Three Stage-Least Square Estimates of Money Demand & Money Supply for Pakistan

By

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Faculty of Basic and Applied Sciences
International Islamic University, Islamabad
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- Money demand -- Pakistan
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بسم الله الرحمن الرحيم

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A Dissertation
Submitted in the Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE
IN
STATISTICS

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Certificate

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A DISSERTATION SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF THE ***MS IN STATISTICS***

We accept this dissertation as conforming to the required standard.

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Pakistan

2014

Dedication

To my mother Shabirran Bibi (Late),

For the endless support and patience.

To my Teachers,

For the constant source of Knowledge and Inspiration.

To my friends,

The ones that are close and the ones that are far.

Forwarding Sheet by Research Supervisor

The thesis entitled **"Three Stage-Least Square Estimates of Money Demand & Money Supply for Pakistan"** submitted by **Saqib Munawar** (Registration # 04-FBAS/MSST/F12) in partial fulfillment of M.S degree in Statistics has been completed under my guidance and supervision. I am satisfied with the quality of his research work and allow him to submit this thesis for further process to graduate with Master of Science degree from Department of Mathematics and Statistics, as per IIU Islamabad rules and regulations.

Dated____________________

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SAQIB MUNAWAR

DECLARATION

I hereby declare that this thesis, neither as a whole nor a part of it, has been copied out from any source. It is further declared that I have prepared this dissertation entirely on the basis of my personal efforts made under the supervision of my supervisor **Dr. Zahid Iqbal.** No portion of the work, presented in this dissertation, has been submitted in the support of any application for any degree or qualification of this or any other learning institute.

Signature: ______________________

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ACRONYMS

ADF	Augmented Dickey-Fuller
AEG	Augmented Engel Granger
AIC	Alkaike Information Criteria
ARDL	Auto Regressive Distributed Lag
ARIMA	Autoregressive Integrated Moving Average
CPI	Consumer Price Index
EX	Exchange Rate (PAK/USD)
ECM	Error Correction Model
GDP	Gross Domestic Product
IFS	International Financial Statistics
IG	Inflation Gap
LM1	Log of Money Demand (M1)
LM2	Log of Money Demand (M2)
LM1 (-1)	1^{st} Lag of Money Demand (M1)
LM2 (-2)	2^{nd} Lag of Money Demand (M2)
M1	Narrow Money
M2	Broad Money
M^d	Money Demand
M^s	Money Supply
NEER	Nominal Effective Exchange Rate
OLS	Ordinary Least Square
RM1	Real Money Demand (M1)
RM2	Real Money Demand (M2)

RGDP	Real Gross Domestic Product
SBP	State Bank of Pakistan
SUR	Seemingly Unrelated Regression
VECM	Vector Error Correction Model
VAR	Vector Auto Regressive
WDI	World Develop Indicators
YG	Output Gap
2SLS	Two Stage-Least Square
3SLS	Three Stage-Least Square

ABSTRACT

The basic objective of this study is to estimate the money demand function (M1, M2) and money supply function (M1, M2), by two stage least square (2SLS) and three stage least square (3SLS) techniques for Pakistan using annually time series data from 1961 to 2013. In this study we compare the results of both techniques, and found that results obtained from (2SLS) are unbiased and consistent but not efficient, and the results obtained by (3SLS) procedure are not only unbiased, and consistent, but also efficient. The Augmented Dickey-Fuller test results shows that all the variables used in our simultaneous equation model are not stationary at their level, but stationary at 1st difference. The Co-Integration test results also reveal that there is long run relationship between all the variables; it means that all the variables have same integrity level. The Hausman test is used to check the simultaneity problem in our model, and found the simultaneity between money demand and interest rate. The order condition and rank condition indicates that both the equations are over identified. (2SLS) estimates for money demand (M1) reveals that real money demand is positively affected by real GDP and negatively associated with interest rate and exchange rate (PAK/USD). The real money supply is positively associated with interest rate and negatively impacted by output gap and inflation gap. The DW statistics for money demand function is 0.57 and 0.39 for money supply equation, which is proving the serial correlation problem in both equations. Again we estimates the money demand function and money supply function for (M2), and found the same expected results, real money demand and real GDP has positive relation, but real money demand has negative effect on interest rate and nominal effective exchange rate (NEER). The real money supply shows the positive effect with interest rate and negative effect with inflation gap and output gap. To remove the problem serial correlation within the equations we regress the money demand and money supply on their own lags. After that LM test clarifies that there is no more autocorrelation problem. But still there is a problem; the residuals of both the equations are strongly correlated to each other, which make the results inefficient. Now we estimates the parameters of our model using (3SLS) technique, the money demand function and money supply function for (M1, M2) have the same relation and expected signs, which is not surprising for us, but there is lot of difference in coefficients. The one notable thing in (3SLS) estimates is that, the residuals of both the equations are not correlated to each other. So the results obtained by (3SLS) are more efficient.

Keywords: Money Demand, Money Supply, 2SLS, 3SLS, Pakistan.

Chapter 1

INTRODUCTION

To study about the relation of money demand and its determinants is a prime issue since the stability of money demand function is helpful to make an effective monetary policy. The money demand function helps to ascertain the liquidity needs of the economy (Handa, 2009). Goldfeld (1994) noted that the relationship between money demand and its main determinants is very important to build macroeconomic theories, and is very crucial component to conduct monetary policy.

According to the studies of (Siklos, Barton and Laidler, 2001) the money demand function is important for effective monetary policy formulation and implementation, irrespective of whether the focus is on developing or developed countries. Among others only consider money demand function for developed countries.

The money stock in an economy is determined by the interaction of forces money demand and money supply Najam us Saqib (1986). But unfortunately, a lot of empirical work in Pakistan has been done only to pertain the separate estimations of demand and supply functions. For example, Abe et al, Akhtar have estimated only money demand function, using alternative approaches. The supply side effectiveness on money stock is ignored in these studies.

But Monetary economics provides one of the important tools, that is monetary policy, to deal with the macroeconomic problems of the economy. It is concerned with the supply of money and the demand for money. The effectiveness of the monetary policy, however, depends on the shape and stability of the estimated demand for money function Abdul Qayyum (2000).

Gross domestic product (GDP), interest rate and other macroeconomic variables are valuable indicators to estimate the money demand function. However quantity of money demand and interest rate are simultaneously obtained through the interaction of money demand and money supply. A single equation method gives unbiased and inconsistent estimates but 3 SLS technique can be a helpful to find efficient estimates. A. M. M. Jamal, Yu Hsing (2011) in U. S. A.

However money demand and money supply dynamics determines interest rates, which consequently impact a country's monetary policy objectives. Because of the premium placed on stable money demand, adequately estimating it makes it easier for policy makers to predict the impact of monetary policy on various macroeconomic aggregates such as inflation, output and interest rates Cziraky D, Gillman M, (2006) and Mishkin FS, (2007).

It is clear evidence that demand for money is affected not only by changes in domestic variables such as permanent income, domestic interest rate and price expectations but also by fluctuations in exchange rate expectations and foreign interest rates and domestic monetary policy is fairly ineffective (Arango and Nadiri 1981).When their exist a relationship between money demand and foreign exchange rate, and traditional variables are added, the impact of foreign exchange rate on money demand function cannot be ignored Mundell, (1963).

As Stability of money demand is essential to make monetary and fiscal policy (Ahmed and Islam 2007) and the determination of the factors which affect the money demand e.g. income (y), Interest rate (r), Inflation (ΔP) and Exchange Rate (e) etc. These factors has affect on long run money demand as well as on short run dynamic adjustment of actual money balances to the desired level (Iqbal & Saghir, 2008).

However Chow (1966) investigated the determinants of long run and short run money demand. His model differentiate between a long run equilibrium and short run adjustment mechanism that relates to the change in money supply to the difference between desired (or equilibrium) and actual money holding. The results by Yu Hsing and M.M.Jamal, (2013) in Canada, lower the Treasury bill rate, has higher real GDP or a depreciation of the dollar would increase real money demand and that a higher Treasury bill rate, a decreased inflation gap, or a decreased output gap would increase real money demand.

S.S.Poloz, (1980) examined the interesting issue of simultaneity with respect to the demand for money arguing (correctly) that different economic environments may call forth different monetary policy instruments, assuming that his model and its error structure would remain constant under different policy regimes. While according to A.W Gregory and M.McAleer, (1981), that the direction and magnitude of bias, where it does occur are empirical.

1.1. Objectives of the study

Objectives of my study are as follows.

1. The problem of identification and choice of econometric methods.
2. Over identification, and Two Stage-Least Square (2SLS) technique, for unbiased and consistent estimates, but may be inefficient.
3. Three Stage-Least Square (3SLS) technique to make results unbiased, consistent and efficient.
4. How we can make stable money demand and money supply in Pakistan. (MD = MS)

Chapter 2

REVIEW OF LITRATURE

M.Ishaq.Nadiri (1976) studied about Demand for money in open economies. He took quarterly postwar data for Canada, U.S, Germany, and U.K to estimate his model covering period 1960 to 1975. Because of endogeniety in the model he used two stage-least square (2SLS) methods to find parameters. In his study, he concluded that real cash balances affected not only by changes in domestic variables such as income, domestic interest rate, and price expectation but also by fluctuations in exchange rate, and foreign interest rate.

Najam-us-Saqib and Ather Maqsood Ahmed, (1986) worked on to estimate money demand and money supply functions for Pakistan using simultaneous equation approach. They took the dependent variable real money demand and independent variables as real income (Y) and interest rate (r). On other side money supply equation contained explanatory variables monetary base (MB) and ratio of currency to demand deposit (cc/dd). The data used in this study covered the period from (1959-60 to 1983-84). The ordinary least square (OLS) and 2 stage-least square (2SLS) procedures were applied on both equations money demand and money supply and results obtained from these two procedures were compared. Both the findings revealed that there is big difference in estimates, which suggest that bias introduce by OLS cannot be ignored. The analysis clearly showed that for forecasting simultaneous equation model performs significantly better as compared to single equation model. From their findings it is concluded that (SEM) have improvement over (OLS).

PROFESSOR: **DR. LOOMIS, (2006)** analyzed the demand for money in Kenya and the economic implications of money. In his paper he discussed the specification of the money demand function as applied to Kenya. The data for each variable was annual time series data from 1965 to 2005 fairly ideal sample size. A two stage least square method is used because of the problem of endogeniety and various estimation techniques coupled with forecasting methods such as exponential smoothing and variable forecasts are applied to the model. According to his analysis estimators for the scale variable real national income were positive and statistically significant. On the other hand, the estimator on nominal interest rate was found to be negative as theory predicts albeit not statistically significant in any of tests. The analysis also produced a 10 year ahead forecast from 2006-2015. He also found that as expected, national income positively influences the level of money demanded in the economy whereas nominal rates negatively impact money demand. This knowledge is useful in making appropriate monetary and fiscal policies. These include decreasing unemployment, stabilizing prices and overall economic development.

Q. A. Samad and H. U. Ahmed (2007) considered a simultaneous model that represents interrelationship between nominal money and real output The OLS and 2SLS estimation methods had been used to obtain the parameters of the model. According to their analysis the estimated real output has shown positive and significant relationship for nominal narrow money. However, the price level of agricultural products shows negative and insignificant effect. The study concluded that in order to develop the stable monetary policy and fiscal policy the wholesale price index of agricultural products, exchange rate, investment volume, the ratio of ODA loan for project development need proper care.

A very short study in Vietnam on money demand function occurred by **NGUYEN Huyen Diu, and Wade Donald PFAU (2009).** They investigated the money demand function in Vietnam from the period (1999) to (2009) by using the co-integration analysis and reduced form short run error correction model (ECM). They found the co-integration relation between the money demand, income, foreign interest rate, and stock exchange. Moreover they conclude that money demand in Vietnam was stable in the period (1999-2009).

An empirical analysis of money demand function in Nepal, (2010) studied by **Birendra Bahadur Budha.** This paper used the annual data of Nepal, for empirical analysis over the period of physical year 1997/1998 to 2009/2010. The data included narrow money (M1) and broad money (M2) as dependent variable, interest rate and real GDP used as scale variable. In his study of money demand function, he found the long run relationship between real money demands (M1, M2) and its determinants interest rate and real GDP using Johnson Co-integration test. The vector error correction model (VECM) also proved the short run relationship between the real money balances and its determinants. Furthermore he concludes that "the velocity of (M2) was observed more stable as compared to (M1). It simply means the Reserve Bank of Nepal (RBN) should focused on (M2) for policy purpose.

A.A.M. Jamal and Yu Hsing, (2011) estimated the money demand and money supply Functions simultaneously in United State, taking quarterly data from 1974:Q4 to 2010:Q2. They found the effects of explanatory variables interest rate (r), income (Y), and exchange rate (EX), on money demand, and money supply equation included the independent variables interest rate, inflation gap (IG), and output gap (YG). Both the equations money demand and money supply estimated using 3 stage-least square techniques (3SLS). The results showed that all coefficients had expected signs and significant at 1% level of significance. In money demand equation a high R^2 indicate 94% variation explained by three variables in which GDP (Y) has stronger relation with money demand as compared to other variables. On the other side in money supply function the real money supply is positively affected by interest rate and negatively affected by inflation gap and output gap. In money supply equation (R^2=.72) was low as compared to money demand function. From their analysis on money demand and money supply they conclude that real money supply is more sensitive to output gap than inflation gap. The estimated results have important implications for conducting macroeconomic policies. Since quantity of money demand is useful for predicting (GDP) and macroeconomic variables.

Ahsan Khan (2012) worked on demand for money in Pakistan. He investigated the long run and short run determinants of money demand (M2) in Pakistan covering the period (1973-2010). Actually two statistical techniques were used in this paper 1st one is autoregressive distributive lag model (ARDL) and 2nd is error correction model (ECM), and their comparison was made. The analysis report indicates that "in long run the determinants interest rate, inflation rate, real income and exchange rate have significant effect on money demand (M2)" in Pakistan. The real income and inflation rate has positive effect on real money demand, but the interest rate and nominal exchange rate have negative impact on real money demand. Furthermore, he found that "the (ARDL) model is more appropriate in stabilizing the money demand function as compared to (ECM).

An empirical investigation in Gambia (2012) done by **Kebba Jammeh**. He studied long and short run money demand and its stability in Gambia using quarterly time series data from 1st quarter 1993 to 4th quarter 2008. The Johnson co-integration test showed a long run relationship between money demand and its determinants. Moreover in long run, the money demand had significant effect with its determinants but in short run there was not a significant relationship between money demand and its determinants. By his study we also knew that money can be considered as luxury in Gambia. Furthermore, the error correction model (ECM) showed that in long run the determinants of money demand have the importance, but in short run these are not significant drivers.

The same technique is used by the same author **Yu Hsing and Abul M. M. Jamal, (2013)** to examine the money demand function and the money supply function for Canada simultaneously. Covering the period from 1968:Q1 to 2011:Q4. The three-stage least squares method was used in estimating regression parameters. Major findings were that a lower interest rate has higher real GDP or depreciation in Canadian dollar would increase real money demand and that a higher interest rate, a decreased inflation gap, or a decreased output gap would increase real money supply. They analyzed three policies from their study. First, in estimating the money demand function, the money supply function should not be treated as exogenous and assumed to be unresponsive to the interest rate. Second, a change in the policy rate and other related interest rates in response to inflation targeting are expected to affect real money supply. Third, while the inflation gap is a major variable in the money supply function, the output gap is also significant in affecting monetary policy.

In (2014) again **A.M.M.Jamal and Yu Hsing** published another paper, about the money demand and money supply of Australia. They used the same technique three stage-least square (3SLS) to find the empirical relations between the variables. They found that money demand was positively associated with Real GDP and Nominal Effective Exchange Rate, but negatively impacted by interest rate. The money supply positively affected by interest rate, and negatively associated with inflation gap and output gap. At the end they suggested that the Federal Reserve Bank reduce money supply if the inflation gap or output gap increase.

Chapter 3

RESEARCH METHODOLOGY

3.1. Source of Data

The data used in this study was obtained from the appendixes of International Financial Statistics (IFS) of Pakistan, and World Development Indicators (WDI). The annual data is used, which covered the period from 1961 to 2013 both inclusive, giving the total of 54 observations. The variables used in this study are real money demand (M1, M2) and real money supply (M1, M2), nominal interest rate, real GDP, exchange rate (PAK/USD), and the nominal effective exchange rate (NEER) is also used as exchange rate, inflation gap and output gap.

3.2. Description of variables

3.2-1 Narrow money (M1)

This is a category of money, which includes all physical money like currency and coins along with demand deposits and other liquid assets held by the Central Bank. We can say "the money which is easily convertible into cash".

3.2-2 Broad Money (M2)

It is type of money which includes M1 plus short-term time deposits in banks and 24-hour money market funds. Usually, time deposits are much larger than both currency in circulation and demand deposits.

3.2-3 Real money demand

Real money balances expresses the quantity of money in terms of the quantity of goods and services, it can buy. This amount M/P is called real money balance. The demand for real money means to desire holding of money. In this study the real money demand is calculated by CPI (2005=100). Data source of money demand (M1, M2) is (WDI) and unit of data is local currency. We will denote this variable in our model as (M^d).

3.2-4 Gross Domestic Product (GDP)

Gross domestic product (GDP) is the market value of all officially recognized final goods and services produced within an economy in a year, or other given specific period of time.

3.2-5 Real GDP

According to the economists the real GDP is the value of goods and services measured using a constant set of prices. Here GDP which is calculated by expenditure approach without making any deductions of natural resources. Real GDP is calculated by CPI (2005=100) i.e. Y/P. data source is (WDI) and GDP unit is local currency unit. The denotation of this variable in model is (Y)

3.2-6 Interest rate

An interest rate is the rate at which interest is paid by a borrower (debtor) for the use of money that they borrow from a lender (creditor). Here unit of interest rate is measured in percentage. There are so many types of interest rate; in our study we use discount rate. The data source of discount rate is State Bank of Pakistan (SBP). In model this variable will be denoted by (R).

3.2-7 Exchange rate

The price of a nation's currency in terms of another currency, or we can say the Rate at which one currency may be converted into another. There are a wide variety of factors which influence the exchange rate, such as interest rates, inflation, and the state of politics and the economy in each country. Here in our study, we will check the effect of exchange rate on money demand (M1, M2). The exchange rate is used in model (PAK/USD). The data source is international financial statistics (IFS).

3.2-8 Nominal effective exchange rate (NEER)

The NEER represents the relative value of a home country's currency compared to the other major currencies being traded e.g. (U.S. dollar, Japanese yen, euro, etc.). Data source is (IFS). In Pakistan the Nominal Effective Exchange Rate (NEER) is calculated using geometric mean as an average.

$$NEER = \frac{I}{\exp\left[\sum_{i=1}^{N} w_i \ln(I_i)\right]}$$

Where;

I = exchange rate index of US dollar per currency of compiling economy (Pakistan)

I_i = exchange rate index of US dollar per currency of trading partner's currency

W_i = trade weights for the countries

N = number of trading partner countries

3.2-9 Output gap

The output gap is the difference between actual GDP and potential GDP. The calculation for the output gap is Y–Y* divided by Y* where Y is actual output and Y* is potential output. (YG) is denotation in the model.

$$YG = \frac{actual(GDP) - potential(GDP)}{potential(GDP)}$$

3.2-10 Potential GDP

Potential GDP is a measure of the real value of the services and goods that can be produced when a country's factors of production are fully employed. It is also known as production capacity of an economy. This potential output is generally higher than the GDP, of a country. The data of potential GDP commonly not available, so it can be calculate. Here we use hodrick-prescot filter method to find potential GDP.

3.2-11 Inflation gap

An inflation gap is the difference between the actual inflation rate and the target inflation rate. By Taylor rule (1993) the target inflation rate should be 2%. After taking the difference of actual and target inflation rate we can get the inflation gap. Inflation rate unit is percentage and data source is (WDI). This variable is denoted by (IG).

3.3. Model specification

Money demand function:

$$\log(M_t^d) = \alpha_0 + \alpha_1 \log(R_t) + \alpha_2 \log(Y_t) + \alpha_3 \log(EX_t) + u_{1t} \quad (1)$$

Money supply function:

$$\log(M_t^s) = \beta_0 + \beta_1 \log(R_t) + \beta_2 (YG_t) + \beta_3 (IG_t) + u_{2t} \quad (2)$$

In equilibrium, we have

$$M^d = M^s \quad (3)$$

Where,

M^d = real demand for money (M1, M2)

M^s = real money supply (M1, M2)

R = Interest Rate

Y = real gross domestic product (GDP)

EX = Nominal Effective Exchange Rate (NEER)

YG = output gap,

IG = inflation gap.

t = time subscript.

In our model, we take all the variables in log form except two variables inflation gap (IG) and output gap (YG).

3.4. Simultaneity problem

There is a strong assumption of least square method to a single equation that explanatory variables are truly exogenous, or we can say that there is only one way causation between dependent variable and explanatory variables. If this assumption is not fulfilled we say that there is simultaneity problem or two way causation between dependent variable and independent variable. The same problem we are facing in our model.

The model is given equation (1), (2) and (3) as below

$$\log(M_t^d) = \alpha_0 + \alpha_1 \log(R_t) + \alpha_2 \log(Y_t) + \alpha_3 \log(EX_t) + u_{1t}$$

$$\log(M_t^s) = \beta_0 + \beta_1 \log(R_t) + \beta_2 YG_t + \beta_3 IG_t + u_{2t}$$

$$M_t^d = M_t^s$$

Here real money demand (M^d) and real money supply (M^s) are explained as dependent variables and R, Y, EX, YG, and IG are explained as independent variables. By theory except interest rate (R) all the variables are considered as exogenous variables but interest rate (R) is not exogenous variable because there is two way causation between interest rate (R) and real money demand (M^d), so we can say there is simultaneity problem as $M^d = f(R)$ but also $R = f(M^d)$. the assumption of truly exogenous is not fulfilled here, so we are not allowed to single equation model for the description of variables between M^d and R. because if we run the OLS (ordinary least square) regression on the above models the results would be biased and inconsistent due to the problem of simultaneity.

3.4-1 Simultaneity test

The above description about the model only based on the econometric theory, now we statistically proved whether there exist simultaneity problem or not. For this purpose we use Hausman test.

3.4-2 Hausman test

As mentioned above that Hausman test is used to check the simultaneity problem, the Hausman test contained two steps.

1. Regress the endogenous variable (R) on all the explanatory variables Y, EX, YG, and IG to obtain the residuals $\hat{v}_t$
2. Regress the (M^d) on estimated (R) and estimated residual v. and perform t-test on the coefficient of estimated residual.

1^{st} step:

Putting both equations (1) and (2) in equation (3).

$$\alpha_0 + \alpha_1 \log(R_t) + \alpha_2 \log(Y_t) + \alpha_3 \log(EX_t) + u_{1t} = \beta_0 + \beta_1 \log(R_t) + \beta_2 YG_t + \beta_3 IG_t + u_{2t}$$

$$(\alpha_1 - \beta_1)\log(R_t) = (\beta_0 - \alpha_0) + \beta_2 YG_t - \alpha_2 \log(Y_t) + \beta_3 IG_t - \alpha_3 \log(EX_t) + u_{2t} - u_{1t}$$

$$\log(R_t) = \frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} + \frac{\beta_2}{\alpha_1 - \beta_1} YG_t - \frac{\alpha_2}{\alpha_1 - \beta_1}\log(Y_t) + \frac{\beta_3}{\alpha_1 - \beta_1} IG_t - \frac{\alpha_3}{\alpha_1 - \beta_1}\log(EX_t) + \frac{u_{2t} - u_{1t}}{\alpha_1 - \beta_1}$$

We can write this equation as

$$\log(R_t) = \prod_{10} + \prod_{11} YG_t + \prod_{12} \log(Y_t) + \prod_{13} IG_t + \prod_{14} \log(EX_t) + v_1 \qquad (4)$$

Where,

$$\prod_{10} = \frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} \quad (4.1)$$

$$\prod_{11} = \frac{\beta_2}{\alpha_1 - \beta_1} \quad (4.2)$$

$$\prod_{12} = -\frac{\alpha_2}{\alpha_1 - \beta_1} \quad (4.3)$$

$$\prod_{13} = \frac{\beta_3}{\alpha_1 - \beta_1} \quad (4.4)$$

$$\prod_{14} = -\frac{\alpha_3}{\alpha_1 - \beta_1} \quad (4.5)$$

$$v_1 = \frac{u_{2t} - u_{1t}}{\alpha_1 - \beta_1} \quad (4.6)$$

Here we get reduced form equation in which (R) is used as endogenous variable at left hand side and Y, EX, YG, and IG are used as explanatory variables on right hand side. Now applying the OLS (ordinary least square) method on equation (4) we get the residuals.

There is no need to discuss the results of OLS estimates because we just run regression on equation (4) to obtain the residuals of the equation. So we can write from equation (4).

$$\log(R_t) = \hat{R}_t + \hat{v}_t \quad (5)$$

Using equation (5) we can find the $\hat{R}_t$ (R-hat) and $\hat{v}_t$ (v-hat) alternatively as

$$\hat{R}_t = \log(R_t) - \hat{v}_t \quad (6)$$

$$\hat{v}_t = \log(R_t) - \hat{R}_t \quad (7)$$

2^{nd} step:

Now putting the equation (5) in equation (1) and (2) we get the concerned equations as follows.

$$\log(M_t^d) = \alpha_0 + \alpha_1(\hat{R}_t + \hat{v}_t) + \alpha_2 \log(Y_t) + \alpha_3 \log(EX_t) + u_{1t}$$

$$\log(M_t^s) = \beta_0 + \beta_1(\hat{R}_t + \hat{v}_t) + \beta_2 YG_t + \alpha_3 IG_t + u_{2t}$$

Let $\alpha_1 = \delta$ and $\beta_1 = \delta$, here δ is considered as special coefficient of error term.

These are our concerned two equations on which we will apply Hausman test to verify the problem of simultaneity.

$$\log(M_t^d) = \alpha_0 + \alpha_1 \hat{R}_t + \delta\hat{v}_t + \alpha_2 \log(Y_t) + \alpha_3 \log(EX_t) + u_{1t} \quad (8)$$

$$\log(M_t^s) = \beta_0 + \beta_1 \hat{R}_t + \delta\hat{v}_t + \beta_2 YG_t + \beta_3 IG_t + u_{1t} \quad (9)$$

If the coefficient of residual term ($\hat{v}_t$) for equation (8) & (9) would be significant we reject the null hypothesis H_0 means that there will be simultaneity.

3.5. Effects of simultaneity problem in our model

It is proved from the above discussion that there is joint dependence between real money demand (M^d) and nominal interest rate (R). Here we can say (R) is random variable which is not independent of error term; it means there is correlation between interest rate and error term. But the OLS assumption is that the explanatory variables should be independent with error term, which is not fulfilling in our model. If we apply OLS on our model in presence of simultaneity between two variables our results would be biased and inconsistent. Now we prove mathematically how can be our results biased and inconsistent.

We have to prove that

$$Cov(R_t, u_{2t}) \neq 0 \qquad (10)$$

By definition

$$Cov(R_t, u_{2t}) = E[\{u_{2t} - E(u_{2t})\}\{R_t - E(R_t)\}] \qquad (10.1)$$

As we know that

$$E(u_{2t}) = 0$$

So

$$Cov(R_t, u_{2t}) = E[u_{2t}R_t - u_{2t}E(R_t)] \qquad (10.2)$$

By equating both the equations (1) and (2), from our model.

$$\alpha_0 + \alpha_1 \log(R_t) + \alpha_2 \log(Y_t) + \alpha_3 \log(EX_t) + u_{1t} = \beta_0 + \beta_1 \log(R_t) + \beta_2 YG_t + \beta_3 IG_t + u_{2t}$$

$$(\alpha_1 - \beta_1)\log(R_t) = (\beta_0 - \alpha_0) + \beta_2 YG_t - \alpha_2 \log(Y_t) + \beta_3 IG_t - \alpha_3 \log(EX_t) + u_{2t} - u_{1t}$$

$$\log(R_t) = \frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} + \frac{\beta_2}{\alpha_1 - \beta_1} YG_t - \frac{\alpha_2}{\alpha_1 - \beta_1}\log(Y_t) + \frac{\beta_3}{\alpha_1 - \beta_1} IG_t - \frac{\alpha_3}{\alpha_1 - \beta_1}\log(EX_t) + \frac{u_{2t} - u_{1t}}{\alpha_1 - \beta_1}$$

Taking expectation

$$E(\log R_t) = E\left[\frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} + \frac{\beta_2}{\alpha_1 - \beta_1} YG_t - \frac{\alpha_2}{\alpha_1 - \beta_1}\log(Y_t) + \frac{\beta_3}{\alpha_1 - \beta_1} IG_t - \frac{\alpha_3}{\alpha_1 - \beta_1}\log(EX_t) + \frac{u_{2t} - u_{1t}}{\alpha_1 - \beta_1}\right]$$

$$E(\log R_t) = \begin{bmatrix} \frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} + \frac{\beta_2}{\alpha_1 - \beta_1} YG_t - \frac{\alpha_2}{\alpha_1 - \beta_1}\log Y_t + \frac{\beta_3}{\alpha_1 - \beta_1} IG_t - \frac{\alpha_3}{\alpha_1 - \beta_1}\log EX_t + \\ \frac{E(u_{2t}) - E(u_{1t})}{\alpha_1 - \beta_1} \end{bmatrix}$$

As, $E(u_{2t}) = 0$ and $E(u_{1t}) = 0$

$$E(\log R_t) = \frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} + \frac{\beta_2}{\alpha_1 - \beta_1} YG_t - \frac{\alpha_2}{\alpha_1 - \beta_1} \log Y_t + \frac{\beta_3}{\alpha_1 - \beta_1} IG_t - \frac{\alpha_3}{\alpha_1 - \beta_1} \log EX_t$$

Now putting log (R_t) and E ($\log R_t$) in equation (10.2)

$$\text{cov}(R_t, u_{2t}) = E\left[u_{2t}\left\{\begin{matrix}\frac{\beta_0 - \alpha_0}{\alpha - \beta_1} + \frac{\beta_2}{\alpha_1 - \beta_1} YG_t - \frac{\alpha_2}{\alpha_1 - \beta_1} \log Y_t + \frac{\beta_3}{\alpha_1 - \beta_1} IG_t - \frac{\alpha_3}{\alpha - \beta_1} \log EX_t \\ + \frac{u_{2t} - u_{1t}}{\alpha_1 - \beta_1}\end{matrix}\right\}\right]$$

$$- E\left[u_{2t}\left\{\frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} + \frac{\beta_2}{\alpha_1 - \beta_1} YG_t - \frac{\alpha_2}{\alpha_1 - \beta_1} \log Y_t + \frac{\beta_3}{\alpha_1 - \beta_1} IG_t - \frac{\alpha_3}{\alpha_1 - \beta_1} \log EX_t\right\}\right]$$

$$\text{cov}(R_t, u_{2t}) = E\left[\frac{u_{2t}}{\alpha_1 - \beta_1}\left\{\begin{matrix}\beta_0 - \alpha_0 + \beta_2 YG_t - \alpha_2 \log Y_t + \beta_3 IG_t - \alpha_3 \log EX_t + u_{2t} - u_{1t} - \beta_0 \\ + \alpha_0 - \beta_2 YG_t + \alpha_2 \log Y_t - \beta_3 IG_t + \alpha_3 \log EX_t\end{matrix}\right\}\right]$$

By simplifying

$$\text{cov}(R_t, u_{2t}) = E\left[\frac{u_{2t}}{\alpha_1 - \beta_1}\{u_{2t} - u_{1t}\}\right]$$

$$\text{cov}(R_t, u_{2t}) = \frac{E(u_{2t}^2)}{\alpha_1 - \beta_1}$$

So $\text{cov}(R_t, u_{2t}) = \frac{\delta^2 u_t}{\alpha_1 - \beta_1} \neq 0$ (biased)

Since the application of Ordinary Least Square (OLS), to an equation belonging to a simultaneous equation or the presence of simultaneity problem yields biased and inconsistent results.

Now the problem is that what we should do to find good estimates which are unbiased and consistent. The obvious solution is to apply other method of estimation, which gives better estimates of our concerned parameters. There are so many methods which can be used under this situation. But here in our study we use following two methods.

1. **Two stage least square method (2SLS)**
2. **Three stage least square method (3SLS)**

These two techniques can be used to obtain unbiased and consistent estimates of our model. But these methods also have some requirements, which are identification of the model. Now we check the identification of our model, whether the model is identified or under identified. If identified whether it's exactly identified or over identified. After that we shall be able to choose a suitable technique for our model. If the model exactly identified then indirect least square method (ILS) would be best to find good estimators, but if model is over identified we will chose two stage least square (2SLS) and three stage least square techniques.

3.6. Over identification by reduced form

Now by equating equations (1) and (2)

$$\alpha_0 + \alpha_1 \log R_t + \alpha_2 \log Y_t + \alpha_3 \log EX_t + u_{1t} = \beta_0 + \beta_1 \log R_t + \beta_2 YG_t + \beta_3 IG_t + u_{2t}$$

$$(\alpha_1 - \beta_1)\log R_t = (\beta_0 - \alpha_0) + \beta_2 YG_t - \alpha_2 \log Y_t + \beta_3 IG_t - \alpha_3 \log EX_t + u_{2t} - u_{1t}$$

$$\log R_t = \frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} + \frac{\beta_2}{\alpha_1 - \beta_1} YG_t - \frac{\alpha_2}{\alpha_1 - \beta_1} \log Y_t + \frac{\beta_3}{\alpha_1 - \beta_1} IG_t - \frac{\alpha_3}{\alpha_1 - \beta_1} \log EX_t + \frac{u_{2t} - u_{1t}}{\alpha_1 - \beta_1}$$

We can write this equation as

$$\log(R_t) = \prod_{10} + \prod_{11} YG_t + \prod_{12} \log(Y_t) + \prod_{13} IG_t + \prod_{14} \log(EX_t) + v_1 \qquad (11)$$

Putting equation (11) in (1)

$$\log M_t^d = \alpha_0 + \alpha_1 \left[\frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} + \frac{\beta_2}{\alpha_1 - \beta_1} YG_t - \frac{\alpha_2}{\alpha_1 - \beta_1} \log Y_t + \frac{\beta_3}{\alpha_1 - \beta_1} IG_t - \frac{\alpha_3}{\alpha_1 - \beta_1} \log EX_t + \frac{u_{2t} - u_{1t}}{\alpha_1 - \beta_1} \right]$$
$$+ \alpha_2 \log Y_t + \alpha_3 \log EX_t + u_{1t}$$

$$\log M_t^d = \frac{\alpha_1 \beta_0 - \alpha_0 \beta_1}{\alpha_1 - \beta_1} + \frac{\alpha_1 \beta_2}{\alpha_1 - \beta_1} YG_t + \frac{\alpha_1 \beta_3}{\alpha_1 - \beta_1} IG_t - \frac{\alpha_2 \beta_1}{\alpha_1 - \beta_1} \log Y_t - \frac{\alpha_3 \beta_1}{\alpha_1 - \beta_1} \log EX_t + \frac{\alpha_1 u_{2t} - \beta_1 u_{1t}}{\alpha_1 - \beta_1}$$

$$\log(M_t^d) = \prod_{20} + \prod_{21} YG_t + \prod_{22} IG_t + \prod_{23} \log Y_t + \prod_{24} \log EX_t + v_2 \qquad (12)$$

Equations (11) and (12) are reduced form equations, where

$$\prod_{10} = \frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} \qquad (11.1) \qquad \prod_{20} = \frac{\alpha_1 \beta_0 - \alpha_0 \beta_1}{\alpha_1 - \beta_1} \qquad (12.1)$$

$$\prod_{11} = \frac{\beta_2}{\alpha_1 - \beta_1} \qquad (11.2) \qquad \prod_{21} = \frac{\alpha_1 \beta_2}{\alpha_1 - \beta_1} \qquad (12.2)$$

$$\prod_{12} = -\frac{\alpha_2}{\alpha_1 - \beta_1} \qquad (11.3) \qquad \prod_{22} = \frac{\alpha_1 \beta_3}{\alpha_1 - \beta_1} \qquad (12.3)$$

$$\prod_{13} = \frac{\beta_3}{\alpha_1 - \beta_1} \qquad (11.4) \qquad \prod_{23} = -\frac{\alpha_2 \beta_1}{\alpha_1 - \beta_1} \qquad (12.4)$$

$$\prod_{14} = -\frac{\alpha_3}{\alpha_1 - \beta_1} \qquad (11.5) \qquad \prod_{24} = -\frac{\alpha_3 \beta_1}{\alpha_1 - \beta_1} \qquad (12.5)$$

$$v_1 = \frac{u_{2t} - u_{1t}}{\alpha_1 - \beta_1} \qquad (11.6) \qquad v_2 = \frac{\alpha_1 u_{2t} - \beta_1 u_{1t}}{\alpha_1 - \beta_1} \qquad (12.6)$$

The money demand function (1) and money supply function (2) contained eight structural coefficients, but there are ten equations to estimate them. The ten reduced form coefficients are given above. So it is clear that numbers of equations are greater than the number of parameters. So unique estimation of our model is not possible, which can be shown easily.

From the above reduced form coefficients we can obtain the coefficient of interest rate (α_1) as

$$\alpha_1 = \frac{\prod_{21}}{\prod_{11}} = \frac{\alpha_1\beta_2}{\alpha_1-\beta_1} * \frac{\alpha_1-\beta_1}{\beta_2} \qquad (12.1.1)$$

And similarly α_1 may be calculated as follows

$$\alpha_1 = \frac{\prod_{22}}{\prod_{12}} = \frac{\alpha_1\beta_3}{\alpha_1-\beta_1} * \frac{\alpha_1-\beta_1}{\beta_3} \qquad (12.1.2)$$

So we can see here that there are two estimates of interest rate coefficient (α_1) in money demand function. Therefore the money demand equation is over-identified.

Now substituting equation (11) into (2)

$$\log M_t^s = \beta_0 + \beta_1\left[\frac{\beta_0-\alpha_0}{\alpha_1-\beta_1} + \frac{\beta_2}{\alpha_1-\beta_1}YG_t - \frac{\alpha_2}{\alpha_1-\beta_1}\log Y_t + \frac{\beta_3}{\alpha_1-\beta_1}IG_t - \frac{\alpha_3}{\alpha_1-\beta_1}\log EX_t + \frac{u_{2t}-u_{1t}}{\alpha_1-\beta_1}\right]$$
$$+\beta_2 YG_t + \beta_3 IG_t + u_{2t}$$

$$\log M_t^s = \frac{\alpha_1\beta_0-\alpha_0\beta_1}{\alpha_1-\beta_1} + \frac{\alpha_1\beta_2}{\alpha_1-\beta_1}YG_t + \frac{\alpha_1\beta_3}{\alpha_1-\beta_1}IG_t - \frac{\alpha_2\beta_1}{\alpha_1-\beta_1}\log Y_t - \frac{\alpha_3\beta_1}{\alpha_1-\beta_1}\log EX_t + \frac{\alpha_1 u_{2t}-\beta_1 u_{1t}}{\alpha_1-\beta_1}$$

$$\log(M_t^s) = \prod_{30} + \prod_{31}YG_t + \prod_{32}IG_t + \prod_{33}\log Y_t + \prod_{34}\log EX_t + v_3 \qquad (13)$$

Here again in money supply equation (α_1) can be calculated by two ways, so money supply function is also over-identified.

There are two estimates for coefficient of interest rate (R) in supply function and there is no guarantee that these two estimates will be same. So here we can conclude that indirect least square (ILS) approach is not appropriate. As we noted from above discussion that our model is identified, now we consider an alternative approach which is less time consuming method of determining whether an equation in simultaneous equation model is identified.

3.7. Identification

A model of equations will be identified when all the structural coefficients can be obtained with the help of reduce form equations. If this cannot be done we say particular equation is under-identified. In econometric theory equations can be identified or under-identified. If an equation is identified then it can be exactly identified or over identified.

3.7-1 Exactly Identified

An equation said to be exactly identified if unique numerical values of the structural parameters can be obtained. In other words we say if the number of unknowns is equal to the number of equations then equation is exactly identified.

3.7-2 Over-identified

If more than one numerical estimate can be obtained for some of the parameters of the structural equations then we say particular equation is over-identified, or if the number of equations are greater than the number of unknowns the particular equation will be over-identified.

3.8. Conditions for identification

There are two conditions which must be fulfilled for an equation to be identified.

1. Order condition (necessary condition)
2. Rank condition (necessary and sufficient condition)

3.8-1 Order condition (necessary condition)

According to order condition the equation will be identified if the total number of variables both endogenous and exogenous excluded from it must be equal to or greater than the number of endogenous variables in the model less one.

The order condition symbolically expressed as follows

$$(K-M)\geq(G-1) \tag{14}$$

Where,

K = Total number of variables in the model (endogenous and pre-determined)

M = Total number of variables in a particular equation (endogenous and exogenous)

G = Total number of equations or total number of endogenous variables.

Now we apply order condition on our simultaneous equation model

As order condition numerically expressed as

$$(K-M)\geq(G-1)$$

For equation (1)

K = 7, M = 4, G = 3,

$$(7-4)\geq(3-1)$$

As we conclude that (3>2), so money demand equation (1) is over identified.

For equation (2)

K = 7, M = 4, G = 3,

$$(7-4)\geq(3-1)$$

Here we see that money supply equation (2) also over identified.

3.8.2 Rank condition (necessary and sufficient condition)

Order condition is necessary condition for identification, but rank condition not only necessary as well as sufficient condition for identification of equations. Because some time it happens that equation is identified by order condition but same equation is not identified by rank condition. So rank condition, required for identification.

In a system of G equations any particular equation is identified if it construct at least one non-zero determinant of order (G-1) from the coefficients of variables excluded from that particular equation but included in other equation of the model.

Identification of an equation by rank condition the following steps will be hold.

1. Write the parameters of all equations in a table, if a variable is excluded from an equation put its value zero.
2. Strike out the equation which is being examined for identification.
3. Strike out the column where the non-zero coefficient appears of particular equation.
4. If at least one determinant is non-zero of order (G-1) the equation is identified.

5. Whether the particular equation is exactly identified or over-identified, we use order condition.

Now we apply rank condition on our model and see that whether our concerning equations are identified or under-identified. From equations (1), (2) and (3) without taking log.

$$M_t^d = \alpha_0 + \alpha_1 R_t + \alpha_2 Y_t + \alpha_3 EX_t + u_{1t}$$

$$M_t^s = \beta_0 + \beta_1 R_t + \beta_2 YG_t + \beta_3 IG_t + u_{2t}$$

$$M_t^d = M_t^s$$

The above model is simultaneous equation model in mathematical form. Here in our model we have three endogenous variables as (M_t^d, M_t^s and R_t) and four exogenous variables as (Y_t, EX_t, YG_t, and IG_t).

Now we convert the equations into structural forms as

$$M_t^d + 0M_t^s - \alpha_0 - \alpha_1 R_t - \alpha_2 Y_t - \alpha_3 EX_t - u_{1t} = 0 \quad (15)$$

$$0M_t^d + M_t^s - \beta_0 - \beta_1 R_t - \beta_2 YG_t - \beta_3 IG_t - u_{2t} = 0 \quad (16)$$

$$M_t^d - M_t^s = 0 \quad (17)$$

Ignoring the random terms (u_1, u_2) and intercepts (α_0, β_0) the table of structural equations coefficients is given bellow.

Table: 3.1 Rank Conditions

Equations	Variables					
	M_t^d	R_t	Y_t	EX_t	YG_t	IG_t	M_t^s
1st	1	$-\alpha_1$	$-\alpha_2$	$-\alpha_3$	0	0	0
2nd	0	$-\beta_1$	0	0	$-\beta_2$	$-\beta_3$	1
3rd	1	0	0	0	0	0	-1

3.8-2-1. Identification for demand equation

We strike out the 1st row and striking out the columns where appearing non-zero coefficient of 1st equation, and get the following matrices.

$$\Delta = \begin{vmatrix} -\beta_2 & -\beta_3 & 1 \\ 0 & 0 & -1 \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} -\beta_2 & -\beta_3 \\ 0 & 0 \end{vmatrix}$$ this implies that $\Delta_1 = 0$

$$\Delta_2 = \begin{vmatrix} -\beta_2 & 1 \\ 0 & -1 \end{vmatrix}$$ this implies that $\Delta_2 = \beta_2$

$$\Delta_3 = \begin{vmatrix} -\beta_3 & 1 \\ 0 & -1 \end{vmatrix}$$ this implies that $\Delta_3 = \beta_3$

According to the rank condition at least one determinant should be non-zero for any equation to be identified. So equation (1) provided that two determinants ($\Delta_2 = \beta_2$) and ($\Delta_3 = \beta_3$) are non-zero, as a result we can say money demand equation (1) is identified. It is also proved above by order condition that money demand equation is over-identified.

3.8-2-2. Identification for supply equation

By striking out the 2nd equation and considering those columns where zero entries exist in 2nd equation. We get the metrics as follow

$$\Delta = \begin{vmatrix} 1 & -\alpha_2 & -\alpha_3 \\ 1 & 0 & 0 \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} 1 & -\alpha_2 \\ 1 & 0 \end{vmatrix} \text{ this implies that } \Delta_1 = \alpha_2$$

$$\Delta_2 = \begin{vmatrix} -\alpha_2 & -\alpha_3 \\ 0 & 0 \end{vmatrix} \text{ this implies that } \Delta_2 = 0$$

$$\Delta_3 = \begin{vmatrix} 1 & -\alpha_3 \\ 1 & 0 \end{vmatrix} \text{ this implies that } \Delta_3 = \alpha_3$$

From the above calculation the values of the 1st and 3rd determinants Δ_1 and Δ_3 are non-zero, so according to the rank or sufficient condition money supply equation is identified. We have proved above the supply equation is over-identified by order condition or necessary condition.

3.9. Choice of econometric method

Because both equations are over identified and simultaneity problem also exist. So here ordinary least square (OLS) method is not applicable because it will give biased and inconsistent estimates even sample size increased. Indirect least square (ILS) technique also not possible here because this technique is only applicable if the equations are exactly identified. Under this situation 2 stage least square method is applicable; it will give unbiased and consistent estimates. If the problem of correlation between residuals will exist then use of 3 stage least square method which is the combination of (SUR) Seemingly Unrelated Regression model and 2 stage least square method, will be applied to remove the problem.

Before we apply our required techniques 2SLS, and 3SLS on our model, 1st we check whether all the variables are stationary at level or at first difference.

3.10 Stationary Test

3.10-1 Unit root

Very first, because the data is time series so to check the stationary is compulsory for us. As the co-integration based on the order of integration of variables, therefore we apply the formal (ADF) test to check the order of integration of our variables. The test provides us whether there exist a unit root or not.

3.10-2 Augmented Dickey-Fuller test

The test is performed by augmenting each variable with its lags. The number of lags to be used in all the pre-estimation and estimation models in this study is determined by using the Alkaike Information Criterion (AIC). The maximum lag length is chosen based on the minimum AIC criterion. The ADF test in this study is conducted by including a constant only. The test could be estimated with the following regression:

$$\Delta Y_t = \beta_1 + \delta Y_{t-1} + \alpha_i \sum_{i=1}^{m} \Delta Y_{t-1} + \varepsilon_t \qquad (18)$$

Where ΔY_t is the individual variable at time t, $\Delta Y_t = Y_{t-1} - Y_t$, and ε_t is a pure white noise error term, β_1 is the constant, m is the number of lags which should be large enough to ensure that the error terms are white noise and small enough to save degree of freedom, t is the trend variable in years and $\delta = (\rho-1)$. The equation above is the ADF with a constant. In each case, the null hypothesis is that δ=0; which means that there is unit root or the time series is nonstationary. The alternative hypothesis is that δ<0; this means that the variable is stationary, using a (τ) statistics. At 99 percent confidence level, if the p-value is less than or equals to 0.01, we reject the null hypothesis otherwise we do not reject the null hypothesis that the variable is nonstationary.

3.11 Co-Integration Test

A Co-integration test is used to determine if there exists an equilibrium or long-run relationship between two or more variables. If two or more variables are non-stationary but a linear combination of them is stationary, then the variables are said to be co-integrated.co-integration is requirement for any econometric model which involve non-stationary time series data, because if the variables do not co-integrate to each other then the model may suffer to spurious regression. According to Granger "co-integration is the pre-test for spurious regression. There are number of methods for testing of co-integration has been proposed in literature. Here we consider only Augmented Engle-Granger (AEG) test.

3.11-1 Augmented Engle-Granger (AEG) test

According to (AEG) test we perform (ADF) test on the estimated residuals of co-integration regression. Let we have number of variables in our model which are individually non-stationary. Now we check whether these variables are co-integrated to each other or not.

$$M_t^d = \alpha_0 + \alpha_1 R_t + \alpha_2 Y_t + \alpha_3 EX_t + YG_t + IG_t + u_t \qquad (19)$$

1st we will regress the money demand (M^d) on all the regressors, and obtain the residuals and use the (ADF) on estimated residuals. If its coefficient δ will be significant we reject the null hypothesis (there is no co-integration) and vice versa. The residual regression can be run as.

$$\Delta \hat{u}_t = \delta \hat{u}_{t-1} \qquad (20)$$

3.12 Procedure of Two stage-least square (2SLS) estimates

In our model there is problem of simultaneity and both the equations are over identified, so we can apply two-stage least square (2SLS) method to find our estimates.

Let we explain the procedure in two stages.

Again we repeat the model of equation (1), (2) and (3) given below.

$$\log(M_t^d) = \alpha_0 + \alpha_1 \log(R_t) + \alpha_2 \log(Y_t) + \alpha_3 \log(EX_t) + u_{1t}$$

$$\log(M_t^s) = \beta_0 + \beta_1 \log(R_t) + \beta_2 YG_t + \beta_3 IG_t + u_{2t}$$

$$M_t^d = M_t^s$$

1st stage:

To overcome the correlation between R_t and u_{1t} 1st regress R_t on all predetermined variables in the whole model we get the reduced form equation for endogenous variable (R_t) the numerical explanation has been explained in section (3.4).

From equation (4)

$$\log(R_t) = \prod_{10} + \prod_{11} YG_t + \prod_{12} \log(Y_t) + \prod_{13} IG_t + \prod_{14} \log(EX_t) + v_t$$

$$\hat{R}_t = \prod_{10} + \prod_{11} YG_t + \prod_{12} \log(Y_t) + \prod_{13} IG_t + \prod_{14} \log(EX_t)$$

$$\log(R_t) = \hat{R}_t + \hat{v}_t$$

The $\hat{R}_t$ variable can be generated as

$$\hat{R}_t = \log(R_t) - \hat{v}_t$$

2^{nd} stage:

After generating the variable $\hat{R}_t$ we put this as an instrument in money demand equation (1) and money supply equation (2). Now the generated variable R-hat is independent of the error term, means uncorrelated with residual. So by this way we get whatever results will be unbiased and consistent. The procedure is follow.

Replacing $\log(R_t) = \hat{R}_t + \hat{v}_t$ in the model of equation (1)

$$\log(M_t^d) = \alpha_0 + \alpha_1(\hat{R}_t + \hat{v}_t) + \alpha_2 \log(Y_t) + \alpha_3 \log(EX_t) + u_{1t}$$

$$\log(M_t^d) = \alpha_0 + \alpha_1\hat{R}_t + \alpha_1\hat{v}_t + \alpha_2 \log(Y_t) + \alpha_3 \log(EX_t) + u_{1t}$$

$$\log(M_t^d) = \alpha_0 + \alpha_1\hat{R}_t + \alpha_2 \log(Y_t) + \alpha_3 \log(EX_t) + u_{1t} + \alpha_1\hat{v}_t$$

We can write money demand equation as

$$\log(M_t^d) = \alpha_0 + \alpha_1\hat{R}_t + \alpha_2 \log(Y_t) + \alpha_3 \log(EX_t) + u_{1t}^* \quad (21)$$

Where

$$u_{1t}^* = u_{1t} + \alpha_1\hat{v}_t \quad (22)$$

Now replacing $\log(R_t) = \hat{R}_t + \hat{v}_t$ in equation (2) in the model

$$\log(M_t^s) = \beta_0 + \beta_1(\hat{R}_t + \hat{v}_t) + \beta_2 YG_t + \beta_3 IG_t + u_{2t}$$

$$\log(M_t^s) = \beta_0 + \beta_1\hat{R}_t + \beta_1\hat{v}_t + \beta_2 YG_t + \beta_3 IG_t + u_{2t}$$

$$\log(M_t^s) = \beta_0 + \beta_1\hat{R}_t + \beta_2 YG_t + \beta_3 IG_t + u_{2t} + \beta_1\hat{v}_t$$

Here the money supply equation can be written in this form

$$\log(M_t^s) = \beta_0 + \beta_1\hat{R}_t + \beta_2 YG_t + \beta_3 IG_t + u_{2t}^{*} \quad (23)$$

Where

$$u_{2t}^{*} = u_{2t} + \beta_1\hat{v}_t \quad (24)$$

By applying OLS on equation (21) we can get the required two stage least square estimates for money demand equation, and applying OLS on equation (23) we will get our estimated results for money supply equation. Now our results will be unbiased and consistent, but we can expect that there may be autocorrelation problem because we ignored the relationship between error terms of both the equations. We can solve this problem by applying three-stage least square (3SLS) method.

3.13 Three-stage least square (3SLS) technique

Three stage-least square (3SLS) is the combination of two-stage least square estimate and seemingly unrelated regression model (SUR). There are following three steps to find the (3SLS) estimates.

1. The first step is identical to the two-stage procedure: regress the endogenous variable (R_t) on all the exogenous variables present in the model, to get the reduced form equation.

Let we start from our model given in equation (1), (2) and (3):

$$\log(M_t^d) = \alpha_0 + \alpha_1\log(R_t) + \alpha_2\log(Y_t) + \alpha_3\log(EX_t) + u_{1t}$$

$$\log(M_t^s) = \beta_0 + \beta_1\log(R_t) + \beta_2 YG_t + \beta_3 IG_t + u_{2t}$$

$$M_t^d = M_t^s$$

From equation (4) reduced form equation can be formed as

$$\log(R_t) = \prod_{10} + \prod_{11} YG_t + \prod_{12} \log(Y_t) + \prod_{13} IG_t + \prod_{14} \log(EX_t) + v_t$$

$$\hat{R}_t = \prod_{10} + \prod_{11} YG_t + \prod_{12} \log(Y_t) + \prod_{13} IG_t + \prod_{14} \log(EX_t)$$

$$\log(R_t) = \hat{R}_t + \hat{v}_t$$

The $\hat{R}_t$ variable can be generated as

$$\hat{R}_t = \log(R_t) - \hat{v}_t$$

2. In the second step, after generating the new variable ($\hat{R}_t$). Both the structural equations money demand (M^d) and money supply (M^s) are to be estimated by putting ($\hat{R}_t$) as independent variable. The residuals of both the equations are saved and labeled as $\hat{u}^*_{1t}$, $\hat{u}^*_{2t}$.

So the money demand and money supply equations can be written as

$$\log(M_t^d) = \alpha_0 + \alpha_1 \hat{R}_t + \alpha_2 \log(Y_t) + \alpha_3 \log(EX_t) + u^*_{1t}$$

$$\log(M_t^s) = \beta_0 + \beta_1 \hat{R}_t + \beta_2 YG_t + \beta_3 IG_t + u^*_{2t}$$

3. To make estimates more efficient re-estimate the structural equations with $\hat{u}^*_{1t}$ and $\hat{u}^*_{2t}$ included as explanatory variables, this stage is also called the seemingly unrelated regression model. Here we use the cross equation residuals.

$$\log(M_t^d) = \alpha_0 + \alpha_1 \hat{R}_t + \alpha_2 \log(Y_t) + \alpha_3 \log(EX_t) + \eta \hat{u}^*_{2t} + u^{**}_{1t} \quad (25)$$

$$\log(M_t^s) = \beta_0 + \beta_1 \hat{R}_t + \beta_2 YG_t + \beta_3 IG_t + \kappa \hat{u}^*_{1t} + u^{**}_{2t} \quad (26)$$

In a multivariate regression model, the errors in different equations may be correlated. In this case the efficiency of the estimation may be improved by taking these cross-equation correlations into account.

Chapter 4

RESULTS AND DISCUSSIONS

In this chapter we analyze and interpret the results of the findings of this thesis. This chapter has been divided into eight sections .In section 4.1 descriptive statistics has been presented. Section 4.2 & 4.3 presents the simultaneity test results. Section 4.3 reports the pre-estimation test results, stationary and co-integration. In section 4.4 we discuss the 2SLS estimates for M1 and section 4.6 presents the 2SLS estimates for M2. Three stage-least square estimates present in section 4.5 & 4.7 for M1 and M2 respectively.

4.1. Descriptive statistics of variables

In this section we describe our variables analytically briefly. Descriptive Statistics are presented on next page table: 4-1. Firstly, we see that the values of mean and median of all variables are not too close to each other except the output gap and interest rate. Secondly, we check the normality of data through the skewness this is not close to zero and kurtosis also away from 3. So we can say that data is not normal. At 3rd step, Jarque-Bera test also proved that only two variables i.e. Output Gap (YG) and Interest Rate (R) are normal, and the data of all other variables are not normal. From the table given below we can see there is significant variation within each variable. So it becomes compulsory for us to modify the variables to give equal weights.

Table: 4.1. Descriptive Statistics

	Real Money (M1)	Nominal Effective Exchange Rate	Inflation Gap	Exchange Rate	Interest Rate	Real GDP	Real Money (M2)	Output Gap
Mean	8960436479	260.5552	6.313482	28.37887	9.481132	30187061809	13086294246	0.035607
Median	6032285765	214.1967	5.336123	16.64751	10	21368600332	9254677837	0.041949
Maximum	29265907796	889.9935	24.66303	93.3952	20	93071624805	37742301782	0.219504
Minimum	1547190476	59.0375	-2.51646	4.7619	4	4752061752	1844482630	-0.14958
Std. Dev.	8122370315	189.1425	5.476553	26.10764	3.755716	25330439560	11131264614	0.082502
Skewness	1.243699522	1.644995	1.304847	0.969308	0.58354	1.014369582	0.9450945	-0.37924
Kurtosis	3.267816952	6.311334	5.094519	2.680014	3.287361	2.84502806	2.53552405	2.744381
Jarque-Bera	13.82169316	39.03856	24.26127	8.525547	3.190275	9.142055908	8.36638647	1.414745
Probability	0.000996913	3.33E-09	5.39E-06	0.014083	0.202881	0.010347318	0.015249734	0.492938
Sum	4.74903E+11	11203.87	328.3011	1504.08	502.5	1.59991E+12	6.93574E+11	1.887156
Sum.Sq.Dev.	3.43E+21	1502545	1529.624	35443.66	733.4811	3.34E+22	6.44E+21	0.353944
Observations	54	44	53	54	54	54	54	54

4.2. Hausman test results

Now we present the Hausman test results to find the simultaneity problem in our model.

Table: 4.2. Hausman test results, Money demand function (OLS Estimates)

Independent Variables	Log (real money M1)			Log (real money M2)		
	coefficient	t-stat	p-values	Coefficient	t-stat	p-values
Constant	2.9648	1.10	0.2754	0.2580	0.1389	0.8902
R-hat	-0.5713	-6.13**	0.000	-0.3764	-5.83**	0.000
V-hat	0.058	0.8058	0.4253	0.1993	3.94**	0.0003
Log(Real GDP)	0.928	10.79**	0.000	1.010	16.9**	0.000
Log(NEER)	-0.2185	-2.219*	0.0325	-0.0895	-1.3128	0.1971

Note: ** show significant at 0.01 & * denotes significant at 0.05 levels
Dependent variable is log of Real Money Demand (M1 & M2)

In table 4.2 there are two types of money (M1, M2) which are separately regressed on estimated residuals using equation (8). Since the t-value of real money demand (M2) is 3.94 (p-value = 0.0003) statistically significant at 1% level of significance. So we can not reject the hypothesis that there is simultaneity between real money demand (M2) and interest rate (R), which should not be surprising. Now on the other side the t-value of real money demand (M1) is 0.8058 and (p-value = 0.4253) which is statistically insignificant even at 10% level, here we can accept the null hypothesis that there is no simultaneity between real money demand (M1) and interest rate (R).

Table: 4.3. Hausman test results, Money supply function (OLS Estimates)

Independent Variables	Log (real money M1)			Log (real money M2)		
	coefficient	t-stat	p-values	Coefficient	t-stat	p-values
Constant	13.4396	18.0**	0.000	13.825	19.9**	0.000
R-hat	4.3477	12.8**	0.000	4.3683	13.8**	0.000
V-hat	0.058	0.213	0.8324	0.1993	1.77*	0.044
Output gap	-10.2648	-10.5**	0.000	-9.810	-10.8**	0.000
Inflation gap	-0.0805	-7.2**	0.000	-0.0859	-8.32**	0.000

Note: ** show significant at 0.01 & * denotes significant at 0.05 level
Dependent variable is log of real money supply (M1 & M2)

Table: 4.3 shows the money supply function, in which dependent variables are real money supply (M1, M2) again separately, regressed on estimated residuals using the equation (9). The t-value is 1.77, (p-value = 0.044) for (M2) which is statistically significant at 5% level of significance. Now at 5% level we can reject the null hypothesis that there is no simultaneity between real money supply (M2) and nominal interest rate (R), it means again we are facing simultaneity problem in supply function as in money demand function in table: 4.2.

On the side of real money supply (M1), the coefficient of residual (v-hat) is 0.058, it means if 1% change occur in residuals the real money supply increase 0.058%. It shows a very low contribution to respondent variable. We can also see it from the t-value = 0.213 which is insignificant, means there is no simultaneity problem between real money supply (M1) and nominal interest rate (R).

4.2-1. Hausman test results, using Exchange Rate (PAK/USD)

A variable nominal effective exchange rate (NEER) has been used in above model, but now we introduce a new variable Exchange Rate (PAK/USD) in place of (NEER) in our model to check whether new variable Exchange Rate (PAK/USD) has significant effect or not.

Table: 4.4. Hausman test results, Money demand function (OLS Estimates)

Independent Variables	Log (real money M1)			Log (real money M2)		
	Coefficient	t-stat	p-values	Coefficient	t-stat	p-values
Constant	-4.440	-2.006*	0.0505	-2.7204	-1.6054	0.1151
R-hat	-0.2347	-2.255*	0.028	-0.2638	-3.31**	0.0018
V-hat	-0.1770	-2.70**	0.0095	0.0509	1.057	0.2957
Log(Real GDP)	1.166	11.3**	0.000	1.10	13.9**	0.000
Log(EX/DOLR	-0.0828	-0.8442	0.4028	0.0074	0.099	0.9213

Note: ** show significant at 0.01, & * denotes significant at 0.05 level
Dependent variable is log of real money demand (M1 & M2)

In table: 4.4 we show money demand function, where real money demand (M1, M2) are dependent variables. 1st we regress real money demand (M1) on estimated residuals, by equation (8). The results show the negative effect. The calculated t-value of residuals (v-hat) is -2.70 which is highly significant; we can reject the null hypothesis that there is no simultaneity. Here we conclude that real money demand (M1) is jointly dependent with nominal interest rate (R). But on the other side if we make a glance on M2, where coefficient of residuals (v-hat) is 0.05 which is very low. The t-value of (v-hat) is 1.05 (p-value = 0.2957) which is insignificant, means there is no simultaneity problem between real money demand (M2) and nominal interest rate (R). We can say that by including a new variable exchange rate (PAK/USD) in place of (NEER) the results are completely opposite. At the above model when we use a variable (NEER) the interest rate show the joint dependence with (M2) but not with (M1). When we replace a variable exchange rate (PAK/USD) then simultaneity occurs between (M1) and interest rate, but not between (M2) and interest rate.

Table: 4.5. Hausman test results, Money supply function (OLS Estimates)

Independent Variables	Log (real money M1)			Log (real money M2)		
	Coefficient	t-stat	p-values	Coefficient	t-stat	p-values
Constant	17.01	115.42**	0.000	17.1711	136.3**	0.000
R-hat	2.8523	39.05**	0.000	2.9668	47.51**	0.000
V-hat	-0.1700	-2.072*	0.0437	0.0509	0.734	0.4715
Output gap	-4.3823	-16.76**	0.000	-4.2876	-19.2**	0.000
Inflation gap	-0.0846	-18.65**	0.000	-0.089	-23.1**	0.000

Note: ** show significant at 0.01, & * denotes significant at 0.05 level
Dependent variable is log of real money demand (M1 & M2)

In money supply function for (M1) the t-value is -2.072 (p-value = 0.043) statistically significant at 5% level of significance. Also show the negative effect between residuals (v-hat) and real money supply (M1). So we can say at 5% level there is simultaneity between real money supply (M1) and nominal interest rate (R) these results are obtained from equation (9) by replacing a new variable Exchange Rate (PAK/USD) in place of (NEER). But for (M2) the t-stat is 0.734 show insignificant, means there is no simultaneity between real money supply (M2) and nominal interest rate (R). These money supply function results also show the opposite direction as in the above model money supply function table:

4.3 Variables selection for (M1) and (M2)

From the above discussion, it is proved that when we include the variable exchange rate (PAK/USD) in the model, the money demand (M1) shows the simultaneity problem with interest rate (R). But by replacing the variable nominal effective exchange rate (NEER) in the model, we see that money demand (M2) and interest rate (R) have simultaneity problem. So for (M1) we use the variable exchange rate (PAK/USD) and for (M2) nominal effective exchange rate (NEER) will be used.

4.4 Unit root & co-integration test results

Before any formal unit root test was conducted, we plotted all the variables used in this study in order to have initial glance about the properties of the variables. In figure: 4.1 shows a visual plot of all the variables, where the variables are not stationary at level. The Augmented Dickey-Fuller (ADF) test of unit root shows that all the variables in the model at 1% level of significance are stationary at first difference. The econometrics result of the ADF test is presented in Table: 4.6. These results show that the null hypothesis of unit root at level cannot be rejected for money demand (M1, M2); nominal interest rate (R), gross domestic product (GDP), nominal effective exchange rate (NEER), and exchange rate (PAK/USD), at 5% level of significance. But at 1st difference all the variables have no unit root (stationary) even at 99% confidence level.

Table: 4.6. Augmented Dickey-Fuller test results

Variables	Level			1st difference		
	p-value	t-stat	Critical value (1%)	p-value	t-stat	Critical value (1%)
Log(RM2)	0.8021	-0.8287	-3.5683	0.0000	-5.6062**	-3.5683
Log(RM1)	0.9601	0.0678	-3.5627	0.0000	-6.1512**	-3.5654
Log(Interest Rate)	0.1136	-2.5344	-3.5683	0.0000	-5.4461**	-3.5654
Log(Real GDP)	0.9396	-0.1360	-3.5627	0.0000	-7.7407**	-3.5654
Log(NEER)	0.6875	-1.1483	-3.5966	0.0000	-5.8267**	-3.5654
Output Gap	0.020	-3.3208*	-3.5654	0.0000	-5.5007**	-3.5654
Inflation Gap	0.021	-3.3177*	-3.5654	0.0000	-7.0437**	-3.5683
Log(PAK/USD)	0.9804	0.3839	-3.5626	0.0000	-5.8269**	-3.5654

Note: ** show significant at 0.01, & * denotes significant at 0.05 level

So from the above discussion we can say that all the variables are stationary at 1st difference with 99 percent confidence level, and have the same integrity level I(1). As all the variables are I (1), now we proceed to determine whether there is long run relationship between all the variables which are included in our model. The Engle granger co-integration test has been used to find co-integration between the variables. The test results are shown on next page in the following table: 4.7. 1st we regress the money demand (M1, M2) dependent variable on all the regressors, then the residuals are obtained by using the e-views software. Now by applying the ADF test on equation (20) the residuals obtained from equation (19). The table: 4.7 results show that residuals (M1) is significant at 5% level of significance, it means all the variables are cointegrated to each other. For residuals (M2) the t-stat is -3.5328 (p-value = 0.0119) which falls in critical region, significant at 95 percent level. So again the variables are co-integrated to each other for (M2) residuals.

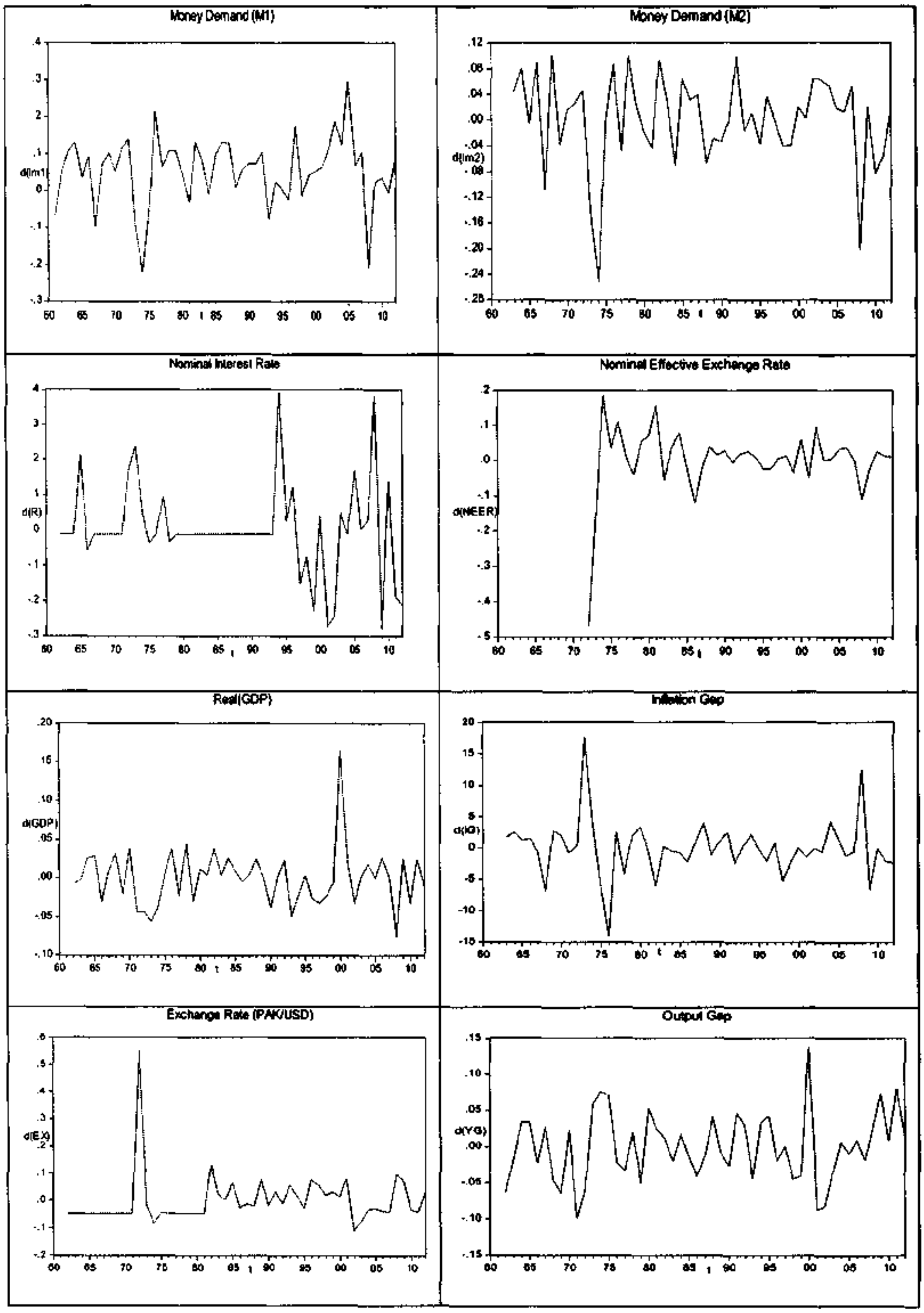
Table: 4.7. Co-integration test results

Variables	p-value	t-stat	Critical values
Residuals (M1)	0.0159	-3.3892**	-3.5654(1%) -2.9199(5%) -2.5979(10%)
Residuals (M2)	0.0119	-3.5328**	-3.6009(1%) -2.9350(5%) -2.6058(10%)

Note: ** show significant at 0.01 levels.

As we see from the above discussion that all the variables are stationary at 1st difference. Now we present the stationary of all the variables graphically on next page in figure 4.1. These graphs are made by using E-Views software; each variable graph shows that they are stationary at 1st difference.

Figure 4.1-Graphical representation of variables stationary at 1st diff



4.5 Two stage-least square estimates for (M1)

Two stage-least square estimates of money demand function (M1) are shown in table: 4.8. All the coefficients of money demand function are showing the expected signs. The nominal interest rate (R) has the negative effect on the respondent variable money demand (M1), and t-stat is -2.1204 indicate that interest rate has significant effect on money demand at 5% level, but insignificant at 1% level. The coefficient of interest rate is -0.2347, indicate a one percent increase in interest rate the demand for real money will decline 0.2347 percent. The real money demand showing positive effect with real GDP, and has stronger effect as compared to interest rate. The t-value is 10.64 for real GDP which is highly significant. The quantity of real money demand (M1) will increase 1.166% if GDP increase 1% and vice versa. The exchange rate (PAK/USD) has negative effect on real money demand (M1) but insignificant even at 10% level of significance.

Table: 4.8. Money Demand Function, (2SLS Estimates)

Variables	Coefficients	t-stat	Std error	p-values
Constant	-4.4407	-1.8858	2.3543	0.065
Log(Interest Rate)	-0.2347	-2.1204*	0.1107	0.0392
Log(Real GDP)	1.166	10.648**	0.1095	0.000
Log(PAK/USD)	-0.0828	-0.7936	0.1044	0.4313

Note: ** show significant at 0.01, & * denotes significant at 0.05 level
Dependent variable is log of real money demand (M1)

Table: 4.8-1. LM test results (Money Demand Function)

Adj (R^2)	F-stat	DW-stat	LM-test F-stat	LM-test p-value
0.9822	943.85**	0.57	13.31**	0.0000

Note: ** show significant at 0.01 levels

In the lower part of the table 4.8-1.There are some statistical terminologies; one of them is adj (R^2). The high value of adj (R^2) indicates that three independent variables used in equation may explain nearly 98 percent variation in dependent variable. The F-stat reveals that overall model is significant, but a low DW-stat and Breusch Godfrey serial correlation langrange-Multiplier (LM) test indicating presence of autocorrelation.

Table: 4.9. Money Supply Function, (2SLS Estimates)

Variables	Coefficients	t-stat	Std error	p-values
Constant	17.010	111.652**	0.1523	0.000
Log(Interest Rate)	2.8523	37.77**	0.0755	0.000
Output Gap	-4.3823	-16.219**	0.2701	0.000
Inflation Gap	-0.08463	-18.049**	0.0046	0.000

Note: ** show significant at 0.01, & * denotes significant at 0.05 level
Dependent variable is log of real money supply (M1)

Table: 4.9-1. LM test results (Money Supply Function)

Adj (R^2)	F-stat	DW-stat	LM-test F-stat	LM-test p-value
0.97	581.18**	0.39	19.51**	0.0000

Note: ** show significant at 0.01 levels.

Table: 4.9 represent the results of money supply function. In the estimated money supply function adjusted $R^2 = 0.97$, it means 97% variation can be explained in real money supply (M1) by three right hand side variables nominal interest rate (R), inflation gap, and output gap. We can see from the above table that all the coefficients are significant at 1 percent level of significance, and have expected signs. The real money supply is positively associated with interest rate and negatively impacted by output gap and inflation gap. One thing is notable here that the coefficient of output gap is larger. So we can say that the real money supply is more sensitive to output gap as compared to interest rate and inflation gap. F-stat is 581.18 which is highly significant, means the model is significant overall. The Durbin Watson stat is too much low showing the presence of autocorrelation in the model.

Again we see the F-stat of LM test is highly significant reveals that there is autocorrelation problem.

As we see from the above table: 4.9-1 that there is problem of autocorrelation in money demand function. Now we regress the real money demand (M1) on its own lag and all the independent variables mentioned in table: 4.9. If we compare the results of both tables, we see that again all the coefficients have expected signs and make no difference in significance. The lag of dependent variable LM1 (-1) has significant effect on response variable (M1).

Table: 4.10. Money Demand Function, (2SLS Estimates)

Variables	Coefficients	t-stat	Std error	p-values
Constant	-2.4562	-1.8978	1.2942	0.0639
Log(Interest Rate)	-0.3481	-5.69**	0.0611	0.0000
Log(Real GDP)	0.4025	4.33**	0.0928	0.0001
Log(PAK/USD)	-0.0012	-0.022	0.05733	0.9823
LM1(-1)	0.7198	10.73**	0.06708	0.0000

Note: ** show significant at 0.01, &* denotes significant at 0.05 level
Dependent variable is log of real money demand (M1)

The adjusted R-square increased from .98 to .99 by including the new variable LM1 (-1). The main thing DW- stat has been increased from 0.57 to 1.94, but here DW stat is not reliable detector of autocorrelation. Because now our model has become autoregressive, so we use h-statistic to detect autocorrelation. The h-statistic is insignificant means autocorrelation has been removed from the model. Also the F-stat of LM test showing that H_0 is accepted means there is no autocorrelation.

Table: 4.10-1. LM test results, (Money Demand Function)

Adj (R^2)	F-stat	DW-stat	DW h-stat	LM-test F-stat	LM-test p-value
0.99	2420.14**	1.94	0.2439	1.046	0.4034

Note: ** show significant at 0.01 levels.

The same problem autocorrelation, we see in money supply function. To remove this problem we take the lag of real money supply (M1) and regress with respondent variable real money supply. The results are shown in table: 5.8. There is little bit difference in results of money supply function by including the lag of dependent variable and without including the lag of dependent variable. Only the variables interest rate and output gap were highly significant in table: 4.8 but in table: 4.10 these variables are significant at 1%. Now if we see the coefficients there is lot of difference in numerical values but signs are same and expected.

Table: 4.11. Money Supply Function (2SLS Estimates)

Variables	Coefficients	t-stat	Std error	p-values
Constant	2.6773	2.4320*	1.1008	0.0189
Log(Interest Rate)	0.4715	2.5365**	0.1858	0.0146
Output Gap	-0.9902	-3.4224**	0.2893	0.0013
Inflation Gap	-0.0238	-4.6288**	0.0051	0.0000
LM1(-1)	0.8458	13.047**	0.0648	0.0000

Note: ** show significant at 0.01, & * denotes significant at 0.05 level
Dependent variable is log (real money supply, M1)

Table: 4.11-1. LM test results, (Money Supply Function)

Adj (R^2)	F-stat	DW-stat	DW h-stat	LM-test F-stat	LM-test p-value
0.99	2015.3**	1.85	0.6022	1.013	0.4218

Note: ** show significant at 0.01 levels.

Again we can notice here that adjusted R-square and Durbin Watson statistic has been improved. The h-statistic is insignificant means there is no autocorrelation more in model. By applying the LM test on money supply function, it is clearly shown that there is no autocorrelation in money supply equation now.

The figure: 4.2 on next page present the two graphs of money demand (M1) and money supply (M1). Actual and fitted lines are close to each other. In residual graph there are some points which are outside the interval, but mostly points are close to zero. It means the residuals of money demand and money supply are stable.

Figure: 4.2- Actual, Fitted, Residual Graphs (2SLS) at level

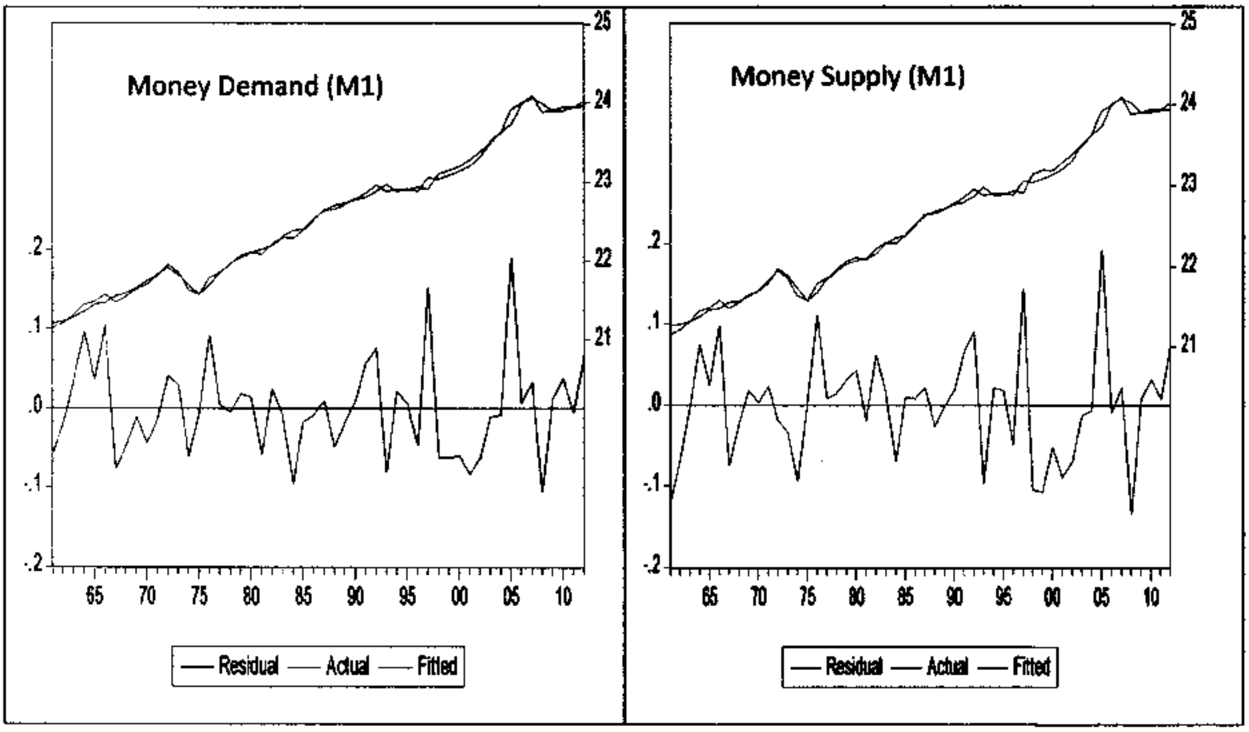
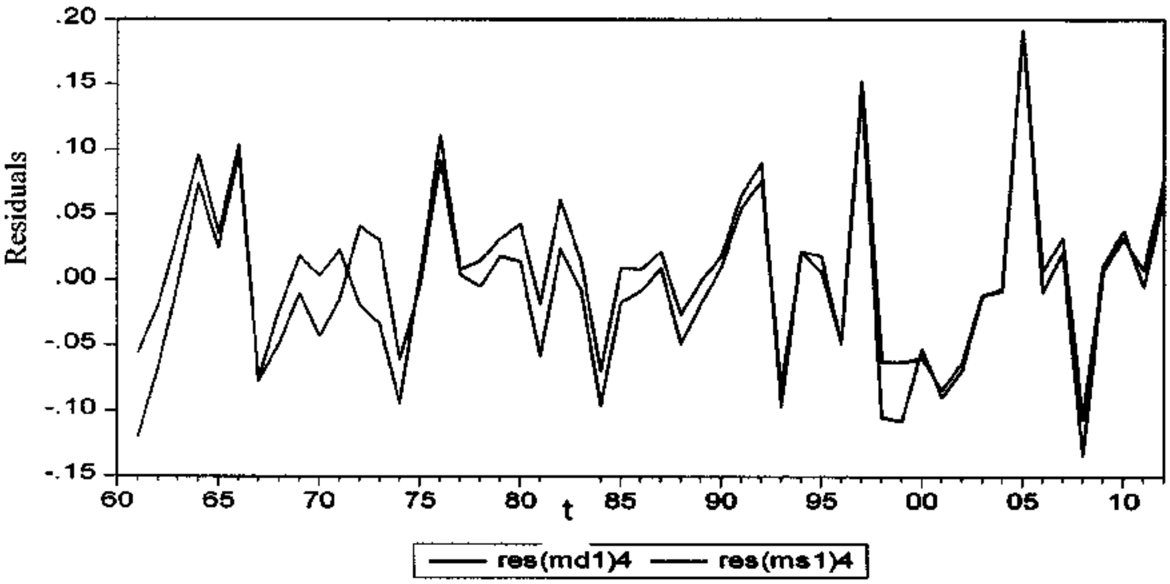


Figure: 4.3- (2SLS) Graph of Correlation b/w two residuals



We see from the above table: 4.11-1 that autocorrelation within the equation errors has been removed by regressing lag of depending variable. But figure: 4.3 shown below are revealing that residuals of money demand (M1) and money supply (M1) are highly correlated to each other. As we can see from the figure: 4.3. So to remove this type of autocorrelation we will use the 3rd stage, which is also called the seemingly unrelated regression model.

4.6 Three stage-least square estimates for (M1)

Both the equations of our model money demand and money supply were estimated simultaneously using three stage-least square (3SLS) procedure. The estimated results obtained from equation (25) of real money demand are reported in table: 4.12. If we examine the results 2SLS and 3SLS for money demand function, there is lot of difference between both the techniques results. Coefficients obtained from 2SLS are comparatively higher than the coefficient calculated by 3SLS method. But one thing is notable here that the variable exchange rate (PAK/USD), which was insignificant in (2SLS), estimates now it becomes significant in (3SLS), estimates. However all the coefficients have expected signs, which is not surprising for us? The real money demand is negatively affected by interest rate, but exchange rate (PAK/USD) and real GDP have positive effect on real money demand (M1). The coefficient of interest rate is -0.4064, if one percent increase in interest rate the real money demand decrease by 0.4064 percent and conversely will happened same. A one percent change in real GDP the demand for money in response will increase by 0.5067% and reverse will happened if real GDP decline. The coefficient of exchange rate (PAK/USD) is 0.5408 which is significant. Here exchange rate has positive effect on money demand, it means a one percent increase in exchange rate the real money demand increase by 0.54%. From the table given below it is clear that interest rate, GDP, and exchange rate are highly significant even at 1% level of significance.

Table: 4.12. Money Demand Function, (3SLS estimates)

Variables	Coefficients	t-stat	Std error	p-values
Constant	9.7915	6.093**	1.6067	0.0000
Log(interest rate)	-0.4064	-7.349**	0.0553	0.0000
Log(real GDP)	0.5076	6.809**	0.0745	0.0000
Log(PAK/USD)	0.5408	7.633**	0.0708	0.0000
Res(2SLS)M^s	0.9799	12.564**	0.0779	0.0000

Note: ** show significant at 0.01, &* denotes significant at 0.05 level.
Dependent variable is log of real money demand (M1)

Table: 4.12-1. LM test results (Money Demand Function)

Adj (R^2)	F-stat	DW-stat	LM-test F-stat	LM-test p-value
0.99	3060.7**	1.96	1.07	0.4134

Note: ** show significant at 0.01 levels

99 percent change can be measured by three right-hand side variables in real money demand (M1). The adjusted R^2 calculated by 2SLS is little bit low as computed from 3SLS technique, but the standard errors for all the variables obtained by 3SLS are lower than the standard errors computed by 2SLS. The residuals obtained in second stage also regressed with response variable money demand (M1), to remove the effect of autocorrelation which can occur between the errors of both the equations. Also there is no more autocorrelation, as Durbin Watson stat has been improved. So the results obtained by 3SLS are more efficient than the estimates of 2SLS.

Table: 4.13 Presents the estimates for real money supply using equation (26). These estimates were obtained by 3SLS procedure. As shown in the table, all the coefficients are significant at 1% level of significance and have expected signs. Approximately 98% variation in real money supply may be explained by three right-hand side variables. The real money supply is positively affected by interest rate, if one percent increases in interest rate the real money supply will be increased 2.92%, and if one percent decreases in interest rate the real money supply will also decrease by 2.92%. We can see here that interest rate is more sensitive with money supply as compared to money demand. The other variables inflation gap and output gap are negatively influenced by real money supply, but the output gap has greater effect on respondent variable as compared to inflation gap. The coefficient of output gap is -3.89 means the real money supply will decline 3.89% if one unit increase in output gap, and results will be reverse if one unit increase in output gap. Inflation gap also has the same effect as output gap, but inflation gap has lower effect than output gap on real money supply.

Table: 4.13. Money Supply Function, (3SLS Estimates)

Variables	Coefficients	t-stat	Std error	p-values
Constant	16.884	149.85**	0.1126	0.0000
Log(interest rate)	2.9180	52.180**	0.0559	0.0000
Output gap	-3.8945	-18.51**	0.2103	0.0000
Inflation gap	-0.0898	-25.61**	0.0035	0.0000
Res(2SLS)M^d	0.9809	6.585**	0.1489	0.0000

Note: ** show significant at 0.01, & * denotes significant at 0.05 level.
Dependent variable is log of real money supply (M1)

Table: 4.13-1. LM test results (Money Demand Function)

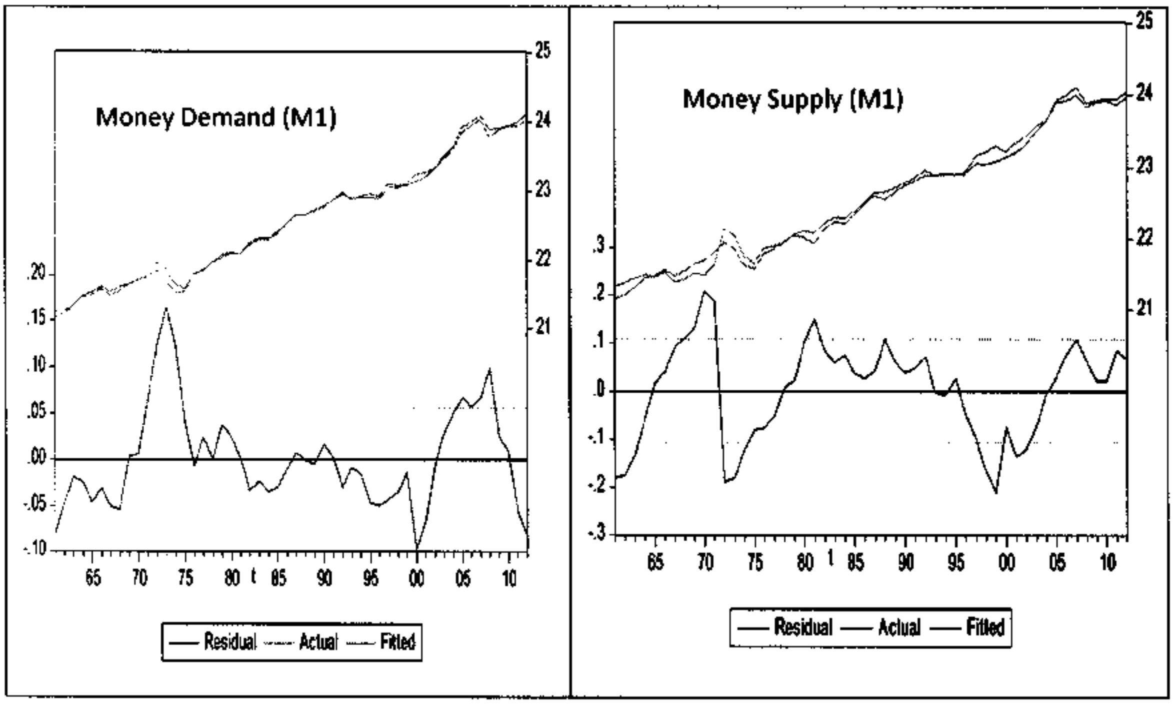
Adj (R^2)	F-stat	DW-stat	LM-test F-stat	LM-test p-value
0.98	831.53**	1.67	0.992	0.2347

Note: ** show significant at 0.01 levels

Now if we compare the 2SLS results output for real money supply reported in table: 4.9 with the 3SLS estimates for real money supply reported in the above table: 4.13. Adjusted R^2 computed by 2SLS is 0.97, which is lower than adjusted R^2 calculated from 3SLS technique because of including the residuals as regressor in the money supply equation. If we see the coefficients of all the variables, there is too much difference of both techniques coefficients except the output gap coefficient which is approximately same computed by 2SLS and 3SLS. Now if we make a glance at the standard errors, we can see from both the tables that the standard errors are comparatively low of 3SLS estimates to 2SLS estimates. So we can say that three stage least square estimates are more efficient than two stage least square estimates.

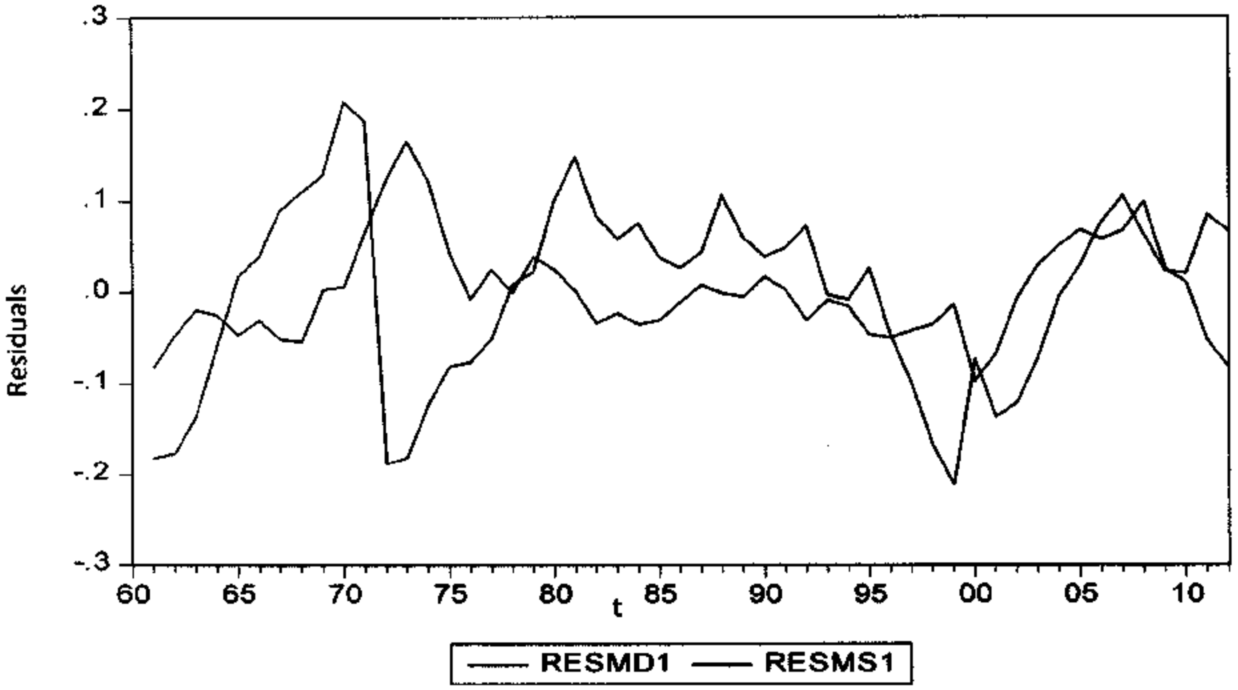
Figure: 4.4 show the actual fitted and residuals graphs of money demand (M1) and money supply (M1) for (3SLS) estimates. In money demand function actual and fitted lines are close to each other and residuals are stable. But in money supply function actual and fitted lines have little bit difference; it means residuals for money supply are not stable. We can see that residuals are not correlated to each other.

Figure: 4.4- Actual, Fitted, Residual Graphs (3SLS) at level



Also we can see from the figure: 4.5 given below that there is no more correlation between two error terms.

Figure: 4.5- (3SLS) Graph of correlation b/w two residuals



The figure: 4.5 above clearly showing that both the residuals have no more relation. Residuals obtained from money demand (M1) equation approximately constant around zero value. But the residuals obtained by money supply (M1) equation have fluctuations; means the correlation between two error terms has been removed. So it is clear evidence that estimates calculated by three stage least square (3SLS) procedure are more efficient as compared to obtained by two stage least (2SLS) square procedure.

4.7. Two stage-least square estimates for (M1) at 1^{st} difference

Table: 4.6 Augmented Dickey-Fuller test results showed that there were two variables output gap (YG) and inflation gap (IG) which was stationary at level at 5% level of significance, but stationary at 1^{st} difference. Now we estimate our results by taking the 1^{st} difference of each variable and check whether the results are meaningful or not. Here D is used as the 1^{st} difference each variable.

Table: 4.14. Money Demand Function, (2SLS Estimates) at 1st diff

Variables	Coefficients	t-stat	Std error	p-values
Constant	0.034	1.4084	0.024	0.1656
D[Log(interest rate)]	-0.3173	-3.722**	0.0852	0.0005
D[Log(real GDP)]	0.4393	1.3446	0.3267	0.1852
D[Log(PAK/USD)]	0.0666	0.5373	0.1241	0.5935

Note: ** show significant at 0.01, & * denotes significant at 0.05 level
Dependent variable is log of real money demand (M1)

In table: 4.14 we presents (2SLS) estimates for money demand function (M1). This also showing the expected signs, interest rate has negative effect on money demand and significant at 1% level. The real GDP has positive relation with money demand but not significant. The exchange rate also insignificantly, showing positive relation with money demand. These results are comparatively too much different from the results table: 4.8.

Table: 4.14-1. LM test results (Money Demand Function) at 1st diff

Adj (R^2)	F-stat	DW-stat	LM-test F-stat	LM-test p-value
0.22	5.67.85**	1.73	0.432	0.6517

Note: ** show significant at 0.01 levels

The adjusted R^2 is 0.22 which is low because all the variables are regressed on their 1st differences. DW stat is close to value 2, its mean there is no autocorrelation problem. LM test is insignificant, means there is no serial correlation problem in money demand equation.

Table: 4.15. Money Supply Function, (2SLS Estimates) at 1st diff

Variables	Coefficients	t-stat	Std error	p-values
Constant	0.0509	2.9728**	0.01715	0.0046
D[Log(interest rate)]	0.2964	0.4186	0.7086	0.6774
D(Output gap)	-1.106	-1.286	0.8604	0.2047
D(Inflation gap)	-0.0148	-0.775	0.0191	0.4418

Note: ** show significant at 0.01, & * denotes significant at 0.05 level
Dependent variable is log of real money supply (M1)

As we noted from table: 4.15 all the coefficients have expected signs but insignificant. Also the coefficients are comparatively low from the coefficients of table: 4.9. So we cannot rely on these results because all the independent variables have major role to contribute in money demand but they are insignificant. The table: 4.15-1 below reveals that there is no more autocorrelation in money supply equation.

Table: 4.15-1. LM test results (Money Supply Function) at 1st diff

Adj (R^2)	F-stat	DW-stat	LM-test F-stat	LM-test p-value
0.25	6.51**	1.88	0.083	0.9203

Note: ** show significant at 0.01 levels.

The adjusted R^2 is 0.25, which means that only 25% variation explained by three right hand side variables in the dependent variable money supply. Breusch Godfrey serial correlation LM test also reveals that serial correlation problem has been removed in money supply equation.

Figure: 4.6- Actual, Fitted, Graphs (2SLS) at 1st diff

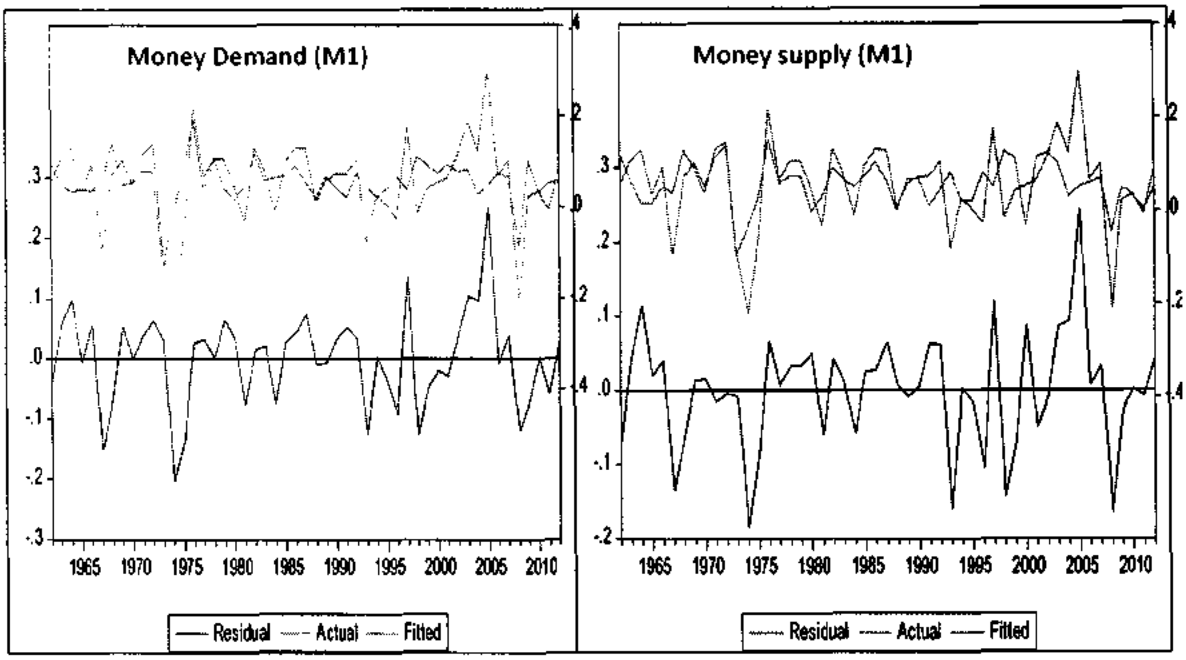


Figure: 4.6 is shown above, the actual, fitted, and residuals of money demand (M1) and money supply (M1) for (2SLS) at 1st difference are compared to each other. We can see from both the equations that actual and fitted lines are not close to each other. The figure: 4.6 clearly showing that both the residuals are correlated to each other.

4.8. Three stage-least square estimates for (M1) at 1st difference

Three stage-least square results for money demand function (M1) are estimated at 1st difference given below table: 4.16. Though the coefficients of interest rate and real GDP have expected signs, but real GDP is insignificant. The exchange rate variable has positive effect on money demand although insignificant, but in table: 4.12 it is significant.

Table: 4.16. Money Demand Function, (3SLS estimates) at 1st diff

Variables	Coefficients	t-stat	Std error	p-values
Constant	0.0597	2.98**	0.0200	0.0046
D[Log(interest rate)]	-0.2836	-4.08**	0.0694	0.0002
D[Log(real GDP)]	0.0117	0.0430	0.2731	0.9659
D[Log(PAK/USD)]	0.0279	0.2808	0.0996	0.7801
D[Res(2SLS)M^s]	0.4839	5.305**	0.0913	0.0000

Note: ** show significant at 0.01, & * denotes significant at 0.05 level.
Dependent variable is log of real money demand (M1)

Again if we make a glance at the results of table: 4.16, the coefficients of all the variables are comparatively low by the results mentioned in table: 4.12. In money demand equation only 51% variation explained by three right hand side variables. The DW stat and LM test also showing that there is autocorrelation problem. We can say these results are comparatively poor as compared to the results that we estimate at level. Here we see that only interest rate is significant, but other two independent variables real GDP and exchange rate are highly insignificant, which is surprising for us.

Table: 4.16-1. LM test results (Money Supply Function) at 1st diff

Adj (R^2)	F-stat	DW-stat	LM-test F-stat	LM-test p-value
0.51	13.83**	0.90	12.08**	0.0000

Note: ** show significant at 0.01 levels

Table: 4.17 present the results of money supply function (M1) at 1st difference. The interest rate has positive effect on real money supply, but output gap and inflation gap have negative effect on money supply; all the variables are insignificant. But in table: 4.13, interest rate is positively affected with money supply and output gap, inflation gap are negatively affected with money supply and they have significant effect on money supply. The coefficients of all the variables also low as compared to the coefficients of table: 4.13. So we cans say that results tabulated in table: 4.13 are meaningful.

Table: 4.17. Money Supply Function, (3SLS Estimates) at 1st diff

Variables	Coefficients	t-stat	Std error	p-values
Constant	0.0551	4.19**	0.0131	0.0001
D[Log(interest rate)]	0.0829	0.1541	0.5377	0.8782
D[Output gap]	-0.6632	-1.014	0.6539	0.3159
D[Inflation gap]	-0.0070	-0.4834	0.0145	0.6311
D[Res(2SLS)M^d]	0.5145	5.997**	0.0857	0.0000

Note: ** show significant at 0.01, & * denotes significant at 0.05 level.
Dependent variable is log of real money supply (M1)

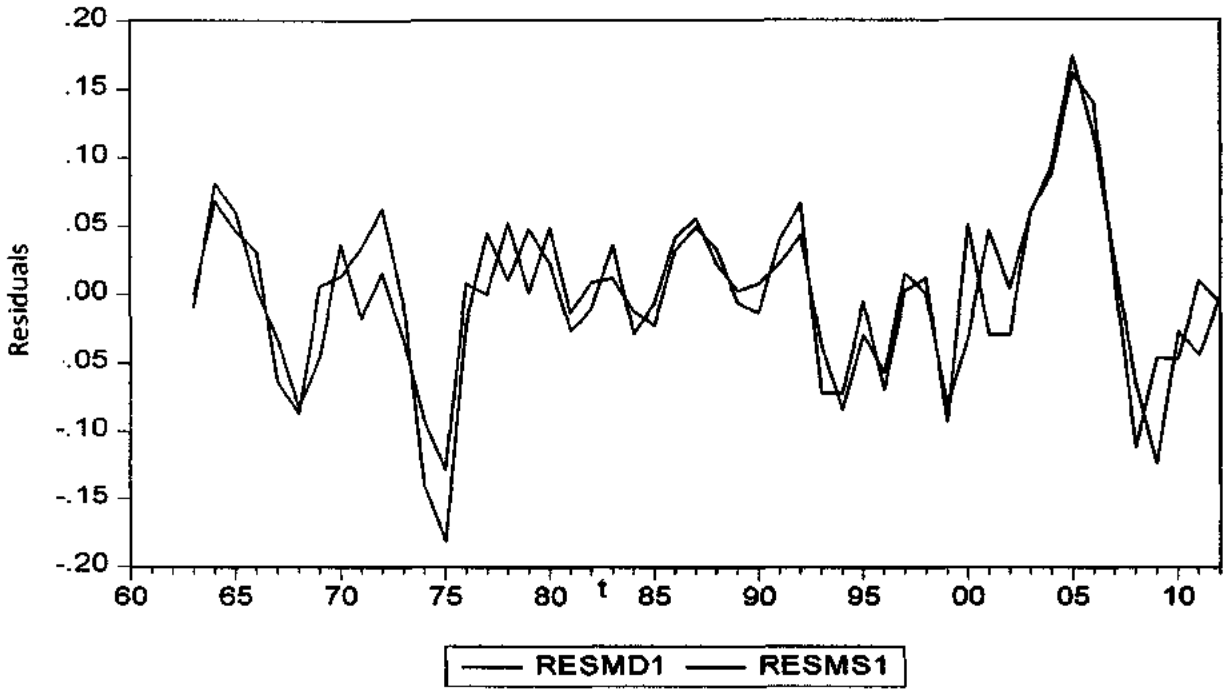
The adjusted R^2 is 0.57, a healthy variation explained by independent variables in response variable. The DW stat is 1.19, which is away from the value 2. The LM test is showing that there is autocorrelation problem in money supply equation.

Table: 4.17-1. LM test results (Money Supply Function) at 1st diff

Adj (R^2)	F-stat	DW-stat	LM-test F-stat	LM-test p-value
0.57	17.91**	1.19	9.08**	0.0000

Note: ** show significant at 0.01 levels

Figure: 4.7- (3SLS) Graph of correlation b/w two residuals (at 1st diff)



We can see from figure: 4.8 that, when three stage least square estimates are estimated by taking the 1st difference of each variable, but there is strong correlation between both residuals of money demand (M1) and money supply (M1) equations. So we can say that the results obtained by (3SLS) at 1st difference are not efficient. Because we use (3SLS) technique to remove the problem of correlation between two error terms but still there is correlation between two error terms.

4.9. Two stage-least square estimates for (M2)

Table: 4.18 indicate the results of two stage-least square estimates for money demand function (M2). As shown, two variables interest rate and GDP are both highly significant, but the nominal effective exchange rate (NEER) is insignificant. From the table we see that coefficient of real GDP is 1.01, which is more sensitive to real money demand as compared to interest rate. However all the coefficients have expected signs, real money demand (M2) is positively associated to real GDP and negatively impacted by interest rate and (NEER).

Table: 4.18. Money Demand Function, (2SLS Estimates)

Variables	Coefficients	t-stat	Std error	p-values
Constant	0.2580	0.1185	2.1762	0.9062
Log(interest rate)	-0.3764	-4.9782**	0.0756	0.000
Log(real GDP)	1.0100	14.4583**	0.0698	0.000
Log(NEER)	-0.089	-1.12009	0.0799	0.2695

Note: ** show significant at 0.01, & * denotes significant at 0.05 level
Dependent variable is log of real money demand (M2)

Table: 4.18-1. LM Test results (Money Demand Function)

Adj (R^2)	F-stat	DW-stat	LM-test F-stat	LM-test p-value
0.99	1486.78**	0.62	9.1639**	0.0000

Note: ** show significant at 0.01 levels.

A very high adjusted R^2 reveals that 99 percent variation occurred in respondent variable real money demand (M2) by three right-hand side variables interest rate, real GDP, and (NEER). The Lagrange-Multiplier LM test and Durbin Watson statistic indicates the presence of autocorrelation in money demand (M2) equation.

The estimates of real money supply (M2) are reported in table: 4.19. We can see from the following table that all the coefficients are significant, and have expected signs. Interest rate is positively related to real money supply (M2), output gap and inflation gap are negatively affected to real money supply. A one percent increase in interest rate the real money supply increase by 4.36% and results will be reversed if one percent decreases in interest rate. The output gap has greater effect on money supply, approximately 9.8% real money supply decrease by increasing one percent output gap. As we have seen inflation gap influenced the real money supply (M1) a very low margin, same results we see for (M2).

0.08% change occurs in real money supply (M2) due to inflation gap. Again we notice here that all the coefficients have expected signs like as for (M1). But there is lot of difference in R squares, adjusted R^2 for (M1) was 0.97, and 0.82 for (M2). Only the 82% variation can be explained by all regressors of money supply equation, which is smaller than adjusted R^2 for the money demand equation. It is just because of variables are modified in money supply equation.

Table: 4.19. Money Supply Function, (2SLS Estimates)

Variables	**Coefficients**	**t-stat**	**Std error**	**p-values**
Constant	13.8250	20.036**	0.6839	0.000
Log(Interest Rate)	4.3683	13.9532**	0.3130	0.000
Output Gap	-9.810	-10.9373**	0.8969	0.000
Inflation Gap	-0.0859	-8.3691**	0.0102	0.000

Note: ** show significant at 0.01, & * denotes significant at 0.05 level
Dependent variable is log of real money supply (M2)

Table: 4.19-1. LM test results, (Money Supply Function)

Adj (R^2)	F-stat	DW-stat	LM-test F-stat	LM-test p-value
0.82	67.99	0.56	9.6832	0.0000

Note: ** show significant at 0.01 levels.

F-stat show that overall model is significant in table: 4.19-1. Again the Durbin Watson stat indicates that there is autocorrelation in the model. LM test also reveals that H_0 is rejected; means there is problem of autocorrelation in money supply (M2) equation.

Now we estimate the parameters of money demand equation (M2) and money supply equation (M2) by regressing there lags dependent variables, to remove the problem of autocorrelation.

Table: 4.20. Money Demand Function, (2SLS estimates)

Variables	Coefficients	t-stat	Std error	p-values
Constant	-1.4059	-0.8612	1.6325	0.3947
Log(Interest Rate)	-0.1765	-2.7854	0.0633	0.0084
Log(Real GDP)	0.6314	6.9145	0.0913	0.0000
Log(NEER)	0.0062	0.1012	0.062	0.9199
LM2(-1)	0.7580	5.8384	0.1298	0.0000
LM2(-2)	-0.3356	-2.9005	0.1157	0.0062

Note: ** show significant at 0.01, & * denotes significant at 0.05 level
Dependent variable is log of real money demand (M2)

Table: 4.20 shows the results of money demand function (M2). As we have seen from the table: 4.18 there is autocorrelation problem in money demand equation. Therefore we regress the dependent variable real money demand (M2) on two lags of dependent variable. We can see from the above table all the coefficients have the expected signs except the constant, which is negative in table: 4.20 but positive in table 4.18. Here we see two lags of dependent variable are significant. Now by including the two lags of dependent variable the adjusted R-square showing 99.5% variation in respondent variable by three right hand side variables and two lags of real money demand (M2). The DW- stat and LM test indicating the absence of autocorrelation. We can say the results have been improved.

Table: 4.20-1. LM test results (Money Demand Function)

Adj (R^2)	F-stat	DW-stat	DW h-stat	LM-test F-stat	LM-test p-value
0.995	1729.40	1.66	1.665	1.23	0.3119

Note: ** show significant at 0.01 levels.

The table below 4.21 shows the results of money supply function by including the lag of dependent variable as independent variable. We have seen from the table: 4.19 of money supply function (M2), the entire variable were highly significant but in present table: 4.21, we see the variables are not highly significant except inflation gap and lag of dependent variable. However the signs are expected as required. The F-stat of LM test indicating that there is no autocorrelation more in money supply (M2) equation.

Table: 4.21. Money Supply Function, (2SLS Estimates)

Variables	Coefficients	t-stat	Std error	p-values
Constant	0.7795	2.0929*	0.3724	0.0431
Log(Interest Rate)	0.2947	2.5600*	0.1247	0.01269
Output Gap	-0.495	-2.3351*	0.2963	0.01898
Inflation Gap	-0.0144	-5.5610**	0.0025	0.0000
LM2(-1)	0.9538	36.84**	0.0258	0.0000

Note: ** show significant at 0.01, & * denotes significant at 0.05 level
Dependent variable is log of real money supply (M2)

Table: 4.21-1 indicates that there is no more autocorrelation problem. Because Durbin h-statistic is insignificant, so we can say that money supply equation has no autocorrelation.

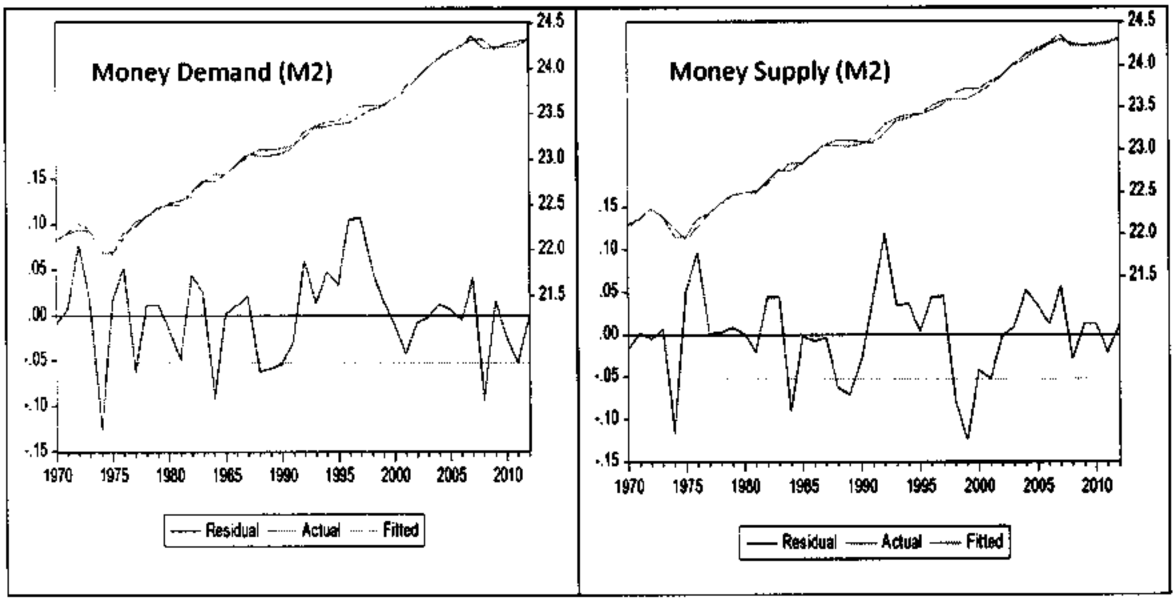
Table: 4.21-1. LM test results (Money Supply Function)

Adj (R^2)	F-stat	DW-stat	DW h-stat	LM-test F-stat	LM-test p-value
0.995	2163.75**	1.46	1.77	0.9478	0.4634

Note: ** show significant at 0.01 levels.

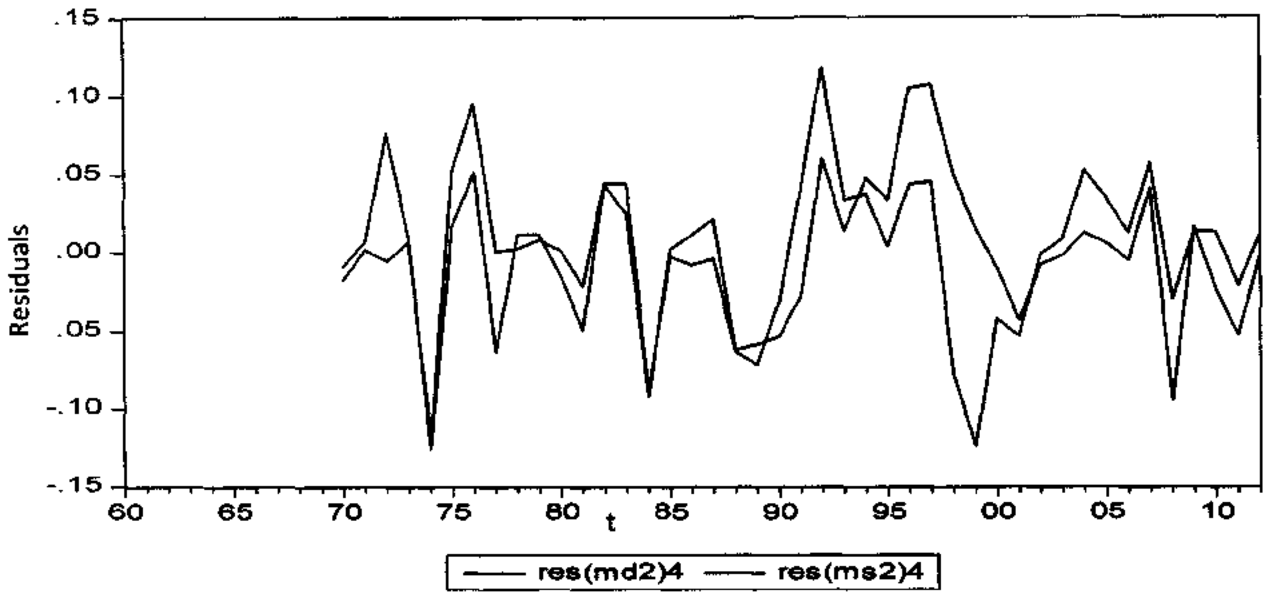
The actual and fitted graphs of money demand (M2) and money supply (M2) for (2SLS) are shown below in figure: 4.8. We see that there is no meaningful difference between both the actual and fitted residuals. It is clearly shown that both the residuals for money demand and money supply are correlated to each other.

Figure: 4.8- Actual, Fitted, Residuals Graph (2SLS) at level



The figure: 4.9 presenting that there is strong correlation between the residuals of money demand function and money supply function. This type of autocorrelation we will remove to applying the 3rd stage, in which we will regress the residuals as independent variable obtained by 2SLS procedure.

Figure: 4.9- (2SLS) Graph of correlation b/w two residuals



4.10. Three stage-least square estimates for (M2)

Money demand (M2) and money supply (M2) were estimated simultaneously using a three stage-least square procedure. The estimates of money demand equation and money supply equations are reported in table (4.22) and (4.23) respectively. The results reveal that all the estimated coefficients of money demand function have expected signs. The interest rate and nominal effective exchange rate (NEER) are positively related to real money demand, but the real GDP has positive effect on real money demand (M2). The interest rate and real GDP are significant at 1% but (NEER) is significant at 5% level of significance. But we have noted that, (NEER) was insignificant for (2SLS) estimates, and now significant for (3SLS) estimates. A relatively high (Adjusted R^2 = 0.99) value indicates that independent variables used in this model may explain 99% variation in real money demand. A one percent increase in interest rate will result in 0.4135 % decline in quantity of real money demand and conversely. The GDP has comparatively stronger effect on money demand. By increasing real GDP 1% the quantity of real money demand increase 0.8287%, however reverse will be true if real GDP decline. If the change occurs 1% in nominal effective exchange rate for Pakistani rupee the inverse affect will be 0.2695% on quantity of real money demand.

Table: 4.22. Money Demand Function, (3SLS Estimates)

Variables	Coefficients	t-stat	Std error	p-values
Constant	5.6639	1.7696	3.2006	0.0848
Log(Interest Rate)	-0.4135	-5.59**	0.0740	0.0000
Log(Real GDP)	0.8287	7.859**	0.1054	0.0000
Log(NEER)	-0.2695	-2.421*	0.1113	0.0204
Res(2SLS)M^s	0.1292	2.217*	0.0582	0.0326

Note: ** show significant at 0.01, &* denotes significant at 0.05 level.
Dependent variable is log of real money demand (M2)

Table: 4.22-1. LM test results (Money Demand Function)

Adj (R^2)	F-stat	DW-stat	LM-test F-stat	LM-test p-value
0.99	1228.37	2.01	0.334	0.9872

Note: ** show significant at 0.01 levels

From table: 4.22-1, we can see DW stat is 2.01 and LM test also reveals that there is no more autocorrelation problem.

Table: 4.23. Money supply function, (3SLS estimates)

Variables	Coefficients	t-stat	Std error	p-values
Constant	13.828	20.21**	0.6839	0.0000
Log(Interest Rate)	4.3654	14.06**	0.3103	0.0000
Output Gap	-9.8114	-11.03**	0.8891	0.0000
Inflation Gap	-0.0854	-8.39**	0.0102	0.0000
Res(2SLS)M^d	0.8882	1.299*	0.6835	0.2016

Note: ** show significant at 0.01, & * denotes significant at 0.05 level.
Dependent variable is log of real money supply (M2)

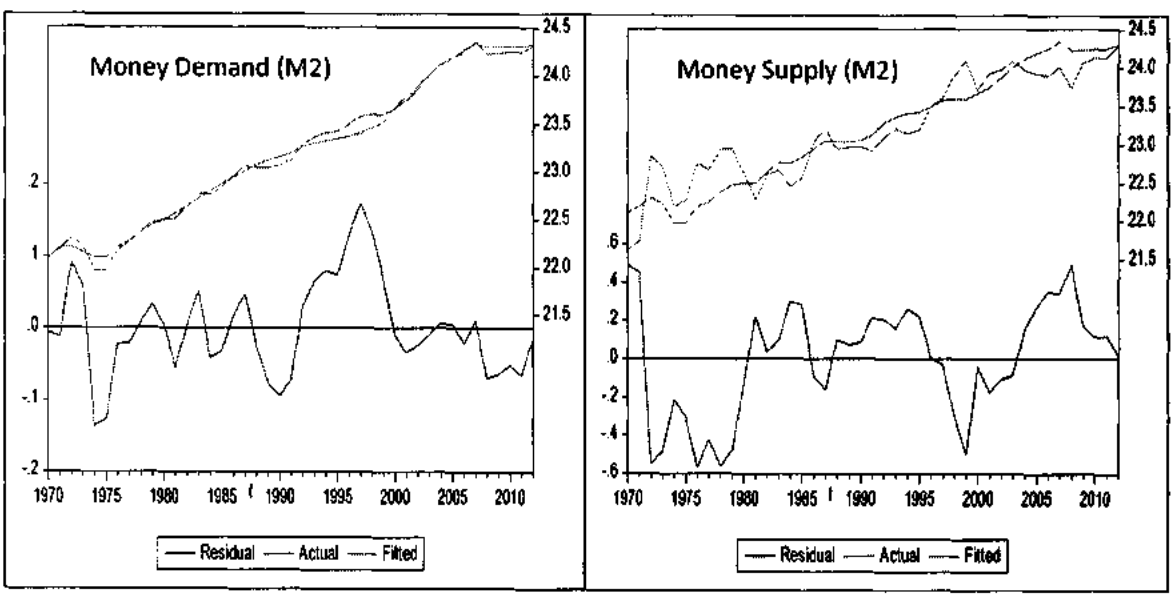
Table: 4.23-1. LM test results (Money Supply Function)

Adj (R^2)	F-stat	DW-stat	LM-test F-stat	LM-test p-value
0.83	52.322**	1.67	0.889	0.4734

Note: ** show significant at 0.01 levels.

Table: 4.23 show the estimates for real money supply (M2). As shown all the coefficients indicates the expected signs. Real money supply is positively associated with interest rate and negatively influenced by inflation gap and output gap. We can see from the table: 4.23 all the coefficients are significant at 1% level of significance. (Adjusted $R^2 = 0.83$) indicates approximately 83% variation in real money supply can be explained by three right hand side variables. The estimated parameters suggest that real money supply is more sensitive to output gap as compared to inflation gap.

Figure: 4.10- Actual, Fitted, Residuals Graphs (3SLS) at level



For (3SLS) estimates of money demand (M2) and money supply (M2) actual fitted and residuals graphs are shown in figure: 4.10. In money demand equation we see that actual and fitted lines are going to close to each other, and residuals are stable only three points are outside of the interval. But in money supply equation there is meaningful difference between actual and fitted lines, and residuals also not stable. Here we can see that both the residuals are not correlated to each other. Now we see the clearly uncorrelated residuals in figure below.

Figure: 4.11- (3SLS) Graph of correlation b/w two residuals

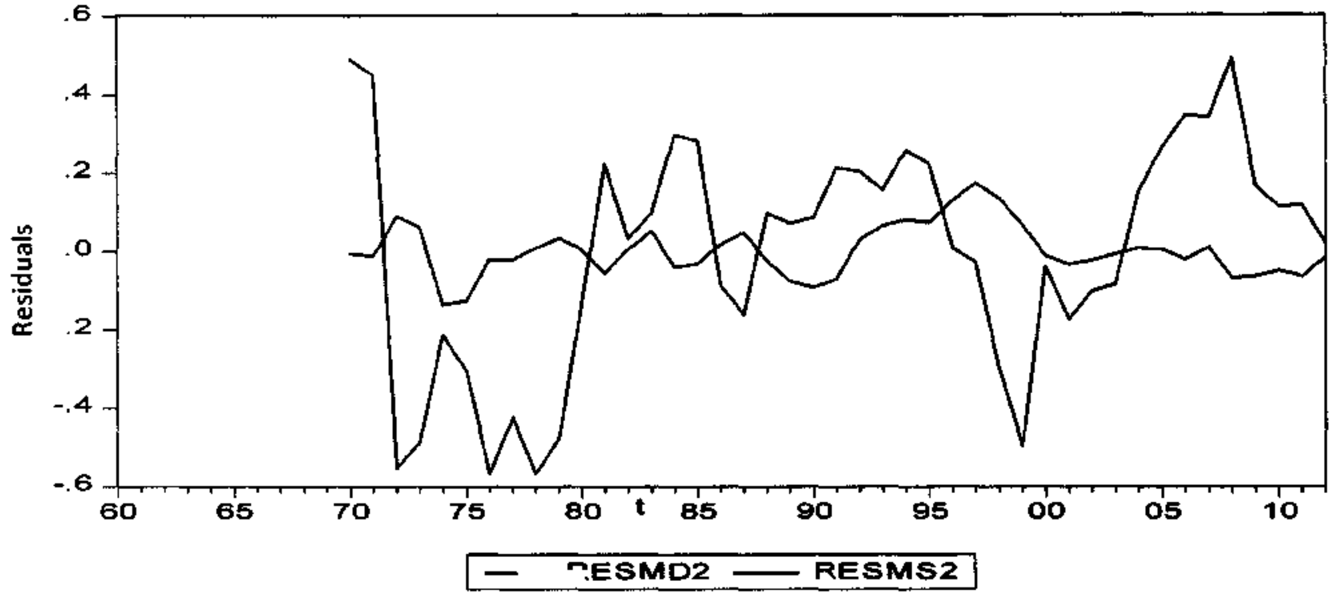


Figure: 4.11 showing that there is no more correlation between both the residuals obtained from money demand equation and money supply equation. So we can say that three stage least square estimates are more efficient as compared to two stage least square estimates.

4.11. Two stage-least square estimates for (M2) at 1st difference

It is to be noted from table: 4.6 Augmented Dickey-Fuller test results that there are two variables output gap (YG), and inflation gap (IG) which was stationary at level at 5% level of significance, but stationary at 1st difference. Now we estimate our results by taking the 1st difference of each variable, to check whether the results obtained by at 1st difference are reliable, meaningful or not.

Table: 4.24. Money Demand Function, (2SLS Estimates) at 1st diff

Variables	Coefficients	t-stat	Std error	p-values
Constant	-0.0101	-0.5802	0.017	0.5652
D[Log(Interest Rate)]	-0.5243	-6.764**	0.0775	0.0000
D[Log(Real GDP)]	0.9784	4.234**	0.2311	0.0000
D[Log(NEER)]	-0.3064	-3.294**	0.0930	0.0021

Note: ** show significant at 0.01, & * denotes significant at 0.05 level
Dependent variable is log of real money demand (M2)

Table: 4.24 represent the (2SLS) estimates of money demand function (M2) at 1st difference. If we compare these results with table: 4.18 results, we see that all the coefficients have same and expected signs except the constant coefficient. Here in table: 4.24 the interest rate and exchange rate are negatively correlated with money demand and real GDP is positively related with money demand. All the variables have significant effect on money demand.

In the lower part of table: 4.25-1 we see that variation explained by three right hand side variables is 55% in respondent variable, which is low as compared to variation listed in table: 4.18. The DW stat and LM test clearly showing that the problem of autocorrelation is not present in money demand equation.

Table: 4.24-1. LM test results (Money Demand Function) at 1st diff

Adj (R^2)	F-stat	DW-stat	LM-test F-stat	LM-test p-value
0.55	17.83**	1.63	0.7545	0.477

Note: ** show significant at 0.01 levels

Now if we make a glance at table: 4.25 listed the results (2SLS) of money supply function (M2). There is lot of difference between the results of table: 4.19 and table: 4.25. Although all the coefficients have same and expected signs, but the results reported in table: 4.25 are insignificant, which surprising for us. Also the coefficients are smaller as compared to the coefficients tabulated in table: 4.19.

Table: 4.25. Money Supply Function, (2SLS Estimates) at 1st diff

Variables	Coefficients	t-stat	Std error	p-values
Constant	0.0541	5.1728**	0.0104	0.0000
D[Log(Interest Rate)]	0.0362	0.1553	0.2330	0.8773
D[Output Gap]	-0.7147	-1.592	0.4489	0.1197
D[Inflation Gap]	-0.0085	-2.316*	0.0036	0.0260

Note: ** show significant at 0.01, & * denotes significant at 0.05 level
Dependent variable is log of real money supply (M2)

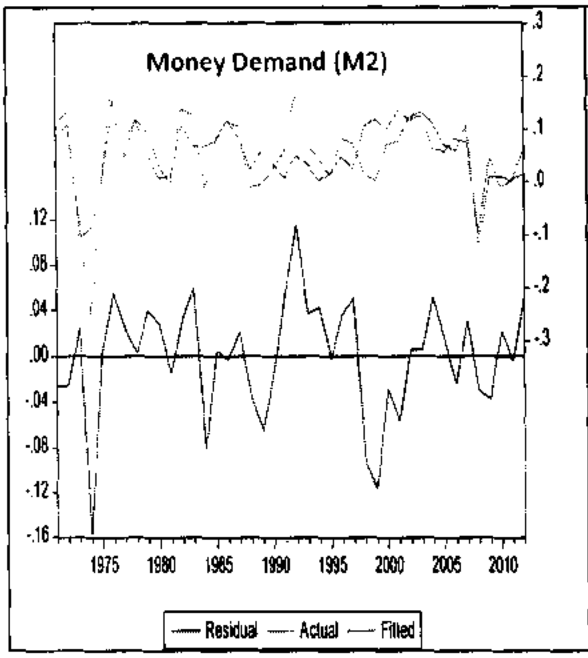
Table: 4.25-1 indicates some indicators of our model. F-stat showing that overall model is significant. LM test report that there is no autocorrelation problem in money supply equation. The adjusted R^2 is 0.38, which is comparatively low.

Table: 4.25-1. LM test results (Money Supply Function) at 1st diff

Adj (R^2)	F-stat	DW-stat	LM-test F-stat	LM-test p-value
0.38	9.39**	1.85	0.8539	0.4342

Note: ** show significant at 0.01 levels.

Figure: 4.12- Actual, Fitted, Residuals Graphs (2SLS) at 1st diff



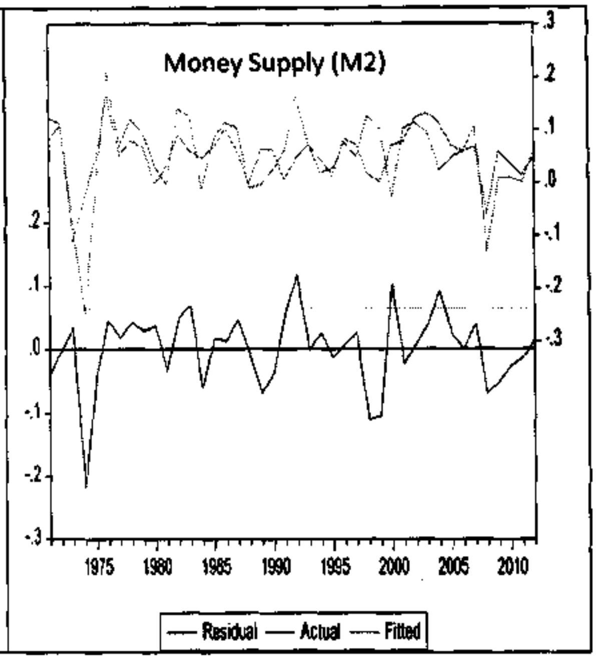


Figure: 4.12 presents the graphs of actual fitted, and residuals of money demand (M2) and money supply (M2) for (2SLS) estimates. Here there is meaningful difference between the actual and fitted lines in both the graphs. So it is clearly shown that residuals for both the equations are correlated to each other.

4.12. Three stage-least square estimates for (M2) at 1st difference

Three stage-least square estimates of money demand function (M2) at 1st difference are listed in table: 4.26 below. Here interest rate and exchange rate are showing negative effect with money demand, and real GDP has positive effect on money demand. The interest rate, exchange rate, and real GDP are significant at 1%, 5% and 10% level of significance respectively. But all of these variables have significant effect on money demand at 1% level of significance, reported in table: 4.26.

Table: 4.26. Money Demand Function, (3SLS estimates) at 1st diff

Variables	Coefficients	t-stat	Std error	p-values
Constant	0.0308	2.013*	0.0153	0.0515
D[Log(Interest Rate)]	-0.5242	-8.631**	0.0607	0.0000
D[Log(Real GDP)]	0.3813	1.8457	0.2066	0.0732
D[Log(NEER)]	-0.1963	-2.679*	0.0732	0.0345
D[Res(2SLS)M^s]	0.4902	5.452**	0.0899	0.0000

Note: ** show significant at 0.01, & * denotes significant at 0.05 level.
Dependent variable is log of real money demand (M2)

Table: 4.26-1 reveals that there is problem of autocorrelation in money demand equation. The overall model is significant. 74% variation is explained by three right hand side variables in dependent variable.

Table: 4.26-1. LM test results (Money Supply Function) at 1st diff

Adj (R^2)	F-stat	DW-stat	LM-test F-stat	LM-test p-value
0.74	30.71**	1.16	5.70**	0.0073

Note: ** show significant at 0.01 levels

Table: 4.27 reveals that inflation gap and output gap negatively associated with money supply (M2) and interest rate also has negative effect on money supply but insignificant. All of these variables we see that have insignificant effect on real money supply. If we compare these results reported in table: 4.27 with table: 4.23, where all the coefficients have same and expected signs, but all the coefficients were significant at 1% level of significance.

Table: 4.27. Money Supply Function, (3SLS Estimates) at 1st diff

Variables	Coefficients	t-stat	Std error	p-values
Constant	0.0572	6.90**	0.0082	0.0000
D[Log(Interest Rate)]	-0.1233	-0.669	0.1840	0.5072
D[Output Gap]	-0.5585	-1.607	0.3474	0.1167
D[Inflation Gap]	-0.0054	-1.869	0.0029	0.0697
D[Res(2SLS)M^d]	0.6137	5.205**	0.1179	0.0000

Note: ** show significant at 0.01, & * denotes significant at 0.05 level.
Dependent variable is log of real money supply (M2)

Here in table: 4.27-1 a healthy variation explained by three right hand side variables in respondent variable. F-stat shows that overall model is significant. Also there is problem of autocorrelation.

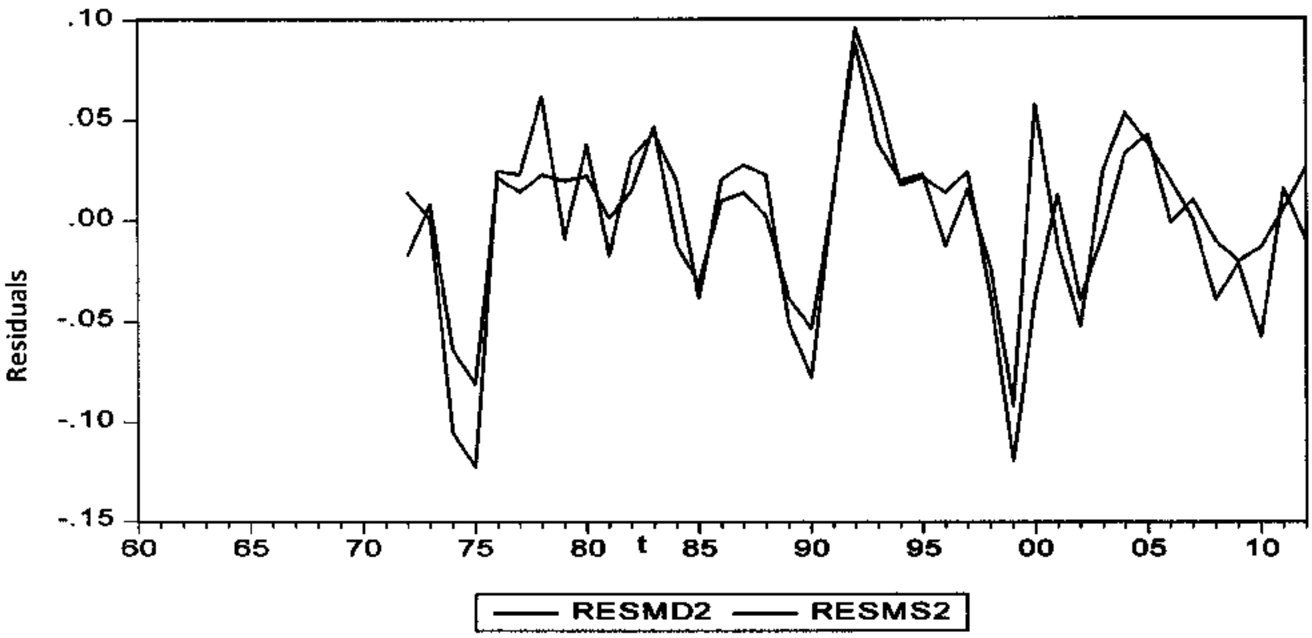
Table: 4.27-1. LM test results (Money Supply Function) at 1st diff

Adj (R^2)	F-stat	DW-stat	LM-test F-stat	LM-test p-value
0.63	18.72**	1.55	1.002	0.9982

Note: ** show significant at 0.01 levels

Here we can see from figure: 4.13 that, when three stage-least square estimates are estimated by taking the 1st difference of each variable, but there is strong correlation between both residuals of money demand (M2) and money supply (M2) equations. So we can say that the results obtained by (3SLS) at 1st difference are not efficient. Because we use (3SLS) technique to remove the problem of correlation between two error terms but still there is correlation between two error terms we can see it on next page figure: 4.13.

Figure: 4.13- (3SLS) Graph of correlation b/w two residuals (at 1st diff)



4.13 Forecasting

It is known fact, that forecasting is an important part of econometric analysis. It is very helpful to the countries to make their future policies. To forecast the econometric variables there are two techniques,

1. **Autoregressive Integrated Moving Average (ARIMA)**
2. **Vector Auto Regression (VAR)**

Here in our model we use the (ARIMA) model to forecast the economic variables, as nominal interest rate (R), real GDP, exchange rate (PAK/USD), and nominal effective exchange rate, to forecast the money demand (M1), and (M2). We forecast the money demand (M1) and (M2) for next six years (2015 to 2020). For this purpose we chose the best models to forecast each variable, using (ARIMA) model dynamically. We forecast each variable one step ahead, dynamically.

4.13-1. Forecasting of money demand (M1)

To forecast the money demand (M1) for next six years we use our (3SLS) estimates given in table: 4.12, because these estimates are unbiased, consistent and efficient. The econometric model can be formed as.

$$M_t^d = 9.7915 - 0.4064R_t + 0.5076Y_t + 0.5408EX_t \quad (26)$$

4.13-2. Forecasting of money demand (M2)

To estimate the future values of money demand (M2) for next six years we use the results of (3SLS) estimates reported in table: 4.22. From this table the equation of money demand (M2) can be formed as

$$M_t^d = 5.6639 - 0.4135R_t + 0.8287Y_t - 0.2695EX_t \quad (27)$$

For next six years, estimated values of money demand (M1) and (M2) in Pakistan are as following table.

Table: 4.28. Forecasting Money Demand (M1) and (M2)

Years	Money Demand (M1) Log(Rupee)	Money Demand (M2) Log(Rupee)
2014	24.27548	24.67293
2015	24.38624	24.69565
2016	24.44523	24.78954
2017	24.50422	24.84787
2018	24.56321	24.90625
2019	24.62215	24.96453
2020	24.68115	25.02286

Figure: 4.14- Actual, Fitted, Residuals Graph of Forecasting Model (M2)

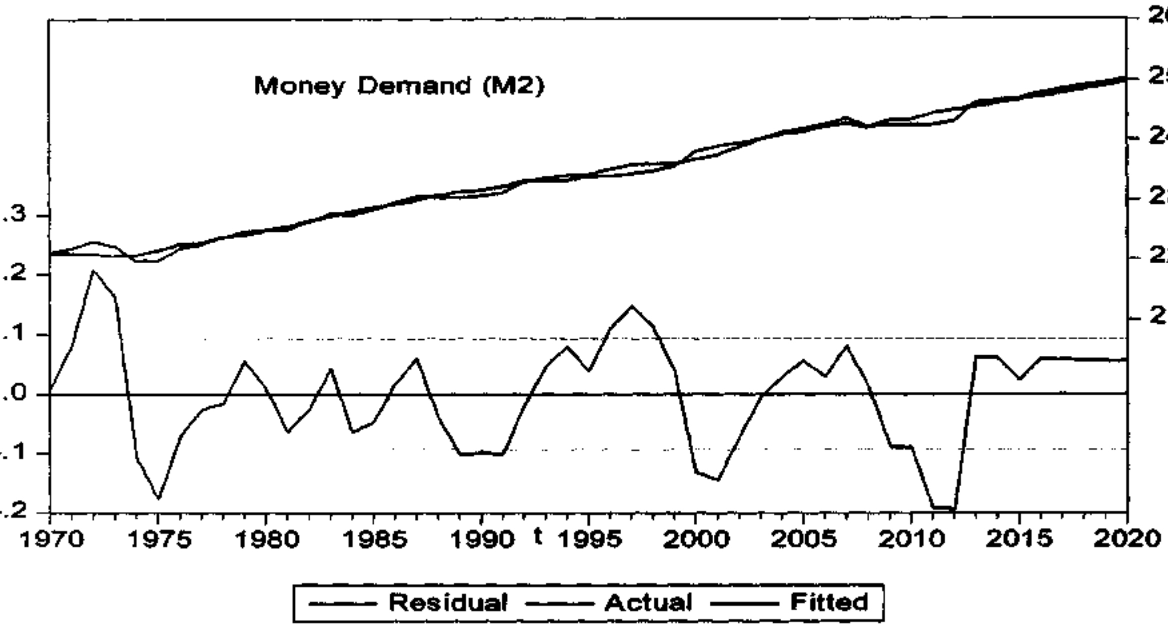


Figure: 4.15- Actual, Fitted, Residuals Graph of Forecasting Model (M1)

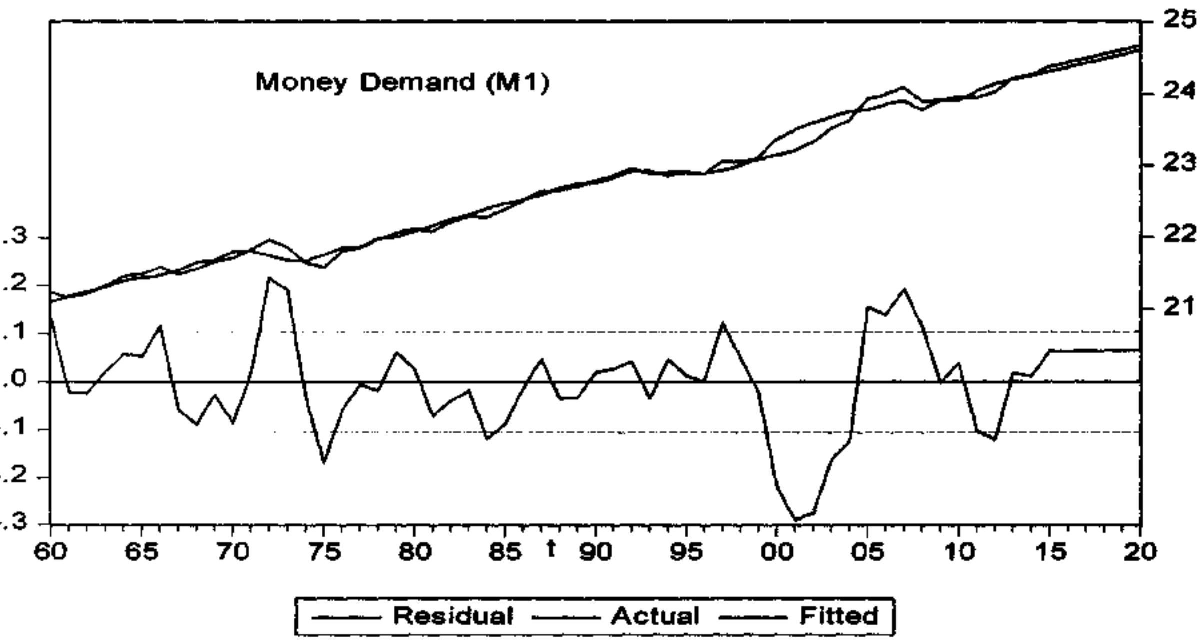


Figure: 4.14 and 4.15 are presenting the actual fitted residuals graphs of forecasting model, here we see that actual and fitted lines become straight and smooth for both money demand function (M1) and (M2). The residuals also become zero.

Figure: 4.16. Forecasting Graph (M1)

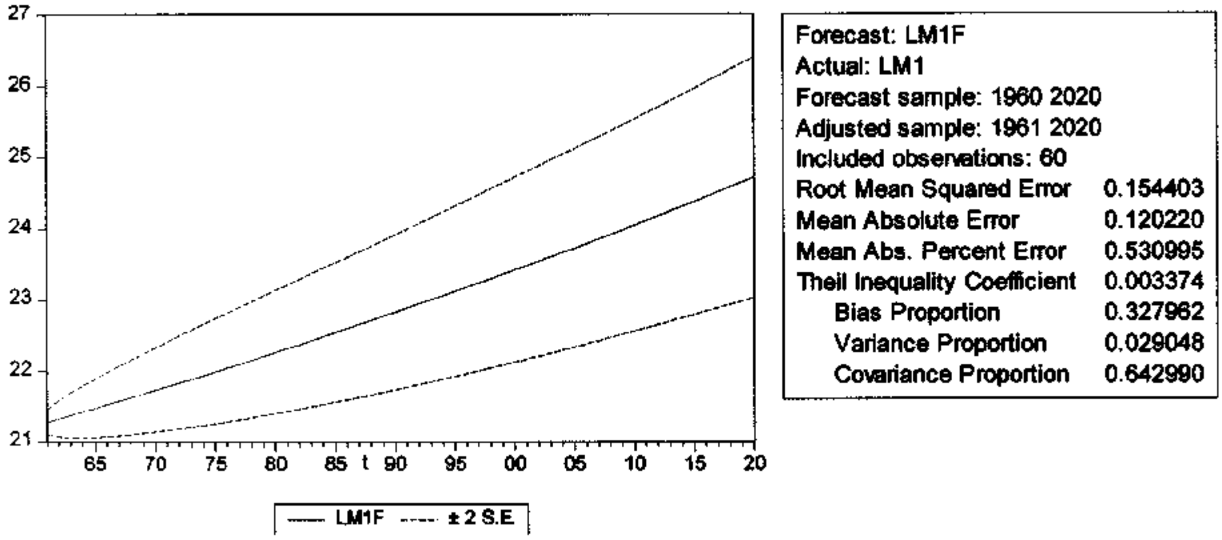
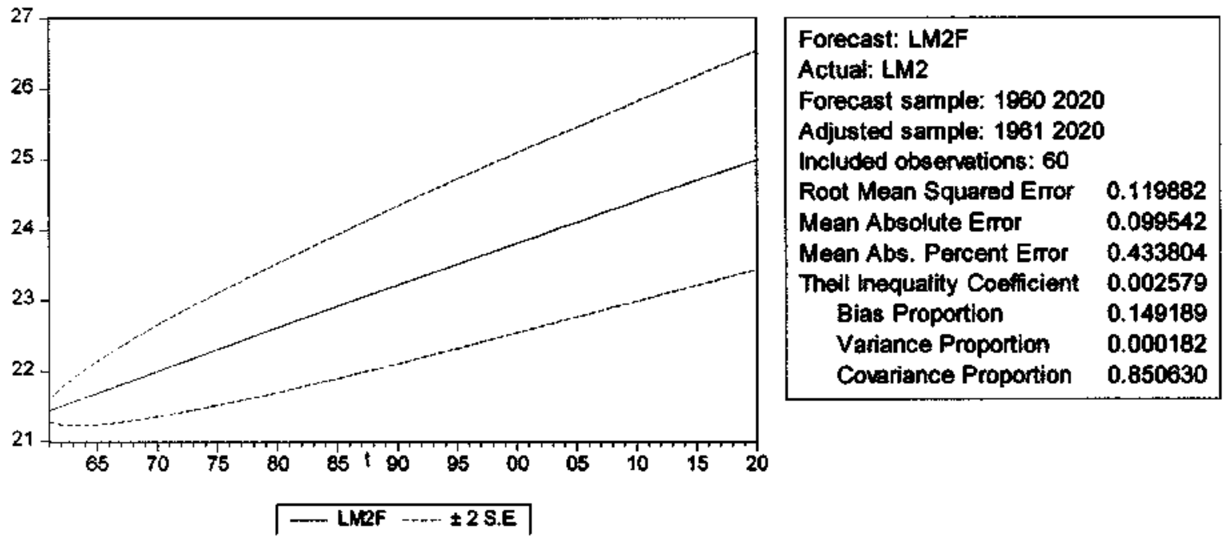


Figure: 4.17. Forecasting Graph (M2)



In figure: 4.16, we present the forecasting graph of money demand (M1) in log form. Theil Inequality and variance proportion are approximately close to zero. But in figure: 4.17 we note that Theil Inequality coefficient, variance proportion, and Mean Absolute Error are close to zero. The forecasting of money demand (M1) and (M2) are between plus and minus 2 standard deviation.

Chapter 5

CONCLUSION & RECOMMENDATIONS

In literature the money demand function has been a central interest. The central bank conducts its monetary policies on the basis of determination of a correctly specified money demand and money supply functions. The stability of money demand function is pre-requisition to make any new policy driven by the State Bank. Usually it is known that a single equation method is used to estimate the parameters of money demand and money supply functions, which are to be likely gives the biased and inconsistent results because of the simultaneity problem.

In this study we present a simple system of simultaneous equation model of money demand and money supply. The first requisition of this model is that whether there is simultaneity problem or not. For this purpose we apply Hausman test, which shows that there is simultaneity problem between Nominal Interest Rate and Real Money Demand (M1, M2).

At second step, we check the identification of both money demand and money supply equations. For this purpose we apply order and rank condition on our model and conclude that both the equations are over identified. At third step, the Augmented Dickey-Fuller test is used to check the stationary of variables, and its clearly indicates that all the variables of money demand function and money supply function are not stationary at level but stationary at their 1st difference at 1 percent level of significance. The Augmented Angel-Granger test is used to check the long run relationship between the variables, and its shows that Nominal Interest Rate, Real GDP, Exchange Rate (PAK/USD) and Real Money Demand (M1) and (M2) have long run equilibrium relationship.

At fourth step, we estimate the parameters of money demand function (M1, M2) and money supply function (M1, M2) using Two Stage-Least Square (2SLS) estimates procedure at level and also at 1st difference. When we estimate the parameters of money demand function (M1) at level, the estimated long run money demand function (M1) has negative effect with nominal interest rate (R), and exchange rate (PAK/USD), but the real (GDP) has positive effect on money demand (M1). In real money supply function (M1) the interest rate is positively associated with real money supply (M1), but the output gap and inflation gap were negatively associated to dependent variable. The output gap is more sensitive to real money supply as compared inflation gap. The results obtained by (2SLS) are unbiased and consistent but not efficient, because the residuals of both the equations were correlated to each other. To overcome this problem, we used the 3rd stage which is also known as Seemingly Unrelated Regression (SUR) model. The results obtained by this process were not only unbiased, and consistent but also efficient.

After that we estimate the parameters of our model at 1st difference, because all the variables were stationary at 1st difference. By applying (2SLS) and (3SLS) techniques on the model of money demand (M1) and money supply (M1) to estimate the results, we were astonished to see that that all the variables have expected signs but insignificant.

At step number five, we estimate the parameters of money demand (M2) and money supply (M2) using 2 stage-least square (2SLS) methods. Again we see that all the coefficients in money demand function (M2) had expected signs. The interest rate and nominal effective exchange rate (NEER) are negatively affected and real GDP is positively associated with real money demand (M2). Again the same results we saw in money supply function (M2) inflation gap and output gap were negatively associated and interest rate was positively affected with real money supply (M2). But again we face a problem that both the equations residuals were correlated to each other, which makes the results inefficient. By applying (3SLS) technique our estimates became more efficient as compared to the results of (2SLS).

Again we estimate the results for (M2) at 1st difference. We saw that nominal interest rate and (NEER) has negative effect on real money demand (M2) and real GDP has positive effect. It is interesting to note that, the entire coefficients were significant for money demand function (M2). But in money supply function (M2) inflation gap has significant effect but all other variables were not significant.

From the analysis report we can conclude that the coefficients for money demand and money supply functions (M2) are greater than the money demand and money supply (M1) functions. So here we can say that "money demand and money supply (M2) are more sensitive as compared to money demand and money supply (M1)".

At the end we realize that, the exchange rate (PAK/USD) has a low impact on money demand; it means that Pakistani people do not desire to hold money if rupees value increased as compared to U.S. dollar, but it's less elastic because the coefficient of exchange rate (PAK/USD) is less than 1. We can also say that depreciation in Pakistani rupee the real money demand would be increased but its effect is very low. There is another important factor "interest rate" which is negatively affected by real money demand (M1) and (M2). We can say that the people of Pakistani want more money if interest rate is low. The real GDP has significant positive effect on real money demand (M1). The estimated coefficient of real GDP is less than 1 for (3SLS) estimates, which implies that money in Pakistan cannot be considered as luxury. We also conclude that State Bank of Pakistan circulate more money if rate of interest higher. In Pakistan a decrease in inflation gap and output gap will accelerate to money supply.

5.1 Recommendations

The money demand function has been a topic of continuing research. Usually parameters of the money demand function are estimated by a single equation method which is likely to be biased and inconsistent. In this study we present a simple system of equations representing money demand and supply relationships in Pakistan. There are number of policy implications main of these are listed as. First, in estimating the money demand function, the money supply function should not be treated as exogenous. Second, a change in monetary policy regarding the interest rate with the aim of controlling inflation is expected to affect real money supply. Third, while the inflation gap has a major role in the money supply function, the output gap could also be a significant tool for monetary policy. Finally, a consistent and unbiased estimate of the quantity of money demand is a useful indicator of GDP.

These results suggest that State Bank of Pakistan would reduce money supply, if inflation gap and output gap increase.

Chapter 6

REFERENCES

Abe, S. et al, (1975). "The Demand for Money in Pakistan: Some Alternative Estimates", *Pakistan Development Review.* Vol. XIV, No. 2. Summer

Ahmed, S. and Islam, M. E. (2007). "A Co-integration Analysis of the Demand for Money in Bangladesh", *Bangladesh Development Studies*, 30(1 & 2), 1-48.

Arango, S. and Nadiri, M. I. (1981). "Demand for Money in Open Economies", *Journal of Monetary Economics* 7, 69-83.

Bashier, A. A. and Dahlan, A. (2011). "The Money Demand Function for Jordan: An Empirical Investigation", *International Journal of Business and Social Science,* Vol. 2, No. 5; [Special Issue- March 2011].

Chow, G. C. (1966). "On the Long-Run and Short-Run Demand for Money", *Journal of Political Economy,* 74, 111-131

Cziraky, D. and Gillman, M. (2006). "Money demand in an EU accession country: a VECM study of Croatia", *Bulletin of Economic Research,* 58(2): 73–159.

Fair, R. C. (1987). "International Evidence on the Demand for Money", *Review of Economics and Statistics,* 69 (3) 473-480.

Fanta, F. (2012). "Macroeconomic Uncertainty, Excess Liquidity and Stability of Money Demand (M3) in Australia", *International Journal of Monetary Economics and Finance*, 5 (4), 325-344.

Gregory, A. W. and McAleer, M. (1981). "Simultaneity and the Demand for Money in Canada: Comments and Extensions", *The Canadian Journal of Economics/Revue Canadienne d'Economique*, Vol. 14, No. 3, pp. 488-496.

Goldfeld, Stephen M. (1994). "Demand for Money: Empirical Studies", *In the New Palgrave Dictionary of Money and Finance. Edited by P. Newman, M. Milgrate and J. Eatwell. London: Macmillan Press.*

Hafer, R. W. and Jansen, D. W. (1991). "The Demand for Money in the United States: Evidence from Cointegration Tests", *Journal of Money, Credit and Banking,* 23 (2), 155-168.

Handa, J. (2009). *Monetary Economics*. New York: Routledge Taylor & Francis Group.

Hising, Y. and Jamal, M. M. (2013). "A Simultaneous Equation Model of Money Demand and Money Supply for Canada", *Modern economy* 4, 32-36.

Iqbal, Z. and Siddiqi, J.S. (2008). "Modeling Broad Demand for money (M2) in Pakistan: A Statistical Investigation", Presented on International Conference on Mathematical Sciences (ICM) held on 3-6 March 2008 at UAE University, Alain, and UAE

Laidler, D. (1999). "The Quantity of Money and Monetary Policy", ***Working Paper*** *Series No. 99–5. Ottawa: Bank of Canada*

Mishkin, F. S. (2007). "The Economics of Money, Banking and Financial Markets", *Boston: Pearson–Addison Wesley*

Mundell, R. A. (1963). "Capital mobility and stabilization policy under fixed and flexible exchange rates", *Canadian Journal of Economics and Political Science,* Volume 29(4), pp. 475-485.

Najam, S. and Ahmed, M.A. (1986). "Monetary and Physical Policy Issues", ***Pakistan*** *Development Review,* Vol. XXV, No. 4.

Poloz, S. S. (1980). "Simultaneity and the Demand for Money in Canada", *The Canadian Journal of Economics/Revue Ca- nadienned'Economique*, Vol. 13, No. 3, pp. 407-420.

Qayyum, A. (2000). "Demand for Real Money Balances by the Business", ***The Pakistan*** *Development Review* 39: 4 Part II (Winter 2000) pp. 857–873

Siklos, P. L. Barton, A. G. (2001). "Monetary aggregates as indicators of economic activity in Canada: empirical evidence", *Canadian Journal of Economics*, 34: 1–17.

Valadkhani, A. (2008). "Long- and Short-Run Determinants of the Demand for Money in the Asian-Pacific Countries: An Empirical Panel Investigation", *Annals of Economics and Finance*, 9(1), 47-60

Wooldridge, J. M. (2003). *Introductory Econometrics: A Modern Approach*. Thomson.