Flow of Ellis fluid in renal tubule

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Certificate

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We accept this dissertation as conforming to the required standard.

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Declaration

I hereby declare and affirm that this research work neither as a whole nor as a part has been copied out from any source. It is further declared that I have developed this research work entirely on the basis of my personal efforts.

Moreover, no portion of the work presented in this thesis has been submitted in support of an application for other degree or qualification in this or any other university or institute of learning.

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DEDICATED

TO

MY PARENTS, TEACHERS AND

FRIENDS.

Preface

An important flow phenomenon that occurs in kidneys is the passage of fluid through renal tubule. The function of nephron in kidneys strongly depends upon the flow through renal tubule that helps in bearing the final products of metabolism. They are also helpful to maintain the body fluids volumes. The transport of fluid in tubule is due to the pressure drop. The understanding of hydrodynamics in tubule will help to understand the functions of nephron. The mathematical model of the renal tubule consists of a permeable tube that allows the fluid to move across the boundaries. The theoretical study of flow in renal tubule was first presented by Macey [1, 2]. They assumed a creeping viscous flow through a narrow permeable tube. They predicted that an exponentially decaying flow rate exist along the tube. An extension of the work presented in [1, 2] for the flow through porous wall duct for small Reynold number is discussed by Kozinski [3]. The effects of the variable cross-section tube for the flow through tube were analyzed by Radhakrishnacharya et al. [4]. The exact closed form solution for a viscous flow through a permeable tubule was presented by Marshall et al. [5]. In [5] they have neglected the inertial terms by supposing a creeping flow situation. Palatt et al. [6] solved the viscous flow through a permeable tube by imposing the assumption that fluid loss across the tube wall is a linear function of the pressure gradient across the wall. Effects of variable wall permeability on the creeping flow of a viscous fluid through a tubule were investigated by Chaturani and Ranganatha [7]. In a recent study Siddiqui et al. [8] analyzed the effects of an external applied magnetic field on the theoretical model of the flow for renal tubule. The literature survey indicates that most of the theoretical studies of flow in renal tubules are investigated for Newtonian fluids. It is now established fact that many physiological and industrial fluids deviates from the Newton's law of viscosity. On the basis of experimental studies many relationships of the apparent viscosity are proposed in the literature. These fluids are generally classified as generalized Newtonian fluids and in such fluids the fluid responses to an applied shear stress at an instant do not depends upon the response at some previous instant. The generalized fluid models are widely applied to discuss the physiological flows such as peristaltic flows [9-16] and blood flows [17-22]. Keeping this fact in mind we have revisited the hydrodynamical model of renal tubule using an Ellis fluid model. Ellis fluid exhibits the shear thinning and thickening characteristics at low, moderate and high shear rates. The advantage of Ellis fluid is that it can predict the Newtonian and power law behavior at small and large shear stress. Javed at al. [23] discussed the theoretical analysis of calendaring of an Ellis fluid based on lubrication approximation theory. Hopke and Slattery [24] provides the upper and lower bounds of the drag coefficient for a sphere moving slowly through Ellis fluid. Chhabra at el. [25] discussed the experimental results as compared to the theory of Hopke and Slattery [24] for creeping motion of sphere through Ellis fluid.

The present dissertation is structured as follows. Chapter 1 is devoted to include basic definitions and governing equations. The detail review of a paper by Palatt et al. [6] is carried out in chapter 2. Chapter 3 extends the analysis of Palatt et al. [6] for a non-Newtonian Ellis fluid model.

Contents

Chapter 1

Chapter 1

Introduction

Some basic definitions and governing equations are included in this chapter.

1.1 Fluid mechanics

Fluid mechanics is the branch of mechanics that concerns with behavior of fluids either in motion or in rest and their interaction with boundaries. It is divided into two main branches fluid statics and dynamics. The discipline that deals with the behavior of fluids in rest is categorized as fluid statics. In contrast the discipline that deals with fluids in motion is classified as fluid dynamics.

1.2 Fundamental concepts

1.2.1 Fluid

Fluid is a substance which cannot sustain a shear force under the static condition.

1.2.2 Density

Density (ρ) is defined as mass (m) per unit volume (V) of the fluid. Mathematically,

$$\rho = \frac{m}{V}. \tag{1.1}$$

1.2.3 Pressure

It is the force (F) per unit area (A) that is applied perpendicular to the surface. Mathematically,

$$p = \frac{F}{A}. \tag{1.2}$$

1.2.4 Viscosity

It is the internal property of fluid that measures the resistance of a fluid against any deformation when different forces are acting upon it. Mathematically, it can be expressed as

$$viscosity(\mu) = \frac{Shear\ stress}{Rate\ of\ shear\ strain}. \tag{1.3}$$

It is convenient to use kinematic viscosity (ν) given by the ratio of dynamic viscosity (μ) to the fluid density as

$$\nu = \frac{\mu}{\rho}. \tag{1.4}$$

1.2.5 Velocity field

Fluid motion cannot be understood without the concept of velocity field. Among the properties of a flow the velocity field $\boldsymbol{V}(\boldsymbol{r}, t)$ is the foremost. By a solution of the flow problem we mean to determine its velocity field. Once a velocity field is determined other properties follow directly from it. If one needs to determine temperature field it can be obtained once a velocity field is known. Mathematically,

$$\boldsymbol{V}(\boldsymbol{r}, t) = [u(\boldsymbol{r}, t), v(\boldsymbol{r}, t), w(\boldsymbol{r}, t)], \tag{1.5}$$

in which **r** is the position vector and u, v and w are components of velocity in three orthogonal directions respectively.

1.2.6 Shear stress

The component of stress tangential to the material cross section and arises from the component of force parallel to that cross section is known as shear stress. It is proportional to the deformation and the constant of proportionality is the dynamic viscosity. All fluids that interact with the solid boundary experience a shear stress. The deformation of fluid element can be expressed as rate of deformation or shear stress

$$\tau_{rz} = \mu \frac{du}{dz}. \tag{1.6}$$

1.2.7 Normal stress

The component of stress that acts in the normal direction on a control volume is known as normal or tensile stress.

1.3 Classification of fluid flows

1.3.1 Steady flow

Flow properties of some fluids do not depend upon time, such flow of fluid is known as steady flow. In steady flow time derivative of any fluid property is zero. i.e.

$$\frac{\partial \eta}{\partial t} = 0. \tag{1.7}$$

Where η represent any fluid property and t is the time

1.3.2 Unsteady flow

In contrast to steady flow, if fluid properties depends upon time, we call the flow as unsteady and therefore

$$\frac{\partial \eta}{\partial t} \neq 0. \tag{1.8}$$

1.3.3 Laminar flow

In laminar flow each particle of fluid has distinct, definite path and never intersect with its own path. If there is interaction between the paths of different fluid particles then the flow is turbulent.

1.3.4 Incompressible flow

In general all fluids are compressible to some extent because the density varies with variations in pressure or temperature. In most of the cases this change is small enough that we can neglect it, in such a case we treat density as constant and flow as an incompressible flow.

1.3.5 Couette flow

It is a flow between two plates, in which one plate remains at rest and the other one is moving with uniform velocity.

1.3.6 Poiseuille flow

In the direction of flow a constant pressure gradient is produced between two plates is known as Poiseuille flow.

1.3.7 One-dimensional flow

A flow which the velocity field depends only on one space variable is called a one-dimensional flow.

1.3.8 Two-dimensional flow

A flow for which the velocity field depends upon two space variables is called a two-dimensional flow.

1.3.9 Three-dimensional flow

A flow for which the velocity fields have three space variables is called a three-dimensional flow.

1.4 Types of fluids

1.4.1 Ideal fluids

Fluids having no viscosity fall in the category of ideal fluids. In this case fluid offers no resistance to the applied shear stress.

1.4.2 Real fluids

All real fluids have some non-zero value of viscosity, and offer resistance to the flow. It also distributed into two main types.

1. Newtonian fluids
2. Non-Newtonian fluids

1.4.3 Newtonian fluids

Fluids that obey the Newtonian law of viscosity are Newtonian or viscous fluids. For one dimensional flow that mathematical expression for Newtonian fluid is given by

$$\tau = \mu \frac{du}{dy}. \tag{1.9}$$

Examples of Newtonian fluid are water, air, oil etc. Viscosity is constant for Newtonian fluid.

1.4.4 Non-Newtonian fluids

Fluids that do not follow the linear relationship between stress and rate of deformation are non-Newtonian fluids and in such case

$$\tau = k \left(\frac{du}{dy}\right)^n, \tag{1.10}$$

where n and k are flow behavior and consistency index. Rewriting Eq. (1.10) one gets

$$\tau = k \left(\frac{du}{dy}\right)^{n-1} \frac{du}{dy} = \eta \frac{du}{dy}, \tag{1.11}$$

where η is apparent viscosity. The common examples are shampoo, paint, tooth paste, blood etc. Viscosity in such a case depends upon shear stress and is no more constant.

1.4.5 Ellis fluid model

The limitations of power-law fluid at low and high shear rates are addressed in the Ellis fluid model. Apparent viscosity of three constant Ellis model is defined as

$$\mu = \frac{\mu_0}{1+\left(\tau/\tau_0\right)^{\alpha-1}}, \tag{1.12}$$

where τ_0 and α are material constant, μ_0 is the viscosity at zero shear rate and τ is the second invariant of deformation tensor given by

$$\tau = |\boldsymbol{\tau}| = \sqrt{\frac{1}{2}(\boldsymbol{\tau}:\boldsymbol{\tau})}. \tag{1.13}$$

Eq. (1.12) predicts Newtonian fluid behavior if $\alpha = 1$.

1.5 Assumptions for fluid flow

To turn the result into equation some basic assumptions have to make. The necessary assumptions to analyze any problem in fluid mechanics are as follow

1. Conservation of mass
2. Conservation of momentum

1.5.1 Conservation of mass

For compressible flow of conservation of mass is given by

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \boldsymbol{V}) = 0, \tag{1.14}$$

Conservation of mass for incompressible steady flow is given by

$$\nabla.\boldsymbol{V} = 0. \tag{1.15}$$

In cylindrical coordinates, the continuity equation is given by

$$\frac{1}{r}\frac{\partial}{\partial r}(rv) + \frac{\partial}{\partial \theta}(rw) + \frac{\partial u}{\partial z} = 0, \tag{1.16}$$

in which v, w, u are velocity components in r, θ, z directions, respectively.

1.5.2 Equation of motion

The equation that gives the conservation of momentum is

$$\rho\left[\frac{\partial \boldsymbol{V}}{\partial t} + (\boldsymbol{V}.\nabla)\boldsymbol{V}\right] = -\nabla p + \nabla.\boldsymbol{\tau} + \rho \boldsymbol{b}, \tag{1.17}$$

where $\boldsymbol{b}$ is the body force and $\boldsymbol{\tau}$ is the Cauchy stress tensor. In absence of body forces the component of equation of motion in cylindrical coordinates are given by

$$\rho\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial r}+\frac{w}{r}\frac{\partial v}{\partial \theta}-\frac{w^2}{r}+u\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial r}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr})+\frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta r}+\frac{\partial}{\partial z}\tau_{zr}-\frac{\tau_{\theta\theta}}{r}\right), \tag{1.18}$$

$$\rho\left(\frac{\partial w}{\partial t}+v\frac{\partial w}{\partial r}+\frac{w}{r}\frac{\partial w}{\partial \theta}+\frac{vw}{r}+u\frac{\partial w}{\partial z}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta})+\frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta\theta}+\frac{\partial}{\partial z}\tau_{z\theta}+\frac{\tau_{\theta r}-\tau_{r\theta}}{r}\right), \tag{1.19}$$

$$\rho\left(\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial r}+\frac{w}{r}\frac{\partial u}{\partial \theta}+u\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz})+\frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta z}+\frac{\partial}{\partial z}\tau_{zz}\right). \tag{1.20}$$

Chapter 2

Viscous flow in a permeable tubule

In this chapter the Newtonian fluid flow in small diameter, porous tube (renal tubule) is considered. The governing equations are solved analytically to obtain an exact solution. The graphical results are presented and discussed for various values of involved parameters. In this chapter we are presenting a detail review of a paper by Palatt et al. [6].

2.1 Mathematical formulation

Consider the steady, axisymmetric creeping flow of a viscous fluid in a long thin permeable tube with small diameter. For the mathematical model a cylindrical coordinate system is used. The flow under consideration is two-dimensional and is represented by the following velocity field.

$$\boldsymbol{V} = [\tilde{v}(\tilde{r},\tilde{z}), 0, \tilde{u}(\tilde{r},\tilde{z})]. \tag{2.1}$$

For Newtonian fluid extra stress tensor $\boldsymbol{\tau}$ is given by

$$\boldsymbol{\tau} = \mu \boldsymbol{A}_1, \tag{2.2}$$

where $\boldsymbol{A}_1$ is the first Rivlin-Erickson tensor given by

$$\boldsymbol{A}_1 = \nabla \boldsymbol{V} + (\nabla \boldsymbol{V})^T, \tag{2.3}$$

For velocity field given in Eq. (2.1), we have

$$\boldsymbol{\tau} = \mu \begin{pmatrix} 2\tilde{v}_{\tilde{r}} & 0 & \tilde{u}_{\tilde{r}} + \tilde{v}_{\tilde{z}} \\ 0 & 2\,\tilde{v}/\tilde{r} & 0 \\ \tilde{u}_{\tilde{r}} + \tilde{v}_{\tilde{z}} & 0 & 2\tilde{u}_{\tilde{z}} \end{pmatrix} \tag{2.4}$$

Substituting Eqs. (2.1) and (2.4) in Eqs. (1.17) and (1.18)-(1.20) respectively, we get

$$\frac{\partial}{\partial \tilde{z}}(\tilde{r}\tilde{u}) + \frac{\partial}{\partial \tilde{r}}(\tilde{r}\tilde{v}) = 0, \tag{2.5}$$

$$\frac{1}{\mu}\frac{\partial \tilde{p}}{\partial \tilde{r}} = \frac{\partial^2 \tilde{v}}{\partial \tilde{r}^2} + \frac{1}{\tilde{r}}\frac{\partial \tilde{v}}{\partial \tilde{r}} - \frac{\tilde{v}}{\tilde{r}^2} + \frac{\partial^2 \tilde{v}}{\partial \tilde{z}^2}, \tag{2.6}$$

$$\frac{1}{\mu}\frac{\partial \tilde{p}}{\partial \tilde{z}} = \frac{\partial^2 \tilde{u}}{\partial \tilde{r}^2} + \frac{1}{\tilde{r}}\frac{\partial \tilde{u}}{\partial \tilde{r}} + \frac{\partial^2 \tilde{u}}{\partial \tilde{z}^2}. \tag{2.7}$$

In above equations we have neglected the inertial term due to the fact that flow in renal tubule is creeping. The appropriate boundary conditions are

$$\tilde{v}(0, \tilde{z}) = 0, \tag{2.8}$$

$$\partial \tilde{u}(0, \tilde{z})/\partial \tilde{r} = 0, \tag{2.9}$$

$$\tilde{v}(a, \tilde{z}) = L_p[\tilde{p}(a, \tilde{z}) - p_e], \tag{2.10}$$

$$\tilde{u}(a, \tilde{z}) = 0, \tag{2.11}$$

$$\tilde{u}(\tilde{r}, 0) = 2\tilde{u}_0[1 - (\tilde{r}/a)^2], \quad \tilde{u}_0 = constant, \tag{2.12}$$

$$\tilde{p}(\tilde{r}, 0) = \tilde{p}_0, \quad \tilde{p}_0 = constant. \tag{2.13}$$

The volumetric flow rate is

$$\tilde{Q}_0 = 2\pi \int_0^a \tilde{r}\tilde{u}(\tilde{r}, 0) d\tilde{r} = \pi a^2 \tilde{u}_0, \tag{2.14}$$

which gives

$$\tilde{u}_0 = \frac{\tilde{Q}_0}{\pi a^2}. \tag{2.15}$$

In above equations, $\tilde{v}$ and $\tilde{u}$ are the radial and axial velocity components, respectively, $\tilde{u}_0$ is the mean axial velocity at $\tilde{z} = 0$, $\tilde{r}$ and $\tilde{z}$ are radial and axial coordinate of tubule, respectively, $\tilde{p}$ is the hydrostatic pressure within tubule, $\tilde{p}_0$ is hydrostatic pressure within tubule at point $\tilde{z} = 0$, μ is the coefficient of viscosity, a is the radius of tubule, L_p is the hydrodynamic coefficient of permeability of the tubule wall, p_e is the external hydrostatic pressure of the tubule and $\tilde{Q}_0$ is the flow rate within tubule at point $\tilde{z} = 0$.

Introducing the normalized variables

$$A = \frac{a}{L}, r = \frac{\tilde{r}}{a}, z = \frac{\tilde{z}}{L}, u(r,z) = \frac{\tilde{u}}{\tilde{u}_0}, \quad v(r,z) = \frac{\tilde{v}}{\tilde{u}_0},$$

$$Q(z) = \frac{\tilde{Q}}{\tilde{Q}_0}, \ p(r,z) = \frac{[\tilde{p}(\tilde{r},\tilde{z}) - p_e]a^2}{\mu L \tilde{u}_0}, k = \frac{L_p \mu L}{a^2}. \tag{2.16}$$

Equations (2.5)-(2.13) takes the form

$$A\frac{\partial}{\partial z}(ru) + \frac{\partial}{\partial r}(rv) = 0, \tag{2.17}$$

$$\frac{\partial p}{\partial r} = A\left[\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2} + A^2\frac{\partial^2 v}{\partial z^2}\right], \tag{2.18}$$

$$\frac{\partial p}{\partial z} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + A^2\frac{\partial^2 u}{\partial z^2}, \tag{2.19}$$

$$v(0,z) = 0, \tag{2.20}$$

$$\frac{\partial}{\partial r}u(0,z) = 0, \tag{2.21}$$

$$v(1,z) = kP(1,z), \tag{2.22}$$

$$u(1,z) = 0, \tag{2.23}$$

$$u(r,0) = 2(1 - r^2), \tag{2.24}$$

$$p(r,0) = p_0. \tag{2.25}$$

2.2 Approximations

In biological tubule of small radius one can assume that the ratio of tubule radius to its length is very small. Furthermore, the outward radial velocity is much smaller than mean axial velocity. Therefore

(1) $A \ll 1$

(2) $V_w \ll U_m$

An order and magnitude analysis of Eqs. (2.17)-(2.19) gives

$$\frac{\partial^2 v}{\partial r^2} \approx \frac{1}{r}\frac{\partial v}{\partial r} \approx \frac{v}{r^2} \approx O[V_w], \tag{2.26}$$

$$A^2 \frac{\partial^2 v}{\partial z^2} \approx O[A^2. V_w], \tag{2.27}$$

$$\frac{\partial^2 u}{\partial r^2} \approx \frac{1}{r}\frac{\partial u}{\partial r} \approx O[U_m], \tag{2.28}$$

$$A^2 \frac{\partial^2 u}{\partial z^2} \approx O[A^2. U_m]. \tag{2.29}$$

The simplified equations for the flow are

$$\frac{\partial p}{\partial r} \approx A[\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2}], \tag{2.30}$$

$$\frac{\partial p}{\partial z} \approx \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}[r\frac{\partial u}{\partial r}], \tag{2.31}$$

From Eqs. (2.30) and (2.31)

$$\frac{\partial p}{\partial r} \approx O[A. V_w] , \tag{2.32}$$

$$\frac{\partial p}{\partial z} \approx O[U_m]. \tag{2.33}$$

Using approximations, we neglect (2.30) in comparison to (2.31) and pressure can be evaluated using continuity equation. The flow is then governed by

$$\frac{\partial p}{\partial r} = 0 , \tag{2.34}$$

$$\frac{dp}{dz} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u}{\partial r}\right]. \tag{2.35}$$

2.3 Analytical solution

Since pressure is not a function of r, therefore one can integrate Eq. (2.35) in the following way

$$r\frac{\partial u}{\partial r} = \frac{dp}{dz}\frac{r^2}{2} + c_1, \tag{2.36}$$

Boundary condition (2.21) suggests that $c_1 = 0$ and hence

$$\frac{\partial u}{\partial r} = \frac{dp}{dz}\frac{r}{2}. \quad (2.37)$$

Integrating Eq. (2.37) and utilizing boundary condition (2.23), we get

$$u = \frac{(r^2-1)}{4}\frac{dp}{dz}. \quad (2.38)$$

Utilizing the value of u obtained in Eq. (2.38) into continuity Eq. (2.18), we get

$$A\frac{r(r^2-1)}{4}\frac{d^2p}{dz^2} + \frac{\partial}{\partial r}(r\,v) = 0. \quad (2.39)$$

The equation that gives the pressure distribution can be obtained by integrating Eq. (2.39) from $R = 0$ to 1 and is given by

$$\frac{d^2p}{dz^2} - \frac{16}{A}v(1,z) = 0, \quad (2.40)$$

Eliminating $v(1,z)$ between Eqs. (2.40) and (2.22) one gets

$$\frac{d^2p}{dz^2} - \frac{16k}{A}p = 0. \quad (2.41)$$

Solution of Eq. (2.41) is then

$$p(z) = \theta_1\,exp(-\beta\,z) + \theta_2\,exp(\beta\,z), \quad (2.42)$$

where $\beta^2 = 16k/A$, and θ_1 and θ_2 are arbitrary constants.

Substituting Eq. (2.42) into (2.38)

$$u = \frac{\beta(r^2-1)}{4}[\theta_1\,exp(-\beta\,z) + \theta_2\,exp(\beta\,z)]. \quad (2.43)$$

Radial velocity component v can be evaluated by integrating Eq. (2.39) w.r.t r and is given by

$$v = \frac{A}{16}(2r - r^3)\frac{d^2p}{dz^2}. \quad (2.44)$$

Upon substituting p from Eq. (2.42), Eq. (2.44) becomes

$$v = \frac{A\beta^2(2r-r^3)}{16}[\theta_1 exp(-\beta\, z) + \theta_2 exp(\beta\, z)]. \tag{2.45}$$

Conditions (2.24) and (2.25) are utilized to give

$$\theta_1 = \frac{p_0}{2} + \frac{4}{\beta}, \tag{2.46}$$

$$\theta_2 = \frac{p_0}{2} - \frac{4}{\beta}. \tag{2.47}$$

Therefore,

$$p(z) = \left(\frac{p_0}{2} + \frac{4}{\beta}\right) exp(-\beta\, z) + \left(\frac{p_0}{2} - \frac{4}{\beta}\right) exp(\beta\, z). \tag{2.48}$$

$$u(r,z) = \frac{\beta(r^2-1)}{4}\left[\left(\frac{p_0}{2} - \frac{4}{\beta}\right) exp(\beta\, z) - \left(\frac{p_0}{2} + \frac{4}{\beta}\right) exp(-\beta\, z)\right]. \tag{2.49}$$

$$v(r,z) = \frac{A\beta^2(2r-r^3)}{16}\left[\left(\frac{p_0}{2} + \frac{4}{\beta}\right) exp(-\beta\, z) + \left(\frac{p_0}{2} - \frac{4}{\beta}\right) exp(\beta\, z)\right]. \tag{250}$$

The non-dimensional pressure at $z = 1$ is given by

$$p(1) = p_0 cosh\, \beta - (8\, cosh\, \beta)/\beta, \tag{2.51}$$

Assuming $p(1) \geq 0$, we get

$$p_0 \geq (8 \tanh \beta)/\beta, \tag{2.52}$$

The case when $\beta = 0$, Eq. (2.51) gives

$$p(1) = p_0 - 8. \tag{2.53}$$

To validate the theoretical results given by the assumed model, we have compared our results with available data existing in the literature.

Table 1: Experimental data for rat proximal convoluted tubule

Quantity	Experimental Value
a	$1.08 \times 10^{-3} cm$
L	$0.67 cm$
L_p	$1.50 \times 10^{-6} \frac{cm}{sec - cm\, H_2O}$
$\tilde{p}_o$	$14.4\ cm\ H_2O$
p_e	$10.3\ cm\ H_2O$
π_e	$16.5\ cm\ H_2O$
$\tilde{Q}_o$	$40.2 \times 10^{-6} \frac{cm^3}{sec}$
μ	$7.37 \times 10^{-6} cm\ H_2O - sec$

2.5 Validity of Approximations

Numerical data presented in table 1 gives

$A = \frac{a}{L} \approx 1.6 \times 10^{-3}$, $k = \frac{L_p \mu L}{a^2} \approx 6.4 \times 10^{-6}$, $\beta = 4\sqrt{\frac{K}{A}} \approx 0.25$,

$p_o = \frac{(\tilde{p}_o - p_e + \pi_e)\pi a^4}{\mu L q_o} \approx 44.4$, $\theta_1 = \frac{\tilde{p}_o}{2} + \frac{4}{\beta} \approx 38.2$, $\theta_2 = \frac{\tilde{p}_o}{2} - \frac{4}{\beta} \approx 6.2$

Since $A \ll 1$, approximation (1) is valid.

Also

$$\frac{V_w}{U_m} = \frac{8k}{\beta}\left[\frac{\theta_1 \exp(-\beta z) + \theta_1 \exp(\beta z)}{\theta_1 \exp(-\beta z) - \theta_1 \exp(\beta z)}\right] \approx 3.6 \times 10^{-4}, \tag{2.54}$$

Thus approximation (2) is valid. The volumetric flow rate gives

$$Q(z) = 2\int_0^1 u(r,z) r\, dr, \tag{2.55}$$

Utilizing value of u and integrating we get

$$Q(z) = \cosh \beta z - \frac{\beta p_0}{8} \sinh \beta z. \tag{2.56}$$

The non-dimensional mean pressure drop is

$$\Delta p = p(0) - p(z), \tag{2.57}$$

$$\Delta p = p_0 \left[1 - \left(\cosh \beta z - \frac{8}{\beta p_0} \sinh \beta z\right)\right]. \tag{2.58}$$

The dimensionless shear stress is obtained as

$$\tau_w(z) = \tau_{rz}|_{r=1}, \tag{2.59}$$

$$\tau_w(z) = 4\cosh \beta z - \frac{\beta p_0}{2} \sinh \beta z. \tag{2.60}$$

The dimensionless fractional reabsorption is obtained as

$$FR = \frac{Q(0) - Q(1)}{Q(0)}, \tag{2.61}$$

$$FR = 1 - \left(\cosh \beta - \frac{\beta p_0}{8} \sinh \beta\right). \tag{2.62}$$

2.10 Results and discussions

This section is devoted to discuss the obtained results for the flow of a Newtonian fluid in a renal tubule. In Fig. 2.1 inlet pressure is plotted against parameter β to differentiate between admissible and inadmissible regions. A decrease in inadmissible region is observed for large values of β. The outlet pressure plotted against inlet pressure is depicted in Fig. 2.2 for three different values of β. The case of $\beta = 0$, corresponds to the Poisseuille flow for a non-porous tube. Eqs. (2.48) and (2.56) respectively can be written as

$$\frac{p(z)}{p_0} = \cosh \beta z - \frac{1}{\gamma} \sinh \beta z, \tag{2.63}$$

$$Q(z) = \cosh \beta z - \gamma \sinh \beta z, \tag{2.64}$$

where $\gamma = 2Kp_0/\beta$ and $K = k/A$. It is observed that γ is strongly effects the behavior of pressure distribution and volume flow rate. To discuss $Q(z)$ in detail Fig. 2.3 is plotted for various values of γ. There are some important cases for various values of γ. For $-\infty < \gamma < 0$, the flow rate monotonically increase from $Q = 1$ at $z = 0$ and Q approaches to $+\infty$ as $z \to \infty$. For some large values of z the expression of flow rate is approximated by

$$Q = \frac{1}{2}(1 + |\gamma|)exp(\beta\, z). \tag{2.64}$$

For $\gamma = 0$, $Q = cosh\, \beta z$ and the flow rate also present the same properties. For $0 < \gamma < 1$, flow rate initially decrease from $Q = 1$ at $z = 0$ to a minimum value of $Q_{min} = \sqrt{1-\gamma^2}$ when $z = \frac{1}{2\beta} ln(1 + \gamma/1 - \gamma)$. The flow rate monotonically increase from that point and $Q \to +\infty$ as $z \to \infty$. When $Q = \frac{1}{2}(1-\gamma)exp(\beta\, z)$ for large z. When $\gamma = 1$, the flow rate monotonically decreases from $Q = 1$ at $z = 0$ and $Q \to 0$ as $z \to \infty$. For the case of $1 < \gamma < \infty$, Q monotonically decreases from $Q = 1$ at $z = 0$ and $Q = 0$ at $z = \frac{1}{2\beta} ln(\gamma + 1/\gamma - 1)$. Negative flow rate is obtained after that point, it decreases monotonically and $Q \to -\infty$ as $z \to \infty$. For large z, flow rate is approximated by $Q = -\frac{1}{2}(\gamma - 1)exp(\beta\, z)$. This predicts a reverse flow phenomena that may not be admissible in many physical situations. Fig. 2.4 is plotted for $p(z)/p_0$ with z, for different values of γ with presents the same properties. Fig. 2.5 presents the variation of radial velocity v for different K by keeping other parameter fixed. The radial velocity is enhanced by increasing K. The velocity component u for different K is shown in Fig. 2.6. The increment in K cause a decrease in u. Mean pressure drop is presented in Fig. 2.7. It is noted that mean pressure drop decreases by increasing K. Fig. 2.8 represent the variation of wall shear stress τ_w. It is clear from figure that the wall shear stress decreases with increasing K. Fig. 2.9 depicts the variation of fractional reabsorption FR with p_0. It can be seen from this figure that the fractional reabsorption is enhanced with increasing parameter K.

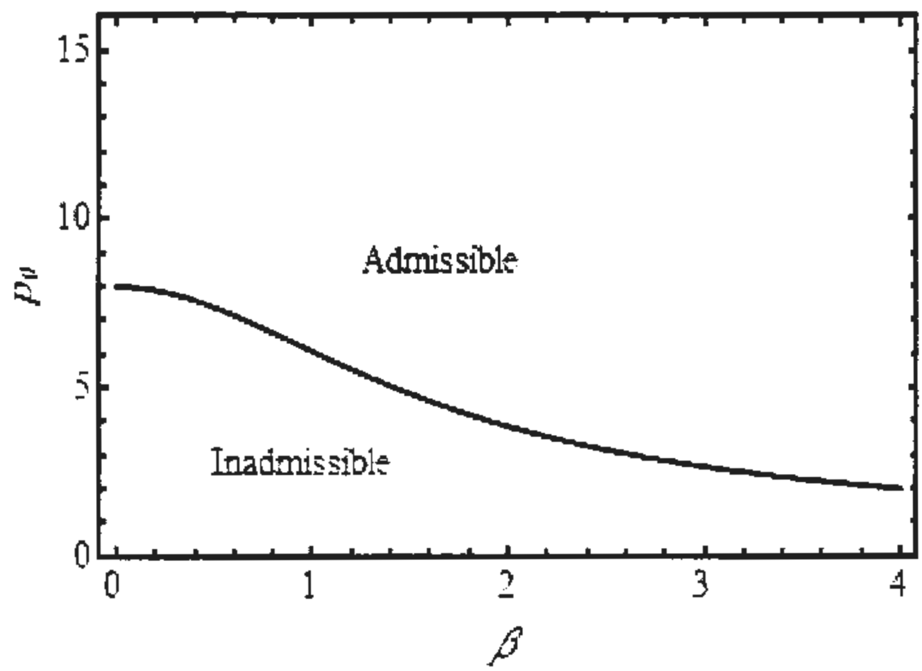

Fig 2.1. Response of admissible region for inlet pressure p_0 with β.

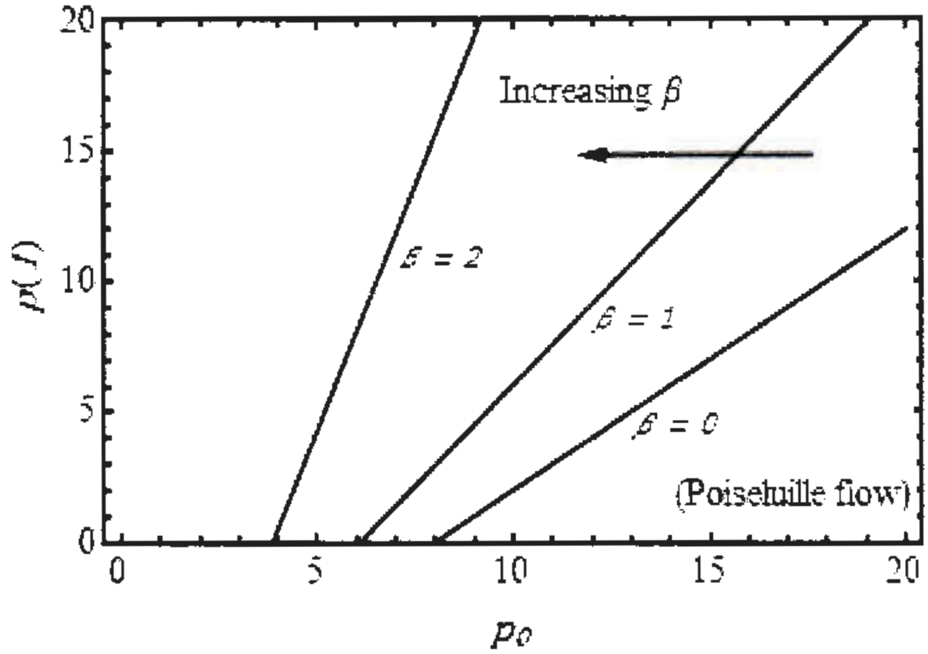

Fig 2.2. Response of non-dimensional pressure at $z = 1$ with non-dimensional inlet pressure p_0 for different β.

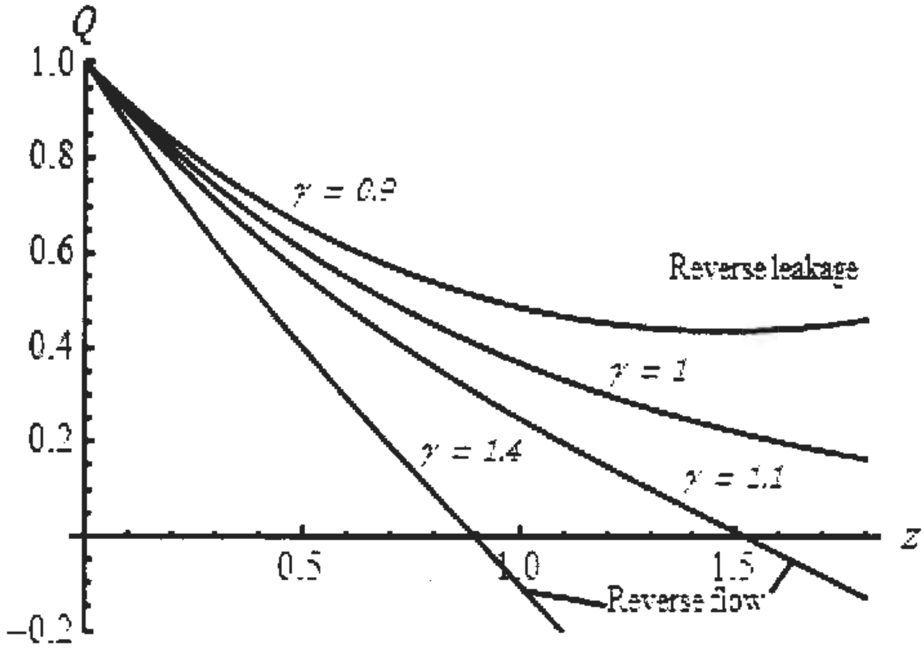

Fig 2.3. Response of flow rate Q for different γ, when $\beta = 1$.

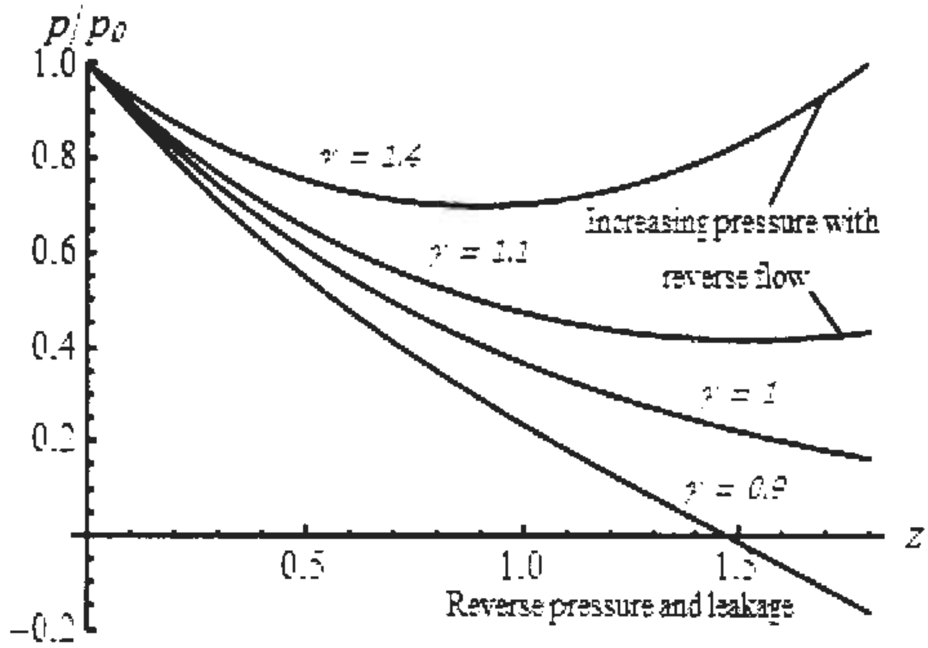

Fig 2.4. Response of pressure distribution p/p_0 for different γ, when $\beta = 1$.

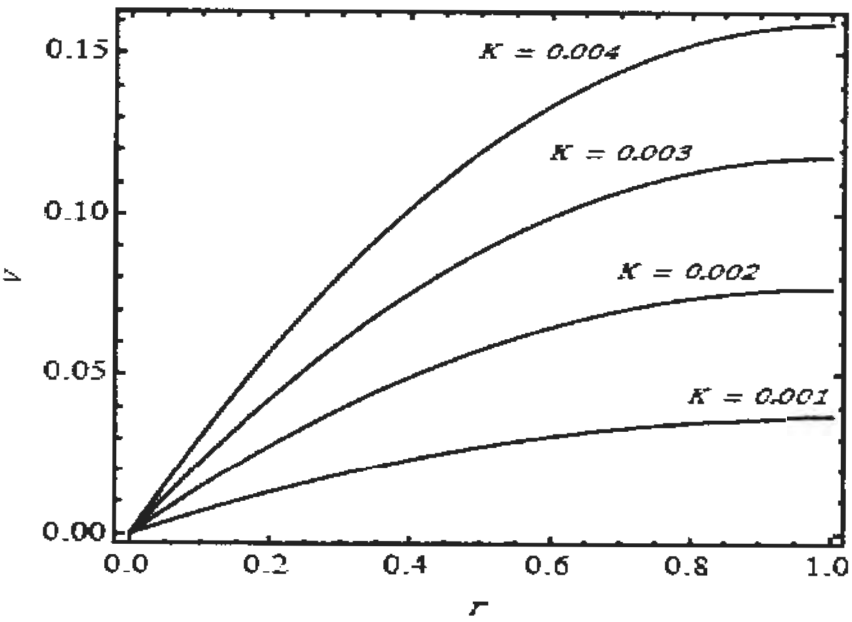

Fig 2.5. Response of radial velocity v for different K, when $z = 0.1$, $p_0 = 43$

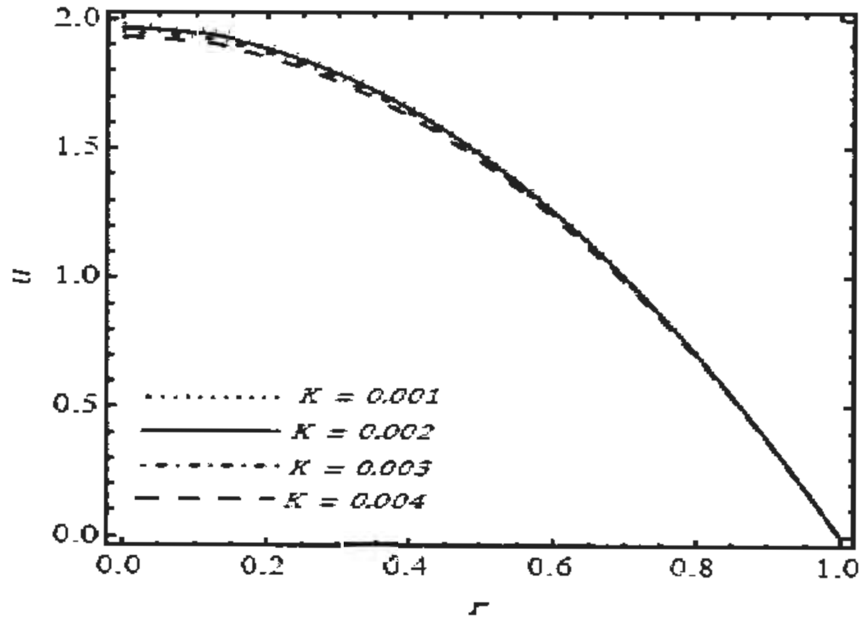

Fig 2.6. Response of axial velocity u for different K, when $z = 0.1$, $p_0 = 43$.

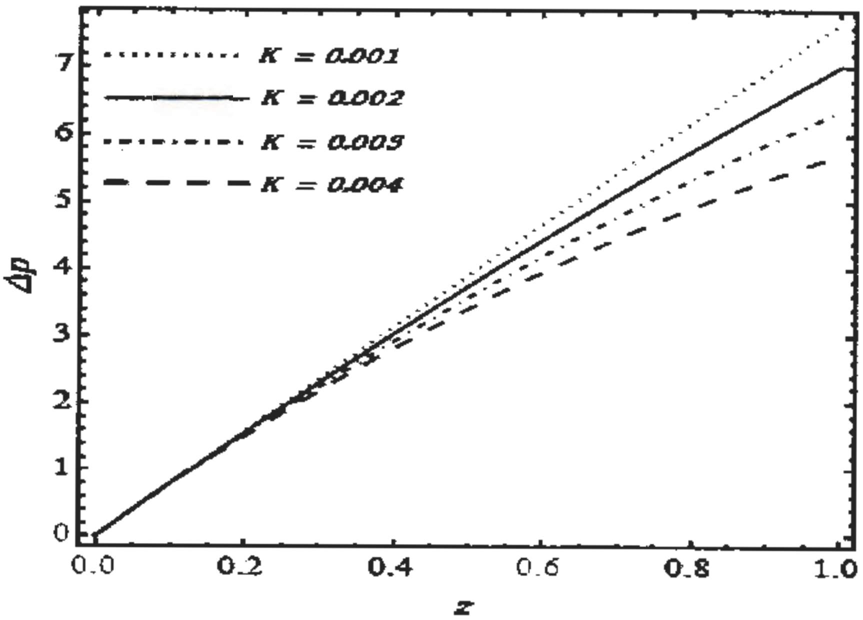

Fig 2.7. Response of mean pressure Δp for different K, when $p_0 = 43$.

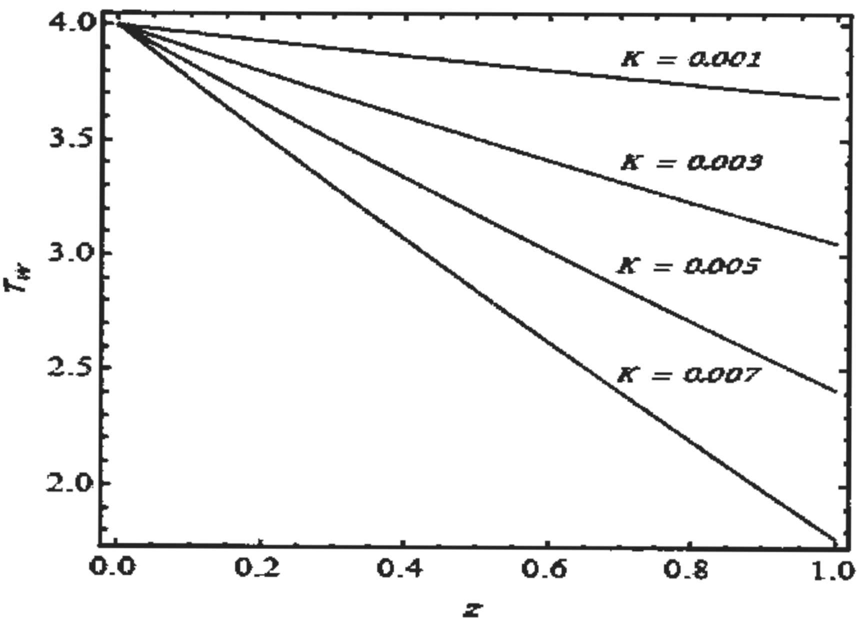

Fig 2.8. Response of wall shear stress τ for different K, when $p_0 = 43$.

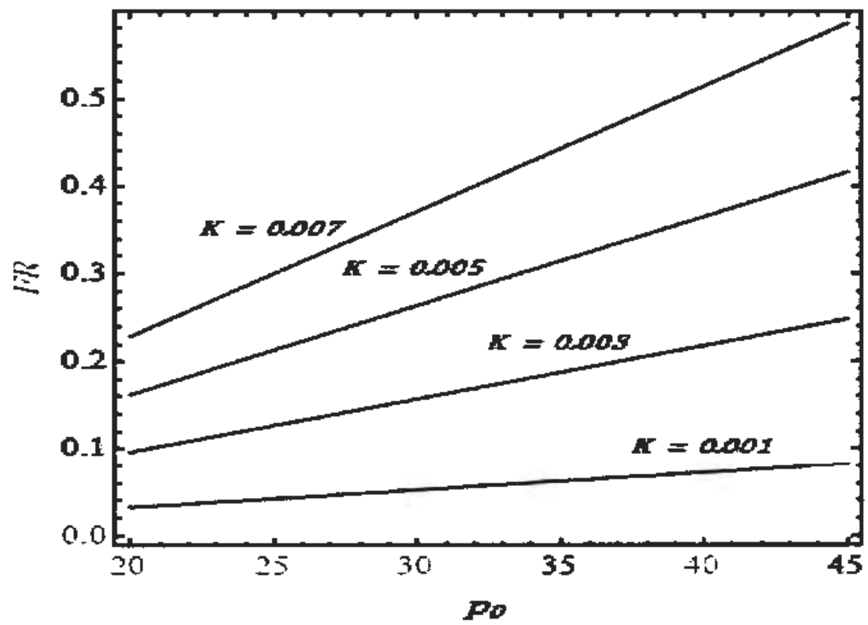

Fig 2.9. Response of fractional reabsorption FR for different K.

Chapter 3

Flow of an Ellis fluid in renal tubule

In this chapter we have extended the analysis of chapter 2 for an Ellis fluid flow in a permeable tube of small diameter with an application to the flow in renal tubule. The modeled equations are solved and results are presented in the graphs to discuss the effects of various parameters.

3.1 Formulation of the problem

For an Ellis fluid $\boldsymbol{\tau}$ is given by

$$\boldsymbol{\tau} = \mu(\dot{\gamma})\boldsymbol{A_1}, \tag{3.1}$$

Since

$$\boldsymbol{\tau} = \begin{pmatrix} \tilde{\tau}_{rr} & 0 & \tilde{\tau}_{rz} \\ 0 & \tilde{\tau}_{\theta\theta} & 0 \\ \tilde{\tau}_{zr} & 0 & \tilde{\tau}_{zz} \end{pmatrix}, \tag{3.2}$$

$$\boldsymbol{\tau^2} = \begin{pmatrix} \tilde{\tau}_{rr}^2 + \tilde{\tau}_{rz}^2 & 0 & \tilde{\tau}_{rz}(\tilde{\tau}_{rr} + \tilde{\tau}_{zz}) \\ 0 & \tilde{\tau}_{\theta\theta}^2 & 0 \\ \tilde{\tau}_{zr}(\tilde{\tau}_{rr} + \tilde{\tau}_{zz}) & 0 & \tilde{\tau}_{rz}^2 + \tilde{\tau}_{zz}^2 \end{pmatrix}. \tag{3.3}$$

Therefore,

$$\mu(\dot{\gamma}) = \frac{2\mu_o}{1+\left(\frac{1}{\tau_o^2}\sqrt{\frac{1}{2}(\tilde{\tau}_{rr}^2+2\tilde{\tau}_{rz}^2+\tilde{\tau}_{\theta\theta}^2+\tilde{\tau}_{zz}^2)}\right)^{\alpha-1}}, \tag{3.4}$$

Substituting Eqs. (2.3) and (3.4) into (3.1), we get

$$\boldsymbol{\tau} = \frac{2\mu_o}{1+\left(\frac{1}{\tau_o^2}\sqrt{\frac{1}{2}(\tilde{\tau}_{rr}^2+2\tilde{\tau}_{rz}^2+\tilde{\tau}_{\theta\theta}^2+\tilde{\tau}_{zz}^2)}\right)^{\alpha-1}} \begin{pmatrix} 2\tilde{v}_{\tilde{r}} & 0 & \tilde{u}_{\tilde{r}} + \tilde{v}_{\tilde{z}} \\ 0 & 2\tilde{v}/\tilde{r} & 0 \\ \tilde{u}_{\tilde{r}} + \tilde{v}_{\tilde{z}} & 0 & 2\tilde{u}_{\tilde{z}} \end{pmatrix}, \tag{3.5}$$

Substituting the values from Eqs. (2.1) and (3.5) in Eqs. (1.16) and (1.18)-(1.20) and neglecting inertial terms and body forces, we obtained the following equations

$$\frac{\partial}{\partial \tilde{z}}(\tilde{r}\tilde{u}) + \frac{\partial}{\partial \tilde{r}}(\tilde{r}\tilde{v}) = 0, \tag{3.6}$$

$$\frac{\partial \tilde{p}}{\partial \tilde{r}} = \frac{1}{\tilde{r}}\frac{\partial}{\partial \tilde{r}}(\tilde{r}\tilde{\tau}_{rr}) + \frac{\partial}{\partial \tilde{z}}\tilde{\tau}_{zr} - \frac{\tilde{\tau}_{\theta\theta}}{\tilde{r}}, \tag{3.7}$$

$$\frac{\partial \tilde{p}}{\partial \tilde{z}} = \frac{1}{\tilde{r}}\frac{\partial}{\partial \tilde{r}}(\tilde{r}\tilde{\tau}_{rz}) + \frac{\partial}{\partial \tilde{z}}\tilde{\tau}_{zz}\,, \tag{3.8}$$

where

$$\tilde{\tau}_{rr} = \frac{4\mu_o \frac{\partial \tilde{v}}{\partial \tilde{r}}}{1+\left[\frac{1}{\tau_o^2}\sqrt{\frac{\tilde{\tau}_{rr}^2+2\tilde{\tau}_{rz}^2+\tilde{\tau}_{\theta\theta}^2+\tilde{\tau}_{zz}^2}{2}}\right]^{\alpha-1}}, \tag{3.9}$$

$$\tilde{\tau}_{rz} = \frac{2\mu_o\left(\frac{\partial \tilde{u}}{\partial \tilde{r}}+\frac{\partial \tilde{v}}{\partial \tilde{z}}\right)}{1+\left[\frac{1}{\tau_o^2}\sqrt{\frac{\tilde{\tau}_{rr}^2+2\tilde{\tau}_{rz}^2+\tilde{\tau}_{\theta\theta}^2+\tilde{\tau}_{zz}^2}{2}}\right]^{\alpha-1}}, \tag{3.10}$$

$$\tilde{\tau}_{\theta\theta} = \frac{4\mu_o \frac{\tilde{v}}{\tilde{r}}}{1+\left[\frac{1}{\tau_o^2}\sqrt{\frac{\tilde{\tau}_{rr}^2+2\tilde{\tau}_{rz}^2+\tilde{\tau}_{\theta\theta}^2+\tilde{\tau}_{zz}^2}{2}}\right]^{\alpha-1}}, \tag{3.11}$$

$$\tilde{\tau}_{zz} = \frac{4\mu_o \frac{\partial \tilde{u}}{\partial \tilde{z}}}{1+\left[\frac{1}{\tau_o^2}\sqrt{\frac{\tilde{\tau}_{rr}^2+2\tilde{\tau}_{rz}^2+\tilde{\tau}_{\theta\theta}^2+\tilde{\tau}_{zz}^2}{2}}\right]^{\alpha-1}}. \tag{3.12}$$

The appropriate boundary conditions are

$$\tilde{v}(0,\tilde{z}) = 0, \tag{3.13}$$

$$\partial \tilde{u}(0,\tilde{z})/\partial \tilde{r} = 0, \tag{3.14}$$

$$\tilde{v}(a,\tilde{z}) = L_p[\tilde{p}(a,\tilde{z}) - p_m], \tag{3.15}$$

$$\tilde{u}(a,\tilde{z}) = 0, \tag{3.16}$$

$$\tilde{p}(\tilde{r},0) = \tilde{p}_o, \tag{3.17}$$

$$\tilde{Q}_o = 2\pi \int_0^a \tilde{r}\tilde{u}(\tilde{r},0)d\tilde{r}, \tag{3.18}$$

where $p_m = p_e - \pi_e$, π_e is the osmotic pressure outside the tubule. Introducing the dimensionless variables and parameters

$$r = \frac{\tilde{r}}{a},\ z = \frac{\tilde{z}}{L},\ u(r,z) = \frac{\pi a^2 \tilde{u}}{\tilde{Q}_o},\ v(r,z) = \frac{\pi a^2 L \tilde{v}}{\tilde{Q}_o},\ A = \frac{a}{L}, Q(z) = \frac{\tilde{Q}(z)}{\tilde{Q}_o},$$

$$p(r,z) = \frac{[\tilde{p}(\tilde{r},\tilde{z}) - p_m]\pi a^4}{\mu_o L \tilde{Q}_o},\ K = \frac{L_p \mu_o L}{a^2 A}, \tau_{ij} = \frac{\pi a^3 \tilde{\tau}_{ij}}{\mu_o \tilde{Q}_o}. \tag{3.19}$$

The transformed system of equations are

$$\frac{\partial}{\partial r}(rv) + \frac{\partial}{\partial z}(ru) = 0, \tag{3.20}$$

$$\frac{\partial p}{\partial r} = A\left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + A\frac{\partial \tau_{rz}}{\partial z} - \frac{\tau_{\theta\theta}}{r}\right], \tag{3.21}$$

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + A\frac{\partial \tau_{zz}}{\partial z}, \tag{3.22}$$

$$\tau_{rr} = \frac{4A\frac{\partial v}{\partial r}}{1+\left[\left(\frac{\mu_o \tilde{Q}_o}{\pi a^3 \tau_o^2}\right)\sqrt{\frac{\tau_{rr}^2 + 2\tau_{rz}^2 + \tau_{\theta\theta}^2 + \tau_{zz}^2}{2}}\right]^{\alpha-1}}, \tag{3.23}$$

$$\tau_{rz} = \frac{2\left(\frac{\partial u}{\partial r} + A^2\frac{\partial v}{\partial z}\right)}{1+\left[\left(\frac{\mu_o \tilde{Q}_o}{\pi a^3 \tau_o^2}\right)\sqrt{\frac{\tau_{rr}^2 + 2\tau_{rz}^2 + \tau_{\theta\theta}^2 + \tau_{zz}^2}{2}}\right]^{\alpha-1}}, \tag{3.24}$$

$$\tau_{\theta\theta} = \frac{4A\frac{v}{r}}{1+\left[\left(\frac{\mu_o \tilde{Q}_o}{\pi a^3 \tau_o^2}\right)\sqrt{\frac{\tau_{rr}^2 + 2\tau_{rz}^2 + \tau_{\theta\theta}^2 + \tau_{zz}^2}{2}}\right]^{\alpha-1}}, \tag{3.25}$$

$$\tau_{zz} = \frac{4A\frac{\partial u}{\partial z}}{1+\left[\left(\frac{\mu_o \tilde{Q}_o}{\pi a^3 \tau_o^2}\right)\sqrt{\frac{\tau_{rr}^2 + 2\tau_{rz}^2 + \tau_{\theta\theta}^2 + \tau_{zz}^2}{2}}\right]^{\alpha-1}}. \tag{3.26}$$

$$v(0,z) = 0, \tag{3.27}$$

$$\frac{\partial u}{\partial r}(0,z) = 0, \tag{3.28}$$

$$v(1,z) = Kp(1,z), \tag{3.29}$$

$$u(1,z) = 0, \tag{3.30}$$

$$p(r,0) = p_o, \tag{3.31}$$

$$2\int_0^1 ru(r,0)dr = 1, \tag{3.32}$$

3.2 Solution of reduced equations

Invoking the appropriate simplification for biological tubules assuming that a fluid loss through the walls. In such a situation $A \ll 1$, $v_w \ll u_m$ and governing equation becomes

$$\frac{\partial p}{\partial r} = 0, \tag{3.33}$$

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}), \tag{3.34}$$

$$\tau_{rr} = \tau_{\theta\theta} = \tau_{zz} = 0, \tag{3.35}$$

$$\tau_{rz} = \frac{2\left(\frac{\partial u}{\partial r}\right)}{1+(\beta\tau_{rz})^{\alpha-1}}, \tag{3.36}$$

where $\beta = \mu_0\tilde{Q}_0/\pi a^3\tau_0^2$. Equation implies (3.33) $p = p(z)$ only.

Integrating Eq. (3.34) from $r = 0$ to r, we get

$$\tau_{rz} = \frac{r}{2}\frac{dp}{dz}, \tag{3.37}$$

Substituting Eq. (3.37) into (3.36), we obtained

$$\frac{\partial u}{\partial r} = \frac{r}{4}\frac{dp}{dz} + \frac{1}{2}(\beta)^{\alpha-1}\left(\frac{1}{2}\frac{dp}{dz}\right)^{\alpha} r^{\alpha}, \tag{3.38}$$

Integrating Eq. (3.38) and utilizing the boundary condition (3.30), we have

$$u = \frac{1}{8}\frac{dp}{dz}(r^2-1) + \frac{(\beta)^{\alpha-1}}{2(\alpha+1)}\left(\frac{1}{2}\frac{dp}{dz}\right)^{\alpha}(r^{\alpha+1}-1). \tag{3.39}$$

Substituting (3.39) into (3.20), we get

$$\frac{\partial}{\partial r}(rv) = -\frac{1}{8}\frac{d^2p}{dz^2}(r^3 - r) - \frac{\alpha(\beta)^{\alpha-1}}{4(\alpha+1)}\left(\frac{1}{2}\frac{dp}{dz}\right)^{\alpha-1}\frac{d^2p}{dz^2}(r^{\alpha+2} - r), \quad (3.40)$$

Integrating equation (3.40) from $r = 0$ to r, we get

$$v(r,z) = -\frac{1}{32}\frac{d^2p}{dz^2}(r^3 - 2r) - \frac{\alpha(\beta)^{\alpha-1}}{8(\alpha+1)(\alpha+3)}\left(\frac{1}{2}\frac{dp}{dz}\right)^{\alpha-1}\frac{d^2p}{dz^2}(2r^{\alpha+2} - (\alpha+3)r). \quad (3.41)$$

Now integrating (3.40) from $r = 0$ to 1, and using the condition (3.29)

$$\frac{d^2p}{dz^2} = \frac{32Kp(z)}{1+\frac{4\alpha}{(\alpha+3)}\left(\frac{\beta}{2}\frac{dp}{dz}\right)^{\alpha-1}}, \quad (3.42)$$

A numerical solution of Eq. (3.42) is obtained subject to boundary conditions (3.31) and (3.32). The numerical values of the appearing parameters for the proximal convoluted tubule are given in table 1.

3.3 Result and discussion

This section is devoted to analyze how the Ellis fluid material parameters effects the pressure gradient, pressure. velocity, leakage flux, flow rate, wall shear stress, mean pressure drop and fractional reabsorption. To investigate the volume flow rate $Q(z)$ in response to the various value of c. Fig. 3.1 is plotted for Newtonian and Ellis fluids. There are important regions of interest for different c for both Newtonian and Ellis fluids. For region 1, $0 < c < 1$ for Newtonian fluid and $0 < c < 0.7$ for Ellis fluid. It is noted that Q decreases from $Q = 1$ at $z = 0$ to a minimum value and then starts increasing monotonically and approaches $+\infty$ as $z \to \infty$. For region 2, c=1 and $c = 0.7$ respectively for Newtonian and Ellis fluids. Q decreases monotonically from $Q = 1$ at $z = 0$ and $Q \to 0$ as $z \to \infty$ in this region. For region 3, $1 < c < \infty$ for Newtonian and $0.7 < c < \infty$ for Ellis fluid a monotonic decrease is observed in the flow rate and for large z, flow rate approaches $-\infty$. Region 3, predicts a reverse flow phenomena that may not be admissible in many physical situations. This figure further illustrate that flow rate decreases by increasing c and α. In Figs. 3.2 and 3.3 pressure normalized with p_0 and leakage flux are plotted against axial coordinate z for Ellis and Newtonian fluids for different values of parameter c. Figs. 3.2 and 3.3 elucidate that pressure and leakage flux respectively decreases and attains a minimum for $c < 1.0$ in the case of Newtonian fluid. However for Ellis fluid with $\alpha = 3$ the pressure and leakage flux increases with the reverse flow even for $c < 1.0$ in this case the pressure and leakage flux decreases and attains minimum for $c < 0.7$. Fig. 3.4 presents the effects of radial velocity v for some values of material parameter α by keeping the other parameter fixed. The radial velocity v is large in magnitude for Ellis fluid as compared to Newtonian fluid. The radial velocity for different K is presented graphically in Fig. 3.5. The radial velocity is an increasing function of the permeability of the wall. The change in axial velocity u for different α and K are presented in Figs. 3.6 and 3.7, respectively. The axial velocity u decreases by increasing both the parameters α and K. However the magnitude of change in the case of K is very small in comparison to the case of material parameter α. The mean pressure drop over the length of the tubule is calculated for different values of α and K and is shown in Figs. 3.8 and 3.9. It is predicted that mean pressure drop decreases by increasing α and permeability coefficient K. The influence of α and wall permeability coefficient K on the flow rate is shown in Figs. 3.10 and 3.11, respectively. These figures Illustrate that Q decreases by increasing material parameter α and wall permeability coefficient K. Variation of wall shears stress τ_w with z for different α and K are shown in Figs. 3.12 and 3.13, respectively. It is clear from figures that τ_w decrease considerably with the increasing value of K and increases with increasing the value of α. The changed in fractional reabsorption FR with p_0 for different α and K are presented in Figs. 3.14 and 3.15, respectively. The fact of K on fractional reabsorption is enhanced with increasing K and α.

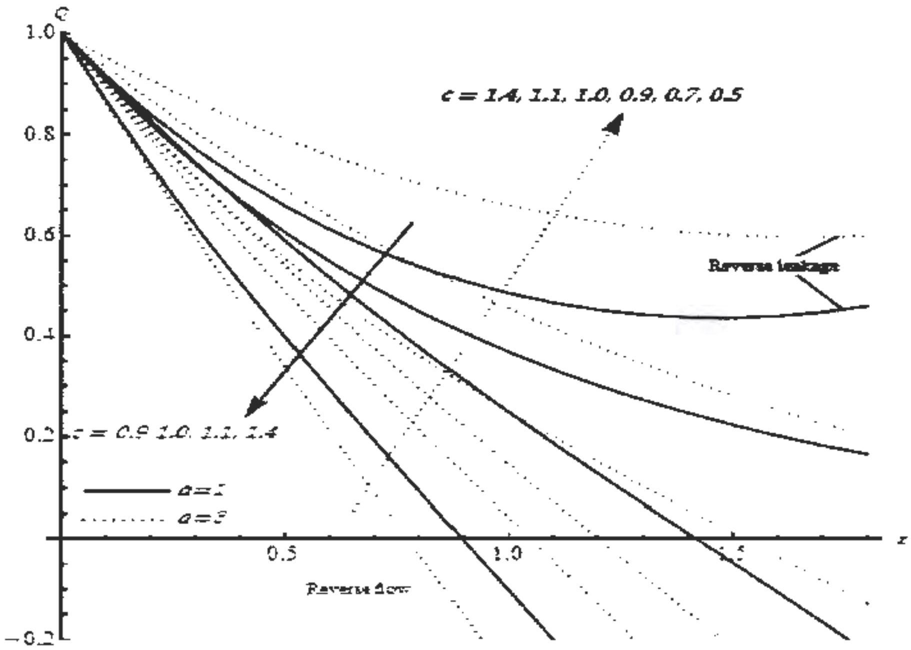

Fig 3.1. Response of flow rate Q for different c, when $\beta = 2$.

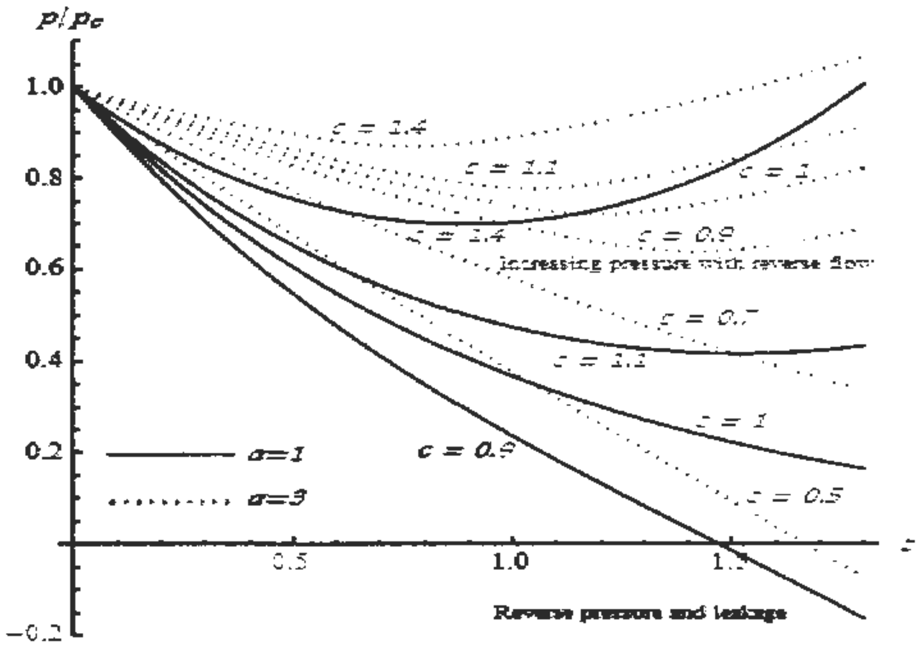

Fig 3.2. Response of pressure p/p_0 for different c, when $\beta = 2$.

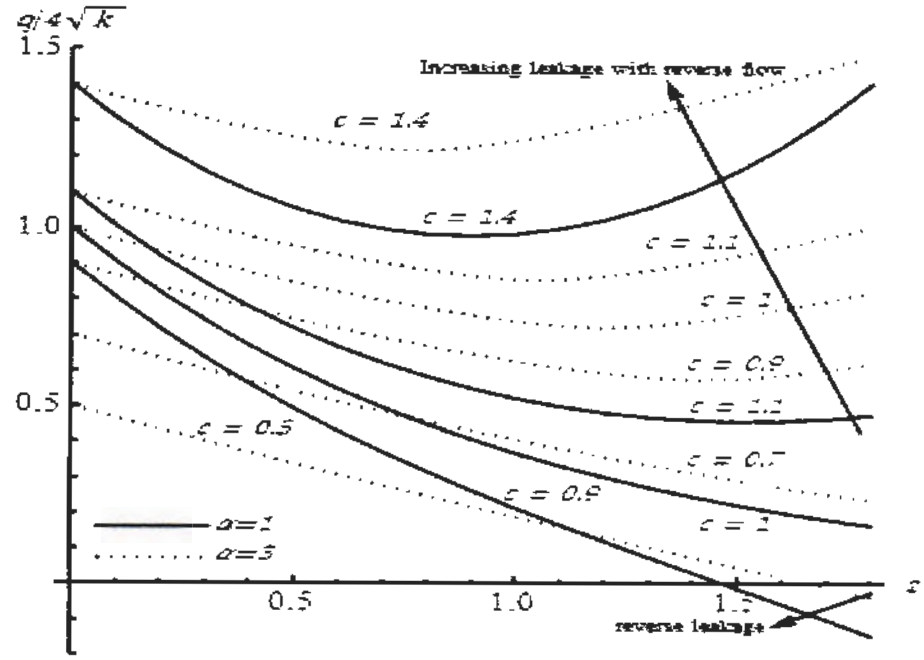

Fig 3.3. Response of leakage flux $q/4\sqrt{K}$ for different c, when $\beta = 2$.

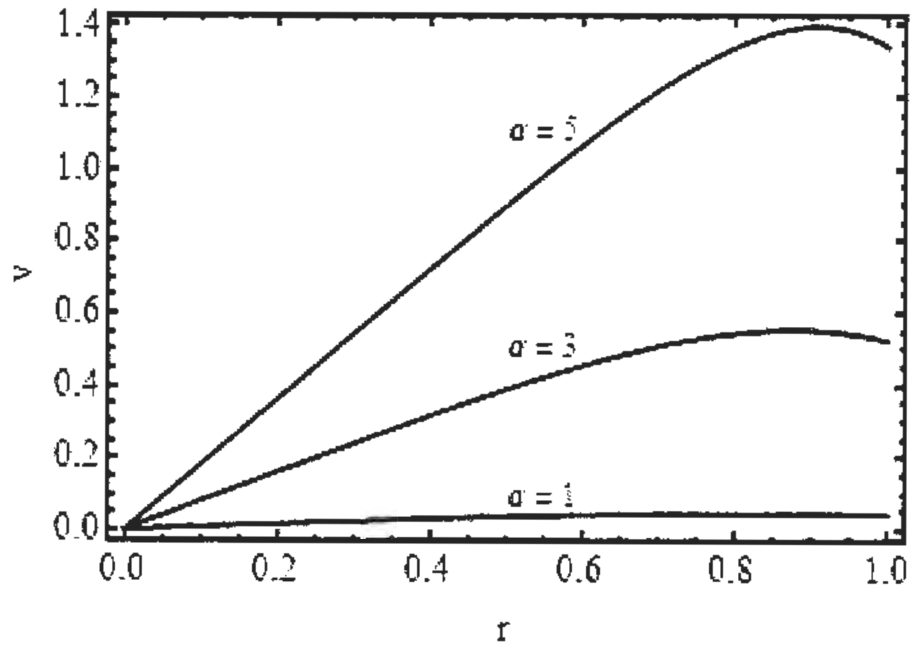

Fig 3.4. Response of axial velocity v for different α, when $p_0 = 43, \beta = 2$, $K = 0.001$.

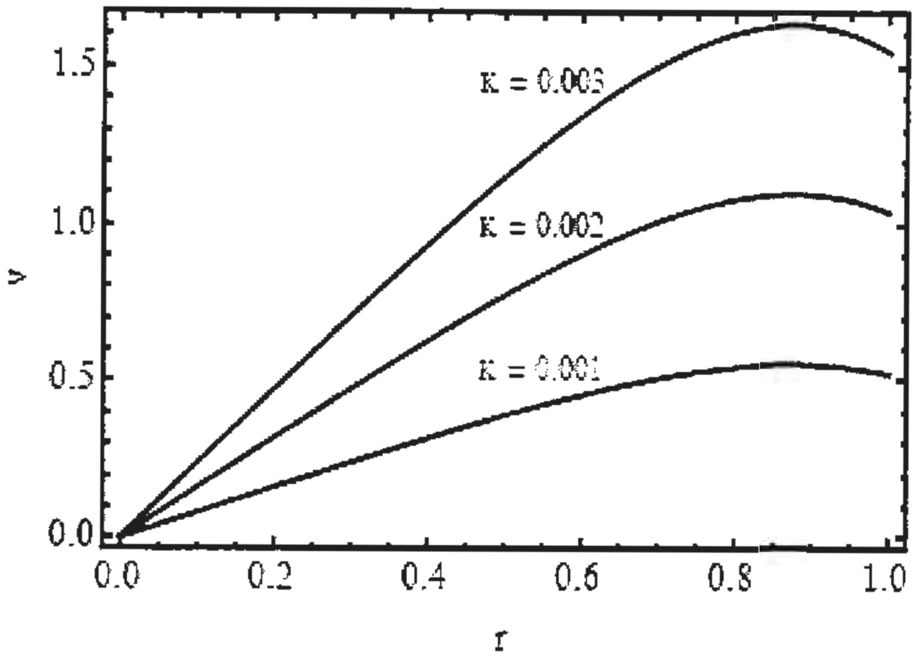

Fig 3.5. Response of axial velocity v for different K, when $p_0 = 43, \beta = 2$, $\alpha = 3$.

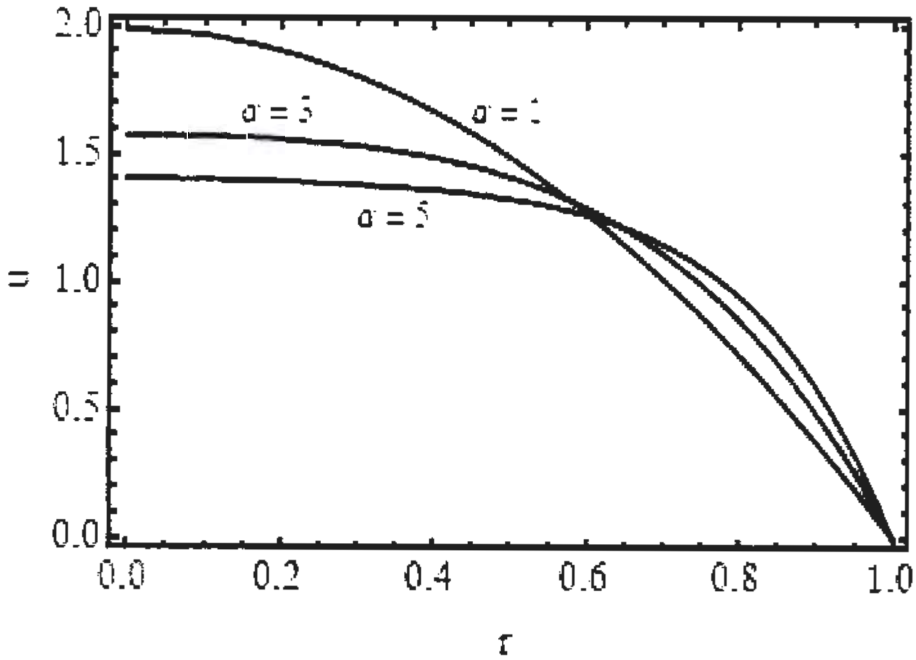

Fig 3.6. Response of radial velocity u for different α, when $p_0 = 43$, $z = 0.1$, $\beta = 2$, $K = 0.001$.

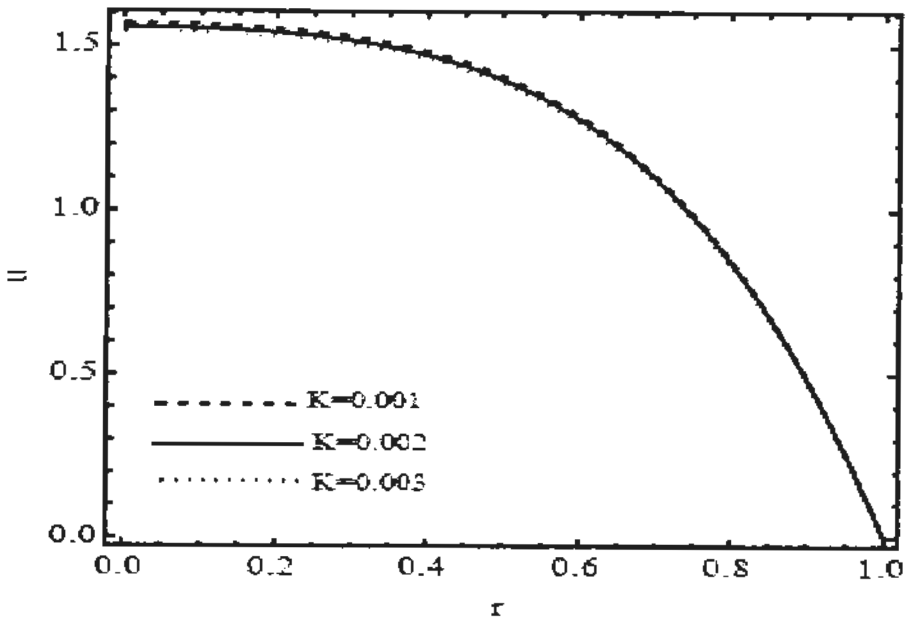

Fig 3.7. Response of radial velocity u for different K, when $p_0 = 43$, $z = 0.1$, $\beta = 2$, $\alpha = 3$.

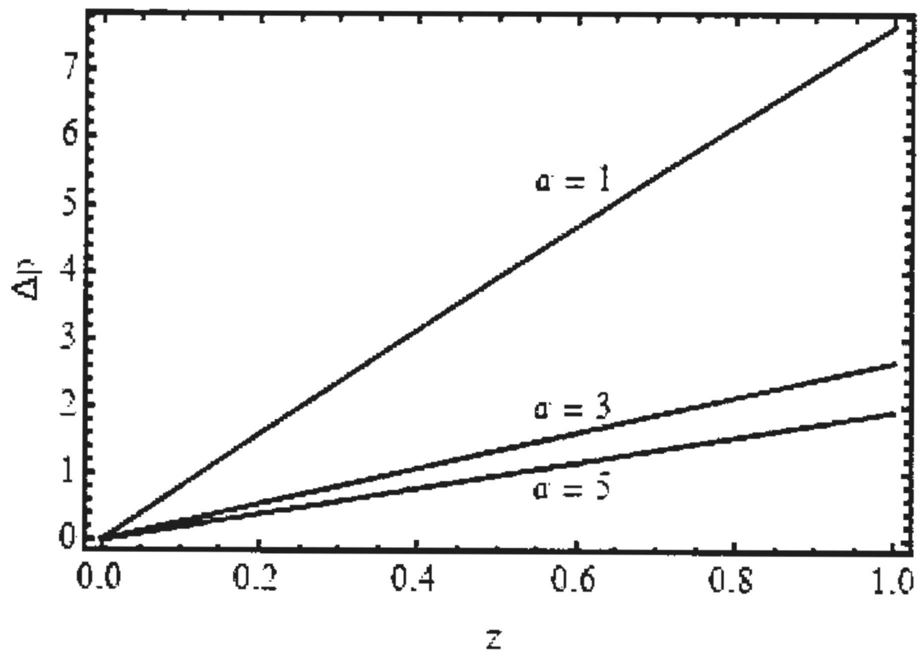

Fig 3.8. Response of mean pressure Δp for different α, when $p_0 = 43$, $\beta = 2$, $K = 0.001$

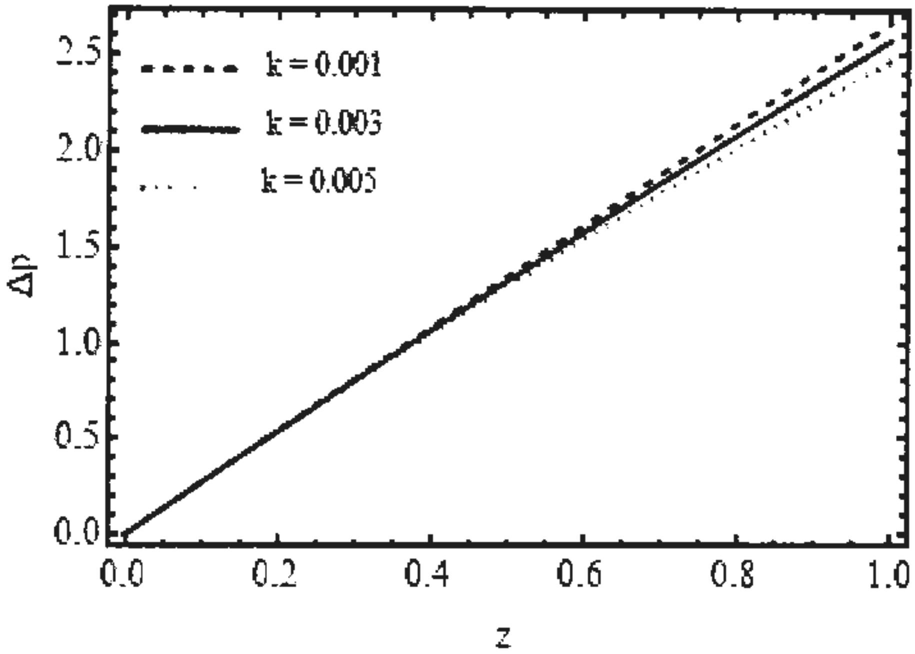

Fig 3.9. Response of mean pressure Δp for different K, $p_0 = 43$, $\beta = 2$, $\alpha = 3$.

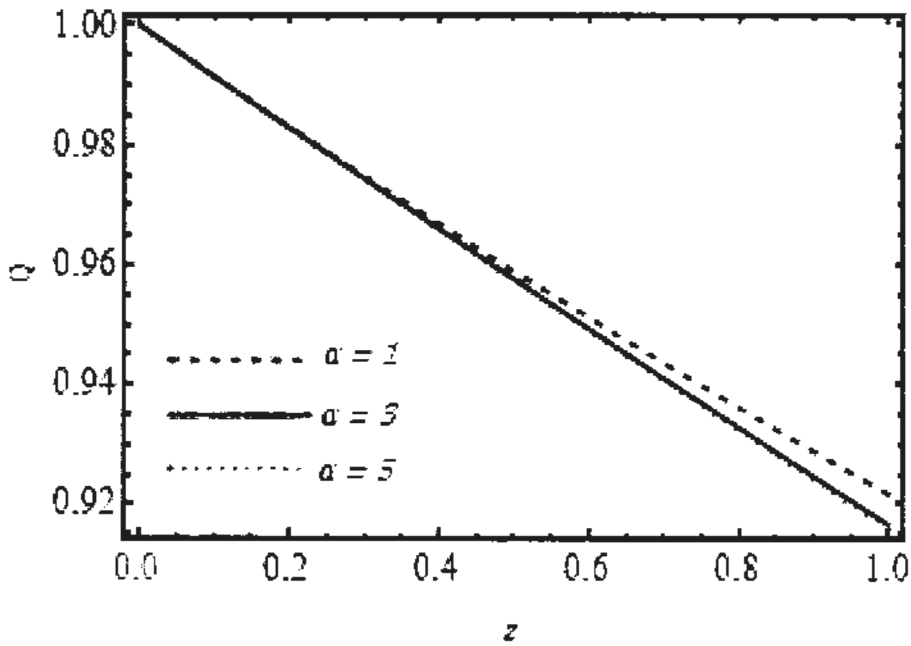

Fig 3.10. Response of flow rate Q for different α, $p_0 = 43$, $\beta = 2$, $K = 0.001$.

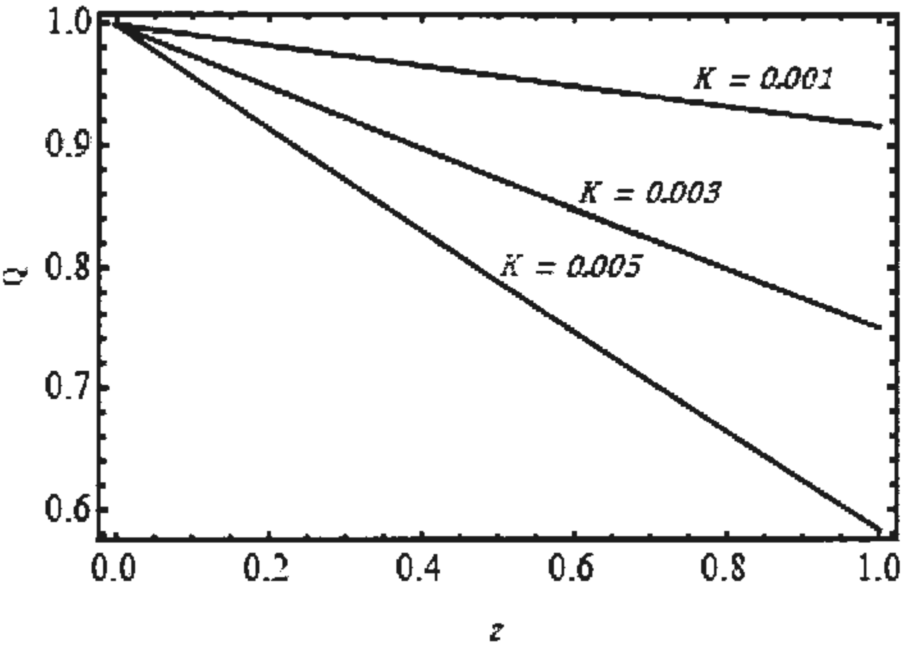

Fig 3.11. Response of flow rate Q for different K, when $p_0 = 43, \beta = 2, \ \alpha = 3$.

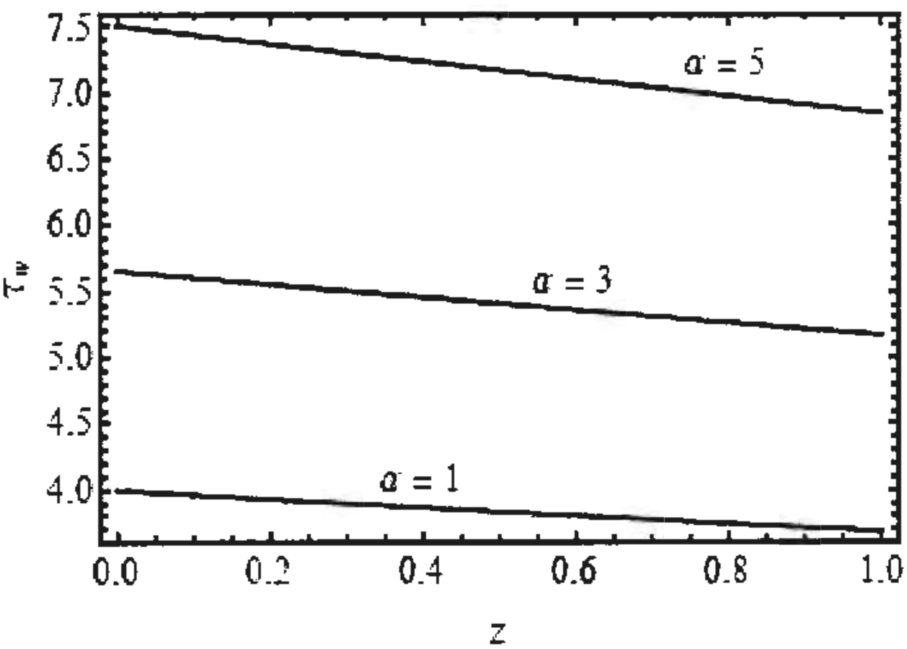

Fig 3.12. Response of wall shear stress τ_w for different α, when $p_0 = 43, \ \beta = 2$, $K = 0.001$.

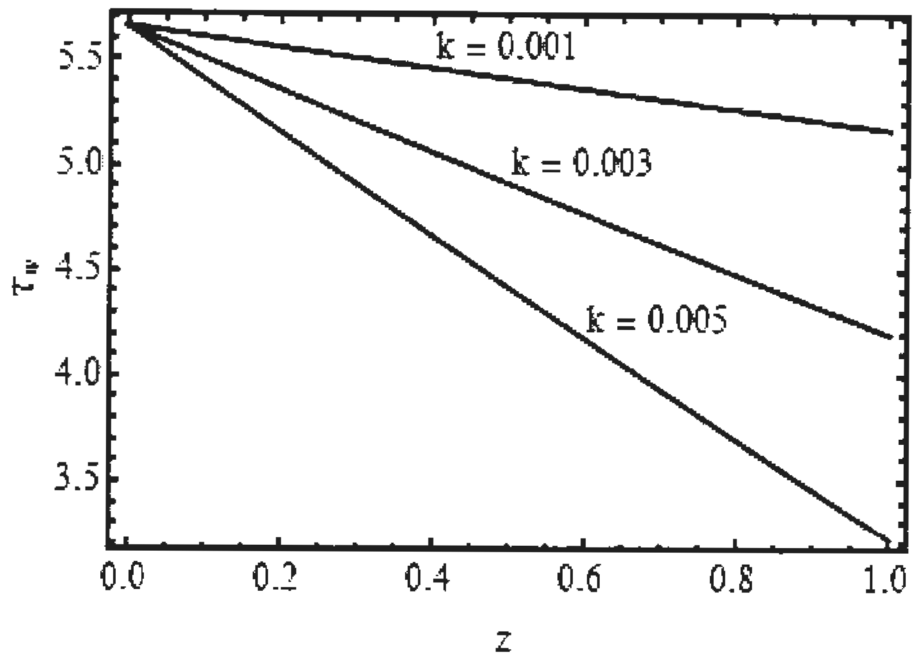

Fig 3.13. Response of wall shear stress τ_w for different K, when $p_0 = 43$, $\beta = 2$, $\alpha = 3$.

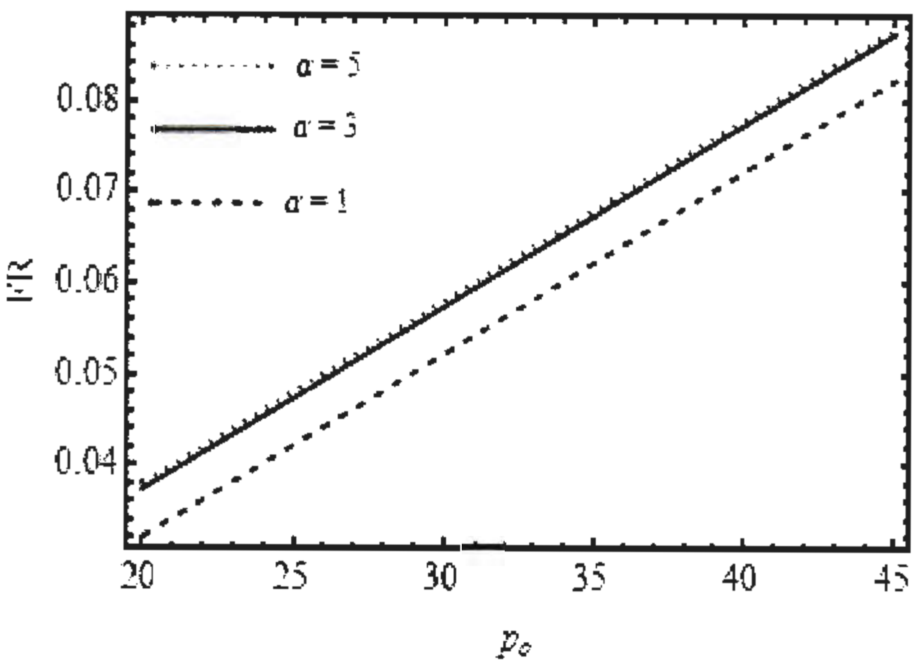

Fig 3.14. Response of fraction reabsorption FR for different α, when $\beta = 2$, $K = 0.001$.

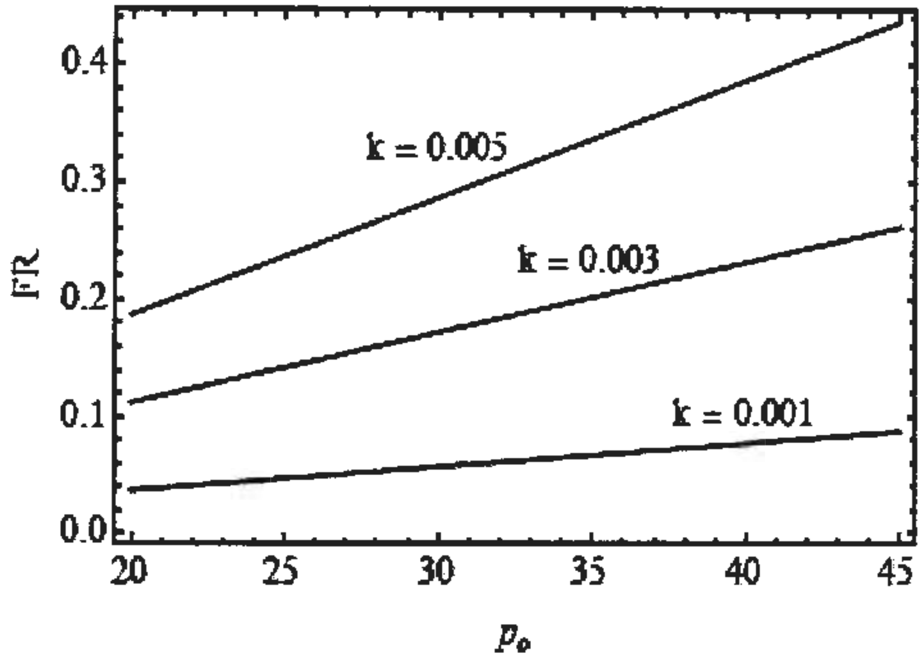

Fig 3.15. Response of fractional reabsorption FR for different K, when $\beta = 2$, $\alpha = 3$.

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