

Multiple Block Detection for Differential Space Time Codes



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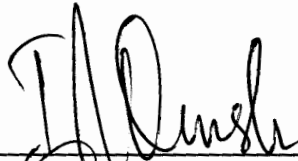
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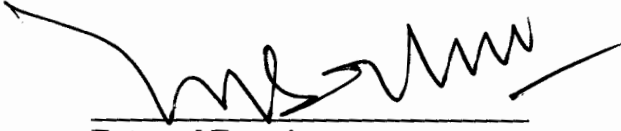
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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

Dedicated To,

THE HOLY PROPHET HAZRAT MUHAMMAD (S.A.W.),

HIS SAHABA, MY PARENTS, TEACHERS & UMMAH.

Abstract

The capacity demand in wireless local area networks has grown in past few years in an explosive manner. The need for internet access through wireless require an increase in information throughput. The use of multiple antennas at the transmitters and receivers in the system will make this increase in data rate possible. A system with multiple transmit and receive antennas is often called a multiple-input multiple-output (MIMO) system. Multiple antenna system helps in fighting the destructive effects of fading and also improves the spectral efficiency of a communication system. Space-time block codes (STBC) present a way of introducing transmit diversity into the communication system. A wide range of research is carried out in STBC. Differential space-time codes are form of STBC that does not need to know the channel impairments at the receiver in order to be able to decode the signal. A detection technique for multiple blocks is proposed for differential space-time block codes, which reduces the performance loss by extending the observation interval for decoding. As, Maximum likelihood (ML) block sequence is the optimal solution. So, ML block sequence detection is used instead of traditional block-by-block detection. The Detection techniques for 2 and 3 blocks for Differential space time codes are already proposed by Jafarkhani[7] and Fan[14] respectively. This work is a generalization to any number of blocks.

Acknowledgement

I humbly praise and thank ALMIGHTY ALLAH, The compassionate and The merciful, Who gave me health, thoughts, affectionate parents, talented teachers, helping friends and opportunity to contribute to the vast body of knowledge. Peace and prayers for His Prophet HAZRAT MUHAMMAD (S.A.W.) Whose incomparable life is the glorious model for humanity.

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CHAPTER 1

Introduction

Guglielmo Marconi demonstrated radio's ability to make contact with with ships sailing the English Channel, as a foundation towards Wireless Communication [1]. New methods introduced and new devices invented during the last ten years. Now, we use wireless devices and networks like mobile phone, wireless internet, etc. Wireless communication is one of the most exciting fields in present engineering. Increase of such products and services create a serious challenge. The challenge is how we can support the exceedingly high data rates and capacity required for these applications?

The difficulties associated with wireless surroundings are difficult to overcome interference from other users and inter-symbol interference (ISI) from multiple paths of one's own signal also create distortion[2]. Also, Doppler Effect will spread the frequency spectrum of received signals when transmit and receive antennas are in relative motion [3]. The outcomes are time varying channel characteristics. Many systems must work without a Line-of-Sight (LOS) between transmit and receive antennas. The additive white Gaussian noise (AWGN) corrupts the signal also.

There are limited bandwidth and strict power limitations on both the mobile unit (for battery conservation) and the base station (to satisfy government safety regulations). Spectral efficiency is maximized by providing as much information as possible into a given bandwidth. A solution to the bandwidth and power problem is the cellular concept, in which low power cells are reused at cells far away and frequency bands are allocated small. Space-time coding increases data rate, capacity, and spectral efficiency.

1.1 Motivations and Background

Space-time block coding is a technique used to transmit multiple copies of a data stream across a number of antennas and to make use of the various received forms of the data to improve the reliability of data transfer. The fact that transmitted data must pass through a potentially difficult environment with scattering, reflection, refraction and so on as well as be corrupted by thermal noise in the receiver means that some of the received copies of the data will be better than others. This redundancy results in a higher chance of being able to use one or more of the received copies of the data to correctly decode the received signal. In fact, space-time coding combines all the copies of the received signal in an optimal way to extract as much information from each of them as possible. Research on space-time coding has made a significant progress in recent years.

Space-time trellis coding scheme was first proposed by Tarokh[4] on a Rayleigh fading channel. The scheme provides a good trade-off between constellation size, data rate, diversity advantage and trellis complexity. Alamouti[5] proposed a much simpler

Space-time block code (STBC) scheme, which provides full diversity advantage, but is not optimized for coding gain. Alamouti's scheme for two transmit antennas supports a maximum likelihood detection scheme based only on linear processing at the receiver. Tarokh [6] generalized the scheme to multiple transmit antennas (three, four or eight), to obtain full diversity for real-valued constellations. Full diversity can be obtained only at the cost of reduced coding rate for complex constellations. Alamouti's scheme [5] has been adopted by 3G standards, such as W-CDMA and cdma2000 due to its relative simplicity of implementation.

The design of the Space-time codes mentioned above is based on the assumption that perfect estimation of channel state information (CSI) is available at the receiver. This is reasonable when channel changes slowly compared with the symbol rates, since the transmitter can send training symbols, which enable the receiver to estimate the channel accurately. For cases when accurate channel estimation is not possible or the effort associated with channel estimation is to be avoided, it is of interest to develop techniques, which do not require CSI.

Tarokh and Jafarkhani [7] first came up with a differential STBC scheme for a slow Rayleigh fading channel with two transmit antennas. The scheme employs block-by-block detection, in which neither transmitter nor receiver knows the CSI. The same authors generalized the differential detection for STBC to more than two transmit antennas [8]. Similar to single antenna channels, a loss of approximately 3dB is always paid for this differential scheme compared to the related coherent scheme. Hochwald and Seldom [9] proposed a new class of differential modulation schemes for multiple transmit antennas based on unitary space-time modulation. At the same

time, a related differential modulation scheme was proposed by Hughes in terms of group codes [10]. These schemes utilize constellations of unitary matrices or group codes to achieve full transmit diversity without knowledge of CSI, but with a loss of about 3 dB in performance. While Tarokh's scheme has properties similar to the 2×2 unitary matrixes in [9] or 2×2 group codes in [10], it outperforms both.

Multiple Symbol Differential detection (MSDD) was first presented for one transmit antenna over the additive white Gaussian noise (AWGN) channel by Divsalar and Simon [11]. By extending the observation interval to more than two symbols, the technique makes use of maximum likelihood sequence detection instead of symbol-by-symbol detection as in conventional differential detection. The performance of MSDD depends on the number of observation symbols. For a moderate number of symbols, MSDD bridges the performance gap between non-coherent and coherent communications. In [12] and [13], MSDD is applied to the flat Rayleigh fading channel.

Fan [14] extended the observation interval of differential STBC [7] to 3 blocks. As a result, a performance improvement of about 0.5 dB for BPSK message was demonstrated. Here, we generalize differential STBC for larger observation intervals. The decision metric for N blocks of observation interval is derived. It is shown that for an observation interval of N blocks there is a gain of 1.5dB over differential STBC with two blocks and a gain of 1.0dB over STBC with three blocks.

1.2 Thesis Outline

This thesis consists of five chapters. Chapter 1 gives the introduction and problem definition with thesis layout. The fundamental concepts related to Wireless MIMO channels are discussed in chapter 2. Chapter 3 discusses Space Time codes; their types and their performance are discussed. Chapter 4 discusses the system model, calculations and simulations for the proposed system. Conclusions are given in Chapter 5.

CHAPTER 2

Wireless MIMO Systems

Multiple antennas can be used either at the transmitter or receiver or at both. These various configurations are referred to as Multiple Input Single Output (MISO), Single Input Multiple Output (SIMO) or Multiple Input Multiple Output (MIMO). The MISO and SIMO architectures are forms of transmit and receive diversity schemes respectively. On the other hand, MIMO architectures can be used for combined transmit and receive diversity, as well as for parallel data transmission, i.e., Spatial Multiplexing (SM). When used for SM, MIMO technology uses the available spectrum efficiently. The presence of multiple transmitted streams, combination of the propagation channel and multiple users transmitting at the same time results in various forms of interfering scenarios at the receive antennas of a wireless system. Therefore, multi-user detection techniques to reduce the interference across the symbols transmitted from different antennas and techniques to avoid the negative effect of the channel distortion are essential parts of a well designed transceiver.

2.1 Introduction

The concept of MIMO was first introduced by Jack Winters [15], in 1987 for two basic communication systems. The first was for communication between multiple mobiles and a base station with multiple antennas and the second for communication between two mobiles each with multiple antennas. Hence, the concept of MIMO can be attributed to any informal system using multiple antennas at both ends, either for a point-to-point link or for radio transmission between multiple units. A basic analysis of MIMO systems is provided for the case of point-to-point transmission, where one user uses all the available resources in the system.

2.2 MIMO System Model

A single point-to-point MIMO system with n_T antennas at the transmitter and n_R antennas at the receiver is considered. A generic block diagram of a linear time-discrete MIMO system is shown in Fig. 2.1.

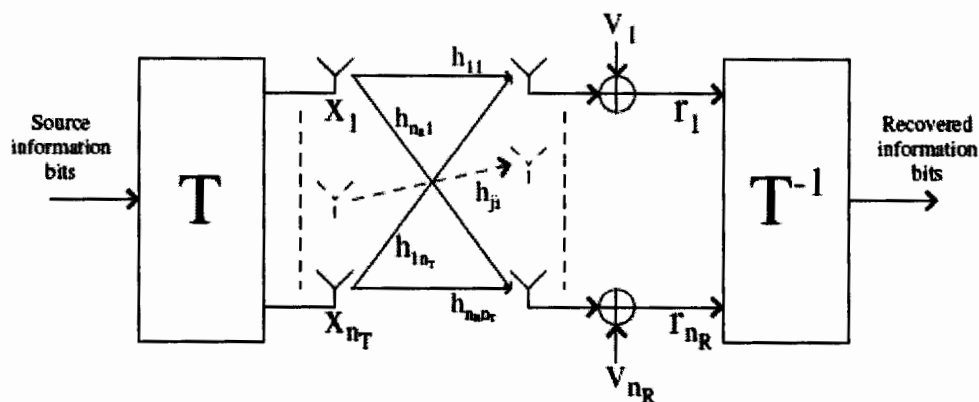


Figure 2.1: MIMO system block diagram

The information bits are first transformed by a generic processor \mathbf{T} to generate the transmitted signal, represented by an $n_T \times 1$ column vector $\mathbf{x} = [x_1, x_2, \dots, x_{n_T}]^T$. The total transmitted power is constrained to P and the elements of x are zero mean, independent and identically distributed (i.i.d.) random variables with equal average power of $P_i = P/n_T$. The signal x_i transmitted from antenna i , where $i = 1; \dots; n_T$, passes through the wireless channel and arrives at receive antenna j , where $j = 1; \dots; n_R$. The total received signal at the receive antenna j is expressed as

$$r_j = \sum_{i=1}^{n_T} h_{ji} x_i + v_j \quad j = 1, 2, \dots, n_R \quad (2.2.1)$$

where h_{ji} is a sample of a complex random variable that describes the channel between transmit antenna i and receive antenna j and v_j is a zero mean i.i.d. Additive White Gaussian Noise (AWGN) complex sample at receive antenna j . The total received signal $\mathbf{r} = [r_1; \dots; r_{n_R}]^T$ at the receiver can be expressed in matrix form as

$$\mathbf{r} = \begin{bmatrix} h_{1,1} & \cdot & \cdot & \cdot & h_{1,n_T} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ h_{n_R,1} & \cdot & \cdot & \cdot & h_{n_R,n_T} \end{bmatrix} \mathbf{x} + \mathbf{v} = \mathbf{H}\mathbf{x} + \mathbf{v} \quad (2.2.2)$$

where the $n_R \times n_T$ complex matrix \mathbf{H} represents the MIMO channel and the vector $\mathbf{v} = [v_1; \dots; v_{n_R}]$ is the noise vector at the receiver. We assume that at each receive antenna the noise has identical variance σ_v^2 . We denote the average power at the output of each receive antenna by P_r .

The channel matrix \mathbf{H} in (2.2.2) describes a narrowband MIMO channel in which individual sub-channels between a given pair of transmit and receive antennas are

modeled by an independent flat fading process, i.e., the channel is memoryless. In the case of wideband channels, each sub-channel has a memory equal to its impulse response. If the impulse response of each sub-channel is finite, i.e., the sub-channel has a finite memory, the discrete-time wideband MIMO channel can be written in a matrix form similar to (2.2.2). It can be shown that for Finite Impulse Response (FIR) channels, each entry of \mathbf{H} is a complex vector with length equal to the memory order of the corresponding sub-channel and can be modeled as a tapped delay line [16]. In particular, let us consider the Single Input Single Output (SISO) sub-system between transmit antenna i and receive antenna j , for $i = 1; \dots; n_T$ and $j = 1; \dots; n_R$, as if it was isolated from the rest of the system. For a transmitted symbol waveform $x_i(t)$ and Channel Impulse Response (CIR) $h_{ji}(t)$, the received waveform at time t is given by

$$r_j(t) = x_i(t) * h_{ji}(t) + v_j(t) \quad (2.2.5)$$

where $v_j(t)$ represents the noise process at the receiver at time t . The discrete-time model of (2.2.5), in the case of a symbol waveform of duration T and a FIR channel of length v_{ji} , can be written as

$$r_j(m) = \sum_{l=1}^{v_{ji}-1} x_i(m-l)h_{ji}(l) + v_j(m) \quad m = 1, 2, \dots, M-1 \quad (2.2.6)$$

where the channel is sampled at the symbol rate, i.e., $h_{ji}(m) = h_{ji}(mT)$, and a transmitted block of M samples is considered. It is clear that if the CIR is shorter than the symbol waveform length, i.e., $v_{ji} = 1$, then (2.2.6) reduces to (2.2.1), for $n_T = 1$. On the other hand, if the CIR is longer than T the received signal in (2.2.6) is the superposition of two or more consecutive transmitted samples, whose effect is known as ISI. In matrix form, (2.2.6) can be written as

$$\mathbf{r}_j = \mathbf{h}_{ji} \mathbf{x}_i + \mathbf{v}_j \quad (2.2.7)$$

where

$$\mathbf{h}_{ji} = \begin{bmatrix} h_{ji}(v_{ji}-1) & \dots & h_{ji}(0) & 0 & \dots & 0 \\ 0 & h_{ji}(v_{ji}-1) & \dots & h_{ji}(0) & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \dots & 0 & h_{ji}(v_{ji}-1) & \dots & h_{ji}(0) \end{bmatrix} \quad (2.2.8)$$

is a $M \times (M + v - 1)$ non-square Toeplitz matrix modeling the SISO discrete sub-channel matrix over a block of M symbols, the $(M + v_{ji} - 1) \times 1$ complex vector $\mathbf{x}_i = [x_i(-v_{ji}+1), \dots, x_i(0), \dots, x_i(M-1)]^T$ is the transmitted block, the $M \times 1$ complex vector $\mathbf{r}_j = [r_j(0), \dots, r_j(M-1)]^T$ is the received signal block, and the $M \times 1$ complex vector $\mathbf{v}_j = [v_j(0), \dots, v_j(M-1)]^T$ is the additive noise at the receiver.

For the frequency selective MIMO channel, the matrix relation between multi-antenna transmitted signal and total received signal can be easily obtained by replacing \mathbf{h}_{ji} with \mathbf{h}_{ji} in (2.2.2), yielding

$$\mathbf{r} = \begin{bmatrix} h_{1,1} & \cdot & \cdot & \cdot & h_{1,n_T} \\ \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot & \\ h_{n_R,1} & \cdot & \cdot & \cdot & h_{n_R,n_T} \end{bmatrix} \mathbf{x} + \mathbf{v} = \mathbf{H}\mathbf{v} + \mathbf{v} \quad (2.2.9)$$

In the MIMO case, the frequency selective channel effectively introduces both TAI and ISI. In a real environment, ISI is generally caused by the superposition of multiple reflected and diffracted electromagnetic waves produced by a single source, i.e., a single transmit antenna, but arriving to a certain sink, i.e., a single receive antenna, at

different times and with diverse phases. Those multiple replicas, or multi-paths, of the signal might add constructively or destructively, resulting in fluctuations of the received waveform amplitude. This type of multi-path fading channel can be modeled as (2.2.8), where the generic element of the channel matrix, i.e., $h_{ji}(l)$, $l=0, \dots, v_{ji}-1$, represents the fade experienced by the l^{th} replica of the signal, i.e., the l^{th} path, with a delay of l time instants relative to the first path. Moreover, the parameter v_{ji} represents the maximum number of resolvable multi-paths in the channel. Hence, ISI is mainly related to the wireless channel characteristics of a given environment, i.e., indoor, urban or suburban, where the effect of multi-paths might or might not be negligible. On the other hand, TAI is a type of interference strictly related to the antenna topology used at the transmitter, i.e., it is a characteristic of MIMO systems. In other words, even in a system without ISI, such as a line-of-sight (LOS) scenario in an open field, the resulting MIMO flat channel would generally cause TAI at each receiver. We refer to the channel \mathbf{H} in (2.2.9) as a MIMO frequency selective channel and we assume that the CSI, i.e., the channel matrix \mathbf{H} , is always perfectly known at the receiver. Moreover, the CSI is considered unknown at the transmitter, unless stated otherwise. The channel coefficients $h_{ji}(l)$, $l=0, \dots, v_{ji}-1$, are modeled as zero mean complex Gaussian random variables, thus their amplitudes follow a Rayleigh distribution and their phases are uniformly distributed in the interval $(0, 2\pi]$. This type of channel will be referred to as a Rayleigh distributed channel. Furthermore, a channel is considered to be quasi-static when the channel coefficients are time-invariant for the duration of the entire block of M symbols. This channel is also called slow fading channel. On the other hand, if the channel coefficients change every

symbol, the channel is called fast fading channel. It will be considered a channel with Rayleigh distribution and slow fading.

The transformation \mathbf{T} in Fig. 2.1 models, in the most generic way, the air interface employed to realize a MIMO wireless communications system. Various solutions and schemes have been devised throughout the years to enable effective transceiver architectures for wireless links. However, although they might appear different to each other, they all aim at transforming the source information bits in a set of one or more symbols to be transmitted through the channel from one or more transmit antennas and finally collected at one or more receive antennas. Once received, the transformation applied at the transmitter, i.e., \mathbf{T} , is reverted at the receiver, i.e., \mathbf{T}^{-1} , in order to recover the information bits. Note that the presence of the channel and the additive noise at the receive antennas does not allow a perfect reconstruction of the information bits, thus introducing errors on the recovered data. Moreover, the transformation \mathbf{T} and \mathbf{T}^{-1} are not necessarily linear transformations.

The fundamental problem is how should the transformations \mathbf{T} and \mathbf{T}^{-1} be designed to optimize the performance of the wireless link? The performance might be expressed by the following goals:

1. The aim is to provide the highest possible data rate per unit bandwidth. This is often defined as the spectral efficiency. The ultimate limit or highest possible data rate that can be achieved, with arbitrarily low bit error probability in a Gaussian noise channel, was derived by Shannon and is used as a reference.
2. Often conflicting with high data rate is the reliability or robustness of the transmission which can be measured using the average BER. Since the radio

channel is time varying, it is important to choose a communication strategy that can withstand the fading dips in the received signal power caused by the multipath propagation.

3. As one end of the wireless link might be battery powered, it is important to have low complexity architectures since an algorithm with higher complexity will be more power consuming. Hence, it is often desirable to design the transmission/reception schemes so that the complexity is asymmetric, i.e., locating the low complexity algorithms at the battery powered side. Hence, the transformations \mathbf{T} and \mathbf{T}^{-1} are usually different on different sides of the channel in a duplex system.

CHAPTER 3

Space Time Codes

A typical communication system consists of a transmitter, a channel, and a receiver. Space-time coding involves use of multiple transmit and receive antennas, as illustrated in Fig. 3.1. Bits entering the space-time encoder serially are distributed to parallel sub-streams. Within each sub-stream, bits are mapped to signal waveforms, which are then emitted from the antenna corresponding to that sub-stream. The scheme used to map bits to signals is called a space-time code. Signals transmitted simultaneously over each antenna interfere with each other as they propagate through the wireless channel. Meanwhile, the fading channel also distorts the signal waveforms. At the receiver, the distorted and superimposed waveforms detected by each receive antenna are used to estimate the original data bits.

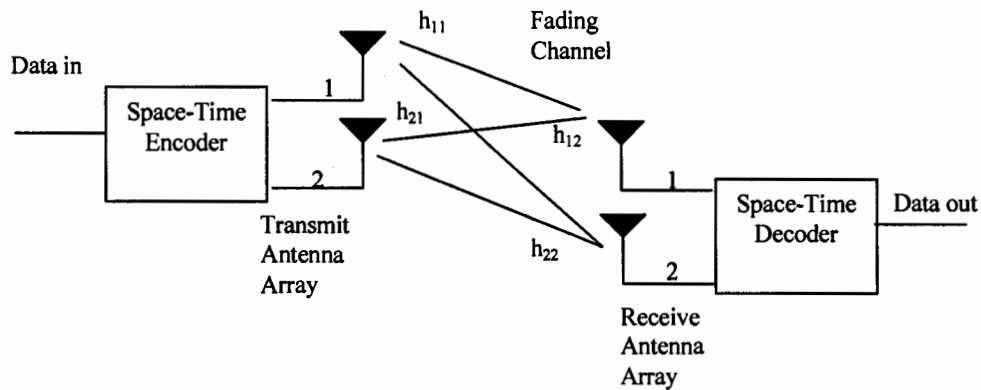


Fig 3.1. A typical communication system utilizing space-time coding.

There are two main types of STCs, namely Space-Time Trellis Codes (STTC) and Space-Time Block Codes (STBC):

- Space-Time Trellis Codes (STTC) operate on one input symbol at a time, producing a sequence of vector symbols whose length represents antennas. Like traditional TCM (trellis coded modulation) for a single-antenna channel, space-time trellis codes provide coding gain. Since they also provide full diversity gain, their key advantage over space-time block codes is the provision of coding gain. Their disadvantage is that they are extremely hard to design and generally require high complexity encoders and decoders.
- Space-Time Block Codes (STBC) operate on a block of input symbols, producing a matrix output whose columns represent time and rows represent antennas. In contrast to single-antenna block codes for the AWGN channel, space-time block codes do not generally provide coding gain, unless concatenated with an outer code. Their main feature is the provision of full diversity with a very simple decoding scheme.

3.1 Space-Time Trellis Codes (STTC)

Space-time code was introduced by Tarokh et al.[4], has the following structure:

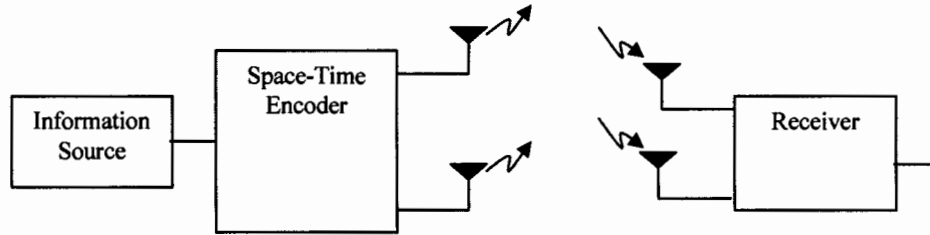


Fig 3.2 Space Time Coding

A mobile communication system with n transmit and m receive antennas is shown in figure 3.2. Data is encoded by the channel encoder, the encoded data goes through a serial to parallel converter, and is divided into n streams of data. Each stream of data is used as the input of a pulse shaper. The output of each shaper is then modulated. At each time slot t , the output of the modulator i is a signal c_t^i that is transmitted using transmit antenna i for $1 \leq i \leq n$. Then signals are transmitted simultaneously each from a different antenna and all of them have the same transmission period T . The signal at each receive antenna is a noisy superposition of the n transmitted signals corrupted by Rayleigh or Rician fading. It is assumed that the elements of the signal constellation are contracted by a factor of $\sqrt{E_s}$ to get the average energy of the constellation as 1.

The received signal at receive antenna j for $1 \leq j \leq m$ is r_t^j and is given by

$$r_t^j = \sqrt{E_s} \sum_{i=1}^n \alpha_{i,j} c_t^i + \eta_t^j \quad (3.1.1)$$

where the noise η_t^j at time t is modeled as independent samples of zero-mean complex Gaussian random variable with variance $N_0/2$ per dimension. The fading

coefficient $\alpha_{i,j}$ is the path gain from transmit antenna i to receive antenna j . The coefficients $\alpha_{i,j}$ are modeled as independent samples of complex Gaussian random variables with possibly nonzero complex mean $E\alpha_{i,j}$ and variance 0.5 per dimension, or equivalently the signals transmitted from different antennas undergo independent fading. The probability of error is considered as the probability that the maximum likelihood receiver decides in favor of a signal

$$e = e_1^1 e_1^2 \dots e_1^n e_2^1 e_2^2 \dots e_2^n e_l^1 e_l^2 \dots e_l^n \quad (3.1.2)$$

Assuming that

$$c = c_1^1 c_1^2 \dots c_1^n c_2^1 c_2^2 \dots c_2^n c_l^1 c_l^2 \dots c_l^n \quad (3.1.3)$$

was transmitted. Assuming ideal channel state information (CSI), the probability of transmitting c and deciding in favor of e at the decoder can be approximated as

$$P(c \rightarrow e | \alpha_{i,j}, i = 1, 2, \dots, n, j = 1, 2, \dots, m) \leq \exp(-d^2(c, e)E_s / 4N_0) \quad (3.1.4)$$

where $N_0/2$ is the noise variance of each dimension and

$$d^2(c, e) = \sum_{j=1}^m \sum_{i=1}^l \left| \sum_{i=1}^n \alpha_{i,j} (c_i^i - e_i^i) \right|^2 \quad (3.1.5)$$

Setting $\Omega_j = (\alpha_{1,j}, \dots, \alpha_{n,j})$ and $A_{pq} = X_p \cdot X_q$

where $X_p = (c_1^p - e_1^p, c_2^p - e_2^p, \dots, c_l^p - e_l^p)$ for $1 \leq p, q \leq n$ and after some manipulations

$$d^2(c, e) = \sum_{j=1}^m \Omega_j A \Omega_j^* \quad (3.1.6)$$

Therefore

$$P(c \rightarrow e | \alpha_{i,j}, i = 1, 2, \dots, n, j = 1, 2, \dots, m) \leq \prod_{j=1}^m \exp(-\Omega_j A(c, e) \Omega_j^* E_s / 4N_0) \quad (3.1.7)$$

Where $A_{pq} = \sum_{i=1}^l (c_i^p - e_i^p)(c_i^q - e_i^q)$ It can be easily seen that the matrix $A(c; e)$ is hermitian. Thus there exists a unitary matrix V and a real diagonal matrix D such that $VA(c,e)V^* = D$. The rows of V are formed a complete orthonormal basis of C^n given by the eigenvectors of A . Then, the matrix $B(c; e)$ constructed as

$$B(c, e) = \begin{bmatrix} e_1^1 - c_1^1 & e_2^1 - c_2^1 & \dots & e_l^1 - c_l^1 \\ e_1^2 - c_1^2 & e_2^2 - c_2^2 & \dots & e_l^2 - c_l^2 \\ \vdots & \vdots & \ddots & \vdots \\ e_1^n - c_1^n & e_2^n - c_2^n & \dots & e_l^n - c_l^n \end{bmatrix} \quad (3.1.8)$$

clearly satisfies $A(c; e) = B(c; e)B^*(c; e)$. Thus, $B(c; e)$ is a square root of $A(c; e)$. Therefore the eigenvalues of $A(c; e)$ are nonnegative real numbers. Set $(\beta_{1,j}, \dots, \beta_{n,j}) = \Omega_j V^*$, then

$$\Omega_j A(c, e) \Omega_j^* = \sum_{i=1}^n \lambda_i |\beta_{i,j}|^2 \quad (3.1.9)$$

Replacing (3.1.9) in (3.1.7) results

$$P(c \rightarrow e | \alpha_{i,j}, i = 1, 2, \dots, n, j = 1, 2, \dots, m) \leq \prod_{j=1}^m (\prod_{i=1}^n \exp(\lambda_i |\beta_{i,j}|^2 E_s / 4N_0)) \quad (3.1.10)$$

Define $K^j = (E\alpha_{1,j}, E\alpha_{2,j}, \dots, E\alpha_{n,j})$ and the rows of V as $\{v_1, v_2, \dots, v_n\}$. Since $\alpha_{i,j}$ are samples of complex Gaussian random variable with mean $E\alpha_{i,j}$ and V is a unitary matrix, $\beta_{i,j}$'s are independent complex random variables with variance 0.5 per dimension and mean $K^j \cdot v_i$. Therefore, $|\beta_{i,j}|$'s are independent Rician distributed. Thus, averaging the upper bound of the probability of error in equation (3.1.10) with respect to independent Rician distributed random variables $|\beta_{i,j}|$ gives

$$P(c \rightarrow e) \leq \prod_{j=1}^m \left(\prod_{i=1}^n \frac{1}{1 + \frac{E_s}{4N_0} \lambda_i} \exp\left(-\frac{K_{i,j} \frac{E_s}{4N_0} \lambda_i}{1 + \frac{E_s}{4N_0} \lambda_i}\right) \right) \quad (3.1.11)$$

where $K_{i,j} = |E\beta_{i,j}|^2$.

$E\alpha_{i,j} = 0$ for Rayleigh fading channel. Thus, $K_{i,j} = 0$ and (3.1.11) can be written as

$$P(c \rightarrow e) \leq \left(\frac{1}{\prod_{i=1}^n (1 + \lambda_i E_s / 4N_0)} \right)^m \quad (3.1.12)$$

If r denotes the rank of matrix \mathbf{A} , then there are r nonzero eigenvalues of \mathbf{A} which are

$$\lambda_1, \lambda_2, \dots, \lambda_r. \text{ Hence } P(c \rightarrow e) \leq \left(\prod_{i=1}^r \lambda_i \right)^{-m} (E_s / 4N_0)^{-rm} \quad (3.1.13)$$

From inequality (3.1.13) the diversity advantage is the negative of the power of $E_s / 4N_0$ which is equal to rm , where r is the rank of matrix $A(c; e)$ and m is the number of receivers. It can easily be shown that $A(c; e)$ and $B(c; e)$ have equal ranks.

The coding advantage of the system is $(\lambda_1, \lambda_2, \dots, \lambda_r)^{1/r}$.

3.2 Space Time Block Codes (STBC)

Alamouti [5] presented a remarkable scheme for transmission using two transmit antennas, which is much less complex than STTC for two transmit antennas and attractive in terms of both simplicity and performance. Tarokh et al. generalized this new scheme to an arbitrary number of transmitter antennas, leading to the concept of space-time block codes (STBC) [17]. STBC can achieve full diversity and have a fast decoding algorithm. The decoding complexity increases linearly, not exponentially with the code size since the transmitted symbols can be decoded separately for STBC. STBC can be classified into real orthogonal designs and complex orthogonal designs. The former deals with real constellations such as PAM, while the latter deals with complex constellations such as PSK and QAM. Real orthogonal designs have been well developed. Tarokh et al [17] proposed systematic constructions of real orthogonal designs for any number of transmit antennas with full rate. However, complex orthogonal designs are not well understood. There exist several different types of space-time block codes from complex orthogonal designs [5] [17] [18] [19] [20].

3.2.1 Alamouti's Scheme

We will first introduce MRC technique, and then discuss Alamouti's Scheme [5] for one receiver and multiple receivers. After that, we will analyze the performance of the scheme over fast fading channels. Finally, we will present the performance results of this new scheme.

3.2.1.1 Maximum Ratio Combining

Multiple antennas are used at the receiver and Maximum Ratio Combining (MRC) of the received signals is employed for improving the performance. MRC technique involving one transmitter and two receivers is shown here.

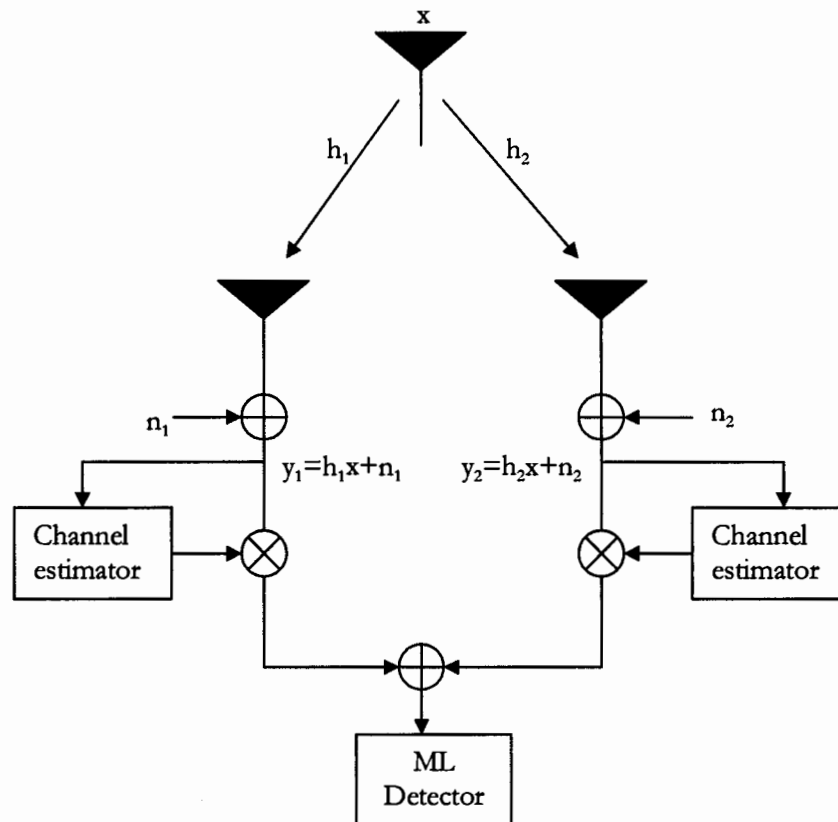


Fig 3.3 Maximum Ratio Combining with 1Tx and 2Rx

$$h_1 = \alpha_1 \exp(j\Theta_1); h_2 = \alpha_2 \exp(j\Theta_2)$$

$$y_1 = h_1x + n_1; y_2 = h_2x + n_2$$

$$\tilde{x} = h_1^* y_1 + h_2^* y_2 = h_1^* (h_1x + n_1) + h_2^* (h_2x + n_2) = (\alpha_1^2 + \alpha_2^2)x + h_1^* n_1 + h_2^* n_2$$

The maximum likelihood decision rule at the receiver is:

$$\text{Choose } x_i, \text{ iff } (\alpha_1^2 + \alpha_2^2 - 1) |x_i|^2 + d^2(\tilde{x}_0, x_i) \leq (\alpha_1^2 + \alpha_2^2 - 1) |x_k|^2 + d^2(\tilde{x}_0, x_k) \forall i \neq k$$

For equal energy constellation, the above decision rule can be simplified to:

Choose x_i , iff

$$d^2(\tilde{x}_0, x_i) \leq d^2(\tilde{x}_0, x_k) \forall i \neq k$$

3.2.1.2 Alamouti's Scheme Using One Receiver

Alamouti [5] proposed a simple transmit diversity scheme in 1998, which is the simplest form of STBC.

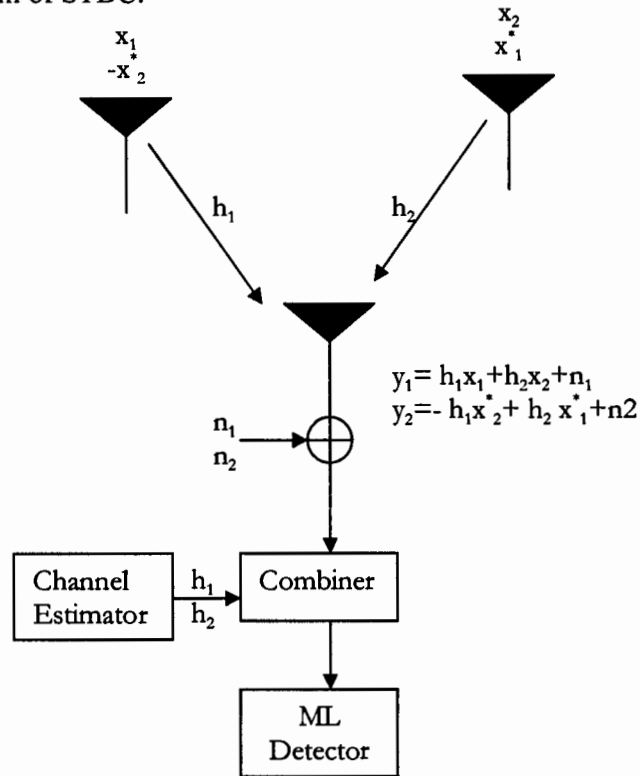


Fig 3.4 Simple twin-transmitter STBC using one receiver

The transition matrix is $G_2^c = \begin{bmatrix} x_1 & x_2 \\ -x_2 & x_1 \end{bmatrix}$ in Alamouti's scheme. Besides that,

quasi-static channel is assumed,

i.e; $h_1(t) = h_1(t+T) = \alpha_1 \exp(j\Theta_1)$; $h_2(t) = h_2(t+T) = \alpha_2 \exp(j\Theta_2)$;

$$y_1 = h_1 x_1 + h_2 x_2 + n_1$$

$$y_2 = -h_1 x_2^* + h_2 x_1^* + n_2$$

$$\tilde{x}_1 = h_1^* y_1 + h_2 y_2^* = (\alpha_1^2 + \alpha_2^2) x_1 + h_1^* n_1 + h_2 n_2^* \quad (3.2.1.2.1)$$

$$\tilde{x}_2 = h_2^* y_1 - h_1 y_2^* = (\alpha_1^2 + \alpha_2^2) x_2 - h_1 n_2^* + h_2^* n_1 \quad (3.2.1.2.2)$$

From equations (3.2.1.2.1) (3.2.1.2.2), we can easily see that the signals x_1 and x_2 are separated by simple multiplications and additions. Due to the orthogonality of the STBC G_2^c , the unwanted signal x_2 , x_1 is removed from \tilde{x}_1, \tilde{x}_2 , respectively. Then signals \tilde{x}_1, \tilde{x}_2 are passed to the maximum likelihood detector for decision and decision rules here are the same with those used earlier in the classic MRC technique.

Choose x_i , iff $(\alpha_1^2 + \alpha_2^2 - 1) |x_i|^2 + d^2(\tilde{x}_0, x_i) \leq (\alpha_1^2 + \alpha_2^2 - 1) |x_k|^2 + d^2(\tilde{x}_0, x_k) \forall i \neq k$

For equal energy constellation:

Choose x_i , iff $d^2(\tilde{x}_0, x_i) \leq d^2(\tilde{x}_0, x_k) \forall i \neq k$

3.2.1.3 Alamouti's Scheme Using Multiple Receivers

Alamouti also extended his scheme to the case of two transmit antennas and multiple receive antennas. The encoding and transmission sequence are identical to the case of a single receiver. If the subscript i in the notation h_{ij} , n_{ij} , y_{ij} represents the receiver index, the subscript j denotes the transmitter index in the h_{ij} denotes the time slot in n_{ij} and y_{ij} , the scheme can be generalized as the following:

$$\tilde{x}_1 = \sum_{i=1}^q (h_{i1}^* y_{i1} + h_{i2} y_{i2}^*) = \sum_{i=1}^q (|h_{i1}|^2 + |h_{i2}|^2) x_1 + h_{i1}^* n_{i1} + h_{i2} n_{i2}^* \quad (3.2.1.3.1)$$

$$\tilde{x}_2 = \sum_{i=1}^q (h_{i2}^* y_{i1} - h_{i1} y_{i2}^*) = \sum_{i=1}^q (|h_{i1}|^2 + |h_{i2}|^2) x_1 + h_{i2}^* n_{i1} - h_{i1} n_{i2}^* \quad (3.2.1.3.2)$$

where $i = 1, \dots, q$ and q is the number of receivers.

Signals \tilde{x}_1, \tilde{x}_2 are finally derived and passed to the maximum likelihood detector.

From equations (3.2.1.3.1) (3.2.1.3.2), we can see that the same decision rules can be used again to determine the maximum likelihood transmitted symbols.

3.2.1.4 Alamouti's scheme in fast fading

We will analyze the performance of Alamouti's scheme in fast fading channels.

$$h_1(t) = h_1 = \alpha_1 \exp(j\Theta_1); h_2(t) = h_2 = \alpha_2 \exp(j\Theta_2)$$

$$h_1(t+T) = h_3 = \alpha_3 \exp(j\Theta_3); h_2(t+T) = h_4 = \alpha_4 \exp(j\Theta_4)$$

$$y_1 = h_1 x_1 + h_2 x_2 + n_1$$

$$y_2 = -h_3 x_2^* + h_4 x_1^* + n_1$$

$$\tilde{x}_1 = h_3^* y_1 + h_2 y_2^* = (h_1 h_3^* + h_2 h_4^*) x_1 + h_3^* n_1 + h_2 n_2^* \quad (3.2.1.4.1)$$

$$\tilde{x}_2 = h_4^* y_1 - h_1 y_2^* = (h_1 h_3^* + h_2 h_4^*) x_2 - h_1 n_2^* + h_4^* n_1 \quad (3.2.1.4.2)$$

$$h_1 h_3^* + h_2 h_4^* = \alpha_1 \alpha_3 \exp(j(\Theta_1 - \Theta_3)) + \alpha_2 \alpha_4 \exp(j(\Theta_2 - \Theta_4))$$

Compare (3.2.1.4.1) (3.2.1.4.2) with (3.2.1.2.1) (3.2.1.2.2), we can see that \tilde{x}_1, \tilde{x}_2 experience more distortions (both amplitude and phase distortions). So we expect that the performance of Alamouti's scheme over the flat fading channels is always better than that over the rapid fading channels.

3.2.1.5 Performance of Alamouti's Scheme

Characteristics of Alamouti's scheme include:

- No bandwidth expansion (since redundancy is applied in space across multiple antennas, not in time or frequency)
- Low complexity decoders
- Added reliability as scheme includes a soft failure mode, where the transmitted signal may still be received with lower quality even if diversity gain was lost.
- Identical performance as MRC if the total radiated power is doubled from that used in MRC, else if transmit power is kept constant, this scheme suffers a 3dB penalty in performance.

3.3 Differential Space-Time Codes

Differential space-time codes are a way of transmitting data in wireless communications. They are a form of space-time code that does not need to know the channel impairments at the receiver in order to be able to decode the signal. They are usually based on space-time block codes, and transmit one block-code from a set in response to a change in the input signal. The differences among the blocks in the set are designed to allow the receiver to extract the data with good reliability. The first differential space-time block code was disclosed by Tarokh et al [7].

Chapter 4

Multiple Block Detection (MBD) for Differential Space Time Codes

4.1 Signal Model

A wireless communication system operating over a slow, flat Rayleigh fading channel in which space-time block coded symbols are sent from two transmit antennas and received by B receive antennas is considered. Transmit diversity is achieved by sending a symbol through a transmit antenna followed by its complex conjugate being sent by the other antenna at the next time slot. The time interval for transmitting the l^{th} symbol is divided into two time slots. The signal received at antenna k , $1 \leq k \leq B$ at time slot t ($t=1,2$) is given by

$$s_{k,l}^{(t)} = \sqrt{E_s} \sum_{j=1}^2 h_{j,k} u_{j,l}^{(t)} + \zeta_{k,l}^{(t)} \quad (4.1.1)$$

$h_{j,k}$ is the Gaussian complex fading path gain from transmit antenna j to receive antenna k . Paths gains are unchanged (over a frame of length X) samples of independent, complex-valued Gaussian random variables with zero-mean and variance $\frac{1}{2}$ per dimension. Path gains vary independently from frame to frame. Noise samples $\zeta_{k,l}^{(t)}$ are modeled as independent, zero-mean complex Gaussian random variables with variance $N_0/2$ per dimension; $u_{j,l}$ is the l^{th} transmitted symbol from

antenna j , E_s is the total symbol energy on two transmit antennas at time slot t ($t=1,2$).

The optimal receiver computes the decision metric [6] when the channel state information is known,

$$\sum_{j=1}^2 \sum_{k=1}^B \left| s_{k,l}^{(t)} - \sqrt{E_s} \sum_{j=1}^2 h_{j,k} u_{j,l}^{(t)} \right|^2 \quad (4.1.2)$$

over all symbols $u_{j,l}^1$ ($l=1,2,\dots,X$) at time slot 1: $z_{1,1}, z_{2,1}, z_{1,2}, z_{2,2}, \dots, z_{1,L}, z_{2,L}$ and all symbols $u_{j,l}^2$ ($l=1,2,\dots,X$) at time slot 2: $-z_{2,1}^*, z_{1,1}^*, -z_{2,2}^*, z_{1,2}^*, \dots, -z_{2,L}^*, z_{1,L}^*$ to decide in favor of the sequences that maximizes the metric.

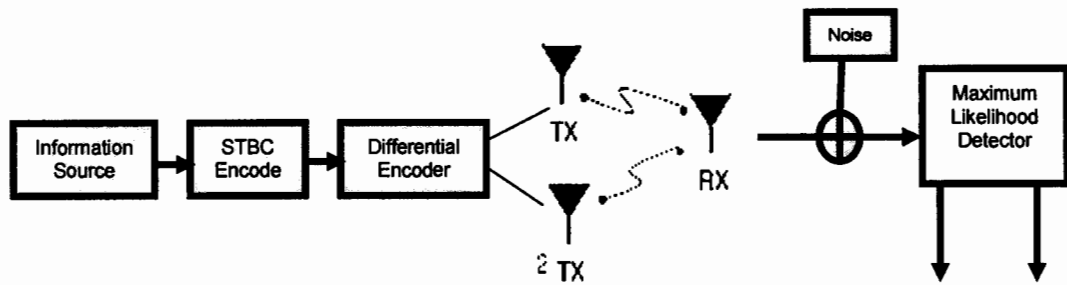


Figure 4.1 System Model

4.2 Differential Encoding

The differential encoding scheme used here is the one proposed by Tarokh et al. [7] and it is based on the Alamouti transmit diversity scheme [5].

The message matrix T_l is defined as:

$$T_l = \begin{bmatrix} t_{1,l} & t_{2,l} \\ -t_{2,l}^* & t_{1,l}^* \end{bmatrix} \quad (4.2.1)$$

The superscript “*” denotes complex conjugation and the symbols $t_{1,l}, t_{2,l}$ belong to an M-PSK constellation. The vectors $[t_{1,l}, t_{2,l}]$ and $[-t_{2,l}^*, t_{1,l}^*]$ have unit length and are orthogonal to each other. It follows that

$$T_l T_l^H = I_2 \quad (4.2.2)$$

The superscript “ H ” denotes Hermitian and I_2 is the 2×2 identity matrix. Transmission of T_l already incorporates transmit diversity [5]. We wish to differentially encode the message T_l . The message T_l can be differentially encoded in a way similar to standard single-antenna DPSK [16]. The transmitter sends a message matrix P_0 , which does not carry any message to initialize transmission. Let

$$P_0 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \quad (4.2.3)$$

The code matrix P_l is defined as;

$$P_l = \begin{bmatrix} P_{1,l} & P_{2,l} \\ -P_{2,l}^* & P_{1,l}^* \end{bmatrix} \quad (4.2.4)$$

The differential STBC scheme consists of sending the code matrix P_l rather than the message matrix T_l directly. The differentially encoded message P_l at time l is obtained by applying the previous code matrix P_{l-1} to the current message T_l :

$$P_l = T_l P_{l-1} \quad (4.2.5)$$

This process is initialized with $P_1 = T_1 P_0$. These relations are similar to single-antenna DPSK. The only difference is that the variables are matrices rather than scalars. The relation in (4.2.5) is consistent with the encoding algorithm in [7]

$$P_1 = T_1 P_{1-1}$$

$$\begin{aligned} \begin{bmatrix} p_{1,j} & p_{2,j} \\ -\dot{p}_{2,j} & \dot{p}_{1,j} \end{bmatrix} &= \begin{bmatrix} t_{1,j} & t_{2,j} \\ -\dot{t}_{2,j} & \dot{t}_{1,j} \end{bmatrix} \begin{bmatrix} p_{1,j-1} & p_{2,j-1} \\ -\dot{p}_{2,j-1} & \dot{p}_{1,j-1} \end{bmatrix} \\ \begin{bmatrix} p_{1,j} & p_{2,j} \\ -\dot{p}_{2,j} & \dot{p}_{1,j} \end{bmatrix} &= \begin{bmatrix} t_{1,j} p_{1,j-1} - t_{2,j} \dot{p}_{2,j-1} & t_{1,j} p_{2,j-1} + t_{2,j} \dot{p}_{1,j-1} \\ -\dot{t}_{2,j} p_{1,j-1} - \dot{t}_{1,j} \dot{p}_{2,j-1} & -\dot{t}_{2,j} p_{2,j-1} + \dot{t}_{1,j} \dot{p}_{1,j-1} \end{bmatrix} \end{aligned} \quad (4.2.6)$$

Consider first row

$$[p_{1,j} \quad p_{2,j}] = [t_{1,j} p_{1,j-1} - t_{2,j} \dot{p}_{2,j-1} \quad t_{1,j} p_{2,j-1} + t_{2,j} \dot{p}_{1,j-1}]$$

$$\begin{bmatrix} p_{1,j} \\ p_{2,j} \end{bmatrix} = \begin{bmatrix} t_{1,j} p_{1,j-1} - t_{2,j} \dot{p}_{2,j-1} \\ t_{1,j} p_{2,j-1} + t_{2,j} \dot{p}_{1,j-1} \end{bmatrix}$$

$$\begin{bmatrix} p_{1,j} \\ p_{2,j} \end{bmatrix} = t_{1,j} \begin{bmatrix} p_{1,j-1} \\ p_{2,j-1} \end{bmatrix} + t_{2,j} \begin{bmatrix} -\dot{p}_{2,j-1} \\ \dot{p}_{1,j-1} \end{bmatrix} \quad (4.2.7)$$

Now consider second row of (4.2.6)

$$[-\dot{p}_{2,j} \quad \dot{p}_{1,j}] = [-\dot{t}_{2,j} p_{1,j-1} - \dot{t}_{1,j} \dot{p}_{2,j-1} \quad -\dot{t}_{2,j} p_{2,j-1} + \dot{t}_{1,j} \dot{p}_{1,j-1}]$$

$$\begin{bmatrix} -\dot{p}_{2,j} \\ \dot{p}_{1,j} \end{bmatrix} = \begin{bmatrix} -\dot{t}_{2,j} p_{1,j-1} - \dot{t}_{1,j} \dot{p}_{2,j-1} \\ -\dot{t}_{2,j} p_{2,j-1} + \dot{t}_{1,j} \dot{p}_{1,j-1} \end{bmatrix}$$

$$\begin{bmatrix} -\dot{p}_{2,j} \\ \dot{p}_{1,j} \end{bmatrix} = -\dot{t}_{2,j} \begin{bmatrix} p_{1,j-1} \\ p_{2,j-1} \end{bmatrix} + \dot{t}_{1,j} \begin{bmatrix} -\dot{p}_{2,j-1} \\ \dot{p}_{1,j-1} \end{bmatrix} \quad (4.2.8)$$

7H-47H

Differential space-time block encoding leads to an expansion of the constellation of the symbols that are differentially encoded. The code matrix P_l has the same unitary property as the message matrix T_l . It is easily verified from the definition (4.2.4) that

$$P_l P_l^H = I_2 \quad (4.2.9)$$

Multiplying (4.2.5) on the left with P_{l-1}^H , based on (4.2.9), we get

$$P_l P_{l-1}^H = T_l P_{l-1} P_{l-1}^H = T_l \quad (4.2.10)$$

It follows that if the code matrices P_l are observable at the receiver, the messages T_l can be decoded from

$$P_l P_{l-1}^H = T_l \quad (4.2.11)$$

$$a > b$$

$$\begin{aligned} P_a P_b^H &= P_a P_{a-1}^H P_{a-1} P_{a-2}^H P_{a-2} P_{a-3}^H \cdots P_{b+2} P_{b+1}^H P_{b+1} P_b^H \\ P_a P_b^H &= T_a T_{a-1} T_{a-2} \cdots T_{b+1} \end{aligned} \quad (4.2.12)$$

4.3 Receiver Model

A single receive antenna is assumed to simplify the notation. We try to find the received signals when the differential code matrix P_I is transmitted. Since only one receive antenna is assumed, from (4.1.1), the corresponding received signals can be written

$$\begin{aligned} s_{1,j}^{(1)} &= \sqrt{E_s} (p_{1,j} h_{1,1} + p_{2,j} h_{2,1}) + \zeta_{1,j}^{(1)} \\ s_{1,j}^{(2)} &= \sqrt{E_s} (-p_{2,j}^* h_{1,1} + p_{1,j}^* h_{2,1}) + \zeta_{1,j}^{(2)} \end{aligned}$$

The above expression can be written in matrix form as:

$$\begin{bmatrix} s_{1,j}^{(1)} \\ s_{1,j}^{(2)} \end{bmatrix} = \sqrt{E_s} \begin{bmatrix} p_{1,j} & p_{2,j} \\ -p_{2,j}^* & p_{1,j}^* \end{bmatrix} \begin{bmatrix} h_{1,1} \\ h_{2,1} \end{bmatrix} + \begin{bmatrix} \zeta_{1,j}^{(1)} \\ \zeta_{1,j}^{(2)} \end{bmatrix} \quad (4.3.1)$$

$s_{1,k}^{(1)}, s_{1,j}^{(2)}$ represent two received signals at time slots 1 and 2, respectively.

Define

$$\begin{aligned} q_{1,j} &= s_{1,j}^{(1)}, q_{2,j} = -s_{1,j}^{(2)*} \\ n_{1,j} &= \zeta_{1,j}^{(1)}, n_{2,j} = -\zeta_{1,j}^{(2)*} \end{aligned}$$

$$\begin{aligned} s_{1,j}^{(1)} &= \sqrt{E_s} (p_{1,j} h_{1,1} + p_{2,j} h_{2,1}) + \zeta_{1,j}^{(1)} \\ q_{1,j} &= \sqrt{E_s} (p_{1,j} h_{1,1} + p_{2,j} h_{2,1}) + n_{1,j} \end{aligned} \quad (4.3.2)$$

$$\begin{aligned} s_{1,j}^{(2)} &= \sqrt{E_s} (-p_{2,j}^* h_{1,1} + p_{1,j}^* h_{2,1}) + \zeta_{1,j}^{(2)} \\ -s_{1,j}^{(2)*} &= \sqrt{E_s} (p_{2,j} h_{1,1}^* - p_{1,j} h_{2,1}^*) - \zeta_{1,j}^{(2)*} \\ -s_{1,j}^{(2)*} &= \sqrt{E_s} (-p_{1,j} h_{2,1}^* + p_{2,j} h_{1,1}^*) - \zeta_{1,j}^{(2)*} \\ q_{2,j} &= \sqrt{E_s} (-p_{1,j} h_{2,1}^* + p_{2,j} h_{1,1}^*) + n_{2,j} \end{aligned} \quad (4.3.3)$$

Joining (4.3.2) and (4.3.3), we have,

$$\begin{aligned}
\begin{bmatrix} q_{1,l} & q_{2,l} \end{bmatrix} &= \left[\sqrt{E_s} (p_{1,l} h_{1,1} + p_{2,l} h_{2,1}) + n_{1,l} \quad \sqrt{E_s} (-p_{1,l} h_{2,1}^* + p_{2,l} h_{1,1}^*) + n_{2,l} \right] \\
\begin{bmatrix} q_{1,l} \\ q_{2,l} \end{bmatrix}^T &= \begin{bmatrix} \sqrt{E_s} (p_{1,l} h_{1,1} + p_{2,l} h_{2,1}) + n_{1,l} \\ \sqrt{E_s} (-p_{1,l} h_{2,1}^* + p_{2,l} h_{1,1}^*) + n_{2,l} \end{bmatrix}^T \\
\begin{bmatrix} q_{1,l} \\ q_{2,l} \end{bmatrix}^T &= \sqrt{E_s} \begin{bmatrix} p_{1,l} h_{1,1} + p_{2,l} h_{2,1} \\ -p_{1,l} h_{2,1}^* + p_{2,l} h_{1,1}^* \end{bmatrix}^T + \begin{bmatrix} n_{1,l} \\ n_{2,l} \end{bmatrix}^T \\
\begin{bmatrix} q_{1,l} \\ q_{2,l} \end{bmatrix}^T &= \sqrt{E_s} \begin{bmatrix} p_{1,l} \\ p_{2,l} \end{bmatrix}^T \begin{bmatrix} h_{1,1} & -h_{2,1}^* \\ h_{2,1} & h_{1,1}^* \end{bmatrix} + \begin{bmatrix} n_{1,l} \\ n_{2,l} \end{bmatrix}^T
\end{aligned} \tag{4.3.4}$$

Also from (4.3.2) and (4.3.3), we get

$$\begin{bmatrix} -q_{2,l}^* \\ q_{1,l}^* \end{bmatrix}^T = \sqrt{E_s} \begin{bmatrix} -p_{2,l}^* \\ p_{1,l}^* \end{bmatrix}^T \begin{bmatrix} h_{1,1} & -h_{2,1}^* \\ h_{2,1} & h_{1,1}^* \end{bmatrix} + \begin{bmatrix} -n_{2,l}^* \\ n_{1,l}^* \end{bmatrix}^T \tag{4.3.5}$$

Putting together (4.3.4) and (4.3.5), we have

$$\begin{bmatrix} q_{1,l} & q_{2,l} \\ -q_{2,l}^* & q_{1,l}^* \end{bmatrix} = \sqrt{E_s} \begin{bmatrix} p_{1,l} & p_{2,l} \\ -p_{2,l}^* & p_{1,l}^* \end{bmatrix} \begin{bmatrix} h_{1,1} & -h_{2,1}^* \\ h_{2,1} & h_{1,1}^* \end{bmatrix} + \begin{bmatrix} n_{1,l} & n_{2,l} \\ -n_{2,l}^* & n_{1,l}^* \end{bmatrix} \tag{4.3.6}$$

Utilizing compact matrix notation for (4.3.6), we obtain

$$Q_l = \sqrt{E_s} P_l H_l + \Omega_l \tag{4.3.7}$$

$$Q_l = \begin{bmatrix} q_{1,l} & q_{2,l} \\ -q_{2,l}^* & q_{1,l}^* \end{bmatrix}$$

$$P_l = \begin{bmatrix} p_{1,l} & p_{2,l} \\ -p_{2,l}^* & p_{1,l}^* \end{bmatrix}$$

$$H_l = \begin{bmatrix} h_{1,1} & -h_{2,1}^* \\ h_{2,1} & h_{1,1}^* \end{bmatrix}$$

$$\Omega_l = \begin{bmatrix} n_{1,l} & n_{2,l} \\ -n_{2,l}^* & n_{1,l}^* \end{bmatrix}$$

For the signal model, consider an observation interval consisting of M blocks of symbols, where, consistent with differential decoding, each block is defined as two symbols at two time slots. A frame consists of X symbol blocks. The channel is assumed constant during a frame, which implies that the channel is fixed during the observation interval. Starting from the l^{th} block Q_l , the received sequence can be expressed as:

$$Q = \sqrt{E_s} PH + \Omega \quad (4.3.8)$$

where

$$\begin{aligned} Q &= [Q_l \quad Q_{l-1} \quad \dots \quad Q_{l-M+1}]^T \\ P &= \text{diag}[P_l \quad P_{l-1} \quad \dots \quad P_{l-M+1}] \\ H &= [H_l \quad H_{l-1} \quad \dots \quad H_{l-M+1}]^T \\ \Omega &= [\Omega_l \quad \Omega_{l-1} \quad \dots \quad \Omega_{l-M+1}]^T \end{aligned}$$

The matrices Q , H and Ω are $2M \times 2$ and P is $2M \times 2M$. For convenience we also define the $2M \times 2M$ matrix $T = \text{diag}[T_l \quad T_{l-1} \quad \dots \quad T_{l-M+1}]$.

4.4 Decision Metric

The observation interval is assumed to be of M blocks. If the CSI is known, from expression (4.3.8), conditioned on the transmitted symbols P and the channel H , the matrix Q is a complex-valued, zero-mean Gaussian random matrix. Its probability density function (PDF) is given by [10]

$$p(Q | H, P) = \frac{1}{\Pi^{4M}} \exp\{-tr[(Q - \sqrt{E_s} PH)^H (Q - \sqrt{E_s} PH)]\} \quad (4.4.1)$$

where “tr” denotes the trace function. If the code matrices are equally likely, the optimal receiver is the maximum-likelihood detector [21], so we can detect P by

$$\begin{aligned} \hat{P} &= \arg \max p(Q | H, \tilde{P}) \\ \hat{P} &= \arg \min tr[(Q - \sqrt{E_s} \tilde{P} H)^H (Q - \sqrt{E_s} \tilde{P} H)] \end{aligned} \quad (4.4.2)$$

If (4.4.2) is spread out, we got the same form as (4.1.2). Expression (4.4.2) is the decision metric for the case that channel is known, but we are concerned with decision metric for situations when the channel is unknown.

Differential encoding can be applied when the channel is unknown, but fixed over some time interval. In this case, the transmitter sends the code matrix P_i instead of sending messages T_i directly. For a block of M observations, the received matrix Q given that message matrix T is transmitted (through code matrix P has a multivariate Gaussian conditional PDF

$$P(Q | T) = \frac{1}{\Pi^{4M} \det R} \exp(-tr(Q^H R^{-1} Q)) \quad (4.4.3)$$

where R is the covariance matrix of Q ,

$$R = E[QQ^H | T]$$

$$R = E[(\sqrt{E_s}PH + \Omega)(\sqrt{E_s}PH + \Omega)^H]$$

$$R = E[E_sPHH^H P^H + \Omega\Omega^H]$$

$$R = E_sPE[HH^H]P^H + E[\Omega\Omega^H]$$

Now consider $E[HH^H]$

$$E[HH^H] = E \begin{bmatrix} h_{1,1} & -h_{2,1}^* \\ h_{2,1} & h_{1,1}^* \\ h_{1,1} & -h_{2,1}^* \\ h_{2,1} & h_{1,1}^* \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ h_{1,1} & -h_{2,1}^* \\ h_{2,1} & h_{1,1}^* \end{bmatrix} \begin{bmatrix} h_{1,1}^* & h_{2,1}^* & h_{1,1}^* & h_{2,1}^* & \cdot & \cdot & \cdot & h_{1,1}^* & h_{2,1}^* \\ -h_{2,1} & h_{1,1} & -h_{2,1} & h_{1,1} & \cdot & \cdot & \cdot & -h_{2,1} & h_{1,1} \end{bmatrix} \quad (4.4.4)$$

$$E[HH^H] = E \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & h_{1,1}h_{2,1}^* - h_{1,1}h_{2,1}^* & \cdot & \cdot & \cdot & \cdot & |h_{1,1}|^2 + |h_{2,1}|^2 & h_{1,1}h_{2,1}^* - h_{1,1}h_{2,1}^* \\ h_{2,1}h_{1,1}^* - h_{2,1}h_{1,1}^* & |h_{1,1}|^2 + |h_{2,1}|^2 & \cdot & \cdot & \cdot & \cdot & h_{2,1}h_{1,1}^* - h_{2,1}h_{1,1}^* & |h_{1,1}|^2 + |h_{2,1}|^2 \\ |h_{1,1}|^2 + |h_{2,1}|^2 & h_{1,1}h_{2,1}^* - h_{1,1}h_{2,1}^* & \cdot & \cdot & \cdot & \cdot & |h_{1,1}|^2 + |h_{2,1}|^2 & h_{1,1}h_{2,1}^* - h_{1,1}h_{2,1}^* \\ h_{2,1}h_{1,1}^* - h_{2,1}h_{1,1}^* & |h_{1,1}|^2 + |h_{2,1}|^2 & \cdot & \cdot & \cdot & \cdot & h_{2,1}h_{1,1}^* - h_{2,1}h_{1,1}^* & |h_{1,1}|^2 + |h_{2,1}|^2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ |h_{1,1}|^2 + |h_{2,1}|^2 & h_{1,1}h_{2,1}^* - h_{1,1}h_{2,1}^* & \cdot & \cdot & \cdot & \cdot & |h_{1,1}|^2 + |h_{2,1}|^2 & h_{1,1}h_{2,1}^* - h_{1,1}h_{2,1}^* \\ h_{2,1}h_{1,1}^* - h_{2,1}h_{1,1}^* & |h_{1,1}|^2 + |h_{2,1}|^2 & \cdot & \cdot & \cdot & \cdot & h_{2,1}h_{1,1}^* - h_{2,1}h_{1,1}^* & |h_{1,1}|^2 + |h_{2,1}|^2 \end{bmatrix}$$

$$E[HH^H] = \begin{bmatrix} E|h_{1,1}|^2 + E|h_{2,1}|^2 & 0 & \cdot & \cdot & E|h_{1,1}|^2 + E|h_{2,1}|^2 & 0 \\ 0 & E|h_{1,1}|^2 + E|h_{2,1}|^2 & \cdot & \cdot & 0 & E|h_{1,1}|^2 + E|h_{2,1}|^2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ E|h_{1,1}|^2 + E|h_{2,1}|^2 & 0 & \cdot & \cdot & E|h_{1,1}|^2 + E|h_{2,1}|^2 & 0 \\ 0 & E|h_{1,1}|^2 + E|h_{2,1}|^2 & \cdot & \cdot & 0 & E|h_{1,1}|^2 + E|h_{2,1}|^2 \end{bmatrix}$$

where

$$E|h_{1,1}|^2 = E|h_{2,1}|^2 = 1/2$$

$$E[HH^H] = \begin{bmatrix} 1 & 0 & 1 & 0 & \dots & 1 & 0 \\ 0 & 1 & 0 & 1 & \dots & 0 & 1 \\ 1 & 0 & 1 & 0 & \dots & 1 & 0 \\ 0 & 1 & 0 & 1 & \dots & 0 & 1 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & 0 & 1 & 0 & \dots & 1 & 0 \\ 0 & 1 & 0 & 1 & \dots & 0 & 1 \end{bmatrix} = I_2 \otimes 1_M \quad (4.4.5)$$

where “ \otimes ” denotes the Kronecker product, and I_M represents an $M \times M$ matrix with all elements equal 1.

$$\begin{aligned} R &= E_s P E[HH^H] P^H + E[\Omega \Omega^H] \\ R &= E_s P (I_2 \otimes 1_M) P^H + N_0 I_{2M} \end{aligned} \quad (4.4.6)$$

It can be shown that $\det R$ is independent of the messages $T_1, T_{l-1}, \dots, T_{l-M+1}$ using the unitary property of the matrix P .

Define the matrices A , B and C :

$$A = N_0 I_{2M} \quad (4.4.7)$$

$$C = E_s I_2 \quad (4.4.8)$$

$$B = P (I_2 \otimes 1_{M \times 1}) \quad (4.4.9)$$

$1_{M \times 1}$ is a vector of ones. Then (4.4.6) can be expressed as

$$\begin{aligned} R &= B C B^H + A \\ R &= E_s P (I_2 \otimes 1_{M \times 1}) I_2 (I_2 \otimes 1_{M \times 1})^H P^H + N_0 I_{2M} \end{aligned} \quad (4.4.10)$$

Using matrix inversion lemma $R^{-1} = A^{-1} - A^{-1}B(C^{-1} + B^H A^{-1}B)^{-1}B^H A^{-1}$ and (4.4.6),

we have

$$\begin{aligned}
R^{-1} &= \frac{I_{2M}}{N_0} - \frac{I_{2M}}{N_0} P(I_2 \otimes 1_{M \times 1}) \left\{ \frac{I_2}{E_s} + (I_2 \otimes 1_{M \times 1})^H P^H \right. \\
&\quad \left. \frac{I_{2M}}{N_0} P(I_2 \otimes 1_{M \times 1}) \right\}^{-1} (I_2 \otimes 1_{M \times 1})^H P^H \frac{I_{2M}}{N_0} \\
R^{-1} &= \frac{I_{2M}}{N_0} - \frac{I_{2M}}{N_0^2} P(I_2 \otimes 1_{M \times 1}) \left\{ \frac{I_2}{E_s} + \frac{M I_2}{N_0} \right\}^{-1} (I_2 \otimes 1_{M \times 1})^H P^H \\
R^{-1} &= \frac{I_{2M}}{N_0} - \frac{I_{2M}}{N_0^2} P(I_2 \otimes 1_{M \times 1}) \frac{N_0 E_s}{N_0 + M E_s} I_2 (I_2 \otimes 1_{M \times 1})^H P^H \\
R^{-1} &= \frac{I_{2M}}{N_0} - \frac{E_s}{N_0(N_0 + M E_s)} P(I_2 \otimes 1_{M \times 1}) I_2 (I_2 \otimes 1_{M \times 1})^H P^H \\
R^{-1} &= \frac{I_{2M}}{N_0} - \frac{E_s}{N_0(N_0 + M E_s)} P(I_2 \otimes 1_M) P^H \tag{4.4.11}
\end{aligned}$$

The natural logarithm is a monotonically increasing function of its argument, maximizing $p(Q|T)$ over T in (4.4.3) is equivalent to maximizing $\ln(p(Q|T))$ over T . Choose the sequence T to maximize logarithm of (4.4.3), which results in the decision metric

$$\begin{aligned}
\ln\{P(Q|T)\} &= \frac{1}{\Pi^{4M} \det R} \exp(-tr(Q^H R^{-1} Q)) \\
\hat{\eta} &= -\ln(\det R) - tr(Q^H R^{-1} Q) \tag{4.4.12}
\end{aligned}$$

$\det R$ is independent of transmitted message

$$\begin{aligned}
\hat{\eta} &= -tr(Q^H R^{-1} Q) \\
\hat{\eta} &= -tr\left\{ \frac{Q^H Q}{N_0} - \frac{E_s}{N_0(N_0 + M E_s)} Q^H P(I_2 \otimes 1_M) P^H Q \right\} \\
\hat{\eta} &= tr\left\{ \frac{E_s}{N_0(N_0 + M E_s)} Q^H P(I_2 \otimes 1_M) P^H Q - \frac{Q^H Q}{N_0} \right\} \tag{4.4.13}
\end{aligned}$$

As $\det R$, $Q^H Q$, N_0 , E_s are independent of transmitted messages, they can be ignored.

Then the decision metric becomes

$$\hat{\eta} = \text{tr}\{Q^H P(I_2 \otimes 1_M)P^H Q\} \quad (4.4.14)$$

Expanding (4.2.14), the metric can be expressed

$$\begin{aligned} \hat{\eta} &= \text{tr}\left\{\sum_{j=0}^{M-1} \sum_{k=0}^{M-1} Q_{l-j}^H P_{l-j} P_{l-k}^H Q_{l-k}\right\} \\ \hat{\eta} &= \text{tr}\left\{\sum_{j=0}^{M-1} Q_{l-j}^H P_{l-j} P_{l-j}^H Q_{l-j}\right\} + \text{tr}\left\{\sum_{j=0}^{M-1} \sum_{k=0}^{M-1} Q_{l-j}^H P_{l-j} P_{l-k}^H Q_{l-k}\right\}_{j \neq k} \end{aligned} \quad (4.4.15)$$

$$P_{l-j} P_{l-j}^H = I_2$$

$$\Rightarrow \text{tr}\left\{\sum_{j=0}^{M-1} Q_{l-j}^H P_{l-j} P_{l-j}^H Q_{l-j}\right\} \text{ is independent of transmitted messages}$$

$$\hat{\eta} = \text{tr}\left\{\sum_{j=0}^{M-1} \sum_{k=0}^{M-1} Q_{l-j}^H P_{l-j} P_{l-k}^H Q_{l-k}\right\}_{j \neq k}$$

$$\hat{\eta} = 2 \text{Retr}\left\{\sum_{j=1}^{M-1} \sum_{k=0}^{j-1} Q_{l-j}^H P_{l-j} P_{l-k}^H Q_{l-k}\right\}$$

$$\hat{\eta} = \text{Retr}\left\{\sum_{j=1}^{M-1} \sum_{k=0}^{j-1} Q_{l-j}^H P_{l-j} P_{l-k}^H Q_{l-k}\right\} \quad (4.4.16)$$

Using the identity for the trace function

$$\text{Retr}\{Q_{l-j}^H P_{l-j} P_{l-k}^H Q_{l-k}\} = \text{Retr}\{Q_{l-k} Q_{l-j}^H P_{l-j} P_{l-k}^H\} \quad (4.4.17)$$

From (4.4.16), (4.4.17) and (4.2.12), we have

$$\hat{\eta} = \text{Retr}\left\{\sum_{j=1}^{M-1} \sum_{k=0}^{j-1} Q_{l-k} Q_{l-j}^H P_{l-j} P_{l-k}^H\right\} \quad (4.4.18)$$

$$\hat{\eta} = \text{Retr}\left\{\sum_{j=1}^{M-1} \sum_{k=0}^{j-1} Q_{l-k} Q_{l-j}^H (T_{l-k} T_{l-k+1} \dots T_{l-j+1})^H\right\} \quad (4.4.19)$$

The differentially encoded message \hat{T} can be detected from

$$\hat{T} = \arg \max \text{Retr}\left\{\sum_{j=1}^{M-1} \sum_{k=0}^{j-1} Q_{l-k} Q_{l-j}^H (\tilde{T}_{l-k} \tilde{T}_{l-k+1} \dots \tilde{T}_{l-j+1})^H\right\} \quad (4.4.20)$$

This is the MBD decision metric for an observation interval of M blocks. Notice that no channel information is required for the signal detection. For an observation interval of M blocks, there are message blocks (the first block P_0 does not contain information). Each block contains two unknown symbols. Hence, for M -PSK

symbols, there are $M^{2(M-1)}$ possible message block sequences $\tilde{T}_l, \tilde{T}_{l-1}, \dots, \tilde{T}_{l-M+2}$. As in single antenna MBD, the complexity of the receiver increases exponentially with the length of the observation interval.

Next, we discuss the special cases of $M=2$ and $M=3$.

4.5 Two Blocks Observation Interval

When $M=2$, (4.4.20) becomes

$$\hat{T}_L = \arg \max \text{Retr}\{Q_l Q_{l-1}^H \tilde{T}_l^H\} \quad (4.5.1)$$

Let T_l be the message matrix that is differentially encoded and transmitted. Then

$$\begin{aligned} Q_l Q_{l-1}^H &= (\sqrt{E_s} P_l H_l + \Omega_l)(\sqrt{E_s} P_{l-1} H_{l-1} + \Omega_{l-1})^H \\ Q_l Q_{l-1}^H &= E_s P_l H_l H_{l-1}^H P_{l-1}^H + \sqrt{E_s} P_l H_l \Omega_{l-1}^H + \sqrt{E_s} \Omega_l H_{l-1}^H P_{l-1}^H + \Omega_l \Omega_{l-1}^H \\ H_l H_{l-1}^H &= \begin{bmatrix} h_{1,1} & -h_{2,1}^* \\ h_{2,1} & h_{1,1}^* \end{bmatrix} \begin{bmatrix} h_{1,1}^* & h_{2,1} \\ -h_{2,1} & h_{1,1} \end{bmatrix} \\ H_l H_{l-1}^H &= \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & h_{1,1} h_{2,1}^* - h_{1,1}^* h_{2,1} \\ h_{2,1} h_{1,1}^* - h_{2,1}^* h_{1,1} & |h_{1,1}|^2 + |h_{2,1}|^2 \end{bmatrix} \\ H_l H_{l-1}^H &= \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & 0 \\ 0 & |h_{1,1}|^2 + |h_{2,1}|^2 \end{bmatrix} \\ H_l H_{l-1}^H &= (|h_{1,1}|^2 + |h_{2,1}|^2) I_2 \\ Q_l Q_{l-1}^H &= (|h_{1,1}|^2 + |h_{2,1}|^2) E_s P_l I_2 P_{l-1}^H + \sqrt{E_s} P_l H_l \Omega_{l-1}^H + \sqrt{E_s} \Omega_l H_{l-1}^H P_{l-1}^H + \Omega_l \Omega_{l-1}^H \\ Q_l Q_{l-1}^H &= (|h_{1,1}|^2 + |h_{2,1}|^2) E_s T_l + \sqrt{E_s} P_l H_l \Omega_{l-1}^H + \sqrt{E_s} \Omega_l H_{l-1}^H P_{l-1}^H + \Omega_l \Omega_{l-1}^H \quad (4.2.1.2) \end{aligned}$$

The last expression is similar to (4.30) in [7]. MBD is a generalization of the differential space-time codes in [7]. Our notation not only enables to express the MBD decision statistic, but also provides a simpler way to express known results for two blocks observation interval.

4.6 Three Blocks Observation Interval

Another special case of interest is an observation interval of $M=3$. A receiver scheme with significant notational complexity was suggested in [14]. Once again, our notation provides for a simple decision statistic expressed as a special case of (4.4.20). We can detect T_l, T_{l-1} by

$$[\hat{T}_l \quad \hat{T}_{l-1}] = \arg \max \text{Re} \{ \text{tr} \{ Q_l Q_{l-1}^H \tilde{T}_l^H + Q_{l-1} Q_{l-2}^H \tilde{T}_{l-1}^H + Q_l Q_{l-2}^H (\tilde{T}_l \tilde{T}_{l-1})^H \} \} \quad (4.6.1)$$

4.7 Simulations and Results

We can detect blocks of differentially encoded signals by observing intervals of different lengths using decision metric in (4.4.20). Fig. 4.2 shows the curve presented by Jafarkhani[7] for BPSK. Fig. 4.3 shows the curve presented by Fan[14] for $M=3$ for BPSK. Fig 4.4 and Fig. 4.5 consist of curves for various observation intervals and for BPSK and 4-PSK modulations, respectively. The curves for $M=2$ correspond to the scheme suggested in [7]. Indeed these curves match those in the reference. The curve for $M=3$ in Fig. 4.2 matches well the results in [14]. Note that there is an almost 0.5 dB improvement by increasing the observation interval from $M=2$ to $M=3$. Since the computation complexity of the decision statistic in (4.2.20) increases exponentially with M , we present results for observation intervals of only 8-blocks (16 symbols). For BPSK, there is about 1.5 dB performance improvement compared to the conventional differential detection. This implies a 1.5 dB and 1 dB gain, respectively, over previously published results for 2 and 3 block observation intervals.

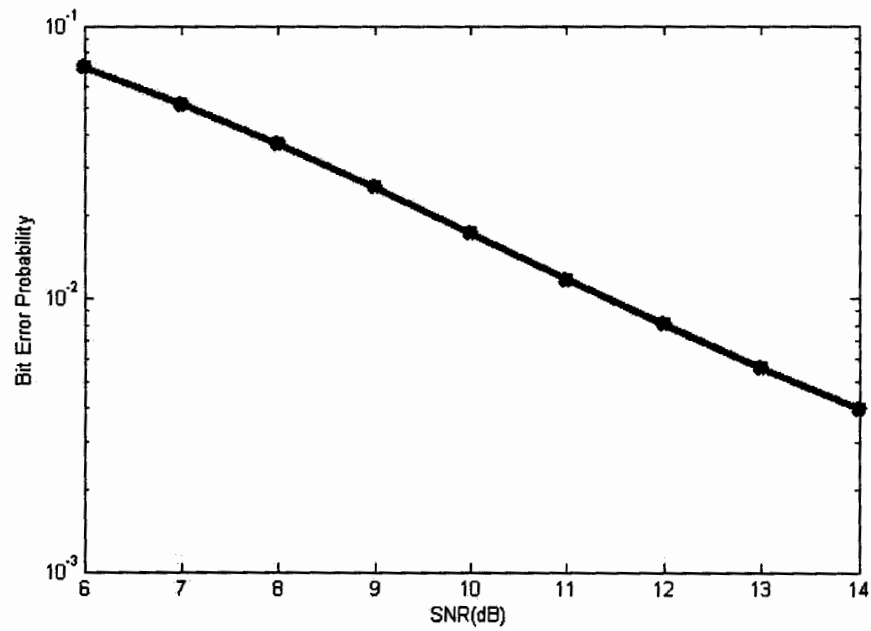


Fig. 4.2 Bit Error Probability versus SNR for $M=2$ using BPSK presented by Jafarkhani[7].

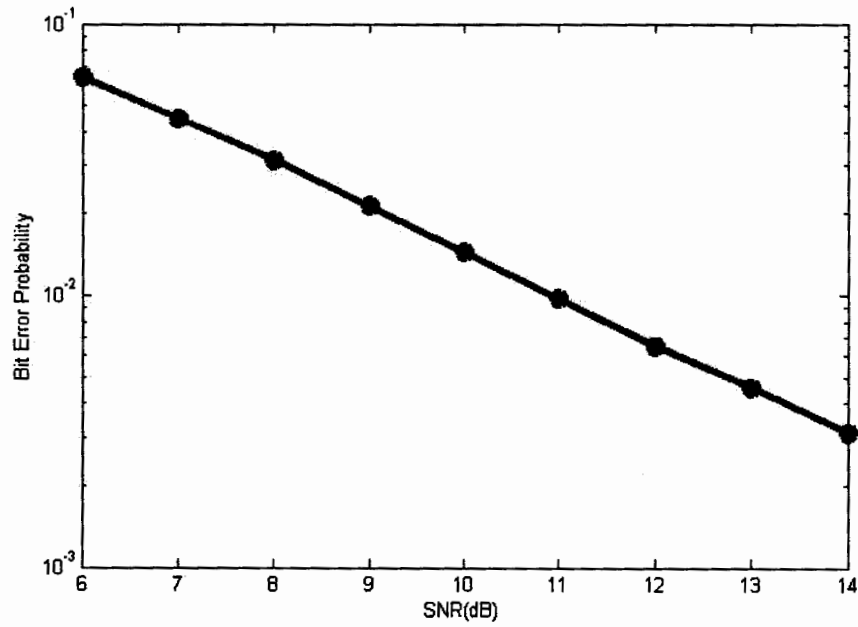


Fig. 4.3 Bit Error Probability versus SNR for $M=3$ using BPSK presented by Fan[14].

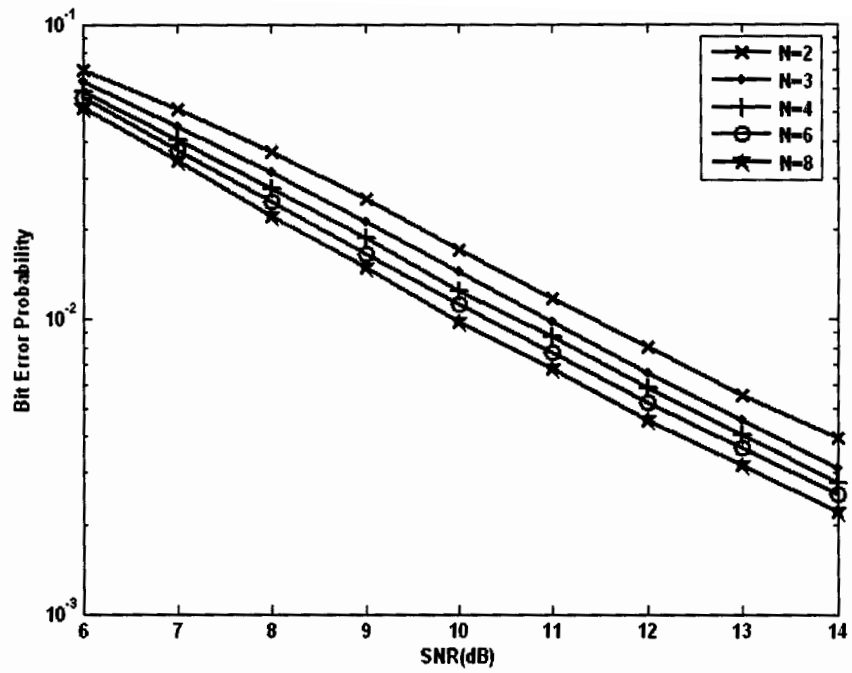


Fig. 4.2. Multiple Block detection for differential space-time code with BPSK signal. Bit Error Probability versus SNR for different length of observation interval, 2 transmit antennas, 1 receive antenna.

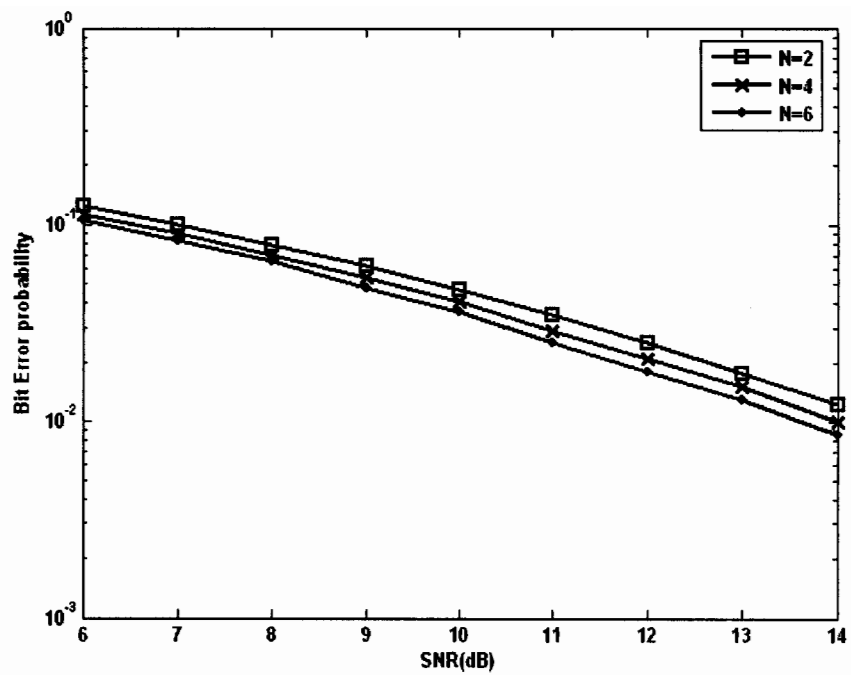


Fig. 4.3. Multiple Block detection for Differential space-time code with QPSK signal. Bit Error Probability versus SNR for different length of observation interval, 2 transmit antennas, 1 receive antenna.

CHAPTER 5

Conclusions and Future Works

A multiple block detector is proposed for differential space-time block codes, in which neither transmitter nor receiver knows the channel state information (CSI). The generalized decision metric for an observation interval of M blocks is derived. Previously published differential STBC schemes' simulations are also presented. It is shown that previously published differential STBC schemes can be obtained as special cases of Multiple Block Detection (MBD). Simulation results demonstrate that MBD can greatly improve the performance of differential STBC. Previously proposed schemes utilizing a two and three block observation interval, incur an SNR performance loss of 3 and 2.5 dB, respectively compared to related coherent detection for BPSK or QPSK modulations.

There are several directions for future work that can be envisioned. One is to increase the size of code (information) matrix by sending more data at one time. Another may be the use of some suboptimal detection technique such as genetic algorithm (GA), Particle swarm optimization (PSO), etc.

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