

A New QPSK/3PSK Differential Coding Scheme for MIMO

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by

Ahsan Tanvir Malik

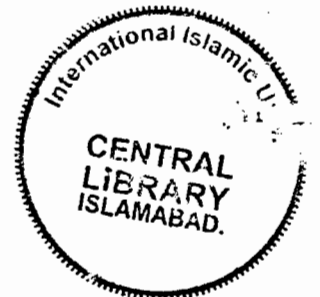
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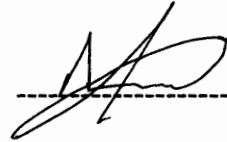
Dedicated To,

The Holy Prophet Hazarat Muhammad (S.A.W), His Sahaba, My Beloved Parents, My
Respected Teachers and All Muslim Umamah.

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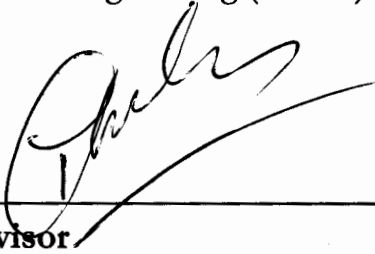
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Abstract

The demand for capacity in cellular and wireless local area networks has grown in a literally explosive manner during the last decade. In particular, the need for wireless internet access and multimedia applications require an increase in information throughput with orders of magnitude compared to the data rates made available by today's technology. One major technological breakthrough that will make this increase in data rate possible is the use of multiple antennas at the transmitter and receivers in the system. A system with multiple transmit and receive antennas is often called a multiple-input multiple-output (MIMO) system.

Multiple antenna system helps in combating the destructive effects of fading as well as improves the spectral efficiency of a communication system. Space-time block codes present a way of introducing two way transmit diversity into the communication system. This kind of system has proven to have a better approach to fight out channel impairments. One major assumption in all space-time block coding (STBC) schemes is that the channel state information is known but in differential space time block codes, there is no need for CSI.

The intention behind this research is to study performance of differential space-time block codes in flat fading channels. The diversity advantage by introducing space-time block codes in a communication system is studied. The other basic problem in wireless channel is the interference due to the other users present and communicating at the same time. To overcome the above said problem we used the inverse channel detection scheme.

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List of Acronyms

| | |
|--------|---|
| AR | auto-regressive |
| AWGN | additive white Gaussian noise |
| BER | bit error rate |
| BPSK | binary phase shift keying |
| CDMA | code division multiple access |
| CSI | channel state information |
| DSSS | direct sequence spread spectrum |
| EGC | equal gain combining |
| FH | frequency hopping |
| GSM | global system for mobile |
| IC | Interference cancellation |
| i.i.d. | Independent and identically distributed |
| LOS | Line-of-sight |
| MRC | maximal ratio combining |
| MIMO | multiple-input, multiple-output |
| ML | Maximum-likelihood |
| MMSE | Minimum Mean Square Error |
| O-STBC | Orthogonal space-time block coding |
| p.d.f. | Probability density function |
| PSK | phase shift keying |
| QPSK | quadrature phase shift keying |
| SC | Selection combining |
| STBC | space-time block coding |
| STC | space-time coding |
| STTC | space time trellis coding |
| SNR | signal-to-noise ratio |

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Chapter 1

INTRODUCTION

Mobile wireless devices are increasingly becoming an essential entity in our daily lives. We use wireless phones at homes, offices and while traveling on the road. Due to the widespread use of wireless devices in every possible scenario, it is important for wireless services to be robust and prone to the errors caused by the channel. Differential coding scheme is that type of scheme in which the difference of angles is taken into account. Differential coding scheme is a powerful technique to improve the reliability of existing communication system. Most of the work in differential coding is done regarding to improve the capacity as well as reliability of telecommunication systems. The modulation scheme, mostly used in differential coding technique like Alamouti scheme, is QPSK. A new 3PSK/QPSK differential coding scheme is presented in this thesis which not only improves the performance of the system but also increases the data rate.

The proposed technique is applied in MIMO. MIMO is that type of telecommunication system in which multiple antennas both at the transmitter and receiver ends is used. Multiple antennas increase the capacity of communication systems without disturbing any other figure of merit like SNR, BER etc. Multiple antennas introduce multipaths and in a wireless channel, multipath can introduce rigorous attenuation into the transmitted signals. Diversity techniques like time diversity, frequency diversity, space diversity etc. provide efficient ways to fight out the destructive nature of fading channels caused by the multipaths. Multiple antennas in a communication system provide diversity by utilizing temporal and spatial characteristics.

The capacity increase due to the use of multiple antennas is studied in [1] [2] [3]. It is shown in these papers that the capacity of a multiple antenna communication system is much better than a single antenna system. Analysis in [2] shows that as long as the number of receive antennas is greater than or equal to the number of transmit antennas, the capacity of multiple antenna systems grows linearly with the number of transmit antennas. Due to the above mentioned benefits the use of multiple antennas in communication systems, working under the constraints of severe fading, is becoming increasingly popular. Cellular service providers use multiple antennas most often to combat fading.

Space-time block codes present a means of introducing transmit diversity into the base station with performance results similar to the receive diversity techniques. With the introduction of space-time block coding the base station can provide reliable communication not only from the mobile user to the base station (receive diversity) but also from the base station to the mobile user (transmit diversity).

We cannot typically use the receive diversity techniques for the mobile station because it has size constraints and power requirements, so it may not be practical to deploy more than a single antenna on it. Furthermore, even if we use multiple antennas on the receiver, we may not get enough separation between the antennas for an effective diversity advantage. Space-time codes provide the communication system engineer effective means to bypass this difficulty, so normally in Space-time codes, transmit diversity is used.

1.1 Problem Statement

A simple orthogonal space-time block code was proposed by Alamouti for wireless communications systems with a very simple linear maximum-likelihood (ML) decoding technique[4]. It was assumed that the channel is invariant over at least two consecutive symbol intervals or the channel is quasi-static (i.e. the channel gains are invariant over the entire signaling frame but may change from one frame to the other frame). Tarokh provided the designs of orthogonal space-time block codes (O-STBC) [5]. O-STBC presented still depend upon the availability of accurate channel state information (CSI) at the receiver. It is shown in [4], [6] and [7] that the O-STBC offer full diversity advantage when the channel is quasi static over a code block.

The differential space-time codes [8], [9] can avoid the need for channel estimation but they still required that the channel is being invariant over a number of symbols. Differential space-time codes are complicated and efforts have been made to simplify the decoding of space-time codes and remove the computational complexity [10], [11]. These works, however, still rely on channel being invariant within a code block. This thesis presents a new differential coding scheme for space time block codes and the quasi-static assumption on space-time block coded communication systems is considered with another assumption that system is synchronous. The data rate versus performance trade-off for a space-time block coded receiver is discussed. To improve the performance of space-time block coded system inverse channel detection scheme is used. Inverse channel scheme is used for interference cancellation. This detection scheme provides tremendous improvements in the results over the existing conventional detector.

1.2 Previous Research and Contributions

Transmit diversity techniques received extensive attention after the introduction of space-time codes. Tarokh introduced space-time trellis coding as a transmit diversity technique.[16] This scheme presents a joint design for transmit diversity, channel coding and modulation. Space-time trellis codes were shown to have good performance but at the cost of higher complexity. For a fixed number of transmit antennas, the decoding complexity of space-time trellis codes was shown to grow exponentially as a function of bandwidth efficiency and the diversity level.

Alamouti introduced a two transmit antenna scheme which utilized very simple transmit and receive processing while still achieving full diversity order without any bandwidth expansion [4]. Further work on this scheme was done by Tarokh [5][6]. The two-antenna scheme introduced by Alamouti was extended to more than two antennas. Tarokh presented the general theory for space-time block code construction [5]. It was shown that for real signals, space-time block codes could be designed to achieve full diversity at the maximum possible transmission rate. For complex constellation signaling, new codes were designed that provided full diversity at half and three-fourths of the maximum transmission rate. Tarokh presented simulation results for the error performance of the newly proposed space-time block codes at different bandwidth efficiencies [6]. Equal rate is considered for all users in the above said work.

1.3 Outline of Thesis

Chapter 2 introduces the concepts of multi path propagation phenomena and then presents different statistical models used for multi path fading channels. It then continues with the discussion of the role of diversity and the different types of diversity combining techniques. Chapter 3 starts with a discussion of general form of a space-time coded system. It then presents simplest scheme by which we can achieve orthogonal space-time block coded system as a case study. The simulation results showing its BER performance in flat fading channel are presented. It is shown how orthogonality and the quasi-static assumption lead to simplified processing at the receiver.

Chapter 4 discusses the implementation of proposed scheme and the block diagram of the system. Chapter 5 describes proposed scheme with other different modulation schemes and presents simulation results showing its BER performance in flat fading channel and synchronous transmission. In the chapter 6, the future enhancements and new ideas for research are presented. It also provides a conclusion of this work.

Chapter 2

MULTIPATH FADING CHANNELS

The signal received at the receiver, contains original signal plus duplicate wave forms that results from reflection of the waves off obstacles. In telecommunications, multipath is defined as the RF signals reaching at the destination by two or more paths. Number of causes for multipath in wireless communication such as ionospheric reflection and refraction, atmospheric ducting, reflection from mountains and high rise buildings. [7] In wireless propagation channel obstacles and reflector are always present, which affect the direction of arrival of signals, not only direct path but the multiplicity of paths. It is a pure probabilistic phenomenon for direct waves and reflected waves. Each wave has its own time delay and degree of attenuation. There are two basic factor of Multipathing.

- **Line-of-Sight:** LOS is a condition where a signal travels over the air directly from a wireless transmitter to a wireless receiver without passing any barrier or obstacles. The transmitter and the receiver are connected directly.
- **Non-Line-of-Sight:** NLOS is a condition where a signal from a wireless transmitter passes a number of obstructions before reaching at a wireless receiver. The signal could be reflected, refracted, scattered and there is a new path due to reflection.

The figure 2.1 shows the difference between LOS and NLOS.

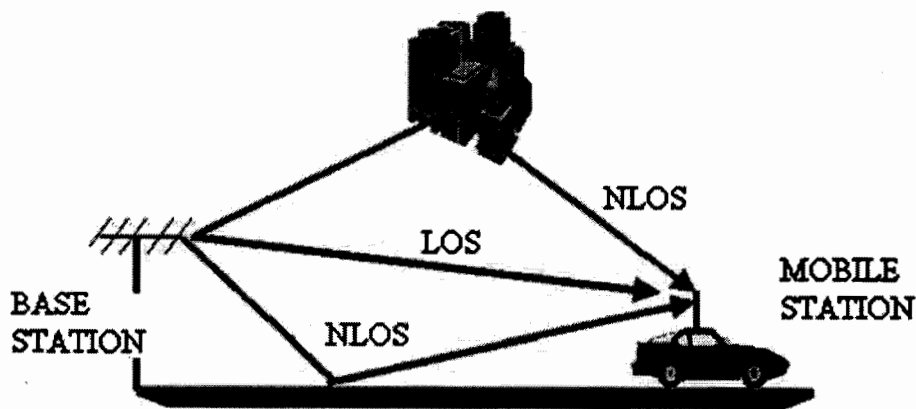


Figure 2.1: Effect of multipath on a mobile station.

The figure 2.1 shows that a signal is transmitted from the base station and reached at the receiver (mobile station) via three different paths. There is a direct path between transmitter and receiver which is mentioned in the figure by arrow head i-e LOS. Instead of LOS, the signal reflected from high rise

building and then reached at the receiver and in the second case signal reflected from the road and then reached to the destination. These two are the examples of NLOS.

2.1 Multipath Interference

Multipath interference is defined as the interference of two or more components of a signal due to multipathing. The components of the waves remain coherent under a necessary condition for Multipath Interference for a time slot. Mainly interference occurs when different components of the wave arise at the receiver with time delay and out of phase with each other.

2.2 Multipath Delay Spread

The multipath delay spread is an important indicator which indicates the performance capabilities of any existing systems. Multipath delay spread occurs when the transmitted signal reached at the receiver through multipath at different times. Multipath delay spread can result in intersymbol interference, the intermingle of bits into one another. Multipath delay spread can be measured by the root mean square (rms) method. For a reliable communication without using adaptive equalization or other anti-multipath techniques, the transmitted data rate should be much smaller than the inverse of the rms delay spread (called coherence bandwidth). There are two types of channels

- 1) - **Narrowband Channel:** The narrowband channel is such a channel in which the transmitted data rate is much smaller than the coherence bandwidth of the channel. This type of wireless channel is also referred as the flat channel.
- 2) - **Wideband Channel:** The wideband channel is defined as the channel in which the transmitted data is closely equal to or larger than the coherence bandwidth of the channel. Another name of this channel is frequency-selective channel.

2.3 Multipath Fading

Fading is defined as the loss of power of signal due to multipathing as well as the change of medium. Such type of fading which is occurred due to multipath is called multipath fading. Fading results in the distortion of the signal. Therefore the signal loses its power due to multipath propagation. This type of fading which occur due to multipath is also called Short-term fading. Fading results from the superposition of transmitted signals that have experienced differences in attenuation, delay and phase shift while traveling from the source to the receiver. It may also be caused by attenuation of a single signal. The destructive nature of multiple interference results in the loss of power of signal due to multiple copies of the signal reached at the receiver from different paths. An easy approach for this

problem is that gathered these signals at the receiver end by minimizing the time delay and make sure that these signals are coherently combined. This target is achieved by applying diversity techniques with the help of multiple antennas. In order to make signal-to-noise ratio a random quantity, the best mathematical approach is to multiplication of the signal with a coefficient which is a random variable and time dependant. Fading channel models are always designed to portray electromagnetic transmission of information over wireless media. These models are applied on cellular phones and used in broadcast communications. However, even for underwater acoustic communications, the notion of fading is useful in understanding the distortion caused by the medium.

There is another type of fading which is called Small-scale fading. It is further divided into two categories i-e multipath time delay spread and Doppler spread. There are two types of fading based on multipath time delay spread:

- **Flat Fading:** Flat fading is that type of fading in which all frequency components of the signal will experience the same magnitude of fading. In flat fading the bandwidth of the signal is less than the coherence bandwidth of the channel or the delay spread is less than the symbol period.
- **Frequency Selective Fading:** Frequency selective fading is that type of fading in which different frequency components of the signal will experience different magnitude of fading. In frequency selective fading the bandwidth of the signal is greater than the coherence bandwidth of the channel or the delay spread is greater than the symbol period.

There are two types of fading on Doppler spread.

- **Fast Fading:** The fading in which the amplitude and phase change imposed by the channel varies considerably over the period of use is called fast fading. Fast fading has a high Doppler spread. The coherence time is less than the symbol period and the channel variations are faster than base band signal variations.
- **Slow Fading:** Slow fading is defined as, when the amplitude and phase change imposed by the channel can be considered roughly constant over the period of use. It has a low Doppler spread. The coherence time is greater than the symbol period and the channel variations are slower than the base band signal variations.

Despite of small scale fading there is another type of fading, shadow fading, in which the change in the signal strength occurs especially in mobile phone frequencies, on the order of a fraction of a meter. This phenomenon is called shadowing. The presence of obstacles and high rise buildings in the line-of-sight of transmitter and receiver causes shadowing. [8]

2.4 Doppler Shift

- A signal transmitted from the transmitter and received by the receiver which is moving relative to the source of the waves and than the change in frequency and wavelength of the wave is called Doppler

Effect. The total Doppler Effect is depending upon the motion of both the transmitter and the receiver. The sound waves that propagate in a wave medium, the velocity of the observer and of the source is suppose relative to the medium in which the waves are transmitted. The waves which require a medium for there traveling, the received frequency f' and emitted frequency f can related according to Doppler effect as

$$f' = \left(\frac{v}{v \pm v_s} \right) f \quad (2.1)$$

Where

v is the speed of waves in the medium

v_s is the velocity of the source

The waves traveling at the speed of light exhibits the following relationship between observed frequency f' and emitted frequency f as

Change in Frequency

$$\Delta f = \frac{fv}{c} = \frac{v}{\lambda} \quad (2.2)$$

Observed Frequency

$$f' = f + \frac{fv}{c} \quad (2.3)$$

Where

f is the transmitted frequency

v is the velocity of the transmitter relative to the receiver in meters/second. This is positive when moving towards one another and negative when moving away

c is the speed of light in a vacuum 3.10^8 m/s

λ is the wavelength of the transmitted wave subject to change [4-6]

2.5 Statistical Modeling of Fading Channels

The constructive and destructive combination of randomly delayed, reflected, scattered and bounced back component of a signal is responsible for Multipath fading. Multipath fading is therefore responsible for the short term signal variations because it is relatively fast. In order to describe the statistical behavior of the multipath fading envelope, there are different models. These models depending upon the nature of the radio propagation environment.

2.5.1 Raleigh Fading Channels

Raleigh fading is because of the consequences of multipath, include constructive and destructive interference and phase shifting of the signal. Raleigh fading is a basic analysis for the effect of a propagation environment on a radio signal, especially used by wireless devices. It assumes that the magnitude of a signal that has passed through such a transmission medium will vary randomly or fade according to a Raleigh distribution. The Rayleigh distribution is a continuous probability distribution and distribution model is made of the assumption that the real and imaginary parts of the response are distributed independently and identically (zero-mean Gaussian processes) so that the amplitude of the response is the sum of two such processes. By taking the modulus of such complex number a Rayleigh-distribution can achieve.

It is a reasonable model for troposphere and ionospheric signal propagation as well as the effect of high rise buildings on radio signals. Raleigh fading is most appropriate when there is no line of sight (NLOS) between the transmitter and receiver. Raleigh is a practical model when there are many objects in the environment that scatter the radio signal before it arrives at the receiver i-e multipathing. If there are many paths due to scattering and reflection the channel impulse response will be look like a Gaussian process irrespective of the distribution of the individual components according to the central limit theorem. If the ratio of a signal component to the scattering is very low, then such the random process will have zero mean. The phase of this random process is uniformly distributed between 0 and 2π radians. The envelope of the channel response will therefore be Raleigh distributed and it will have a pdf

$$P_R(r) = \frac{2r}{\Omega} e^{-r^2/\Omega}, r \geq 0 \quad (2.4)$$

where r is random variable and Ω is the expected value of R^2

2.5.2 Rician Fading Channel

Raleigh fading will become the Rician fading if there is strong line of sight content between transmitter and receiver. Rician fading is usually occurred due to partial cancellation of a radio signal by itself. There are two paths for a signal in its whole traveling towards the receiver and these two paths are different to each other. Rician fading occurs when one of the paths, typically a line of sight signal, is much stronger than the others because at least one of the paths is changing. In Rician fading, the amplitude gain is described by a Rician distribution. The Rician distribution has probability density function as follows

$$f(x|\nu, \sigma) = \frac{x}{\sigma^2} \exp\left(\frac{-(x^2 + \nu^2)}{2\sigma^2}\right) I_0\left(\frac{x\nu}{\sigma^2}\right) \quad (2.5)$$

where I_0 is the Bessel function with order zero. When $\nu = 0$, the distribution reduces to a Raleigh distribution.

2.6 Diversity Scheme

Diversity scheme refers to a technique which utilizing two or more communication channels with different properties. The diversity technique requires multiple replicas of the transmitted signals at the receiver, all carrying the same information but with small correlation in fading statistics. It is based on the fact that individual channels experience different levels of fading and interference. Multiple copies of the same signal may be transmitted and received and combined in the receiver. Alternatively, a redundant forward error correction code may be added and different parts of the message transmitted over different channels. Diversity techniques can minimize the multipath propagation. A diversity gain is measured in decibel.

Wireless communication channels are used a number of diversity methods to get the required performance. The following classes of diversity schemes can be applied

2.6.1 Time Diversity

Multiple copies of the same signal are transmitted at different time slots. Alternatively, a redundant forward error correction code is added and the message is spread in time by means of bit-interleaving before it is transmitted. Hence error bursts are avoided, which simplifies the error correction. The required time separation is at least the Coherence time of the channel, or the reciprocal of the fading rate $1/f_d = c/vf_c$

2.6.2 Frequency Diversity

In case of frequency selective fading channels the signal is transferred using several frequency channels or spread over a wide spectrum that is affected by frequency selective fading. In frequency diversity, the space between the carriers is more than the coherence bandwidth.

2.6.3 Space Diversity

Space diversity can be applied in such a scenario, when a signal is transmitted from the transceiver and due to multipath propagation then signal is transmitted over several different propagation paths. Wired networks achieve space diversity by transmitting via multiple wires but in the case of wireless transmission, it can be achieved by antenna diversity using multiple transmitter antennas. It is also called the transmit diversity. There are different types of transmit diversity as given below

- If the antennas are at far distance then it is called **macro diversity**.
- If the antennas are at a distance in the order of one wavelength, this is called **micro diversity**.
- A special case is **phased antenna arrays**, which also can be utilized for beam forming.

2.6.4 Polarization Diversity

Polarization diversity can be applied by placing two different polarized antennas at transmitter end as well as at the receiver end. Orthogonally polarized waves with independent fading characteristics can be used as a source of diversity. In urban environment, where space is limited, polarization diversity is particularly advantageous because we can place more than one antenna together.

2.7 Diversity Combining Methods

Diversity combining is the technique applied to combine the multiple received signals of a diversity reception device into a single improved signal. The main advantage of diversity combining method is to reduce one possible single-point failure, i.e. any single receiver failure or local interference to a single receiver. It will not block reception of the entire system. Generally, the performance of communication systems, employing diversity techniques, highly depends upon the diversity combining method used at the receiver to maximize the overall received signal-to-noise ratio (SNR). Various diversity combining techniques are given on the next page.

2.7.1 Selection Combining Method:

Selection combining technique is applied by placing multiple receive antennas at large enough distance so that fading is independent in each receive antenna. The signal, which has the largest signal to noise ratio at every symbol interval, is selected as the output from all the receive antenna for detection.

2.7.2 Maximal Combining

Maximal-ratio combining is a method of diversity combining which performs three functions.

- the signals from each channel are added together
- the gain of each channel is made proportional to the rms signal level and inversely proportional to the mean square noise level in that channel
- different proportionality constants are used for each channel.

It is also known as ratio-squared combining and predetection combining. Maximal-ratio-combining is the optimum combiner for independent AWGN channels. The output signal from the combiner is a linear combination of a weighted replica of all the received signals. It is given by

$$r = \sum_{i=1}^{n_R} \partial_i \cdot r_i \quad (2.6)$$

where r_i is the received signal at receive antenna i and ∂_i is the weighing factor for receive antenna i and is dependent upon the channel estimates in maximum ratio combining.

2.7.3 Equal Gain Combining

Equal gain combining (EGC) is a simple linear combining method. EGC applies equal weight to the receiver channels. The weights do not depend on the channel estimates. Signal from each receive antenna are multiplied by the same weight so as to give a lower SNR performance when compared to MRC. Even though the performance for CGC is lower than for MRC, no channel estimate needs to be done in EGC.

Chapter 3

MULTIPLE INPUT MULTIPLE OUTPUT COMMUNICATIONS

Communication system consists of three major parts transmitter, channel and receiver. There are different types of communication system according to transmitter and receiver and these types are given below:

- **SISO** or single-input single-output, a system which has only single antenna both at transmitter and receiver.
- **SIMO** or single-input multiple-output, when the transmitter has a single antenna but there are multiple antennas at receiver end.
- **MISO** or multiple-input single-output, when the receiver has a single antenna but transmitter has multiple antennas.
- **MIMO** or Multiple-input multiple-output, such a technique in which multiple antennas both at the transmitter and receiver are used.

The wireless communications employing MIMO system technology has got enormous attention in the last decades because it offers significantly higher data rates and attractive BER, SNR and link range without requiring additional power and bandwidth. These systems also achieve higher spectral efficiency (i-e more no. of bits per second, per Hertz of bandwidth) and link reliability. Therefore, the upcoming 4G systems will include MIMO technology. MIMO technology is also named as Smart antennas, which performs spatial information processing with multiple antennas

3.1.1 Different Categories

MIMO can be divided into three main categories, Precoding, Spatial multiplexing, or SM, and Diversity Coding.

- **Precoding:** It is basically the beam forming applied on multi-layers. Commonly, in beam forming, multiple antennas are placed at the transmitter end and the same signal is transmitted from each of the transmit antennas with appropriate phase so that the signal power is maximized at the receiver output. In order to increase the signal gain from constructive combining and to reduce the multipath fading effect, beam forming is a good choice and, in the

absence of scattering, beam forming results in a well defined directional pattern but in typical cellular conventional systems, beam forming is not a good correspondence. When the receiver has multiple antennas, the transmit beam forming cannot simultaneously maximize the signal level at all of the receive antenna and it requires precoding. The necessary condition for proceeding is that it requires prior knowledge of the channel state information (CSI) at the transmitter.

- **Spatial Multiplexing:** In spatial multiplexing, a high rate signal stream is sub-divided into multiple lower rate streams. Each stream has the same frequency channel but transmitted from a different transmit antenna. The receiver can distinguish between these streams if these streams arrive at the receiver antenna with sufficiently different spatial signatures. The receiver also can create parallel channels for free. Spatial multiplexing is very powerful technique for increasing channel capacity and making higher Signal to Noise Ratio (SNR). The maximum number of spatial streams is limited by the lesser in the number of antennas at the transmitter or receiver. Spatial multiplexing can be use with or without transmit channel knowledge but it requires MIMO antenna configuration
- **Diversity Coding:** In diversity coding scheme, the coding is required and the signal is coded using techniques called Space-time coding. As compared with spatial multiplexing in which multiple streams are transmitted, a single stream is transmitted. The signal is transmitted from each of the transmit antennas using certain principle of full or near orthogonal coding. Diversity, minimizes the multipath independent fading in the multiple antenna networks to enhance signal reliability. These techniques are used when there is no prior channel knowledge at the transmitter and because, there is no channel knowledge, there is no beam forming or array gain from diversity coding. There are also some merger of spatial multiplexing and precoding when the channel is known at the transmitter. The merger of spatial multiplexing with diversity coding when decoding reliability is in trade-off.

3.1.2 MIMO Limitation

1. The physical antenna spacing are selected to be large enough
2. Multiple wavelengths at the base station.
3. The antenna separation at the receiver is heavily space constrained in hand sets, though at least 0.3 wavelengths are needed.

3.1.3 Application of MIMO

Spatial multiplexing techniques makes the receivers very complex, so it can not be operated exclusively. Therefore it is typically combined with orthogonal frequency-division multiplexing (OFDM) or with Orthogonal Frequency Division Multiple Access (OFDMA) modulation, where the troubles created by multipath channel are removed competently. The IEEE 802.16e standard includes MIMO-OFDMA. The IEEE 802.11n standard, which is estimated to be firm up soon, propose MIMO-OFDM. MIMO is also intended to be used in Mobile radio telephone standards such as recent 3G and 3.5G standards. In 3G, High-Speed Packet Access plus (HSPA+) and Long Term Evolution (LTE) standards take MIMO into account. Moreover, to fully support cellular environments MIMO research consortiums including IST-MASCOT, propose to developed advanced MIMO communication techniques such as cross-layer MIMO, multi-user MIMO and ad-hoc MIMO. These techniques are explained as below:

- **Cross-layer MIMO:** Cross-layer techniques improve the performance of MIMO networks by solving cross-layer problems. These problems usually occur when the MIMO configuration is working in the system. Cross-layer techniques have been enhancing the performance of SISO links as well. Examples of cross-layer techniques are joint source-channel coding, link adaptation, or adaptive modulation and coding (AMC).
- **Multi-user MIMO:** Multi user MIMO, as compared to the conventional or single-user MIMO, which uses only the multiple antenna dimensions, multi-user MIMO can make use of multiple user interference powers as a spatial resource at the cost of advanced transmit processing. Examples of advanced transmit processing for multi-user MIMO are interference conscious precoding and SDMA-based user arrangement.
- **Adhoc MIMO:** It is a technique useful for future cellular networks which considers wireless mesh networking or wireless adhoc networking. In wireless adhoc networks, multiple transmit nodes communicate with multiple receive nodes. In order to optimize the capacity of adhoc channels, MIMO concept and techniques can be applied to multiple links between the transmit and receive node clusters. Unlike multiple antennas at the single-user MIMO transceiver, multiple nodes are located as a distributed manner. In order to achieve the capacity of this network, techniques to manage distributed radio resources are essential such as node cooperation and dirty paper coding (DPC) concept based network coding.

3.1.4 Mathematical Description

In MIMO systems, a transmitter sends multiple streams by multiple transmit antennas. The transmit streams go through a matrix channel which consists of multiple paths between multiple transmit antennas at the transmitter and multiple receive antennas at the receiver. Then, the receiver gets the

received signal vectors by the multiple receive antennas and decodes the received single vectors into the original information. Mathematically a MIMO system can be expressed as follow:

$$s = Hr + n \quad (3.1)$$

where s is the receive vector and r is the transmit vector. H is the channel matrix and n is the noise vector [2]

3.2 Transmit Diversity

Transmit diversity is radio communication using signals that originate from two or more independent sources that have been modulated with identical information bearing signals and that may vary in their transmission characteristics at any given instant. Diversity Coding is the spatial coding techniques for a MIMO system in wireless channels. Wireless channels harshly undergo from fading phenomena, which responsible for unreliability in data decoding. Basically, in order to improve the reliability of the data reception, diversity coding sends multiple copies through multiple transmit antennas. If one of the copies is failed to receive, other copies are used as a candidate for the data decoding. Transmit diversity can help to overcome upon the effects of fading, outages, and circuit failures. When using diversity techniques, both at the transmission and reception ends, the improvement of received signal depends on the two factors which are given below:

- the independence of the fading characteristics of the signal
- circuit outages and failures

3.2.1 Antenna Diversity

Antenna diversity involves the use of multiple antennas to receive multiple instances of the same signal and then make use of the otherwise redundant data contained within these signals. This allows the system to be more robust against the many factors that degrade signal reliability. A single antenna may not be able to receive a signal for several reasons and it could be inappropriate. The antenna may be oriented in the wrong direction or the signal could be blocked by obstacles. In all these cases a second antenna would clearly improve the probability of receiving the signal. The use of multiple antennas is not a new idea and most cellular base stations employ antenna diversity techniques. There are two main reasons for employing antenna diversity techniques which are as follow:

1. Placing of very small multiple antennas, like a handset, is indeed difficult.
2. The capabilities of such a small antenna, like the handset antenna, will be degraded by the presence of inter antenna coupling, cross-correlation envelope and coupling due to biological tissue in the user's head and hand. Antenna diversity techniques generally fall into four categories.

1. **Spatial Diversity** involves the use of physically separated identical antennas. The phase centre of each antenna is also spatially separated.
2. **Pattern Diversity or beam diversity** uses co-located antennas. These antennas are of different size, shape, orientation and material. These antennas have dissimilar radiation patterns and their signals are combined in phase due to their collocation.
3. **Polarization Diversity** uses two antennas oriented at 90° to each other. The result is mutually orthogonal polarization states, such as horizontal and vertical, left-hand circular and right-hand circular or $\pm 45^\circ$ slants. The antennas used in polarization diversity schemes are often identical.
4. **Transmit Diversity and receive diversity** schemes employ separate antennas for transmit and receive functions, so frequency filtering is not needed.

Antenna diversity can also be introduced by manipulating the way that outputs from multiple antennas are processed. Designers can choose a simple combination of signals from both antennas and a switched selection, which chooses the antenna with the best signal-to-noise ratio (SNR).

To illustrate the antenna-diversity design process, it is useful to consider a generic system. A dual diversity system could process two input signals, $r_1(t)$ and $r_2(t)$, to create an improved signal, $r_c(t)$. This would reduce fading and co-channel interference. The signal strength depends on the cross-correlation. Cross-correlation is a statistical value which has the following two factors

1. The level of similarity between the voltages received at the two antennas.
2. Relative strengths of two received signals.

p_e (envelope cross-correlation coefficient) is another important performance indicator. p_e is proportional to the square of the complex cross-correlation. In general, good diversity gain is possible when the p_e is less than 0.5. It is also possible to determine the performance of diversity antennas by looking at the radiation patterns.

3.3 Space-Time Coding

The use of multiple antennas at both transmits and receives results diversity in a multiple-input multiple-output (MIMO) system. The use of diversity techniques at both ends of the link is termed space-time coding. A space time code (STC) is a method employed to improve the reliability of data transmission in wireless communication systems using multiple transmit antennas. Space time code depends on transmitting multiple, redundant copies of a data stream to the receiver and there is probability that, at least some of them, may remain constant in the physical path between transmission and reception in a good enough state to allow reliable decoding. Space time codes may be split into two main types:

Space Time Trellis Codes (STTCs): Space time trellis codes (STTCs) are a type of space time code used in multiple antenna wireless communications. It distribute a trellis code over multiple antennas and multiple time slots and provide both coding gain and diversity gain. This scheme transmits multiple, redundant copies of a trellis (or convolution) code distributed over time and a number of antennas. These multiple, diverse copies of the data are used by the receiver to attempt to reconstruct the actual transmitted data. The necessary condition for space time code is that there must be multiple, but only a single receive antennas is required. Although multiple receive antennas are often used since the performance of the system is improved by so doing. In contrast to space-time block codes (STBCs), they are able to provide both coding gain and diversity gain and have a better bit-error rate performance. In the context of encoding and decoding trellis codes are more complex than STBCs. They depend on Viterbi coder at the receiver where STBCs need only linear processing.

Space Time Block Codes (STBCs) are just like blocks codes process a block of data at once. They provides only diversity gain and fewer complexes in implementation terms than STTCs.

STC may be further subdivided according to whether the receiver knows the channel impairments. In coherent STC, the receiver knows the channel impairments through training or some other form of estimation. These codes have been studied more widely because they are less complex than their non-coherent counterparts. In non coherent STC the receiver does not know the channel impairments but knows the statistics of the channel. In differential space-time codes neither the channel nor the statistics of the channel are available [11]

3.4 Space Time Block Code

Space-time block coding is a very popular coding technique used in telecommunications to transmit multiple copies of a data stream over a number of antennas and to take advantage of the various received copies of the data to get better the consistency of data-transfer. The fact, that transmitted signal face a absolutely difficult environment with scattering, reflection, refraction and so on as well as be corrupted by thermal noise in the receiver means that some of the received copies of the data will be better than others. This redundancy increase the chance of being able to use one or more of the received copies of the data to correctly decode the received signal. In fact, space-time coding combines all the copies of the received signal in an optimal way to extract as much information from each of them as possible.

Most work on wireless communications had focused on having an antenna array at only one end of the wireless link usually at the receiver. Seminal papers by Gerard J. Foschini and Michael J. Gans [1], Foschini [2] and Emre Telatar [3] enlarged the scope of wireless communication possibilities by showing that for the highly scattering environment substantial capacity gains are enabled when antenna arrays are used at both ends of a link. An alternative approach to utilizing multiple antennas

relies on having multiple transmit antennas and only optionally multiple receive antennas. Proposed by Vahid Tarokh, Nambi Seshadri and Robert Calderbank, these space time codes [4] (STCs) achieve significant error rate improvements over single-antenna systems. Their original scheme was based on trellis codes but the simpler block codes were utilized by Siavash Alamouti, and later Vahid Tarokh, Hamid Jafarkhani and Robert Calderbank to develop space time block codes (STBCs). STC involves the transmission of multiple redundant copies of data to compensate for fading and thermal noise in the hope that some of them may arrive at the receiver in a better state than others. In the case of STBC in particular, the data stream to be transmitted is encoded in blocks, which are distributed among spaced antennas and across time. While it is necessary to have multiple transmit antennas, it is not necessary to have multiple receive antennas, although to do so improves performance. This process of receiving diverse copies of the data is known as diversity reception and is what was largely studied until Foschini's 1998 paper.

An STBC is usually represented by a matrix. Each row represents a time slot and each column represents one antenna's transmissions over time.

$$\begin{bmatrix} y_{11} & y_{12} & \dots & y_{1nr} \\ y_{21} & y_{22} & \dots & y_{2nr} \\ \vdots & \vdots & \dots & \vdots \\ y_{\tau 1} & y_{\tau 2} & \dots & y_{\tau nr} \end{bmatrix} \quad (3.2)$$

Here, y_{ij} is the modulated symbol to be transmitted in time slot i from antenna j . There are to be T time slots and nr transmits antennas as well as nr receives antennas. This block is usually considered to be of 'length' T . The code rate of an STBC is defined as the measures of symbols per time slot, it transmits on average over the entire course of one block. If a block encodes k symbols, the code-rate is

$$x = \frac{k}{T} \quad (3.3)$$

Only one standard STBC can achieve full-rate which is Alamouti's code.

3.4.1 Orthogonality

STBCs as originally introduced and as usually studied are orthogonal. This means that the STBC is designed such that the vectors representing any pair of columns taken from the coding matrix are orthogonal. The result of this is simple, linear and optimal decoding at the receiver. Its most serious drawback is that all but one of the codes that satisfy this criterion must sacrifice some proportion of their data rate. There are also 'quasi-orthogonal STBCs' that allow some inter-symbol interference but can achieve a higher data rate and even a better error-rate performance in harsh conditions

Quasi Orthogonal STBC

These codes exhibit partial orthogonality and provide only part of the diversity gain mentioned above. An example reported by Hamid Jafarkhani is

$$C_{4,1} = \begin{bmatrix} y_1 & y_2 & y_3 & y_4 \\ -y_2^* & y_1^* & -y_4^* & y_3^* \\ -y_3^* & -y_4^* & y_1^* & y_2^* \\ y_4 & -y_3 & -y_2 & y_1 \end{bmatrix} \quad (3.4)$$

The orthogonality criterion only holds for columns (1 and 2), (1 and 3), (2 and 4) and (3 and 4). Crucially, however, the code is full-rate and still only requires linear processing at the receiver, although decoding is slightly more complex than for orthogonal STBCs. Results show that this Q-STBC outperforms (in a bit-error rate sense) the fully-orthogonal 4-antenna STBC over a good range of signal-to-noise ratios (SNRs). At high SNRs, though (above about 22dB in this particular case), the increased diversity offered by orthogonal STBCs yields a better BER. Beyond this point, the relative merits of the schemes have to be considered in terms of useful data through

3.4.2 Design of STBC

The design of STBCs is based on the so called diversity criterion derived by Tarokh et. al in their earlier paper on space-time trellis codes. Orthogonal STBCs can be shown to achieve the maximum diversity allowed by this criterion.

3.4.3 Diversity Criterion

Call a codeword

$$c = c_1^1 c_1^2 \dots c_1^{n_T} c_2^1 c_2^2 \dots c_2^{n_T} \dots c_T^1 c_T^2 \dots c_T^{n_T} \quad (3.5)$$

and call an erroneously decoded received codeword

$$\varepsilon = e_1^1 e_1^2 \dots e_1^{n_T} e_2^1 e_2^2 \dots e_2^{n_T} \dots e_T^1 e_T^2 \dots e_T^{n_T} \quad (3.6)$$

Then the matrix

$$B(c, \varepsilon) = \begin{bmatrix} e_1^1 - c_1^1 & e_2^1 - c_2^1 & \dots & e_T^1 - c_T^1 \\ e_1^2 - c_1^2 & e_2^2 - c_2^2 & \dots & e_T^2 - c_T^2 \\ \vdots & \vdots & \vdots & \vdots \\ e_1^{n_T} - c_1^{n_T} & e_2^{n_T} - c_2^{n_T} & \dots & e_T^{n_T} - c_T^{n_T} \end{bmatrix} \quad (3.7)$$

has to be full-rank for any pair of distinct code words c and e to give the maximum possible diversity order of $nTnR$. If instead, $B(c, e)$ has minimum rank b over the set of pairs of distinct code words, then the space-time code offers diversity order bnR . An examination of the example STBCs shown below reveals that they all satisfy this criterion for maximum diversity. STBCs offer only diversity gain (compared to single-antenna schemes) and not coding gain. There is no coding scheme included here the redundancy purely provides diversity in space and time. This is contrast with space-time trellis codes which provide both diversity and coding gain since they spread a conventional trellis code over space and time.

3.4.4 Encoding (Alamouti's Code)

Alamouti scheme is the simplest of all the STBCs in 1998, although he did not create the term "space-time block code" himself. It was originally designed for a two-transmit antenna system and has the coding matrix:

$$C = \begin{bmatrix} y_1 & y_2 \\ -y_2^* & y_1^* \end{bmatrix} \quad (3.8)$$

where $*$ denotes complex conjugate. The signals are transmitted in two consecutive time intervals. In first time slot t_1 signals y_1 and y_2 are transmitted from antenna one and two and in the time slot $t + \Delta t$, $-y_2^*$ is transmitted from antenna one and y_1^* from antenna two. Encoding is done in time domain as well as in space domain. The transmit pattern from antenna one and two is

$$c_1 = [y_1 \quad -y_2^*] \quad (3.9)$$

$$c_2 = [y_2 \quad y_1^*] \quad (3.10)$$

As, it is obvious that inner product of two sequence is zero so the two transmit sequence are orthogonal to each other

$$c_1 \cdot c_2 = y_1 y_2^* - y_2^* y_1 = 0 \quad (3.11)$$

The code matrix is

$$C \cdot C^H = |y_1|^2 + |y_2|^2 \quad (3.12)$$

It is eagerly evident that this is a rate-1 code. It takes two time-slots to transmit two symbols. Using the optimal decoding scheme discussed below, the bit-error rate (BER) of this STBC is equivalent to $2nX$ -branch maximal ratio combining (MRC). This is a result of the perfect orthogonality between the symbols after receive processing. There are two copies of each symbol transmitted and nX copies received. This is a very special STBC. It is the only orthogonal STBC that achieves rate-1. That is to

say that it is the only STBC that can achieve its full diversity gain without needing to sacrifice its data rate. Strictly, this is only true for complex modulation symbols. Since almost all constellation diagrams rely on complex numbers however, this property usually gives Alamouti's code a significant advantage over the higher-order STBCs even though they achieve a better error-rate performance.

Higher Order STBC

Tarokh et al. discovered a set of STBCs that are particularly straightforward, and coined the scheme's name. They also proved that no code for more than 2 transmit antennas could achieve full-rate. Their codes have since been improved upon (both by the original authors and by many others). Nevertheless, they serve as clear examples of why the rate cannot reach 1, and what other problems must be solved to produce 'good' STBCs. They also demonstrated the simple, linear decoding scheme that goes with their codes under perfect channel state information assumption.

3 Transmit Antennas

Two straightforward codes for 3 transmit antennas are:

$$C_{3,1/2} = \begin{bmatrix} y_1 & y_2 & y_3 \\ -y_2 & y_1 & y_4 \\ -y_3 & y_4 & y_1 \\ -y_4 & -y_3 & y_2 \\ y_1^* & y_2^* & y_3^* \\ -y_2^* & y_1^* & y_4^* \\ -y_3^* & y_4^* & y_1^* \\ -y_4^* & -y_3^* & y_2^* \end{bmatrix} \quad (3.13)$$

and

$$C_{3,3/4} = \begin{bmatrix} y_1 & y_2 & \frac{y_3}{\sqrt{2}} \\ -y_2^* & y_1^* & \frac{y_3}{\sqrt{2}} \\ \frac{y_3^*}{\sqrt{2}} & \frac{y_3^*}{\sqrt{2}} & \frac{(-y_1 - y_1^* + y_2 - y_2^*)}{2} \\ \frac{y_3^*}{\sqrt{2}} & \frac{-y_3^*}{\sqrt{2}} & \frac{(y_2 + y_2^* + y_1 - y_1^*)}{2} \end{bmatrix} \quad (3.14)$$

These codes achieve rate-1/2 and rate-3/4 respectively. These two matrices give examples of why codes for more than two antennas must sacrifice rate. It is the only way to achieve orthogonality. One particular problem with $C_{3,3/4}$ is that it has uneven power among the symbols it transmits. This means

that the signal does not have a constant envelope and that the power each antenna must transmit has to vary, both of which are undesirable. Modified versions of this code that overcome this problem have since been designed.

4 Transmit Antennas

Two straightforward codes for 4 transmit antennas are:

$$C_{4,1/2} = \begin{bmatrix} y_1 & y_2 & y_3 & y_4 \\ -y_2 & y_1 & y_4 & y_3 \\ -y_3 & y_4 & y_1 & -y_2 \\ -y_4 & -y_3 & y_2 & y_1 \\ y_1^* & y_2^* & y_3^* & y_4^* \\ -y_2^* & y_1^* & y_4^* & y_3^* \\ -y_3^* & y_4^* & y_1^* & -y_2^* \\ -y_4^* & -y_3^* & y_2^* & y_1^* \end{bmatrix} \quad (3.15)$$

and

$$C_{4,3/4} = \begin{bmatrix} y_1 & y_2 & \frac{y_3}{\sqrt{2}} & \frac{y_3}{\sqrt{2}} \\ -y_2^* & y_1^* & \frac{y_3}{\sqrt{2}} & -\frac{y_3}{\sqrt{2}} \\ \frac{y_3^*}{\sqrt{2}} & \frac{y_3^*}{\sqrt{2}} & \frac{(-y_1 - y_1^* + y_2 - y_2^*)}{2} & \frac{(-y_2 - y_2^* + y_1 - y_1^*)}{2} \\ \frac{y_3^*}{\sqrt{2}} & -\frac{y_3^*}{\sqrt{2}} & \frac{(y_2 + y_2^* + y_1 - y_1^*)}{2} & -\frac{(y_1 + y_1^* + y_2 - y_2^*)}{2} \end{bmatrix} \quad (3.16)$$

These codes achieve rate-1/2 and rate-3/4 respectively, as for their 3-antenna counterparts. $C_{4,3/4}$ exhibits the same uneven power problems as $C_{3,3/4}$. An improved version of $C_{4,3/4}$ is

$$C_{4,3/4} = \begin{bmatrix} y_1 & y_2 & y_3 & 0 \\ -y_2^* & y_1^* & 0 & y_3 \\ -y_3^* & 0 & y_1^* & -y_2 \\ 0 & -y_3^* & y_2^* & y_1 \end{bmatrix} \quad (3.17)$$

which has equal power from all antennas in all time-slots.

3.4.5 Decoding

One particularly attractive feature of orthogonal STBCs is that maximum likelihood decoding can be achieved at the receiver with only linear processing. In order to consider a decoding method, a model

of the wireless communications system is needed. At time t , the signal $x_{\max} = \frac{n_0 + 1}{2n_0}$ received at antenna j is

$$x_t^j = \sum_{i=1}^{n_T} \partial_{ij} y_t^i + n_t^j \quad (3.18)$$

where ∂_{ij} is the path gain from transmit antenna i to receive antenna j and n_t^j is a sample of additive white Gaussian noise (AWGN). The maximum-likelihood detection rule is to form the decision variables

$$X_i = \sum_{t=1}^{n_T} \sum_{j=1}^{n_R} x_t^j \partial_{ij} \delta_t(i) \quad (3.19)$$

where $\delta_t(i)$ is the sign of y_t in the k th row of the coding matrix, $e_k(p) = q$ denotes that $s(p)$ is (up to a sign difference), the (k,q) element of the coding matrix, for $i = 1, 2, \dots, n_T$ and then decide on constellation symbol X_i that satisfies

$$y_i = \arg \min_{s \in A} |X_i - y|^2 + \left(-1 + \sum_{k,l} |\partial_{kl}|^2 \right) |y|^2 \quad (3.20)$$

with A is the constellation alphabet. Despite its appearance, this is a simple, linear decoding scheme that provides maximal diversity.

3.4.6 Rate Limits

Apart from there being no full-rate, complex, orthogonal STBC for more than 2 antennas, it has been further shown that, for more than three antennas, the maximum possible rate is $3/4$ [9]. Codes have been designed which achieve a good proportion of this, but they have very long block-length and are unsuitable for practical use. This is because decoding cannot proceed until all transmissions in a block have been received, so a longer block-length, T results in a longer decoding delay. One particular example, for 16 transmit antennas, has rate- $9/16$ and a block length of 22 880 time-slots.

It has been conjectured, but not proven, that the highest rate any n_T -antenna code can achieve is

$$x_{\max} = \frac{n_0 + 1}{2n_0} \quad (3.21)$$

Where $n_T = 2n_0$ or $n_T = 2n_0 - 1$.

3.4.7 Time Varying Channel

Let us assume now that the channel is not quasi-static. In such a scenario the channel gains $h_j(i)$ will not behave same over entire codeword length i.e. $2T$. Let's suppose that $h_j(i)$ is

constant for only one symbol interval such that.

$$h_1(1) \neq h_1(2) \quad (3.22)$$

$$h_2(1) \neq h_2(2) \quad (3.23)$$

Under these conditions the channel matrix H is given as

$$H = \begin{bmatrix} h_1(1) & h_2(1) \\ h_2^*(2) & -h_1^*(2) \end{bmatrix} \quad (3.24)$$

Now what happens to the orthogonality property of the H matrix

$$H^H H = \begin{bmatrix} h_1^*(1) & h_2(2) \\ h_2^*(1) & -h_1(2) \end{bmatrix} \begin{bmatrix} h_1(1) & h_2(1) \\ h_2^*(2) & -h_1^*(2) \end{bmatrix} \quad (3.25)$$

$$= \begin{bmatrix} |h_1(1)|^2 + |h_2(2)|^2 & h_1^*(1)h_2(1) - h_2(2)h_1^*(2) \\ h_2^*(1)h_1(1) - h_1(2)h_2^*(2) & |h_2(1)|^2 + |h_1(2)|^2 \end{bmatrix} \quad (3.26)$$

That is, H is no more an orthogonal matrix. The orthogonality of the H matrix is destroyed due to the channel variation with time. Figure 3.1 shows that when there is no diversity technique applied BER is very high as compared to the case when there is one antenna at the receiver end and two antennas at the transmitter end. The BER is much better in the case when diversity technique is applied both in transmission and reception.

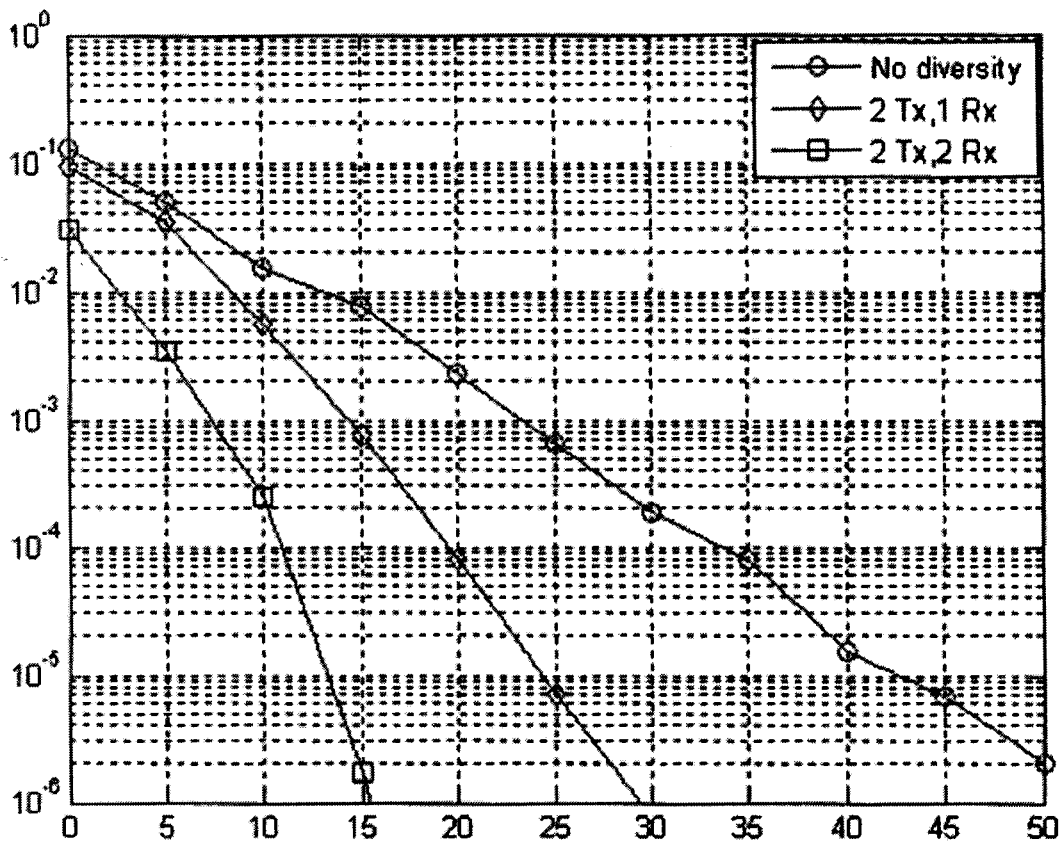


Figure 3.1: BER performance comparison of STBC for two branch diversity)

Chapter 4

SPACE-TIME CODING FOR QPSK/3PSK DIFFERENTIAL MODULATION SCHEME

O-STBC technology has attracted enormous interest over the past few years due to its high diversity order and low decoding complexity at the receiver end. Orthogonal space-time block codes (OSTBC) are an efficient mean in order to exploit the diversity offered by the wireless multiple-input multiple-output. The low decoding complexity of O-STBC results from the linear ML decoder at the receiver. The linear ML decoder, however, relies on the assumption that channel remains constant over the length of entire code word.

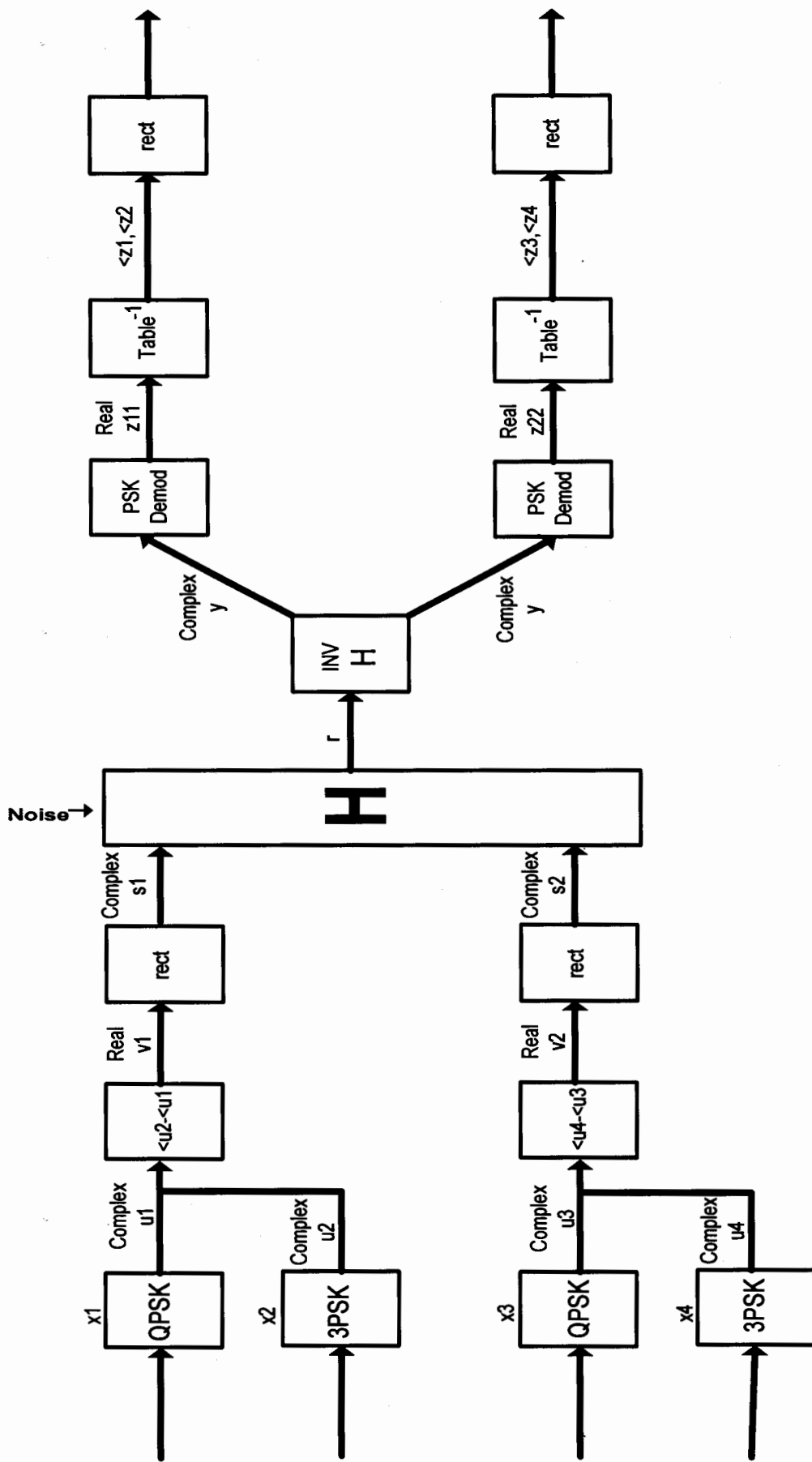
In this chapter a new scheme using QPSK/3PSK differential modulation has been proposed which exploits the orthogonality of the channel matrix. Inverse channel detection scheme is used to detect the symbols. This scheme is based on certain assumption in order to create and to detect the STBC system model. The model assumes that we have perfect estimate of the channel using any of the channel estimation scheme, the noise is AWGN and symbol of the modulation scheme having same probability of occurrence. A block diagram of the proposed model is given on the next page and the it is further describes on the next pages of this chapter.

4.1 QPSK/3PSK Differential Modulation

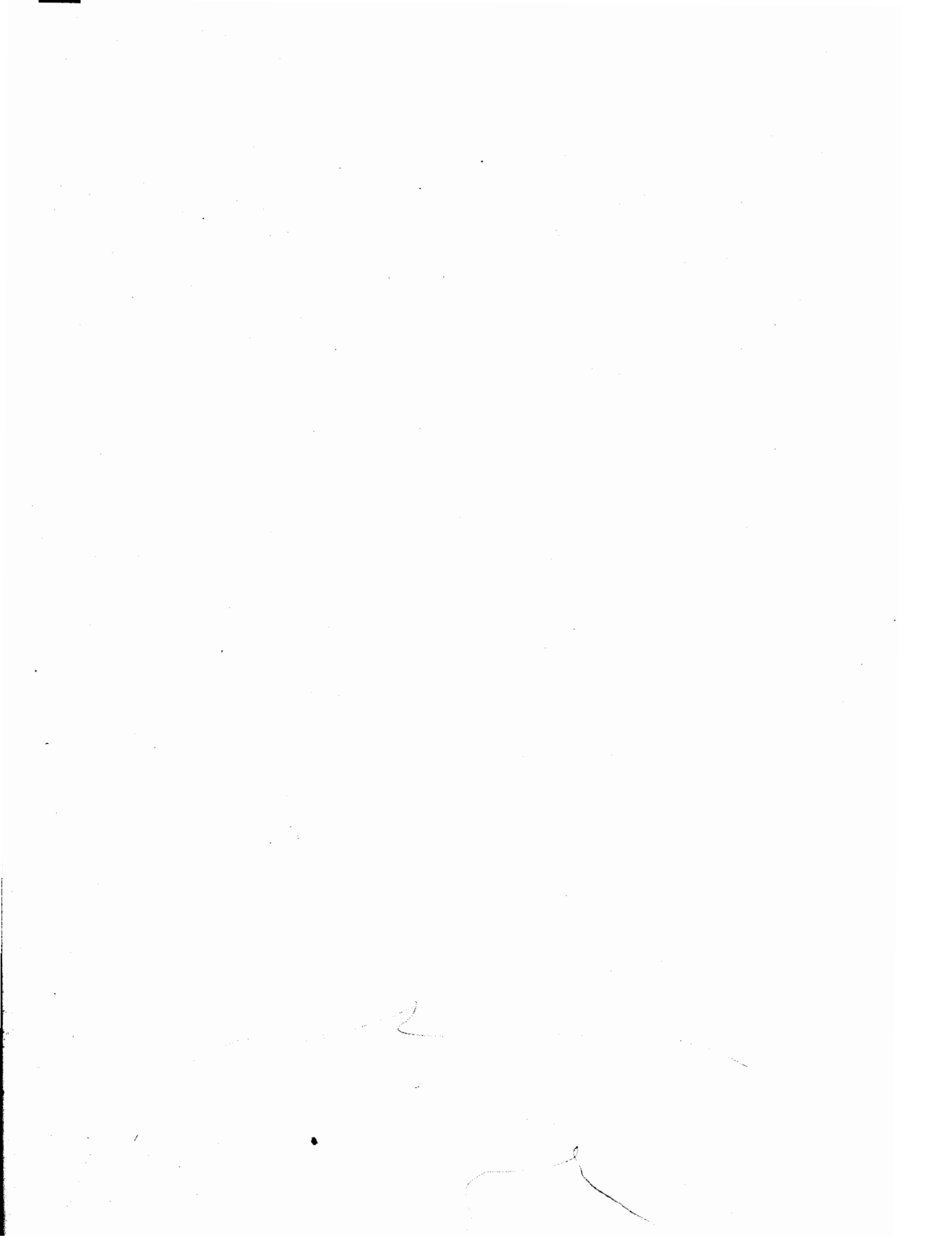
Consider an O-STBC system having two user and each user is equipped with two transmit antennas and there are one receiving antenna as shown in figure 4.1. A unique modulation scheme is used for modulation. The first two bits sending by each user is modulated by using QPSK modulation scheme and for next two bits the modulation scheme is 3PSK.

The code matrix is

$$A = \begin{bmatrix} y_1 & y_2 \\ -y_2^* & y_1^* \end{bmatrix} \quad (4.1)$$



Figur 4.1: Block diagram of model



where y_1 and y_2 are sending at time t and $-y_2^*$ and y_1^* are sending at time $t + \Delta t$

$$AA^H = \begin{bmatrix} y_1 & y_2 \\ -y_2^* & y_1^* \end{bmatrix} \begin{bmatrix} y_1^* & -y_2 \\ y_2^* & y_1 \end{bmatrix} \quad (4.2)$$

$$= \begin{bmatrix} |y_1|^2 + |y_2|^2 & 0 \\ 0 & |y_2|^2 + |y_1|^2 \end{bmatrix} = (|y_1|^2 + |y_2|^2) I_2 \quad (4.3)$$

Assuming that the fading coefficients are constant across two consecutive symbol period which are given as

$$h_2 = |h_2| e^{j\theta_2} \quad (4.4)$$

$$h_2 = |h_2| e^{j\theta_2} \quad (4.5)$$

The received signals can be expressed as

$$x_2 = -h_1 y_2^* + h_2 y_1^* + n_2 \quad (4.6)$$

$$x_2 = -h_1 y_2^* + h_2 y_1^* + n_2 \quad (4.7)$$

as the CSI is known

$$\hat{S}_1 = h_1^* x_1 + h_2^* x_2 = |h_1|^2 y_1 + h_1^* h_2 y_2 + h_1^* n_1 - h_2^* h_1 y_2 + |h_2|^2 y_1 + h_2^* n_2 \quad (4.8)$$

$$\hat{Y}_1 = (|h_1|^2 + |h_2|^2) y_1 + h_1^* n_1 + h_2^* n_2 \quad (4.9)$$

and

$$\hat{Y}_2 = h_2^* x_1 - h_1^* x_2 \quad (4.10)$$

$$\hat{Y}_2 = (|h_1|^2 + |h_2|^2) y_2 - h_1^* n_2 + h_2^* n_1 \quad (4.11)$$

The minimum distance metric is

$$d^2 \left(x_1, h_1 \hat{y}_1 + h_2 \hat{y}_2 \right) + d^2 \left(x_2, -h_1 y_2^* + h_2 y_1^* \right) \quad (4.12)$$

should be minimized

$$= \underbrace{\left| x_1 - h_1 \hat{y}_1 - h_2 \hat{y}_2 \right|^2}_{\langle I \rangle} + \underbrace{\left| x_2 + h_1 y_2^* - h_2 y_1^* \right|^2}_{\langle II \rangle} \quad (4.13)$$

$$\langle I \rangle = \left(\begin{array}{c} (h_1 y_1 + h_2 y_2 + n_1) - h_1 \hat{y}_1 - h_2 \hat{y}_2 \\ x_1 \end{array} \right) \left(\begin{array}{c} (h_1^* \hat{y}_1 + h_2^* \hat{y}_2 + n_1^*) - h_1^* \hat{y}_1 - h_2^* \hat{y}_2 \\ x_1^* \end{array} \right) \quad (4.14)$$

$$= |h_1|^2 |y_1|^2 + |h_2|^2 |y_2|^2 + |n_1|^2 + |h_1|^2 |\hat{y}_1|^2 + |h_2|^2 |\hat{y}_2|^2 + CT_s \quad (4.15)$$

$$\langle II \rangle = \left(\begin{array}{c} -h_1 y_2 + h_2 y_1 + n_2 + h_1 \hat{y}_2 - h_2 \hat{y}_1 \\ x_2 \end{array} \right) \left(\begin{array}{c} -h_1^* \hat{y}_2 + h_2^* \hat{y}_1 + n_2^* + h_1^* \hat{y}_2 - h_2^* \hat{y}_1 \\ x_2^* \end{array} \right) \quad (4.16)$$

$$= |h_1|^2 |y_2|^2 + |h_2|^2 |y_1|^2 + |n_2|^2 + |h_1|^2 |\hat{y}_2|^2 + |h_2|^2 |\hat{y}_1|^2 + CT_s \quad (4.17)$$

$$(\hat{y}_1, \hat{y}_2) = \arg \min_{(\hat{y}_1, \hat{y}_2) \in c} \left\{ d^2 \left(x_1, h_1 \hat{y}_1 + h_2 \hat{y}_2 \right) + d^2 \left(x_2, -h_1 \hat{y}_2 + h_2 \hat{y}_1 \right) \right\} \quad (4.18)$$

$$= \arg \min_{(\hat{y}_1, \hat{y}_2) \in c} \left\{ (|h_1|^2 + |h_2|^2 - 1) \left(|\hat{y}_1|^2 + |\hat{y}_2|^2 \right) + d^2 \left(\hat{y}_1, \hat{y}_1 \right) + d^2 \left(\hat{y}_2, \hat{y}_2 \right) \right\} \quad (4.19)$$

Let for 3PSK the received symbol is

$$R_i = e^{j(\theta_i)} e^{-j(\phi_i)} + n_i \quad (4.20)$$

and for QPSK

$$S_j = e^{j(\theta_j)} e^{-j(\phi_j)} + n_j \quad (4.21)$$

May be absorbed in n_i and n_j^*

$$e^{j(\theta_i - \theta_j)} e^{-j(\phi_i - \phi_j)} \quad (4.22)$$

$$|\Delta\phi_{ij}| < 15 \quad (4.23)$$

$$\Delta\theta_{ij} = \theta_i - \theta_j \quad (4.24)$$

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TABLE 4.1 CONVERSION OF CODED BITS TO 3-PSK SYMBOL. THE NUMBERS 0, 1, AND 2 REPRESENT THE DIFFERENT SIGNAL POINTS OF A 3-PSK SYMBOL.

| Coded Bits | Ternary Symbols | Coded Bits | Ternary Symbols |
|------------|-----------------|------------|-----------------|
| 00 | 0 | 01 | 2 |
| 11 | 1 | 10 | 1 |

TABLE 4.2 CONVERSIONS OF CODED BITS TO Q-PSK SYMBOL. THE NUMBERS 0, 1, 2 AND 3 REPRESENT THE DIFFERENT SIGNAL POINTS OF A Q-PSK SYMBOL.

| Coded Bits | Ternary Symbols | Coded Bits | Ternary Symbols |
|------------|-----------------|------------|-----------------|
| 00 | 0 | 01 | 2 |
| 11 | 1 | 10 | 3 |

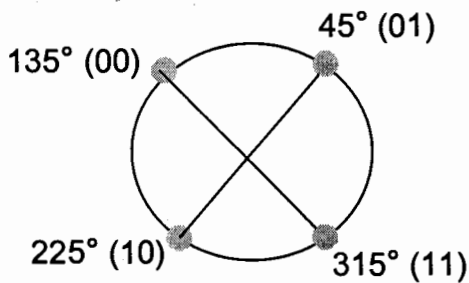


Figure 4.2: QPSK

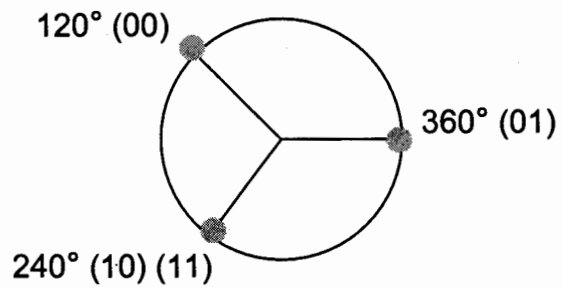


Figure 4.3: 3PSK

4.2 Transmission of O-STBC

x_1 and x_2 are sending to modulator blocks from user1 and x_3, x_4 respectively from user2. now for user1 x_1 is modulated by QPSK Scheme and the output is u_1 and x_2 is by 3PSK scheme. And the resultant complex modulated symbols are u_2 and same as for user2. Now for user1 there are two different modulated symbols and we are calculating the difference in the next block as shown in the figure 4.2. The novel feature of this system is that instead of sending the modulated bits by QPSK/3PSK, the output difference of these bits are transmitted. There are twelve unique points as shown in the table 4.3 and there graphical representation is also shown in figure 4.4

Table 4.3: Twelve different Points for the 3 PSK/QPSK Modulation Scheme

| Sr No | Angle Difference |
|-------|-------------------------------------|
| 1 | $360^\circ - 45^\circ = 315^\circ$ |
| 2 | $360^\circ - 135^\circ = 225^\circ$ |
| 3 | $360^\circ - 225^\circ = 135^\circ$ |
| 4 | $360^\circ - 315^\circ = 45^\circ$ |
| 5 | $120^\circ - 45^\circ = 75^\circ$ |
| 6 | $120^\circ - 135^\circ = 345^\circ$ |
| 7 | $120^\circ - 225^\circ = 255^\circ$ |
| 8 | $120^\circ - 315^\circ = 165^\circ$ |
| 9 | $240^\circ - 45^\circ = 195^\circ$ |
| 10 | $240^\circ - 135^\circ = 105^\circ$ |
| 11 | $240^\circ - 225^\circ = 15^\circ$ |
| 12 | $240^\circ - 315^\circ = 285^\circ$ |

4.3 Graphical Output

On the receiver side we are detecting symbols by the phase difference. The graphical output of the angle differences is as shown below

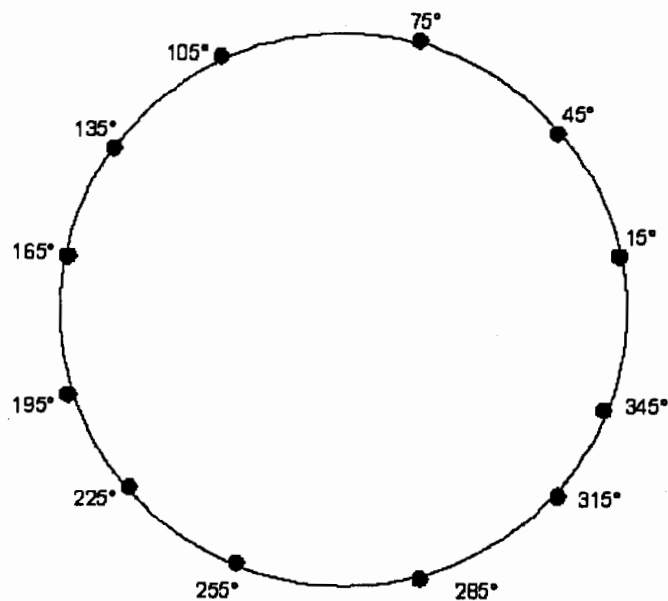


Figure 4.4: Output angle difference

After calculating the difference between angles we have

$$v_1 = \angle u_2 - \angle u_1 \quad (4.25)$$

$$v_2 = \angle u_4 - \angle u_3 \quad (4.26)$$

As these are real no. so in the next block of rect these are becoming complex and the s_1 and s_2 are transmitted at the same time interval but from two separate antennas.

4.4 Inverse Channel Detection Scheme

We can detect and decode user data by the help of inverse channel detection scheme. Referring to the equation

$$R = C.H \quad (4.27)$$

Where H = channel parameters and

$$c = [y_1, y_2] \quad (4.28)$$

As we have perfect knowledge of Channel State Information (CSI) so

$$HI^{-1} r = HI^{-1} HI C + HI^{-1} \quad (4.29)$$

$$y = C + I \quad (4.30)$$

$$\hat{C} = q[y] \quad (4.31)$$

Where \hat{C} is the estimate of the transmitted vector. And $q[p]$ is hard decision on p . The complex signal y is demodulated and then passes through the Table⁻¹ in which the angle of these symbols is calculated.

Chapter 5

SIMULATIONS AND RESULTS

Simulation results are provided in this section, which shows the performance gained by the proposed method. The concentration is given to the performance issue. Data is modulated for each symbol using the QPSK/3PSK modulation scheme. All the simulations, in this section, are done in Matlab according to block diagram shown in figure 4.1. A simple AWGN channel is used as channel model. Inverse channel detection scheme is applied. For all the results the value of $M = 4$, i-e, 4 points in the constellation and no. of bits per symbol is $k = \log_2(M)$, which is 2 bits. For 3PSK $M = 3$, there are 3 points in the constellation and no. of bits per symbol is $k = \log_2(M)$, which is 1.58 bits for 3PSK. Initially the $E_b N_0 = 1$ and goes up till $E_b N_0 \leq 21$. The other three schemes, which are used for comparison, are as follow:

- 1. Alamouti Scheme:** The Alamouti scheme is the first space time block codes scheme, which provide the full rate transmit diversity for system, by using two transmit antennas. The modulation scheme used by Alamouti is QPSK on a slow fading Raleigh channel. Maximum likelihood detection scheme is applied.
- 2. Space Time Block Codes:** The space time block codes are the generalized form of Alamouti codes. They are used for higher order and no. of antennas for diversity is used at least three are above. Space time block code can also achieve full transmit diversity. Coherent detection scheme is applied. The modulation scheme used by space time block codes are 8PSK on a slow fading Raleigh channel.
- 3. Differential Space Time Block Codes:** The differential space time block codes are very useful because they do not require channel estimate either at the receiver end or at the transmitter end. They are widely used in practical cellular mobile communication. The modulation scheme used by differential space time block code is QPSK on a slow fading Raleigh channel. Differential detection scheme is applied.

5.1 Scatter Plot

Scatter plot is a pictorial way of representing data. Scatter plot uses Cartesian coordinates to display values for two variables. The data is displayed as a collection of points, each having one coordinate on the horizontal axis and one on the vertical axis. The scatter plot for 3PSK and twelve output angle difference is presenting below and the variables used for these scatter plots are Quadrature on y-axis and In-phase on x-axis.

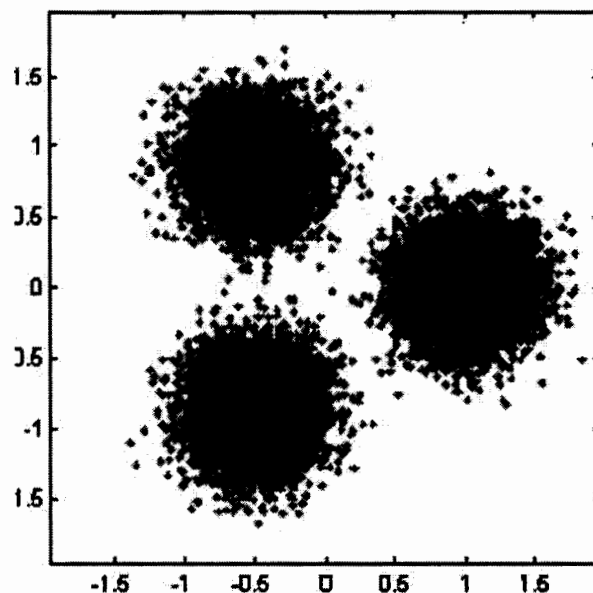


Fig 5.1: Scatter plot for 3PSK modulation

The figure 5.1 shows the three constellation points for 3PSK modulation at the receiver end. There is ambiguity in 3PSK symbols for decoding, as shown in table 4.1 that two symbols are started with the same bit. In order to remove this ambiguity and decode the 3PSK symbols efficiently, a unique scheme is applied. The symbols which are starting with the bit 1, only first bit is taken, instead of complete symbol and add the other bit to the remaining sequence. Conversely, the symbols which are stated with 0 are taken completely in to account. If this practice is done in the sequence for odd no. of times, then add the dummy bit at the end and for even no. there is no need to add the dummy bit.

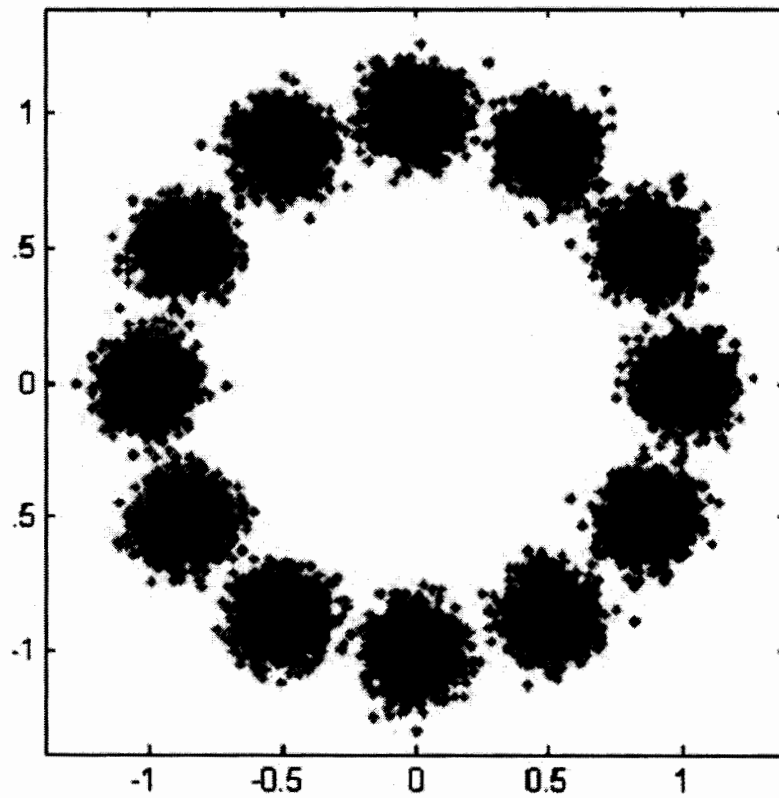


Fig 5.2: Scatter plot for twelve Output angle difference

The figure 5.2 shows the twelve different points for the QPSK/3PSK scheme. These points are unique and the distance between these points is constant. The angle difference for any two consecutive points is 30° , as shown in the table 4.3. The threshold level is, therefore, $\pm 15^\circ$.

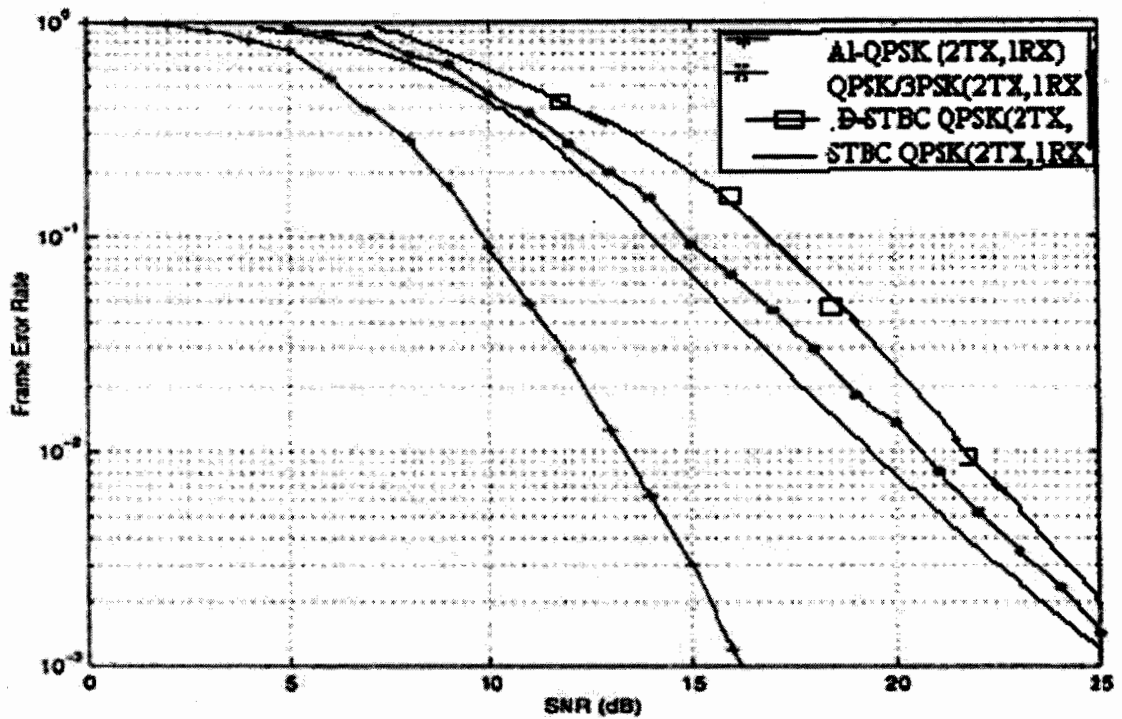
5.2 BER Performance

Bit Error rate (BER) is the percentage of bits that have errors relative to the total number of bits received in a transmission. It is usually expressed as ten to a negative power. The BER is a very powerful indicator, which indicates the performance capabilities of the existing system. It also represents that how often a packet or other data unit has to be retransmitted because of an error. A very high BER may indicate that a slower data rate would actually improve overall transmission time for a given amount of transmitted data. Since the BER might be reduced, lowering the number of packets that had to be resent.

A BERT (bit error rate test or tester) is a procedure or device that measures the BER for a given transmission.

The BER of the proposed scheme QPSK/3PSK is plotted along with BER curves of the Alamouti, space time block codes and differential STBC schemes in the figures 5.3, 5.4 and 5.5. BER performance of the proposed scheme is lower than the other schemes, which are compared, because, in this proposed scheme, there are twelve points. These twelve points make the BER high as compared to others. It is more difficult to recover the phase for QPSK/3PSK scheme, as its provide the stronger coding then the others schemes. Since, the angular difference between signal points is smaller. At a low E_b / N_0 , the BER performance of proposed scheme is better and it performs reasonably well, when compared to higher E_b / N_0 .

The figure 5.3 shows the BER performance of the proposed scheme with other schemes. The modulation scheme used by D-STBC, STBC and Almouti is QPSK with two transmit and one receive antenna. It is obvious that the BER performance of other three schemes are better than the proposed scheme and the conclusion drawn by viewing this graph is that as there are twelve points in 3PSK/QPSK, so threshold level is minimum as compared to simple QPSK where we have four points.



BER performance of the proposed scheme with D-STBC QPSK, STBC QPSK and Alamouti QPSK

The figure 5.4 shows the BER performance of the proposed scheme with other schemes. The modulation scheme used by D-STBC, STBC and is 8PSK with two transmit and one receive antenna. It is obvious that the BER performance of other three schemes are better than the proposed scheme and the conclusion drawn by viewing this graph is that as there are twelve points in 3PSK/QPSK, so threshold level is minimum as compared to simple QPSK where we have eight points.

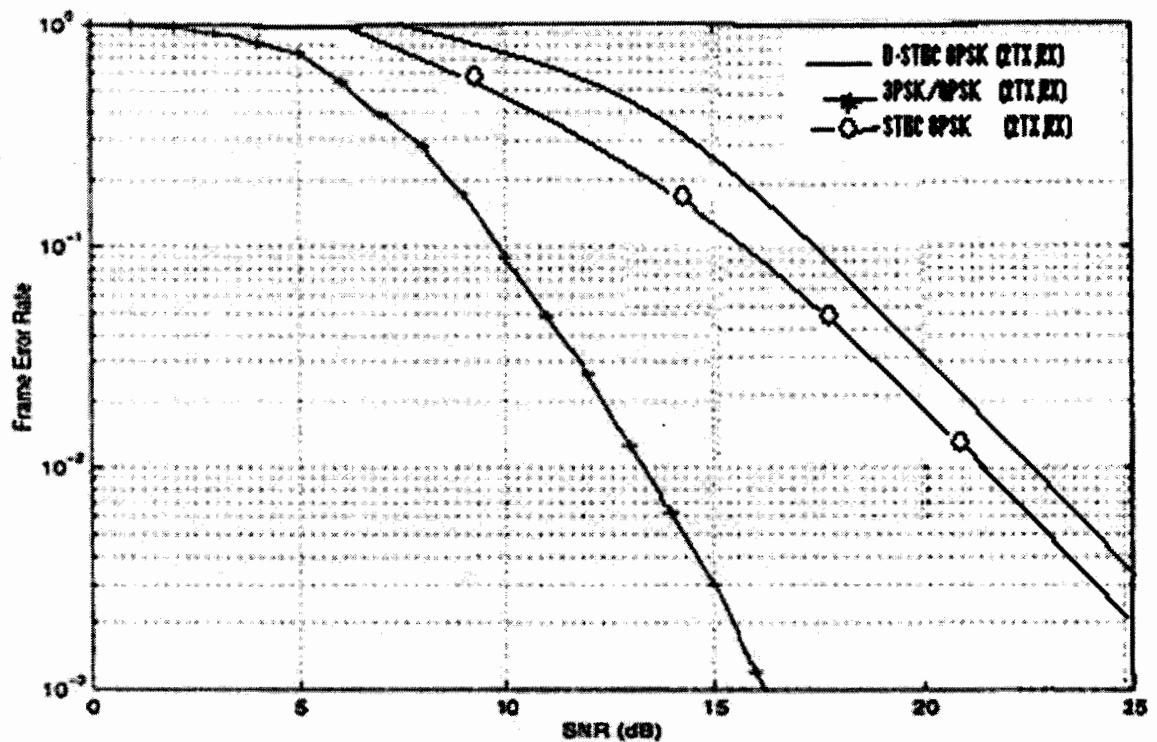


Figure 5.4: BER performance of the proposed scheme with D-STBC 8PSK, STBC 8PSK

The figure above shows the BER performance of the proposed scheme with other schemes. The modulation scheme used by D-STBC, STBC and is BPSK with two transmit and one receive antenna. It is obvious that the BER performance of other three schemes are better than the proposed scheme and the conclusion drawn by viewing this graph is that as there are twelve points in 3PSK/QPSK, so threshold level is minimum as compared to simple QPSK where we have two points.

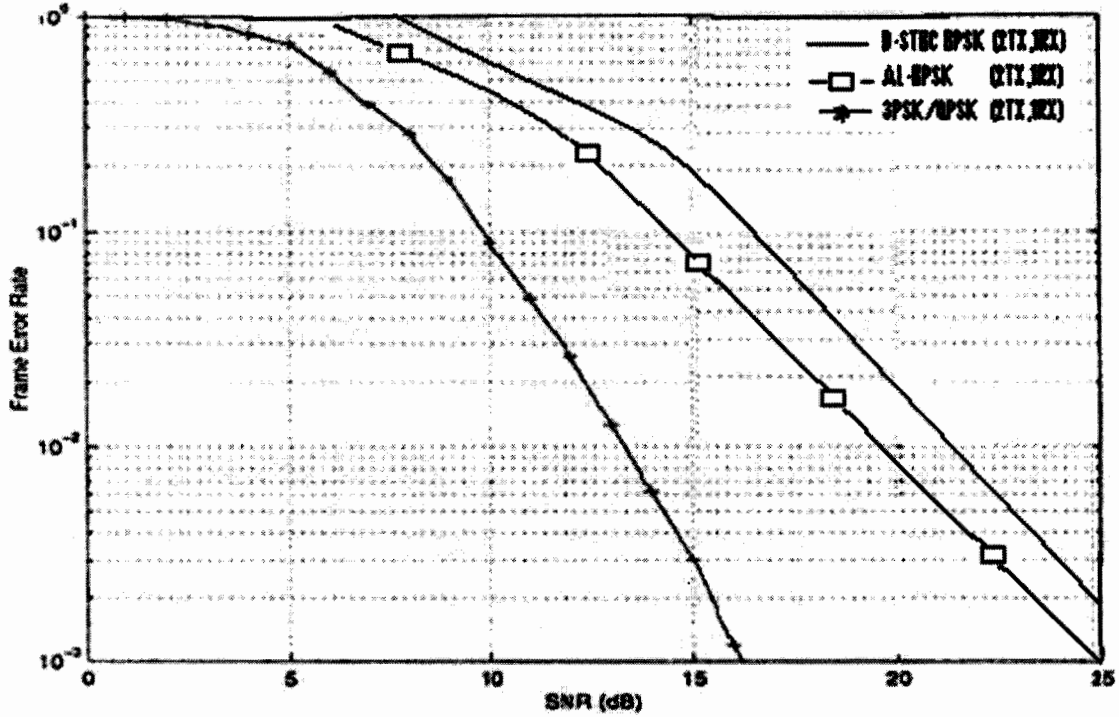


Figure 5.5: BER performance of proposed scheme with D-STBC and Alamouti scheme

5.3: Data Rate

The data transfer rate (DTR) is another important measure to analysis the performance of any transmission scheme. In telecommunication the DTR is referred as the amount of digital data that is moved from one place to another in a given time. The data transfer rate can be viewed as the speed of travel of a given amount of data from one place to another. In general, the greater the bandwidth of a given path, the higher the data transfer rate. In telecommunications, data transfer is usually measured in bits per second. An efficient transmission scheme is that which increase the data rate without requiring the additional bandwidth.

The proposed scheme has much better data rate as compare to Alamouti, D-STBC and STBC schemes or any other scheme which is using QPSK modulation. The 3PSK/QPSK is the merger of the two different schemes, so it has data rate, almost doubled to other schemes. A table for the analysis is given below:

Table:5.1 Data Rate analysis

| Coding Scheme/Modulation scheme | QPSK | 8PSK | BPSK | 3PSK/QPSK |
|------------------------------------|------|------|------|------------|
| Proposed scheme 3PSK/QPSK | - | - | - | 3.5—4 bits |
| Alamouti scheme | 2 | 3 | 1 | - |
| Space time block codes | 2 | 3 | 1 | - |
| Differential space time block code | 2 | 3 | 1 | - |

It is obvious from the table above that only the proposed scheme achieved the data rate of at least 3.5 and at most 4 bits.

Chapter 6

CONCLUSION AND FUTURE WORK

The main theme of this research thesis is to study the performance of space time codes for multi-user environment in flat fading channels. A unique modulation scheme is presented in which the merger of 3PSK/QPSK modulation is used with inverse channel detection scheme. Simulation results demonstrate the performance of the proposed scheme. The results clearly indicate the feasibility of the proposed scheme. Advantage of this scheme is that, as compared to any other coding scheme which uses the QPSK modulation in which we can detect 2 bits when we detect the phase, at least 3 and at most 4 bits are detected once the angle is find in the differential 3PSK/QPSK and in average it achieve the data rate of 3.5 Bits. So it almost doubled the data rate but on the other hand BER increased since the threshold is minimized as there are now 12 points instead of 4 points in QPSK. So there is a trade off between data rate and BER.

Future Work

- It is suggested that we can use the other method of detection for the transmitted symbols like QR Decomposition and ML.
- 5PSK/QPSK and 7PSK/8PSK scheme can used
- Instead of Phase modulation we can use Amplitude modulation

REFERENCES

- [1] A. Narula, M. Trott, and G. Wornell (Sept. 1996). "Information theoretic analysis of multiple antenna transmission diversity". Proc. Int. Symp. Inform. Th. Appl., Canada 9: 230-231.
- [2] G. J. Foschini, Jr. and M. J. Gans (1998). "On limits of wireless communication in a fading environment when using multiple antennas". Wireless Personal Communication 6: 311-322.
- [3] E. Telatar (June 1995). "Capacity of multiple antenna Gaussian channels". AT&T-Bell Labs Internal Tech. Memo 3: 114-115.
- [4] S.M. Alamouti (October 1998). "A simple transmit diversity technique for wireless communications". IEEE Journal on Selected Areas in Communications 16 (8): 1451-1458.
- [5] Vahid Tarokh, Nambi Seshadri, and A. R. Calderbank (March 1998). "Space-time codes for high data rate wireless communication: Performance analysis and code construction". IEEE Transactions on Information Theory 44 (2): 744-765.
- [6] M. Enescu and V. Koivunen (May 2002). "Time-varying channel tracking for space-time block coding". IEEE VTC 2002-5 1: 294-297.
- [7] Federal Standard 1037C, which, as a work of the United States Government
- [8] T.S. Rappaport, Wireless Communications: Principles and practice, Second Edition, Prentice Hall
- [9] Gerard J. Foschini and Michael. J. Gans (January 1998). "On limits of wireless communications in a fading environment when using multiple antennas". Wireless Personal Communications 6 (3): 311-335.

- [10] Gerard J. Foschini (autumn 1996). "Layered space-time architecture for wireless communications in a fading environment when using multi-element antennas". Bell Labs Technical Journal 1: 41–59.
- [11] A. Paulraj, R. Nabar and D. Gore, Introduction to Space-time Communications.
- [12] Vahid Tarokh, Hamid Jafarkhani, and A. R. Calderbank (July 1999). "Space-time block codes from orthogonal designs". *IEEE Transactions on Information Theory* 45 (5): 744–765.
- [13] Vahid Tarokh, Hamid Jafarkhani, and A. Robert Calderbank (March 1999). "Space-time block coding for wireless communications: performance results". *IEEE Journal on Selected Areas in Communications* 17 (3): 451–460.
- [14] Bernard Sklar (July 1997). "Rayleigh Fading Channels in Mobile Digital Communication Systems Part I: Characterization". *IEEE Communications Magazine* 35 (7): 90–100.
- [15] T. S. Rappaport (December 31, 2001). *Wireless Communications: Principles and Practice*, 2nd edition, Prentice Hall PTR.
- [16] John G. Proakis (1995). *Digital Communications*, 3rd edition, Singapore: McGraw-Hill Book Co, 767–768.
- [17] D. H. Evans and W. N. McDicken (2000). *Doppler Ultrasound*, Second Edition, John Wiley and Sons.
- [18] P. Dent, G. E. Bottomley and T. Croft (24 June 1993). "Jakes Fading Model Revisited". *Electronics Letters* 29 (13): 1162–1163.

