(1) = 0, \\ \widetilde{\theta}_2(0) = 1, \ \widetilde{\theta}_2(1) = 0.\end{array}\right\} \tag{3.54}$$

Solving equation (3.52) – (3.53) we obtain the series solution for velocity profile and temperature distribution as follows

$$\left.\begin{aligned} u &= \lim_{p\to 1}\left(\widetilde{u}_0 + p\widetilde{u}_1 + p^2\widetilde{u}_2 + ...\right), \\ v &= \lim_{p\to 1}\left(\widetilde{v}_0 + p\widetilde{v}_1 + p^2\widetilde{v}_2 + ...\right), \\ \theta &= \lim_{p\to 1}\left(\widetilde{\theta}_0 + p\widetilde{\theta}_1 + p^2\widetilde{\theta}_2 + ...\right), \end{aligned}\right\} \tag{3.55}$$

where

$$\begin{aligned} \widetilde{u}_0 &= U_0 z, \\ \widetilde{v}_0 &= V_0 z, \\ \widetilde{\theta}_0 &= 1 + (m-1)z. \end{aligned}$$

$$\begin{aligned} \widetilde{u}_1 = {} & \frac{A_{13}A_3}{A_2} - \frac{A_{12}A_3}{A_2^2} - \frac{A_{12}A_4}{A_1^2} + \frac{A_{13}A_4}{A_1} + \left(\frac{A_{12}A_4}{A_1^2} - \frac{A_{13}A_4}{A_1}\right)e^{A_1 z} \\ & + \left(\frac{A_{12}A_3}{A_2^2} - \frac{A_{13}A_3}{A_2}\right)e^{A_2 z} + (-\frac{A_{10}}{3} - \frac{A_{11}}{2} + \frac{A_{12}A_3}{A_2^2} - \frac{A_{13}A_3}{A_2} + \frac{A_{12}A_4}{A_1^2} \\ & - \frac{A_{13}A_4}{A_1} - \frac{A_{12}A_4 e^{A_1}}{A_1^2} + \frac{A_{13}A_4 e^{A_1}}{A_1} - \frac{A_{12}A_3 e^{A_2}}{A_2^2} + \frac{A_{13}A_3 e^{A_2}}{A_2} - A_{10}m + 3A_{11}m \\ & + (-\frac{A_6}{2} + \frac{A_7}{6})U_0 - \frac{(A_8 - 3A_9)V_0}{6})z + \frac{1}{2}(A_{10} + A_{11} - A_{11}m + A_6 U_0 - A_9 V_0)z^2 \\ & \frac{1}{6}(A_{10}(-1+m) - A_7 U_0 + A_8 V_0)z^3. \end{aligned}$$

$$\widetilde{v}_1 = \frac{1}{6}((A_8U_0 - 3A_9U_0 - 3A_6V_0 + A_7V_0)\,z + (3A_9U_0 + 3A_6V_0)\,z^2 - (A_8U_0 + A_7V_0)\,z^3).$$

$$\begin{aligned} \widetilde{\theta}_1 = {} & \left(\frac{A_4 e^{A_1 z}}{A_1^2} + \frac{A_3 e^{A_2 z}}{A_2^2}\right) EcGm \Pr \operatorname{Re} U_0 \lambda - \frac{(A_1^2 A_3 + A_2^2 A_4) EcGm \Pr \operatorname{Re} U_0 \lambda}{A_1^2 A_2^2} \\ & - \frac{1}{6A_1^2A_2^2} Pr \begin{pmatrix} 6A_2^2A_4(-1+e^{A_1})EcGm \operatorname{Re} U_0\lambda + A_1^2(6A_3(-1+e^{A_2})EcGm \operatorname{Re} U_0\lambda \\ +A_2^2(Re1(3 + 2Q + 2EcGrU_0\lambda + 3EcU_0^2\lambda + 3EcV_0^2\lambda \\ +m(-3 + Q + EcGrU_0\lambda)) - Ec(U_0^2 + V_0^2)(3 + \lambda M)) \end{pmatrix} z \\ & + \left(\frac{1}{2}Pr(-Ec(U_0^2 + V_0^2) + \operatorname{Re}(1 - m + Q + EcGrU_0\lambda + EcU_0^2\lambda + EcV_0^2\lambda)\right) z^2 \\ & + \left(\frac{Pr((-1+m)Q \operatorname{Re} + Ec\lambda(Gr(-1+m) \operatorname{Re} U_0 + (U_0^2 - V_0^2)\lambda)}{6}\right) z^3. \end{aligned}$$

3.5 Result and discussion

In order to get the physical meaning of the problems, the effect of various parameters on velocity field, temperature field and concentration field are discussed.

The figures (3.3)-(3.12) describes the behavior of velocity field versus z due to the variation of Reaction parameter R, heat sink and source parameter Q, rotating parameter Ω, Magnetic parameter M and Maxwell parameter λ for $Sc = .6$, $\Pr = .71$, $\mathrm{Re} = 2$, $M = Gr = Gm = 2$, $U_0 = V_0 = R = Q = m = n = 1$, $Ec = .001$, $\Omega = 1$, $\lambda = 0.1$. From figures (3.3) – (3.4) and (3.11) – (3.12) we observe that velocity u in x-direction decreases and velocity v in y-direction increases as increasing reaction parameter R and magnetic parameter M. The effect of heat sink and source Q is shown in Figs. (3.5) – (3.6). It is observed that velocity u in x-direction decreases and velocity v in y-direction increases as increasing heat sink/source parameter. The effect of rotating parameter Ω on the velocity component u and v is depicted in Figs. (3.7)-(3.8). It is observed that the velocity u in x-direction increases and velocity v in y-direction decreases by increasing Ω. The effect of Maxwell parameter λ is shown in Figs (3.9) – (3.10) in which velocity u in x-direction increases and velocity v in y-direction decreases by increasing Maxwell parameter.

The effect of heat sink and source parameter Q and prandtl number Pr for $Sc = .6$, $\Pr = .71$, $\mathrm{Re} = M = 2$, $Gr = Gm = U_0 = V_0 = Q = R = m = n = \Omega = 1$, $Ec = .001$, $\lambda = 0.1$ on the temperature field depicted in the Figs. (3.13) and (3.14). Fig. (3.13) show that the fluid temperature decrease with the increase of heat sink parameter and increases by increasing heat sink. The effect of prandtl number is shown in Fig. (3.14), it is observed that as we increase the prandtl number the temperature of the fluid decreases.

The variation of the concentration field C versus z under the influence of Schmidt number Sc and chemical reaction parameter R for $R = 1$, $Sc = 0.66$, $\mathrm{Re} = 1$, $n = 2$ is presented in the figures (3.15) and (3.16). The concentration decreases by increasing the chemical reaction and Schmidt parameter.

In all figures it is observed that the magnitude of velocity and temperature is lesser than in the case of Newtonian fluid as compared by the Maxwell fluid.

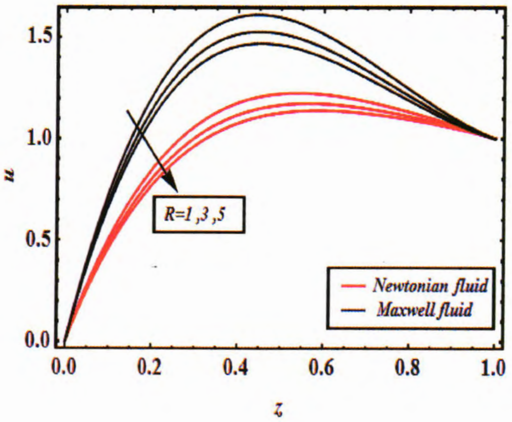

Fig. 3.3: Effect of R on $u\,(z)$.

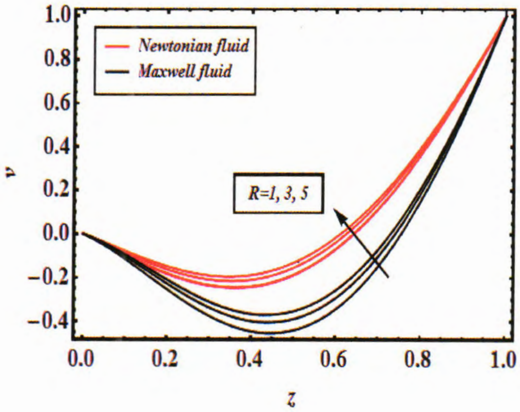

Fig. 3.4: Effect of R on $v\,(z)$.

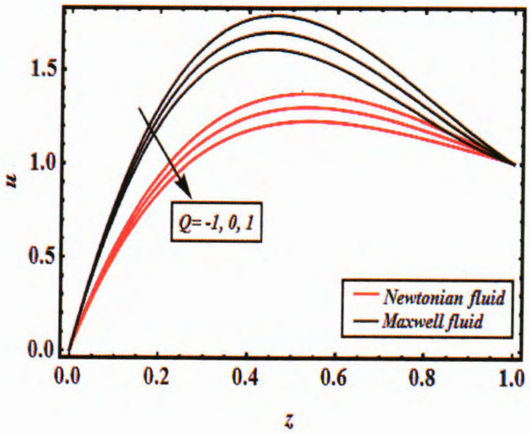

Fig. 3.5: Effect of heat Q on $u\,(z)$

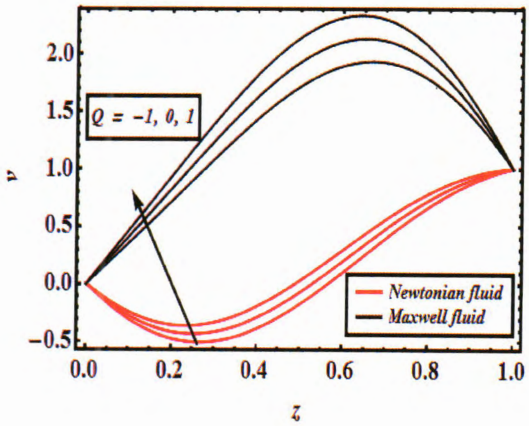

Fig. 3.6: Effect of Q on $v(z)$

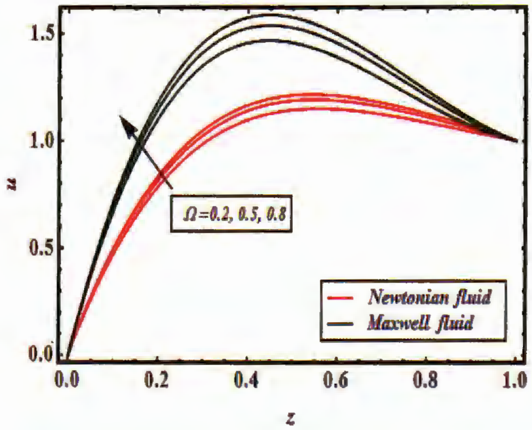

Fig. 3.7: Effect of Ω on $u(z)$.

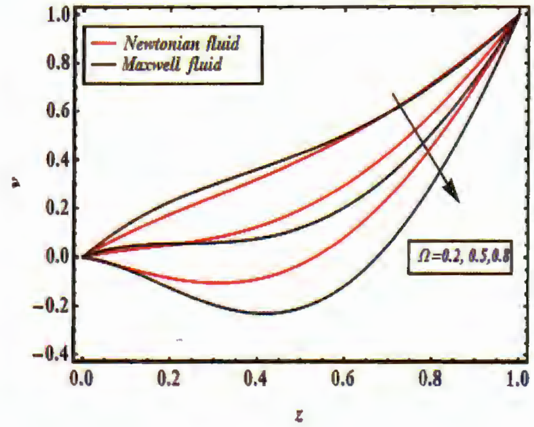

Fig. 3.8: Effect of Ω on $v(z)$.

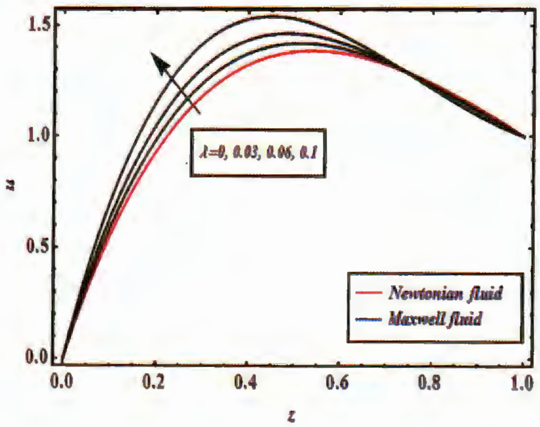

Fig. 3.9: Effect of λ on $u(z)$

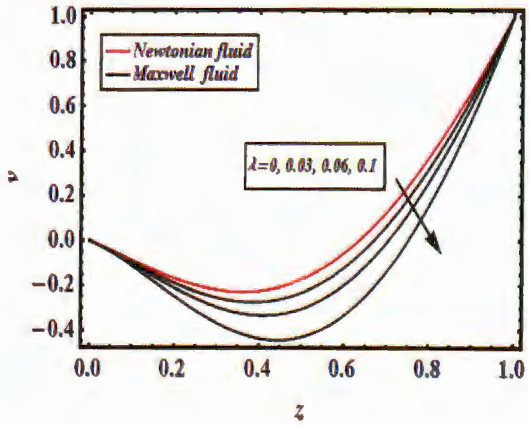

Fig. 3.10: Effect of λ on $v(z)$

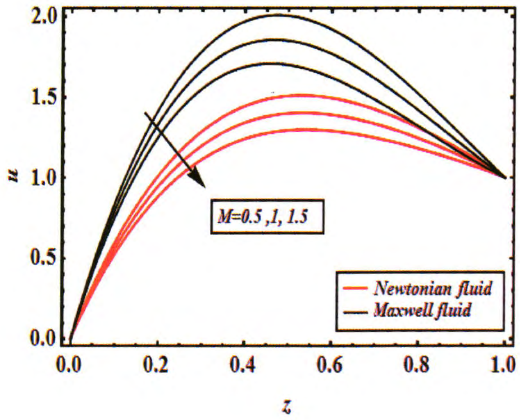

Fig. 3.11: Effect of M on $u(z)$.

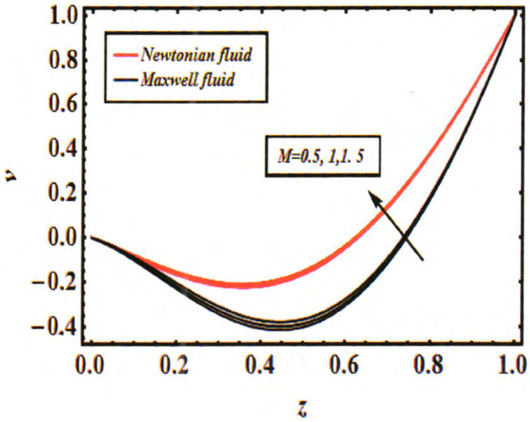

Fig. 3.12: Effect of M on $v(z)$.

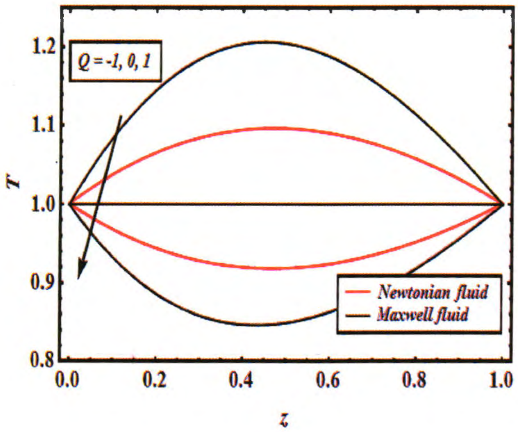

Fig. 3.13: Effect of Q on $T(z)$.

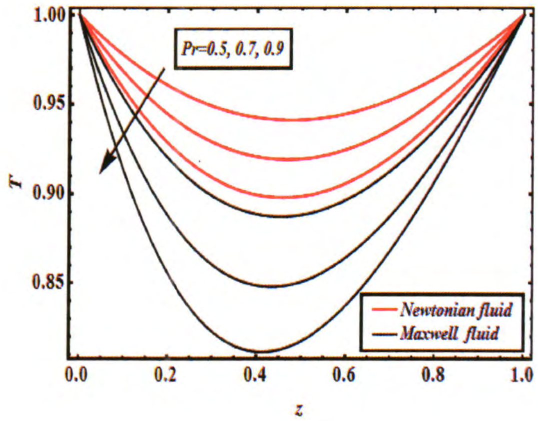

Fig. 3.14: Effect of Pr on $T(z)$.

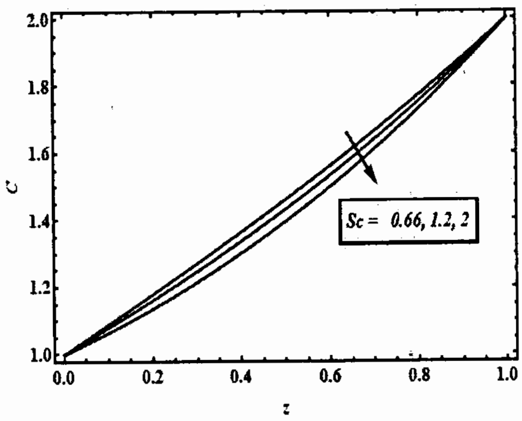

Fig. 3.15: Effect of Sc on $C(z)$.

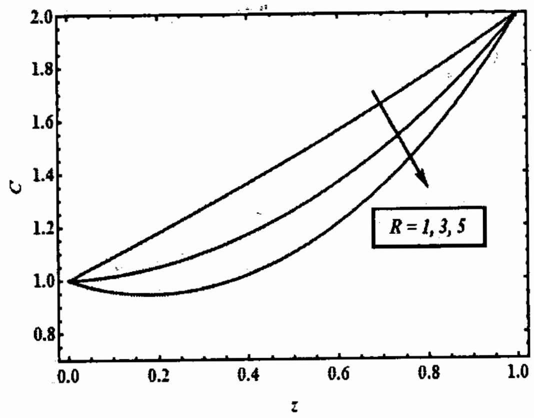

Fig. 3.16: Effect of R on $C(z)$.

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