Equilibrium and Stability Of Spherical Tokamak With Flow

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It is certified that the work presented in this thesis entitled "Equilibrium and Stability of Spherical Tokamaks with Flow" by **Rizwan Afzal Registration No. 15-FBAS/MSPHY/F10** is of sufficient standard in scope and quality for the award of degree of MS Physics from International Islamic University, Islamabad.

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I am filled with the praise and glory to All Mighty Allah, the most merciful and benevolent, who created the universe with the idea of beauty, symmetry and harmony, with regularity and without any chaos, and gave the abilities to discover what He thought.

Bless Muhammad Peace be upon Him) the seal of the prophets, and His pure and pious progeny.

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I can never thank enough my family for everything they have done for me. Even then, I wish to record my deepest obligations to my mother who always loves me and for her endless contribution to make me stay on the track. No one tolerates me more than they do. My father is a great man and great father. I am forever in debated to the love and care of my brothers M.Imran Afzal and M.Kamran Afzal and my loving Arshia.

Rizwan Afzal

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Abstract

The purpose of the present work is the analysis of Equilibrium and Stability of the spherical Tokamaks with flow. Spherical Tokamaks' (ST's) are potentially more efficient way of producing energy from fusion plasmas. They hold plasmas in tighter magnetic fields, forming a more compact, cored apple shape with high β and large q value and with low aspect ratio. Here there are two main task to study, firstly the plasma equilibrium and secondly its stability. First, the formulation of the Equilibrium problem is review. Toroidal flow produces a modification in the equilibrium profile due to the centrifugal force produced by the rotation in curved geometry. The most challenging problem in magnetic confinement fusion is posed by plasma instability control. In order to obtain confinement, the plasma needs not only to be in equilibrium, but also to be in stable equilibrium state. If that was not the case, small perturbations would grow indefinitely, likely causing the premature termination of the discharge.

Particle drifts are discussed. The effects of poloidal and toroidal flows on tokamak plasma equilibria are examined in themagnetohydrodynamic limit. "Transonic" poloidal flows of the order of the sound speed multiplied by the ratio of poloidal magnetic field to total field$\frac{B_\theta}{B}$can cause the normally elliptic Grad–Shafranov(GS) equation to become hyperbolic in part of the solution domain. It has been recognized that the GS equation in the absence of flows is elliptic,the combined system of Grad–Shafranov–Bernoulli equations can be hyperbolic in at least part of the solution domain in the presence of "transonic"poloidal flows of theorder of the sound speed $c_s$times the ratio of poloidal magneticfield $B_\theta$to total field B; throughout this research analysis Bernoulli equation and magnetosonic waves has been derived numerically under the light of which flow code is discussed.

Chapter 1

Introduction

1.1Fusion

This dissertation will start with a brief review of controlled thermonuclear fusion. Large amount of energy is produced during nuclear fusion that takes place continuously in the sun and stars. In the fusion, nuclei of light elements are fused together at very high temperature to produce more tightly bound heavier nuclei, realizing a huge amount of energy in the process. Hence it is natural for scientist to wonder whether fusion might be employed as terrestrial energy source.

In view of the worlds growing energy need and rapid exhaustion energy resources, making controlled fusion into a practical reality is of fundamental importance for the future of the modern world. Thus the struggle to achieve controlled fusion has been underway for the last five decades. various fusion schemes have been investigated, the most promising one is the magnetic confinement fusion(MCF) scheme [1].In this concept a fully ionized gas, so called plasma, is confined by magnetic field ad heated to extreme temperature, so that the thermal velocities of the nuclei are high enough to overcome the repulsive forces and produce the required reactions. The most promising device is tokamak which has shown steady progress over the years to achieve controlled fusion.

Nuclear reactions for fusion reactors are as follows [2],

$$D_1^2 + D_1^2 \rightarrow He_2^3(0.82MeV) + n_0^1(2.45MeV) \qquad (1.1)$$

$$D_1^2 + D_1^2 \rightarrow T_1^3(1.01MeV) + H_1^1(3.03MeV) \qquad (1.2)$$

$$D_1^2 + T_1^3 \rightarrow He_2^4(3.52MeV) + n_0^1(14.06MeV) \qquad (1.3)$$

The deuterium- deuterium reactions are the most difficult to achieve. A large containment vessel, higher temperature and stronger magnetic field coils would be needed. on the other hand deuterium-tritium reaction is considered to be the ideal energy source for a fusion reactor. The excess neutrons in the "D" and "T" nuclei make them larger and less tightly bound, and the result is that the cross-section for the D-T reaction is the largest .Also, because they are only singly charged hydrogen isotopes, the electrical repulsion between them is relatively small. So it is relatively easy to fuse the nuclei of deuterium-tritium. The reaction yields 17.6MeV of energy but requires a temperature of approximately 40 million Kelvin to over come the coulomb barrier and ignition. The deuterium fuel is abundant because about 1 part in 5000 of the hydrogen in sweater is deuterium. But the tritium must be produced either from lithium or in the operation of the deuterium cycle. The breeding is done by the bombardment of slow neutrons on accordingly,

$$n_0^1(slow) + Li_3^6 \rightarrow T_1^3 + He_2^4 + 4.8MeV \qquad (1.4)$$

Slow neutrons mean neutrons of having energy in the range of 0.001-1KeV.Lithium is widely distributed in minerals and is very chemically active. Natural lithium has two isotopes and has abundance of 7.5%.

$$n_0^1 + Li_3^7 \rightarrow T_1^3 + He_2^4 + n_0^1 + 2.4MeV \qquad (1.5)$$

With fast neutrons tritium can be breaded from the more abundant isotope of lithium, .It has natural abundance of 92.5%.Fast neutrons are those having energy range of 0.1-10MeV [3].

1.2 Plasma Confinement

The major problems involved in a fusion reactor are Plasma Confinement and Plasma heating. There exist two different approaches which are being widely used in carrying out controlled fusion on the earth. One approach is to use a plasma having electron and ion number density of the order of $10^{20}\,m^{-3}$, and therefore slow down the fusion reaction rate so that it can be reasonably controlled. In this approach the plasma is confined by applying appropriate magnetic fields. This approach is referred to as magnetic confinement fusion.

The other approach is to bring the plasma to extremely high density (typically more than a thousand times that of a liquid hydrogen densities) and temperature by uniformly by uniformly shining laser on the surface of fuel pellet. Explosions similar to those occurring in hydrogen bomb can be produced. Repeating this process can lead to nuclear energy production analogous to that of internal combustion engine. We shall only deal with magnetic confinement fusion in this dissertation.

1.2.1 Magnetic Confinement

In the controlled thermonuclear fusion reaction, basic research start after the World War II in the United States, the Soviet Union, and England. There are many evidences of research in controlled thermonuclear fusion even in 1940,s .But the first official announcement of successful experiments was made in 1951 by the president of Argentina, Peron, who declared that on February 16, 1951,the German Physicist Richter ,in his laboratory on the island of Huemul (Argentina),had succeeded in realizing a controlled thermonuclear fusion reaction[4].

Now a days , the major research is on the confinement of hot plasmas by means of strong magnetic fields. The most important and widely used magnetic configurations for plasma confinement are mirror and toroidal configurations. Linear mirror fieldsconfinement has advantages over toroidal confinements with respect to stability and anomalous diffusion across the magnetic field. However, particles leaving from the ends along lines of magnetic force is the main problem for confinement. Therefore it is necessary to find ways either to suppress the end loss or to increase the conversion efficiency of the kinetic energy of the escaping particles to usable electrical energy. Toroidal field configurations are used to remove these end losses, because they have no open ends. So, however the problems of end loses are solved by toroidal configuration but, due to magnetic field gradients and curvature, different drifts are produced which destroy the confinement. Now we briefly discuss the magnetic confinement of single charged particle and then group of charged particles plasma) for the purpose of fusion.

Single Charged Particle Motion in Magnetic field

Motion of a single charged particle constant or uniform magnetic field, describes a helical orbit around the magnetic field lines (Fig1.1).If a charged particle enters perpendicularly to the magnetic field, it will circulate around the field line and is thus confined. However, if it enters in magnetic field in such a way that that one of its velocity component is along the field line then it will move freely along that line and is thus not confined in this direction. Confinement of the particle along the magnetic field can be achieved in at least two ways. First, at each end of a region of constant uniform magnetic field, the field strength can be increased so that the magnetic field lines are squeezed close together, as shown in figure(1.2),to form a magnetic bottle. From the Lorentz force $F = qv \times B$ (q=charge on the particle ,v=particle velocity) on the particle, it is easy to see that a particle moving into a region of increasing magnetic field experiences a decelerating force directed along the axis because of its orbital velocity and the radial magnetic field component,

Thus, if the particle does not have excessive kinetic energy directed along the magnetic field, it will be reflected from the high field region return back to the region of uniform magnetic field. For this reason, the regions of increased magnetic field are called magnetic mirrors. Although the mirrors are not perfectly reflecting (since a particle can penetrate them if it has sufficient large kinetic energy directed along the magnetic field),they can however, improve the confinement along the magnetic field.

A single charged particle can also be confined by bending the magnetic field lines into circles as shown in figure(1.3).In this configuration there are no open ends, so magnetic mirrors are not required. Confinement in this toroidal configuration is not perfect, because of the bending of magnetic field lines into circles, means, that the magnetic field is stronger near the inner radius(r_1)side than the outer radius(r_2) side. This gradient in the magnetic field causes

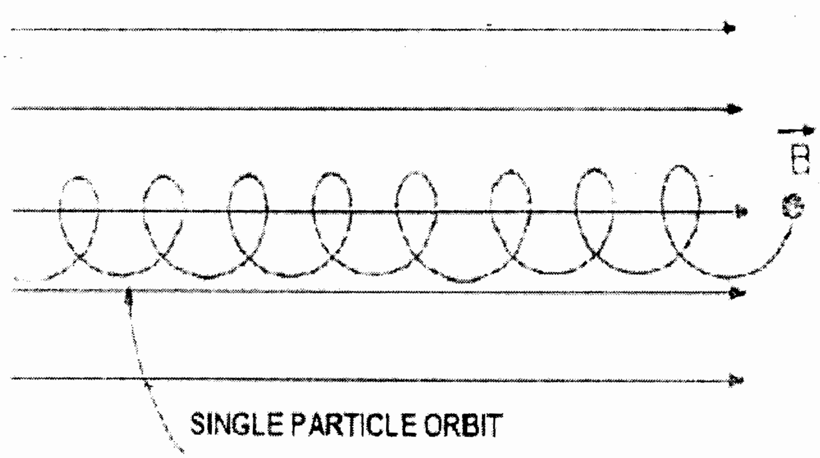


Figure 1.1:Single Particle Orbit in a uniform Magnetic Field

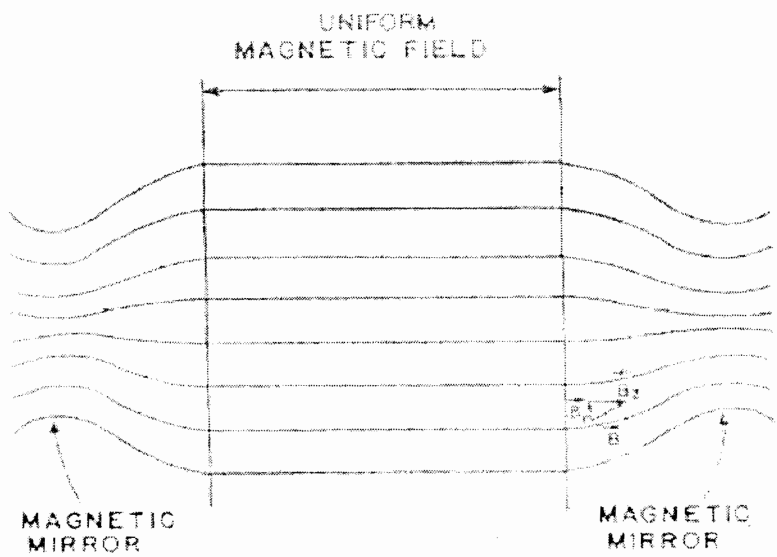


Figure 1.2:Magnetic Mirror Field Configuration

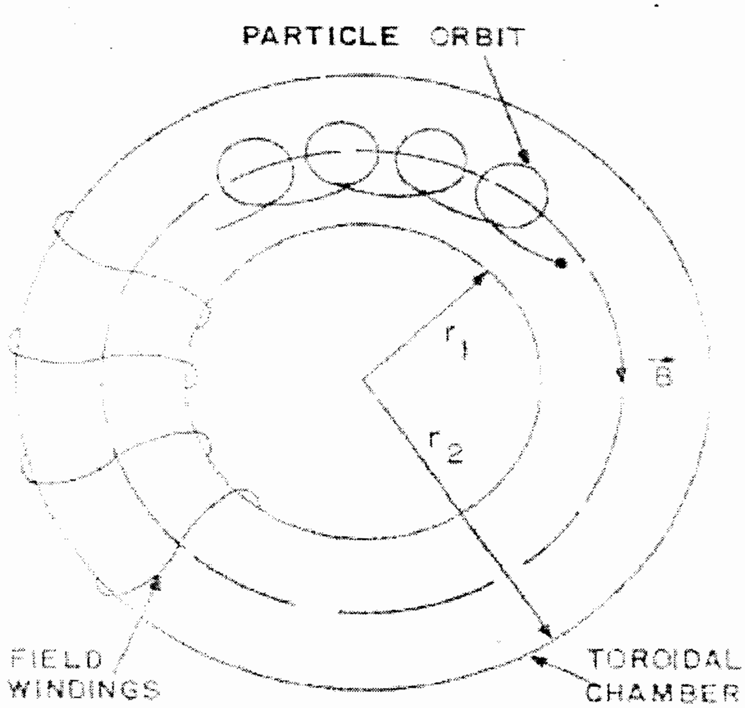


Figure 1.3: Simple Toroidal Magnetic Field Configuration

the particle to drift perpendicularly to the magnetic field, and eventually to be lost as shown in figure(1.4).this transverse drift can be made to average to zero for most particles by making the magnetic field lines spiral around the torus, shows the point of intersection of one of the helical field lines with the plane of the figure after one, two, three,------eight trips around the torus .the angular displacement $\imath$(iota) of the line after each trip around the torus is constant and is known as the rotational transform$\imath$ .unless$\imath$=(2π)/n n a rational number, the points of intersection fill in closed curve. Since a particle tends to follow magnetic lines of force, it tends to move along the curve C shown in figure (1.5) as it travels around the torus and thus moves alternately toward the top and bottom of the torus. Consequently, the transverse drift motion(vertical in figure(1.4) of the particle moves it alternately toward and away from the magnetic axis M figure(1.5) near the center of magnetic field region .If the particle moves around the torus quick enough and the rotational transform is large enough, then the drifts toward and away from the magnetic axis cancel, on the average ,and the particle is confined by the toroidal field .A single particle that moves too slowly along the magnetic field lines will be lost, however.

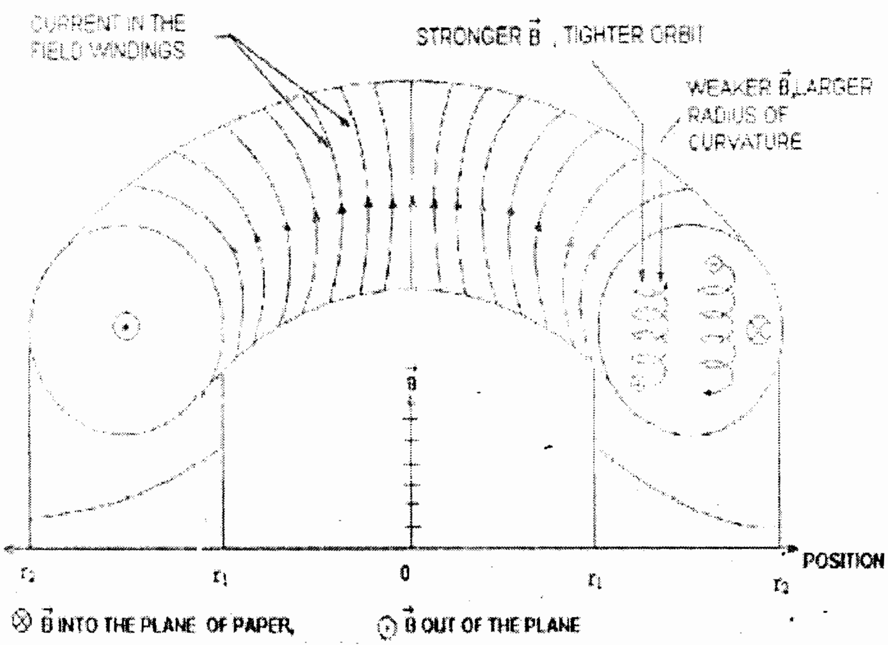


Figure 1.4:Drift particles across a simple Toroidal Confining Magnetic Field

It is interesting to note that those particles which have more kinetic energy along the field lines are lost from magnetic mirror fields but those less kinetic energy along the field lines are lost from toroidal fields. Thus, although both magnetic mirror and toroidal magnetic field configurations can confine single particles effectively, the confinement is not perfect in either case[5].

Motion of Number of Charged Particles in Magnetic field

So far, we have considered confining only a single particle in a magnetic field. When we tries to confine a group of particles, for the purpose of nuclear fusion, additional complications arises which reduces the confinement compared with that for a single particle. For a one thing, if we try to put several positive nuclei into a magnetic bottle, the coulomb repulsion tries to push the particles out. However, this repulsion can be reduced by adding electrons to neutralize the positive charges on the nuclei. The effect is minimized when enough electrons have been added to make the total collection of charged particles neutral .this means that we can create the proper mixture of nuclei (ions)and electrons simply by ionizing the particles in a neutral gas.

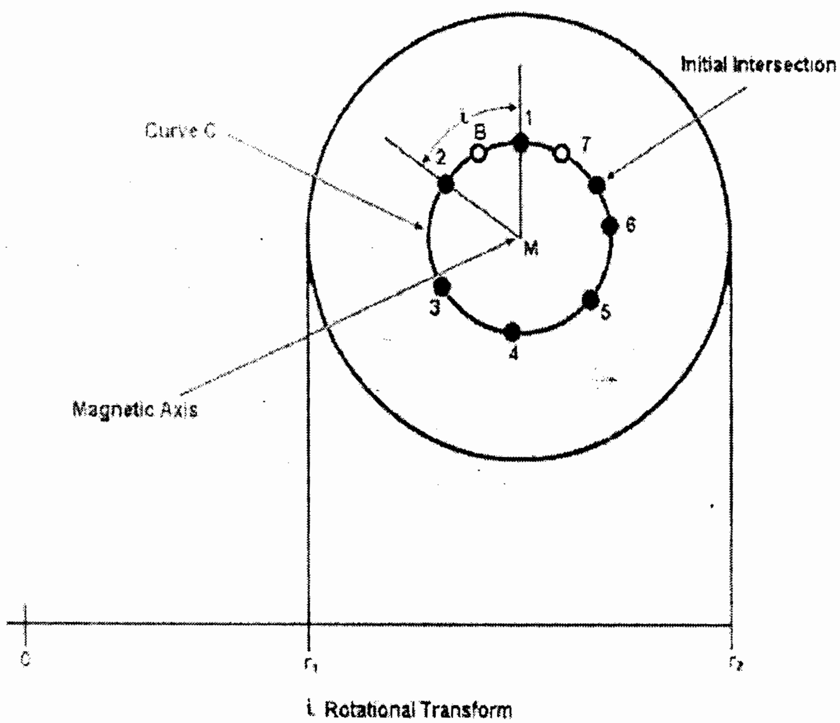


Figure 1.5:Rotational Transform Introduce by spiraling magnetic Field lines (Filled circles represents the intersection of a given helical field line with the plane of a right section of the torous and open circles signify intersections after the field lines have made one complete rotation)

Such a collection of electrons and ions form a plasma.

Collisions between the particles create problems for confinement because they enable the particles to move across the magnetic field lines and be lost. The collisions thus lead to diffusion of the charged particles across the magnetic field lines. Simple calculations shows that the number of particles per unit area per second lost across the confining magnetic field lines due to collisions should be proportional to $1/B^2$. Thus, as the confining magnetic field is increased, the particle losses caused by collisions should decreases. This result is, in part, due to the fact that as B is increased, the particle orbits become smaller, so that the distance a particle move across the magnetic field as a result of a single collision is correspondingly reduced.

A more serious complication that arises when we try to confine a group of particles in a magnetic field is particle losses due to oscillations of the plasma particles, or plasma instabilities. The energy necessary to derive some instabilities comes from forces applied to confine the plasma. The lowest energy state for a plasma of given temperature occurs when the plasma is uniformly distributed through the space. Clearly, plasma confined by external forces, is not in this lowest energy state, and thus has some additional internal energy. Generally the plasma tries to reach the lowest energy state by using the extra energy to derive instabilities that will let the plasma to move across the magnetic field and spread out uniformly [5].

Two things are required for plasma instabilities, first some kind of restoring force must be available .In a plasma, the restoring force can be provided either by the confining magnetic field or by coulomb forces between the charged particles. Second, some kind of inertia must be present. In a plasma, the inertia of ions and electrons is involved. We will further discuss instabilities in plasma latter in this chapter.

1.3 Plasma Parameters for a Thermonuclear Reactor

We now estimate the ranges of density, temperature, and confinement time for plasmas in thermonuclear reactors [2].

1.3.1 Power Density and Particle Density in a Thermonuclear Reactor

Power densities for D-T and D-D thermonuclear reactions versus deuteron density are shown in figure (1.6)

For temperatures of 10KeV AND 100keV.As these curves are not very sensitive to temperature by considering the thermonuclear power density produced in the plasma. If the power density is chosen too low, a large volume of plasma, and hence a spatially large magnetic field, would be required to produce a reasonable output power. The capital cost of the power plant would then be excessive. For example, a deuteron density of $10^{18}m^{-3}$produces less than $1kW/m^{3}$ .Since typical power plant produces more than 10^{8}W.A plasma volume of $10^{5}m^{3}$would be required. This volume corresponds to linear dimension of about fifty meters. The cost of producing a magnetic field strong enough to confine even a10KeV plasma with a density of $10^{18}m^{-3}$and a

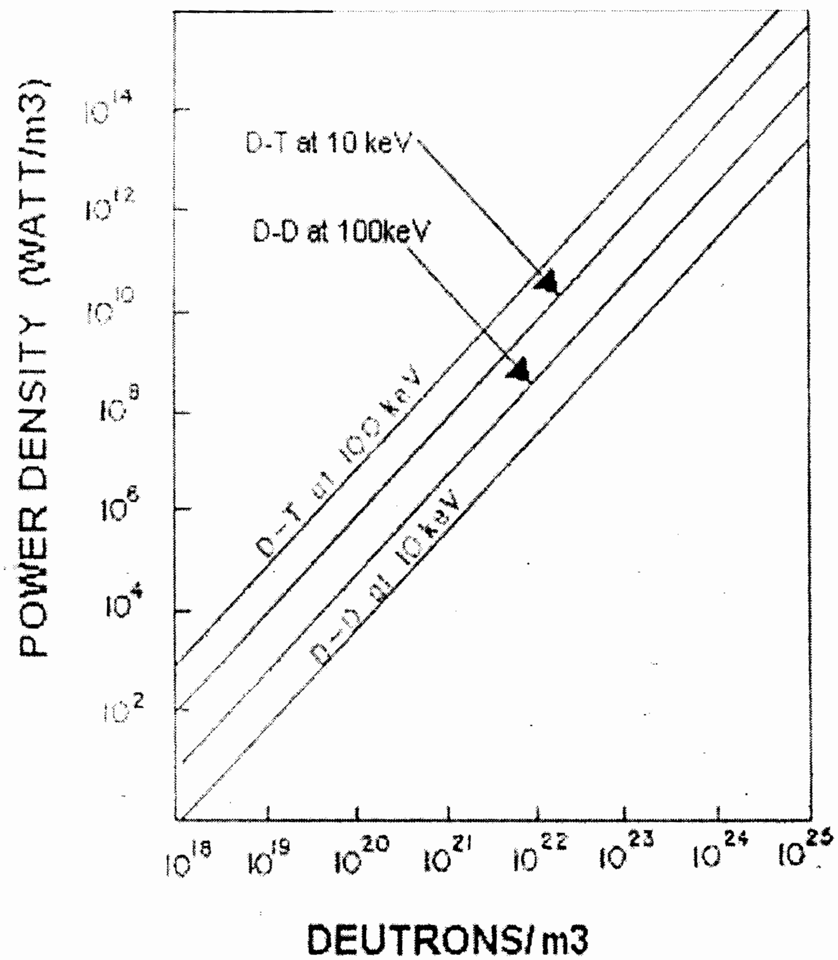


Figure 1.6:Thermonuclear Power Density versus Particle Density For D-T and D-D Reaction

volume of $10^5 m^3$ is enough in itself to rule out any possibility of economicpower production. If, on the other hand, the power density is chosen too high, material problems caused by the intense neutron flux in the reactor are impossible to solve.

In nuclear fission reactors, the design value of the power density in the core is typically 20-60MW/m^3. Although this value is chosen by considering materials problems somewhat different from those in a nuclear fusion reactor,40 MW/m^3 is not likely to be greatly different from the value a careful design of a fusion reactor would give. For one thing, the volume required to produce10^9W is $25m^3$, corresponding to linear dimension of about 3m.Magnetically confining a 10-KeV plasma of this volume is not out of question economically. Referring to figure(1.6),we therefore expect the plasma density in a fusion reactor to be in the range 10^{20} -$10^{21}m^{-3}$ if the power density is about 40 MW/m^3 . These densities are only small fractions $10^{-5}$of the density of a gas at standard temperature (27^0C) and pressure (1 atmosphere). the plasma region of a fusion reactor is thus essentially a vacuum.

1.3.2 The Ideal Break-Even Temperature

The minimum operating temperature for a self-sustaining thermonuclear reactor is that at which the energy is released by nuclear fusion reactor just exceeds that loss from the plasma as a result of radiation loses, mostly due to bremsstrahlung .This temperature, which we call the ideal break down temperature [2], is ideal in the sense that it is assumed no energy lost by means of escaping particles.

Figure (1.7) shows fusion power density released and bremsstrahlung loss as a function of temperature for both D-D and D-T reactions for an ion density of $10^{21}m^{-3}$ .It is evident from this figure that the ideal break down temperature for the D-D and D-T reaction is about 4KeV and 40 KeV, respectively. The relatively low ideal break down temperature of the D-T reaction is a very desirable property for at least two reasons. First, the lower temperature means that it should be easier to heat a D-T plasma to a point at which significant energy is released by fusion. Second, the lower temperature means lower plasma kinetic pressure and hence lower magnetic field to confine it.

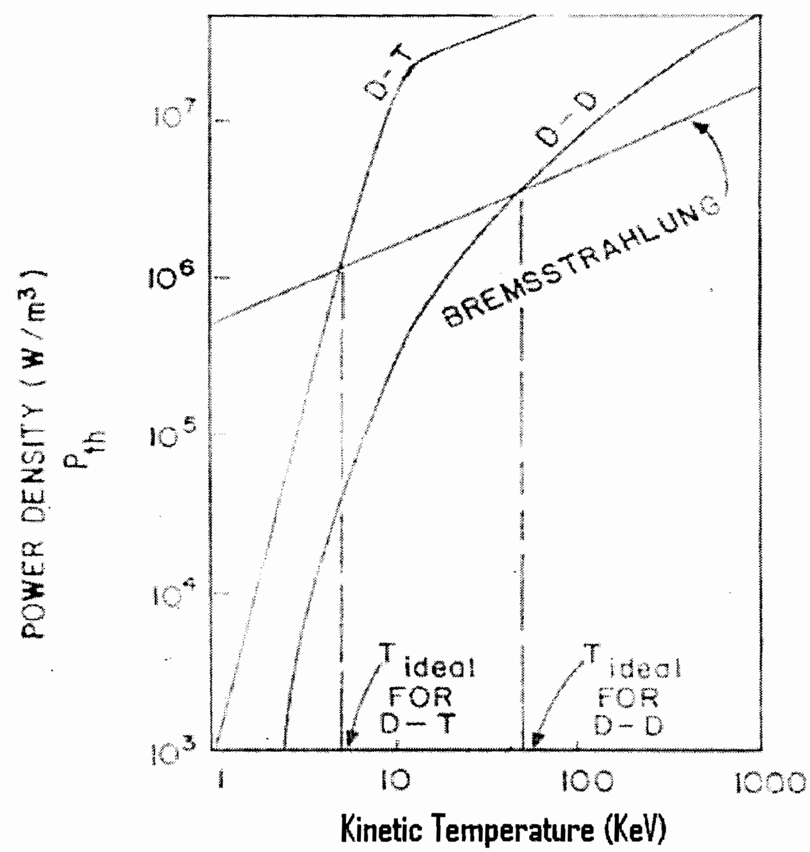


Figure 1-7: Ideal Break-Even conditions For D-D and D-T Fusion Reaction

1.3.3 The Lawson Criterion

we now consider the requirements that must be met in order to produce a plasma in a controlled, self- sustained nuclear reactions. To obtain a net energy gain from fusion reactor at a temperature higher than ignition , it is necessary that during confinement time the released fusion energy should be equal to the energy lost from the reactor in the form of radiations plus the energy needed to raise the temperature of the plasma. Therefore, we can write

$$\tau P_f \geq 3KT + \tau P_r$$

where P_f is the power radiated from fusion reactor ,P_r is the power lost by emission of radiations , T is the temperature of plasma , n be the number density (number of ions per unit volume) and τ is the energy confinement time. By making use of known expression forP_f, the above condition may be written in the following form:

$$n\tau \geq f(T)$$

Where f(T) is a function of plasma temperature T alone. The above expression represents, in a simplified form, the fusion criterion formulated in Ref [3]. The function f(T) for a D-T plasma is shown in figure(1.8). The fusion condition is verified in he plotted curve. At a temperature of (10-20) KeV,or a D-T plasma the Lawson condition is simplify.

$$n\tau \geq 2 \times 10^{20} m^{-3} s$$

This condition can be approached in different ways. The value of n and required by the Lawson criterion will depend upon the confinement method which is used. In magnetic confinement configurations, for instance, the values of n$\sim 10^{20}\, m^{-3}$and $\tau \sim 1s$ are appropriate[2].

1.4 Tokamaks

ThewordTokamak comes from Russian word for toroidal chamber and magnetic field coil, such machines were first built in that country in the late 1950s.the toroidal field configurations

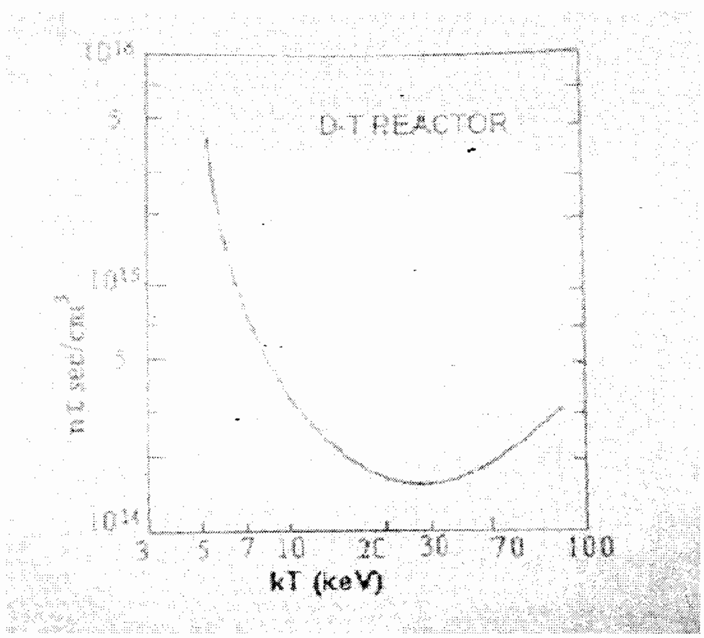


Figure 1.8 Lawson Criterion for D-T Reactor

offer the advantage in the plasma confinement because they have no "ends" through which plasma particles can escape .Furthermore, we know that if magnetic field lines are made to spiral around the torus, the drift of the particles across the magnetic field averages to zero. Therefore, a magnetic field whose lines are toroidal helices seems to be good for confining plasma. How can we, in practice, realize such a field configuration? Figure(1.3) shows the field lines with no spiraling that can be produced by simply passing current through a coil wound on a toroidal form. To make these lines to spiral as they pass around the torus, current is passed through a plasma(which is an excellent conductor) located inside the toroidal coil. The field produced by the plasma current alone is shown in figure(1.9). When this field is superimposed with the field produced by the toroidal coil, we get desired field whose lines are toroidal helices. In terms of toroidal coordinate system the toroidal coil produces a magnetic field in the toroidal (orϕdirection), while the plasma produces a field B_p in a poloidal (or θ)direction. A toroidal confinement device such as the one just as described in which thepolidal current in the plasma producestoroidal component of themagnetic field is called a Tokamak [6].The basic tokamak machine is shown in the figure(1.10). Magnetic field configuration in the tokamak

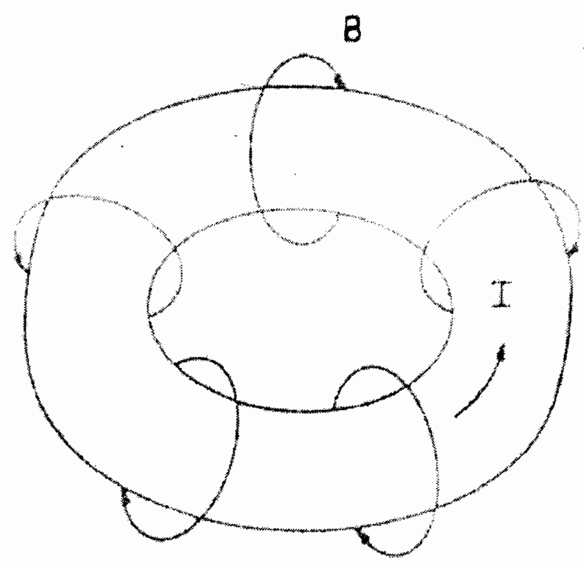


Figure 1.9:Field Lines Produced by Toroidal Current

has a shear and is a minimum B configuration as well. Shear occurs because the plasma current produces a poloidal field which is small near the minor axis of the torus and increases to maximum near the edge of the plasma while the toroidal field varies slowly across the cross section. The net result is the change in the pitch of the helical field lines as we move away from the minor axis of the torus-that is, that is the magnetic field is shared. The average minimum B property also follows from the fact that the poloidal field is small near the minor axis and increases near the plasma edge so that the strength of the magnetic field $B=(B_p^2 + B_t^2)^{1/2}$,has a minimum near the minor axis .Portion of each field line, however, pass through regions that are not in the magnetic well near the minor axis. The particles that tend to follow these field lines spend a part of their time inside the magnetic well and the part of their time outside it. However, in the tokamak field configuration, they spend more time inside the well than on the outside. The tokamak is therefore said to be an average minimum B device. This property together with shear, makes the tokamak device a stable to most MHD instabilities in normal operating regime [5].

An important feather of tokamak is that they are simple in construction, such that a relatively complicated magnetic field configuration required for toroidal confinement is produced

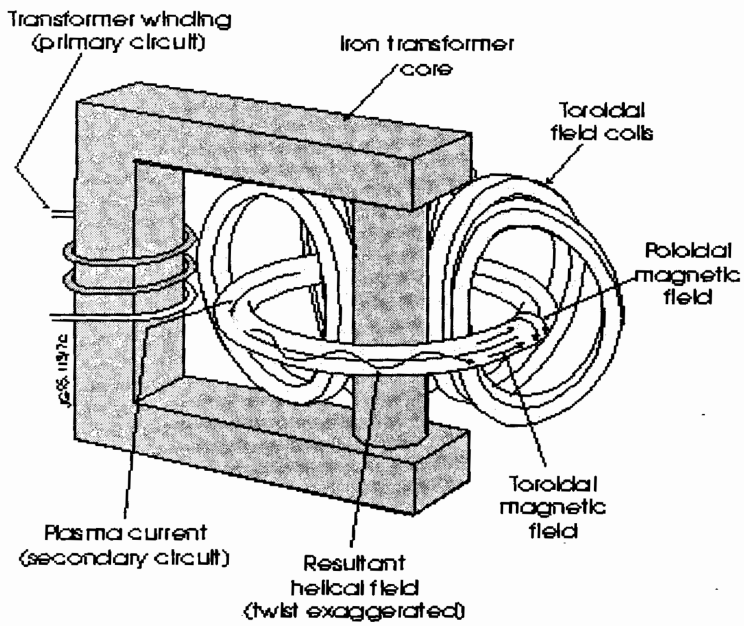


Figure1.10: Schematic Diagram of a Tokamak

by using a very simple coil system to produce hetoroidal field and using a transformer to introduce the toroidal plasma current which also heat the plasma. The operation of the tokamaks, on the other hand, is relatively complicated. For example, if due to some reasons, plasma parameters are changed then they can affect the toroidal current which can then change the poloidal field and hence the confinement. Thus, the tokamak equilibrium is more complicated than the equilibrium for devices in which the confining fields are produced by external coils and are hence subject to direct external control that is more or less independent of the plasma.

The most important feature of tokamak, however, is not simplicity of construction or their complexity of operation. Rather, it is that they seem, at present time, to work better than other plasma confinement devices and to be scalable to reactor sizes.

1.5 Basic Problem of Toroidal Equilibrium

- Fluid description of equilibrium problem is started by considering purely toroidal field
- Such magnetic configuration is difficult to be held in equilibrium
- Consider a plasma of circular cross-section, constant pressure inside the plasma and perfect plasma diamagnetism (i.e. the magnetic field vanishes in the plasma)
- Magnetic field will create a force acting on the external surface of the plasma which is proportional to the square of the magnetic field strength at any point in the plasma surface.
- By Maxwell's equation the magnetic field has a 1/R dependence, inner surface is smaller then the outer surface of the toroidal geometry. Net effect of the magnetic pressure is then and outward pointing force, since $\frac{1}{R^2}$ dependence in the magnetic pressure dominate over all dependence which disturbs equilibrium.

1.6 Magnetic Confinement Fusion and Tokamak Concepts

Plasma being constituted of charged particles, is affected by electric and magnetic fields.

Charged particle motion is mostly influenced by the electromagnetic forces (the Coulomb and Lorentz forces)

$$m\frac{dv}{dt} = q(E + v \times B)$$

Basis for plasma confinement is that particle moving across a magnetic field will experience a force perpendicular to both the direction of motion and the direction of the field.

1.7 Tokamak Stability

- In order to obtain confinement, the plasma needs not only to be in equilibrium, but also to be in a stable equilibrium state. We classify stability with respect to modes as internal mode (without perturbing the plasma surface) and external mode that perturb the plasma boundary

1.8 Layout of Dissertation

The work done in this dissertation is organized as follows. In the first chapter , basic nuclear fusion ,confinement of a single and group of charged particles(plasma),parameters for controlled fusion , configuration of magnetic field in tokamaks and types of plasma instabilities are briefly reviewed. At the end of this chapter we have also discussed ballooning modes.

In the second chapter, different types of drifts in magnetoplasma are discussed. These drifts are produced in the body of the plasma which leads to the non linear effects. Drifts play an important role in understanding the phenomena of different instabilities. At the end of this chapter we also study in the magnetic field configurations how the charged particle is trapped? The chapter is closed with the brief review of drift waves.In the third chapter we shall discuss mathematical derivations of some important relations related to Tokamaks.

Chapter 2

Drifts in Magneto Plasma

Plasma is an assembly of charged and neutral particles which exhibit collective behavior. Plasma consist of a collection of a free moving electrons and ions , whose motion can be controlled by applying electric and magnetic fields. Plasma contains charge particles, as these charges move around; they can generate local concentration of positive and negative charges which give rise to electric fields. Motion of these charges also generates currents and hence magnetic field [7]. In plasmas various kinds of drift -motion are possible which are believed to be responsible for the generation of instabilities in non uniformmagnetoplasmas. These may also couple with Alfven type perturbations and gives rise to various interesting structures in space as well as in laboratory plasmas. In this chapter, we explain various kinds of possible drifts which can be produced in the body of the plasma. At the end of this chapter, we shall also discuss particle trapping in tokamak field configuration and about the drift waves.

2.1 Motion of a charged particle in a Constant Magnetic Field

The equation of motion of a charged particle can be written as

$$m\frac{dv}{dt} = q(v \times B) \qquad (2.1)$$

We can resolve velocity of a charged particle into two component i.e.,

$\boldsymbol{v} = v_{\parallel}\hat{z} + \boldsymbol{v}_{\perp}$ (2.2)

Where $\hat{z} = \frac{\boldsymbol{B}}{B}$ is the unit vector in the direction of B.? The Lorentz force only effects perpendicular motion.

.The centrifugal force $\frac{mv^2}{r}$ balances the Lorentz force $qv_{\perp}\boldsymbol{B}$

$\rho = \frac{mv_{\perp}}{|q|B}$ (2.3)

As $\frac{mv_{\perp}^2}{2} = kT$, we obtained $\rho = \frac{(2mkT)^{1/2}}{|q|B}$.

The frequency of the gyration called the cyclotron frequency Ω_c is given by

$$\Omega_c = \frac{\mathrm{qB}}{\mathrm{m}} \quad (2.4)$$

In fusion experiments the electron's cyclotron frequency is of the same order of magnitude as the plasma frequency [8].Fig.2.1. Illustrate that the particle position x is given by guiding center position R and a rotating gyration radius vector

$x = R + \rho$ (2.5)

$\rho = -\frac{m}{qB^2}\boldsymbol{V} \times \boldsymbol{B}$(2.6)

2.2 Drift Due to an Additional Force

The equation of motion takes the following form when a force acts on a charged particle in addition to Lorentz force

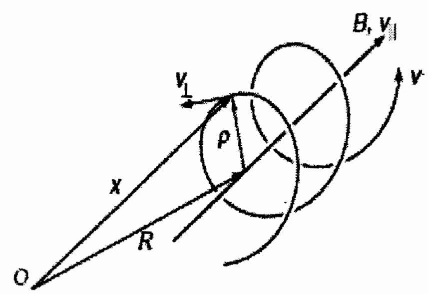


Figure2.1: Definition of the guiding center

$$m\frac{dv}{dt} = q(\boldsymbol{v} \times \boldsymbol{B}) + \boldsymbol{F} \qquad (2.7)$$

velocity of the guiding center can be obtained by doing the following procedure.

$$R = x - \rho$$

$$v_g = \frac{d\boldsymbol{R}}{dt} = \frac{d\boldsymbol{x}}{dt} - \frac{d\rho}{dt}$$

$$= \boldsymbol{v} + \frac{m}{qB^2}\frac{d\boldsymbol{v}}{dt} \times \boldsymbol{B}$$

Putting the value of $m\,{}^{dv}/_{dt}$ from equation (2.7),we get

$$\boldsymbol{v}_g = \boldsymbol{v} + \frac{1}{qB^2}(q\boldsymbol{v} \times \boldsymbol{B} + \boldsymbol{F}) \times \boldsymbol{B}$$

Using

$$(v \times B) \times B = -v_\perp B^2$$

And

$$\boldsymbol{v} - \boldsymbol{v}_\perp = v_\parallel \hat{z}$$

we obtain

$$\boldsymbol{v}_g = v_\parallel \hat{\boldsymbol{z}} + \frac{\boldsymbol{F} \times \boldsymbol{B}}{qB^2} \qquad (2.8)$$

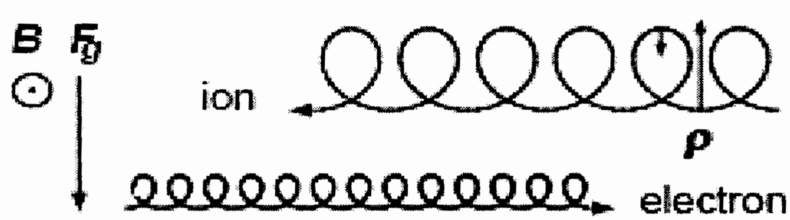


Figure2.2:Drift Of Ion and Electron in a gravitational field

to both F and B. The periodic variation of the gyro-radiuscauses the drift in this direction. when a particle accelerate in the force field its gyro radius increases, and its gyro radius decreaseswhen it slows down, this lead to the un closed trajectories shown in the figure (2.2). Summarizing,

$$v_{g,\perp} = \frac{\boldsymbol{F}_{\perp}\times\boldsymbol{B}}{qB^2} \quad , \frac{d\boldsymbol{v}_{g,\parallel}}{dt} = \frac{\boldsymbol{F}_{\parallel}}{m} \qquad (2.9)$$

For the constant gravitational force we get

$$v_g = \frac{mg}{qB}$$

2.3 E×B Drift

An E×B drift occurs when a constant electric and magnetic fields (in different directions) are act on a charged particle. The electric force due to electric field is $q\boldsymbol{E}$, and then from equation (2.9) the resulting drift velocity is,

$$\boldsymbol{v}_E = \frac{\boldsymbol{E}\times\boldsymbol{B}}{B^2} \qquad (2.10)$$

Let us see what happens physically to the particle trajectory. As particle gains velocity in the y direction, it will feel a magnetic force in the direction $\boldsymbol{v}\times\boldsymbol{B}$ and therefore move in the x diection. The particle will be forced by the magnetic field to return to the positive x axis, at which the velocity of the particle will again be zero. The trajectory of the motion will be cycloidal. The cycloidal loop will be repeated over and over .If the initial y velocity of the particle is not zero, the trajectory will be trochoid. A trochoid is the curve described by a point on a spoke of a wheel as the wheel rolls along a horizontal straight line without slipping [9]. When the tracing point is on the circumference of the wheel, the trochoidis called a cycloid. Figure (2.3) depicts the trajectory of the particle if it is given an initial velocity in the y direction.

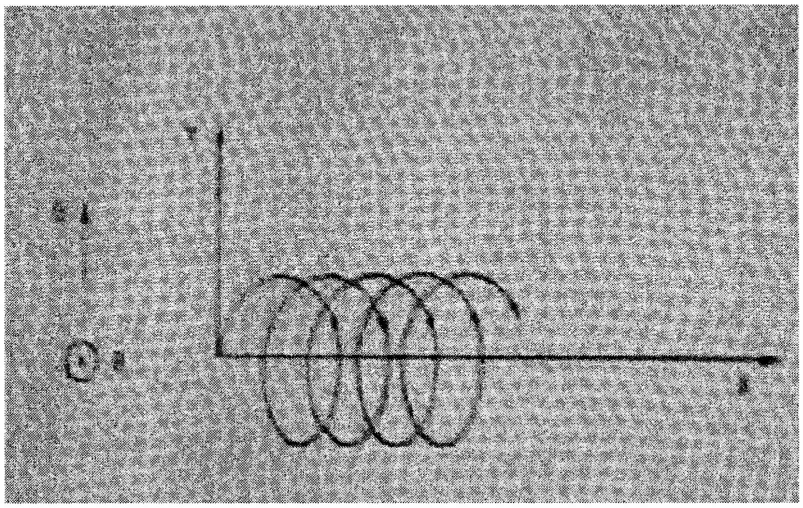


Figure 2.3: Thetrochoidal trajectory of positively charged particle released with an initial velocity in Y direction in the presence of perpendicular electric and magnetic field

2.4 Polarization Drift

For the spatially constant but the time dependent electric field, $\frac{\partial E}{\partial t} \neq 0$

$$\boldsymbol{F} = m\frac{dv_E}{dt} = \frac{m}{B^2}\frac{\partial \boldsymbol{E}}{\partial t} \times \boldsymbol{B} \qquad (2.11)$$

The above force causes an acceleration perpendicular to the magnetic field.

This force will yield another drift,

$$\boldsymbol{v}_p = \frac{\boldsymbol{F}\times\boldsymbol{B}}{qB^2} = \frac{m}{qB^2}\frac{\partial \boldsymbol{E}}{\partial t} \qquad (2.12)$$

The direction of v_E depends on the signature of q and its magnitude depends on mass m. So ions and electrons donot have equal velocities, hence a net current is driven in the plasma which is called polarization current i.e.,

$$\boldsymbol{J}_p = \frac{\rho_m}{B^2}\frac{\partial \boldsymbol{E}}{\partial t} \qquad (2.13)$$

Where

$$\rho_m = m_e n_e + m_i n_i$$

is the overall mass density. This current density is smaller for electrons and larger for ions[8].

The physical reason for this polarization drift is simple. Consider an ion at rest in a magnetic field. If a field E is applied, the ion will move in the direction of E. Only after picking up a velocity v_i does the ion feels a Lorentz force i.e. $e\boldsymbol{v}_i \times \boldsymbol{B}$

and begin to move downwards. If E is now kept constant, there is no further $v_p$drift but only a v_E drift. However, if E is reversed there is again a momentary drift, this time to the left. Thusv_p is a start-up drift due to inertia and occurs only in the first half cycle of each gyration during which $\boldsymbol{E}$change.

2.5 Diamagnetic Drift

In a thermal plasma $< {mv_\perp}^2/2 >= T$and therefore$< m\mu >= \frac{T}{B}$and$M = \frac{\hat{z}p}{B}$.The diamagnetic current produced by magnetization is given by

$$J_D = \nabla \times \boldsymbol{M} = -\frac{\nabla p \times \boldsymbol{B}}{B^2} \qquad (2.14$$

The $\mathbf{J}\times \boldsymbol{B}$force that arises due to this current provides the force balance in a conducting fluid i.e. $\nabla p = \boldsymbol{J} \times \boldsymbol{B}$(Fig 2.4).If we treat ions and electrons as separate fluids in plasma, the diamagnetic velocities of ions and electrons are

$$\boldsymbol{v}_{Di} = -\frac{\nabla p_i \times \boldsymbol{B}}{q_i n B^2}, \boldsymbol{v}_{De} = \frac{\nabla p_e \times \boldsymbol{B}}{q_e n B^2} \qquad (2.15)$$

Which resemble the drift velocities of the form given in equation (2.9). Their relation to the diamagnetic currents is

$$\boldsymbol{J}_D = n_i q_i \boldsymbol{v}_{Di} + n_e q_e \boldsymbol{v}_{De} \qquad (2.16)$$

If we imagine a cylindrical plasma in a uniform magnetic field, with the high pressure in the center of the plasma, it is easy to see that both the electron (q=-e) and the ion (q=e) diamagnetic drifts gives rise to current in the plasma that serve to reduce the magnetic field inside the plasma. Hence this drift is named as diamagnetic drift.

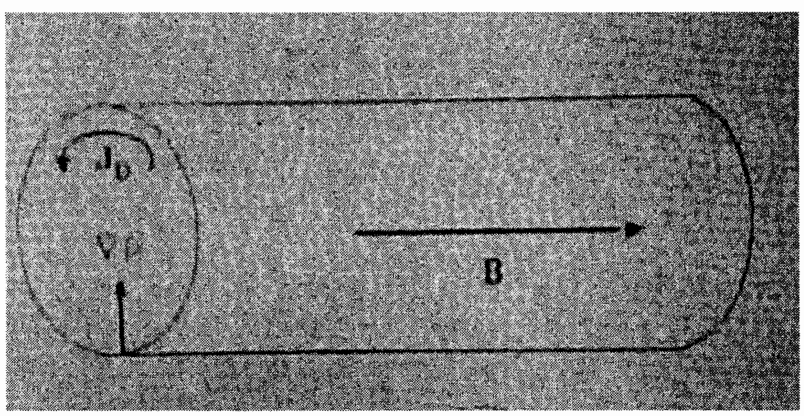


Figure 2.4: Cylindrical Plasma in which the inward $\boldsymbol{J} \times \boldsymbol{B}$force balance the outward force from the pressure gradient

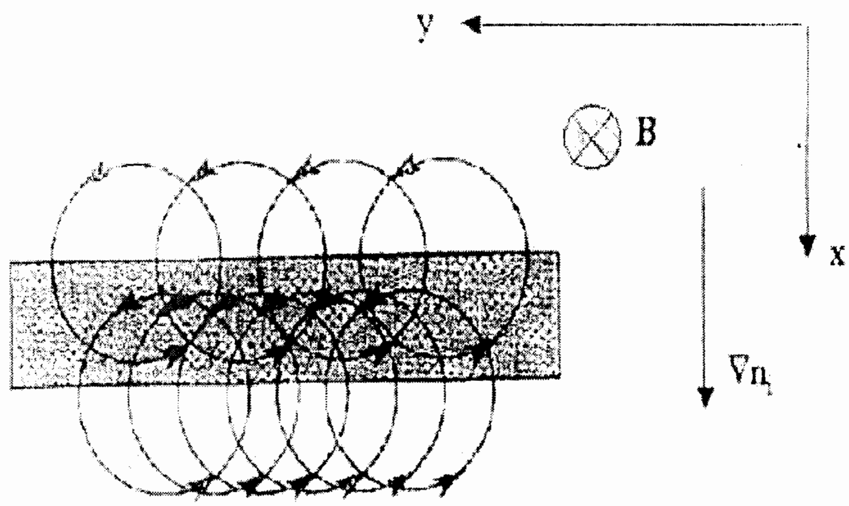


Figure 2.5:Larmor Orbits of ion in the presence of density gradient.In the shaded region there is a net current to the left, even though the guiding centers have no net motion.

The physical reason for this drift can be seen from the figure (2.5). Here the Larmor orbits of ions, gyrating in the magnetic field directed into the paper, are drawn. There is a density gradient towards down, as indicated by the density of orbits. We did notethat the ion and electron Larmor orbits themselves were intrinsically diamagnetic. The diamagnetic drift in a uniform magnetic field is a result of adding together these Larmor orbits in the presence of a density or temperature gradients [10].

2.6 Nonuniform Electric Field Drift

If we have a uniform magnetic field and non uniform electric field then the usual is modified by the in homogeneity in the following form:

$$v_E = (1 + \frac{1}{4}\rho^2 K^2)\frac{E \times B}{B^2} \qquad (2.17)$$

The physical reason for this is easy to see. An ion with its guiding center at a maximum of E actually spends more time in the regions of weaker E. Its average drift, therefore is less than E/B evaluated at guiding center. In a linearly varying E field, on one side of the orbit the ion would be in the stronger field and in a weaker field by the same amount on the other side; the correction to v_E cancels out. From this it is clear that the correction term depends on the second derivative of E. For the sinusoidal distribution, the second derivative is always negative with respect to E. The second term is called the finite Larmor radius effect. Since $\rho$is much larger for ions than for electrons, $v_E$is no longer independent of species.

If a density clump occurs in a plasma, an electric field can cause the ions and electrons to separate, generating another electric field. If there is a feedback mechanism that causes the second electric field to enhance the first one, E grows indefinitely, and the plasma is unstable Thenonuniform electric field effect is, therefore, important at relatively large k, or small scale lengths of the in homogeneity [11].

2.7 Particle Drifts in the Inhomogeneous Magnetic Fields

2.7.1 Curvature Drift

Curvature of the magnetic field lines gives a drift known as curvature drift. The curvature is given by

$$\nabla_{\parallel}\hat{\boldsymbol{z}} = -\frac{\boldsymbol{R_c}}{R_c^2} \qquad (2.18)$$

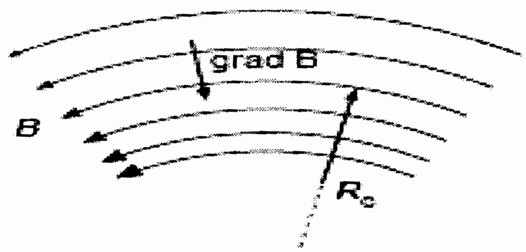


Figure2.6: The magnetic field gradient and radius of curvature.

Which is a unit vector perpendicular to B. Here$\nabla_{\parallel}= \hat{z}$. $\boldsymbol{R}_c$is the curvature radius shown in the fig. (2.6). A particle which follows the curved field line with velocity $v_{\parallel}$experiences a centrifugal force

$$\boldsymbol{F}_c = mv_{\parallel}^2 \frac{\boldsymbol{R}_c}{R_c^2} \qquad (2.19)$$

Which, according to equation(2.9),gives a drift velocity

$$v_c = \frac{mv_{\parallel}^2}{qB^2} B \times \nabla_{\parallel}\hat{z} \qquad (2.20)$$

2.7.2 Grad B Drift

The drift given by the gradient of the magnetic field is known as Grad B drift . A constant force does not cause this drift, and hence equation (2.9) cannot be directly applied.

The magnetic moment is the product of area and current. Since the area encompassed by the gyro-orbit equals $\pi\rho^2$, the magnetic moment per unit particle mass is

$$\mu = \pi\rho^2 \frac{I}{m} = \pi\rho^2 \frac{q^2 B}{2\pi m^2} = \frac{v_\perp^2}{2B} \qquad (2.21)$$

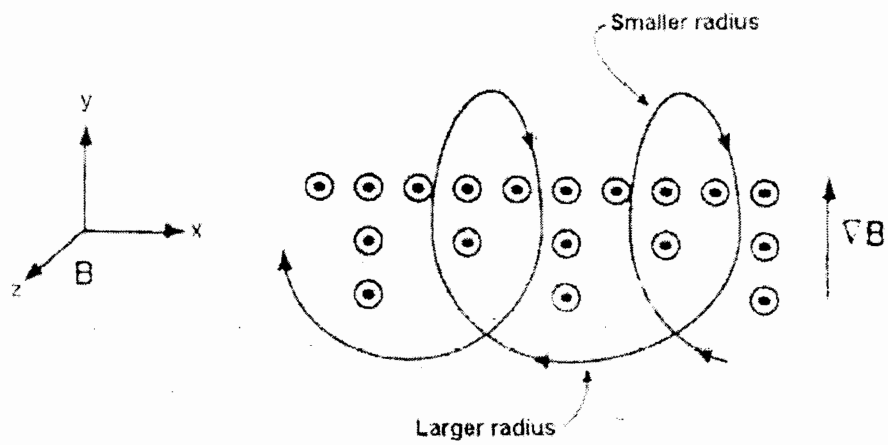


Figure2.7:Ion∇Bdrift motion.

Equating the gyro-averaged force to the force on a magnetic dipole in a magnetic field gradient,

$$F_{\nabla B} = M\mu\nabla B \qquad (2.22)$$

Application of equation (2.9) to this force yields the drift,

$$v_{\nabla B} = \frac{m v_\perp^2}{2qB^3} \boldsymbol{B} \times \nabla \mathrm{B} \qquad (2.23)$$

There is a simple physical picture for the ∇B drift, which follows from the fact that the local radius-of-curvature of the gyro-orbit is smaller on the larger magnetic-field side of the orbit, and correspondingly larger on the smaller magnetic-field side. If we construct a continuous trajectory from smaller orbits on one side, and the larger orbits on the other hand, we find a net drift perpendicular to both B and ∇B, as illustrated in figure(2.7) [15].

Chapter 3

Mathematical Algebra for Tokamaks

3.1 Bernoulli's Principle

Bernoulli's principle is being stated in fluid dynamics as, with a decrease in pressure or a decrease in the fluid's potential energy causes increase in the speed of the fluid.Due to the Dutch-Swiss mathematician Daniel Bernoulli it is named as Bernoulli's principle.[13]

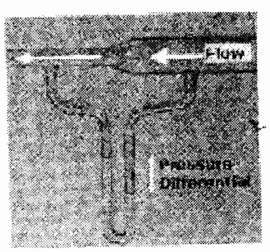


Figure 3.1: The Kinetic Energy is being increased by the application of pressure which is cleared by the difference of height of two columns

3.2 Incompressible Flow Equation

A common form of Bernoulli's equation, valid at an arbitrary point along a streamline where gravity is constant, is:

$$\frac{v^2}{2} + gz + \frac{p}{\rho} = constant \qquad (3.1)$$

Where **V** is the fluid flow speed, g is the acceleration due to gravity, z is the elevation of the point above a reference point, p is the pressure at the chosen point, andρ is the fluid desity.

By multiplying with the fluid densityρ, equation (3.1) can be rewrite as:

$\frac{\rho V^2}{2} + \rho g z + p = constant$ (3.2)

Substituting the following values in equation (3.2) one by one.

$q = \frac{1}{2}\rho V^2$is dynamic pressure,

$h = z + \frac{p}{\rho g}$is the piezometric head or hydraulic head(the sum of the elevation z and the pressure head)

$p_0 = p + q$ Is the total pressure(the sum of the static pressure p and dynamics pressure)

As$z = h - \frac{p}{\rho g}$

We will get first

$$q + \rho g\left(h - \frac{p}{\rho g}\right) + p = constant$$

$$q + \rho g h - p + p = constant$$

$q + \rho g h = constant$ (3.3)

Now again in equation (3.2) substituting for $p = p_0 - q$ and $q = \frac{1}{2}\rho V^2$

$q + \rho g z + p_0 - q = constant$

$p_0 + \rho g z = constant$ (3.4)

From equation (3.3) and (3.4) we can write

$q + \rho g h = p_0 + \rho g z = constant$ (3.5)

Inserting value for h in equation (3.3) we get

$$q + \rho g\left(z + \frac{p}{\rho g}\right) = constant$$

Also resubsituting value of q

$\frac{1}{2}\rho V^2 + \rho g z + p = constant$ (3.6)

Dividing equation (3.6) by ρg we get

$$\frac{V^2}{2g} + z + \frac{p}{\rho g} = constant$$

$= \mathrm{H}(say)$ (3.7)

Where H is energy head

As speed Vforcentury effect is $V = \sqrt{\frac{2(p_1 - p_2)}{\rho}}$ common tradition to take $(p_1 - p_2)$ equal to product of flux function ϕ and magnetic field B.

So from equation (3.7), we get

$\frac{1}{2g}\left[\frac{2\phi(\Psi)B}{\rho}\right] + z + \boldsymbol{W} = \mathrm{H}(\Psi)$ (3.8)

We replaced $\left(\frac{p}{\rho g}\right)by\boldsymbol{W}$, because$\boldsymbol{W} = \mathrm{W}(\rho, B, \Psi)$is the enthalpy function,which depends on closure equations relating pressure,density and temperature.

Using $g = 2\mu_0$ in equation (3.8) we get

$$\frac{1}{2\mu_0}\left[\frac{\phi(\Psi)B}{\rho}\right] + z + \boldsymbol{W} = H(\Psi)$$

Required Bernoulli's equation.

In general, ideal MHD studies the interactions between an ideal (perfectlyconducting) plasma and magnetic, pressure and inertial forces. A complete derivation of the MHD equations starts from Maxwell's equations and a kinetic model of the plasma, with each species of particles being described by a Boltzmann equation.

The basic set of MHD equations is

$$\nabla.(\rho v) = 0$$

$$\rho\frac{\partial \mathrm{v}}{\partial \mathrm{t}} = j \times B - \nabla.P$$

$$\nabla.B = 0$$

$$\nabla \times B = j$$

$$\nabla \times E = 0$$

$$E + v \times B = 0$$

The Grad-Shafranov (GS) equation

$$R\frac{\partial}{\partial R}\frac{1}{R}\frac{\partial \Psi}{\partial R} + \frac{\partial^2 \Psi}{\partial z^2} = -\mu_0 R^2 P'(\Psi) - \mu_0^2 f(\Psi) f'(\Psi)$$

is themost successful instrument describing tokamak plasma equilibria in the frameof MHD theory.

$3.3 Derivation of \rho_k$:

Depending on the choice of kinetic closure we will derive kinetic density. In order to get this derivation we will use following couple of equations which are

$div\, H = 0$ $[\nabla^2 H=0]$ (3.9)

$p = f(\rho)$ (3.10)

$\nabla p + \rho \nabla \Phi = -\frac{1}{4\pi} H \wedge curl\, H$ (3.11)

Where H is enthalpy function, P for pressure and for density [14].

It is again impossible to give a complete solution, but it is possible to reduce the equations to an equation for H alone, that is to eliminate $P$and .

On taking curl of (3.11) we get

$\nabla \rho \wedge \nabla \Phi = curl\, K$ (3.12)

Where

$K = -\frac{1}{4\pi} H \wedge curl\, H$ (3.13)

Thus $curl\, K$ is normal to and; and because of also to .It therefore follows from (3.11) that

$$K.curlK = 0 \qquad (3.14)$$

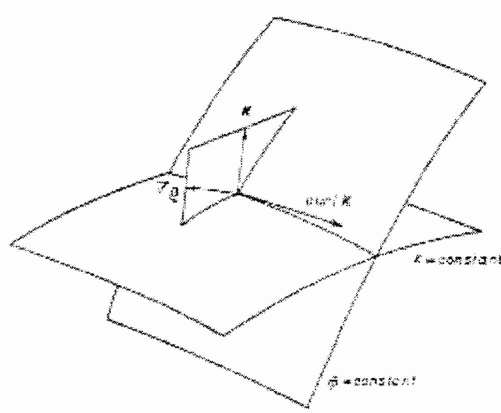
This identity is the condition for K to be represent able in the form

$$K = \lambda gradX$$

With two scalar fields λ and X .Thus K is normal to the surfaces X=constant so that $curl\, K$ is tangent to them.

(3.14) is the condition that the differential has an integrating factor of (Elements of the partial differential equation)

The situation is as follows, in the figure 3.1.there are two sets of surfaces, $\Phi = constant$and $X = constant$.The intersections are the field lines of curl K.On account of equation (3.14) , and hence also P , is constant along each field line, so that ρ and P may be written as functions of the two variables $\Phi$and$X$.The same applies to λ ,because it follows from equation(3.15)



That $grad\, \lambda$ is normal to$curl\, K$. Hence equation(3.11) takes the form of

$$(\frac{\partial P}{\partial \Phi} + \rho)\nabla\Phi + \frac{\partial P}{\partial X}\nabla X = \lambda \nabla X$$

Unless $\nabla\Phi$and ∇Xhappen to parallel it follows that

$\frac{\partial P}{\partial \Phi} = -\rho$ (3.16)

$\frac{\partial P}{\partial X} = \lambda$ (3.17)

The general solution of equation (3.16) is

$P(\rho) = -\Phi + \Omega(X)$ (3.18)

Here denotes an arbitrary function

$$P(\rho) = \int \frac{dP}{P} = \int_0^\rho \frac{f'(\rho)}{\rho} d\rho$$

In order to satisfy equation (3.17) one must have

$$\frac{d\Omega(x)}{dx} = \frac{1}{\rho}\frac{\partial P}{\partial x} = \frac{\lambda}{\rho}$$

Thus equation (3.15) becomes

$K = \rho \nabla \Omega$ (3.19)

According to equation (3.18) it is possible to writeρ as a function of P.

$\rho = F(P) = F(-\Phi+\Omega)$ (3.20)

Consequently we have found that K must have the form

$K = F(\Omega - \Phi) grad\Omega$ $(putting \rho = F(\Omega - \Phi) in$ 3.20)

where$F$ is uniquely determined by $f$whereas Ω is an arbitrary scalar field.

In case of an ideal gas the condition (3.20) simplifies considerably.

One then $P = C_p$has so that $P(\rho) = Clog_{10}\,\rho$

and corresponding kinetic density is

$\rho_K = exp\frac{P}{C} = \exp[\frac{\Omega-\Phi}{C}]$from (3.19) (3.22)

On putting $C\exp\left(\frac{\Omega}{C}\right) = \Xi$one gets from equation (3.22)

$\rho_K = \exp\left(\frac{\Omega}{C}-\frac{\Phi}{C}\right) = \exp\left[\frac{\Omega}{C}\right]\exp[\frac{-\Phi}{C}]$ (3.23)

From kinetic theory we have $C = \frac{kT}{m}$

Equation(3.23) becomes

$\rho_K = \frac{m}{kT}\left(e^{-\frac{m\Phi}{kT}}\right).\Xi$ (3.24)

whereΞ is partition function for ρ_K

$$\Xi = \frac{B^2kT}{4\pi mg}(\frac{kT}{mg}-z)e^{\frac{mgz}{kT}}$$

Putting above value in equation (3.24)

$$\rho_K = \frac{m}{kT}.e^{-\frac{m\Phi}{kT}}.\frac{B^2kT}{4\pi mg}(\frac{kT}{mg}-z)e^{\frac{mgz}{kT}}$$

$$= \frac{B^2}{4\pi g}\left(\frac{kT}{mg}\right)\left(\frac{mg}{kT}\right)\left[\frac{kT}{mg}-z\right].\exp[\frac{mgz-m\Phi}{kT}]$$

$$\rho_K = \frac{B^2kT}{4\pi mg^2}\left[1-\frac{mgz}{kT}\right]\exp[\frac{mgz-m\Phi}{kT}]$$

3.4 Derivation of W_{MHD}:

Basic equations are

$\nabla.(\rho v) = 0$ (3.25)

$\rho(v.\nabla)v + \nabla.P = j\times B$(3.26)

$\nabla.B = 0$ (3.27)

$\nabla\times B = j$ (3.28)

$\nabla\times E = 0$ (3.29)

$E + v\times B = 0$ (3.30)

Where

$\rho = n(m_e + m_i)$ (3.31)

$n$is the particle no. density. $\vec{v}, \vec{E}, \vec{B}and\vec{j}$are the velocity ,electric field, magnetic field and plasma current density respectively .

When plasma is subjected to a strong magnetic field with the ions having a small Larmor radius with a rapid gyro motion , the pressure tensor $\vec{P}$may be no longer isotropic.we can introduce chew-Guldberger-low form of the pressure tensor

$P = P_{\perp}\mathbb{I} + \sigma_{-}\vec{B}\vec{B}$ (3.32)

Where

$\sigma_{-} = \frac{P_{\parallel} - P_{\perp}}{B^2}$ (3.33)

is a measure of the pressure anisotropy,$\mathbb{I}$ is the identity tensor and $B^2 = \vec{B}.\vec{B}$

In order to get to a close system of equations we must introduce some constitutive thermo dynamical assumption so as to obtain an energy equation In earlier work where the pressure was supposed isotropic it has been assumed that either the entropy S or the temperature T are surface quantities. In the former case an adiabatic equation of state has been used

$P\rho^{-\gamma} = A(S)$ (3.34)

The specific enthalpy h satisfies the thermodynamic relation

$dh = Tds + \frac{1}{\rho}dP$ (3.35)

After integration we get from equation (3.35)

$$h = TS + \frac{P}{\rho}$$

now by using equation (3.34) above equation becomes

$$h = TS + \frac{A(S)}{\rho^{-\gamma}\rho}$$

$$h = TS + \frac{A(S)}{\rho^{-\gamma+1}}$$

$h = TS + A(S)\rho^{\gamma-1}andA(S) = S$

Taking natural log on both sides of of above equation

$$lnh = ln[TS + A(S)\rho^{\gamma-1}]$$

$$lnh = \ln[(TS)A(S)\rho^{\gamma-1}]$$

After taking antilog of above equation we will get

$$e^{lnh} = e^{\ln[(TS)A(S)\rho^{\gamma-1}]}$$

$$h = (TS)A(S)\rho^{\gamma-1} \qquad (3.36)$$

From basic statistical mechanics we use a relation

Product of temperature and entropy S always takes as

$$TS = \frac{\gamma}{\gamma - 1}$$

Put in equation(3.36)

$$h = \frac{\gamma}{\gamma - 1} A(S)\rho^{\gamma-1}$$

Taking usual symbols for enthalpy h as W because work is done due to plasma motion and $A(S)$= $S(\Psi)$

$$W = \frac{\gamma}{\gamma - 1} S(\Psi)\rho^{\gamma-1}$$

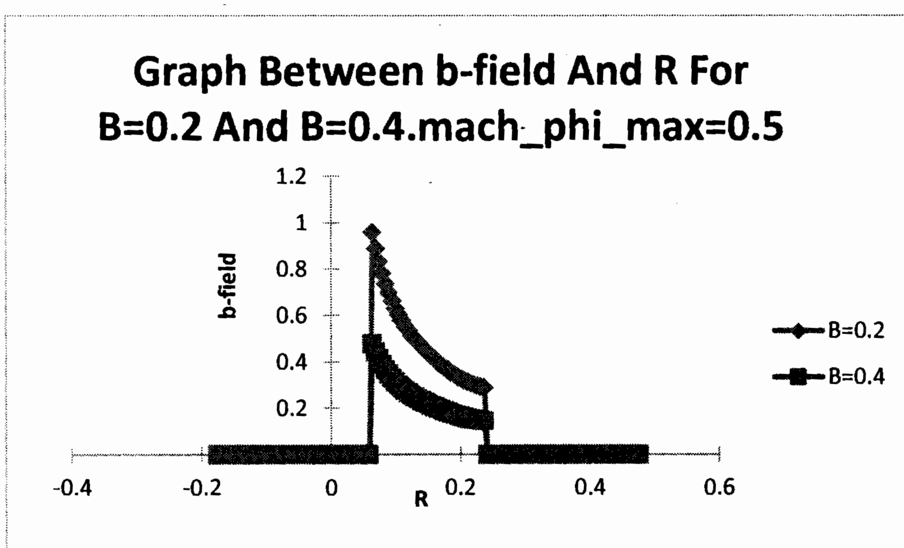
$\Psi$is also free function.

We consider an axisymmetrictoroidal plasma described by the ideal magnetohydrodynamics(MHD) model. Now further in this section we will discuss graphs of bphi (toroidal field), bpol (poloidal field), beta for magnetic field B=0.2

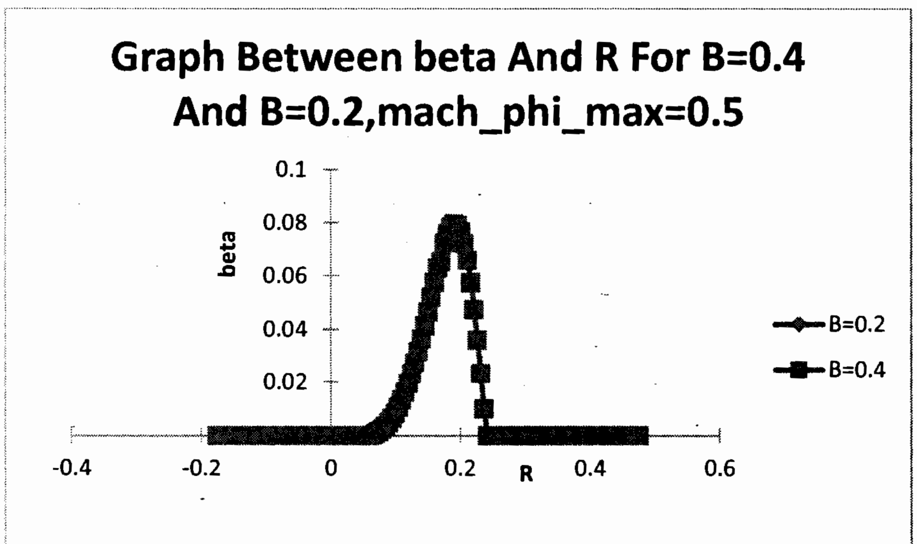
and B=0.4 and for mach-phi-max=0.0, mach-phi-max=0.5.Now we will show that when we increase magnetic field from 0.2 to 0.4 β decreases with increasing magnetic field, temperature remains the same, poloidal field increases current density increases and pressure also increases. The effects of poloidal and toroidal flows on tokamak plasma equilibria are examined in the magnetohydrodynamic limit. "Transonic" poloidal flows of the order of the sound speed multiplied by the ratio of poloidal magnetic field to total field $\frac{B_\theta}{B}$can cause the (normally elliptic) Grad–Shafranov (GS) equation to become hyperbolic in part of the solution domain. It is pointed out that the range of poloidal flows for which the GS equation is hyperbolic increases with plasma beta and $\frac{B_\theta}{B}$, thereby complicating the problem of determining spherical tokamak plasma equilibria with transonic poloidal flows.

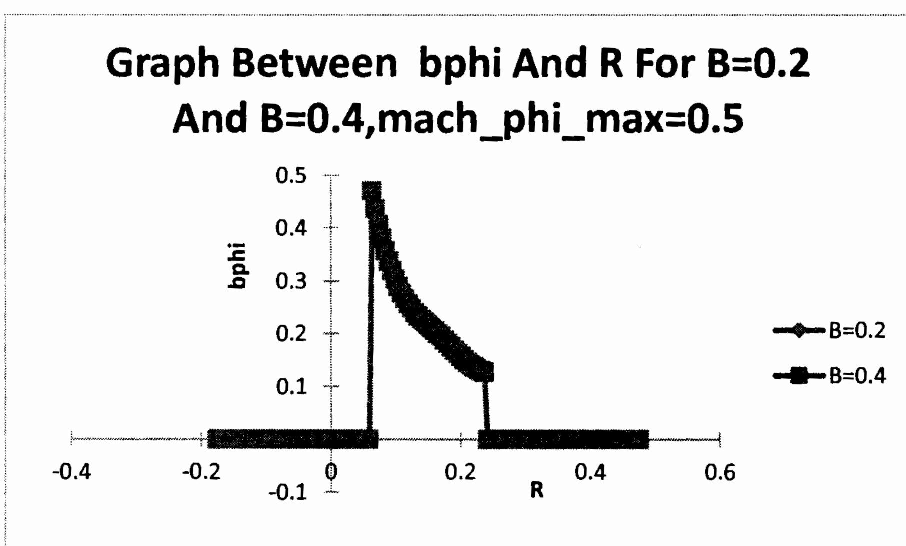
Following graphs are obtained

The graph of b-field is shown below the first straight line corresponds to inner side of the torus showing at inner side b-field is maximum when we move towards the central part of torus a decay is observed in b-field. When we move towards outer part of torus again straight line is obtained but this straight line is smaller as compared to the line obtained for inner part of torus showing b-field is less at outer part due to large distance from field coil.

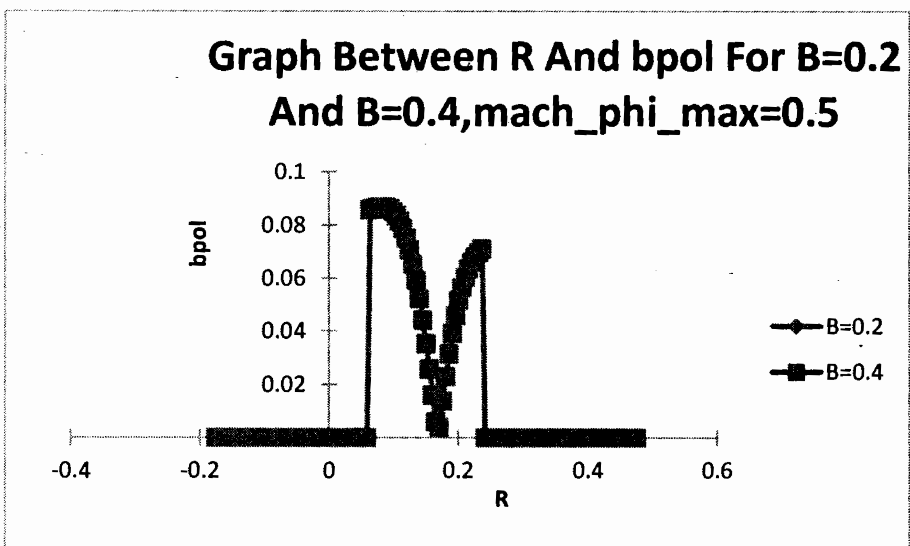


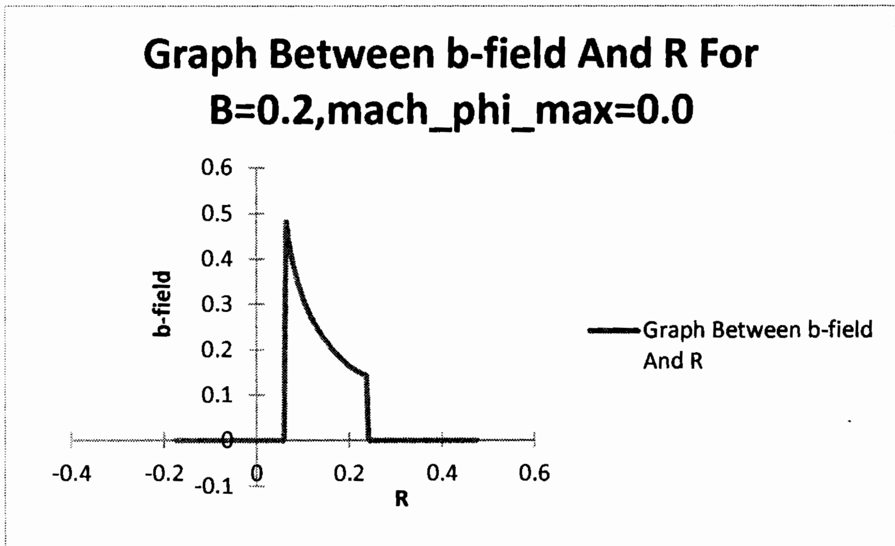
The following graph of beta is showing in the inner side of the torus due to maximum charge accumulation density is greater or plasma is dense in the center due to which kinetic pressure is larger leading to increase in beta at the middle of torus as can be seen from the peak.

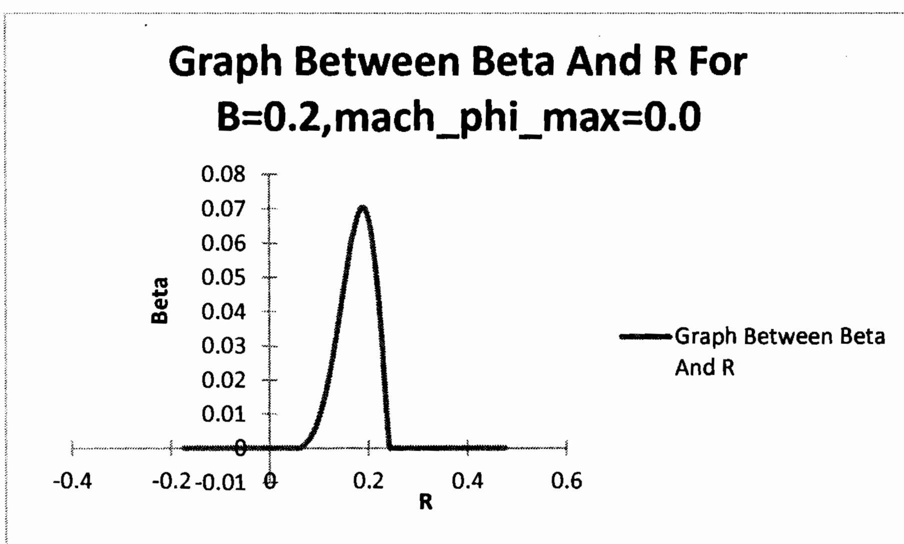


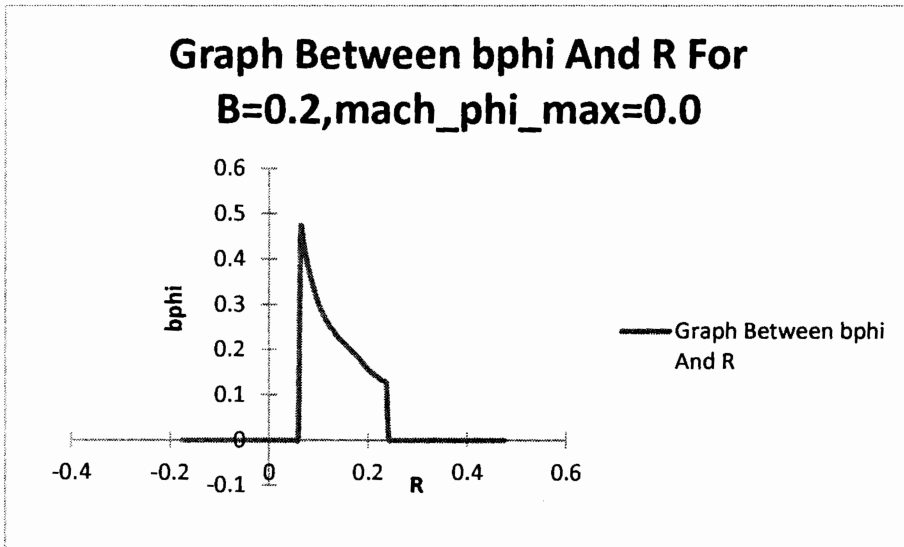


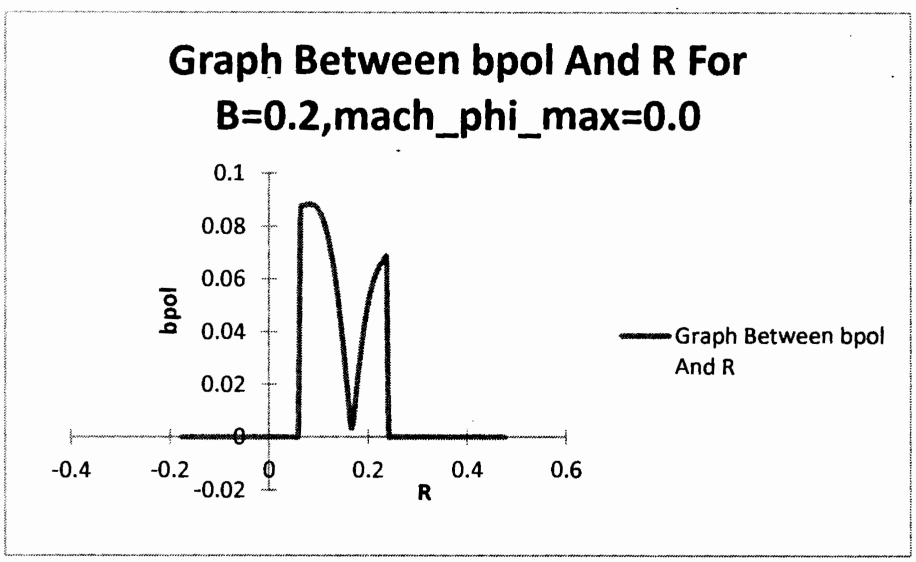
The first peak is showing poloidal field at inner part of the torus and the second peak is showing the poloidal field outer part of the torus at the middle there is a dip because at the middle of the plasma column there will be more effect of toroidal field than that of poloidal field.




Graph Between b-field And R For B=0.2,mach_phi_max=0.0
b-field
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.4
-0.2
0
0.2
0.4
0.6
R
Graph Between b-field And R


Graph Between Beta And R For B=0.2,mach_phi_max=0.0
Beta
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.4
-0.2
0
0.2
0.4
0.6
R
Graph Between Beta And R


Graph Between bphi And R For
B=0.2,mach_phi_max=0.0
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
bphi
-0.4
-0.2
0
0.2
0.4
0.6
R
Graph Between bphi And R


Graph Between bpol And R For
B=0.2,mach_phi_max=0.0
0.1
0.08
0.06
0.04
0.02
0
-0.02
bpol
-0.4
-0.2
0
0.2
0.4
0.6
R
Graph Between bpol And R

3.5 Magneto sonic wave(Low Frequency Waves)

A **magneto sonic wave** (also **magneto acoustic wave**) can be defined as a longitudinal wave of ions (and electrons) propagating perpendicular to the stationary magnetic field in a magnetized plasma..

We consider low frequency electromagnetic waves propagating across B we take

$$\overrightarrow{B_0} = B_0\hat{z}$$

$$\overrightarrow{E_1} = E_1\hat{x}$$

$$\vec{k} = k\hat{y}$$

By using Maxwell equation

$\vec{\nabla} \times \vec{E} = -\dot{\vec{B}}$ (3.37)

$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c^2}\vec{J} + \frac{1}{c^2}\dot{\vec{E}}$ (3.38)

Taking differential of equation (3.38)

$$\vec{\nabla} \times \dot{\vec{B}} = \frac{4\pi}{c^2}\dot{\vec{J}} + \frac{1}{c^2}\ddot{\vec{E}}$$

Substituting the value of $\dot{\vec{B}}$

$$-[\vec{\nabla} \times (\vec{\nabla} \times \vec{E})] = \frac{4\pi}{c^2}\dot{\vec{J}} + \frac{1}{c^2}\ddot{\vec{E}}$$

By using

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{4\pi}{c^2}\dot{\vec{J}} - \frac{1}{c^2}\ddot{\vec{E}}$$

$$\vec{\nabla}(\vec{\nabla}.\vec{E}) - \nabla^2\vec{E} = -\frac{4\pi}{c^2}\dot{\vec{J}} - \frac{1}{c^2}\ddot{\vec{E}}$$

Applying sinusoidal approximation

$$i\vec{k}(i\vec{k}.\vec{E}) - (ik)^2\vec{E} = -\frac{4\pi}{c^2}(-i\omega)\vec{J} - \frac{(-i\omega)^2}{c^2}\vec{E}$$

$$-\vec{k}(\vec{k}.\vec{E}) + k^2\vec{E} = \frac{4\pi i\omega}{c^2}\vec{J} + \frac{\omega^2}{c^2}\vec{E}$$

$$\vec{k}(\vec{k}.\vec{E}) - k^2\vec{E} = -\frac{4\pi i\omega}{c^2}\vec{J} - \frac{\omega^2}{c^2}\vec{E}$$

$$\vec{k}(\vec{k}.\vec{E}) - k^2\vec{E} + \frac{\omega^2}{c^2}\vec{E} = -\frac{4\pi i\omega}{c^2}\vec{J}$$

$$c^2\vec{k}(\vec{k}.\vec{E}) + (\omega^2 - c^2k^2)\vec{E} = -4\pi i\omega\vec{J}$$

In linearized form

$$(\omega^2 - c^2k^2)\vec{E}_1 + c^2\vec{k}(\vec{k}.\vec{E}_1) = -4\pi i\omega\vec{J}_1$$

$$\vec{J} = n_0e(\vec{v}_{i1} - \vec{v}_{e1})$$

Substituting value of $\vec{J}$ in above equation

$(\omega^2 - c^2k^2)\vec{E}_1 + c^2\vec{k}(\vec{k}.\vec{E}_1) = -4\pi i\omega n_0e(\vec{v}_{i1} - \vec{v}_{e1})$(A)

To calculate $\vec{v}_{i1}$and $\vec{v}_{e1}$ we use equation of motion

The linearized equation of motion for electrons is

$$m_en_0\frac{\partial\vec{v}_{e1}}{\partial t} = -en_0(\vec{E}_1 + \vec{v}_{e1}\times\vec{B}_0) - \nabla p$$

$$m_e\frac{\partial\vec{v}_{e1}}{\partial t} = -e(\vec{E}_1 + \vec{v}_{e1}\times\vec{B}_0) - \frac{\nabla p}{n_0}$$

$$m_e\frac{\partial\vec{v}_{e1}}{\partial t} = -e(\vec{E}_1 + \vec{v}_{e1}\times\vec{B}_0) - \frac{\gamma_ek_BT_e\nabla n_{e1}}{n_0}$$

$$(-\mathrm{i}\omega)m_e\vec{v}_{e1} = -e(\vec{E}_1 + \vec{v}_{e1}\times\vec{B}_0) - \frac{\gamma_e k_B T_e \mathrm{i}\vec{k} n_{e1}}{n_0}$$

$$\vec{v}_{e1} = \frac{e}{\mathrm{i}\omega m_e}\left(\vec{E}_1 + \vec{v}_{e1}\times\vec{B}_0\right) + \frac{\gamma_e k_B T_e \vec{k} n_{e1}}{\omega m_e n_0}$$

In component form

$$v_{ex} = \frac{e}{\mathrm{i}\omega m_e}\left(E_x + B_0 v_{ey}\right) + \frac{\gamma_e k_B T_e k_x n_{e1}}{\omega m_e n_0}$$

Since $k_x = 0$

$$v_{ex} = \frac{e}{\mathrm{i}\omega m_e}\left(E_x + B_0 v_{ey}\right) + 0 \qquad (3.39)$$

$$v_{ey} = \frac{e}{\mathrm{i}\omega m_e}\left(E_y - B_0 v_{ex}\right) + \frac{\gamma_e k_B T_e k_y n_{e1}}{\omega m_e n_0}$$

Since $E_y = 0$

$$v_{ey} = \frac{e}{\mathrm{i}\omega m_e}(-B_0 v_{ex}) + \frac{\gamma_e k_B T_e k n_{e1}}{\omega m_e n_0} \quad (3.40)$$

Now to calculate $\frac{n_{e1}}{n_0}$ we use equation of countiuity in linearized form

$$\frac{\partial n_{e1}}{\partial t} + n_0 \vec{\nabla}.\vec{v}_{e1} = 0$$

Using sinusoidal approximation

$$-\mathrm{i}\omega n_{e1} + n_0\, \mathrm{i}\vec{k}.\vec{v}_{e1} = 0$$

$$\frac{n_{e1}}{n_0} = \frac{k v_{ey}}{\omega}$$

Put in equation (3.40)

$$v_{ey} = -\frac{eB_0 v_{ex}}{i\omega m_e} + \frac{\gamma_e k_B T_e}{\omega m_e} k^2 \frac{v_{ey}}{\omega}$$

$$v_{ey} = \frac{ieB_0 v_{ex}}{\omega m_e} + \frac{\gamma_e k_B T_e}{\omega^2 m_e} k^2 v_{ey}$$

$$v_{ey}(1 - \frac{\gamma_e k_B T_e}{\omega^2 m_e} k^2) = \frac{ieB_0 v_{ex}}{\omega m_e}$$

$$v_{ey} = \frac{ieB_0 v_{ex}}{\omega m_e} (1 - \frac{\gamma_e k_B T_e}{\omega^2 m_e} k^2)^{-1}$$

Put in equation (3.39)

$$v_{ex} = \frac{e}{i\omega m_e}\left(E_x + B_0 \frac{ieB_0 v_{ex}}{\omega m_e} (1 - \frac{\gamma_e k_B T_e}{\omega^2 m_e} k^2)^{-1}\right)$$

$$v_{ex} = -\frac{ie}{\omega m_e}\left(E_x + \frac{ieB_0{}^2 v_{ex}}{\omega m_e} (1 - \frac{\gamma_e k_B T_e}{\omega^2 m_e} k^2)^{-1}\right)$$

$$v_{ex} = -\frac{ieE_x}{\omega m_e} + \frac{e^2 B_0{}^2 v_{ex}}{\omega^2 m_e{}^2} (1 - \frac{\gamma_e k_B T_e}{\omega^2 m_e} k^2)^{-1}$$

$$v_{ex}(1 - \frac{\gamma_e k_B T_e}{\omega^2 m_e} k^2) = -\frac{ieE_x}{\omega m_e} (1 - \frac{\gamma_e k_B T_e}{\omega^2 m_e} k^2) + \frac{e^2 B_0{}^2 v_{ex}}{\omega^2 m_e{}^2}$$

$$v_{ex}(1 - \frac{\gamma_e k_B T_e}{\omega^2 m_e} k^2 - \frac{\omega_c{}^2}{\omega^2}) = -\frac{ieE_x}{\omega m_e} (1 - \frac{\gamma_e k_B T_e}{\omega^2 m_e} k^2)$$

$$v_{ex} = \frac{-\frac{ieE_x}{\omega m_e}\left(1 - \frac{\gamma_e k_B T_e}{\omega^2 m_e} k^2\right)}{1 - \frac{\gamma_e k_B T_e}{\omega^2 m_e} k^2 - \frac{\omega_c{}^2}{\omega^2}} \qquad (3.41)$$

Similarly we can write v_{ix} by changing e by $-e$

$$v_{ix} = \frac{\frac{ieE_x}{\omega m_i}(1 - \frac{\gamma_i k_B T_i}{\omega^2 m_i}k^2)}{1 - \frac{\gamma_i k_B T_i}{\omega^2 m_i}k^2 - \frac{\Omega_c{}^2}{\omega^2}} \qquad (3.42)$$

Writing equation A in x-components

Since $\vec{k}$ and $\vec{E}$ are perpendicular thus $\vec{k}.\vec{E} = 0$ and we have

$$(\omega^2 - c^2k^2)E_x + 0 = -4\pi i\omega n_0 e(v_{ix} - v_{ex}) \qquad (3.43)$$

Taking the limit $\omega_c^2 \gg \omega^2$ and as mass of electron is small so neglect it

$$\left(1 - \frac{\gamma_e k_B T_e}{\omega^2 m_e}k^2\right) \ll \frac{\omega_c^2}{\omega^2}$$

Equation (3.41) becomes

$$v_{ex} = \frac{-\frac{ieE_x}{\omega m_e}\left(1 - \frac{\gamma_e k_B T_e}{\omega^2 m_e}k^2\right)}{-\frac{\omega_c{}^2}{\omega^2}}$$

$$v_{ex} = \frac{ieE_x\omega}{m_e\omega_c{}^2}(1 - \frac{\gamma_e k_B T_e}{\omega^2 m_e}k^2)$$

For equation (3.42) $\Omega_c^2 \gg \omega^2$ this implies that $1 - \frac{\gamma_i k_B T_i}{\omega^2 m_i}k^2 = 0$

$$v_{ix} = -\frac{ieE_x\omega}{\Omega_c^2 m_i}(1 - \frac{\gamma_i k_B T_i}{\omega^2 m_i}k^2)$$

Substituting values of v_{ex}andv_{ix} in equation(3.43)

$$(\omega^2 - c^2k^2)E_x = -4\pi i\omega n_0 e(-\frac{ieE_x\omega}{\Omega_c^2 m_i}(1-\frac{\gamma_i k_B T_i}{\omega^2 m_i}k^2) - \frac{ieE_x\omega}{m_e\omega_c{}^2}(1-\frac{\gamma_e k_B T_e}{\omega^2 m_e}k^2))$$

$$(\omega^2 - c^2k^2) = \frac{4\pi i^2\omega^2 n_0 e^2}{\Omega_c^2 m_i}\left(1-\frac{\gamma_i k_B T_i}{\omega^2 m_i}k^2\right) + \frac{4\pi i^2\omega^2 n_0 e^2}{m_e\omega_c{}^2}(1-\frac{\gamma_e k_B T_e}{\omega^2 m_e}k^2)$$

$$(\omega^2 - c^2k^2) = -\frac{4\pi\omega^2 n_0 e^2}{\Omega_c^2 m_i}\left(1-\frac{\gamma_i k_B T_i}{\omega^2 m_i}k^2\right) - \frac{4\pi\omega^2 n_0 e^2}{m_e\omega_c{}^2}(1-\frac{\gamma_e k_B T_e}{\omega^2 m_e}k^2)$$

$$= -\frac{4\pi\omega^2 n_0 e^2 m_i^2}{e^2 B_0^2 m_i} + \frac{4\pi\omega^2 n_0 e^2 m_i^2}{e^2 B_0^2 m_i}\frac{\gamma_i k_B T_i}{\omega^2 m_i}k^2 - \frac{4\pi\omega^2 n_0 e^2 m_e^2}{m_e e^2 B_0^2} + \frac{4\pi\omega^2 n_0 e^2 m_e^2}{m_e e^2 B_0^2}$$

$$= -\frac{4\pi\omega^2 n_0 m_i}{B_0^2} + \frac{4\pi n_0\gamma_i k_B T_i k^2}{B_0^2} - \frac{4\pi\omega^2 n_0 m_e}{B_0^2} + \frac{4\pi n_0\gamma_e k_B T_e k^2}{B_0^2}$$

$$1 - \frac{c^2k^2}{\omega^2} = -\frac{4\pi n_0 m_i}{B_0^2} + \frac{4\pi n_0\gamma_i k_B T_i k^2}{B_0^2\omega^2} - \frac{4\pi n_0 m_e}{B_0^2} + \frac{4\pi n_0\gamma_e k_B T_e k^2}{B_0^2\omega^2}$$

Since $\frac{4\pi n_0 m_e}{B_0^2} \ll \frac{4\pi n_0 m_i}{B_0^2}$ due to heavy mass ion therefore we neglect this term

$$1 - \frac{c^2k^2}{\omega^2} = -\frac{4\pi n_0 m_i}{B_0^2} + \frac{4\pi n_0\gamma_i k_B T_i k^2}{B_0^2\omega^2} + \frac{4\pi n_0\gamma_e k_B T_e k^2}{B_0^2\omega^2}$$

Since

$$\frac{v_A}{c} = \sqrt{\frac{B^2}{4\pi\rho_m}}$$

As $n_0 m_i = \rho_{mi}$

$$1 - \frac{c^2k^2}{\omega^2} = -\frac{4\pi\rho_{mi}}{B_0^2} + \frac{4\pi n_0 k^2}{B_0^2\omega^2}(\frac{\gamma_i k_B T_i + \gamma_e k_B T_e}{m_i})m_i$$

$$1 - \frac{c^2k^2}{\omega^2} = -\frac{c^2}{v_A^2} + \frac{k^2}{\omega^2}\frac{c^2}{v_A^2}c_s^2$$

Where $c_s^2 = \frac{\gamma_i k_B T_i + \gamma_e k_B T_e}{m_i}$

$$\frac{c^2k^2}{\omega^2} = 1 + \frac{c^2}{v_A^2} - \frac{k^2}{\omega^2}\frac{c^2}{v_A^2}c_s^2$$

$$\frac{c^2k^2}{\omega^2} + \frac{k^2}{\omega^2}\frac{c^2}{v_A^2}c_s^2 = 1 + \frac{c^2}{v_A^2}$$

$$\frac{c^2k^2}{\omega^2}\left(1 + \frac{c_s^2}{v_A^2}\right) = 1 + \frac{c^2}{v_A^2}$$

$$\frac{c^2k^2}{\omega^2} = \frac{1 + \frac{c^2}{v_A^2}}{1 + \frac{c_s^2}{v_A^2}}$$

$$\frac{c^2k^2}{\omega^2} = \frac{v_A^2 + c^2}{v_A^2 + c_s^2}$$

$$\frac{\omega^2}{k^2} = c^2\left[\frac{v_A^2 + c_s^2}{v_A^2 + c^2}\right]$$

This is the dispersion relation for the magnetosonic wave propagating perpendicular to B_0.

When $v_A^2 \ll c^2$ i.e. when the velocity of Alfven wave is less than velocity of light)

$$\frac{\omega^2}{k^2} = c^2\left[\frac{v_A^2 + c_s^2}{c^2}\right] = v_A^2 + c_s^2$$

The phase velocity of the magnetosonic mode is always greater than v_A, for this reason it is called"fast hydromagnetic wave).

Since

$$\frac{\omega^2}{k^2} = c^2\left[\frac{v_A^2 + c_s^2}{v_A^2 + c^2}\right]$$

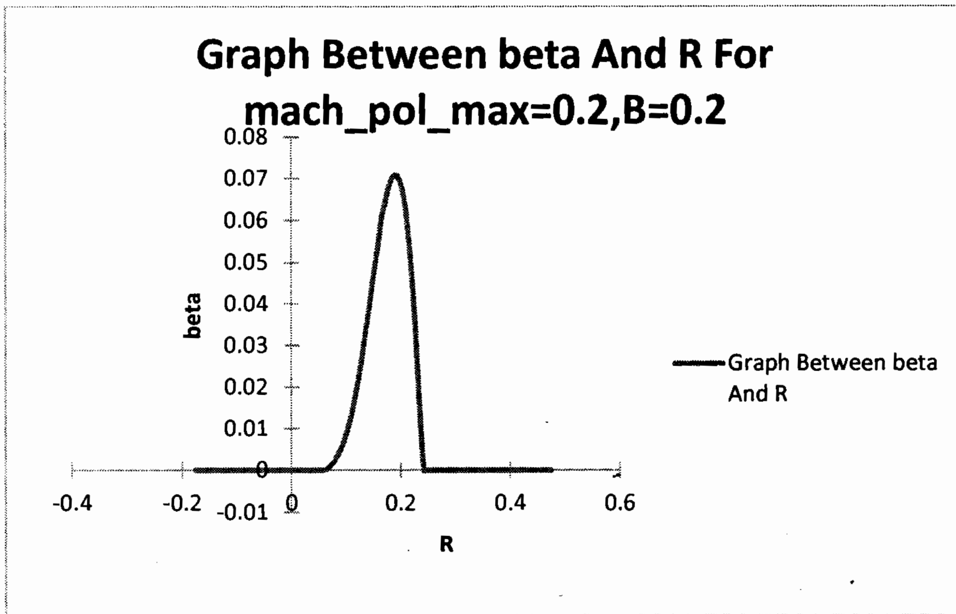
It is an acoustic wave in which compression and rarefactions are produced not by motions along$\vec{E}$,but by $\vec{E} \times \vec{B}$ drifts across$\vec{E}$.

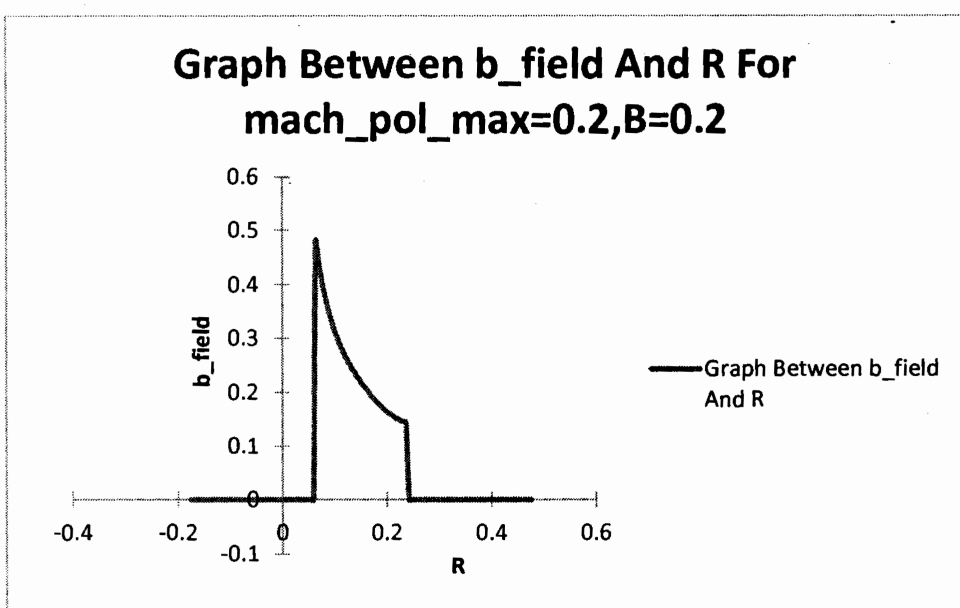
When $\vec{B}_0 \to 0$, $v_A \to 0$, the magnetosonic wave turns into an ordinary ion acoustic wave.

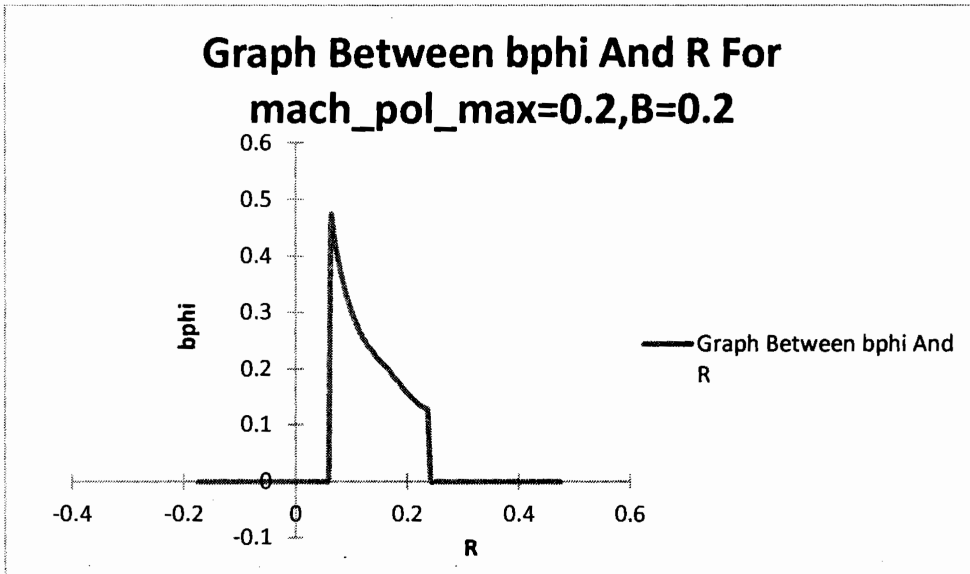
In the limit$kT \to 0, c_s \to 0$, the pressure gradient forces vanishes, and the wave becomes a modified Alfven wave

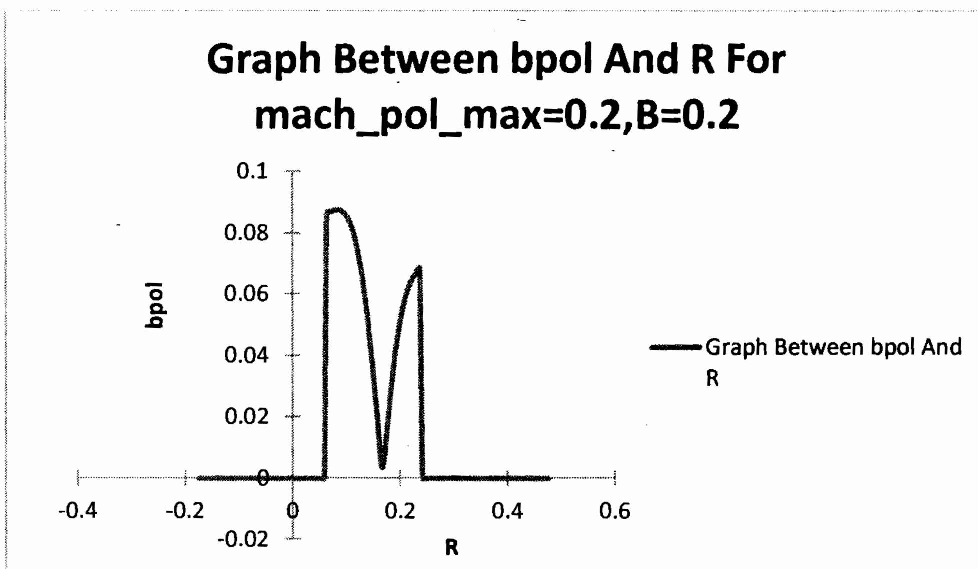
Now we will use code flow in order to investigate the effect of poloidal flow on tokamakequilibria which is of the order of the poloidal sound speed c_{sp}.When the poloidal velocity changes from subsonic to supersonic with respect to the poloidal sound speed then radial discontinuous develop in tokamak plasma.

The following graphs shows effect of poloidal velocity on bphi,bfield,beta,bpol


Graph Between beta And R For
mach_pol_max=0.2,B=0.2
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
beta
-0.4
-0.2
0
0.2
0.4
0.6
R
Graph Between beta And R


Graph Between b_field And R For
mach_pol_max=0.2,B=0.2
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
b_field
-0.4
-0.2
0
0.2
0.4
0.6
R
Graph Between b_field And R


Graph Between bphi And R For mach_pol_max=0.2,B=0.2
bphi
R
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.4
-0.2
0
0.2
0.4
0.6
Graph Between bphi And R


Graph Between bpol And R For mach_pol_max=0.2,B=0.2
bpol
R
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.4
-0.2
0
0.2
0.4
0.6
Graph Between bpol And R

Chapter 4

Summary And Conclusion

Previous chapter contains a set of equations consists of the algebraic Bernoulliequation for plasma density for tokamaks, discussed equation of kinetic density and enthalpy function W which depends upon the closure equations relating pressure density and temperature.

It has been shown that that due to plasma flow equilibrium characteristics of tokamak plasma is changed. Purely toroidal rotation only produces an outward shift of the plasma column due to centrifugal effects. Equilibrium with poloidal velocity of the order of the poloidal sound speed c_{sp} has also been studied. Radial discontinuity develop when poloidal velocity changes from subsonic to supersonic with respect to the poloidal sound speed. Particle drifts are discussed. We discussed tokamakequilibria by using graphs with toroidal and poloidal flow.

It has been recognized that the GS equation in the absence of flows is elliptic, the combined system of Grad–Shafranov–Bernoulli equations can be hyperbolic in at least part of the solution domain in the presence of "transonic" poloidal flows of the order of the sound speed c_s times the ratio of poloidal magnetic fieldB_θ to total field B; throughout this research analysis Bernoulli equation and magnetosonic waves has been derived numerically under the light of which flow code is discussed.

In future flow code can be modified to get more improved temperature to get large amount of fusion energy under controlled situation.

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