MODELING OF MINIMUM STREAM FLOW IN PAKISTAN USING FOUR PARAMETER KAPPA DISTRIBUTION BASED ON LINEAR ORDER STATISTICS

By

Sayyad Anwar

Reg. No. 26-FBAS/MSST/S-13

Department of Mathematics & Statistics
Faculty of Basic and Applied Sciences
International Islamic University, Islamabad
Pakistan

2016

1. Mathematical models

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A dissertation
Submitted in the partial fulfillment of the
Requirements for the degree of
MS IN STATISTICS

SUPERVISED BY:

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Certificate

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A DISSERTATION SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF THE **MS IN STATISTICS**

We accept this dissertation as conforming to the required standard.

1. ______________________
Prof. Dr. Noor Muhammad Larik
(External Examiner)

2. ______________________
Dr. Muhammad Akbar
(Internal Examiner)

3. ______________________
Dr. Ishfaq Ahmad
(Supervisor)

4. ______________________
Prof. Dr. M. Arshad Zia
(Chairman)

DEPARTMENT OF MATHEMATICS & STATISTICS
FACULTY OF BASIC AND APPLIED SCIENCES
INTERNATIONAL ISLAMIC UNIVERSITY, ISLAMABAD
PAKISTAN

2016

In dedication to my worthy parents for making me be

Who I am

Thank you for allowing me to follow my heart

and

encouraging me to pursue my dreams.

Forwarding Sheet by Research Supervisor

The thesis entitled "Modeling of Minimum Stream Flow in Pakistan using four Parameter Kappa Distribution based on Linear Order Statistics" submitted by Sayyad Anwar, Registration No: 26-FBAS/MSST/S-13, in partial fulfillment of MS degree in Statistics has been completed under my guidance and supervision. I am satisfied with the quality of his research work and allow him to submit this thesis for further process to graduate with Master of Science Degree from Department of Mathematics and Statistics, as per IIU Islamabad rules and regulations.

Dated_______________

Dr. Ishfaq Ahmed,
Assistant Professor
Department of Mathematics & Statistics,
International Islamic University,
Islamabad.

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SAYYAD ANWAR

DECLARATION

I hereby declare that this dissertation neither as a whole nor a part of it has been copied out from any source. It is further declare that I have prepared this dissertation entirely on the basis of my personal efforts made under the supervision of my supervisor Dr. Ishfaq Ahmed. No portion of the work, presented in this dissertation has been submitted in the support of any application for any degree or qualification of this or any other learning institute.

Signature: ______________________

Date: ______________________

Sayyad Anwar

MS (Statistics)

Reg. No. 26-FBAS/MSST/S-13

Department of Mathematics & Statistics,

Faculty of Basic & Applied Sciences,

International Islamic University Islamabad, Pakistan.

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Abstract

Four Parameter Kappa Distribution using Linear Moments as method of estimation is being used in this study for 10 days average Annual Minimum Flow Series (AMFS) of each of 09 gauging sites at Indus basin of Pakistan. Four selected distributions Generalized Extreme Value (GEV) Distribution, Generalized Logistic (GLO) Distribution, Generalized Pareto distribution and Exponential distributions being special cases of Kappa distribution are fitted for each of 09 sites, which are commonly used in low flow Analysis (LFA). The fitting of these distributions is also evaluated by using different goodness-of-fit methods, such as Anderson darling test, Kolmogorov test. Further L-Moments Ratio Diagram (LMRD) is also being used to confirm the goodness- of-fit for the above three distributions. Finally the results show that Exponential distribution is the most suitable distribution for the AMS flows for the majority of the sites in Pakistan followed by GEV,GLO and GPA distributions respectively, when the parameters are estimated by using L-moments technique. The Return Periods from 2 to 1000 years and corresponding quantiles are also provided for best fitted distributions. These quantiles are useful for water resources management.

CHAPTER 1

1.1 Introduction

A very few countries in the world have sufficient water resources, Pakistan is one of them. Himalayas and Karakorum heights contain the largest glaciers of the world and it makes them the great source of fresh water for plains of Pakistan. This blessing of nature dampens over 16 million hectors of cultivable land of Pakistan. Being an agricultural country Pakistan is highly dependent on this water for irrigation. Pakistan earns 24% of its GDP out of agriculture. This agriculture is mainly the subject to the Indus Basin water. Himalaya and Arabian Sea make its boundary outline. It touches India and Pakistan on its eastern edge. Indus basin mainly relies on Indus River for its irrigation but it could be insufficient without receiving a contribution from River Jhelum, Chenab, Ravi, and Sutlej. The River Indus is amongst the largest rivers of the world. The quantity of water carried by River Indus is three times the Nile's, ten times the Colorado River in United States of America and Mexico, and equal to the Columbia River's in Canada and United States of America" (Gillani and Azam, 1996). The irrigation system developed in Pakistan through canals and other tools lies among the best and largest irrigation setups in the world. The main canals and branches measure about 62,500 km which irrigate an area of 16 million through 1.44 million km length of water courses diverting 75% of its water. Consequent to the Indus Water Treaty 1960, Pakistan constructed and built seven new link canals simultaneously improving three of its earlier links, six new barrages were provided while four of the barrages which were built before pre-partition period were remodeled for the facilitation and regulation of the waters. It must be noted that under Indus Basin Development Fund two large dams namely Tarbela at River Indus, Mangla at River Jhelum and one medium size dam Warsak were built to sustain and irrigate the agricultural land in Pakistan (Fahlbusch and Thatte 2004). However the Agriculture and Hydro-Electric Generation of Pakistan suffered heavily due to low flows of water of Indus River Basin especially in the Rabi Season (October to March). The last decade has been considered a period of droughts conditions due to the low flows of water of Indus River Basin. The low-flows below the normal are extremely hazardous and perilous for an agricultural country like Pakistan. Some surplus water is stuck in lakes and other water containing bodies. Water quality highly depends upon stream flow.

1.2 Stream Flow

The quantity of water that runs through any explicit point at any specific span of time is called "stream flow" or "discharge". Velocity and Volume are considered two important constituents of stream flow. Velocity refers to the speed of water and volume refers to that quantity of water which runs across the reference point. This "Flow" is usually enumerated as "discharge" Hence the stream flow or discharge can be defined as "the volume of water flowing in a stream channel expressed as unit per time (cusec)" or "the rate of flow or the volume of water that passes through a channel cross section in a specific period of time". The constituents define the energy of running water. Curves and lines of stream are formed due to water energy. The chemical and biological characteristics of water also depend upon water energy.

1.3 Low Stream Flow

From above definitions of stream flow it can be said that low stream flow is "smaller volume of water flowing in a stream per unit time with lesser speed". It means low flow refers to the amount of water that is lesser than the standard volume, with the minimum speed in per unit time. Hence Minimum stream flow witnessed per unit time is "low flow". The measurement of Low stream flow is needed to manage the quality and quantity of water at estimated and unestimated sites. Scales are used by Hydrologists to measure the stream flow. Continuous low flow can result in droughts. Droughts occur in absence of precipitation and indicate the lowermost level of water in reservoirs and underground water. During droughts, stream flow can reach its lowest position and even flow can get converted in static stock of water. Dams and other water reservoirs are used to stock larger amounts of water for industrial, agricultural and civic use. The amount of water stored in a reservoir is subject to the volume and flow of water coming from the source. Low flows represent a crucial component of the natural river flow regimes. The spatial and temporal variability of river low-flow characteristics can be considerable. The statistical data is gathered and observed to predict and manage the stream flow. Because of its high economic concerns Low-flow frequency analysis is recommended to observe before any decision making about water stocking. Because great financial worth is interconnected to the deeds of forecasting and investigation of low-flows and as a result it affects long-term droughts. It is important to mention that droughts inflict more crucial consequences on economy, society and agriculture. Smakhtin (2001) presented a comprehensive review of low-flow hydrology covering such issues as generating mechanisms, estimation methods and applications. Burn et al. (2008) have summarized the processes and patterns of low flows in Canada, specifically. The availability of reliable low- flow occurrence and magnitude estimates is crucial for a wide array of engineering applications such as aquatic ecosystem modeling. Bradford and Heinonen, (2008), environmental impact analysis, water supply assessment for potable and irrigation purposes.

1.4 Introduction Linear –Moments (L-Moments)

The uncertainty events such as minimum flows are uncommon and happen in a short amount of time. Therefore, it is crucial for probability distribution to study its characteristics. Outlier values are generally to be found in the minimum flow data which are logically danger. In the presence of outlier the standard estimation methods are tremendously affected. These methods may contain Least Square (LS), Maximum Likelihood Estimation (MLE) and Method of Moments (MOM). MLE is not good choice when sample is small and in presence of extreme observations. As the outlier observation has extreme effect on these methods, we need a strong method of estimation, to reduce the effect of outliers on the estimates.

L-moments were introduced by Hosking (1990) as the summary statistics for "probability distributions and data samples". They share similarity to conventional moments by delivering "measures of location", "skewness", "kurtosis", "dispersion" and additionally they deliver features of "shape of probability distributions or data samples". The difference lies in their calculation i.e. the linear arrangement of the sample ordered statistics for L-Moments. The Sample L-moments are taken as unbiased estimates of the population L-moments.

Conventional moments cannot offer as many advantages as offered and performed by L-moments e.g. probability distribution for L-moments is meaningful when the distribution has finite mean, requires finite variance for finite standard errors Hosking (1990). Asymptotic estimations applied to sampling distributions are more helpful for L-moments as compared to when used for ordinary moments (Hosking, 1990). L-moments provide a better tool to identify the parent distribution generating data sample (Hosking, 1990). Low flow analysis is based on two steps which are "choice of a probability distribution and estimation of the probability distribution".

1.5 Kappa Distribution

Extreme values in a random sample have many important applications. Natural disasters such as floods, wind storms or heavy rains and Low flow; are the result, of extreme observations. While exact distributions of maxima or minima may sometimes be derived, extreme values are more often modeled by the generalized extreme value (GEV) distribution since theory has shown this distribution to be the limiting form of the distributions of extremes. In reality, however, results are sometimes unsatisfactory when the GEV distribution is fitted to finite samples (Paradia 2006). The four-parameter kappa (KAP) distribution is a generalization of the Generalized Extreme Value Distribution (GEV), Generalized Logistic Distribution (GLO), Generalized Pareto Distribution (GPA), Exponential Distribution, Gumbel Distribution, 2-P Logistic Distribution, Uniform Distribution , 2-P Reverse Exponential Distribution and are useful when the above mentioned 2-P and 3-P distributions not performing well. Hosking (1994) estimated the parameters of Kappa distribution by using method of linear moments (L-Moments).

The leading benefits behind the practice of this technique are the "reliable estimation of parameters" (that ultimately turns in reliable quantiles) mainly out of smaller sample size. L-moments rely on linear order statistic as an alternative of conventional moments. Linear order statistics are resilient to outliers which may occur in presence of any extreme event. That's why this method is reliable.

One of the recent developments in the modeling of minimum flow is 4-P Kappa distribution (Hosking 1994). Four-parameter (4-P) distribution is being used as highly reliable and advantageous tool because it includes the characteristics and benefits of 2 and 3-P distributions as well. 4-P distribution fits well to the data where 2 or 3-P distributions provide poor fit. This study is going to consume 4-P Kappa distribution with the L-Moments procedure, as a tool of study to obtain reliable Quantile estimates, particularly at higher return periods such as 5, 10, 15, 20, 25 and 500 years. The study will rely on the Pakistan flood Commission data observed between 1982 and 2013 at 9 gauging stations across Pakistan as source of data. Quantiles are aimed to make for planning and designing of policy in this regard.

Where significant measures of efficiently watched flow information exist low-flow frequency investigation should be utilized in choice making in light of potential financial effects. A high financial worth is connected with the exercises of forecast and investigation of low-flows and the subsequent long term dry seasons. It must be captioned that dry spells have more extreme results and are regularly more expenses.

1.6 OBJECTIVES OF STUDY

Low flow analysis at Indus basin adversely affects agriculture, environment and economy of Pakistan. So there is a dire need of Frequency Analysis (FA) of low flow at Indus basin in Pakistan.

- To minimize the problems in identification of parent distribution that describes variability of 10-days Annual low flow series by using the 4-P Kappa distribution.
- To predict 10-days Annual low flow (low flow quantiles) in Pakistan for different return periods.

- To address the consequences of low flows in the country and give some solutions to mitigate the after effects / causes of low flow in Pakistan.
- To estimate Quantiles for the Hydrological projects and water resources management such as amount of low flow for hydropower generation, water quality management, designing of irrigation system and impact of prolonged droughts on aquatic ecosystems in the country.

CHAPTER 2

Literature Review

Many researchers of different areas including USA, UK, Malaysia, Pakistan, Iran, India, China and many other countries worked on low streamflow. In order to support this study, we reviewed the work of these researchers. In this chapter, we include the references which explain the methodology of modeling of minimum stream flow using four parameter kappa distribution based on linear order statistics.

2.1 Review of Methodology

Gulhati (1972) discussed the entire Indus Basin in the light of Indus Waters Treaty 1960, signed between India and Pakistan due to efforts of World Bank to resolve the issues of Waters of Indus Basin between two countries. The view point of both the countries and their claims on Waters of rivers on Indus Basin were reflected in details. The impacts of the treaty and distribution of waters of the Indus Basin were also discussed.

Hosking (1990) derived L-moments as Linear Combination of Order Statistics. Author also provided basic definition, properties, methods of estimation of L-moments and estimation of L-moments ratios. The paper also discusses the method of identification of suitable distribution and its parameter estimation and hypothesis testing using L-moments technique.

Ahmed (1993) described Status of Irrigated Agriculture, Surface Water Resources, Construction of Barrages and Canals, Utilization and Management of the Surface Water Resources, Seepage from Irrigation Unlined and Lined Canals and Ground Water Resources of Pakistan. Salient features of major Dams, Barrages, Canals and

Link Canals together with their command area, Rivers along with their catchment area and water quality were also mentioned.

Vogel and Fennessey (1993) concluded that product moment ratio estimators are biased estimators for small samples in case of hydrological applications. On the other hand L-moment estimators are useful estimators for small and large sample sizes and for highly skewed probability distributions. A useful comparison of both kinds of moments is mentioned in the paper.

Guttman et al. (1993) calculated quantile estimates of low precipitation data for the United States. The objective of such estimates is to prepare national drought atlas keeping in view Low streamflow and precipitation data. The quantile values were calculated for 111 regions across the U.S using L-moments algorithm. Pearson Type III and Wakeby were considered the most suitable distributions for the study. The results of the study indicate that for low return periods the quantile estimates are low and for high return periods the quantile estimates are high for both the probability distributions.

Vogel and Wilson (1996) studied Probability Distributions of Minimum, Maximum and Mean stream flows in United States with the help of L-moment Ratio Diagrams. It was found that L-moment Ratio Diagrams are most suitable as compared to Ordinary Moments Diagrams as these can reflect several distributions on the same graph paper. It is concluded that Pearson Type 3 is best fitted distribution for minimum and mean stream flow in the area.

Hosking and Wallis (1997) explained steps to work out the Regional Frequency Analysis of extreme events in hydrology which include (i) Screening the data (ii) Selection of appropriate probability distribution and (iii) Estimation of probability distribution based on L-moments procedure.

Parida (1999) worked on Indian monsoon data which was collected over 50 stations across the India of 41 years from 1940 to 1980 by using four parameter kappa distribution. To obtain reliable quantiles estimates he used linear moments procedure. By using quantiles estimates, he developed isopluvial maps for some generally used return periods such as20, 50, 100, 200, 500 and 1000 years that may be used by meteorologists.

Sankarasubramanian and Srinivasan (1999) provided useful evaluation of sampling characteristics of Linear moments and Conventional moments. It is illustrated C-moments are suitable for lower skewness for small samples, while L-moments are advantageous for high skewness for both small and large samples.

Caruso (2000) carried out Low Flow Frequency Analysis of 21 rivers in New Zealand. The candidate distributions for the homogeneous regions were found GEV, EV I and LN3. However, the GEV was robust distribution. In the paper the author also suggested methods to treat the Zero flow in the data series.

Connie Winchester (2000) extreme values in a random sample are, in many important applications. The most crucial observations. Natural disasters such as floods, wind storms or heavy rain for example, are frequently the result, of extreme observations. While exact distributions of maxima or minima may sometimes be derived, extreme values are more often modeled by the generalized extreme value (GEV) distribution since theory shows this distribution limiting form of the distributions of extremes. In reality, however, results are sometimes unsatisfactory when the GEV distribution is fitted to finite samples. Among other common three-parameter distributions, four-parameter kappa (KAP) distribution is unique due to its generalization of the generalized extreme value distribution and works well when the GEV distribution is not performing. Linear functions of expected order statistics viz. L moment estimates (LMEs) are used by under study method of estimation. However, provided that they are computable, are often nonfeasible. Additionally, for the KAP distribution, the asymptotic distributions of the LMEs are not easily tractable. The well-known maximum likelihood estimation is proposed as an alternative method of parameter estimation. Since this method consists of optimizing non-linear utility of four variables, problem is computationally difficult. A model research is conducted to link the performance of maximum likelihood estimates (MLEs) and the LMEs. Results show the MLEs to be comparable and sometimes more desirable estimates- The variance of the MLEs is further investigated. Finally the fit of the KAP distribution versus the GEV distribution is considered for real-life extreme value wind data.

Peel et al. (2001) attempted to analyze the utility of graphical methods and L-moment ratio diagram for selection of suitable probability distribution. The author concluded that use of graphical technic together with L-moment ratio diagram for the selection of distribution is not appropriate and rather suggested heterogeneity tests together with L-moment ratio diagrams for the selection of distribution.

Bari and Sadek (2002) conducted Low stream flow Frequency Analysis of rivers in North West of Bangladesh. Ten daily low flow data of thirteen gaging stations was used in the study. The quantile estimates of low flow using Regional Frequency Analysis are compared with those of at-site analysis. Weibull and Pearson Type 3 were found most suitable choices to estimate low flow of the rivers in North West part of the country.

Kroll and Vogel (2002) used L-moment diagrams based on L-moment ratios to study probability distribution of Low Stream Flow Series in the United States. The paper indicates that 1-day, 7-days and 30-days annual minimum stream flow data was used in the study. The paper suggested for low stream flow LN3 (Lognormal 3 parameters) is best fitted at non-perennial sites and P3 (Pearson Type-III) for perennial sites in the United States. It is concluded that L-moment ratios behaved differently for perennial and non-perennial sites and recommended to analyse both kind of sites separately.

Reilly and Kroll (2003) they used baseflow correlation to estimate the low streamflow at ungagged sites of US. A provincial assessment of baseflow correlation estimators is made by using daily streamflow data at more than 1300 sites. They investigated that baseflow correlation gives better results when baseflow data are almost independent and are located inside 200 km They compare baseflow correlation and provincial regression and conclude that for low streamflow estimation, baseflow correlation gives good results.

Kumar et al. (2003) developed Flood Frequency relationship according to regions by use of L-moments method. The study area comprises Middle Ganga Plains of India. They found that GEV is the best fitted distribution for the study area. Growth curves for 2, 5, 10, 25, 50, 100, 200, 500 and 1000 years return periods were constructed.

YUE and WANG (2004) used L-moment technique to find distribution of Annual Average Stream Flow of Canada. All the sites included in the study having more than 20 years of data of stream flow. They conclude in their paper that GEV distribution, LN3 and P3 distributions are best fitted for the estimation of annual average streamflow data of Canada.

Bekoe et al (2005) discussed and used various probability distributions to find out the best fitted distribution on low streamflow data also find out the shold of river Ayensu where low streamflow exist. Their results indicate that Okyereko station at basin had some trend to yield unexpected minimum low streamflow with threshold amount of water 0.20 m^3/s that is equal to 95% of time. The chance of existence of low streamflows in basin is small and that amount of water abstraction in terms of water supply for industrial, agricultural and domestic requirement is sustainable and sufficient.

Rakesh Kumar and Chandranath Chatterjee (2005) analyzed data of low streamflow 13 gauging sites of India. They use basic assumptions for the screening of data for regional low streamflow analysis. Also used simulations procedure to check heterogeneity of data by applying kappa distribution. They observed that 10 sites out of 13 having constitute as homogeneous region. They also conclude that the identified distribution GEV is quit robust for study area.

Yurekli et al. (2005) conducted Regional Frequency Analysis of Low Flow data from Cekerek River Basin. In the paper Drought is defined as a phenomena caused by shortage of water due to low stream flow. Three gauged stations were selected from the said basin and 7-days low flow readings were noted. L-moments technique was used for regional frequency analysis to find out the best fitted probability distribution. Amongst the selected candidate distributions the GPA was found most suitable distribution for the study area.

Chen *et al* (2006) used the L-moment procedure to study the regional frequency of low streamflows. They used five distributions: generalized extreme value, generalized logistic, generalized lognormal, Pearson type III, and generalized Pareto to study low streamflows for Dongjiang basin. L-moment procedure was used to estimate the parameters of above mentioned distributions. For each return period they calculate low streamflow estimates by applying method of index flood.

Gustard and Demuth (2008) prepared a Manual on Low –flow Prediction and Estimation. The manual discusses in detail the Estimation, Forecasting and Prediction of Low-flow, Hydrological data, Processes and Regimes, Low-flow Indices, the Flow-Duration Curves, Extreme Value Analysis, Streamflow deficit, and Estimating Low-flow at ungagged sites and artificially influenced rivers. The manual provide useful guidelines for low streamflow studies.

Hussain and Pasha (2009) conducted regional flood frequency by using L-moments method on 7 stations of Punjab Pakistan. They used discordancy measure to screen the data of seven sites. Homogeneity was check by simulations procedure. Generalized extreme-value, Generalized logistic, generalized normal, generalized Pareto, Pearson type III and five parameter Wakeby distributions were used for the selection of most appropriate distribution for quantile estimates.

Seckin *et al* (2010) conducted comparison of maximum likelihood method and probability weighted moments to carry out flood frequency analysis of Ceyhan River Basin in Turkey by using streamflow data. They concluded that for estimation of parameters of lognormal three parameters (LN3) and generalized extreme value (GEV) distribution PWM method is a better choice rather MLE method.

Saf (2009) evaluated regional probability distribution for the annual maximum flood series observed at 45 stream flow gauging river basins in Turkey using index flood method. Seven sites out of 45were removed from the analysis because there was a trend in the series. A regional analysis was performed on the remaining 38 sites. Discordancy measure was used for screening of the data. The homogeneity of regions was identified by using 500 simulations by applying 4-P kapp distribution. To estimate the results on the basis of the relative bias and relative root mean square error, they used Monte Carlo simulation.

Christopher et al (2009) conducted low flow frequency analysis in Canada and northwestern Washington. They used 12 regional regression equations for estimating low flow statistics. They computed adjused R^2 and root-mean-squared error (RMSE) and conclude that gaging stations could be removed from the network without significant loss of information

Gubareva and Gartsman (2010) estimated the parameters of extreme hydrometer logical Characteristics by using streamflow data by L-moment method and found for distribution with heavy tails L-moment can be calculated whereas conventional moments do not exist. The paper reflects that L-moment method give more stable computation procedure as compared to maximum likelihood method (MLE).

Santos et al. (2011) conducted Regional Frequency Analysis of Droughts in Portugal. Assumptions of independence and homogeneity were tested. Paper suggested that the Kappa Distribution model should be used for drought assessment analysis in Portugal.

Devi and Choudhurt (2013) investigated rainfall frequency analysis for Meteorological Division of India. To detect discordant sites they used discordancy measure. They used 4 parameter Kappa distribution to check regional homogeneity by comparing generated homogeneous regions. L-moments ratio diagram and ZDIST statistics were used to select best fitted distribution. Log- Normal type III, Pearson type-3, generalized Pareto, generalized extreme-value and generalized logistic was best fit distributions

Rostami (2013) used flood data to perform frequency analysis based on L-moment approach in west Azerbaijan province basins. Ward hierarchical cluster method was used for the identification of homogeneous regions. The west Azerbaijan province was divided into four regions. By the help of L-moment ratios the parameters of regional frequency distribution were estimated in these regions. For the selection of appropriate distributions L-moment diagram, Z statistic goodness-of-fit test and plotting position methods were used.

Ahmad *et al* (2013) analyzed the data of annual 27 meteorological stations of Pakistan during the period of 1960-2006. The random behavior of monsoon rainfall was investigated through Kappa probability distribution. L-moment technique was used to estimate the parameters of this distribution. These estimates were used to calculated quantiles for different T-years return periods such as for 2, 5, 10, 20, 50,100, 200 and 500 years.

Osman et al. (2013) performed Regional Flood Frequency Analysis at West Coast of Malaysia using L-moments methodology. It was found that it was smaller difference for low return periods between fitted and observed values as compared to high return periods.

Ayesha et al (2013) they conduct at-site flood frequency analysis on Australian annual maximum flood data. They used fifteen different candidate probability distributions for the selection of best fitted distribution. Four goodness-of-fit methods were used to check the performance of distributions i.e., the Bayesian information criterion, the Akaike information criterion, Kolmogorov–Smirnov test and Anderson–Darling test. They conclude that a single distribution cannot specified all the information related to all the Australian states

Agwata et al. (2014) suggested model to study hydrological drought in Upper Tana Basin of Kenya using L-moments and other techniques. The Generalized Normal was best fitted distribution to measure duration and severity of drought for the study area.

CHAPTER 3

Material and Methods

3.1 Study Area and Data

For the current study the data of 09 different sites would be used. The daily Minimum flow data taken from Pakistan Flood Commission is measured by flow gauges in cusecs from which 10 days average AMFS will be constructed for the proposed study. These 09 sites are located in province Sindh, Punjab, and KPK. The record length of AMFS varies from 30 to 74 years.

Table 1.Basic Information about sites used in the study.

Name of sites	River	Latitude (North)	Longitude (East)	No of year (n)	Mean	Standard deviation	Skewness	Coefficient of variation
	Indus	33.99	72.61	30	13650.0	2813.2184	-0.0974	0.2060
Kalabag		32.95	71.50	52	21136.5	3774.6229	0.1729	0.1785
Chashm		32.43	71.38	43	12609.3	4474.6401	-0.0410	0.3548
Taunsa		30.50	70.80	30	14390.0	2918.0460	-0.1196	0.2027
Guddu		28.30	69.50	30	15556.6	4716.6667	0.1672	0.3031
Sukkar		27.72	68.79	74	823.513	614.5168	0.5727	0.7462
Nowshe	Kabul	34.01	72.00	53	6960.37	1107.3294	0.0559	0.1590
Marala	Chenab	32.68	74.43	47	5161.70	2003.3302	0.1477	0.3881
Mangla	Jhelum	33.15	73.65	26	1465.38	597.0769	0.0131	0.4074

Fig. 3.1 Map of Pakistan with the sites location on rivers

Himalaya and Karakorum is the origination of Pakistani rivers system. There are five rivers, Chenab, Indus, Sutlej, Ravi, and Jhelum. They are mostly pass through Punjab province. The word 'pani' means five and 'aab' means water (in Persian language) that's why Punjab is called the land of five rivers. The irrigation system of Pakistan is the widest system of irrigation in the world. This system has a great importance in the agriculture field of the country. These river's water is the biggest source of water in Pakistan.

Indus River is originating from Himalayan region (Tibetan plateau near Lake Mansarovar) in China. In Asia the Indus River is a major river which flows through Pakistan and also the longest river in Pakistan. Length of Indus River is 3180 Kilometers. Balram River, Gilgit River, Tanubal River, Astor River, Kabul River, and Zanskar River are the tributaries of Indus River. Indus River consist of 16 Barrages, 3 reservoirs, 2 Siphons across major river 12 inter link canals, 2 head works, 44 canal system: 23 in Punjab, 14 in Sindh, 5 in KPK, and 2 in Baluchistan.

River Jhelum originates from the south-eastern part of valley Kashmir. River Jhelum is the tributary of River Chenab and 774 kilometer long. Mangla is one of the world largest dam which is constructed on Jhelum in 1967. The storage capacity of Mangla dam is 5.9 million acre-feet. Many other dams and Barrages are also constructed on Jhelum like Rasul Barrage and Trimmu Barrage.

Chenab River is joined by Chandra and Bhaga rivers in the upper Himalayas. It flows through Jammu and Kashmir and at Trimmu Barrage Chenab rivers joined by the Jhelum River. Chenab River is 960 Kilometers long. Under the Indus waters treaty the water of Chenab is allocated to Pakistan and after Indus this is the strongest river of Pakistan.

The Ravi River also known as Parushani or Iravati, Hydraotes in Indian Vedic, and an ancient Greeks. River Ravi originates from Himalayas near Chamba in Himachal Pradesh State, northern India. It flows in Indian Punjab south-west region and flows beside with indo-Pak border and enters into Pakistan and merges with Chenab. The length of River Ravi is nearly 720 kilometers long and in Pakistan its length is 675 kilometer. On Ravi River Important Engineering's headworks are constructed e.g. Sidhani and Balloki.

Sutlej River is also known as Red River and it is 550 kilometers long. It flows through northern Punjab area of Pakisn and India. The location of Sutlej is east of the central Suleiman range in Pakistan, south of the Hindu Kush and north of the Vindhya Range segment of the Himalayas. On Sutlej Important Engineering Barrages are constructed e.g. Sulemanki Barrage and Islam Barrage.

3.2 Basic assumptions of Low streamflow Analysis

There are different fundamental assumptions which are essential in Low Streamflow Analysis. The reason for these assumptions is to test observations for stationarity/independence and Homogeneity. These assumptions are checked for various types of information for all intents and purposes for Low streamflow Analysis, rainfalls, dry spells and so forth. Time series plots are used to check trend behavior in the collected hydrological data for different time periods, Mann-Whitney test to check homogeneity, Ljung-Box Q test to check the stationarity and also independence, while Kendall's tau test also for trend analysis. For example see Laux at el (2001), Zaidman at el (2002), Ahmad et al (2014, 2015 & 2016) and Sadri et al (2016) were used these tests for stationarity/independency and homogeneity.

3.3 Selection of Parameter's Estimation Methods

For the estimation of parameters, there are some techniques including the MOM, MLE, L-moments, TL-moments, LS, Generalized Method of Moments (GMM), Maximum Entropy (MENT), Mixed Moments (MIXM), and Incomplete Means (ICM) Rao et al (2000). The MLE is generally considered as the most efficient technique, because it provides the minimum sampling variance of the estimated parameters as compared to other techniques. But in the presence of outlier, small sample size and large number of parameter MLE gives inappropriate results and also biased estimates. For the parameter estimation another method is MOM which is relatively simple for calculation. The MOM is not efficient as compared to MLE method in the case of large number of parameters of the distributions. In the presence of small sample size the higher order moments may be extremely biased. Hence, as the outlier observation and small sample size has extreme effect on these methods, we need a robust method of estimation to reduce the effect of outliers on the estimates. The L-moment method Hosking (1986) are more robust in the presences of outlier observation and small sample size as compared to other methods. However our data contain extreme observation and small sample size therefore we use the L-moment methods for estimation.

3.3.1 Method of Linear-Moments (L-Moments)

As linear moments have been defined by Hosking (1990), L-moments are expectations of certain linear combinations/arrangements of order statistic. They can be explained for any random variable that has finite mean. "Let

$X_1, X_2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . . X_r$ be the random sample of magnitude r, with cumulative distribution Function $F(X)$ and quantile function $X(F)$. Let $X_{1:r} \leq X_{2:r} \leq X_{3:r} \leq \cdots \ldots\ldots\ldots\ldots\ldots . . \leq X_{r:r}$ be the order statistic of random sample. For the random variable X, the r^{th} population Linear moments" as explained by Hosking (1990) is:

$$\lambda_r = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k} E(X_{r-k:r}) \qquad r = 1,2,\ldots.. \qquad (3.3.1.1)$$

$$\lambda_1 = E(X_{1:1}) \qquad (3.3.1.2)$$

$$\lambda_2 = \frac{1}{2}\mathrm{E}(X_{2:2} - X_{1:2}) \qquad (3.3.1.3)$$

$$\lambda_3 = \frac{1}{3}\mathrm{E}(X_{3:3} - 2X_{2:3} + X_{1:3}) \qquad (3.3.1.4)$$

$$\lambda_4 = \frac{1}{4}\mathrm{E}(X_{4:4} - 3X_{3:4} + 3X_{2:4} + X_{1:4}) \qquad (3.3.1.5)$$

The L-moments ratio has defined as:

$$\tau = \lambda_2/\lambda_1 \qquad (3.3.1.6)$$

$$\tau_3 = \lambda_3/\lambda_2 \qquad (3.3.1.7)$$

$$\tau_4 = \lambda_4/\lambda_2 \qquad (3.3.1.8)$$

In the above mentioned equations λ_1 is measure of location, λ_2 is variance, λ_3 and λ_4 are higher order moments. Where as τ, τ_3 and τ_4 represents Linear-coefficient of variation (L-CV) , Linear- Skewness (L-Skewness) and Linear-Kurtosis (L-Kurtosis) respectively.

3.3.2 Estimation of Linear-Moments (L-Moments)

In practice, L-moments need commonly be estimated after a random sample drawn from an anonymous distribution. As λ_r is a meaning of the expected order statistics of a sample of size r. "Let $x_1, x_2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . . x_n$ be the sample and

$x_{1:n} \leq x_{2:n} \leq x_{3:n} \leq \cdots \ldots \ldots \ldots \ldots \ldots \leq x_{n:n}$ is the order statistics of the samples, then we can define the r^{th}sample L-moments" as by Asquith (2007).

$$l_r = \frac{1}{r}\sum_{i=1}^{n}\left[\sum_{j=0}^{r-1}\frac{(-1^j)\binom{r-1}{j}\binom{i-1}{r-1-j}\binom{n-1}{j}}{\binom{n}{r}}\right]X_{i:n} \qquad r = 1,2,\ldots\ldots (3.3.2.1)$$

By using Wang (1996) direct estimation method of L-moments, the first four sample L-moments

are defined as:

$$l_1 = \sum_{i=1}^{n}\left[\frac{x_{i:n}}{\binom{n}{1}}\right] \qquad (3.3.2.2)$$

$$l_2 = \frac{1}{2}\sum_{i=1}^{n}\left[\frac{\binom{i-1}{1}-\binom{n-i}{1}}{\binom{n}{2}}\right]x_{i:n} \qquad (3.3.2.3)$$

$$l_3 = \frac{1}{3}\sum_{i=1}^{n}\left[\frac{\binom{i-1}{2}-2\binom{i-1}{1}\binom{n-i}{1}+\binom{n-i}{2}}{\binom{n}{3}}\right]x_{i:n} \qquad (3.3.2.4)$$

$$l_4 = \frac{1}{4}\sum_{i=1}^{n}\frac{\binom{i-1}{3}-3\binom{i-1}{2}\binom{n-i}{1}+3\binom{i-1}{1}\binom{n-i}{2}-\binom{n-i}{3}}{\binom{n}{4}}x_{i:n} \qquad (3.3.2.5)$$

The sample L-ratios are defined as:

$$t = {}^{l_2}/_{l_1} \qquad (3.3.2.6)$$

$$t_3 = {}^{l_3}/_{l_2} \qquad (3.3.2.7)$$

$$t_4 = {}^{l_4}/_{l_2} \qquad (3.3.2.8)$$

l_1 is the mean of distribution, t is linear coefficient of variation, t_3 is linear skewness and t_4 is linear kurtosis.

3.3.3 Relationship between Probability Weighted Moments and L-Moments

L-Moments as linear function of Probability Weighted Moments earlier determined by Greenwood et al. (1979) as defined by Hosking (1997).

$$M_{p,r,s} = E[x^p\{F(x)\}^r\{1-F(x)\}^s] = \int_0^1 x^p F(x)^r \left(1-F(x)\right)^s dF \ , \qquad r = 0,1,..$$

If distribution function $F(x) = u$, then the quantile function is $x(u)$ and

$$M_{p,r,s} = E[x(u)^p u^r (1-u)^s] = \int_0^1 x(u)^p u^r (1-u)^s \ du \ , \qquad r = 0,1, \dots \dots ...$$

For a distribution x(u) is a quantile function, then Probability Weighted Moments are:

$\alpha_r = M_{1,0,r}$ and $\beta_r = M_{1,r,0}$

$$\alpha_r = \int_0^1 x(u)(1-u)^r \ du \qquad r = 0,1, \dots \dots ... \qquad (3.3.3.1)$$

$$\beta_r = \int_0^1 x(u)(u)^r \ du \qquad r = 0,1, \dots \dots ... \qquad (3.3.3.2)$$

The relationship between Probability Weighted Moments and L-Moments is as follows:

$$\lambda_{r+1} = (-1)^r \textstyle\sum_{m=0}^{r} p_{r,m}^{*} \, \alpha_m \qquad (3.3.3.3)$$

$$\lambda_{r+1} = \textstyle\sum_{m=0}^{r} p_{r,m}^{*} \, \beta_m \qquad (3.3.3.4)$$

$$\text{Where } p_{r,m}^{*} = (-1)^{r-m} \binom{r}{m} \binom{r+m}{m} \qquad (3.3.3.5)$$

The first four L-Moments and Probability Weighted Moments are related as follows:

$$\lambda_1 = \alpha_0 = \beta_0 \qquad (3.3.3.6)$$

$$\lambda_2 = \alpha_0 - 2\alpha_1 = 2\beta_1 - \beta_0 \qquad (3.3.3.7)$$

$$\lambda_3 = \alpha_0 - 6\alpha_1 + 6\alpha_2 = 6\beta_2 - 6\beta_1 + \beta_0 \tag{3.3.3.8}$$

$$\lambda_4 = \alpha_0 - 12\alpha_1 + 30\alpha_2 - 20\alpha_3 = 20\beta_3 - 30\beta_2 + 12\beta_1 + \beta_0 \tag{3.3.3.9}$$

Then linear moment ratios:

$$\tau = \lambda_2/\lambda_1 \tag{3.3.3.10}$$

$$\tau_3 = \lambda_3/\lambda_2 \tag{3.3.3.11}$$

$$\tau_4 = \lambda_4/\lambda_2 \tag{3.3.3.12}$$

3.3.4 Estimation of L-Moments by Probability Weighted Moments:

Let $x_{1:n} \le x_{2:n} \le x_{3:n}$ ... ... $\le x_{n:n}$ be the ordered sample. It is appropriate to initiate with an estimator of the probability weighted moment β_r . An unbiased estimator of β_r is

$$b_0 = n^{-1} \sum_{j=1}^{n} x_{j:n} \tag{3.3.4.1}$$

$$b_1 = n^{-1} \sum_{j=2}^{n} \frac{(j-1)}{(n-1)} x_{j:n} \tag{3.3.4.2}$$

$$b_2 = n^{-1} \sum_{j=3}^{n} \frac{(j-1)(j-2)}{(n-1)(n-2)} x_{j:n} \tag{3.3.4.3}$$

and in general

$$b_r = n^{-1} \sum_{j=r+1}^{n} \frac{(j-1)(j-2)(j-3)\ldots\ldots\ldots(j-r)}{(n-1)(n-2)(n-3)\ldots\ldots\ldots\ldots(n-r)} x_{j:n} \tag{3.3.4.4}$$

where $r = 0,1,2, \ldots \ldots \ldots \ldots \ldots n-1$

For an ascending series of x, i.e. $x_1 \leq x_2 \leq x_3$ ……………x_n, with x_1 as the lowest and x_n as the largest value, the r^{th} L-Moment of x (λ_r) (Hosking, 1990; Parida et al., 1998) can be defined in terms of the linear combination of the probability weighted moments as:

$$l_1 = b_0 \tag{3.3.4.5}$$

$$l_2 = 2b_1 - b_0 \tag{3.3.4.6}$$

$$l_3 = 6b_2 - 6b_1 + b_0 \tag{3.3.4.7}$$

$$l_4 = 20b_3 - 30b_2 + 12b_1 - b_0 \tag{3.3.4.8}$$

By using equatios (3.3.4.6)-(3.3.4.8) we can calculate the L-Coefficient of Variance (L-C_v) = (t), L-Skewness (L-S_k) = (t_3) and L-Kurtosis (L-C_k) = (t_4) defined by

$$t = l_2/l_1 \tag{3.3.4.9}$$

$$t_3 = l_3/l_2 \tag{3.3.4.10}$$

$$t_4 = l_4/l_2 \tag{3.3.4.11}$$

3.3.5 Four Parameters KAPA DISTRIBUTION

By using the 4-P Kappa distribution with the L-Moments procedure, on the 10 days Annual Minimum flow series (AMFS) collected from Pakistan flood Commission between 1940 and 2013 at 9 gauging stations across Pakistan will be used in this study to achieve reliable quantile estimates, especially at higher return periods. As other estimation methods like MOM and MLE provides extremely biased higher quantile in the presence of small sample and outliers. The R package is used to estimate the parameters of four parameter kappa distribution for each site of study, as it produce different shapes of distributions by changing its shape parameters. Four parameter Kappa distribution with a probability distribution function, cumulative distribution function and quantile function defined by Hosking, 1994 as:

$$f(x) = \alpha^{-1}\{1 - k(x - \xi)/\alpha\}^{\frac{1}{k}-1}\{F(X)\}^{1-h} \quad (3.3.5.1)$$

$$F(x) = [1 - h\{1 - k(x - \xi)/\alpha\}^{\frac{1}{k}-1}]^{\frac{1}{h}} \quad (3.3.5.2)$$

$$x\,(F) = u + \alpha\,\frac{\left\{1 - \frac{1-F(x)^{h}}{h}\right\}^{k}}{k} \quad (3.3.5.3)$$

where ξ and α are the location and scale parameters, h and k denote the shape parameters of the 4-P kappa distribution. Method of L-moments is used to estimate the parameters of four parameter kappa distribution. The Kappa distribution is a generalized distribution and it produces many distributions, if its shape parameter values are changed. For example, when h=0 and k $\neq$ 0 it becomes generalized Extreme value distribution(GEV) if h=-1 and k $\neq$ 0 it becomes generalized Logistic distribution(GLO), if h= 1 and k = 0 Kappa distribution becomes an 2-p exponential distribution(2P Exp), similarly when h=1 and k $\neq$ 0 Generalized Pareto distribution(GPA) arises. Parida (1999) and Ahmed et al (2013) used 4-P kappa distribution for the modeling of monsoon rainfall.

In the current study, kappa distribution produces the following four distributions.

Table 2. Family of distributions generated by the 4-P generalized Kappa distribution

Shape		*Distribution*	*f(x)*
h	*k*		
0	$\neq 0$	3-P geneneralized extreme value	$f(x) = \alpha^{-1}\ e^{=(1-k)y-e^{-y}}$
-1	$\neq 0$	3-P generalized logistic distribution	$f(x) = \dfrac{\alpha^{-1}e^{-(1-k)y}}{(1+e^{-y})^2}$
1	$= 0$	2-P exponential distribution	$f(x) = \alpha^{-1}exp\{-(x-\alpha)/\alpha\}$
1	$\neq 0$	3-P generalized pareto distribution	$f(x) = \alpha^{-1}e^{-(1-k)y}$

3.3.6 Generalized Extreme Value Distribution

The probability distribution function, cumulative distribution function and quantile function of Generalized Extreme Value distribution are with parametersε, α and k. Here $'\xi'$ is location parameter, is $'\alpha'$ scale para parameter and $'k'$ is shape parameter.

$$f(x) = \alpha^{-1}e^{=(1-k)y-e^{-y}} \qquad -\infty < x \le \xi + {}^{\alpha}/_{k} \qquad if \quad k > 0$$

$$-\infty < x \le \infty \qquad if \quad k = 0$$

$$\xi + {}^{\alpha}/_{k} \le x < \infty \qquad if \quad k < 0 \qquad (3.3.6.1)$$

Where

$$y = \begin{cases} -k^{-1}log\{1 - k(x-\xi)/\alpha\} & k \neq 0 \\ {}^{(x-\xi)}/_{\alpha} & k = 0 \end{cases}$$

$$F(x) = e^{-e^{-y}} \qquad (3.3.6.2)$$

$$x(F) = \begin{cases} \xi + \frac{\alpha\{1-(-logF)^k\}}{k}, & k \neq 0 \\ \xi - \alpha log(-logF) & k = 0 \end{cases} \quad (3.3.6.3)$$

3.3.7 Generalized Logistic Distribution

The probability distribution function, cumulative distribution function and quantile function of Generalized Logistic Distribution are with parameters ξ, α and k. Here 'ξ' is location parameter, is 'α' scale para parameter and 'k' is shape parameter.Range of x is as under

$$-\infty < x \leq \xi + {}^{\alpha}/_{k} \qquad if \quad k > 0$$

$$-\infty < x \leq \infty \qquad if \quad k = 0$$

$$\xi + {}^{\alpha}/_{k} \leq x < \infty \qquad if \quad k < 0$$

$$f(x) = \frac{\alpha^{-1}e^{-(1-k)y}}{(1+e^{-y})^2} \quad (3.3.7.1)$$

Where $$y = \begin{cases} -k^{-1}log\{1 - k(x-\xi)/\alpha\} & k \neq 0 \\ {}^{(x-\xi)}/_{\alpha} & k = 0 \end{cases}$$

$$F(x) = {}^{1}/_{(1+e^{-y})} \quad (3.3.7.2)$$

$$x(F) = \begin{cases} \xi + \frac{\alpha[1-\{(1-F)/F\}^k}{k}, & k \neq 0 \\ \xi - \alpha\log\{\frac{1-F}{F}\} & k = 0 \end{cases} \quad (3.3.7.3)$$

3.3.8 Exponential Distribution

The probability distribution function, cumulative distribution function and quantile function of Exponential Distribution are given as under. Its two parameters $\xi(lower\ endpoint\ of\ dist.)$ and α $is\ scale\ parameter.$

$$f(x) = \alpha^{-1}exp\{-(x-\alpha)/\alpha\} \qquad \varepsilon \leq x < \infty \qquad (3.3.8.1)$$

$$F(x) = 1 - exp\{-(x-\xi)/\alpha\} \qquad (3.3.8.2)$$

$$x(F) = \xi - \alpha log(1-F) \qquad (3.3.8.3)$$

3.3.9 Generalized Pareto Distribution.

The probability distribution function, cumulative distribution function and quantile function of Generalized Pareto Distribution with parameter $\xi(location)$, $\alpha(scale)$ and $k(shape)$. Range of x is $\xi \leq x \leq \xi + {}^{\alpha}/_{k}$ if $k > 0$ and $\xi \leq x < \infty$ if $k \leq 0$

$$f(x) = \alpha^{-1}e^{-(1-k)y} \qquad \propto \leq x \leq \beta \qquad (3.3.9.1)$$

$$F(x) = 1 - e^{-y} \qquad (3.3.9.2)$$

$$x(F) = \begin{cases} \xi + \alpha[1 - \{(1-F)\}^{k} /k\ , & k \neq 0 \\ \xi - \alpha \log(1-F) & k = 0 \end{cases} \qquad (3.3.9.3)$$

3.4 Comparison of the Probability distributions using Goodness-of-fit Criteria

The Selected distribution models are also fitted to the observed 10 days Annual Minimum flow series (AMS) by goodness-of-fit tests, Anderson-Darling test, and Kolmogorov-Smirnov test. On the basis of the results of these tests we will be able to determine which distribution is best fitted to the current data among the four selected distributions. For example see Palynchuk and Guo (2008), Calenda el at (2009), Laio et al (2009), Haddad and Rahman (2011) and Beskow et al (2015) were used these tests for goodness-of-fits. We are also using L- ratio diagram to confirm the distribution that is best fitted to the current data.

3.5 Quantiles of Best Fitted Distribution

After estimating the parameters of best fitted distribution we have to find out the Quantile estimates corresponding to different return periods (T). Annual low flows do not occur with any fixed pattern with respect to time or magnitude. The relationship between return periods and occurrence of an extreme event (e.g. amount of low flow less than or equal to some threshold value at a site) may be established through notation of geometric random variable as: $P(Q_T \leq q) = 1 - \frac{1}{T}$ where P is probability of occurrence of T year return period event i.e. $X \leq x_t$, $F(Q_T) = P(Q_T \leq q) = 1 - P(Q_T > q) = 1 - \frac{1}{T}$ (3.5.1)

Where, q is some threshold value, under which low flow will occur. The above equation is the basis for estimating the magnitude of a low flow. Equation (3.6.2 to 3.6.5) is the quantiles functions of GEV, GLO, GPA and Exp. distributions. By using these equations we will find quantile estimates for different return periods.

$$x(F) = \begin{cases} \xi + \frac{\alpha\{1-(-logF)^k\}}{k}, & k \neq 0 \\ \xi - \alpha log(-logF) & k = 0 \end{cases} \tag{3.5.2}$$

$$x(F) = \begin{cases} \xi + \frac{\alpha[1-\{(1-F)/F\}^k}{k}, & k \neq 0 \\ \xi - \alpha \log\{\frac{1-F}{F}\} & k = 0 \end{cases} \tag{3.5.3}$$

$$x(F) = \xi - \alpha log(1-F) \tag{3.5.4}$$

$$x(F) = \alpha + (\beta - \alpha)F \tag{3.5.5}$$

The quantile function of the probability distribution shows the magnitude of an event in terms of non-exceedance probability as well as exceedance probability (whatever we prefer because total probability is unity, and random variable has two categories "occur or not occur"). For example, a 5 years return period event yields a probability of exceedance (occurence of low flow) equal to 0.2 and the probability of non-exceedance is 0.8 and the corresponding quantile value is based on "P" probability and the selected distributions (GEV, GLO, GPA and Exp.).

CHAPTER 4

Results and Discussion

4.1 Basic Assumption:

Before we examine the information, at first we check the basic assumption of Low Streamflow Analysis which are stationarity, homogeneity and independence of the values. To begin with we apply time series plot to identify the patterns in the 10 days Annual Minimum flow series (AMSF). The AM stream flow of all 09 sites demonstrated that there is no efficient bounced or pattern. So we conclude that there is no consistent increasing/decreasing trend in the data of 09 stations of Pakistan. Time series plots of all stations are shown in the fig 4.1 to 4.9. Next for Stationarity, homogeneity and independency we apply Ljung-Box Q- Statistics, Mann-Whitney U and Mann-Kendall tests respectively.

4.1.1Time Series Trend Analysis

The time series plots are important tool of statistics which are used to observe the pattern of inconsistency in a time series data. The graphical assessment is always useful to provide a basic hint about the likely nature of the sequence. When the data of the same variable over a long run is recorded, and then it is hard to determine any trend or pattern. However, the graphical display of the same data points make easier to spot trends. The trends are very significant as they can be used to plan into the future. The principal assumption of Low stream flow Analysis is stationarity, homogeneity and independency which suggests that there would not be any pattern conduct in the collected hydrological data for different time periods. Time series plots of all nine sites are drawn by using 10-days low stream flow annual data:

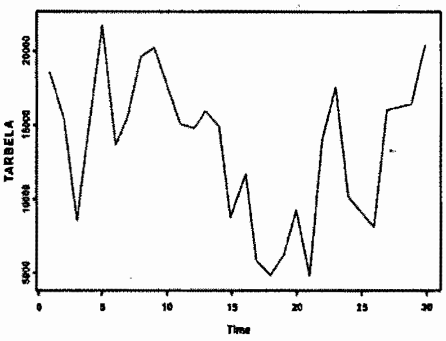

Fig. 4.1 Time series plot of Tarbela

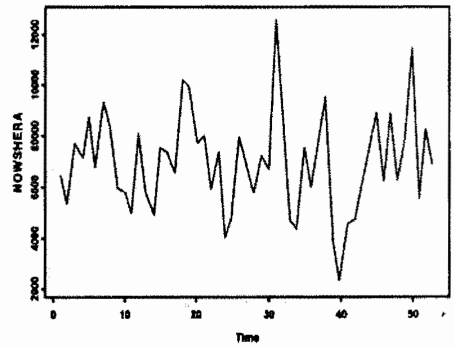

Fig. 4.2 Time series plot of Nowshera

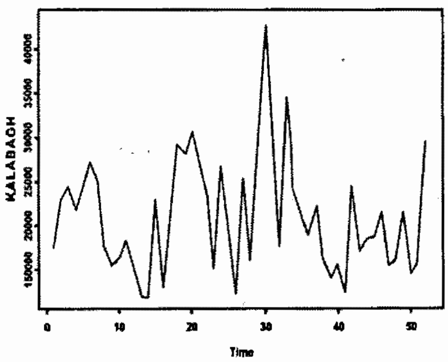

Fig. 4.3 Time series plot of Kalabagh

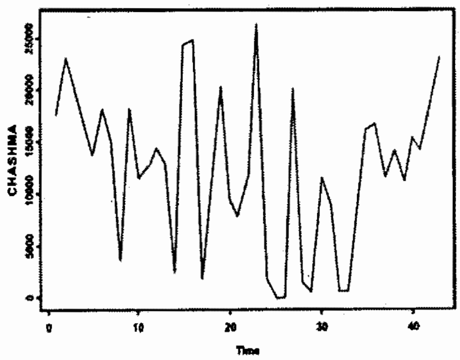

Fig. 4.4 Time series plot of Chashma

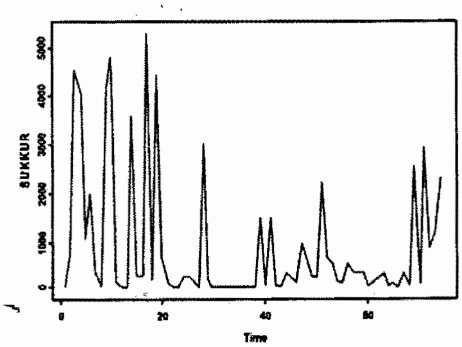

Fig. 4.5 Time series plot of Sukkur

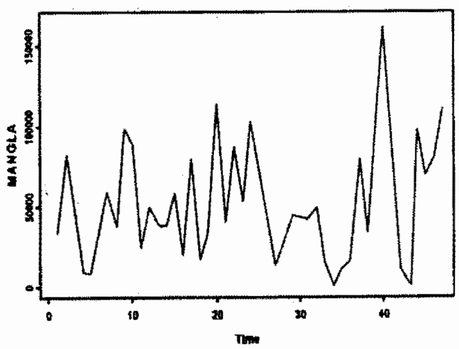

Fig. 4.6 Time series plot of Mangla

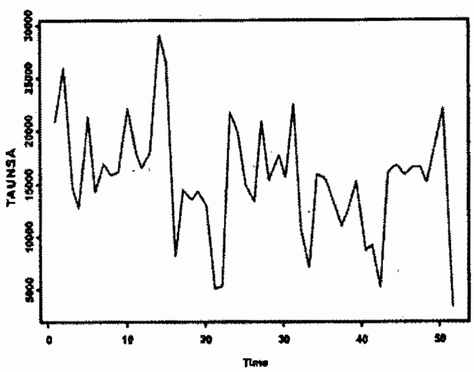

Fig. 4.7 Time series plot of Taunsa

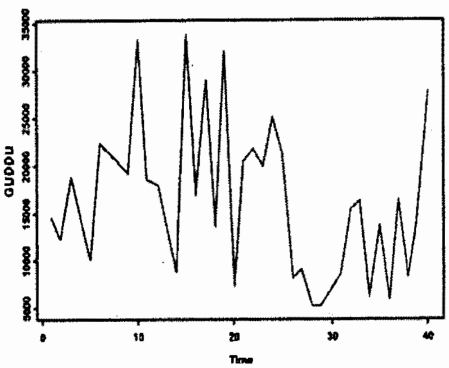

Fig. 4.8 Time series plot of Guddu

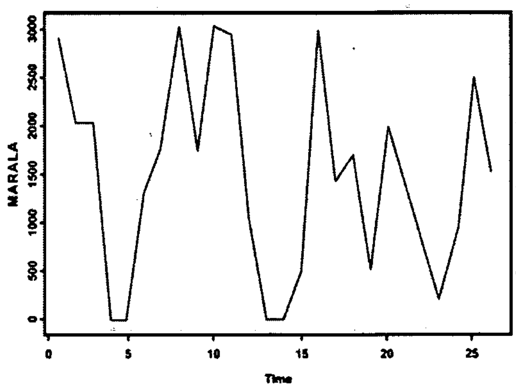

Fig. 4.9 Time series plot of Marala

The Time Series plots presented in Figures 4.1 to 4.9 show that the series of all nine (09) sites have uniform increasing/decreasing trend which indicate that there is randomness in the observation of sites and the time series data is stationary.

4.1.2 Mann-Whitney U Test (Test for homogeneity)

The Mann-Whitney U test (1947) a non-parametric test and is used to check the homogeneity whether the two samples n_1 and n_2 drawn from populations having identical distributions. To carry out test first we arrange the observation in ascending order of magnitude then assign the ranks 1,2,3,..... $n_1 + n_2$ to the arranged observations. We add the ranks given to sample 1 and sample 2 separately and denote the aggregates by P and Q respectively. For both samples we find the values of U as fallow:

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - P \qquad (4.1.2.1)$$

$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - Q \qquad (4.1.2.2)$$

We select the minimum value calculated for U_1 and U_2 as the value for U statistic for Mann-Whitney test. We reject our null hypothesis if the calculated value of U that is $\min[U_1, U_2] \leq$ the smaller value or $\geq$ larger value given in table.

Hypothesis

Ho: Populations have the identical distribution

H1: Populations do not have the identical distribution

Level of significance

$\alpha = 0.05$

The test results together with conclusion drawn of all nine (09) sites are shown in the following table:

Table 4.1 Results of Mann-Whitney test

Sites	**P-value**	**Result**	**Conclusion**	**Homogeneity**
Tarbela	0.06	P-value >0.05	Accept Ho	Homogeneous
Nowshera	0.789	P-value >0.05	Accept Ho	Homogeneous
Kalabagh	0.949	P-value >0.05	Accept Ho	Homogeneous
Chashma	0.101	P-value >0.05	Accept Ho	Homogeneous
Taunsa	0.201	P-value >0.05	Accept Ho	Homogeneous
Guddu	0.352	P-value >0.05	Accept Ho	Homogeneous
Sukkur	0.953	P-value >0.05	Accept Ho	Homogeneous
Mangla	0.890	P-value >0.05	Accept Ho	Homogeneous
Marala	0.297	P-value >0.05	Accept Ho	Homogeneous

From the above table, results show that the information of Annual Average Low Flow of all (09) nine stations are consistent and identically distributed.

4.1.3 Kendall's tau Test (Test for independence/Stationrity)

This test is based on rank correlation method. It was developed by Maurice Kendall (1938). It is a nonparametric test which is employed to estimate the influence of one estimated quantiles on another and vice versa. This test is also used to check the trend over time in the data series (observations are either increasing or decreasing consistently). This test points out the direction of the trend whether it is positive or negative. Such trend analysis are required to check whether observations are temporarily stationary, which is basic assumption of Low flow Analysis application in environmental sciences. The procedure of the test is as follows:

Hypothesis

H_o: there is no trend in the series

H_1: there is trend in the series

Level of significance

$\alpha = 0.05$

Test Statistic

$$\tau = 1 - \frac{4D}{n(n-1)} \tag{4.1.3.1}$$

Here

D= number of discordant pairs

n= Sample size

The test results together with conclusion drawn of all nine (09) sites are shown in the table 4.1.

Table 4.2 Test Results of Kendall's tau test

Site	tau	p-value	Result	Conclusion
Tarbela	-0.14	0.2438	p-value >0.05	Accept Ho
Nowshera	-0.0283	0.55057	p-value >0.05	Accept Ho
Kalabagh	-0.072	0.33002	p-value >0.05	Accept Ho
Chasmha	-0.152	0.17374	p-value >0.05	Accept Ho
Taunsa	-0.135	0.14612	p-value >0.05	Accept Ho
Guddu	-0.211	0.060932	p-value >0.05	Accept Ho
Sukkur	-0.032	0.87639	p-value >0.05	Accept Ho
Mangla	0.0601	0.60468	p-value >0.05	Accept Ho
Marala	-0.109	0.50471	p-value >0.05	Accept Ho

From the above table the results demonstrate that there is no pattern present in the data series of Annual Average Low Flow of all nine sites.

4.1.4 Ljung-Box Q-Statistics

The Ljung-Box test is developed by Ljung and Box (1978) which is modification and extension of Q test earlier developed by Box and Pierce. The Ljung-Box test performed well as compared to Box and Pierce test. It is designed to check the stationarity in time series data. The procedure of Ljung -Box test is as follows:

Hypothesis

H_o: There is no autocorrelation in the data series.

$\rho_1=0, \rho_2=0, \rho_3=0 \ldots\ldots\ldots\ldots\ldots \rho_k=0$

H_1: There is problem of autocorrelation in the data series.

Level of significance

$\alpha = 0.05$

Test Statistic

$$Q = n\,(n+2)\sum_{k=1}^{h}\frac{\hat{\rho}_k^2}{(n-k)} \quad (4.1.4.1)$$

Here

n= sample size

h= number of lags

$\hat{\rho}_k^2$ = sample autocorrelation at lag k

Table 4.3 Test Results of Ljung-Box test

Sites	LB	P –value	Result	
Tarbela	0.621	0.570	P-value	Accept Ho
Nowshera	4.737	0.139	P-value	Accept Ho
Kalabagh	5.040	0.327	P-value	Accept Ho
Chasmha	12.673	0.734	P-value	Accept Ho
Taunsa	11.959	0.615	P-value	Accept Ho
Guddu	16.833	0.695	P-value	Accept Ho
Sukkur	14.324	0.708	P-value	Accept Ho
Mangla	6.308	0.457	P-value	Accept Ho
Marala	8.055	0.532	P-value	Accept Ho

The test results of all nine (09) stations indicated no autocorrelation in the data series. Hence the data series of Annual Average Minimum Low Flow of all nine stations independently distributed.

4.2 Linear moments and Linear moments ratios

Linear moments and Linear moments ratios of 09 sites are calculated by using the R softwere. The results are presented in table 4.4. The values of linear moments and linear moments ratios will be used for the calculation of quantiles function for the four selected distributions by using Method of L-moments.

Table 4.4 Linear moments and Linear moments ratios

Station	l_1	l_2	l_3	l_4	t	t_3	t_4
Tarbela	13650.00	2813.2184	-274.236	169.6862	0.2060	-0.0974	0.0603
Nowshera	6960.377	1107.3294	61.9013	178.2788	0.1590	0.0559	0.1609
Kalabagh	21136.53	3774.6229	652.8643	389.9608	0.1785	0.1729	0.1033
Chashma	12609.30	4474.6401	-183.550	273.2477	0.3548	-0.0410	0.0610
Taunsa	14390.00	2918.0460	-349.261	511.5782	0.2027	-0.1196	0.1753
Guddu	15556.66	4716.6667	789.0805	281.2899	0.3031	0.1672	0.0596
Sukkur	823.5135	614.5168	351.9721	160.8738	0.7462	0.5727	0.2617
Mangla	5161.702	2003.3302	295.9482	185.6839	0.3881	0.1477	0.0926
Marala	.1465.384	597.0769	7.8461	0.6220	0.4074	0.0131	0.0010

It is observed from the table 4.4 that Sukkur site has the smallest average minimum flows, while Kalabagh has the largest average minimum flows. The l_2 ranging from

597.0769 to 4716.6667, l_3 ranging from -349.261 to 789.0805, l_4 ranging from 0.622 to 511.5782. Now comparing the results of Lcv from table 4.4 we found that the site Nowshera has the smallest L-cv 0.159 and the site sukkur has the largest L-cv 0.7462. The range of t_3 is -0.1196 to 0.5727 and of t_4 is from 0.001 to 0.2617.

4.3 Selection of Best-fitted Probability Distribution

Method of L-moments is used to estimate the parameters of four parameter kappa distribution. The Kappa distribution is a generalized distribution and it produces many distributions, if its shape parameter values are changed. For example, when h=0 and k $\neq 0$ it becomes generalized Extreme value distribution(GEV) if h=-1 and k $\neq 0$ it becomes generalized Logistic distribution(GLO), if h= 1 and k = 0 Kappa distribution becomes an 2-p exponential distribution(2P Exp), similarly when h=1 and k $\neq 0$ Generalized Pareto distribution(GPA) arises. Parameters of Kappa distribution and its producing distributions is presented in table 4.5.

Table 4.5. Parameters estmates of 4-P Kapp Distribution

Station	ε	α	k	h	Distribution.
Tarbela	10800.08	8235.56	1.0000	0.0003	GEV
Nowshera	6695.50	1244.69	-0.0056	-1.0000	GLO
Kalabagh	15581.59	8206.075	0.0003	1.0000	2P exp
Chashma	7707.32	12245.65	0.3704	0.0001	GEV
Taunsa	14836.51	2988.22	0.1423	-1.0000	GLO
Guddu	4137.35	16293.17	0.0006	1.0000	2P exp
Sukkur	-9977.50	8830.19	0.0008	1.0000	2P exp
Mangla	2229.74	4578.11	0.0002	1.0000	2P exp
Marala	-346.09	3529.10	0.0190	1.0000	GPA

From the table 4.5 and according to the above mentioned conditions of kappa distribution, GEV is best fitted distribution for Terbala and Chashma, GLO is best fitted for Nowshera and Taunsa, 2P exponential distribution is best fitted for Kalabagh, Guddu, Sukkur and Mangla while GPA distribution is best fitted for marala site. The purpose is not only to specify the best fitted distribution, but also to detect the distribution that will provides correct quantile estimates for each site.

4.4 Testing the Goodness of Fit Measure

Three methods have been used to check the goodness of fit measure i.e. Anderson darling test, Kolmogrove-Smirknov test, and L-moments ratio diagram. Actually, here we verify the results of Table 4.5.

4.4.1 Anderson-Darling Test

The Anderson-Darling test is used to compare the fit observed distribution function to expected distribution function. Anderson-Darling test pays extra weight to the tails than Kolmogorov-Smirnov Test. Test statistic used in this test is:

$$A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)[lnF(X_i) + \ln(1 - F(X_{n-i+1}))] \qquad (4.4.1.1)$$

Here; n = sample size; F = distribution function.

If the calculated value of A^2 greater than the tabulated value we reject the null hypothesis at given level of significance. By using Easy Fit package we calculate AD values for all candidate models then select the distribution having minimum AD value.

4.4.2 Kolmogorov-Smirnov Test

Kolmogorov-Smirnov test is used to make a decision whether two samples come from identical populations. This is based on the distribution function. When we apply this test, we focus on observed cumulative distribution function and hypothesized cumulative distribution function. Let suppose that we have a sample $x_1, x_2, \ldots \ldots x_n$ from a distribution with distribution function F(x). This test follows the steps as under:

Hypothesis:

$H_0: F(x) = F_0(x) \text{ for all values of } x$

$H_1: F(x) \neq F_0(x) \text{ for at least one values of } x$

Level of significance: $\alpha = 0.05$;

Test Statistic:

$$D = \sup_x |S(x) - F_0(x)| \tag{4.4.2.1}$$

Where "D is supremum, over all , of the absolute value of difference $S(x) - F_0(x)$" D is the largest difference between $F_0(x)$ and $S(x)$ when we graphically represent the two functions. We will reject H_0 at specific level of significance if the value of D exceeds the 1-α tabulated value. By using Easy Fit package we calculate Kolmogorov-Smirnov Test values for all candidate models then select the distribution having minimum Kolmogorov-Smirnov Test value.

Table 4.6 Goodness of fit results

Station Name	Kolmogorov Smirnov				Anderson Darling			
	2P Exp	GEV	GLO	GPA	2P Exp	GEV	GLO	GPA
Tarbela	0.2035	0.0993	0.1081	0.1114	6.1966	0.4254	0.5074	0.4848
Nowshera	0.3142	0.0572	0.0556	0.0636	10.517	0.1892	0.1395	0.2352
Kalabagh	0.0710	0.1587	0.0859	0.1264	0.2909	3.4151	0.4572	0.7943
Chashma	0.2797	0.1101	0.1189	0.1206	6.7857	0.5547	0.6359	0.7133
Taunsa	0.3028	0.1456	0.1388	0.1878	6.0249	0.5224	0.4358	0.6242
Guddu	0.1232	0.1566	0.1391	0.1417	0.4032	2.4891	0.5092	0.6194
Sukkur	0.1883	0.3600	0.1932	0.3047	4.2693	5.683	4.4662	10.492
Mangla	0.0829	0.1854	0.0987	0.1060	0.2901	0.5277	0.4204	0.6419
Marala	0.2035	0.1114	0.1081	0.0993	6.1966	0.4848	0.5074	0.4254

4.4.3 Linear-Moments Ratio Diagram

Linear-moment ratio diagram is based on the relationships between L-moment ratios of theoretical probability distributions and data samples. L-moment ratio diagram is used to determine the best fitted distribution using data. This is simplest method to find out the best fitted distribution. Hence, from the L-moment ratio diagram the identification of parent distribution can also be achieved. The associations among $\tau_3$and $\tau_4$are used for the four selected distribution containing GEV, GLO, GPA and Exponential used in this study. For each distribution the sample L-moment ratios take the range $-1 \leq \tau_3 \leq 1$. For this interval, τ_4 is computed for the four selected distribution using their relationships with τ_3. Then the sample L-moment ratios are plotted in the diagram as (τ_3, τ_4) the distribution for which their L-moment ratios are close to the values of sample ratios are considered to be the best distribution for fitting the observed data. Three parameters distributions are presented with line and two parameters distributions presented with dots in L-ratio diagram. L-moments ratio diagram / plot for all nine sites are shown in Fig. 4.1.

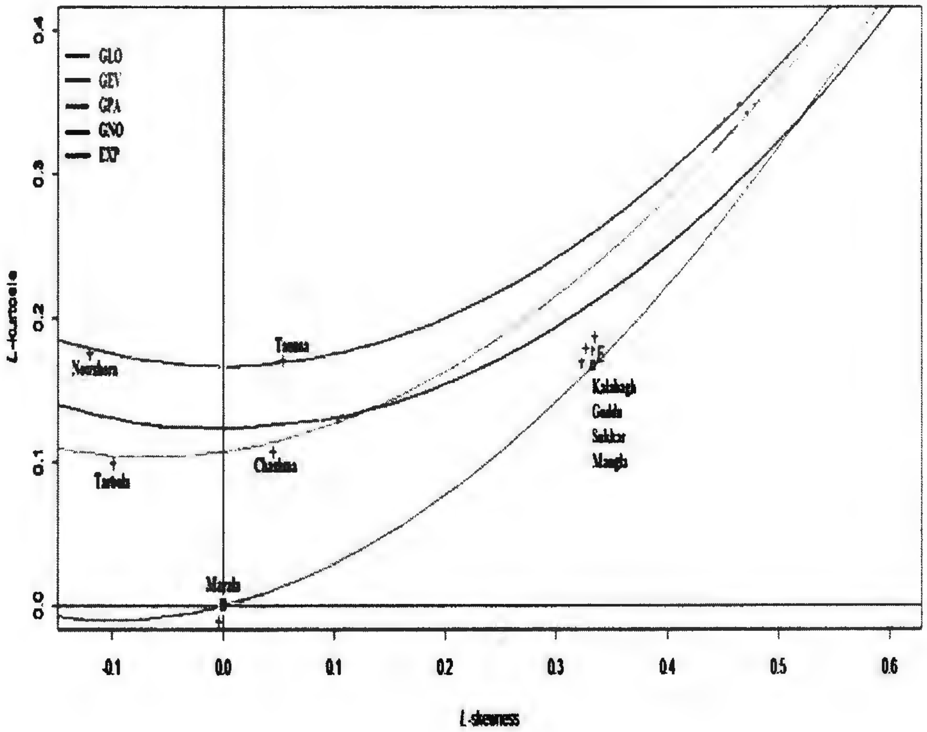

Fig. 4.1 L-moment ratio diagram for nine sites

4.4.4 Best fitted distributions based on AD Test, Kolmogorov-Smirnov Test and LMRD

According to the results of AD Test, Kolmogorov-Smirnov Test (Table 4.6) and L-Moment Ratio Diagram (LMRD), we conclude that GEV distribution is best fitted for two sites, GLO distribution is best fitted for two sites, and GPA distribution is best fit for one site where as Exponantial distribution is best fitted for four sites.

4.4. Quantiles Estimation and Return Periods

The quantile estimates $q^(F)$ with return periods and non-exceedance probabilities for each site are presented in Table 4.7 calculated by using linear moment procedure.

Quantile estimates has been calculated for the best fitted distribution which is relatively best for the site mentioned against the distribution. For Tarbela site GEV is the best fitted distribution and we can interpret it as, for example $q^{\wedge}_{GEV} = 23188.62$ is the threshold value of flow which may occurs once in 500 years on the average. In other words, there are only 0.2% chances that in a return periods of 500 years, one time discharge (low flow) will be greater than the threshold value 23188.62 and consequently low flow will not occur and 99.8% are the chances that in a return periods of 500 years, one time discharge (low flow) will be lower than the threshold value 23188.62 indicating a situation of occurrence of the drought. Here point to be noted is that a 500 year low flow event occurrence in one year has no effect on the probability of its happening in next year. Another way to interpret that the probability of occurrence at least once of a T year event in N next years $p_N = 1 - (1-p)^N$. The probability that 50 year low flow will occur within next 100 year is $p_N = 1 - (1 - 1/50)^{100} = 0.867$. Therefore the probability of a 50 year event occurring atleast once in next 100 year period is about 87%.

Table 4.6 Quantile Estimates for Nine sites

Sites Name	Best Distribution	Quantile estimates with non-exceedance probability F							
		*0.5 **2	0.8 5	0.9 10	0.95 20	0.98 50	0.99 100	0.998 500	0.999 1000
Tarbela	GEV	14142.77	18120.38	19806.15	20951.39	21964.22	22480.19	23188.62	23363.34
Nowshera	GLO	6858.06	8446.418	9434.095	10384.19	11648.19	12630.3	15041.38	16144.6
Kalabagh	2 P exp	18819.97	25735.85	30969.26	36202.66	43118.54	48351.95	60501.23	65734.63
Chashma	GEV	12918.23	19604.94	22749.7	25067.29	27300.4	28547.46	30472.9	31013.84
Taunsa	GLO	14959.84	18600.51	20465.46	22031.09	23825.52	25032.84	27450.36	28352.62
Guddu	2 P exp	12661.57	21304.86	27843.32	34381.79	43026.63	49565.1	64746.85	71285.31
Sukkur	2 P exp	446.3443	1572.499	2424.424	3276.266	4402.421	5254.346	7232.425	8084.267
Mangla	2 P exp	3932.185	7603.187	10380.7	13157.69	16828.7	19606.21	26054.72	28831.72
Marala	GPA	13909.45	23123.06	28053.76	31721.94	35171.06	37016.86	39681.22	40372.2

*non exceedance probability F ** Return periods

Summary and Conclusion

10 days Annual Minimum flow analysis has been performed on 09 sites of Pakistan. The 10 days Annual Minimum flow data was taken from Pakistan flood Commission. The average minimum streamflows was being measured in cusec. The record length of average minimum streamflows varies from 30 to 74 year.

The average minimum streamflows of 09 sites of Pakistan being studied are located in three provinces of Pakistan namely Punjab, Sindh and Khyber Pakhtunkhwa (KPK). Initially the basic assumptions of Low streamflow analysis are tested by different tests that are time series plots, Mann-Whitney test, Mann-Kendall's tau test, Ljung-Box-Statistics. All sites satisfied these tests, which means that observations at any site are independent, stationary and homogeneous. Therefore data of all the 09 sites were used for further analysis.

The sample Linear moments and Linear moments ratios are very useful for summarizing statistical properties of hydrological data and can be used for parameters estimation of distribution and choice of best fit distribution. The four selected distributions GEV, GLO, Exp. and GPA are being applied to the average 10 days Annual Minimum flow of 09 sites. Then the distributions are compared by using goodness-of-fit tests as Anderson darling test and Kolmogrove-Smirknov test. While for graphical representation of best fit distributions, the L-Moment ratio diagram is being used. In this study of L-moments the result showed that for the most of the sites Exponential is best fitted distribution, followed by GEV, GLO, and GPA. For policy implication and practical purposes at least these four distributions can be used for minimum streamflows at these sites. In the study quantile estimates are found for that distribution which is best fitted for that site. It can be suggested that gauging networking system should be applied and increased in the country to improve national water resources planning and development. These return periods and subsequently estimates of quantiles are very significant in the design of Hydrological projects. Frequency Analysis of low flows is of immense importance in water resources management such as amount of low flow for hydropower generation, water quality management, designing of irrigation system and impact of prolonged droughts on aquatic ecosystems in the country.

Recommendations for the Future Study

1. For future study the method of L-moment may be compared with other estimation methods such as Maximum likelihood method or the Method of Moments.
2. The study may be conducted using other estimation methods like TL-moments, LQ-moments, partial L-moments and LH-moments then estimated quantiles can be compared with L-moments.
3. The estimates of study can be used to assess the feasibility of construction of new water structures in future.

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