Homogeneous-Heterogeneous Reactions in Stagnation Point Flow of Casson Fluid

By

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Fluid mechanics.
Newtonian fluids.

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A DISSERTATION SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF THE MASTER OF SCIENCE IN MATHEMATICS

We accept this dissertation as conforming to the required standard.

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Declaration

I hereby declare that this thesis, neither as a whole nor as a part thereof, has been copied out from any source. It is further declared that I have prepared this thesis entirely on the basis of my personal efforts made under the sincere guidance of my supervisor. No portion of the work, presented in thesis, has been submitted in the support of any application for any degree or qualification of this or any other institute of learning.

Signature: ________ _______

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Dedicated to

My Family

Acknowledgement

To begin with the name of Almighty **ALLAH,** the creator of the universe, who bestowed his blessing on me to complete this dissertation. I offer my humblest, sincerest and million Darood to the **Holy Prophet Hazrat MUHAMMAD (s.a.w)** who exhorts his followers to seek for knowledge from cradle to grave.

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Preface

The flow problem of Newtonian/non-Newtonian fluids in the boundary layer induced by a continuously moving or stretching surface is important in many manufacturing processes. In industry, polymer sheets and filaments are manufactured by continuous extrusion of the polymer from a die to a windup roller, which is located at a finite distance away. The thin polymer sheet constitutes a continuously moving surface with a non-uniform velocity through an ambient fluid [1]. Crane [2] has discussed in very detail about the effects of flow past a stretching plate, and he obtained a numerical solution for his flow problem with the help of Runge-Kutta method. Heat and mass transfer has been discussed by P.S. Gupta [3]. He also considered a stretching sheet for his flow problem with the extra effects of suction and blow. In stagnation point flow, a rigid wall or a stretching surface occupies the entire domain $y > 0$ for the horizontal axes and the flow impinges on the wall either orthogonal or at an arbitrary angle of incidence. Heat transfer in stagnation point towards a stretching sheet has been discussed by T.R. Mahapatra, A.S Gupta [4].

Due to increasing interest in the flow of fluids, a number of materials are utilized whose flow qualities are not analyzed with the help of Newtonian fluid model. In this situation, non-Newtonian fluid are very important because of their applications in polymer processing industries, petroleum drilling and biofluids dynamics and many others. The most popular subclass of these fluids is Casson fluid, which displays yield stress impact. Oka [5] discussed the characteristic of Casson fluid model in tubes and considered a generalized model for flow of Non-Newtonian fluid in tube from which the Casson fluid model was constructed as a special case. Boundary layer flow of different fluids in the region of stagnation point on a stretching/shrinking surface has attracted many scientists and engineers due to its real world applications in industry engineering processes [6-12]. Motivated by the above mentioned studies, the thesis is arranged as follow as follow.

Chapter 1 includes some basic definitions and prerequisites [13-14] for the convenience and better understanding of the reader. Chapter 2 comprises the study of non-orthogonal stagnation point flow towards a stretching sheet [19]. The model equations are solved with the help of Runge-Kutta method of order 4^{th} [15]. In chapter 3, we revised the work related to homogenous-heterogeneous reactions in stagnation point flow of Casson fluid [20].

Contents

Chapter 1

Preliminaries

In this chapter, some basic definitions and concepts [15] which are used directly or indirectly in the next chapters are discussed. The equations that govern the flow of any fluid namely continuity and momentum equations are also presented. Shooting method with Runge-Kutta fourth order [18] integration for the general second order boundary value problem also presented in detail.

1.1 Fundamental Concepts

Some basic definitions and fundamental concept for better understanding of the next chapters are as follows:

1.1.1 Deformation

Change of body from a reference shape to a current shape is known as deformation. A deformation may cause by external loads, body force (such as gravity or electromagnetic force) or change in temperature.

1.1.2 Shear stress

Shear stress is defined as the ratio of shear force to the cross-sectional area. It is denoted by τ, and mathematically it can be represented as

$$\tau = \frac{shear\ force}{cross - sectional\ area} \tag{1.1}$$

1.1.3 Fluid

Fluid is a substance that deforms continuously when acted upon by shear stress of any magnitude and in this way liquids and gasses are considered as fluids. Water, oil and air are common examples of fluid.

1.1.4 Flow

A phenomenon of continuous deformation under the action of applied force is called flow.

1.1.5 Fluid mechanic

Fluid mechanic is a branch of continuous mechanic in which we deal with the behaviors of fluids in the state of rest and motion.

1.1.6 Stagnation point

A stagnation point is a point in a flow field where the local velocity of the fluid is zero. Stagnation point exists at the surface of object in the flow field where the fluid is brought to rest by the object.

1.2 Properties of fluid

The fluid can be analyzed based on following defined properties.

1.2.1 Density

Density is define as the amount of mass per unit volume of the fluid. Mathematically it is denoted by

$$\rho = \frac{mass\ of\ fluid}{volume\ of\ fluid} = \frac{m}{v} \tag{1.2}$$

1.2.2 Viscosity

It is defined as the resistance offered to a layer of fluid when it moves over another layer of fluid. It is denoted by μ and mathematically denoted as

$$viscosity = \frac{shear\ stress}{rate\ of\ shear\ strain}$$

or

$$\mu = \frac{\tau_{yx}}{du/dy} \tag{1.3}$$

1.2.3 Temperature

Temperature is a physical quantity that measures the degree of hotness and coldness in an object on a numerical scale. Temperature does not depend on the number of particles in an object as it is an average measurement.

1.2.4 Kinematic viscosity

Kinematic viscosity is the ratio of dynamic viscosity to density (mass density) of a fluid. It is denoted by υ and mathematically it can be defined as

$$\upsilon = \frac{\mu}{\rho}. \tag{1.4}$$

1.2.5 Velocity field

Fluid motion cannot be visualized without the concept of velocity field. Among the properties of a flow, the velocity field $V(r,t)$ is the foremost. By a solution of the flow problem we mean to find its velocity field, once a velocity field is determined other properties follow directly from it. If one needs to determine temperature field, it can be obtained once a velocity field is known. Generally, velocity is a vector point function of position and times and has three component in a three dimensional space. Mathematically

$$V(r,t) = [\, u(r,t), v(r,t), w(r,t)], \qquad (1.5)$$

here r denote the position vector and $u, v \; and \; w$ are components of velocity in three orthogonal directions respectively.

1.3 Classification of fluid

The fluid can be classified into following two classes based on the viscosity of the fluid.

1.3.1 Ideal fluid

A fluid which is incompressible and in which viscosity is zero then, it is called an ideal fluid. The occurrence of such fluid in real world is rare. Ideal fluid is also known as inviscid fluid.

1.3.2 Real fluid

A fluid which is incompressible but viscous is called a real fluid. Real fluid is also known as viscous fluid. Real fluid is further divided into two categories.

(i) Newtonian fluid

(ii) Non-Newtonian fluid

(i) Newtonian fluid

Fluids which obey the Newton's law of viscosity are called Newtonian fluids. Newton's law of viscosity is expressed by

$$\tau = \mu \frac{dv}{dy} \qquad (1.6)$$

where τ is shear stress, μ is the viscosity of the fluid and $\frac{dv}{dy}$ is the rate of strain. Examples of Newtonian fluids are water, glycerin, light, hydrocarbon oil and silicon oil etc.

(ii) Non-Newtonian fluid

Fluids which do not obey the Newton's law of viscosity are called non-Newtonian fluids. Examples of Non-Newtonian fluids are ketchup, tooth paste, blood, paints and greases etc.

1.3.3 Compressible fluid

Compressible fluids are those in which fluid density changes with the change in pressure or temperature. In general all gasses are treated as compressible fluid.

1.3.4 Incompressible fluid

Incompressible fluids are those in which fluid density remain independent of the pressure or temperature.

1.4 Types of flow

The flows are distinguished on the basis of time dependence.

1.4.1 Steady flow

For steady flow, all the properties of fluid flow (e.g. velocity, pressure and density etc.) are independent of time. Mathematically it can be written as

$$\frac{\partial \eta}{\partial t} = 0 \tag{1.7}$$

where η represents the any fluid property.

1.4.2 Unsteady flow

For unsteady flow, the properties of fluid are function of time. Mathematically it can be represented as

$$\frac{\partial \eta}{\partial t} \neq 0 \tag{1.8}$$

where η represents any of fluid property.

1.4.3 Uniform flow

Flow is called a uniform flow if the velocity of flow does not alter either in sense of magnitude or in direction for a given time at any point in a flowing fluid.

1.4.4 Non-Uniform flow

Flow is called a non-uniform flow if the change is occur in the velocity of the flow for a given time at various points in a flowing fluid.

1.4.5 Laminar flow or stream flow

A flow is called a laminar flow if the fluid particle move in a smooth parallel layers as shown in figure 1.1.

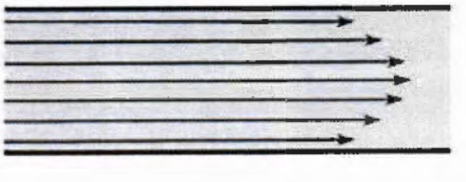

Figure 1.1: Laminar flow pattern

1.4.6 Turbulent flow

A flow is said to be turbulent flow if the particles of the fluid mix with each other and do not have specific path is known as turbulent flow. In turbulent flow, the velocity of the fluid at each point changes randomly both in direction and in magnitude.

Figure 1.2: Turbulent flow

1.4.7 Rotational flow

If each particle of the fluid rotate about its own axis then this kind of flow is called rotational flow.

1.4.8 Irrotational flow

An irrotational flow is defined as the flow in which fluid particles do not rotate about their own axes.

1.5 Boundary layer

Ludwig Prandtl, a German astronomer provided the idea of boundary layer in 1904, through his paper which he presented in mathematical congress. It is a layer adjacent to the solid surface, where the viscosity effects are dominant in determine the flow field. Thus a fluid flow is retarded in the vicinity of the wall and a slow moving boundary layer is formed. The thickness of the boundary layer is taken to be the distance from the wall to the point at which the velocity is equal to 99% of the free-stream velocity. As the solution of the Navier-Stoke equation is expensive, so this approach helps us to reduce and simplify the equations.

1.6 Dimensionless numbers

Dimensionless quantities are those quantities which have no physical dimension (length, mass, time). The relevant dimensionless numbers which arise in the study of fluid mechanics are as follows;

1.6.1 Reynold's number

Reynold's number is defined as the ratio of inertial forces to the viscous forces, and is denoted by Re. Mathematically it can be expressed as

$$Re = \frac{inertial\, forces}{viscous\, forces} \tag{1.9}$$

1.6.2 Prandtl number

The ratio of momentum diffusivity to thermal diffusivity is called Prandtl number. It is denoted by Pr, and can be expressed as

$$Pr = \frac{c_p \mu}{k} \tag{1.10}$$

where c_p denote heat capacity per unit volume of the fluid, μ denote the kinematic viscosity and k is its thermal conductivity.

1.8 Governing equations

The laws which govern the motion of fluid are described as follow:

1.8.1 Continuity equation

The partial differential equation which represent the law of conservation of mass is called continuity equation. The general form of continuity equation is

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho V) = 0. \tag{1.11}$$

For incompressible flow i.e. if the density is independent of time and space the above equation take the simplified form

$$\nabla.V = 0. \tag{1.12}$$

In cylindrical coordinates, the continuity equation is of the form

$$\frac{\partial u}{\partial z} + \frac{1}{r}\frac{\partial (vr)}{\partial r} + \frac{1}{r}\frac{\partial w}{\partial \phi} = 0, \tag{1.13}$$

here u, v and w are velocity component respectively.

1.8.2 Energy equation

The energy equation used to analyze heat transfer within the fluid. Energy equation can be construct by using first law of thermodynamics under the statement that energy can neither be created nor be destroyed. In Cartesian coordinate system, the energy equation is

$$u\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} + w\,\frac{\partial T}{\partial z} = \left(\frac{k}{pc_p}\right)\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + \left(\frac{\mu}{pc_p}\right)\varphi, \tag{1.14}$$

where, the dissipation function for three-dimensional case is

$$\varphi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right]^2 + \left[\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right]^2 + \left[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right]^2 \tag{1.15}$$

The energy equation for steady fluid flow, in term of the cylindrical coordinate system is

$$u\frac{\partial T}{\partial z} + v\,\frac{\partial T}{\partial r} + \frac{w}{r}\frac{\partial T}{\partial \varphi} = \left(\frac{k}{pc_p}\right)\left[\frac{\partial^2 T}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \varphi^2}\right] + \left(\frac{\mu}{pc_p}\right)\varphi, \tag{1.16}$$

where the dissipation function is

$$\varphi = 2\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial w}{\partial \varphi} + \frac{v}{r}\right)^2\right] + \left[\frac{1}{r}\frac{\partial u}{\partial \varphi} + \frac{\partial w}{\partial z}\right]^2 + \left[\frac{\partial v}{\partial z} + \frac{\partial u}{\partial r}\right]^2 + \left[\frac{1}{r}\frac{\partial v}{\partial \varphi} + \frac{\partial w}{\partial r} - \frac{w}{r}\right]^2. \tag{1.15}$$

1.8.3 Momentum equation

The equation of motion representing the law of conservation of momentum in vector form is

$$\rho\frac{\partial V}{\partial t} = \nabla.\mathrm{T} + \rho b, \tag{1.16}$$

here b is the body force per unit mass and Cauchy stress T is given by

$$\mathrm{T} = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix}, \tag{1.17}$$

where τ_{xx}, τ_{yy} and τ_{zz} are the normal stresses and τ_{xy}, τ_{xz}, and τ_{zy} are the shear stresses.

1.9 Numerical methods

Finding the exact solution of governing partial differential equations is hardly possible due to its nonlinearity, but numerical solution of such equations can be computed. Although a lot of numerical method are being used to find numerical solution of these types of problems, fourth-order Runge-Kutta method (RK) and shooting method are more frequently used. The details of these methods are as follows:

1.9.1 Fourth-order Runge-Kutta method

There are many numerical techniques available for solving initial value problems in term of ordinary differential equations. However, the most effective techniques were developed around 1900 by two German mathematicians C. Runge and M.W. Kutta. These methods are famous as Runge-Kutta (RK) methods and distinguished due to their orders, because they agree with Taylors series solution up to the term h^r, where r is order of method. Runge-Kutta 4th order method (RK4) is a numerical technique used to solve linear and non-linear ordinary differential equations. Let us consider the second order initial value problem of the form

$$\frac{d^2y}{dx^2} = f\left(x, y, \frac{dy}{dx}\right), \tag{1.18}$$

subject to initial conditions

$$y(x_o) = a, \ \frac{dy}{dx}(x_o) = b. \tag{1.19}$$

To solve above problem, it is required to convert the second order initial value problem to the first order initial value problems. Introducing new dependent variable z as

$$\left.\begin{aligned} \frac{dy}{dx} &= z = g\,(x, y, z) \\ \frac{dz}{dx} &= f\,(x, y, z) \end{aligned}\right\}, \tag{1.20}$$

the initial conditions Eq. (1.19) become

$$y(x_o) = a, \quad z(x_0) = b. \tag{1.21}$$

Now the solution of the system of two first order ordinary differential equations given in Eq. (1.20) subject to initial conditions given in Eq. (1.21) can be computed explicitly by the formula of Runge-Kutta 4th order method as follow

$$y_{n+1} = y_n + \frac{1}{6}(p_1 + 2p_2 + 2p_3 + p_4), \tag{1.22}$$

$$z_{n+1} = z_n + \frac{1}{6}(q_1 + 2q_2 + 2q_3 + q_4), \tag{1.23}$$

where

$$\left.\begin{aligned} p_1 &= h\,g(x_n, y_n, z_n) \\ q_1 &= h\,f(x_n, y_n, z_n) \\ p_2 &= h\,g\left(x_n + \frac{h}{2}, y_n + \frac{p_1}{2}, z_n + \frac{q_1}{2}\right) \\ q_2 &= h\,f\left(x_n + \frac{h}{2}, y_n + \frac{p_1}{2}, z_n + \frac{q_1}{2}\right) \\ p_3 &= h\,g\left(x_n + \frac{h}{2}, y_n + \frac{p_2}{2}, z_n + \frac{q_2}{2}\right) \\ q_3 &= h\,f\left(x_n + \frac{h}{2}, y_n + \frac{p_2}{2}, z_n + \frac{q_2}{2}\right) \\ p_4 &= h\,g(x_n + h, y_n + p_3, z_n + q_3) \\ q_4 &= h\,f(x_n + h, y_n + p_3, z_n + q_3) \end{aligned}\right\}, \tag{1.24}$$

Where n is number of steps, h is uniform step size obtained by $h = \frac{b-a}{n}$ and x_n ($n = 1,2, \dots, N$) are uniform grid points with $x_0 = a, x_N = b$.

1.9.2 Shooting method

Shooting method is very popular to solve the boundary value problems given in terms of ordinary differential equations. The idea of shooting technique is to reduce the boundary value problem to the initial value problem. This method is applicable to almost all types of boundary value problems containing different forms of the boundary conditions. In shooting method we start to find the solution of the boundary value problem at one end and "shoot" to the other end with the initial value solver, unless the boundary condition at the other end converge to its correct value. The two point boundary value problem is a second order differential equation with one condition specified at $x = a$ and another at $x = b$. Let us consider the general second order boundary value problem as

$$y'' = f(x, y, y'), \;\; y(a) = \alpha, \;\; y(b) = \beta. \qquad (1.25)$$

It is required to reduce the above boundary value problem into the initial value problem in shooting method. The boundary value problem Eq. (1.25) is reduced to an initial value problem as

$$y'' = f(x, y, y'), \;\; y(a) = \alpha, \;\; y'^{(a)} = u^{(i)}. \qquad (1.26)$$

Here the missing initial condition $u^{(i)}$is needs to be determined, that might be done by supposing the value of $u^{(i)}$ as an initial guess by$u^{(0)} = s$. Now we have to calculate the solution of initial value problem Eq. (1.26) from $x = a$ to $x = b$. One can obtain it through Ruge-Kutta 4[th] order method as explained in the previous section. Hence the value of $y(b)$ can be obtained at this stage, if the value is β which is our boundary condition $y(b) = \beta$, then it is right, otherwise we have to readjust the value of initial guess s such that $y(b) = \beta$ is satisfied. Instead of doing let and tried with initial guesses of $u^{(i)}$, Newton-Raphson formula is used for this purpose as follows

$$u^{(i+1)} = u^{(i)} - \frac{(y(b) - \beta)}{\frac{dy}{dx}(b)}. \qquad (1.27)$$

Chapter 2

Study of non-orthogonal stagnation point flow towards a stretching sheet

This chapter focuses the analysis of steady flow of viscous and incompressible and viscus fluid towards a non-orthogonal stagnation point over a stretching sheet investigated by [19]. The system of governing coupled partial differential equations (PDE's) are converted to the system of the coupled ordinary differential equation which are exposed to fourth-order Runge-Kutta method with shooting techniques (for two unknown initial condition) to solve numerically. The results are presented in the form of tables and graphs. The detail discussion is made at the end of this chapter.

2.1 Mathematical formulation

Let us take an incompressible flow of fluid nearby a non-orthogonal stagnation point on the surface happen together with the plane represented by $y = 0$, where flow is two dimensional, steady and always hold the condition, $y > 0$. The surface is kept a full stretched by applying a parallel and equal force on both the ends of the x-axis. It is supposed that the fluid having some velocity $V_e(u_e, v_e)$is colliding on the stretching surface with a random choice of angle of incidence γ as shown in figure 2.1.

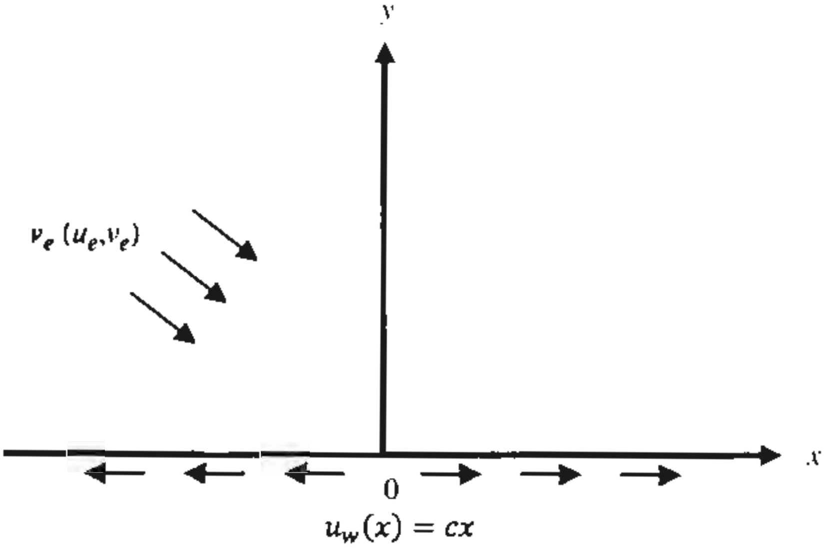

Figure 2.1: Physical model and coordinates system

Under the above assumptions, the flow can be described in the form of following mathematical equations.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\nabla^2 u, \tag{2.2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \upsilon\nabla^2 v. \tag{2.3}$$

In above equations u and v denote the velocity components along the $\bar{x}$ and $\bar{y}$-axis respectively, p in place of pressure, ρ in place of density and υ is the kinematic viscosity of the fluid. The boundary conditions of the described problem are defined as

$$\begin{aligned} u &= u_w(x) = cx, v = 0, \text{at } y = 0, \\ u &= u_e = axsin\gamma + bysin\gamma, \\ v &= v_e = aysin\gamma, \text{as } y \to \infty, \end{aligned} \tag{2.4}$$

where a, b and c are positive constants and γ is the parameter. For $0 < \gamma < {}^{\pi}/_{2}$ and ${}^{\pi}/_{2} < \gamma < \pi$ the flow model is favorable flow and unfavorable flow respectively. The external flow is identified as a mixture of potential stagnation point flow and linear shear flow. We introduce non-dimensional variables and eliminating pressure p from prescribed governing Eqs. (2.2) and (2.3), the non-dimensional variables are given as

$$x = (c/\nu)^{1/2}\bar{x}, \quad y = (c/\nu)^{1/2}\bar{y}, \quad \psi = \frac{\bar{\psi}}{\nu}, \tag{2.5}$$

in above equations ψ is the stream function defined through the usual relations $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$ in Cartesian plane. The function ψ is also called non-dimensional stream function. The Eqs. (2.2) and (2.3) turn into non-dimensional form after using non-dimensional variables in them, which is defined as

$$\nabla^2(\nabla^2\psi) + \frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}(\nabla^2\psi) - \frac{\partial \psi}{\partial y}\frac{\partial}{\partial x}(\nabla^2\psi) = 0. \tag{2.6}$$

The appropriate boundary conditions given in Eq. (2.4) are also transformed (after applying non-dimensional variables) into the following form

$$\psi = 0, \qquad \frac{\partial \psi}{\partial y} = x \quad \text{at} \quad y = 0, \tag{2.7a}$$

$$\psi = \lambda xysin\gamma + \frac{1}{2}ky^2cos\gamma \quad as \quad y \to \infty, \tag{2.7b}$$

where λ =a/c and k = b/c are taken as positive constants. The stream function ψ defined in Eq. (2.7b) exactly satisfies Eq. (2.6). Solution of Eq. (2.6) can also derived subject to the boundary condition Eqs. (2.7a) and (2.7b) in the form

$$\psi(x, y) = xf(y) + g(y). \tag{2.8}$$

From Eq. (2.8), we have

$$\begin{aligned} &\frac{\partial\psi}{\partial x} = f(y), \frac{\partial\psi}{\partial y} = xf'(y) + g'(y), \\ &\frac{\partial^2\psi}{\partial x^2} = 0, \qquad \frac{\partial^2\psi}{\partial y^2} = xf''(y) + g''(y). \end{aligned} \tag{2.9}$$

After putting the expressions from Eq. (2.9) into Eq. (2.6) and after simplification, we get

$$\begin{aligned} &f^{iv} + ff''' - f'f'' = 0, \\ &g^{iv} + fg''' - g'f'' = 0. \end{aligned} \tag{2.10}$$

Integrating once the above equations with respect to the following coupled ordinary differential equations (ODEs) are formed

$$f''' + ff'' - f'^2 + c_1 = 0, \tag{2.11}$$

$$g''' + fg'' - g'f' + c_2 = 0, \tag{2.12}$$

where c_1 and c_2 denote the constant of integration and the value of it can be completed by applying the following boundary conditions

$$f(0) = 0, f'(0) = 1, f'(\infty) = \lambda sin\gamma, \tag{2.13a}$$

$$g(0) = 0, g'(0) = 0, g''(\infty) = kcos\gamma, \tag{2.13b}$$

in above equations, where prime sign indicates the differentiation w.r.t. y. In order to determine constant c_1, we take limit y approaches to ∞ in equation (2.11) and use the boundary condition defined as $f'(\infty) = \lambda sin\gamma$, and we find value of $c_1 = \lambda^2 sin^2\gamma$. To find the value of the other constantc_2, assume $f(y) \sim y\lambda sin\gamma + \alpha$ as y tends to ∞, here $\alpha = f - y\lambda sin\gamma$ is taken as real constant, by incorporating $g'(\infty) = y\,k\,cos\gamma$ and $g''(\infty) = kcos\gamma$, we get the value of $c_2 = -\alpha kcos\gamma$. By putting values of c_1 and c_2 into Eqs. (2.11) and (2.12), we get

$$f''' + ff'' - f'^2 + \lambda^2 sin^2\gamma = 0, \tag{2.14}$$

$$g''' + fg'' - f'g' - \alpha kcos\gamma = 0, \tag{2.15}$$

with boundary conditions given in Eqs. (2.13a) and (2.13b). The linearity of Eq. (2.15) allow us to find the solution of the form

$$g(y) = kcos\gamma h(y) \quad (2.16)$$

where $h(y)$ can be obtained from Eq. (2.15)

$$h''' + fh'' - f'h' - \alpha = 0, \quad (2.17a)$$

$$h(0) = 0, h'(0) = 0, h''(\infty) = 1. \quad (2.17b)$$

It is noticed that $\alpha = \alpha(\lambda, \gamma)$ is obtained by solving Eq. (2.14) subject to boundary conditions Eq. (2.13a). Some values of α for different values of λ *and* γ are given in table 2.1. From Eq. (2.8), the stream function ψ becomes

$$\psi(x, y) = xf(y) + kcos\gamma h(y). \quad (2.18)$$

In non-dimensional form, the skin friction is given by

$$\tau_w = \frac{\partial^2\psi}{\partial y^2} = xf''(y) + g''(y), \quad (2.19)$$

As $g(y) = kcos\gamma h(y)$, then above equation become

$$\tau_w = \left(\frac{\partial^2\psi}{\partial y^2}\right)_{y=0} = xf''(0) + kcos\gamma h''(0). \quad (2.20)$$

The curve $u = 0$ and the dividing streamlines $\psi = 0$ connect the wall where $\tau_w = 0$ at the stagnation point. Thus the stagnation point x_s is defined by following relation

$$x_s = \frac{kcos\gamma h''(0)}{f''(0)}. \quad (2.21)$$

2.2 Numerical solution

Since the boundary value problems Eqs. (2.14 & 2.17a) are non- linear, so it is impossible to find the exact solution of it. To construct its numerical solution, we use Runge-Kutta 4th order method with shooting technique. For this purpose, we need to convert modelled system of boundary value problem into the system of first order initial value problem as follow

$$f = y_1, \quad (2.22)$$

$$f' = y_2, \quad (2.23)$$

$$f'' = y_3, \quad (2.24)$$

$$f_3' = -y_1y_3 + y_2^2 - \lambda^2\sin(\gamma), \quad (2.25)$$

$$h = y_4, \quad (2.26)$$

$$h' = y_5, \quad (2.27)$$

$$h'' = y_6, \quad (2.28)$$

$$y_6' == -y_1y_6 + y_2y_5 + \alpha, \quad (2.29)$$

with given initial conditions

$$\begin{aligned} &y_1(0) = 0, y_2(0) = 1, y_3(0) = u_1, \\ &y_4(0) = 0, y_5(0) = 0, y_6(0) = u_2, \end{aligned} \quad (2.30)$$

where u_1 and u_2 are two missing conditions, which can be found in such a way that solution satisfy the boundary conditions (2.13a) and (2.17b) at ∞. We construct both the solution with the help of Runge-Kutta 4th order method with shooting technique.

2.3 Result and discussion

The pair of ordinary differential Eqs. (2.14) and (2.17a) subject to boundary conditions given in Eqs. (2.13a) and (2.17b) are solved numerically for different values of the participated parameter λ and angle γ with the help of shooting techniques based on Runge-Kutta 4th order method. The parameter k is considered to be one in all cases and considered the situation of favorable flow i.e $0 < \gamma < \pi/2$. The case of unfavorable flow i.e. $\pi/2 < \gamma < \pi$ is not discussed here as it is symmetric about y −axis. The computed results are compared with the results represented by Mahapatra and Gupta [4], Nazar et al. [10] for orthogonal flow i.e. ($\gamma = \pi/2$). It is found that the result are quite similar which prove the accuracy of our code. The comparison of the values are given in table 2.2. It is noticed from the table that the magnitude of $f''(0)$ fell down continuously with increasing value of λ. Nonetheless the magnitude of $f''(0)$ rises when $\lambda > \lambda_c(\gamma)$, here λ_c denote critical value of parameter λ. It means that boundary layer is thinning progressively with increasing value of λ. The values of $\lambda_c = 1/\sin\gamma$ with respect to γ are shown in table 2.3. For $\lambda < \lambda_c$, and inverted boundary layer is formed, and with the increasing value of λ the boundary layer's thickness decrease, this phenomena is shown in figures (2.2 - 2.4). It is also notice from the figure that when γ is small, the structure of stagnation flow imposing obliquely on a fixed wall plane, which has been discussed by Tamada [12]. The values for the stagnation point x_s with λ for several values of γ are shown in table 2.4. It is clearly notice from table 2.1 and 2.4 that for λ=2 when $\gamma = \pi/6$, it is hard to find the exact values of stagnation point x_s and α, because of the numerical uncertainty near to the $\lambda_c(\gamma)$ i.e. critical value of λ. Generally the magnitude of α is approximately very small, while on the other hand the magnitude of stagnation point x_s is too large. Figs. (2.5-2.9) demonstrates the shape of streamlines for the orthogonal flow, when the flow is favorable i.e. $(0 < \gamma < \pi/2)$ for λ=2.5 and for some values of γ. To construct the graphs of streamlines, we considered the positive constant k to be 1.0 in all calculation. It is seen from the Figs. (2.5-2.9) that the stagnation point x_s are positive for the small values of the angle of incidence $\gamma = \pi/15$ and $\gamma = \pi/12$. On the other hand, the stagnation point x_s is negative for large values of the angle of incidence $\gamma = \pi/6$ and $\gamma = \pi/4$.

Table 2.1: Values of α for several values of λ and γ.

λ	α				
	$\gamma = \frac{\pi}{15}$	$\gamma = \frac{\pi}{12}$	$\gamma = \frac{\pi}{6}$	$\gamma = \frac{\pi}{4}$	$\gamma = \frac{\pi}{3}$
0.1	0.9488	0.9371	0.8852	0.8445	0.8152
0.2	0.9025	0.8811	0.7917	0.7245	0.6776
0.3	0.8606	0.8316	0.7115	0.6246	0.5654
0.4	0.8215	0.7857	0.6406	0.5387	0.4700
0.5	0.7849	0.7432	0.5772	0.4627	0.3862
0.6	0.7506	0.7036	0.5192	0.3941	0.3122
0.7	0.7182	0.6662	0.4663	0.3325	0.2438
0.8	0.6874	0.6312	0.4165	0.2739	0.1832
0.9	0.6580	0.5981	0.3710	0.2233	0.1274
1.0	0.6301	0.5666	0.3283	0.1743	0.0748
1.5	0.5063	0.4277	0.1456	-0.0316	-0.1431
2.0	0.4018	0.3138	-	-0.1946	-0.3181
2.5	0.3123	0.2146	-0.1211	-0.3310	-0.4630
3.0	0.2311	0.1289	-0.2277	-0.4482	-0.5882
4.0	0.0952	-0.0181	-0.4104	-0.6498	-0.8022

Table 2.2: Comparison of computed $f''(0)$ with that of Mahapatra & Gupta [4] and Nazar et al. [10] for several values of λ and γ.

λ	$f''(0)$					
	$\gamma=\pi/12$	$\gamma=\pi/4$	$\gamma=\pi/3$	$\gamma=\pi/2$		
				Present	Mahapatra & Gupta [4]	Nazar et al. [10]
0.1	-0 .9943	-0.9807	-0.9747	-0.9693	-0.9693	-0.9694
0.2	-0.9870	-0.9504	-0.9336	-0.9181	-0.9181	-0.9181
0.5	-0.9562	-0.8062	-0.7344	-0.6672	-0.6672	-0.6673
1.0	-0.8796	-0.4243	-0.2050	—	—	—
2.0	-0.6486	0.7384	0.4010	0.0176	2.0175	2.0175
3.0	-0.3319	0.3131	0.5666	0.7296	2.7293	4.7296
4.0	0.0568	0.2218	0.1840	0.0013	—	—
5.0	0.5089	0.4180	0.1899	11.7537	—	—

Table 2.3: Values of $\lambda=\lambda_c$ for different value of γ.

γ	λ_c
$\pi/15$	4.8097
$\pi/12$	3.8637
$\pi/6$	2.0000
$\pi/4$	1.4142
$\pi/3$	1.1547
$\pi/2$	1.0000

Table 2.4: Values of stagnation point x_s for several values of λ and γ.

λ	x_s			
	$\gamma=\pi/12$	$\gamma=\pi/6$	$\gamma=\pi/4$	$\gamma=\pi/3$
0.1	0.0137	0.0967	0.1296	0.1164
0.2	0.1141	0.2353	0.2682	0.2264
0.3	0.1984	0.3462	0.3793	0.3167
0.4	0.2719	0.4425	0.4804	0.4039
0.5	0.3377	0.5315	0.5818	0.4993
0.6	0.3981	0.6181	0.6915	0.6153
0.7	0.4544	0.7060	0.8193	0.7712
0.8	0.5079	0.7989	0.9780	1.0052
0.9	0.5595	0.9006	1.1892	1.4133
1.0	0.6099	1.0157	1.4939	2.3375
1.5	0.8660	2.1542	-7.2924	-1.0317
2.0	1.1830	—	-1.0437	-0.4053

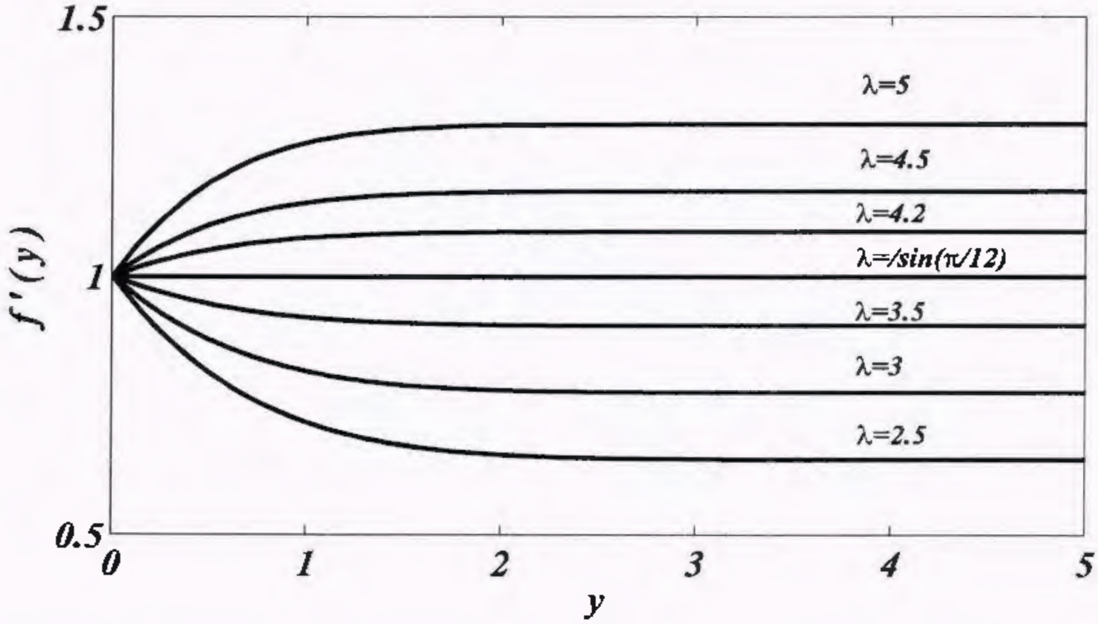

Figure 2.4: Variation in velocity profile $f'(y)$ with y for different values of λ when $\gamma = \pi/3$.

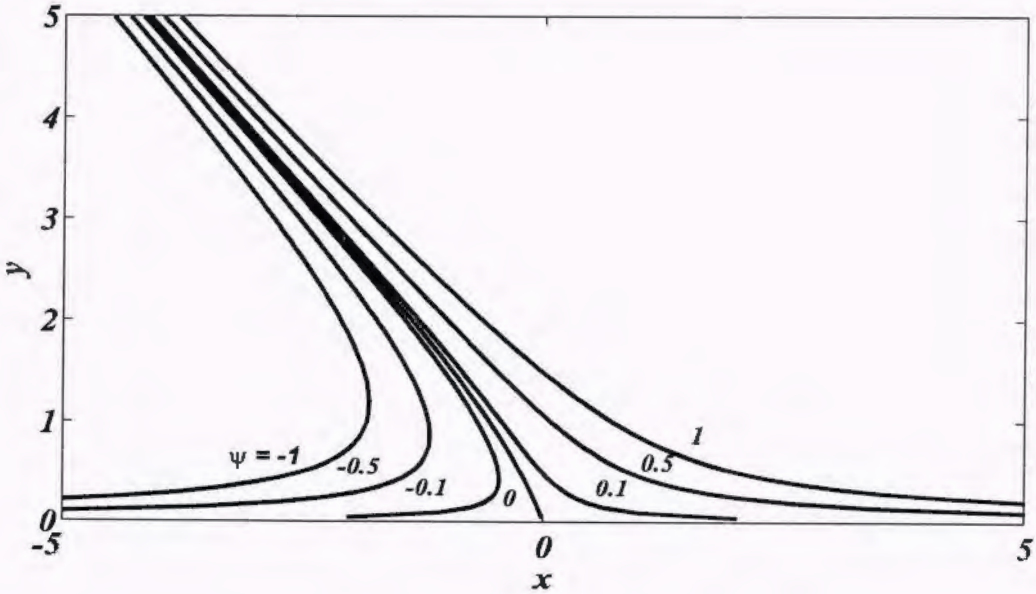

Figure 2.5: Streamlines pattern for $\lambda = 2.5$ when $\gamma = \pi/15$.

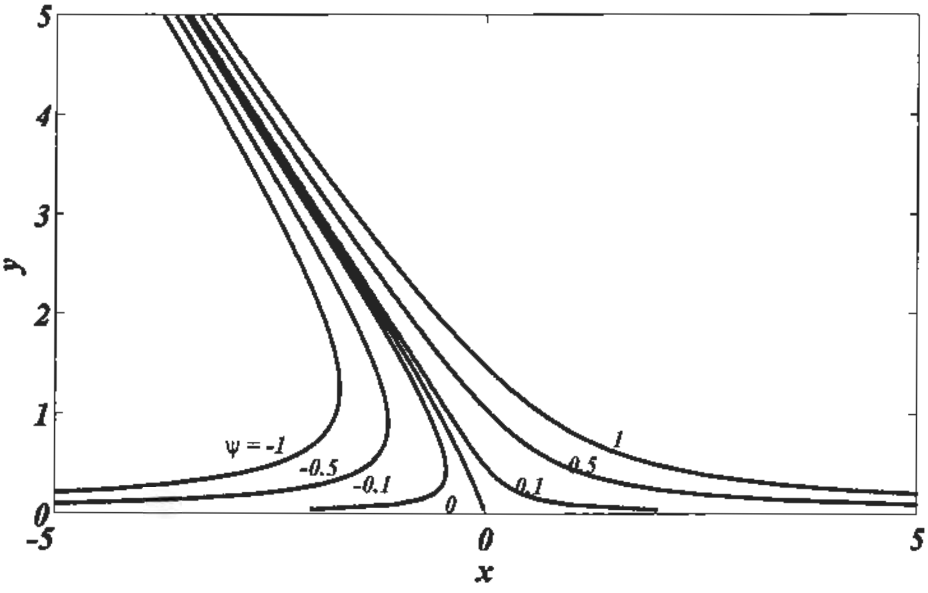

Figure 2.6: Streamlines pattern for $\lambda = 2.5$ when $\gamma = \pi/12$.

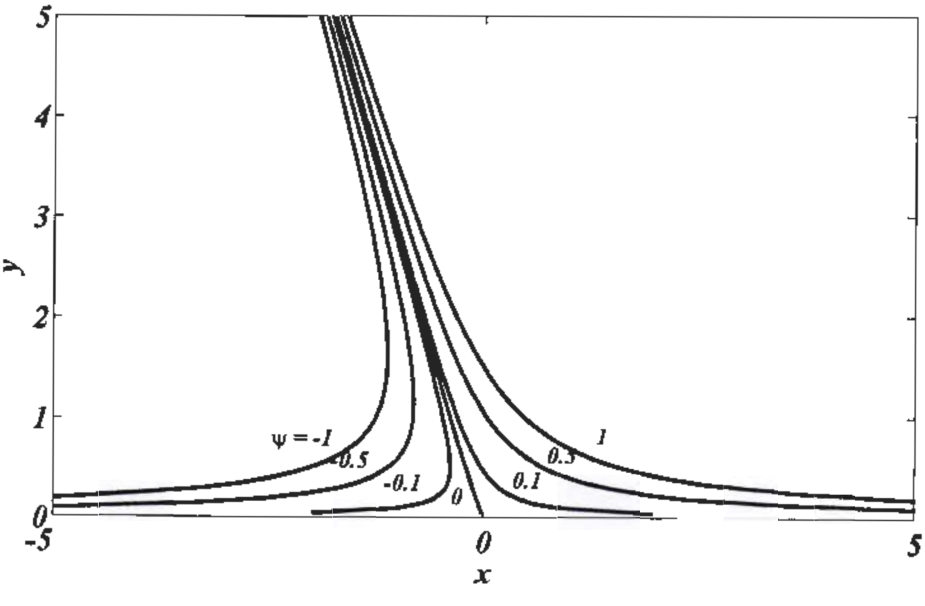

Figure 2.7: Streamlines pattern for $\lambda = 2.5$ when $\gamma = \pi/6$.

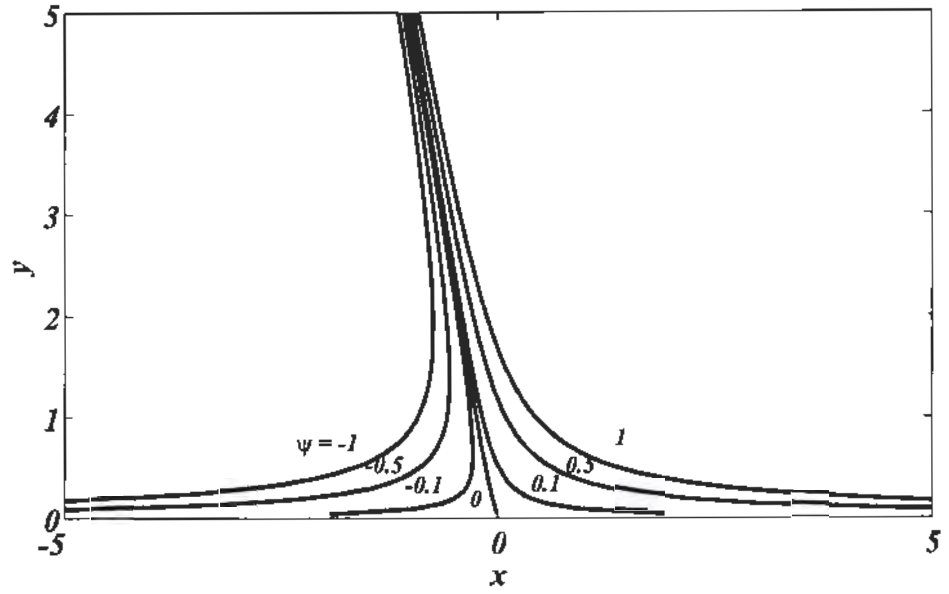

Figure 2.8: Streamlines pattern for $\lambda = 2.5$ when $\gamma = \pi/4$.

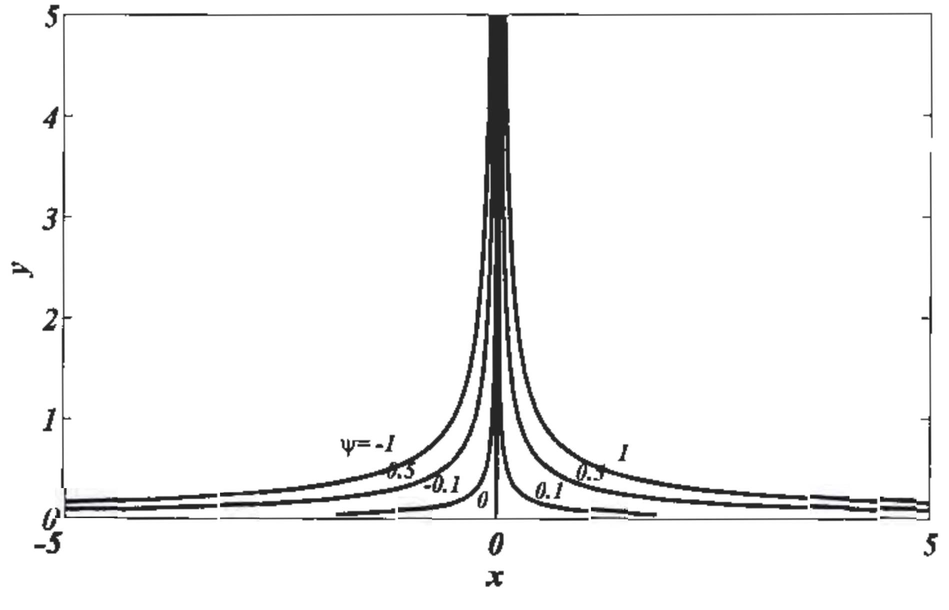

Figure 2.9: Streamlines pattern for $\lambda = 2.5$ when $\gamma = \pi/2$.

Chapter 3

Study of uniform suction and slip effects on homogenous-heterogeneous reactions on the flow of non-Newtonian Casson fluid

3.1 Introduction

In this chapter, we revised the simultaneous influences of homogenous – heterogeneous reactions on non-Newtonian Casson fluid in the region of stagnation point studied by [20]. The slippery wall are having a uniform phenomenon of suction on a porous stretching /shrinking sheet. The governing partial differential equations, are initially converted to couple ordinary differential equations, with the help of suitable transformation. Later, a numerical solution for these ordinary differential equation is sought by using the well-known shooting technique method (for two unknown initial condition) with the addition of Rung-Kutta method of 4^{th} – order described in detail in initial chapter. It is noted that unique solution is displayed to observe the variation in the stretching parameters. On the other hand, variation in the shrinking parameter, yields a dual solution for the shrinking sheet case. Finally, the velocity of concerned fluid is displayed, graphically against dual solution.

3.2 Mathematical formulation

Here, we assume the steady, incompressible two-dimensional flow of a non-Newtonian Casson fluid in the region of stagnation point caused by the linear shrinking/stretching of the surface. Flow is restricted to the upper half plane represented by the region $y > 0$ above the plane that analogous with $(y = 0)$ on which $x = 0$ is the fixed stagnation point. The x −axis is taken along the surface and y −axis is taken perpendicular to it as presented in figure 2.1. The surface is being stretched in both the positive and negative x −directions by applying two equal and opposite forces which produces a linear surface velocity defined by expression $u_w(x) = mx$, in which $m > 0$ corresponds to stretching of sheet and m less than zero represents shrinking of sheet. The velocity outside or above the boundary layer is represented by the expression $u_e(x) = cx$, here $c > 0$ represents the strength of the stagnation flow. The equations that govern the flow are written in non-dimension form as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{du_e}{dx}+\nu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}, \tag{3.2}$$

$$u\frac{\partial a}{\partial x}+v\frac{\partial a}{\partial y}=D_A\frac{\partial^2 a}{\partial y^2}-k_c ab^2, \tag{3.3}$$

$$u\frac{\partial b}{\partial x}+v\frac{\partial b}{\partial y}=D_B\frac{\partial^2 b}{\partial y^2}+k_c ab^2, \tag{3.4}$$

where u and v represents the components of velocity in x and y −direction respectively, a and b are the concentrations of chemical species A and B, k_i $(i=c,s)$ are the rate constant, ν denotes the kinematic viscosity, $\beta=\mu_\beta\sqrt{2\pi_c}/p_y$ is in place of Casson parameter, diffusion coefficients are presented by D_A *and* D_B .The boundary conditions for modelled problem defined below

$$\left.\begin{aligned} u(0)=u_w(x)=mx+L\left(1+\frac{1}{\beta}\right)\frac{\partial u}{\partial y}, v(0)=-v_w \\ D_A\frac{\partial a}{\partial y}\Big|_{y=0}=k_s a(0), D_B\frac{\partial b}{\partial y}\Big|_{y=0}=-k_s a(0) \\ u(\infty)=u_e(x)=cx, v(\infty)=0, a(\infty)=a_0, b(\infty)=0 \end{aligned}\right\}, \tag{3.5}$$

where m and c are the dimensional constants of dimension $1/time$, L represent the velocity slip length parameter, v_w symbolized by mass transfer which corresponds to suction velocity and a_0 is also assumed as a constant and the no slip condition can be deduced by assuming $L=0$. By incorporating the following non-dimensional variables

$$\eta=\sqrt{\frac{c}{\vartheta}}y, u=cxf'(\eta),\ \ v=-\sqrt{c\nu}f(\eta), g(\eta)=\frac{a}{a_0}, h(\eta)=\frac{b}{a_0}. \tag{3.6}$$

In Eq. (3.6), the continuity equations represented in Eq. (3.1) is satisfied identically and the Eqs. (3.2)−(3.4) after calculation reduce to

$$\left(1+\frac{1}{\beta}\right)f'''+ff''-f'^2+1=0. \tag{3.7}$$

Using Eq. (3.6) in Eq. (3.3), we get

$$\frac{1}{Sc}g''+fg'-kgh^2=0, \tag{3.8}$$

where $Sc=\frac{\nu}{D_A}$and $K=k_c a_0^2/c$.

By using the non-dimensional variables Eq. (3.4) reduce to

$$\frac{\delta}{sc}h'' + fh' + kgh^2 = 0. \tag{3.9}$$

where $\delta = D_B/D_A$ and $K = k_c a_0^2/c$.

And by using the non-dimensional variables the boundary condition Eq. (3.5) becomes

$$\left.\begin{array}{c} f(0) = S, f'(0) = \lambda + \left(1 + \frac{1}{\beta}\right)\gamma f''(0),\ f'(\infty) = 1 \\ g'(0) = k_s g(0), \quad g(\infty) = 1 \\ \delta h'(0) = -k_s g(0), \quad h(\infty) = 0 \end{array}\right\}, \tag{3.10}$$

here prime represents the differentiation w.r.t. the independent variable η, $S = v_w/(cv)^{1/2}$ (> 0) corresponds to the mass suction parameter. The parameter λ represents the ratio of the stretching to the external flow rate defined as $\lambda = m/c$, where the λ greater than zero is for the stretching of sheet while less then zero is for the shrinking of sheet and equal to zero is for fixed sheet. The parameter $\gamma = L\sqrt{c/_{v}}$ denotes the velocity slip parameter, the non-dimensional Schmidt number is defined by $Sc = \nu/D_A$, the ratio of diffusion coefficients is $\delta = D_B/D_A$, $K = k_c a_0^2/c$ is used to measure the amount of the homogenous reaction, $K_s = k_s Re^{-1/2}/D_A$ is used to measure the amount of heterogeneous reaction and $Re = c\backslash\nu$ is known as the dimensionless Reynolds number. Following Chaudary and Merkin [11], we assumed that coefficients D_A *and* D_B which defines the diffusion are equal and by assuming δ to be one, the assumption deduced the following relation

$$h(\eta) = 1 - g(\eta), \tag{3.11}$$

thus Eq.(3.8) and Eq. (3.9) reduce to

$$\frac{1}{Sc}g'' - kg(1-g)^2 + fg' = 0, \tag{3.12}$$

equipped with specified boundary conditions defined as

$$g'(0) = K_s g(0) \text{and } g(\infty) = 1. \tag{3.13}$$

The important physical quantity of our interest is the coefficient of the skin friction symbolized by C_f and defined as

$$C_f = \frac{\tau_w}{\rho u^2{}_w}. \tag{3.14}$$

In above equation τ_w is the shear stress defined via relation

$$\tau_w = \left(\mu_B + \frac{P_y}{\sqrt{2\pi_c}}\right)\left(\frac{\partial u}{\partial y}\right)_{y=0}. \tag{3.15}$$

After incorporating Eq. (3.5) and (3.15) into Eq. (3.14) we get

$$Re_{x^{1/2}} C_f = \left(1 + \frac{1}{\beta}\right) f''(0), \tag{3.16}$$

where $Re_x = xu_w/v$ represents the Reynolds number.

3.3 Graphical results and discussion

The governing partial differential equation are transformed to ordinary differential equation (ODE) with the help of suitable transformation introduced in chapter 2. The boundary layer non-linear Eqs. (3.7) and (3.12) together with given boundary condition Eqs. (3.10) and (3.13) are solved numerically by Runge-Kutta 4th-order method with well-known Shooting technique method (for two unknown initial conditions). Here, the values of $f''(0)$ and $\theta'(0)$ are calculated for which velocity profile as well as temperature profile satisfies the boundary conditions defined on the infinity for several values of the participated physical parameters. To get the accurate analytical solution, the process is repeated until the error is reduced to 10^{-6}. The numerical computations have been done for possible important range of the involved parameters in the governing equations. The important governing parameters are Casson parameter β, slip parameter γ, stretching/shrinking parameter λ and suction parameter S. The strength of homogeneous and heterogeneous parameter are represented by K and K_s respectively, and the dimensionless Schmidt number is represented by Sc. Figure 3.1 shows the velocity profile $f'(\eta)$ for different values of the stretching and shrinking parameters λ by keeping other parameter fixed. From the figure, it is clear that for stretching sheet case which corresponds to $\lambda > 0$, the solution obtained for all values and the fluid velocity increases greater as compared to free stream velocity but fluid velocity decreases as the value of η increases, while for shrinking sheet which corresponds $\lambda < 0$, the existence of dual solution is noted shown only for $\lambda = -1.52, = -1.42, \lambda = -1.32$ and at initial phases the velocity is in the region of negative values but gradually velocity increases with increasing values of the parameter η and becomes positive. Figure 3.2 illustrates the dual solution of the velocity profile $f'(\eta)$ for various values of non-Newtonian Casson fluid parameter β for $\lambda = -1.52$ and by keeping the other main parameters fixed. From the figure, it is also note that the $f'(\eta)$ fluid velocity increase by increasing the Casson Parameter β which elucidates that for higher values of non-Newtonian (Casson) parameter β, the fluid behaves like Newtonian fluid. Furthermore, figure exhibits the decrease in momentum boundary layer thickness for magnifying values of Casson parameter β. Figure 3.3 exhibits the effects of the slip parameter γ for shrinking case with values$\lambda = -1.52$ on the velocity profile $f'(\eta)$ with $S = 5$ and assuming $\beta = 1$. From this figure, it also seen that for first solution the momentum boundary layer thickness and the values of fluid velocity decreases due to increase in the values of γ. So one can say that by providing the retracting forces the development of momentum can be controlled and causes the deceleration of the boundary layer flow with shrinking of the sheet and in the case of second solution it is in reverse trend. Figure 3.4 represents the effect on the fluid velocity profile $f'(\eta)$ for variation in the mas suction parameter S for fixed vales of other parameters as $\beta = 1, \gamma = 0.3$ and $\lambda = -1.52$. The same effects are noted as that of velocity slip parameter on the fluid velocity as well as on momentum boundary layer thickness because suction is also a source of producing resistance against the flow of the fluid.

Figure 3.5 shows the variation of concentration profile $g(\eta)$ against η and is presented for numerous values of the λ representing stretching-shrinking of the sheet. The dual solution is obtained as shown in graphs for concentration profile $g(\eta)$ for different values of the $\lambda = -1.52, -1.42, -1.32$. All the curves obeying 'S' shape trajectory which generated from the origin and increases to the value at unity satisfies the boundary conditions that also agrees with results of Shaw et al. [13]. Furthermore, it is noted that the behavior of concentration profile for first and second solution is opposite due to increase in concentration of boundary thickness with increase in λ for first solution, while decrease in concentration of boundary thickness is observed with increase in λ as predicted by the second solution. Figure 3.6 contains the effects on concentration profile due to variation in the non-Newtonian Casson fluid parameter β by keeping other involved parameter fixed. This figure elucidated that the increase in Casson fluid parameter causes the increase in concentration profile while shrinks the concentration boundary layer thickness because the enhanced values of Casson fluid parameter magnifies the strength of elasticity stress. Figure 3.7 shows the effects on the concentration profile for variation in the slip parameter γ in shrinking case by assuming $\lambda = -1.52$ and fixed values of other involved parameters.

It is noted that the concentration profile is the increasing function of the slip parameter however increasing values of slip parameter shrinks the concentration boundary layer thickness for first solution while decrease in fluid velocity is because of existence of slip condition located at the wall and makes a sense that enhanced in the solute concentration and thus opposite behavior is noticed for second solution. Figure 3.8 shows the variation in the concentration profiles $g(\eta)$ in concentration boundary thickness by varying the values of the parameter S which represents the mass suctionand fixed values of other involved parameters. The same effects are reported due to variation in the suction parameter S on the concentration profile as in that of variation in the velocity slip parameter and this is because of the fact that the increase in the friction between the layers of fluid causes resistance in the flow of the fluid. Figure 3.9 shows the behavior of the gradient of concentration $g'(0)$ against non-Newtonian Casson parameter β at the surface due to different values of the mass suction parameter S along with $\lambda = -1.52$. It is noted from the figure that the gradient of concentration is an increasing function of both S and Casson fluid parameter β for as predicted by both first and second solution while gradient of concentration at $S = 1,2$ coincide for second solution.

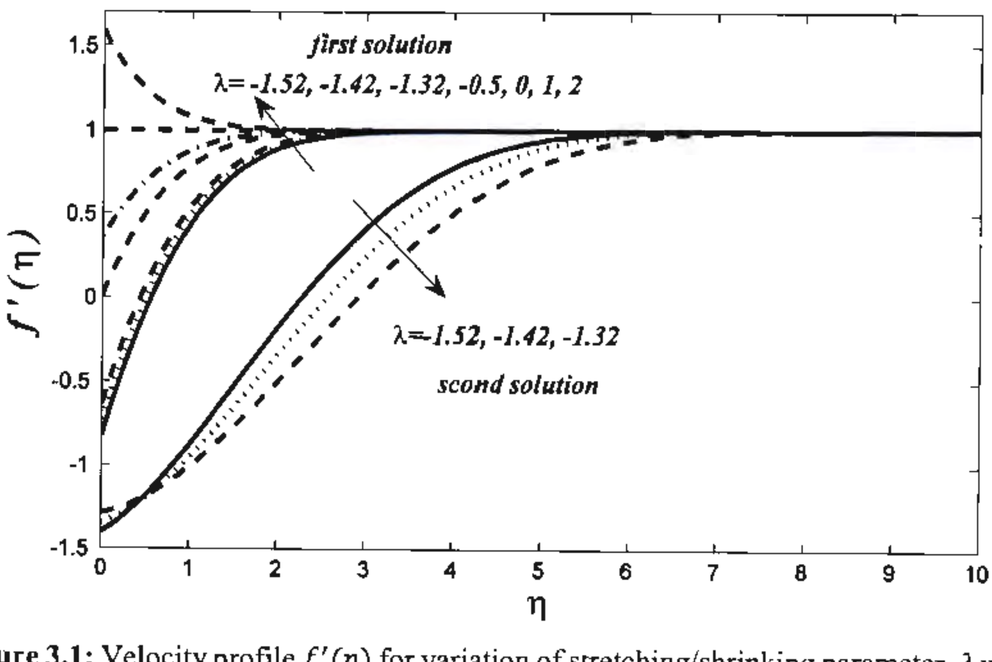

Figure 3.1: Velocity profile $f'(\eta)$ for variation of stretching/shrinking parameter λ with fixed values of $\gamma = 0.3, \beta = 3$ and $S = 0$.

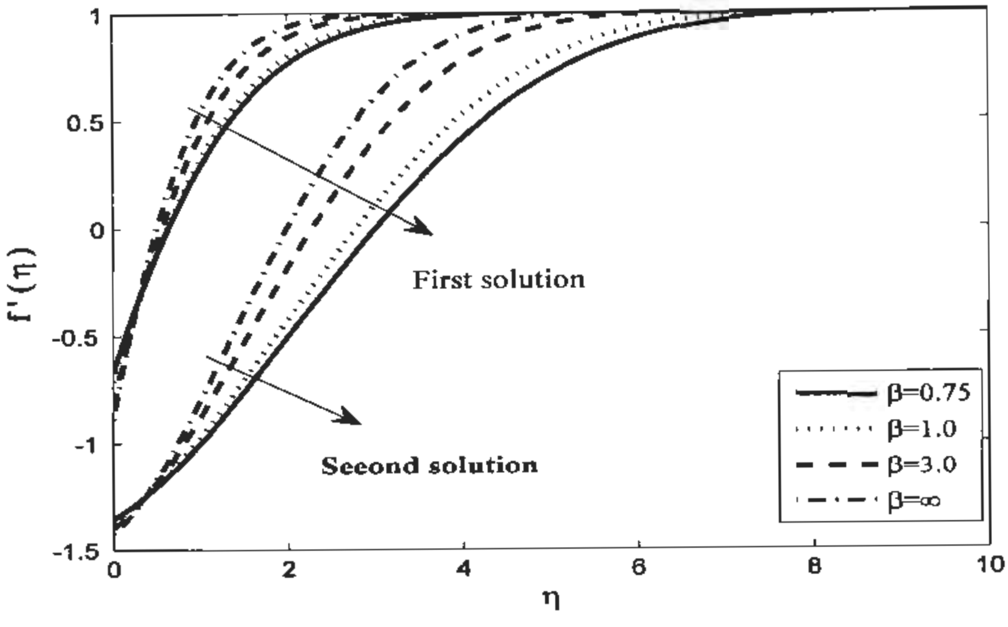

Figure 3.2: Velocity profile $f'(\eta)$ for variation of the Casson parameter β with fixed values of $\gamma = 0, \lambda = -1.52$ and $S = 0.5$.

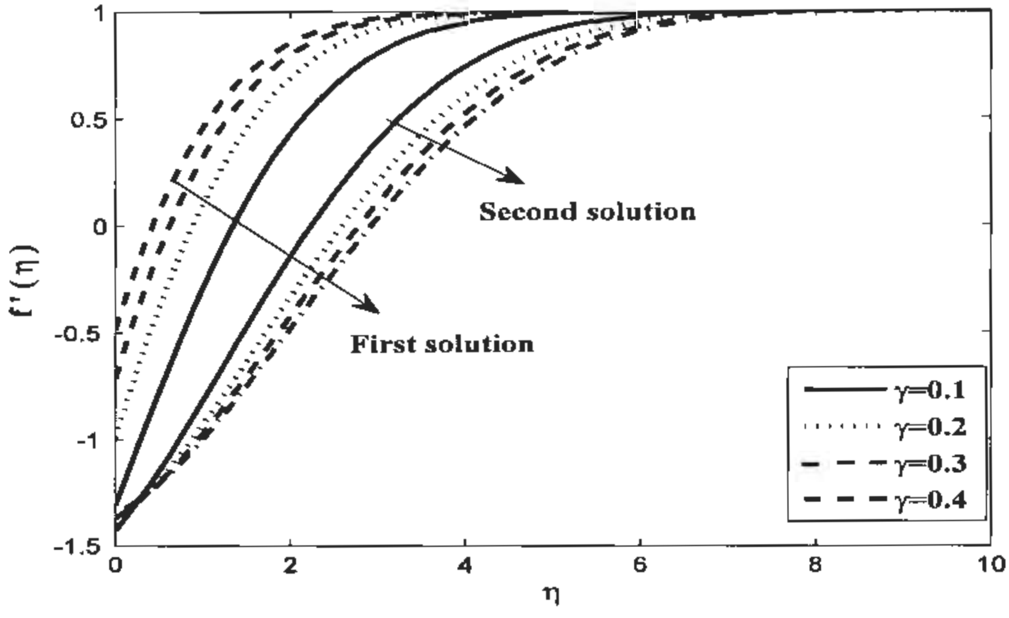

Figure 3.3: Velocity profile $f'(\eta)$ for variation of the slip parameter γ with fixed values of $, \lambda = -1.52, S = 0.5$ and $\beta = 1$.

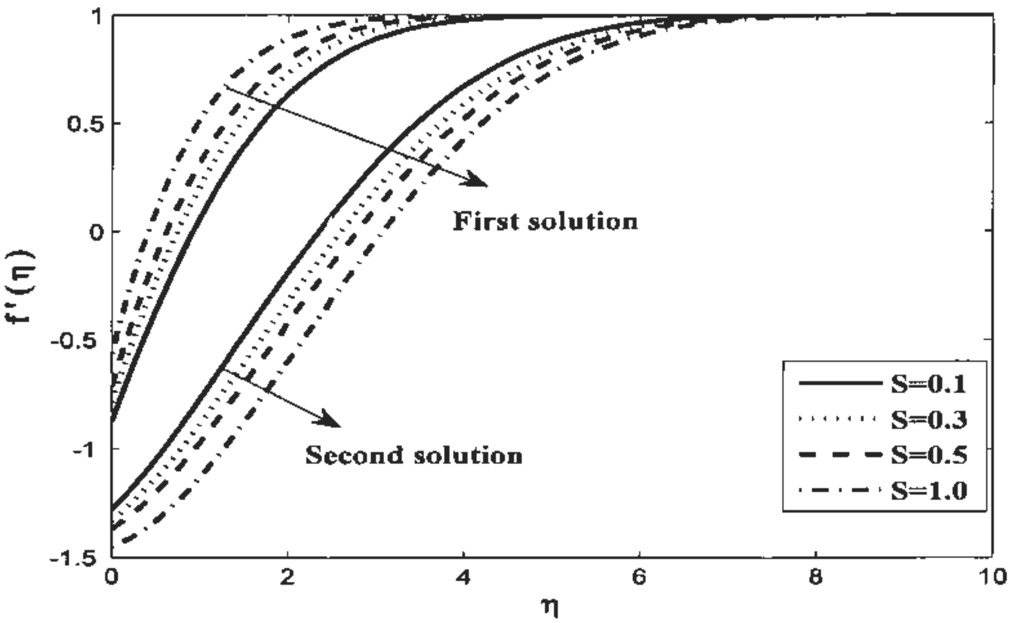

Figure 3.4: Velocity profile $f'(\eta)$ for the variation of the suction parameter S with fixed values of $\gamma = 0.3,\ \lambda = -1.52$ and $\beta = 1$.

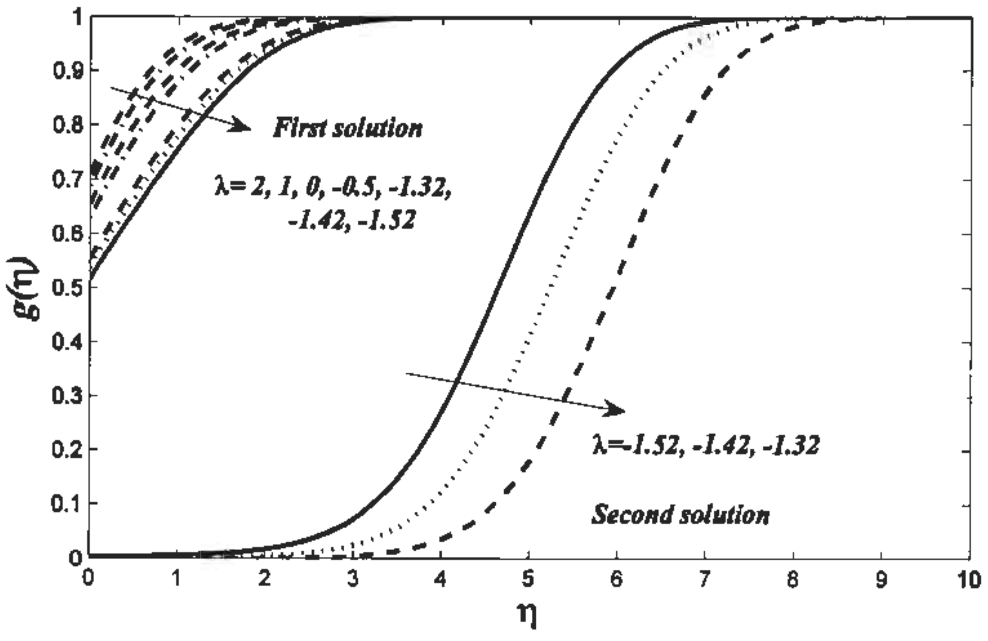

Figure 3.5: Concentration profiles $g(\eta)$ for variation of the velocity parameter λ with fixed values of $\beta = 3, K_s = 0.5, S = 0.5, \gamma = 0.3, Sc = 1$ and $K = 0.5$.

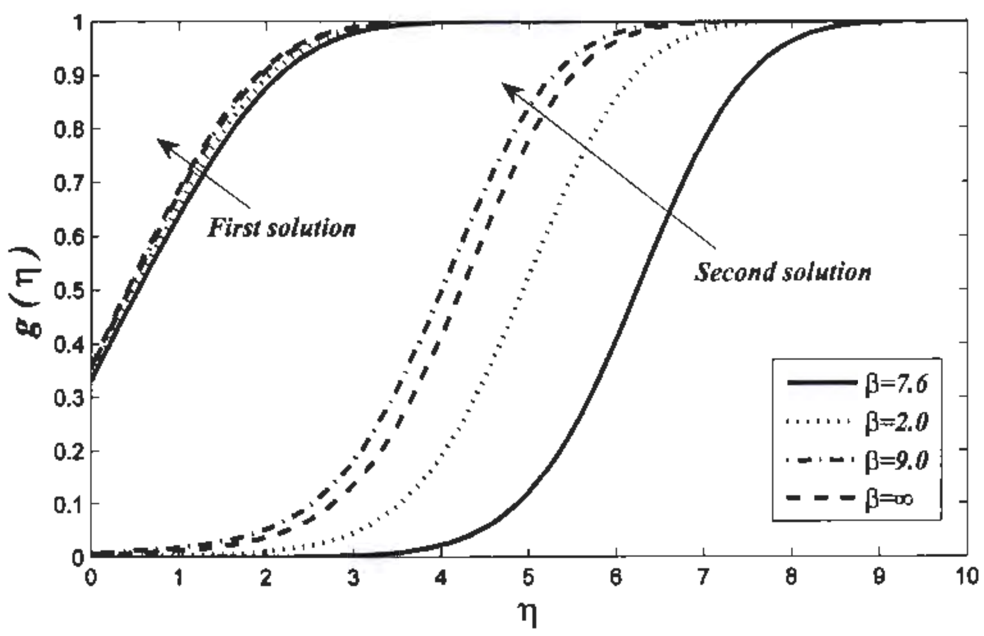

Figure 3.6: Concentration profiles $g(\eta)$ for variation of the Casson parameter β with fixed values of $\lambda = -1.52, K = 0.5, \gamma = 0.3, S = 0.5, K_s = 1$ and $Sc = 1$.

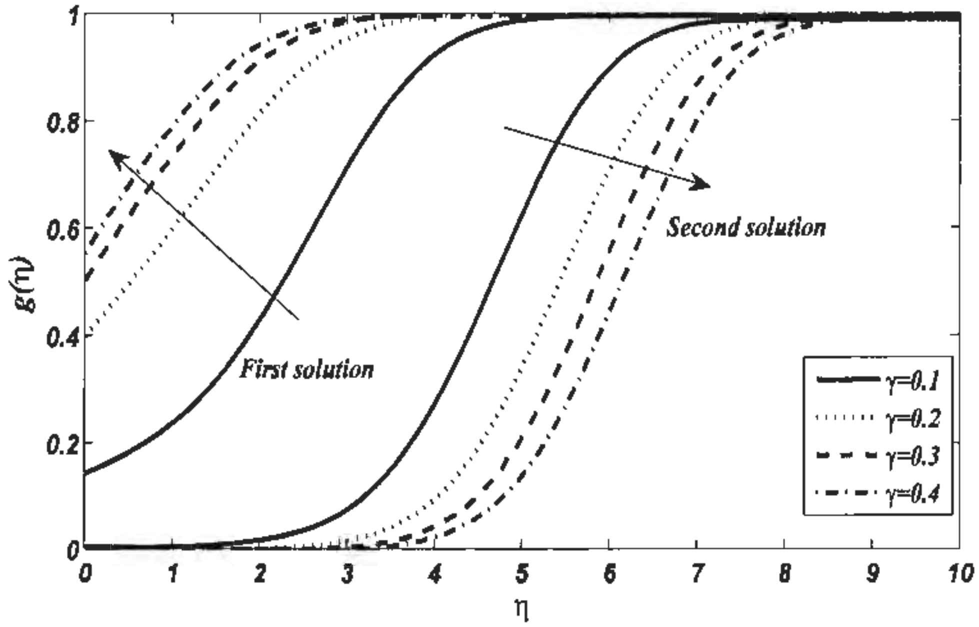

Figure 3.7: Concentration profiles $g(\eta)$ for variation of the slip parameter γ with fixed values of $\lambda = -1.52, K_s = 0.5, \beta = 1, K = 0.5, S = 0.5$ and $Sc = 1$.

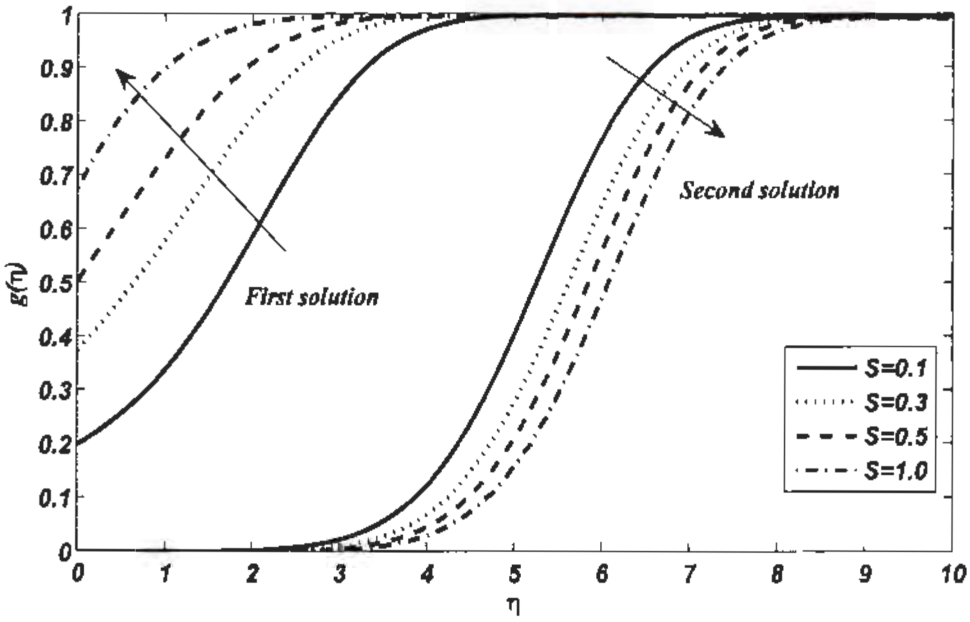

Figure 3.8: Concentration profiles $g(\eta)$ for the variation of the suction parameter S with fixed values of $\lambda = -1.52, K_s = 0.5, \beta = 1, \gamma = 0.3, K = 0.5$, and $Sc = 1$.

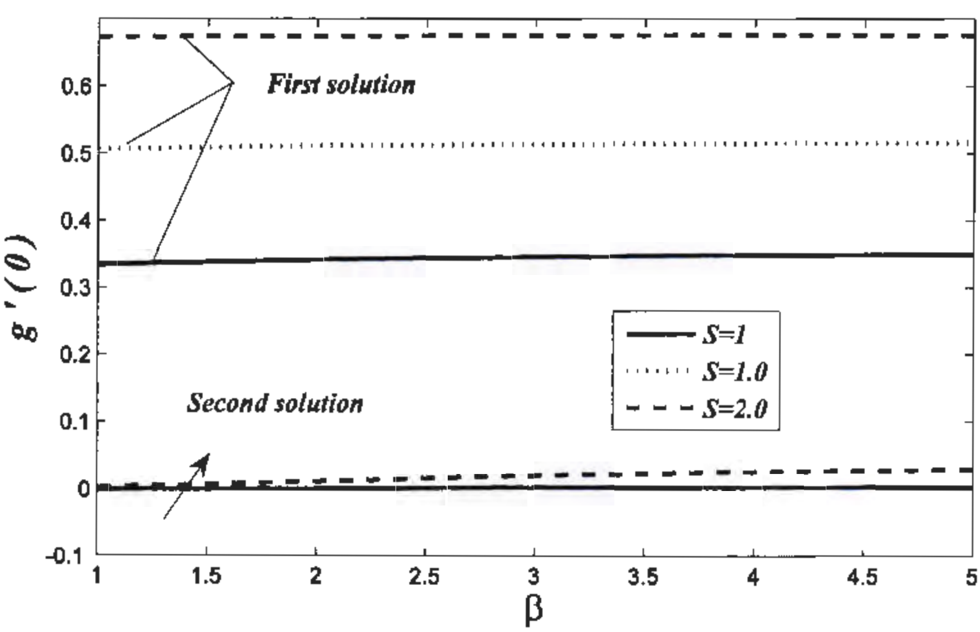

Figure 3.9: Gradient of the Concentration profile $g'(0)$ against Casson fluid parameter β for variation of the suction parameter S along with fixed values of $\lambda = -1.52,\ \gamma = 0.3, K = 0.5, Sc = 1$ and $K_s = 1$.

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