Comparison of Pre-Coding MIMO System in Rayleigh Fading Environment

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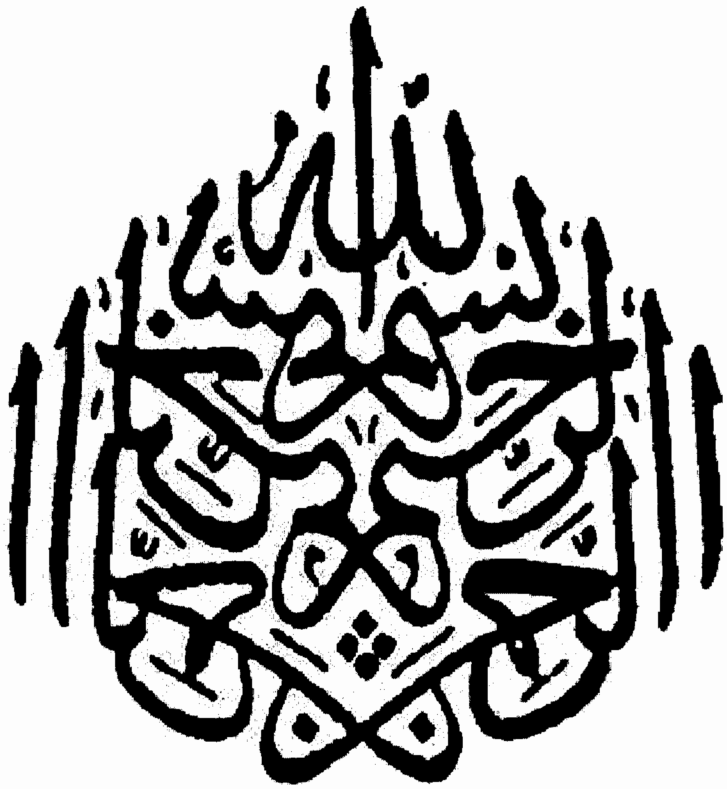

In the Name of

ALLAH

Who is the Most Gracious and Merciful

Dedicated to Prophet Muhammad (PBUH) and My Parents

Who Always Love Us the Most

Declaration

I certify that except where due acknowledgments has been made, the work is that of my alone: the work has not been submitted previously, in whole or in the part, to qualify for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program; and any editorial work, paid or unpaid, carried out by a third party is acknowledged.

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May Allah help us in following the true spirit & principles of Islam write down in the Holy Quran and Sunna! (Aameen)

Mujtaba Qureshi

Abstract

In this work we discuss the performance comparisons of different space-time codes over Rayleigh fading environment. We present an analysis of performance comparisons of space-time trellis codes (STTC), exploiting the generator sequences from Tarokh/ Seshadri/ Calderbank (TSC), Baro/ Bauch/ Hansmann (BBH) and the best code proposed by Vucetic/ Yuan (VY). The comparisons are made on the basis of symbol error rate (SER), bit error rate (BER), and frame error rate (FER) performance metrics. The pre-coding MIMO (multiple-input multiple-output) system is analyzed in block fading and fast fading environments using the above three codes with QPSK modulation scheme. In both fading environments the performance comparison are between the bit error rate (BER) Vs SNR (signal to noise ratio), symbol error rate (SER) Vs SNR, and frame error rate (FER) Vs SNR. Results show that these codes show optimum/suboptimum behavior under different fading conditions.

Result in case of BER Vs SNR clearly show that in block fading environment best codes do not give optimum result while TSC code gives best result among the three. While in case of SER Vs SNR best codes give better performance than BBH codes and TSC codes. In this case best codes give optimum result while TSC code gives worst result among the three. Simulation results in case of FER Vs SNR show that all the three codes show almost same behavior. However, with little margin BBH codes give better performance among the three.

For fast fading environment where the results in case of BER Vs SNR clearly show that BBH and best codes outperform the TSC codes in QPSK modulation scheme. In this case best codes and BBH codes give optimum result while TSC code gives worst result among the three. Results for SER Vs SNR clearly show that the best codes give better performance than BBH codes and TSC codes. In this case best codes gives optimum result while TSC code gives worst result among the three. Final results in case of FER Vs SNR shows that best codes and BBH codes outperform the TSC codes. So in this case best and BBH codes give optimum results in fast fading channel.

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Space-time Coding Notation

n_T	Number of transmit antennas
n_R	Number of receive antennas
$\boldsymbol{S}$	Transmitted signals
$\propto_{i,j}(t)$	Path gain of i^{th} transmit and j^{th} receive antenna
$\boldsymbol{r}$	Received signal
$\boldsymbol{H}$	Channel
$\boldsymbol{Z}$	Receiver noise
R	Receiver noise covariance
σ^2	Noise power
C	Capacity
W	Bandwidth
B_s	Signal bandwidth
B_c	Coherence bandwidth
τ	Coherence time
$\emptyset$	Phase
a	Amplitude
σ_s^2	Variance of signal
D	Peak amplitude of the direct signal
$I_o(.)$	Modified Bessel function of the first kind and of zero-order
K	Rician factor
Ω	Weighting factor
$\hat{\boldsymbol{S}}$	Estimate erroneous sequence

B	Codeword difference matrix
A	Codeword distance matrix
Δ	Diagonal element matrix
λ	Eigen-value
δ_H	Minimum Hamming distance
d_p^2	Minimum product distance
$\boldsymbol{B}$	Input binary data stream
G	Generator sequences
v	Total memory order
$\tilde{S}$	Decision statistics

Abbreviations

LOS	Line-Of-Sight
SNR	Signal-to-Noise Ratio
MIMO	Multi-Input Multi-Out
STC	Space-Time Code
STTC	Space-Time Trellis Code
QPSK	Quadrature Phase Shift Keying
PEP	Pair-wise Error Probability
FER	Frame Error Rate
BER	Bit Error Rate
SER	Symbol Error Rate
TSC	Tarokh/Seshadri/Calderbank
BBH	Baro/Bauch/Hansmann
VY	Vucetic and Yuan
MRRC	Maximal Ratio Receive Combining
STBC	Space-Time Block Code
NLOS	Non-Line-Of-Sight
iid	independent identically distributed
MRC	Maximum Ratio Combining
MLSE	Maximum Likelihood Sequence Estimator
MMSE	Minimum Mean Square Error
CSI	Channel State Information
A-STC	Alamouti Space-Time Code

1

Introduction

This chapter discusses space time coding and its important techniques used in wireless communication. We also discuss different methods to improve the performance of communication system. In literature review we cover important work done in the field of space time coding.

1.1 Space Time Coding

This is very important coding technique in wireless communication which is developed for multiple transmit antennas in order to increase the reliability of transmitted data. In this method multiple redundant copies of data stream are transmitted in a good enough state in order to allow reliable decoding at the receiver end. Generally in spatial and temporal domain coding is performed to introduce correlation between the signals which are transmitted at various time periods from various antennas.

The spatial and temporal domain correlation is used for minimizing the transmission errors at the receiver end. By performing coding we can get power gain as well as transmit diversity without sacrificing any bandwidth over spatially uncoded systems [1].

In order to perform space time coding, we use different coding structures which can be divided in two main types

- **Space time trellis codes:** A trellis code is transmitted in different time slots from different antennas and can provide diversity gain as well as coding gain.
- **Space time block codes:** It is applied to a group of data simultaneously and can provide diversity gain only but is comparatively less complex than space time trellis codes.

These two types of codes discussed above are the most commonly used and important techniques in space-time coding (STC). Details of these two codes are given in chapter 1 and chapter 4 respectively.

1.2 Problem Statement

In the new era wireless communications is now a challenging and exciting field. With the passage of time, new technologies are emerging very rapidly due to which high data rate requirements in wireless communication has been increased.

Two main challenges that we face in modern wireless communications are:

- To increase data rates using limited bandwidth,
- To increase the performance of a system without any increase in power or bandwidth.

Main problem in order to achieve better performance is its degradation because of the rapid motion of the scattering objects and mobile unit in the environment. Such movements make the radio channel unstable and cause instantaneous decrease in signal-to-noise ratio. As a result of this reduction in signal-to-noise ratio, error bursts occur which in turn give rise to significant degradation in the performance of wireless communication system.

Although Vucetic et al. [1] compared the designed codes, with Tarokh/Seshadri/Calderbank (TSC) [2] and Baro/Bauch/Hansmann (BBH) [3] codes for frame error rate but

- Comparison under same modulation scheme and based on slow and fast fading have not been presented for different design.
- No comparisons of BBH, TSC and Vucetic and Yuan codes have been provided for Bit Error Rate (BER) and Symbol Error Rate (SER).
- The authors in [1] discussed different codes but did not provide any comparison of these codes with others coding techniques in a same plate form.
- Comparisons under same modulation scheme but for different number of received antennas have also not been presented.

1.3 Proposed Method

Let us consider a person sitting in a bullet train wants to use internet, live video streaming, video conferencing and other high rate data services. The train is running at a speed of 400 *km/hr*, which causes a shift of 333 *Hz* in its frequency (Assume the carrier frequency is 900 *MHz*). The coherence time during which the channel is considered to be uniform, is only 3 *msec* which is a very short time as compared with the time of a transmitted block. It means that the channel will remain constant only for a few symbols rather than for a block. We thus will experience a highly time-varying fast fading environment. In such fading conditions, only through the exploitation of diversity techniques a reliable communication system can be developed where the receiver can overcome different replicas of the signals transmitted under varying fading environments. Such kind of the diversity techniques is always useful to decrease the probability of errors and save the transmitted signals from strong attenuation. In MIMO (Multi Input Multi Output) systems, we can achieve a much higher degree of reliability using different coding techniques.

After an extensive study of the literature to analyze the performance of space-time codes, we find that a unified approach for a thorough comparison of the various performance analyzing techniques for space-time codes which is not done before. Moreover, in order to implement such a unified approach, a common platform of evaluation is also needed. We thus intend to

- Analyze space time codes under same fading conditions, using the existing analyzing techniques available in the literature.

- Present a comparison of the above-mentioned analyzing techniques to achieve a unified decision for the tightness of the performance bounds.

1.4 Space Time Trellis Code

This code was introduced first time by Tarokh, Seshadri and Colderbank. In the literature it is discussed in a broad sense and also known as STTC. It can combat the effects of fading. At the same time it can provide diversity improvement, spectral efficiency and a substantial coding gain on flat fading environment. By the effect of STC (space time code) construction criterion, most favorable space time trellis coded minimum phase shift keying techniques for multiple numbers of antennas at the transmitter side and spectral efficiencies can be designed for fast as well as slow fading environments [1].

1.4.1 Encoder Structure

For space time trellis code, the encoder assigns binary data to every modulation symbols, as shown in figure 1.

Here we analyze an encoder structure of space time trellis coded minimum phase shift keying modulation having total n_T antennas at the transmitter side as given in the figure 1. In figure, c denotes the input message stream and is given by

$$c = (c_0, c_1, c_2, c_3, \dots \dots \dots \dots c_t \dots \dots), \tag{1.1}$$

where a group of bits of information $m = \log_2 M$ is denoted by c_t at time t and is given by

$$c_t = (c_t^1, c_t^2, c_t^3, \dots \dots \dots \dots \dots \dots \dots c_t^m) \tag{1.2}$$

The diagrammatic representation of space-time trellis encoder is given in figure 1.

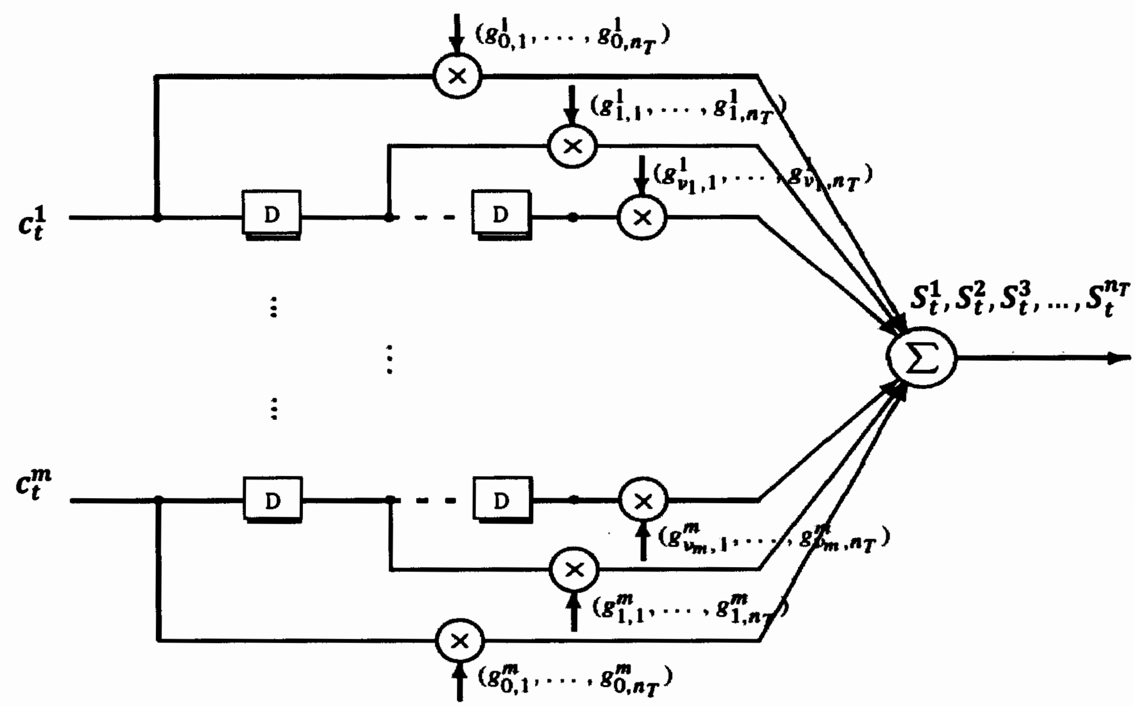

Figure 1: Space-Time Trellis Code Encoder

The encoder is used in order to map the input stream into minimum phase shift keyed modulated signal stream, which can be written as

$$s = (s_0, s_1, s_2, \dots \dots \dots \dots, s_t, \dots \dots .), \tag{1.3}$$

where at time interval t the space time symbol is denoted by s_t and can be written as

$$s_t = (s_t^1, s_t^2, \dots \dots \dots \dots \dots \dots s_t^{nT})^T \tag{1.4}$$

Total of n_T antennas at the transmitter side is used in order to transmit the modulated signals.

1.4.2 Generator Description

In the space time trellis encoder m binary sequences of input $c^1, c^2, \dots \dots, c^m$ are thrown into the encoder, here we have m feed forward shift registers. The k^{th} input sequence

$$c^k = \left(c_0^k, c_1^k, c_2^k, \dots \dots \dots ., c_t^k, \dots \dots .\right), \quad k = 1, 2, \dots . m, \tag{1.5}$$

is then moved to the k^{th} shift register where it is multiplied by coefficient set of an encoder. From every shift registers the multiplier outputs are then added modulo M, where it gives the result of the encoder

$$s = (s^1, s^2, \dots \dots \dots \dots \, s^{nT}) \tag{1.6}$$

The connection between the modulo M adder and the shift register elements are more often determined by the m multiplication coefficient sets given below in matrix form

$$\begin{bmatrix} g^1 \\ g^2 \\ \cdot \\ \cdot \\ g^m \end{bmatrix} = \begin{bmatrix} \left(g_{0,1}^1, g_{0,2}^1, \dots, g_{0,nT}^1\right), \left(g_{1,1}^1, g_{1,2}^1, \dots, g_{1,nT}^1\right), \dots, \left(g_{v1,1}^1, g_{v1,2}^1, \dots, g_{v1,nT}^1\right) \\ \left(g_{0,1}^2, g_{0,2}^2, \dots, g_{0,nT}^2\right), \left(g_{1,1}^2, g_{1,2}^2, \dots, g_{1,nT}^2\right), \dots, \left(g_{v2,1}^2, g_{v2,2}^2, \dots, g_{v2,nT}^2\right) \\ \vdots \\ \left(g_{0,1}^m, g_{0,2}^m, \dots, g_{0,nT}^m\right), \left(g_{1,1}^m, g_{1,2}^m, \dots, g_{1,nT}^m\right), \dots, \left(g_{vm,1}^m, g_{vm,2}^m, \dots, g_{vm,nT}^m\right) \end{bmatrix} \tag{1.7}$$

The encoder result at time t , for the i^{th} transmit antenna (n_T) , denoted by s_t^i can be determined as

$$s_t^i = \left(\sum_{k=1}^{m} \sum_{j=0}^{v_k} g_{j,i}^k \, c_{t-j}^k \right) \text{mod M}, \tag{1.8}$$

where $g_{j,i}^k$, $k = 1,2,\ldots,m$, $j = 1,2,\ldots,v_k$, $i = 1,2,\ldots,n_T$, is an element of $M-PSK$ constellation set; v_k is the memory order of the k^{th} encoder branch. The value of v_k for $M-PSK$ constellations is computed as

$$v_k = \left\lfloor \frac{v+k-1}{\log_2 M} \right\rfloor, \tag{1.9}$$

where the encoder has total memory order, denoted by v , which is sum of all v_k, i.e;

$$v = \sum_{k=1}^{m} v_k \tag{1.10}$$

In the trellis encoder we have total 2^v states. The *generator sequences* are also known as m multiplication coefficient set sequences ($g_{m,i}^k$), and these generator sequences completely describe the structure of encoder.

The transmitted signals x_t , passed through Rayleigh flat fading channel. The path gain between i^{th} transmit antenna (n_T^i) and the j^{th} received antenna (n_R^j) at time t is represented by $\alpha_{i,j}(t)$ as shown in figure 2. These path gains are complex Gaussian random variables having zero mean and their variance is unity.

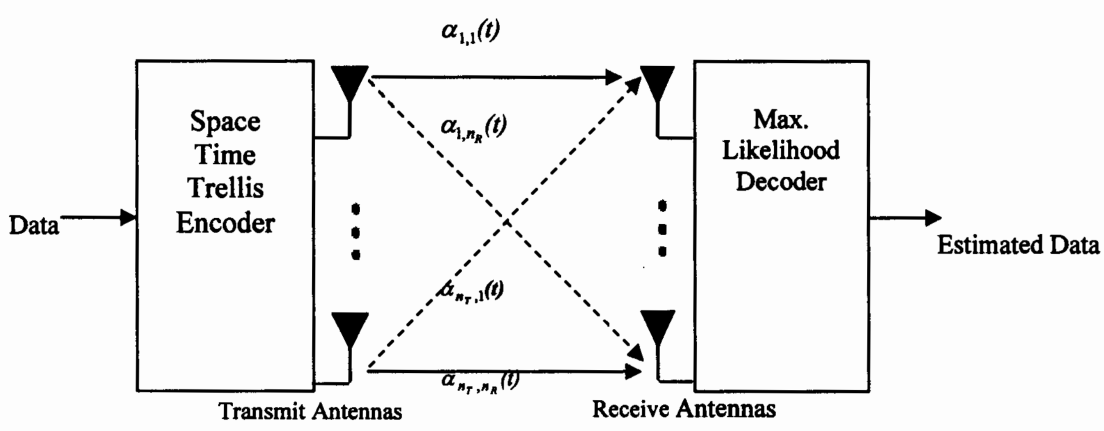

Figure 2: Model of ST Trellis Encoder and Decoder with n_T Transmit & n_R Receive Antennas

The signal received at j^{th} receive antenna at time t is

$$r_t^j = \sum_{i=1}^{n_T} \sqrt{E_o/n_T}\ \alpha_{i,j}(t)\, S_t^i + z_t^j, \tag{1.11}$$

where $\sqrt{E_o/n_T}$ is the average transmitted energy from each transmit antenna and z_t^j is the j^{th} receiver noise, which is independent Gaussian random variable with zero mean and variance $N_o/2$ per dimension.

The received signals are then moved by the decoder and because of the decoding error the received signals may differ from the transmitted symbol sequences. The erroneously decoded symbol sequence from i^{th} transmit antenna at time t will be denoted as $\hat{x}_t^i$ and the decoded symbols at same time will be

$$\hat{S}_t=(\hat{S}_t^1, \hat{S}_t^2, \hat{S}_t^3, \dots, \hat{S}_t^i, \dots, \hat{S}_t^{n_T}) \tag{1.12}$$

At decoder side, the *Viterbi algorithm* is utilized in order to accomplish the maximum likelihood decoding, with assumption that complete information about the channel is given to the receiver site. The branch metric is determined as the squared Euclidean distance between the actual received signals and the conjectured received symbols (r_t^j) and the path is selected according to lesser path metric as,

$$\sum_{j=1}^{n_R} \left| r_t^j - \sum_{i=1}^{n_T} \alpha_{i,j}(t) S_t^i \right|^2 \tag{1.13}$$

The Viterbi algorithm is used in space time trellis codes in order to select the path according to lesser path metric as the decoded sequence [1].

1.4.3 Features of Space-time Trellis Code

This code have the following features:

- Complex structure
- Joint design of error control to achieve maximum possible diversity, Modulation, transmit and receive diversity
- Spectral efficiency, Coding gain and diversity improvement

1.5 Different Methods to Improve Performance

Performance of communication system can be increased by

1. **Diversity Techniques**
2. **Coding Techniques**
3. **Exploiting highly complex adaptive signal processing algorithms**
4. **Increasing Signal to Noise ratio**

In order to achieve reliable communication in wireless communication we use different diversity techniques, where the receiver is able to overcome multiple replicas of the transmitted signals under varying fading environment. If these different replicas fade independently, then it is less probable to have all copies of the transmitted signal in deep fade simultaneously. The probability of all the samples being simultaneously below a given level is much lower than the probability of any individual sample being below that level. Therefore, the receiver can reliably decode the transmitted signal by using copies of these received signals. Different diversity techniques used in wireless communication are discussed in chapter 3.

In wireless communication Performance can also be increased by pre-coding the transmitted signal. For this purpose we exploit coding methods before data transmission, the erroneously received data can recovered at the receiver end. In communication system we used STC to decrease the error rate. Most commonly used coding techniques has been discussed earlier other well known coding techniques are:

- DSTBCs (Differential space-time block codes)
- STTTCs (Space-time turbo trellis codes)
- LSTCs (Layered space-time codes)

In practical communication system at detectors some very highly sophisticated adaptive algorithms are utilized in order to increase the performance of a system. These signal processing algorithms are typically dependent on channel estimation. For this purpose we estimate the channel by sending some *training bits*. These bits are also called known bits. These algorithms regularly change channel information at receiver on the basis of error probability.

In practical communication system, the performance can be increased by increasing signal to noise ratio which demands high power, but here we face some restrictions. This limit is due to safety reason as radio waves are harmful to human being and also in order to ignore the interference of others radio signal working on the same frequency. Signal to noise ratio is the best of all performance measure of a digital communication system. It is the most understandable performance measure parameter in practical digital communication system [4]. The denominator in the SNR term should be minimum so that we can get higher value of signal to noise ratio and thus we get better performance in practical communication system.

1.6 Literature Review

In the last decade, many researchers have explored the field of space-time coding with gradual improvements in the performance of MIMO communication links. These researchers have also analyzed the existing techniques with the establishment of performance mile-stone for the future researchers. Some of the pioneering research works performed in the field of space-time coding include S*hah et al.* [5], *Hedayat et al.* [6], *Byun et al.* [7], *Safar et al.* [8] and *Zummo et al.* [9].

Shah, et al., have analyzed the performance of an important class of MIMO (multiple-input multiple-output) systems, that of orthogonal space time block codes concatenated with channel coding. They have studied this system under the umbrella of correlated fading and spatially independent fading may arise from transmit or receive antennas or unfavorable scattering conditions. In their analysis, they have considered the effects of time correlation and presented an analysis for temporal and spatial correlations [5].

Hedayat, et al., have worked under the umbrella of correlated fading environment on MIMO system. They have calculated PEP (pair-wise error probability) equations under different fading conditions. The PEP expressions has been derived under temporally correlated fading, block fading, fast fading, as well as quasi-static fading environment considering both Rician as well as Rayleigh fading environments. They have used the expressions of pair-wise error probability in order to determine union bounds on the behavior of different codes like diagonal algebraic codes, STTCs, linear-dispersion code, and super orthogonal codes [6].

Byun, et al., have worked on different space–time codes and observe their performance by calculating a set of equations of FEP (frame error probabilities). For FEP a new approximation has been derived over quasi-static Rayleigh fading environment for space-time-trellis-coded modulations. They have taken the benefit of two important schemes: the limiting-before-averaging techniques and the modified bounding techniques and thus the upper bound have been tightened [7].

Safar, et al., has derived different criteria about the performance of wireless communication system in order to analyze space-time coded system in both domains i.e; temporal as well as spatial domain. They have shown that if we have full rank correlation matrix, then the problems of space-time codes construction for correlated environments are decreased to the codes construction problems for fast fading environments [8].

Zummo, et al. have considered the behavior of binary coded system in order to observe the performance of such system over BF (block fading) environments. The data bits were interleaved at the transmitter side before the transmission take place in order to scatter errors occur in bursts because of strongly faded environments. The PEP (pair wise error probability) has a function of union bound. He determined the union bound expressions on BEP (bit error probability) by the utilization of weight enumerator of the code considering a uniform interleaving. The suggested limits were obtained for both the codes i.e; turbo codes and convolutional codes [9].

In order to improve the performance of wireless communication a lot of work has been done to suggest different methods. Space-time code (STC) design methods mostly consider perfect models for channel: either fast fading or quasi-static fading. In [10] the design method for two transmit antennas is suggested for quasi-static environment in order to achieve the maximum diversity benefit. Later work on construction methods of systematic trellis code in [11], have shown that maximum diversity advantage can also be achieved by using multiple number of transmits antenna in MIMO communication system.

The authors in [12] have explored STTC design method for fast fading environment initially for the first time. They have constructed space time code having multiple antennas at the transmitter side exploiting the concept of single set division using QPSK (quadrature phase shift keying) modulation scheme. Tarokh et al. suggested a STTC for communication system having multiple antennas at the transmitter side. W. firmanto and J. Yuan has proposed Space time codes designed and optimized for fast fading environment using two transmit antennas and for both 8-PSK and QPSK modulation schemes by searching on computer [13]. For fast fading environment construction method for systematic trellis code has been suggested [14]. Moreover, Code construction problem for correlated fading has been addressed [15].

In order to construct and analyze the performance of space time codes, N. Seshadri and A. calderbank have proposed important guideline for STTC based on PEP for slow fading environment and have explained a much easier but untight upper bound for pair-wise error probability using chernoff bounds. Although, the bound derived in [10] can minimize pair-wise error probability but unable to minimize FER. Later work by S. zummo, S. Al-Semari, A. R. hammons and H. El Gamal have successively derived tighter upper bounds for space-time codes utilizing distance spectrum [12] [16].

Another most commonly used technique of STC is STBC (space-time block code) [17]. STBC and STTC [10] techniques can provide useful diversity advantage like any other commonly used MRRC (Maximal Ratio Receive Combining) technique. In STBC, a block of input symbols producing a matrix output over antennas and time where as in STTC, a signal input symbol producing a sequence of spatial vector outputs at given time. STBCs with simple decoding and encoding algorithms have the advantage of full diversity. But with full rate code it does not give any coding gain. On the other hand both coding gain as well as diversity gain is provided by STTCs. Alamouti briefly discussed STCs having multiple antennas at the transmitter side which are historically the first STBC [18]. Later STBC is designed for more transmit antennas [19]. Vucetic et al. have discussed different STTCs but do not provide any comparison of these codes with STBCs for different number of received antennas. They have designed optimum generting sequences for STTCs through computer search [1]. They have taken BBH and TSC codes as reference. Generator sequences have the similar minimum rank as that of BBH and TSC but have greater coding gain. The authors has also compared FER for the designed codes, with TSC and BBH codes. But no comparisons have been provided for SER and BER. Comparison under same modulation scheme and based on slow and fast fading have not been presented for different design.

Along with the above-mentioned literature, we will keep on searching updated research in the field of space-time coding for improvement in our approach.

1.7 Organization of the Work

This work focuses on the performance comparison of pre-coding MIMO (multi-input multi-output) system in Rayleigh fading environment. In Chapter 2 we review MIMO system. In Chapter 3 we discuss different types of Diversity techniques used in wireless communication. We also present performance analysis of different STTCs in fast fading environment in chapter 3. In chapter 4 we discuss space time coding techniques and their encoder structure. Here we also present the performance comparison of different STTCs in block fading environment. Chapter 5 summarizes the conclusion of the work.

1.8 Scope of the Work

In this work we present performance comparison of different space time codes, exploiting the generator sequences from TSC codes, BBH and Vucetic and Yuan (VY) codes. The performance comparisons have been made on the basis of SER, FER and BER. The performance comparisons have been done in fast as well as in block fading environment.

In both fading environments the performance comparison are between the BER Vs SNR, SER Vs SNR and FER Vs SNR. Simulation results show that these codes show optimum/suboptimum behavior in Rayleigh fading environment.

2

Multi-Input Multi-Output System

In this chapter an overview of MIMO system has been given. Here communication system and different types of channels have been discussed. At the end, statistical models for fading channels have been described.

2.1 Multi-input Multi-output (MIMO) System

Consider a MIMO system with n_T transmit and n_R receive antennas shown in figure 3.

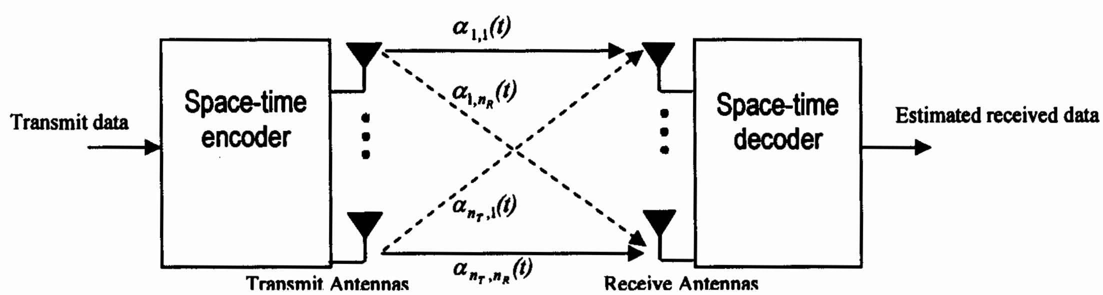

Figure 3: Model of MIMO System with n_T Transmit and n_R Receive Antennas

The transmitted signals S can be represented as $T \times n_T$ matrix, whose column corresponds to number of transmits antennas and rows to time for transmission of block.

$$S = \begin{bmatrix} S_{1,1} & \cdots & S_{1,n_T} \\ \vdots & \ddots & \vdots \\ S_{T,1} & \cdots & S_{T,n_T} \end{bmatrix} \tag{2.1}$$

The transmitted signals from n_T transmit antennas can be written as

$$S = (S_1, S_2, S_3, \dots, S_i, \dots, S_{n_T}), \tag{2.2}$$

where S_i in the transmitted signal from i^{th} transmitted antenna and is

$$S_i = [S_{1,i}\, S_{2,i}\, \dots\, S_{T,i}]^T \tag{2.3}$$

The transmitted signal covariance is given as [20]

$$R_{s_i s_i} = E\{S_i S_i^H\} \tag{2.4}$$

The transmitted power can be calculated as

$$Transmitted\ power = tr(R_{s_i s_i}), \tag{2.5}$$

where $tr(R_{s_i s_i})$ is the trace of $R_{s_i s_i}$, obtained by the sum of the diagonal elements of $R_{s_i s_i}$.

The channel $\boldsymbol{H}$ is represented by $n_T \times n_R$ complex matrix. The path gain between i^{th} transmit antenna (n_T^i) and the j^{th} received antenna (n_R^i) at time t is described by $\propto_{i,j}(t)$. The total $\boldsymbol{H}$ matrix can be represented as

$$\boldsymbol{H} = \begin{bmatrix} \propto_{1,1}(t) & \cdots & \propto_{1,n_R}(t) \\ \vdots & \ddots & \vdots \\ \propto_{n_T,1}(t) & \cdots & \propto_{n_T,n_R}(t) \end{bmatrix} \tag{2.6}$$

The elements of channel matrix can either be random or deterministic. In wireless communication we mostly focus on Rician and Rayleigh distribution of channel matrix. In most cases we assume the Rayleigh distribution, as it is most practical and represents *non-line-of-sight (NLOS)* radio propagation. In the Rayleigh fading channel, the path gains are represented by independent sophisticated Gaussian orbitrary variables at each time slot. Thus we can say that in the Rayleigh fading channel real and imaginary parts of the path gains at each time slot are independent identically distributed (iid) Gaussian random variables.

For time duration *T*, the received signal with dimension of $T \times n_R$ matrix can be written as

$$\boldsymbol{r} = \begin{bmatrix} r_{1,1} & \cdots & r_{1,n_R} \\ \vdots & \ddots & \vdots \\ r_{T,1} & \cdots & r_{T,n_R} \end{bmatrix} \tag{2.7}$$

By using the linear model, the received signal can be written as

$$\boldsymbol{r} = \boldsymbol{SH} + \boldsymbol{Z}, \tag{2.8}$$

where $\boldsymbol{Z}$ is the receiver noise, which is sophisticated and independent Gaussian variable with zero mean. With dimension $T \times n_R$, noise matrix $\boldsymbol{Z}$ is defined by

$$\boldsymbol{z} = \begin{bmatrix} z_{1,1} & \cdots & z_{1,n_R} \\ \vdots & \ddots & \vdots \\ z_{T,1} & \cdots & z_{T,n_R} \end{bmatrix} \tag{2.9}$$

The covariance matrix of the receiver noise is given as [20]

$$R_{zz} = E\{z^H z\} \tag{2.10}$$

For the case of uncorrelation among components of z , the covariance matrix becomes

$$R_{zz} = \sigma^2 I_{n_R}, \tag{2.11}$$

where I_{n_R} is the identity matrix and σ^2 is the noise power.

For normalization purpose one approach is that transmission power is normalized to one. The transmitted symbols average power is normalized to $E_o = {}^1/_{n_T}$, then if the variance of the noise samples is ${}^1/_{2\gamma}$ per complex dimension that is $N_o = {}^1/_{\gamma}$. At every antenna at the receiver side, the average power of signals is 1 and the received SNR is γ.

Another approach for normalizing E_o and N_o is that we take average power of one for transmission symbols and unit-power noise samples. Then the received signals linear model can be described as

$$r = \sqrt{\frac{\gamma}{n_T}} SH + Z, \tag{2.12}$$

where γ is the received SNR.

2.2 The types of random channels

According to varying channel coefficients, we will differentiate in to three types, in all three cases we assume that channel matrix H is random.

Fast fading: For one symbol period entries of matrix H are constant and randomly changed at the start of every symbol period T.

Block fading: For specific number of symbol periods entries of matrix H are constant, as compared to total transmission time this time is much shorter.

Slow fading: For all symbols entries of H are kept constant throughout, once it is calculated at the begging of transmission then its constant for all the time.

2.2 Fading Channels Models

Based on the random phases formed due to multipaths, the received signal can meet either in a destructive or constructive way. The resultant sum of these multipath components forms a spatially varying standing wave field. If the mobile unit is fixed and the surrounding objects are moving then due to this movement the received signal may have different amplitude levels and different phases. Thus fading channels are classified into following types

- Based on **Multipath time delay** into
  - Flat fading
  - Frequency selective fading

- on **Doppler spread** into
  - Slow fading
  - Fast fading

As these two phenomena are independent therefore we get following four types of fading channels:

- ***Frequency Selective Fast Fading:*** Here channel coherence time (τ) is smaller than the signal duration (T_s) and channel coherence bandwidth (B_c) is smaller than the signal bandwidth (B_s).

- ***Frequency Selective Slow Fading:*** Here channel coherence time (τ) is larger than the signal duration (T_s) and channel coherence bandwidth (B_c) is smaller than signal bandwidth (B_s).

- ***Flat Fast Fading:*** Here channel coherence time (τ) is smaller than the signal duration (T_s) and channel coherence bandwidth (B_c) is larger than the signal bandwidth (B_s)

- ***Flat Slow Fading:*** In this case channel coherence time (τ) is larger than the signal duration (T_s) and the channel coherence bandwidth (B_c) is larger than signal bandwidth (B_s).

2.3 Slow fading condition

Consider the transmitted signals $\boldsymbol{S}$ can be represented as $T \times n_T$ matrix, whose rows correspond to time for transmission and column number of transmits antennas.

$$\boldsymbol{S} = \begin{bmatrix} S_{1,1} & \cdots & S_{1,n_T} \\ \vdots & \ddots & \vdots \\ S_{T,1} & \cdots & S_{T,n_T} \end{bmatrix} \tag{2.13}$$

For the case of slow fading condition the fading coefficients within each frame are constant for all, that is

$$\propto_{i,j}= constant\,;\;\; i = 1,2,\dots,n_T\,, i = 1,2,\dots,n_R \tag{2.14}$$

Assume that decoded estimate erroneous sequence $\hat{\boldsymbol{S}}$, and then $T \times n_T$ estimate erroneous sequence matrix is given as

$$\hat{\boldsymbol{S}} = \begin{bmatrix} \hat{S}_{1,1} & \cdots & \hat{S}_{1,n_T} \\ \vdots & \ddots & \vdots \\ \hat{S}_{T,1} & \cdots & \hat{S}_{T,n_T} \end{bmatrix} \tag{2.15}$$

The *codeword difference matrix* is defined as

$$\boldsymbol{B}(\boldsymbol{S},\hat{\boldsymbol{S}}) = \boldsymbol{S} - \hat{\boldsymbol{S}} \tag{2.16}$$

And $T \times n_T$ codeword difference matrix is given as

$$\boldsymbol{B}(\boldsymbol{S},\hat{\boldsymbol{S}}) = \begin{bmatrix} (S_{1,1} - \hat{S}_{1,1}) & \cdots & (S_{1,n_T} - \hat{S}_{1,n_T}) \\ \vdots & \ddots & \vdots \\ (S_{T,1} - \hat{S}_{T,1}) & \cdots & (S_{T,n_T} - \hat{S}_{T,n_T}) \end{bmatrix} \tag{2.17}$$

The *codeword distance matrix* $\boldsymbol{A}(\boldsymbol{S},\hat{\boldsymbol{S}})$ can be construct as

$$A(\boldsymbol{S},\hat{\boldsymbol{S}}) = \boldsymbol{B}(\boldsymbol{S},\hat{\boldsymbol{S}}).B^H(\boldsymbol{S},\hat{\boldsymbol{S}}), \tag{2.18}$$

where H denotes the Hermitian of a matrix. It is notable that $A(\boldsymbol{S},\hat{\boldsymbol{S}})$ is nonnegative definite Hermitian, as $A(\boldsymbol{S},\hat{\boldsymbol{S}}) = A^H(\boldsymbol{S},\hat{\boldsymbol{S}})$ and the eigenvalues of $A(\boldsymbol{S},\hat{\boldsymbol{S}})$ are nonnegative real numbers. Let the rank of matrix $A(\boldsymbol{S},\hat{\boldsymbol{S}})$ is denoted by r.

Let unitary matrix $\boldsymbol{V}$ and Δ is the real diagonal matrix such that

$$\boldsymbol{V}\mathrm{A}(\mathrm{S},\hat{\mathrm{S}})\boldsymbol{V}^{\boldsymbol{H}} = \Delta\,, \tag{2.19}$$

where the diagonal element Δ is given as

$$\Delta= \begin{bmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_{n_T} \end{bmatrix} \tag{2.20}$$

The $\lambda_1, \lambda_2, \lambda_3, \dots \lambda_{n_T}$ are the eigenvalues of *codeword distance matrix* $A(\boldsymbol{S}, \widehat{\boldsymbol{S}})$ and $\boldsymbol{V} = (v_1, v_2, \dots, v_{n_T})$ are the eigenvectors of $A(\boldsymbol{S}, \widehat{\boldsymbol{S}})$.

2.4 Fast fading condition

Now we consider the case of fast fading condition in which the fading coefficients are constant for each symbol duration. Then like slow fading case, space-time symbol difference vector $F(S_t, \hat{S}_t)$ as

$$F(S_t, \hat{S}_t) = [S_t^1 - \hat{S}_t^1, S_t^2 - \hat{S}_t^2, \dots, S_t^{n_T} - \hat{S}_t^{n_T}]^T \quad (2.21)$$

and

$$C(S_t, \hat{S}_t) = F(S_t, \hat{S}_t). F^H(S_t, \hat{S}_t) \quad (2.22)$$

$C(S_t, \hat{S}_t)$ is the Hermitian and its eigenvalues and eigenvectors are calculated in similar manner as calculated for slow fading.

2.5 Statistical models for fading channels

Different statistical models are prepared to describe the varying behavior of channels. Two important models describing the *non-line-of-sight (NLOS)* and *line-of-sight (LOS)* propagation of radio signal respectively are

- Rayleigh Fading Model
- Rician Fading Model

2.5.1 Rayleigh fading model

In wireless communication we consider a situation where only reflected waves are received from different paths to the mobile unit. The transmitted waves do not find any direct path to the receiver. Consider the k number of reflected waves reaches the receiver, then transmitting a signal over the carrier frequency f_c results in receiving the sum of k components from different paths plus a Gaussian noise as follows:

$$r(t) = \sum_{i=1}^{k} a_i \cos(2\pi f_c t + \emptyset_i) + z(t), \tag{2.23}$$

where a_i is the amplitude and $\emptyset_i$ is the phase of the i^{th} component, and $z(t)$ is the Gaussian noise. Then at any time the envelope of the received signal has a Rayleigh distribution between $-\pi$ and π . The probability density function (pdf) of the Rayleigh distribution is

$$p(r) = \begin{cases} \frac{r}{\sigma_s^2} e^{-\frac{-r^2}{2\sigma_s^2}} & r \geq 0 \end{cases} \tag{2.24}$$

and

$$p(r) = 0 \qquad r < 0.$$

The pdf in equation (2.24) is normalized so that the signal average power is unity, after normalization we have

$$p(r) = \begin{cases} 2re^{-r^2} & r \geq 0 \end{cases} \tag{2.25}$$

and

$$p(r) = 0 \qquad r < 0.$$

The pdf for normalized Rayleigh distribution is shown in figure 4.

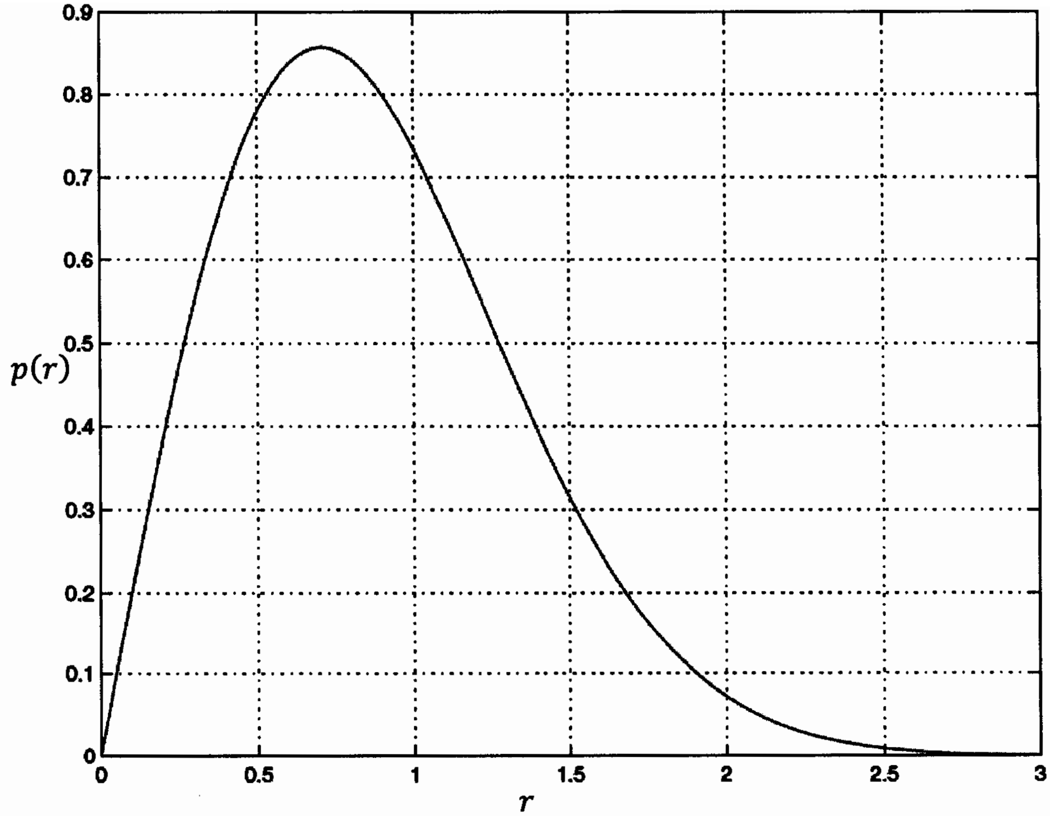

Figure 4: The pdf of Rayleigh distribution

2.5.2 Rician fading model

Rician fading model describes for the situation when the receive signal consists of a direct wave and a number of reflected waves. In some cases their is no obstruction between sender and reciever for example in microcellular mobile or satellite communications. The *line-of-sight (LOS)* wave is a stationary non-fading signal with constant amplitude. While the reflected waves are orbitrary signals reaches from different paths to the reciever. These received reflected waves collectively called the *scattered component.*

The Rayleigh distributed scattered signals and the *line-of-sight (LOS)* constant amplitude direct signal collectively results in a signal with a *Rician distribution.* The probability density function (pdf) of the Rician distribution is given by

$$p(r) = \begin{cases} \frac{r}{\sigma_s^2} e^{-\frac{(r^2+D^2)}{2\sigma_s^2}} I_o \frac{(rD)}{\sigma_s^2} & r \geq 0 \end{cases} \quad (2.26)$$

and

$$p(r) = 0 \qquad\qquad r < 0$$

where D denotes the peak amplitude of the dominant signal (direct signal) and $I_o(.)$ is the modified Bessel function of the first kind and of zero-order.

The normalized pdf can be obtained by assuming that the power of signals is one. The normalized Rayleigh distribution can be written as

$$p(r) = \left\{2r(1+K)e^{-K-(1+K)r^2}\, I_o\left(2r\sqrt{K(K+1)}\right) \qquad r \geq 0 \qquad (2.27)\right.$$

and

$$p(r) = 0 \qquad\qquad r < 0$$

where K is the ratio of power of the reflected components and the direct components of tranmitted signals called the *Rician factor*. The Rician factor can be described as

$$K = \frac{D^2}{2\sigma_s^2} \qquad (2.28)$$

The Rician distributions with various values of K are shown in figure 5.

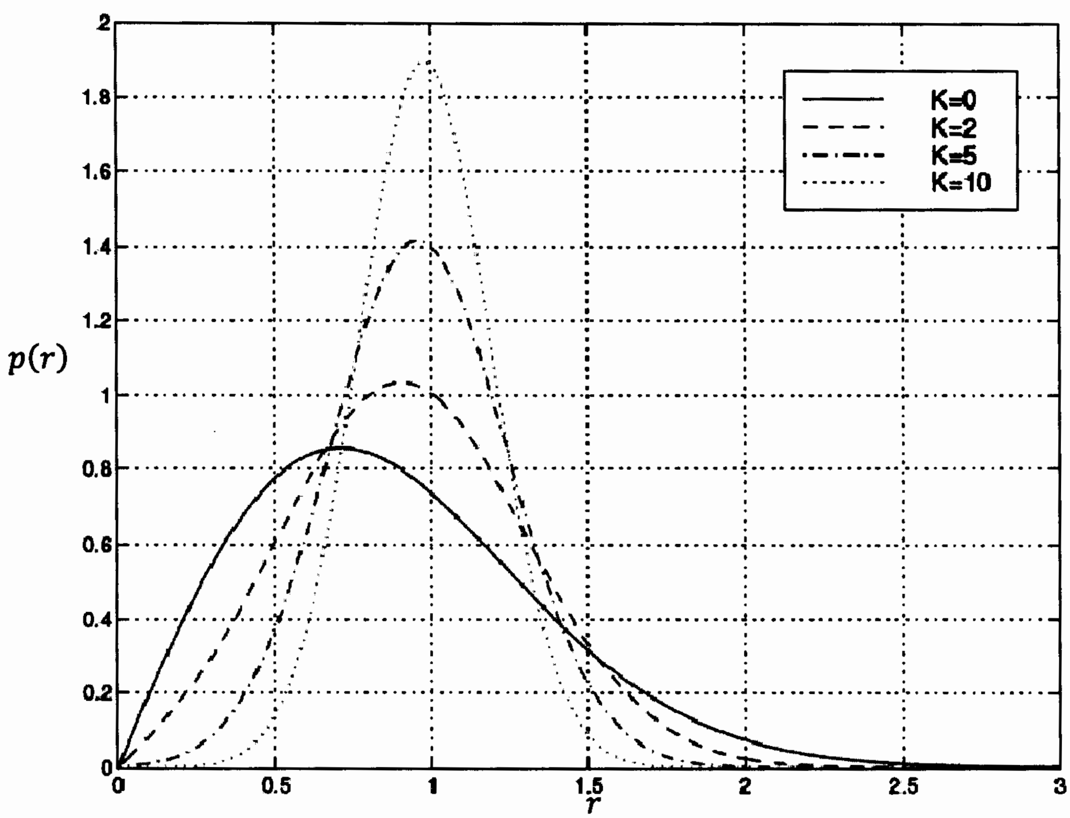

Figure 5: The pdf of Rician Distribution for different values of *K* [10]

3

Diversity Techniques

To decrease the effects of multipath fading in wireless communications, different diversity methods has been widely introduced, these techniques without any increase in transmission power or without any bandwidth sacrificing can improve the reliability of transmitted signals [21].

Diversity methods need group of replicas of the signals at the receiver, having small correlation but all holding same information. Basically the concept behind the diversity is that if different samples of a signal are taken then some samples will be minimum attenuated while other are severely faded, these samples fade in an uncorrelated manner. Therefore there is a great decrease in severity of fading statistics because of suitable combination of various samples and therefore improved reliability of transmission.

Diversity techniques can be divided into space, frequency and time depending on the domain where diversity is introduced [21].

3.1 Time Diversity

Time diversity is one of the diversity techniques where same messages are transmitted on different time periods. Because of this at the receiver end there is uncorrelated fading signals.

Channel coherence time should be at least equal to the required time separation. Within coherence time the fading process of the channel is correlated.

In digital communication system in order to provide coding gain the phenomenon of error control coding is also used. To achieve transmit diversity interleaving is added with error control coding. In such conditions at the receiver end signals replicas in the time domain are given in the shape of redundancy introduced by error control coding.

The time separation is given by time interleaving between the signal replicas to get independent fades at the input of the decoder. However, decoding delays is the result of time interleaving. For fast fading environment this method is usually effective where we have small channel coherence time. For slow fading environment, in result of large interleave there is a significant delay which is unbearable in delay sensitive case such as transmission of voice. That is why for some mobile radio system time diversity has its own limitations e.g. if in case a mobile radio station is stationary then in such situation time diversity cannot help to decrease the effect of fades.

One of the limitations of using time diversity is loss in bandwidth efficiency because of the redundancy introduced in time domain [21].

3.2 Frequency Diversity

In this technique identical message is transmitted on different frequencies. In order to guarantee uncorrelated fading attached with every frequency the frequencies are required to be separated. The frequency separation will ensure that the statistical measure of fading for different frequencies is uncorrelated.

For different propagation environment the coherence bandwidth is also different. In wireless communications, in the frequency domain at the receiver end replicas of the transmitted signal are given in the shape of redundancy introduced by spread spectrum like DSSS (direct sequence spread spectrum), frequency hopping and multicarrier modulation. However when the channel has small coherence bandwidth then in this case spread spectrum methods are useful. While when the channel has larger coherence bandwidth as compared to spreading bandwidth, in such situation the multipath delay spread is smaller than the symbol period. In order to provide frequency diversity in this situation spread spectrum is not helpful.

Similar to time diversity, because of redundancy introduced in the frequency domain frequency diversity also induces a loss in bandwidth efficiency [21].

3.3 Space Diversity

In mobile microwave communications another famous technique is space diversity. This is also known as antenna diversity. This technique is generally implemented by utilizing multiple antennas for reception and/or transmission.

In order to introduce uncorrelation among the signals all the antennas are separated from one another by a suitable distance. However, the distance requirements are dependent on the height of antennas, frequency and propagation environment. In order to achieve uncorrelated signals distance of few wavelengths is enough. In this method, in space domain the transmitted replicas from multiple antennas are given in a shape of redundancy. Irrespective of the other two techniques discussed above, in this method there is not any loss in the bandwidth efficiency. Thus therefore in order to achieve higher data rate in wireless communications this scheme is very much attractive for future.

Angle diversity and Polarization diversity is an example of space diversity [21].

3.3.1 Polarization Diversity

In this scheme for transmission and reception two polarized antennas are used. Here by two different polarized antennas vertical and horizontal signals are transmitted and received by two different polarized antennas. Here in this scheme we see that the signals are uncorrelated because of different polarization there is no need to place these antennas at far distance from each other [21].

3.3.2 Angle Diversity

Angel diversity is used in a situation where transmission with carrier frequency is bigger than 10GHz. In such conditions, after transmission the signals are spread in a space, at the receiver end the signals are independent from each other which are received from different angles at the receiver. Therefore, to introduce uncorrelation between the transmitted replicas of the signals at the receiver, we must point two or more antennas in different angles [21].

3.3.3 Classification of Space Diversity

Space diversity can be divided into two types that is; transmit diversity and receive diversity. These classifications are based upon multiple antennas that are utilized whether for transmission or reception.

- **Receive Diversity**

In order to get uncorrelated copies of the transmitted signals, at the receiver end multiple numbers of antennas are used. During transmission transmitted signals replicas are properly added with each other in order to increase the overall received signal to noise ratio and mitigate multipath fading.

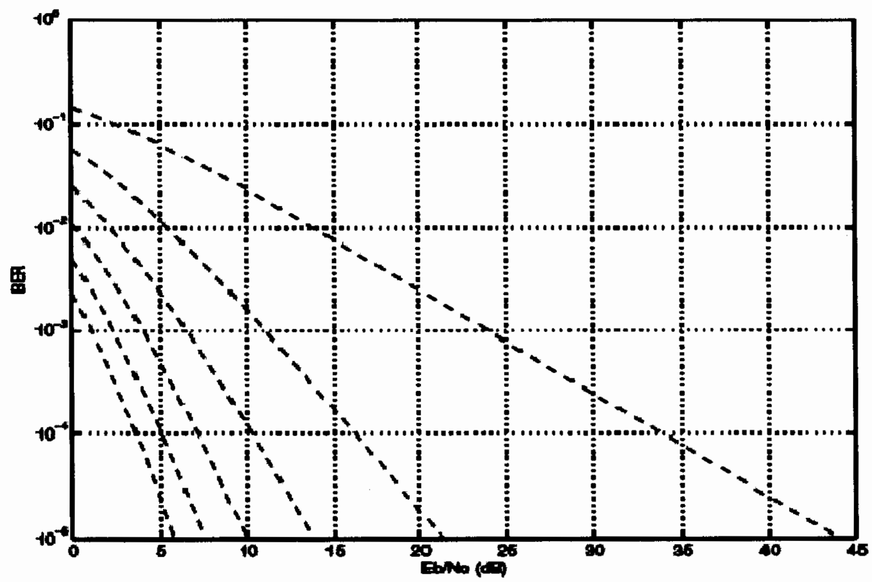

Figure 6: Performance with receive diversity; the bottom curves show diversity with 2, 3, 4, 5, and 6 antennas respectively at the receiver side. While top curve shows the performance without diversity

- **Transmit Diversity**

Here multiple numbers of antennas are used at the transmitter side. Message after processing at the transmitter side are then scattered into the space by multiple antennas.

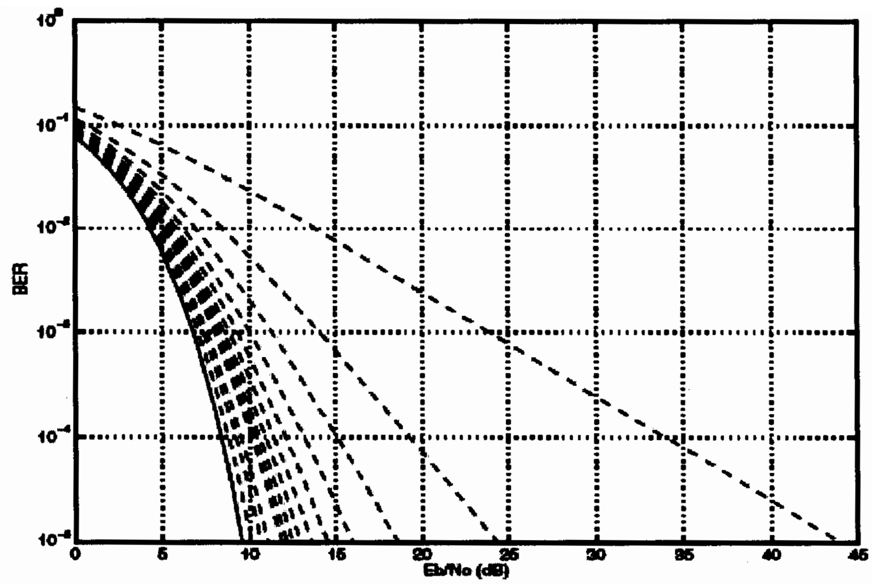

Figure 7: Performance with transmit diversity; the bottom curves show diversity with 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 20, and 40 antennas respectively at the transmitter side. While top curve shows the performance without diversity

3.4 Comparison of Space Time Trellis Codes in Fast Fading

In order to increase the performance of space-time trellis code, different generating sequences are considered. We can get the optimum results by using the generator sequences used in STTCs. In this section we compared the performance of different STTCs. We compared these performances results into fast fading environment. In these comparisons we have concluded that these codes show optimum/suboptimum behavior under different conditions. In this comparison, we exploit the generator sequences from Vucetic and Yuan (VY) [1], TSC (Tarokh/ Seshadri/ Calderbank) [2] and BBH (Baro/ Bauch/ Hansmann) codes [3]. The performance comparisons are made on the basis of symbol error rate (SER), frame error rate (FER) and bit error rate (BER) performance metrics, fast fading environment are simulated using the above three codes with QPSK modulation scheme. We use a frame size of 12 bits and simulate for 70000 iterations. Hence we prefer a small sized frame of 12 bits as the relative comparison results are not affected by frame size. At the receiver end we use maximum-likelihood decoding algorithm with complete information about the channel.

From figure (8) to figure (10) the graphs are plotted for fast fading environment where the performance comparison are between the BER (bit error rate) Vs SNR (signal to noise ratio), SER (symbol error rate) Vs SNR and FER (frame error rate) Vs SNR are shown. Figure 8 clearly shows that BBH and best codes outperform the TSC codes in QPSK modulation scheme. In this case BBH and best codes give almost same performance among the three. In this comparison TSC code gives the worst performance among the three. Thus we conclude that in case of BER Vs SNR best codes and BBH codes gives optimum result while TSC code gives worst result among the three. Figure 9 clearly shows that the best code outperform the BBH codes and TSC codes. In this case best codes give best performance among the three if we compare the performance of TSC codes and the BBH codes, we see that performance of BBH codes are better than that of TSC codes. So TSC codes give worst performance among the three. Figure 10 shows that best codes and BBH codes outperforms the TSC codes. In Figure 10 TSC codes give worst performance among the three and best codes give optimum result in fast fading channel.

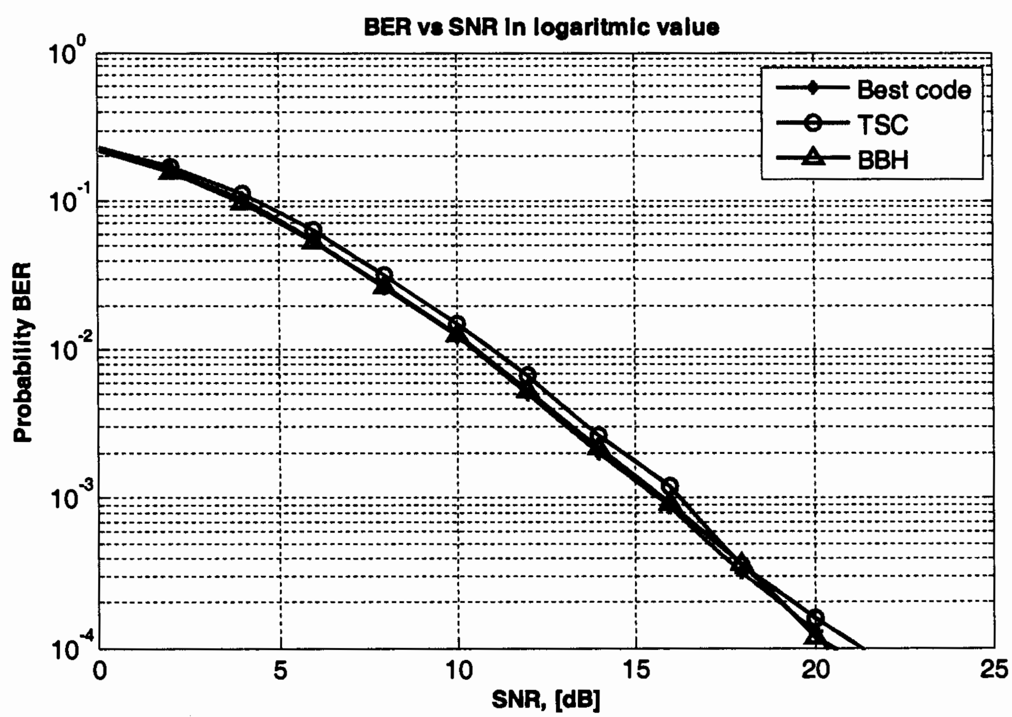

Figure 8: Comparison of BER probability Vs SNR in fast fading

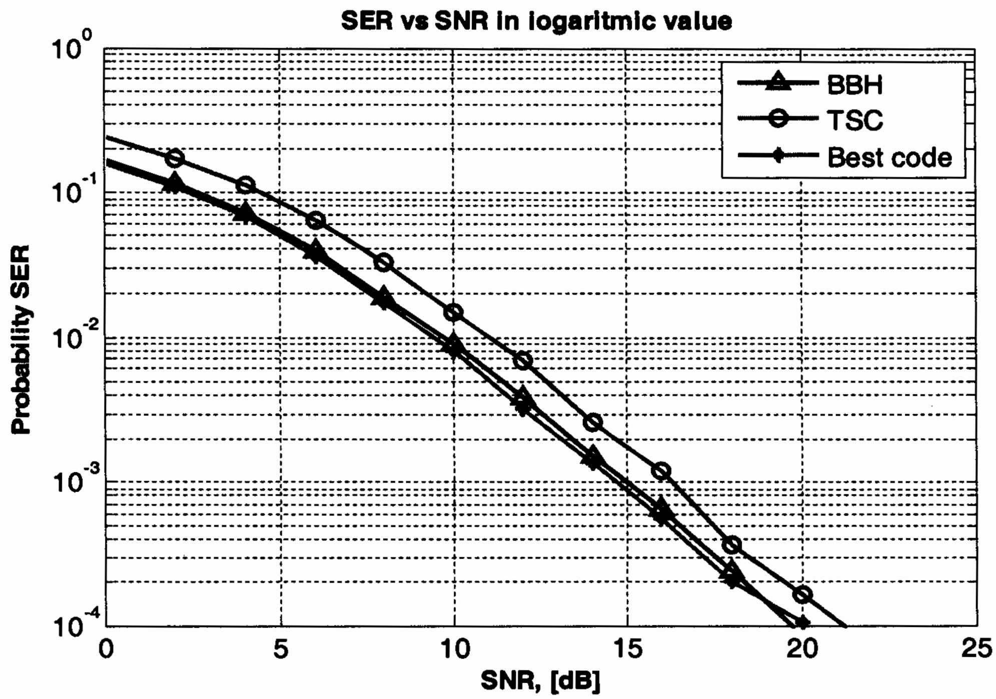

Figure 9: Comparison of SER probability Vs SNR in fast fading

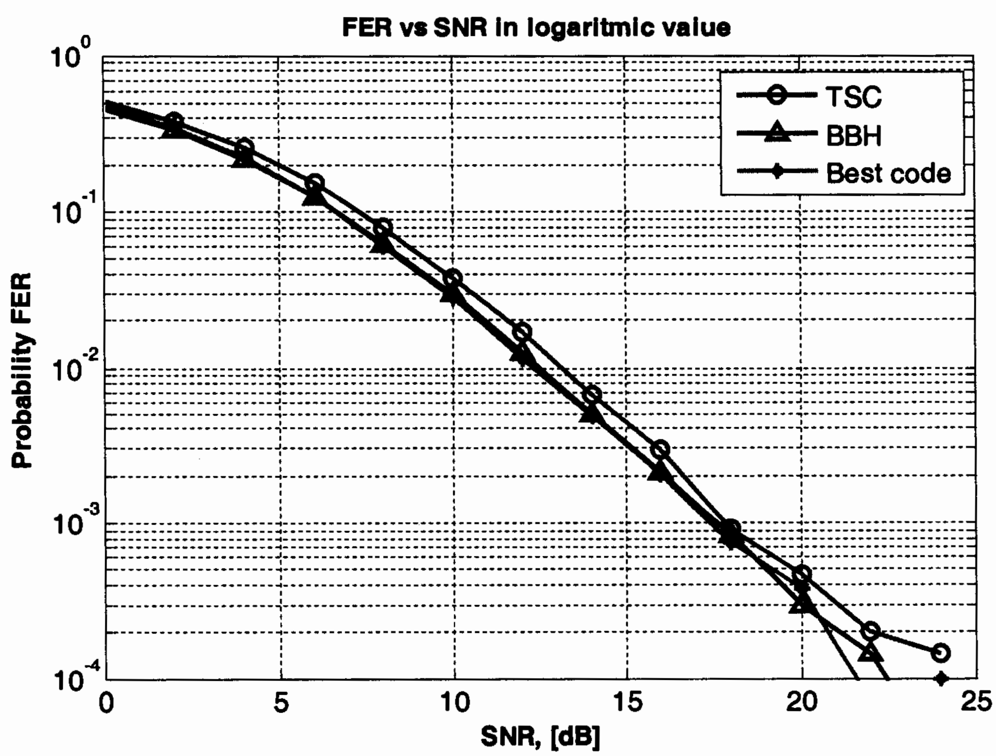

Figure 10: Comparison of FER probability Vs SNR in fast fading

4

Space Time Codes

This chapter discusses space time codes, their encoder structure and compares different codes in block fading environment.

4.1 Space Time Block Codes (STBC)

Historically Alamouti is considered as the first STBC that gives full transmit diversity for the system having two antennas at the transmitter side [22].

The most important character of Alamouti scheme is orthognality between the signal sequences provided by the two transmit antennas [23]. To this scheme theory of orthogonal designs is applied to some arbitrary number of transmit antennas. Now this generalized scheme is known as STBCs. So, if the number of transmit antennas n_T are specified then we can get full transmit diversity advantage by STBCs, but here much simpler maximum likelihood decoding algorithm is used which depends just on linear processing of the received signals .

Maximum possible diversity benefit can be achieved by STBC with simple decoding algorithm. So because of this key characteristics and simplicity much attention is given to STBCs but on the other hand STBCs provide no coding gain, however bandwidth expansion can be controlled by using non full rate STBCs [24].

4.1.1 Space Time Block Encoder

Generally $n_T \times p$ transmission matrix x represents space time block code. Where the number of time intervals of one block of coded symbol transmission is represented by p and number of transmit antennas is represented by n_T.

Now let us consider that total 2^m points represent the signal constellation. Into this constellation in order to choose k modulated signals $x_1, x_2, x_3, \cdots\cdots, x_k$ a block of km information bits are assigned at every operation of encoding, where constellation signal is selected by every group of m bits. According to the transmission matrix X in order to produce n_T parallel signal sequences of p length the Space time block encoder encode the k modulated signals. From n_T transmit antennas these signal sequences are then simultaneously in p time intervals transmitted. In the STBC, k numbers of symbols are taken by encoder as its input in every encoding operation. Multiple transmit antennas transmit the coded symbols in p transmission intervals, or we can say that for every block of k input symbols there are p space time symbols which are transmitted from every antenna. The ratio between the input symbols taken by the encoder and the coded symbols transmitted from every antenna is called the rate of STBC. It can be written as

$$R = k/p, \tag{4.1}$$

where STBC's spectral efficiency is given by

$$\eta = \frac{r_b}{B} = \frac{r_s m R}{r_s} = \frac{km}{p} \frac{\frac{bits}{s}}{hz}, \tag{4.2}$$

where r_s is the symbol rate and r_b is the bit rate, while B shows the bandwidth.

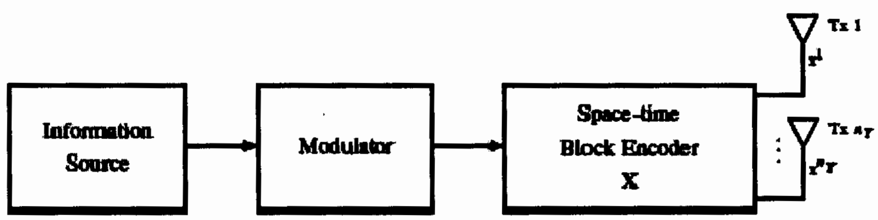

Figure 11: Encoder structure for STBC

x is the transmission matrix whose entries are linear combination of the total k modulated symbols $x_1, x_2, x_3, \dots\dots\dots\dots x_k$ and their conjugates. Here full transmit diversity of n_T is achievable by making transmission matrix x based on orthogonal designs such that [24]

$$x.x^H = c(|x_1|^2 + |x_2|^2 \dots\dots\dots\dots |x_k|^2) I_{nT} \tag{4.3}$$

where I_{nT} is an $n_T \times n_T$ identity matrix, c is constant here and x^H is the Hermitian of x. At time j the space time symbol transmitted is regarded as j^{th} column of x. At time j the transmit antenna simultaneously transmit the symbols which is represented by j^{th} column of x. While in total p transmission intervals i^{th} antenna at the transmitter side transmit the symbols which is represented by the ith row of x. The element of the transmission matrix x in the j^{th} column and i^{th} row, shows the signal transmitted at time j from the antenna i.

$$x_{i,j}, i = 1,2,3, \ldots \ldots.., n_T, j = 1,2,3, \ldots \ldots \ldots p,$$

With full transmit diversity STBCs has the rate of less than or equal to one, $R \leq 1$ [24]. However no bandwidth expansion is needed for the code with a full rate R = 1. On the other hand bandwidth expansion of 1/R is needed for the code with rate R<1. We denote the transmission matrix by x_{nT} for the STBC having n_T transmit antennas. Such code is known as STBC with size n_T.

In order to construct STBCs for this purpose orthogonal designs are used. Each rows of the transmission matrix x_{nT} are orthogonal to every other row. Thus we concluded that in every block, from any two transmit antennas the sequences of the signal are orthogonal to each other e.g. if the transmitted sequence is $x_i = (x_{i,1}, x_{i,2}, \ldots \ldots \ldots \ldots \ldots x_{i,p})$, transmitted from ith antenna $i = 1, 2, 3, \ldots \ldots n_T$ then we have

$$x_i . x_j = \sum_{t=1}^{p} x_{i,t} . x^*_{j,t} = 0; \quad i \neq j; \quad i,j \in \{1,2,3, \ldots \ldots n_T\} \qquad (4.4)$$

where $x_i . x_j$ represents the inner product of the sequence x_i and x_j. For a given number of transmit antennas in order to get full transmit diversity is possible because of orthognality. Moreover, by this receiver is allowed to decouple the signals which are transmitted from different antennas [21].

4.2 Alamouti Space Time Code

This is historically the first STBC [22]. As another scheme like delay diversity schemes can also provide full transmit diversity, but complex detectors is needed at the receiver end and interference is introduced between symbols [23]. Alamouti code is simple two branch transmit diversity scheme having the advantage that with a simple maximum-likelihood decoding algorithm it can achieve a full diversity gain.

Thus we can say that Alamouti space-time codes have two important characteristics.

- **Simple Decoding:**
  In Alamouti space-time codes every symbol is decoded uniquely using only linear processing technique.
- **Maximum Diversity:**
  The rank criterion is satisfied by the Alamouti space-time code and therefore it can achieve maximum possible diversity.

The encoding structure of Alamouti space-time code is given below.

4.2.1 Encoder structure

In the encoder structure of Alamouti STBC we observe that M-ary modulation technique has been utilized, where every block of m data bits are initially modulated, here $m = \log_2 M$. Then the group of two symbols after modulation that is s_1 and s_2 is taken by the encoder in every operation of encoding and assign them to the transmit antennas according to the code matrix, for example,

$$s = \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix} \tag{4.5}$$

By using two transmit antennas the encoder transmit the outputs in two consecutive time intervals. During the first time interval, signal s_1 and signal s_2 are transmitted from first and second antenna respectively. While during the second time interval, signals $-s_2^*$ are transmitted from first antenna while s_1^* are transmitted from second antenna, here s_1 has complex conjugate i.e; s_1^*.

Here we assume that in both domains the process of encoding is performed that is; in space domain and time domain.

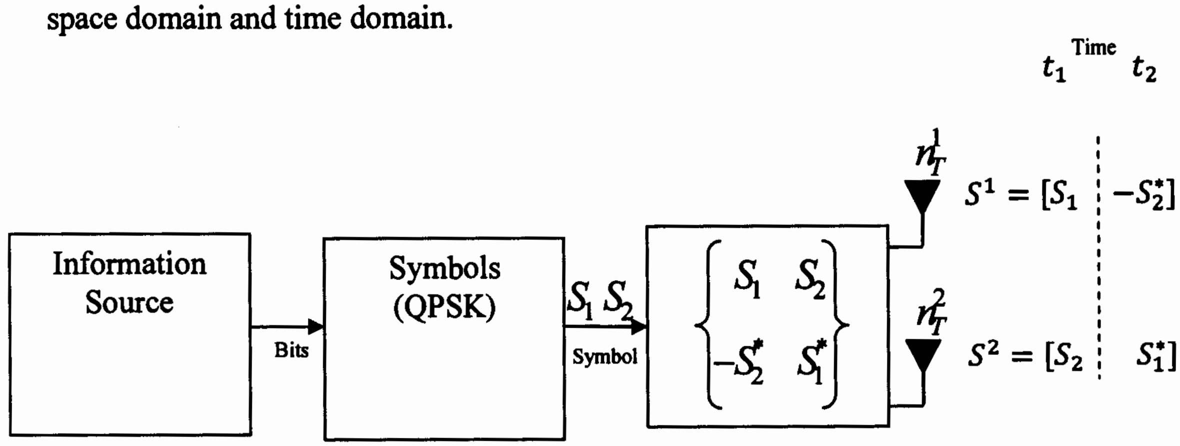

Figure 12: Block diagram of Alamouti Space-time encoder with two Transmit antennas $(n_T = 2)$

Now let us assume the transmitted sequence from first antenna by s_1 and transmitted sequence from second antenna by s_2;

$$s^1 = [s_1, \ -s_2{}^*]$$
$$s^2 = [s_2, \ s_1{}^*] \tag{4.6}$$

The key characteristic of Alamouti code is that we have orthogonal sequences at the end of transmission from different antennas. So, the product of two sequences s^1 and s^2 is zero that is

$$s^1.s^2 = s_1 s_2{}^* - s_2{}^* s_1 = 0 \tag{4.7}$$

Now let us assume that path gains are uniform across two consecutive symbols transmission intervals. Then the path gain from first and second transmit antennas to j^{th} received antenna at time t_1 and t_2 are constant, are $\alpha_{1,j}(t) = \alpha_{1,j}(\mathrm{t}_1) = \alpha_{1,j}(\mathrm{t}_2)$ and $\alpha_{2,j}(t) = \alpha_{2,j}(\mathrm{t}_1) = \alpha_{2,j}(\mathrm{t}_2)$ respectively. Then the received signals are,

$$r_{t_1}^j = \alpha_{1,j}(t)S_1 + \alpha_{2,j}(t)S_2 + z_{t_1}^j \tag{4.8}$$

$$r_{t_2}^j = -\alpha_{1,j}(t)S_2^* + \alpha_{2,j}(t)S_1^* + z_{t_2}^j$$

where $r_{t_1}^j$ are received signal at time t_1 and $r_{t_2}^j$ at time t_2 respectively and $z_{t_1}^j$ and $z_{t_2}^j$ are the j^{th} receiver noise at time t_1 and t_2 respectively, which are sophisticated, Gaussian variable with variance $N_o/2$ per dimension. In term of linear combination of the received signal the decision statistics are

$$\tilde{S}_1 = \sum_{j=1}^{n_R} \left[r_{t_1}^j \, \alpha_{1,j}^*(t) + \left(r_{t_2}^j\right)^* \alpha_{2,j}(t) \right] \tag{4.9}$$

$$\tilde{S}_2 = \sum_{j=1}^{n_R} \left[r_{t_1}^j \alpha_{2,j}^*(t) - \left(r_{t_2}^j\right)^* \alpha_{1,j}(t) \right]$$

where $\tilde{S}_1$ and $\tilde{S}_2$ are the decision statistics and for decoding the transmitted signals, the maximum likelihood decoder finds the closest symbol to $\tilde{S}_1$ in the constellation for decoding S_1 . Similarly, for decoding S_2 decoder finds the closest symbol to $\tilde{S}_2$ in the constellation. For independent signals S_1 and S_2 , maximum likelihood decoder rules are

$$\hat{S}_q = \arg_{(\hat{S}_1,\hat{S}_2)\in S} min \left[\left(\sum_{j=1}^{n_R} \left(|\alpha_{1,j}(t)|^2 + |\alpha_{2,j}(t)|^2 \right) - 1 \right) |\hat{S}_q|^2 + d^2(\tilde{S}_q, \hat{S}_q) \right] \quad (4.10)$$

where $q = 1, 2$ Thus $\hat{S}_q$ are given as,

$$\hat{S}_1 = \arg_{(\hat{S}_1,\hat{S}_2)\in S} min \left[\left(\sum_{j=1}^{n_R} \left(|\alpha_{1,j}(t)|^2 + |\alpha_{2,j}(t)|^2 \right) - 1 \right) |\hat{S}_1|^2 + d^2(\tilde{S}_1, \hat{S}_1) \right] \quad (4.11)$$

$$\hat{S}_2 = \arg_{(\hat{S}_1,\hat{S}_2)\in S} min \left[\left(\sum_{j=1}^{n_R} \left(|\alpha_{1,j}(t)|^2 + |\alpha_{2,j}(t)|^2 \right) - 1 \right) |\hat{S}_2|^2 + d^2(\tilde{S}_2, \hat{S}_2) \right]$$

where $(\hat{S}_1, \hat{S}_2) \in \boldsymbol{S}$ is the set of all possible modulated symbol pair. In case of single received antenna then equation (4.11) can be simplified as

$$\hat{S}_1 = \arg_{(\hat{S}_1,\hat{S}_2)\in S} min\left[d^2(\tilde{S}_1, \hat{S}_1)\right] \quad (4.12)$$

$$\hat{S}_2 = \arg_{(\hat{S}_1,\hat{S}_2)\in S} min\left[d^2(\tilde{S}_2, \hat{S}_2)\right]$$

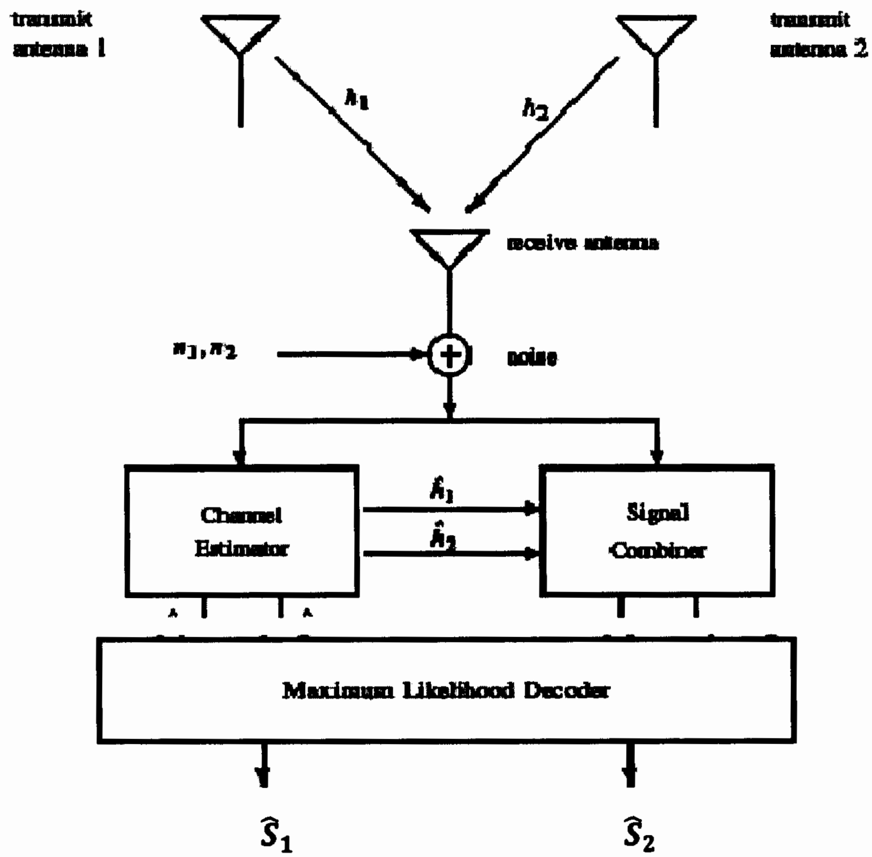

Figure 13: receiver of Alamouti scheme

4.2.2 Features of Alamouti Code

This codes has the following key features

- Simple to implement
- maximum possible diversity is achievable
- No coding gain and bandwidth expansion

4.4 Comparison of Space Time Trellis Codes in Block Fading

In order to increase the performance of STTCs, different generating sequences are considered. We can get the optimum results by using the generator sequences used in STTCs. In this section we compared the performance of different STTCs. We compared these performances results in block fading environment.

In these comparisons we have concluded that these codes show optimum/suboptimum behavior under different conditions. In this comparison, we exploit the generator sequences from Vucetic and Yuan (VY) [1], TSC (Tarokh/ Seshadri/ Calderbank) [2] and BBH (Baro/ Bauch/ Hansmann) codes [3]. The performance comparisons are made on the basis of symbol error rate (SER), frame error rate (FER) and bit error rate (BER) performance metrics, block fading environment are simulated using the above three codes with QPSK modulation scheme. We use a frame size of 12 bits and simulate for 70000 iterations. Hence we prefer a small sized frame of 12 bits as the relative comparison results are not affected by frame size. At the receiver end we use maximum-likelihood decoding algorithm with complete information about the channel.

From Figure (14) to Figure (16) the graphs are plotted for block fading environment where the performance comparison are between the BER Vs SNR, SER Vs SNR and FER Vs SNR are shown. Figure 14 clearly shows that TSC codes outperform the BBH codes and the best codes. In this case TSC codes give best result while best codes give worst result among the three. Thus we conclude that in case of BER Vs SNR best codes does not give optimum result while TSC code gives best result among the three. Figure 15 clearly shows that the best codes outperform the BBH codes and TSC codes. In this case best code give best performance among the three if we compare the performance of TSC codes and the BBH codes, we see that performance of BBH codes are better than that of TSC codes. So TSC codes give worst performance among the three. Thus we conclude that in case of SER Vs SNR best codes gives optimum result while TSC code gives worst result among the three. Figure 16 shows that all the three codes shows almost same behavior. However, with little margin BBH codes give better performance among the three. So in case of FER Vs SNR all the three codes have almost same behavior in block fading channel.

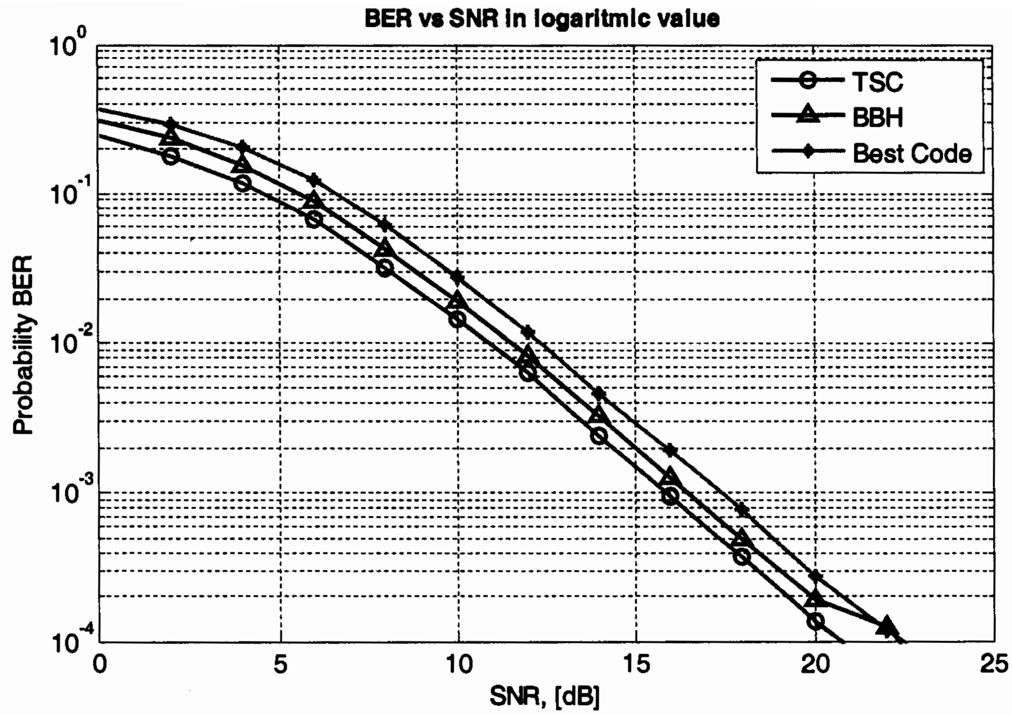

Figure 14: Comparison of BER probability Vs SNR in block fading channel

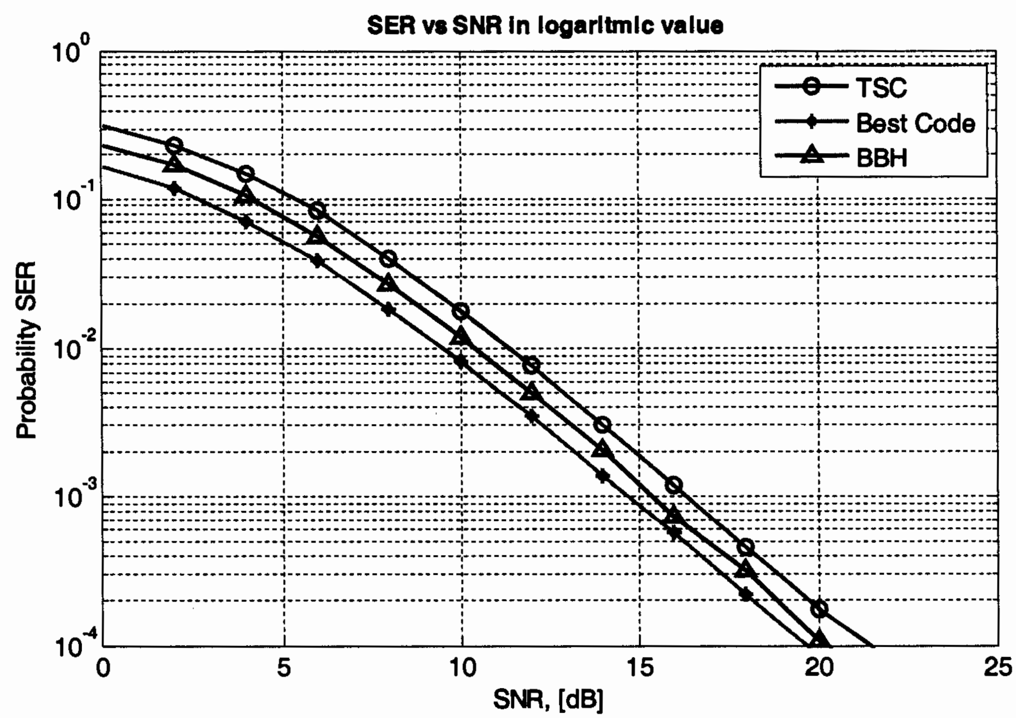

Figure 15: Comparison of SER probability Vs SNR in block fading channel

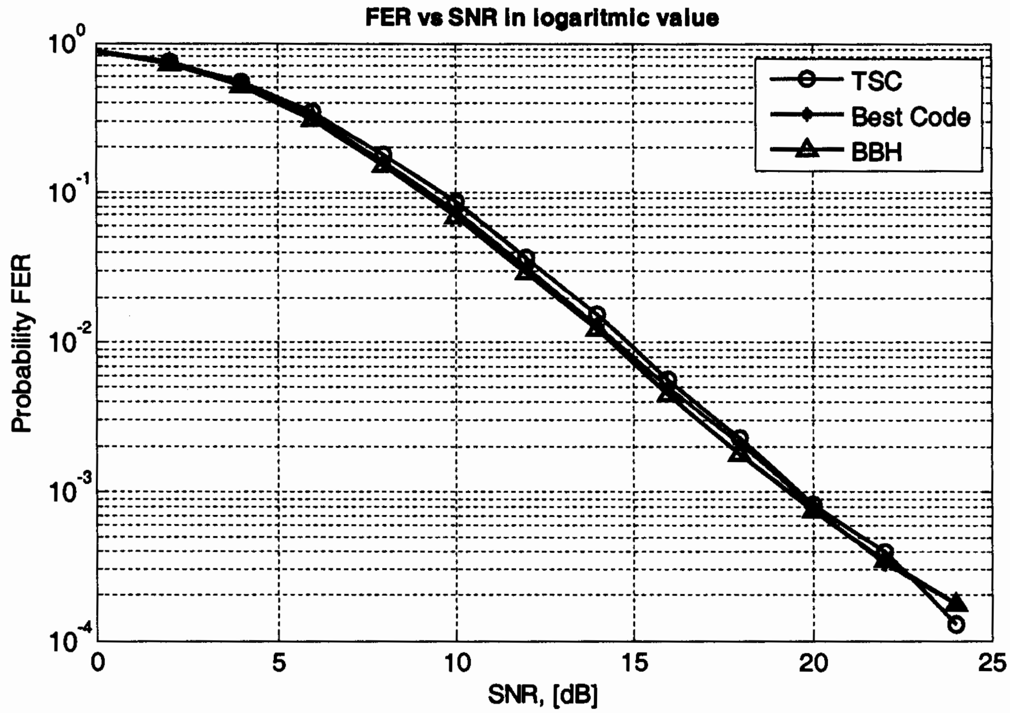

Figure 16: Comparison of FER probability Vs SNR in block fading channel

5

Conclusion

In this work we have analyzed the performance comparison of space-time trellis codes. We performed these analyses in two parts. In first part we analyze the performance of STTCs in fast fading environment, exploiting the generator sequences from Vucetic and Yuan (VY) [1], TSC (Tarokh/ Seshadri/ Calderbank) [2] and BBH (Baro/ Bauch/ Hansmann) codes [3]. In second part we analyze the performance of these STTCs in block fading environment.

The performance comparisons are made on the basis of SER (symbol error rate), FER (frame error rate) and BER (bit error rate) performance metrics. The performance comparison of pre-coding MIMO (multiple-input multiple-output) system is analyzed in block and fast fading environments using the above three codes with QPSK modulation scheme. In block fading and fast fading environment the performance comparison are between the BER Vs SNR, SER Vs SNR and FER Vs SNR. Simulation results show that these codes show optimum/suboptimum behavior under different fading conditions.

In fast fading environment results clearly show that in case of BER Vs SNR, BBH and best codes give better performance than TSC codes in QPSK modulation scheme. In this case BBH and best codes give almost same performance among the three. In this comparison TSC code gives the worst performance among the three codes in fast fading environment. Thus we conclude that in case of BER Vs SNR best codes and BBH codes give optimum result while TSC code gives worst result among the three. Results for SER clearly shows that the best code outperform the BBH codes and TSC codes. In this case best code give best performance among the three if we compare the performance of TSC codes and the BBH codes, we see that performance of BBH codes are better than that of TSC codes. So TSC codes give worst performance among the three. Thus we conclude that in case of SER Vs SNR best codes gives optimum result while TSC code gives worst result among the three. Final result in case of FER Vs SNR in fast fading environment shows that best codes and BBH codes outperform the TSC codes. In this case TSC codes give worst performance among the three. So in case of FER Vs SNR best and BBH codes give optimum results in fast fading environment.

A simulation result in case of BER Vs SNR clearly shows that in block fading environment TSC codes gives better performance than BBH codes and the best code in QPSK modulation scheme. In this case TSC code give best performance among the three but if we compare the performance of BBH codes and the optimum best codes, we see that performance of BBH codes are better than that of best codes. We conclude in this comparison that best code gives the worst performance among the three. Thus we conclude that in case of BER Vs SNR best codes does not give optimum result while TSC code gives best result among the three. Results in case of SER Vs SNR show that the best codes outperform the BBH codes and TSC codes. In this case best code give best performance among the three but if we compare the performance of TSC codes and the BBH codes, we see that performance of BBH codes are better than that of TSC codes. So TSC codes give worst performance among the three. Thus we conclude that in case of SER Vs SNR best codes gives optimum result while TSC code gives worst result among the three. Simulation result in case of FER Vs SNR also shows that all the three codes shows almost same behavior. However, with little margin BBH codes give better performance among the three. So in case of FER Vs SNR all the three codes have almost same behavior in block fading channel.

Hence, the claim of optimality made by Vucetic and Yuan (VY) for slow fading stands valid in different fading conditions, in QPSK modulation scheme. As these codes show optimum/suboptimum behavior under different fading conditions, so we have to do tradeoff among these space-time codes according to the situation for implementation.

Future Work

In this work we used generator sequences 'g' as fixed values and in future work we can make this generator sequences as adoptive according to the channels. We can check channel state by sending some training bits after some interval during the communication. When channel states changes we changes generator sequences according to that. Weights of the generator sequence become adaptive according to channel.

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```
p% Space Time Trellis Code (STTC)
% ======================================================

clc,close all
iteration = 1000;
Ntx=2 ;Nrx=2;
SNR_max = 25;
snr=0:2:SNR_max;
if Nrx == 1
    IM = 1; % identity matrix
elseif Nrx == 2
    IM = [1 0; 0 1]; % identity matrix
end
loop1=1;
for loop1=1:3
    if loop1==1
% --- Parameter STTC ---TSC(Tarokh/Seshadri/Calederbank) Approach
g1=[0 2;2 0]; % 1-st generator
g2=[0 1;1 0]; % 2-nd generator
    end % end of if
    if loop1==2
% --- Parameter STTC ---BBH(Baro/Bauch/Hansmann) Approach
g1=[2 2;1 0]; % 1-st generator
g2=[0 2;3 1]; % 2-nd generator
    end% end of if
    if loop1==3
% --- Parameter STTC --- Optimum Approach by kab
    transmitted_signal=H*input_kanal;
g1=[0 2;1 0]; % 1-st generator
g2=[2 2;0 1]; % 2-nd generator
    end% end of if
M = 4; % M-PSK = QPSK
% ----------------------

% ****************************** Transmitter ******************************
% ==== Data ====
data=randint(1,48,2);  % Taken Random

% ==== Encoder STTC ====
[s,data_mod,v] = encsttc(g1,g2,data,M);
input_kanal = data_mod;

for jum=1:iteration
    jum;
   H=[randn(Nrx,Ntx) + i*randn(Nrx,Ntx)];%Taken Random for fast fading
    singular(:,jum)=svd(H);

    for oye = 1:length(snr)

        % ==== Noise Addition ====
        received_signal=awgn(transmitted_signal,snr(oye),'measured');   %
Taken Random

        % ==== STTC Detection ====
        map = symbolref(g1,g2,M);
```

```
        allsum = ML(received_signal,map,H,Nrx);
        [mindist,ind,survpath,dist,srx,data_topi] = viterbi
(allsum,v,M,g1,g2);
        % ==== Performance Evaluation ====
        bit_error(1,oye) = sum(data~=data_topi); % BER
        simbol_error(1,oye) = sum(sum(s~=srx)); % SER
        pair_err1 = sum(s~=srx);
        for q = 1:length(pair_err1)
            if pair_err1(q) ~= 0
                pair_err2(q) = 1;
            else
                pair_err2(q) = 0;
            end
        end
        Eb_No=snr(oye)-3; EbNo=10^(Eb_No/10);
        pair_err3(1,oye) = sum(pair_err2); % PEP
        cap(1,oye)=log2(det(IM + (EbNo/Ntx)*H*H')); % MIMO channel capacity
   end
    for yipi=1:length(bit_error)
        if bit_error(yipi)==0
            fer(yipi)=0;
        else
            fer(yipi)=1;
        end
    end % end of for
    total_fer(jum,:)=fer;
    total_bit_error(jum,:) = bit_error;
    total_simbol_error(jum,:) = simbol_error;
    total_pep(jum,:) = pair_err3;
    total_cap(jum,:) = cap;
 clc
loops_left=3-loop1
itrations_left=iteration-jum
end

BER = sum(total_bit_error)/(length(data)*iteration);
SER = sum(total_simbol_error)/((size(s,1))*(size(s,2))*iteration);
FER = sum(total_fer)/iteration;


if loop1==1
% --- Parameter STTC ---TSC(Tarokh/Seshadri/Calederbank) Approach
ER1=BER;
ER11=SER;
ER21=FER;
    end
    if loop1==2
% --- Parameter STTC ---BBH(Baro/Bauch/Hansmann) Approach
ER2=BER;
ER12=SER;
ER22=FER;
    end
    if loop1==3
% --- Parameter STTC --- Optimum Approach
```

```
ER3=BER;
ER13=SER;
ER23=FER;
    end
    loop1=loop1+1;
end


figure
semilogy(snr,ER1,'r-o');hold on
semilogy(snr,ER2,'b-^');hold on
semilogy(snr,ER3,'g-*');hold on
legend('TSC','BBH','Best code')
title('BER vs SNR in logaritmic value')
xlabel('SNR, [dB]');
ylabel('Probability BER');
axis([0,25,.0001,1]);
grid

figure
plot(snr,ER1,'r-o');hold on
plot(snr,ER2,'b-^');hold on
plot(snr,ER3,'g-*');hold on
legend('TSC','BBH','Best code')
title('BER vs SNR in linear value')
xlabel('SNR, [dB]');
ylabel('Probability BER');
grid

figure
semilogy(snr,ER11,'r-o');hold on
semilogy(snr,ER12,'b-^');hold on
semilogy(snr,ER13,'g-*');hold on
legend('TSC','BBH','Best code')
title('SER vs SNR in logaritmic value')
xlabel('SNR, [dB]');
ylabel('Probability SER');
axis([0,25,.0001,1]);
grid

figure
plot(snr,ER11,'r-o');hold on
plot(snr,ER12,'b-^');hold on
plot(snr,ER13,'g-*');hold on
legend('TSC','BBH','Best code')
title('SER vs SNR in linear value')
xlabel('SNR, [dB]');
ylabel('Probability SER');
grid

figure
semilogy(snr,ER21,'r-o');hold on
semilogy(snr,ER22,'b-^');hold on
semilogy(snr,ER23,'g-*');hold on
legend('TSC','BBH','Best code')
title('FER vs SNR in logaritmic value')
```

```
xlabel('SNR, [dB]');
ylabel('Probability FER');
axis([0,25,.0001,1]);
grid

figure
plot(snr,ER21,'r-o');hold on
plot(snr,ER22,'b-^');hold on
plot(snr,ER23,'g-*');hold on
legend('TSC','BBH','Best code')
title('FER vs SNR in linear value')
xlabel('SNR, [dB]');
ylabel('Probability FER');
grid
function [y] = bit2num(x)

y = 0; mul = 1;
for i=(length(x):-1:1)
    y = y + mul*x(i);
    mul = mul*2;
end
function [s,data_mod,v] = encsttc(g1,g2,input,M)
[a b]= size(input);
v1 = length(g1)-1;
v2 = length(g2)-1;
v = v1+v2; % memory degree
m =log2 (M);
% serial to paralel
n_kol = b/m;
for index=0:n_kol-1
    c(:,index+1) = input(1,(m * index)+1:m*(index+1));
end
temp = zeros (m,v);
s = mod(shat,M);
[n m]=size(s);
data_mod=[];

% Mapper QPSK from symbol STTC
for a=1:n
    for b=1:m
        if s(a,b)==[0]
            data_mod(a,b) = 1+i;
        elseif s(a,b)==[1]
            data_mod(a,b) = 1-i;
        elseif s(a,b)==[2]
            data_mod(a,b) = -1-i;
        elseif s(a,b)==[3]
            data_mod(a,b) = -1+i;
        end;
    end;
end;
function [trellis1,trellis2,statego]=gen2trellis(g1,g2,M)

% function to change generator code to trellis
% trellis1 = trellis antenna 1
% trellis2 = trellis antenna 2
```

```
% statego  = state moving

v1 = length(g1)-1;
v2 = length(g2)-1;
v = v1+v2;
jumlahstate = 2^v;
m =log2 (M);
state = 0:jumlahstate-1;
for a=1:jumlahstate
    for b=1:4
        input = [getbits(state (1,a),jumlahstate/2),getbits(state(1,b),2)];

        % serial to paralel
        n_kol = length (input)/2;
        for index=0:n_kol-1
            c(:,index+1) = input(1,(m * index)+1:m*(index+1));
        end
        temp = zeros (m,v);



            temp;
        end
        s = mod(shat,M);
        trellis1(a,b) = s(1,n_kol);
        trellis2(a,b) = s(2,n_kol);
        statego(a,b) = bit2num(input(1,(length(input)-v2):length(input)));
    end
end

function [bits] = getbits(x, n)
% dec to bin

bits = zeros(1, n);
ind  = 1;
while (x~=0)
    bits(ind) = mod(x,2);
    x = floor(x/2);
    ind = ind + 1;
end
bits = fliplr(bits);
function dist = ML(symrec,symref,kanal,numrx);

% maximum likelyhood function
%     dist = ML
%     symrec = received symbol
%     symref = reference symbol
%     kanal = channel response


for a=1:length(symrec)
    for p=1:size(symref,1)
        for b=0:numrx-1
            disthat(1,b+1)=norm((kanal(b+1,:)*symref(p,:).' -
symrec(b+1,a)),'fro');
        end
```

```
        dist(p,a) = sum(disthat)/numrx;
    end
end

function symbol = symbolref(g1,g2,M)

[trellis1,trellis2,statego]=gen2trellis(g1,g2,M);
[a b] = size (trellis1);
[n m]=size(ref);
ref;
symbol = [];
for a=1:n
    for b=1:m
        if ref(a,b)== 0
            symbol(a,b) = 1+i;
        elseif ref(a,b)== 1
            symbol(a,b) = 1-i;
        elseif ref(a,b)== 2
            symbol(a,b) = -1-i;
        elseif ref(a,b)== 3
            symbol(a,b) = -1+i;
        end;
    end;
end;

function [mindist,ind,survpath,dist,srx,data] = viterbi (distance,v,M,g1,g2)
% It survivor path in trellis
% distance = Tx symbol to Rx symbol distance
% v = memory on generator in over case 2
% M = M-psk
% mindist = minimum distance
% ind = matrix index of survivor path
% survpath = state on survivor path
% disthat = inter node distance
% dist = minimum distance at each node each state

numstate = 2^v;
dist = [];
[brs kol]=size (distance);
[trellis1,trellis2,statego]=gen2trellis(g1,g2,M);
for a = 1:kol
    if a == 1
        dist(:,a) = distance(1:numstate,a);
    else
        disthat = [];
        for b=0:M-1
            for bb=1:numstate
                if bb < 5
                    disthat(bb,b+1) = dist(b*(v-1)+1,a-1)+
distance(b*numstate+bb,a);
                else
                    disthat(bb,b+1) = dist(b*(v-1)+2,a-1)+
distance(b*numstate+bb,a);
                end
            end
        end
```

```
        disthat;
        for b=1:numstate
            dist(b,a)=min (disthat(b,:));
        end
    end
end
state= 0;
for a = 1:kol
    p = statego(state+1,1)+1;
    q = statego(state+1,4)+1;
    [mindist(:,a) ind(:,a)] = min (dist(p:q,a));
    if p ~= 1
        ind(:,a)=ind(:,a)+4;
    end
    state = ind(:,a)-1;

end
survpath = ind -1;
demap = [0 0;0 1;1 0;1 1]';
for a = 1:kol
    if a == 1
        srx(1,a) = trellis1(1,ind(1,a));
        srx(2,a) = trellis2(1,ind(1,a));
        data(1,2*(a-1)+1:2*(a-1)+2)= demap(:,ind(1,a))';

    end
end
```