THESIS FOR THE DEGREE OF MASTER OF PHYSICS

Analysis of Left Right Symmetric Model with Additional Family Symmetry

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MASTER OF PHYSICS

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Certificate

This is to certify that the work contained in this research entitled " Analysis of Left Right Symmetric Model with Additional Family Symmetry" has been carried out by Maria Altaf under my supervision. She has equal contribution in this research work. In my opinion, this work is fully adequate in scope and quality for the degree of MS Physics.

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The undersigned certify that they have read this master's thesis of **Maria Altaf**, Registration No. **377-FBAS/MSPHY/S16** and that in their opinion it is satisfactory in scope and quality as a thesis for the degree of Master of Physics.

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Abstract

A natural avenue to further extend the Standard Model (SM) is to establish it into a more symmetric framework. Hence, Left-Right (LR) Models have been focused mainly, which treat Left- and Right-handed chiralities on identical footing. Left-Right symmetric model has been taken into consideration where the quark multiplets and scalar fields transform under the influence of an additional family dependent global $U(1)$ symmetry. The new charges have been assigned, rendering masses for top and bottom quarks. A viable pattern resulting in peculiar textures for up and down-type Yukawa matrices were obtained. Analysis of the model has been provided using the kaon mixing.

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Chapter 1

Introduction

The Standard Model (SM) in particle physics is built upon top of primary requirements such as Lorentz covariance and renormalizability, and offers a trite framework for the explication of all known microscopic interactions in terms of local gauged symmetries. This theory was the outcome for the struggle of many generations of accomplished physicists, and is surely one of the most magnificent achievements of sciences. For example, one has tracked a long road to attain the formulation of the theory of electroweak interactions, merging Electromagnetism and Weak processes. Actually, first the weak interactions were presented as a new fundamental interaction in the 30′s by Fermi [1], originated at that time as a contact interaction. Later, as exactly 60 years ago, parity violation in weak decays was indicated [2], provoking doubts about charge-conjugation and time reversion symmetries [3]. The observation of parity violation [4–6] in the succeeding year firmly established such a hypothesis and was of utmost importance to understand the weak interactions (see [7] for historical details). Following the exploration of parity violation, they believed as a $V - A = \gamma^\mu - \gamma^\mu\gamma_5$ interaction [8,9], indicating that the exchange of vector bosons was the underlying reason for the weak force.

The exchange of heavy gauge bosons, was affectively interpreted as the low-energy limit of a symmetric and fundamental theory, called the Electroweak interaction of Glashow-Salam-Weinberg. The Electroweak (EW) symmetry is immediately broken through vacuum expectation value (VEV) of a scalar field ϕ, which also introduces the Higgs boson of the SM. At the same time that this mechanism, named Brout-Englert-Higgs (BEH) [10], illustrates the short-distance nature of weak interactions by ceding masses to the $W^\pm$ and

Z^0 bosons, the particles which are responsible for the weak forces. It also offers an origin for charged leptons and quarks masses, through their interaction with the similar scalar field ϕ.

It is interesting to take account that the SM gives hints towards the possibility of having something more fundamental beyond itself. Indeed, the hierarchical structure of CKM matrix, together with strong hierarchy of masses of the quarks and leptons, claim for a deeper understanding and questioning. Moreover, the values of the gauge couplings g_s, g_L, g_Y are roughly similar. On top of that, though very triumphant in describing comprehensively extensive particle physics phenomena, the SM leaves unexplained some properties of nature.

The class of models reinstating parity, the Left-Right Symmetric Models (LRSMs), has been first perceived in the seventies [11–14], and since then it is at the origin of some fruitful investigations. This is surely due to the flexibility it has regarding its specific realization, a property exploited for addressing a wide range of phenomenological problems, including the smallness of neutrino masses [15] and strong CP violation [16, 17]. Meanwhile, the LR Model may result from Grand Unified gauge groups [18], as part of their instinctive breaking pattern. From these aspects then, investigating the violation of parity symmetry may be a window for handling with other questions in particle physics.

Left Right Symmetric Model (LRSM) renders a natural extension of the Standard Model (SM) elucidating the Left handed form of the SM through the presence of a larger gauge group i.e. $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_Y$, that is broken initially at a scale μ_R which is of the order of the TeV (instigating a distinction between Left and Right sectors) accompanied by an electroweak symmetry hreaking appearing at the scale μ_W. This extension of SM initiates the existence of heavy spin-1 Z' and W' bosons coupled primarily to Right-handed fermions, originating a new Right handed CKM-like matrix for Right-handed quarks in conjunction with charged and neutral heavy Higgs bosons having a impressive pattern of flavor changing currents [19, 20]. When considering parity restoration in the LHC energy reach, the mentioned framework is restored in recent years [18, 21] for its potential collider implications.

In the second chapter, a review of Electroweak theory and Higgs mechanism is provided. In chapter 3, the existing Left-Right symmetric models (LRSMs) is extended further to construct a model with the application of an additional symmetry, the so called horizontal symmetry. In chapter 4, analysis of LR model is discussed by considering the $K^0 - \bar{K}^0$ mixing.

Chapter 2

Standard Model (SM)

All kinds of matter that exists in the universe are formed of three types of elementary particles which includes leptons, quarks, and force mediators. Quark and lepton are the fundamental constituent of matter. There are six quarks also known as flavors, forming three families. The up (u) and down (d) quarks are included in first family quarks whereas second family contains strange (s) and charm (c) quarks while top (t) and the bottom (b) belong to third family quarks. The first family have the lowest masses of all quarks, more stability and are found most commonly in the universe as compared with the other quarks, whereas the second and third family of quarks need high energy collisions for production. The quarks carry fractional electric charge and are always confined in hadronic states. The up type quarks possess $+2/3$ of the electronic charge while the down quarks are having $-1/3$ of the electronic charge. For every quark flavor there is a corresponding anti-quark having electric charge contrary to analogous quark, i.e. reversed in sign. The quarks and anti-quarks, each carry three colors namely, green(g), red(r) and blue(b). Taking into account charges and colors, it can be said that there exists 36 quarks in the Standard Model.

Likewise there exists six leptons, forming three generations or families. These leptons are categorized conforming to their electron number, muon number, taun number and the electric charges. First family consists of electronic leptons, comprising the electron (e^-) along with corresponding electron neutrino (ν_e). Muonic leptons are specified as the second family of leptons, incorporating the muon (μ^-) and the corresponding neutrino (ν_μ) and the third family consists of tauonic leptons, comprising the tau (τ^-)

and the corresponding neutrino (ν_τ). Among all the charged leptons, the electrons possess minimum mass. For every lepton, there is a corresponding anti-lepton. There are six anti-leptons which have equal magnitude but possess a charge that is opposite to the commensurate lepton, i-e. positron, an anti-particle of electron carries an electric charge +1 contrary to electron. In this manner, there exist twelve leptons in the standard model.

Force between particles is mediated through force carriers. All the particles interacts with each other through the mediators of the fundamental forces. The electromagnetic (EM) force is mediated by the photons. Mediators for weak nuclear force are the two W bosons, i.e. $W^{\pm}$ having mass of 80.22 $GeV/(c^2)$, a neutral Z boson having mass of 91.187 $GeV/(c^2)$. Mediating particle for strong force are gluons. Like quarks, the gluons do not exist freely in nature and carries colors. There are eight gluons in the Standard Model. Hence the SM contains tweleve mediators, two charged $W^{\pm}$ bosons, a Photon, one electrically neutral Z^0 boson and the eight gluons.

In the past few decades, The Standard Model is an ingenious theoretical model that is developed by the particle physicists. This theoretical framework gives the insight of the fundamental particles and the forces of nature. The SM has a major ingredient, a hypothetical field, that is believed to render masses to the particles. This field is recognized as the Higgs field and has an associated particle recognized as the Higgs particle, as a consequence of wave-particle duality. The Higgs boson has a spin zero and generates the masses of all the SM fermions along with the masses for Z and $W^{\pm}$ bosons. The Large Hadron Collider (LHC) has tested and confirmed almost all the predictions of the SM with an eminent accuracy. There are a total of sixty one particles in the SM. After the discovery of the Higgs bosons, all the particles of SM are in our hands.

The SM being a Quantum Field Theory is having all elementary matter particles represented through fermionic fields with spin one-half covering three fundamental interactions: under the symmetry group $SU(3)_C$ is the strong interaction while under $SU(2)_L$ it possess the weak interaction and the electromagnetic interaction is possessed by the $U(1)_Y$ group. The complete symmetry group of the SM is then given by

$$SU(3)_c \times \ SU(2)_L \times \ U(1)_Y \tag{2.1}$$

in which the index C represents color, L stands for left, as only Left-handed particles participates in weak interaction, and Y denotes hypercharge. The $SU(3)_C$ has color quantum number, for $SU(2)_L$ is the weak isospin, and for the $U(1)_Y$ it is hypercharge.

The following subsections contains the two eminent features of the SM which are evaluated in detail .

1. Electroweak theory (The Glashow-Salam-Weinberg theory)
2. The Higgs-mechanism

2.1 Electroweak Theory

Even though the electromagnetic and the weak force appear distinct from each other at low energies, it was found that at a scale of around 246 GeV they unify to a single force, known as the Electroweak(EW) force. In this theory the symmetry is broken spontaneously i.e. $SU(2)_L \times\ U(1)_Y \rightarrow U(1)_{EM}$, where gauge group $SU(2)_L \times\ U(1)_Y$, specifies the weak and electromagnetic interactions. The electric charge Q is connected to Hypercharge (Y) and third component of isospin I_3 by Gell Mann-Nishijimaas relation as:

$$Q = I_3 + Y/2 \tag{2.2}$$

The $SU(2)_L$ symmetry distinguishes between fields with different chiralities and plays a remarkable character in the Glashow-Salam-Weinberg model. The $SU(2)$ doublets are assigned to Left handed fields, whereas their Right handed counterparts transform trivially under the EW symmetry. The three copies of $SU(2)_L$ doublets accommodates the charged leptons and neutrinos, categorizing the Standard Model fermions in three generations or families. Similarly, one down-type quark is paired with each up-type quark, forming three generations or families of $SU(2)_L$ doublets. Left handed $SU(2)$ doublets of fermionic fields are represented as,

$$\begin{aligned}\Psi_L^q &= \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L \\ \Psi_L^l &= \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L\end{aligned} \tag{2.3}$$

with third component of isospin $I_3 = +1/2$ for up-type quarks and $I_3 = -1/2$ for the down-type quarks.
The Right-handed particles acts as singlets and are represented as,

$$\begin{aligned}\psi_L^q &= u_R, d_R, c_R, s_R, t_R, b_R, \\ \psi_L^l &= e_R, \mu_R, \tau_R\end{aligned} \tag{2.4}$$

with third component of isospin $I_3 = 0$

Neutrinos are assumed massless in the electroweak section of SM suggested by Abdus Salam, Glashow and Weinberg, so neutrinos do not have a Right handed state but only the Left handed state.
The Langrangian for the free fermions can be written as

$$\mathcal{L}_f = i(\bar{\psi}^i_L \gamma^\mu \partial_\mu \psi^i_L + \bar{\psi}^i_R \gamma^\mu \partial_\mu \psi^i_R) \tag{2.5}$$

Mass term in not included in above equation because it explicitly violates the gauge invariance by mixing the Left-and Right-handed fields.
Left-and Right-handed components are transformed under the gauge $SU(2)_L$ transformations as,

$$\psi_L \rightarrow \psi'_L = e^{ig\alpha(x)\frac{\sigma}{2}} \psi_L \qquad \psi_R \rightarrow \psi'_R = \psi_R \tag{2.6}$$

with σ regarded as pauli matrices.
Similarly, fermions transforms under the $U(1)_Y$ gauge transformation given below,

$$\psi_L \rightarrow \psi'_L = e^{ig'\beta(x)\frac{Y}{2}} \psi_L \qquad \psi_R \rightarrow \psi'_R = e^{ig'\beta(x)\frac{Y}{2}} \psi_R \tag{2.7}$$

Coupling parameters g and g' are real numbers and describe the strength of the interaction associated with gauge transformation. For having the Lagrangian to be also invariant under local gauge transformations, letting the α and β depend on position i.e. $\alpha^i = \alpha^i(x)$, $\beta = \beta(x)$. This symmetry requirement is satisfied by changing the fermion derivatives with the covariant objects, so covariant derivative for Left handed fermion is,

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + i\frac{g}{2}\sigma \cdot W_\mu(x) + ig'YB_\mu(x) \tag{2.8}$$

and the covariant derivatives for the Right handed fermions is written as,

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + ig'YB_\mu(x) \tag{2.9}$$

Since four gauge parameters are present, $\alpha^i(x)$ and $\beta(x)$, four distinguished real gauge fields are introduced $W^1_\mu(x), W^2_\mu(x), W^3_\mu(x)$, and $B_\mu(x)$.

From the condition of gauge invariance, these gauge fields transforms like

$$\begin{aligned} &B_\mu(x) \rightarrow B'(x)_\mu = B_\mu(x) - \partial_\mu \beta(x) \\ &W^\alpha_\mu(x) \rightarrow W'^\alpha_\mu(x) = W^\alpha_\mu(x) - \partial_\mu \alpha^\alpha(x) - \epsilon^\alpha_{bc} \alpha^b(x) W^c_\mu(x) \end{aligned} \tag{2.10}$$

The mass term of gauge bosons are precluded by the gauge symmetry. Also the masses for fermions are unattainable, because they would communicate with Left-handed and Right-handed fields, possessing unalike transformation properties, hence explicitly breaks the gauge symmetry. Thus the EW Lagrangian only contains massless terms owing to the fact that by inserting mass terms in Lagrangian will give rise non invariance. So in terms of massless particles, the EW Lagrangian can be written as,

$$\mathcal{L}_{EW} = i\bar{\psi}_L^i \gamma^\mu D_\mu \psi_L^i + i\bar{\psi}_R^i \gamma^\mu D_\mu \psi_R^i - \frac{1}{4} B^{\mu v}(x) B_{\mu v}(x) - \frac{1}{4} W^{\alpha\mu v}(x) W^{\alpha}_{\mu v}(x)$$

The last two terms in the Lagrangian describes the self-interaction terms for gauge fields. For constructing gauge invariant kinetic term for the new gauge fields, the strength tensor gauge fields are defined as follows,

$$\begin{aligned} B^{\mu v}(x) &= \partial^\mu B^v(x) - \partial^v B^\mu(x) \\ W^{\alpha\mu v}(x) &= \partial^\mu W^{\alpha v}(x) - \partial^v W^{\alpha\mu}(x) - g\epsilon^{\alpha}_{bc} W^{b\mu}(x) W^{cv}(x) \end{aligned} \tag{2.11}$$

In the Langrangian of Eq.(2.11) the fields are unphysical. The physical fields of charged vector bosons $W_\mu^\pm$ are superpositions of the $W_\mu^1(x)$ and $W_\mu^2(x)$ fields as given below,

$$W_\mu^\pm(x) = \frac{1}{\sqrt{2}}(W_\mu^1(x) \pm iW_\mu^2(x)) \tag{2.12}$$

Rotation of the gauge fields $W_\mu^3(x)$ and $B^\mu(x)$ with the Weinberg angle also called mixing angle θ_W gives the neutral vector boson field $Z_\mu(x)$ and the photon field $A_\mu(x)$.

Weak mixing angle, elementary charge and gauge coupling g' and g are related as,

$$\begin{aligned} \sin\theta_w &= \frac{g'}{\sqrt{g'^2 - g^2}}, \cos\theta_w = \frac{g}{\sqrt{g'^2 - g^2}}, \\ \tan\theta_w &= \frac{g'}{g}, e = g'\cos\theta_w, e = g\sin\theta_w \end{aligned} \tag{2.13}$$

In electroweak theory, for the local invariance the masses of all gauge bosons in addition to fermionic mass are instructed to be zero, but experimentally all particles actually are pretty massive except for the gluons and photon. So for the generation of fermionic and bosonic masses without violating the EW theory, one needs a fundamental process which is recognized as Higgs mechanism.

2.2 The Higgs Mechanism

The disclosure of massive Z and W bosons tackled the notion of local gauge invariance with a problem. The introduced gauge fields for making the Lagrangian invariant under a local gauge transformation have to be massless for the reason that, a mass term for the gauge fields in the Lagrangian is not gauge invariant. Hence, the Higgs mechanism is the simplest and the best possible solution for generating the fermionic and bosonic masses. For Higgs sector [23–25] a complex scalar doublet is specified and is written as,

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \qquad Y = -1 \tag{2.14}$$

where ϕ^0 and ϕ^+ specifies the complex neutral and charged fields. Now for a complex gauge invariant spin zero particle field ϕ, the Lagrangian can be written as

$$L_{Higgs} = (D^\mu \Phi)(D_\mu \Phi)^\dagger + V(\Phi\Phi^\dagger) \tag{2.15}$$

where V is the Higgs potential which is invariant under $SU(2)_L$ transformation, and is written as

$$V(\Phi\Phi^\dagger) = \mu^2 V \Phi\Phi^\dagger + \lambda(\Phi\Phi^\dagger)^2 \tag{2.16}$$

For the potential energy density V to be bounded from below to have a ground state, λ must be greater than zero ($\lambda > 0$). Also, when $\mu^2 < 0$, scaler field possess a vacuum expectation value (VEV) which is non-vanishing.

$$\langle \Phi\Phi^\dagger \rangle = -\frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2} > 0 \tag{2.17}$$

where $\frac{v}{\sqrt{2}}$ is the VEV of the Higgs field.

Due to $SU(2)_L$ symmetry, there is a continuum of non-vanishing absolute minima for the Higgs potential. By choosing one specific value as ground state, the symmetry gets broken spontaneously. After the SSB, the expanded value of the Higgs field around this chosen VEV yields

$$\Phi(x) = e^{i\Theta_j(x)\frac{\sigma_j}{2}} \begin{pmatrix} 0 \\ \frac{[v+\phi(x)]}{\sqrt{2}} \end{pmatrix} \tag{2.18}$$

where $\Theta_j (j = 1, 2, 3)$ represents the Goldstone bosons which have three degrees of freedom.

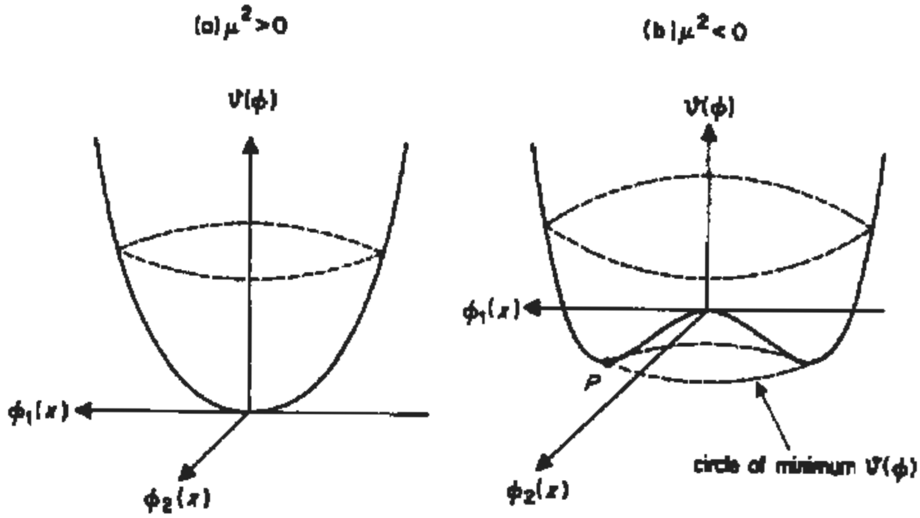

Figure 2.1: The potential energy density Eq.(2.17), (a) for $\mu^2 > 0$ (b), $\mu^2 < 0$, for vacuum ground state corresponding to point labeled as P on the circle

However, by inserting the Higgs field into the Lagrangian with covariant derivatives, illustrates that gauge bosons gain masses by absorbing these degrees of freedom. The Goldstone bosons can be completely removed by an $SU(2)$ gauge transformation as,

$$\Phi(x) \rightarrow \Phi' = e^{-i\Theta_j(x)\frac{\sigma_j}{2}}\Phi(x) = \begin{pmatrix} 0 \\ \frac{[v+\phi(x)]}{\sqrt{2}} \end{pmatrix} \tag{2.19}$$

The remaining field is the Higgs field.

The EW symmetry is broken concurrently through Higgs mechanism to $U(1)_{EM}$ at low energies, while photon remains massless.

2.2.1 Fermion masses

By introducing an additional scaler higgs doublet $SU(2)$, an $SU(2)$ invariant interaction of fermions could possibly be written accompanied by Higgs field, so the Lagrangian plngged with interaction term for the fermions is

$$\mathcal{L}_{int} = g(\bar{\psi}_L \Phi l_R^- + \phi^\dagger \bar{l^-}_R \psi_L) \tag{2.20}$$

Now ϕ is substituted by

$$\phi = \begin{pmatrix} 0 \\ \frac{[v+H]}{\sqrt{2}} \end{pmatrix} \tag{2.21}$$

where v is VEV and H is neutral physical Higgs particle. In the unitary gauge, the Yukawa type Lagrangian attains the simpler form

$$\mathcal{L}_{int} = \frac{gv}{\sqrt{2}}(\bar{l}_L l_R + \bar{l}_R l_L) + \frac{g}{\sqrt{2}}(\bar{l}_L l_R + \bar{l}_R l_L)H \tag{2.22}$$

so the SSB mechanism generates leptonic mass

$$m_l = \frac{gv}{\sqrt{2}} \tag{2.23}$$

It is speculated that, there exists no Right-handed neutrino state ν_R so one can not write corresponding mass term $\bar{\nu}_R \nu_L$ leading neutrino mass to be zero, $m_\nu = 0$.

Yukawa coupling of Left and Right handed fields introduces mass term to $SU(2)_L$ Higgs doublet. The interaction term added to the Lagrangian for first generation of quarks is

$$\mathcal{L}_{yukawa} = -\tilde{g}_d(\bar{q}_L \Phi q_R + \bar{d}_L \Phi^\dagger q_R) - \tilde{g}_u(\bar{q}_L \Phi_C u_R + \bar{u}_R \Phi_C^\dagger q_L) + h.c \tag{2.24}$$

$\tilde{g}_u$,$\tilde{g}_d$ in above equations are couplings of Higgs field. The field Φ_C represented as

$$\Phi_C(x) = -\iota\sigma_2 \Phi^\dagger(x) = \begin{pmatrix} \frac{[v+\phi(x)]}{\sqrt{2}} \\ 0 \end{pmatrix} \tag{2.25}$$

By using the relations of Higgs field in the above Lagrangian one gets,

$$\mathcal{L}_{yukawa} = -\tilde{g}_d \frac{v}{\sqrt{2}} \bar{d}d - \tilde{g}_u \frac{v}{\sqrt{2}} \bar{u}u - \tilde{g}_d \frac{\Phi(x)}{\sqrt{2}} \bar{d}d - \tilde{g}_u \frac{\Phi(x)}{\sqrt{2}} \bar{u}u \tag{2.26}$$

The first two terms in the above equation gives masses to quarks, while the other two terms specify the quark and Higgs coupling with a coupling strength that is proportional to the quark masses. So the term giving the quark masses is

$$m_q = \tilde{g}_q \frac{v}{\sqrt{2}} \tag{2.27}$$

The mass generating interactions of scalar field ϕ with gauge fields arises from the extra terms in kinetic energy part of the Higgs Lagrangian. These terms generate 'spin one' bosons masses. In terms of VEV of Higgs field and couplings, the W and Z bosons masses are written as

$$M_Z = \frac{v}{2}\sqrt{(g'^2 + g^2}, \qquad M_W = \frac{vg}{2} \tag{2.28}$$

Term like $mA_\mu(x)A^\mu(x)$ in not present, so the mass of photon is zero.

$$M_\gamma = 0 \tag{2.29}$$

Electro-weak mixing angle are written as,

$$\sin^2\theta_w = 1 - \frac{M_W^2}{M_Z^2}, \qquad \cos^2\theta_w = \frac{M_W^2}{M_Z^2} \tag{2.30}$$

Remaining terms in the Higgs Lagrangian represents the kinetic energy and self-coupling of Higgs. Mass of Higgs boson obtained is,

$$M_H = \sqrt{-2\mu^2} = \sqrt{2\lambda}v, \qquad v = 246GeV \tag{2.31}$$

The Higgs boson mass is described by the parameter λ.

In spite of so much success of the SM, there exist still many unanswered questions left by SM like CP. Hence its compulsory to extend the SM for having better understanding of the features which are not explained by the SM. For physics beyond SM a number of influential extensions of the SM have been proposed, and among them is Left-Right symmetric model analyzed in third chapter.

2.3 The flavor sector of the SM

The Lagrangian of the SM, $\mathcal{L}_{SM}$, can be splitted in three parts:

$$\mathcal{L}_{SM} = L_{kinetic} + L_{Higgs} + L_{Yukawa} \tag{2.32}$$

The Yukawa part of the Lagrangian is splitted into the baryonic and leptonic parts. The lepton-Yukawa interactions, at the renormalizable level are given by the equation:

$$-\mathcal{L}_{Yukawa}^{leptons} = Y_{ij}^e \bar{L}'_{Li}\phi E'_{Rj} + h.c \tag{2.33}$$

These terms gives the masses to charged lepton after Higgs acquires a VEV. As the SM perceives massless neutrinos so the three physical parameters that

are involved in lepton-Yukawa terms, usually specified as the masses for the three charged lepton.
Likewise, the quark-Yukawa interactions could be written as

$$-\mathcal{L}^{quarks}_{Yukawa} = Y^d_{ij}\bar{Q}'_{Li}\phi D'_{Rj} + Y^u_{ij}\bar{Q}'_{Li}\tilde{\phi}U'_{Rj} + h.c \tag{2.34}$$

with Y^d and Y^u representing complex yukawa coupling constants. This part of yukawa Langrangian permits quarks masses and flavor. Ten physical parameters illustrates quark-yukawa interactions and are specified as the six quark masses and the four CKM matrix parameters.

2.3.1 The CKM matrix

When the Higgs doublet is introduced, the Yukawa interactions result in the mass terms:

$$-\mathcal{L}^q_M = (M_d)_{ij}\bar{D}'_{Li}D'_{Rj} + (M_u)_{ij}\bar{U}'_{Li}U'_{Rj} + h.c \qquad M_q = \frac{v}{\sqrt{2}}Y^q \tag{2.35}$$

here q represents quarks as only quark term from the langrangian is taken.

One can diagonalize the mass matrices using unitary transformation matrices, V_{qL} and V_{qR} as

$$V_{qL}M_qV_{qR}{}^\dagger = M^{diag}_q = \begin{pmatrix} m_q & 0 & 0 \\ 0 & m_q & 0 \\ 0 & 0 & m_q \end{pmatrix} \; q = u, d \tag{2.36}$$

The mass eigenstates for quarks are recognized as

$$q'_{Li} = (V_{qL})_{ij}q_{Lj}, \quad q'_{Ri} = (V_{qR})_{ij}q_{Rj} \quad q = u, d \tag{2.37}$$

Furthermore, the charged current interactions could be written as

$$\mathcal{L}^q_{W^\pm} = \frac{g}{\sqrt{2}}\bar{u}_{Li}\gamma^\mu(V_{uL}V^\dagger_{dL})_{ij}d_{Lj}W^+_\mu + h.c \tag{2.38}$$

The unitary matrix is 3×3 matrix and defined as

$$V = V_{uL}V_{dL}{}^\dagger, \qquad (VV^\dagger = 1) \tag{2.39}$$

is the Cabibbo-Kobayashi-Maskawa (CKM) or quark mixing matrix. The flavor changing interactions of quarks within SM is provoked through coupling of $W^{\pm}$ gauge bosons to mass eigenstates quarks of non-identical families. The CKM matrix contains one physical phase known as the Kobayashi-Maskawa phase, mainly denoted by δ_{KM}.

The elements of V could be written as

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{2.40}$$

There are uncountable ways of expressing V but the standard parametrization of the CKM-matrix [26], with three rotational angles (θ_{ij}) and a complex phase (δ) is given as

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ -s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{2.41}$$

with $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$. The $\sin\theta_{ij}$ are the three real mixing parameters while δ denotes the Kobayashi-Maskawa (KM)-phase.

A strongly hierarchical pattern is exhibited by off-diagonal elements of CKM matrix. The elements $|V_{cb}|$ and $|V_{ts}|$ are of the order of 0.22, while $|V_{ub}|$ and $|V_{td}|$ are close to 4×10^{-2} and $|V_{us}|$ and $|V_{cd}|$ approximately equal to 5×10^{-3}. The expansion of the CKM matrix elements in powers of the small parameter $\lambda = |V_{us}| \approx 0.22$ is pointed out by the Wolfenstein parametrization [27] to evince this particular hierarchy in a more straightforward way.

$$V = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) \tag{2.42}$$

with four mixing parameters (λ, A, ρ, η), and η representing CP violating phase.

The unitarity of CKM matrix suggests following relation among elements

$$\sum_{k=1}^{3} V_{ki}V_{kj}^{\star} = \delta_{ij} \qquad for i, j = 1, 2, 3 \tag{2.43}$$

This relation is a distinguishing feature of the SM, in which the only origin of quark flavor mixing is the CKM matrix. Such relations are six in total and

$$V_{ud}V_{ub}^{\star} + V_{cd}V_{cb}^{\star} + V_{td}V_{tb}^{\star} = 0 \tag{2.44}$$

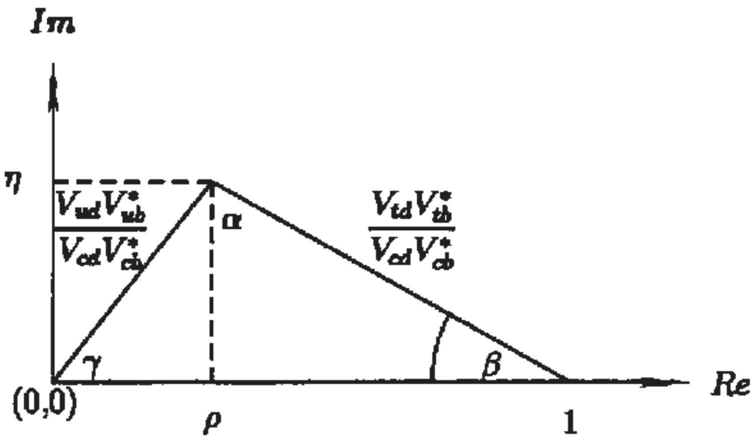

Figure 2.2: Unitarity Triangle

for $i \neq j$ requiring sum of three complex quantities to become zero. Hence, their geometric representation could be done as a triangle in the complex plane which is known as "unitarity triangles". All unitarity triangles are having equal areas which is termed as a major attribute of the CKM matrix. Among the relations for $i \neq j$, the one obtained for $i = 1$ and $j = 3$ is of unique interest for the reason that it involves the sum of three terms all of the comparable magnitude in λ and is normally represented as a unitarity triangle in the complex plane.

Now defining the rescaled unitarity triangle by specifying a phase convention such that $(V_{cd}V^*_{cb})$ is real and dividing lengths of all sides by $|V_{cd}V^*_{cb}|$. This rescaled form of unitarity triangle is almost identical to the unitarity triangle and has two vertices exist at $(0,0)$ and $(1,0)$. The remaining vertex co-ordinates correlates with the Wolfenstein parameters (ρ, η). The unitarity triangle is given in Fig.(2.2)

$$\frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}} + 1 + \frac{V_{td}V^*_{tb}}{V_{cd}V^*_{cb}} = 0 \tag{2.45}$$

The three angles are defined as below

$$\begin{aligned} \alpha &= arg\Big[-\frac{V_{td}V^*_{tb}}{V_{ud}V^*_{ub}}\Big], \\ \beta &= arg\Big[-\frac{V_{cd}V^*_{cb}}{V_{td}V^*_{tb}}\Big], \\ \gamma &= arg\Big[-\frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}}\Big] \end{aligned} \tag{2.46}$$

Flavour Changing Neutral Currents (FCNCs)

In SM, the $W^{\pm}$ bosons mediates the flavour changing charged currents, which is solely origin of flavor changing interaction specifically of generation changing interaction. It is known already that the two interactions of flavor changing charged current can give the outcome as FCNC interaction.

The flavour changing neutral current (FCNC) processes like certain rare and radiative meson decays, particle-antiparticle mixing, and also CP violating decays rendered an eminent role in constructing the SM and are governed by the Glashow-Iliopoulos-Maiani (GIM) mechanism [28] ensuring that these processes are suppressed naturally which is an experimentally observed fact. As a outcome of this mechanism, FCNC processes existence at tree level is abrogated and the major contributions result due to the one-loop diagrams, the box diagrams and the penguin diagrams.

Parity, Charge conjugation and CP violation

In particle physics, CP violation is contravention of the combined conservation laws linked with charge conjugation (C) and parity (P) by the weak force, accountable for reactions such as the radioactive decay of atomic nuclei. The particle-antiparticle conjugation operator C acting on one particle state transforms the particle into its antiparticle, allowing space coordinates, time and spin unaltered. A parity operation connects and object and its mirror image. The elementary processes involving the the strong and weak forces and electromagnetic force were assumed to exhibit symmetry concerning both parity and charge conjugation (these two properties were always conserved in any particle interactions).

In 1949 Powell spotted a cosmic ray particle disintegrated into three pions which he dubbed "the tau meson". Another particle called the "theta meson" was also detected which disintegrated into two pions. Both of the particles disintegrated via the weak force.

$$\begin{aligned} \theta^+ &\to \pi^+\pi^- \\ \tau^+ &\to \pi^-\pi^+\pi^- \end{aligned} \tag{2.47}$$

An evident deficit of parity conservation in decay of theta and tau into two and three pi-mesons prompted the physicists to inspect the experimental basis of parity conservation. In 1956, it was showed that there was no affirmation supporting the invariance of parity in weak interactions [2]. It was demonstrated conclusively from the experiments conducted the following year that parity was not conserved in particle decays, including nuclear beta decay that occur through the weak force. This allowed the identification of both θ^+ and τ^+ with the K^+ meson.

The Glashow, Iliopoulos, and Maiani mechanism (GIM)

An immediate consequence of the Cabibbo theory is the presence, in the Lagrangian, of the term

$$\bar{d}'_L\gamma_a d'_L = \cos^2\theta_C \bar{d}_L\gamma_a d_L + \sin^2\theta_C \bar{s}_L\gamma_a s_L + \cos\theta_C\sin\theta_C[\bar{d}_L\gamma_a s_L + \bar{s}_L\gamma_a d_L] \tag{2.48}$$

which describes neutral-current transitions. In particular, the last term implies neutral currents that change strangeness (SCNC, strangeness-changing neutral currents) because they connect s and d quarks. However, the corresponding physical processes are strongly suppressed. For example, the two NC and CC decays

$$K^+ \to \pi^+ + v_e + \bar{v}_e, \quad K^+ \to \pi^+ + v_e + e^+ \tag{2.49}$$

should proceed with similar probabilities. On the contrary, the former decay is strongly suppressed.

S. Glashow, I. Iliopoulos, and L. Maiani observed in 1970 [28] that the d' and u states can be thought of as the members of the doublet $\begin{pmatrix} u \\ d' \end{pmatrix}$. Now, they thought, a fourth quark might exist, the 'charm' c as the missing partner of s', to form a second similar doublet $\begin{pmatrix} c \\ s' \end{pmatrix}$.

Since s' is orthogonal to d' we have

$$s' = -d\cos\theta_C + s\cos\theta_C. \tag{2.50}$$

Clearly, the relationship between the two bases is the rotation

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} \tag{2.51}$$

From historical point of view this was the prediction of a new flavour.

To see how the 'GIM' mechanism succeeds in suppressing the strangeness-changing neutral currents, in addition to the terms Eq.(2.48) now one has,

$$\bar{s}'_L\gamma_a s'_L = \sin^2\theta_C \bar{d}_L\gamma_a d_L + \cos^2\theta_C \bar{s}_L\gamma_a s_L - \cos\theta_C \sin\theta_C[\bar{d}_L\gamma_a s_L + \bar{s}_L\gamma_a d_L] \tag{2.52}$$

summing the two. one gets

$$\bar{s}'_L\gamma_a s'_L + \bar{d}'_L\gamma_a d'_L = \bar{d}_L\gamma_a s_L + \bar{s}_L\gamma_a d_L \tag{2.53}$$

The SCNC cancel out. However, a NC term remains in the Lagrangian, namely the NC between equal quarks or, in other words, the strangeness-conserving neutral current. It is observed here that the Cabibbo rotation is irrelevant for the NC term. In other words this term is the same in the two bases.

Chapter 3

Left Right Symmetric Model

Regardless of the fact that the SM stays unchallenged by the effect of impressive body of precision electroweak measurements. It lacks those attributes which are pointed out as deficient by the Physicists. As it lacks the recognition of the number of generations, the root of quantum number assignments and also there exist some complications like the strong CP-violating parameter and the small CKM matrix off-diagonal elements is one of the most important question and also the lacking of its leptonic counterpart PMNS. Hence it is compulsory to look beyond SM to answer all the ambiguities.

Success of the gauge theories have highly influenced the theoretical thoughts regarding physics beyond the SM. For example, GUTs (grand unified theories) has been contemplated as Standard model's natural extension but to take into consideration the three families GUTs still simply triplicate the particle content. Apart from the fact that fermions are termed as constituent of the mated multiplet of the group that is utilized for the grand unification, hence at GUT Scale their masses have to be equal. It is not possible to get any ansatz that might give the answers to all or at least some of the above questions from these theories.

Super Symmetric theories (SUSY) are the other most eminent theories to consider and explore the physics beyond Standard Model which are extensively used to signify the high energy physics data. Concerning to the mixing of flavors and symmetry-breaking sector, the condition in a super symmetric theory happens to be much detrimental than in SM. Except the fact that three families are established through hand conforming to SM hence the scalar sector must be expanded further to cope up with super symmetry.

In addition when the SM particles introduces its existing, super symmetry particles contributes many more sources of CP violation and flavor mixing, hence its necessary to reconcile the theory to be consistent with data. Above all, these noted small CP violations which are only noticed in the charged sectors are not capable enough to be introduced into a super symmetric theory in a natural way. Through all this understanding it is possible to consider that super symmetry undoubtly has flavor problem.

While considering Physics beyond the SM, an idea of simply extending existing SM is drawn without even disturbing its structure. There exists numerous concepts considered and discussed in the literature regarding how to acheive the extension of SM, one out of those is Left Right Symmetric Models [13,29,30]. SM gets to so called Left Right symmetry because LRSMs are based on the notion that along with the Left handed doublets Right-handed ones should also be present. The most astonishing feature of LRSMs is the existence of the Right handed neutrinos consisting Yukawa couplings, can generate small neutrino masses through see-saw mechanism. One more distinction of LRSMs is that they can be unified together in the form of grand unified schemes.

3.1 Left Right Symmetric Models (LRSMs)

SM does not have the Left Right symmetry of basic Lagrangian as only Left handed neutrinos are introduced without any exemplary elucidation then the phenomenological fact that neutrinos are mass less or very light. The gauge group on which the LRSMs are just based is $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, with the symmetric treatment of both Left and Right handed fermion fields. A parity-violating theory like SM at the low energies is generated by breaking Left Right symmetry at a high scale.

When SM is protracted further to the LRSMs, the hypercharge quantum number Y now turns out to be the difference between the baryonic and leptonic number [25,31]. Two special postulated scenarios in the weak sector have generally been discussed historically.

- The first one "the manifest LR symmetry" [13] assumes that CP violation do not arise from the spontaneous symmetry breaking. The mass matrices for quarks are thus hermitian and the Right- and Left-handed quark mixing come to be identical.

- The other is known as "pseudo-manifest LR symmetry" [32] suggests that violation of CP arises utterly from the spontaneous symmetry breaking of the vacuum and the entire Yukawa couplings are real. Furthermore, it assumes the quark mass matrices are symmetric and complex, suggesting that the right handed quark mixing is proportional to the complex conjugate of CKM matrix multiplied by the additional phases.

The LRSM constructed originally by an $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry. The couplings of $SU(2)_L$ and $SU(2)_R$ are equal, because the $L-R$ invariance is considered, i.e. $g_L = g_R = g$. The quarks are allocated in the below doublets

$$\begin{aligned} Q_{iL} &= \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix} \equiv (2, 1, \frac{1}{3}) \\ Q_{iR} &= \begin{pmatrix} u_{iR} \\ d_{iR} \end{pmatrix} \equiv (1, 2, \frac{1}{3}) \end{aligned} \tag{3.1}$$

or leptons it could be written as

$$\begin{aligned} L_{iL} &= \begin{pmatrix} v_{iL} \\ e_{iL} \end{pmatrix} \equiv (2, 1, -1) \\ L_{iR} &= \begin{pmatrix} v_{iR} \\ e_{iR} \end{pmatrix} \equiv (1, 2, -1) \end{aligned} \tag{3.2}$$

with $i = 1, 2, 3$ representing generation index.

Two triplets are introduced for gauge bosons:

$$\begin{aligned} W_{\mu L} &= \begin{pmatrix} W^+_{\mu L} \\ Z^0_{\mu L} \\ W^-_{\mu L} \end{pmatrix} \equiv (3, 1, 0) \\ W_{\mu R} &= \begin{pmatrix} W^+_{\mu R} \\ Z^0_{\mu R} \\ W^-_{\mu R} \end{pmatrix} \equiv (1, 3, 0) \end{aligned} \tag{3.3}$$

and one singlet

$$B_\mu = B^0_\mu \equiv (1, 1, 0) \tag{3.4}$$

The charge introduced in LRSMs is given below

$$Q = I_{3L} + I_{3R} + \frac{B-L}{2} \tag{3.5}$$

where $(B-L)$ specifying gauge symmetry of LRSMs. At $E > M_W$, charge(Q) and I_{3L} remain preserved, so local $B-L$ invariances and parity are broken extemporaneously. The symmetry breaking intelligible form can be written as:

$$\begin{gathered} SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ \downarrow M_{W_R} \\ SU(3)_C \times SU(2)_L \times U(1)_Y \\ \downarrow M_{W_L} \\ SU(3)_C \times U(1)_{e.m} \end{gathered}$$

One of the salient subject in the LR models has been the scale at which Right handed current interactions become notable. The constraints on the Right handed W bosons mass M_{W_R} has been explored in [33]. The present experimental bound on M_{W_R} in direct collider search is about $800GeV$ [34]. Given these values, it could be relevant enough to have a Right handed gauge boson with mass of order $1-2$ TeV.

For the extension of gauge sector, the number of scalar particle are extended further to apply the symmetry breaking mechanism. A bi-doublet scalar boson is needed to implement symmetry breaking process because both leptons and quarks are present in doublets. The doublet iutroduced comprises Higgs doublet of SM and is Left Right symmetric.

$$H \sim (2,2,0) \sim \begin{pmatrix} h_1^0 & h_1^+ \\ h_2^- & h_2^0 \end{pmatrix}, \quad \tilde{H} = \tau_2 H^* \tau_2 \sim (2,2,0) \tag{3.6}$$

However one needs to have some additional Higgs multiplets to arrive at a satisfactory symmetry pattern. The simplest possibility is to choose a doublet [29].

$$\chi_R \sim (1,2,1) \sim \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix} \tag{3.7}$$

Due to LR invariance Left handed doublet is introduced as well

$$\chi_L \sim (2,1,1) \sim \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix} \tag{3.8}$$

The Right doublet $(SU(2)_R)$, then broken at the M_{W_R} (Right handed scale) by the VEVs of χ_R and VEVs of H breaks EW symmetry. Same masses and similar charges for both Left-and the Right-weak bosons is rendered by H (bi-doublet) hence it is beneficial to consider χ_R.

Higgs doublets χ_L and χ_R introduced are singlets of either Left or Right gauge groups. In order to overcome the problem of giving masses to neutrinos, two Higgs triplets are introduced which is very useful to explain the couplings of leptons with triplets. Hence the outcome is that neutrinos become Majorana particles [15, 24, 35]. The two more triplets introduced are given as

$$\begin{aligned} \Delta_L \sim (3,1,2) \sim \begin{pmatrix} \delta_L^+/\sqrt{2} & \delta_L^{++} \\ \delta_L^0 & -\delta_L^+/\sqrt{2} \end{pmatrix}. \\ \Delta_R \sim (1,3,2) \sim \begin{pmatrix} \delta_R^+/\sqrt{2} & \delta_R^{++} \\ \delta_R^0 & -\delta_R^+/\sqrt{2} \end{pmatrix} \end{aligned} \tag{3.9}$$

Due to LR invariance the symmetry requires

$$Q_L \leftrightarrow Q_R, \qquad H \leftrightarrow H^\dagger \tag{3.10}$$

$$\Delta_L \leftrightarrow \Delta_R, \qquad \chi_L \leftrightarrow \chi_R \tag{3.11}$$

By this means that $g_L = g_R$ as it was declared above. So its clear that [13,36] if exactly symmetric potential exists for Higgs with reference to the discrete transformation, one can chose the vacuum as $v_R \gg v_L$ explaining parity violation at low energies as an outcome of SSB. One can also define it as; smallness of neutrino masses and parity non-conservation at low energies has same origin in a Left-Right symmetric model.

While discussing the two cases about break symmetry impulsive, it is significant to give the reference about most of the literature about LRSMs. The first choice which specified a Higgs bi-doublet and triplet, and the second in which Higgs doublets χ_L, χ_R are noted along with bi-doublet H. Nonetheless both form of LRSMs which are discussed makes significance of a bi-doublets H, two doublets χ_L, χ_R and two Higgs triplet Δ_L, Δ_R.

A special case of LRSM will be discussed in coming section with an additional family symmetry to give the importance of using so many scalars. Symmetry breaking scales could be altered to make it stay away from FCNCs existence at tree level (to replicate the SM like texture) even though it is very difficult to achieve.

One can write the most common scalar potential as below as below [37–40]

$$V = V_H + V_\chi + V_\Delta + V_{H\chi} + V_{H\Delta} + V_{\chi\Delta} + V_{H\chi\Delta} \tag{3.12}$$

It is beyond the point of discussion to take into the consideration, corresponding potential for spontaneous symmetry breaking. It is thoroughly mentioned in [13] (taking into consideration only one H and two χ_L, χ_R). The VEV for the scalar field consistent with the Higgs potential minima, requisite for $L-R$ symmetry breaking are given below accompanied by certain imposed assumptions on them.

$$\langle H\rangle = \begin{pmatrix} v\sin\beta & 0 \\ 0 & v\cos\beta \end{pmatrix}, \quad \langle \tilde{H}\rangle = \begin{pmatrix} v\cos\beta & 0 \\ 0 & v\sin\beta \end{pmatrix} \tag{3.13}$$

Similarly

$$\langle \chi_L\rangle = \begin{pmatrix} 0 \\ \lambda_L \end{pmatrix}, \qquad \langle \chi_R\rangle = \begin{pmatrix} 0 \\ \lambda_R \end{pmatrix} \tag{3.14}$$

$$\langle \Delta_L\rangle = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}, \qquad \langle \Delta_R\rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \tag{3.15}$$

v is the SM Higgs VEV, since $SU(2)_L$ is broken through v_L which in the first step of symmetry breaking is kept preserved so with this constraint one essentially has to specify $v_R \gg v_L$ which infact provides really heavy masses to the Right handed weak boson W_R^-, W_R^+, and Z_R^0. [41] suggests further that v_R ought to be not less than 2.7×10^7 GeV to match the experimental restrictions from neutrinos [34,39]. Furthermore, Δ_L is a triplet under $SU(2)_L$ so v_L ought to be much less than v_R not to spoil the eminent condition from experiments $M_{W_L}^2/M_{Z_L}^2 \simeq \cos^2\theta_W, \rho = 1$ relation. In the same way, $\lambda_L\lambda_R = \mathcal{O}(v^2)$ and $\tan\beta \gg 1$ which is contemplated for unraveling the mass difference between the third family (top,bottom) quarks.

3.2 LRSMs with additional $U(1)_{family}$ symmetry

Among the foremost aims to go beyond SM is acquiring some perception concerning hierarchy between the quark masses. It is enticing to use the symmetry arguments for elucidating the flavor structure of SM. A family symmetry on the LRSMs is applied here to get some understanding of flavor physics. However there is neither a predictive nor an approved framework for flavor till now. Certain constraints have to be satisfied by the applied family symmetry. The general inference is that the observed structure to the quark mass matrices is given by such a symmetry. Among the significant aspect of this symmetry, one is that it cannot be exact, so it need to be broken. Furthermore, the scalar sector needs to be enlarged because this symmetry cannot be spontaneously broken by the VEV (vacuum expectation value)

	u	d	s
m_q	$[1.5-3.3]MeV$	$[3.5-6.0]MeV$	$[70-135]MeV$
$\log_\lambda(m_q/m_t)$	$7-8$	$6-7$	$4-5$
	c	b	t
m_q	$[1.16-1.34]GeV$	$[4.13-4.37]GeV$	$\sim172GeV$
$\log_\lambda(m_q/m_t)$	$3-4$	$2-3$	1

Table 3.1: Quark masses in SM with imprecise scaling with Wolfenstein parameter $\lambda \sim 0.2$

of single scalar field. This has been elucidated before in foregoing section considering more than one scalar Higgs specifically, two Higgs doublet, one bi-doublet, and triplets.

The gauge group under consideration is $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_{family}$. Quark multiplets and scalar fields are contemplated to undergo a marked change under an additional family-dependent global $U(1)$ symmetry. Accompanied by necessity of values assigned to the new charges, the attainable patterns giving rise to distinct textures for masses of up and down type quarks can be obtained.

By applying the befitting $U(1)_{family}$ charges, intention here is to permit or disallow definite entries in Yukawa matrices to attain the observed hierarchy in the CKM elements and quark mass matrices.

The approximate scaling of quark masses with the Wolfenstein parameters "$\lambda \sim \epsilon$" are stated in Table.(3.1). The right handed quarks exists as doublets under the $SU(2)_R$ gauge group in LRSM, therefore an analogous CKM matrix exists for Right handed V^R_{CKM}, consonant to SM CKM matrix V^L_{CKM}. As LRSM is set up with axial $U(1)_{family}$ charges, the same power counting i.e. $V^R_{CKM} = V^L_{CKM}$ is expected for Left Right CKM matrices.

$$V^R_{CKM} \sim V^L_{CKM} \sim \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} \tag{3.16}$$

The power counting which is used for the effective SM Yukawa-matrices and the CKM elements for the up-type quarks is ,

$$
\begin{aligned}
Y_U &= V_L diag[y_u, y_c, y_t] V_R^\dagger \\
&\sim V_L diag[\epsilon^7, \epsilon^3, 1] V_R^\dagger \\
&\sim \begin{pmatrix} \epsilon^7 & \epsilon^4 & \epsilon^3 \\ \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}
\end{aligned}
\tag{3.17}
$$

and for the down-type quarks one can write

$$
\begin{aligned}
Y_D &= \tilde{V}_L diag[y_u, y_c, y_t] \tilde{V}_R^\dagger \\
&\sim \tilde{V}_L diag[\epsilon^7, \epsilon^3, 1] \tilde{V}_R^\dagger \\
&\sim \begin{pmatrix} \epsilon^{6(7)} & \epsilon^{5(6)} & \epsilon^5 \\ \epsilon^{5(6)} & \epsilon^{4(5)} & \epsilon^4 \\ \epsilon^5 & \epsilon^4 & \epsilon^2 \end{pmatrix}
\end{aligned}
\tag{3.18}
$$

with the assumption that generically not only $V_{CKM} \sim V_L \sim V_R$ but some of the elements of Y_U and Y_D could also be small then the represented value or it can even be zero.

3.2.1 Viable Gauge Couplings

The $L-R$ gauge symmetry strongly constrains the possible scalar couplings to fermionic fields. All the viable operators that are satisfying the $L-R$ symmetry to the dimension-8 are contemplated and are analyzed one by one as follow.

Dimension-4

The Higgs bi-doublet H, $\tilde{H}$ couples with the quarks and gives the dimension-4 terms given below,

$$
\begin{aligned}
O^{(4)} &= (\bar{Q}_L H Q_R) \rightarrow v\mathrm{sin}\beta(\bar{U}_L U_R) + v\mathrm{cos}\beta(\bar{D}_L D_R), \\
\tilde{O}^{(4)} &= (\bar{Q}_L \tilde{H} Q_R) \rightarrow v\mathrm{cos}\beta(\bar{U}_L U_R) + v\mathrm{sin}\beta(\bar{D}_L D_R)
\end{aligned}
\tag{3.19}
$$

The contributions from Yukawa couplings rendered to the up and down type quarks from the dimension-4 are eminent by condition $\tan\beta \gg 1$. Taking into account the viable power counting, it could be written as

$$
\sin\beta = \mathcal{O}(1), \qquad \cos\beta = \mathcal{O}(\epsilon^2) \tag{3.20}
$$

such that the $3-3$ elements of the Y_U and Y_D (i.e. $m_t = v\sin\beta$ and $m_b = v\cos\beta$) could be generated from the operator $\mathcal{O}^4$, while $\bar{\mathcal{O}}$ operator is not permitted.

Dimension-5

Coupling of doublets χ_L and χ_R with the quarks gives dimension−5 .

$$\begin{aligned} O^{(5)} &= \frac{1}{\Lambda}(\bar{Q}_L\chi_L^c)(\chi_R^T Q_R) \to v\frac{\lambda_L\lambda_R}{v\Lambda}(\bar{U}_L U_R), \\ \tilde{O}^{(5)} &= \frac{1}{\Lambda}(\bar{Q}_L\chi_L)(\chi_R^\dagger Q_R) \to v\frac{\lambda_L\lambda_R}{v\Lambda}(\bar{D}_L D_R) \end{aligned} \tag{3.21}$$

where χ_R^T denotes transposed fields including anti-symmetric tensor in doublets ($SU(2)$), i.e. $\chi_R^T \equiv \epsilon_{ij}\chi_R^i Q_R^j$. Here it is mandatory to note that contributions from the dimension−5 to quark mass matrix are all suppressed by the $\mathcal{O}(\frac{v}{\Lambda})$.

The ϵ^2 contribution to the $2-3$ element of Y_U is assumed to arise via the dimension−5 terms which restricts

$$\frac{v}{\Lambda} \sim \mathcal{O}(\epsilon^2) \tag{3.22}$$

with the further suggestion

$$\frac{v_R}{\Lambda} \sim \frac{\lambda_R}{\Lambda} \sim \mathcal{O}(\epsilon), \qquad \frac{v_L}{\Lambda} \sim \frac{\lambda_L}{\Lambda} \sim \mathcal{O}(\epsilon^3) \tag{3.23}$$

which gives

$$\mathcal{O}_5 \sim \bar{\mathcal{O}}_5 \sim \epsilon^2 \tag{3.24}$$

Dimension-6

The following operators contribute into dimension−6:

$$
\begin{aligned}
O_a^{(6)} &= \frac{1}{\Lambda^2}(\bar{Q}_L\chi_L^c)(\chi_R^\dagger\Delta_R Q_R) \to v\frac{v_R}{\Lambda}\frac{\lambda_L\lambda_R}{v\Lambda}(\bar{U}_L U_R),\\
\tilde{O}_a^{(6)} &= \frac{1}{\Lambda^2}(\bar{Q}_L\chi_L)(\chi_R^T\Delta_R^\dagger Q_R) \to v\frac{v_R}{\Lambda}\frac{\lambda_L\lambda_R}{v\Lambda}(\bar{D}_L D_R),\\
O_b^{(6)} &= \frac{1}{\Lambda^2}(\bar{Q}_L\Delta_L^\dagger\chi_L)(\chi_R^T Q_R) \to v\frac{v_L}{\Lambda}\frac{\lambda_L\lambda_R}{v\Lambda}(\bar{U}_L U_R),\\
\tilde{O}_b^{(6)} &= \frac{1}{\Lambda^2}(\bar{Q}_L\Delta_L\chi_L^c)(\chi_R^\dagger Q_R) \to v\frac{v_L}{\Lambda}\frac{\lambda_L\lambda_R}{v\Lambda}(\bar{D}_L D_R),\\
O_c^{(6)} &= \frac{1}{\Lambda^2}(\bar{Q}_L\Delta_L^\dagger\tilde{H}\Delta_R Q_R) \to v\sin\beta\frac{v_L v_R}{\Lambda^2}(\bar{U}_L U_R),\\
\tilde{O}_c^{(6)} &= \frac{1}{\Lambda^2}(\bar{Q}_L\Delta_L^\dagger\Delta_R^\dagger Q_R) \to v\sin\beta\frac{v_L v_R}{\Lambda^2}(\bar{D}_L D_R),\\
O_d^{(6)} &= \frac{1}{\Lambda^2}(\bar{Q}_L\Delta_L^\dagger H\Delta_R Q_R) \to v\cos\beta\frac{v_L v_R}{\Lambda^2}(\bar{U}_L U_R),\\
\tilde{O}_d^{(6)} &= \frac{1}{\Lambda^2}(\bar{Q}_L\Delta_L\tilde{H}\Delta_R^\dagger Q_R) \to v\cos\beta\frac{v_L v_R}{\Lambda^2}(\bar{D}_L D_R)
\end{aligned} \tag{3.25}
$$

Also one gets

$$
\begin{aligned}
O_e^{(6)} &= \frac{1}{\Lambda^2}(\bar{Q}_L H Q_R)[tr(H^\dagger\tilde{H} + \tilde{H}^\dagger H)]\\
&\to v\frac{v^2\sin 2\beta}{\Lambda^2}(\sin\beta(\bar{U}_L U_R) + \cos\beta(\bar{D}_L D_R),
\end{aligned} \tag{3.26}
$$

$$
\begin{aligned}
\tilde{O}_e^{(6)} &= \frac{1}{\Lambda^2}(\bar{Q}_L\tilde{H}Q_R)tr[H^\dagger\tilde{H} + \tilde{H}^\dagger H]\\
&\to v\frac{v^2\sin 2\beta}{\Lambda^2}(\cos\beta(\bar{U}_L U_R) + \sin\beta(\bar{D}_L D_R)
\end{aligned}
$$

Dim−6 operators also includes some contributions which are not appropriate having same combinations as dim−4 and hence will not be able to create new entries in the quark-Yukawa matrices. The analyzed power counting from above thus can be given as

$$
\begin{aligned}
\mathcal{O}_{6a} &\sim \bar{\mathcal{O}}_{6a} \sim \epsilon^3\\
\mathcal{O}_{6c} &\sim \tilde{\mathcal{O}}_{6c} \sim \epsilon^4\\
\mathcal{O}_{6b} &\sim \tilde{\mathcal{O}}_{6b} \sim \epsilon^5\\
\mathcal{O}_{6e} &\sim \tilde{\mathcal{O}}_{6e} \sim \epsilon^6 + \epsilon^8\\
\mathcal{O}_{6d} &\sim \tilde{\mathcal{O}}_{6d} \sim \epsilon^6
\end{aligned} \tag{3.27}
$$

Dimension-7

Dimension−7 operators for Quarks are given as under

$$\begin{aligned}
O_a^{(7)} &= \frac{1}{\Lambda^3}(\bar{Q}_L H Q_R)(\chi_R^\dagger \Delta_R \chi_R^C + \chi_L^T \Delta_L^\dagger \chi_L) \\
&\to v\frac{v_R\lambda_R^2 + v_L\lambda_L^2}{\Lambda^3}(\sin\beta(\bar{U}_L U_R) + \cos\beta(\bar{D}_L D_R)), \\
\tilde{O}_a^{(7)} &= \frac{1}{\Lambda^3}(\bar{Q}_L \tilde{H} Q_R)(\chi_R^\dagger \Delta_R \chi_R^C + \chi_L^T \Delta_L^\dagger \chi_L) \\
&\to v\frac{v_R\lambda_R^2 + v_L\lambda_L^2}{\Lambda^3}(\cos\beta(\bar{U}_L U_R) + \sin\beta(\bar{D}_L D_R)), \\
O_b^{(7)} &= \frac{1}{\Lambda^3}(\bar{Q}_L H Q_R)(\chi_L^C \Delta_R \chi_R^\dagger + \chi_L \chi_R^T \Delta_R^\dagger) \\
&\to v\frac{v_R\lambda_R\lambda_L}{\Lambda^3}(\sin\beta(\bar{U}_L U_R) + \cos\beta(\bar{D}_L D_R)), \\
\tilde{O}_b^{(7)} &= \frac{1}{\Lambda^3}(\bar{Q}_L \tilde{H} Q_R)(\chi_L^C \Delta_R \chi_R^\dagger + \chi_L \chi_R^T \Delta_R^\dagger) \\
&\to v\frac{v_R\lambda_R\lambda_L}{\Lambda^3}(\cos\beta(\bar{U}_L U_R) + \sin\beta(\bar{D}_L D_R))
\end{aligned} \tag{3.28}$$

The terms arising in power counting due to the above mentioned operators are

$$\begin{aligned}
\mathcal{O}_{7a} &\sim \tilde{\mathcal{O}}_{7a} \sim \epsilon^3 + \epsilon^5 + \text{higher order terms} \\
\mathcal{O}_{7b} &\sim \tilde{\mathcal{O}}_{7b} \sim \epsilon^3 + \epsilon^6 + \text{higher order terms}
\end{aligned} \tag{3.29}$$

Dimension-8

Dimension−8 operators for Quarks are given as under

$$\begin{aligned}
O_a^{(8)} &= \frac{1}{\Lambda^4}(\bar{Q}_L H H^\dagger Q_R)(\chi_L^\dagger \Delta_R \chi_R^C) \\
&\to v^2\frac{v_R\lambda_L\lambda_R}{\Lambda^4}(\sin^2\beta(\bar{U}_L U_R) + \cos^2\beta(\bar{D}_L D_R)), \\
\tilde{O}_a^{(8)} &= \frac{1}{\Lambda^4}(\bar{Q}_L \tilde{H} \tilde{H}^\dagger Q_R)(\chi_L^\dagger \Delta_R \chi_R^C) \\
&\to v^2\frac{v_R\lambda_L\lambda_R}{\Lambda^4}(\cos^2\beta(\bar{U}_L U_R) + \sin^2\beta(\bar{D}_L D_R))
\end{aligned} \tag{3.30}$$

The terms arising in power counting due to the above mentioned operators are

$$\mathcal{O}_{8a} \sim \tilde{\mathcal{O}}_{8a} \sim \epsilon^7 + \text{higher order terms} \tag{3.31}$$

giving the capability to reproduce the hierarchies in the quark mass matrices. Also, for taking into the consideration all viable contributions from quark masses to a given order ϵ^n, one has to contemplate all the operators to dimension $d = 4 + n$. In short, the terms of the order ϵ^6 or higher are not considered.

3.2.2 Specifying appropriate $U(1)_{family}$ charges

Except for the power counting explained in previous section, for producing eminent hierarchies in mass matrices of quarks, below mentioned initial suppositions are also applied:

- To break the discrete L-R symmetry, $Z(Q^i_L) = -Z(Q^i_R)$ charges are taken throughout.
- A charged Higgs bi-doublet H is taken and by means of this consideration, H and $\tilde{H}$ can be eminent, so the top-bottom splitting can be described by $\tan\beta \gg 1$. Additionally, some terms will also be debarred in the scalar potential . So the new $U(1)_{family}$ charges are fixed as

$$Z(H) = -Z(\tilde{H}) = +2 \tag{3.32}$$

- With this assumption, the third family charges are confirmed as

$$Z(Q^{III}_L) = +1, \qquad Z(Q^{III}_R) = -1 \tag{3.33}$$

  and also the allowed dim-4 Yukawa term is,

$$y_{33}\bar{Q}^{III}_L H Q^{III}_R + h.c \tag{3.34}$$

  while $\bar{Q}^{III}_L \tilde{H} Q^{III}_R$ are debarred.

- For still existing inter-generational combinations do not participate at dim-4 Yukawa terms, one needs that $Z(Q^{I,II}) \pm Z(Q^{III}) \neq \pm Z(H)$ due to which $Z(Q^{I,II}) \neq \pm 1, \pm 3$ is obtained.
- which allows

$$Z(Q^{II}_L) = 0 \quad Z(Q^{II}_R) = 0 \tag{3.35}$$

  a nominee for charge assignment for the second family. The charges are established for the doublet fields $\chi_{L,R}$ in such a manner, so that dim-5 terms give mass to second family. This gives

$$Z(\chi_L) = 0 \;\; Z(\chi_R) = 0 \tag{3.36}$$

and the term of dim−5

$$y_{22}\bar{Q}_L^{II}\chi_L\chi_R^\dagger Q_R^{II} + \tilde{y}_{22}\bar{Q}_L^{II}\chi_L^C\chi_R^T Q_R^{II} + h.c, \tag{3.37}$$

is permitted.

- For the still available inter-generational combinations do not participate at dim−4 or the dim−5, $Z(Q^I) \neq \pm 0, 1, 2, 3$ should be eliminated, following from the constraints $Z(Q^I) \pm Z(Q^{II,III}) \neq \pm Z(H)$ and $Z(Q^I) \pm Z(Q^{II,III}) \neq \pm Z(\chi_L\chi_R^\dagger) = 0$

  This leaves the simplest possible charge assignment for the first generation of quarks as:

$$Z(Q_L^I) = -4, \quad Z(Q_R^I) = +4 \tag{3.38}$$

- For fixing the charges for triplet fields Δ_L, Δ_R viable dim-6 contributions to up and down masses can be described now one of the simplest choice is

$$Z(\Delta_L) = -3, \qquad Z(\Delta_R) = +3, \tag{3.39}$$

  permitting

$$y_{11}(\bar{Q}_L^I\tau^a\tilde{H}\tau^b Q_R^I)\Delta_L^a(\Delta_R^\dagger)^b + h.c. \tag{3.40}$$

- It can be further incite generation-mixing terms at dim-6 level through the operators

$$y_{12}(\bar{Q}_L^I\tau^a H\tau^b Q_R^{II})\Delta_L^a(\Delta_R^\dagger)^b + y_{21}(\bar{Q}_L^{II}\tau^a H\tau^b Q_R^I)\Delta_L^a(\Delta_R^\dagger)^b + h.c + (L \leftrightarrow R) \tag{3.41}$$

  and

$$y_{13}(\bar{Q}_L^I\chi_L\chi_R^T\tau^a Q_R^{III})(\Delta_R^\dagger)^a + y_{31}(\bar{Q}_L^{III}\chi_L\chi_R^T\tau^a Q_R^I)(\Delta_R^\dagger)^a + h.c + (L \leftrightarrow R) \tag{3.42}$$

  Now the thing to observe here is that for this specific case, no off-diagonal mass matrix elements exists between second and the third generation. Likely the dim-7 and the dim-8 contributions can be taken into account with noticing above mentioned constraints.

By using above mentioned constraints, different values to $U(1)_{family}$ charges can be ascribed. The permitted values are given in Table.(3.2). It should be noticed that examples are given for charges that are less or identical 4 and

	$Z_{Q^I_L} = -Z_{Q^I_R}$	$Z_{Q^{II}_L} = -Z_{Q^{II}_R}$	$Z_{\chi_L} = -Z_{\chi_R}$	$Z_{\Delta_L} = -Z_{\Delta_R}$
Case I	+3	0	0	+1
Case II	+3	0	0	+3
Case III	4	0	0	1
Case IV	−4	0	0	−3
Case V	−2	+3	−3	+1
Case VI	0	3	−3	+3
Case VII	0	−4	−4	−3
Case VIII	0	4	−4	+1
Case IX	+2	+3	−3	+3
Case X	0	+3	−3	+1

Table 3.2: Viable charge assignments for quark and scaler fields, and its classification.

also the cases where charges that differs trivially by relative signs are not mentioned. The charges for third generation and the Higgs bi-doublet are determined as $Z_H = +2$ and $Z_{Q^{III}_L} = -Z_{Q^{III}_R} = +1$. After taking all these suppositions under consideration the possible values of $U(1)_{family}$ charges can be shown in the Table.(3.2).

Chapter 4

Phenomenology of the Left-Right models

At tree level, the appearance of FCNC processes is prohibited because of flavor diagonal structure in the basic vertices which includes Z, G and γ. However, through flavor changing $W^{\pm}$-vertex, the higher order diagrams and the one-loop which mediate FCNC processes could be constructed. It is a fact that such processes arises as loop effects only, forms them eminently fruitful for the testing of quantum structure of the theory and also in exploring physics beyond the SM. At the one-loop level, they can be elucidated through a set of basic triple (penguin diagrams) and also quartic (box-diagrams) effective vertices.

4.1 Effective Vertices

4.1.1 Penguin Vertices

The effective penguin vertices shown in Fig.(4.1) involve only quarks, where the charge of i and j is same but flavor is different and k signifies internal quark having unlike charge than i and j. By making use of elementary vertices and propagators, these effective vertices can be calculated.

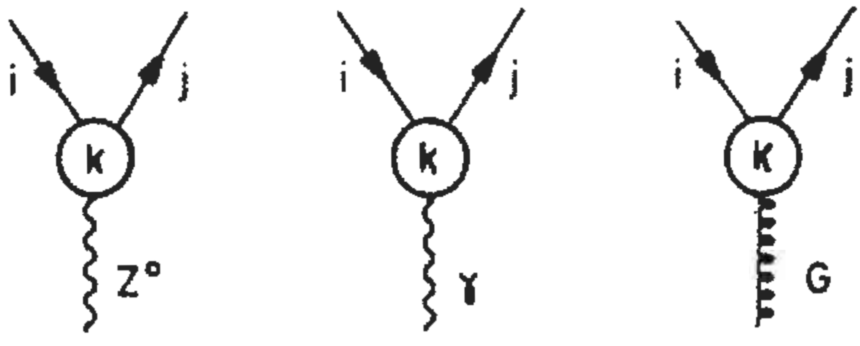

Figure 4.1: The Penguin vertices

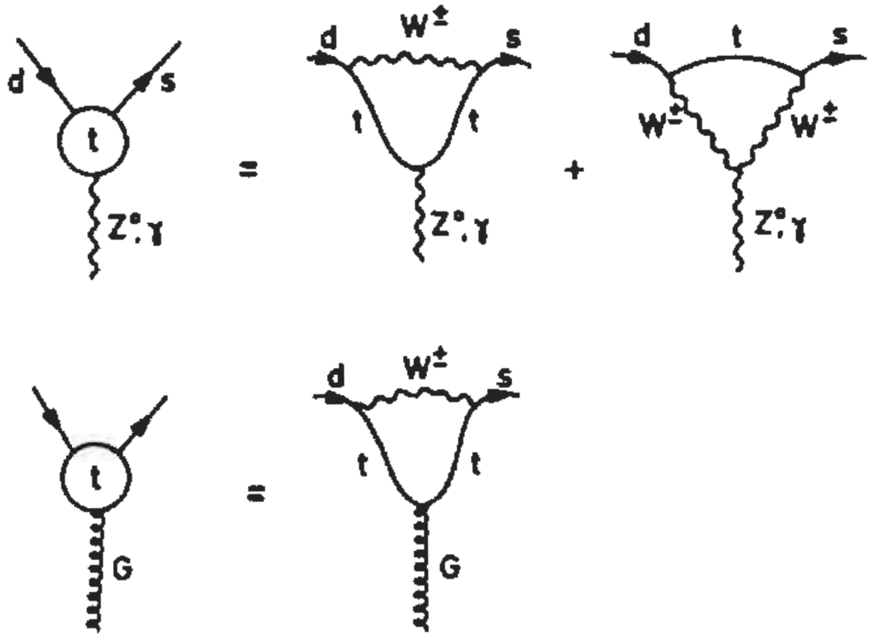

Figure 4.2: The Penguin vertices resolved in terms of basic vertices

4.1.2 Box Vertices

The box vertices mainly involve both the leptons and quarks and it could be described as in Fig.(4.3) and i, j, m, n represents leptons or external quarks and k, l signifies leptons and internal quarks. The vertex (a) shows the flavor violation arises on both sides (left and right) of the box, while in (b) the right hand side acts as flavor conserving. By using the elementary propagators and vertices such effective quartic vertices can calculated. For example the vertices shown in Fig.(4.4) are contributing to $B_d^0 - \bar{B}_d^0$ mixing and also to $K^+ \to \pi^+ v\bar{v}$.

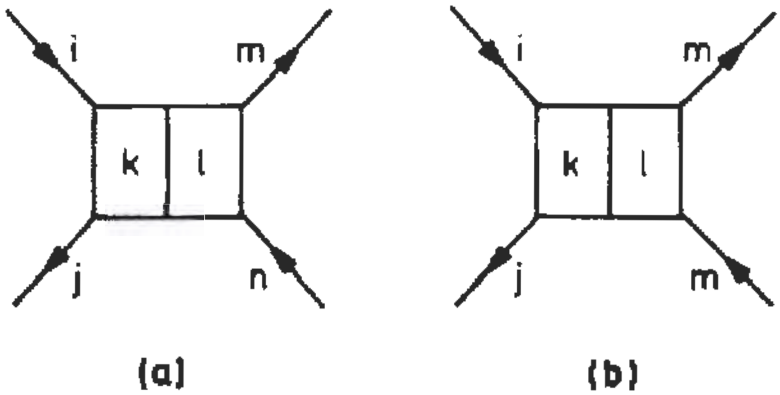

Figure 4.3: The Box vertices

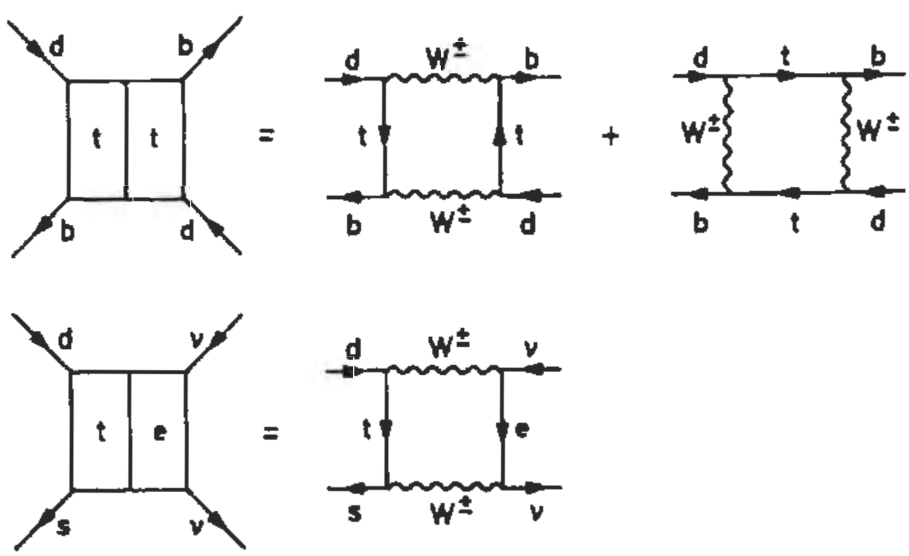

Figure 4.4: The Box vertices resolved in terms of basic vertices

4.1.3 The Effective Feynman Rules

Feynman rules regarding the effective vertices mentioned in [42] for the $W^{\pm}$ propagator are,

$$Box(\Delta S=2)=\lambda_i^2\frac{G_F^2}{16\pi^2}M_W^2S_0(x_i)(\bar{s}d)_{V-A}(\bar{s}d)_{V-A} \tag{4.1}$$

$$Box(T_3=1/2)=\lambda_i\frac{G_F}{\sqrt{2}}\frac{\alpha}{2\pi\sin^2\Theta_W}[-4B_0(x_i)](\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A} \tag{4.2}$$

$$Box(T_3=-1/2)=\lambda_i\frac{G_F}{\sqrt{2}}\frac{\alpha}{2\pi\sin^2\Theta_W}B_0(x_i)(\bar{s}d)_{V-A}(\bar{\mu}\mu)_{V-A} \tag{4.3}$$

$$\bar{s}Zd=i\lambda_i\frac{G_F}{\sqrt{2}}\frac{e}{2\pi^2}M_Z^2\frac{\cos\Theta_W}{\sin\Theta_W}C_0(x_i)\bar{s}\gamma_\mu(1-\gamma_5)d \tag{4.4}$$

$$\bar{s}\gamma d=-i\lambda_i\frac{G_F}{\sqrt{2}}\frac{e}{8\pi^2}D_0(x_i)\bar{s}(q^2\gamma_\mu-q_\mu\not{q})(1-\gamma_5)d \tag{4.5}$$

$$\bar{s}G_d^a=-i\lambda_i\frac{G_F}{\sqrt{2}}\frac{g_s}{8\pi^2}E_0(x_i)\bar{s}_\alpha(q\gamma_\mu-q_\mu\not{q})(1-\gamma_5)T^a_{\alpha\beta}d_\beta \tag{4.6}$$

$$\bar{s}\gamma' b=i\bar{\lambda}_i\frac{G_F}{\sqrt{2}}\frac{e}{8\pi^2}D_0'(x_i)\bar{s}[i\sigma_{\mu\lambda}q^\lambda[m_b(1+\gamma_5)]]b \tag{4.7}$$

$$\bar{s}G_d'^a=-i\bar{\lambda}_i\frac{G_F}{\sqrt{2}}\frac{g_s}{8\pi^2}E_0'(x_i)\bar{s}_\alpha[i\sigma_{\mu\lambda}q^\lambda[m_b(1+\gamma_5)]]T^a_{\alpha\beta}b_\beta \tag{4.8}$$

where ,

$$\lambda_i=V_{is}^\star V_{id}\qquad \bar{\lambda}_i=V_{is}^\star V_{ib} \tag{4.9}$$

In the above rules, q_μ indicates the momentum of outgoing gluon or of photon and T_3 shows whether l^+l^- or $\nu\bar{\nu}$ is leaving box diagram. Only quarks are included in first rule and the last two rules iuvolves the on-shell gluon and photon. These rules are written with the condition $m_s=0$.

The basic functions present in Eqs.(4.1-4.9) were calculated by different authors, in particular by Inami and Lim [43] given below ,

$$B_0(x_t)=\frac{1}{4}\Big[\frac{x_t}{1-x_t}+\frac{x_t\ln x_t}{(x_t-1)^2}\Big] \tag{4.10}$$

$$C_0(x_t)=\frac{x_t}{8}\Big[\frac{x_t-6}{x_t-1}+\frac{3x_t+2}{(x_t-1)^2}\ln x_t\Big] \tag{4.11}$$

$$D_0(x_t)=-\frac{4}{9}\ln x_t+\frac{19x_t^3+25x_t^2}{36(x_t-1)^3}+\frac{x_t^2(5x_t^2-2x_t-6)}{18(x_t-1)4}\ln x_t \tag{4.12}$$

$$E_0(x_t) = -\frac{2}{3}\ln x_t + \frac{x_t^2(15-16x_t+4x_t^2)}{6(1-x_t)^4\ln x_t} + \frac{x_t(18-11x_t-x_t^2)}{12(1-x_t)^3} \tag{4.13}$$

$$D_0(x_t) = -\frac{(8x_t^3+5x_t^2-7x_t)}{12(1-x_t)^3} + \frac{x_t^2(23x_t)}{2(1-x_t)^4}\ln x_t \tag{4.14}$$

$$E_0'(x_t) = -\frac{x_t(x_t^2-5x_t-2)}{4(1-x_t)^3} + \frac{3}{2}\frac{x_t^2}{(1-x_t)^4}\ln x_t \tag{4.15}$$

$$S_0(x_t) = \frac{4x_t-11x_t^2+x_t^3}{4(1-x_t)^2} - \frac{3x_t^3\ln x_t}{2(1-x_t)^3} \tag{4.16}$$

$$S_0(x_c) = x_c \tag{4.17}$$

$$S_0(x_c,x_t) = x_c\left[\ln\frac{x_t}{x_c} - \frac{3x_t}{4(1-x_t)} - \frac{3x_t^2\ln x_t}{4(1-x_t)^2}\right] \tag{4.18}$$

- In the last two expressions only terms linear in $x_c \ll 1$ are kept, but of course all orders in x_t. The function $S_0(x_t)$ in Eq.(4.16) incorporate box diagrams with simultaneous charm- and top-quarks exchanges.

- The "0" in subscript shows that functions do not comprise the QCD corrections to the related penguin and box diagrams.

- The x_t-independent terms are excluded having no contribution to decays because of the GIM mechanism. Moreover

$$S_0(x_t) \approx F(x_t,x_t) + F(x_u,x_u) - 2F(x_t,x_u) \tag{4.19}$$

Also

$$S_0(x_i,x_j) = F(x_i,x_j) + F(x_u,x_u) - F(x_i,x_u) - F(x_j,x_u) \tag{4.20}$$

with $F(x_i,x_j)$ specifying the true function which corresponds to i and j quark exchanges of a given box diagram. These distinct combinations could be obtained by drawing all the attainable box diagrams (also with u-quark exchanges), and taking $m_u = 0$ and by using unitarity of the CKM-matrix which suggests the following relation:

$$\lambda_u + \lambda_c + \lambda_t = 0. \tag{4.21}$$

Hence for FCNC transitions, only summing over t and c quarks the effective Hamiltonians is obtained [44, 45].

$K^0 - \bar{K}^0$-mixing	$S_0(x_t), S_0(x_c, x_t)$
$B^0 - \bar{B}^0$-mixing	$S_0(x_t)$
$K \to \pi v \bar{v}, B \to X_{d,s} \bar{v}$	$X_0(x_t)$
$K_L \to \mu\bar{\mu}, B \to ll$	$Y_0(x_t)$
$K_L \to \pi^0 e^+ e^-$	$E_0(x_t), Z_0(x_t), Y_0(x_t)$
$B \to X_s \gamma$	$D_0'(x_t), E_0'(x_t)$
$B \to X_s \mu^+ \mu$	$E_0(x_t), E_0'(x_t), Y_0(x_t), D_0'(x_t), Z_0(x_t),$

In this manner, a set of gauge independent functions controlling the FCNC process is written as

$$S_0(x_t), E_0(x_t), X_0(x_t), Y_0(x_t), Z_0(x_t), D_0'(x_t), E_0'(x_t) \tag{4.22}$$

There exist many decays depending only on a single function e.g $K^+ \to \pi^+ v \bar{v}$. Yet, mostly various basic functions can contribute to a given decay. More specifically, the correspondence exist among the most interesting FCNC processes and the basic functions is shown in Table.(4.1.3).

4.2 $K^0 - \bar{K}^0$ mixing

The investigation of neutral Kaon mixing is highly significant to understand the SM in particle physics. Initially, CP-violation was discovered in K_S regeneration experiments [46] and small value of $K_L - K_S$ mass difference laid the foundation for predicting the charm quark at the GeV scale [28, 47]. Within the SM only W-exchange dominates the Neutral Kaon mixing while beyond the SM, both Left handed and Right handed currents may contribute in the $K^0 - \bar{K}^0$ mixing process and the CP-violation parameter is also sensitive to new CP violating phases which are generically predicted by these models.

Except for he WW box found in the SM earlier, one important class of contributions comprises the exchange of a single W' in a box together with a W boson.

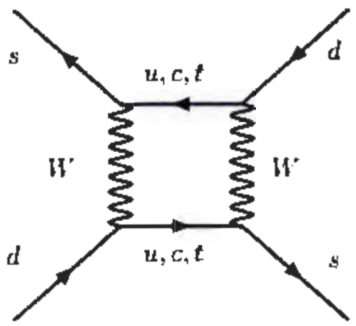

Figure 4.5: Diagram for kaon mixing in SM and LR models

4.2.1 SM

In SM, the single contribution for the kaon mixing is shown in Fig.(4.5) where two W bosons are exchanged in a way that the internal flavors can be u, c, t. The final expression is found for instance in [48], and is given below as

$$\begin{aligned} H_{SM} = \frac{G_F^2 M_W^2}{4\pi^2}\Big[&\lambda_c\lambda_c\eta_{cc}S(x_c) + \lambda_t\lambda_t\eta_{tt}S(x_t) \\ &+ 2\lambda_c\lambda_t\eta_{ct}S(x_c, x_t)\Big]Q_1 + h.c, \end{aligned} \tag{4.23}$$

$\eta_{cc}, \eta_{tt}, \eta_{ct}$ specifies the QCD corrections. In the absence of QCD correction $\eta_{cc}, \eta_{tt}, \eta_{ct} = 1$. In the above equation $\lambda_i = V_{is}V_{id}^*$ joins the two CKM matrix element with s, d representing the external quark flavours. The functions $S(x_i)$ are the related Inami-Lim functions [43] relying on the quark masses by $x_i = m_i^2/M_W^2$. Using the unitarity relation $\sum_{U=u,c,t}\lambda_U = 0$, the contribution from box diagrams with exchange of i-th and j-th up-quarks calculated in Appendix A can be written as,

$$\begin{aligned} H_{SM} = \frac{G_F^2 M_W^2}{4\pi^2}\Big[&\lambda_c\lambda_c S(x_c) + \lambda_t\lambda_t S(x_t) \\ &+ 2\lambda_c\lambda_t S(x_c, x_t)\Big]Q_1 + h.c \end{aligned} \tag{4.24}$$

with the only one operator mediating the transition is

$$Q_1 = \bar{s}\gamma^\mu P_L d\bar{s}\gamma_\mu P_L d \tag{4.25}$$

The functions $S(x_i), S(x_i, x_j)$ calculated in Appendix A are written as

$$
\begin{aligned}
&S(x_c) = x_c \\
&S(x_t) = x_t\Big(\frac{1}{4} + \frac{9}{4(1-x_t)} - \frac{3}{2(1-x_t)^2}\Big) - \frac{3}{2}\Big(\frac{x_t}{1-x_t}\Big)^3 \log x_t, \\
&S(x_c, x_t) = -x_c \log x_c + \frac{x_c(x_t^2 - 8x_t + 4)}{4} 1 - x_t)^2 + \frac{3}{4}\frac{x_t}{(x_t - 1)}
\end{aligned}
\tag{4.26}
$$

The above three contributions alluded to charm-charm, top-top and charm-top, subsequently.

4.2.2 LR Models

The Left-Right model generates the correction for kaon mixing compared to SM case. Except for he WW box found in the SM earlier, one important class of contributions comprises the exchange of a single W' in a box together with a W. The other classes of diagrams found in LR models consist of charged Higgs and tree level neutral Higgs exchanges [49, 50] shown in Fig.(4.6). Considering the contribution from box diagrams of Fig.(4.6).

The contribution from WW box diagram of Fig.(4.5) calculated in Appendix A is written as,

$$
\begin{aligned}
H_{LR} = \frac{G_F^2 M_W^2}{4\pi^2}\Big[&\lambda_c\lambda_c S(x_c) + \lambda_t\lambda_t S(x_t) \\
&+ 2\lambda_c\lambda_t S(x_c, x_t)\Big] Q_1 + h.c
\end{aligned}
\tag{4.27}
$$

with the only one operator mediating the transition is

$$
Q_1 = \bar{s}\gamma^\mu P_L d \bar{s}\gamma_\mu P_L d \tag{4.28}
$$

The functions $S(x_i), S(x_i, x_j)$ calculated in Appendix A are written as

$$
\begin{aligned}
&S(x_c) = x_c \\
&S(x_t) = x_t\Big(\frac{1}{4} + \frac{9}{4(1-x_t)} - \frac{3}{2(1-x_t)^2}\Big) - \frac{3}{2}\Big(\frac{x_t}{1-x_t}\Big)^3 \log x_t, \\
&S(x_c, x_t) = -x_c \log x_c + \frac{x_c(x_t^2 - 8x_t + 4)}{4} 1 - x_t)^2 + \frac{3}{4}\frac{x_t}{(x_t - 1)}
\end{aligned}
\tag{4.29}
$$

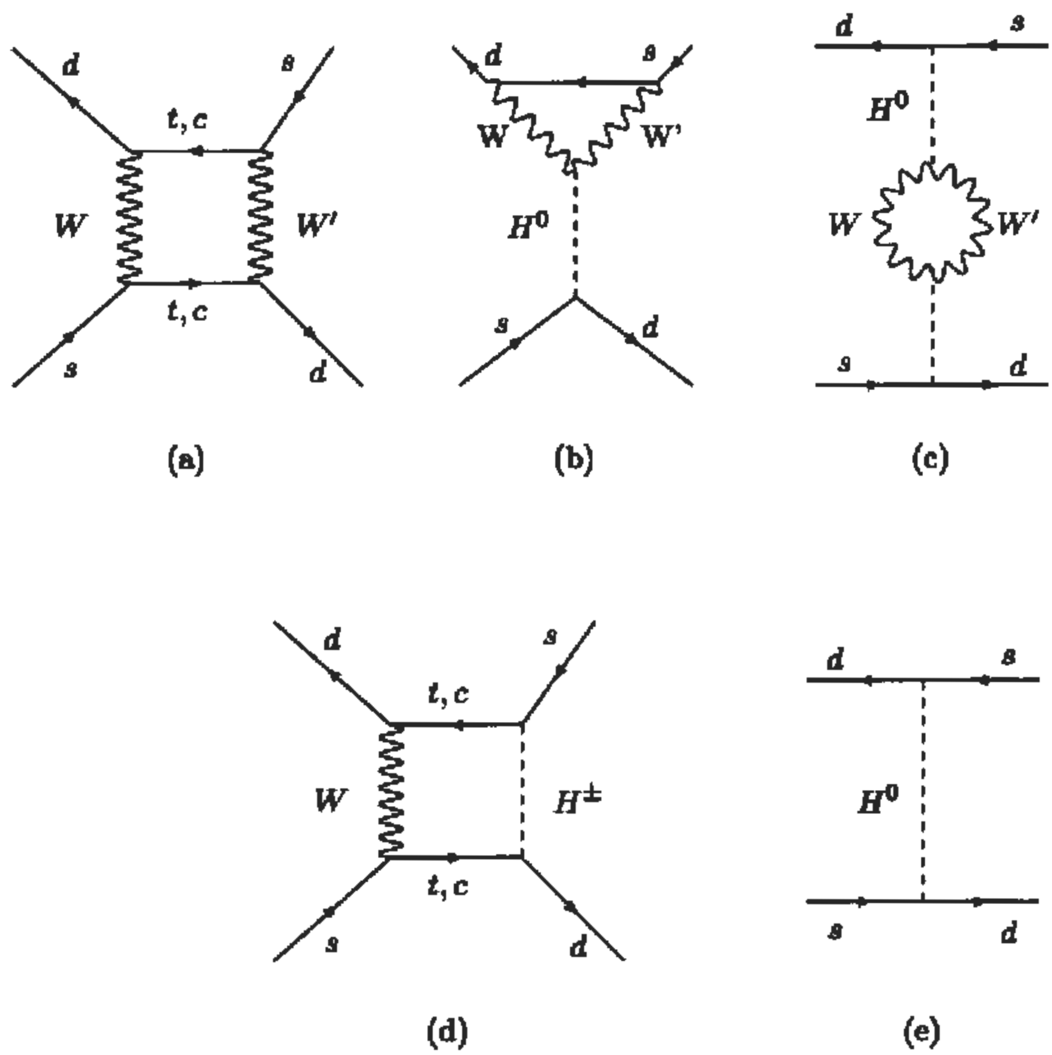

Figure 4.6: Diagrams showing kaon mixing in LR models

Except for the WW box, also WW' boxes are present. The heavy character of the W' suggests the need to consider only its right-handed coupling [51,52] . The WW' box diagrams are calculated in Appendix B and thus written as .

$$A^{(box)} = \frac{G_F^2 M_W^2}{4\pi^2} 2\beta Q_2^{LR} \Big[\lambda_c^{LR}\lambda_c^{RL} S^{(box)}(x_c, x_c, \beta) + \lambda_t^{LR}\lambda_t^{RL} S^{(box)}(x_t, x_t, \beta) + (\lambda_c^{LR}\lambda_t^{RL} + \lambda_t^{LR}\lambda_c^{RL}) S^{(box)}(x_c, x_t, \beta) \Big] \tag{4.30}$$

The different handednesses of the main couplings of the W, W' apply chiral flips leading to the overall mass term, as seen in the first line of Eq.(4.30). Note that the operator calculated in Appendix B consists a very different structure when comparing to the SM operator, Q_1^{VLL} and is defined as:

$$Q_2^{LR} = \bar{s}P_R d \cdot \bar{s}P_L d. \tag{4.31}$$

It is also to be noted that in literature, Eq.(4.30) also contains gauge couplings comprised in h i.e. $h = g_R/g_L$ but due to the L-R invariance here, this factor goes to 1. Here the loop functions in the absence of QCD corrections are calculated as,

$$\begin{aligned} S^{(box)}(x_c, x_t, \beta) &= \sqrt{x_c x_t}\Big[\frac{x_t - 4}{x_t - 1}\log(x_t) + \log(\beta)\Big] \\ S^{(box)}(x_t, x_t, \beta) &= x_t\Big(\frac{x_t^2 - 2x_t + 4}{(x_t - 1)^2}\log(x_t) + \frac{x_t - 4}{x_t - 1} + \log(\beta)\Big) \\ S^{(box)}(x_c, x_c, \beta) &= x_c(4\log(x_c) + 4 + \log(\beta)) \end{aligned} \tag{4.32}$$

Note that because of the overall mass factors, x_u, diagrams involving an up-quark are much suppressed and thus can be ignored. The term $\log(\beta)$ is also present in the above equations.

Other than the contributions given in Eqs.(4.27-4.30), the last set of diagrams, shown in Fig.4.6(d) comprises of boxes $WH_i^{\pm}, i = 1, 2$, where $H_i^{\pm}$ is a heavy, electrically charged Higgs (as discussed in [53] [22]). In this case, the hamiltonian calculated is written as,

$$A^{(H^{\pm}box)} = \frac{G_F^2 M_W^2}{4\pi^2}\langle Q_2^{LR}\rangle \sum_{U,V=c,t} \lambda_U^{LR}\lambda_V^{RL} S_{LR}^H(x_U, x_V, \beta\omega), \tag{4.33}$$

with the function

$$\begin{aligned} S_{LR}^H(x_c, x_t, \omega\beta) &= \Big(\frac{x_t}{x_t - 1}\log(x_t) + \log(\omega\beta)\Big) + \beta \\ S_{LR}^H(x_t, x_t, \omega\beta) &= \Big(\frac{2x_t}{x_t - 1}\log(x_t) - x_t + \log(\omega\beta)\Big) \\ S_{LR}^H(x_c, x_c, \omega\beta) &= \log(\omega\beta) + \beta \end{aligned} \tag{4.34}$$

A concluding comment is in order. In all cases involving physical scalars, when considering the limit where ω goes to zero the contributions from the particles $H^{\pm}$ go to zero and this scalar decouples from the meson-mixing phenomenology.

Chapter 5

Conclusion

Among the paramount problems of the particle physics, understanding the flavour mixing in fermionic (leptons and quarks) sectors is one of the major issue. The gauge symmetries put forward an effective way to understand the basic interaction with respect to the coupling constants. The SM is a fundamental and quiet sophisticated gauge theory developed to date. All the parameters of SM have experimentally been confirmed. SM is formulated on the spontaneously broken $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry grounds, where $SU(3)_C$ deals with the strong interactions and $SU(2)_L \times U(1)_Y$ corresponds to electroweak sector. The electroweak theory assumed the gauge bosons and the fermions to be massles for the gauge invariance. Through the introduction of Higgs mechanism in the SM, the masses of fermions and bosons are acquired. The SM has 12 generatros mediating the strong, weak and electromagnetic interactions. These are eight gluons(g), a photon(γ) and three weak bosons ($W^{\pm}, Z^0$).

Regardless of the fact that the standard model stays unchallenged, it lacks tbose attributes pointed out as unsatisfactory by the Physicists. A good example is the violation of CP symmetry. Also the small off-diagonal elements of CKM matrix is one of most important question. Hence it is compulsory to look beyond the Standard Model for the better understanding of these aspects of the SM. A class of extensions of the SM called Left-Right Models is considered here. which not only extended the scalar sector but also gives new gauge bosons Z', W'. The W' boson produces in LR model couples to the right-handed fields with strengths described by a mixing-matrix corresponding to the CKM matrix of the SM. This is particularly interesting due to the possibility of introducing new sources of CP violation. In full generality, other CP-violating phases could also come from the VEV triggering the spontaneous breaking of LR gauge group down to electromagnetism at low energies.

A model is developed taking into consideration, an extra family symmetry, $U(1)_{family}$. By applying the restrictions not to have very high quantum numbers, few physically realistic connections of CKM matrix elements with the VEVs of various Higgs fields are obtained. One can also develop the connection between the observed power counting, CKM matrix elements and VEVs by proceeding this work further and can obtain the quark masses.

$K^0 - \bar{K^0}$ mixing is considered to discuss the Left-Right model phenomenology. Box diagram is the only contribution found in the SM for $K^0 - \bar{K^0}$ mixing whereas the LR model possesses some other contributions apart from the box diagram found in the SM: WW' box diagram, vertex diagram, self-energy diagram, $WH^{\pm}$ box diagram, and a tree level neutral Higgs exchange diagram. In all cases involving physical scalars, when considering the limit where ω goes to zero the contributions from the particles $H^{\pm}, H^0$ go to zero and this scalar sector decouples from the meson-mixing phenomenology.

Appendix A

A.1

$$
\begin{aligned}
M = \int \frac{d^4k}{(2\pi)^4} \sum_{i,j} \bar{s} V_{is}^* \Big(\frac{-ig}{\sqrt{2}} \gamma_\mu \frac{1-\gamma_5}{\sqrt{2}}\Big) \Big(\frac{ig^{\mu\sigma} + ik^\mu k^\sigma / M_W^2}{k^2 - M_W^2}\Big) \times \\
V_{id} \Big(\frac{-ig}{\sqrt{2}} \gamma_\nu \frac{1-\gamma_5}{\sqrt{2}}\Big) d \times d V_{jd}^* \Big(\frac{-ig}{\sqrt{2}} \gamma_\sigma \frac{1-\gamma_5}{\sqrt{2}}\Big) i \Big(\frac{g^{\nu\rho} + k^\nu k^\rho / M_W^2}{k^2 - M_W^2}\Big) \\
\times V_{js} \Big(\frac{-ig}{\sqrt{2}} \gamma_\rho \frac{1-\gamma_5}{\sqrt{2}}\Big) \bar{s} \Big[\frac{\not{k} + m_i^2}{k^2 - m_i^2}\Big] \Big[\frac{\not{k} + m_j^2}{k^2 - m_j^2}\Big]
\end{aligned}
\tag{A.1}
$$

$$
\begin{aligned}
M = \int \frac{d^4k}{(2\pi)^4} \sum_{i,j} \bar{s} V_{is}^* \Big(\frac{-ig}{\sqrt{2}} \gamma_\mu \frac{1-\gamma_5}{\sqrt{2}}\Big) \Big[\frac{\not{k} + m_i^2}{k^2 - m_i^2}\Big] V_{id} \Big(\frac{-ig}{\sqrt{2}} \gamma_\nu \frac{1-\gamma_5}{\sqrt{2}}\Big) d \times \\
d V_{jd}^* \Big(\frac{-ig}{\sqrt{2}} \gamma_\sigma \frac{1-\gamma_5}{\sqrt{2}}\Big) \Big[\frac{\not{k} + m_j^2}{k^2 - m_j^2}\Big] V_{js} \Big(\frac{-ig}{\sqrt{2}} \gamma_\rho \frac{1-\gamma_5}{\sqrt{2}}\Big) \bar{s} \Big(\frac{ig^{\mu\sigma} + ik^\mu k^\sigma / M_W^2}{k^2 - M_W^2}\Big) \\
i \Big(\frac{g^{\nu\rho} + k^\nu k^\rho / M_W^2}{k^2 - M_W^2}\Big)
\end{aligned}
\tag{A.2}
$$

$$
\begin{aligned}
M = \frac{g^4}{4} \int \frac{d^4k}{(2\pi)^4} \frac{\sum_{i,j} \bar{s} V_{is}^* V_{id} V_{jd}^* V_{js} \Big(\frac{\gamma_\mu (1-\gamma_5) \not{k}}{2} + \frac{\gamma_\mu (1-\gamma_5) m_i}{2}\Big)}{(k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_W^2)(k^2 - M_W^2)} \times \\
\frac{\frac{\gamma_\nu (1-\gamma_5)}{2} d * \bar{s} \Big(\frac{\gamma_\sigma (1-\gamma_5) \not{k}}{2} + \frac{\gamma_\sigma (1-\gamma_5) m_j}{2}\Big) \Big(\frac{\gamma_\rho (1-\gamma_5)}{2}\Big) d}{(k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_W^2)(k^2 - M_W^2)} \times \\
\frac{g^{\mu\sigma} g^{\nu\rho} + g^{\mu\sigma} k^\nu k^\rho / M_W^2 + k^\mu k^\sigma / M_W^2 g^{\nu\rho} + k^\mu k^\sigma k^\nu k^\rho}{(k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_W^2)(k^2 - M_W^2)}
\end{aligned}
\tag{A.3}
$$

Taking terms with gamma matrices and solving one by one

$$\begin{aligned}
G_1 &= \bar{s}\gamma_\mu \frac{(1-\gamma_5)}{2}\gamma_\nu m_i \frac{(1-\gamma_5)}{2} d = 0 \\
G_2 &= \bar{s}\gamma_\sigma \frac{(1-\gamma_5)}{2}\gamma_\rho m_j \frac{(1-\gamma_5)}{2} d = 0 \\
G_3 &= \bar{s}\gamma_\mu \frac{(1-\gamma_5)}{2}\not{k}\gamma_\nu \frac{(1-\gamma_5)}{2} d = \frac{1}{2}\bar{s}\gamma_\mu(1-\gamma_5)\not{k}\gamma_\nu d \\
G_4 &= \bar{s}\gamma_\sigma \frac{(1-\gamma_5)}{2}\not{k}\gamma_\rho \frac{(1-\gamma_5)}{2} d = \frac{1}{2}\bar{s}\gamma_\sigma(1-\gamma_5)\not{k}\gamma_\rho d
\end{aligned} \tag{A.4}$$

Now putting Eqs.(A.4) in the amplitude M of Eq.(A.2) and solving it, one gets

$$\begin{aligned}
M = \frac{g^4}{4}\int \frac{d^4k}{(2\pi)^4} & \frac{\sum_{i,j} V_{is}^* V_{id} V_{jd}^* V_{jd}\Big(\bar{s}\gamma_\mu \frac{(1-\gamma_5)}{2}\not{k}\gamma_\nu d * \bar{s}\gamma_\sigma \frac{(1-\gamma_5)}{2}\not{k}\gamma_\rho d\Big)}{(k^2-m_i^2)(k^2-m_j^2)(k^2-M_W^2)(k^2-M_W^2)} \\
& \times \frac{g^{\mu\sigma}g^{\nu\rho} + g^{\mu\sigma}k^\nu k^\rho/M_W^2 + k^\mu k^\sigma/M_W^2 g^{\nu\rho} + k^\mu k^\sigma k^\nu k^\rho}{(k^2-m_i^2)(k^2-m_j^2)(k^2-M_W^2)(k^2-M_W^2)}
\end{aligned} \tag{A.5}$$

solving above equation further, one gets

$$\begin{aligned}
M = \frac{g^4}{16} & \frac{\sum_{i,j} V_{is}^* V_{id} V_{jd}^* V_{jd}(\bar{s}\gamma_\mu(1-\gamma_5)\gamma_\alpha d\gamma_\nu \cdot \bar{s}\gamma_\sigma(1-\gamma_5)\gamma_\beta\gamma_\rho d)}{(k^2-m_i^2)(k^2-m_j^2)(k^2-M_W^2)^2} \\
& \int \frac{d^4k}{(2\pi)^4}\Big[\frac{k^\alpha k^\beta g^{\mu\sigma}g^{\nu\rho} + k^\alpha k^\beta g^{\mu\sigma}k^\nu k^\rho/M_W^2}{(k^2-m_i^2)(k^2-m_j^2)(k^2-M_W^2)^2} \\
& + \frac{k^\alpha k^\beta k^\mu k^\sigma g^{\nu\rho}/M_W^2 + k^\alpha k^\beta k^\mu k^\sigma K^\nu K^\rho/M_W^4}{(k^2-m_i^2)(k^2-m_j^2)(k^2-M_W^2)^2}\Big]
\end{aligned} \tag{A.6}$$

Decomposing above equation and solving it

$$\begin{aligned}
A_1 = {} & \bar{s}\gamma_\mu(1-\gamma_5)\gamma_\alpha\gamma_\nu d\bar{s}\gamma_\sigma(1-\gamma_5)\gamma_\beta\gamma_\rho d k^\alpha k^\beta g^{\mu\sigma}g^{\nu\rho} \\
= {} & \bar{s}\gamma_\mu(\gamma_\alpha\gamma_\nu - \gamma_\alpha\gamma_\nu)d\bar{s}\gamma_\mu(\gamma_\alpha\gamma_\nu - \gamma_\alpha\gamma_\nu)\frac{k^2}{4}d \\
= {} & \bar{s}\gamma_\mu\gamma_\alpha\gamma_\nu(1-\gamma_5)d\bar{s}\gamma_\mu\gamma_\alpha\gamma_\nu(1-\gamma_5)d\frac{k^2}{4} \\
= {} & \bar{s}(S_{\mu\alpha\nu\sigma}\gamma^\sigma - i\varepsilon_{\mu\alpha\nu\sigma}\gamma^\sigma\gamma^5)(1-\gamma_5)d\bar{s}(S_{\mu\alpha\nu\sigma}\gamma^\sigma - i\varepsilon_{\mu\alpha\nu\sigma}\gamma^\sigma\gamma^5) * \\
& (1-\gamma_5)d \\
= {} & k^2(\bar{s}\gamma_\sigma(1-\gamma_5)d\bar{s}\gamma^\sigma(1-\gamma_5)d)
\end{aligned} \tag{A.7}$$

$$
\begin{aligned}
A_2 &= \bar{s}\gamma_\mu(1-\gamma_5)\gamma_\alpha\gamma_\nu d\bar{s}\gamma_\sigma(1-\gamma_5)\gamma_\beta\gamma_\rho d k^\alpha k^\beta g^{\mu\sigma} k^\nu k^\rho / M_W^2 \\
&= \frac{k^4}{M_W^2}(\bar{s}\gamma_\nu(1-\gamma_5)d\bar{s}\gamma^\mu(1-\gamma_5)d) \\
A_3 &= \bar{s}\gamma_\mu(1-\gamma_5)\gamma_\alpha\gamma_\nu d\bar{s}\gamma_\sigma(1-\gamma_5)\gamma_\beta\gamma_\rho d k^\alpha k^\beta k^\mu k^\sigma g^{\nu\rho} / M_W^2 \\
&= \frac{k^4}{M_W^2}(\bar{s}\gamma_\nu(1-\gamma_5)d\bar{s}\gamma^\nu(1-\gamma_5)d) \\
A_4 &= \bar{s}\gamma_\mu(1-\gamma_5)\gamma_\alpha d\gamma_\nu\bar{s}\gamma_\sigma(1-\gamma_5)\gamma_\beta\gamma_\rho d k^\alpha k^\beta k^\mu k^\sigma k^\nu k^\rho / M_W^4 \\
&= \frac{k^6}{4M_W^4}(\bar{s}\gamma_\mu(1-\gamma_5)d\bar{s}\gamma^\nu(1-\gamma_5)d)
\end{aligned}
\tag{A.8}
$$

Hence complete M can be written as

$$
\begin{aligned}
M = \frac{g^4}{16}\sum_{i,j}\lambda_i\lambda_j[(\bar{s}\gamma_\mu(1-\gamma_5)d\bar{s}\gamma^\nu(1-\gamma_5)d)] \\
\int\frac{d^4k}{(2\pi)^4}\Big\{\frac{k^2}{(k^2-m_i^2)(k^2-m_j^2)(k^2-M_W^2)^2} - \\
\frac{2k^4}{M_W^2(k^2-m_i^2)(k^2-m_j^2)(k^2-M_W^2)^2} \\
+\frac{k^6}{M_W^2(k^2-m_i^2)(k^2-m_j^2)(k^2-M_W^2)^2}\Big\}
\end{aligned}
\tag{A.9}
$$

Term wise power analysis

$$
\begin{aligned}
P_1 &= \sim\frac{d^4\cdot k^2}{k^2k^2k^4}\sim\frac{k^6}{k^8}\sim\frac{1}{k^2} \\
P_2 &= \sim\frac{d^4\cdot k^4}{k^2k^2k^4}\sim\frac{k^8}{k^8}\sim 1 \\
P_3 &= \sim\frac{d^4\cdot k^6}{k^2k^2k^4}\sim\frac{k^10}{k^8}\sim k^2
\end{aligned}
\tag{A.10}
$$

applying unitarity of CKM matrix

$$
\begin{aligned}
&\sum_{i,j}\lambda_i\lambda_j[E(m_i,m_j)-E(0,m_j)-E(m_i,0)+E(0,0)] \\
=&\sum_{i,j}\lambda_i\lambda_j\Big[\frac{1}{(k^2-m_i^2)(k^2-m_j^2)(k^2-M_W^2)^2}-\frac{1}{k^2(k^2-m_j^2)(k^2-M_W^2)^2} \\
&-\frac{1}{k^2(k^2-m_i^2)(k^2-M_W^2)^2}+\frac{1}{k^4(k^2-M_W^2)^2}\Big] \\
=&\sum_{i,j}\frac{m_i^2m_j^2}{k^4(k^2-m_i^2)(k^2-m_j^2)(k^2-M_W^2)^2}
\end{aligned}
$$

Now the Eq.(A.9) becomes

$$
\begin{aligned}
M = & \frac{g^4}{16}\sum_{i,j}[(\bar{s}\gamma_\mu(1-\gamma_5)d\bar{s}\gamma^\nu(1-\gamma_5)d)]\lambda_i\lambda_j \\
& \int \frac{d^4k}{(2\pi)^4}\frac{m_i^2 m_j^2}{M_W^2(k^2-m_i^2)(k^2-m_j^2)(k^2-M_W^2)^2}\times \\
& \left(k^2-\frac{2k^4}{M_W^2}+\frac{k^6}{M_W^4}\right)
\end{aligned} \tag{A.11}
$$

Solving the integrals in above equation will give expression for M as below

$$
\begin{aligned}
M = & \frac{g^4}{16}\sum_{i,j}[(\bar{s}\gamma_\mu(1-\gamma_5)d\bar{s}\gamma^\nu(1-\gamma_5)d)]\lambda_i\lambda_j\times \\
& \frac{-i\pi^2}{M_W^2(2\pi)^4}\frac{-x_i x_j}{x_i-x_j}\left[\frac{\ln x_i}{4}-\frac{3}{2}\frac{\ln x_i}{x_i-1}-\frac{3}{4}\frac{\ln x_i}{x_i-1)^2}\right]\times \\
& -\frac{3x_i x_j}{4x_i x_j}\frac{1}{(x_i-1)(x_j-1)}-\frac{3}{2}\left(\frac{x_i}{x_i-1}\right)^3\ln x_i-\frac{x_i}{4} \\
& +\frac{9x_j}{4(x_i-1)}-\frac{3}{2(x_j-1)^2}
\end{aligned} \tag{A.12}
$$

with $x_i = m_i/M_W^2$. In above equation is the four-quark operator Q_{12} is defined as,

$$
Q_{12} = \bar{s}\gamma_\mu\frac{(1-\gamma_5)}{2}d\bar{s}\gamma^\nu\frac{(1-\gamma_5)}{2}d \tag{A.13}
$$

The effective Hamiltonian can be written as

$$
H_{ij} = \frac{G_F^2 M_W^2}{4\pi^2}\sum_{i,j}\lambda_i\lambda_j S_{ij}Q_{12} \tag{A.14}
$$

It can be written as

$$
\begin{aligned}
H_{SM} = & \frac{G_F^2 M_W^2}{4\pi^2}\Big[\lambda_c\lambda_c S(x_c)+\lambda_t\lambda_t S(x_t) \\
& +2\lambda_c\lambda_t S(x_c,x_t)\Big]Q_1 + h.c
\end{aligned} \tag{A.15}
$$

Appendix B

B.1

$$
\begin{aligned}
M = & \int \Big\{ \frac{d^4k}{(2\pi)^4} \sum_{i,j} \bar{s} V_{is}^{*} \Big(\frac{-ig_L}{\sqrt{2}} \gamma_\mu \frac{1-\gamma_5}{\sqrt{2}} \Big) \Big(\frac{ig^{\mu\sigma} + ik^\mu k^\sigma / M_W^2}{k^2 - M_W^2} \Big) \times \\
& V_{id} \Big(\frac{-ig_L}{\sqrt{2}} \gamma_\nu \frac{1-\gamma_5}{\sqrt{2}} \Big) d \times d V_{jd}^{*} \Big(\frac{-ig_L}{\sqrt{2}} \gamma_\sigma \frac{1-\gamma_5}{\sqrt{2}} \Big) i \Big(\frac{g^{\nu\rho} + k^\nu k^\rho / M_{W'}^2}{k^2 - M_{W'}^2} \Big) \\
& \times V_{js} \Big(\frac{-ig_L}{\sqrt{2}} \gamma_\rho \frac{1-\gamma_5}{\sqrt{2}} \Big) \bar{s} \Big[\frac{\not{k} + m_i^2}{k^2 - m_i^2} \Big] \Big[\frac{\not{k} + m_j^2}{k^2 - m_j^2} \Big] \Big\} + \\
& \int \Big\{ \frac{d^4k}{(2\pi)^4} \sum_{i,j} \bar{s} V_{is}^{*} \Big(\frac{-ig_R}{\sqrt{2}} \gamma_\mu \frac{1+\gamma_5}{\sqrt{2}} \Big) \Big(\frac{ig^{\mu\sigma} + ik^\mu k^\sigma / M_W^2}{k^2 - M_W^2} \Big) \times \\
& V_{id} \Big(\frac{-ig_R}{\sqrt{2}} \gamma_\nu \frac{1+\gamma_5}{\sqrt{2}} \Big) d \times d V_{jd}^{*} \Big(\frac{-ig_R}{\sqrt{2}} \gamma_\sigma \frac{1+\gamma_5}{\sqrt{2}} \Big) i \Big(\frac{g^{\nu\rho} + k^\nu k^\rho / M_{W'}^2}{k^2 - M_{W'}^2} \Big) \\
& \times V_{js} \Big(\frac{-ig_R}{\sqrt{2}} \gamma_\rho \frac{1+\gamma_5}{\sqrt{2}} \Big) \bar{s} \Big[\frac{\not{k} + m_i^2}{k^2 - m_i^2} \Big] \Big[\frac{\not{k} + m_j^2}{k^2 - m_j^2} \Big] \Big\}
\end{aligned}
\tag{B.1}
$$

$$M = \int \Big\{ \frac{d^4k}{(2\pi)^4} \sum_{i,j} \bar{s} V^*_{is} \Big(\frac{-ig_L}{\sqrt{2}} \gamma_\mu \frac{1-\gamma_5}{\sqrt{2}} \Big) \Big[\frac{\not{k} + m_i^2}{k^2 - m_i^2} \Big] V_{id} \Big(\frac{-ig_L}{\sqrt{2}} \gamma_\nu \frac{1-\gamma_5}{\sqrt{2}} \Big) d$$

$$\times \, d V^*_{jd} \Big(\frac{-ig_L}{\sqrt{2}} \gamma_\sigma \frac{1-\gamma_5}{\sqrt{2}} \Big) \Big[\frac{\not{k} + m_j^2}{k^2 - m_j^2} \Big] V_{js} \Big(\frac{-ig_L}{\sqrt{2}} \gamma_\rho \frac{1-\gamma_5}{\sqrt{2}} \Big) \bar{s}$$

$$\Big(\frac{ig^{\mu\sigma} + ik^\mu k^\sigma / M_W^2}{k^2 - M_W^2} \Big) i \Big(\frac{g^{\nu\rho} + k^\nu k^\rho / M_{W'}^2}{k^2 - M_{W'}^2} \Big) \Big\} +$$

$$\int \Big\{ \frac{d^4k}{(2\pi)^4} \sum_{i,j} \bar{s} V^*_{is} \Big(\frac{-ig_R}{\sqrt{2}} \gamma_\mu \frac{1+\gamma_5}{\sqrt{2}} \Big) \Big[\frac{\not{k} + m_i^2}{k^2 - m_i^2} \Big] V_{id} \Big(\frac{-ig_R}{\sqrt{2}} \gamma_\nu \frac{1+\gamma_5}{\sqrt{2}} \Big) d$$

$$\times \, d V^*_{jd} \Big(\frac{-ig_R}{\sqrt{2}} \gamma_\sigma \frac{1+\gamma_5}{\sqrt{2}} \Big) \Big[\frac{\not{k} + m_j^2}{k^2 - m_j^2} \Big] V_{js} \Big(\frac{-ig_R}{\sqrt{2}} \gamma_\rho \frac{1+\gamma_5}{\sqrt{2}} \Big) \bar{s}$$

$$\Big(\frac{ig^{\mu\sigma} + ik^\mu k^\sigma / M_W^2}{k^2 - M_W^2} \Big) i \Big(\frac{g^{\nu\rho} + k^\nu k^\rho / M_{W'}^2}{k^2 - M_{W'}^2} \Big) \Big\} \tag{B.2}$$

$$M = \frac{g^4}{4} \int \frac{d^4k}{(2\pi)^4} \frac{\sum_{i,j} \bar{s} V^*_{is} V_{id} V^*_{jd} V_{js} (\frac{\gamma_\mu(1-\gamma_5)\not{k}}{2} + \frac{\gamma_\mu(1-\gamma_5)m_i}{2})}{(k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_W^2)(k^2 - M_{W'}^2)} \times$$

$$\frac{\frac{\gamma_\nu(1-\gamma_5)}{2} d * \bar{s} (\frac{\gamma_\sigma(1-\gamma_5)\not{k}}{2} + \frac{\gamma_\sigma(1-\gamma_5)m_j}{2})(\frac{\gamma_\rho(1-\gamma_5)}{2}) d}{(k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_W^2)(k^2 - M_{W'}^2)} \times$$

$$\frac{g^{\mu\sigma} g^{\nu\rho} + g^{\mu\sigma} k^\nu k^\rho / M_{W'}^2 + k^\mu k^\sigma / M_W^2 g^{\nu\rho} + k^\mu k^\sigma k^\nu k^\rho / M_W^2 M_{W'}^2}{(k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_W^2)(k^2 - M_{W'}^2)} \tag{B.3}$$

$$+ \frac{g^4}{4} \int \frac{d^4k}{(2\pi)^4} \frac{\sum_{i,j} \bar{s} V^*_{is} V_{id} V^*_{jd} V_{js} (\frac{\gamma_\mu(1+\gamma_5)\not{k}}{2} + \frac{\gamma_\mu(1+\gamma_5)m_i}{2})}{(k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_W^2)(k^2 - M_{W'}^2)} \times$$

$$\frac{\frac{\gamma_\nu(1+\gamma_5)}{2} d * \bar{s} (\frac{\gamma_\sigma(1+\gamma_5)\not{k}}{2} + \frac{\gamma_\sigma(1+\gamma_5)m_j}{2})(\frac{\gamma_\rho(1+\gamma_5)}{2}) d}{(k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_W^2)(k^2 - M_{W'}^2)} \times$$

$$\frac{g^{\mu\sigma} g^{\nu\rho} + g^{\mu\sigma} k^\nu k^\rho / M_{W'}^2 + k^\mu k^\sigma / M_W^2 g^{\nu\rho} + k^\mu k^\sigma k^\nu k^\rho / M_W^2 M_{W'}^2}{(k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_W^2)(k^2 - M_{W'}^2)}$$

$$
\begin{aligned}
M = &\frac{g^4}{4}\int \frac{d^4k}{(2\pi)^4}\sum_{i,j} V_{is}^* V_{id} V_{jd}^* V_{js} \times \\
&\Big\{\Big(\bar{s}\frac{\gamma_\mu(1-\gamma_5)\not{k}}{2} + \frac{\gamma_\mu(1-\gamma_5)m_i}{2}\Big) \times \Big(\frac{\gamma_\nu(1-\gamma_5)}{2}\Big)d \\
&\bar{s}\Big(\frac{\gamma_\sigma(1-\gamma_5)\not{k}}{2} + \frac{\gamma_\sigma(1-\gamma_5)m_j}{2}\Big)\Big(\frac{\gamma_\rho(1-\gamma_5)}{2}\Big)d\Big\} \\
&+\Big\{\bar{s}\Big(\frac{\gamma_\mu(1+\gamma_5)\not{k}}{2} + \frac{\gamma_\mu(1+\gamma_5)m_j}{2}\Big) \times \Big(\frac{\gamma_\nu(1+\gamma_5)}{2}\Big)d* \\
&*\,\bar{s}\Big(\frac{\gamma_\sigma(1+\gamma_5)\not{k}}{2} + \frac{\gamma_\sigma(1+\gamma_5)m_j}{2}\Big)\Big(\frac{\gamma_\rho(1+\gamma_5)}{2}\Big)d\Big\}* \\
&\frac{g^{\mu\sigma}g^{\nu\rho} + g^{\mu\sigma}k^\nu k^\rho/M_{W'}^2 + k^\mu k^\sigma/M_W^2 g^{\nu\rho} + k^\mu k^\sigma k^\nu k^\rho/M_W^2 M_{W'}^2}{(k^2-m_i^2)(k^2-m_j^2)(k^2-M_W^2)(k^2-M_{W'}^2)}
\end{aligned} \tag{B.4}
$$

Taking terms with gamma matrices and solving one by one

$$
\begin{aligned}
G_1 =& \bar{s}\gamma_\mu\frac{(1-\gamma_5)}{2}\gamma_\nu m_i\frac{(1-\gamma_5)}{2}d = 0 \\
G_2 =&\bar{s}\gamma_\sigma\frac{(1-\gamma_5)}{2}\gamma_\rho m_j\frac{(1-\gamma_5)}{2}d = 0 \\
G_3 =&\bar{s}\gamma_\mu\frac{(1-\gamma_5)}{2}\not{k}\gamma_\nu\frac{(1-\gamma_5)}{2}d = \frac{1}{2}\bar{s}\gamma_\mu(1+\gamma_5)\not{k}\gamma_\nu d \\
G_4 =&\bar{s}\gamma_\sigma\frac{(1-\gamma_5)}{2}\not{k}\gamma_\rho\frac{(1-\gamma_5)}{2}d = \frac{1}{2}\bar{s}\gamma_\sigma(1-\gamma_5)\not{k}\gamma_\rho d
\end{aligned} \tag{B.5}
$$

Similarly

$$
\begin{aligned}
G_1 =& \bar{s}\gamma_\mu\frac{(1+\gamma_5)}{2}\gamma_\nu m_i\frac{(1+\gamma_5)}{2}d = 0 \\
G_2 =&\bar{s}\gamma_\sigma\frac{(1-\gamma_5)}{2}\gamma_\rho m_j\frac{(1-\gamma_5)}{2}d = 0 \\
G_3 =&\bar{s}\gamma_\mu\frac{(1-\gamma_5)}{2}\not{k}\gamma_\nu\frac{(1-\gamma_5)}{2}d = \frac{1}{2}\bar{s}\gamma_\mu(1+\gamma_5)\not{k}\gamma_\nu d \\
G_4 =&\bar{s}\gamma_\sigma\frac{(1-\gamma_5)}{2}\not{k}\gamma_\rho\frac{(1-\gamma_5)}{2}d = \frac{1}{2}\bar{s}\gamma_\sigma(1-\gamma_5)\not{k}\gamma_\rho d
\end{aligned} \tag{B.6}
$$

Adding the above equation,one gets

$$
\begin{aligned}
G =& \frac{1}{2}\bar{s}\gamma_\mu(1+\gamma_5)\not{k}\gamma_\nu d + \frac{1}{2}\bar{s}\gamma_\sigma(1-\gamma_5)\not{k}\gamma_\rho d + \\
&\frac{1}{2}\bar{s}\gamma_\mu(1+\gamma_5)\not{k}\gamma_\nu d + \frac{1}{2}\bar{s}\gamma_\sigma(1-\gamma_5)\not{k}\gamma_\rho d \\
=&\bar{s}\gamma_\mu(1+\gamma_5)\not{k}\gamma_\nu d + \bar{s}\gamma_\sigma(1-\gamma_5)\not{k}\gamma_\rho d
\end{aligned} \tag{B.7}
$$

Now putting Eqs.(B.7) in the amplitude M of Eq.(B.4) and solving it, one gets

$$M = \frac{g^4}{4}\int \frac{d^4k}{(2\pi)^4} \frac{\sum_{i,j} V_{is}^* V_{id} V_{jd}^* V_{jd}\Big(\bar{s}\gamma_\mu(1+\gamma_5)\not{k}\gamma_\nu d * \bar{s}\gamma_\sigma(1-\gamma_5)\not{k}\gamma_\rho d\Big)}{(k^2-m_i^2)(k^2-m_j^2)(k^2-M_W^2)(k^2-M_{W'}^2)} \times \frac{g^{\mu\sigma}g^{\nu\rho} + g^{\mu\sigma}k^\nu k^\rho/M_{W'}^2 + k^\mu k^\sigma g^{\nu\rho}/M_W^2 + k^\mu k^\sigma k^\nu k^\rho/M_W^2 M_{W'}^2}{(k^2-m_i^2)(k^2-m_j^2)(k^2-M_W^2)(k^2-M_{W'}^2)} \tag{B.8}$$

using the relation

$$\not{k} = \gamma_\alpha k_\alpha \quad \not{k} = \gamma_\beta k_\beta \tag{B.9}$$

putting this in Eq.(B.8) and solving further one gets

$$M = \frac{g^4}{4}\frac{\sum_{i,j} V_{is}^* V_{id} V_{jd}^* V_{jd}(\bar{s}\gamma_\mu(1+\gamma_5)\gamma_\alpha d\gamma_\nu \cdot \bar{s}\gamma_\sigma(1-\gamma_5)\gamma_\beta\gamma_\rho d)}{(k^2-m_i^2)(k^2-m_j^2)(k^2-M_{W'}^2)^2} \int \frac{d^4k}{(2\pi)^4}\Big[\frac{k^\alpha k^\beta g^{\mu\sigma}g^{\nu\rho} + k^\alpha k^\beta g^{\mu\sigma}k^\nu k^\rho/M_{W'}^2}{(k^2-m_i^2)(k^2-m_j^2)(k^2-M_W^2)(k^2-M_{W'}^2)} + \frac{k^\alpha k^\beta k^\mu k^\sigma g^{\nu\rho}/M_{W'}^2 + k^\alpha k^\beta k^\mu k^\sigma K^\nu K^\rho/M_W^2 M_{W'}^2}{(k^2-m_i^2)(k^2-m_j^2)(k^2-M_W^2)(k^2-M_{W'}^2)}\Big] \tag{B.10}$$

Decomposing above equation and solving it

$$\begin{aligned} A_1 &= \bar{s}\gamma_\mu(1+\gamma_5)\gamma_\alpha\gamma_\nu d\bar{s}\gamma_\sigma(1-\gamma_5)\gamma_\beta\gamma_\rho d k^\alpha k^\beta g^{\mu\sigma}g^{\nu\rho} \\ &= \bar{s}\gamma_\mu(\gamma_\alpha\gamma_\nu + \gamma_\alpha\gamma_\nu\gamma_5)d\bar{s}\gamma_\mu(\gamma_\alpha\gamma_\nu - \gamma_\alpha\gamma_\nu\gamma_5)\frac{k^2}{4}d \\ &= \bar{s}\gamma_\mu\gamma_\alpha\gamma_\nu(1+\gamma_5)d\bar{s}\gamma_\mu\gamma_\alpha\gamma_\nu(1-\gamma_5)d\frac{k^2}{4} \end{aligned}$$

using the property

$$\gamma_\mu\gamma_\alpha\gamma_\nu = (S_{\mu\alpha\nu\sigma}\gamma^\sigma - i\varepsilon_{\mu\alpha\nu\sigma}\gamma^\sigma\gamma^5) \tag{B.11}$$

$$
\begin{aligned}
A_1 = &\bar{s}(S_{\mu\alpha\nu\sigma}\gamma^\sigma - i\varepsilon_{\mu\alpha\nu\sigma}\gamma^\sigma\gamma^5)(1+\gamma_5)d\bar{s}(S_{\mu\alpha\nu\sigma}\gamma^\sigma - i\varepsilon_{\mu\alpha\nu\sigma}\gamma^\sigma\gamma^5)* \\
&(1-\gamma_5)d \\
=&k^2(\bar{s}\gamma_\sigma(1+\gamma_5)d\bar{s}\gamma^\sigma(1-\gamma_5)d) \\
A_2 =&\bar{s}\gamma_\mu(1+\gamma_5)\gamma_\alpha\gamma_\nu d\bar{s}\gamma_\sigma(1-\gamma_5)\gamma_\beta\gamma_\rho dk^\alpha k^\beta g^{\mu\sigma}k^\nu k^\rho/M_{W'}^2 \\
=&\frac{k^4}{M_{W'}^2}(\bar{s}\gamma_\nu(1+\gamma_5)d\bar{s}\gamma^\mu(1-\gamma_5)d) \\
A_3 =&\bar{s}\gamma_\mu(1+\gamma_5)\gamma_\alpha\gamma_\nu d\bar{s}\gamma_\sigma(1-\gamma_5)\gamma_\beta\gamma_\rho dk^\alpha k^\beta k^\mu k^\sigma g^{\nu\rho}/M_W^2 \\
=&\frac{k^4}{M_W^2}(\bar{s}\gamma_\nu(1+\gamma_5)d\bar{s}\gamma^\nu(1-\gamma_5)d) \\
A_4 =&\bar{s}\gamma_\mu(1+\gamma_5)\gamma_\alpha d\gamma_\nu\bar{s}\gamma_\sigma(1-\gamma_5)\gamma_\beta\gamma_\rho dk^\alpha k^\beta k^\mu k^\sigma k^\nu k^\rho/M_W^2 M_{W'}^2 \\
=&\frac{k^6}{4M_W^2 M_{W'}^2}(\bar{s}\gamma_\mu(1+\gamma_5)d\bar{s}\gamma^\nu(1-\gamma_5)d)
\end{aligned}
\tag{B.12}
$$

Hence complete M can be written as

$$
\begin{aligned}
M = &\frac{g^4}{4}\sum_{i,j}\lambda_i\lambda_j[(\bar{s}\gamma_\mu(1+\gamma_5)d\bar{s}\gamma^\nu(1-\gamma_5)d)] \\
&\int\frac{d^4k}{(2\pi)^4}\Big\{\frac{k^2}{(k^2-m_i^2)(k^2-m_j^2)(k^2-M_W^2)(k^2-M_{W'}^2)}+ \\
&\frac{k^4}{M_{W'}^2(k^2-m_i^2)(k^2-m_j^2)(k^2-M_W^2)(k^2-M_{W'}^2)}+ \\
&\frac{k^4}{M_W^2(k^2-m_i^2)(k^2-m_j^2)(k^2-M_W^2)(k^2-M_{W'}^2)} \\
&+\frac{k^6}{4M_W^2 M_{W'}^2(k^2-m_i^2)(k^2-m_j^2)(k^2-M_W^2)(k^2-M_{W'}^2)}\Big\}
\end{aligned}
\tag{B.13}
$$

Term wise power analysis

$$
\begin{aligned}
P_1 = &\sim\frac{d^4\cdot k^2}{k^2k^2k^4}\sim\frac{k^6}{k^8}\sim\frac{1}{k^2} \\
P_2 = &\sim\frac{d^4\cdot k^4}{k^2k^2k^4}\sim\frac{k^8}{k^8}\sim 1 \\
P_3 = &\sim\frac{d^4\cdot k^4}{k^2k^2k^4}\sim\frac{k^8}{k^8}\sim 1 \\
P_4 = &\sim\frac{d^4\cdot k^6}{k^2k^2k^4}\sim\frac{k^10}{k^8}\sim k^2
\end{aligned}
\tag{B.14}
$$

applying unitarity of CKM matrix

$$\begin{aligned}
&\sum_{i,j} \lambda_i \lambda_j [E(m_i, m_j) - E(0, m_j) - E(m_i, 0) + E(0,0)] \\
&= \sum_{i,j} \lambda_i \lambda_j \Big[\frac{1}{(k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_W^2)(k^2 - M_{W'}^2)} \\
&\quad + \frac{1}{k^2(k^2 - m_j^2)(k^2 - M_W^2)(k^2 - M_{W'}^2)} \\
&\quad + \frac{1}{k^2(k^2 - m_i^2)(k^2 - M_W^2)(k^2 - M_{W'}^2)} + \frac{1}{k^4(k^2 - M_W^2)(k^2 - M_{W'}^2)} \Big] \\
&= \sum_{i,j} \frac{m_i^2 m_j^2}{k^4(k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_W^2)(k^2 - M_{W'}^2)}
\end{aligned}$$

Now the Eq.(B.13) becomes

$$\begin{aligned}
M = \frac{g^4}{4} \sum_{i,j} &[(\bar{s}\gamma_\mu (1+\gamma_5) d \bar{s} \gamma^\nu (1-\gamma_5) d)] \lambda_i \lambda_j \\
&\int \frac{d^4k}{(2\pi)^4} \frac{m_i^2 m_j^2}{M_W^2 (k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_W^2)(k^2 - M_{W'}^2)} \times \\
&\left(k^2 + \frac{k^4}{M_{W'}^2} + \frac{k^4}{M_W^2} + \frac{k^6}{4 M_W^4 M_{W'}^2} \right)
\end{aligned} \tag{B.15}$$

$$\begin{aligned}
M = \frac{g^4}{4} \sum_{i,j} &[(\bar{s}\gamma_\mu (1+\gamma_5) d \bar{s} \gamma^\nu (1-\gamma_5) d)] \lambda_i \lambda_j \\
&\int \frac{d^4k}{(2\pi)^4} \Big[\frac{k^2 m_i^2 m_j^2}{M_W^2 (k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_W^2)(k^2 - M_{W'}^2)} + \\
&\frac{k^4 m_i^2 m_j^2}{M_W^2 M_{W'}^2 (k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_W^2)(k^2 - M_{W'}^2)} + \\
&\frac{k^4 m_i^2 m_j^2}{(M_W^2)^2 (k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_W^2)(k^2 - M_{W'}^2)} + \\
&\frac{k^6 m_i^2 m_j^2}{4 M_W^5 M_{W'}^2 (k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_W^2)(k^2 - M_{W'}^2)} \Big]
\end{aligned} \tag{B.16}$$

solving the integrals in above equation one gets,

$$M = \frac{g^4}{4}\sum_{i,j}[(\bar{s}\gamma_\mu(1+\gamma_5)d\bar{s}\gamma^\nu(1-\gamma_5)d)]\lambda_i\lambda_j$$
$$\frac{-i\pi^2}{M_W^2 M_{W'}^2(2\pi)^2}\sqrt{x_c x_t}\Big[\frac{x_t-4}{x_t-1}\log(x_t)+\log(\beta)\Big]+$$
$$x_t\Big(\frac{x_t^2-2x_t+4}{(x_t-1)^2}\log(x_t)+\frac{x_t-4}{x_t-1}+\log(\beta)\Big)$$
$$+\,x_c(4\log(x_c)+4+\log(\beta)) \tag{B.17}$$

The effective hamiltonian in this case is written as

$$A^{(box)} = \frac{G_F^2 M_W^2}{4\pi^2}2\beta Q_2^{LR}\Big[\lambda_c^{LR}\lambda_c^{RL}S^{(box)}(x_c,x_c,\beta)+\lambda_t^{LR}\lambda_t^{RL}S^{(box)}(x_t,x_t,\beta)$$
$$+\,(\lambda_c^{LR}\lambda_t^{RL}+\lambda_t^{LR}\lambda_c^{RL})S^{(box)}(x_c,x_t,\beta)\Big] \tag{B.18}$$

with $\beta = M_W^2/M_{W'}^2$ and the quark operator given by the equation

$$Q_2^{LR} = \bar{s}P_R d\cdot\bar{s}P_L d \tag{B.19}$$

containing both the left and right handed contributions.

Similarly the calculation for the box diagram including Charged Higgs is carried giving the effective hamiltonian given below

$$A^{(H^\pm box)} = \frac{G_F^2 M_W^2}{4\pi^2}(Q_2^{LR})\sum_{U,V=c,t}\lambda_U^{LR}\lambda_V^{RL}S_{LR}^H(x_U,x_V,\beta\omega) \tag{B.20}$$

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