Effects of Uniform Magnetic Fields on Peristaltic Transport of a Particle-Fluid Suspension in a Planar Channel

By

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- Uniform magnetic field
- Peristaltic transport
- Particle fluid Suspension

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A Dissertation
Submitted in the Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE
In
MATHEMATICS

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بسم الله الرحمن الرحيم

Certificate

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A DISSERTATION SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF THE ***MASTER OF SCIENCE in MATHEMATICS***

We accept this dissertation as conforming to the required standard.

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Declaration

The work described in this thesis is carried out under the supervision and guidance of Dr. Ahmed Zeeshan, Department of Mathematics and Statistics, International Islamic University, Islamabad. No portion of the work referred in this thesis is submitted in support of another degree or qualification either in this university or any other institute. This thesis is my original work except where due references are given.

Signature: ________________

Kinza Gilani
MS (Mathematics)

Dedicated to my

loving

Mother.

Acknowledgements

First of all, I pay my special thanks to everlasting Allah, the creator of mankind, who granted us health, knowledge and intelligence to extract the hidden realities in the universe through scientific and critical approach. I offer salutations upon ***Prophet Hazrat Muhammad (Peace be upon him)*** who has lightened the life of mankind with His guidance.

I express my profound gratitude to my supervisor ***Dr. Ahmed Zeeshan*** for his regardless and inspirational efforts and moral support throughout my research carrier. His sound advices and lots of good ideas were very helpful to me. I would have been lost without him. May ***ALLAH*** bless him with all kinds of happiness and success in his life and may all his wishes come true. I also pay my regards to my all teachers who always directed me to right dimensions and made it possible for me to achieve an attractive goal.

My deepest gratitude to my Mother, my phupo, my brother and my husband who contributed and supported alot my prosperous life and to arrive in this stage. They always encouraged me and showed their everlasting love, care and support throughout my life. . I extend my gratitude to my little son Muhammad Haider Ali. The continuous encouragement and humble prayers support (both financially and moral) from my mother is unforgettable.

I am also thankful to my MS friends for their emotional support, entertainment, love and care they provided.

Finally, I would like to thanks to everybody who was important in successful realization of thesis as well as expressing my apology that I could not mention personally all of them.

Kinza Gilani

Preface

Peristaltic pumping has been the object of scientific and engineering research during past few decades. It is wave like involuntary contraction and expansion of elastic wall. It is an important biological mechanism, which is later implemented. In many engineering applications *Latham* [1] was perhaps the first to study the phenomenon theoretically and experimentally.

Jaffrin and *Shapiro* [2] give a detail review of earlier literature. A large amount of literature is available now ever since [3-10]. Particulate suspension in Fluid Dynamics is now of interest of scientist since Pre-historic times. The suspension like solid particles, liquid droplets and gas bubbles etc. are very useful in understanding various engineering applications [11-14]. The particulate nature of blood has become the scientific research [15-19]. The purpose of this thesis is to discuss the effects of MHD of particle-fluid suspension in a planar channel.

This thesis is composed in three chapters. First chapter is constructed to give the brief introduction of Fluid Dynamics and perturbation method which is use to solve mathematical model.

Chapter two is the review work of *Mekheimer* et.al [20]. In this chapter he studies the peristaltic pumping of particle fluid suspension in a planar channel. The mathematical model is developed, the model is then simplified using stream function and transforming from fixed to wave frame. The model is then solved using perturbation method and graphical results are displayed for pressure rise and stream lines.

In chapter three the effect of uniform magnetic field is considered. The particles are non-conducting whereas fluid is electrically conducting. The solution is developed and graphical results are discussed.

Contents

Chapter 1

Preliminaries

In this chapter, some basic definitions and parameters are defined, which are helpful in the subsequent chapters. The phenomenon of peristaltic transport has enjoyed increased interest from investigators in several engineering disciplines. The word peristaltic terms from the Greek word peristaltikos, which means clasping and compressing. It occurs due to the action of progressive waves which propagates along the length of distensible tube containing liquid. Peristalsis is a natural mechanism of fluid transport for many physiological fluids. This is achieved by passage of progressive waves of area contraction and expansion over flexible walls of tube containing fluid. The peristaltic pump is based on alternating compression and relaxation of tube drawing the contents into tube, operating in a similar way to our throat and intestines. Peristalsis offers the opportunity of constructing pumps in which the transported medium does not come in direct contact with any moving parts such as valves, plungers and rotors. This can be of great benefit in cases where the medium is either highly abrasive or decomposable under stress. This has led to the development of finger and roller pumps which work according to the principle of peristalsis.

In fluid mechanics, the study of peristaltic transport start with the assumption that fluid is either Newtonian or non-Newtonian. The equation concerning the law of conservation of mass and momentum with constitutive equation for a Newtonian fluid provide the well known Navier -Stokes equations, which justifies the mathematical treatment of a motion of fluid after deformation by applied stress. In the beginning, analyses of periodic flows incorpoorated by theoretical assumptions such as periodic, sinusoidal wave trains in infinitely long tubes or channels, having long wavelength or low Reynolds number. Several theoretical and experimental attempts have been made to understand peristaltic action in different situations from the first investigation of Latham [1]. A review of much of early literature is presented by Jaffrin and Shapiro [2]. The particulate suspension theory of blood has become the object of scientific research Hill and Bedford [16]. Theoretical study of this fluid system is concerned with powder technology, sedimentation, in medicine, rain erosion in guided missiles and in oceanography.

Most of analytical studies use perturbation series in a small parameter such as amplitude ratio or dimensionless wavenumber, but appears that no rigorous attempt has been made to study the effects of Reynolds number, wave number and concentration of particles on pressure rise, peristaltic pumping, augmented pumping, and backward pumping for a particle-fluid suspension. The purpose of this paper is to study peristaltic pumping of a particle-fluid suspension in planar channel with and without MHD.

A regular perturbation series is used to solve present problem; variables are expanded in a power series of wave number α, which is defined as ratio of half-width of channel to wavelength of peristaltic wave, closed form solutions upto order α^2 are presented. The pressure rise per wavelength is obtained as a function of time-averaged flow rate.

1.1 Fluid

A substance that deforms continuously under the action of applied shear stress is known as fluid i.e liquids and gases.

1.2 Types of fluid

1.2.1 Ideal fluid

A fluid that has no viscosity is called ideal fluid. Such type of fluids do not exist in reality. It is incompressible in nature.

1.2.2 Real Fluid

Real fluids are compressible in nature. They have some viscosity. Examples: Kerosene, Petrol, Castor oil.

1.2.3 Newtonian fluid

In a fluid if the viscous stresses that arises from its flow, at every point, are propotional to the strain rate then the fluid is said to be Newtonian. For a Newtonian fluid, viscosity is entirely dependent upon the temperature and pressure of the fluid. Mathematically, it can be written as,

$$\tau_{yx} = \mu \frac{du}{dy}, \tag{1.1}$$

where τ_{yx} is shear stress acting on the plane normal to y axis and μ is the viscosity of fluid. Water and gasoline are examples of Newtonian fluids under normal conditions. The Newtonian fluid is an idealized fluid that approximates the behavior of water, air and many other fluids.

1.2.4 Non-Newtonian fluid

Fluids that do not obey Newton's law of viscosity are non-Newtonian fluids. Most commonly the viscosity of non-Newtonian fluids is dependent on shear rate. Mathematically, it can be written as,

$$\tau_{yx} = \mu (\frac{du}{dy})^{n_1}, n_1 \neq 1, \tag{1.2}$$

where n1 denote the own behavior index and consistency index respectively. Paints, blood, shampoo etc. are the common examples of non-Newtonian fluids.

1.2.5 Compressible fluid

A compressible fluid is one in which the fluid density changes when it is subjected to high pressure gradients. For gases, changes in density are accompanied by changes in temperature, and this complicates considerably the analysis of compressible flow.

1.2.6 Incompressible fluid

If the density of fluid is constant then it is known as incompressible fluid. Incompressibility is that the divergence of the flow velocity is zero. All liquids are assumed as incompressible fluid.

1.3 Viscosity

The force which resist the motion of a fluid is called viscosity. Mathematically, viscosity is the ratio of shear stress to the shear strain i.e.

$$\text{viscosity} = \mu = \frac{\text{shear stress}}{\text{shear strain}},$$

where, μ is called the coefficient of viscosity.

1.4 Flow

A material goes under deformation when different forces act on it. If deformation increases continuously without any limit then the phenomenon is known as flow.

1.5 Types of flow

1.5.1 Uniform flow

The flow is defined as uniform flow when in the flow field the velocity and other hydrodynamic parameters do not change from point to point at any instant of time.

For a uniform flow, the velocity is a function of time only, which can be expressed in Eulerian description as

$$V = V(t). \tag{1.3}$$

1.5.2 Non-uniform flow

A flow in which fluid particles possess through different velocities at each section of a channel or a pipe is called non-uniform flow.

1.5.3 Laminar flow

A flow in which each particle has a definite path and the path of individual particles do not cross each other is called laminar flow. Laminar flow generally happens when dealing with small pipes and low flow velocities. Laminar flow can be regarded as a series of liquid cylinders in the pipe, where the innermost parts flow the fastest, and the cylinder touching the pipe isn't moving at all. Shear stress depends almost only on the viscosity μ and is independent of density ρ.

1.5.4 Turbulent flow

In turbulent flow vortices, eddies and wakes make the flow unpredictable. Turbulent flow happens in general at high flow rates and with larger pipes. Shear stress for turbulent flow is a function of the density ρ.

1.5.5 Steady flow

A flow in which properties associated with the motion of fluid are independent of time or flow pattern and remains unchanged with the time is called a steady flow. Mathematically it can be written as

$$\frac{\partial v}{\partial t} = 0, \tag{1.4}$$

where, v represents the velocity.

1.5.6 Unsteady flow

All those flows in which properties associated with the motion of fluid depend on time so that flow pattern varies with the time are called unsteady flows. Flow in ocean tides is an example of unsteady flow. Mathematically, it can be expressed as,

$$\frac{\partial v}{\partial t} \neq 0. \tag{1.5}$$

1.6 Pressure

The magnitude of force per unit area is known as pressure. It is a scalar quantity.

1.7 Density

Density of a fluid is defined as mass per unit volume. Mathematically, density ρ at a point P may be defined as

$$\rho = \frac{m}{V}, \tag{1.6}$$

where, V is total volume element around the point and m is the mass of fluid.

1.8 Dimensionless numbers

A dimensionless number is a number without any unit associated with it. It is ratio of the quantities having same unit. There is a lot of dimensionless number but here we mention only those being used in this work.

1.8.1 Reynolds number

Ratio of inertial force to the viscous force is said to be the Reynolds number. It is denoted by the symbol Re. Mathematically,

$$\text{Re} = \frac{\text{inertial forces}}{\text{viscous forces}}. \tag{1.7}$$

1.8.2 Wave number

Wave number is defined as total number of waves to wavelength. Mathematically,

$$\alpha = \frac{2\pi b}{\lambda}. \tag{1.8}$$

1.9 Boundary conditions

The set of conditions specied for the behavior of the solution to a set of differential equations at the boundary of its domain. Boundary conditions are important in determining the mathematical solutions to many physical problems.

1.10 Governing equations for fluid motion

In order to describe physical behaviour of fluid flow, one needs to have some mathematical relations. In fluid mechanics, we have three basic laws which account for motion of fluid and those are recognized as law of conservation of mass, momentum and energy.

1.10.1 Law of conservation of mass

This law states that mass of closed system always remains constant with time, as mass of sytem cannot change quantity except being added or removed. The mathematical relation expressing law of conservation of mass is known as continuity equation. For compressible fluid, it is defined as,

$$\frac{d\rho}{dt} + \rho\nabla.V = 0, \tag{1.9}$$

ρ is fluid density, t is time, V is the flow velocity vector field.

For an incompressible fluid, the density remains stable and therefore, the continuity equation becomes

$$\nabla.V = 0. \tag{1.10}$$

1.10.2 Law of conservation of momentum

This law is defined as the total momentum of an isolated system is always conserved. The equations which describe this law mathematically are called as Navier-Stokes equations. In general, these equations are composed in subsequent form

$$\rho\left(\frac{\partial V}{\partial t} + V.\nabla V\right) = -\nabla P + \mu\nabla^2 V + \rho b, \tag{1.11}$$

where, P is pressure, V is the velocity field, μ is viscosity and b represents the body force.

1.11 Magnetohydrodynamics

The word magnetohydrodynamics (MHD) comes from "magneto" meaning magnetic field, "hydro" means liquid and "dynamics" means movement. The fundamental theme of MHD is that magnetic fields can induce currents in a moving conductive fluid, which in response impose forces on the fluid and also effects the magnetic field itself. The basic equations which describe MHD are a combination of Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism.

In the presence of MHD, the momentum equation will be

$$\rho\left(\frac{\partial V}{\partial t} + V.\nabla V\right) = -\nabla P + \mu\nabla^2 V + J \times B, \tag{1.12}$$

where the term $J \times B$ is the Lorentz force and can be written as

$$J \times B = \frac{(B.\nabla)\,B}{\mu_0} - \nabla\left(\frac{B_0^2}{2\mu_0}\right). \tag{1.13}$$

Maxwell's equations can be described by following expressions.

Solenoidal nature of magnetic field B

$$\nabla.B = 0. \tag{1.14}$$

Faradays law

$$\nabla \times E = \frac{-\partial B}{\partial t}. \tag{1.15}$$

Ampere equation

$$\nabla \times B = \mu_0 J. \tag{1.16}$$

Charge conservation

$$\nabla.J = 0. \tag{1.17}$$

Lorentz force

$$F = J \times B. \tag{1.18}$$

Ohm's law

$$J = \sigma (E + V \times B) \tag{1.19}$$

In these equations, B is total magnetic field, B_0 is magnetic field strength, E is electric field, μ_0 is permeability of free space, J is current density and σ is conductivity.

1.12 Method of solution

Most of the problem encountered in fluid mechanics are highly nonlinear. To find the exact solution of these nonlinear problems is very difficult and sometimes impossible. Therefore various methods have been developed to solve nonlinear differential equations. Among these perturbation is the widely used analytical technique. We have used this technique in the subsequent chapter to obtain the solution of the problem.

1.12.1 Perturbation solution

The mathematical methods used to find out the approximate solution to a problem by starting from the exact solution of a related problem are studied in perturbation theory. In this method, the solution is given by few terms of an expansion. These expansions may be carried out in term of small or large parameter which appear in the equations.

1.13 Problem definition

Peristaltic flow problems are unsteady moving boundary value problems. Mathematical modeling of peristaltic transport deals with a prescribed train of waves moving with constant speed on the flexible boundaries. The fluid motion is studied in either a fixed frame of reference, (X, Y), or a wave frame of reference, (x, y), moving with constant velocity of the wave. The longitudinal direction is parallel to the direction of the wave progression. Here the two dimensional flow of a mixture of small, spherical, rigid particles in an incompressible Newtonian viscous fluid in an infinite channel of width 2b is considered. We choose a rectangular coordinate system for the channel with X along the centerline in the direction of wave propogation and Y transverse to it. There exist two geometrical ratios. The first, $\phi = b/a$, is the amplitude ratio, which is the amplitude of the wave divided by the total height or radius. The second ratio is wave number, which is the ratio of the total radius or height divided by the wavelength and multiplied by π, so it represents the number of repeating units of a propagating wave.

The only restriction on the waveshape, H, of the wall is that it be a function of the quantity X -ct for two-dimensional plane or Z -ct for axisymmetric. This form of the waveshape allows for an easy, direct change frame of reference from the fixed frame (laboratory frame) to the moving frame (wave frame), in which the observer moves with the wave at the wavespeed c.

Chapter 2

Peristaltic Transport of a Particle-Fluid Suspension in a Planar Channel

This chapter is a detailed review of a research article peristaltic motion of a particle-fluid suspension in a planar by Makheimer et al. [20]. This chapter focuses on peristaltic transport of a particle-fluid suspension in a planar channel. The analysis has been carried out under the assumption of long wavelength and low Reynolds number. The analytic solutions are obtained for velocity, pressure gradient and stream function using perturbation method. The mathematical modeling is discussed in detail. The graph of parameter of interest are drawn and analyzed.

2.1 Mathematical formulation

Consider the two dimensional particle fluid suspension in a planar channel of width 2b. The flow is generated by wave motion of channel walls. A rectangular coordinate system (X, Y) is taken, where X-axis is along the center line of channel and Y-axis is perpendicular. The geometry of the wall surface is defined as,

$$\bar{h} = b + a \sin\left[\frac{2\pi\left(\overline{\mathbf{X}} - ct\right)}{\lambda}\right], \tag{2.1}$$

and the b.c's are

$$\begin{aligned} \frac{\partial \overline{U}_f}{\partial \overline{Y}} &= \frac{\partial \overline{U}p}{\partial \overline{Y}} = 0, \ \ \overline{V}_f = \overline{V}_p = 0 \ \text{ at } \bar{Y} = 0, \\ \overline{U}_f &= 0, \qquad \text{at } \bar{Y} = \bar{h}, \end{aligned} \tag{2.2}$$

where a is the wave amplitude, c is the velocity of the wave, λ is the wavelength, b is the half width. The drag coefficient is as

$$S = \frac{9\mu_0}{2a'^2}\lambda'(C) \tag{2.3}$$

and

$$\lambda'(C) = \frac{4 + 3[8C - 3C]^{1/2} + 3C}{[2 - 3C]^2}, \tag{2.4}$$

where μ_0 is fluid viscosity and a' is radius of particles.

The emperical relation for viscosity of suspension suggested by Charm and Kurland [2] :

$$\mu_s(C) = \mu_0 \frac{1}{1 - qC}, \tag{2.5}$$

$$q = 0.07 \exp[2.49C + \frac{1107}{T}\exp(-1.69C)], \tag{2.6}$$

where, T is absolute temperature (K). The velocity profile are defined as

$$\left\{W_f = [\overline{U}(x,y,t), \overline{V}(x,y,t), 0], \ W_p = [\overline{U}(x,y,t), \overline{V}(x,y,t), 0]\right\}. \tag{2.7}$$

The governing equations are,

$$\frac{\partial \overline{U}_f}{\partial \overline{X}} + \frac{\partial \overline{V}_f}{\partial \overline{Y}} = 0, \tag{2.8}$$

$$(1-C)\rho_f\left[\frac{\partial\overline{U}_f}{\partial t}+\overline{U}_f\frac{\partial\overline{U}_f}{\partial\overline{X}}+\overline{V}_f\frac{\partial\overline{U}_f}{\partial\overline{Y}}\right]=-(1-C)\frac{\partial\overline{P}}{\partial\overline{X}}+(1-C)\mu_S\left[\frac{\partial\overline{U}_f^2}{\partial\overline{X}^2}+\frac{\partial\overline{U}_f^2}{\partial\overline{Y}^2}\right] \quad (2.9)$$
$$+CS\left(\overline{U}_p-\overline{U}_f\right),$$
$$(1-C)\rho_f\left[\frac{\partial\overline{V}_f}{\partial t}+\overline{V}_f\frac{\partial\overline{V}_f}{\partial\overline{X}}+\overline{V}_f\frac{\partial\overline{V}_f}{\partial\overline{Y}}\right]=-(1-C)\frac{\partial\overline{P}}{\partial\overline{Y}}+(1-C)\mu_S\left[\frac{\partial\overline{V}_f^2}{\partial\overline{X}^2}+\frac{\partial\overline{V}_f^2}{\partial\overline{Y}^2}\right]$$
$$+CS\left(\overline{V}_p-\overline{V}_f\right),$$

$$\frac{\partial\overline{U}_p}{\partial\overline{X}}+\frac{\partial\overline{V}_p}{\partial\overline{Y}}=0, \quad (2.10)$$

$$C\rho_p\left[\frac{\partial\overline{U}_p}{\partial t}+\overline{U}_p\frac{\partial\overline{U}_p}{\partial\overline{X}}+\overline{V}p\frac{\partial\overline{U}_p}{\partial\overline{Y}}\right] = -C\frac{\partial\overline{P}}{\partial\overline{X}}+CS\left(\overline{U}_f-\overline{U}_p\right), \quad (2.11)$$
$$C\rho_p\left[\frac{\partial\overline{V}_p}{\partial t}+\overline{U}_p\frac{\partial\overline{V}_p}{\partial\overline{X}}+\overline{V}p\frac{\partial\overline{V}_p}{\partial\overline{Y}}\right] = -C\frac{\partial\overline{P}}{\partial\overline{X}}+CS\left(\overline{V}_f-\overline{V}_p\right).$$

Introducing the transformation for conversion from fix to wave frame,

$$\overline{x}=\overline{X}-ct, \overline{y}=\overline{Y},\ \overline{u}_f=\overline{U}_f-c,\ \overline{v}_f=\overline{V_f}, \overline{u}_p=\overline{U}p-c,\ \overline{v}_p=\overline{V_p}, \quad (2.12)$$

where $\overline{U}, \overline{V}, \overline{P}$ are the velocity components and pressure in the laboratory frame and $\overline{u}, \overline{v}, \overline{p}$ are the velocity components and pressure in the wave frame respectively. Using the eq. (2.12) into the eqs. (2.8) – (2.11).

For fluid phase

$$\frac{\partial\overline{u}_f}{\partial\overline{x}}+\frac{\partial\overline{v}_f}{\partial\overline{y}}=0, \quad (2.13)$$

$$(1-C)\rho_f\left[\overline{u}_f\frac{\partial\overline{u}_f}{\partial\overline{x}}+\overline{v}_f\frac{\partial\overline{u}_f}{\partial\overline{y}}\right]=-(1-C)\frac{\partial\overline{p}}{\partial\overline{x}}+(1-C)\mu_S\left[\frac{\partial\overline{u}_f^2}{\partial\overline{x}^2}+\frac{\partial\overline{u}_f^2}{\partial\overline{y}^2}\right]$$
$$+CS\left(\overline{u}_p-\overline{u}_f\right), \quad (2.14)$$
$$(1-C)\rho_f\left[\overline{u}_f\frac{\partial\overline{v}_f}{\partial\overline{x}}+\overline{v}_f\frac{\partial\overline{v}_f}{\partial\overline{y}}\right]=-(1-C)\frac{\partial\overline{p}}{\partial\overline{y}}+(1-C)\mu_S\left[\frac{\partial\overline{v}_f^2}{\partial\overline{x}^2}+\frac{\partial\overline{v}_f^2}{\partial\overline{y}^2}\right]$$
$$+CS\left(\overline{v}_p-\overline{v}_f\right).$$

For particulate phase

$$\frac{\partial\overline{u}_p}{\partial\overline{x}}+\frac{\partial\overline{v}_p}{\partial\overline{y}}=0, \quad (2.15)$$

$$C\rho_p\left[\overline{u}_p\frac{\partial\overline{u}_p}{\partial\overline{x}}+\overline{v}p\frac{\partial\overline{u}_p}{\partial\overline{y}}\right]=-C\frac{\partial\overline{p}}{\partial\overline{x}}+CS\left(\overline{u}_f-\overline{u}_p\right), \quad (2.16)$$
$$C\rho_p\left[\frac{\partial\overline{v}_p}{\partial\overline{x}}+\overline{v}p\frac{\partial\overline{v}p}{\partial\overline{y}}\right]=-C\frac{\partial\overline{p}}{\partial\overline{y}}+CS\left(\overline{v}_f-\overline{v}_p\right).$$

Now, Introducing the nondimensional quantities as follows,

$$\left.\begin{array}{c} x=\frac{2\pi\overline{x}}{\lambda}, y=\frac{\overline{y}}{b}, u_f=\frac{\overline{u_f}}{c}, u_p=\frac{\overline{u}_p}{c} \\ v_f=\frac{\overline{v}_f}{c}, v_p=\frac{\overline{v}_p}{c}, h=\frac{\overline{h}(\overline{x})}{b}, P=\frac{2\pi b^2}{\lambda c\mu_s}\overline{p(x)} \end{array}\right\}. \tag{2.17}$$

Defining the stream function ψ,

$$u_f=\frac{\partial\psi_f}{\partial y}, v_f=-\alpha\frac{\partial\psi_f}{\partial x}, u_p=\frac{\partial\psi_p}{\partial y}, v_p=-\alpha\frac{\partial\psi_p}{\partial x}. \tag{2.18}$$

Using eqs.(2.17) and (2.18) into eqs. (2.13) – (2.16), we find that eqs. (2.13) and (2.15) are satisfied identically and eqs. (2.14) and (2.16) yields,

$$\left.\begin{array}{c} (1-C)\operatorname{Re}\alpha\left[\alpha^2\frac{\partial\psi_f}{\partial y}\frac{\partial^3\psi_f}{\partial x^3}+\frac{\partial\psi_f}{\partial y}\frac{\partial^2\psi_f}{\partial x\partial y^2}+\left(\alpha^2\frac{\partial\psi_f}{\partial x}\frac{\partial^3\psi_f}{\partial x^2\partial y}+\frac{\partial\psi_f}{\partial x}\frac{\partial^3\psi_f}{\partial y^3}\right)\right]=\left[\alpha^4\frac{\partial^4\psi_f}{\partial x^4}+\frac{\partial^4\psi_f}{\partial y^4}\right] \\ +\alpha^2\frac{\partial^4\psi_f}{\partial x^2\partial y^2}+\frac{\partial^4\psi_f}{\partial x^2\partial y^2}+CM\left(\alpha^2\frac{\partial^2\psi_p}{\partial x^2}+\frac{\partial^2\psi_p}{\partial y^2}-\alpha^2\frac{\partial^2\psi_f}{\partial x^2}+\frac{\partial^2\psi_f}{\partial y^2}\right) \end{array}\right\}, \tag{2.19}$$

$$\left.\begin{array}{c} C\alpha\operatorname{Re}\left[-\left(\alpha^2\frac{\partial\psi_p}{\partial y}\frac{\partial^3\psi_p}{\partial x^3}+\frac{\partial\psi_p}{\partial y}\frac{\partial^3\psi_p}{\partial x\partial y}\right)+\left(\alpha^2\frac{\partial\psi_p}{\partial x}\frac{\partial^3\psi_p}{\partial x^2\partial y}+\frac{\partial\psi_p}{\partial x}\frac{\partial^3\psi_p}{\partial y^3}\right)\right] \\ =CN\left(-\left(\alpha^2\frac{\partial^2\psi_f}{\partial x^2}+\frac{\partial^2\psi_f}{\partial y^2}\right)+\alpha^2\frac{\partial^2\psi_p}{\partial x^2}+\frac{\partial^2\psi_p}{\partial y^2}\right) \end{array}\right\}. \tag{2.20}$$

Simplifying,

$$\begin{aligned} (1-C)\operatorname{Re}\alpha\left[\Psi_{fy}\nabla^2\,\Psi_{fx}-\Psi_{fx}\nabla^2\,\Psi_{fy}\right] &= \nabla^2\,\nabla^2\,\Psi_f+CM\left(\nabla^2\,\Psi p-\nabla^2\,\Psi_f\right), \quad (2.21) \\ C\alpha\operatorname{Re}\left(\Psi_{py}\nabla^2\,\Psi_{px}-\Psi_{px}\nabla^2\,\Psi_{py}\right) &= CN\left(\nabla^2\,\Psi_f-\nabla^2\,\Psi_p\right), \end{aligned}$$

where

$$\alpha=\frac{2\pi b}{\lambda}, M=\frac{\mu_S(C)\,Sb^2}{(1-C)\,\mu_s}, \mathbf{N}=\frac{Sb^2\rho_f}{(1-C)\,\mu_s}, Re=\frac{cb\rho_f}{(1-C)\,\mu_S}, \nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}.$$

2.2 Rate of volume flow and boundry conditions

The instantaneous volume flow rate in fixed frame is given by

$$Q_f = (1 - C) \int_0^{\hbar} \bar{U}_f\left(\bar{X}, \bar{Y}, t\right) d\bar{Y}, \tag{2.22}$$

$$Q_p = C \int_0^{\hbar} \bar{U}_p\left(\bar{X}, \bar{Y}, t\right) d\bar{Y}, \tag{2.23}$$

$$Q_m = Q_f + Q_p, \tag{2.24}$$

where Q_f, Q_p, and Q_m are volume flow rate for fluid phase, particulate phase, and the mixture. $\hbar$ is a function of $\bar{X}$ and t.

The instantaneous volume flow rate in wave frame is given by

$$q_f = (1 - C) \int_0^{\hbar} \bar{u}_f\left(\bar{x}, \bar{y}\right) d\bar{y}, \tag{2.25}$$

$$q_p = C \int_0^{\hbar} \bar{u}_p\left(\bar{x}, \bar{y}\right) d\bar{y}, \tag{2.26}$$

$$q_m = q_f + q_p, \tag{2.27}$$

where $\hbar$ is a function of $\bar{x}$.

We are interested only with volume flow rate of fluid in this study. By using (2.12) into (2.22) and making use of (2.25), we find that

$$Q_f = q_f + (1 - C)\, c\hbar. \tag{2.28}$$

The time-mean flow over a period T at a fixed position $\bar{X}$ is defined as

$$\bar{Q}_f = \frac{1}{T} \int_0^{T} Q_f dt. \tag{2.29}$$

Substituting (2.27) into (2.28), and integrating, we get

$$\bar{Q}_f = q_f + (1 - C)\, ac. \tag{2.30}$$

On defining the dimensionless time-mean flows θ and F in the fixed and wave frame as

$$\theta = F + 1, \tag{2.31}$$

where

$$F = \int \frac{\partial \Psi_f}{\partial y} dy = \Psi_f(h) - \Psi_f(0). \tag{2.32}$$

If we choose the zero value of streamline at $(y = 0)$, then

$$\Psi_f(h) = F. \tag{2.33}$$

The b.c's for dimensionless stream function in the wave frame are

$$\begin{aligned} \psi_f &= F, \frac{\partial \psi_f}{\partial y} = -1 \quad \text{at} \quad y = h, \\ \psi_f &= \psi_p = 0, \frac{\partial^2 \psi_f}{\partial y^2} = \frac{\partial^2 \psi_p}{\partial y^2} = 0 \text{ at } y = 0. \end{aligned} \tag{2.34}$$

2.3 Solution of problem

To obtain the solution, we expand the flow quantities in a power series of small parameter α as follows,

$$\left\{ \begin{array}{c} \psi_f = \psi_{f0} + \alpha\psi_{f1} + O(\alpha^2), \psi_p = \psi_{p0} + \alpha\psi_{p1} + O(\alpha^2), F = F_0 + \alpha F_1 + O(\alpha^2) \\ \frac{\partial p}{\partial x} = \frac{\partial p_0}{\partial x} + \alpha \frac{\partial p_1}{\partial x} + O(\alpha^2) \end{array} \right\}. \tag{2.35}$$

After substituting eq. (2.35) into eqs. (2.19) – (2.20) and equating the coefficients of like powers of α, we obtain a system of equations of different orders.

2.3.1 Zeroth order problem

$$\frac{\partial^4 \psi_{f0}}{\partial y^4} + CM\left(\frac{\partial^2 \psi_{p0}}{\partial y^2} - \frac{\partial^2 \psi_{f0}}{\partial y^2}\right) = 0, \tag{2.36}$$

$$CN\left(\frac{\partial^2\psi_{f0}}{\partial y^2}-\frac{\partial^2\psi_{p0}}{\partial y^2}\right)=0, \tag{2.37}$$

along with the boundary conditions

$$\begin{aligned}\psi_{f0} &= \psi_{p0}=0, \psi_{f0yy}=\psi_{p0yy}=0 \text{ at y=0,} \\ \psi_{f0} &= F_0, \psi_{f0y}=-1 \text{ at } y=h.\end{aligned} \tag{2.38}$$

2.3.2 First order problem

$$(1-C).\operatorname{Re}\left[\frac{\partial\psi_{f0}}{\partial y}\frac{\partial^3\psi_{f0}}{\partial x\partial y^2}-\frac{\partial\psi_{f0}}{\partial x}\frac{\partial^3\psi_{f0}}{\partial y^3}\right]=\frac{\partial^4\psi_{f1}}{\partial y^4}+CM\left(\frac{\partial^2\psi_{p1}}{\partial y^2}-\frac{\partial^2\psi_{f1}}{\partial y^2}\right), \tag{2.39}$$

$$C.\operatorname{Re}\left[\frac{\partial\psi_{p0}}{\partial y}\frac{\partial^3\psi_{p0}}{\partial x\partial y^2}-\frac{\partial\psi_{p0}}{\partial x}\frac{\partial\psi_{p0}}{\partial y^3}\right]=CN\left(\frac{\partial^2\psi_{f1}}{\partial y^2}-\frac{\partial^2\psi_{p1}}{\partial y^2}\right), \tag{2.40}$$

the corresponding boundary conditions are

$$\begin{aligned}\psi_{f1} &= \psi_{p1}=0, \psi_{f1yy}=\psi_{p1yy}=0 \text{ at y=0,} \\ \psi_{f1} &= F_1, \psi_{f1y}=0 \text{ at } y=h.\end{aligned} \tag{2.41}$$

2.3.3 Second order problem

$$\begin{aligned}(1-C).\operatorname{Re}\left[\frac{\partial\psi_{f1}}{\partial y}\frac{\partial^3\psi_{f0}}{\partial x\partial y^2}+\frac{\partial\psi_{f0}}{\partial y}\frac{\partial^3\psi_{f1}}{\partial x\partial y^2}-\frac{\partial\psi_{f1}}{\partial x}\frac{\partial^3\psi_{f0}}{\partial y^3}-\frac{\partial\psi_{f0}}{\partial x}\frac{\partial^3\psi_{f1}}{\partial y^3}\right]=\frac{\partial^4\psi_{f1}}{\partial y^4} \\ +CM\left(\frac{\partial^2\psi_{p1}}{\partial y^2}-\frac{\partial^2\psi_{f1}}{\partial y^2}\right),\end{aligned} \tag{2.42}$$

$$\begin{aligned}C.\operatorname{Re}\left[\frac{\partial\psi_{p1}}{\partial y}\frac{\partial^3\psi_{p0}}{\partial x\partial y^2}+\frac{\partial\psi_{p0}}{\partial y}\frac{\partial^3\psi_{p1}}{\partial x\partial y^2}-\frac{\partial\psi_{p1}}{\partial x}\frac{\partial^3\psi_{p0}}{\partial y^3}-\frac{\partial\psi_{p0}}{\partial x}\frac{\partial^3\psi_{p1}}{\partial y^3}\right] \\ =CN\left(\frac{\partial^2\psi_{f1}}{\partial y^2}-\frac{\partial^2\psi_{p1}}{\partial y^2}\right),\end{aligned} \tag{2.43}$$

along boundry conditions

$$\begin{aligned}\psi_{f2} &= \psi_{p2}=0, \psi_{f2yy}=\psi_{p2yy}=0 \text{ at y=0,} \\ \psi_{f1} &= F_2, \psi_{f2y}=0 \text{ at } y=h.\end{aligned} \tag{2.44}$$

2.3.4 Zeroth order solution

The solution in terms of stream function, is given by

$$\psi_{f0} = \frac{-F0y^3 - y^3h + 3F0yh^2 + yh^3}{2h^3}, \tag{2.45}$$

and

$$\psi_{p0} = \frac{-6F0y - 6yh}{2CMh^3} + \frac{-F0y^3 - y^3h + 3F0yh^2 + yh^3}{2h^3}. \tag{2.46}$$

The axial velocity is

$$\begin{aligned} u_{f0} &= \frac{-3F0y^2 - 3y^2h + 3F0h^2 + h^3}{2h^3}, \\ u_{p0} &= \frac{-6F0 - 6h}{2CMh^3} + \frac{-3F0y^2 - 3y^2h + 3F0h^2 + h^3}{2h^3}. \end{aligned} \tag{2.47}$$

2.3.5 First order solution

$$\psi_{f1} = y^7A1 + y^5A2 + y^3A3 + yA4, \tag{2.48}$$

$$\psi_{p1} = y^7B1 + y^5B2 + y^3B3, \tag{2.49}$$

and the velocities are

$$u_{f1} = 7y^6A1 + 5y^4A2 + 3y^2A3 + A4, \tag{2.50}$$

$$u_{p1} = 7y^6B1 + 5y^4B2 + 3y^2B3, \tag{2.51}$$

where

$$A1 = \frac{1}{280N1h^7}y^7 \begin{pmatrix} -3CF0^2M\,\mathrm{Re}\,h' - 3F0^2N1\,\mathrm{Re}\,h' \\ +3CF0^2\,\mathrm{Re}\,h' - 5CF0M\,\mathrm{Re}\,hh' \\ -5F0N1\,\mathrm{Re}\,hh' + 5CF0N1\,\mathrm{Re}\,hh' \\ -2CM\,\mathrm{Re}\,h^2h' - 2N1\,\mathrm{Re}\,h^2h' \\ +2CN1\,\mathrm{Re}\,h^2h' \end{pmatrix},$$

$$A2 = \frac{1}{280N1h^7}y^5 \begin{pmatrix} 21CF0^2M\,\mathrm{Re}\,h^2h' + 21F0^2N1\,\mathrm{Re}\,h^2h' \\ -21CF0^2N1\,\mathrm{Re}\,h^2h' + 21CF0M\,\mathrm{Re}\,h^3h' \\ +21F0N1M\,\mathrm{Re}\,h^3h' - 21F0^2CM\,\mathrm{Re}\,h^3h' \\ +7CM\,\mathrm{Re}\,h^4h' + 7N1\,\mathrm{Re}\,h^4h' \\ -7CN1\,\mathrm{Re}\,h^4h' \end{pmatrix},$$

$$A3 = \frac{1}{280N1h^7}y^3 \begin{pmatrix} -140F1N1h^4 - 33CF0^2M\,\mathrm{Re}\,h^4h' \\ +33CF0^2N1\,\mathrm{Re}\,h^4h' - 27CF0M\,\mathrm{Re}\,h^5h' \\ -27F0N1\,\mathrm{Re}\,h^5h' + 27CF0N1\,\mathrm{Re}\,h^5h' \\ -8CM\,\mathrm{Re}\,h^5h' - 8N1\,\mathrm{Re}\,h^5h' \\ +8CN1\,\mathrm{Re}\,h^5h' \end{pmatrix},$$

$$A4 = \frac{1}{280N1h^7}y \begin{pmatrix} 420F1N1h^6 + 15CF0^2M\,\mathrm{Re}\,h^6h' \\ -15C1F0^2N1\,\mathrm{Re}\,h^6h' + 11CF0M\,\mathrm{Re}\,h^7h' \\ +11F0N1\,\mathrm{Re}\,h^7h' - 11CF0N1\,\mathrm{Re}\,h^7h' \\ +3CM\,\mathrm{Re}\,h^8h' + 3N1\,\mathrm{Re}\,h^8h' \\ -3CN1\,\mathrm{Re}\,h^8h' \end{pmatrix},$$

$$B1 = \frac{1}{280N1h^7}y^7 \begin{pmatrix} -3CF0^2M\,\mathrm{Re}\,h' - 3F0^2N1\,\mathrm{Re}\,h' \\ +3CF0^2\,\mathrm{Re}\,h' - 5CF0M\,\mathrm{Re}\,hh' \\ -5F0N1\,\mathrm{Re}\,hh' + 5CF0N1\,\mathrm{Re}\,hh' \\ -2CM\,\mathrm{Re}\,h^2h' - 2N1\,\mathrm{Re}\,h^2h' \\ +2CN1\,\mathrm{Re}\,h^2h' \end{pmatrix},$$

$$B2 = \frac{1}{280N1h^7}y^5 \begin{pmatrix} 126F0^2\,\mathrm{Re}\,h' + 210F0\,\mathrm{Re}\,hh' \\ +84\,\mathrm{Re}\,h^2h' + 21CF0^2M\,\mathrm{Re}\,h^2h' \\ +21F0^2\,\mathrm{Re}\,N1h^2h' - 21N1F0^2C\,\mathrm{Re}\,h^2h' \\ +21CF0M\,\mathrm{Re}\,h^3h' + 21F0N1\,\mathrm{Re}\,h^3h' \\ +7N1\,\mathrm{Re}\,h^4h' - 7CN1\,\mathrm{Re}\,h^4h' \end{pmatrix},$$

$$B3 = \frac{1}{280N1h^7}y \begin{pmatrix} 420F1N1h^6 + 15CF0^2M\,\mathrm{Re}\,h^6h' \\ -15CF0^2N1\,\mathrm{Re}\,h^6h' + 11CF0M\,\mathrm{Re}\,h^7h' \\ +11F0N1\,\mathrm{Re}\,h^7h' - 11CF0N1\,\mathrm{Re}\,h^7h' \\ +3CM\,\mathrm{Re}\,h^8h' + 3N1\,\mathrm{Re}\,h^8h' \\ -3CN1\,\mathrm{Re}\,h^8h' \end{pmatrix}.$$

2.3.6 Second order Solution

$$\begin{aligned} \psi_{f2} &= y^{11}A5 + y^9A6 + y^7A7 \\ &\quad + y^5A8 + yA9 + yA10, \end{aligned} \tag{2.52}$$

and

$$\psi_{p2} = y^{11}B4 + y^9B5 + y^7B6 + y^5B7 + y^3B8, \tag{2.53}$$

where

$$\begin{aligned} A5 &= \frac{1}{166320N1h^3}(21CMG5\,\mathrm{Re}\,h^3 + 21G1N1\,\mathrm{Re}\,h^3 \\ &\quad -21CG1N1\,\mathrm{Re}\,h^3), \end{aligned}$$

$$A6 = \frac{1}{166320N1h^3}\begin{pmatrix} 55G6Mh^3 + 55G2N1\,\mathrm{Re}\,h^3 \\ -55C1G2N1\,\mathrm{Re}\,h^3 \end{pmatrix},$$

$$A7 = \frac{1}{166320N1h^3}\begin{pmatrix} 198G7Mh^3 + 198G3N1\,\mathrm{Re}\,h^3 \\ -198CG3N1\,\mathrm{Re}\,h^3 \end{pmatrix},$$

$$A8 = \frac{1}{166320N1h^3}\begin{pmatrix} 1386G8h^3 + 1386G4N1\,\mathrm{Re}\,h^3 \\ -1386CG4N1\,\mathrm{Re}\,h^3 \end{pmatrix},$$

$$A9 = \frac{1}{166320N1h^3}\begin{pmatrix} -83160F2N1 - 2772Mh^5 - 2772G4N1\,\mathrm{Re}\,h^5 \\ +2772CG4N1\,\mathrm{Re}\,h^5 - 594G7Mh^7 \\ -594G3N1\,\mathrm{Re}\,h^7 + 594CG3N1\,\mathrm{Re}\,h^7 \\ -220G6Mh^9 - 220G2N1\,\mathrm{Re}\,h^9 \\ +220CG2\,\mathrm{Re}\,h^9 - 105CG5M\,\mathrm{Re}\,h^{11} \\ -105G1N1\,\mathrm{Re}\,h^{11} + 105CG1N1\,\mathrm{Re}\,h^{11} \end{pmatrix},$$

$$A10 = \frac{1}{166320N1h^3}\begin{pmatrix} 249480F2N1h^2 + 1386G8Mh^7 \\ +1386G4N1\,\mathrm{Re}\,h^7 - 1386CG4N1\,\mathrm{Re}\,h^7 \\ +396G7Mh^9 + 396G3N1\,\mathrm{Re}\,h^9 \\ -396CG3N1\,\mathrm{Re}\,h^9 + 165G6Mh^{11} + \\ 165G2N1\,\mathrm{Re}\,h^{11} - 165CG2N1\,\mathrm{Re}\,h^{11} \\ +84CG5M\,\mathrm{Re}\,h^{13} + 84G1N1h^{13} \\ -84CG1N1h^{13} \end{pmatrix},$$

where

$$B4 = \frac{1}{166320N1h^3}(21C^2MG5\,\mathrm{Re}\,h^3 + 21CG1N1\,\mathrm{Re}\,h^3 - 21C^2G1N1\,\mathrm{Re}\,h^3),$$

$$B5 = \frac{1}{166320N1h^3}\begin{pmatrix} 55CG6Mh^3 - 2310CG5\,\mathrm{Re}\,h^3 \\ +55CG2N1\,\mathrm{Re}\,h^3 - 55C^2G2N1\,\mathrm{Re}\,h^3 \end{pmatrix},$$

$$B6 = \frac{1}{166320N1h^3}\begin{pmatrix} -3960G6h^3 + 198CG7Mh^3 \\ +198CG3N1\,\mathrm{Re}\,h^3 - 198C^2G3N1\,\mathrm{Re}\,h^3 \end{pmatrix},$$

$$B7 = \frac{1}{166320N1h^3}\begin{pmatrix} -8316G7h^3 + 1386CG8h^3 \\ +1386CG4N1\,\mathrm{Re}\,h^3 - 1386C^2G4N1\,\mathrm{Re}\,h^3 \end{pmatrix},$$

$$B8 = \frac{1}{166320N1h^3}\left(\begin{array}{c} -83160CF2N1 - 27720G8h^5 \\ -2772CG8Mh^3 - 2772CG4N1\,\mathrm{Re}\,h^5 \\ +2772C^2G4N1\,\mathrm{Re}\,h^5 - 594CG7Mh^7 \\ -594CG3N1\,\mathrm{Re}\,h^7 + 594C^2G3N1\,\mathrm{Re}\,h^7 \\ -220CG6Mh^9 - 220CG2N1\,\mathrm{Re}\,h^9 + \\ 220C^2G2\,\mathrm{Re}\,h^9 - 105C^2G5M\,\mathrm{Re}\,h^{11} \\ -105CG1N1\,\mathrm{Re}\,h^{11} + 105C^2G1N1\,\mathrm{Re}\,h^{11} \end{array}\right),$$

where

$$\begin{aligned} G1 &= \frac{-252F0\phi A1Cosx}{(1+\phi\,\mathrm{Si}\,nx)^4} - \frac{168\phi A1Cosx}{(1+\phi\,\mathrm{Si}\,nx)^3} - \frac{63F0A1'}{(1+\phi\,\mathrm{Si}\,nx)^3} \\ &\quad - \frac{63A1'}{(1+\phi\,\mathrm{Si}\,nx)^2} - \frac{(-6F0 - 6\,(1+\phi\,\mathrm{Si}\,nx)\,A1')}{2\,(1+\phi\,\mathrm{Si}\,nx)^3}, \end{aligned}$$

$$\begin{aligned} G2 &= \frac{-45F0\phi A2Cosx}{(1+\phi\,\mathrm{Si}\,nx)^4} - \frac{30\phi A2Cosx}{(1+\phi\,\mathrm{Si}\,nx)^3} + \frac{63F0A1'}{(1+\phi\,\mathrm{Si}\,nx)} \\ &\quad + \frac{315F0\phi A1Cosx}{(1+\phi\,\mathrm{Si}\,nx)^2} + 21A11' - \frac{30F0A2'}{(1+\phi\,\mathrm{Si}\,nx)^2} \\ &\quad - \frac{(-6F0 - 6\,(1+\phi\,\mathrm{Si}\,nx)\,A2')}{2\,(1+\phi\,\mathrm{Si}\,nx)^3}, \end{aligned}$$

$$\begin{aligned} G3 &= \frac{18F0\phi A3Cosx}{(1+\phi\,\mathrm{Si}\,nx)^4} + \frac{12\phi A3Cosx}{(1+\phi\,\mathrm{Si}\,nx)^3} + \frac{90F0\phi A2Cosx}{(1+\phi\,\mathrm{Si}\,nx)} \\ &\quad + 10A2' + \frac{30F0A2'}{1+\phi\,\mathrm{Si}\,nx} - \frac{9F0A3'}{(1+\phi\,\mathrm{Si}\,nx)^3} \\ &\quad - \frac{9A3'}{(1+\phi\,\mathrm{Si}\,nx)^2} - \frac{(-6F0 - 6(1+\phi\,\mathrm{Si}\,nx)A3')}{2(1+\phi\,\mathrm{Si}\,nx)^3}, \end{aligned}$$

$$
\begin{aligned}
G4 &= \frac{9F0\phi A4Cosx}{(1+\phi\,\mathrm{Si}\,nx)^4} - \frac{36\phi^2 Cosx}{(1+\phi\,\mathrm{Si}\,nx)^4} + \frac{6\phi A4Cosx}{(1+\phi\,\mathrm{Si}\,nx)^3} \\
&- \frac{6\phi\,\mathrm{Si}\,nx}{(1+\phi\,\mathrm{Si}\,nx)^3} + \frac{9F0\phi A3Cosx}{(1+\phi\,\mathrm{Si}\,nx)^2} \\
&+6F0\left(\frac{12\phi^2 Cosx^2}{(1+\phi\,\mathrm{Si}\,nx)^5} + \frac{3\phi\,\mathrm{Si}\,nx}{(1+\phi\,\mathrm{Si}\,nx)^4}\right) \\
&+6\,(1+\phi\,\mathrm{Si}\,nx)\left(\frac{12\phi^2 Cosx^2}{(1+\phi\,\mathrm{Si}\,nx)^5} + \frac{3\phi\,\mathrm{Si}\,nx}{(1+\phi\,\mathrm{Si}\,nx)^4}\right) \\
&+3A3' + \frac{9F0A3'}{1+\phi\,\mathrm{Si}\,nx} - \frac{(-6F0-6(1+\phi\,\mathrm{Si}\,nx)A4')}{2(1+\phi\,\mathrm{Si}\,nx)^3},
\end{aligned}
$$

$$
\begin{aligned}
G5 &= -\frac{252F0\phi B1Cosx}{(1+\phi\,\mathrm{Si}\,nx)^4} - \frac{168\phi B1Cosx}{(1+\phi\,\mathrm{Si}\,nx)^3} - \frac{63F0B1'}{(1+\phi\,\mathrm{Si}\,nx)^3} \\
&- \frac{63B1'}{(1+\phi\,\mathrm{Si}\,nx)^2} - \frac{(-6F0-6(1+\phi\,\mathrm{Si}\,nx)B1')}{2(1+\phi\,\mathrm{Si}\,nx)^3},
\end{aligned}
$$

$$
\begin{aligned}
G6 &= \frac{1890F0\phi B1Cosx}{CM(1+\phi\,\mathrm{Si}\,nx)^4} - \frac{45F0B2Cosx}{(1+\phi\,\mathrm{Si}\,nx)^4} \\
&+ \frac{1260\phi B1Cosx}{CM(1+\phi\,\mathrm{Si}\,nx)^3} - \frac{30\phi B2Cosx}{(1+\phi\,\mathrm{Si}\,nx)^3} \\
&+ \frac{315F0\phi B1Cosx}{(1+\phi\,\mathrm{Si}\,nx)^2} + 21B1' + \frac{63F0B1'}{1+\phi\,\mathrm{Si}\,nx} \\
&- \frac{21(-6F0-6(1+\phi\,\mathrm{Si}\,nx)B1')}{CM(1+\phi\,\mathrm{Si}\,nx)^3} \\
&- \frac{30F0B2'}{(1+\phi\,\mathrm{Si}\,nx)^3} - \frac{(-6F0-6(1+\phi\,\mathrm{Si}\,nx)B2')}{2(1+\phi\,\mathrm{Si}\,nx)^3},
\end{aligned}
$$

$$
\begin{aligned}
G7 &= \frac{540F0\phi B2Cosx}{CM(1+\phi\,\mathrm{Si}\,nx)^4} + \frac{18F0\phi B3Cosx}{(1+\phi\,\mathrm{Si}\,nx)^4} + \frac{360\phi B2Cosx}{CM(1+\phi\,\mathrm{Si}\,nx)^4} \\
&+ \frac{12\phi B3Cosx}{(1+\phi\,\mathrm{Si}\,nx)^3} + \frac{90F0\phi B2Cosx}{(1+\phi\,\mathrm{Si}\,nx)^2} + 10B1' \\
&+ \frac{30F0B2'}{1+\phi\,\mathrm{Si}\,nx} - \frac{10(-6F0-6(1+\phi\,\mathrm{Si}\,nx)B2')}{C1M(1+\phi\,\mathrm{Si}\,nx)^3} \\
&- \frac{9F0B3'}{(1+\phi\,\mathrm{Si}\,nx)^3} - \frac{(-6F0-6(1+\phi\,\mathrm{Si}\,nx)B3')}{2(1+\phi\,\mathrm{Si}\,nx)^3},
\end{aligned}
$$

$$
\begin{aligned}
G8 &= \frac{54F0\phi B3Cosx}{CM(1+\phi \mathrm{Si}\,nx)^4} + \frac{36\phi B3Cosx}{CM(1+\phi \mathrm{Si}\,nx)^4} + \frac{9F0\phi B3Cosx}{(1+\phi \mathrm{Si}\,nx)^2} + 3B3' \\
&+ \frac{9F0B3'}{1+\phi \mathrm{Si}\,nx} - \frac{3(-6F0-6(1+\phi \mathrm{Si}\,nx)B3')}{CM(1+\phi \mathrm{Si}\,nx)^3},
\end{aligned}
$$

$$
\begin{aligned}
u_{f2} &= 11y^{10}A5 + 9y^8A6 + 7y^6A7 + 5y^4A8 \\
&+3y^2A9 + A10,
\end{aligned} \tag{2.54}
$$

$$
\begin{aligned}
u_{p2} &= 11y^{10}B4 + 9y^8B5 + 7y^6B6 + 5y^4B7 \\
&+3y^2B8.
\end{aligned} \tag{2.55}
$$

The result of our analyses can be expressed to second order flow rate by defining

$$
F^{(2)} = F_0 + \alpha F_1 + \alpha^2 F_2. \tag{2.56}
$$

Then substituting F_0 into Ψ_f and neglecting the terms greater than $O(\alpha^2)$, we obtain second order expression for stream function $\Psi^2_{f,p}$ in terms of the second flow rate $F^{(2)}$.

where

$$
F_0 = F^{(2)} - \alpha F_1 + \alpha^2 F_2. \tag{2.57}
$$

2.4 Pressure gradient

By subtituting (2.35) in dimensionless equation of motion and equating like powers of α on both sides of equations, we get a set of partial differential equations for $\frac{\partial p_0}{\partial x}$, $\frac{\partial p_1}{\partial x}$ and $\frac{\partial p_2}{\partial x}$.

We define the dimensionless pressure rise per wavelength in wave frame as

$$
\nabla P_\lambda = \int \frac{dp}{dx} dx, \tag{2.58}
$$

by using (2.35) in equation (2.58),we obtain

$$
\nabla P_\lambda = \nabla P_{\lambda 0} + \alpha \nabla P_{\lambda 1} + \alpha^2 \nabla P_{\lambda 1} + ..., \tag{2.59}
$$

and then we compute the pressure rise per wavelength by using zeroth, first and second order solution and integrating from 0 to 2π.

2.5 Graphs and discussion

In order to see quantitative effects of various emerging parameters involved, velocity distribution and pressure rise, the result of our analysis are presented as by graphical presentation of obtained solution.

The results of our analytical solution are presented for the pressure rise and flow rate for the various parameters such as Re, C, and small parameter α. It is also represented for the stream lines and trapping regions for several parameters like Re, C, and α. Figure 2.1 represents the pressure change per wavelength ΔP and observed flow rate θ for the various values of concentration particle C. We notice that an increase in C results a decrease in pumping rate. Figure 2.2 is a graph of ΔP and peristaltic pumping rate θ shows that an increase in Reynolds number Re results an increase in pumping rate if all other parameters are held fixed. Figure 2.3 shows that peristaltic pumping rate increases for the various values of α.

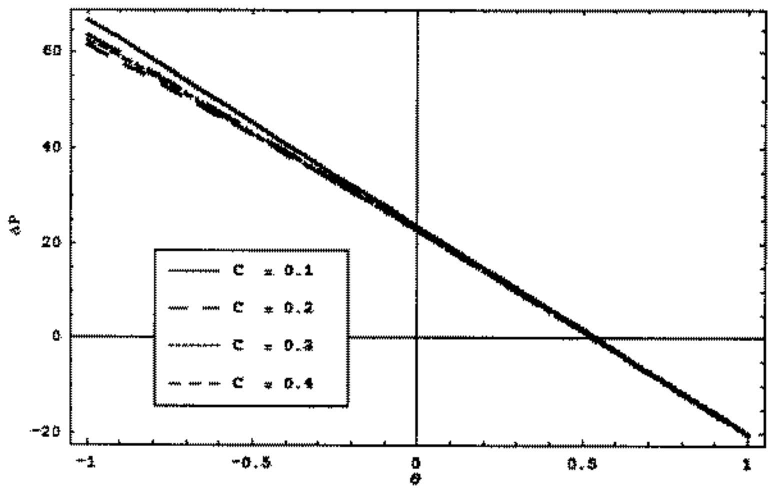

Figure 2.1: Graph of dimensionless pressure gradient per wave length ΔP and flow rate θ for fixed $Re = 1$, $\alpha = 0.06$ and for various values of concentration C.

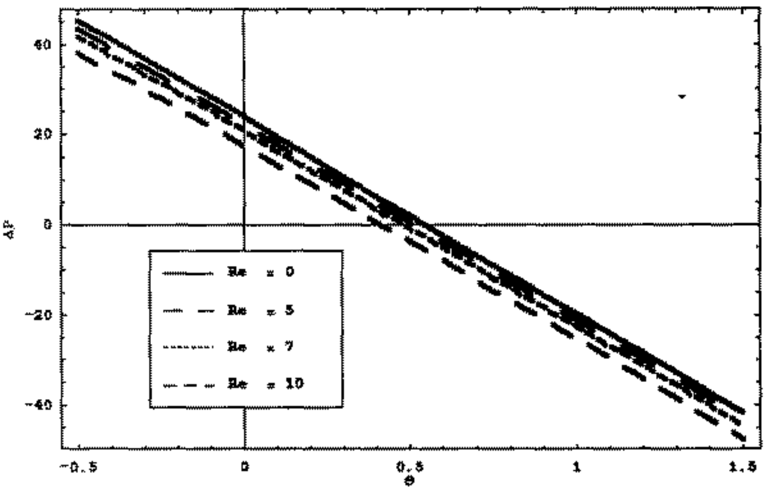

Figure 2.2: Graph of dimensionless pressure gradient per wave length ΔP and flow rate θ for fixed α=0.2, $C = 0.4$, and for various values of Reynolds number Re.

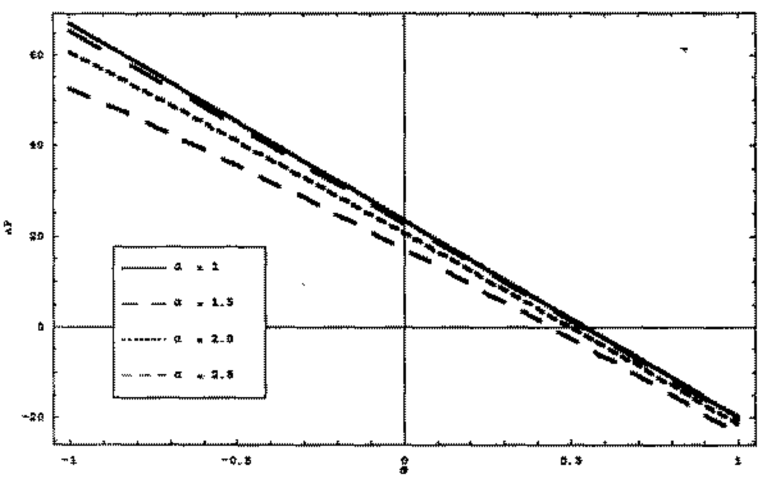

Figure 2.3: Graph of dimensionless pressure gradient per wave length ΔP and flow rate θ for fixed $Re = 1, C = 0.4$, and for various values of α.

Stream lines and fluid trapping

Stream lines are the geometrical representation of flow velocity. Stream line is a curve tangent to velocity vector. The trapping phenomenon, whereby a bolus (defined as a volume of fluid bounded by closed stream lines in the wave frame) is transported at wave speed, has been examined by several investigators. Fig. 2.4 – 2.8 are graphs of streamlines for the conditions $Re = 1$, $\alpha = 0.06$, $C = 0$, $\phi = 0.4$ and $\theta = 0.5, 0.7, 1, 2$. Figure 2.4 shows that there is no trapping for peristaltic pumping when flow rate is small at low Reynold number. Figure 2.5-2.8 represents that the streamlines from all the ends of the tube direct themselves in such a way that all are clustered along the central part .

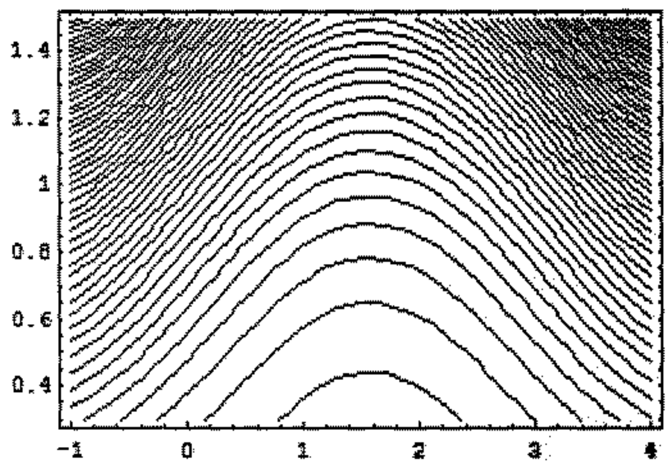

Figure 2.4: Contour streamlines for α=0.06, C=0, ϕ=0.4, Re=1 and θ=0.5.

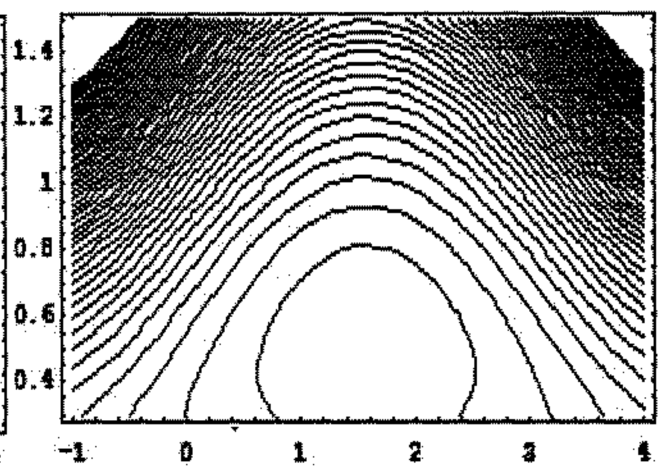

Figure 2.5: Contour streamlines for α=0.06, C=0 , ϕ=0.4 , Re=1 and θ=0.7.

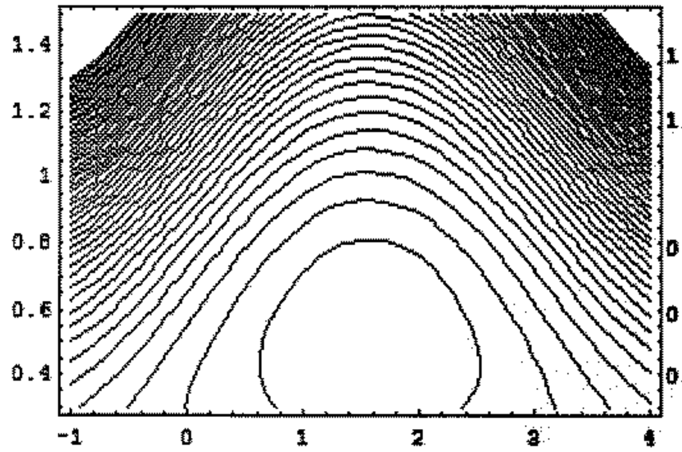

Figure 2.6: Contour streamlines for α=0.06, C=0, ϕ=0.4, Re=1 and θ=1.

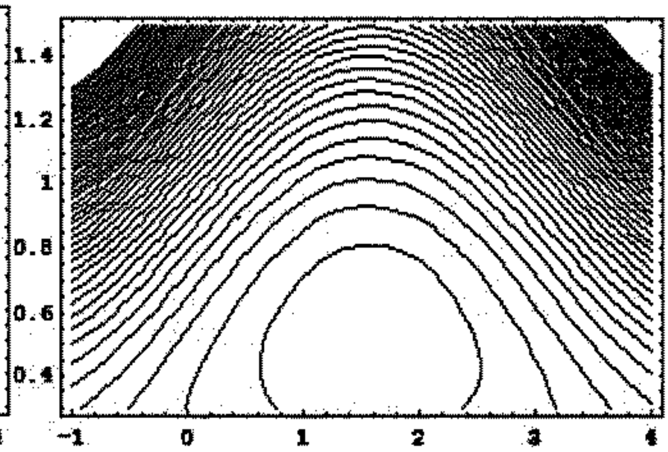

Figure 2.7: Contour streamlines for α=0.06, C=0, ϕ=0.4, Re=1 and θ=2.

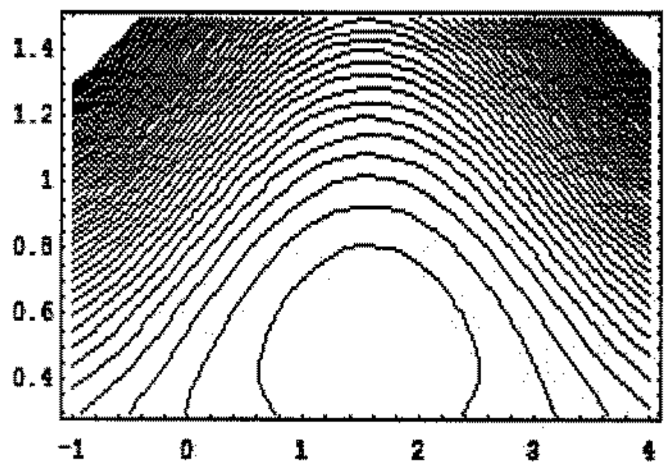

Figure 2.8: Contour streamlines for α=0.06, C=0, ϕ=0.4, Re=1 and θ=5.

Chapter 3

Effects of Uniform Magnetic Fields on Peristaltic Transport of a Particle-Fluid Suspension in a Planar Channel

Mekheimer [20] discussed the effect of peristaltic motion of a particle fluid suspension in a planar channel. In this chapter we will analyze the mechanics of peristaltic pumping of a particle-fluid suspension in a planar channel with uniform magnetic field applied perpendicularly. The approximations of low Reynolds number and long wavelength have been employed to reduce highly nonlinear partial differential equations. Perturbation method is used to obtain solution for velocities, stream lines and pressure gradient in terms of flow rate. The solutions are graphical displayed to see the effects of physical parameters like Reynolds number, concentration of particles, wave number, flow rate and concentration is discussed graphically. The streamlines are also drawn to discuss the trapping bolus discipline.

3.1 Mathematical formulation

We consider the peristaltic flow with variable viscosity in an infinite channel having width $2b$. The fluid is moving through infinite wave with velocity c along the walls. The X-axis and Y-axis are selected along and transverse to the channel walls. It is assumed that fluid conducts electricity, whereas particles are non magnetic in nature. The governing equations of fluid and particulate phase with transverse magnetic field is

$$(1-C)\rho_f \frac{dW_f}{dt} = -(1-C)\nabla P + (1-C)\mu_S(C)\nabla^2 W_f + CS(W_P - W_f) + J \times B, \quad (3.1)$$

$$C\rho_p \frac{dW_p}{dt} = -C\nabla P + CS(W_f - Wp), \quad (3.2)$$

where **J** is the current density and **B** is the total magnetic field.

The equations governing the two dimensional motion of this model are

$$\frac{\partial \overline{U}_f}{\partial \overline{X}} + \frac{\partial \overline{V}_f}{\partial \overline{Y}} = 0, \quad (3.3)$$

$$\left.\begin{array}{l} (1-C)\rho_f\left[\frac{\partial \overline{U}_f}{\partial t} + \overline{U}_f\frac{\partial \overline{U}_f}{\partial \overline{X}} + \overline{V}_f\frac{\partial \overline{U}_f}{\partial \overline{Y}}\right] = -(1-C)\frac{\partial \overline{P}}{\partial \overline{X}} \\ +(1-C)\mu_S\left[\frac{\partial \overline{U}_f^2}{\partial \overline{X}^2} + \frac{\partial \overline{U}_f^2}{\partial \overline{Y}^2}\right] + CS\left(\overline{U}_p - \overline{U}_f\right) - \sigma B_0^2 \overline{U}_f \end{array}\right\},$$

$$\left.\begin{array}{l} (1-C)\rho_f\left[\frac{\partial \overline{V}_f}{\partial t} + \overline{V}_f\frac{\partial \overline{V}_f}{\partial \overline{X}} + \overline{V}_f\frac{\partial \overline{V}_f}{\partial \overline{Y}}\right] = -(1-C)\frac{\partial \overline{P}}{\partial \overline{Y}} \\ +(1-C)\mu_S\left[\frac{\partial \overline{V}_f^2}{\partial \overline{X}^2} + \frac{\partial \overline{V}_f^2}{\partial \overline{Y}^2}\right] + CS\left(\overline{V}_p - \overline{V}_f\right) \end{array}\right\}, \quad (3.4)$$

$$\frac{\partial \overline{U}_p}{\partial \overline{X}} + \frac{\partial \overline{V}_p}{\partial \overline{Y}} = 0, \quad (3.5)$$

$$\begin{aligned} C\rho_p\left[\frac{\partial \overline{U}_p}{\partial t} + \overline{U}_p\frac{\partial \overline{U}_p}{\partial \overline{X}} + \overline{V}p\frac{\partial \overline{U}_p}{\partial \overline{Y}}\right] &= -C\frac{\partial \overline{P}}{\partial \overline{X}} + CS\left(\overline{U}_f - \overline{U}_p\right), \quad (3.6) \\ C\rho_p\left[\frac{\partial \overline{V}_p}{\partial t} + \overline{U}_p\frac{\partial \overline{V}_p}{\partial \overline{X}} + \overline{V}p\frac{\partial \overline{V}_p}{\partial \overline{Y}}\right] &= -C\frac{\partial \overline{P}}{\partial \overline{X}} + CS\left(\overline{V}_f - \overline{V}_p\right), \end{aligned}$$

using eqs.(2.12) into (3.3) − (3.6), we get for fluid phase,

$$\frac{\partial \overline{u}_f}{\partial \overline{x}} + \frac{\partial \overline{v}_f}{\partial \overline{y}} = 0, \tag{3.7}$$

$$\left.\begin{array}{c}(1-C)\rho_f\left[\overline{u}_f\frac{\partial \overline{u}_f}{\partial \overline{x}} + \overline{v}_f\frac{\partial \overline{u}_f}{\partial \overline{y}}\right] = -(1-C)\frac{\partial \overline{p}}{\partial \overline{x}} + \\ (1-C)\mu_S\left[\frac{\partial \overline{u}_f^2}{\partial \overline{x}^2} + \frac{\partial \overline{u}_f^2}{\partial \overline{y}^2}\right] + CS(\overline{u}_p - \overline{u}_f) - \sigma B_0^2\overline{u}_f\end{array}\right\},$$

$$\left.\begin{array}{c}(1-C)\rho_f\left[\overline{u}_f\frac{\partial \overline{v}_f}{\partial \overline{x}} + \overline{v}_f\frac{\partial \overline{v}_f}{\partial \overline{y}}\right] = -(1-C)\frac{\partial \overline{p}}{\partial \overline{y}} + \\ (1-C)\mu_S\left[\frac{\partial \overline{v}_f^2}{\partial \overline{x}^2} + \frac{\partial \overline{v}_f^2}{\partial \overline{y}^2}\right] + CS(\overline{v}_f - \overline{v}_f)\end{array}\right\}, \tag{3.8}$$

and for particulate phase we have

$$\frac{\partial \overline{u}_p}{\partial \overline{x}} + \frac{\partial \overline{v}_p}{\partial \overline{y}} = 0, \tag{3.9}$$

$$\begin{aligned} C\rho_p\left[\overline{u}_p\frac{\partial \overline{u}_p}{\partial \overline{x}} + \overline{v}p\frac{\partial \overline{u}_p}{\partial \overline{y}}\right] &= -C\frac{\partial \overline{p}}{\partial \overline{x}} + CS(\overline{u}_f - \overline{u}_p), \\ C\rho_p\left[\frac{\partial \overline{v}_p}{\partial \overline{x}} + \overline{v}p\frac{\partial \overline{v}p}{\partial \overline{y}}\right] &= -C\frac{\partial \overline{p}}{\partial \overline{y}} + CS(\overline{v}_f - \overline{v}_p), \end{aligned} \tag{3.10}$$

using nondimensional quantities from eq. (2.16) and stream function as (2.17) into eqs. (3.7) – (3.10).

The equations takes the form,

$$\left.\begin{array}{c}(1-C)\operatorname{Re}\alpha\left[\Psi_{fy}\nabla^2\ \Psi_{fx} - \Psi_{fx}\nabla^2\ \Psi_{fy}\right] = \nabla^2\ \nabla^2\ \Psi_f + CM\left(\nabla^2\ \Psi p - \nabla^2\ \Psi_f\right) - \gamma^2\Psi_{fyy}, \\ C\alpha\operatorname{Re}\left(\Psi_{py}\nabla^2\ \Psi_{px} - \Psi_{px}\nabla^2\ \Psi_{py}\right) = CN\left(\nabla^2\ \Psi_f - \nabla^2\ \Psi_p\right),\end{array}\right\} \tag{3.11}$$

along with b.c's as described in previous chapter.

3.2 Solution of problem

3.2.1 Zeroth order problem

$$\frac{\partial^4\psi_{f0}}{\partial y^4} + CM\left(\frac{\partial^2\psi_{p0}}{\partial y^2} - \frac{\partial^2\psi_{f0}}{\partial y^2}\right) = 0, \tag{3.17}$$

$$CN\left(\frac{\partial^2\psi_{f0}}{\partial y^2} - \frac{\partial^2\psi_{p0}}{\partial y^2}\right) = 0, \tag{3.18}$$

along with the boundary conditions

$$\begin{aligned}\psi_{f0} &= \psi_{p0}=0, \psi_{f0yy}=\psi_{p0yy}=0 \text{ at y=0}, \\ \psi_{f0} &= F_0, \psi_{f0y} = -1 \text{ at } y=h.\end{aligned} \tag{3.19}$$

3.2.2 First order problem

$$\begin{aligned}(1-C)\,.\,\mathrm{Re}\left[\frac{\partial\psi_{f0}}{\partial y}\frac{\partial^3\psi_{f0}}{\partial x\partial y^2}-\frac{\partial\psi_{f0}}{\partial x}\frac{\partial^3\psi_{f0}}{\partial y^3}\right] &= \frac{\partial^4\psi_{f1}}{\partial y^4}+CM\left(\frac{\partial^2\psi_{p1}}{\partial y^2}-\frac{\partial^2\psi_{f1}}{\partial y^2}\right) \\ &\quad -\gamma^2\frac{\partial^2\psi_{f0}}{\partial y^2},\end{aligned} \tag{3.20}$$

$$C.\,\mathrm{Re}\left[\frac{\partial\psi_{p0}}{\partial y}\frac{\partial^3\psi_{p0}}{\partial x\partial y^2}-\frac{\partial\psi_{p0}}{\partial x}\frac{\partial\psi_{p0}}{\partial y^3}\right]=CN\left(\frac{\partial^2\psi_{f1}}{\partial y^2}-\frac{\partial^2\psi_{p1}}{\partial y^2}\right), \tag{3.21}$$

the corresponding boundary conditions are

$$\begin{aligned}\psi_{f1} &= \psi_{p1}=0, \psi_{f1yy}=\psi_{p1yy}=0 \text{ at y=0}, \\ \psi_{f1} &= F_1, \psi_{f1y} = 0 \text{ at } y=h.\end{aligned} \tag{3.22}$$

3.2.3 Second order problem

$$\begin{aligned}&(1-C)\,.\,\mathrm{Re}\left[\tfrac{\partial\psi_{f1}}{\partial y}\tfrac{\partial^3\psi_{f0}}{\partial x\partial y^2}+\tfrac{\partial\psi_{f0}}{\partial y}\tfrac{\partial^3\psi_{f1}}{\partial x\partial y^2}-\tfrac{\partial\psi_{f1}}{\partial x}\tfrac{\partial^3\psi_{f0}}{\partial y^3}-\tfrac{\partial\psi_{f0}}{\partial x}\tfrac{\partial^3\psi_{f1}}{\partial y^3}\right]= \\ &\tfrac{\partial^4\psi_{f1}}{\partial y^4}+CM\left(\tfrac{\partial^2\psi_{p1}}{\partial y^2}-\tfrac{\partial^2\psi_{f1}}{\partial y^2}\right)-\gamma^2\tfrac{\partial^2\psi_{f1}}{\partial y^2},\end{aligned} \tag{3.23}$$

$$\begin{aligned}&C.\,\mathrm{Re}\left[\tfrac{\partial\psi_{p1}}{\partial y}\tfrac{\partial^3\psi_{p0}}{\partial x\partial y^2}+\tfrac{\partial\psi_{p0}}{\partial y}\tfrac{\partial^3\psi_{p1}}{\partial x\partial y^2}-\tfrac{\partial\psi_{p1}}{\partial x}\tfrac{\partial^3\psi_{p0}}{\partial y^3}-\tfrac{\partial\psi_{p0}}{\partial x}\tfrac{\partial^3\psi_{p1}}{\partial y^3}\right] \\ &=CN\left(\tfrac{\partial^2\psi_{f1}}{\partial y^2}-\tfrac{\partial^2\psi_{p1}}{\partial y^2}\right),\end{aligned} \tag{3.24}$$

along b.c's

$$\begin{aligned}\psi_{f2} &= \psi_{p2}=0, \psi_{f2yy}=\psi_{p2yy}=0 \text{ at y=0}, \\ \psi_{f1} &= F_2, \psi_{f2y} = 0 \text{ at } y=h.\end{aligned} \tag{3.25}$$

3.2.4 Zeroth order solution

The solution of zeroth order system gives the stream function as

$$\psi_{f0} = \frac{-F0y^3 - y^3h + 3F0yh^2 + yh^3}{2h^3}, \tag{3.26}$$

$$\psi_{p0} = \frac{-6F0y - 6yh}{2CMh^3} + \frac{-F0y^3 - y^3h + 3F0yh^2 + yh^3}{2h^3}. \tag{3.27}$$

The axial velocity is

$$\begin{aligned} u_{f0} &= \frac{-3F0y^2 - 3y^2h + 3F0h^2 + h^3}{2h^3}, \\ u_{p0} &= \frac{-6F0 - 6h}{2CMh^3} + \frac{-3F0y^2 - 3y^2h + 3F0h^2 + h^3}{2h^3}. \end{aligned} \tag{3.28}$$

3.2.5 First order solution

The solution of first order system gives the stream function and axial velocity as

$$\psi_{f1} = y^7 d_{11} + y^5 d_{12} + y^3 d_{13} + y d_{14}, \tag{3.29}$$

where

$$d_{11} = \frac{1}{280N1h^7} \begin{pmatrix} -3CF0^2 M \operatorname{Re} h' - 3F0^2 N1 \operatorname{Re} h' \\ +3CF0^2 \operatorname{Re} h' - 5CF0M \operatorname{Re} hh' \\ -5F0N1 \operatorname{Re} hh' + 5CF0N1 \operatorname{Re} hh' \\ -2CM \operatorname{Re} h^2 h' - 2N1 \operatorname{Re} h^2 h' \\ +2CN1 \operatorname{Re} h^2 h' \end{pmatrix},$$

$$d_{12} = \frac{1}{280N1h^7}\begin{pmatrix} 7F0N1\gamma^2 \operatorname{Re} h^4 - 7CF0N1\gamma^2 \operatorname{Re} h^4 \\ +7N1\gamma^2 \operatorname{Re} h^5 - 7CN1\gamma^2 \operatorname{Re} h^5 \\ +21F0^2N1 \operatorname{Re} h^2h' - 21CF0^2N1 \operatorname{Re} h^2h' \\ +21CF0^2M \operatorname{Re} h^2h' + 21CF0M \operatorname{Re} h^3h' \\ -21F0N1 \operatorname{Re} h^3h' - 21CF0N1 \operatorname{Re} h^3h' \\ +7CM \operatorname{Re} h^4h' + 7N1 \operatorname{Re} h^4h' \\ -7CN1 \operatorname{Re} h^4h' \end{pmatrix},$$

$$d_{13} = \frac{1}{280N1h^7}\begin{pmatrix} -140F1N1h^4 - 14F0\gamma^2N1 \operatorname{Re} h^6 \\ +14CF0\gamma^2N1 \operatorname{Re} h^6 - 14N1\gamma^2 \operatorname{Re} h^7 \\ -14CN1\gamma^2 \operatorname{Re} h^7 - 33CF0^2M \operatorname{Re} h^4h' \\ +33CF0^2N1 \operatorname{Re} h^4h' - 27CF0M \operatorname{Re} h^5h' \\ -27F0N1 \operatorname{Re} h^5h' + 27CF0N1 \operatorname{Re} h^5h' \\ -8CM \operatorname{Re} h^6h' - 8N1 \operatorname{Re} h^6h' \\ +8CN1 \operatorname{Re} h^6h' \end{pmatrix},$$

$$d_{14} = \frac{1}{280N1h^7}\begin{pmatrix} 420F1N1h^6 + 7F0N1\gamma^2 \operatorname{Re} h^8 \\ -7CF0N1\gamma^2 \operatorname{Re} h^8 + 7N1 \operatorname{Re} \gamma^2h^9 \\ -7CN1 \operatorname{Re} \gamma^2h^9 + 15CF0^2M \operatorname{Re} h^6h' \\ +15F0^2N1 \operatorname{Re} h^6h' - 15CF0^2N1 \operatorname{Re} h^6h' \\ +11CF0M \operatorname{Re} h^7h' + 11F0N1 \operatorname{Re} h^7h' \\ -11CF0N1 \operatorname{Re} h^7h' + 3CM \operatorname{Re} h^8h' \\ +3N1 \operatorname{Re} h^8h' - 3CN1 \operatorname{Re} h^8h' \end{pmatrix},$$

$$\psi_{p1} = y^7 d_{15} + d_{16}y^5 + y^3 d_{17}, \tag{3.30}$$

$$d_{15} = \frac{1}{280N1h^7}\begin{pmatrix} -3CF0^2M \operatorname{Re} h' - 3F0^2N1 \operatorname{Re} h' + \\ 3CF0^2 \operatorname{Re} h' - 5CF0M \operatorname{Re} hh' \\ -5F0N1 \operatorname{Re} hh' + 5CF0N1 \operatorname{Re} hh' - \\ 2CM \operatorname{Re} h^2h' - 2N1 \operatorname{Re} h^2h' + \\ 2CN1 \operatorname{Re} h^2h' \end{pmatrix},$$

$$d_{16}=\frac{1}{280N1h^7}\begin{pmatrix} 7F0N1\gamma^2\,\mathrm{Re}\,h^4-7CF0N1\gamma^2\,\mathrm{Re}\,h^4 \\ +7N1\gamma^2\,\mathrm{Re}\,h^5-7CN1\gamma^2\,\mathrm{Re}\,h^5 \\ +126F0^2\,\mathrm{Re}\,h'+210F0\,\mathrm{Re}\,hh' \\ +84\,\mathrm{Re}\,h^2h'+21F0^2N1\,\mathrm{Re}\,h^2h' \\ -21CF0^2N1\,\mathrm{Re}\,h^2h'+21CF0^2M\,\mathrm{Re}\,h^2h' \\ +21CF0M\,\mathrm{Re}\,h^3h'-21F0N1\,\mathrm{Re}\,h^3h' \\ -21CF0N1\,\mathrm{Re}\,h^3h'+7CM\,\mathrm{Re}\,h^4h' \\ +7N1\,\mathrm{Re}\,h^4h'-7CN1\,\mathrm{Re}\,h^4h' \end{pmatrix},$$

$$d_{17}=\frac{1}{280N1h^7}\begin{pmatrix} -140F1N1h^4-14F0\gamma^2N1\,\mathrm{Re}\,h^6 \\ +14CF0\gamma^2N1\,\mathrm{Re}\,h^6-14N1\gamma^2\,\mathrm{Re}\,h^7 \\ -14CN1\gamma^2\,\mathrm{Re}\,h^7-420F0^2\,\mathrm{Re}\,h^2h' \\ -420F0\,\mathrm{Re}\,h^3h'-140\,\mathrm{Re}\,h^4h' \\ -33CF0^2M\,\mathrm{Re}\,h^4h'+33CF0^2N1\,\mathrm{Re}\,h^4h' \\ -27CF0M\,\mathrm{Re}\,h^5h'-27F0N1\,\mathrm{Re}\,h^5h' \\ +27CF0N1\,\mathrm{Re}\,h^5h'-8CM\,\mathrm{Re}\,h^6h' \\ -8N1\,\mathrm{Re}\,h^6h'+8CN1\,\mathrm{Re}\,h^6h' \end{pmatrix},$$

$$u_{f1}=7y^6d_{11}+5y^4d_{12}+3y^2d_{13}+d_{14}, \tag{3.31}$$

$$u_{p1}=7y^6d_{15}+5y^4d_{16}+3y^2d_{17}. \tag{3.32}$$

3.2.6 Second order solution

$$\psi_{f2}=y^{11}d_{18}+y^9d_{19}+y^7d_{20}+y^5d_{21}+y^3d_{22}+yd_{23}, \tag{3.33}$$

$$d_{18}=\frac{1}{166320N1h^3}\begin{pmatrix} 21CEG5M\,\mathrm{Re}\,h^3+21EG1N1\,\mathrm{Re}\,h^3 \\ -21CEG1N1\,\mathrm{Re}\,h^3 \end{pmatrix},$$

$$d_{19}=\frac{1}{166320N1h^3}\begin{pmatrix} 55CEG6M\,\mathrm{Re}\,h^3 \\ +55EG2N1\,\mathrm{Re}\,h^3-55CEG2N1\,\mathrm{Re}\,h^3 \end{pmatrix},$$

$$d_{20}=\frac{1}{166320N1h^3}\begin{pmatrix} 198CEG7M\,\mathrm{Re}\,h^3 \\ +198EG3N1\,\mathrm{Re}\,h^3-198CEG3N1\,\mathrm{Re}\,h^3 \end{pmatrix},$$

$$d_{21} = \frac{1}{166320N1h^3}\left(\begin{array}{c} 1386CEG8M\,\mathrm{Re}\,h^3 \\ +1386EG4N1\,\mathrm{Re}\,h^3 - 1386CEG4N1\,\mathrm{Re}\,h^3 \end{array}\right),$$

$$d_{22} = \frac{1}{166320N1h^3}\left(\begin{array}{c} -83160F2N1 - 2772CEG8M\,\mathrm{Re}\,h^5 \\ -2772EG4N1\,\mathrm{Re}\,h^5 + 2772CEG4N1\,\mathrm{Re}\,h^5 \\ -594CEG7M\,\mathrm{Re}\,h^7 - 594EG3N1\,\mathrm{Re}\,h^7 \\ +594CEG3N1\,\mathrm{Re}\,h^7 - 220CEG6M\,\mathrm{Re}\,h^9 \\ -220EG2N1\,\mathrm{Re}\,h^9 + 220CEG2N1\,\mathrm{Re}\,h^9 \\ -105EG5M\,\mathrm{Re}\,h^{11} - 105EG1N1\,\mathrm{Re}\,h^{11} \\ +105CEG1N1\,\mathrm{Re}\,h^{11} \end{array}\right),$$

$$d_{23} = \frac{1}{166320N1h^3}\left(\begin{array}{c} 249480F2N1h^2 + 1386CEG8M\,\mathrm{Re}\,h^7 \\ +1386EG4N1\,\mathrm{Re}\,h^7 - 1386CEG4N1\,\mathrm{Re}\,h^7 \\ +396CEG7M\,\mathrm{Re}\,h^9 + 396EG3N1\,\mathrm{Re}\,h^9 \\ -396CEG3N1\,\mathrm{Re}\,h^9 + 165CEG6M\,\mathrm{Re}\,h^{11} \\ +165EG2N1\,\mathrm{Re}\,h^{11} - 165CEG2N1\,\mathrm{Re}\,h^{11} \\ +84CEG5M\,\mathrm{Re}\,h^{13} + 84EG1N1\,\mathrm{Re}\,h^{13} \\ -84CEG1N1\,\mathrm{Re}\,h^{13} \end{array}\right),$$

$$\begin{aligned} \psi_{p2} &= y^{11}d_{24} + y^9 d_{25} + y^7 d_{27} \\ &\quad + y^5 d_{28} + y^3 d_{29} + y d_{30}, \end{aligned} \tag{3.34}$$

$$d_{24} = \frac{1}{166320N1h^3}\left(\begin{array}{c} 21CEG5M\,\mathrm{Re}\,h^3 + 21EG1N1\,\mathrm{Re}\,h^3 \\ -21CEG1N1\,\mathrm{Re}\,h^3 \end{array}\right),$$

$$d_{25} = \frac{y^9}{166320N1h^3}\left(\begin{array}{c} -2310EG5\,\mathrm{Re}\,h^3 + 55CEG6M\,\mathrm{Re}\,h^3 \\ +55EG2N1\,\mathrm{Re}\,h^3 - 55CEG2N1\,\mathrm{Re}\,h^3 \end{array}\right),$$

$$d_{26} = \frac{1}{166320N1h^3}\left(\begin{array}{c} -3960EG6\,\mathrm{Re}\,h^3 + 198CEG7M\,\mathrm{Re}\,h^3 \\ +198EG3N1\,\mathrm{Re}\,h^3 - 198CEG3N1\,\mathrm{Re}\,h^3 \end{array}\right),$$

$$d_{28} = \frac{1}{166320N1h^3}\begin{pmatrix} -8316EG7\,\mathrm{Re}\,h^3 + 1386CEG8M\,\mathrm{Re}\,h^3 \\ +1386EG4N1\,\mathrm{Re}\,h^3 - 1386CEG4N1\,\mathrm{Re}\,h^3 \end{pmatrix},$$

$$d_{29} = \frac{1}{166320N1h^3}\begin{pmatrix} -83160F2N1 - 2772CEG8M\,\mathrm{Re}\,h^5 \\ -27720EG8\,\mathrm{Re}\,h^3 - 2772EG4N1\,\mathrm{Re}\,h^5 \\ +2772CEG4N1\,\mathrm{Re}\,h^5 - 594CEG7M\,\mathrm{Re}\,h^7 \\ -594EG3N1\,\mathrm{Re}\,h^7 + 594CEG3N1\,\mathrm{Re}\,h^7 \\ -220CEG6M\,\mathrm{Re}\,h^9 - 220EG2N1\,\mathrm{Re}\,h^9 \\ +220CEG2N1\,\mathrm{Re}\,h^9 - 105EG5M\,\mathrm{Re}\,h^{11} \\ -105EG1N1\,\mathrm{Re}\,h^{11} + 105CEG1N1\,\mathrm{Re}\,h^{11} \end{pmatrix},$$

$$d_{30} = \frac{y}{166320N1h^3}\begin{pmatrix} 249480F2N1h^2 + 1386CEG8M\,\mathrm{Re}\,h^7 \\ +1386EG4N1\,\mathrm{Re}\,h^7 - 1386CEG4N1\,\mathrm{Re}\,h^7 \\ +396CEG7M\,\mathrm{Re}\,h^9 + 396EG3N1\,\mathrm{Re}\,h^9 \\ -396CEG3N1\,\mathrm{Re}\,h^9 + 165CEG6M\,\mathrm{Re}\,h^{11} \\ +165EG2N1\,\mathrm{Re}\,h^{11} - 165CEG2N1\,\mathrm{Re}\,h^{11} \\ +84CEG5M\,\mathrm{Re}\,h^{13} + 84EG1N1\,\mathrm{Re}\,h^{13} \\ -84CEG1N1\,\mathrm{Re}\,h^{13} \end{pmatrix},$$

the axial velocity is

$$u_{f2} = 11y^{10}d_{18} + 9y^8d_{19} + 7y^6d_{20} + 5y^4d_{21} + 3y^2d_{22} + d_{23}, \tag{3.35}$$

$$\begin{aligned} u_{p2} &= 11y^{10}d_{24} + 9y^8d_{25} + 7y^6d_{27} \\ &\quad +5y^4d_{28} + 3y^2d_{29} + d_{30}. \end{aligned} \tag{3.36}$$

3.3 Graphs and discussion

In order to analyze the effects of pertinent parameter, namely perturbation parameter α, the volume fraction density C, the amplitude ratio ϕ and volume flow rate θ on pressure rise represented as by graphs of solution. Figure 3.1 shows that for positive values of peristaltic pumping rate θ, pressure decreases and for negative values pressure increases for case of various values of Re. Figure 3.2 is a graph of peristaltic pumping rate θ vs pressure change per wavelength ΔP for the case of α represents that as α increases θ also increases. Figure 3.3 shows the effect of concentration particle C on pumping rate. We observe that an increase in C results a decrease in pumping rate. Also backward pumping increases with increasing concentration of particles. Figure 3.4 is a graph of pumping rate θ vs pressure change per wavelength shows that as MHD γ increases then flow rate also increases. Fig 3.5 represents that for various values of amplitude ratio ϕ results an increase in peristaltic pumping rate θ. Fig 3.6 is a graph of pressure change per wavelength vs pumping rate θ in the presence of MHD shows that an increase in Re results a decrease in flow rate θ.

Fig 3.7-3.11 are graphs of streamlines for the conditions Re=1, ϕ=0.4, C=0, α=0.0628, θ=0.5, 0.7, 1, 2, 5. Figure 3.7-3.11 represents that the streamlines from all the ends of the tube direct themselves in such a way that all are clustered along the central part and bolus size decreases as θ increases.

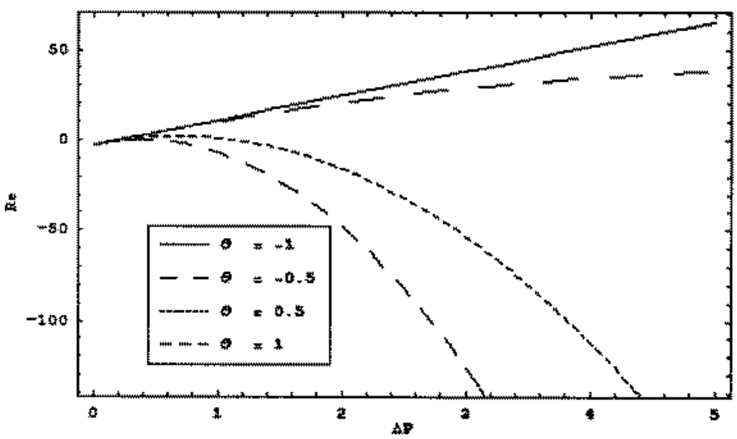

Figure 3.1: Graph of dimensionless pressure gradient per wave length ΔP and flow rate θ for fixed α=0.2, $C = 0.3$, M=1, N=1 and for various values of Reynolds number.

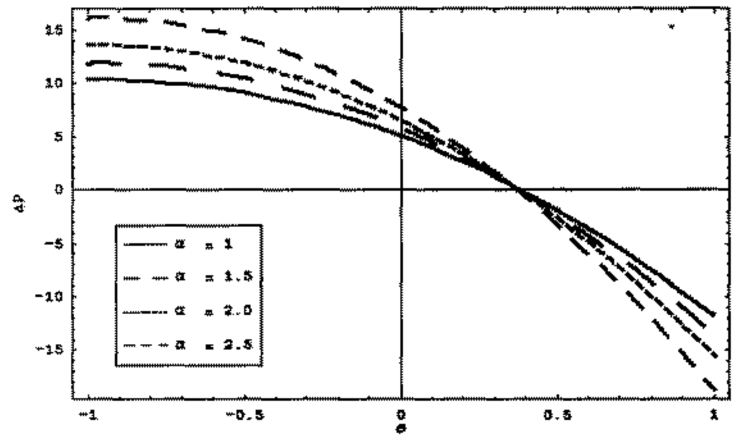

Fig 3.2: Graph of dimensionless pressure gradient per wave length ΔP and flow rate θ for fixed , Re = 0.3, $\gamma = 1$, M=1, N=1 and for various values of α.

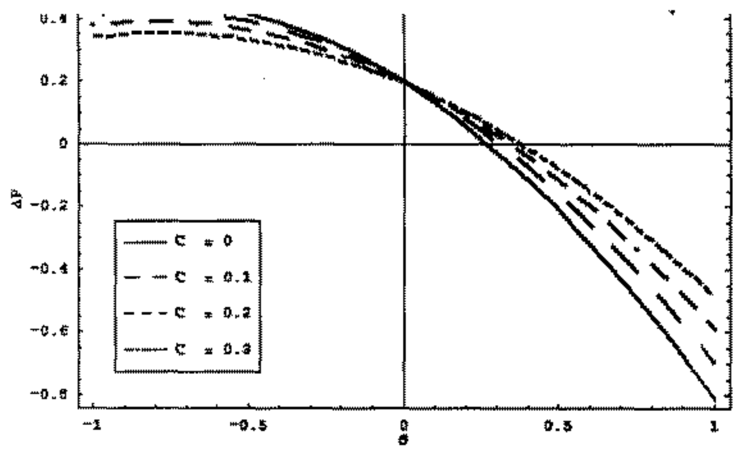

Figure 3.3: Graph of dimensionless pressure gradient per wave length ΔP and flow rate θ for fixed α=-03.5 , M=1, N=1, $\phi = 0.3$, $Re = 1$ and for various values of C.

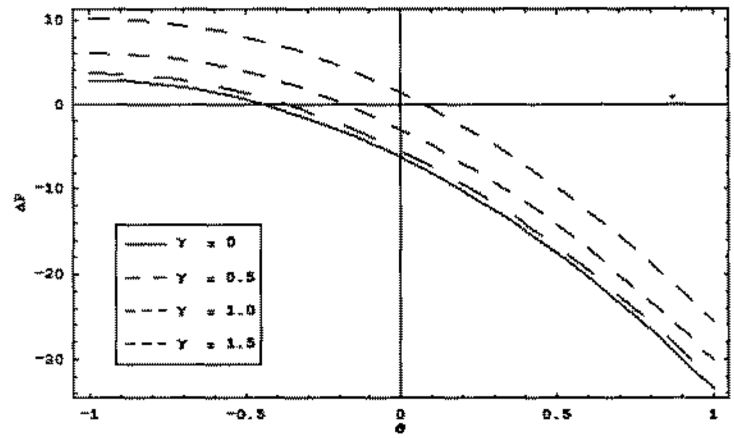

Figure 3.4: Graph of dimensionless pressure gradient per wave length ΔP and flow rate θ for fixed α=-03.5 , $C = 0.3$, $Re = 1$ and for various values of γ.

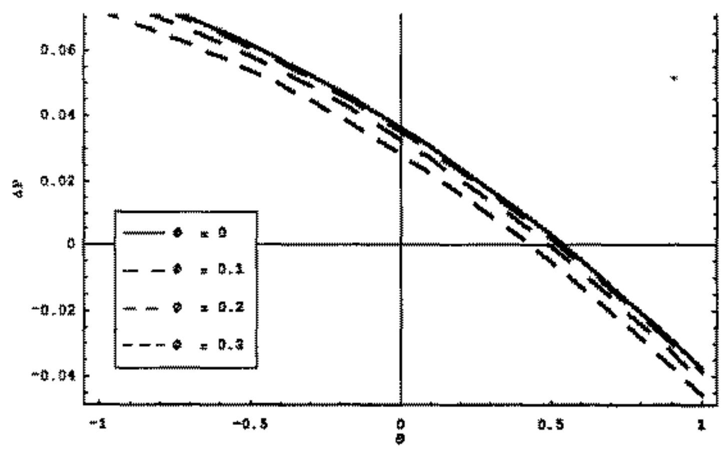

Figure 3.5: Graph of dimensionless pressure gradient per wave length ΔP and flow rate θ for fixed α=0.2, $C = 0.3$, M=1, N=1, Re=1 and for various values of ϕ.

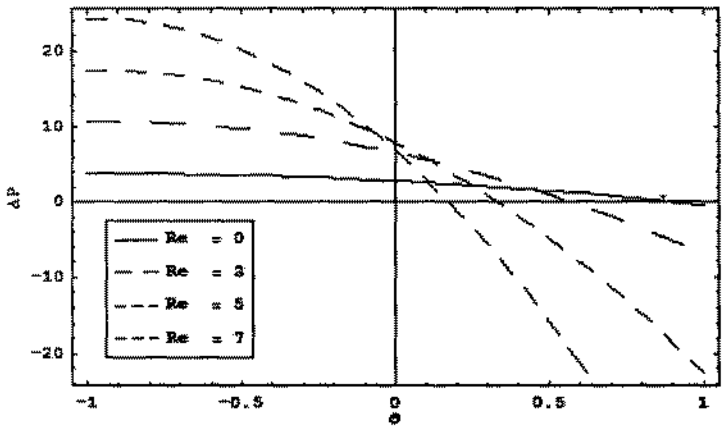

Figure 3.6: Graph of dimensionless pressure gradient per wave length ΔP and flow rate θ for fixed α=-03.2, $C = 0.3$, M=1, N=1 and for various values of Re.

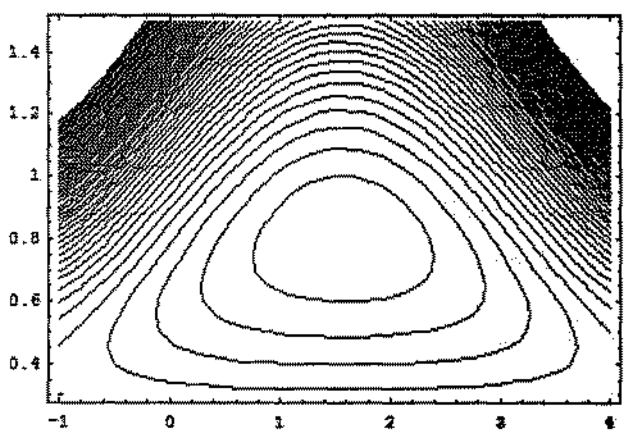

Figure 3.7: For R=1, θ =1, α =0.06, ϕ= 0.4, $\gamma = 4$.

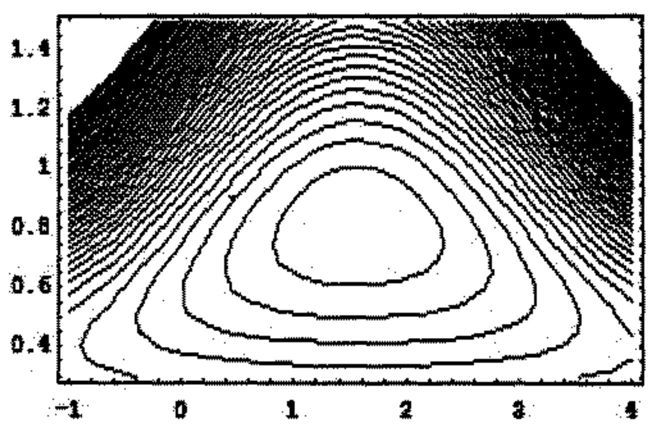

Figure 3.8: For R=1, θ =1, α =0.06, ϕ= 0.4, $\gamma = 4$.

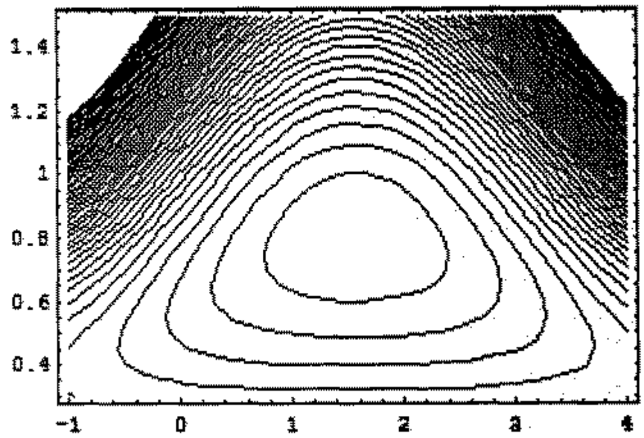

Figure 3.9: For R=1, θ =2, $\alpha = 0.06, \phi$= 0.4, $\gamma = 2$.

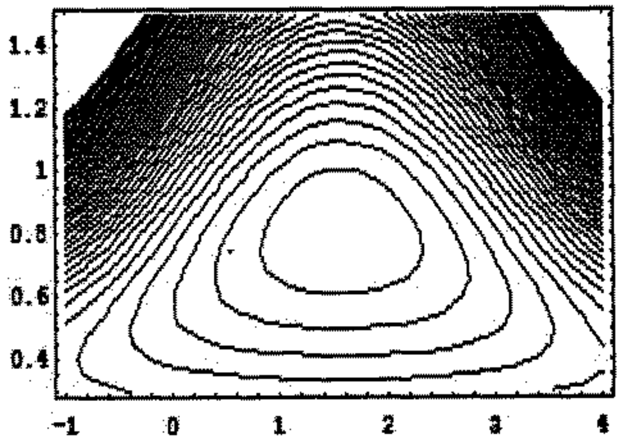

Fig 3.10: For R=1, θ =5, $\alpha = 0.06, \phi$= 0.4, $\gamma = 2$.

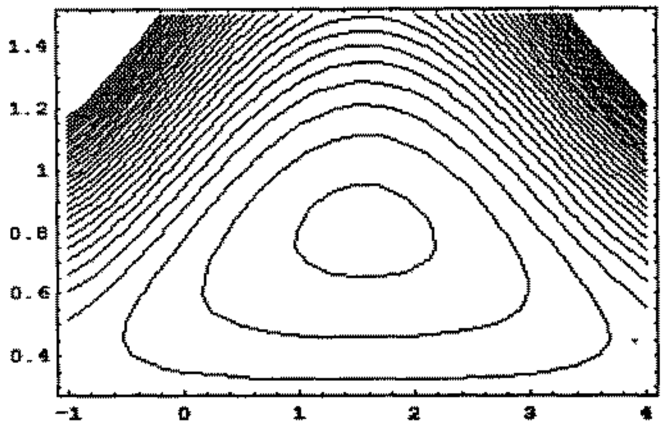

Figure 3.11: For R=1, θ =7, $\alpha = 0.06, \phi$= 0.4, $\gamma = 1$.

3.4 Conclusion

In this thesis the peristaltic transport of a particle-fluid suspension in a planar channel is discussed with and without MHD. Chapter two is a detailed review work of Mekheimer et al. [1].The small MHD effects are discussed in chapter three. The problem is formulated under the implementation of long wavelength and low Reynolds number. The analytical results are developed using perturbation technique. All the results are described graphically by observing the variation of various physical parameters. The results of ΔP and streamline have displayed using graphs. The main results evaluated from the above discussion are summarized as follows.

- It is found that peristaltic pumping rate increases with an increase in small parameter α.
- It is observed that peristaltic flow rate decreases with an increase in concentration C.
- It is measured that peristaltic pumping rate increases with an increase in Re in the presence of MHD but results a decrease in pumping rate without MHD.
- It is seen that peristaltic pumping rate increases with an increase in amplitude ratio ϕ.
- It is noticed that the effect of γ is the increase in pumping rate.
- It is observed that trapped bolus is being small with an increase in peristaltic pumping rate θ.
- It is found that streamlines from all the ends of tube direct themselves in such a way that all are clustered along the central part.

3.5 References

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