1- Communication engineering.
2- Wireless communication systems.

Title Page?

بسم الله الرحمن الرحيم

In the name of Allah (SWT) the most benificient and the most merciful.

Dedicated...

To a Person who is
"The Rehmat" for all the Universe,
and
To Our Parents,
and
To whom we love and respect.

Robust RLS Based Detector for MC-CDMA

Dated: 25-02-10

It is certified that we have read the thesis submitted by **Mr. Muhammad Adnan Khan Reg. No. 78-FET/MSEE/F07**. It is our judgment that this thesis is of sufficient standard to warrant its acceptance by the International Islamic University, Islamabad for Masters Degree in Electronic Engineering (MSEE).

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Declaration

I hereby declare that this thesis, neither as a whole nor as a part thereof has been copied out from any source. It is further declared that I have developed this thesis entirely on the basis of my personal effort made under the sincere guidance of my supervisor (Dr.Aamer Saleem). No portion of the work presented in this thesis has been submitted in support of any application for any other degree or qualification of this or any other university or institute of learning.

Muhammad Adnan Khan

78-FET/MSEE/F07

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Muhammad Adnan Khan

Abstract

Multi-Carrier Code Division Multiple Access (MC-CDMA) is one of the effective multicarrier transmission schemes for supporting the multiple access communication. MC-CDMA is a combination of two techniques: Code Division Multiple Access (CDMA) and Orthogonal Frequency Division Multiplexing (OFDM). It's compensation in frequency diversity and multipath fading resilience paid it good consideration. Like the CDMA system, MC-CDMA is a interference restricted system. Multiple Access Interference (MAI) is the problem of the MC-CDMA system same as CDMA system. MAI occur due to the sharing of same channel among multiple users. The Blind CMOE (Constraint Minimum Output Energy) detector is effective to mitigate MAI in the absence of the Decision Directed Mode. But, in the presence of the channel modeling errors, the performance of Blind CMOE detector degrades considerably. In this case, Robust RLS based CMOE detector with quadratic constraint is good one to come over. In this thesis, Robust RLS based detector is implemented to analyze the performance of MC-CDMA system. Further, on the basis of the simulations the performance of the Robust RLS based Detector is compared with the Blind CMOE detector for the MC-CDMA.

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Introduction

Chapter 1

Introduction

1.1. Introduction

One of the major deficiencies of the Multi-Carrier transmission is Multiple Access Interference (MAI). MAI occur from the sharing of same channel between multiple users concurrently. Multi-Carrier Code Division Multiple Access (MC-CDMA) is a Multi-Carrier (MC) transmission system. In MC-CDMA, same bandwidth is shared by the multiple users concurrently and the data division is done by applying the separate user particular codes. In MC-CDMA, multicarrier modulation is also applied for reduction in symbol rate. Its effect reduces Inter Symbol Interference (ISI) for each sub channel by adding cyclic prefix. The difficulty is to reduce the MAI. Least Minimum Mean Square Error (LMMSE) detectors are effective to reduce the MAI.

LMMSE detector known as Blind Constrained Minimum Output Energy (CMOE) detector is effective for the reduction of MAI in the MC-CDMA system [1]. In favorable condition, the signal to interference plus noise ratio (SINR) is capitalized for the CMOE detector. Recursive Least Squares (RLS) updating of the correlation matrix is the basis of the Blind CMOE detection. The efficiency of the blind CMOE detector decreases significantly in the presence of signal modeling errors such as errors in channel and data correlation matrix estimation. This was found in adaptive beam forming [3]-[5] and Direct Sequence (DS)-CDMA multiple user detection [6].

To avoid this reduction in MC-CDMA performance, in [1] a RLS based Robust CMOE detector also called Robust RLS based detector with quadratic constraint is purposed. This Robust RLS based technique improves the robustness of detector especially for channel modeling errors. In this method, the minimum eigenvector related to channel information is calculated. For decreasing the complexity in calculating the minimum eigenvector relevant to channel information, the inverse iteration method is required to recursive approximation of minimum eigenvector [7].

1.2. Objective

In this thesis Robust RLS Based detector is studied and implemented in Matlab. The efficiency of Robust RLS based detector is further compared with old Blind CMOE detector in various conditions. The performance of RLS based detector and Blind CMOE detector was observed with respect to average SINR, number of data samples, SNR, average MSE and BER. This helps in making a decision to utilize RLS based detector or Blind CMOE detector according to requirement. CMOE detector has aptitude to provide good efficiency by RLS updating of the correlation matrix and the channel is estimated by finding the least eigenvector of a data associate matrix. But, after the incident of signal modeling errors, the efficiency of CMOE detector degrades significantly. In the absence of signal modeling errors, the CMOE detector is the effective one. In company of signal modeling errors like in approximation of channel and data correlation matrix. the Robust RLS based detector do better than Blind CMOE detector. The imitations and coding work was done in MATLAB 7.3 and detectors were coded in form of functions.

1.3. Span of Thesis

The understanding of the MC-CDMA system and Blind CMOE detector is very important before Robust RLS based detector. So, to have the understanding of Robust

RLS based detector, the MC-CDMA system and Blind CMOE detector is also discussed. As Robust RLS based detector needs the understanding of Blind CMOE detector. Therefore, the study of the MC-CDMA system is given in the start followed by the Blind CMOE detector. After the detailed study of the Blind CMOE detector, the Robust RLS based detector is studied. Later on, the mathematical model for MC-CDMA system, Blind CMOE detector and Robust RLS based detector has been presented. Finally, MC-CDMA system and Robust RLS based detectors have been studied and mathematical equations are derived.

1.4. Thesis Organization

The thesis is organized as follows. Chapter 1 is consists of the introduction, objectives and span of work. Chapter 2, which is essentially the literature review, it covers the relevant literature which was studied during thesis. In Chapter 3, problem statement has been presented and system design of MC-CDMA system and Robust RLS based detector has been discussed. Chapter 4 presents general algorithm and simulation results for Robust RLS based detector which are compared with the Blind CMOE detector. The comparison is followed by the conclusion. In the end, Bibliography section is added.

Chapter 2
Introductory Literature

Chapter 2

Introductory Literature

In this thesis, the RLS Based detector has been applied on the MC-CDMA system. Therefore, a good knowledge of MC-CDMA system is required before the study of Robust RLS based Detector for the MC-CDMA can be initiated.

2.1 Multi Carrier Code Division Multiple Access System

The Code Division Multiple Access (CDMA) permits concurrent usage of identical bandwidth between multiple users by unique user signature assigned to each user that is orthogonal to supplementary user signatures. The Orthogonal Frequency Division Multiplexing (OFDM) scheme is powerful against the multipath effect and guarantees high-quality spectral efficiency. The CDMA and OFDM communication schemes jointly forms the Multi-Carrier Code Division Multiple Access system. The CDMA and OFDM systems are discussed first for clear understanding of MC-CDMA system [9].

2.1.1. Code Division Multiple Access System

In case of CDMA, more than one user is permitted to divide a channel or sub channel by the method of Direct-Sequence Spread Spectrum Signals (DS-SSS) in CDMA system [8]. Code sequence or Signature sequence which are distinct are allocated to each user. Every user modulates the data signal by using these code sequences along the allocated frequency band. The cross correlation with every user code sequence is used for multiple users signals separation. There is small cross correlation among the above mentioned code sequences. As crosstalk rise at this point due to multiple users received signals which should be mitigated [9].

In CDMA system, the users use the channel randomly. This consequences more than one user in total overlie of signals broadcast of numerous users together in frequency domain as well as time domain. The separation and demodulation of signals at receiver are done by code sequence which is used for each user signal modulation.

CDMA Signal and Channel Model

Assume K users are concurrently sharing CDMA system. Each user has a distinct code waveform $g_k(t)$ of period T, where T is the duration of symbol. Hence, code waveform is

$$g_k(t)=\sum_{n=o}^{L-1}a_k(n)p(t-nT_c), \quad 0\le t\le T \tag{2.1}$$

The code sequence $\{a_k(n), 0\le n\le L-1\}$ is consist of total L chips with significances ± 1. These code sequences are Pseudo Noise (PN), $p(t)$ is pulse with interval T_c. The interval of chip is T_c. This shows that each symbol has L chips and

$$T = LT_c$$

with assumption that K code waveforms have energy nearly equivalent to unity, i.e.

$$\int_0^T g_k^2(t)dt = 1 \tag{2.2}$$

The $\rho_{ij}(0)$ is the cross correlation which is necessary for synchronous transmission given by

$$\rho_{ij}(0) = \int_0^T g_i(t)g_j(t)dt \tag{2.3}$$

For easiness, the information is transmitted by using the binary antipodal signal. The k^{th} user information sequence is presented as $\{b_k(m)\}$. Each bit has values ± 1. The bits block are with duration N. So, the k^{th} user data block is given by

$$\mathbf{b}_k = [b_k(1).....b_k(N)]' \tag{2.4}$$

The low pass equivalent waveform relevant to it is

$$s_k(t) = \sqrt{\varepsilon_k} \sum_{i=1}^{N} b_k(i) g_k(t - iT - \tau_k) \quad (2.5)$$

Where, signal bit energy is ε_k. The aggregate signal of K users is represented as

$$s(t) = \sum_{k=1}^{K} \sqrt{\varepsilon_k} \sum_{i=1}^{N} b_k(i) g_k(t - iT - \tau_k) \quad (2.6)$$

where τ_k are the transmission holdups with values $0 \le \tau_k < T$. In synchronous transmission $\tau_k = 0$ for $1 \le k \le K$.

The dispatched signal is supposed to be abused by AWGN. The received signal after AWGN corruption is

$$r(t) = s(t) + n(t) \quad (2.7)$$

where $s(t)$ is same as above and $n(t)$ is the noise having the power spectral density $\frac{1}{2} N_0$.

Synchronous CDMA Receiver

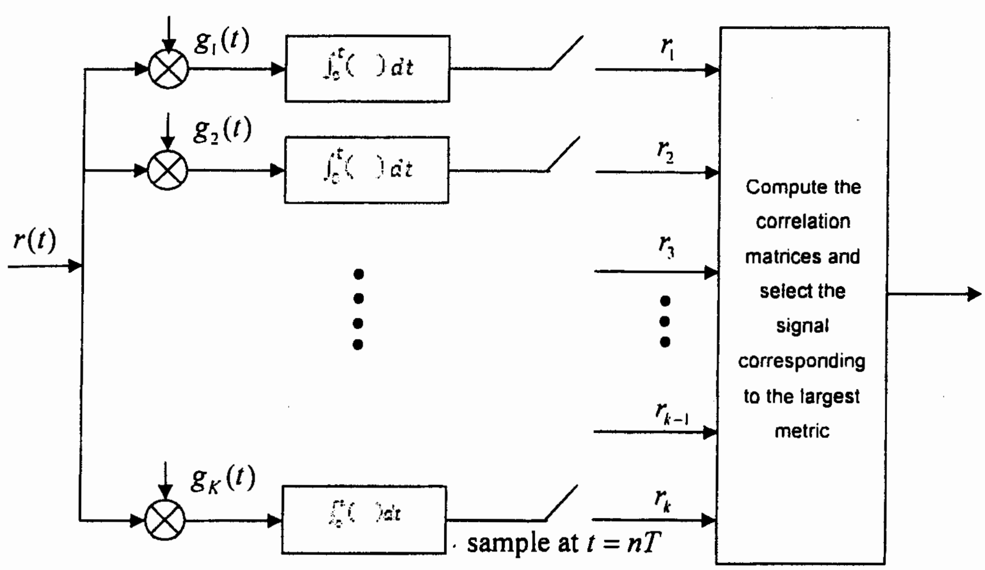

Figure.2.1 Receiver of synchronous CDMA broadcast system

where the received signal $r(t)$, the code waveform of every user is $g_k(t)$ and the correlation matrix is

$$r(t) = \sum_{k=1}^{K} \sqrt{\varepsilon_k} b_k(t) + n(t), \qquad 0 \le t \le T$$

The probabilistic log detection function is merged by maximum-likelihood receiver. This is presented in form of correlation matrix

$$c(\mathbf{r}_k ; \mathbf{b}_k) = 2\mathbf{b}_k^{'} \mathbf{r}_k - \mathbf{b}_k^{'} \mathbf{R}_s \mathbf{b}_k$$

where the correlation matrix R_s has components $\rho_{jk}(0)$ and

$$\mathbf{r}_k = [r_1 \ r_2 \ \ldots . \ r_k]^{'}$$

$$b_k^{'} = [\sqrt{\varepsilon_1} b_1(1) \ldots . \sqrt{\varepsilon_k} b_k(1)]^{'}$$

There possible available bits in information sequence are 2^k. The computation of correlation matrix is done for every sequence and the sequence with largest correlation matrix is chosen. The complexity of CDMA system rises exponentially with the increase in users.

2.1.2 Orthogonal Frequency Division Multiplexing System

A unique frequency slot is allocated to every user in OFDM. Each user is allowed to modulate the information or data signal across the assigned frequency band only. These frequency slots are distinct to each other.

The stream of serial data is split to a parallel number of channels after moving through the Serial to Parallel (S/P) converter. The N channels of frequencies $f_0, f_1, \ldots\ldots, f_{n-1}$ has N modulators because every channel data is applied in such a way. The separation between the channels is Δf. The whole adapted bandwidth W of N channels is $N\Delta f$. These N adapted carriers are compositely giving the OFDM signal [9].

OFDM Signal and Channel Model

Suppose a communication system that modulates on the basis of multi-carrier. This system transmits N_c source symbols $S_n, n = 0, \ldots, N_c - 1$ which are complex valued [9]. These source symbols are parallel after transmission on N_c sub-carriers which can be acquired by source and channel coding, interleaving and symbol mapping. The source symbol interval is T_d. After S/P transfer the OFDM symbol interval is

$$T_s = N_c T_d \quad (2.8)$$

The subcarriers have the separation of $F_s = \frac{1}{T_s}$ for attaining the orthogonality between the signals on these subcarriers by using the rectangular pulse shape. These adjacently transmitted symbols are OFDM symbol. The complex envelop by rectangular pulse shaping is

$$x(t) = \frac{1}{N_c} \sum_{n=0}^{N_c-1} S_n e^{j2\pi f_n t}, \quad 0 \le t \le T_s \quad (2.9)$$

The N_c sub-carriers frequencies are placed on

$$f_n = \frac{n}{T_s}, \quad n = 0, 1, \ldots\ldots, N_c - 1$$

The Inverse Fast Fourier Transform (IFFT) or Inverse Discrete Fourier Transform (IDFT) could be used for applying the multi-carrier transmission in OFDM system. The complex envelop $x(t)$ sampling of single OFDM symbol at rate of $\frac{1}{T_d}$. The samples are presented as

$$x_v = \frac{1}{N_c} \sum_{n=0}^{N_c-1} S_n e^{j2\pi n v / N_c}, \quad v = -L_g, \ldots\ldots N_c - 1 \quad (2.10)$$

where x_v is sampled sequence resulted from the IDFT or IFFT of the S_n.

The increase in subcarriers results that the interval OFDM symbol T_s grows larger as compared to period of impulse response τ_{max} of channel which reduces Inter Symbol Interference (ISI). For complete prevention of the ISI, the guard interval of period $T_g \ge \tau_{max}$ is placed among OFDM symbols close to each other. The recurring addition of every OFDM symbol for guard interval is gotten by expanding the OFDM symbol interval to

$$T_s' = T_g + T_s \qquad (2.11)$$

The modeled OFDM succession including guard interval is

$$x_v = \frac{1}{N_c}\sum_{n=0}^{N_c-1} S_n e^{j2\pi n v/N_c}, \quad v = -L_g, ...N_c - 1 \qquad (2.12)$$

This succession need to passes along a converter which is Digital to Analog (D/A). The final waveform $x(t)$ is with augmented interval T_s'. After that RF conversion. transmission of signal to the channel is done.

The RF back conversion gives received signal waveform y(t). The convolution of $x(t)$ with channel impulse response $h(\tau,t)$ including the noise gives the $y(t)$.

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau,t)d\tau + n(t) \qquad (2.13)$$

The frequency domain received symbol R_n is also represented

as $R_n = H_n S_n + N_n, \quad n = 0,......, N_c - 1 \qquad (2.14)$

Basic Multi-Carrier OFDM transmission system

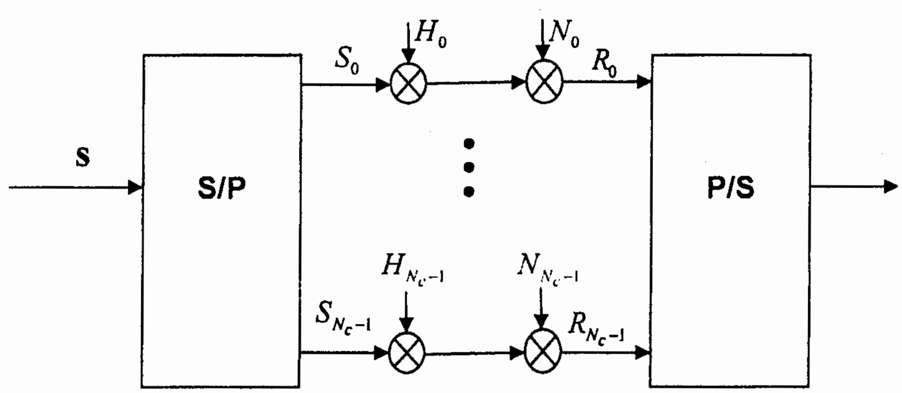

Figure.2.2 Basic Multi-Carrier OFDM transmission system

The source symbol is $\mathbf{s}$, $N_c \times N_c$ channel matrix is $\mathbf{H}$ and the additive noise is $\mathbf{N}$.

The vector of received symbols $\mathbf{r}$ resulted following the inverse OFDM is

$$\mathbf{r} = (R_0, R_1, \ldots R_{N_c-1})^T \tag{2.15}$$

is gotten via

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{2.16}$$

where

$$\mathbf{H} = \begin{bmatrix} H_{0,0} & 0 & . & . & . & 0 \\ 0 & H_{1,1} & & & & 0 \\ . & & & . & & . \\ . & & & & . & . \\ 0 & 0 & . & . & . & H_{N_c-1,N_c-1} \end{bmatrix}$$

and

$$\mathbf{s} = \left(S_0, S_1, \ldots, S_{N_c-1}\right)^T$$

and

$$\mathbf{n} = \left(N_0, N_1, \ldots, N_{N_c-1}\right)^T$$

2.1.3 Detailed Multiple Carrier Code Division Multiple Access System

Since the focus is on MC-CDMA system, it should be discussed in detail. The signal structure of MC-CDMA is described first [9].

Signal Structure

According to the previous discussion in chapter, MC-CDMA combines the CDMA with OFDM. Each direct sequence chip modulating the data symbol is drawn on a distinct subcarrier. Therefore, the spread data symbol chips are sent equivalently on distinct subcarriers of MC-CDMA system. The concurrent active users are K in MC-CDMA transmission scheme.

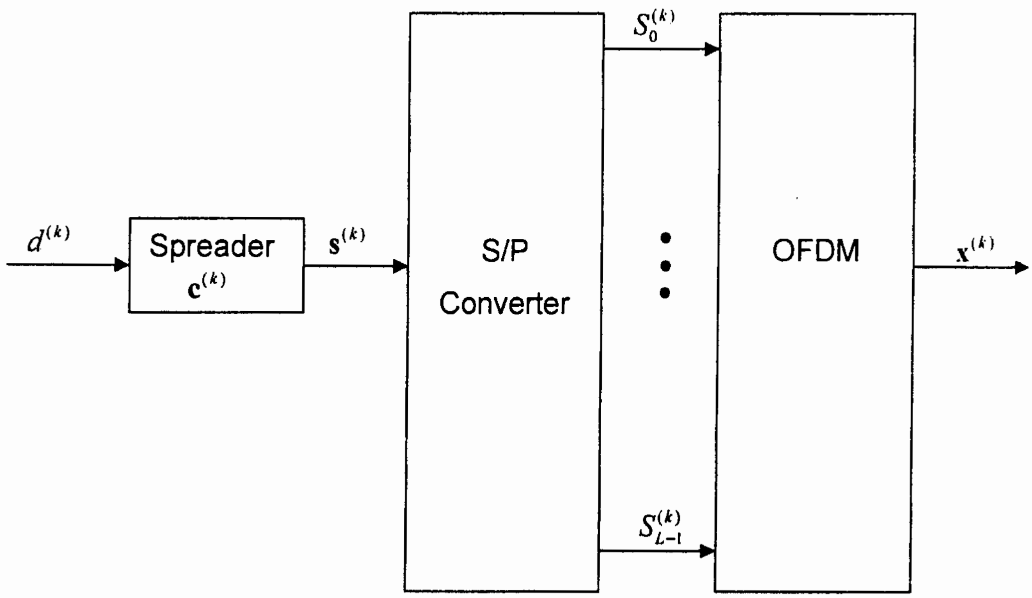

Figure.2.3 Signal generation through multi-carrier spread spectrum

The figure 2.3 illustrates one complex valued data symbol $d^{(k)}$ is allocated k user spread by multi-carrier spectrum. The symbol rate of serial data is $1/T_d$.

For simplification, the generation of MC-CDMA signal is focused pro one data symbol for each user. The complex valued data symbol $d^{(k)}$ is multiplied with user precise spreading code

$$\mathbf{c}^{(k)} = (c_0^k, c_1^k, \ldots, c_{L-1}^{(k)})^T \tag{2.17}$$

The length of spreading code is $L = P_G$. Before S/P alteration, the serial spreading code $\mathbf{c}^{(k)}$ chip rate is given by

$$\frac{1}{T_c} = \frac{L}{T_d} \tag{2.18}$$

This chip rate is L times larger than rate of data symbol $1/T_d$. The vector form of complex valued sequence spreading is

$$\mathbf{s}^{(k)} = d^{(k)}\mathbf{c}^{(k)} = (S_0^{(k)}, S_1^{(k)}, ..., S_{L-1}^{(k)})^T \quad (2.19)$$

The components $s_l^{(k)}$ modulation on subcarriers adjacently results in multicarrier spread spectrum signal. If subcarriers N_c of an OFDM symbol is equivalent to length L of spreading code, the OFDM symbol interval with multicarrier spread spectrum including a guard interval as well is

$$T_s' = T_g + LT_c \quad (2.20)$$

T_g is guard interval, length of the spreading code L and the spreading chip is T_c. One data symbol per user is sent on each OFDM symbol in above discussion.

Downlink Signal

The OFDM operation in synchronous transmission is performed after adding the spread signals of total K users. The K sequences $s^{(k)}$ assignment result in sequence given by

$$\begin{aligned}\mathbf{s} &= \sum_{k=0}^{K-1} \mathbf{s}^{(k)} \\ &= (S_0, S_1, ..., S_{L-1})^T\end{aligned} \quad (2.21)$$

Another downlink presentation for **s** is

$$\mathbf{s} = \mathbf{C}\mathbf{d} \quad (2.22)$$

where

$$\mathbf{d} = (d^{(0)}, d^{(1)}, ..., d^{(k-1)})^T$$

There are total K users transmitted symbols in vector **d**. The spreading matrix **C** is

$$\mathbf{C} = (\mathrm{c}^{(0)}, \mathrm{c}^{(1)}, ..., \mathrm{c}^{(k-1)})$$

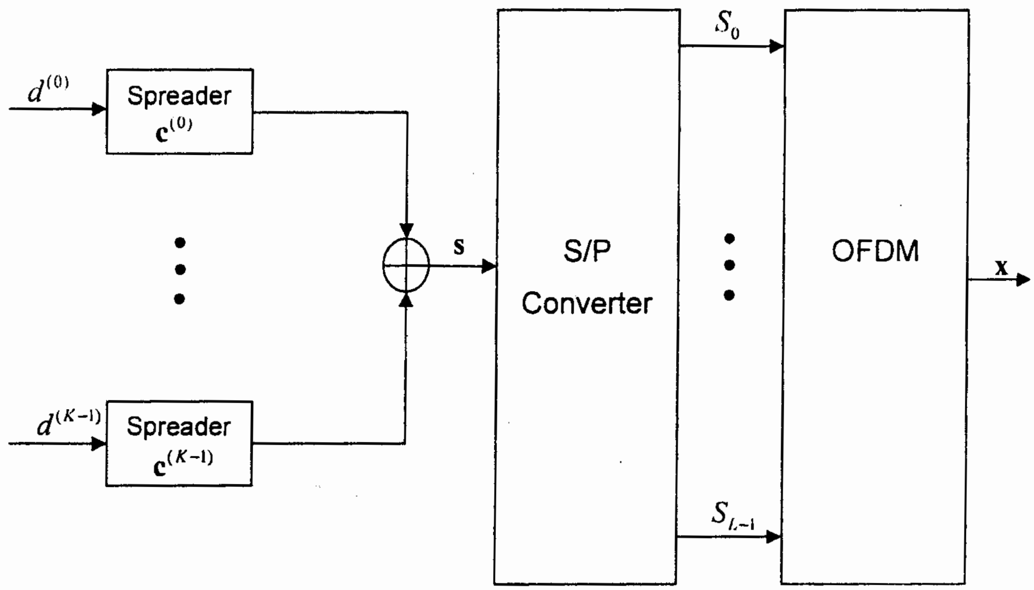

Figure.2.4. MC-CDMA Downlink transmitter

MC-CDMA signal for downlink attained after sequence **s** processing in OFDM block by (2.9)

$$x(t) = \frac{1}{N_c} \sum_{n=0}^{N_c - 1} S_n e^{j2\pi f_n t}, \quad 0 \le t \le T_s$$

Suppose all the echoes are sop up because of long enough guard interval. Hence inverse OFDM and de-interleaving of received vector transmitted sequence **s** is

$$\mathbf{r} = \mathbf{Hs} + \mathbf{n} = (R_0, R_1, ..., R_{L-1})^T \quad (2.23)$$

The $L \times L$ channel matrix is **H** and the noise vector **n** has length L. the **r** is set to the data detector for getting hard or soft estimation of the spread data. In multiuser detection, the system matrix **A** is utilized rather than **H**. So (2.23) can also given as

$$\mathbf{r} = \mathbf{Ad} + \mathbf{n} = (R_0, R_1, ..., R_{L-1})^T \quad (2.24)$$

In addition to system matrix A is

$$\mathbf{A} = \mathbf{H} \ \mathbf{C}$$

Uplink Signal

The MC-CDMA signal is resulted following processing of succession $s^{(k)}$ in the OFDM block by (2.9)

$$x(t) = \frac{1}{N_c} \sum_{n=0}^{N_c-1} S_n e^{j2\pi f_n t}, \quad 0 \le t \le T_s$$

in uplink. The inverse OFDM and frequency de-interleaving of acknowledged vector of the broadcasted successions $s^{(k)}$ is given by

$$\mathbf{r} = \sum_{k=0}^{K-1} \mathbf{H}^{(k)}\mathbf{s}^{(k)} + \mathbf{n} = (R_0, R_1, \ldots, R_{L-1})^T \tag{2.25}$$

$\mathbf{H}^{(k)}$ consists of the sub-channels coefficients allocated to user k. The requirement is to suppose the uplink synchronous for attaining the high spectral efficiency of OFDM. The vector $\mathbf{r}$ is fed to the data detector for getting hard or soft estimate of the transmitted data. The system matrix is $\mathbf{A} = (\mathbf{a}^{(0)}, \mathbf{a}^{(1)}, \ldots, \mathbf{a}^{(k-1)})$ comprises of K user specific vectors

$$\mathbf{a}^{(k)} = \mathbf{H}^{(k)}\mathbf{c}^{(k)} = (H_{0,0}^{(k)}c_0^{(k)}, H_{1,1}^{(k)}c_1^{(k)}, \ldots, H_{L-1,L-1}^{(k)}c_{L-1}^{(k)})^T \tag{2.26}$$

Spreading Codes

There are a number of spreading codes that are capable for the downlink and uplink signals transmission. The synchronous and asynchronous transmission have the different spreading codes. The Walsh codes are utilized in our MC-CDMA transmission system.

2.2. Multiple User Detection

There are two types of multiuser detection

1- Optimum detection

2- Sub-optimum detection

The brief description for both of them is as following:

1-Optimum detection:

In this detection scheme, the most probable sequence bits such that $\{b_k(n), 1 \le n \le N, 1 \le k \le K\}$ over received signal $r(t)$ detected for time duration $0 \le t \le NT + 2T$ [8]. This technique utilizes the a posteriori (MAP) principle or maximum likelihood (ML) principle. Some of the good Maximum likelihood optimum detection techniques are Maximum Likelihood Sequence Estimate (MLSE) and Maximum Likelihood Symbol-by-Symbol Estimation (MLSEE).

One of the good optimum multiuser detection methods is the Maximum Likelihood (ML) Detection. The two ML techniques are briefly described here. First is Maximum Likelihood Sequence Estimate (MLSE) [8] that guesses the transmitted data sequence $\mathbf{d} = (d^{(0)}, d^{(1)}, ..., d^{(k-1)})^T$. The second is the Maximum Likelihood Symbol-by-Symbol Estimation (MLSEE) that guesses the transmitted data symbol $d^{(k)}$ [10]. The conceivable transmitted data symbol vectors are $\mathbf{d}_\mu$, $\mu = 0, ..., M^K - 1$ M^K is the number of the conceivable transmitted data symbol vectors and M is the number of possible realizations of $d^{(k)}$.

2-Sub-optimum Detectors

The intricacy of the optimum detector gives the rise in power of integers on increasing users [9]-[10]. But, the suboptimum detectors are less complex than optimum detectors that arise linearly by increasing users.

One common sub-optimum method for detection is Block Linear equalization. The block linear equalization needs information of system matrix **A** in detector. There are many ways for using system matrix **A** for data detection in receiver, i.e. MMSE Block Linear Equalizer, Linear Minimum Mean Square Error Block detector etc. The LMMSE Block detectors are used in this thesis. Therefore, the LMMSE technique is discussed [11].

Linear MMSE Block Linear Equalizer

The MC-CDMA transmitter modulates the unique data stream through dissimilar subcarriers by a given spreading code in frequency domain [9]-[10]. Each subcarrier has dissimilar amplitude levels and phase shifts in the frequency selective fading channel. This reduces the orthogonality between users and produces MAI. The received sequence have to equalized by utilizing single tap equalizer for each subcarrier to induce for the amplitude and phase distortion effected by mobile radio channel after Fast Fourier Transform (FFT) and frequency de-interleaving. To induce for MAI, MMSE principle pertained separately to attain improved efficiency. LMMSE detection uses MMSE principle applied for each user which results in high-quality efficiency.

In LMMSE detection, the mean square error is minimized between transmitted symbol b_k and estimated symbol $\hat{b}_k$. Suppose optimal weight vector be

$$\mathbf{W}_k^T = [W_k^0, W_k^1, ..., W_k^N] \tag{2.32}$$

The size of b_k is N and overal weights in W_k^T are N. The k^{th} user predictable symbol is given as

$$\hat{b}_k = W_j^H \mathbf{R} = \mathbf{C}_k^H \mathbf{HR} \tag{2.33}$$

The spreading code of user k is **C**, **H** is impulse response and **R** is received vector. The weight vector **W** is calculated in different ways in different MMSE algorithms. In next section, **W** for Robust RLS based detector is discussed.

2.3 Robust RLS based Detection

We will formulate a robust RLS based detector with quadratic constraint that is based on the Constrained Minimum Output Energy detector [1]. The efficiency of the regular CMOE detector decreases in company of channel and data correlation matrix estimation errors. This insufficiency was detected in adaptive beam-forming [3]-[5] and

Direct Sequence (DS) CDMA multiuser detection [6]. In Robust RLS based Detection, the CMOE detector is prepared more robust beside above insufficiencies. As Robust RLS based Detection is based on the CMOE detector, therefore a good knowledge of the CMOE detector is required. We will first discuss the CMOE detector and than Robust RLS based Detector.

2.3.1 CMOE Detection

The Recursive Least Square (RLS) updating of correlation matrix is utilized in it. The channel is guessed after calculating the least Eigen-vector of data sub-matrix after calculation of the correlation matrix. Therefore, here is the need to summarize the RLS algorithm first.

2.3.1.1 Recursive Least Square Algorithm

[11] RLS algorithm is modeled such that least squares estimation of tap weight vector of filter at iteration $n-1$ be utilized to compute renew approximation of vector at iteration n on influx of new statistics. The RLS algorithm is as described below.

Initialization

$$\hat{\mathbf{w}}(0) = 0 \tag{2.34}$$

$$\mathbf{P}(0) = \delta^{-1}\mathbf{I} \tag{2.35}$$

The procedure for the initialization process of RLS is given above. where

$$\delta = \begin{pmatrix} \text{is the small positive constant for the high SNR} \\ \text{is the large positive constant for the low SNR} \\ \end{pmatrix}$$

$\hat{\mathbf{w}}(0)$ be the preferred tap weight vector on first iteration and $\mathbf{P}(0)$ is inverse correlation matrix on first iteration. The δ be regularization parameter which is positive real number and the Identity matrix is $\mathbf{I}$.

From here, each immediate is computed for intervals, $n = 1, 2, 3, \ldots$

Gain Vector

$$\pi(n) = \mathbf{P}(n\text{-}1)\mathbf{u}(n) \tag{2.36}$$

$$\mathbf{k}(n) = \frac{\pi(n)}{\lambda + \mathbf{u}^H(n)\pi(n)} \tag{2.37}$$

$\mathbf{k}(n)$ is M-by-1 gain vector. The gain vector value is updated by $\mathbf{k}(n)$, λ is positive constant with value less than one.

Filtering Operation

$$\xi(n) = d(n) - \hat{\mathbf{w}}^H(n-1)\mathbf{u}(n) \tag{2.38}$$

The priori evaluation error $\xi(n)$ assist in filtering process of algorithm. $\hat{\mathbf{w}}^H(n-1)\mathbf{u}(n)$ is desired response $d(n)$ estimation.

Adaptive Operation

$$\hat{\mathbf{w}}(n) = \hat{\mathbf{w}}(n-1) + \mathbf{k}(n)\xi^*(n) \tag{2.39}$$

The preferred tap weight vector $\hat{w}(n)$ presents adaptive process for RLS algorithm, where the tap weight vector updating by increasing its previous value by total equal to product of complex conjugate of apriori channel approximation error $\xi(n)$ and timely changeable gain vector $\mathbf{k}(n)$.

Inverse correlation matrix

$$\mathbf{P}(n) = \lambda^{-1}\mathbf{P}(n\text{-}1) - \lambda^{-1}\mathbf{k}(n)\mathbf{u}^H(n)\mathbf{P}(n\text{-}1) \tag{2.40}$$

The inverse correlation matrix P(n) assists for allowing personal updating of gain vector. Now, the CMOE detector is formulated.

2.3.1.2 CMOE Detection and Channel Estimation

Generally, "The CMOE detector minimizes the variance of the receiver on the whole output by keeping preferred signals with the gain equal to unity". The subscript will be fall from signal model in MC-CDMA system model for covariance. The preferred user is zero. The functioning of CMOE detector is following:

Weight optimization

$$\mathbf{w}_0 = (\mathbf{d}_0^H(n)\mathbf{R}_y^{-1}(n)\mathbf{d}_0(n))^{-1}\mathbf{R}_y^{-1}(n)\mathbf{d}_0(n) \qquad (2.41)$$

The solution to (2.41) is constraint optimization problem given by [12]. $\mathrm{d}_0(n)$ is the effective signature waveform of the desired user given as

$$\mathbf{d}_0(n) = \mathbf{C}_0\mathbf{h}_0(n) \text{ with } \mathbf{C}_0 = diag[C_{0,0}, C_{0,1}, \ldots, C_{0,M-1}]$$

with $\mathbf{h}_0(n)$ is the impulse response of desired user. $\mathbf{R}_y(n)$ is the covariance matrix given as

$$\mathbf{R}_y(n) = \lambda\mathbf{R}_y(n-1) + \mathbf{y}(n)\mathbf{y}^H(n) \qquad (2.42)$$

The value of the forgetting factor is $0<\lambda<1$ and $\mathbf{y}(n)$ is baseband MC-CDMA signal.

Minimum Output Variance

The minimum output variance of detector can also presented as [13]

$$\mathbf{P}_{\min} = \mathbf{w}_0^H\mathbf{R}_y(n)\mathbf{w}_0 = (\mathbf{d}_0^H(n)\mathbf{R}_y^{-1}(n)\mathbf{d}_0(n))^{-1} \qquad (2.43)$$

Effective Signature Waveform Estimation

The maximizing of (2.43) with respect to $\mathbf{d}_0(n)$ results in estimate of effective signature waveform. The result is that the signal gears at the receiver output are maximized later than the interference repression. The effective signature waveform approximation equation is

$$\hat{\mathbf{d}}_0(n) = \arg\min_{\mathbf{d}_0(n)} \mathbf{d}_0^H(n)\mathbf{R}_y^{-1}(n)\mathbf{d}_0(n) \tag{2.44}$$

Channel Impulse Response

Let $g_0(n)$ is the channel impulse response of first user with L paths. The impulse response of $g_0(n)$ is $\mathbf{h}_0(n) = \mathbf{F}_m\mathbf{g}_0(n)$ of first user (desired user). $\mathbf{F}_m$ is matrix with dimensions $M \times L$. The values of $\mathbf{F}_m$ is given by

$$[\mathbf{F}_m]_{m,l} = \exp(-j2\pi((P_M + p)l)/(M)), m = 0,\ldots, M-1, l = 0,\ldots, L-1$$

The reduction of the waveform estimation equation is given by

$$\hat{\mathrm{g}}_0(\mathrm{n}) = \arg\min_{\mathrm{g}_0(\mathrm{n}) \in C^{L\times 1}} \mathrm{g}_0^H(n)\mathbf{\Omega}\,\mathrm{g}_0(n) \tag{2.45}$$

where

$$\mathbf{\Omega} = \mathbf{F}_m^H \mathbf{C}_0^H \mathbf{R}_y^{-1}(n)\mathbf{C}_0\mathbf{F}_m$$

The channel estimation $\hat{\mathrm{g}}_0(n)$ is associated with the smallest Eigen-value of the $\mathbf{\Omega} \in C^{L\times L}$.

Eigen Value Computation

Many methods exists to find the minimum Eigen-vector of $\mathbf{\Omega}$, like Singular Value Decomposition (SVD), Eigen-value decomposition (EVD) etc. But it is difficult to pursue the preferred eigenvector of $\mathbf{\Omega}$. But here, the inverse iteration method is utilized calculate the minimum Eigen-vector of the matrix $\mathbf{\Omega}$.

Inverse Iteration method

The inverse iteration method is used since of its little complexity and good tracking performance [6]. The inverse iteration method is

Initialization: $\hat{\mathbf{g}}(0)$;

$\boldsymbol{\Omega}(n)\mathbf{a}(n) = \hat{\mathbf{g}}(n-1)$; LU decomposition to get $\mathbf{a}(n)$

$\hat{\mathbf{g}}(n) = \mathbf{a}(n) / \| \mathbf{a}^H(n)\mathbf{a}(n) \|^{1/2}$; Normalization

2.3.2 Robust RLS based Detector

As discussed before that CMOE detector minimizes the overall variance of the receiver output. The solution of weight optimization for the CMOE detector is achieved very rarely. Firstly, the estimated $\tilde{d}_0(n)$ contains errors, i.e., $\tilde{d}_0(n) \neq d_0(n)$. Next, the estimated covariance matrix $\tilde{\mathbf{R}}_y(n)$ is to be getting from a limited observable data. The CMOE detector degrades in the presence of the signal modeling errors due to these two factors. Therefore it is necessary that the CMOE detector must come over these deficiencies. To overcome these deficiencies a quadratic constraint is forced. How it is forced is given below.

Quadratic Constraint

The quadratic constraint is forced on the norm of the weight vector **w**

$$\mathbf{w}^H\mathbf{w} \leq T_0 \tag{2.46}$$

The theme is that the weight vector **w** is not allowed to be too big because the desired signal is badly affected by the model errors.

Diagonal Loading

The way of forcing the constraint on the weight vector **w** given above has the same result as adding diagonal loading of the data covariance matrix $\mathbf{R}_y$ [3].

$$\mathbf{w} = [\mathbf{d}_0^H(\mathbf{R}_y + \beta\mathbf{I}_{M\times M})^{-1}\mathbf{d}_0]^{-1}(\mathbf{R}_y + \beta\mathbf{I}_{M\times M})^{-1}\mathbf{d}_0 \tag{2.47}$$

There is no fine way to load β with closed expression. An adaptive algorithm to compute the diagonal loading β given with a quadratic constraint T_0 is used [1]. Here the need is guarantee that the constraint $\mathbf{w}^H(n)\mathbf{d}_0(n) = 1$.

Projection

The above assurance is given by defining the

$$\mathbf{w}_q(n) = \mathbf{d}_0(n)(\mathbf{d}_0^H(n)\mathbf{d}_0(n))^{-1} \tag{2.48}$$

The $w_q(n)$ vector helps to venture the received signal vector on to the constraint subspace. A fair value for the constraint is

$$T_0 = 1.5 \,||\, \mathbf{w}_q(n) \,||$$

Weight Updation

$$\mathbf{w} \approx (\tilde{\mathbf{w}} - \beta \mathbf{v})/(1 - r\beta \tilde{\mathbf{w}}^H \tilde{\mathbf{w}}) \tag{2.49}$$

The derivation of weight updation equation (2.49) involves the Taylor series. The Taylor series is discussed here briefly and discussion about the weight updation with its components will be shown in the next chapter.

2.3.2.1 Taylor Series

Taylor series expansion of a function is about a point [14]. A one dimensional Taylor series is an expansion of the real function $f(x)$ about a point x=a is given by

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \ldots \tag{2.50}$$

The Theorem states that "Any function satisfying certain condition can be expressed as Taylor series"

Chapter 3
Design

Chapter 3

Design

3.1 Introduction to proposed system

Due to interference limitation in MC-CDMA system, it is alert to the multiple access interference (MAI). The MAI arises from the sharing of the same frequency band among the multiple users. It is essential to conquer the MAI for well performing MC-CDMA system. The LMMSE detectors are projected to be the effective in MAI reduction [15]-[17].

The task is to study RLS based detector for the MC-CDMA system in which MAI is mitigated in the company of the signal modeling errors. The Robust RLS based Detector is based on ordinary CMOE detector for the MC-CDMA. As our purposed technique is for the MC-CDMA, therefore the sequence of the description for this chapter is as follow: "Firstly, a brief problem statement is given. After that a special attention is paid to the MC-CDMA system model. In the end, the system model of the Robust RLS based Detector for MC-CDMA system is given".

3.2 Problem Declaration

As in MC-CDMA system, same frequency band is used by the multiple users concurrently. Due to this, the Multiple Access Interference (MAI) occurs. MAI minimizes the data rate and channel efficiency. The target is to study such a MC-CDMA system that mitigates the MAI in the company of the white Gaussian noise as well as signal modeling errors. In the absence of the signal modeling errors the ordinary CMOE detector for the MC-CDMA system performs better [1]. But, when the signal modeling errors take place, the proposed Robust RLS based Detector for MC-CDMA is efficient than the regular CMOE detector for MC-CDMA system [1].

3.3.MC-CDMA System Model

Let a K user synchronous MC-CDMA transmission system. In the system, the uplink transmission is to be taking place from the terminal user to base station. The figure 3.1 illustrates the block diagram of the MC-CDMA transmitter.

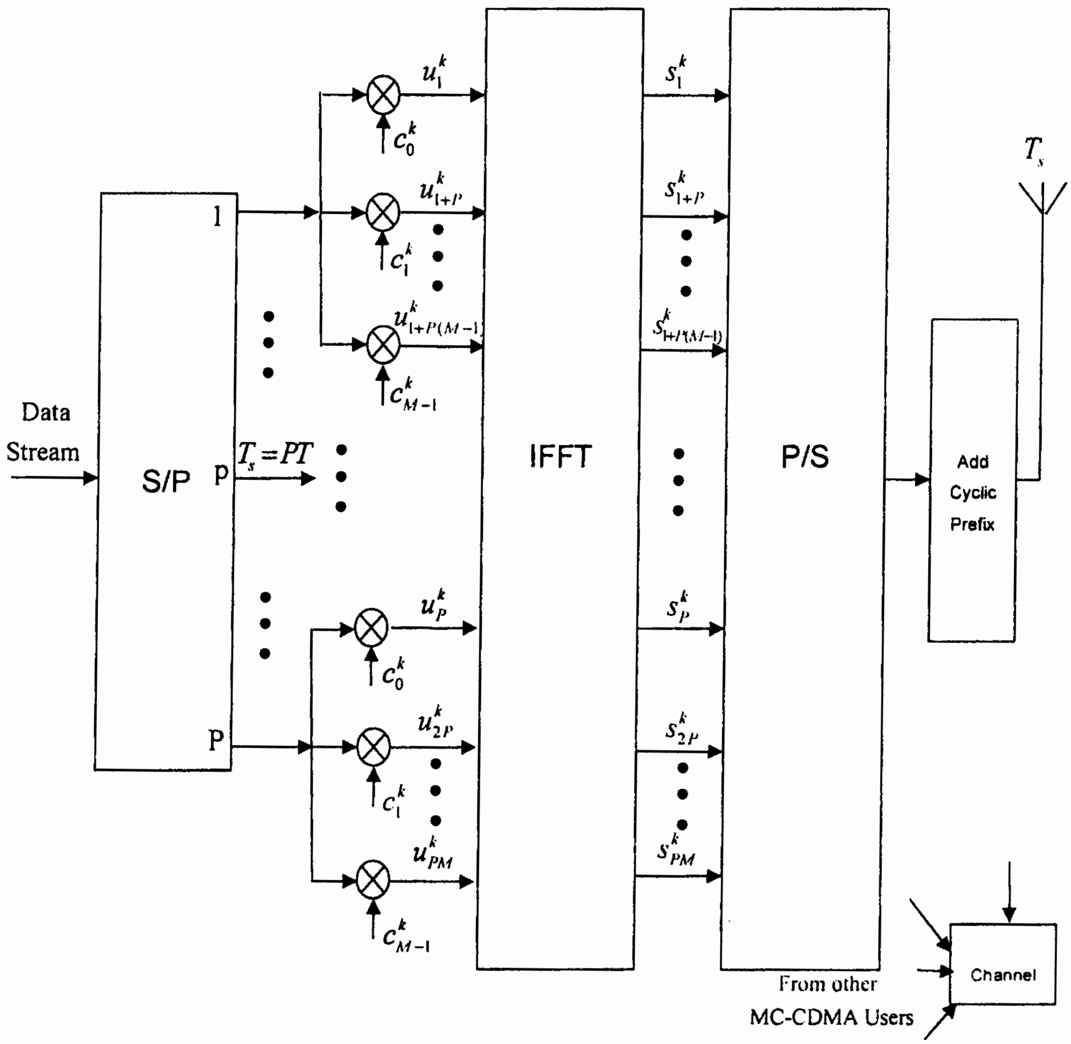

Figure.3.1. The Transmitter of MC-CDMA system

All the data streams given below are for the i^{th} time interval. The 1^{st} user data stream is

$$\mathbf{b}^1(i) = [b_0^1(i), b_1^1(i), b_2^1(i), \ldots, b_{P-1}^1(i)] \quad (3.1)$$

The 2^{nd} user data stream is

$$\mathbf{b}^2(i) = [b_0^2(i), b_1^2(i), b_2^2(i), \ldots, b_{p-1}^2(i)] \tag{3.2}$$

Correspondingly, the P^{th} user data stream is

$$\mathbf{b}^P(i) = [b_0^P(i), b_1^P(i), b_2^P(i), \ldots, b_{p-1}^P(i)] \tag{3.3}$$

Total K users data stream is

$$\mathbf{b}(i) = [\mathbf{b}^1(i), \mathbf{b}^2(i), \mathbf{b}^3(i), \ldots, \mathbf{b}^K(i)] \tag{3.4}$$

The K^{th} user data stream is changed to P similar data sequences $\mathbf{b}(i)$ at the i^{th} time. Let T and T_s be the symbols period prior to and later than the serial to parallel (S/P) conversion. However, $T_s = PT$. If symbol period T_s at subcarrier is longer as compare to channel multipath holdup spread, than each subcarrier almost practice flat fading.

Suppose the user spreading code is $\mathbf{c}_k = [c_0^k, c_1^k, \ldots, c_{M-1}^k]^T, k = 0, \ldots, K-1$. $\mathbf{c}_k$ is used for spreading the output of the each user after the S/P conversion. The data chips are changed to M subcarriers after S/P.

The 1st user output modulated with c_k is

$$b_0^1(i)[c_0^1, c_1^1, \ldots, c_{M-1}^1]^T = [b_0^1 c_0^1, b_0^1 c_1^1, \ldots, b_0^1 c_{M-1}^1]^T$$
$$b_1^1(i)[c_0^1, c_1^1, \ldots, c_{M-1}^1]^T = [b_1^1 c_0^1, b_1^1 c_1^1, \ldots, b_1^1 c_{M-1}^1]^T$$

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$$b_{p-1}^k(i)[c_0^1, c_1^1, \ldots, c_{M-1}^1]^T = [b_{p-1}^k c_0^1, b_{p-1}^k c_1^1, \ldots, b_{p-1}^k c_{M-1}^1]^T$$

The k^{th} user output modulated with c_k is

$$b_0^k(i)[c_0^k, c_1^k, \ldots, c_{M-1}^k]^T = [b_0^k c_0^k, b_0^k c_1^k, \ldots, b_0^k c_{M-1}^k]^T$$
$$b_1^k(i)[c_0^k, c_1^k, \ldots, c_{M-1}^k]^T = [b_1^k c_0^k, b_1^k c_1^k, \ldots, b_1^k c_{M-1}^k]^T$$

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$$b_{p-1}^k(i)[c_0^k, c_1^k, \ldots, c_{M-1}^k]^T = [b_{p-1}^k c_0^k, b_{p-1}^k c_1^k, \ldots, b_{p-1}^k c_{M-1}^k]^T$$

The separation among consecutive subcarriers is frequency $\Delta f = 1/T_s$. The upper limit of frequency diversity is achieved by sending the data bit b_p^k on the subcarrier with frequencies $f_1 + (p+mP)\Delta f, m = 0,...,M-1$. The resultant chips are $N = PM$. These are

$$\mathbf{u}_k(i) = [b_0^k(i)c_0^k, b_0^k(i)c_1^k, ..., b_0^k(i)c_{M-1}^k, b_1^k(i)c_0^k, b_1^k(i)c_1^k, ..., \\ b_1^k(i)c_{M-1}^k, ..., b_{P-1}^k(i)c_0^k, b_{P-1}^k(i)c_1^k, ..., b_{P-1}^k(i)c_{M-1}^k]^T_{N\times 1} \quad (3.5)$$

Suppose $u_0^1 = b_0^1 c_0^1$. Equally, $u_{k+Pk}^1 = b_k^1 c_k^1$. So it can be written

$$\mathbf{u}_0^1 = [u_0^1, u_{0+p}^1, ..., u_{0+p(M-1)}^1]^T$$

$$\mathbf{u}_1^1 = [u_1^1, u_{1+p}^1, ..., u_{1+p(M-1)}^1]^T$$

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$$\mathbf{u}_{p-1}^1 = [u_{p-1}^1, u_{2p+1}^1, ..., u_{p(M-1)}^1]^T$$

Correspondingly,

$$\mathbf{u}_0^P = [u_0^P, u_{0+p}^P, ..., u_{0+p(M-1)}^P]^T$$

$$\mathbf{u}_1^P = [u_1^P, u_{1+p}^P, ..., u_{1+p(M-1)}^P]^T$$

.

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$$\mathbf{u}_{p-1}^P = [u_{p-1}^P, u_{2p+1}^P, ..., u_{p(M-1)}^P]^T$$

The data vector with dimensions $N \times 1$ is

$$\mathbf{u}^k = \begin{bmatrix} \mathbf{u}_0^k \\ \mathbf{u}_1^k \\ \vdots \\ \mathbf{u}_{P-1}^k \end{bmatrix} = \begin{bmatrix} u_0^k, u_{0+P}^k, \cdots, u_{0+P(M-1)}^k, \cdots, u_1^k, u_{1+P}^k, \cdots \\ , u_{1+P(M-1)}^k, \cdots, u_{P-1}^k, u_{2P-1}^k, \cdots, u_{PM-1}^k \end{bmatrix}^T_{N\times 1} \quad (3.6)$$

$u_k(i)$ is than modulated by Inverse Fast Fourier Transform (IFFT). The IFFT of k users results in data samples at i^{th} time are

$$\mathbf{s}_k(i) = \mathbf{F}_1 \mathbf{u}_k(i) \quad (3.7)$$

$\mathbf{F}_1 \in \mathbf{C}^{N\times N}$ is fourier matrix utilized for Inverse Discrete Fourier Transform (IDFT). The $(u,v)^{th}$ component is $\exp(j2\pi uv/N)/N$. The 1^{st} component of k^{th} user at the i^{th} time is

$$\begin{aligned} s_0^K(i) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} u_0^k e^{jwi} di \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} b_0^k(i) c_0^k e^{jwi} di \\ &= \frac{c_0^k}{2\pi}\int_{-\infty}^{\infty} b_0^k(i) e^{jwi} di \\ &= \frac{c_0^k}{2\pi} \end{aligned} \tag{3.8}$$

Correspondingly, $s_1^K,...,s_{PM-1}^k$ can be computed. The $s^k(i)$ vector is given by

$$\mathbf{s}^k(i) = \begin{bmatrix} s_0^K(i), s_{0+P}^K(i), ..., s_{P(M-1)}^K(i), \\ s_1^K(i), s_{1+P}^K(i), ..., s_{1+P(M-1)}^K(i), \\ ..., s_{P-1}^K(i), s_{2P-1}^K(i), ..., s_{PM-1}^K(i) \end{bmatrix}_{N\times 1} \tag{3.9}$$

Hence, the explanation given above can be used to derive (3.7).

This user signal has been transmit through a frequency selective channel with L paths. The channel is presented by $(L-1)^{th}$ Finite Impulse Response (FIR) filter [18]. The channel has impulse response

$$h_k(t) = \sum_{l=0}^{L-1} g_{k,l}(t)\delta(t-\tau_{k,l}) \tag{3.10}$$

Where k is user indicator and $g_{k,l}(t)$ is l^{th} path gain. The $g_{k,l}(t)$ is autonomous zero mean complex Gaussian random process that has variance $\sigma_{k,l}^2$. The delay spread of l^{th} path is $\tau_{k,l}$. The power wait profile is the follower $\sigma_{k,l}^2\ (l = 0,...,L-1)$.

At the same time the Inter Symbol Interference (ISI) also occurs. So, the elimination of ISI is essential at the same time. To get rid of ISI, a cyclic prefix of samples N_g is further to every symbol, where $N_g \geq L-1$. The MC-CDMA signal is transmitted to the channel after that.

The figure (3.2) shows the receiver for MC-CDMA system.

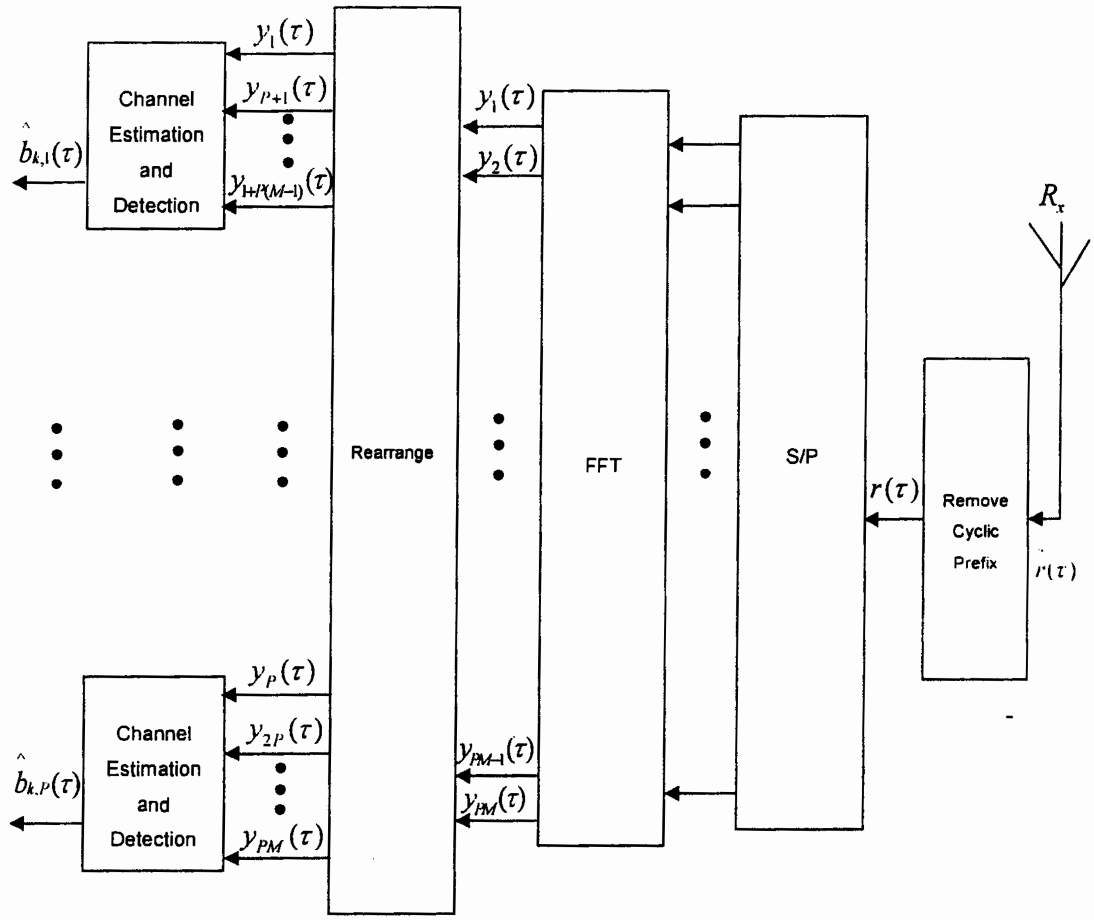

Figure 3.2. Receiver for MC-CDMA system

Suppose that channel is constant during MC-CDMA symbol. At the receiver, the signal is sampled at rate of $(N+N_g)/T_s$. The received signal for k^{th} user at τth sample is

$$\hat{\mathbf{r}}_k(\tau) = \int h_k(t)\hat{\mathbf{s}}_k(t-l)dt + \boldsymbol{\eta}$$
$$= \int \sum_{l=0}^{L-1} g_{k,l}(t)\delta(t-\tau_{k,l})\hat{\mathbf{s}}_k(t-l)dt + \boldsymbol{\eta} \qquad (3.11)$$

So, the baseband signal received for k^{th} user on τth sample

$$\hat{\mathbf{r}}_k(\tau) = \sum_{l=0}^{L-1} \sqrt{P_k}\, g_{k,l}(\tau)\mathbf{s}_k(\tau-l)dt + \boldsymbol{\eta}_k \qquad (3.12)$$

$\tau_{k,l} = t$ and $\int \delta(t-\tau_{k,l})dt = 1$. The received signal on τth sample is

$$
\begin{aligned}
\hat{\mathbf{r}}(\tau) &= \sum_{k=0}^{K-1} \hat{\mathbf{r}}_k(\tau) \\
&= \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} \sqrt{P_k}\, g_{k,l}(\tau) \hat{\mathbf{s}}_k(\tau - l)dt + \hat{\boldsymbol{\eta}}(\tau)
\end{aligned} \tag{3.13}
$$

$\hat{\boldsymbol{\eta}}(\tau) = [\eta_1 \quad \eta_2 \quad \cdots \quad \eta_k]^T$ refers to the white Gaussian noise that has zero mean and variance σ_v^2, $g_{k,l}(\tau)$ is for complex envelop on i^{th} path, $\hat{\mathbf{s}}_k(\tau) \in \mathbf{C}^{N_{tot} \times 1}$, $N_{tot} = N + N_g$ is the broadcasted signal vector along with cyclic prefix, the chip energy P_k is taken equal to 1 in implementation.

Suppose time and frequency harmonization is achieved on receiver and ISI is mitigated with condition $N_g \geq L-1$. All samples relevant to cyclic prefix are removed. Therefore, the harmonized τth MC-CDMA symbol after eliminating cyclic prefix is

$$
\mathbf{r}(\tau) = \sum_{k=0}^{K-1} \sqrt{P_k}\, \mathbf{G}_k(\tau) \mathbf{s}_k(\tau) + \boldsymbol{\eta}(\tau) \tag{3.14}
$$

with

$\mathbf{s}_k(\tau) = \left[\hat{\mathbf{s}}_k(\tau N_{tot} + N_g), \ldots, \hat{\mathbf{s}}_k(\tau N_{tot} + N_{tot} - 1) \right]^T$,

$\boldsymbol{\eta}(\tau) = \left[\hat{\boldsymbol{\eta}}(\tau N_{tot} + N_g), \ldots, \hat{\boldsymbol{\eta}}(\tau N_{tot} + N_{tot} - 1) \right]^T$. $G_k(\tau)$ is the Toeplitz matrix like

$$
\mathbf{G}_k = \begin{bmatrix}
g_{k,0} & 0 & 0 & \cdots & g_{k,l} \\
g_{k,l} & g_{k,0} & 0 & \cdots & g_{k,2} \\
g_{k,2} & g_{k,l} & g_{k,0} & \cdots & g_{k,3} \\
. & . & . & & . \\
. & . & . & . & . \\
. & . & . & & . \\
0 & 0 & 0 & . & g_{k,0}
\end{bmatrix}
$$

(If entire elements on main diagonal of the matrix are same and the elements on whichever diagonal similar to the major diagonal are also like, than matrix is called Toeplitz Matrix).

In the end, the fast FFT with size N is executed on receiver. Let $\mathbf{F} \in \mathbf{C}^{N\times N}$ is Fourier matrix. $\exp(-j2\pi uv/N)$ is $(u,v)th$ component of $\mathbf{F}$. The baseband signal in frequency domain in matrix form

$$\begin{aligned}\mathbf{y}(\tau) &= \mathbf{F}\mathbf{r}(\tau)\\ &= \mathrm{F}\left[\sum_{k=0}^{K-1}\sqrt{P_k}\,\mathbf{G}_k(\tau)\mathbf{s}_k(\tau)+\boldsymbol{\eta}(\tau)\right]\end{aligned} \tag{3.15}$$

$\mathbf{s}_k(\tau) \underset{F}{\overset{F^{-1}}{\rightleftarrows}} \mathbf{u}_k(\tau)$ *and* $\boldsymbol{\eta}(\tau) \underset{F^{-1}}{\overset{F}{\rightleftarrows}} \mathbf{n}(\tau)$, $\mathbf{G}_k(\tau)$ is Toeplitz matrix described previously. $\mathbf{H}_k(\tau) = diag\left[h_{k,0}(\tau),\ldots,h_{k,N-1}(\tau)\right]$ moreover

$$h_{k,n}(\tau) = \sum_{l=0}^{L-1} g_{k,l}(\tau)\exp(-j2\pi(nl)/N),\; n = 0,\ldots,N-1$$

It can be written as

$$h_n^k(\tau) = \sum_{l=0}^{L-1} g_l^k(\tau)\exp(-j2\pi(nl)/N),\; n = 0,\ldots,N-1 \tag{3.16}$$

$h_0^k(\tau), h_1^k(\tau), \ldots, h_{N-1}^k$ can be computed from (3.16)

$$h_0^k(\tau) = \sum_{l=0}^{L-1} g_l^k(\tau)\exp(-j2\pi((0)l)/N) = \sum_{l=0}^{L-1} g_l^k(\tau) \tag{3.17}$$

$$h_1^k(\tau) = \sum_{l=0}^{L-1} g_l^k(\tau)\exp(-j2\pi l/N)$$

.

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$$h_{N-1}^k(\tau) = \sum_{l=0}^{L-1} g_l^k(\tau)\exp(-j2\pi(N-1)l/N)$$

Because $\mathbf{G}_k(\tau) \xrightarrow{FFT} \mathbf{H}_k(\tau)$. Therefore, the equation (3.15) can be written as

$$\mathbf{y}(\tau) = \sum_{k=0}^{k-1} \sqrt{P_k}\,\mathbf{H}_k(\tau)\mathbf{u}_k(\tau) + \mathbf{n}(\tau) \tag{3.18}$$

$\mathbf{n}(\tau)$ is white Gaussian noise having zero mean and variance σ_n^2.

Consider, every user transmits the p^{th} data stream. The $M \times 1$ data vector $\mathbf{y}_p(\tau) = \left[y_p(\tau), y_{P+p}(\tau), \ldots, y_{P(M-1)+p}(\tau)\right]^T$ corresponding to b_p^k is

$$\begin{aligned}\mathbf{y}_p(\tau) &= \sum_{k=0}^{K-1} \sqrt{P_k}\,\mathbf{C}_k \mathbf{h}_p^k(\tau) b_p^k(\tau) + \mathbf{n}_p(\tau) \\ &= \sum_{k=0}^{K-1} \sqrt{P_k}\,\mathbf{d}_p^k(\tau) b_p^k(\tau) + \mathbf{n}_p(\tau)\end{aligned} \tag{3.19}$$

$\mathbf{h}_p^k(\tau) = [h_p^k(\tau), h_{P+p}^k(\tau), \ldots, h_{P(M-1)+p}^k(\tau)]^T$ is channel coefficient on subcarriers $(Pm+p)th$.

$\mathbf{C}_k = diag\{c_0^k, c_1^k, \ldots, c_{M-1}^k\}$ is code matrix, $\mathbf{d}_p^k(\tau) = \mathbf{C}_k \mathbf{h}_p^k(\tau)$ is the effective signature of user k and $\mathbf{n}_p(\tau)$is the white Gaussian noise with zero mean and variance .

3.4. Robust RLS based Detector

As CMOE detector diminish variance on the whole at receiver output. The optimal weight solution of CMOE Detector is given by

$$\mathbf{w}_0 = (\mathbf{d}_0^H(n)\mathbf{R}_y^{-1}(n)\mathbf{d}_0(n))^{-1}\mathbf{R}_y^{-1}(n)\mathbf{d}_0(n) \tag{3.20}$$

This solution of weight optimization is attained very infrequently. Firstly, the waveform form estimation equation have errors, i.e., $\mathrm{d}_0(n) \neq \hat{\mathrm{d}}_0(n)$. The other thing is that estimated covariance matrix $\bar{\mathbf{R}}_y(n)$ is attained from a finite observable data. Therefore, in realistic applications the adaptive weight vector is given by

$$\mathbf{w}_0 = (\tilde{\mathbf{d}}_0^H(n)\tilde{\mathbf{R}}_y^{-1}(n)\tilde{\mathbf{d}}_0(n))^{-1}\tilde{\mathbf{R}}_y^{-1}(n)\tilde{\mathbf{d}}_0(n) \tag{3.21}$$

The efficiency of CMOE detector decreases in the company of signal modeling errors because of above two issues. Therefore it is required that the CMOE detector should over come these deficiencies.

To overcome these deficiencies, in Robust CMOE detector also known as Robust RLS based detector a quadratic restriction is forced [1]. The quadratic constraint is forced on the norm of weight vector **w**.

$$\mathbf{w}^{H}\mathbf{w} \leq T_0 \tag{3.22}$$

The theme is that the weight vector $\|\mathbf{w}\|$ is evaded from to be too outsized because the desired signal is badly affected by model errors. This way of forcing constraint on the weight vector $\mathbf{w}$ given above has the same effect as adding diagonal packing of the data covariance matrix $\mathbf{R}_y$ [3].

$$\mathbf{w} = [\bar{\mathbf{d}}_0^{H}(\mathbf{R}_y + \beta\mathbf{I}_{M\times M})^{-1}\mathbf{d}_0]^{-1}(\mathbf{R}_y + \beta\mathbf{I}_{M\times M})^{-1}\mathbf{d}_0 \tag{3.23}$$

It is not possible to find optimal expression to find β [6]. But here an adaptive algorithm to find the diagonal packing of β by forcing a quadratic constraint T_0 is proposed [1]. This is assured by constraint $\mathbf{w}^{H}(n)\mathbf{d}_0(n) = 1$. This assurance is given by defining the

$$\mathbf{w}_q(n) = \mathbf{d}_0(n)(\mathbf{d}_0^{H}(n)\mathbf{d}_0(n))^{-1} \tag{3.24}$$

The constraint subspace is projected by the received signal vector by $\mathbf{w}_q(n)$ vector. A fair value for the constraint is

$$T_0 = 1.5\,\|\mathbf{w}_q(n)\|$$

The (3.23) can be presented as

$$\mathbf{w} = [(\mathbf{I} + \beta\mathbf{R}_y^{-1})^{-1}\mathbf{R}_y^{-1}\mathbf{d}_0]^{-1}\mathbf{d}_0^{H}(\mathbf{I} + \beta\mathbf{R}_y^{-1})^{-1}\mathbf{R}_y^{-1}\mathbf{d}_0 \tag{3.25}$$

For the small value of the β, $(\mathbf{I}+\beta\mathbf{R}_y^{-1})^{-1}$ can be examined by using the Taylor series. First two terms of the Taylor series are sufficient to approximate $(\mathbf{I}+\beta\mathbf{R}_y^{-1})^{-1}$. Suppose that

$$g(\beta)=(\mathbf{I}+\beta\mathbf{R}_y^{-1})^{-1} \tag{3.26}$$

So, by Taylor series expansion

$$g(\beta)=g(0)+\beta g'(0) \tag{3.27}$$

Therefore, first order derivation of (3.27) is given by

$$\begin{aligned} g'(\beta) &= -(\mathbf{I}+\beta\mathbf{R}_y^{-1})^{-2}\mathbf{R}_y^{-1} \\ &= -(\mathbf{I}+\beta\mathbf{R}_y^{-1})^{-1}\mathrm{R}_y^{-1}(\mathbf{I}+\beta\mathbf{R}_y^{-1})^{-1} \end{aligned} \tag{3.28}$$

And

$$\begin{aligned} g'(0) &= -(\mathbf{I})\mathbf{R}_y^{-1}(\mathbf{I}) \\ &= \mathbf{I} \end{aligned} \tag{3.29}$$

From the equation (3.27), we can write that

$$g(\beta)=\mathbf{I}-\beta\mathbf{R}_y^{-1} \tag{3.30}$$

Approximately

$$(\mathbf{I}+\beta\mathbf{R}_y^{-1})^{-1} \approx \mathbf{I}-\beta\mathbf{R}_y^{-1} \tag{3.31}$$

The (3.25) can be written approximately equivalent to

$$\begin{aligned} \mathbf{w} &= [(\mathbf{I}-\beta\mathbf{R}_y^{-1})\mathbf{R}_y^{-1}\mathbf{d}_0]^{-1}\mathbf{d}_0^H(\mathbf{I}-\beta\mathbf{R}_y^{-1})\mathbf{R}_y^{-1}\mathbf{d}_0 \\ &= \frac{(\mathbf{I}-\beta\mathbf{R}_y^{-1})\mathbf{R}_y^{-1}\mathbf{d}_0}{\mathbf{d}_0^H[(\mathbf{I}-\beta\mathbf{R}_y^{-1})\mathbf{R}_y^{-1}\mathbf{d}_0]} \\ &= \frac{\mathbf{R}_y^{-1}\mathbf{d}_0-\beta\mathbf{R}_y^{-1}\mathbf{R}_y^{-1}\mathbf{d}_0}{\mathbf{d}_0^H\mathbf{R}_y^{-1}\mathbf{d}_0-\beta\mathbf{d}_0^H\mathbf{R}_y^{-1}\mathbf{R}_y^{-1}\mathbf{d}_0} \end{aligned} \tag{3.32}$$

Suppose $r=\mathbf{d}_0^H\mathbf{R}_y^{-1}\mathbf{d}_0$, $\mathbf{v}=\mathbf{R}_y^{-1}\mathbf{R}_y^{-1}\mathbf{d}_0/r=\mathbf{R}_y^{-1}\tilde{\mathbf{w}}$ and $\tilde{\mathbf{w}}=(\mathbf{R}_y^{-1}\mathbf{d}_0)/r$, then we will get from the (3.32)

$$\begin{aligned} \mathbf{w} &= \frac{\mathbf{R}_y^{-1}\mathbf{d}_0-\beta\mathbf{R}_y^{-1}\mathbf{R}_y^{-1}\mathbf{d}_0}{\mathbf{d}_0^H\mathbf{R}_y^{-1}\mathbf{d}_0[\mathbf{I}-\beta\mathbf{R}_y^{-1}]} \\ &= \frac{\mathbf{R}_y^{-1}\mathbf{d}_0-\beta\mathbf{R}_y^{-1}\mathbf{R}_y^{-1}\mathbf{d}_0}{\mathbf{d}_0^H\mathbf{R}_y^{-1}\mathbf{d}_0}\left[\frac{1}{[\mathbf{I}-\beta\mathbf{R}_y^{-1}]}\right] \end{aligned} \tag{3.33}$$

Taking the part before the brackets in (3.33)

$$\frac{\mathbf{R}_y^{-1}\mathbf{d}_0 - \beta\mathbf{R}_y^{-1}\mathbf{R}_y^{-1}\mathbf{d}_0}{\mathbf{d}_0^H\mathbf{R}_y^{-1}\mathbf{d}_0} = \frac{\mathbf{R}_y^{-1}\mathbf{d}_0}{\mathbf{d}_0^H\mathbf{R}_y^{-1}\mathbf{d}_0} - \frac{\beta\mathbf{R}_y^{-1}\mathbf{R}_y^{-1}\mathbf{d}_0}{\mathbf{d}_0^H\mathbf{R}_y^{-1}\mathbf{d}_0}$$

We know that

$$\mathbf{v} = \frac{\mathbf{R}_y^{-1}\mathbf{R}_y^{-1}\mathbf{d}_0}{\tilde{\mathbf{d}}_0^H\tilde{\mathbf{R}}_y^{-1}\tilde{\mathbf{d}}_0},$$

$$\& \ where$$

$$r = \mathbf{d}_0^H\mathbf{R}_y^{-1}\mathbf{d}_0$$

and

$$\tilde{\mathbf{w}} = \frac{\mathbf{R}_y^{-1}\mathbf{d}_0}{\mathbf{d}_0^H\tilde{\mathbf{R}}_y^{-1}\tilde{\mathbf{d}}_0}$$

So, we can write

$$\frac{\mathbf{R}_y^{-1}\mathbf{d}_0 - \beta\mathbf{R}_y^{-1}\mathbf{R}_y^{-1}\mathbf{d}_0}{\mathbf{d}_0^H\mathbf{R}_y^{-1}\mathbf{d}_0} = (\tilde{\mathbf{w}} - \beta\mathbf{v})$$

Now taking the second part of the (3.33), that is $\frac{1}{\mathbf{I} - \beta\mathbf{R}_y^{-1}}$. We can write

$$\beta\mathbf{R}_y^{-1} = r\beta\tilde{\mathbf{w}}^H\mathbf{w}$$

$$= \mathbf{d}_0^H\mathbf{R}_y^{-1}\mathbf{d}_0\beta\frac{\mathbf{d}_0^H\mathbf{R}_y^{-H}}{\tilde{\mathbf{d}}_0^H\tilde{\mathbf{R}}_y^{-H}\tilde{\mathbf{d}}_0}\square\frac{\mathbf{R}_y^{-1}\mathbf{d}_0}{\mathbf{d}_0^H\mathbf{R}_y^{-1}\mathbf{d}_0}$$

$$= \frac{\beta\mathbf{d}_0^H\mathbf{R}_y^{-H}\mathbf{d}_0\mathbf{R}_y^{-1}}{\mathbf{d}_0^H\mathbf{R}_y^{-H}\mathbf{d}_0} = \beta\mathbf{R}_y^{-1}$$

$$or$$

$$\beta\mathbf{R}_y^{-1} = \beta\mathbf{R}_y^{-1}$$

Then the second part of the (3.33) can be written as

$$\frac{1}{\mathbf{I} - \beta\mathbf{R}_y^{-1}} = \frac{1}{\mathbf{I} - r\beta\tilde{\mathbf{w}}^H\mathbf{w}}$$

The equation (3.33) will now become

$$\mathbf{w} = (\tilde{\mathbf{w}} - \beta \mathbf{v}) \left(\frac{1}{1 - r\beta \tilde{\mathbf{w}}^H \mathbf{w}} \right)$$

$$= \frac{(\tilde{\mathbf{w}} - \beta \mathbf{v})}{1 - r\beta \tilde{\mathbf{w}}^H \mathbf{w}} \tag{3.34}$$

In the case, $\tilde{\mathbf{w}}$ do not convince norm constraint, we can put the (3.34) in to (3.22). this will help us to solve the equality for the β .As by (3.22)

$$\mathbf{w}^H \mathbf{w} \leq T_0$$

or

$$\mathbf{w}^H \mathbf{w} - T_0 = 0 \tag{3.35}$$

(3.35) can be formulated as

$$\left(\frac{(\tilde{\mathbf{w}} - \beta \mathbf{v})}{1 - r\beta \tilde{\mathbf{w}}^H \mathbf{w}} \right)^H \left(\frac{(\tilde{\mathbf{w}} - \beta \mathbf{v})}{1 - r\beta \tilde{\mathbf{w}}^H \mathbf{w}} \right) - T_0 = 0$$

$$\frac{(\tilde{\mathbf{w}} - \beta \mathbf{v})^H}{(1 - r\beta \tilde{\mathbf{w}}^H \mathbf{w})^H} \frac{(\tilde{\mathbf{w}} - \beta \mathbf{v})}{(1 - r\beta \tilde{\mathbf{w}}^H \mathbf{w})} - T_0 = 0$$

$$\frac{(\tilde{\mathbf{w}} - \beta \mathbf{v})^H (\tilde{\mathbf{w}} - \beta \mathbf{v}) - T_0 (1 - r\beta \tilde{\mathbf{w}}^H \mathbf{w})^H (1 - r\beta \tilde{\mathbf{w}}^H \mathbf{w})}{(1 - r\beta \tilde{\mathbf{w}}^H \mathbf{w})^H (1 - r\beta \tilde{\mathbf{w}}^H \mathbf{w})} = 0$$

$$\| (\tilde{\mathbf{w}} - \beta \mathbf{v}) \|^2 - T_0 \| (1 - r\beta \tilde{\mathbf{w}}^H \mathbf{w}) \|^2 = 0$$

$$\| \tilde{\mathbf{w}} \|^2 + \beta^2 \| \mathbf{v} \|^2 - 2\tilde{\mathbf{w}}^H \beta \mathbf{v} - T_0 [1 + r^2 \beta^2 \| \mathbf{w} \|^4 - 2r\beta \| \tilde{\mathbf{w}} \|^2] = 0$$

$$\| \tilde{\mathbf{w}} \|^2 + \beta^2 \| \mathbf{v} \|^2 - 2\tilde{\mathbf{w}}^H \beta \mathbf{v} - T_0 - T_0 r^2 \beta^2 \| \mathbf{w} \|^4 + T_0 2r\beta \| \tilde{\mathbf{w}} \|^2 = 0$$

$$\beta^2 (\| \mathbf{v} \|^2 - T_0 r^2 \| \mathbf{w} \|^4) + \beta (T_0 2r \| \tilde{\mathbf{w}} \|^2 - 2\tilde{\mathbf{w}}^H \mathbf{v}) + (\| \tilde{\mathbf{w}} \|^2 - T_0) = 0 \tag{3.36}$$

where

$$a = \| \mathbf{v} \|^2 - T_0 r^2 \| \mathbf{w} \|^4,$$

$$b = 2[-\mathrm{Re}(\tilde{\mathbf{w}}^H v) + rT_0 \| \tilde{\mathbf{w}} \|^2]$$

&

$$c = \| \tilde{\mathbf{w}} \|^2 - T_0$$

Therefore, the (3.36) will become

$$a\beta^2 + b\beta + c = 0 \tag{3.37}$$

To make sure the constancy, the solution of β in (3.37) by using the quadratic equation can be written as

$$\beta = \frac{-b - \mathrm{Re}(\sqrt{b^2 - 4ac})}{2(a)} \tag{3.38}$$

The estimate to optimal packing level is the solution of β. The β has real and non negative value. The $\mathbf{R}_y$ can be rotten in conditions of the eigen-vectors $\mathbf{u}_i$ & eigen-values Λ_i, $(i = 1, \ldots, M)$

$$\mathbf{R}_y = \sum_{i=1}^{M} \Lambda_i u_i u_i^H \tag{3.39}$$

$(\mathbf{R}_y + \beta \mathbf{I}_{M\times M})^{-1}$ is afterward decomposed as following

$$(\mathbf{R}_y + \beta \mathbf{I}_{M\times M})^{-1} = \sum_{i=1}^{M} \mathbf{u}_i \mathbf{u}_i^H / (\Lambda_i + \beta) \tag{3.40}$$

where eigen-values Λ_i is the positive since $\mathbf{R}_y$ is the positive exactly.

The data bit of the preferred user is demodulated by

$$\hat{b}_0 = \mathrm{sgn}(z) = \mathrm{sgn}\{\Re e(\mathbf{w}^H \mathbf{y})\} \tag{3.41}$$

The table given below summarizes the RLS algorithm for revising the weight vector $\mathbf{w}$(n) with $\mathbf{I}_{M\times M}$ identity matrix and δ has small stable positive value. The table given below is the suitable adaptive realization of the Robust RLS based detector or in other words Robust CMOE detector.

Robust RLS algorithm

$initialization: \mathbf{w}(0) = 0, \mathbf{P}[0]\delta^{-1}\mathbf{I}_{M \cdot M}$

$Updation: K[n] = \frac{\mathbf{P}[n-1]\mathbf{y}[n]}{\lambda + \mathbf{y}^H[n]\mathbf{P}[n-1]\mathbf{y}[n]}$

$\mathbf{P}[n] = \frac{1}{\lambda}\{\mathbf{P}[n-1] - \mathbf{K}[n]^* \mathbf{y}^H[n]\mathbf{P}[n-1],$

$\mathbf{w}[n] = (\mathbf{d}_0^H[n]\mathbf{P}[n]\mathbf{d}_0[n])^{-1}\mathbf{P}[n]\mathbf{d}_0[n];$

$\mathbf{w}_q[n] = \mathbf{d}_0[n](\mathbf{d}_0^H[n]\mathbf{d}_0[n])^{-1}, \ T_0 = \| \mathbf{w}_q[n] \|^2,$

$if \| \mathbf{w}_q[n] \|^2 > T_0,$

$r = (\mathbf{d}_0^H[n]\mathbf{P}[n]\mathbf{d}_0[n]), \mathbf{v}[n] = r\mathbf{P}[n]\mathbf{w}[n],$

$a = \| \mathbf{v}[n] \|^2 - T_0 r^2 \| \mathbf{w}[n] \|^4,$

$b = 2[-\Re e(\mathbf{w}[n]^H \mathbf{v}[n]) + rT_0 \| \mathbf{w}[n] \|^2],$

$c = \| \mathbf{w}[n] \|^2 - T_0$

$\beta = \frac{-b - \mathrm{Re}(\sqrt{b^2 - 4ac})}{2(a)}$

$\mathbf{w}[n] = (\mathbf{w}[n] - \beta\mathbf{v}[n]) / (1 - r\beta\mathbf{w}[n]^H \mathbf{w}[n]).$

The complexity of the update procedure for the w(n) to satisfy the norm is O(M). Hence, the complexity of the Robust RLS base detector is determined by RLS updating. Therefore, its complexity is $O(M^2)$.

Chapter 4
Imitations and Results

CHAPTER 4

Imitations and Results

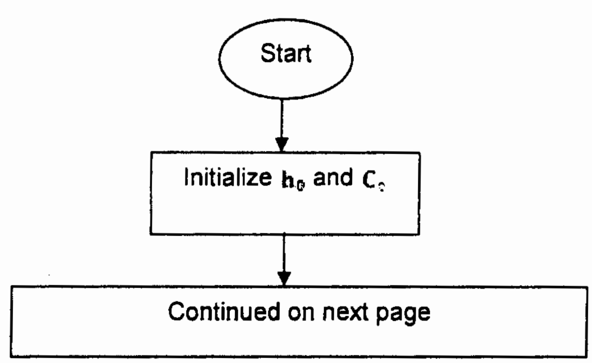
4.1 Introduction:

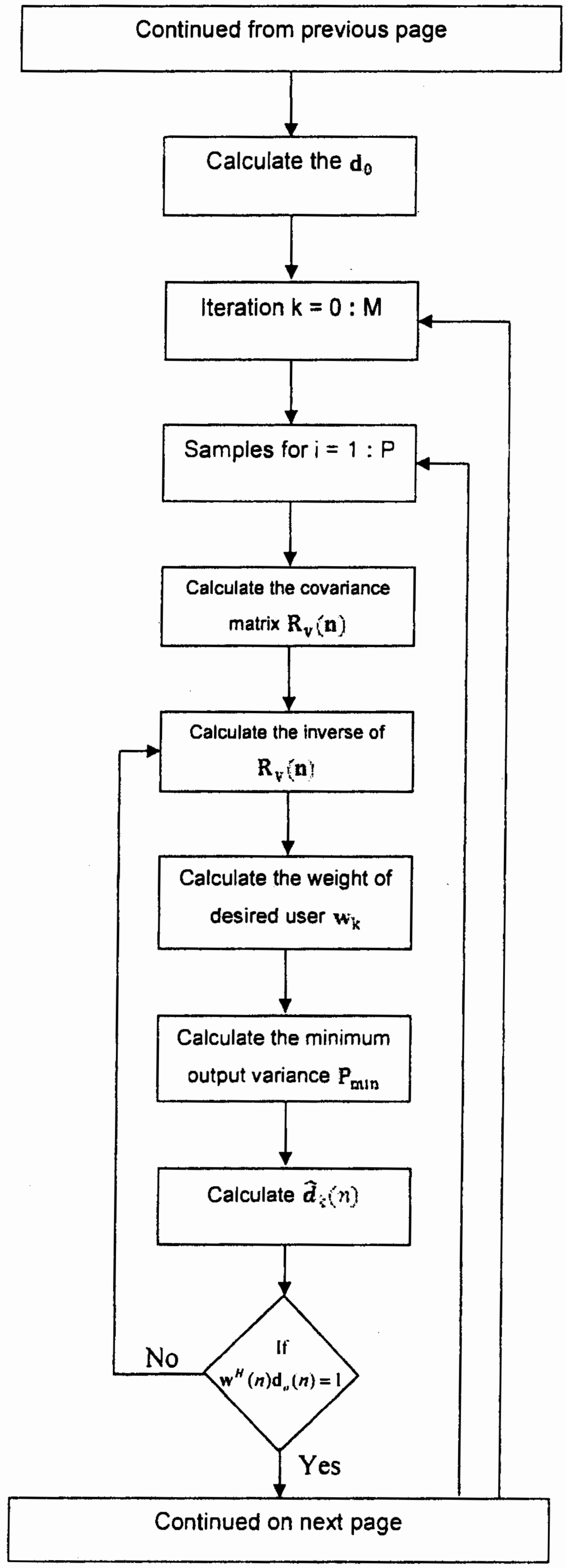
This chapter will show the simulation and consequences of the work shown in the system design chapter. The focused detection is Robust RLS based detection for the MC-CDMA but for comparison CMOE detection was also implemented programmatically. This chapter also gives these two specific algorithms. The simulations have been developed in Matlab 7.3 with functions and programs written for CMOE and Robust RLS based Detector.

Both CMOE and Robust RLS based Detector were pertained on the MC-CDMA revealing process individually. The evaluation of these consequences is stands on diverse parameters such as number of iterations, SNR, SINR, BER etc.

4.2 The Algorithm for CMOE and Robust RLS based Detection:

The algorithm of CMOE Detection is shown in figure 4.1. This flow chart evidently gives details of the functioning for the CMOE Detection of MC-CDMA System.




Continued from previous page
Calculate the $\mathbf{d}_0$
Iteration k = 0 : M
Samples for i = 1 : P
Calculate the covariance matrix $\mathbf{R_v(n)}$
Calculate the inverse of $\mathbf{R_v(n)}$
Calculate the weight of desired user $\mathbf{w}_k$
Calculate the minimum output variance $\mathbf{P}_{min}$
Calculate $\hat{d}_k(n)$
If $\mathbf{w}^H(n)\mathbf{d}_u(n) = 1$
No
Yes
Continued on next page

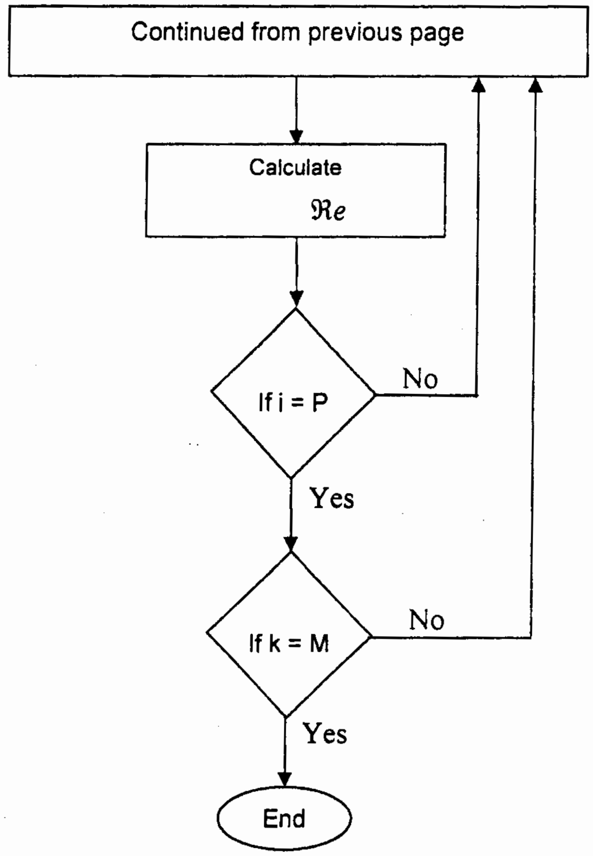
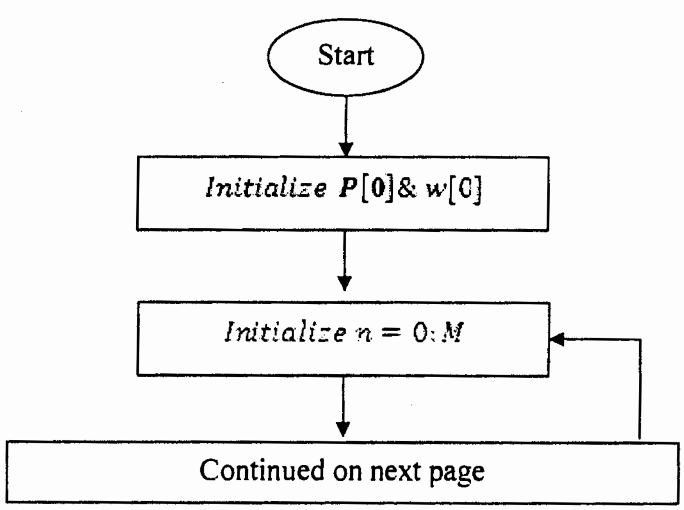
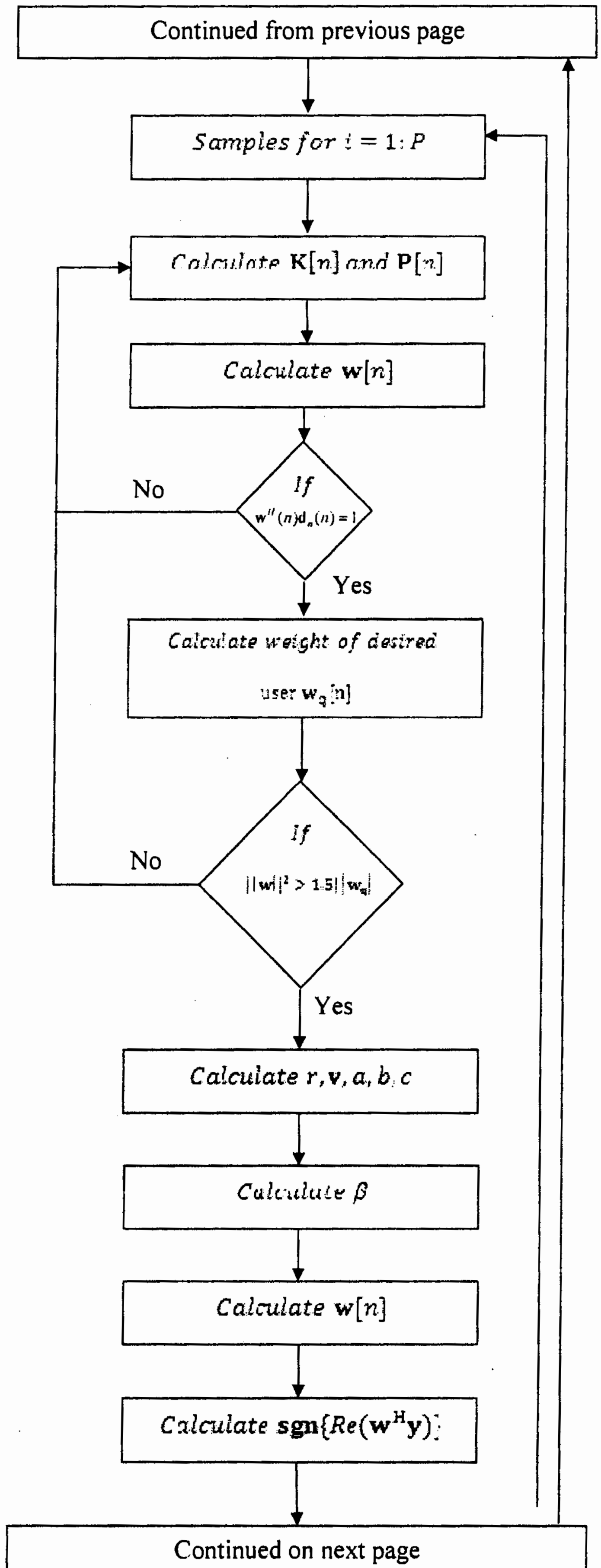


Figure.4.1. General Algorithm for CMOE Detection of MC-CDMA

The algorithm of Robust RLS based Detection is shown in figure 4.2. This flow chart evidently gives details of the functioning for the Robust RLS Detection of MC-CDMA System.





Continued from previous page
Samples for i = 1: P
Calculate K[n] and P[n]
Calculate w[n]
No
If
w^H(n)d_a(n) = 1
Yes
Calculate weight of desired
user w_q[n]
No
If
||w||² > 1.5||w_q||
Yes
Calculate r, v, a, b, c
Calculate β
Calculate w[n]
Calculate sgn{Re(w^H y)}
Continued on next page

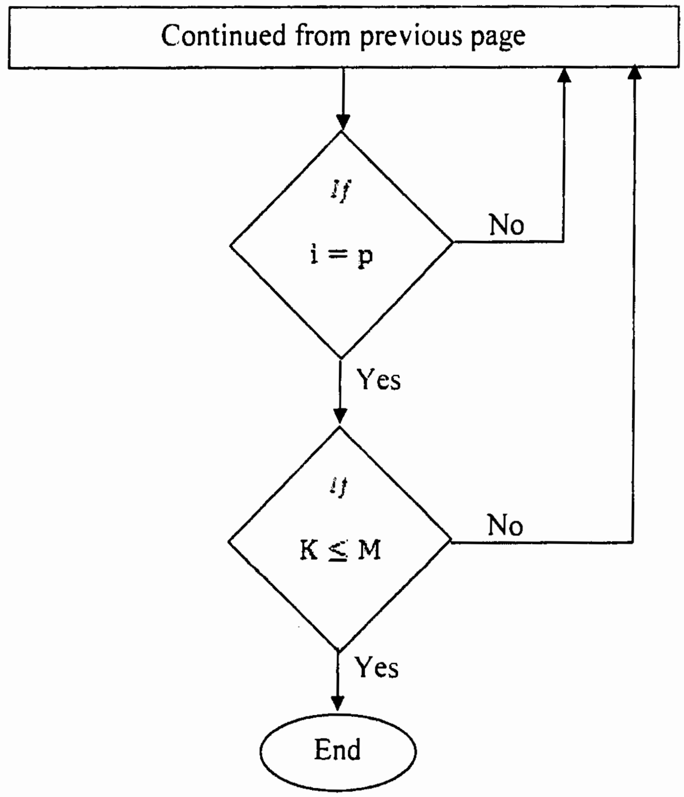


Figure.4.2. General Algorithm for Robust RLS based detection of MC-CDMA

4.3. Imitations

The simulations help for the comparison among the CMOE detection and Robust RLS based detection of the MC-CDMA system. The CMOE and Robust RLS Based detection were implemented for distinct values of SNR and many functional results are gotten from it.

In this MC-CDMA system, the values are M=16, P=8, K=8, λ=0.995 and δ=0.1. Hence, the original data sequence is first S/P converted to 8 data streams. Then extra guard intervals of size 8 were additional to mitigate the ISI. There are 8 concurrent users in channel and the preferred user is zero. The interference signal ratio of MAI is 10 dB superior than the preferred user, i.e.; $P_k / P_0 = 10$. By Rayleigh Fading supposition, the channel coefficients are produced randomly following to a complex Gaussian distribution. The SINR analysis is applied on basis of SINR analysis for LMMSE detectors in [1].

Based on the above data, figure 4.3 shows the average SINR versus number of data samples for the CMOE and Robust RLS based detection. When the numbers of samples are small the Robust RLS based detector is better than the CMOE detector. It can be easily seen from the graph that when the number of users increases by 2500, the performance of both detectors is nearly equivalent.

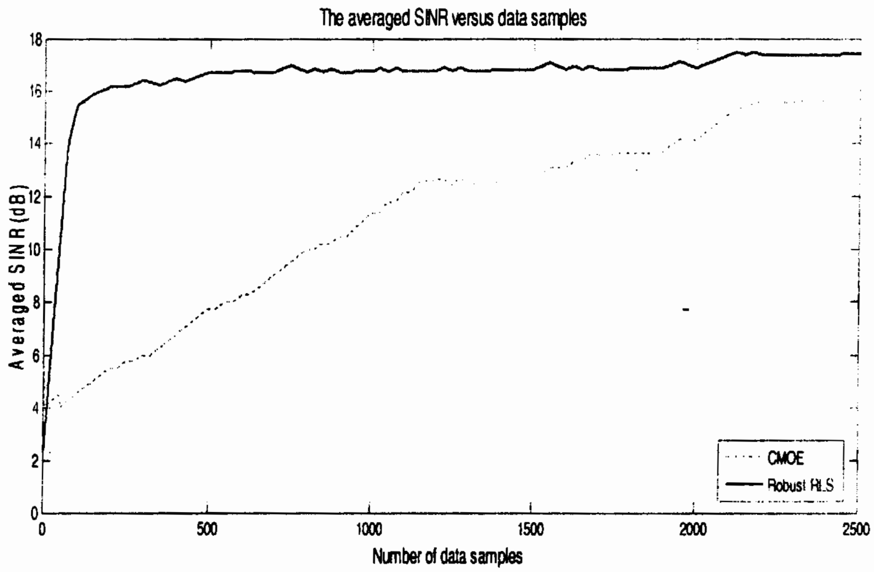


Figure.4.3. The average SINR versus data samples

At the low number of samples the Robust RLS detector performs better than the CMOE detector. But at large number of samples the performance of CMOE detector is nearly equivalent to the Robust RLS detector.

Figure 4.4(a) and (b) shows the Average SINR and MSE Versus SNR for the CMOE and Robust RLS based Detector.

The performance of the Robust RLS based detector is better than the CMOE detector from figures 4.4(a) and (b).At low SNR Robust RLS detector is better than the CMOE detector. But, the performance of the CMOE detector is nearly equivalent to the Robust RLS based detector as the SNR increases.

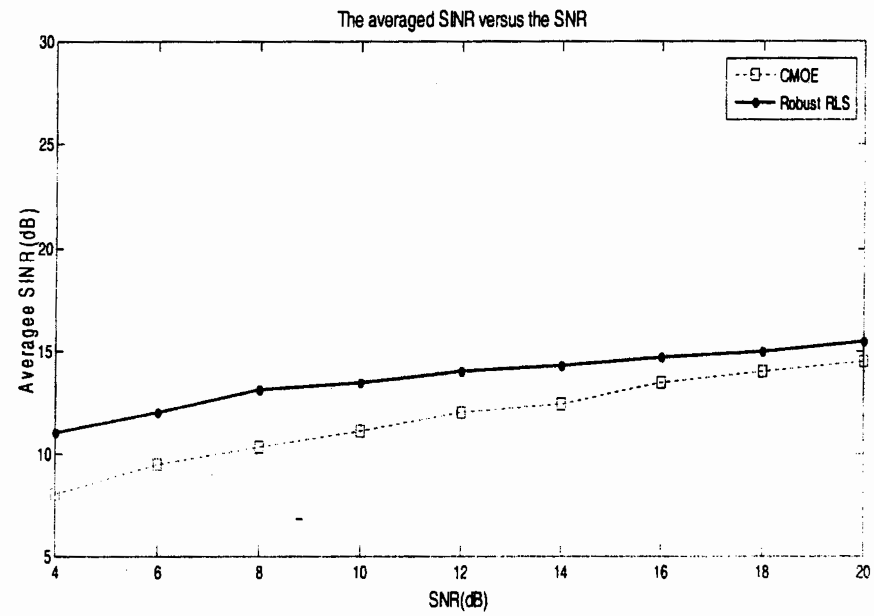


Figure.4.4 (a). The average SINR versus SNR

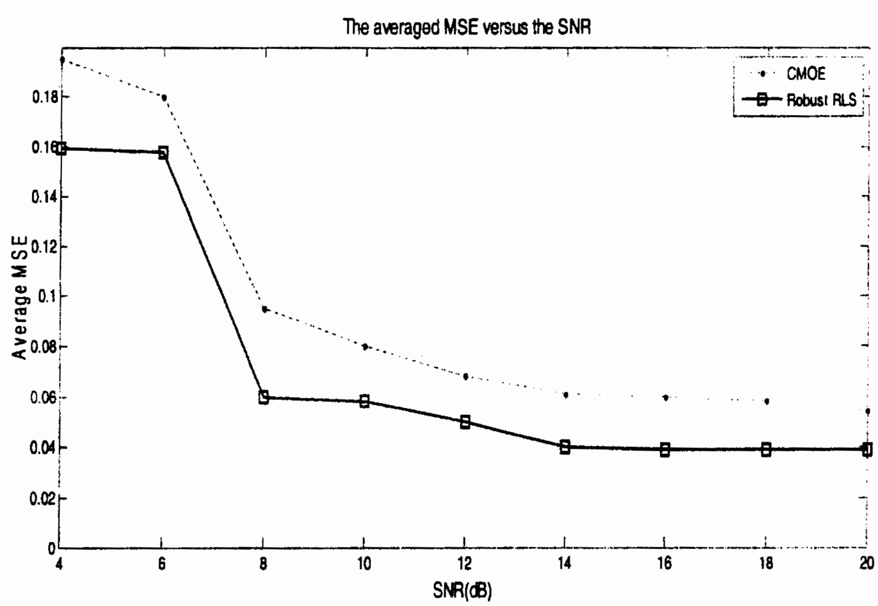


Figure.4.4 (b). The MSE versus SNR

Figure 4.5 shows the Average SINR versus number of sample for the CMOE detector and Robust RLS based detector.

The performance of CMOE detector is nearly equivalent to the Robust RLS detector for large number of samples from figure 4.5. When the numbers of samples are small the Robust RLS based detector is better than the CMOE detector. It can be easily seen from the graph that when the number of users increases by 2500, the performance of both detectors is nearly equivalent.

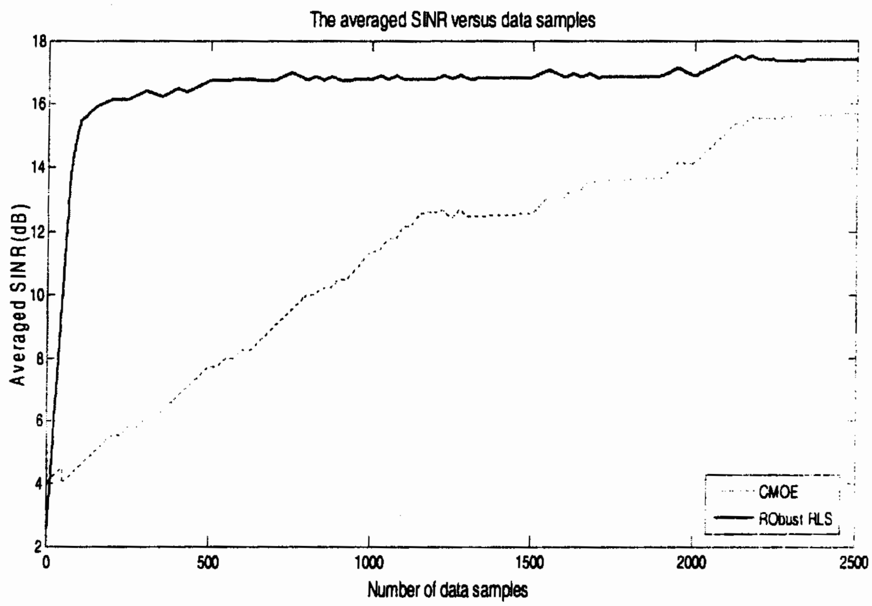


Figure.4.5. The average SINR versus number of samples

Figure 4.6 shows the BER versus the SNR for the CMOE detector and Robust RLS based detector. At low SNR the bit error rate for Robust RLS based detect is smaller than the CMOE. At high SNR, the performance of both detectors is equivalent. The Robust RLS based detector performs better than the CMOE detector with respect to the bit error rate.

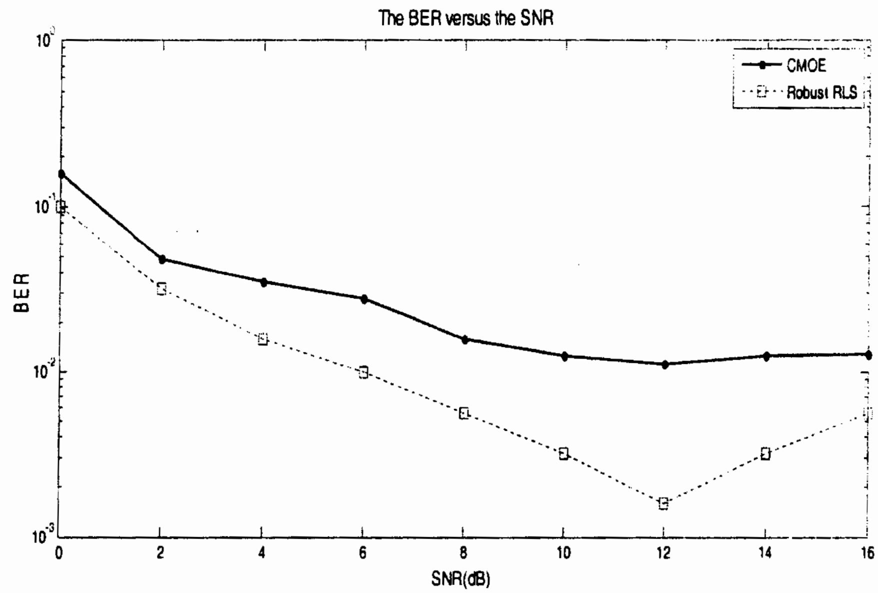


Figure.4.6. The BER versus SNR

Figure 4.7 shows the mean square error versus SNR for the Robust RLS based detector and CMOE detector. At low SNR the mean square error for Robust RLS based detect is smaller than the CMOE. At high SNR, the performance of both detectors is equivalent.

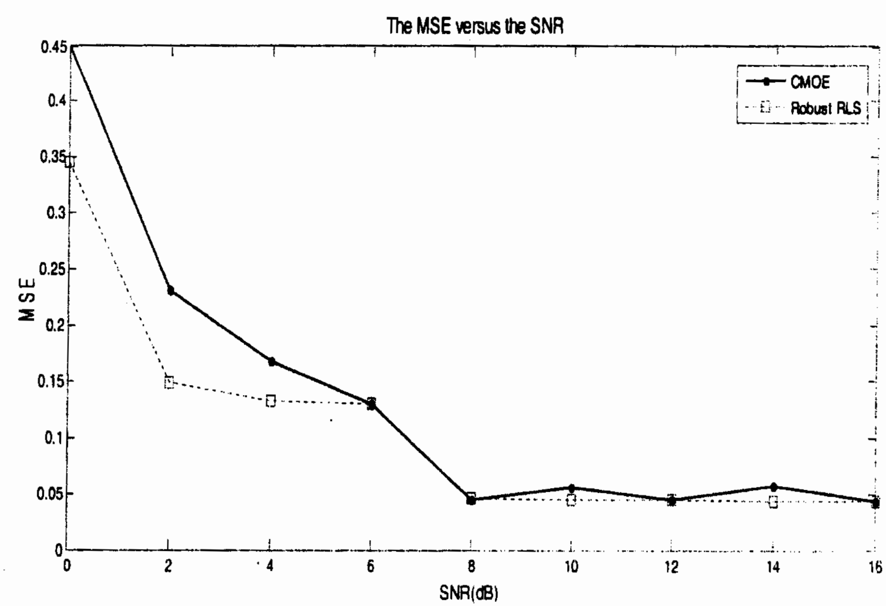


Figure.4.7. The MSE versus SNR

The performance of the Robust RLS based detector is nearly equivalent to the CMOE detector at the medium SNR.

4.4. Conclusion

In this thesis the Robust RLS detector based on the CMOE detection of MC-CDMA system has been studied and implemented. The Robust RLS based detector is efficient than the CMOE detector in case of the signal modeling errors occurrence. The results show that at medium SNR, the performance of the Robust RLS based detector is similar to the CMOE detector as they have same MSE at the middle SNR.

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Appendix

```
SINRmoe=[];
SNR=[4,6,8,10,12,14,16,18];
Rini=[];
y=[];
b=1;
U=[];
S=[];
u=[];
it=700;
p=[];
w=[];
wq=[];
del=0.1;
idel=inv(del);
p=idel*(eye(8,8));

temp=[];
go_est=[];
ohm=[];
 SINRopt=[];
for l=1:it
  for i=1:((it*8*8)-1)
     if(rand(1)<0.5)
         b_tem=-1;
     else
         b_tem=1;
     end
         b=[b;b_tem];
  end
 b=reshape(b,8,8,it);


 C=[1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1;1,1,1,1,1,1,1,1,-1,-1,-1,-1,-1,-1,
-1,-1;1,1,-1,-1,-1,-1,1,1,1,1,-1,-1,-1,-1,1,1;1,1,-1,-1,-1,-1,1,1,-1,
-1,1,1,1,1,-1,-1;-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1;-1,
-1,-1,-1,1,1,1,1,-1,-1,-1,-1,1,1,1,1;-1,-1,-1,-1,1,1,1,1,1,1,1,1,-1,
-1,-1,-1;-1,1,1,-1,-1,1,1,-1,-1,1,1,-1,-1,1,1,-1;];
 for l=1:it
 for i=1:8
     for j=1:8
         u(j,:,i,it)=b(i,j,it)*C(i,:);
    end
 end
 end

 for l=1:it
    for i=1:8
           U(i,:,l)= reshape(u(:,:,i,l)',128,1);
           S(i,:,l)=ifft(U(i,:,l));
     end
 end

 c=ceil(rand(128,1)*10);
 r=ceil(rand(128,1)*10);
 G=toeplitz(r,c);
 rs=[];

 for snrit=1:8
for l=1:it
```

```
temp(:,:,l)=S(:,:,l)*G(:,:);
rs(:,:,l)=awgn(temp(:,:,l),SNR(snrit));
end

 for l=1:it
    for i=1:8
        y(i,:,l)=fft(rs(i,:,l));
     end
 end

 pgain=G(:,1);
 N=128;
 go=pgain(1,1);
 Fm=zeros(16,8);
 for i=1:16
     for k=1:8
            Fm(i,k)=exp((-j*2*pi*i*k)/N)/N;
     end
 end

 Fmh=Fm.';
  Co=Coh.';
  go=rand(1);
  goh=go.';


 for l=1:it
    Ryi(:,:,l) = inv(cov(y(:,:,l)));
    ohm(:,:,l)=Fmh*Coh*Ryi(:,:,l)*Co*Fm;
    [Vectors values]=eig(ohm(l));
    [a b1]=min(diag(values));
    go_est(l)=goh*a'*go;
 end

 err=[];

 for l=1:it
    err(l)=go-go_est(l);
 end

%----------------------------------------
M=16;
lam=0.995;
dlam=1/lam;
po=1;
h=[];
dk=[];
 v1=((1-lam)*(M-1))/(2*lam);
N=128;
%----------------------------------------------------------


   for l=1:it
      for i=1:8
         for k=1:128
                h(k)=pgain(k)*exp((-j*2*pi*(i-1)*(k-1))/N)/N;
         end
      end
   end
```

```
v=[];
 d0=[0.0834;0.0834;0.0834;0.0834;0.0834;0.0834;0.0834;0.0834;];
w=zeros(M,1);
for l=1:it
  k=((p*y(:,:,l))/(lam+(y(:,:,l)'*p*y(:,:,l))));
  %error for dimension 16*16 because y 8*128
  p=dlam*(p-(k*y(:,:,l)'*p));
  %-----------error for dimension 8*8 p
  w=(inv(d0'*p*d0))*p*d0;
  wq=d0*inv(d0'*d0);
%-------------------------------------
    t0=norm(wq);
    t1=norm(w);
    t2=t1*t1;

    while(t1>t0)
       r=d0'*p*d0;
       v=r*p*w;
       nv=norm(v);
       a=nv-t0*r*r*t2;
       b=2*((-1*real(w'*v))+(r*t0*t1));
       c=t1-t0;
       beta=(-b-real(sqrt((b*b)-4*a*c)))/(2*a);
       w=(w-beta*v)/(1-r*beta*w'*w);

        t1=norm(w);

     end
end


    Col=reshape(C,1,128);
    Cohl=diag(Col);
     for l=1:it
       Cohl=eye(128,128);
       d=Cohl*h';
       Rs(:,:,l)=d*d';
     end


for l=1:it
b2(:,:,l)=w.'*yy(:,:,l);
end

ber=b2(8,8,l)-b(8,8,l);
end
t=[1:it];
plot(ber,SNR)



SINRmoe=[];
Rini=[];
SNR=15;
y=[];
b=1;
```

```
U=[];
S=[];
u=[];
it=700;
p=[];
w=[];
wq=[];
del=0.1;
idel=inv(del);
p=idel*(eye(8,8));
SINR=[];
SINRmoe=[];
temp=[];
go_est=[];
ohm=[];
 SINRopt=[];

  for i=1:((it*8*8)-1)
     if(rand(1)<0.5)
         b_tem=-1;
     else
         b_tem=1;
     end
         b=[b;b_tem];
  end
 b=reshape(b,8,8,it);


 C=[1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1;1,1,1,1,1,1,1,1,-1,-1,-1,-1,-1,-1,
-1,-1;1,1,-1,-1,-1,-1,1,1,1,1,-1,-1,-1,-1,1,1;1,1,-1,-1,-1,-1,1,1,-1,
-1,1,1,1,1,-1,-1;-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1;-1,
-1,-1,-1,1,1,1,1,-1,-1,-1,-1,1,1,1,1;-1,-1,-1,-1,1,1,1,1,1,1,1,1,-1,
-1,-1,-1;-1,1,1,-1,-1,1,1,-1,-1,1,1,-1,-1,1,1,-1;];
 for l=1:it
 for i=1:8
     for j=1:8
         u(j,:,i,it)=b(i,j,it)*C(i,:);
    end
 end
 end

 for l=1:it
    for i=1:8
            U(i,:,l)= reshape(u(:,:,i,l)',128,1);
            S(i,:,l)=ifft(U(i,:,l));
     end
 end

 c=ceil(rand(128,1)*10);
 r=ceil(rand(128,1)*10);
 G=toeplitz(r,c);
 rs=[];

 for l=1:it
        temp(:,:,l)=S(:,:,l)*G(:,:);
        rs(:,:,l)=awgn(temp(:,:,l),SNR);
 end

 for l=1:it
    for i=1:8
```

```
        y(i,:,l)=fft(rs(i,:,l));
    end
end

pgain=G(:,1);
N=128;
go=pgain(1,1);
Fm=zeros(16,8);
for i=1:16
    for k=1:8
           Fm(i,k)=exp((-j*2*pi*i*k)/N)/N;
    end
end

Fmh=Fm.';
 Co=Coh.';
 go=rand(1);
 goh=go.';


for l=1:it
   Ryi(:,:,l) = inv(cov(y(:,:,l)));
   ohm(:,:,l)=Fmh*Coh*Ryi(:,:,l)*Co*Fm;
   [Vectors values]=eig(ohm(l));
   [a b1]=min(diag(values));
   go_est(l)=goh*a'*go;
end

err=[];

for l=1:it
   err(l)=go-go_est(l);
end

M=16;
lam=0.995;
dlam=1/lam;
po=1;
h=[];
dk=[];
 v1=((1-lam)*(M-1))/(2*lam);
N=128;
%-------------------------------------------------------


   for l=1:it
      for i=1:8
         for k=1:128
                h(k)=pgain(k)*exp((-j*2*pi*(i-1)*(k-1))/N)/N;
         end
      end
    end


v=[];
 d0=[0.0834;0.0834;0.0834;0.0834;0.0834;0.0834;0.0834;0.0834;];
w=zeros(M,1);
for l=1:it
  k=((p*y(:,:,l))/(lam+(y(:,:,l)'*p*y(:,:,l))));
```

```
    %&error for dimension 16*16 becauze y 8*128
    p=dlam*(p-(k*y(:,:,l)'*p));
    %----------error for dimension 8*8 p
    w=(inv(d0'*p*d0))*p*d0;
    wq=d0*inv(d0'*d0);
    t0=norm(wq);
    t1=norm(w);
    t2=t1*t1;

     while(t1>t0)
         r=d0'*p*d0;
         v=r*p*w;
         nv=norm(v);
         a=nv-t0*r*r*t2;
         b=2*((-1*real(w'*v))+(r*t0*t1));
         c=t1-t0;
         beta=(-b-real(sqrt((b*b)-4*a*c)))/(2*a);
         w=(w-beta*v)/(1-r*beta*w'*w);
         t1=norm(w);

      end
end

     Col=reshape(C,1,128);
     Cohl=diag(Col);
      for l=1:it
     Cohl=eye(128,128);
      d=Cohl*h';
      Rs(:,:,l)=d*d';

      end

  for l=1:it
       Rini(:,:,l)=(cov(y(:,:,l))-Rs(:,:,l));
       SINR(l)=(w'*Rs(:,:,l)*w)/(w'*Rini(:,:,l)*w);

    end
t=[1:it];
plot(t,(SINR/it))


SINRmoe=[];
SNR=[4,6,8,10,12,14,16,18];
Rini=[];
y=[];
b=1;
U=[];
S=[];
u=[];
it=700;
p=[];
w=[];
wq=[];
del=0.1;
idel=inv(del);
p=idel*(eye(8,8));
temp=[];
go_est=[];
ohm=[];
```

```
 SINRopt=[];
  for i=1:((it*8*8)-1)
      if(rand(1)<0.5)
          b_tem=-1;
      else
          b_tem=1;
      end
          b=[b;b_tem];
  end
 b=reshape(b,8,8,it);


 C=[1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1;1,1,1,1,1,1,1,1,-1,-1,-1,-1,-1,-1,
-1,-1;1,1,-1,-1,-1,-1,1,1,1,1,-1,-1,-1,-1,1,1;1,1,-1,-1,-1,-1,1,1,-1,
-1,1,1,1,1,-1,-1;-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1;-1,
-1,-1,-1,1,1,1,1,-1,-1,-1,-1,1,1,1,1;-1,-1,-1,-1,1,1,1,1,1,1,1,1,-1,
-1,-1,-1;-1,1,1,-1,-1,1,1,-1,-1,1,1,-1,-1,1,1,-1;];
 for l=1:it
  for i=1:8
     for j=1:8
         u(j,:,i,it)=b(i,j,it)*C(i,:);
    end
  end
 end

 for l=1:it
    for i=1:8
            U(i,:,l)= reshape(u(:,:,i,l)',128,1);
            S(i,:,l)=ifft(U(i,:,l));
     end
 end

 c=ceil(rand(128,1)*10);
 r=ceil(rand(128,1)*10);
 G=toeplitz(r,c);
 rs=[];

 for snrit=1:8
  for l=1:it
    temp(:,:,l)=S(:,:,l)*G(:,:);
    rs(:,:,l)=awgn(temp(:,:,l),SNR(snrit));
  end

 for l=1:it
    for i=1:8
        y(i,:,l)=fft(rs(i,:,l));
     end
 end

 pgain=G(:,1);
 N=128;
 go=pgain(1,1);
 Fm=zeros(16,8);
 for i=1:16
     for k=1:8
            Fm(i,k)=exp((-j*2*pi*i*k)/N)/N;
     end
 end
```

```
Fmh=Fm.';
Co=Coh.';
go=rand(1);
goh=go.';

for l=1:it
   Ryi(:,:,l) = inv(cov(y(:,:,l)));
   ohm(:,:,l)=Fmh*Coh*Ryi(:,:,l)*Co*Fm;
   [Vectors values]=eig(ohm(l));
   [a b1]=min(diag(values));
   go_est(l)=goh*a'*go;
end

err=[];

for l=1:it
   err(l)=go-go_est(l);
end


M=16;
lam=0.995;
dlam=1/lam;
po=1;
h=[];
dk=[];
 v1=((1-lam)*(M-1))/(2*lam);
N=128;

  for l=1:it
     for i=1:8
        for k=1:128
               h(k)=pgain(k)*exp((-j*2*pi*(i-1)*(k-1))/N)/N;
        end
     end
   end

v=[];
 d0=[0.0834;0.0834;0.0834;0.0834;0.0834;0.0834;0.0834;0.0834;];

w=zeros(M,1);
for l=1:it
   k=((p*y(:,:,l))/(lam+(y(:,:,l)'*p*y(:,:,l))));
   %      error for dimension 16*16 becauze y 8*128
   p=dlam*(p-(k*y(:,:,l)'*p));   %----------error for dimension 8*8 p
   w=(inv(d0'*p*d0))*p*d0;
   wq=d0*inv(d0'*d0);
   t0=norm(wq);
   t1=norm(w);
   t2=t1*t1;

   while(t1>t0)
       r=d0'*p*d0;
       v=r*p*w;
       nv=norm(v);
       a=nv-t0*r*r*t2;
       b=2*((-1*real(w'*v))+(r*t0*t1));
       c=t1-t0;
       beta=(-b-real(sqrt((b*b)-4*a*c)))/(2*a);
```

```
        w=(w-beta*v)/(1-r*beta*w'*w);
        t1=norm(w);

      end
  end

    Col=reshape(C,1,128);
    Coh1=diag(Col);
     for l=1:it
       Coh1=eye(128,128);
       d=Coh1*h';
       Rs(:,:,l)=d*d';
      end
SINR=[];
SINRmoe=[];

    for l=1:it
      Rini(:,:,l)=(cov(y(:,:,l))-Rs(:,:,l));
      SINR(l)=(w'*Rs(:,:,l)*w)/(w'*Rini(:,:,l)*w);
      SINRmoe(l)=SINR/((1+v1)+(v1*SINR));
    end
end
t=[1:it];
plot(SINR,SNR)
```