MHD Non-Darcian Flow of Nano Fluid in a Wavy Channel

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1. Fluid dynamics
2. Numerical analysis

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A Dissertation Submitted in the Partial Fulfillment of the Requirements for the degree of

MASTER OF SCIENCE IN MATHEMATICS

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Certificate

Entropy Generation in Non-Darcian Natural Convection Flow Over Wavy Surface

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A DISSERTATION SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF THE MASTER OF SCIENCE IN MATHEMATICS

We accept this dissertation as conforming to the required standard.

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Mr. Aftab Ahmad

Dedicated

To

My

Honorable MOTHER and Loving FATHER

SUCCESS

belong to those who believes in the beauty of their

DREAMS

Declaration

I hereby declare that the original work of this thesis neither as a whole nor as a part has been copied out from any source. It is further declared that I have prepared this thesis entirely on the basis of my personal efforts made under the sincere guidance of my supervisor. No portion of the work presented in this thesis has been submitted in the support of any application for any degree or qualification of this or any other institute of learning.

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Nomenclature

a	Wave amplitude (m)	d	Width of channel (m)
L	Length of channel (m)	λ	Wavelength
H_1, H_2	Lower and upper walls of channel	g	Gravitational acceleration (ms^{-2})
$\bar{V}$	Dimensional fluid velocity	$\bar{T}$	Dimensional fluid temperature
$\bar{p}$	Pressure (Nm^{-2})	p	Dimensionless pressure
ρ	Density $(kg\ m^{-3})$	μ	Viscosity $(N\ s\ m^{-2})$
k	Thermal conductivity $(\mathrm{W\ m^{-1}}\ K^{-1})$	σ	Electrical conductivity
c_p	Specific heat at constant pressure $(kJ\ kg^{-1}\ \mathrm{K}^{-1})$	Φ	Viscous dissipation
β	Volumetric volume expansion coefficient (k^{-1})	ϕ	Nanoparticle volume fraction
K	Porous medium permeability coefficient	F^*	Non Darcy parameter
T_1	Lower wall temperature	T_2	Upper wall temperature
T^*	Mean value of $\mathrm{T_1}$ and $\mathrm{T_2}$	$\bar{J}$	Joule current
B_0	Uniform transverse magnetic field	E_0	Uniform electric field
$\bar{u}$, $\bar{v}$	Dimensional $\bar{x}$ and $\bar{y}$ components of velocity (ms^{-1})	δ	Dimensionless wave number
u, v	Dimensional x and y components of velocity (ms^{-1})	θ	Dimensionless temperature
Re	Local Reynolds number	q_r	Radiative heat flux
Pr	Prandtl number	Gr	Grashof number
E_c	Eckert number	Ha	Hartmann number
E_1	Local electromagnetic parameter	D_a	Porosity parameter
α	Radiation absorption coefficient	N	Radiation parameter
m	Temperature scale	h	Embedding parameter
C_f	Skin friction coefficient	Nu	Nusselt number

Subscripts

f	Fluid
p	Particle
nf	Nanofluid

Preface

The flow of nanofluids has taken considerable importance in recent times due to its thermal properties and engineering applications. These fluids are mentioned first by Choi [1 , 2] who described their extraordinary properties, which gave a rise even up to 50% of thermal conductivity of liquids which are less conductive such as water or ethylene glycol. Ever since a huge literature in both experimental and theoretical domains appears [3-12]. It is vitally important that who is discussing the behavior of the fluid to keep an eye on entropy generation or heat loss during the flow.[13-12] The heat loss or thermal irreversibility must be minimized in order to generate a thermally ideal system. It is established that the thermal irreversibility is due to viscosity, electric field and conduction properties of the fluid.

The work aim to discuss the entropy generation of Ag-water nanofluid flowing through a wavy channel with simultaneous effects of viscous dissipation, mixed convection and for a reverse magnetic field while flowing through a non Darcy porous space.

This thesis is based on three chapters. Chapter one provides some basic concepts and definitions for the subsequent chapters. Chapter two deals with the MHD non-Darcian flow of nanofluid in a wavy channel. Third chapter is the extension of second chapter with the discussion of entropy generation in MHD non-Darcian flow of nanofluid in a wavy channel.

Contents

Chapter 1

Preliminaries

In this chapter, some basic definitions and parameters are defined, which are helpful in the subsequent chapters.

1.1 Fluid Mechanics

Fluid mechanics is that branch of applied mathematics that deals with the behavior of fluids (liquids or gases) at rest or in motion.

Fluid mechanics may be divided into three categories: fluid statics, Fluid kinematics, fluid dynamics. Fluid statics deal with the study of fluids at rest, while fluid kinematics is the study of fluid in motion without considering the forces which cause or accompany the motion. On the other hand fluid dynamics is the study of fluids in motion considering the forces acting on the fluid.

1.2 Fluid

A fluid is a substance that deforms continuously when subjected to a shear stress, no matter how small that shear stress may be. In simple words, a fluid is a substance which is capable of flowing and which conforms to the shape of containing vessel.

1.3 Some Physical Properties of the Fluid

1.3.1 Viscosity

The viscosity of a fluid is a measure of its resistance to deformation i.e. resistance to a shearing force when the fluid is in motion. For liquids, it corresponds to the informal notion of "thickness". For example, honey has a higher viscosity than water. Mathematically, it is defined as the ratio of shear stress to the rate of shear strain i.e.

$$Viscosity = \mu = \frac{shear\ stress}{Rate\ of\ shear\ strain},$$

where μ is called dynamic viscosity.

1.3.2 Density

The density of a fluid denoted by ρ is defined as the mass per unit volume. Thus if m is the mass enclosed in a volume v, then

$$Density = \frac{mass\ of\ fluid}{volume\ of\ fluid},$$

or

$$\rho = \frac{m}{v}.$$

The unit of density is kgm^{-3}.

1.3.3 Kinematic Viscosity

The kinematic viscosity (also called "momentum diffusivity") is the ratio of the absolute viscosity μ to the density of the fluid ρ. It is usually denoted by the Greek letter nu (υ).

$$\upsilon = \frac{\mu}{\rho},$$

Its unit is m^2s^{-1}.

1.3.4 Temperature

Temperature of a body is defined as a measure of the intensity of heat. Heat always flows from a region of higher temperature to one of lower temperature. Physical state of a substance change with temperature. For example, water at low temperature is ice, at higher temperature is water and at still a higher temperature is steam. Temperature can be measure by different scales. Three common used temperature scales are the Celsius (or centigrade), Kelvin (or absolute) and the Fahrenheit scale.

1.3.5 Energy

Energy is the capacity of a physical system to perform work. Energy exists in several forms such as heat, kinetic or mechanical energy, light, potential energy, electrical, or in other form.

1.3.6 Stress

The stress is defined as the force per unit area on which it acts. If the stress is uniformly distributed over the plane area A, the stress called the average stress.The stress at any point P in the fluid is defined as

$$\text{Stress at any point P} = \underset{\Delta s \to 0}{Lt} \frac{\Delta \vec{F}}{\Delta S},$$

where $\Delta \vec{F}$ is the force acting on an element of surface area ΔS enclosing the point P.

1.3.7 Entropy Generation

Entropy is the measure of system's thermal energy per unit temperature that is unavailable for doing useful work. Work is obtained from ordered molecular motion, so the amount of entropy is also the measure of molecular disorder. For reversible process entropy is zero.

1.3.8 Enthalpy

Enthalpy is a measure of total energy of a thermodynamic system. It is a state function and an extensive quantity.

1.4 Types of Fluids

1.4.1 Compressible and Incompressible Fluids.

It is usual to divide fluids into two groups: liquids and gases. All known liquids are slightly compressible and their density varies little with temperature and pressure. For most practical purposes, liquids are considered to be incompressible. But for situation involving either sudden or great changes in pressure or temperature, their compressibility becomes important.

1.4.2 Inviscid Fluids

An in viscid fluid is that fluid having zero viscosity. With zero viscosity the fluid offers no internal resistance to a change in shape. Thus an inviscid fluid, whether at rest or in motion, can exert only a normal stress (i.e. pressure) or any surface with which it is in contact. Consequently the shear stress in this case is zero.

1.4.3 Ideal Fluids

A fluid which is both inviscid and incompressible is called an ideal fluid.

1.4.4 Real Fluid

A real fluid is one which has finite viscosity and thus can exert a tangential stress on a surface with which it is in contact. The flow of real fluid is called a viscous flow. Real fluids can further be subdivided into Newtonian fluids and non-Newtonia n fluids.

1.4.5 Newtonian Fluid

The Newtonian fluid is a fluid in which shear stress is directly and linearly proportional to the rate of deformation. For example water, air, emulsions. Mathematically, it can be described as:

$$\tau_{yx} = \mu \frac{du}{dy},$$

Where τ_{yx} is the shear stress acting on the plane normal to the $y-axis$ and μ is the viscosity of fluid. Water and gasoline are the examples of Newtonian fluids under normal conditions. The Newtonian fluid is an idealized fluid that approximates the behavior of water, air and many other fluids.

1.4.6 Non-Newtonian Fluid

The Newtonian fluid is a fluid in which shear stress is directly but not linearly proportional to the rate of deformation. For example, paints, blood, shampoo, flubber, (suspension of starch in water)

Mathematically it can be expressed as:

$$\tau_{yx} = \mu \left(\frac{du}{dy} \right)^{m_1}, m_1 \neq 0,$$

Where m_1 denotes the flow behavior index and consistency index respectively.

1.5 Types of Flows

1.5.1 Uniform Flow

A flow is said to be uniform when the velocity vector as well as other fluid properties do not change from point to point in the fluid. Thus

$$\frac{\partial \vec{V}}{\partial s} = 0, \frac{\partial \rho}{\partial s} = 0.$$

i.e. the partial derivative w. r. t 'distance' of any quantity vanishes.

1.5.2 Non Uniform Flow:

A flow is said to be non-uniform when velocity, density, pressure, etc. change from point to point in the fluid flow i.e.

$\frac{\partial \vec{V}}{\partial s} \neq \vec{\upsilon}$ For example, a liquid flow through a long straight pipe of constant diameter is s uniform flow. On the other hand, a liquid flow through a pipe of reducing section or through a curved pipe is a non-uniform flow.

1.5.3 Steady Flow

A flow is said to be steady (i.e. stationary) when the velocity vector and other fluid properties at every point in a fluid do not change with time so that flow pattern remains unchanged i.e.

$$\frac{\partial \vec{V}}{\partial t} = 0, \qquad \frac{\partial \rho}{\partial t} = 0,$$

i.e. the partial derivative w.r.t time of any quantity vanishes. Thus the velocity is constant w.r.t. time but it may vary from point to point. Hence the steady flow may be uniform or non-uniform.

1.5.4 Unsteady Flow:

A flow is said to be unsteady when fluid properties and conditions at any point in a fluid change with time i.e. $\frac{\partial \vec{V}}{\partial t} \neq \vec{\upsilon}$ etc.

1.5.5 Laminar Flow

A flow is said to be laminar if the fluid particles move along straight parallel path in layers or laminar. Thus in this flow, the curves traced out by any two different fluid particles do not intersect.

1.5.6 Turbulent Flow

A flow is said to be turbulent if the particle of the fluid move in irregular fashion in all directions.

1.5.7 Irrotational Flow

A flow is said to be irrotational if the fluid particles do not rotates about their own axes during the flow.

1.5.8 Rotational Flow

A flow is said to be rotational if the fluid particles go on rotating about their own axis during the flow.

1.5.9 Internal Flow

Internal flows are those where fluid flow through confined spaces such as pipes, open channel, and fluid mechanics. The internal flow of liquids in which the channel does not flow full is called an open channel flow. For example, flow in rivers and irrigation canals.

1.5.10 External Flow

External flows occur over bodies immersed in an unbounded fluid, such as atmosphere through which airplanes, missiles, and space vehicle travel, or the ocean water through which submarines and torpedoes.

1.6 Dimensionless Numbers

A dimensionless number is a number without any unit associated with it. It is the ratio of the quantities having same dimensions. They fully characterize the fluid under flow and processing. There are many dimensionless numbers. Which are used in fluid mechanics, few of them are presented as follows:

1.6.1 Prandtl Number

The Prandtl number (Pr) is a dimensionless number, named after the German physicist Ludwig Prandtl, defined as the ratio of momentum diffusivity to thermal diffusivity. That is, the Prandtl number is given as

$$Pr = \frac{\upsilon}{\alpha} = \frac{\textit{viscous diffusion rate}}{\textit{thermal diffusion rate}} = \frac{\mu / \rho}{k / c_p \rho} = \frac{c_p \mu}{k}.$$

1.6.2 Reynolds Number

In fluid mechanics, the Reynolds number (Re) is a dimensionless quantity that is used to help predict similar flow patterns in different fluid flow situations. The concept was introduced by George Gabriel Stokes in 1851, but the Reynolds number is named after Osborne Reynold (1842–1912), who popularized its use in 1883.

1.6.3 Grashof Number

The Grashof number (Gr) is a dimensionless number in fluid dynamics and heat transfer which approximates the ratio of the buoyancy to viscous force acting on a fluid. It frequency arises in the study of situations involving natural convection.

1.6.4 Eckert Number

The non-dimensional number which expresses the relationship between flow's kinetic energy and enthalpy is called the Eckert number E_c: Mathematically, it is expressed as

$$E_c = \frac{v^2}{c_p.A}.$$

1.6.5 Bejan Number

The Bejan number is the ratio of heat transfer irreversibility to total irreversibility due to heat transfer and fluid frictio

1.6.6 Hartman Number

It is a dimensionless number which is expressed by the ratio of electromagnetic force to the viscous force. It is denoted by Ha and mathematically given by

$$Ha = BL\sqrt{\frac{\sigma}{\rho\nu}}.$$

1.6.7 Nusselt Number

A Nusselt number close to one , namely convection and conduction of similar magnitude, is characteristic of "slug flow" or laminar flow. A large Nusselt number corresponds to more active convection, with turbulent flow typically in the 100-1000 range. The convection and conduction heat flows are parallel to each other and to the surface normal of the boundary surface, and are all perpendicular to the mean fluid flow in the simple case.

$$Nu = \frac{Convective\ heat\ transfer}{Conductive\ heat\ transfer} = \frac{hL}{k},$$

Where h is the convective heat transfer coefficient of the flow, L is the characteristic length, k is the thermal conductivity of the fluid.

1.6.8 Skin Friction

Skin friction arises from the friction of the fluid against the "skin" of the object that is moving through it. Skin friction arises from the interaction between the fluid and the skin of the body, and is directly related to the area of the surface of the body that is in contact with the fluid. Skin friction follows the drag equation and rises with the square of the velocity. Skin friction is caused by viscous drag in the boundary layer around the object. There are two ways to decrease skin friction, the first is to shape the moving body so that smooth flow is possible, like an airfoil. The second method is to decrease the length and cross section of the moving object as much as is practicable

1.7 Convection

Convection is the transfer of heat by the actual movement of the warmed matter. Heat leaves the coffee cup as the current of the stream and air rise. Convection is the transfer of heat energy in a gas or liquid by movement of currents. It can also happen in some solids like sand. The heat moves with the fluid. Consider this: convection is responsible for making macaroni rise and fall in a pot of heated water. The warmer portion of the water is less dense and therefore, they rise. Meanwhile the cooler portions of the water fall because they are denser.

1.7.1 Natural Convection

Natural convection is a mechanism, or a type of heat transport, in which the fluid motion is not generated by any external source (like a pump , fan, suction device, etc) but only by density differences in the fluid occurring due to temperature gradients.

1.7.2 Forced Convection

Forced convection is a mechanism, or type of heat transport in which fluid motion is generated by an external source (like a pump, fan, suction device, etc). It should be considered as one of the main methods of useful heat transfer as significant amounts of heat energy can be transported very efficiently.

1.7.3 Mixed Convection

Combined forced convection and natural convection, or mixed convection, occurs when natural convection and forced convection mechanism act together to transfer heat. This is also defined in situations where both pressure forces and buoyant forces interact.

1.8 Homotopy Analysis Method

Homotopy Analysis Method (HAM) allows perturbation solution to be valid for moderate to large value of parameter. HAM has been developed by Liao in 1992.This method has been successfully applied to solve m any types of non-linear problems. The basic idea of HAM is to produce a succession of approximate solution tend to the exact solution from any initial guess of the problem. The presence of auxiliary parameter and functions in the approximate solution results in a production of a family of approximation solution rather than the single solution produced by traditional perturbation methods by varying these auxiliary functions and parameters. It is possible to adjust the region and rate of convergence of series solution.

1.8.1 General Approach of HAM

Consider non-linear equation

$$N\left[u(x)\right]=0 \tag{1.1}$$

Subject to some initial condition or boundary condition. The first step in the HAM solution of the equation is construct the homotopy.

$$H\left[\phi(x;q);\phi_0(x),H(x),\hbar \qquad -q)L\left[\phi(x;q)-\phi_0(x)\right] -q\hbar \qquad \phi(x;q)\right]$$

where $\hbar$ is an auxiliary parameter, $H(x)\neq 0$ is an auxiliary function, $q\in\left[0,1\right]$ is embedding parameter, $\phi_0(x)$ is an initial approximation to the solution that satisfies the given initial condition or boundary conditions, $\phi(x;q)$ satisfies the initial or boundary conditions and L is some linear operator. The linear operator L should normally be of the same order as the non linear operator *N*. Setting homotopy equal to zero so that

$$(1-q)L\left[\phi(x;q)-\phi_0(x)\right] = q\hbar \qquad \phi(x;q)\right] \tag{1.2}$$

Equation (1.2) is known as the zero-order deformation equation. By letting $q= 0$ in this equation we obtain

$$L\left[\phi(x;0)-\phi_0(x)\right] = 0 \tag{1.3}$$

It follows from our definition of

$L\left[\phi(x)\right]$, $\phi(x;q)$, and $\phi_0(x)$ that

$$\phi(x;0) = \phi_0(x) \tag{1.4}$$

Now letting q=1 then

$$N\left[\phi(x;1)\right] = 0$$

It is clear that $\phi(x;q)$ satisfy the initial and boundary condition of the problem an

$$\phi(x;1) = \phi(x) \tag{1.5}$$

So $\phi(x;q)$ varies continuously from initial approximation to the required solution

$\phi(x)$ As q increases 0 to 1. Now we define the terms

$$\phi_m(x) = \frac{1}{m!}\frac{\partial^m \phi(x,q)}{\partial q^m}\Big|_{q=0} \tag{1.6}$$

By Tailor's theorem we can write

$$\phi(x;q) = \phi(x;0) + \sum_{m=1}^{\infty} \frac{1}{m!} \frac{\partial^m \phi(x,q)}{\partial q^m}\bigg|_{q=0} \,, q^m = \phi_0(x) + \sum_{m=1}^{\infty} \phi_m(x) q^m \tag{1.7}$$

Now we differentiate Eq (1.4) with respect to q and setting $q = 0$ and finally divided *by* $m!$ Then so called mth-order deformation equation become

where,

$$L\left[\phi_m(x) - \chi_m \phi_{m-1}(x)\right] = \hbar \qquad \frac{1}{(m-1)!} \frac{\partial^{m-1} N\left[\phi(x;q)\right]}{\partial q^{m-1}}\bigg|_{q=0} \tag{1.8}$$

$$\chi_m = \begin{cases} 0, \ldots\ldots\ldots m \le 1 \\ 1, \ldots\ldots\ldots\ldots else \end{cases}$$

Thus this equation is valid for all $m \geq 1$. The right hand side of Eq.(1.8) will depend on term $\phi_m(x)$ with n < m. As a result the term $\phi_m(x)$ can be obtained in order of increasing m by solving the linear deformation equation in succession. The solution to the mth order deformation equation can be written as,

$$\phi_m(x) = \phi^h(x) + \phi^p{}_m(x), \tag{1.9}$$

Where $\phi^h(x)$ satisfies the homogenous equation

$$L\left[\phi^h(x)\right] = 0 \tag{1.10}$$

and $\phi^p{}_m(x)$ is particular solution of Eq.(1.7) we can express it as

$$\phi^{P}{}_{m}(x) = \chi_m \phi_{m-1}(x) + L^{-1}\left(\frac{1}{(m-1)!}\frac{\partial^{m-1} N[\phi(x;q)]}{\partial q^{m-1}}\Big|_{q=0}\right) \tag{1.11}$$

L^{-1} is the inverse operator of the linear operator L. The m[th] partial sum of the term $\phi_m(x)$ as

$$\phi_m(x) = \sum_{k=o}^{\infty} \phi_k(x) \tag{1.12}$$

Thus solution can be expressed as

$$\phi(x) = \phi(x\ ;1 =) \sum_{k=o}^{\infty} \phi_k(x) = \lim_{m\to\infty} \phi^m(x) \tag{1.13}$$

This solution will be valid where ever the series converges.

1.8.2 Advantages of Homotopy Analysis Method

HAM provides the liberty in how to develop the solutions to non linear problems. The liberty endure several benefits over ordinary perturbation methods such as,

1. It is always valid no matter whether there exit small physical parameter or not.
2. The HAM technique can be used to develop valid solution even to problems that are highly nonlinear.
3. The HAM provides a convenient way to guarantee the convergence of approximation series.
4. The HAM provides great freedom to choose the equation type of linear sub-problem and the base function of solution.

Chapter 2

MHD Non-Darcian Flow of Nanofluid in a Wavy Channel

In this chapter, we discussed the flow of nanofluid through non Darcian porous wavy channel. Constant magnetic field is applied perpendicular to flow and sinusoidal wavy walls of channel considered with long wavelength. A nonlinear system of equations comprising mass, momentum and thermal energy equations for nanofluid with mixed convection completes the flow model. The coupled ODE's are solved using Homotopy Analytical Method (HAM). Graphs for velocity and heat are drawn to observe the effects of various parameters like Darcy number, porosity parameter, Magnetic number, Grashof number and Eckret number.

2.1 Problem Formulation

The 2-D steady flow of a nanofluid through the horizontal symmetric channel bounded by wavy walls is considered in the flow direction and y-axis is perpendicular to it. The configuration of the walls is defined by the following equations

$$H_1 = -d - a\cos\left(\frac{2\pi}{L}\bar{x}\right) \;, H_2 = d + a\cos\left(\frac{2\pi}{L}\bar{x}\right) \tag{2.1}$$

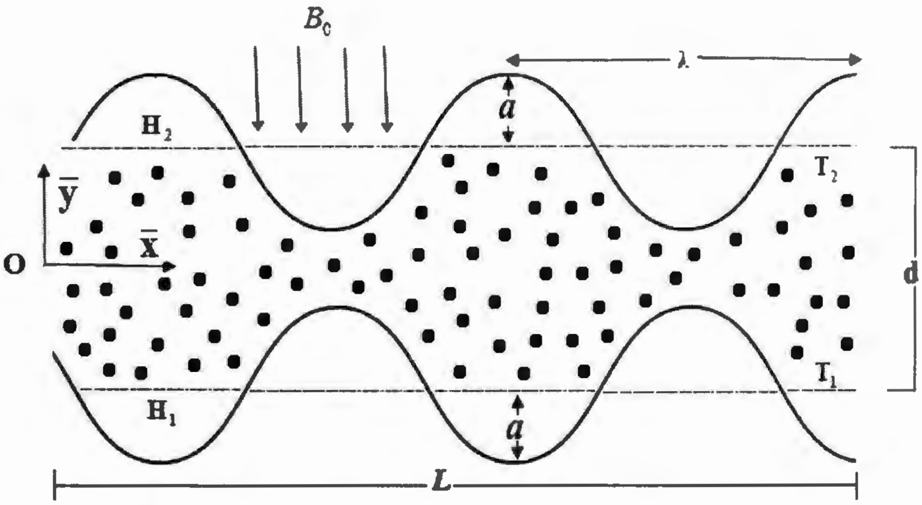

Fig. 2.1 Schematic figure of the physical model

where a is wave amplitude of wavy wall, d the mean width of channel and L the length of wavy channel. Following equations represent the conservation of mass, momentum and energy respectively representing in components form can be written as,

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0 \tag{2.2}$$

$$\rho_{nf}\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right)=-\frac{\partial \bar{p}}{\partial \bar{x}}+\mu_{nf}\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right)-\sigma_{nf}B_0^2\bar{u}-\frac{\mu_{nf}}{k}\bar{u}+(\rho\beta)_{nf}\,g\left(\bar{T}-T^*\right)-\bar{u}^2F^* \tag{2.3}$$

$$\left(\rho C_p\right)_{nf}\left(\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{T}}{\partial \bar{y}}\right)=K_{nf}\left(\frac{\partial^2 \bar{T}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{T}}{\partial \bar{y}^2}\right)+(\mu)_{nf}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2+\sigma_{nf}B_0^2\bar{u}^2 \tag{2.4}$$

The forces involved in the equation of conservation of momentum can be written as:

Inertial term $=\rho_{nf}\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right)$, Pressure gradient $=\left(-\frac{\partial \bar{p}}{\partial \bar{x}}\right)$,

Viscous forces $=\mu_{nf}\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right)$, Lorentz force $=\sigma_{nf}B_0^2\bar{u}$, Darcy forces $=\left(\frac{\mu_{nf}}{k}+F^{*}\ \bar{u}\right)\bar{u}$ and Convection$=(\rho\beta)_{nf}\, g\left(\bar{T}-T^{*}\right)$

where ρ_{nf} is the density, μ_{nf} is the viscosity and σ_{nf} is electric conductivity of the nanofluid, while B_o is magnetic field strength and F^{*} is the Frochhiemers correction.

The terms present in the energy equation can be classified as follows:

Inertial term$=\left(\rho C_p\right)_{nf}\left(\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{T}}{\partial \bar{y}}\right)$, Heat conductivity$=K_{nf}\left(\frac{\partial^2 \bar{T}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{T}}{\partial \bar{y}^2}\right)$,

viscous dissipation$=\mu_{nf}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2$ and Joule's heating$=\sigma_{nf}B_0^2\bar{u}^2$,

Where C_p is the specific heat of the nanofluid, The effective density of the nanofluid is given by

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_p$$

where ϕ is the solid volume fraction. The effective dynamic viscosity of the nanofluid given by Brinkman [17] is

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}}$$

The heat capacitance of the nanofluid is

$$\left(\rho C_p\right)_{nf}=(1-\phi)\left(\rho C_p\right)_f+\phi\left(\rho C_p\right)_p$$

The electrical conductivity of nanofluid is

$$\frac{\sigma_{nf}}{\sigma_f}=1+\frac{3\left(\frac{\sigma_p}{\sigma_f}-1\right)\phi}{\left(\frac{\sigma_p}{\sigma_f}+2\right)-\left(\frac{\sigma_p}{\sigma_f}-1\right)\phi}$$

The effective thermal conductivity of the nanofluid given by Maxwell Grant model

[18] is

$$\frac{k_{nf}}{k_f}=\frac{k_p+2k_f-2\phi\left(k_f-k_p\right)}{k_p+2k_f+\phi\left(k_f-k_p\right)}$$

The corresponding boundary conditions are:

$$\begin{aligned}&\bar{T}=T_1,\quad \bar{u}=0,\quad \bar{v}=0,\ \text{at}\ \bar{y}=H_1\\&\bar{T}=T_2,\quad \bar{u}=0,\quad \bar{v}=0\ \ \text{at}\ \bar{y}=H_2\end{aligned}\tag{2.5}$$

In order to reduce the governing equations and boundary conditions into similar form, following similarity transforms are deployed:

$$\left.\begin{aligned}&\bar{x}=x\lambda,\ \bar{y}=yd,\ \bar{u}=uc,\ \delta=\frac{d}{\lambda},\ h_1=\frac{H_1}{d},\\&h_2=\frac{H_2}{d},\ m=\frac{T_2-T^*}{T_1-T^*},\quad \theta=\frac{\bar{T}-T^*}{T_1-T^*}.\end{aligned}\right\}\tag{2.6}$$

Where δ is the dimensionless wave number, θ is dimensionless temperature and m is the temperature scale, governing the variable temperature difference between the two channel walls. To reduce Eqs. (2.2) to (2.5) in dimensionless form we use the non dimensionless variables and then we have

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,\tag{2.7}$$

$$A_2Re\delta\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=A_1\left[-\frac{\partial p}{\partial x}+\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\right]-A_3Mu-A_1\frac{u}{Da}+A_4G_r\theta-A_2\,\mathrm{Re}\,F^*u^2\tag{2.8}$$

$$A_5RePr\delta\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right)=A_6\left(\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)+A_3EcPrMu^2+A_1EcPr\left(\frac{\partial u}{\partial y}\right)^2.\tag{2.9}$$

$$\left.\begin{aligned}\theta=1,\ \ u=0,\ v=0,\ at\ y=h_1\\ \theta=n,\ u=0,\ v=0,\ at\ y=h_2\end{aligned}\right\} \qquad (2.10)$$

The corresponding dimensionless boundary conditions are:

$$h_1=-1-\frac{a}{d}\text{Cos}\left(\frac{2\pi\lambda}{L}x\right)\text{and } h_2=1+\frac{a}{d}\text{Cos}\left(\frac{2\pi\lambda}{L}x\right)$$

u and v are the components of velocity in x and y direction respectively and

$$\left.\begin{aligned}&Gr=\frac{(\rho\beta)_f gd^2(T_1-T^*)}{\mu_f c}, Re=\frac{\rho_f cd}{\mu_f},\ \ Pr=\frac{\mu_f(C_p)_f}{K_f},\ Ec=\frac{c^2}{(C_p)_f(T_1-T^*)},\\ &E_1=\frac{E_0}{B_0 c}, M=\frac{\sigma_f B_0{}^2 d^2}{\mu_f}, A_1=\frac{\mu_{nf}}{\mu_f},\ A_2=\frac{\rho_{nf}}{\rho_f},\ A_3=\frac{\sigma_{nf}}{\sigma_f},\ A_4=\frac{(\rho\beta)_{nf}}{(\rho\beta)_f},\\ &A_5=\frac{(\rho C_p)_{nf}}{(\rho C_p)_f},\ A_6=\frac{K_{nf}}{K_f}.\end{aligned}\right\} \qquad (2.11)$$

The physical properties of the nanofluid and particle are mentioned in Table 2.1 and the values of different involved ratios $(A_1, A_2, A_3, A_4, A_5, A_6)$ are mentioned in the Table 2.2

Table 2. 1
Physical properties of water and nano-particles.

Properties	**Water**	**Silver**
$C_p\left(JKg^{-1}k^{-1}\right)$	4179	235
$\rho\left(Kgm^{-3}\right)$	997.1	10500
$\sigma\left(m^{-1}\right)$	5×10^{-4}	6.30×10^{7}
$\beta\times10^{-5}\left(k^{-1}\right)$	21	1.89
$\mu\left(Nsm^{-2}\right)$	1.62×10^{-2}	-----

Table 2.2

Parametric values for different volume fraction						
ϕ	A1	A2	A3	A4	A5	A6
1%	1.30135	1.95305	0.33333	0.99477	0.95922	1.33175
2%	1.32157	1.98489	1.11453	1.33546	1.00987	1.77878

In view of the said dimensionless numbers, the Eqs. (2.7) to (2.9) with long wave length approximation take the following form,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.12}$$

$$-A_1\frac{\partial p}{\partial x}+A_1\frac{\partial^2 u}{\partial y^2}-A_3Mu-A_1\frac{u}{Da}+A_4G_r\theta-A_2\operatorname{Re}F^*u^2=0, \tag{2.13}$$

$$A_6\frac{\partial^2\theta}{\partial y^2}+A_3EcPrMu^2+A_1EcPr\left(\frac{\partial u}{\partial y}\right)^2=0. \tag{2.14}$$

The Skin friction along the walls can be expressed as

$$C_f=\frac{2\tau_w}{\rho_f c^2\omega}, where\ \tau_w=\mu_{nf}\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)_{\bar{y}=H_1\text{ and }H_2} \tag{2.15}$$

τ_w is wall sharing stress at $\bar{y}=H_1\ and\ \bar{y}=H_2$.

The dimensionless Skin friction at the wavy wall $y=h_1\ and\ y=h_2$ is given by

$$C_f=\frac{2A_1}{\operatorname{Re}}u'(y)\bigg|_{y=h_1\text{ and }h_2} \tag{2.16}$$

The Nusselt number along the walls can be expressed as

$$Nu=\frac{dq_w}{k_{nf}\left(T_1-T^*\right)}\quad where\ q_w=-k_{nf}\left(\frac{\partial\bar{T}}{\partial\bar{y}}\right)_{\bar{y}=H_1\text{ and }H_2}, \tag{2.17}$$

q_w is the rate of heat transfer at $\bar{y}=H_1\ and\ \bar{y}=H_2$

The dimensionless Nusselt number at the wavy wall $y=h_1\ and\ y=h_2$ is given by

$$u(y,0)=u_0(\mathrm{y}) \quad \text{and } \theta(y,0)=\theta_0(\mathrm{y}) \tag{2.23}$$

For q= 1

$$u(y,1)=u(\mathrm{y}) \quad \text{and } \theta(y,1)=\theta(\mathrm{y}) \tag{2.24}$$

When embedding parameter q diverges from 0 to 1, then $u(y,q)$, $\theta(y,q)$ varies from initial guess $u_0(y)$,$\theta_0(y)$ to final $u(y)$, $\theta(y)$ solution. Let us expand $u(y,q)$,$\theta(y,q)$ by Taylor series as

$$\left.\begin{aligned} u(y,q)&=u_0(\mathrm{y})+\sum_{l=1}^{\infty}u_1(y)\,q^l \\ \theta(y,q)&=\theta_0(\mathrm{y})+\sum_{l=1}^{\infty}\theta_1(y)\,q^l \end{aligned}\right\} \tag{2.25}$$

In which

$$\left.\begin{aligned} u_1(y)&=\frac{1}{l!}\frac{\partial^l u(y,q)}{\partial q^l}\Big|_{q=0} \\ \theta_1(y)&=\frac{1}{l!}\frac{\partial^l \theta(y,q)}{\partial q^l}\Big|_{q=0} \end{aligned}\right\} \tag{2.26}$$

Differentiating l– times to zero$^{\text{th}}$-order deformation Eq. (2.12) with respect to the q and dividing it by l! then putting $q=0$ and gain 1$^{\text{st}}$order deformation expression for $u_1(y)$ and $\theta_1(y)$ as follows

$$\left.\begin{aligned} £_1\left[u_l(y)-\chi_l u_{l-1}(y)\right]&=\hbar \\ £_2\left[\theta_l(y)-\chi_l \theta_{l-1}(y)\right]&=\hbar \end{aligned}\right\} \tag{2.27}$$

$$\chi=\left\{\begin{matrix} 0, & \mathrm{m}\le 1 \\ 1, & \mathrm{m}>1 \end{matrix}\right\}$$

$$\begin{aligned} u_l(y,q)&=0, \quad \theta_l(\mathrm{y},q)=1 \; at \; y=h_1 \\ u_l(y,q)&=0, \quad \theta_l(\mathrm{y},q)=m \; at \; y=h_2 \end{aligned} \tag{2.28}$$

$$
\left.\begin{aligned}
N_1\left[u(y,q),\theta(y,q)\right] &= -A_1P + A_1u_l'' - A_3Mu_l - A_1\frac{u_l}{Da} + A_4G_r\theta_l - A_2ReF^* \\
N_2\left[u(y,q),\theta(y,q)\right] &= A_6\theta_l'' + A_3EcPrHa^2u_l^2 + A_1EcPr\left(u_l'\right)^2
\end{aligned}\right\} \quad (2.29)
$$

The solution of lth - order approximation can be stated as

$$
\left.\begin{aligned}
u(y) &= u_0(\mathrm{y}) + \sum_{k=1}^{l} u_k(y) \\
\theta(y) &= \theta_0(\mathrm{y}) + \sum_{k=0}^{l} \theta_k(y)
\end{aligned}\right\} \quad (2.30)
$$

For velocity and temperature the solution expressions for the best understanding of readers at first and second iteration are given as

$$
\left.\begin{aligned}
u_1(y) = &-1 - \frac{13}{20}\mathrm{h}_1 - \frac{9Gr}{400}\mathrm{h}_1 + \frac{1859M}{5000Da}\mathrm{h}_1 + \frac{209F}{500}\mathrm{h}_1 + \frac{3Gr}{400}\mathrm{h}_1y + y^2 + \\
&\frac{13M}{20}\mathrm{h}_1y^2 + \frac{9Gr}{400}\mathrm{h}_1y^2 - \frac{507M}{1000Da}\mathrm{h}_1y^2 - \frac{57F}{100}\mathrm{h}_1y^2 - \frac{3Gr}{400}\mathrm{h}_1y^3 + \\
&\frac{169M}{1000Da}\mathrm{h}_1y^4 + \frac{19F}{100}\mathrm{h}_1y^4 - \frac{169M}{5000Da}\mathrm{h}_1y^6 - \frac{19F}{500}\mathrm{h}_1y^6.
\end{aligned}\right\} \quad (2.31)
$$

$$
\left.\begin{aligned}
\theta_1(y) = &-\frac{13E_C}{3}\mathrm{h}_2 - \frac{1E_CM}{10}\mathrm{h}_2 + \frac{1-y}{2} + \frac{3E_CM}{2}\mathrm{h}_2y^2 + \frac{13E_C}{3}\mathrm{h}_2y^4 - \\
&\frac{1E_CM}{2}\mathrm{h}_2y^4 + \frac{1E_CM}{10}\mathrm{h}_2y^6
\end{aligned}\right\} \quad (2.32)
$$

And

$$
\left.
\begin{aligned}
u(y) = {} & -1-\frac{13}{10}h_1-\frac{9Gr}{200}h_1+\frac{91E_CGr}{500}h_1h_2+\frac{1859M}{2500}h_1+ \\
& \frac{40842M}{40000Da}h_1^4+\frac{2217E_CGrM}{56000}h_1h_2-\frac{1885M^2}{1055Da^2}h_1^4- \\
& \frac{43519GrM^2}{35600Da^2}h_1^4+\frac{12211M^2F}{17500Da^2}h_1^4y-\frac{18823GrMF^2}{35000Da^2}h_1^4- \\
& \frac{13615M^4}{65450Da^4}h_1^4+y^2+\frac{93866MF}{154000Da}h_1^4y+\frac{16742GrM^2}{35600Da^2}h_1^4 \\
& \frac{13}{10}h_1y^2+\frac{9Gr}{200}h_1y^2-\frac{39E_CGr}{200}h_1h_2y^2-\frac{99E_CGrM}{2000}h_1h_2y^2+ \\
& \frac{535775M^2}{20000Da^2}h_1^4y^2+\frac{13E_CGr}{1000}h_1h_2y^6+\frac{169M}{25000Da}h_1y^6 \\
& -\frac{37129M}{4000Da}h_1^4y^6-\frac{77711GrM}{8000Da}h_1^4y^6-\frac{25704GrM}{16000Da}h_1^4y^6+ \\
& \frac{627485E_CGrM}{10000Da^2}h_1h_2y^2+\frac{43748M^2}{10000}h_1^4y^6-\frac{169M}{2500Da}h_1y^6- \\
& \frac{3E_CGrM}{2000}h_1h_2y^6-\frac{169M}{25000Da}h_1y^6-\frac{37129M}{4000Da}h_1^4y^6- \\
& \frac{77711GrM}{8000Da}h_1^4y^6-\frac{25704GrM}{16000Da}h_1^4y^6+\frac{627485E_CGrM}{10000Da^2}h_1h_2y^2+ \\
& \frac{43748M^2}{10000}h_1^4y^6
\end{aligned}
\right\} \qquad (3.33)
$$

$$
\left.
\begin{aligned}
\theta(y) = & -\frac{26E_C}{3}\mathrm{h}_2 - \frac{169E_C}{30}\mathrm{h}_1\mathrm{h}_2 - \frac{39E_CGr}{200}\mathrm{h}_1\mathrm{h}_2 - \frac{169E_C}{30}\mathrm{h}_2{}^2 - \\
& \frac{11E_CM}{5}\mathrm{h}_2 - \frac{143E_CM}{100}\mathrm{h}_1\mathrm{h}_2 + \frac{35811E_CM}{1050}\mathrm{h}_1\mathrm{h}_2 - \\
& \frac{99E_CGrM}{2000}\mathrm{h}_1\mathrm{h}_2 - \frac{143E_CM}{100}\mathrm{h}_2{}^2 + \frac{8313E_CMF}{10500}\mathrm{h}_1\mathrm{h}_2 + \\
& \frac{4026E_CF}{10500Da}\mathrm{h}_1\mathrm{h}_2 - \frac{1-y}{2} - \frac{13E_CGr}{2000}\mathrm{h}_1\mathrm{h}_2y - \frac{3E_CGrM}{400Da}\mathrm{h}_1\mathrm{h}_2y^3 + \\
& \frac{261E_CM^2}{10550Da}\mathrm{h}_2y^4 + \frac{16993E_C}{30Da}\mathrm{h}_1\mathrm{h}_2y - \frac{9346E_CM^2}{10500Da}\mathrm{h}_1\mathrm{h}_2y^2 + \\
& \frac{57E_CGrM}{14000}\mathrm{h}_1\mathrm{h}_2y + 3E_CM\mathrm{h}_2y^2 + \frac{39E_CM}{20}\mathrm{h}_1\mathrm{h}_2y^2 + \\
& \frac{27E_CGrM}{400}\mathrm{h}_1\mathrm{h}_2y^2 + \frac{39E_CM}{20}\mathrm{h}_2{}^2y^2 - \frac{5577E_CM^2}{2000}\mathrm{h}_1\mathrm{h}_2y^2 - \\
& \frac{627E_CMF}{500}\mathrm{h}_1\mathrm{h}_2y^2 + \frac{13E_CGr}{200}\mathrm{h}_1\mathrm{h}_2y^3 - \frac{3E_CGrM}{400}\mathrm{h}_1\mathrm{h}_2y^3 + \\
& \frac{26E_C}{3}\mathrm{h}_2y^4 + \frac{169E_C}{30}\mathrm{h}_1\mathrm{h}_2y^4 - E_CM\mathrm{h}_2y^4 - \frac{13E_CM}{20}\mathrm{h}_1\mathrm{h}_2y^4 - \\
& \frac{2197E_CGr}{200}\mathrm{h}_1\mathrm{h}_2y^3 - \frac{9E_CGrM}{400}\mathrm{h}_1\mathrm{h}_2y^4 + \frac{494E_CF}{375}\mathrm{h}_1\mathrm{h}_2y
\end{aligned}
\right\}
\tag{3.34}
$$

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2.3 Results and Discussion

This section provides the behaviour of parameters present in the expression of flow, heat and mass transfer characteristics. In particular, the influence of Eckert number (Ec), permeability (porosity) parameter (Da), Grashof number (Gr), Magnetic Field parameter (M), Reynolds number (Re), and non Darcy parameter (F^{*}). In order to bring out the addition of dissimilar up-and-coming parameters, software Mathematica is used to imagine the behaviour of involved parameters through graphs.

Figs. 2.2 to 2.5 Represents the impact of Eckert number, Grashof number, Magnetic field parameter, non Darcy parameter on velocity profile. It represent from Fig. 2.2 indicates that as the Eckert number increases velocity also increases. It can be seen from Fig. 2.3 that when the Grashof number increases velocity also increases and the Fig. 2.4 indicates that when magnetic field is applied, then it disagree the flow due to the effect of Lorentz force and hence the velocity of the fluid reduced. Fig 2.5 indicates that as the non Darcy parameter increases velocity decreases.

Figs. 2.6 to 2.9 represents the impact of Grashof number , Eckert number non Darcy parameter and magnetic field parameter on dimensionless temperature profiles. It represent from Fig. 2.6 that when the Grashof number increases temperature decreases while Fig. 2.7 indicates that temperature increased by increasing the values of Eckert number. Fig. 2.8 indicates that when the porosity parameter increases temperature will also be increased, same is the case in Fig. 2.9 it as the Magnetic field parameter increases temperature also increases. Fig. 2.10 (bottom) shows the effects of skin friction with the various values of non Darcy parameter. It can be noted that in Fig. 2.10 (bottom) the skin friction decreases with increasing the non Darcy parameter. Fig. 2.11 (top) shows the the impact of Eckret number on skin friction coefficient for various values of Grashof number. From Fig. 2.11it can be seen that Skin friction decreases with the increases values of Grashof number. In Fig. 2.12 (top) shows the the impact of Magnetic field parameter on skin friction efficient for various values of non Darcy parameter, it can be seen from the fig that the Skin friction decreases with the increases values non Darcy parameter. While Fig. 2.13(bottom) shows the the impact of Magnetic parameter on skin friction for various values of non Darcy parameter, it can be seen from the fig that the Skin friction increases with the increases values of non Darcy parameter. Fig.2.14(bottom) shows variation of Nusselt number for different values of non Darcy parameter. It can be seen that Nusselt number increases with the increase in the non Darcy Parameter. Fig.2.15(bottom)shows variation of Nusselt number along the wavy channel for different values of magnetic parameter. The graph exposed that Nusselt number decreases with the increase in magnetic field parameter.

Fig. 2.16(bottom) shows the impact of magnetic parameter on Nusselt number for various values of non Darcy parameter. It is obevious that the Nusselt number decreases with the increases values of non Darcy parameter.

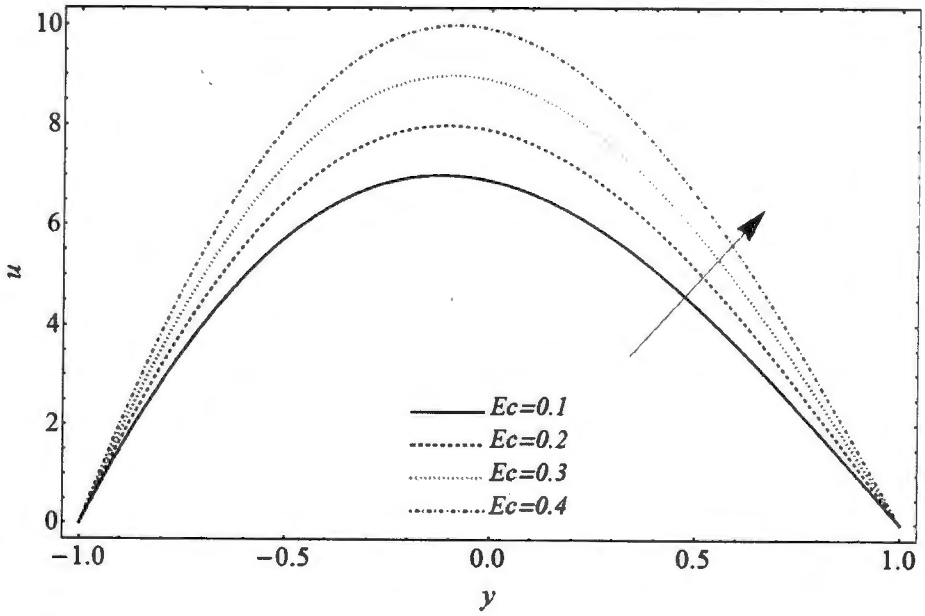

Fig. 2.2 The impact of Eckert number on the velocity profiles.

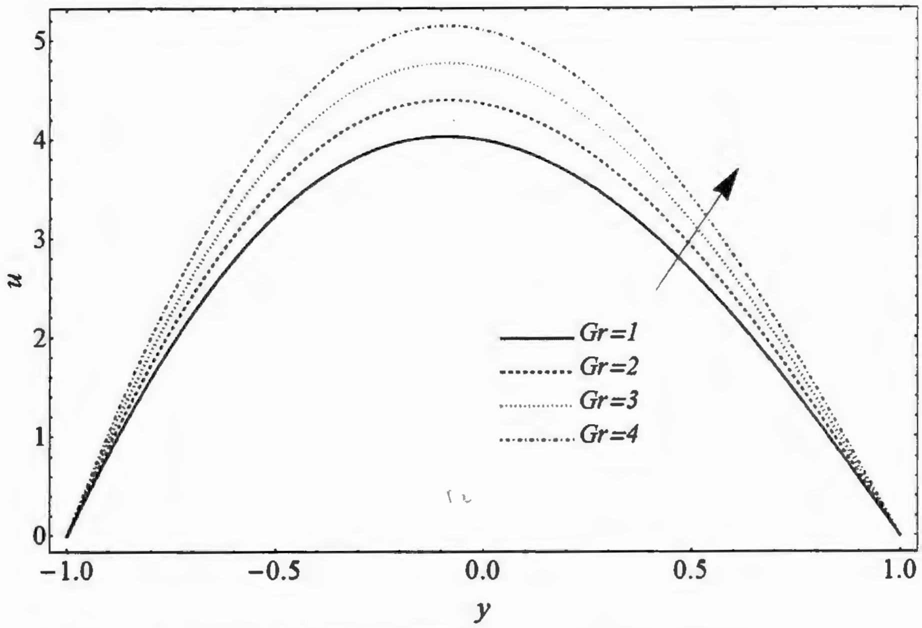

Fig.2.3 The impact of Grashof number on velocity profiles

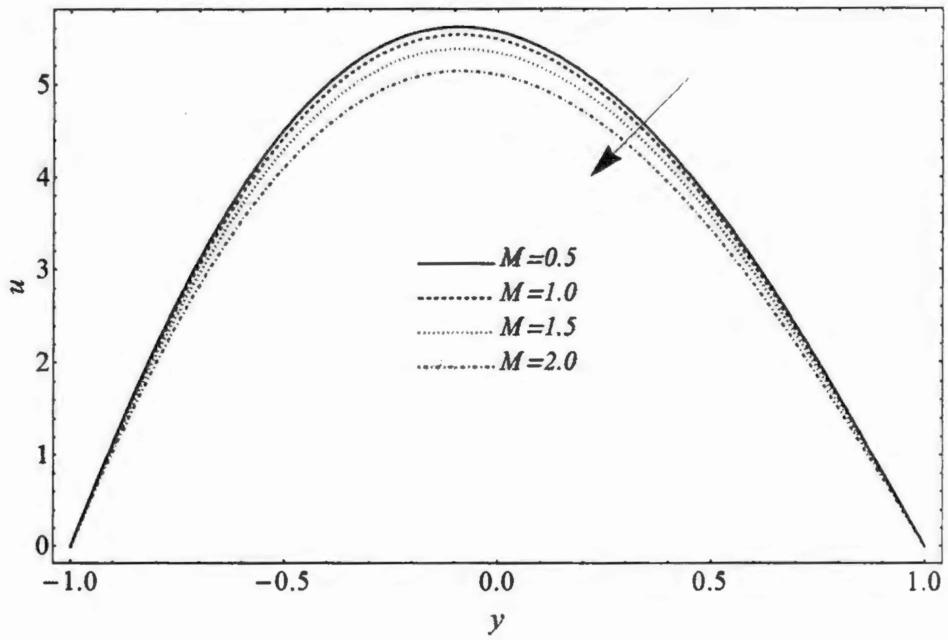

Fig. 2.4 The impact of the magnetic parameter on velocity profile

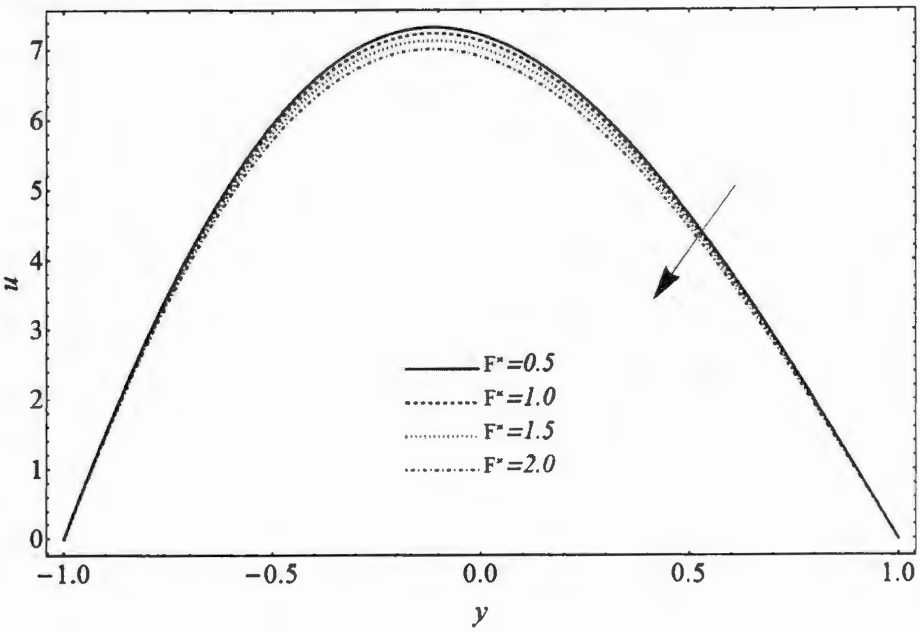

Fig. 2.5 The impact of non Darcy parameter on velocity profile.

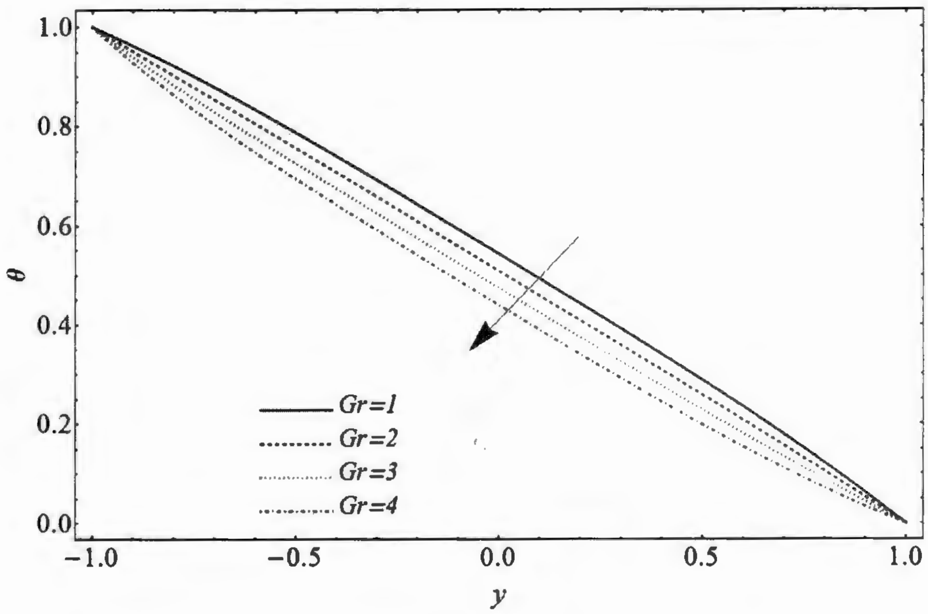

Fig. 2.6 The impact of Grashof number on temperature profiles

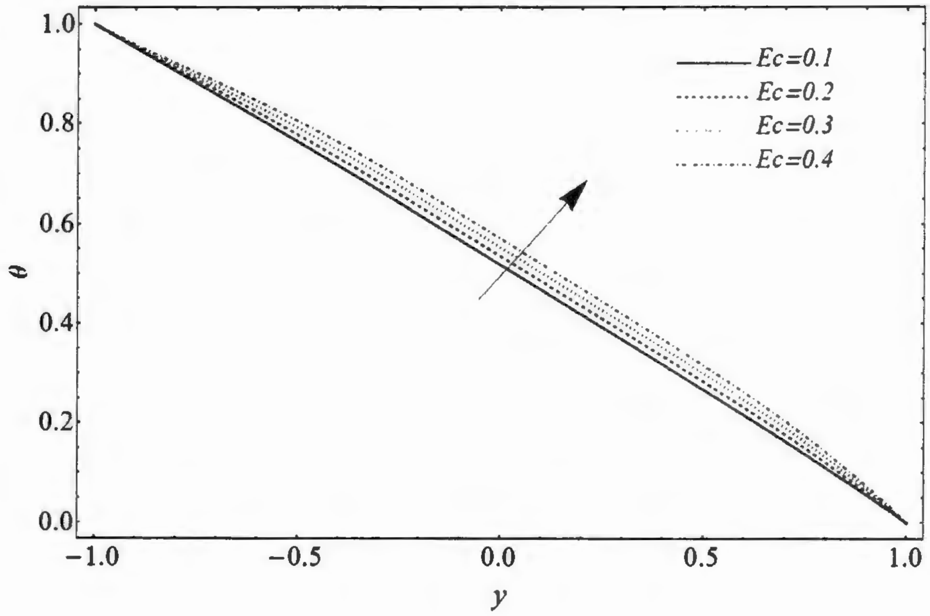

Fig. 2.7 The impact of Eckert number on temperature profile

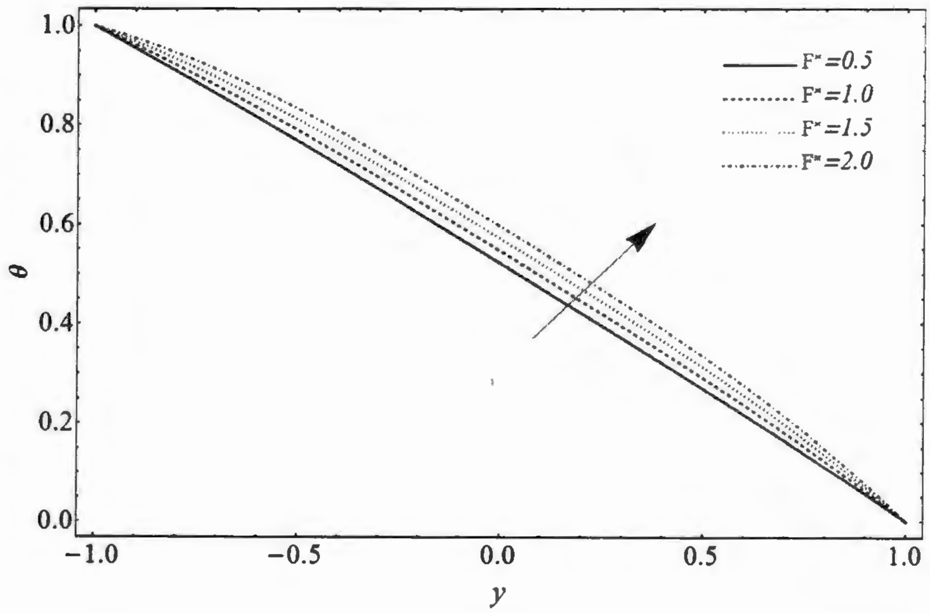

Fig. 2.8 The impact of non Darcy parameter on temperature profiles

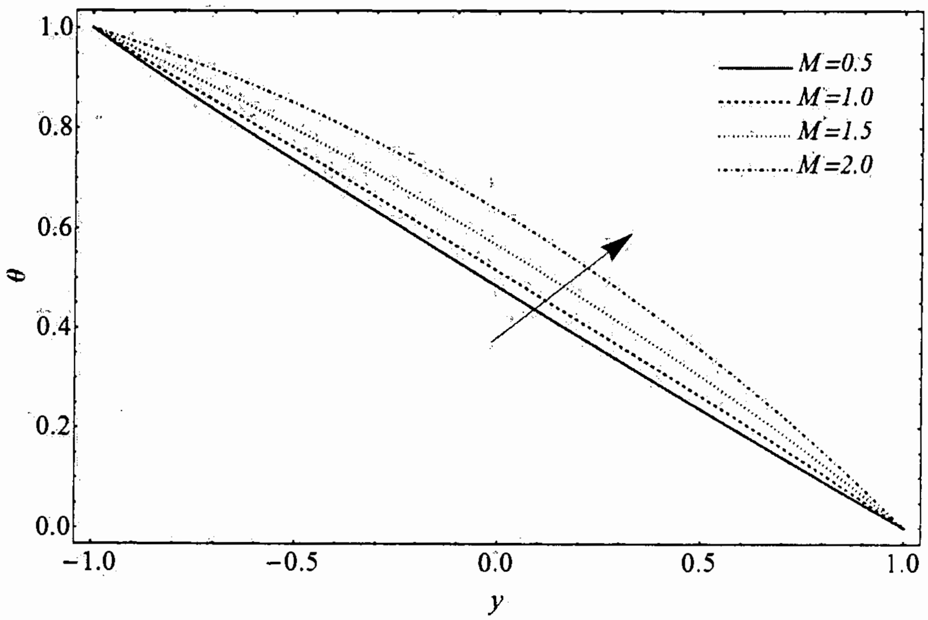

Fig. 2.9 The impact of Magnetic parameter on temperature profiles

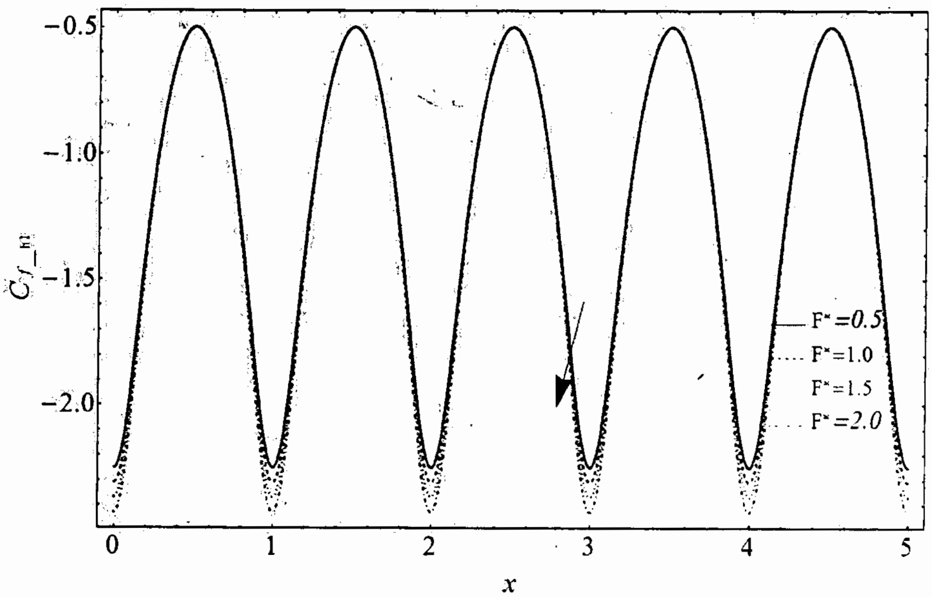

Fig. 2.10 (Bottom)The impact of non Darcy parameter on skin friction coefficient

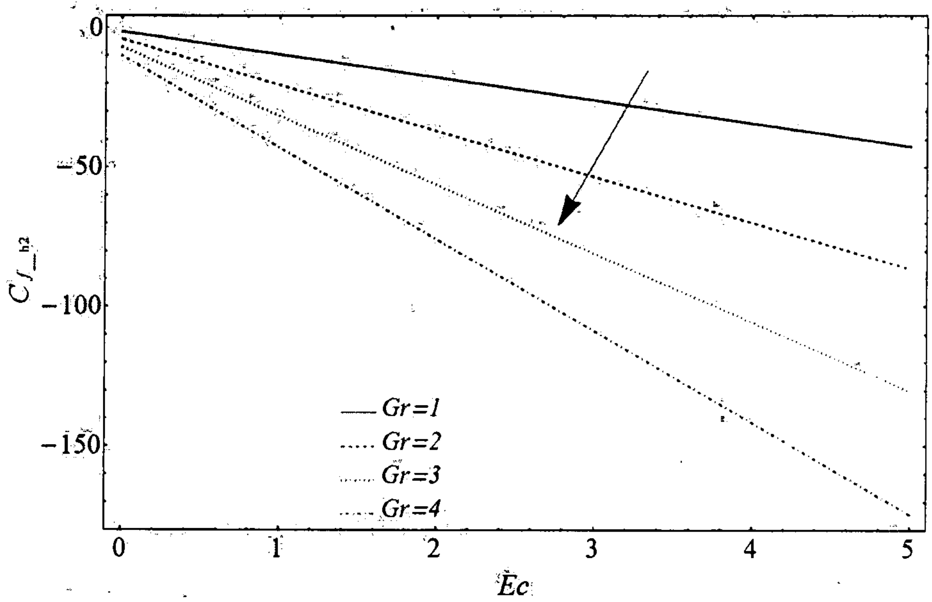

Fig. 2.11(Top)The impact of Eckret number on skin friction coefficient for various values of Grashof number

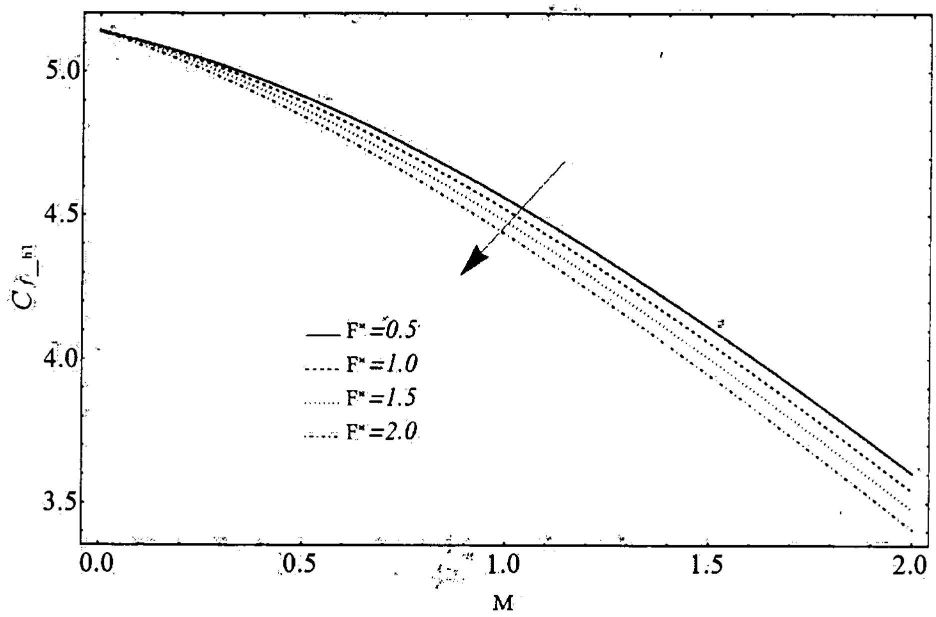

Fig. 2.12 (Top)The impact of Magnetic field parameter on skin friction coefficient for various values of non Darcy parameter

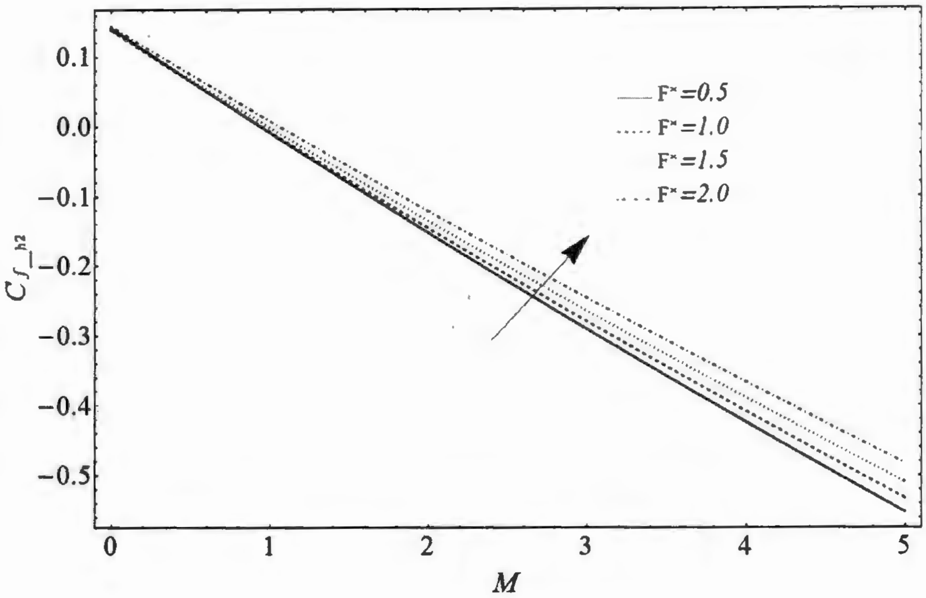

Fig. 2.13(Bottom)The impact of Magnetic parameter on skin friction coefficient for various values of non Darcy parameter

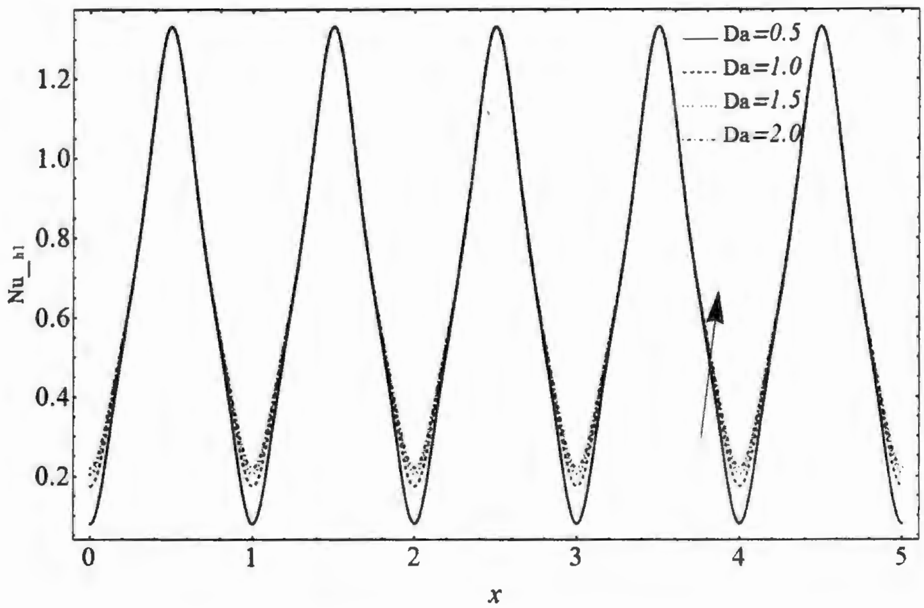

Fig.2.14 (bottom)The Impact of porosity parameter on Nusselt number

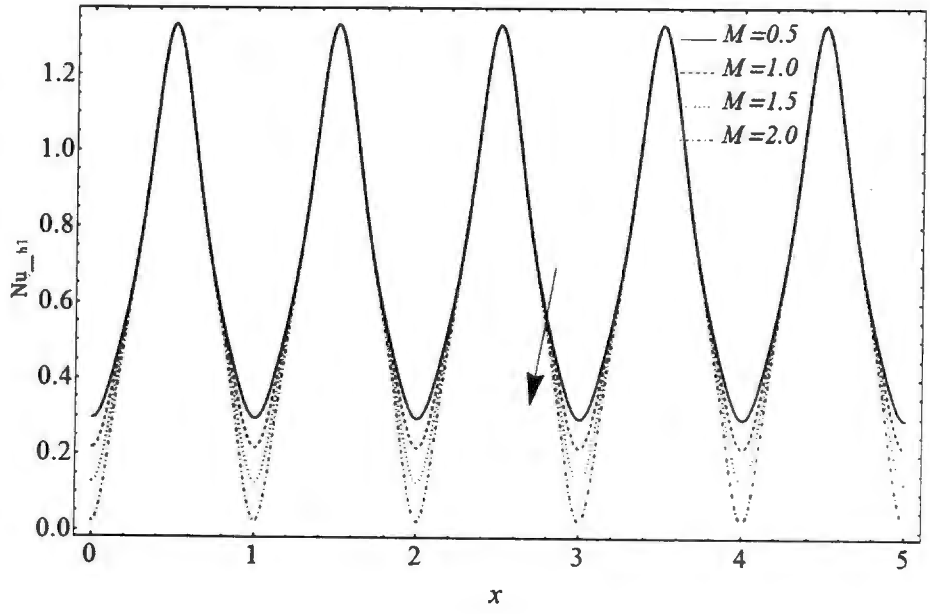

Fig.2.15(bottom)The Impact of magnetic parameter on Nusselt number

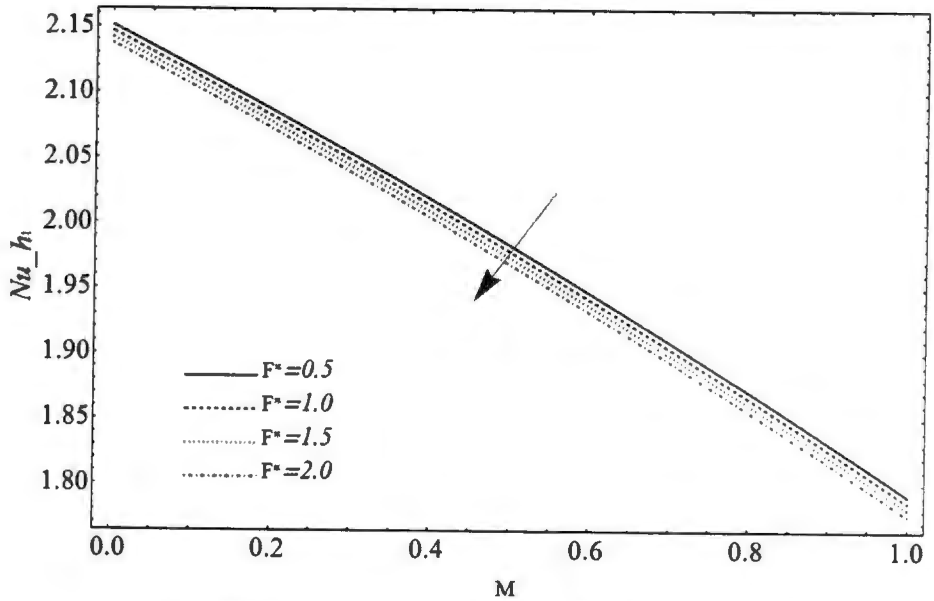

Fig.2.16(bottom)The Impact of Magnetic parameter on local Nusselt number for various values of non Darcy number

2.4 Conclusion

The following points are observed and concluded as:

It is noticed that as the Eckert number and Grashof number increases velocity also increases. It is observed that by applying magnetic field it disagree the flow due to the effect of Lorentz force and so the velocity of the fluid reduced, also when the non Darcy parameter increases velocity decreases.

Temperature decreases for the increasing values of Grashof number and increases as the Eckert number and porosity parameter and magnetic parameter increases. It is measured that Skin friction efficient decreases for increasing values of non Darcy parameter. It is noted that skin fiction coefficient decrease when the impact of Eckret number has been seen on skin friction coefficient for various values of Grashof number. The impact of magnetic field parameter on skin friction efficient for various values of non Darcy parameter has been seen, it results in decrease of skin friction efficient.

The Impact of porosity parameter and magnetic field parameter on Nusselt number is noticed and observed that Nusselt number increases as the values of porosity parameter increases and decreases as the values of magnetic field parameter increases. The Impact of magnetic parameter on Nusselt number for various values of non Darcy parameter has been observed and noticed that when the magnetic field applied Nusselt number decreases for different values of non Darcy parameter.

Chapter 3

Entropy Generation in MHD Non-Darcian Flow of Nanofluid in a Wavy Channel

In this chapter, entropy generation analysis of non-Darcian natural convection MHD nanofluid flow over wavy surface has been analyzed. The solutions of velocity and the temperature distributions are used from the previous chapter. The equations for entropy generation are modeled and get dimensionless using similarity transformations. The influence of numerous involving parameters on entropy generation and on Bejan number are displayed through graphs and discussed in details.

3.1 Mathematical Formulation of Problem

The entropy generation rate of nanofluid, in the presence magnetic field and according to the above assumptions, can be described as

$$S'''_{gen} = \frac{K_{nf}}{T^{*}}(\nabla T)^{2} + \frac{\mu_{nf}}{T^{*}}\Phi + \frac{1}{T^{*}}\frac{J\cdot J}{\sigma} \tag{3.1}$$

Where

$$\begin{aligned}(\nabla T)^{2} &= \left(\frac{\partial \bar{T}}{\partial \bar{x}}\right)^{2} + \left(\frac{\partial \bar{T}}{\partial \bar{y}}\right)^{2}\\ \Phi &= \left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^{2}\\ J &= \sigma\left(E + V \times B\right)\end{aligned} \tag{3.2}$$

The dimensionless form of equation (3.1) via similarity transformation defined in equation (2.6) can be written as

$$S'''_{gen} = \frac{K_{nf}}{T^{*2}} \Delta T^2 \left[\frac{1}{\lambda^2} \left(\frac{\partial \theta}{\partial x} \right)^2 + \frac{1}{d^2} \left(\frac{\partial \theta}{\partial y} \right)^2 \right] + \frac{\mu_{nf}}{T^*} \frac{c^2}{d^2} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\sigma_{nf} B^2{}_0 c^2 u^2}{T^*} \tag{3.3}$$

$$S'''_{gen} = \frac{K_{nf}}{T^{*2}} \left(T_1 - T^* \right)^2 \left[\frac{1}{\lambda^2} \left(\frac{\partial \theta}{\partial x} \right)^2 + \frac{1}{d^2} \left(\frac{\partial \theta}{\partial y} \right)^2 \right] + \frac{\mu_{nf}}{T^*} \frac{c^2}{d^2} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\sigma_{nf} B^2{}_0 c^2 u^2}{T^*} \tag{3.4}$$

In Eq. (3.1) the right hand side consists of three parts: the first part is entropy generation due to contribution of thermal irreversibility that comprises the entropy generation by heat transfer due to axial conduction from the wavy surface; the second part is fluid friction irreversibility and the third part denotes joule dissipation irreversibility, which is due to the movement of electrically conducting fluid under the consideration of magnetic field, thus inducing electric current that circulate in the fluid. The entropy generation number N_G is the similar form of the entropy generation rate, which shows the ratio between actual entropy generation rate S'''_{gen} and characteristic entropy generation rate S'''_o which is given below.

$$S'''_o = \frac{k_f (\Delta T)^2}{d^2 T^{*2}} \tag{3.5}$$

Now dividing equation (3.3) by equation (3.4), we get

$$N_G = \frac{S'''_{gen}}{S'''_o}$$

$$N_G = \frac{d^2 T^{*2}}{k_f (\Delta T)^2} \times \left[\begin{array}{l} \dfrac{k_{nf}}{T^{*2}} (\Delta T)^2 \left\{ \dfrac{1}{\lambda^2} \left(\dfrac{\partial \theta}{\partial x} \right)^2 + \dfrac{1}{d^2} \left(\dfrac{\partial \theta}{\partial y} \right)^2 \right\} + \\ \dfrac{\mu_{nf}}{T^*} \dfrac{c^2}{d^2} \left(\dfrac{\partial u}{\partial y} \right)^2 + \dfrac{\sigma_{nf} B_o^2 c^2 u^2}{T^*} \end{array} \right] \tag{3.6}$$

The non-dimensional form of entropy generation number N_G can be written as,

$$N_G = \frac{k_{nf}}{k_f}\left[\delta^2\left(\frac{\partial\theta}{\partial x}\right)^2 + \left(\frac{\partial\theta}{\partial y}\right)^2\right] + \frac{\mu_{nf}}{\mu_f}\frac{B_r}{\Omega}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma_{nf}}{\sigma_f}\frac{MB_r}{\Omega}u^2 \qquad (3.7)$$

Where

$$\Omega = \frac{\Delta T}{T^*}, \quad Br = \frac{\mu_f c^2}{k_f \Delta T} \qquad (3.8)$$

The dominance of the irreversibility procedure is essential, since the entropy generation number is not capable to conquer this problem. The Bejan number Be is employed to comprehend the entropy generation mechanism. Mathematically it can be written as:

$$Be = \frac{N_H}{N_H + N_F}, \qquad (3.9)$$

$$N_H = \frac{k_{nf}}{k_f}\left[\delta^2\left(\frac{\partial\theta}{\partial x}\right)^2 + \left(\frac{\partial\theta}{\partial y}\right)^2\right], \quad N_F = \frac{\mu_{nf}}{\mu_f}\frac{B_r}{\Omega}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma_{nf}}{\sigma_f}\frac{MB_r}{\Omega}u^2. \qquad (3.10)$$

Now using (3.10) into (3.9), we get

$$Be = \frac{\frac{k_{nf}}{k_f}\left[\delta^2\left(\frac{\partial\theta}{\partial x}\right)^2 + \left(\frac{\partial\theta}{\partial y}\right)^2\right]}{\frac{k_{nf}}{k_f}\left[\delta^2\left(\frac{\partial\theta}{\partial x}\right)^2 + \left(\frac{\partial\theta}{\partial y}\right)^2\right] + \frac{\mu_{nf}}{\mu_f}\frac{B_r}{\Omega}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma_{nf}}{\sigma_f}\frac{MB_r}{\Omega}u^2} \qquad (3.11)$$

It is clear from Eq. (3.11) that $0 < Be < 1$. When $Be = 0$, fluid friction and joule dissipation irreversibility dominate entropy generation, when $Be = \frac{1}{2}$, the contribution of heat transfer irreversibility is equal to sum of fluid friction and Joule heating and when $Be = 1$, the irreversibility mechanism is dominated by heat transfer effects. The dimensionless volumetric entropy rate and also average Bejan number, which tend to be a good important measure connected with total global entropy, will be evaluated with all current following formula:

$$N_{G,avg} = \frac{1}{\forall}\int_0^m\int_0^1 2\pi r N_G dr d\eta,\ Be_{avg} = \frac{1}{\forall}\int_0^m\int_0^1 2\pi r N_G dr d\eta \qquad (3.12)$$

where $\forall$ will be control volume.

3.2 Results and Discussion.

This section provides the behaviour of parameters involved in the expressions of entropy generation. In particular, the influence of the parameters Br, Da, Ec, F^*, Gr *and* M on entropy generation number N_G is discussed graphically, in addition to this the impact of, Br, Da, M, Gr on Bejan number Be is also discussed. In order to bring out the addition of dissimilar up-and-coming parameters, computational software Mathematica has been used to imagine the behaviour of all the parameters mentioned above through graphs.

Figs. 3.1 to 3.6 are drawn to show the effects of involving parameter such as Brickman number *Br*, porosity *Da*, Eckert number Ec, non Darcy parameter F^*, Grashof number *Gr* and magnetic field parameter *M* on entropy generation number N_G

In Fig 3.1it can be seen that the entropy generation rate increases with increasing the values of *Br*. Fig 3.2 is showing that increase in the values of *Da* results in the increase of entropy generation rate. And *Ec*, but the fig 3.4 Shows that entropy generation rate decreases with increasing values of F^*, Again from figs. 3.5, 3.6 indicate that the entropy generation rate increases with increasing the values of *Gr* and F^*. In Figs. 3.7 to 3.9, we observed that the Bejan number decreases with an increase in the parameter values of *Br*, *Da* and *M* while Fig 3.10 indicates that Bejan number increases with the increasing values of *Gr*.

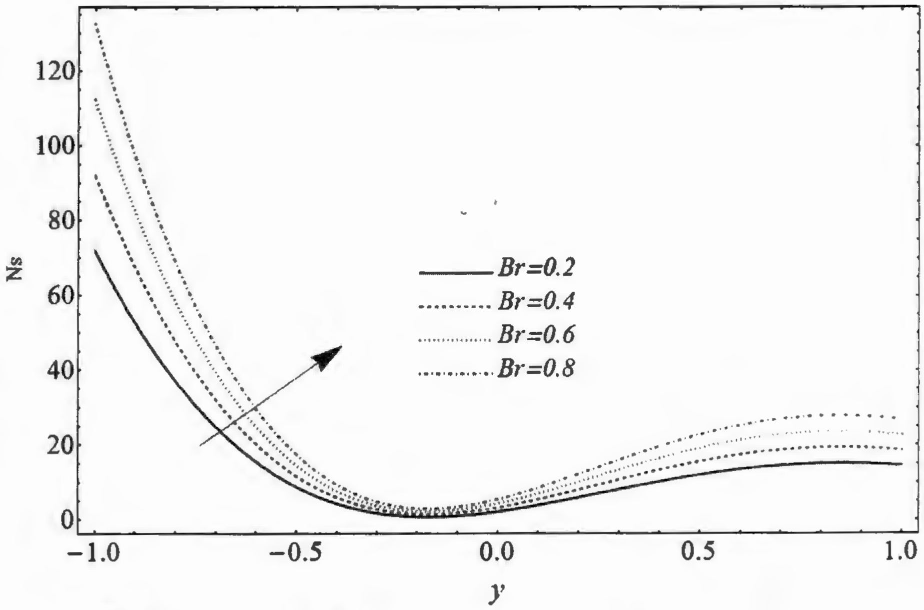

Fig 3.1 Impact of *Br* on entropy generation number N_G.

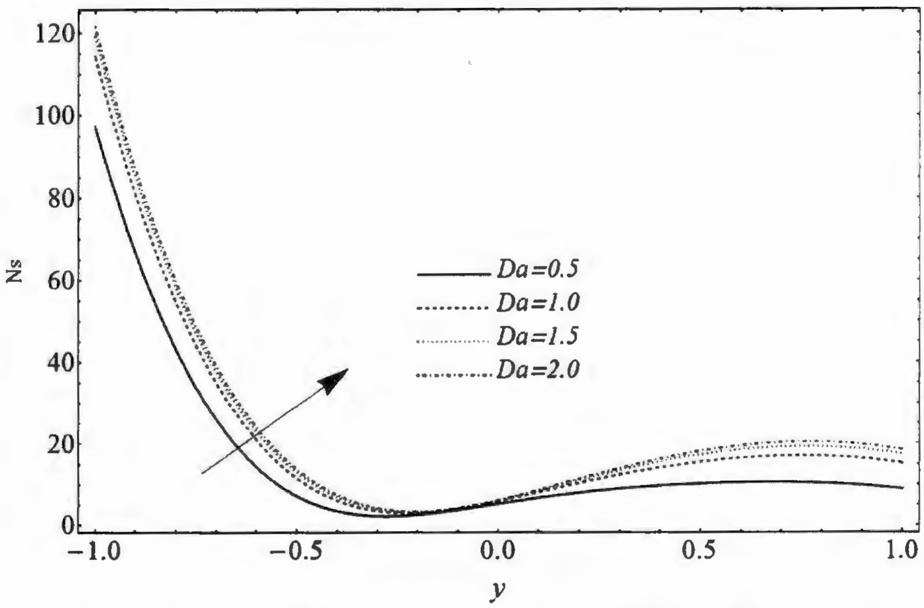

Fig. 3.2 Impact of *Da* on entropy generation number N_G

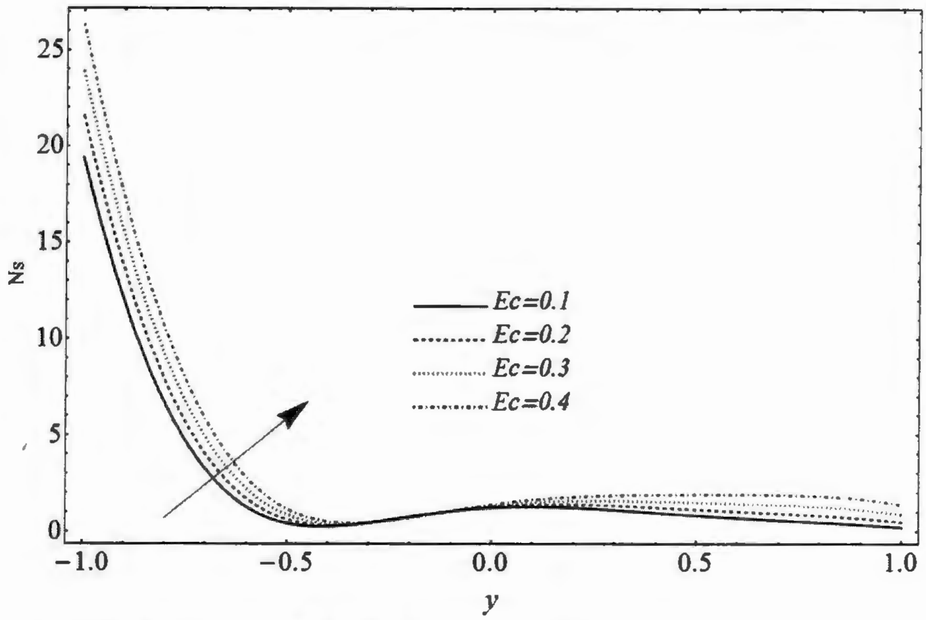

Fig. 3.3 Impact of Ec on entropy generation number N_G

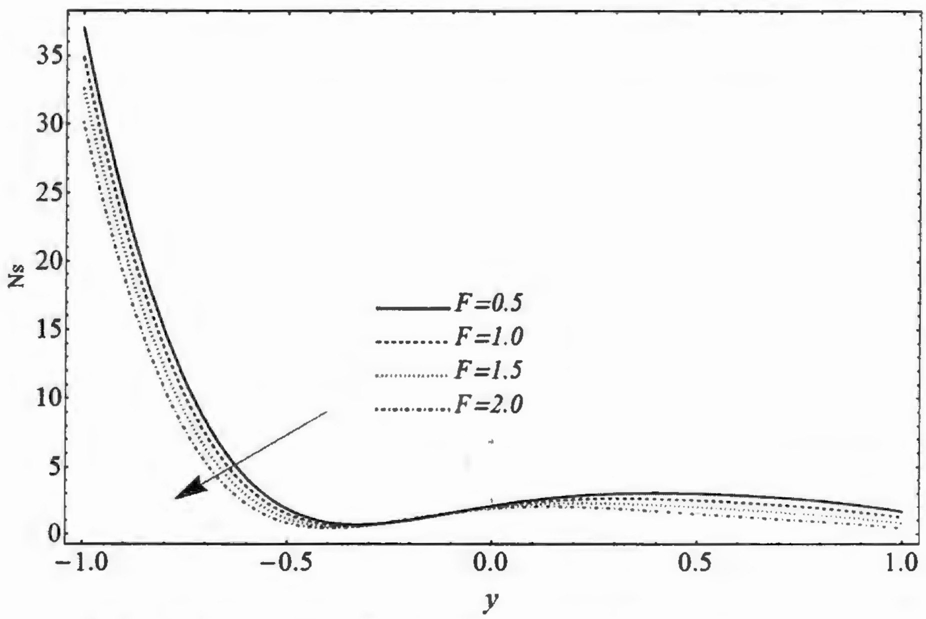

Fig. 3.4 Impact of F^{*}, on entropy generation number N_G

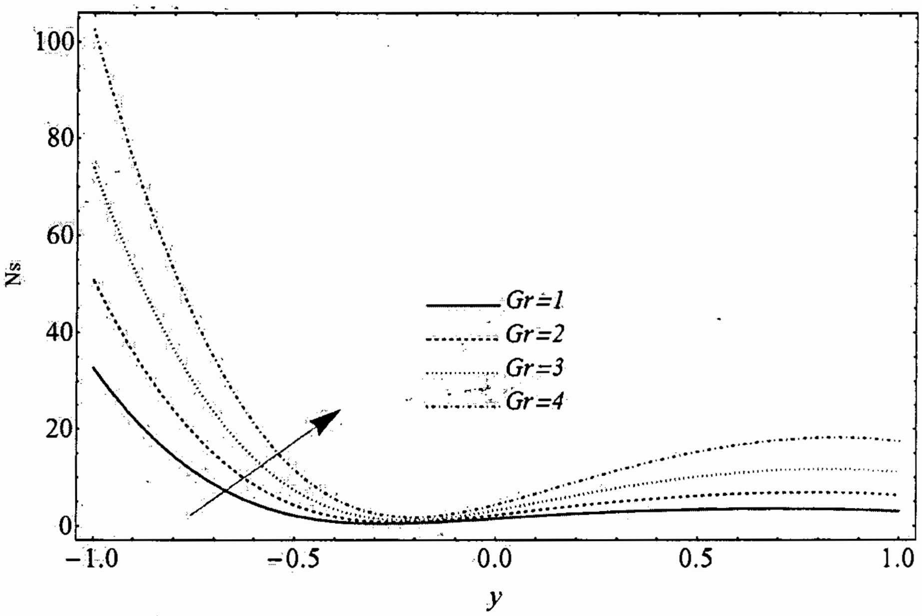

Fig.3.5 Impact of *Gr* on entropy generation number

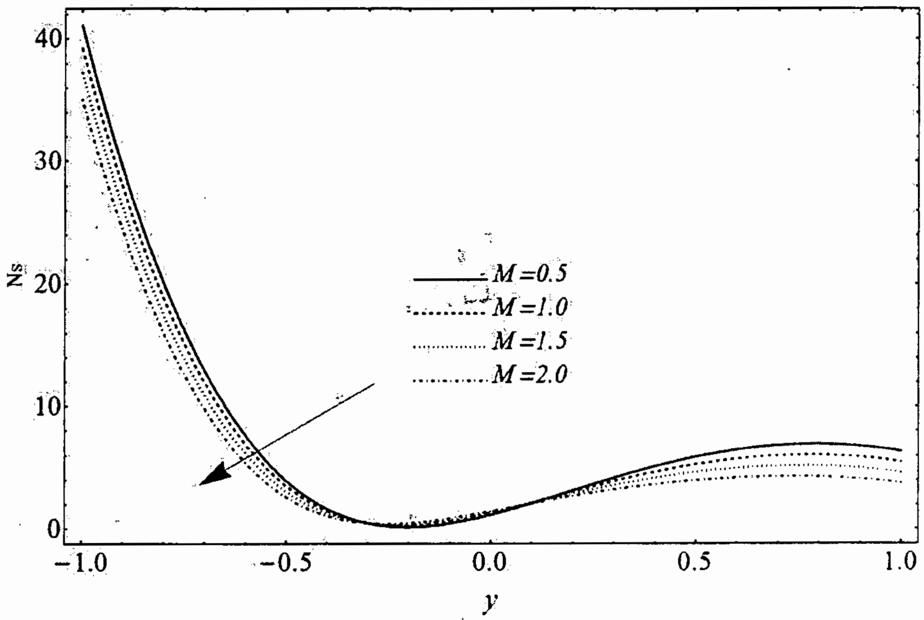

Fig 3.6 Impact of M on entropy generation number N_G

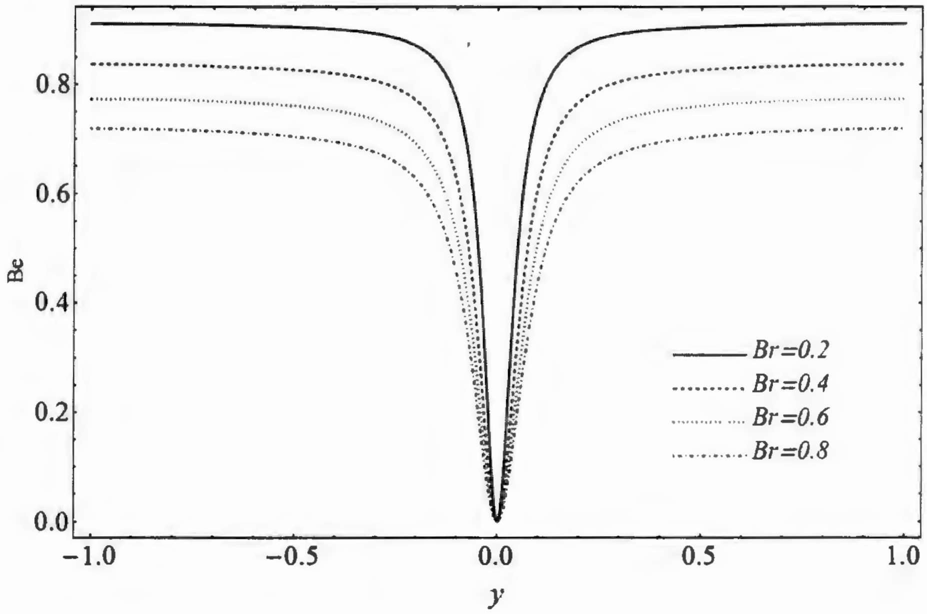

Fig 3.7 Impact of *Br* on Bejan number *Be*

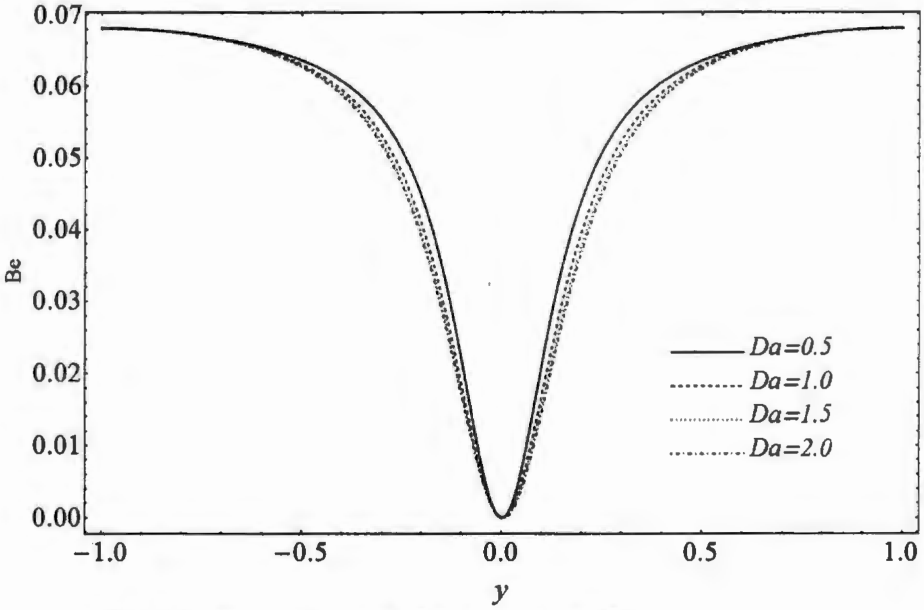

Fig 3.8 Impact of *Da* on Bejan number *Be*

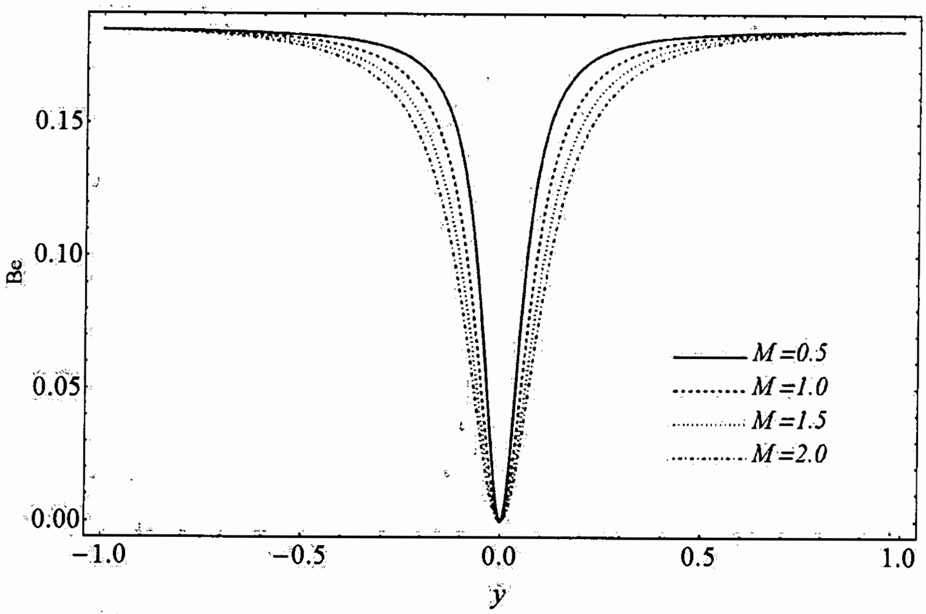

Fig 3.9 Impact of M on Bejan number Be.

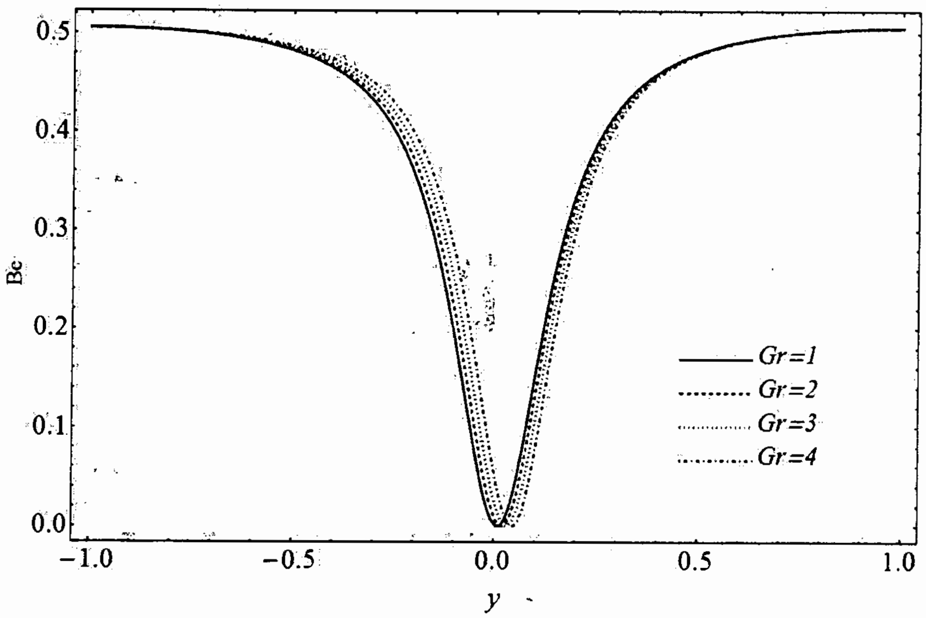

Fig 3.10 Impact of Gr on Bejan number Be

3.3 Conclusion

In Fig 3.1 to 3.3 it can be observed that the entropy generation rate increases with increasing values of *Br*, *Da* and *Ec*, but the fig 3.4 Shows that entropy generation rate decreases with increasing values of *F*. Again from figs. 3.5, 3.6 indicates that the entropy generation rate increases with increasing the values of *Gr* and *F*. In Figs. 3.7 to 3.9, we observed that the Bejan number decreases with an increase in the parametric values of *Br*, *Da* and *M*, while Fig 3.10 indicates that Bejan number increases with an increase in the parameter values of *Gr*.

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