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Study of Poloidal Field Coils and Their Control Systems in Tokamaks

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Dedicated to;

My Father & Mother

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List of Abbreviations

CS	Central Solenoid
TF	Toroidal Field
PF	Poloidal Field
PFC	Poloidal Field Coil
GS equation	Grad-Shafranov equation
MHD	Magnetohydrodynamics
OH	Ohmic Heating
ITER	International Thermonuclear Experimental Reactor
LCFS	Last Closed Flux Surface
TOQAMEQ	Tokamak Equilibrium

Abstract

Poloidal field coils are extremely significant for plasma confinement in tokamak. These Poloidal field coils are act as electromagnetic control for controlling the electric and magnetic fields, which sustain or change the plasma position, shape, current and control plasma discharge. These also contribute to the radial and toroidal pressure stability required to sustain the plasma in equilibrium. This thesis is associated with the design of poloidal field coils and control of their currents. The mathematical model for calculation of currents in poloidal field coils as well as forces on poloidal field coils are build up by using Maxwell's equations, magnetohydrodynamics (MHD) equations and Grad-Shafranov (GS) equation. GS equation contains a poloidal flux function. We have computed poloidal flux function in expression of vector potential for circular current loop, elliptical integral and Green function. Plasma section has a field which is controlled by GS equation, but in vacuum section have a field which is achieved by solving the Laplace's equation. GS equation is derived using MHD equations. TOKAMEQ code is used as virtual design tools for poloidal field coils and control of currents in them. The TOKAMEQ code is dependent on the numerically solved GS equation. The poloidal flux cross section as an output result obtained from the TOKAMEQ code. In this cross section plasma shape is elongated. Elongation is very significant for safety factor and stability of plasma.

CHAPTER 1

1. Introduction

1.1 Tokamak Plasma Fusion

Nuclear fusion is foundation in the contact of two light nuclei which merge into a heavier and extra stable nucleus generating a huge quantity of energy. Two light nuclei are contained into an ionized gas called plasma. This plasma can be confined by means of electromagnetic forces produced by exterior magnetic fields, which is identified as magnetic confinement. Currently, the most talented magnetic confinement system is the Tokamak.

A Tokamak fundamentally is a toroidal machine that confines the hot plasma by means of a helical magnetic field [1].

The objective of controlled nuclear fusion research is to produce energy by merging two light mass nuclei to form an extra huge nucleus. This reaction is the power cause of the sun and other stars, where confinement and heating take place through compression below massive gravitational forces. On earth, probable candidates for using fusion energy are the subsequent reactions:

$$^{2}D + {}^{2}D \rightarrow {}^{3}T\ (1.01Mev) + {}^{1}p\ (3.03MeV) \tag{1.1}$$

$$^{2}D + {}^{2}D \rightarrow {}^{3}He\ (0.82MeV) + {}^{1}n\ (2.46MeV) \tag{1.2}$$

$$^{2}D + {}^{3}T \rightarrow {}^{4}He\ (3.57MeV) + {}^{1}n\ (14.06MeV) \tag{1.3}$$

$$^{2}D + {}^{3}He \rightarrow {}^{4}He\ (3.67MeV) + {}^{1}p\ (14.67MeV) \tag{1.4}$$

Beyond a doubt the most available and capable reaction for fusion reactors is a reaction in which Deuterium (D) and Tritium (T) combine, producing a Helium nucleus (He) and a neutron (n). This reaction has the biggest cross section at the least energy [2].

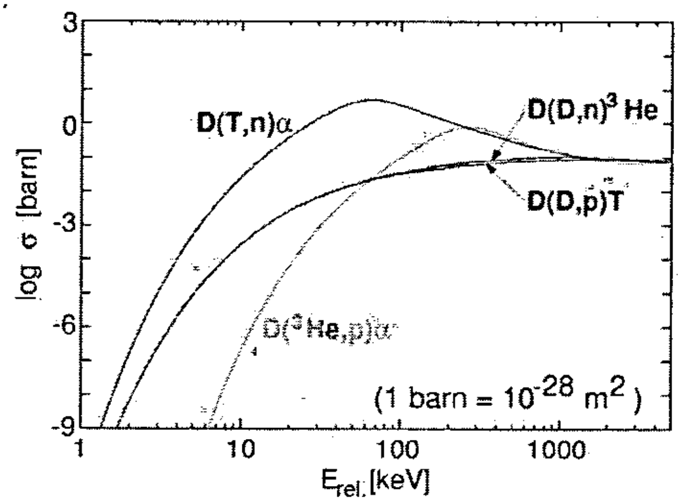

Figure-1.1: Cross section for the reactions D-T, D-D and D-^{3}He. The two D-D reactions have similar cross sections [2].

Both nuclei have to overcome the existing repulsive Coulomb force for a fusion reaction to take place. The nuclear force is energetic only for distances in the order of the nucleus size (10^{-15} m). For bigger distances, the repulsive Coulomb force dominates where potential barrier is numerous 100keV. A Deuterium and Tritium fuel merge must be confined for an enough time at an adequately tall temperature in a state where ions and electrons are separated, called a plasma state. For break-even, so-called Lawson criterion, the fusion energy unconfined equals the quantity of energy functional to heat the plasma; fusion gain $Q = \frac{P_{fus}}{P_{in}} = 1$, where P_{fus} and P_{in} are the fusion and input heating power, in that order. A step advance is the fusion ignition, where the supplementary heating can be turned off. For the Deuterium/Tritium reaction an essential condition, the so-called fusion triple product, for the ignition is:

$$nT\tau_E > 3\times10^{21} m^{-3} keVoltSecond \qquad (1.5)$$

Where n is the mean density over the plasma volume, T the mean temperature and τ_E is the energy confinement time; proportion between the energy in the plasma $W = \frac{3}{2}(nT)$

and the input heating power P_{in}, i.e. $\tau_E = \frac{W}{P_{in}}$. The necessary temperature is in the order of 10^8 K that is analogous to about one hundreds to two hundreds million degree centigrade [2].

1.2 Tokamak Machine

In view of the fact that an enormously high temperature is wanted for confinement of hot plasma is not a minor problem. At recent, two most important approaches exist; inertial and magnetic fusion.

In inertial fusion, thick, hot plasma is created and confined just for an extremely small time (nanoseconds) dictated by its inertia. In fusion reaction, influential lasers or particle beams at the same time converge on a small target (D-T fuel pellet), powerfully heating the exterior and squeezing the fuel into the centre of the pellet. The powerful heat and stress force the fuel to fuse, to a large extent similar to within a star. The fuel pellet reaches the necessary temperature and at last the burning pellet ignites.

In magnetic fusion, warm plasmas are confined by means of magnetic fields. Converse to inertial fusion, plasma densities are moderate, however the energy confinement time can be greatly longer, of the order of one second in the recent fusion machines. Magnetic fusion exploits the information that the charged particles in a magnetic field are joined to the field lines. For a toroidal machine, the magnetic field lines are closed. But, in addition to the motion of particles adjacent to the field lines and the gyro motion in the region of the field lines, the particles contain a drift velocity in the direction upright to the magnetic field and its gradient [2].

For this cause, additional magnetic field components are summed, forming helically winding field lines in the region of the centre of the torus. The helicity of the magnetic field lines stops the particles from escaping confinement due to the upright drift. In order to wind the magnetic field lines, two unlike principles are utilized; the stellarator and the tokamak.

For a stellarator, exterior coils create both the toroidal and the poloidal magnetic field components. All the magnetic fields are controlled from exterior and can stream continually, so steady state circumstances are essentially present

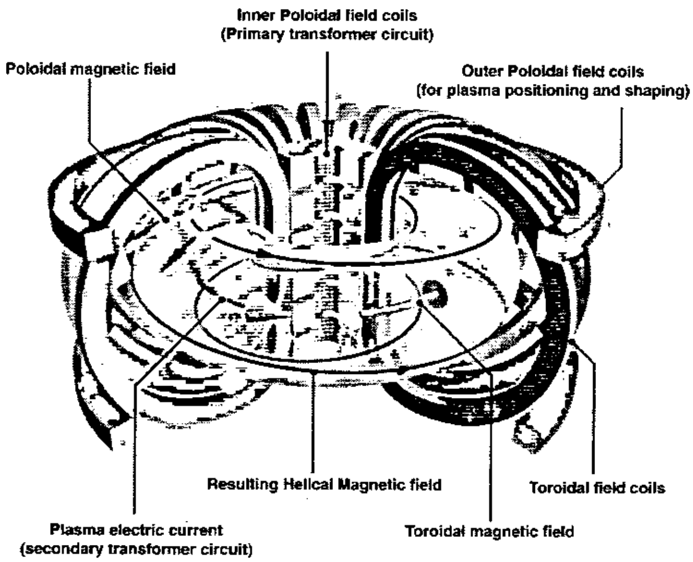

Figure-1.2: Schematic view of a tokamak [3].

For a tokamak, exterior coils construct the toroidal field component, at the same time as a toroidal current flowing inside the plasma itself creates a poloidal field component. This current is created by induction, the plasma performing as the secondary winding of a transformer through the primary winding in the centre of the torus. The magnetic field is axisymmetric in toroidal way.

The poloidal field is principally created by current in the plasma itself, this current is flowing in the toroidal way. The current also supply for plasma build-up and heating [2].

1.3 Toroidal Field

Toroidal coordinate system exposed in Figure. The toroidal magnetic field created by current carrying wire has magnitude B_0 at $R = R_0$, and drops off in power with R^{-1} is described by:

$$B_r = 0 \tag{1.6}$$

$$B_\phi = \frac{B_0 R_0}{R} \tag{1.7}$$

$$B_z = 0 \tag{1.8}$$

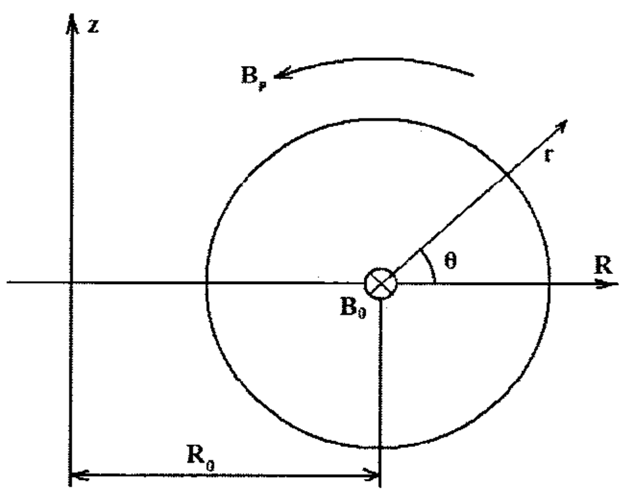

Figure-1.3: Toroidal Co-ordinates [4].

The guiding centre of the Larmor motion moves with velocity given by Equation (1.9), where $\hat{\phi}$ and $\hat{Z}$ are unit vectors.

$$V_g = v_\parallel \hat{\phi} + \frac{m}{qB_\phi R}(v_\parallel^{\,2} + \frac{v_\perp^{\,2}}{2})\hat{Z} \tag{1.9}$$

Electrons and ions round the torus in Larmor movement, but the second term in equation involves that ions and electrons gradually drift in reverse directions along the z-axis. This happens due to the field power lessening inversely with R. Since a particle moves into a weaker field section, its Larmor radius amplify and foundations upright $\hat{Z}$ drift when the particle travels reverse into stronger fields. This drift generates charge separation, which outcomes in an upright electric field ($\vec{E} = E_0\hat{z}$). This foundations ions and electrons to drift radially externals, by means of a radial drift velocity agreed by Equation

$$\vec{v}_g = \frac{\vec{E}\times\vec{B}}{B^2}. \tag{1.10}$$

For confine plasma in the toroidal arrangement, one necessity stops the charge disconnection that creates the electric field causing the radial drift. Count an orthogonal part to the magnetic field is individual technique of doing that [4].

1.4 Poloidal Field

If a current is contain in the toroidal ($\hat{\phi}$) direction, a magnetic field in the poloidal ($\hat{\theta}$) direction is formed. The radial coordinate in the toroidal cross section is specified as r in Figure 1.3. Suppose the amount of the poloidal field be B_p and the amount of the toroidal field be B_ϕ. Then the revolving transform angle, which explains the quantity the magnetic field revolves in the poloidal cross-section, when ϕ development through 2π, is:

$$\varepsilon = \frac{2\pi R B_p}{r B_\phi} \tag{1.11}$$

The frequency at which a particle revolves about the small axis of the torus, each time θ increments by 2π, is:

$$\omega = \frac{\varepsilon v_{\parallel}}{2\pi R_0} \tag{1.12}$$

This revolution in the poloidal direction contain the effect of removing the charge separation that led to radial drift velocities when just toroidal fields are present [4].

1.5 Limiter and Divertor

The powerful interactions arise between the extremely hot plasma and the at once neighbouring material that make up the plasma chamber. Electrons, Ions, and radiation from the plasma are occurrence on the neighbouring material surfaces, heating them and creating molecules and neutral atoms of plasma and barrier materials which come again to the plasma and which together dilute and cool the plasma fuel. Two dissimilar approaches present to reduce and organize the plasma material interactions in a tokamak. The initial opportunity is to materially limit the radius of plasma by introducing a so-called limiter in the vacuum vessel.

The limiter describes the LCFS short form for Last Closed Flux Surface, which is the border between the central part plasma where each and every one magnetic surfaces close back on themselves and the SOL short form for Scrape-Off Layer plasma where field lines are open and finish on the neighbouring material structures (called primary wall).

The other opportunity to describe the LCFS is to utilize an exterior magnetic coil creating a current parallel to the plasma current. These parallel current outcomes in the formation of an X-point where the poloidal magnetic field disappears. This diverts the poloidal field lines to toroidally symmetric plates: the divertor targets. Therefore, the name diverter.

A limiter is extremely close to the confined hot plasma; the plasma-surface interaction being localized to the foremost edge of the limiter. The limiter can so suffer from harsh heating, erosion and melting. Furthermore, the nearness to the confined plasma implies that any impurity free from the limiter can without difficulty penetrate into the plasma and infect the core. When ingoing in the core plasma, the impurities can chill it down by radiation which is to be avoided to favour fusion reactions. A number of modern tokamaks still utilize the limiter arrangement similar to TEXTOR (Germany) and Tore Supra (France) [5].

The majority modern tokamaks favour the divertor arrangement, where the LCFS is defined exclusively by the magnetic field and plasma-surface interactions are contained near the divertor target plates.

The impurities free from the target are ionized and possibly swept reverse to the target by the plasma flow earlier than they can go into the confined plasma. The section under the X-point and within the separatrix is called the Private Flux Region, it contains a slim layer of plasma lying beside the two separatrix arms and ending at the target [5].

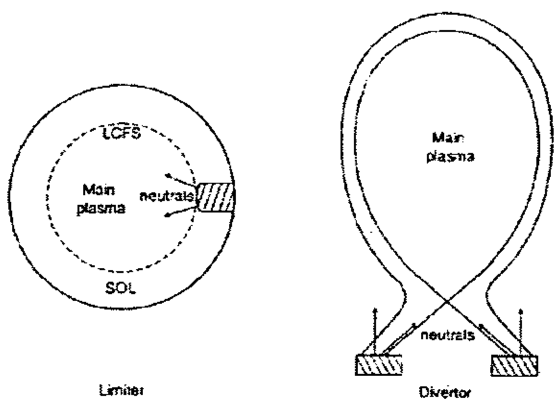

Figure-1.4: Poloidal cross-sections of a tokamak illustrating the limiter (left) and divertor (right) configurations [5].

The first aim of a divertor design is to reduce the impurity content of the plasma by keeping the plasma surface interactions isolated from the confined plasma, and avoiding any impurities formed at the target to go into the confined plasma (by the divertor particle flow)

The second aim of a divertor design is to eliminate the alpha particle power by heat relocate through a solid surface to a chilling fluid,

The third aim of a divertor design is to create a high helium neutral density section to simplicity tire out of the helium vestiges formed by the fusion reactions.

By reason of the localization of the plasma-surface interactions close to the target plates, erosion of the target surface, in addition to momentous power deposition on the target plates, can take place and be a serious difficulty for their life span. A probable move toward to decrease this difficulty is to create a "detached divertor plasma". For sufficiently elevated plasma density (which depends on the power input), a fall of the plasma temperature close to the targets is experiential. Temperature can go down low sufficient for electron-ion recombination to become significant, therefore eliminating charged particles and extinguishing nearby the plasma flow. This is habitually accompanied by an important reduce in the incident power to the targets and plasma flux density [5].

1.6 Plasma Heating System

One of the major necessities for fusion is to warmth the plasma particles to extremely tall temperatures or energies. The subsequent techniques are characteristically used to warmth the plasma [3].

1.6.1 Ohmic Heating

The primary heating in all tokamaks arrives from the ohmic heating sourced by the toroidal current. At small temperatures ohmic heating is quite influential and, in huge tokamaks, makes temperature of a little keV. The current intrinsically warmth the plasma by refreshing plasma electrons and ions in a particular toroidal direction. Only some mega watt of heating power is supplied in this way [3].

According to Ohm's Law heat energy is

$$Energy = I_p^{\ 2} R \tag{1.13}$$

1.6.2 Neutral Beam Heating

An extensive method of the extra plasma heating is foundation on the vaccination of influential beams of unbiased atoms into ohmically preheated plasma. The beam atoms hold a huge only single directional kinetic energy. In the plasma, beam atoms baggy electrons due to collisions, i.e. they obtain ionized and as an outcomes are arrested by the magnetic field of tokamak. These fresh ions are greatly quicker then mean plasma particles. In a sequence of collisions, the group velocity of beam atoms is relocated into an amplified average velocity of the disordered motion of every plasma particles. In fusion research, the unbiased beams are habitually created by atoms of hydrogen isotopes (hydrogen, deuterium or tritium). The energy of the beam should be enough to arrive at the plasma centre. If the beam atoms were too sluggish, they would get ionized at once at the plasma border. Simultaneously, the beam is hypothetical to have sufficient power to distribute important quantity of speedy atoms into plasma; otherwise the heating result would not be manifest [3].

1.6.3 Radio Frequency Heating

Since the plasma ions and electrons are confined to revolve in the region of the magnetic field lines (gyro-motion) in the tokamak, electromagnetic waves of a frequency coordinated to the ions or electrons gyrofrequency are capable to resonate or humid their wave power into the plasma particles.

Ion cyclotron resonant heating (ICRH) is regularly applied on Tokamak. It is resonant by means of the second harmonic frequency of ion gyration of major plasma ions (deuterium) or by means of a base frequency of gyration of minority kinds (tritium, helium). There are numerous additional resonant frequencies in tokamak plasmas but experiments have established a few to be incompetent or not practical while others just cannot penetrate through the plasma border section. Even though the lesser hybrid frequency can obtain into the plasma, regrettably it has an incompetent heating effect. However one more important application of lesser hybrid frequency has evolved: the equivalent lower hybrid wave can coerce electric current credit to the information that it has an electric component parallel to magnetic field lines [3].

1.7 Basic Tokamak Variable

1.7.1 Toroidal Beta

The ratio of plasma kinetic pressure $p = \sum nkT$ and toroidal magnetic pressure $p_m = \frac{B_\phi^{\ 2}}{2\mu_0}$ is equal to toroidal beta (β) [6].

$$\beta = \frac{\sum nkT}{\frac{B_\phi^{\ 2}}{2\mu_0}} \tag{1.14}$$

1.7.2 Poloidai Beta

The ratio of plasma kinetic pressure $p = \sum nkT$ and poloidal magnetic pressure $p_m = \frac{B_\theta^{\ 2}}{2\mu_0}$ is equal to poloidal beta (β_p) [7].

$$\beta_p = \frac{\sum nkT}{\frac{B_\theta^{\ 2}}{2\mu_0}} \tag{1.15}$$

1.7.3 Aspect Ratio

The ratio of major horizontal radius to minor horizontal radius is called aspect ratio A. From figure-1.5

$$A = \frac{R}{a} \tag{1.16}$$

1.7.4 Inverse Aspect Ratio

The ratio of minor horizontal radius to major horizontal radius is called inverse aspect ratio ε. From figure-1.5

$$\varepsilon = \frac{a}{R} \tag{1.17}$$

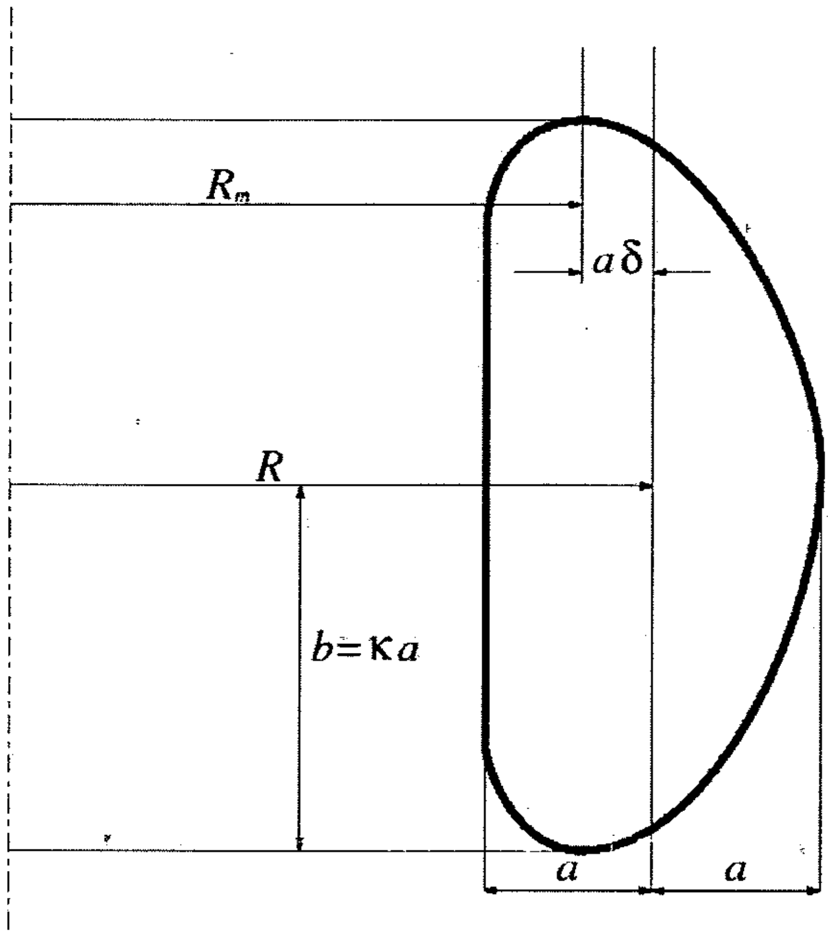

Figure-1.5: D-Shaped plasma shape parameters: k is elongation and δ is triangularity [8]

1.7.5 Elongation

The ratio of minor vertical radius to minor horizontal radius is called elongation k. From figure-1.5

$$b = ka \tag{1.18}$$

$$k = \frac{b}{a} \tag{1.19}$$

1.7.6 Triangularity

Triangularity δ is describe as from figure-1.5

$$a\delta = R - R_m \tag{1.20}$$

$$\delta = \frac{R - R_m}{a}, \tag{1.21}$$

where R_m is the horizontal distance to the highest point of the external plasma flux surface.

1.7.7 Safety Factor

The kink safety factor q_* is define as [9]

$$q_* = \frac{2\pi B_0 a^2}{\mu_o R_o I}\left(\frac{1+k^2}{2}\right) \tag{1.22}$$

1.7.8 Stability Factor

The stability factor f_s is define as

$$f_s = 1 + \frac{\tau_g}{\frac{\tau_L}{R}} \tag{1.23}$$

Where τ_g is the vertical instability growth time and $\frac{\tau_L}{R}$ is the longest up-down asymmetric time constant of the surrounding structure [10].

1.8 Scheme of Tokamaks Poloidal Field Coils

The plan and location of the Central Solenoid (CS) and Poloidal Field (PF) coils was based on the necessities of plasma physics.

The poloidal field coil organization offers the balance, shaping and manage fields for the plasma. Near the beginning round side view tokamaks contain upright field scheme but because perpendicularly elongated plasma in tokamak attain superior presentation for specified tokamak plan constraint like toroidal magnetic field, main and small radii, due to amplified plasma current. Poloidal magnetic field schemes are consider to attain superior confinement time and presentation. Tokamaks contain a physically powerful toroidal magnetic field, measure in tesla, but the force applied by toroidal field is not sharp towards the plasma and can not avoid external spreading out of toroidal shaped plasma by reason of the ring force, consequently a poloidal field is necessary [11-20].

In a characteristic tokamak, poloidal magnetic field is significantly minor than the toroidal magnetic field. A lot of practical constraints are available in poloidal coil locations plan. A number of them are admittance to the torus, its protection, assembly and limitations on the consumption of gap in the internal tokamak column. Poloidal field coils plan involves complicated exchange between every one of these thoughts. Also, this exchange is subjective by confinement time of plasma and performance. A great deal of variables are available associated to poloidal field coils plan of tokamaks similar to beta, ion and electron temperature, plasma current, plasma confinement time, plasma volume and much more. And diverse variable in plan procedure similar to entity location, currents and quantity of coil turns and summation of every current in poloidal field coils. Ambition of the poloidal magnetic field plan is to maintain the plasma stability. So above plasma variable are essential for the confinement of plasma. The plan procedure required a number of hypotheses like plasma temperature, plasma, current, toroidal magnetic field, plasma shape, numeral of poloidal magnetic field coils and much more. Additionally, pre-postulation for entity location, currents and numeral of coil turns are utilized to acquire the time for plasma confinement and modify these variables until come up to the greatest time of confinement. The peak time of confinement provides the numeral of turns, locations and currents of poloidal field coils. The outcomes are required to reconcile inside universal tokamak stability restrictions, i.e. greatest plasma density, greatest plasma beta and greatest plasma current.

Precise calculation of the currents in poloidal field coils are made by exact explanation of Maxwell's equations used for the magnetic fields in neighbouring vacuum segment and in the plasma segment. Above calculations shows that the current in the poloidal field coil is transfer to the entire ampere-turns in the same coil. So simply single power supply is required for whole the poloidal field coils. If the numeral of turns of the coils is amplified then it creates the necessary current in the poloidal field coils. The Grad–Shafranov (GS) equation is used to control the plasma area field while solution of Laplace's equation control the vacuum area field.

The function of poloidal field coils are very significance for tokamak machines. Poloidal field coils are utilized in different scheme for all tokamaks [11-20].

1.9 Outline of the Thesis

In the present chapter, we describe fundamental of fusion, plasma physics, tokamak plasma, tokmak machines, tokamak variables and the scheme of poloidal field coils for different tokamaks.

In second chapter, we derive the tokamak equilibrium, poloidal fiux and Grad-Shafranov equation.

In third chapter, we construct the mathematical model for poloidal field coils currents solver and force calculations.

In fourth chapter, we write the result of simulation of poloidal field coils draw out put flux contours.

At last in chapter five we write the summary and conclusion of the simulation of poloidal field coils.

CHAPTER 2

2. Equilibrium and Grad Shafranov Equation

2.1 Tokamak Equilibrium

Tokamak equilibrium can be measured as an interior balance between the forces from the magnetic field and plasma pressure. This offer grows to the shape and location of the plasma, controlled by the currents in the exterior poloidal field coils.

According to single fluid magnetohydrodynamics (MHD) equation of motion.

$$\rho\frac{\partial\vec{v}}{\partial t}=\vec{J}\times\vec{B}-\nabla p+\rho g \tag{2.1}$$

Above equation express the mass flow.

For steady state conditions $\frac{\partial}{\partial t}=0$ and $g=0$

Equation (1) becomes

$$\vec{J}\times\vec{B}=\nabla p \tag{2.2}$$

This is plasma equilibrium equation in which $\vec{J}\times\vec{B}=$magnetic pressure and $\nabla p=$Plasma Kinetic Pressure [21].

Taking dot product of $\vec{B}$ and Equation (2.2)

$$\vec{B}\bullet\vec{J}\times\vec{B}=\vec{B}\bullet\nabla p$$

$$\vec{J}\bullet\vec{B}\times\vec{B}=\vec{B}\bullet\nabla p$$

$$0=\vec{B}\bullet\nabla p \tag{2.3}$$

Also taking dot product of $\vec{J}$ and Equation (2.2)

$$\vec{J} \bullet \vec{J} \times \vec{B} = \vec{J} \bullet \nabla p$$

$$\vec{B} \bullet \vec{J} \times \vec{J} = \vec{J} \bullet \nabla p$$

$$0 = \vec{J} \bullet \nabla p \tag{2.4}$$

Now Ampere law in differential form is

$$\nabla \times \vec{B} = \mu_0 \vec{J} \tag{2.5}$$

Equation (2.5) in θ and Z components

$$J_\theta = -\frac{1}{\mu_0} \frac{dB_Z}{dr} \tag{2.6}$$

and $$J_Z = \frac{1}{\mu_0 r} \frac{d(rB_\theta)}{dr} \tag{2.7}$$

Gauss's Law in Magnetism can be written as

$$\nabla \bullet \vec{B} = 0 \tag{2.8}$$

Screw pinch consists of a cylindrical plasma by means of both angular and axial components of J and B. So Gauss's Law in Magnetism for screw pinch cylindrical coordinates is [22]

$$\nabla \bullet \vec{B} = \frac{1}{r} \frac{\partial B_\theta}{\partial \theta} + \frac{\partial B_Z}{\partial z} = 0 \tag{2.9}$$

Equation (2.2) in cylindrical coordinates is

$$J_\theta B_Z - J_Z B_\theta = \frac{dp}{dr} \tag{2.10}$$

By using Equation (2.6) and (2.7), we have

$$-\frac{1}{\mu_0}B_Z\frac{dB_Z}{dr}-\frac{1}{\mu_0 r}B_\theta\frac{d(rB_\theta)}{dr}=\frac{dp}{dr}$$

$$-\frac{1}{2\mu_0}\frac{dB_Z^{\,2}}{dr}-\frac{1}{\mu_0 r}B_\theta\left(B_\theta\frac{dr}{dr}+r\frac{dB_\theta}{dr}\right)=\frac{dp}{dr}$$

$$-\frac{1}{2\mu_0}\frac{dB_Z^{\,2}}{dr}-\frac{1}{2\mu_0}\frac{dB_\theta^{\,2}}{dr}-\frac{B_\theta^{\,2}}{\mu_0 r}=\frac{dp}{dr}$$

$$-\frac{d}{dr}\left(\frac{B_\theta^{\,2}+B_Z^{\,2}}{2\mu_0}\right)-\frac{B_\theta^{\,2}}{\mu_0 r}=\frac{dp}{dr}$$

$$\frac{dp}{dr}+\frac{d}{dr}\left(\frac{B_\theta^{\,2}+B_Z^{\,2}}{2\mu_0}\right)+\frac{B_\theta^{\,2}}{\mu_0 r}=0$$

$$\frac{d}{dr}\left(P+\frac{B_\theta^{\,2}+B_Z^{\,2}}{2\mu_0}\right)+\frac{B_\theta^{\,2}}{\mu_0 r}=0 \qquad (2.11)$$

Equation (2.11) indicate that information of the profiles of the plasma pressure and one of the magnetic field parts constructs it achievable to decide the profile of the other magnetic field part. Consider that the axial part of the field is known, the azimuthal part can be determined from [22]

$$r\frac{dp}{dr}+\frac{r}{2\mu_0}\frac{dB_\theta^{\,2}}{dr}+\frac{r}{2\mu_0}\frac{dB_Z^{\,2}}{dr}+\frac{B_\theta^{\,2}}{\mu_0}=0$$

$$r\frac{dp}{dr}+r\frac{d}{dr}\left(\frac{B_\theta^{\,2}}{2\mu_0}\right)+r\frac{d}{dr}\left(\frac{B_Z^{\,2}}{2\mu_0}\right)+\frac{B_\theta^{\,2}}{\mu_0}=0$$

$$r\frac{d}{dr}\left(\frac{B_\theta^{\,2}}{2\mu_0}\right)+\frac{B_\theta^{\,2}}{\mu_0}=-r\frac{dp}{dr}-r\frac{d}{dr}\left(\frac{B_Z^{\,2}}{2\mu_0}\right) \qquad (2.12)$$

For discover this part of the field accurately, however, needs information of the boundary conditions.

Overall, the plasma is holed in equilibrium by an outwardly applied magnetic field. This field is created by a set of current hauling conductors neighbouring the plasma. Additionally, a vacuum section is assumed to be present between the plasma and the conductors. From this, it is achievable to define the generally non circular plasma surface like the curve beside which the plasma pressure is efficiently zero [22].

2.2 Boundary Conditions

At the plasma surface, boundary conditions are specified by

$$\hat{n} \bullet \vec{B}|_a = \hat{n} \bullet \hat{B}|_a \tag{2.13}$$

$$\hat{n} \times \vec{B}|_a = \hat{n} \times \hat{B}|_a \tag{2.14}$$

$$B^2|_a = \hat{B}^2|_a \tag{2.15}$$

Where $\vec{B}$ =magnetic field inside the plasma

$\hat{B}$ =vacuum magnetic field

$\hat{n}$ =outward pointing unit vector normal to the plasma surface.

Above boundary conditions guarantee that the normal and tangential parts of the magnetic field and the magnetic pressure are incessant across the plasma surface and it is supposed that no surface current flow. Also by using $\vec{B} \bullet \nabla P = 0$ then [22]

$$\hat{n} \bullet \vec{B}|_a = 0 \tag{2.16}$$

$$\hat{n} \bullet \hat{B}|_a = 0 \tag{2.17}$$

The vacuum field $\hat{B}$ is found from

$$\hat{B} = \vec{B}_a + \widetilde{B} \tag{2.18}$$

Where $\bar{B}_a$ = magnetic field of conductor and $\widetilde{B}$ = vacuum magnetic field of the plasma [22].

2.3 Poloidal Flux Function, Current Flux Function and Grad Shafranov Equation

Consider Cylindrical (r,ϕ,z) and Quasi Cylindrical (ρ,ω,ϕ) coordinates systems.

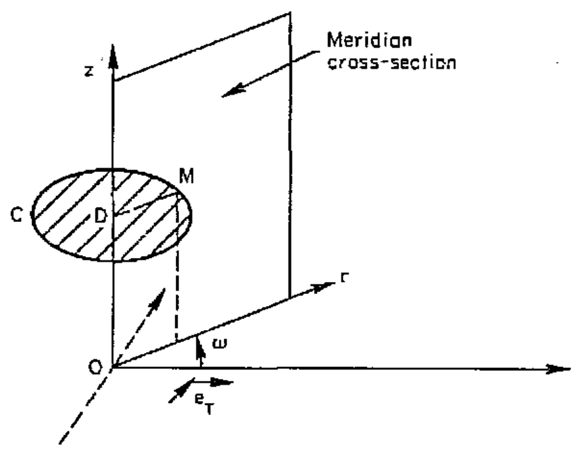

Figure-2.1: Cylindrical (r,ϕ,z) and quasi cylindrical (ρ,ω,ϕ) coordinates systems [23]

Poloidal flux $\psi(r,z)$ is define as

$$\psi(r,z)=\int_0^r r'B_z dr' \tag{2.19}$$

$$\psi(r,z)=\frac{1}{2\pi}\int_D B_z d(\pi r'^2)$$

Where $Area = a = \pi r'^2$

$$\psi(r,z)=\frac{1}{2\pi}\int_D B_z da$$

$$\psi(r,z)=\frac{1}{2\pi}\int_D \vec{B}\bullet d\vec{a} \tag{2.20}$$

Up to a factor of 2π the magnitude ψ is therefore equivalent to the poloidal flux (across D) of the magnetic field $\vec{B}$.

$$\psi(r,z)=\int_0^r r'B_z dr' \tag{2.21}$$

Taking partial derivative of equation (2.21) with respect to r

$$\frac{\partial\psi}{\partial r}=\frac{\partial}{\partial r}\int_0^r r'B_z dr'$$

$$\frac{\partial\psi}{\partial r}=\int_0^r \frac{d}{dr}r'B_z dr'$$

$$\frac{\partial\psi}{\partial r}=rB_z$$

$$B_z=\frac{1}{r}\frac{\partial\psi}{\partial r} \tag{2.22}$$

Now taking partial derivative of equation (2.21) with respect to z [23]

$$\frac{\partial\psi}{\partial z}=\frac{\partial}{\partial z}\int_0^r r'B_z dr'$$

$$\frac{\partial\psi}{\partial z}=\int_0^r \frac{\partial}{\partial z}(r'B_z)dr'$$

$$\frac{\partial\psi}{\partial z}=\int_0^r -\frac{\partial}{\partial r'}(r'B_r)dr'$$

$$\frac{\partial\psi}{\partial z} = \int_0^r \frac{d}{dr'}(-r'B_r)dr'$$

$$\frac{\partial\psi}{\partial z} = -rB_r$$

$$B_r = -\frac{1}{r}\frac{\partial\psi}{\partial z} \tag{2.23}$$

In cylindrical coordinates (r,ϕ,z), Gauss's Law in magnetism can be written as

$$\nabla\bullet\vec{B} = \frac{1}{r}\frac{\partial rB_r}{\partial r} + \frac{1}{r}\frac{\partial B_\phi}{\partial\phi} + \frac{\partial B_Z}{\partial z} = 0 \tag{2.24}$$

According to hypothesis of axisymmetric B is independent of ω and $\frac{\partial}{\partial\phi} = 0$, so

Equation (2.24) becomes [23]

$$\nabla\bullet\vec{B} = \frac{1}{r}\frac{\partial(rB_r)}{\partial r} + \frac{\partial B_Z}{\partial z} = 0 \tag{2.25}$$

$$\frac{\partial(rB_r)}{\partial r} + r\frac{\partial B_Z}{\partial z} = 0$$

$$\frac{\partial(rB_r)}{\partial r} = -\frac{\partial(rB_Z)}{\partial z} \tag{2.26}$$

Now gradient of ψ in cylindrical coordinates can be written as

$$\nabla\psi = \frac{\partial\psi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\psi}{\partial\phi}\hat{\phi} + \frac{\partial\psi}{\partial z}\hat{z} \tag{2.27}$$

For hypothesis of axisymmetric $\frac{\partial}{\partial\phi} = 0$, we get [23]

$$\nabla\psi = \frac{\partial\psi}{\partial r}\hat{r} + \frac{\partial\psi}{\partial z}\hat{z} \tag{2.28}$$

By using equation (2.22) and (2.23) in equation (2.28), we get

$$\nabla\psi = rB_z\hat{r} - rB_r\hat{z}. \tag{2.29}$$

Magnetic field $\vec{B}$ in r and z coordinate can be written as

$$\vec{B} = B_r\hat{r} + B_z\hat{z}. \tag{2.30}$$

Taking vector product of Equations (2.29) and (2.30) then

$$\vec{B}\times\nabla\psi = (B_r\hat{r} + B_z\hat{z})\times(rB_z\hat{r} - rB_r\hat{z})$$

$$\vec{B}\times\nabla\psi = 0 \tag{2.31}$$

Now, taking scalar product of equation (2.29) and (2.30) then

$$\vec{B}\bullet\nabla\psi = (B_r\hat{r} + B_z\hat{z})\bullet(rB_z\hat{r} - rB_r\hat{z})$$

$$\vec{B}\bullet\nabla\psi = 0 \tag{2.32}$$

By equation (2.3) and (2.32) we can write

$$p = p(\psi) \tag{2.33}$$

Where p is pressure as a function of ψ

Then a current flux function f also exists, and can be written as [24]

$$f = f(\psi) \tag{2.34}$$

So current density can be written in r and z components as

$$J_r = -\frac{1}{r}\frac{\partial f}{\partial z} \tag{2.35}$$

$$J_z = \frac{1}{r}\frac{\partial f}{\partial r} \tag{2.36}$$

According to Ampere's Law in r and z components

$$\mu_0(J_r\hat{r}+J_z\hat{z})=-\frac{\partial B_\phi}{\partial z}\hat{r}+\frac{1}{r}\frac{\partial(rB_\phi)}{\partial r}\hat{z} \tag{2.37}$$

Now comparing r and z components of equation (2.37), we have

$$\mu_0 J_r=-\frac{\partial B_\phi}{\partial z} \tag{2.38}$$

$$\mu_0 J_z=\frac{1}{r}\frac{\partial(rB_\phi)}{\partial r} \tag{2.39}$$

Putting equation (2.35) in Equation (2.38) then

$$-\frac{1}{r}\frac{\partial f}{\partial z}=-\frac{1}{\mu_0}\frac{\partial B_\phi}{\partial z}$$

$$\frac{f}{r}=\frac{B_\phi}{\mu_0}$$

$$f=\frac{rB_\phi}{\mu_0} \tag{2.40}$$

The function f contains the whole current in the windings creating the toroidal field [24]

Now equation (2.4) in r and z components as

$$(J_r\hat{r}+J_z\hat{z})\bullet\left(\frac{\partial p}{\partial r}\hat{r}+\frac{\partial p}{\partial z}\hat{z}\right)=0$$

$$(-\frac{1}{r}\frac{\partial f}{\partial z}\hat{r}+\frac{1}{r}\frac{\partial f}{\partial r}\hat{z})\bullet\left(\frac{\partial p}{\partial r}\hat{r}+\frac{\partial p}{\partial z}\hat{z}\right)=0$$

$$-\frac{1}{r}\frac{\partial f}{\partial z}\frac{\partial p}{\partial r}+\frac{1}{r}\frac{\partial f}{\partial r}\frac{\partial p}{\partial z}=0$$

$$\frac{\partial f}{\partial z}\frac{\partial p}{\partial r} - \frac{\partial f}{\partial r}\frac{\partial p}{\partial z} = 0$$

$$\nabla f \times \nabla p = 0, \tag{2.41}$$

this shows that f is a function of P, so we can write

$$p = p(\psi) \text{ and } f = f(\psi).$$

Consider e_ϕ is unit vector in toroidal ϕ direction. Now taking dot product of e_ϕ with equation (2.29) then

$$e_\phi \bullet \nabla \psi = 0 \tag{2.42}$$

and also similarly

$$e_\phi \bullet \nabla f = 0 \tag{2.43}$$

so we can write

$$e_\phi \bullet \nabla \psi = e_\phi \bullet \nabla f = 0. \tag{2.44}$$

Now equation (2.2) in component can be written as [25]

$$J_p \times e_\phi B_\phi + J_\phi e_\phi \times B_p = \nabla p, \tag{2.45}$$

where ∇p = plasma pressure

J_p = poloidal current density

B_p = poloidal magnetic field.

Now, poloidal magnetic field can be written as

$$B_p = \frac{1}{r}(\nabla \psi \times e_\phi) \tag{2.46}$$

and poloidal current density can written as

$$J_p = \frac{1}{r}(\nabla f \times e_\phi). \tag{2.47}$$

Put equations (2.46) and (2.47) in equation (2.45), then

$$\frac{1}{r}(\nabla f \times e_\phi) \times e_\phi B_\phi + J_\phi e_\phi \times \frac{1}{r}(\nabla \psi \times e_\phi) = \nabla p$$

$$-\frac{B_\phi}{r} e_\phi \times (\nabla f \times e_\phi) + \frac{J_\phi}{r} e_\phi \times (\nabla \psi \times e_\phi) = \nabla p. \tag{2.48}$$

Now $e_\phi \times (\nabla f \times e_\phi) = \nabla f (e_\phi \bullet e_\phi) - (e_\phi \bullet \nabla f) e_\phi$,

where $(e_\phi \bullet \nabla f) e_\phi = 0$,

so $e_\phi \times (\nabla f \times e_\phi) = \nabla f$. (2.49)

Similarly $e_\phi \times (\nabla \psi \times e_\phi) = \nabla \psi (e_\phi \bullet e_\phi) - (e_\phi \bullet \nabla \psi) e_\phi$

Where $(e_\phi \bullet \nabla \psi) e_\phi = 0$ so

$$e_\phi \times (\nabla \psi \times e_\phi) = \nabla \psi \tag{2.50}$$

Put equation (2.49) and (2.50) in equation (2.48) then [25]

$$-\frac{B_\phi}{r} \nabla f + \frac{J_\phi}{r} \nabla \psi = \nabla p. \tag{2.51}$$

Consider

$$\nabla f(\psi) = \frac{df}{d\psi} \nabla \psi \tag{2.52}$$

and $\nabla P(\psi) = \frac{dP}{d\psi} \nabla \psi$. (2.53)

Put equations (2.52) and (2.53) in equation (2.51) then

$$-\frac{B_\phi}{r}\frac{df}{d\psi}\nabla\psi+\frac{J_\phi}{r}\nabla\psi=\frac{dP}{d\psi}\nabla\psi$$

$$J_\phi=r\frac{dP}{d\psi}+B_\phi\frac{df}{d\psi} \quad . \tag{2.54}$$

Equation (3.40) can be written as

$$B_\phi=\frac{\mu_0 f}{r} \tag{2.55}$$

Put equation (2.55) in equation (2.54) then [25]

$$J_\phi=r\frac{dP}{d\psi}+\frac{\mu_0 f}{r}\frac{df}{d\psi} \tag{2.56}$$

Only J_ϕ component of Ampere's law can be written as

$$\mu_0 J_\phi=\frac{\partial B_r}{\partial z}-\frac{\partial B_z}{\partial r} \tag{2.57}$$

Put equation (2.22) and (2.23) in equation (2.57) then

$$\mu_0 J_\phi=\frac{\partial}{\partial z}(-\frac{1}{r}\frac{\partial\psi}{\partial z})-\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial\psi}{\partial r})$$

$$\mu_0 J_\phi=-\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial\psi}{\partial r})-\frac{1}{r}\frac{\partial^2\psi}{\partial z^2}$$

$$-r\mu_0 J_\phi=r\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial\psi}{\partial r})+\frac{\partial^2\psi}{\partial z^2} \tag{2.58}$$

Put equation (2.56) in equation (2.58) then

$$-r^2\mu_0\frac{dP}{d\psi}-\mu_0^{\,2}f\frac{df}{d\psi}=r\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial\psi}{\partial r})+\frac{\partial^2\psi}{\partial z^2}$$

$$r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right)+\frac{\partial^2 \psi}{\partial z^2}=-r^2\mu_0\frac{dP}{d\psi}-\mu_0{}^2 f\frac{df}{d\psi} \tag{2.59}$$

Let $F=\mu_0 f$ (2.60)

Then equation (2.59) becomes

$$r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right)+\frac{\partial^2 \psi}{\partial z^2}=-r^2\mu_0\frac{dP}{d\psi}-\mu_0 F\frac{df}{d\psi}$$

$$r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right)+\frac{\partial^2 \psi}{\partial z^2}=-r^2\mu_0\frac{dP}{d\psi}-F\frac{d(\mu_0 f)}{d\psi}$$

$$r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right)+\frac{\partial^2 \psi}{\partial z^2}=-r^2\mu_0\frac{dP}{d\psi}-F\frac{dF}{d\psi}$$

$$\Delta^*\psi=-r^2\mu_0\frac{dP}{d\psi}-F\frac{dF}{d\psi}. \tag{2.61}$$

Equation (2.61) is GRAD SHAFRANOV equation [25],

where

$$\Delta^*=r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\right)+\frac{\partial^2}{\partial z^2} \tag{2.62}$$

CHAPTER 3

3. Model for Poloidal Field Coil Current Solver and Force Computation

Precise calculation of the poloidal field coil currents needed the self reliable result of Maxwell's equation for the vacuum sections and the magnetic fields in the tokamak plasma. Solution of Grad-Shafranov equation gives plasma field, and the solution of Laplace's equation gives vacuum field. Boundary conditions required at the plasma surface join the two solutions [22].

3.1 Parameters of Plasma Surface

The toroidally axisymmetric plasma surface is describe by

$$r = r_p(\mu) \tag{3.1}$$

$$z = z_p(\mu), \tag{3.2}$$

where μ is a random angular coordinate.

The unit normal vector is describe by

$$e_n = \frac{1}{Q}[z_\mu e_r - r_\mu e_z] \tag{3.3}$$

And also unit tangent vector is described by

$$e_t = \frac{1}{Q}[r_\mu e_r + z_\mu e_z] \tag{3.4}$$

Where e_r and e_z are units vector in r and z direction [22].

The magnitude of both unit normal and tangential vector is

$$Q = \sqrt{r_{\mu}^{2} + z_{\mu}^{2}} \tag{3.5}$$

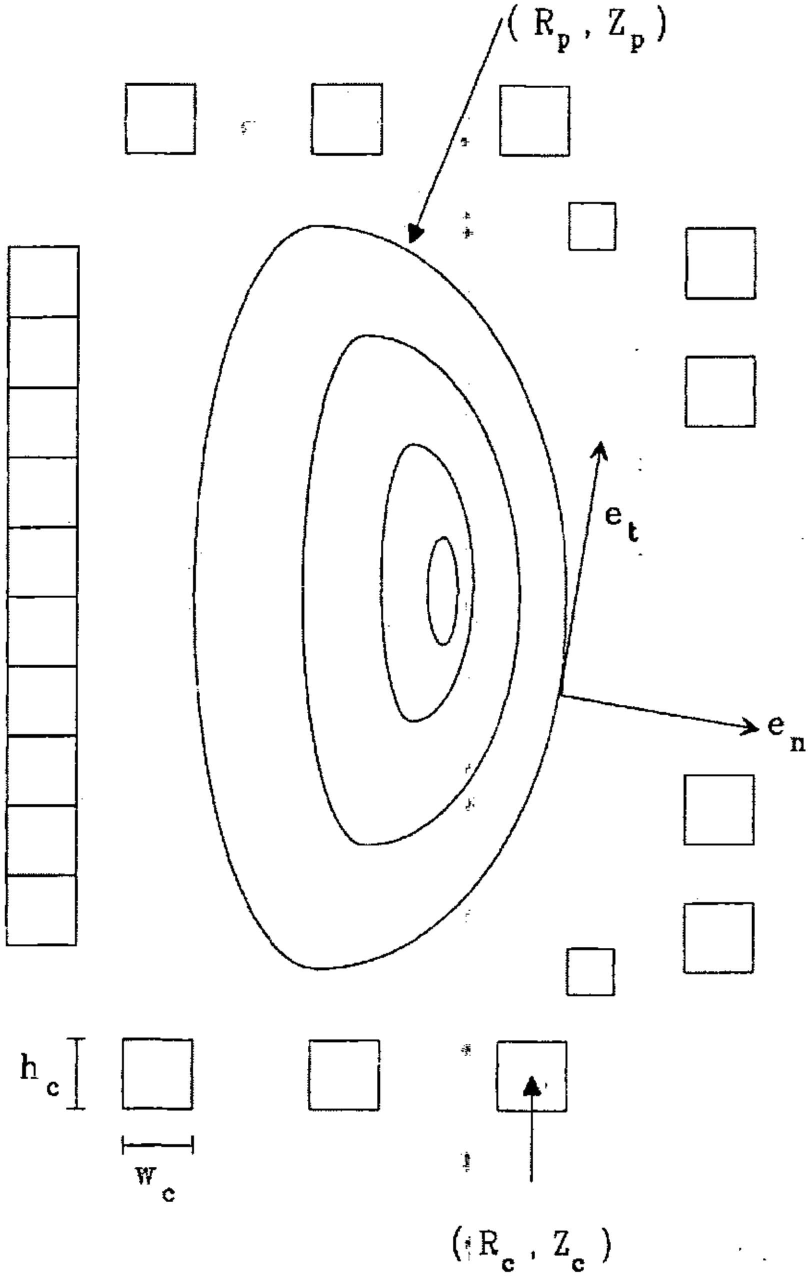

Figure-3.1: Geometry of plasma surface and conductors for use in the poloidal field coil current calculation [22].

Normal derivative is describe by

$$Qe_n \bullet \nabla = \frac{\partial}{\partial n} = z_\mu \frac{\partial}{\partial r} - r_\mu \frac{\partial}{\partial z} \tag{3.6}$$

And also Tangential derivative is describe by

$$Qe_t \bullet \nabla = \frac{\partial}{\partial \mu} = r_\mu \frac{\partial}{\partial r} + z_\mu \frac{\partial}{\partial z} \tag{3.7}$$

The differential component of arc length is work out from

$$ds^2 = dr_p{}^2 + dz_p{}^2 \tag{3.8}$$

$$ds^2 = r_\mu{}^2 d\mu + z_\mu{}^2 d\mu = Q^2 d\mu^2 \tag{3.9}$$

$$ds = Qd\mu \tag{3.10}$$

And differential surface area is

$$ds' = rdsd\phi = rQd\mu d\phi \tag{3.11}$$

The plasma represented in figure is imagined to be bounded by vacuum. Total number of j toroidally axisymmetric conductors are present in vacuum section. The location of every conductor is described by (r_j, z_j). These conductors are act as filaments to make simpler the calculation of the currents. Every conductor is supposed to contain a rectangular cross-section by means of a height h_j and width ω_j [22].

Vacuum magnetic field can be written as

$$\vec{B}_v = \vec{B}_p + \vec{B}_c, \tag{3.12}$$

where

$\vec{B}_v$ = vacuum magnetic field

$\vec{B}_p$ = plasma magnetic field

$\vec{B}_c$ = magnetic field due to external coils

The field in the vacuum region due to the plasma satisfies

$$\nabla \bullet \vec{B}_p = 0 \tag{3.13}$$

$$\nabla \times \vec{B}_p = 0 \tag{3.14}$$

Plasma magnetic field can be expressed as

$$\vec{B}_p = \nabla\phi + \vec{B}_i \tag{3.15}$$

ϕ is scalar magnetic potential, $\vec{B}_i$ is field due to single filament or coil. $\vec{B}_i$ is located at magnetic axis.

Scalar magnetic potential ϕ satisfied Laplace Equation. Taking divergence of equation (3.15) then [22]

$$\nabla \bullet \vec{B}_p = \nabla \bullet \nabla\phi + \nabla \bullet \vec{B}_i$$

$$0 = \nabla^2\phi + 0$$

$$\nabla^2\phi = 0 \tag{3.16}$$

3.2 Poloidal Flux in Elliptical Integral

By using distance formula and cosine law of triangle [22]

$$|\vec{r} - \vec{r}'| = (z-z')^2 + r'^2 + r^2 - 2rr'\cos(\phi - \phi') \tag{3.17}$$

According to relation

$$\nabla \times \vec{E} = 0 \tag{3.18}$$

$$\vec{E} = -\nabla \phi \tag{3.19}$$

Similarly

$$\nabla \bullet \vec{B} = 0 \tag{3.20}$$

$$\vec{B} = \nabla \times \vec{A} \tag{3.21}$$

and $$\vec{B} = -\nabla \phi , \tag{3.22}$$

where $\vec{E}$ is electric field,

ϕ is electric scalar potential,

$\vec{A}$ is magnetic vector potential.

In cylindrical coordinate (r, ϕ, z), $\vec{B} = \nabla \times \vec{A}$ can be written as [26]

$$B_r = \frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \tag{3.23}$$

$$B_\phi = \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \tag{3.24}$$

$$B_z = \frac{1}{r}\frac{\partial}{\partial r}(rA_\phi) - \frac{1}{r}\frac{\partial A_r}{\partial \phi} \tag{3.25}$$

$$\vec{E} = -\frac{d\vec{A}}{dt} \tag{3.26}$$

According to Faraday's Law

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{3.27}$$

By using equation (3.21) then

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}(\nabla \times \vec{A})$$

$$\nabla \times \vec{E} + \frac{\partial}{\partial t}(\nabla \times \vec{A}) = 0$$

$$\nabla \times \vec{E} + \nabla \times \frac{\partial \vec{A}}{\partial t} = 0$$

$$\nabla \times (\vec{E} + \frac{\partial \vec{A}}{\partial t}) = 0$$

By using equation (3.18) and (3.19) then

$$\vec{E} + \frac{\partial \vec{A}}{\partial t} = -\nabla \phi$$

$$\vec{E} = -\nabla \phi - \frac{\partial \vec{A}}{\partial t} \tag{3.28}$$

According to Ampare Law

$$\mu_0 \vec{J} = \nabla \times \vec{B}$$

By using equation (3.21) then

$$\mu_0 \vec{J} = \nabla \times (\nabla \times \vec{A})$$

$$\mu_0 \vec{J} = (\nabla \bullet \vec{A})\nabla - (\nabla \bullet \nabla)\vec{A} \tag{3.29}$$

Since $\qquad \nabla \bullet \vec{A} = 0 \qquad (3.30)$

Then equation (3.29) becomes [26]

$$\mu_0 \vec{J} = -(\nabla \bullet \nabla)\vec{A}$$

$$\mu_0 \vec{J} = -\nabla^2 \vec{A} \tag{3.31}$$

Solution of this equation (3.31) can be written as

$$A(r) = \frac{\mu_0}{4\pi} \int_{V'} \frac{J(r')}{|r - r'|} dV' \tag{3.32}$$

where
$$J(r') = \frac{current}{Area}, \tag{3.33}$$

By using Gauss's and Stake's theorem, equation (3.32) becomes

$$A_\phi = \frac{\mu_0}{4\pi} \int_0^{2\pi} \frac{I dl_\phi}{|r - r'|} \tag{3.34}$$

where
$$dl_\phi = r' \cos(\phi - \phi') d(\phi - \phi'). \tag{3.35}$$

Then equation (3.34) becomes

$$A_\phi = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{r' \cos(\phi - \phi') d(\phi - \phi')}{\sqrt{(z - z')^2 + r'^2 + r^2 - 2rr' \cos(\phi - \phi')}} \tag{3.36}$$

Let
$$\beta = \phi - \phi' \tag{3.37}$$

So
$$A_\phi = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{r' \cos(\beta) d(\beta)}{\sqrt{(z - z')^2 + r'^2 + r^2 - 2rr' \cos(\beta)}} \tag{3.38}$$

If circular loop so small then β can be written as

$$\beta = \pi + 2\theta \tag{3.39}$$

Now half angle identity

$$\cos\beta = 2\cos^2 \frac{\beta}{2} - 1 \tag{3.40}$$

Equation (3.39) can be written as

$$\frac{\beta}{2} = \frac{\pi}{2} + \theta \tag{3.41}$$

Taking ' cos ' on both sides of equation (3.41) then

$$\cos\left(\frac{\beta}{2}\right) = \cos\left(\frac{\pi}{2} + \theta\right)$$

$$\cos\left(\frac{\beta}{2}\right) = \cos\left(\frac{\pi}{2}\right)\cos(\theta) - \sin\left(\frac{\pi}{2}\right)\sin(\theta)$$

$$\cos\left(\frac{\beta}{2}\right) = -\sin(\theta) \tag{3.42}$$

Put equation (3.42) in equation (3.40) then

$$\cos\beta = 2\sin^2\theta - 1 \tag{3.43}$$

For Lower limit of equation (3.38), if $\beta = 0$ then equation (3.39) becomes

$$0 = \pi + 2\theta$$

$$\theta = -\frac{\pi}{2} \tag{3.44}$$

For Upper limit of equation (3.38), if $\beta = 0$ then equation (3.39) becomes

$$2\pi = \pi + 2\theta$$

$$\theta = \frac{\pi}{2} \tag{3.45}$$

Taking differential of equation (3.39) then

$$d\beta = 2d\theta \tag{3.46}$$

By using equations (3.46), (3.45), (3.44), (3.43) in equation (3.38) then

$$A_\phi = \frac{\mu_0 I}{4\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{r'(2\sin^2\theta - 1)2d\theta}{\sqrt{(z-z')^2 + r'^2 + r^2 - 2rr'\left(2\sin^2\theta - 1\right)}}$$

$$A_\phi = \frac{\mu_0 I r'}{2\pi} \int_0^{\pi} \frac{(2\sin^2\theta - 1)d\theta}{\sqrt{(z-z')^2 + r'^2 + r^2 - 2rr'(2\sin^2\theta - 1)}}$$

$$A_\phi = \frac{\mu_0 I r'}{2\pi} \int_0^{\pi} \frac{(2\sin^2\theta - 1)d\theta}{\sqrt{(z-z')^2 + r'^2 + r^2 + 2rr' - 4rr'\sin^2\theta}}$$

$$A_\phi = \frac{\mu_0 I r'}{2\pi} \int_0^{\pi} \frac{(2\sin^2\theta - 1)d\theta}{\sqrt{(z-z')^2 + (r+r')^2 - 4rr'\sin^2\theta}}$$

$$A_\phi = \frac{\mu_0 I r'}{2\pi\sqrt{(z-z')^2 + (r+r')^2}} \int_0^{\pi} \frac{(2\sin^2\theta - 1)d\theta}{\sqrt{1 - \frac{4rr'\sin^2\theta}{(z-z')^2 + (r+r')^2}}} \tag{3.47}$$

Where

$$A^2 = (z-z')^2 + (r+r')^2 \tag{3.48}$$

$$k_p^{\,2} = \frac{4rr'}{(z-z')^2 + (r+r')^2} = \frac{4rr'}{A^2} \tag{3.49}$$

Put equations (3.49) and (3.48) in equation (3.47) then

$$A_\phi = \frac{\mu_0 I r'}{2\pi\sqrt{\frac{4rr'}{k_p^{\,2}}}} \int_0^{\pi} \frac{(2\sin^2\theta - 1)d\theta}{\sqrt{1 - k_p^{\,2}\sin^2\theta}}$$

$$A_\phi = \frac{\mu_0 I r' k_p}{4\pi\sqrt{rr'}} \int_0^{\pi} \frac{(2\sin^2\theta - 1)d\theta}{\sqrt{1 - k_p^{\,2}\sin^2\theta}}$$

$$A_\phi = \frac{\mu_0 I}{2\pi} k_p \sqrt{\frac{r'}{r}} \left[\int_0^{\frac{\pi}{2}} \frac{(2\sin^2\theta - 1)d\theta}{\sqrt{1-k_p^{\,2}\sin^2\theta}} \right]$$

$$A_\phi = \frac{\mu_0 I}{2\pi} k_p \sqrt{\frac{r'}{r}} \left[\int_0^{\frac{\pi}{2}} \frac{2\sin^2\theta d\theta}{\sqrt{1-k_p^{\,2}\sin^2\theta}} - \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-k_p^{\,2}\sin^2\theta}} \right]$$

$$A_\phi = \frac{\mu_0 I}{2\pi} k_p \sqrt{\frac{r'}{r}} \left[-\frac{2}{k_p^{\,2}} \int_0^{\frac{\pi}{2}} \frac{-k_p^{\,2}\sin^2\theta d\theta}{\sqrt{1-k_p^{\,2}\sin^2\theta}} - \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-k_p^{\,2}\sin^2\theta}} \right]$$

$$A_\phi = \frac{\mu_0 I}{2\pi} k_p \sqrt{\frac{r'}{r}} \left[-\frac{2}{k_p^{\,2}} \int_0^{\frac{\pi}{2}} \frac{(1-k_p^{\,2}\sin^2\theta - 1)d\theta}{\sqrt{1-k_p^{\,2}\sin^2\theta}} - \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-k_p^{\,2}\sin^2\theta}} \right]$$

$$A_\phi = \frac{\mu_0 I}{2\pi} k_p \sqrt{\frac{r'}{r}} \left[\begin{array}{l} -\dfrac{2}{k_p^{\,2}} \displaystyle\int_0^{\frac{\pi}{2}} \sqrt{1-k_p^{\,2}\sin^2\theta}\, d\theta - \dfrac{2}{k_p^{\,2}} \displaystyle\int_0^{\frac{\pi}{2}} \frac{-d\theta}{\sqrt{1-k_p^{\,2}\sin^2\theta}} \\ -\displaystyle\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-k_p^{\,2}\sin^2\theta}} \end{array} \right]$$

$$A_\phi = \frac{\mu_0 I}{2\pi} k_p \sqrt{\frac{r'}{r}} \left[\begin{array}{l} -\dfrac{2}{k_p^{\,2}} \displaystyle\int_0^{\frac{\pi}{2}} \sqrt{1-k_p^{\,2}\sin^2\theta}\, d\theta + \dfrac{2}{k_p^{\,2}} \displaystyle\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-k_p^{\,2}\sin^2\theta}} \\ -\displaystyle\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-k_p^{\,2}\sin^2\theta}} \end{array} \right] \tag{3.50}$$

Where $$K = K(k_p) = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-k_p^{\,2}\sin^2\theta}} \tag{3.51}$$

and $$E = E(k_p) = \int_0^{\frac{\pi}{2}} \sqrt{1 - k_p^{\,2} \sin^2 \theta d\theta} \tag{3.52}$$

are Elliptical integrals of first and second kind respectively and k_p modulus of elliptical integral [27]. So equation (3.50) becomes

$$A_\phi = \frac{\mu_0 I}{2\pi} k_p \sqrt{\frac{r'}{r}} \left[-\frac{2}{k_p^{\,2}} E + \frac{2}{k_p^{\,2}} K - K \right]$$

$$A_\phi = \frac{\mu_0 I}{\pi} \frac{1}{k_p} \sqrt{\frac{r'}{r}} \left[-E + K - \frac{k_p^{\,2}}{2} K \right]$$

$$A_\phi = \frac{\mu_0 I}{\pi} \frac{1}{k_p} \sqrt{\frac{r'}{r}} \left[(1 - \frac{k_p^{\,2}}{2}) K - E \right] \tag{3.53}$$

According to hypothesis of axisymmetric, magnetic field B is independent of ω and $\frac{\partial}{\partial \phi} = 0$, So $\vec{B} = \nabla \times \vec{A}$ can be write in cylindrical coordinate as [23]

$$B_r = -\frac{\partial A_\phi}{\partial z} \tag{3.54}$$

$$B_z = \frac{1}{r'} \frac{\partial}{\partial r'} (r' A_\phi) \tag{3.55}$$

Above equation can be written as

$$\partial (r' A_\phi) = r' B_z \partial r' \tag{3.56}$$

Integrate on both sides of equation (3.56) then

$$r A_\phi = \int_0^r r' B_z dr'$$

$$rA_\phi = \frac{1}{2\pi}\int_0^r 2\pi r' B_z dr'$$

$$rA_\phi = \frac{1}{2\pi}\int_0^r B_z d(\pi r'^2)$$

$$rA_\phi = \frac{1}{2\pi}\int_0^r B_z da$$

$$rA_\phi = \frac{1}{2\pi}\int_0^r \vec{B} \bullet d\vec{a}$$

$$rA_\phi = \frac{1}{2\pi}\int_D \vec{B} \bullet d\vec{a} \tag{3.57}$$

Where $\quad$ area $= a = \pi r'^2$

By comparing equation (2.20) and equation (3.57) then

$$rA_\phi = \psi(r,z)$$

$$\psi(r,z) = rA_\phi \tag{3.58}$$

Where $\psi(r,z)$ is poloidal flux

Equation (3.58), represent the poloidal flux in term of vector potential.

Putting the value of vector potential from equation (3.53) in equation (3.58) then

$$\psi(r,z) = r\frac{\mu_0 I}{\pi}\frac{1}{k_p}\sqrt{\frac{r'}{r}}\left[(1-\frac{k_p^{\ 2}}{2})K - E\right]$$

$$\psi(r,z) = \frac{\mu_0 I}{\pi}\frac{1}{k_p}\sqrt{rr'}\left[(1-\frac{k_p^{\ 2}}{2})K - E\right] \tag{3.59}$$

Equation (3.59) represent poloidal flux in term of elliptical integrals [22]

$$\psi(r,z) = \mu_0 I \psi_p \tag{3.60}$$

Where

$$\psi_p = \frac{\sqrt{rr'}}{\pi k_p}\left[(1-\frac{k_p^{\ 2}}{2})K - E\right] \tag{3.61}$$

$$\psi_p = \frac{\sqrt{rr'}}{2\pi k_p}\left[(2-k_p^{\ 2})K - 2E\right] \tag{3.62}$$

Now by using equation (2.46) and (3.60), filamentary magnetic field B_i can be written as

$$\vec{B}_i = \frac{1}{r}(\nabla \mu_0 I \psi_p \times e_\phi)$$

$$\vec{B}_i = \frac{\mu_0 I(\nabla \psi_p \times e_\phi)}{r} \tag{3.63}$$

$\vec{B}_i$ = filamentary magnetic field [22].

3.3 Green Function in Term of Elliptical Integral

The technique utilized to conclude the vacuum field B_V is foundation on an application of Green's theorem for the scalar magnetic potential ϕ, which is describe as follow[22]

$$\sigma\phi(r) + \int_{S_p}[\phi(r')(e_n' \bullet \nabla'\hat{G}(r,r')) - \hat{G}(r,r')(e_n' \bullet \nabla'\phi(r'))]ds' = 0 \tag{3.64}$$

$$\frac{1}{2}\phi(\mu) + \int_0^{2\pi}[\phi(\mu')\frac{\partial G(\mu,\mu')}{\partial n'} - G(\mu,\mu')\frac{\partial \phi(\mu')}{\partial n'}]r'd\mu' = 0 \tag{3.65}$$

Where σ is a coefficient which depends on the position of the inspection point comparative to the plasma surface. If r is outside the plasma then value of $\sigma = 1$, if r is on the plasma surface then value of $\sigma = \frac{1}{2}$ and if r is inside the plasma then value of $\sigma = 0$.

Green function can be describe as

$$\hat{G}(r,r') = -\frac{1}{4\pi|\bar{r}-\bar{r}'|} \tag{3.66}$$

And reduced Green function G can be describe as [22]

$$G = \int_0^{2\pi} \hat{G} d(\phi - \phi') \tag{3.67}$$

$$G = \int_0^{2\pi} -\frac{1}{4\pi|\bar{r}-\bar{r}'|} d(\phi - \phi') \tag{3.68}$$

Put $\beta = \pi + 2\theta$

and $d\beta = 2d\theta$

then
$$G = -\frac{1}{4\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{2d\theta}{|\bar{r}-\bar{r}'|} \tag{3.69}$$

By using equation (3.17) then

$$G = -\frac{1}{4\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{2d\theta}{\sqrt{(z-z')^2 + r'^2 + r^2 - 2rr'\left(2\sin^2\theta - 1\right)}}$$

$$G = -\frac{1}{4\pi} \int_0^{\pi} \frac{2d\theta}{\sqrt{(z-z')^2 + (r+r')^2 - 4rr'\sin^2\theta}}$$

$$G=-\frac{1}{\pi\sqrt{(z-z')^2+(r+r')^2}}\int_0^{\frac{\pi}{2}}\frac{d\theta}{\sqrt{1-\frac{4rr'\sin^2\theta}{(z-z')^2+(r+r')^2}}}$$

$$G=-\frac{k_p}{2\pi\sqrt{rr'}}\int_0^{\frac{\pi}{2}}\frac{d\theta}{\sqrt{1-k_p{}^2\sin^2\theta}}$$

$$G=-\frac{k_p K}{2\pi\sqrt{rr'}} \tag{3.70}$$

Equation (3.70) is Green's function in term of elliptical integrals [22].

3.4 Properties for Derivative of Elliptical Integrals

Some properties for derivative of elliptical integrals are as follows [27]

$$\frac{dK}{dk_p}=\frac{1}{k_p}[\frac{E}{1-k_p{}^2}-K] \tag{3.71}$$

$$(k_p')^2=1-k_p{}^2 \tag{3.72}$$

$$\frac{dE}{dk_p}=\frac{1}{k_p}[E-K] \tag{3.73}$$

$$\frac{\partial k_p}{\partial r'}=\frac{k_p}{2r'}-\frac{k_p{}^3}{4r'}-\frac{k_p{}^3}{4r} \tag{3.74}$$

3.5 Normal derivative of Green's Function in term of Elliptical Integrals

Now normal derivative of Green function can be finding by using the properties for derivative of elliptical integrals [22].

$$\frac{\partial G}{\partial n'} = z'_{\mu}\frac{\partial G}{\partial r'} - r'_{\mu}\frac{\partial G}{\partial z'} \tag{3.75}$$

$$\frac{\partial G}{\partial r'} = -\frac{\partial}{\partial r'}\frac{k_p K}{2\pi\sqrt{rr'}} \tag{3.76}$$

$$\frac{\partial G}{\partial r'} = -\frac{k_p K}{2\pi}\frac{\partial}{\partial r'}\left(\frac{1}{\sqrt{rr'}}\right) - \frac{1}{2\pi\sqrt{rr'}}\frac{\partial}{\partial r'}(k_p K)$$

$$\frac{\partial G}{\partial r'} = \frac{k_p K}{4\pi r'\sqrt{rr'}} - \frac{1}{2\pi\sqrt{rr'}}\frac{\partial}{\partial k_p}(k_p K)\frac{\partial k_p}{\partial r'}$$

$$\frac{\partial G}{\partial r'} = \frac{k_p K}{4\pi r'\sqrt{rr'}} - \frac{1}{2\pi\sqrt{rr'}}\left[K + k_p\frac{\partial K}{\partial k_p}\right]\frac{\partial k_p}{\partial r'}$$

$$\frac{\partial G}{\partial r'} = \frac{k_p K}{4\pi r'\sqrt{rr'}} - \frac{1}{2\pi\sqrt{rr'}}\left[\frac{E}{1-k_p^{\,2}}\right]\left[\frac{k_p}{2r'} - \frac{k_p^{\,3}}{4r'} - \frac{k_p^{\,3}}{4r}\right]$$

$$\frac{\partial G}{\partial r'} = \frac{k_p}{4\pi r'\sqrt{rr'}}\left[K - \frac{E}{1-k_p^{\,2}}\left\{1 - \frac{k_p^{\,2}}{2}\left(1+\frac{r'}{r}\right)\right\}\right] \tag{3.78}$$

Now

$$\frac{\partial G}{\partial z'} = -\frac{\partial}{\partial z'}\frac{k_p K}{2\pi\sqrt{rr'}} \tag{3.79}$$

$$\frac{\partial G}{\partial z'} = -\frac{1}{2\pi\sqrt{rr'}}\frac{\partial}{\partial z'}(k_p K)$$

$$\frac{\partial G}{\partial z'} = -\frac{1}{2\pi\sqrt{rr'}}\frac{\partial}{\partial k_p}(k_p K)\frac{\partial k_p}{\partial z'}$$

$$\frac{\partial G}{\partial z'} = -\frac{1}{2\pi\sqrt{rr'}}\left(k_p\frac{\partial K}{\partial k_p} + K\right)\frac{\partial k_p}{\partial z'}$$

$$\frac{\partial G}{\partial z'}=-\frac{1}{2\pi\sqrt{rr'}}\left(\frac{E}{1-k_p^{\,2}}\right)\frac{\partial k_p}{\partial z'} \tag{3.80}$$

Now

$$\frac{\partial k_p}{\partial z'}=\frac{\partial}{\partial z'}\frac{2\sqrt{rr'}}{\sqrt{(z-z')^2+(r+r')^2}} \tag{3.81}$$

$$\frac{\partial k_p}{\partial z'}=\frac{k_p(z-z')}{A^2}$$

$$\frac{\partial k_p}{\partial z'}=\frac{k_p^{\,3}(z-z')}{4rr'} \tag{3.82}$$

Put equation (3.82) in equation (3.80) then

$$\frac{\partial G}{\partial z'}=-\frac{1}{2\pi\sqrt{rr'}}\left(\frac{E}{1-k_p^{\,2}}\right)\frac{k_p^{\,3}(z-z')}{4rr'} \tag{3.83}$$

Substitute equation (3.83) and (3.78) in equation (3.75) then multiply by r' so

$$r'\frac{\partial G}{\partial n'}=r'z'_\mu\frac{k_p}{4\pi r'\sqrt{rr'}}\left[K-\frac{E}{1-k_p^{\,2}}\left\{1-\frac{k_p^{\,2}}{2}\left(1+\frac{r'}{r}\right)\right\}\right]-r'r'_\mu\left[-\frac{1}{2\pi\sqrt{rr'}}\left(\frac{E}{1-k_p^{\,2}}\right)\frac{k_p^{\,3}(z-z')}{4rr'}\right]$$

$$r'\frac{\partial G}{\partial n'}=\frac{k_p}{4\pi\sqrt{rr'}}\left[z'_\mu K-\frac{z'_\mu E}{1-k_p^{\,2}}\left\{1-\frac{k_p^{\,2}}{2}\left(1+\frac{r'}{r}\right)\right\}\right]-\frac{k_p}{4\pi\sqrt{rr'}}\left(\frac{r'_\mu E}{1-k_p^{\,2}}\right)\frac{k_p^{\,2}(z'-z)}{2r}$$

$$r'\frac{\partial G}{\partial n'}=\frac{k_p}{4\pi\sqrt{rr'}}\left[z'_\mu K-\frac{z'_\mu E}{1-k_p^{\,2}}\left\{1-\frac{k_p^{\,2}}{2}\left(1+\frac{r'}{r}\right)\right\}-\left(\frac{r'_\mu E}{1-k_p^{\,2}}\right)\frac{k_p^{\,2}(z'-z)}{2r}\right]$$

$$r'\frac{\partial G}{\partial n'}=\frac{k_p}{4\pi\sqrt{rr'}}\left[z'_\mu K-\frac{z'_\mu E}{1-k_p^{\,2}}+\frac{z'_\mu E}{1-k_p^{\,2}}\frac{k_p^{\,2}}{2}\left(1+\frac{r'}{r}\right)-\left(\frac{r'_\mu E}{1-k_p^{\,2}}\right)\frac{k_p^{\,2}(z'-z)}{2r}\right] \tag{3.84}$$

Now equation (3.49) can be written as

$$1-k_p^{\,2}=1-\frac{4rr'}{(z-z')^2+(r+r')^2}$$

$$1-k_p^{\,2}=\frac{(z-z')^2+(r+r')^2-4rr'}{(z-z')^2+(r+r')^2}$$

$$1-k_p^{\,2}=\frac{(z-z')^2+(r-r')^2}{(z-z')^2+(r+r')^2}$$

$$\frac{k_p^{\,2}}{1-k_p^{\,2}}=\frac{4rr'}{(z-z')^2+(r-r')^2} \tag{3.85}$$

Putting equation (3.85) in (3.84) then

$$r'\frac{\partial G}{\partial r'}=\frac{k_p}{4\pi\sqrt{rr'}}\left[z'_\mu K-\frac{z'_\mu E\left\{(z-z')^2+(r+r')^2\right\}}{(z-z')^2+(r-r')^2}+\frac{z'_\mu E}{2}\left(1+\frac{r'}{r}\right)\frac{4rr'}{(z-z')^2+(r-r')^2}-\left(\frac{r'_\mu E}{2r}\right)\frac{4rr'(z'-z)}{(z-z')^2+(r-r')^2}\right]$$

$$r'\frac{\partial G}{\partial r'}=\frac{r'}{2\pi\sqrt{rr'}}\left[\frac{z'_\mu k_p K}{2r'}-\frac{z'_\mu Ek_p\left\{(z-z')^2+(r+r')^2\right\}}{2r'\left\{(z-z')^2+(r-r')^2\right\}}+z'_\mu Ek'_p\left(1+\frac{r'}{r}\right)\frac{r}{(z-z')^2+(r-r')^2}-\frac{r'_\mu Ek_p(z'-z)}{(z-z')^2+(r-r')^2}\right]$$

$$r'\frac{\partial G}{\partial r'}=\frac{1}{2\pi}\sqrt{\frac{r'}{r}}\left[\frac{z'_\mu k_p K}{2r'}-\frac{z'_\mu Ek_p\left\{(z-z')^2+(r+r')^2\right\}}{2r'\left\{(z-z')^2+(r-r')^2\right\}}+\frac{z'_\mu Ek_p(r+r')}{(z-z')^2+(r-r')^2}-\frac{r'_\mu Ek_p(z'-z)}{(z-z')^2+(r-r')^2}\right]$$

$$r'\frac{\partial G}{\partial r'}=\frac{1}{2\pi}\sqrt{\frac{r'}{r}}\left[\frac{z'_\mu k_p K}{2r'}-\frac{r'_\mu Ek_p(z'-z)}{(z-z')^2+(r-r')^2}-\frac{z'_\mu Ek_p\left\{(z-z')^2+(r+r')^2\right\}}{2r'\left\{(z-z')^2+(r-r')^2\right\}}+\frac{z'_\mu Ek_p(r+r')}{(z-z')^2+(r-r')^2}\right]$$

$$r'\frac{\partial G}{\partial r'}=\frac{1}{2\pi}\sqrt{\frac{r'}{r}}\left[\frac{z'_\mu k_p K}{2r'}-\frac{r'_\mu Ek_p(z'-z)}{(z-z')^2+(r-r')^2}-\frac{z'_\mu Ek_p}{(z-z')^2+(r-r')^2}\left\{\frac{(z-z')^2+(r+r')^2}{2r'}-(r+r')\right\}\right]$$

$$r'\frac{\partial G}{\partial r'}=\frac{1}{2\pi}\sqrt{\frac{r'}{r}}\left[\frac{z'_\mu k_p K}{2r'}-\frac{r'_\mu Ek_p(z'-z)}{(z-z')^2+(r-r')^2}-\frac{z'_\mu Ek_p}{(z-z')^2+(r-r')^2}\left\{\frac{(z-z')^2+(r+r')^2-2r'(r+r')}{2r'}\right\}\right]$$

$$r'\frac{\partial G}{\partial n'}=\frac{1}{2\pi}\sqrt{\frac{r'}{r}}\left[\frac{z'_\mu k_p K}{2r'}-\frac{r'_\mu E k_p(z'-z)}{(z-z')^2+(r-r')^2}-\frac{z'_\mu E k_p}{(z-z')^2+(r-r')^2}\left\{\frac{(z-z')^2+(r+r')^2-2rr'-2r'^2}{2r'}\right\}\right]$$

$$r'\frac{\partial G}{\partial n'}=\frac{1}{2\pi}\sqrt{\frac{r'}{r}}\left[\frac{z'_\mu k_p K}{2r'}-\frac{r'_\mu E k_p(z'-z)}{(z-z')^2+(r-r')^2}-\frac{z'_\mu E k_p}{(z-z')^2+(r-r')^2}\left\{\frac{(z-z')^2+(r-r')^2+2rr'-2r'^2}{2r'}\right\}\right]$$

$$r'\frac{\partial G}{\partial n'}=\frac{1}{2\pi}\sqrt{\frac{r'}{r}}\left[\frac{z'_\mu k_p K}{2r'}-\frac{r'_\mu E k_p(z'-z)}{(z-z')^2+(r-r')^2}-\frac{z'_\mu E k_p}{(z-z')^2+(r-r')^2}\left\{\frac{(z-z')^2+(r-r')^2}{2r'}+\frac{2rr'-2r'^2}{2r'}\right\}\right]$$

$$r'\frac{\partial G}{\partial n'}=\frac{1}{2\pi}\sqrt{\frac{r'}{r}}\left[\frac{z'_\mu k_p K}{2r'}-\frac{r'_\mu E k_p(z'-z)}{(z-z')^2+(r-r')^2}-\frac{z'_\mu E k_p}{(z-z')^2+(r-r')^2}\left\{\frac{(z-z')^2+(r-r')^2}{2r'}+(r-r')\right\}\right]$$

$$r'\frac{\partial G}{\partial n'}=\frac{1}{2\pi}\sqrt{\frac{r'}{r}}\left[\frac{z'_\mu k_p K}{2r'}-\frac{r'_\mu E k_p(z'-z)}{(z-z')^2+(r-r')^2}-\frac{z'_\mu E k_p}{(z-z')^2+(r-r')^2}\left\{\frac{(z-z')^2+(r-r')^2}{2r'}-(r'-r)\right\}\right]$$

$$r'\frac{\partial G}{\partial n'}=\frac{1}{2\pi}\sqrt{\frac{r'}{r}}\left[\frac{z'_\mu k_p K}{2r'}-\frac{r'_\mu E k_p(z'-z)}{(z-z')^2+(r-r')^2}-\frac{z'_\mu E k_p}{2r'}+\frac{z'_\mu E k_p(r'-r)}{(z-z')^2+(r-r')^2}\right]$$

$$r'\frac{\partial G}{\partial n'}=\frac{1}{2\pi}\sqrt{\frac{r'}{r}}\left[\frac{z'_\mu E k_p(r'-r)}{(z-z')^2+(r-r')^2}-\frac{r'_\mu E k_p(z'-z)}{(z-z')^2+(r-r')^2}+\frac{z'_\mu k_p K}{2r'}-\frac{z'_\mu E k_p}{2r'}\right]$$

$$r'\frac{\partial G}{\partial n'}=\frac{1}{2\pi}\sqrt{\frac{r'}{r}}\left[\left\{\frac{z'_{\mu_p}(r'-r)}{(z-z')^2+(r-r')^2}-\frac{r'_\mu(z'-z)}{(z-z')^2+(r-r')^2}\right\}E k_p+\frac{z'_\mu}{2r'}\left(k_p K-E k_p\right)\right]$$

$$r'\frac{\partial G}{\partial n'}=\frac{1}{2\pi}\sqrt{\frac{r'}{r}}\left[\left\{\frac{z'_{\mu_p}(r'-r)-r'_\mu(z'-z)}{(z-z')^2+(r-r')^2}\right\}E k_p+\frac{z'_\mu}{2r'}\left(k_p K-E k_p\right)\right]$$

$$r'\frac{\partial G}{\partial n'}=\frac{1}{2\pi}\sqrt{\frac{r'}{r}}\left[\left\{\frac{z'_{\mu_p}(r'-r)-r'_\mu(z'-z)}{(z-z')^2+(r-r')^2}\right\}k_p E+\frac{z'_\mu}{2r'}\left(K k_p-E k_p\right)\right] \tag{3.86}$$

Where
$$\Gamma=\frac{z'_\mu}{2r'} \tag{3.87}$$

and
$$\Lambda = \frac{z'_{\mu_p}(r'-r) - r'_\mu(z'-z)}{(z-z')^2 + (r-r')^2} \tag{3.88}$$

then
$$r'\frac{\partial G}{\partial n'} = \frac{1}{2\pi}\sqrt{\frac{r'}{r}}\left[\Lambda k_p E + \Gamma\left(Kk_p - Ek_p\right)\right]$$

$$r'\frac{\partial G}{\partial n'} = \frac{1}{2\pi}\sqrt{\frac{r'}{r}}\left[\Lambda k_p E + \Gamma k_p\left(K - E\right)\right] \tag{3.89}$$

Equation (3.89) is the normal derivative of Green's function in terms of elliptical integrals [22].

3.6 Vector Potential in Terms of Green's Function

Vector potential can also be written in volume Integral as (Distributed currents)[22]

$$A(r) = e_\phi \frac{\mu_0 J}{4\pi} \int_{Z-\frac{h}{2}}^{Z+\frac{h}{2}} \int_0^{2\pi} \int_{R-\frac{w}{2}}^{R+\frac{w}{2}} \frac{\cos(\phi'-\phi) r' dr' dz d(\phi'-\phi)}{\sqrt{(z-z')^2 + r'^2 + r^2 - 2rr'\cos(\phi-\phi')}} \tag{3.90}$$

$$A(r) = e_\phi \frac{\mu_0 J}{4\pi} \int_{Z-\frac{h}{2}}^{Z+\frac{h}{2}} \int_0^{2\pi} \int_{R-\frac{w}{2}}^{R+\frac{w}{2}} \frac{\cos(\phi'-\phi) r' dr' dz' d(\phi'-\phi)}{\sqrt{r'^2 + r^2 - 2rr'\cos(\phi-\phi') + (z-z')^2}} \tag{3.91}$$

Where
$$J = \frac{I}{Area} \tag{3.92}$$

Coils with rectangular crossection of width w, height h, major radius R and elevation Z earring a uniform current density J .and e_ϕ is unit vector direction.

Put $\beta = \phi' - \phi$ then equation (3.91) becomes

$$A(r) = e_\phi \frac{\mu_0 J}{4\pi} \int_{Z-\frac{h}{2}}^{Z+\frac{h}{2}} \int_0^{2\pi} \int_{R-\frac{w}{2}}^{R+\frac{w}{2}} \frac{\cos(\beta) r' dr' dz' d(\beta)}{\sqrt{r'^2 + r^2 - 2rr'\cos(\phi-\phi') + (z-z')^2}} \tag{3.93}$$

In term of Green function (G)

$$A(r) = e_\phi \frac{\mu_0 J}{4\pi} \int r' G(r,z;r',z') dr' dz' \tag{3.94}$$

Where $$G(r,z;r',z') = \int_0^{2\pi} \frac{\cos(\beta) d\beta}{\sqrt{r'^2 + r^2 - 2rr'\cos(\phi - \phi') + (z-z')^2}} \tag{3.95}$$

By using equations from equation (3.39) to equation (3.46) then equation (3.95) becomes [22]

$$G(r,z;r',z') = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{(2\sin^2\theta - 1) 2 d\theta}{\sqrt{(z-z')^2 + r'^2 + r^2 - 2rr'\left(2\sin^2\theta - 1\right)}}$$

$$G(r,z;r',z') = 2\int_0^{\pi} \frac{(2\sin^2\theta - 1) d\theta}{\sqrt{(z-z')^2 + (r+r')^2 - 4rr'\sin^2\theta}}$$

$$G(r,z;r',z') = \frac{2}{\sqrt{(z-z')^2 + (r+r')^2}} \int_0^{\pi} \frac{(2\sin^2\theta - 1) d\theta}{\sqrt{1 - \frac{4rr'\sin^2\theta}{(z-z')^2 + (r+r')^2}}}$$

Where $k_p{}^2 = \dfrac{4rr'}{(z-z')^2 + (r+r')^2} = \dfrac{4rr'}{A^2}$

$$G(r,z;r',z') = \frac{2}{A} \int_0^{\pi} \frac{(2\sin^2\theta - 1) d\theta}{\sqrt{1 - k_p{}^2 \sin^2\theta}}$$

$$G(r,z;r',z') = \frac{2}{A} \int_0^{\pi} \frac{2\sin^2\theta d\theta}{\sqrt{1 - k_p{}^2 \sin^2\theta}} - \frac{2}{A} \int_0^{\pi} \frac{d\theta}{\sqrt{1 - k_p{}^2 \sin^2\theta}}$$

$$G(r,z;r',z') = \frac{4}{A} \int_0^{\frac{\pi}{2}} \frac{2\sin^2\theta d\theta}{\sqrt{1 - k_p{}^2 \sin^2\theta}} - \frac{4}{A} \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1 - k_p{}^2 \sin^2\theta}}$$

Where $K = K(k_p) = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-k_p^2\sin^2\theta}}$ is first Elliptical integral.

$$G(r,z;r',z') = \frac{8}{Ak_p^2}\int_0^{\frac{\pi}{2}} \frac{k_p^2\sin^2\theta d\theta}{\sqrt{1-k_p^2\sin^2\theta}} - \frac{4}{A}K$$

$$G(r,z;r',z') = \frac{-8}{Ak_p^2}\int_0^{\frac{\pi}{2}} \frac{(1-k_p^2\sin^2\theta-1)d\theta}{\sqrt{1-k_p^2\sin^2\theta}} - \frac{4}{A}K$$

$$G(r,z;r',z') = \frac{-8}{Ak_p^2}\int_0^{\frac{\pi}{2}} \frac{(1-k_p^2\sin^2\theta)d\theta}{\sqrt{1-k_p^2\sin^2\theta}} + \frac{8}{Ak_p^2}\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-k_p^2\sin^2\theta}} - \frac{4}{A}K$$

$$G(r,z;r',z') = \frac{-8}{Ak_p^2}\int_0^{\frac{\pi}{2}} \sqrt{1-k_p^2\sin^2\theta}\,d\theta + \frac{8}{Ak_p^2}K - \frac{4}{A}K$$

$$G(r,z;r',z') = \frac{-8}{Ak_p^2}E + \frac{8}{Ak_p^2}K - \frac{4}{A}K$$

Where $E = E(k_p) = \int_0^{\frac{\pi}{2}} \sqrt{1-k_p^2\sin^2\theta}\,d\theta$ is second Elliptical integral.

$$G(r,z;r',z') = \frac{8}{Ak_p^2}\left[-E + K - \frac{k_p^2}{2}K\right]$$

$$G(r,z;r',z') = \frac{8}{Ak_p^2}\left[-E + \left(1-\frac{k_p^2}{2}\right)K\right]$$

$$G(r,z;r',z') = \frac{8}{Ak_p^2}\left[\left(1-\frac{k_p^2}{2}\right)K - E\right]$$

By using $k_p{}^2 = \dfrac{4rr'}{A^2}$

$$G(r,z;r',z') = \frac{2A}{rr'}\left[\left(1-\frac{k_p{}^2}{2}\right)K - E\right] \tag{3.96}$$

A current J flowing in a very thin loop can be considered filamentary so that [22]

$$J(r) = I\delta(r'-R)\delta(z'-Z)e_\phi, \tag{3.97}$$

where δ is the Dirac Dalta function

So vector potential $A(r) = e_\phi \dfrac{\mu_0 J}{4\pi}\int r'G(r,z;r',z')dr'dz'$ can be written as

$$A(r) = e_\phi \frac{\mu_0}{4\pi}\int r'G(r,z;r',z')Jdr'dz' \tag{3.98}$$

$$A(r) = e_\phi \frac{\mu_0}{4\pi}\int r'G(r,z;r',z')I\delta(r'-R)\delta(z'-Z)dr'dz' \tag{3.99}$$

By using property of Dirac Dalta function [26]

$$f(x)\delta(x-a) = f(a)\delta(x-a) \tag{3.100}$$

$$\int_{-\infty}^{\infty} f(x)\delta(x-a)dx = f(a) \tag{3.101}$$

Then vector potential can be written as

$$A(r) = e_\phi \frac{\mu_0 I}{4\pi} RG(r,z;R,Z) \tag{3.102}$$

Equation (3.102) represents vector potential in terms of Green's function. [22]

3.7 Magnetic Energy of a System of Currents

The amount of charge per unit time passing down the wire is current I so the total work done per unit time is [26]

$$\frac{dW}{dt} = -I\varepsilon = L\frac{dI}{dt}I, \tag{3.103}$$

where ε = Electromotive force=electric potential for self inductance L

$$\varepsilon = -L\frac{dI}{dt} \tag{3.104}$$

Put equation (3.104) in equation (3.103) then

$$dW = LIdI \tag{3.105}$$

Integrate on both sides

$$W = \int LIdI$$

$$W = L\int IdI$$

$$W = \frac{1}{2}I^2L \tag{3.106}$$

Magnetic flux ϕ_m is

$$\phi_m = \int \bar{B} \bullet d\bar{a} \tag{3.107}$$

Put $\bar{B} = \nabla \times \bar{A}$

Then $\phi_m = \int_S (\nabla \times \bar{A}) \bullet d\bar{a}$ (3.108)

Applying Stokes Theorem on equation (3.108)

$$\phi_m = \int_l \vec{A} \bullet d\vec{l} \tag{3.109}$$

Magnetic flux can also be written as

$$\phi_m = LI \tag{3.110}$$

Where L is called self inductance.

Comparing equation (3.110) and equation (3.109) then [26]

$$LI = \int_l \vec{A} \bullet d\vec{l} \tag{3.111}$$

Equation (3.106) can be written as

$$W = \frac{1}{2} I(LI) \tag{3.112}$$

By using equation (3.111) in equation (3.112) then

$$W = \frac{1}{2} I \int_l \vec{A} \bullet d\vec{l}$$

$$W = \frac{1}{2} \int_l I\vec{A} \bullet d\vec{l} \tag{3.113}$$

The relation between current and current density is [26]

$$I = \int_s \vec{J} \bullet d\vec{a} \tag{3.114}$$

By using equation (3.114) in equation (3.113) then

$$W = \frac{1}{2} \int_l \int_s (\vec{J} \bullet d\vec{a}) \vec{A} \bullet d\vec{l}$$

$$W = \frac{1}{2}\int_l \int_s (\vec{J} \bullet \vec{A}) d\vec{a} \bullet d\vec{l}$$

$$W = \frac{1}{2}\int_{V'} (\vec{J} \bullet \vec{A}) dV' \tag{3.115}$$

The above work done is equal to the magnetic energy i.e. $W = U$, so

$$U = \frac{1}{2}\int_{V'} (\vec{J} \bullet \vec{A}) dV' \tag{3.116}$$

This magnetic energy associated with current density $\vec{J}$ and vector potential $\vec{A}$ [22].

3.7.1 Magnetic Energy for Two Distributed Currents

Let the set of coils shown in fig. One of them coil has height h_i, radial width w_i, major radius R_i and height given by Z_i. The properties of the other coils are indicated with a subscript j. The energy linked with the interaction of J_i with magnetic field formed by J_j is establish from

$$U_{j \to i} = \frac{1}{2}\int_{V'} \vec{J}_i(r) \bullet \vec{A}_j(r) dV' \tag{3.117}$$

Where $\vec{A}_j(r)$ is the vector potential linked with J_j and V' is volume surrounding J_i [26].

3.7.2 Magnetic Energy for a Single Distributed Currents

Current also interrelate with its own magnetic field so[22]

$$U_{i \to i} = \frac{1}{2}\int_{V'} \vec{J}_i(r) \bullet \vec{A}_i(r) dV' \tag{3.118}$$

3.7.3 Magnetic Energy for Two Filamentary Currents

The current density for ith and jth current is [22]

$$J_i(r) = I_i\delta(r - R_i)\delta(z - Z_i)e_\phi \tag{3.119}$$

$$J_j(r) = I_j\delta(r' - R_j)\delta(z' - Z_j)e_\phi \tag{3.120}$$

The vector potential for jth and ith current is

$$A_j(r) = e_\phi \frac{\mu_0 I_j}{4\pi} R_j G(r, z; R_j, Z_j) \tag{3.121}$$

$$A_i(r) = e_\phi \frac{\mu_0 I_i}{4\pi} R_i G(R_i, Z_i; r', z') \tag{3.122}$$

Above equations using in equation (3.117) then

$$U_{j\to i} = \frac{1}{2}\int_{V'} e_\phi \frac{\mu_0 I_j}{4\pi} R_j G(r, z; R_j, Z_j) \bullet I_i\delta(r - R_i)\delta(z - Z_i)e_\phi dV' \tag{3.123}$$

By using the properties of Dirac Delta function then

$$U_{j\to i} = \frac{\mu_0 I_i I_j}{4} R_i R_j G(R_i, Z_i; R_j, Z_j) \tag{3.124}$$

The magnetic energy of a set of two filamentary currents is described by above equation [22].

3.7.4 Magnetic Energy for a Single Filamentary Current

If expands the elliptical integrals for k_p approaches to 1 in equation (3.96) and (3.99) then the vector potential A_ϕ can be written as [22]

$$A_{\phi i} = \frac{\mu_0 I R_i}{2\pi}\left[\ln\left(\frac{8R_i}{a}\right) - 2\right] \tag{3.125}$$

Where a is the comparable radius of the coil, given by

$$a = \sqrt{\frac{h_i \omega_i}{\pi}} \tag{3.126}$$

Substitute equation (3.125) in equation (3.118), then we get magnetic energy for a single filamentary current as [22]

$$U_{i \to i} = \frac{\mu_0 I_i^{\,2} R_i}{2} \left[\ln\left(\frac{8R_i}{a} \right) - 2 \right] \tag{3.127}$$

3.8 Computation of Force

The net force acting on the on the poloidal field coils system can be written as negative gradient of magnetic potential energy [22].

$$F = -\nabla \sum_{i,j} U_{j \to i} \tag{3.128}$$

For two coil system total energy is given by

$$U_t = U_{i \to i} + U_{i \to j} + U_{j \to i} + U_{j \to j} \tag{3.129}$$

$$U_t = U_{i \to i} + U_{i \to j} + U_{j \to i} + U_{j \to j}$$

When $\quad U_{i \to i} = U_{j \to i} \quad$ then

$$U_t = U_{i \to i} + 2U_{i \to i} + U_{j \to j}$$

To compute force on coil i the ∇ operator is replaced with ∇_i

$$\nabla = \nabla_i = e_R \frac{\partial}{\partial R_i} + e_Z \frac{\partial}{\partial Z_i} \tag{3.131}$$

Magnetic energy for mutual induction can be written as

$$W_{ij} = \frac{1}{2} M_{ij} I_i I_j$$

And for self induction

$$W = \frac{1}{2}LI^2 = \frac{1}{2}LII = \frac{1}{2}\Phi I$$

Where $\Phi = flux$

So magnetic energy $U_{j\to i}$ coils is

$$U_{j\to i} = \frac{1}{2}M_{ij}I_iI_j$$

$$M_{ij} = \frac{2U_{j\to i}}{I_iI_j}$$

Since $flux = \Phi_{j\to i} = M_{ij}I_j$

Total flux through each coil as [22]

$$\Phi_i = M_{ii}I_i + M_{ij}I_j$$

$$\Phi_j = M_{ji}I_i + M_{jj}I_j$$

$$\Phi_j = M_{ij}I_i + M_{jj}I_j,$$

where $M_{ij} = M_{ji}$.

Now since the flux is constant so

$$\nabla_i\Phi_i = M_{ii}\nabla_iI_i + I_i\nabla_iM_{ii} + M_{ij}\nabla_iI_j + I_j\nabla_iM_{ij} = 0$$

$$\nabla_i\Phi_j = M_{ij}\nabla_iI_i + I_i\nabla_iM_{ij} + M_{ij}\nabla_iI_j = 0$$

Total energy can be written as

$$U_t = \frac{1}{2}\left(\Phi_iI_i + \Phi_jI_j\right)$$

$$\nabla_i U_t = -\frac{1}{2} I_i^2 \nabla_i M_{ii} - I_i I_j \nabla_i M_{ij}$$

By using equation (3.128) then force on coil i is

$$\vec{F}_i = -(\nabla_i U_t)_\phi = \frac{1}{2} I_i^2 \nabla_i M_{ii} + I_i I_j \nabla_i M_{ij}, \tag{3.132}$$

where ϕ specify that the energy gradient has been calculated with respect to the constant flux requirement.

This expression can be created to an N coil system [22].

$$\vec{F}_i = I_i^2 \nabla_i \left(\frac{U_{i\to i}}{I_i^2} \right) + I_i \sum_{j=1, j\neq 1}^{N} 2 I_j \nabla_i \left(\frac{U_{j\to i}}{I_i I_j} \right) \tag{3.133}$$

3.8.1 Force Between Two Filamentary Currents

The equation (4.124) represents the magnetic energy of a set of two filamentary currents i.e. [22]

$$U_{j\to i} = \frac{\mu_0 I_i I_j}{4} R_i R_j G(R_i, Z_i; R_j, Z_j)$$

Where $$G(R_i, Z_i; R_j, Z_j) = \frac{2A}{R_i R_j} \left[\left(1 - \frac{k_p^2}{2} \right) K - E \right]$$

Where $A^2 = (Z_i - Z_j)^2 + (R_i + R_j)^2$ and $k_p^2 = \dfrac{4 R_i R_j}{(Z_i - Z_j)^2 + (R_i + R_j)^2} = \dfrac{4 R_i R_j}{A^2}$

So magnetic energy of a set of two filamentary currents is given by

$$U_{j\to i} = \frac{\mu_0 I_i I_j}{2} A \left[\left(1 - \frac{k_p^2}{2} \right) K - E \right] \tag{3.134}$$

By using equation (3.128), (3.131) and (3.134) then magnetic force between two filamentary currents can be written as

$$\bar{F}_{j\to i} = e_R \frac{\partial U_{j\to i}}{\partial R_i} + e_Z \frac{\partial U_{j\to i}}{\partial Z_i} \tag{3.135}$$

The e_R component of magnetic force can be finding by using the derivative properties of elliptical integrals as follow: [22]

$$\frac{\partial U_{j\to i}}{\partial R_i} = \frac{\mu_0 I_i I_j}{2} \frac{\partial}{\partial R_i}\left[A\left\{\left(1-\frac{k_p{}^2}{2}\right)K - E\right\}\right]$$

$$\frac{\partial U_{j\to i}}{\partial R_i} = \frac{\mu_0 I_i I_j}{2}\left[\frac{\partial A}{\partial R_i}\left\{\left(1-\frac{k_p{}^2}{2}\right)K - E\right\} + A\frac{\partial}{\partial k_p}\left\{\left(1-\frac{k_p{}^2}{2}\right)K - E\right\}\frac{\partial k_p}{\partial R_i}\right]$$

Where $\dfrac{\partial A}{\partial R_i} = \dfrac{(R_i + R_j)}{\sqrt{(Z_i - Z_j)^2 + (R_i + R_j)^2}} = \dfrac{R_i + R_j}{A}$

and

$$\frac{\partial k_p}{\partial R_i} = \frac{\sqrt{(Z_i - Z_j)^2 + (R_i + R_j)^2}\,\dfrac{R_j}{\sqrt{R_i R_j}} - \dfrac{2(R_i + R_j)\sqrt{R_i R_j}}{\sqrt{(Z_i - Z_j)^2 + (R_i + R_j)^2}}}{(Z_i - Z_j)^2 + (R_i + R_j)^2}$$

Multiply and divide by k_p

$$\frac{\partial k_p}{\partial R_i} = \frac{2R_j - \dfrac{4R_i R_j (R_i + R_j)}{(Z_i - Z_j)^2 + (R_i + R_j)^2}}{k_p\left\{(Z_i - Z_j)^2 + (R_i + R_j)^2\right\}}$$

$$\frac{\partial k_p}{\partial R_i} = \frac{2R_j - \dfrac{4R_i R_j (R_i + R_j)}{A^2}}{k_p A^2}$$

$$\frac{\partial k_p}{\partial R_i} = \frac{2A^2R_j - 4R_iR_j(R_i + R_j)}{k_pA^4}$$

And

$$\frac{\partial}{\partial k_p}\left\{\left(1-\frac{k_p^{\,2}}{2}\right)K - E\right\} = -k_pK + \left(1-\frac{k_p^{\,2}}{2}\right)\frac{\partial K}{\partial k_p} - \frac{\partial E}{\partial k_p}$$

By using

$$\frac{dK}{dk_p} = \frac{1}{k_p}\left[\frac{E}{1-k_p^{\,2}} - K\right] \text{ and } \frac{dE}{dk_p} = \frac{1}{k_p}[E-K] \text{ then}$$

$$\frac{\partial}{\partial k_p}\left\{\left(1-\frac{k_p^{\,2}}{2}\right)K - E\right\} = -k_pK + \left(1-\frac{k_p^{\,2}}{2}\right)\frac{1}{k_p}\left[\frac{E}{1-k_p^{\,2}} - K\right] - \frac{1}{k_p}[E-K]$$

$$\frac{\partial}{\partial k_p}\left\{\left(1-\frac{k_p^{\,2}}{2}\right)K - E\right\} = -k_pK + \frac{E}{k_p(1-k_p^{\,2})} - \frac{K}{k_p} - \frac{k_pE}{2(1-k_p^{\,2})} + \frac{k_pK}{2} - \frac{E}{k_p} + \frac{K}{k_p}$$

$$\frac{\partial}{\partial k_p}\left\{\left(1-\frac{k_p^{\,2}}{2}\right)K - E\right\} = -k_pK + \frac{E}{k_p(1-k_p^{\,2})} - \frac{k_pE}{2(1-k_p^{\,2})} + \frac{k_pK}{2} - \frac{E}{k_p}$$

$$\frac{\partial}{\partial k_p}\left\{\left(1-\frac{k_p^{\,2}}{2}\right)K - E\right\} = -\frac{k_pK}{2} + \frac{E}{k_p(1-k_p^{\,2})} - \frac{k_pE}{2(1-k_p^{\,2})} - \frac{E}{k_p}$$

$$\frac{\partial}{\partial k_p}\left\{\left(1-\frac{k_p^{\,2}}{2}\right)K - E\right\} = -\frac{k_pK}{2} + \frac{2E - k_p^{\,2}E - 2(1-k_p^{\,2})E}{2k_p(1-k_p^{\,2})}$$

$$\frac{\partial}{\partial k_p}\left\{\left(1-\frac{k_p^{\,2}}{2}\right)K - E\right\} = -\frac{k_pK}{2} + \frac{k_p^{\,2}E}{2k_p(1-k_p^{\,2})}$$

$$\frac{\partial}{\partial k_p}\left\{\left(1-\frac{k_p^{\ 2}}{2}\right)K-E\right\}=-\frac{k_pK}{2}+\frac{k_pE}{2(1-k_p^{\ 2})}$$

$$\frac{\partial}{\partial k_p}\left\{\left(1-\frac{k_p^{\ 2}}{2}\right)K-E\right\}=\frac{k_p}{2}\left[\frac{E}{1-k_p^{\ 2}}-K\right]$$

$$\frac{\partial U_{j\to i}}{\partial R_i}=\frac{\mu_0 I_i I_j}{2}\left[\frac{R_i+R_j}{A}\left\{\left(1-\frac{k_p^{\ 2}}{2}\right)K-E\right\}+A\frac{k_p}{2}\left(\frac{E}{1-k_p^{\ 2}}-K\right)\frac{2A^2R_j-4R_iR_j(R_i+R_j)}{k_pA^4}\right]$$

$$\frac{\partial U_{j\to i}}{\partial R_i}=\frac{\mu_0 I_i I_j}{2}\left[\frac{R_i+R_j}{A}\left\{\left(1-\frac{k_p^{\ 2}}{2}\right)K-E\right\}+\left(\frac{E}{1-k_p^{\ 2}}-K\right)\frac{A^2R_j-2R_iR_j(R_i+R_j)}{A^3}\right]$$

$$\frac{\partial U_{j\to i}}{\partial R_i}=\frac{\mu_0 I_i I_j}{2}\left[\frac{R_i+R_j}{A}\left\{\left(1-\frac{k_p^{\ 2}}{2}\right)K-E\right\}+\left(\frac{E}{1-k_p^{\ 2}}-K\right)\left\{\frac{R_j}{A}-\frac{2R_iR_j(R_i+R_j)}{A^3}\right\}\right]$$

$$\frac{\partial U_{j\to i}}{\partial R_i}=\frac{\mu_0 I_i I_j}{2}\left(\frac{R_i+R_j}{A}\right)\left[\left\{\left(1-\frac{k_p^{\ 2}}{2}\right)K-E\right\}+\left(\frac{E}{1-k_p^{\ 2}}-K\right)\left\{\frac{R_j}{R_i+R_j}-\frac{2R_iR_j}{A^2}\right\}\right]$$

$$\frac{\partial U_{j\to i}}{\partial R_i}=\frac{\mu_0 I_i I_j}{2}\left(\frac{R_i+R_j}{A}\right)\left[\left\{\left(1-\frac{k_p^{\ 2}}{2}\right)K-E\right\}+\left(\frac{E}{1-k_p^{\ 2}}-K\right)\left\{\frac{R_j}{R_i+R_j}-\frac{k_p^{\ 2}}{2}\right\}\right]$$

$$\frac{\partial U_{j\to i}}{\partial R_i}=\frac{\mu_0 I_i I_j}{2}\left(\frac{R_i+R_j}{A}\right)\left[\left(1-\frac{k_p^{\ 2}}{2}\right)K-E+\frac{E}{1-k_p^{\ 2}}\left\{\frac{R_j}{R_i+R_j}-\frac{k_p^{\ 2}}{2}\right\}-K\left\{\frac{R_j}{R_i+R_j}-\frac{k_p^{\ 2}}{2}\right\}\right]$$

$$\frac{\partial U_{j\to i}}{\partial R_i}=\frac{\mu_0 I_i I_j}{2}\left(\frac{R_i+R_j}{A}\right)\left[\left(1-\frac{R_j}{R_i+R_j}\right)K+\frac{E}{1-k_p^{\ 2}}\left\{\frac{R_j}{R_i+R_j}-\frac{k_p^{\ 2}}{2}\right\}-E\right]$$

$$\frac{\partial U_{j\to i}}{\partial R_i}=\frac{\mu_0 I_i I_j}{2}\left(\frac{R_i+R_j}{A}\right)\left[\left(\frac{R_i}{R_i+R_j}\right)K+\frac{E}{1-k_p^{\ 2}}\left\{\frac{R_j}{R_i+R_j}-\frac{k_p^{\ 2}}{2}-1+k_p^{\ 2}\right\}\right]$$

$$\frac{\partial U_{j\to i}}{\partial R_i}=\frac{\mu_0 I_i I_j}{2}\left(\frac{R_i+R_j}{A}\right)\left[\left(\frac{R_i}{R_i+R_j}\right)K+\frac{E}{1-k_p^{\,2}}\left\{k_p^{\,2}-\frac{k_p^{\,2}}{2}+\frac{R_j}{R_i+R_j}-1\right\}\right]$$

$$\frac{\partial U_{j\to i}}{\partial R_i}=\frac{\mu_0 I_i I_j}{2}\left(\frac{R_i+R_j}{A}\right)\left[\left(\frac{R_i}{R_i+R_j}\right)K+\frac{E}{1-k_p^{\,2}}\left\{\frac{k_p^{\,2}}{2}-\frac{R_i}{R_i+R_j}\right\}\right] \tag{3.136}$$

Similarly

$$\frac{\partial U_{j\to i}}{\partial R_j}=\frac{\mu_0 I_i I_j}{2}\left(\frac{R_i+R_j}{A}\right)\left[\left(\frac{R_j}{R_i+R_j}\right)K+\frac{E}{1-k_p^{\,2}}\left\{\frac{k_p^{\,2}}{2}-\frac{R_j}{R_i+R_j}\right\}\right]. \tag{3.137}$$

Now the e_Z component of magnetic force can be finding by using the derivative properties of elliptical integrals as follow [22]

$$\frac{\partial U_{j\to i}}{\partial Z_i}=\frac{\mu_0 I_i I_j}{2}\frac{\partial}{\partial Z_i}\left[A\left\{\left(1-\frac{k_p^{\,2}}{2}\right)K-E\right\}\right]$$

$$\frac{\partial U_{j\to i}}{\partial Z_i}=\frac{\mu_0 I_i I_j}{2}\left[\frac{\partial A}{\partial Z_i}\left\{\left(1-\frac{k_p^{\,2}}{2}\right)K-E\right\}+A\frac{\partial}{\partial Z_i}\left\{\left(1-\frac{k_p^{\,2}}{2}\right)K-E\right\}\right]$$

$$\frac{\partial U_{j\to i}}{\partial Z_i}=\frac{\mu_0 I_i I_j}{2}\left[\frac{\partial A}{\partial Z_i}\left\{\left(1-\frac{k_p^{\,2}}{2}\right)K-E\right\}+A\frac{\partial}{\partial k_p}\left\{\left(1-\frac{k_p^{\,2}}{2}\right)K-E\right\}\frac{\partial k_p}{\partial Z_i}\right]$$

$$\frac{\partial A}{\partial Z_i}=\frac{Z_i-Z_j}{\sqrt{(Z_i-Z_j)^2+(R_i+R_j)^2}}$$

$$\frac{\partial A}{\partial Z_i}=\frac{Z_i-Z_j}{A}$$

$$\frac{\partial k_p}{\partial Z_i}=\frac{-k_p(Z_i-Z_j)}{(Z_i-Z_j)^2+(R_i+R_j)^2}$$

$$\frac{\partial k_p}{\partial Z_i}=\frac{-4R_iR_j(Z_i-Z_j)}{k_p\{(Z_i-Z_j)^2+(R_i+R_j)^2\}\{(Z_i-Z_j)^2+(R_i+R_j)^2\}}$$

$$\frac{\partial k_p}{\partial Z_i}=\frac{-4R_iR_j(Z_i-Z_j)}{k_pA^2A^2}$$

$$\frac{\partial k_p}{\partial Z_i}=\frac{-4R_iR_j(Z_i-Z_j)}{k_pA^4}$$

$$\frac{\partial U_{j\to i}}{\partial Z_i}=\frac{\mu_0I_iI_j}{2}\left[\frac{Z_i-Z_j}{A}\left\{\left(1-\frac{k_p^{\ 2}}{2}\right)K-E\right\}+A\frac{k_p}{2}\left(\frac{E}{1-k_p^{\ 2}}-K\right)\left(\frac{-4R_iR_j(Z_i-Z_j)}{k_pA^4}\right)\right]$$

$$\frac{\partial U_{j\to i}}{\partial Z_i}=\frac{\mu_0I_iI_j}{2}\left[\frac{Z_i-Z_j}{A}\left\{\left(1-\frac{k_p^{\ 2}}{2}\right)K-E\right\}-\left(\frac{E}{1-k_p^{\ 2}}-K\right)\left(\frac{2R_iR_j(Z_i-Z_j)}{A^3}\right)\right]$$

$$\frac{\partial U_{j\to i}}{\partial Z_i}=\frac{\mu_0I_iI_j}{2}\left(\frac{Z_i-Z_j}{A}\right)\left[\left\{\left(1-\frac{k_p^{\ 2}}{2}\right)K-E\right\}-\left(\frac{E}{1-k_p^{\ 2}}-K\right)\left(\frac{2R_iR_j}{A^2}\right)\right]$$

$$\frac{\partial U_{j\to i}}{\partial Z_i}=\frac{\mu_0I_iI_j}{2}\left(\frac{Z_i-Z_j}{A}\right)\left[\left(1-\frac{k_p^{\ 2}}{2}\right)K-E-\frac{E}{1-k_p^{\ 2}}\left(\frac{2R_iR_j}{A^2}\right)+K\left(\frac{2R_iR_j}{A^2}\right)\right]$$

$$\frac{\partial U_{j\to i}}{\partial Z_i}=\frac{\mu_0I_iI_j}{2}\left(\frac{Z_i-Z_j}{A}\right)\left[\left(1-\frac{k_p^{\ 2}}{2}\right)K-E-\frac{E}{1-k_p^{\ 2}}\left(\frac{k_p^{\ 2}}{2}\right)+K\left(\frac{k_p^{\ 2}}{2}\right)\right]$$

$$\frac{\partial U_{j\to i}}{\partial Z_i}=\frac{\mu_0I_iI_j}{2}\left(\frac{Z_i-Z_j}{A}\right)\left[\left(1-\frac{k_p^{\ 2}}{2}\right)K+K\left(\frac{k_p^{\ 2}}{2}\right)-E-\frac{E}{1-k_p^{\ 2}}\left(\frac{k_p^{\ 2}}{2}\right)\right]$$

$$\frac{\partial U_{j\to i}}{\partial Z_i}=\frac{\mu_0I_iI_j}{2}\left(\frac{Z_i-Z_j}{A}\right)\left[K+\frac{E}{1-k_p^{\ 2}}\left(-1+k_p^{\ 2}-\frac{k_p^{\ 2}}{2}\right)\right]$$

$$\frac{\partial U_{j\to i}}{\partial Z_i}=\frac{\mu_0 I_i I_j}{2}\left(\frac{Z_i-Z_j}{A}\right)\left[K+\frac{E}{1-k_p^{\,2}}\left(-1+\frac{k_p^{\,2}}{2}\right)\right]$$

$$\frac{\partial U_{j\to i}}{\partial Z_i}=\frac{\mu_0 I_i I_j}{2}\left(\frac{Z_i-Z_j}{A}\right)\left[K+\frac{E}{1-k_p^{\,2}}\left(\frac{k_p^{\,2}}{2}-1\right)\right] \tag{3.138}$$

Similarly

$$\frac{\partial U_{j\to i}}{\partial Z_j}=-\frac{\mu_0 I_i I_j}{2}\left(\frac{Z_i-Z_j}{A}\right)\left[K+\frac{E}{1-k_p^{\,2}}\left(\frac{k_p^{\,2}}{2}-1\right)\right] \tag{3.139}$$

By comparing equation (3.138) and equation (3.139) it is proved that

$$\frac{\partial U_{j\to i}}{\partial Z_j}=-\frac{\partial U_{j\to i}}{\partial Z_i}. \tag{3.140}$$

Substitute equations (3.136) and (3.139) in equation (3.135) then total magnetic force on coil i due to coil j is given by [22]

$$\begin{aligned}\bar{F}_{j\to i}&=\frac{\mu_0 I_i I_j}{2}\left(\frac{R_i+R_j}{A}\right)\left[\left(\frac{R_i}{R_i+R_j}\right)K+\frac{E}{1-k_p^{\,2}}\left\{\frac{k_p^{\,2}}{2}-\frac{R_i}{R_i+R_j}\right\}\right]e_R\\&+\frac{\mu_0 I_i I_j}{2}\left(\frac{Z_i-Z_j}{A}\right)\left[K+\frac{E}{1-k_p^{\,2}}\left(\frac{k_p^{\,2}}{2}-1\right)\right]e_Z\end{aligned} \tag{3.141}$$

and similarly total magnetic force on coil j due to coil i is given by

$$\begin{aligned}\bar{F}_{i\to j}&=\frac{\mu_0 I_i I_j}{2}\left(\frac{R_i+R_j}{A}\right)\left[\left(\frac{R_j}{R_i+R_j}\right)K+\frac{E}{1-k_p^{\,2}}\left\{\frac{k_p^{\,2}}{2}-\frac{R_j}{R_i+R_j}\right\}\right]e_R\\&-\frac{\mu_0 I_i I_j}{2}\left(\frac{Z_i-Z_j}{A}\right)\left[K+\frac{E}{1-k_p^{\,2}}\left(\frac{k_p^{\,2}}{2}-1\right)\right]e_Z\end{aligned} \tag{3.142}$$

3.8.2 Self Force of a Filamentary Current

The magnetic self force of a filamentary coil interrelating by means of its own magnetic field is expressed by taking negative gradient of equation (3.127) so [22]

$$\bar{F}_{i\to i} = -\nabla_i U_{i\to i} = e_R \frac{\mu_0 I_i^{\,2}}{2}\left[\ln\left(\frac{8R_i}{a}\right)-1\right] \tag{3.143}$$

3.8.3 Force Between Two Distributed Currents

The interaction energy of a distributed current J_i in the magnetic field created by a current J_j is given by [22]

$$U_{j\to i} = \frac{1}{2}\int_{V'} \bar{J}_i(r)\bullet \bar{A}_j(r)dV' \tag{3.144}$$

Where $$I = I_i = J_i \int_{Z_i-\frac{h_i}{2}}^{Z_i+\frac{h_i}{2}} \int_{R_i-\frac{w_i}{2}}^{R_i+\frac{w_i}{2}} drdz$$

and $$I' = I_j = J_j \int_{Z_j-\frac{h_j}{2}}^{Z_j+\frac{h_j}{2}} \int_{R_j-\frac{w_j}{2}}^{R_j+\frac{w_j}{2}} drdz$$

then $$U_{j\to i} = \frac{\mu_0 J_i J_j}{4} \int_{Z_i-\frac{h_i}{2}}^{Z_i+\frac{h_i}{2}} \int_{R_i-\frac{w_i}{2}}^{R_i+\frac{w_i}{2}} \int_{Z_j-\frac{h_j}{2}}^{Z_j+\frac{h_j}{2}} \int_{R_j-\frac{w_j}{2}}^{R_j+\frac{w_j}{2}} rr'G(r,z;r',z')drdzdr'dz' \tag{3.145}$$

By using $\bar{F}_{j\to i} = -\nabla U_{j\to i}$

$$\bar{F}_{j\to i} = e_R \frac{1}{2}\frac{\partial}{\partial R_i}\int_{V'} \bar{J}_i(r)\bullet \bar{A}_j(r)dV' + e_Z \frac{1}{2}\frac{\partial}{\partial Z_i}\int_{V'} \bar{J}_i(r)\bullet \bar{A}_j(r)dV' \tag{3.146}$$

$$\bar{F}_{j\to i} = e_R \frac{\mu_0 J_i J_j}{4} \int_{Z_i-\frac{h_i}{2}}^{Z_i+\frac{h_i}{2}} \int_{Z_j-\frac{h_j}{2}}^{Z_j+\frac{h_j}{2}} \int_{R_j-\frac{w_j}{2}}^{R_j+\frac{w_j}{2}} \frac{\partial}{\partial R_i} \int_{R_i-\frac{w_i}{2}}^{R_i+\frac{w_i}{2}} rr'G(r,z;r',z')drdzdr'dz'$$
$$+ e_Z \frac{\mu_0 J_i J_j}{4} \int_{R_i-\frac{w_i}{2}}^{R_i+\frac{w_i}{2}} \int_{Z_j-\frac{h_j}{2}}^{Z_j+\frac{h_j}{2}} \int_{R_j-\frac{w_j}{2}}^{R_j+\frac{w_j}{2}} \frac{\partial}{\partial Z_i} \int_{Z_i-\frac{h_i}{2}}^{Z_i+\frac{h_i}{2}} rr'G(r,z;r',z')drdzdr'dz' \tag{3.147}$$

$$\bar{F}_{j\to i} = e_R \frac{\mu_0 J_i J_j}{4} \int_{Z_i-\frac{h_i}{2}}^{Z_i+\frac{h_i}{2}} \int_{Z_j-\frac{h_j}{2}}^{Z_j+\frac{h_j}{2}} \int_{R_j-\frac{w_j}{2}}^{R_j+\frac{w_j}{2}} (R_i+\frac{w_i}{2})r'G(R_i+\frac{w_i}{2},z;r',z')dzdr'dz'$$
$$- e_R \frac{\mu_0 J_i J_j}{4} \int_{Z_i-\frac{h_i}{2}}^{Z_i+\frac{h_i}{2}} \int_{Z_j-\frac{h_j}{2}}^{Z_j+\frac{h_j}{2}} \int_{R_j-\frac{w_j}{2}}^{R_j+\frac{w_j}{2}} (R_i-\frac{w_i}{2})r'G(R_i-\frac{w_i}{2},z;r',z')dzdr'dz'$$
$$+ e_Z \frac{\mu_0 J_i J_j}{4} \int_{R_i-\frac{w_i}{2}}^{R_i+\frac{w_i}{2}} \int_{Z_j-\frac{h_j}{2}}^{Z_j+\frac{h_j}{2}} \int_{R_j-\frac{w_j}{2}}^{R_j+\frac{w_j}{2}} rr'G(r,Z_i+\frac{h_i}{2};r',z')drdr'dz'$$
$$- e_Z \frac{\mu_0 J_i J_j}{4} \int_{R_i-\frac{w_i}{2}}^{R_i+\frac{w_i}{2}} \int_{Z_j-\frac{h_j}{2}}^{Z_j+\frac{h_j}{2}} \int_{R_j-\frac{w_j}{2}}^{R_j+\frac{w_j}{2}} rr'G(r,Z_i-\frac{h_i}{2};r',z')drdr'dz' \tag{3.148}$$

Equation (3.148) represents the magnetic force between two distributed currents [22].

CHAPTER 4

4. Simulation of Poloidal Field Coils

The TOKAMEQ Code (Tokamak Equilibrium) is utilized in this thesis to design the poloidal field coils and control the currents in them. This code was developed in Moscow State University for calculations of the magnitudes of the exterior coils currents. This code has foundation on the numerical solving of GRAD-SHAFRANOV equation [28].

4.1 Poloidal Field Coils Design and Currents Controls

Adjusting the initial parameters, coils currents and positions in TOKAMEQ Codes then obtain following are possible Poloidal Cross Sections and poloidal flux contours as outputs.

4.1.1 Case-1

Domain centre on R = 15 centimetre

Domain magnitude R = 20 centimetre, Z = 50 centimetre

Mesh magnitude R = 45 Z = 62

Symmetry on Z

Poloidal Beta = 0.1

Total current = 40 Kilo Ampere

Co-ordinates of magnetic axis R = 15.7809 centimetre, Z = 0 centimetre

Plasma diameter R = 18.1251 centimetre, Z = 28.3058 centimetre

Elongation = 1.561691

Flux at the axis	= 5.910 millivolt seconds
Flux at the boundary	= 8.25 millivolt seconds
Flux at the separatrix	= 5.938 millivolt seconds
Cord triangularity	average=0.2341, top=0.2341, bottom=0.2341
95% region triangularity	average =0.2063, top=0.2063, bottom=0.2063

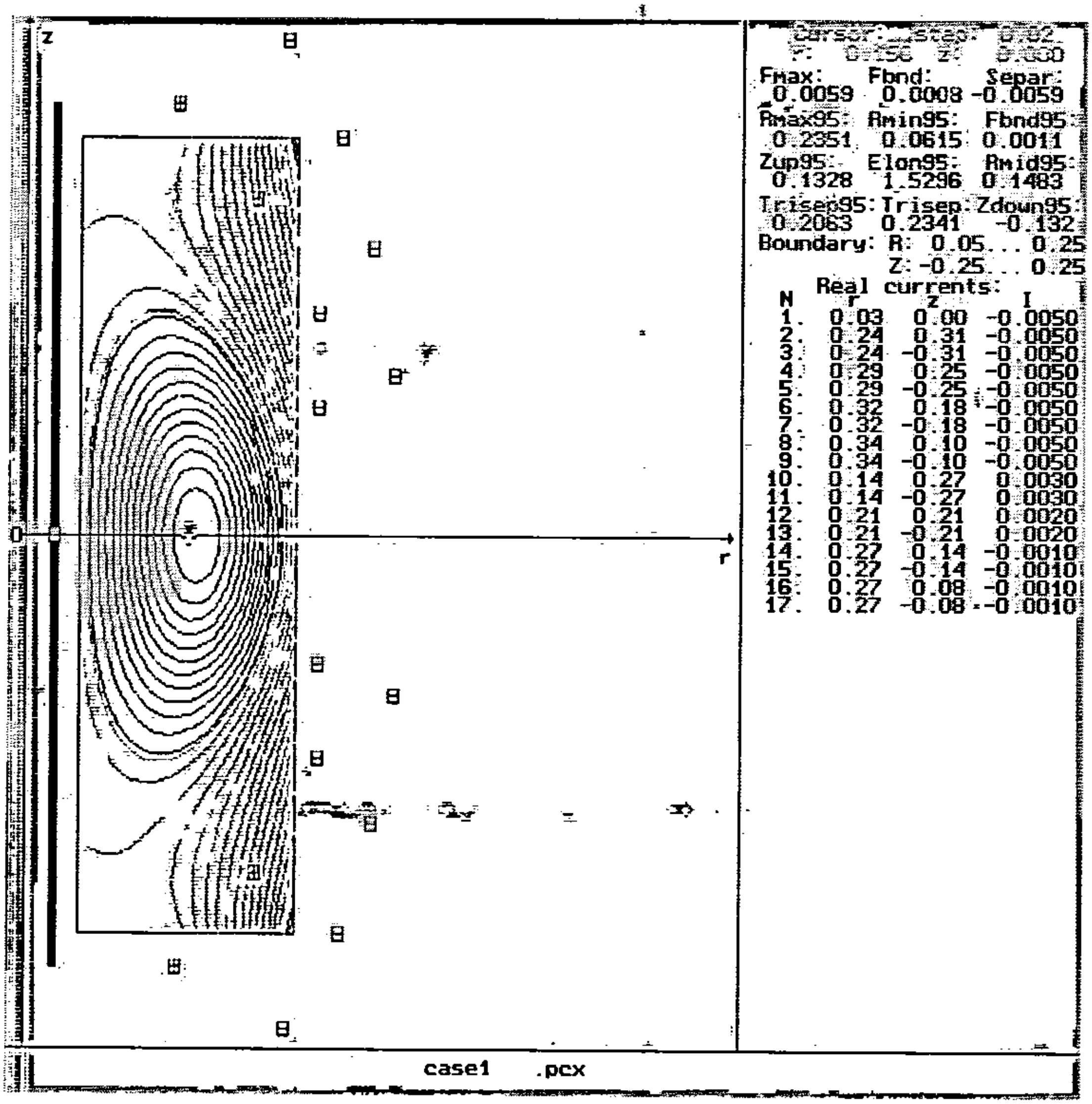

Figure-4.1: Position of poloidal field coils and poloidal cross-section.

In above case adjusting the center of domain on $R=$ 15 centi metre, Domain magnitude R=20 centimetre and Z=50 centi metre, Poloidal β =0.1, Total plasma current = 40 Kilo Amperes, and also adjust cuurents in Kilo Amperes and position of poloidal field coils as shown in figure, the magnetic axis is approximately at the center of the rectangular region.

Total seventeen poloidal field coils are used. The poloidal field coil-1 represents central solenoid, coil10 and 11 are elongation poloidal field coils to control of elongation and remaining poloidal field coils are shaping coil for plasma.

Elongation =1.470373 is obtained by adjusting above parameters.

4.1.2 Case-2

Domain centre on R = 15 centimetre

Domain magnitude R = 20 centimetre, Z = 50 centimetre

Mesh magnitude R = 45, Z = 62 centimetre

Symmetry on Z

Poloidal Beta = 0.1

Total current = 35 Kilo Ampere

Co-ordinates of magnetic axis R = 15.5928 centimetre, Z = 0 centimetre

Plasma diameter on R = 18.2688 centimetre, Z = 28.7086 centimetre

Elongation = 1.571459

Flux at the axis = 4.964 millivolt seconds

Flux at the boundary = 0.6 millivolt seconds

Flux at the separatrix = 5.678 millivolt seconds

Cord triangularity average = 0.2554 , top = 0.2554 , bottom = 0.2554

95% region triangularity average = 0.2273 , top = 0.2273 , bottom = 0.2273

In case-2 adjusting the Poloidal Beta =0.1, Total plasma current = 35 Kilo Ampere, and also change coils cuurents in Kilo Ampere and changed position of poloidal field coils as shown in fig.. The magnetic axis is approximately at the center of the rectangular region.

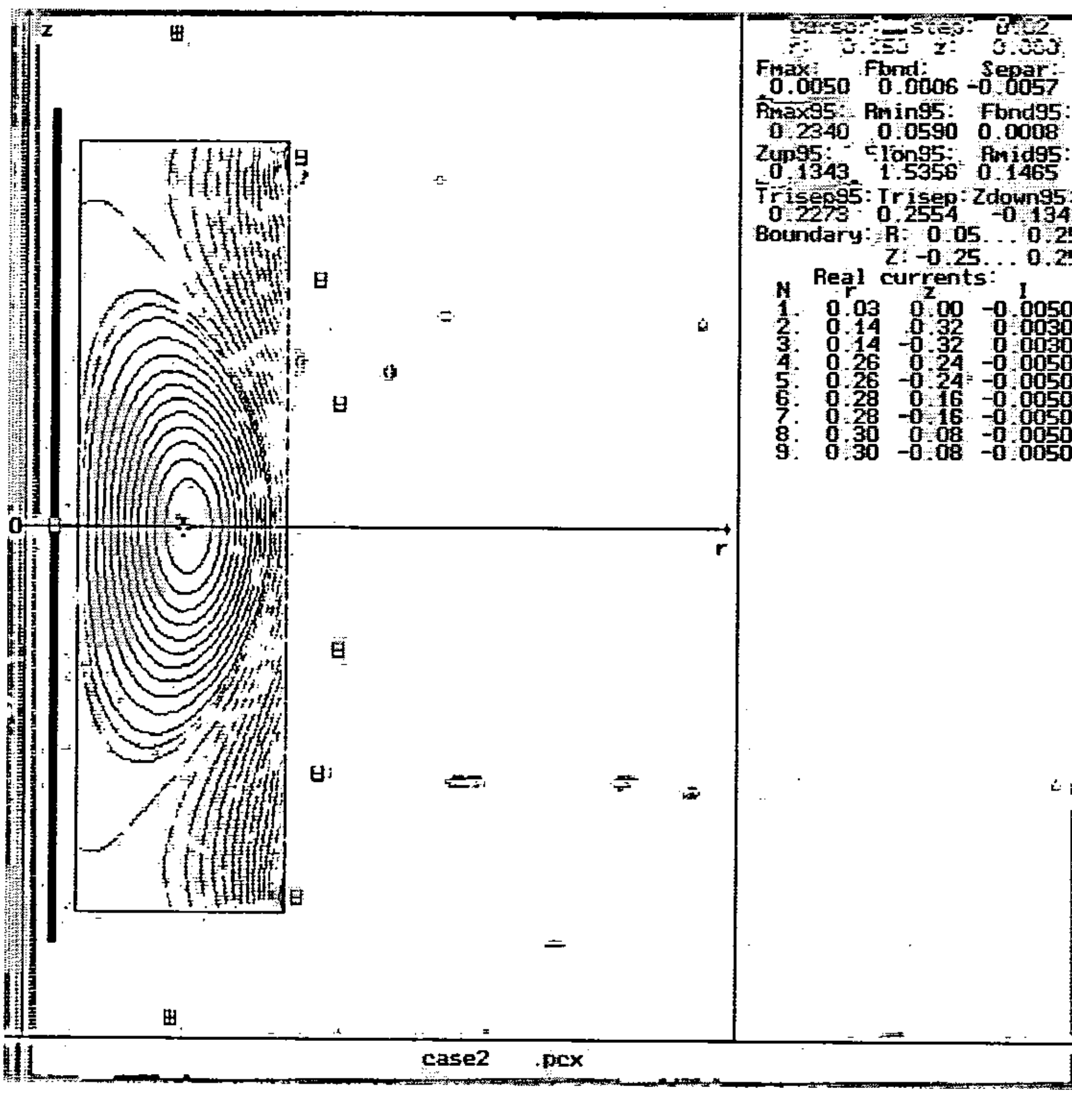

Figure-4.2: Position of poloidal field coils and poloidal cross-section.

Total nine poloidal field coils are used. The poloidal field coil-1 represents central solenoid, coil-2 and 3 are elongation poloidal field coils to control of elongation and remaining poloidal field coils are shaping coil for plasma.

Elongation=1.571459 is obtained by adjusting above parameters.

The toroidal field is generated by a series of coils in an even way spaced around the torus, and the poloidal field is generated by a strong electric current flowing through the plasma.

Elongation and poloidal beta is important for stability, safety factor and confinement of plasma. If elongation is equal to one then plasma shape is circular and if elongation is greater than one plasma shape is elongated. Large value of elongation is needed for large confinement time of plasma. Poloidal beta should be less than one for stability and large time of confinement. Elongation and poloidal beta are obtained by control of currents in the poloidal field coils.

CHAPTER 5

5. Summary and Conclusion

The goal of this thesis was the design of poloidal field coils and their control of currents. For this purpose inputs was given to the TOKAMEQ Code and out put results was obtained in the form of poloidal flux cross section and poloidal flux contours. In the output of TOKAMAQ Code, the poloidal flux cross section and poloidal flux contours are distributed around the horizontal and symmetrical vertical axis and within the rectangular boundary. The position of the poloidal field coils is keep near rectangular boundary inside or out side. The shape of the coil is adjusted by size of horizontal width and vertical width. The poloidal field coil on the left side of rectangular boundary is act as central solenoid and its function is like the primary of transformers. The plasma is present inside the rectangular boundary and act as secondary of the transformers. The distance between the vertical axis and central solenoid is keep very small. The current in the central solenoid and the coils on the right side is negative. These poloidal field coils induced positive current in the plasma. The poloidal field coils on the top and bottom of the rectangular boundary are act as elongation coils and current in them is positive as in plasma. The magnetic field is produced due to changing the currents in the poloidal field coils. The magnetic field produced magnetic force act on the plasma by poloidal field coils. The pressure on the center of the plasma is greater than the pressure towards the out side of plasma boundary. The last close flux contour which make X-point is called separatrix. All the close flux contour of the plasma is present in side the boundary.

The force of interaction is also present between the coils. Plasma will occupy all the geometrical space accessible, because of the collisions between the particles. Magnetic fields are utilized for confine a plasma, because the electrons and ions of which it consists will pursue helical paths in the region of the magnetic field lines.

If a vessel containing plasma is located in a rectilinear magnetic field, the particles of plasma cannot get in touch with the side walls, but they will hit the ends of the vessel. Tokamak poloidal field configuration is used to put off the particles from coming into contact with the material walls in this way. For tokamak configuration, the danger of losses is separated by curving the magnetic lines just about to form a closed loop. Theoretical study of particle trajectories demonstrates that, if the particles are to be confined, the toroidal field be required to have superimposed upon it a field component perpendicular to it. This component is poloidal field. The force lines of the whole field thus become helical paths, around and along which the plasma particles are guided.

Different out put elongation are obtained by changing position of coils, changing their shapes, changing their currents, changing total plasma current, changing poloidal beta. These values of elongation are importance for plasma stability. An elongated plasma contain a higher poloidal beta than a rounded plasma by means of the same safety factor and aspect ratio Elongated plasma provides the greater value of plasma current density. Increasing the value of plasma elongation then increasing the value of safety factor. Large value of safety factor shows the more stability of the elongated plasma. Stability of plasma is essential for large amount of plasma confinement time. Greater plasma confinement time is required for confinement of plasma in the tokamak machine.

Above conversation point out that the poloidal field coils are compulsory to stabilize the upright instability linked by means of extremely elongated equilibria. Because the plasma floats in the upright direction, current is provided to poloidal field coils in sequence to push the plasma reverse to the center of the tokamak. The design goal is achieved by satisfactory results.

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