

**Enhancement of Angiographic Images based on
Multi-resolution Techniques**



Undertaken by
Nida Akhter [322-FAS/MSCS/F06]

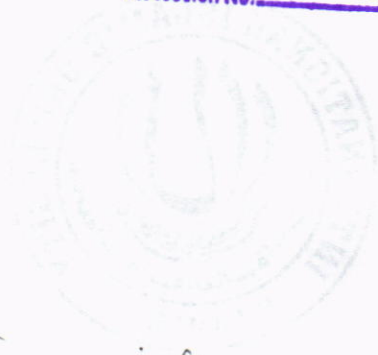
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Multi-resolution Techniques

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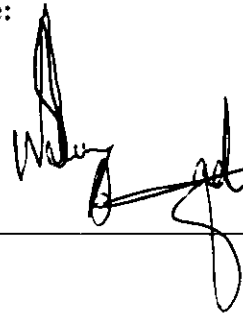
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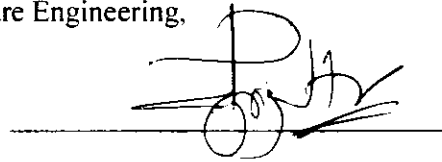
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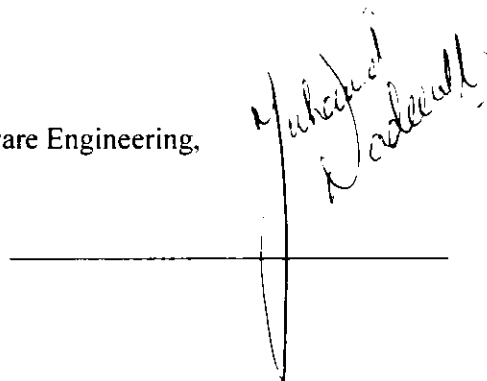
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A dissertation submitted to the
Department of Computer Science,
Faculty of Applied Sciences,
International Islamic University, Islamabad
as a partial fulfillment of the requirements
for the award of the degree of
MS in Computer Science

**Dedicated To
To My Parents**

Declaration

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Nida Akhter

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Nida Akhter
322-FAS/MSCS/F06

Project In Brief

Project Title:	Enhancement of Medical Images based on Multi-resolution Techniques
Undertaken By:	Nida Akhter
Supervised By:	Mr. Muhammad Nadeem
Tool Used:	Matlab 7.0
System Used:	HP Pavilion Entertainment PC
Operating System Used:	Microsoft Windows XP
Starting Date:	March, 2008
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Abstract

In medical imaging, angiography is of primary interest to radiologists. Unfortunately, sometimes these images are of poor quality. The poor distribution of gray levels and addition of noise can be the distortion factors. These distortions reduce the overall contrast and the texture differences between individual organs, and pose a serious visualization problem since radiologists need clear visual representations of organs to produce proper diagnoses.

In our research work we proposed a method for the enhancement of angiogram images by combining two existing multi-resolution techniques. The merging of 2D continuous wavelets and laplacian pyramid is used. This technique is applied on various angiogram images which showed satisfactory results.

The objective of this research is to select a method for enhancement of angiogram images and to provide enhanced medical information to medical doctors for accurate diagnosis.

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CHAPTER 1
INTRODUCTION

1. Introduction

Radiology is the branch of medicine that deals with medical imaging. It uses radioactive substances, electromagnetic radiation, and sound waves to create images of the body, its organs, and structures for the purpose of diagnosis and treatment. Images can also show how effectively the body and its internal organs and structures are functioning.

The modern imaging technologies which are being used now-a-days are **Radiographs, Fluoroscopy, Computed tomography, Magnetic resonance imaging, Ultrasound, Creation of three-dimensional images (CT, MRI and Ultrasound), Angiography, SPECT** etc.

1.1. Angiography

Angiography injects a liquid dye so that the arteries should be clear in the image. In recent days angiography has been changed, as different methods are now been introduced like ultrasound, CT scans and MRI scans.

Angiography basically shows a clear picture of the blood vessels. Angiogram is used to identify any blood clots for on an artery that will cause blockage in the vessels or make the vessels weak. Angiogram is also very helpful in identifying a tumor which is caused by cerebral bleed. Angiogram is a very beneficial discovery done in the medical field it can help the doctor to decide if the patient requires any operation. Similarly, it also detects any abnormalities in the arteries or blood vessels.

Before taking an X-ray, a liquid dye is injected into the blood vessels. When the test is on the arteries of the heart, the carotid artery, or the major arteries coming from the aorta, the catheter is inserted into the groin, or occasionally the arm.

Before a catheter can be inserted into an artery, the surrounding area has to be numbed with a local anesthesia. A short, thin wire with a rounded tip is then carefully inserted into the artery using a needle. It is guided with the help of fluoroscopy (X-ray images) to the spot where the dye is needed. The needle is then removed and a vascular sheath inserted around the wire. A catheter may then be inserted along the guide wire. When the catheter is in the correct position, the wire is pulled out and dye is inserted through the catheter.

The patient may experience a feeling of warmth in the area, but this will disappear after a few seconds.

Angiography does involve certain risk factors like a patient can be allergic to the liquid dye similarly, a patient suffering from liver, heart or kidney diseases must be very careful before going through the procedure. Now a days, modern techniques are reducing the risk factors but still sometimes it is quite harmful this is why the medical imaging is introduced so that the patient may not have to pass through the process again and again.[1]

1.2. Image Enhancement

Image enhancement is a technique that reduces image noise, removes artifacts, and preserves details. A principal objective of enhancement is to process an image so that it provides a better presentation than the original image. The image enhancement algorithm used depends upon the desired objective as well as the application. For example, a method that is useful to enhance X-ray images may not be the best to enhance images of Mars transmitted by a space probe. Regardless of the method, image enhancement is one of the most interesting and visually appealing areas of image processing.

Image enhancement falls into two broad categories:

1.2.1. Spatial Domain

It refers to the image plane itself, means pixels of image are directly manipulated. The popular methods include histogram equalization, scaling of gray-values to maximum extent and transformation of gray-scale range.

The methods in spatial domain are still in use, therefore we also compared our proposed method with the above mentioned techniques afterwards. After comparing our technique with these methods it is proved that even though these methods work well with the original image i.e. non-noisy image but the results are very different when applied on the de-noised image.

Following results will show the techniques when applied on the original image i.e. the non-noisy image.

Power Law Transform

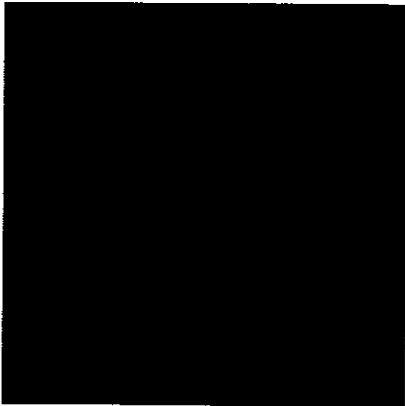


Figure 1.1: Original Image

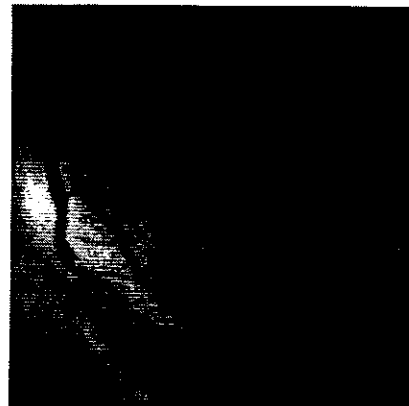


Figure 1.2: After applying power transform

Since the actual image was quite darker, hence in order to produce some bright effect we use the value 0.5 in the power that is less than 1. Power law is much more versatile for compressing/spreading of gray levels in an image.

Contrast Stretching

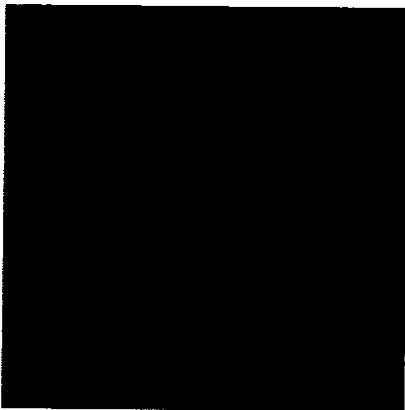


Figure 1.3: Original Image

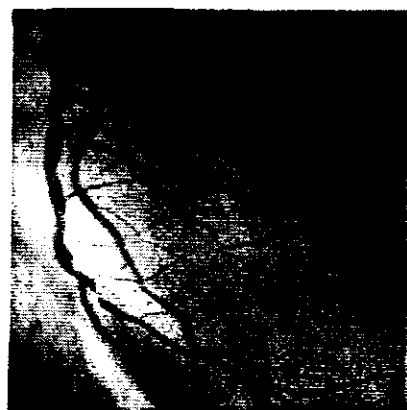


Figure 1.4: After Applying Contrast Stretching

The basic idea behind contrast stretching is to increase the dynamic range of the gray levels in the range being processed. The resulting image contains almost all the gray levels between the range of 0 and 1 (normalized form) as is clear from the histogram. That's why this image is more enhanced

Histogram Equalization

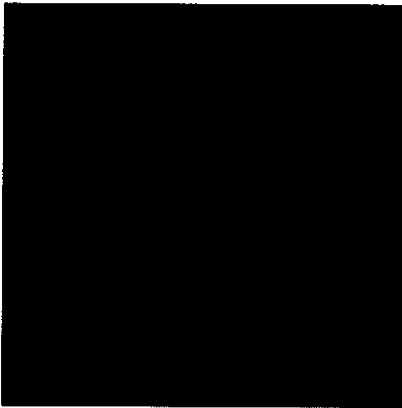


Figure 1.5: Original Image



Figure 1.6: After Applying Histogram Equalization

Histogram equalization basically doesn't work well with all sort of images, specifically in this image histogram equalization is not giving the desired results. This application has enhanced the dark pixels and bright pixels unnecessarily and hence some important part of the image is not visible to the viewer.

1.2.2. Frequency Domain

In this domain, processing techniques are based on modifying the Fourier Transform of an image. It is performed to attenuate the certain frequencies whose removal introduces appealing effects in the image. The popular methods include ideal filtering, butter-worth and homomorphic filtering.

Multi-resolution image enhancement uses both spatial and frequency domain. The laplacian pyramid technique will use Gaussian low pass filter for decreasing noise components. Therefore, these techniques are also implemented on the original image.

Butterworth Low Pass Filter

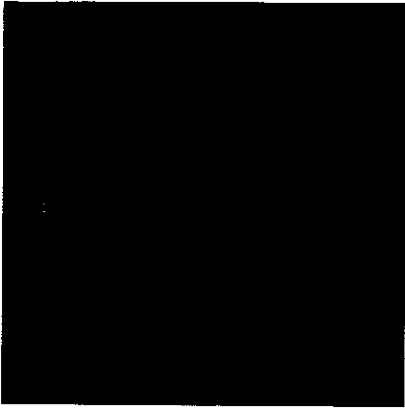


Figure 1.7: Original Image



Figure 1.8: After Applying Butterworth Filter

The Butterworth Low Pass Filter (BLPF) transfer function does not have sharp discontinuity that establishes a clear cutoff between passed and filtered frequencies. Butterworth Filter is designed in a way that it design smooth transfer function, defining a cutoff frequency locus at points for which the filter is down to a certain fraction of its maximum value is customary.

We have applied butterworth filters of different orders ranging from 1 to 5, of which order 2 is our choice. Butterworth of order1 has no ringing but it also results in not effective low pass filtering. Butter worth of orders greater than 2 results in more severe ringing. In general, BLPF of order 2 is a good compromise between effective low pass filtering and acceptable ringing characteristics.

Gaussian Low Pass Filter



Figure 1.9: Original Image



Figure 1.10: After Applying Gaussian Filter

The Gaussian low pass filter (GLPF) did not achieve as much smoothing as the butterworth of order 2 for the same value of cutoff frequency, because the profile of GLPF is not as “tight” as the profile of BLPF of order 2. However, the results are quite comparable and in general, and we are assured of no ringing in the case of GLPF. This is an important characteristic in practice

1.3. Multi-resolution Image Enhancement

Several techniques are being used in image enhancement and multi-resolution image enhancement is one of them. In the recent years, multi-resolution techniques are increasingly being applied to image processing problems. Global features are quickly and efficiently extracted from images through these techniques. However, the idea of analyzing images at different scales of resolution is not new. **Wavelets** and **Pyramid algorithms** are the two main categories of multi-resolution image enhancement. Research in several fields such as mathematics, physics, signal processing, and data analysis has a common foundation in wavelets and multi-resolution analysis. The most interesting characteristic of wavelets is their simplicity. A single function is translated and scaled to generate a basis for analyzing all functions of a certain class.

The main advantage of using wavelet-based techniques is the fact that wavelet-based techniques achieve good localization properties in both the signal and the transform domain.

Other than wavelets, pyramid algorithms incorporate a multi-resolution strategy that is not wavelet-based. Pyramid algorithms use a multi-resolution architecture to reduce the computational complexity of image processing operations.

1.3.1 Wavelets

The wavelet transform or wavelet analysis is probably the most recent solution to overcome the shortcomings of the Fourier transform. In wavelet analysis the use of a fully scalable modulated window solves the signal-cutting problem. The window is shifted along the signal and for every position the spectrum is calculated. Then this process is repeated many times with a slightly shorter (or longer) window for every new cycle. In the end the result will be a collection of time-frequency representations of the signal, all with different resolutions. Because of this collection of representations we can speak of a

multiresolution analysis. In the case of wavelets we normally do not speak about time-frequency representations but about time-scale representations, scale being in a way the opposite of frequency, because the term frequency is reserved for the Fourier transform.[2]

1.3.2 Image Pyramids

The predicted value for each pixel is computed as a local weighted average, using a unimodal Gaussian-like (or related trimodal) weighting function centered on the pixel itself. The predicted values for all pixels are first obtained by convolving this weighting function with the image. The result is a lowpass filtered image which is then subtracted from the original.

In image pyramid low pass filter is first applied to the original image. The desired result along with the prediction error is encoded. This results in a net data compression. The low pass filtered image can be encoded at a reduced sample rate. This process is repeated in order to get further data compression. The reduced image which is a low pass filtered image yields another image which results in another error image. These steps are then repeated we can get a sequence of two dimensional arrays. In image pyramid each image is smaller than its predecessor by a scale factor of half. These images are when placed over one other, gives a shape of a pyramid structure. The value at each node in the pyramid represents the difference between two Gaussian-like or related functions convolved with the original image. The difference between these two functions is similar to the "Laplacian" operators commonly used in image enhancement.[3]

1.4. Problem Formulation

Medical imaging is taking on an increasingly critical role in healthcare, as the industry strives to lower patient costs and achieve earlier disease prediction. To provide the functionality needed to meet these industry goals, equipment developers are turning to programmable logic devices.

The poor distribution of gray levels in a typical DICOM format of a Computed Tomography (CT) and MRI scan reduces the overall contrast and the texture differences between individual organs, and poses a serious visualization problem since radiologists need clear visual representations of organs to produce proper diagnoses. To achieve lower

patient costs and earlier disease prediction, different enhancement techniques are being used. This research work will also be dealing with the same problem that medical imaging is facing now a days.



Figure 1.11: Noisy Angiogram Image

1.5. Objective of the Project

This research project aims at developing a system that will enhance the poor quality image so that the radiologist can perform correct coronary artery operation. The objective is to process and analyze medical images obtained using Coronary angiography to extract veins or arteries structure. The theme of the project is to design and develop new and advanced algorithms and techniques to enhance those medical images which are not only poor in contrast but are also having the noise content in them.

1.6. Scope and Plan of the Project

It is divided into two categories.

- **Pyramid construction**

Laplacian and Gaussian pyramids are to be constructed in order to get desired coefficients.

- **Wavelet analysis**

Wavelets are constructed using the mapping functions. The mapping function will be determined based on the following considerations:

- a) The coefficients having great values are heavily weighted, since they carry more effective information.
- b) The coefficients at low levels are heavily weighted, since they carry edge information.
- c) The scaling coefficients of level *max* are not manipulated for preventing image distortion.

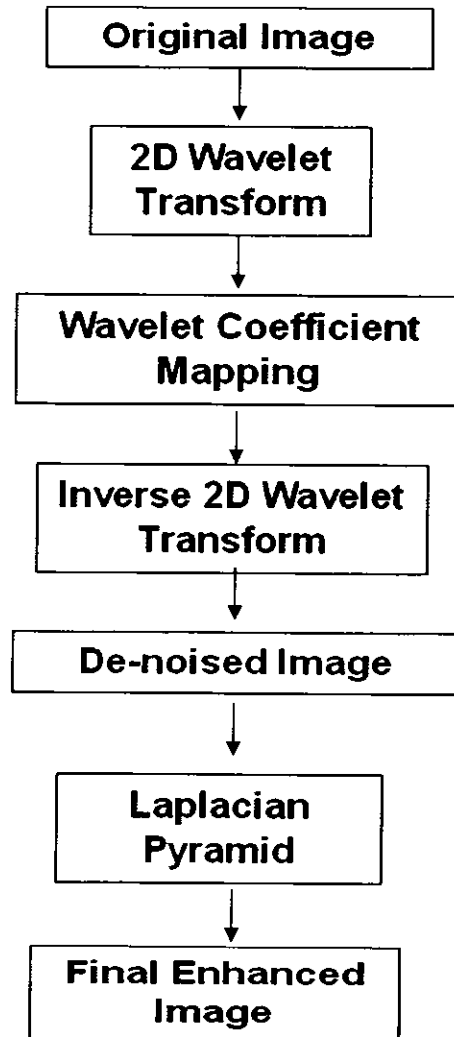


Fig. 1.12 Flow chart of image enhancement using the wavelet transform

CHAPTER 2
MEDICAL IMAGING

2. Medical Imaging

Medical imaging methods are the techniques and processes used to create images of the human body (or parts of the body) for clinical purposes or medical science (including the study of normal anatomy and function). The advancement in medical imaging presents a useful way to examine the inner structure of the human body. However, due to the different characteristics of medical imaging, a single source image cannot provide overall information for different needs. For example, CT shows the clear structure on denser tissue with less distortion, such as bone; while MRI reveals better pattern on soft tissue, such as brain. Some of the modern imaging technologies are discussed below are the some of the major issues of the reliability issues.

2.1. Angiography

Angiography is a medical test that helps physicians diagnose and treat medical conditions. Angiography uses one of three imaging technologies and, in some cases, a contrast material to produce pictures of major blood vessels throughout the body.

Angiography is performed using:

- x-rays with catheters
- computed tomography (CT)
- magnetic resonance imaging (MRI)

In magnetic resonance angiography (MRA), a powerful magnetic field, radio waves and a computer produce the detailed images. MR angiography does not use ionizing radiation (x-rays).

MR angiography may be performed with or without contrast material. If needed, the contrast material is usually injected using a vein in the arm.[1]



Figure 2.1: Angiogram Image

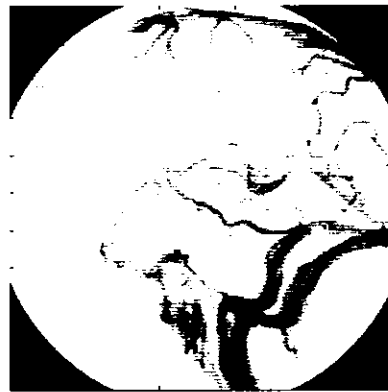


Figure 2.2: Angiogram Image

Procedure of Angiography

X-rays are a form of radiation like light or radio waves. X-rays pass through most objects, including the body. Once it is carefully aimed at the part of the body being examined, an x-ray machine produces a small burst of radiation that passes through the body, recording an image on photographic film or a special digital image recording plate.

X-rays are absorbed by the body accordingly, dense bones have high rate of absorption for the radiations while the soft tissues passes x-rays through them. As a result, the images of soft tissues are always in shades of grey while the background appears to be black.

Now days the images are saved in electronic form i.e. the images are in digital form. This makes easy accessibility to the image and are then enhanced and maintained easily and a lot of money is also saved.

In fluoroscopy pulsed x-ray beam is used to create sequence of images onto a screen, or television-like monitor. This is when used with an oral contrast material, clearly defines the area being examined and make it appear bright white, this technique helps the physicians to view internal organs in motion. [1]

Uses of Angiography

Physicians use the procedure to:

- Identify disease and aneurysms in the aorta, both in the chest and abdomen, or in other major blood vessels.
- Detect atherosclerosis disease in the carotid artery of the neck, which may limit blood flow to the brain and cause a stroke.
- Identify a small aneurysm or arteriovenous malformation inside the brain.
- Detect atherosclerotic disease that has narrowed the arteries to the legs and help prepare for endovascular intervention or surgery.
- Detect disease in the arteries to the kidneys or visualize blood flow to help prepare for a kidney transplant.[1]

Equipment

The equipment typically used for this examination consists of a radiographic table, an x-ray tube and a television-like monitor that is located in the examining room or in a nearby room. When used for viewing images in real time (called fluoroscopy), the image intensifier (which converts x-rays into a video image) is suspended over a table on which the patient lies. When used for taking still pictures, the image is captured either electronically or on film.

The catheter used in angiography is a long plastic tube about as thick as a strand of spaghetti.

Benefits

Angiography may eliminate the need for surgery. If surgery remains necessary, it can be performed more accurately.

- Catheter angiography is helpful for the surgical procedure or percutaneous intervention.
- By selecting the arteries through which the catheter passes, it is possible to assess vessels in several specific body sites. In fact, a smaller catheter may be passed

through the larger one into a branch artery supplying a small area of tissue or a tumor; this is called super selective angiography.

- In angiography use of a catheter combines diagnosis and treatment in a single procedure, e.g. angioplasty and placement of a stent.
- The degree of detail displayed by catheter angiography may not be available with any other noninvasive procedures.
- No radiation remains in a patient's body after an x-ray examination.
- X-rays usually have no side effects in the diagnostic range.[1]

Risks

- There is always a slight chance of cancer from excessive exposure to radiation. However, the benefit of an accurate diagnosis far outweighs the risk.
- A patient having allergy to x-rays must be advised to take medicine before angiography so that the allergic reaction should be reduced.
- Leaking of the x-ray material can be dangerous and can cause skin damage. Severe pain should be informed to the technologist.
- The risk of serious allergic reaction to contrast materials that contain iodine is extremely rare, and radiology departments are well-equipped to deal with them.
- Some risks are quite rare but they are also being observed e.g. blood clotting which might need an operation to reopen the vessels.
- The diabetes patient should be a little careful as they can face kidney diseases or the kidney may be injured.
- Rarely, the catheter punctures the artery, causing internal bleeding. It also is possible that the catheter tip will separate material from the inner lining of the artery, causing a block downstream in the blood vessel.[1]

CHAPTER 3
LITERATURE SURVEY

3. Literature Survey

Enhancement of medical images is a significant area of research in the field of medical imaging. The main issue for medical imaging is the quality of the image specifically in Pakistan where old machines are still being used. The noisy images, when de-noised verily have different contrast enhancement problems. Contrast enhancement techniques are when applied to de-noised image do not give same results as when applied to normal image, therefore a method is to be applied that can give good results for de-noised image.

Considering the above mentioned problem a literature survey is to be done that will deal with the method for de-noising and then a method to be searched that can further enhance the de-noised image without much information loss and most of the original contents of the image are to be restored.

3.1. Pyramid Methods in Image Processing

E. H. Adelson, C. H. Anderson, J. R. Bergen, P. J. Burt and J. M. Ogden(1984)[3] described that the pyramid offers a useful image representation for a number of tasks that includes contrast enhancement and sharpening. Pyramid method in image processing is quite efficient as compare to the fast Fourier transforms as it is faster and gives more appropriate results that are required. Different levels that are constructed by using the pyramid technique includes information that is localized in both space and spatial frequency domains, this is why the technique is very easy to implement.

This method is also quite effective for data compression, analysis and graphics. This method is quite effective in making certain changes according to ones own application to the low pass and high pass bands. The Gaussian pyramid constructs low pass bands while the laplacian pyramid constructs high pass filtering.

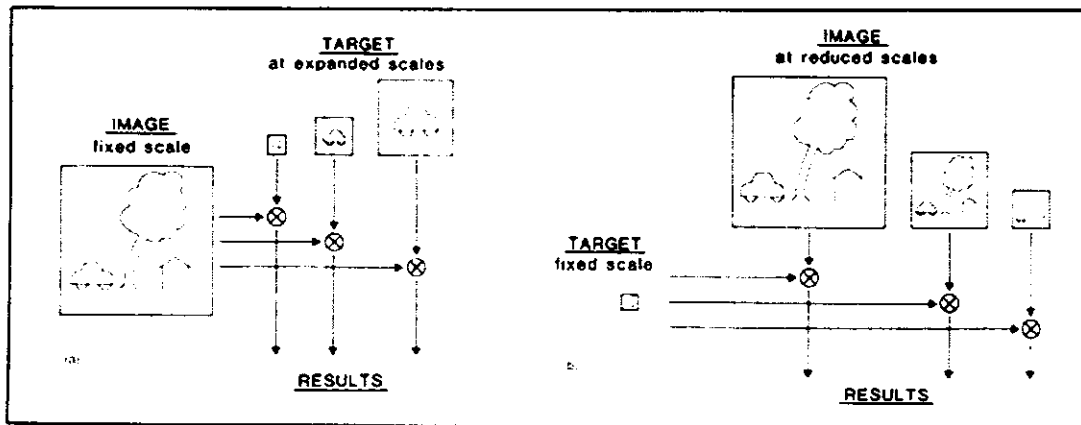


Fig. 3.1 Two methods of searching for a target pattern over many scales

3.2. Wavelet-based Methods for the Enhancement of Medical Images

M.J. Lado, P.G. Tahoces and A.J. Mendez [4] proposed a wavelet-based method for the enhancement of medical images. In their proposed method, non-linear mapping functions are introduced for weighting wavelet coefficients when multi-resolution levels are low and absolute values of the wavelet coefficients are great, thereby resulting in edge enhancement of images. They applied their research to chest radiographic images and mammograms. The results were compared to the FFT-based method and a conventional WT-based method. The results demonstrated that edges of chest images and microclassifications of mammograms could be enhanced by using the proposed method without a significant enhancement of noise.

3.3. Wavelet Feature Enhancement of Small Airways Disease

Guang-Zhong Yang* and David M. Hansell(1997)[5] enhances the CT features of small airways disease, by presenting a wavelet based feature enhancement technique. The technique enhances the very small density differences in the lung parenchyma of high resolution CT images. The basic idea was to introduce a technique which can effectively enhance the feature differences between normal and abnormal lung parenchyma. This can help in the diagnosis of the small airways disease. A hybrid structural filtering technique has been used. This technique removes the pulmonary vessels appearing in the CT cross-sectional images. Even though during the process the small intensity details of the lung parenchyma are not disturbed, but still possible distortion done by the hybrid filtering is

then restored by using a feature localization process. The feature localization process is also based on the wavelet reconstruction.

3.4. Directional Selectivity of Wavelet Transform

Lihua Li, Fei Mao, Wei Qian and Laurence P. Clarke(1997) [6] did an analysis and comparison on directional selectivity of wavelet transforms with three different frequency decomposition were taken. The WTs with radial-angular decomposition were used for two medical imaging problems: mass detection in digital mammography and lung nodule detection in chest radiography. Their results demonstrated that directional feature extraction using wavelet transform is an efficient approach for improving the diagnosis in medical imaging.

3.5. Applying Multi-resolution Methods to Medical Image Enhancement

Monica A. Trifas, John M. Tyler and Oleg S. Pinykh(2006) [7] applied Laplacian pyramid technique to the images for image enhancement. In this paper it is concluded that the laplacian pyramid technique not only enhances the subtle details but it is also helpful in improving the contrast of the image, while suppressing the noise elements. The contrast is improved by modifying the coefficients obtained from the decomposed image. The compressed image is formed by encoding the error image which remains when an expanded one is subtracted from the original image. This image is said to be the bottom level of the laplacian pyramid. The Laplacian pyramid is a sequence of error images L_0, L_1, \dots, L_N .

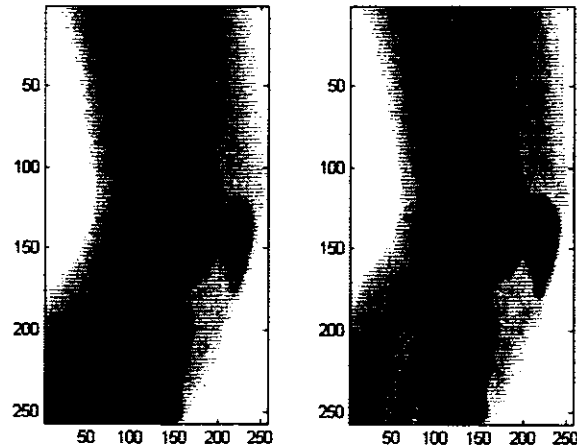


Fig. 3.2 Improving a blurred image using Laplacian Pyramid

3.6. Image Enhancement by Using Directional Wavelet Transform

D. Heric, B. Potocnik(2006) [8] describes directional wavelet transform. Directional wavelet transform divides an image into four dimensional space. The four dimensional space gives the scale and directional information. The results showed that the directional wavelet is very efficient in removing noise while retaining very less information loss.

The enhancement is performed by selecting a threshold value that helps in detecting the singularities via maximum entropy measure.

The use of scalable modulated window is helpful in solving the signal-cutting problem. The window moves with the signal and the process repeats by reducing the window size for a new cycle. This forms time-frequency representation with different resolution.

The wavelet transform represents the time scale representation, as opposite of Fourier transform The wavelet analysis described in the introduction is known as the *continuous wavelet transform* or *CWT*.

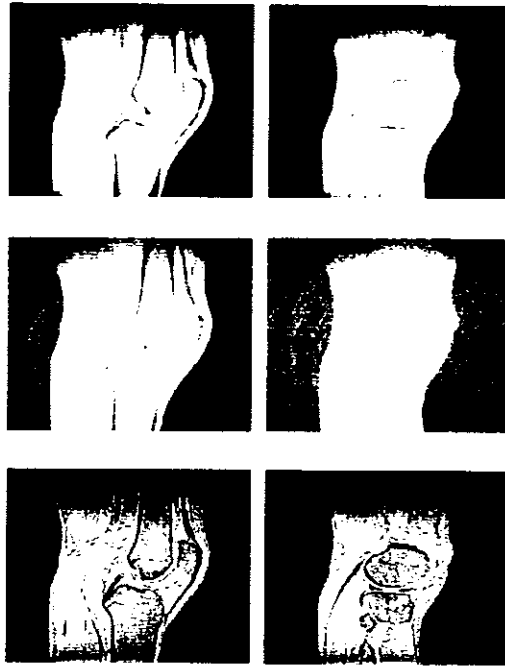


Fig. 3.3 Original Images (1st row), Histogram equalized images (2nd row), Wavelet transform (3rd row)

3.7. A wavelet-Based Multi-resolution Reconstruction Method for Fluorescent Molecular Tomography

Wei Zou, Jiajun Wang, Kongpei Wu, and David Dagan Feng (2009) [9] introduced an image reconstruction of fluorescent molecular tomography (FMT) often involves repeatedly solving large-dimensional matrix equations, which are computationally expensive, especially for the case where there are large deviations in the optical properties between the target and the reference medium. In this paper, a wavelet-based multiresolution reconstruction approach is proposed for the FMT reconstruction in combination with a parallel forward computing strategy, in which both the forward and the inverse problems of FMT are solved in the wavelet domain. Simulation results demonstrate that the proposed approach can significantly speed up the reconstruction process and improve the image quality of FMT.

3.8. The Laplacian Pyramid as a Compact Code

Peter J. Burt and Edward H. Adelson [10] has described the Laplacian pyramid technique for data compression. This paper describes the versatility of the technique. As the quantization applied on the technique will result in data compression.

It is also a great significance of the Laplacian pyramid that it is localized in both spatial frequency as well as in space.

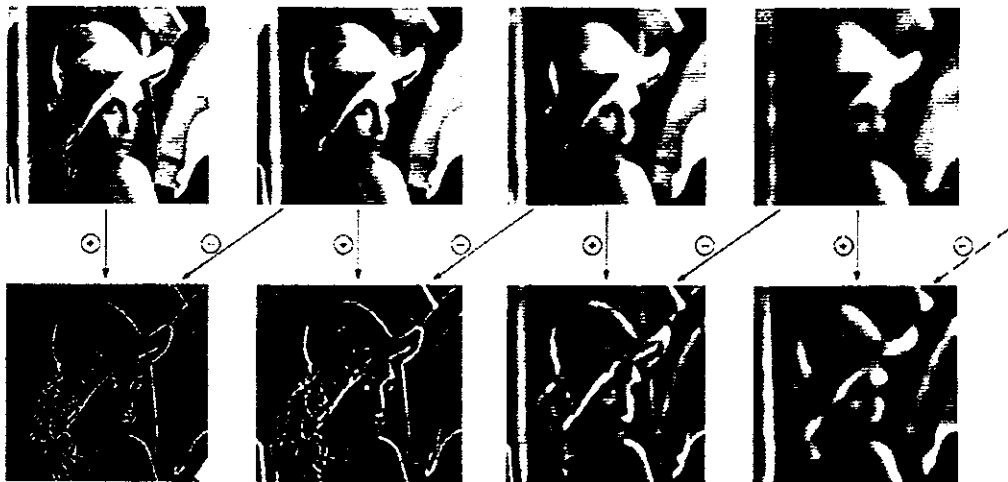


Fig. 3.4 First Four Levels of the Gaussian and Laplacian Pyramid

3.9. Computational Complexity of the Continuous Wavelet Transform in 2D[11]

The paper describes the basic concept behind the 2D continuous wavelet transform. The basic concept will remain the same in all applications while the mother wavelet chosen can affect the results either to be used for removing noise or any other application.

The paper basically deals with the two basic densities. One is the position or aspect-angle and the second is the scale-angle densities. The CWT can also be interpreted as the space-frequency representation as well as the spatial-frequency. The translation factor corresponds to the space-frequency while the inverse of the scale and the rotation taken together, corresponds to the spatial-frequency variable.

3.10. Study on Methods of Noise Reduction in a Stripped Image

Chi Chang-Yan and Zhang Ji-xian [12] has done comparison of different noise reduction methods. The paper made comparison between three different techniques, that are the low pass filter, the grey value substitution and the wavelet transformation. The results are compared by visual analysis as well as by MSE and PSNR values. In low pass filter the noise is removed well but much detailed information is lost.

The grey value substitution is better in saving information while small stripes still exists and the brightness is also enhanced. The wavelet transform method is much more helpful in saving the required information, but when stripped noise is concerned the usual wavelet decomposition cannot remove the stripped noise. The horizontal part is still effected by the noise while the vertical and cross part of the image does not have much noise. Therefore, it is suggested to de-noise the LL and HL parts by using ordinary noise reduction method and the HH and LH should be treated by wavelet decomposition.



Fig. 3.5 Image De-noising using Wavelet Transform

3.11. Image Enhancement of Nano Tubes Smoothing and Sharpening[13]

The images of the nano-tubes requires more enhancing because of low contrast and low SNR. In this paper two methods for image enhancement are proposed, one is image sharpening and other is image smoothing.

Image sharpening is done by high pass filtering while the soothing part is acquired by using low pass filtering. In low pass filtering, choosing the media value gives better results for suppressing noise and preserving edges than the mean filter. While for sharpening, the low frequency image is to be subtracted from the original image. The butterworth filter provides more improvement in SNR and CNR values as compared to Gaussian filters.

3.12. Multi-scale Contrast Enhancement Laplacian Pyramid versus Wavelet Transform

Sabine Dippel, Martun Stabl and Rafael Wiemker [14] describes the difference between laplacian pyramid and wavelet transform for contrast enhancement. There are certain drawbacks while using wavelet transform technique. Because of the additional overshooting of the signal the high pass filter is back transformed once more, this emphasizes the edges even more, this results in the ringing artifacts. Secondly, because of the number of iterations the time complexity increases and that consumes a lot of time which is not affordable in medical imaging.

On the other hand, laplacian pyramid is a technique where smoothing operation is done in the back transform. As the edge enhancement performed at lower scales, therefore it gives a soft image impression while avoiding boosting noise too strongly.

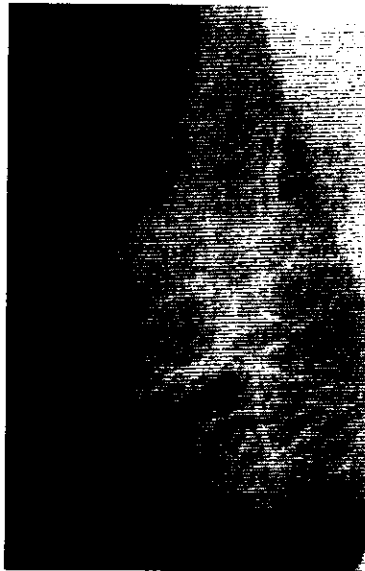


Fig. 3.6 Mammogram processed by Laplacian Pyramid

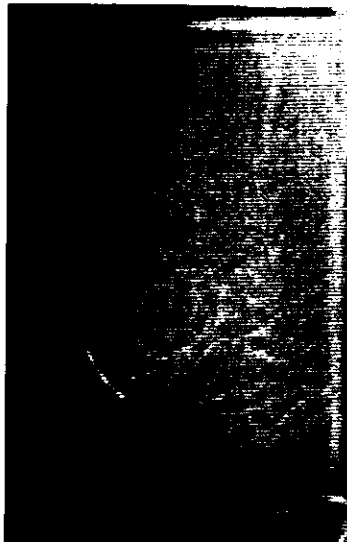


Fig. 3.7 Mammogram processed by Wavelet Transform

3.13. Speckle Noise Reduction Ultrasound Images by Wavelet Thresholding

S.Sudha, G.R. Suresh and R. Sukanesh [15] proposed a wavelet based method for de-noising ultrasound images. It is to be noted that the value of the threshold plays a significant role in achieving good results, specifically when medical images are concerned. A small threshold value can leave noisy coefficients while a large threshold value can destroy some details, therefore an optimum value should be considered.

In this paper universal thresholding function is used. The results are then compared to the other de-noising techniques. Both the qualitative and quantitative results show that wavelet transform is far better in denoising.

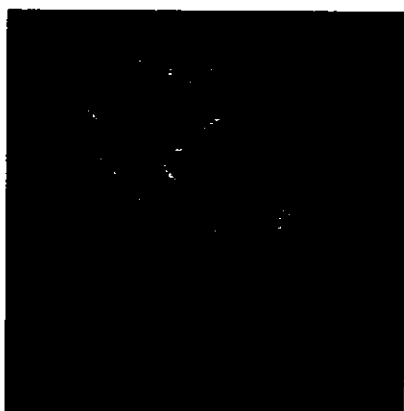


Fig. 3.8 Noisy Image

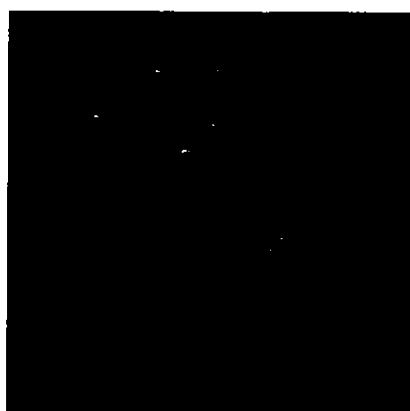


Fig. 3.9 Kaun Filter

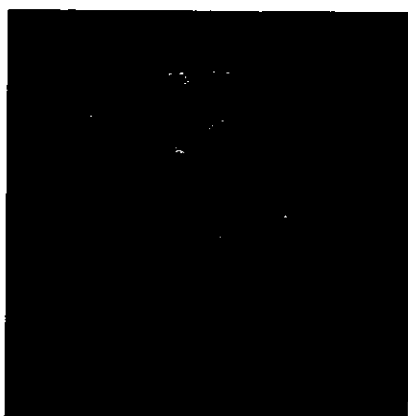


Fig. 3.10 Frost Filter

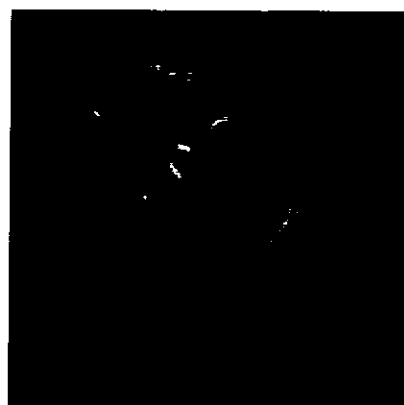


Fig. 3.11 Wavelet Transform

CHAPTER 4
RESEARCH METHODOLOGY

4. Research Methodology

The main idea of this research work is to figure out the best possible techniques to enhance the noisy angiogram images. This research proposes a solution which leads to a denoised image with improved contrast and sharpening of the required data that are the veins and arteries for diagnosis. The first technique resolves the problem of noisiness while the second technique helps to improve the non uniform illumination while doing less information loss. Later these techniques are compared with other ordinary techniques to prove its efficiency.

4.1. 2D Continuous Wavelets Transform

The first method we choose is the “2D continuous wavelet transform”. This method is applied for removing noise distracting the image.

4.1.1. Reasons for selecting the technique

There are few good reasons for using 2D continuous wavelets transform for denoising the angiogram images considering the fact that in medical images it is important to retrieve maximum information as possible:

- 1) The angiogram images are not specifically bound to the direction aspect that is the vessels can be present at any direction horizontal, vertical or even diagonal. As the continuous wavelets are rotational invariant therefore it will recover the data from all the main directions which will help in enhancing even the subtle details in an image.
- 2) Secondly, the multi-resolution property of wavelets will help to retrieve maximum information by enhancing the image in high as well as lower resolutions.
- 3) The idea behind using the Mexican hat function is its “Gaussian of Laplacian” property. The function will first suppress the noise through Gaussian and then helps in sharpening the edges mainly, by using laplacian. When we talk about

angiogram images it is important to note that mainly every blood vessel will be considered as an edge and therefore the laplacian will help in retrieving the maximum information. The sharpened areas will be saved as singularities and the rest will be deleted as noise.

- 4) Hard thresholding gives best results by completely removing the noise, but as in angiogram images we have very subtle details which are to be restored. Therefore it is important to select correct standard deviation value which will not only gives good results in removing noise but should also be helpful in restoring the subtle details which are important for the diagnosis.

4.1.2. Methodology

In 2D continuous wavelet transform a noisy angiogram image was taken as an input image. This input data was be then divided into wavelet coefficients. These coefficients were divided into three directional components as horizontal, vertical and diagonal. These directional components were formed by applying a mother wavelet kernel. In this method we used Mexican hat kernel as a mother wavelet.

A threshold is to be applied by considering the standard deviation and was applied to the coefficients. After applying the threshold the main features of the kernel function were termed as singularities while the rest was deleted as noise. This data is then decomposed in the original format.

2-D wavelets can only provide three directional components, namely horizontal, vertical, and diagonal. The 45o and 135o directions are mixed in diagonal subbands. The wavelets are then further enhanced by introducing directional property in them. The directional wavelet resolves the mixing problem in the diagonal subbands and is very helpful in extracting the subtle details, which in turn de-noises the image quite effectively as compare to simple 2-D wavelet transform.

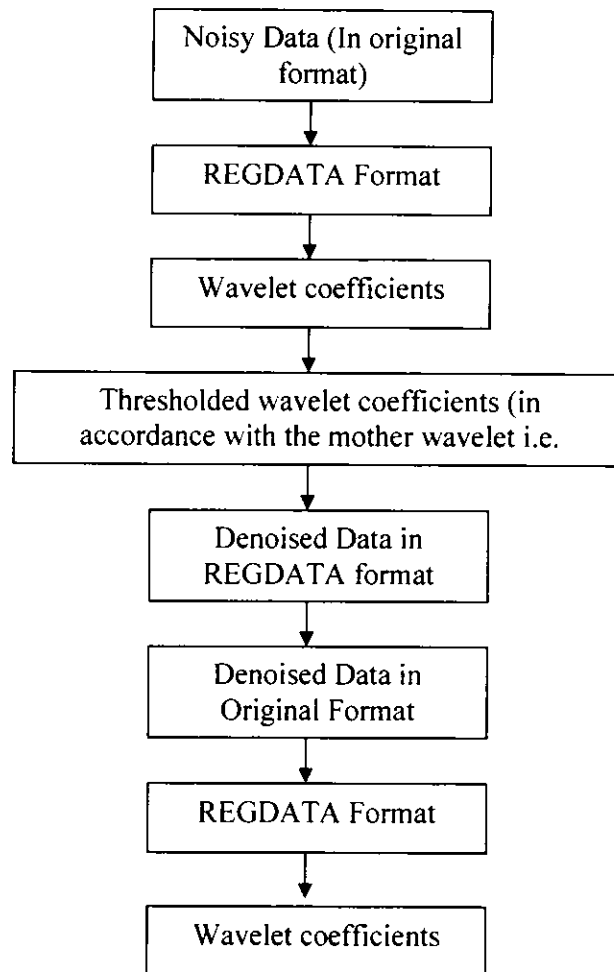


Figure 4.1 Flow chart of denoising of data by 2D continuous wavelet transform

4.1.3. Comments

After applying this technique on several images few disadvantages were seen other than noticing that this technique is much better than the rest. The main disadvantage is that after applying 2D continuous wavelets transform the image turned out to be low in contrast because of which non-uniform illumination occurs. This non-uniform illumination makes it difficult for the radiologist to view the details in that specific region. The second disadvantage to be noted is the aliasing factor but after showing the

drawn results to some doctors, they were quite comfortable with this aliasing problem as it did not have any effect in diagnosis.

4.2. Laplacian Pyramid

The second method is “Laplacian pyramid”. This method was proposed by Burt and Adelson in 1981 in order to compress images. Its another feature of contrast improvement is discussed by Monica A. Trifas, John M. Tyler and Oleg S. Pinykh, in their paper “Applying multiresolution methods to medical image enhancement” Published by *ACM SE'06*, March, 10-12, 2006, Melbourne, Florida, USA.

4.2.1. Reasons for selecting the technique

After de-noising, the image will generously lose its contrast in some regions, which will result in non-uniform illumination. Considering this fact a method is to be proposed which will increase the illumination factor by enhancing even the smallest blood vessel in an image as possible. Few good reasons for selecting laplacian pyramid technique is:

- 1) Considering the above mentioned problems, Laplacian pyramid was supposed to be better than any other ordinary techniques. As it has the ability to improve contrast as well as sharpening the details in an image. In laplacian pyramid technique the edge enhancement is performed at lower scales, therefore it gives a soft impression while avoiding boosting noise too strongly.
- 2) In laplacian pyramid subtle details are amplified to improve the visibility of the corresponding details, which is actually required for angiogram images.

4.2.2. Methodology

The Laplacian Pyramid decomposition, originally developed by Burt and Adelson in order to compress an image but later this technique was implemented for many other applications like enhancement of medical images. In this method we took a noisy

angiogram image as an input data. The Laplacian Pyramid then decomposed the original image into a hierarchy of images such that each level corresponds to a different band of image frequencies. This was done by taking the difference of levels in the Gaussian pyramid. The original image was then reconstructed from its Laplacian pyramid by reversing this process.

To obtain a compressed representation, we encode the error image which remains when an expanded g_1 is subtracted from g_0 . This image becomes the bottom level of the Laplacian pyramid. The next level is generated by encoding g_1 in the same way. We now give a formal definition for the Laplacian pyramid.

The Laplacian pyramid is a sequence of error images L_0, L_1, \dots, L_N . Each is the difference between two levels of the Gaussian pyramid. Thus, for $0 < i < N$,

$$L_i = g_i - \text{EXPAND}(g_{i+1}) \quad \text{----- eq(1)}$$

$$= g_i - g_{i+1} \quad \text{----- eq(2)}$$

Since there is no image g_{N+1} to serve as the prediction image for g_N , we say $L_N = g_N$.

4.2.3. Comments

After applying Laplacian pyramid to the de-noised image it was noted that the low contrast region has improved by addition of illumination factor as well as the details are also visible. On the other hand the other techniques were unable to provide that perfection. At the same time we configured a drawback of this method also. At reaching a certain low resolution the image was forming a blocky effect, if that blocky effect would not be there, the results would have been better than what we were having. After applying this method onto several images it was noted that the images having much worst illumination problems, required to construct a pyramid till the lowest resolution i.e. if we are having a worst 512x512 image, it required a lowest resolution of 16x16 to get best results, but because of blocky image we had to stop it till 32x32. The results were not the best but were also not disappointing.

The main idea behind the project is to develop a technique that can remove noise from the angiographic image while avoiding much loss of the subtle details, and then the required contrast enhancement should be made while emphasizing on the small details.

CHAPTER 5
EXPERIMENTAL RESULTS

5. Results

In our experiment we chose 3 different angiogram images and tested our results on them. The images are first de-noised by using wavelet transform and then laplacian pyramid is applied for contrast enhancement. The three images are of better to worst quality. In the worst case the image was not only noisy but it was also very low in contrast. The results were quite satisfactory for all the three images.

The parameters used in the process are as follows

- Mother wavelet = maxican hat function
- Threshold type = hard
- Noise Added = 10%
- Standard deviation of noise estimated to $\sigma = 10.103$

Afterwards, the proposed method is also compared with the other techniques and their quantitive and qualitative results are shown. The proposed technique is compared with:

- Global Histogram Equalization
- Local Histogram Equalization
- Laplacain

The above mentioned techniques are applied on the de-noised image and the results showed that the laplaian pyramid gives much better results on the de-noised image as compare to these techniques while it is shown in chapter 1 that these techniques works good with the non-noisy image.

5.1. Noise Removal Results

The technique of 2D continuous wavelets transform is applied on different angiogram images to remove noise. Consider the angiogram image containing noise as shown in figure 5.1. 2D continuous wavelets transform were applied onto it, while using Mexican hat function as mother wavelet and we get a denoised image as shown in fig 5.2.



Figure 5.1: Noisy Angiogram Image



Figure 5.2: Denoised Angiogram Image

5.2. Applying Laplacian Pyramid

The non-uniform illumination was tackled by using laplacian pyramid. The laplacian pyramid improved the contrast of the image and also sharpened the required diagnosis part of the angiogram. Fig 5.4. shows the required results after applying laplacian pyramid over the noisy image.



Figure 5.3: Angiogram Image



Figure 5.4: Applying Laplacian Pyramid

5.3. Combining Both Techniques

Combining both the techniques helped the image to enhance properly. After applying both the techniques the noise was removed, the image got better in contrast, and at the same time the arteries were also enhanced. But there was a problem as what technique should be applied first. We experimented both ways, fig 5.5. shows the image where wavelets were applied first and then the laplacian pyramid technique, while fig 5.6. shows the image where wavelets were applied after improving the contrast by using laplacian pyramid. Results have shown that fig 5.6. is better as compare to fig 5.5. because of the fact that the wavelets cause aliasing problem and laplacian enhances it as it sharpens the image, therefore it is advisable to use the laplacian technique first so that the aliasing factor doesn't increase.



Figure 5.5: Applying 2D wavelets first

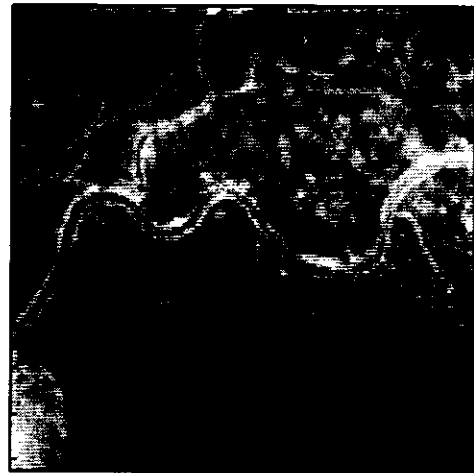


Figure 5.6: Applying Laplacian pyramid first

5.4. Results for Worst Image

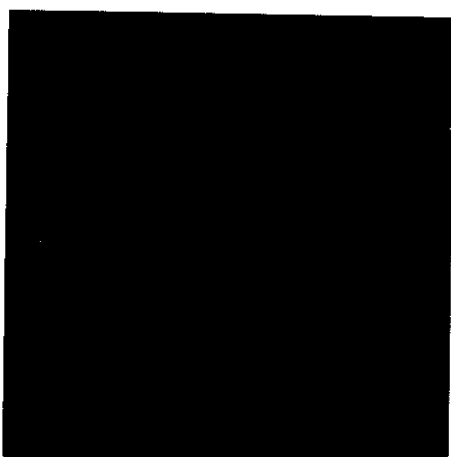


Figure 5.7: Original Noisy image

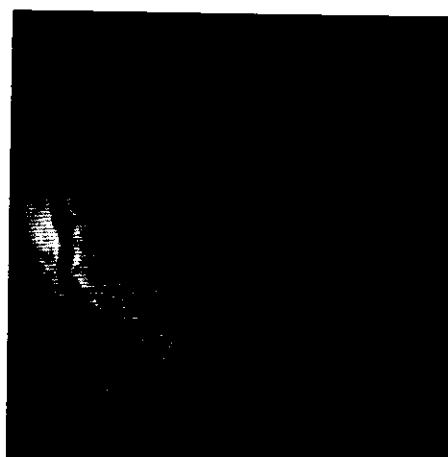


Figure 5.8: Image after Applying Laplacian Pyramid and Wavelets

5.5. Comparison

After de-noising the image the results are compared with some other useful contrast enhancement techniques e.g. Laplacian, Local histogram equalization and Global histogram equalization. As we can see visually the results of laplacian pyramid gives much better results. The images are much clearer and the subtle details are also enhanced with less noise addition, whereas the noise is enhanced in histogram equalization either local or global which makes it difficult to understand the real image. Similarly, the SNR values also show that laplacian pyramid is better than the rest. The results are also compared for the worst image, it is clear from the figure and SNR values that laplacian even works better for this image.

Fig 5.9 and fig. 5.10 shows that all the other three techniques are having some main issues regarding enhancement. The three techniques are quite efficient in enhancing the contrast when it was tested on the non-noisy image, but the results are quite different when tested on the noisy image. As far as qualitative issues are concerned the simple laplacian technique is giving a blurred image where the details are more suppressed, but the noise is not further enhanced. In global histogram equalization the noisy elements are further enhanced while giving a much bright effect to the image which was not required as the brightness is affecting the diagnosis. Similarly, the local histogram equalization technique is also enhancing the noisy elements while giving much bright effect to the whole image

which is again making it difficult for the expert to diagnose. It is very clear from the figures that the proposed method not only enhances the subtle details without introducing much noise but is also helpful in avoiding the brightening effect to the image.

Figure 5.10 also shows that even in case of a worst image the details are restored in a very reluctant way by the proposed method as compare to the rest of the three techniques. The final result of this image is further enhanced by using high boosting which gives an equal light effect and the subtle details are also not disturbed.

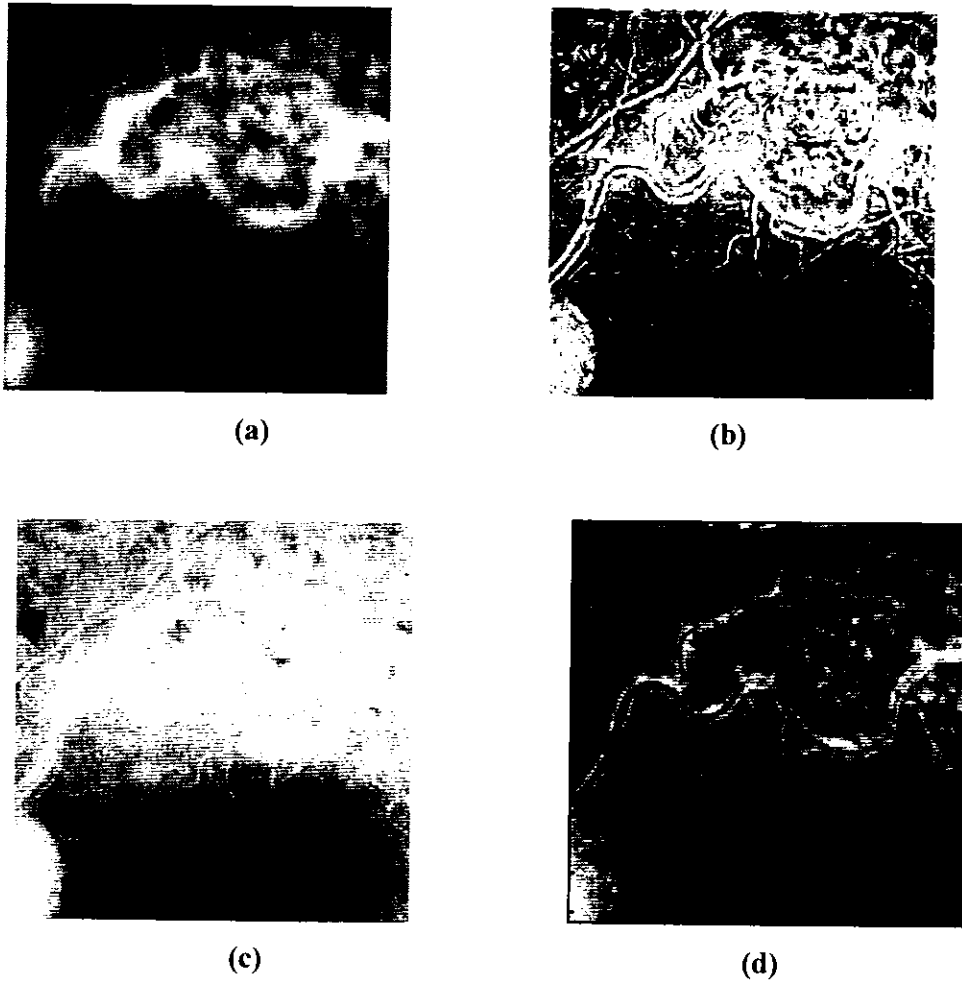
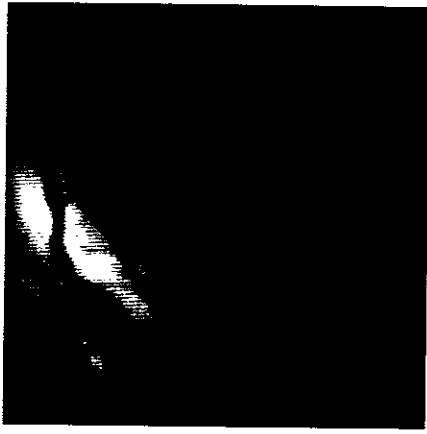


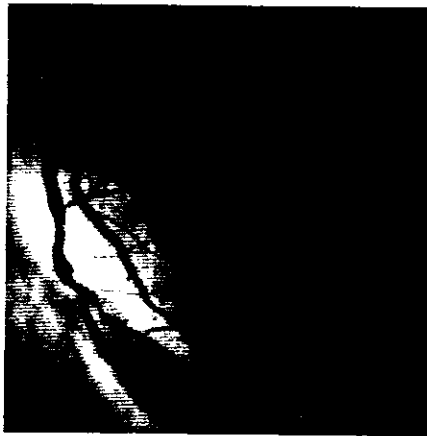
Figure 5.9: (a) Laplacian (b) Global Histogram Equalization (c) Local Histogram Equalization (d) Laplacian Pyramid



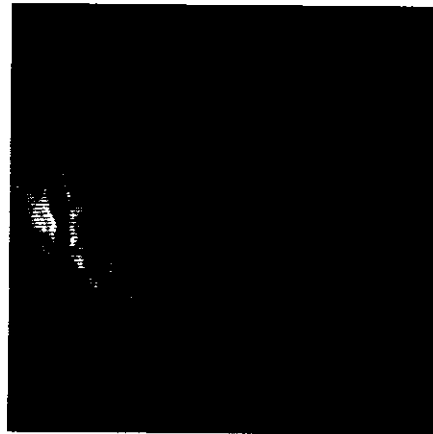
(a)



(b)



(c)



(d)

Figure 5.10: (a) Laplacian (b) Global Histogram Equalization (c) Local Histogram Equalization (d) Laplacian Pyramid

METHODS USED	SNR VALUE For fig.1	SNR VALUE For fig.2
Global histogram equalization	3.87	3.45
Local histogram equalization	3.45	3.04
Laplacian Filter	4.55	4.07
Proposed Method	5.88	5.09

Table

e 5.11: SNR values

The SNR values also show that laplacian pyramid is better than the rest. The results are also compared for the worst image, it is clear from the figure and SNR values that laplacian even works better for this image.

CHAPTER 6

CONCLUSION AND FUTURE ENHANCEMENTS

6. Conclusion and Future Enhancements

Based upon the experimental results the following conclusion is drawn and few directions for further study are also given.

6.1. Conclusion

During the acquisition process, every image up to some extent has noise or poor contrast. For proper diagnosis it is important to improve these distortion factors, as their complete removal is not possible. Wavelets are proven to be the best method for the removal of noise specifically when we are talking about the Gaussian noise. It is important to understand the effectiveness of using a proper mother wavelet. A mother wavelet should be selected according to the image.

This report applies the technique of 2D continuous wavelets transform for noise removal from angiogram images, and then laplacian pyramid is used for further enhancement of the image. The noise removal method is not helpful in removing poor contrast in fact the threshold process in wavelets further decreases the contrast. In order to enhance the inadequacies of wavelets laplacian pyramid is used.

6.2. Future Enhancements

There is a lot of scope in medical imaging. Some of the directions of further study are as follows:-

- 1) This technique should be tested on some other type of images other than the angiogram images. The changes can be done accordingly.
- 2) The technique can be tested on the colored images and enhancements can be made accordingly.

System Implementation

All the modules of the project are implemented in Matlab 7.0. The sample code and explanation for all the major modules is given in this chapter.

Image Loading

In the loading stage a noisy image is to be taken as an input data. This image will be of any resolution, we have implemented the technique over an image of size 512x512.

```
% xx: input angiogram image
xy=imread('ang5.jpg');
xx=imresize(xy,[512,512],'bilinear');
subplot(2,2,1),yashow(xx,'cmap','gray');
```

Denoising the image

In the denoising stage we are dealing with the Gaussian noise. In this stage we are using 2D continuous wavelet transform to denoise the image. We have used the toolbox known as YWTB which is not yet part of Matlab, therefore we have to load some main files from this toolbox into matlab's directory.

```
% xx: input angiogram image of size 512x512
% Adding Gaussian noise to the image
% Denoising the image
nX = double(xx) + 5*randn(512,512);
s1=yasnr((double(xx)),nX);
dX = fwt2d_denoise(fft2(nX), 'cmw', 4, 8, 'mexican2d', 255/10);
s2=yasnr(dX,(double(xx)));
```

Edge Sharpening and Contrast Enhancement

After denoising the image it is important to enhance the vessels so that the image gets clearer and should be helpful for diagnosis. Edge sharpening is done by using Laplacian

pyramid. The laplacian pyramid technique will not only sharpens up the main features in an image but it also tackles with the non-uniform illumination to improve contrast.

```

% xx: input angiogram image of size 512x512
% Applying laplacian pyramid
[height,width]=size(rX);
for r=1:height
    for c=1:width
        % ourimage1(r,c)=imag((r-1)*width+c);
    end
end
% %ourimage2=flipud(ourimage1);

% Fourier Transform Image
FT=fft2(rX);

% Shifting image
SHF=fftshift(FT);
GLP = ones(height,width);
D = 27;    % 5% of 512
for i=1:height
    for j=1:width
        D1=sqrt((i-(height/2))^2 + (j-(width/2))^2);
        GLP(i,j)=exp((-D1^2)/(2*D^2));

    end
end
% %
% % % Point by point multiplication
Fnew=SHF .* GLP;
% % % Inverse shifting and Fourier
YY = ifftshift(Fnew);
YY1 = ifft2(YY);
%image(YY1);

```

```
S=real(YY1);
minv=min(min(S));
maxv=max(max(S));
newimag1=((S-minv)/(maxv-minv))*256;

[height,width]=size(newimag1);
for I=1:height
    for J=1:width
        outimage1(I,J)=nX(I,J)-newimag1(I,J)+90;

%     if(outimage1(I,J)>255)
%         outimage1(I,J)=255;
%     end
    end
end

[height,width]=size(newimag1);
for r1=1:height/2
    for c1=1:width/2
        newimag2(r1,c1)=newimag1(2*r1,2*c1);
    end
end

[height,width]=size(newimag2);
for r=1:height
    for c=1:width
        % newimag3(r,c)=newimag2((r-1)*width+c);
    end
end

% %ourimage2=flipud(ourimage1);

% Fourier Transform Image
FT=fft2(newimag2);
```

```

% Shifting image
SHF=fftshift(FT);
GLP = ones(height,width);
D = 27;    % 5% of 512
for i=1:height
    for j=1:width
        D1=sqrt((i-(height/2))^2 + (j-(width/2))^2);
        GLP(i,j)=exp((-D1^2)/(2*D^2));

    end
end
% %
% % % Point by point multiplication
Fnew=SHF .* GLP;
% % % Inverse shifting and Fourier
YY = ifftshift(Fnew);
YY1 = ifft2(YY);
%image(YY1);
S=real(YY1);
minv=min(min(S));
maxv=max(max(S));
newimag4=((S-minv)/(maxv-minv))*256;

[height,width]=size(newimag4);
for I=1:height
    for J=1:width
        outimage2(I,J)=newimag2(I,J)-newimag4(I,J)+90;

%         if(outimage(I,J)>255)
%             outimage(I,J)=255;
%         end
    end
end
end
outimage17=rX+outimage8+outimage9+outimage11+outimage12+outimage13;

```

```
subplot(2,2,3),yashow(rX,'cmap','gray');title(['SNR = ',num2str(s3)]);  
subplot(2,2,4),yashow(outimage17,'cmap','gray');
```

References

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