Peristaltic motion in a curved channel

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In the name of almighty ALLAH,
the most beneficent, the most merciful

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Dedicated to

My Father, Mother and all family members.

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Foremost, I am always grateful to Almighty ALLAH, who made human being, the best creation of all the living species and made them understand to write with pen. He provided me the boldness and capability to achieve this task. I offer countless Darood and Salaams to my beloved Holy Prophet Hazrat Muhammad (PBUH), for whom this universe has been manifested. ALLAH has shown His existence and oness by sending him as a messenger of Islam and born me as a Muslim.

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Khurram Javid

DECLARTION

I hereby declare that this thesis, neither as a whole nor as a part thereof, has been copied out from any source. It is further declared that I have prepared this thesis entirely on the basis of my personal efforts made under the sincere guidance of my supervisor. No portion of the work, presented in this thesis, has been submitted in the support of any application for any degree or qualification of this or any other institute of learning.

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Preface

The most vital mechanism observed for transportation for biofluids in human beings is the peristaltic mechanism. According to this mechanism, the contents in the vessel or tube are transported due to the sinusoidal motion of the boundary. The initial studies regarding the fluid mechanics of this mechanism were performed by Latham [1], Shapiro et al. [2], Jaffrin and Shapiro [3], Fung and Yih [4], Yin and Fung [5] and many others.

Later on, it was realized by the researchers that in investigating peristaltic motion, it is not always appropriate to consider the nature of the fluid as Newtonian. This fact was first realized by Raju and Devanathan [6] and they studied peristaltic flow of a power-law fluid and obtained the solution for the stream function as a power series in terms of the amplitude of deformation. In another paper, they analyzed the peristaltic motion of a viscoelastic fluid with fading memory in a tube [7]. After that a number of investigators attempted to analyze peristaltic flows of different non-Newtonian fluids under various assumptions. Some important contributions in this area can be found in refs. [8-18].

In all the above mentioned studies, the peristaltic flow is considered in a straight channel or tube. However, this is an assumption and in reality the tube or duct may be curved. Therefore, one has to incorporate the effects of curvature in analyzing peristaltic flow. Motivated by this fact, Sato el al. [19] discussed two dimensional peristaltic flows in a curved channel. Following them, Ali et al. [20] studied peristaltic flow in a curved channel in wave frames of reference. They have used the no-slip condition at the upper and lower walls of the channel and obtained the expressions for stream function, axial velocity and pressure gradient.

Peristaltic flows in a straight channels/tubes employing slip condition has been studied by a number of authors. Mention may be made to the works, Chu and Fang [21], El-Shehawy et al. [22], Hayat et al. [23, 24] and Ali et al. [25].

To the best of my knowledge, no attempt has been made yet to study the peristaltic flow in curved channel when no-slip condition is inadequate. Therefore, the main aim of this thesis is to provide such a study. The brief layout of thesis is as follows. It consists of three chapters. Basic definitions and governing equations (i.e. continuity and momentum equations) for a viscous fluid in curvilinear coordinates are included in chapter 1. Chapter 2 presents the detailed review of ref [20]. In chapter 3, the work of ref [20] is extended by taking the slip condition at the walls of the channel. The effects of slip parameter on various features of peristaltic motion are discussed in detail.

Contents

Chapter 1

Preliminaries

In this chapter, some basic definitions and equations are provided which will be used in the next chapters. All the material of this chapter is based on the book by Resrick, Halliday and Krane and web.

1.1 Flow

A material goes under deformation when certain forces acts upon it. If the deformation exceed continuously without limit, then the phenomena is known as flow.

1.2 Fluid

The word fluid comes from a Latin word meaning to flow. Fluid will flow, for example, to take the shape of any container that holds them; in a fluid, the particles/molecules can move relative to one another.

1.3 Fluid mechanics

Fluid mechanics is the study of fluids and the forces acts upon them (Fluids include liquids, gases and plasmas). Fluid mechanics can be divided into fluid kinematics, the study of fluid motion, and fluid dynamics, the study of the effect of forces on fluid motion, which can further be divided into fluid statics, the study of fluids at rest, and fluid kinetics, the study of fluids in motion.

1.4 Fluid kinematics

Kinematics is the branch of mechanics that deals with quantities involving space and time only. It is used to describe the motions of particles and objects, but does not take the forces that cause these motions into account.

1.5 Fluid dynamics

The fluid dynamics is a sub-discipline of fluid mechanics that deals with fluid flow and the natural science of fluids (liquids and gases) in motion.

1.6 Fluid statics

Fluid statics is a sub-discipline of fluid mechanics that deals with fluids at rest.

1.7 Velocity field

In dealing with fluid in motion, we shall necessarily be concerned with the description of a velocity field. If we define a fluid particle as a small mass of fluid of fixed identity of volume δv, then the velocity at point C is defined as the instantaneous velocity of the fluid particles which, at a given instant, is passing through point C. At a given instant, the velocity field $\mathbf{V}$ is a function of the space coordinates (x, y, z). The velocity at any point in the flow field might vary from one instant to another. Thus the complete representation of velocity is given by

$$\mathbf{V} = \mathbf{V}(x, y, z, t). \tag{1.1}$$

1.8 Pressure

The magnitude of the normal force per unit surface area is called the pressure. Pressure is a scalar quantity; it has no directional properties. Mathematically:

$$p = \frac{F}{A}. \tag{1.2}$$

where F is the magnitude of force acting in the direction perpendicular to the surface of the fluid and A is the area of the surface of the fluid. Pressure has dimensions of force divided by area, and common unit for pressure is N/m^2. This unit is given the SI designation Pascal (abbreviation Pa: $1Pa = 1N/m^2$).

1.9 Density

The density ρ of a small element of any material is the mass δm of the element divided by its volume δV:

$$\rho = \lim_{\delta V -> \delta V'} \left(\frac{\delta m}{\delta V} \right). \tag{1.3}$$

where $\delta V'$ is the small volume over which the substance can be considered as a continuum. If the density of an object has the same value at all points, the density of the object is equal to the mass of the entire object divided by its volume:

$$\rho = \frac{m}{V}. \tag{1.4}$$

1.10 Viscosity

Viscosity in fluid flow is similar to friction in the motion of solid bodies. The measure of resistance to the motion of the fluid is called viscosity. It is also known as absolute or dynamics viscosity. Mathematically, viscosity is the ratio of shear stress to the rate of shear strain.

$$\mu = \frac{\text{Shear stress}}{\text{Rate of deformation}}. \tag{1.5}$$

1.11 Kinematic viscosity

Kinematic viscosity is the ratio of absolute viscosity μ to the density ρ. It is denoted by v and given by

$$v = \frac{\mu}{\rho}. \tag{1.6}$$

1.12 Curvature

The curvature of a curve at a point is a measure of how sensitive its tangent line is to moving the point to the other nearby point.

1.13 Peristalsis

Successive waves of involuntary contraction passing along the walls of hollow muscular structure (as the esophagus or intension) and forcing the contents onward.

1.14 Coefficient of viscosity

The ratio between stress and strain in the fluid is called the coefficient of viscosity of the fluid. It is denoted by Greek letter η (eta).

$$\eta = \frac{\frac{F}{A}}{\frac{dv}{dy}}. \tag{1.7}$$

where F is the magnitude of force acting in the direction perpendicular to the surface of the fluid. A is the area of the surface of the fluid and dv/dy is velocity gradient.

1.15 Types of flow

1.15.1 Uniform flow

It is a flow in which the velocity of fluid particles are same at each layer.

1.15.2 Non-uniform flow

It is a flow in which the velocity of fluid particles are different at different layers.

For a uniform flow, by its definition, the area of the cross section of the flow should remain constant. For example, the uniform flow is the flow of a liquid through a pipeline of constant diameter. And contrary to this, the flow through a pipeline of variable diameter would be necessarily non-uniform.

1.15.3 Steady flow

We describe the flow in terms of the values of such variables as pressure, density and flow velocity at every point of the fluid. If these variables are constant in time, the flow is said to be steady. Mathematically:

$$\frac{\partial \eta}{\partial t} = 0. \tag{1.8}$$

where η represents any fluid property and t is the time.

1.15.4 Unsteady flow

If these variables (pressure, density and flow velocity at every point of the fluid) are the functions of time, the flow is said to be unsteady. Mathematically:

$$\frac{\partial \eta}{\partial t} \neq 0. \tag{1.9}$$

1.15.5 Rotational flow

If an element of the moving fluid rotates about an axis through the centre of mass of the element, the flow is said to be rotational. Mathematically, for rotational flow

$$\nabla \times \mathbf{V} \neq 0. \tag{1.10}$$

1.15.6 Irrotational flow

If an element of the moving fluid does not rotate about an axis through the centre of mass of the element, the flow is said to be irrotational. Mathematically, for irrotational flow

$$\nabla \times \mathbf{V} = 0. \tag{1.11}$$

1.15.7 Laminar flow

Fluid flow in which the speed varies layer-by-layer is called laminar flow. In laminar flow, viscous shear stresses acts between these layers of the fluid which defines the velocity distribution among these layers of flow.

1.15.8 Turbulent flow

Fluid flow in which the velocities vary erratically from point to point as well as from time to time is called turbulent flow.

In a viscous fluid, the flow at low speed can be described as laminar, which suggests layers sliding smoothly over one another. When the flow speed is sufficiently large, the motion becomes disordered and irregular: this is turbulent flow.

1.15.9 Compressible flow

If the density ρ of a fluid is a not constant, dependent of x, y, z and t, its flow is called compressible flow.

1.15.10 Incompressible flow

If the density ρ of a fluid is a constant, independent of x, y, z and t, its flow is called incompressible flow.

Liquids can usually be considered as flowing incompressibly. But even for a highly compressible gas, the variation in density may be insignificant, and for practical purposes, we can consider its flow to be incompressible. For example, in flight at speeds much lower than the speed of sound in air (described by subsonic aerodynamics), the flow of the air over the wings is nearly incompressible.

1.15.11 Couette flow

It is a flow between two plates, in which one plate remains at rest and other plate is moving with uniform velocity.

1.15.12 Poiseuille flow

A flow between two plates produced by a constant pressure gradient in the direction of the flow is called Poiseuille flow.

1.15.13 One dimensional flow

A flow for which the velocity field depends only on one space variable is called one-dimensional flow.

1.15.14 Two dimensional flow

A flow for which the velocity field depends upon two space variables is called two-dimensional flow.

1.16 Classification of fluids

1.16.1 Ideal fluids

A non-existent, assumed fluid without either viscosity or compressibility is called an ideal fluid or perfect fluid. In nature, this type of fluid does not exist. Furthermore, a gas subject to Boyle's-Charle's law is called a perfect or an ideal gas. It is the hypothetical form of fluids. However, the fluid with negligible viscosity may be considered as an ideal fluid.

1.16.2 Real fluids

Real fluids are those in which fluid friction has significant effects on the fluid motion. In other words, we can not neglect the viscosity effects on the motion. Real fluids are further classified into two classes on the basis of Newton's law of viscosity. According to this law, "shear stress is directly proportional to the rate of deformation". For one dimensional flow, it can be written as

$$\tau_{yx} = \mu \frac{du}{dy}. \tag{1.12}$$

where τ_{yx} is the shear stress and du/dy is the rate of deformation.

1.16.3 Newtonian fluid

A Newtonian fluid (named after *Isaac Newton*) is a fluid whose stress versus strain (deformation) rate curve is linear and passes through the origin. i.e., Newtonian fluid obeys Newton's law of viscosity. Water, gasoline and mercury are some examples of Newtonian fluids.

1.16.4 Non-Newtonian fluid

A non-Newtonian fluid is a fluid whose flow properties are not described by a single constant value of viscosity, i.e., it does not satisfy Newton's law of viscosity. For non-Newtonian fluids

$$\tau_{yx} = k \left(\frac{du}{dy}\right)^n, \qquad n \neq 1 \tag{1.13}$$

or

$$\tau_{yx} = \eta \left(\frac{du}{dy}\right), \tag{1.14}$$

where

$$\eta = k \left(\frac{du}{dy}\right)^{n-1}. \tag{1.15}$$

is the apparent viscosity. Examples of non-Newtonian fluids are tooth paste, ketchup, gel, shampoo, blood, soaps etc.

1.16.5 Time independent non-Newtonian fluids

Such fluids where apparent viscosity does not depend upon time are known time independent non-Newtonian fluids. These are further sub-divided in the following types.

1.16.6 Time dependent non-Newtonian fluids

Such fluids where apparent viscosity depends upon time are known time dependent non-Newtonian fluids. These are further sub-divided in the following types.

1.16.7 Thixotropic fluids

Such fluids show a decrease in η under a constant applied shear stress. An example of such a fluid is yogurt.

1.16.8 Rheopatic fluids

Such fluids show an increase in η with time under a constant applied shear stress. An example of such a fluid is blood.

1.16.9 Viscoelastic fluids

After deformation when applied stress is released, some fluids partially come to their original shape or position, such fluids are called viscoelastic fluids.The examples of such fluids are nylon, flour dough etc.

1.17 Equation of continuity

The mathematical form of the law of conservation of mass for a fluid is known as equation of continuity. It has the following form

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \mathbf{V}) = 0. \tag{1.16}$$

and for an incompressible fluid, it reduces to the form

$$\boldsymbol{\nabla} \cdot \mathbf{V} = 0. \tag{1.17}$$

1.18 Equation of motion

The motion of fluid is governed by law of conservation of momentum. The application of this law to an arbitrary control volume in flowing fluid yields the following equation commonly known as equation of motion

$$\rho \frac{d\mathbf{V}}{dt} = -\boldsymbol{\nabla} p + \operatorname{div} \mathbf{T} + \rho \mathbf{b}. \tag{1.18}$$

In above equation, $\mathbf{T}$ is Cauchy stress tensor and $\mathbf{b}$ is body force per unit mass.

The above equation can be written in a more convenient form as

$$\rho\left[\frac{\partial \mathbf{V}}{\partial t} - \mathbf{V}\times(\nabla\times\mathbf{V}) + \nabla\left(\frac{V^2}{2}\right)\right] = \rho k + \operatorname{div}\mathbf{T}. \tag{1.19}$$

1.19 Curvilinear coordinates

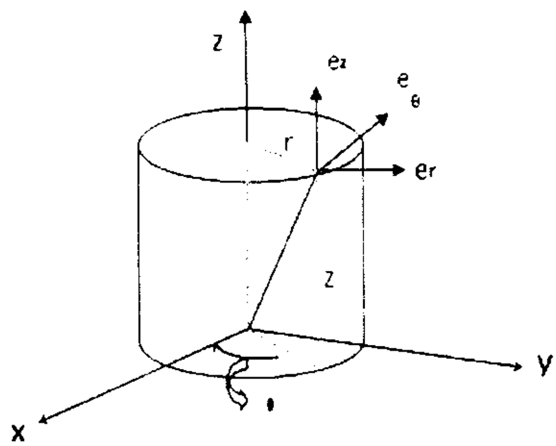

Fig. (1.1). Cylindrical coordinates system with coordinates (r, θ, z).

An orthogonal system is a system for which the coordinate surfaces are mutually perpendicular. For the cylindrical system (Fig. (1.1)), the coordinate surfaces are r = constant, θ = constant and z = constant. These three coordinate surfaces intersect through a given point at right angles. The three curves of intersection of the coordinate surfaces in pair intersect at right angles. These curves are called coordinate lines or directions. We draw unit basis vectors tangent to the coordinate directions. For the cylindrical system (Fig. (1.1)), we might call them $\mathbf{e}_r$, $\mathbf{e}_\theta$ and $\mathbf{e}_z$. These unit vectors form an orthogonal triad like $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. We refer to such a coordinate systems as curvilinear coordinate systems when the coordinate surfaces are not planes and the coordinate lines are curves other than straight lines.

We consider the curvilinear orthogonal coordinates q_1, q_2, q_3, which can be calculated from the cartesian coordinates (x_1, x_2, x_3) as:

$$q_1 = q_1(x_1, x_2, x_3), \tag{1.20}$$

$$q_2 = q_2(x_1, x_2, x_3), \tag{1.21}$$

$$q_3 = q_3(x_1, x_2, x_3), \tag{1.22}$$

or in short

$$q_i = q_i(x_j). \tag{1.23}$$

We assume that Eq. (1.23) has unique inverse:

$$x_i = x_i(q_j). \tag{1.24}$$

or

$$\mathbf{x} = \mathbf{x}(q_j). \tag{1.25}$$

If q_2 and q_3 are kept constant, the vector $\mathbf{x} = \mathbf{x}(q_1)$ describes a curve in a space which is the coordinates curve q_1. $\partial\mathbf{x}/\partial q_1$ is the tangent vector to this curve. The corresponding unit vector in the direction of increasing q_1 reads:

$$\mathbf{e}_1 = \frac{\frac{\partial \mathbf{x}}{\partial q_1}}{\left|\frac{\partial \mathbf{x}}{\partial q_1}\right|}. \tag{1.26}$$

If we let $|\partial\mathbf{x}/\partial q_1| = b_1$, then we see that

$$\frac{\partial \mathbf{x}}{\partial q_1} = \mathbf{e}_1 b_1, \tag{1.27}$$

and in the same way

$$\frac{\partial \mathbf{x}}{\partial q_2} = \mathbf{e}_2 b_2, \tag{1.28}$$

$$\frac{\partial \mathbf{x}}{\partial q_3} = \mathbf{e}_3 b_3, \tag{1.29}$$

with $|\partial\mathbf{x}/\partial q_2| = b_2$ and $|\partial\mathbf{x}/\partial q_3| = b_3$.

Since $\mathbf{x} = \mathbf{x}(q_j)$, we can write the line element as

$$d\mathbf{x} = \frac{\partial \mathbf{x}}{\partial q_1} dq_1 + \frac{\partial \mathbf{x}}{\partial q_2} dq_2 + \frac{\partial \mathbf{x}}{\partial q_3} dq_3. \tag{1.30}$$

or

$$d\mathbf{x} = b_1 dq_1 \mathbf{e}_1 + b_2 dq_2 \mathbf{e}_2 + b_3 dq_3 \mathbf{e}_3. \tag{1.31}$$

and the square of the line element is

$$d\mathbf{x} \cdot d\mathbf{x} = b_1^2 dq_1^2 + b_2^2 dq_2^2 + b_3^2 dq_3^2. \tag{1.32}$$

Further, the volume element (Fig.(1.2)) is given by

$$dV = b_1 b_2 b_3 dq_1 dq_2 dq_3. \tag{1.33}$$

The expression of q_1 surface element of the volume element dV (i.e. the surface element perpendicular to the q_1 direction) is

$$dS_1 = b_2 b_3 dq_2 dq_3, \tag{1.34}$$

and similarly the other surface elements are:

$$dS_2 = b_1 b_3 dq_1 dq_3. \tag{1.35}$$

$$dS_3 = b_1 b_2 dq_1 dq_2. \tag{1.36}$$

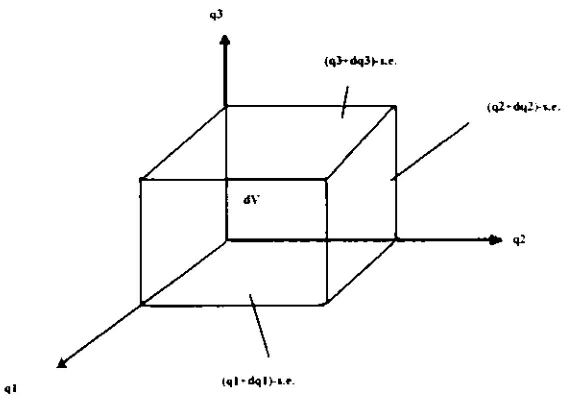

Fig. (1.2). Volume element in the curvilinear coordinate system.

1.19.1 Gradient, Divergence and Curl in curvilinear coordinates

In this subsection, we will discuss the components of gradient and curl of a vector, divergence of a tensor and rate of deformation tensor along with the expression of the divergence of a tensor.

If Φ is a scalar function, then the components of vector $\nabla\Phi$ are:

$$\text{Along } q_1 \quad : \qquad (\nabla\Phi)_1 = \frac{1}{b_1}\frac{\partial\Phi}{\partial q_1}. \tag{1.37}$$

$$\text{Along } q_2 \quad : \qquad (\nabla\Phi)_2 = \frac{1}{b_2}\frac{\partial\Phi}{\partial q_2}. \tag{1.38}$$

$$\text{Along } q_3 \quad : \qquad (\nabla\Phi)_3 = \frac{1}{b_3}\frac{\partial\Phi}{\partial q_3}. \tag{1.39}$$

The divergence of a vector $\mathbf{V}$ (here considered as fluid velocity) having components u_1, u_2 and u_3 in the direction of the increasing q_1, q_2 and q_3 is given by

$$\boldsymbol{\nabla}.\mathbf{V} = \frac{1}{b_1 b_2 b_3}\left[\frac{\partial}{\partial q_1}(b_2 b_3 u_1) + \frac{\partial}{\partial q2}(b_3 b_1 u_2) + \frac{\partial}{\partial q_3}(b_1 b_2 u_3)\right]. \tag{1.40}$$

The components of curl $\mathbf{\nabla} \times \mathbf{V}$ are:

$$\text{Along } q_1: \quad (\mathbf{\nabla} \times \mathbf{V})_1 = \frac{1}{b_2 b_3}\left[\frac{\partial}{\partial q_2}(b_3 u_3) - \frac{\partial}{\partial q_3}(b_2 u_2)\right]. \tag{1.41}$$

$$\text{Along } q_2: \quad (\mathbf{\nabla} \times \mathbf{V})_2 = \frac{1}{b_1 b_3}\left[\frac{\partial}{\partial q_3}(b_1 u_1) - \frac{\partial}{\partial q_1}(b_3 u_3)\right]. \tag{1.42}$$

$$\text{Along } q_3: \quad (\mathbf{\nabla} \times \mathbf{V})_3 = \frac{1}{b_1 b_2}\left[\frac{\partial}{\partial q_1}(b_2 u_2) - \frac{\partial}{\partial q_2}(b_1 u_1)\right]$$

The components of the divergence of stress tensor are given by

$$\text{Along } q_1: \quad (\mathbf{\nabla}.\mathbf{T})_1 = \frac{1}{b_1 b_2 b_3}\left[\frac{\partial}{\partial q_1}(b_2 b_3 T_{11}) + \frac{\partial}{\partial q_2}(b_3 b_1 T_{21}) + \frac{\partial}{\partial q_3}(b_1 b_2 T_{31})\right] + \frac{T_{21}}{b_1 b_2}\frac{\partial b_1}{\partial q_2} + \frac{T_{31}}{b_1 b_3}\frac{\partial b_1}{\partial q_3} - \frac{T_{22}}{b_1 b_2}\frac{\partial b_2}{\partial q_1} - \frac{T_{33}}{b_1 b_3}\frac{\partial b_3}{\partial q_1}. \tag{1.43}$$

$$\text{Along } q_2: \quad (\mathbf{\nabla}.\mathbf{T})_2 = \frac{1}{b_1 b_2 b_3}\left[\frac{\partial}{\partial q_1}(b_2 b_3 T_{12}) + \frac{\partial}{\partial q_2}(b_3 b_1 T_{22}) + \frac{\partial}{\partial q_3}(b_1 b_2 T_{32})\right] + \frac{T_{32}}{b_2 b_3}\frac{\partial b_2}{\partial q_3} + \frac{T_{12}}{b_2 b_1}\frac{\partial b_2}{\partial q_1} - \frac{T_{33}}{b_2 b_3}\frac{\partial b_3}{\partial q_2} - \frac{T_{11}}{b_1 b_2}\frac{\partial b_1}{\partial q_2}. \tag{1.44}$$

$$\text{Along } q_3: \quad (\mathbf{\nabla}.\mathbf{T})_3 = \frac{1}{b_1 b_2 b_3}\left[\frac{\partial}{\partial q_1}(b_2 b_3 T_{13}) + \frac{\partial}{\partial q_2}(b_3 b_1 T_{23}) + \frac{\partial}{\partial q_3}(b_1 b_2 T_{33})\right] + \frac{T_{13}}{b_1 b_3}\frac{\partial b_3}{\partial q_1} + \frac{T_{23}}{b_2 b_3}\frac{\partial b_3}{\partial q_2} - \frac{T_{11}}{b_1 b_3}\frac{\partial b_1}{\partial q_2} - \frac{T_{22}}{b_3 b_2}\frac{\partial b_2}{\partial q_3}. \tag{1.45}$$

The Cauchy-stress tensor in symbolic form is

$$\mathbf{T} = (-p + \lambda * \mathbf{\nabla}.\mathbf{V})\,\mathbf{I} + 2\mu\mathbf{E}. \tag{1.46}$$

which for an incompressible fluid reduces to

$$\mathbf{T} = -p\mathbf{I} + 2\mu\mathbf{E}. \tag{1.47}$$

where **I** is the identity tensor and **E** the rate of deformation tensor.

The components of the rate of deformation tensor in curvilinear coordinates are given by

$$e_{11} = \frac{1}{b_1}\frac{\partial u_1}{\partial q_1} + \frac{u_2}{b_1 b_2}\frac{\partial b_1}{\partial q_2} + \frac{u_3}{b_1 b_3}\frac{\partial b_1}{\partial q_3}. \tag{1.48}$$

$$e_{22} = \frac{1}{b_2}\frac{\partial u_2}{\partial q_2} + \frac{u_1}{b_1 b_2}\frac{\partial b_2}{\partial q_1} + \frac{u_3}{b_2 b_3}\frac{\partial b_2}{\partial q_3}. \tag{1.49}$$

$$e_{33} = \frac{1}{b_3}\frac{\partial u_3}{\partial q_3} + \frac{u_1}{b_1 b_3}\frac{\partial b_3}{\partial q_1} - \frac{u_2}{b_2 b_3}\frac{\partial b_3}{\partial q_2}. \tag{1.50}$$

$$2e_{12} = \frac{b_1}{b_2}\frac{\partial}{\partial q_2}\left(\frac{u_1}{b_1}\right) + \frac{b_2}{b_1}\frac{\partial}{\partial q_1}\left(\frac{u_2}{b_2}\right) = 2e_{21}. \tag{1.51}$$

$$2e_{32} = \frac{b_2}{b_3}\frac{\partial}{\partial q_2}\left(\frac{u_2}{b_2}\right) + \frac{b_3}{b_2}\frac{\partial}{\partial q_1}\left(\frac{u_3}{b_3}\right) = 2e_{23}. \tag{1.52}$$

$$2e_{31} = \frac{b_1}{b_3}\frac{\partial}{\partial q_3}\left(\frac{u_1}{b_1}\right) + \frac{b_3}{b_1}\frac{\partial}{\partial q_1}\left(\frac{u_3}{b_3}\right) = 2e_{13}. \tag{1.53}$$

1.20 Equation of continuity in curvilinear coordinates

Using the formulas given in section (1.19.1), we can write the equation of continuity (1.16) in terms of curvilinear coordinates as

$$\frac{\partial \rho}{\partial t} + \frac{1}{b_1 b_2 b_3}\left[\frac{\partial}{\partial q_1}(b_2 b_3 \rho u_1) + \frac{\partial}{\partial q_2}(b_3 b_1 \rho u_2) + \frac{\partial}{\partial q_3}(b_1 b_2 \rho u_3)\right] = 0. \tag{1.54}$$

If ρ is constant, then we get

$$\frac{\partial}{\partial q_1}(b_2 b_3 u_1) + \frac{\partial}{\partial q2}(b_3 b_1 u_2) + \frac{\partial}{\partial q_3}(b_1 b_2 u_3) = 0. \tag{1.55}$$

Eq. (1.55) is counterpart of Eq. (1.17) in curvilinear coordinates and valid for incompressible flows.

1.21 Equation of motion in curvilinear coordinates

In the same manner, the application of results in section (1.19.1) yields the following components of equation of motion in curvilinear coordinates q_1, q_2 and q_3.

Along q_1 :

$$\rho\left[\begin{array}{c}\frac{\partial u_1}{\partial t} - \frac{u_2}{b_1 b_2}\left(\frac{\partial}{\partial q_1}(b_2 u_2) - \frac{\partial}{\partial q_2}(b_1 u_1)\right)\\ + \frac{u_3}{b_1 b_3}\left(\frac{\partial}{\partial q_3}(b_1 u_1) - \frac{\partial}{\partial q_1}(b_3 u_3)\right) + \frac{u_1}{b_1}\frac{\partial u_1}{\partial q_1}\end{array}\right]$$
$$= \rho k_1 + \frac{1}{b_1 b_2 b_3}\left[\frac{\partial}{\partial q_1}(b_2 b_3 T_{11}) + \frac{\partial}{\partial q_2}(b_3 b_1 T_{21}) + \frac{\partial}{\partial q_3}(b_1 b_2 T_{31})\right]$$
$$+ \frac{T_{21}}{b_1 b_2}\frac{\partial b_1}{\partial q_2} + \frac{T_{31}}{b_1 b_3}\frac{\partial b_1}{\partial q_3} - \frac{T_{22}}{b_1 b_2}\frac{\partial b_2}{\partial q_1} - \frac{T_{33}}{b_1 b_3}\frac{\partial b_3}{\partial q_1}. \quad (1.56)$$

Along q_2 :

$$\rho\left[\begin{array}{c}\frac{\partial u_2}{\partial t} - \frac{u_3}{b_2 b_3}\left(\frac{\partial}{\partial q_2}(b_3 u_3) - \frac{\partial}{\partial q_3}(b_2 u_2)\right)\\ + \frac{u_1}{b_1 b_2}\left(\frac{\partial}{\partial q_1}(b_2 u_2) - \frac{\partial}{\partial q_2}(b_1 u_1)\right) + \frac{u_2}{b_2}\frac{\partial u_2}{\partial q_2}\end{array}\right]$$
$$= \rho k_2 + \frac{1}{b_1 b_2 b_3}\left[\frac{\partial}{\partial q_1}(b_2 b_3 T_{12}) + \frac{\partial}{\partial q_2}(b_3 b_1 T_{22}) + \frac{\partial}{\partial q_3}(b_1 b_2 T_{32})\right]$$
$$+ \frac{T_{32}}{b_2 b_3}\frac{\partial b_2}{\partial q_3} + \frac{T_{12}}{b_2 b_1}\frac{\partial b_2}{\partial q_1} - \frac{T_{33}}{b_2 b_3}\frac{\partial b_3}{\partial q_2} - \frac{T_{11}}{b_1 b_2}\frac{\partial b_1}{\partial q_2}. \quad (1.57)$$

Along q_3 :

$$\rho\left[\begin{array}{c}\frac{\partial u_3}{\partial t} - \frac{u_1}{b_1 b_3}\left(\frac{\partial}{\partial q_3}(b_1 u_1) - \frac{\partial}{\partial q_1}(b_3 u_3)\right)\\ + \frac{u_2}{b_2 b_3}\left(\frac{\partial}{\partial q_2}(b_3 u_3) - \frac{\partial}{\partial q_3}(b_2 u_2)\right) + \frac{u_3}{b_3}\frac{\partial u_3}{\partial q_3}\end{array}\right]$$
$$= \rho k_3 + \frac{1}{b_1 b_2 b_3}\left[\frac{\partial}{\partial q_1}(b_2 b_3 T_{13}) + \frac{\partial}{\partial q_2}(b_3 b_1 T_{23}) + \frac{\partial}{\partial q_3}(b_1 b_2 T_{33})\right]$$
$$+ \frac{T_{13}}{b_1 b_3}\frac{\partial b_3}{\partial q_1} + \frac{T_{23}}{b_2 b_3}\frac{\partial b_3}{\partial q_2} - \frac{T_{11}}{b_1 b_3}\frac{\partial b_1}{\partial q_2} - \frac{T_{22}}{b_3 b_2}\frac{\partial b_2}{\partial q_3}. \quad (1.58)$$

Chapter 2

Long wavelength flow analysis in a curved channel

2.1 Introduction

This chapter is wholly based on the material of a paper published by Ali et al. [20]. All the mathematical derivations and graphical results given in this paper are reproduced by the author of this thesis. The aim of this chapter is to understand the phenomenon of peristalsis in a curved channel and to provide the necessary material on which the contents of chapter (3) are based.

2.2 Flow geometry and governing equations

Let us consider a curved channel of half width a coiled in a circle with center O and radius R^* (Fig. (2.1)). We choose curvilinear coordinates $\overline{R}$, $\overline{S}$ and $\overline{Z}$ such that $\overline{R}$ is along the radial direction, $\overline{S}$ is along the axial direction and $\overline{Z}$ perpendicular to both $\overline{R}$ and $\overline{S}$. The geometry of the upper and lower walls is given as:

$$\overline{H}(\overline{S},\bar{t}) = a + b\sin\left[\frac{2\pi}{\lambda}(\overline{S} - c\bar{t})\right], \quad \text{upper wall}, \tag{2.1}$$

$$-\overline{H}(\overline{S},\bar{t}) = -a - b\sin\left[\frac{2\pi}{\lambda}(\overline{S} - c\bar{t})\right], \quad \text{lower wall}. \tag{2.2}$$

In the above equations, c is the speed, λ is the wavelength, b is the amplitude of the wave and $\bar{t}$ is the time. Having described the geometry of the problem, we will now move to derive the flow equations. In deriving the flow equations, the result of sections (1.19 and (1.20) will be used.

Identifying $q_1 = \overline{R}$, $q_2 = \overline{S}$ and $q_3 = \overline{Z}$ and flow as two-dimensional, we can write the velocity field $\mathbf{V}$ as

$$\mathbf{V} = \left[\overline{V}(\overline{R}, \overline{S}, \bar{t}),\ \overline{U}(\overline{R}, \overline{S}, \bar{t}),\ 0\right]. \tag{2.3}$$

where $\overline{V}$ is the components of velocity along the $\overline{R}-$direction and $\overline{U}$ is along the $\overline{S}-$direction.

The appropriate transformations between curvilinear coordinates $(\overline{R}, \overline{S}, \overline{Z})$ and cartesian coordinates $(\overline{X}, \overline{Y}, \overline{Z})$ are:

$$\overline{X} = (R^* + \overline{R})\cos\left(\frac{\overline{S}}{R^*}\right), \tag{2.4}$$

$$\overline{Y} = (R^* + \overline{R})\sin\left(\frac{\overline{S}}{R^*}\right), \tag{2.5}$$

$$\overline{Z} = \overline{Z}. \tag{2.6}$$

Using the transformation (2.4) – (2.6) and results in section (1.19), the following values of b_1, b_2 and b_3 are obtained.

$$b_1 = 1, \tag{2.7}$$

$$b_2 = \left(\frac{R^* + \overline{R}}{R^*}\right), \tag{2.8}$$

and

$$b_3 = 1. \tag{2.9}$$

Having in hand, the scale factors b_1, b_2 and b_3 and taking $u_1 = \overline{V}$, $u_2 = \overline{U}$ and $u_3 = 0$, the continuity equation (1.16) takes the form

$$\frac{\partial}{\partial \overline{R}}\left\{(R^* + \overline{R})\overline{V}\right\} + R^*\frac{\partial \overline{U}}{\partial \overline{S}} = 0. \tag{2.10}$$

Similarly the components of the rate of deformation tensor are:

$$e_{11} = \frac{\partial \overline{V}}{\partial \overline{R}}. \tag{2.11}$$

$$e_{22} = \left(\frac{R^*}{R^* + \overline{R}}\right)\frac{\partial \overline{U}}{\partial \overline{S}} + \frac{\overline{V}}{R^* + \overline{R}}. \tag{2.12}$$

$$2e_{12} = \left(\frac{R^*}{R^* + \overline{R}}\right)\frac{\partial \overline{V}}{\partial \overline{S}} + \left(\frac{R^* + \overline{R}}{R^*}\right)\frac{\partial}{\partial \overline{R}}\left[\left(\frac{R^*}{R^* + \overline{R}}\right)\overline{U}\right] = 2e_{21}. \tag{2.13}$$

and thus the components of $\mathbf{T}$ become

$$T_{11} = -p + 2\eta\frac{\partial \overline{V}}{\partial \overline{R}}. \tag{2.14}$$

$$T_{22} = -p + 2\eta\left[\left(\frac{R^*}{R^* + \overline{R}}\right)\frac{\partial \overline{U}}{\partial \overline{S}} + \frac{\overline{V}}{R^* + \overline{R}}\right]. \tag{2.15}$$

$$T_{12} = T_{21} = \eta(R^* + \overline{R})\frac{\partial}{\partial \overline{R}}\left(\frac{\overline{U}}{R^* + \overline{R}}\right) + \eta\left(\frac{R^*}{R^* + \overline{R}}\right)\frac{\partial \overline{V}}{\partial \overline{S}}. \tag{2.16}$$

After substitution of b_1, b_2 and b_3 and the components of $\mathbf{T}$ in Eqs. (1.56) and (1.57), the results are given in the following equations for the flow under consideration in absence of body forces.

$$\rho\left[\frac{\partial \overline{U}}{\partial \overline{t}} - \overline{U}\left(\frac{R^*}{R^*+\overline{R}}\right)\left[\frac{\partial}{\partial \overline{R}}\left(\frac{R^*+\overline{R}}{R^*}\right)\overline{U} - \frac{\partial \overline{V}}{\partial \overline{S}}\right] + \frac{(\nabla u^2)}{2}\right] = \frac{2\eta}{(R^*+\overline{R})}\frac{\partial}{\partial \overline{R}}\left[(R^*+\overline{R})\frac{\partial \overline{V}}{\partial \overline{R}}\right] + \mu\left[\left(\frac{R^*}{R^*+\overline{R}}\right)^2\frac{\partial^2 \overline{V}}{\partial \overline{S}^2} - \frac{\partial p}{\partial \overline{R}} - 2\eta\frac{R^*}{(R^*+\overline{R})^2}\frac{\partial \overline{U}}{\partial \overline{S}} - \frac{2\eta \overline{V}}{(R^*+\overline{R})^2}\right]. \tag{2.17}$$

$$\rho\left[\frac{\partial \overline{U}}{\partial \overline{t}} + \overline{V}\frac{\partial \overline{U}}{\partial \overline{R}} + \frac{\overline{V}\overline{U}}{(R^*+\overline{R})} + \left(\frac{R^*}{R^*+\overline{R}}\right)\overline{U}\frac{\partial \overline{U}}{\partial \overline{S}}\right] = -\left(\frac{R^*}{R^*+\overline{R}}\right)\frac{\partial p}{\partial \overline{S}} + \mu\left[\frac{1}{(R^*+\overline{R})}\frac{\partial}{\partial \overline{R}}\left[(R^*+\overline{R})\frac{\partial \overline{U}}{\partial \overline{R}}\right] + \left(\frac{R^*}{R^*+\overline{R}}\right)^2\frac{\partial^2 \overline{U}}{\partial \overline{S}^2} + \frac{2R^*}{(R^*+\overline{R})^2}\frac{\partial \overline{V}}{\partial \overline{S}} - \frac{\overline{U}}{(R^*+\overline{R})^2}\right]. \tag{2.18}$$

In the laboratory frame $(\overline{R}, \overline{S})$, the flow in the curved channel is unsteady. However, we can treat this system as steady in the wave frame $(\overline{r}, \overline{s})$ moving with the speed of wave. The transformations relating the two frames are:

$$\overline{s} = \overline{S} - c\overline{t},\ \overline{r} = \overline{R},\ \overline{u} = \overline{U} - c,\ \overline{v} = \overline{V}, \tag{2.19}$$

where $\overline{v}$ and $\overline{u}$ are the velocity components along the $\overline{r}$ and $\overline{s}$−directions in the wave frame. Utilizing transformations (2.19), Eqs. (2.10), (2.17) and (2.18) can be casted as

$$\frac{\partial}{\partial \overline{r}} \left\{ (\overline{r} + R^*)\overline{v} \right\} + \overline{r}\frac{\partial \overline{u}}{\partial \overline{s}} = 0. \tag{2.20}$$

$$-c\frac{\partial \overline{v}}{\partial \overline{s}} + \overline{v}\frac{\partial \overline{v}}{\partial \overline{r}} + \frac{R^*(\overline{u}+c)}{\overline{r}+R^*}\frac{\partial \overline{v}}{\partial \overline{s}} - \frac{(\overline{u}+c)^2}{\overline{r}+R^*} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial \overline{r}} + \upsilon \left[\begin{array}{c} \frac{1}{\overline{r}+R^*}\frac{\partial}{\partial \overline{r}}\left\{ (\overline{r}+R^*)\frac{\partial \overline{v}}{\partial \overline{r}} \right\} \\ + \left(\frac{R^*}{\overline{r}+R^*}\right)^2 \frac{\partial^2 \overline{v}}{\partial \overline{s}^2} \\ - \frac{v}{(\overline{r}+R^*)^2} - \frac{2R^*}{(\overline{r}+R^*)^2}\frac{\partial \overline{u}}{\partial \overline{s}} \end{array} \right]. \tag{2.21}$$

$$\begin{array}{c} -c\frac{\partial \overline{u}}{\partial \overline{s}} + \overline{v}\frac{\partial \overline{u}}{\partial \overline{r}} + \frac{R^*(\overline{u}+c)}{\overline{r}+R^*}\frac{\partial \overline{u}}{\partial s} \\ + \frac{(\overline{u}+c)\overline{v}}{\overline{r}+R^*} \end{array} = -\frac{R^*}{\rho(\overline{r}+R^*)}\frac{\partial \overline{p}}{\partial \overline{s}} + \upsilon \left[\begin{array}{c} \frac{1}{\overline{r}+R^*}\frac{\partial}{\partial \overline{r}}\left\{ (\overline{r}-R^*)\frac{\partial \overline{u}}{\partial \overline{r}} \right\} \\ + \left(\frac{R^*}{\overline{r}+R^*}\right)^2 \frac{\partial^2 \overline{u}}{\partial \overline{s}^2} \\ - \frac{(u+c)}{(\overline{r}+R^*)^2} + \frac{2R^*}{(\overline{r}+R^*)^2}\frac{\partial \overline{v}}{\partial s} \end{array} \right]. \tag{2.22}$$

Defining the dimensionless variables and stream functions as:

$$\begin{aligned} s &= \frac{2\pi\overline{s}}{\lambda},\ \eta = \frac{\overline{r}}{a},\ u = \frac{\overline{u}}{c},\ v = \frac{\overline{v}}{c},\ \mathrm{Re} = \frac{\rho c a}{\mu}, \\ p &= \frac{2\pi a^2}{\lambda \mu c}\overline{p},\ h = \frac{\overline{H}}{a},\ \delta = \frac{2\pi a}{\lambda},\ k = \frac{R^*}{a}. \end{aligned} \tag{2.23}$$

$$u = -\frac{\partial \Psi}{\partial \eta},\ v = \delta\frac{k}{\eta + k}\frac{\partial \Psi}{\partial s}. \tag{2.24}$$

Eqs. (2.21) and (2.22) can be written in the following form

$$\mathrm{Re}\,\delta\left(\frac{k}{k+\eta}\right)\left[\begin{array}{c} -\delta^2\frac{\partial^2\Psi}{\partial s^2}+\delta^2\frac{\partial\Psi}{\partial s}\frac{\partial}{\partial\eta}\left(\frac{k}{k+\eta}\frac{\partial\Psi}{\partial s}\right) \\ +\delta^2\frac{k}{k+\eta}\left(1-\frac{\partial\Psi}{\partial\eta}\right)\frac{\partial^2\Psi}{\partial s^2} \\ -\frac{1}{k}\left(1-\frac{\partial\Psi}{\partial\eta}\right)^2 \end{array}\right] = \begin{array}{c} -\frac{\partial p}{\partial\eta} \\ +k\delta^2\left[\begin{array}{c} \frac{\partial}{\partial\eta}\left\{\frac{1}{(k+\eta)}\frac{\partial^2\Psi}{\partial\eta\partial s}\right\}+\delta^3\frac{k}{k+\eta}\frac{\partial^3\Psi}{\partial s^3} \\ +\delta\frac{2k}{(k+\eta)^2}\frac{\partial^2\Psi}{\partial\eta\partial s}\end{array}\right]\end{array}. \quad (2.25)$$

$$\mathrm{Re}\,\delta\left[\begin{array}{c} -\delta\frac{k+\eta}{k}\frac{\partial^2\Psi}{\partial\eta\partial s}-\frac{\partial\Psi}{\partial s}\frac{\partial^2\Psi}{\partial\eta^2} \\ -\left(1-\frac{\partial\Psi}{\partial\eta}\right)\frac{\partial^2\Psi}{\partial\eta\partial s}+\frac{1}{k+\eta}\left(1-\frac{\partial\Psi}{\partial\eta}\right)\end{array}\right] = \begin{array}{c} -\frac{\partial p}{\partial s}-\frac{1}{k}\frac{\partial}{\partial\eta}\left\{(k+\eta)\frac{\partial^2\Psi}{\partial\eta^2}\right\} \\ -\delta^2\left(\frac{k}{k+\eta}\right)\frac{\partial^3\Psi}{\partial s^2\partial\eta}-\frac{1}{k+\eta}\left(1-\frac{\partial\Psi}{\partial\eta}\right) \\ -\delta^2\frac{2k}{(k+\eta)^2}\frac{\partial^2\Psi}{\partial s^2}.\end{array} \quad (2.26)$$

In above equations. Re and δ are Reynolds number and wave number respectively. Note that the definition of stream function makes the continuity equation (1.16) identically satisfied.

Now applying long wavelength and low Reynolds number approximations, the above equations reduce to

$$\frac{\partial p}{\partial\eta}=0, \quad (2.27)$$

$$-\frac{k}{k+\eta}\frac{\partial p}{\partial s}+\frac{1}{k+\eta}\frac{\partial}{\partial\eta}\left\{(k+\eta)\frac{\partial u}{\partial\eta}\right\}-\frac{(u+1)}{(k+\eta)^2}=0. \quad (2.28)$$

Differentiating Eq. (2.28) w.r.t. η and using Eq. (2.27), we get

$$\frac{\partial^2}{\partial\eta^2}\left\{(k+\eta)\frac{\partial^2\Psi}{\partial\eta^2}\right\}+\frac{\partial}{\partial\eta}\left\{\frac{1}{k+\eta}\left(1-\frac{\partial\Psi}{\partial\eta}\right)\right\}=0. \quad (2.29)$$

The dimensional volume flow rate in laboratory frame is define as

$$Q=\int_{-\overline{H}}^{\overline{H}}\overline{U}d\overline{R}. \quad (2.30)$$

in which H is a function of $\overline{S}$ and $\bar{t}$. The above expression in wave frame becomes

$$F=\int_{-\bar{H}}^{\bar{H}}\overline{u}d\overline{r}, \quad (2.31)$$

where H is a function of $\overline{s}$ alone. From Eqs. (2.19) and (2.31), we can write

$$Q = F + 2c\overline{H}. \tag{2.32}$$

The time-averaged flow over a period T at a fixed position $\overline{S}$ is

$$\overline{Q} = \frac{1}{T}\int_0^T Q dt. \tag{2.33}$$

Invoking Eq. (2.32) into (2.33) and then integrating, one has

$$\overline{Q} = F + 2ca. \tag{2.34}$$

If we define the dimensionless mean flows Θ, in the laboratory frame, and q, in the wave-frame, according to

$$\Theta = \frac{\overline{Q}}{ca}, \quad q = \frac{F}{ca}, \tag{2.35}$$

Eq. (2.34) reduces to

$$\Theta = q + 2, \tag{2.36}$$

in which

$$q = -\int_{-h}^{h} \frac{\partial \Psi}{\partial \eta} d\eta = -(\Psi(h) - \Psi(-h)). \tag{2.37}$$

Selecting $\Psi(h) = -q/2$, we have $\Psi(-h) = q/2$ and the appropriate boundary conditions in the wave frame are

$$\Psi = -\frac{q}{2}, \quad \frac{\partial \Psi}{\partial \eta} = 1 \quad \text{at} \quad \eta = h = 1 + \Phi \sin s, \tag{2.38}$$

$$\Psi = \frac{q}{2}, \quad \frac{\partial \Psi}{\partial \eta} = 1 \quad \text{at} \quad \eta = -h = -1 - \Phi \sin s, \tag{2.39}$$

where $\Phi = b/a$ is the amplitude ratio.

2.2.1 Solution of the problem

Eq. (2.29) can also be expressed in the following form

$$\frac{\partial}{\partial\eta}\left\{(k+\eta)\frac{\partial^3\Psi}{\partial\eta^3}+\frac{\partial^2\Psi}{\partial\eta^2}-\frac{1}{k+\eta}\frac{\partial\Psi}{\partial\eta}+\frac{1}{k+\eta}\right\}=0. \tag{2.40}$$

or

$$\frac{\partial}{\partial\eta}\left\{(k+\eta)\frac{\partial}{\partial\eta}\left[\frac{\partial^2\Psi}{\partial\eta^2}+\frac{1}{(k+\eta)}\frac{\partial\Psi}{\partial\eta}\right]\right\}=\frac{1}{(k+\eta)^2}. \tag{2.41}$$

By three times integration of above equation w.r.t. η, we can write

$$\frac{\partial\Psi}{\partial\eta}=1-\frac{k}{k+\eta}+c_1\frac{(k+\eta)}{2}\left\{\ln(k+\eta)-\frac{1}{2}\right\}+\frac{(k+\eta)}{2}c_2+\frac{c_3}{k+\eta}. \tag{2.42}$$

Eq. (2.42) multiplied by a minus sign represents the velocity component u as a function of η and k, i.e.,

$$u=-1+\frac{k}{k+\eta}-c_1\frac{(k+\eta)}{2}\left\{\ln(k+\eta)-\frac{1}{2}\right\}-\frac{(k+\eta)}{2}c_2-\frac{c_3}{k+\eta}. \tag{2.43}$$

Another integration of Eq. (2.42) results in the following expression for the stream function

$$\Psi=\eta-k\ln(k+\eta)+c_1\frac{(k+\eta)^2}{4}\{\ln(k+\eta)-1\}+\frac{(k+\eta)^2}{4}c_2+c_3\ln(k+\eta)+c_4. \tag{2.44}$$

where c_1, c_2, c_3 and c_4 are constants of integration. Using the boundary conditions, we get the following values of c_1, c_2, c_3 and c_4.

$$\begin{aligned} c_1 \;=\; & \Bigg[hk\Bigg(\ln[k-h]\left\{\frac{h^7}{8}+h^3k^3\left(-\frac{k}{8}-\frac{q}{4}\right)+h^4k^2\left(-\frac{k}{2}-\frac{q}{16}\right)+h^2k^4\left(\frac{k}{4}-\frac{q}{16}\right)+\right. \\ & h^6\left(\frac{k}{4}-\frac{q}{16}\right)+\frac{h^5k}{8}(-k-q)+\frac{hk^5}{8}(k+q)+\frac{k^6q}{16}-\left(-\frac{h^7}{2}-\frac{hk^5}{2}(k+q)+\right. \\ & \frac{h^5k}{2}(k-q)-h^6\left(k+\frac{q}{4}\right)+h^2k^4\left(\frac{q}{4}-k\right)+h^4k^2\left(2k+\frac{q}{4}\right) \end{aligned}$$

$$
-\frac{k^6q}{4} + h^3k^3\left(\frac{k}{2}+q\right)\Bigg)\ln[k+h] + \left\{\frac{h^7}{2} - h^3k^3\left(\frac{k}{2}+q\right) - h^4k^2\left(2k+\frac{q}{4}\right)\right.
$$

$$
\left. -h^2k^4\left(\frac{q}{4}-k\right) + h^6\left(\frac{q}{4}+k\right) + \frac{h^5k}{2}(-k+q) + \frac{hk^5}{2}(k+q) + \frac{k^6q}{4}\right\}\ln[k-h]^2\Bigg\}
$$

$$
+\ln[k+h]\left\{-\frac{h^7}{8} - \frac{h^5k}{8}(k+q) + \frac{hk^5}{8}(k-q) - h^6\left(\frac{k}{4}+\frac{q}{16}\right) + h^2k^4\left(-\frac{k}{4}+\frac{q}{16}\right)\right.
$$

$$
+h^4k^2\left(\frac{k}{4}+\frac{q}{16}\right) + h^3k^3\left(\frac{k}{8}+\frac{q}{4}\right) - \frac{k^6q}{16} + \left\{\frac{h^7}{2} - h^3k^3\left(\frac{k}{2}+q\right)\right.
$$

$$
\left. -h^4k^2\left(2k+\frac{q}{4}\right) + h^6\left(\frac{q}{4}+k\right) + \frac{h^5k}{2}(-k+q) + \frac{hk^5}{2}(k+q) + \frac{k^6q}{4}\right\}\ln[k-h]^2\Big\}
$$

$$
+\ln[k+h]\left\{-\frac{h^7}{8} - \frac{h^5k}{8}(k+q) + \frac{hk^5}{8}(k-q) - h^6\left(\frac{k}{4}+\frac{q}{16}\right) + h^2k^4\left(-\frac{k}{4}+\frac{q}{16}\right)\right.
$$

$$
+h^4k^2\left(\frac{k}{4}+\frac{q}{16}\right) + h^3k^3\left(\frac{k}{8}+\frac{q}{4}\right) - \frac{k^6q}{16} + \left\{\frac{h^7}{2} - h^3k^3\left(\frac{k}{2}+q\right) - h^4k^2\left(2k+\frac{q}{4}\right)\right.
$$

$$
\left. -h^2k^4\left(\frac{q}{4}-k\right) + h^6\left(\frac{q}{4}+k\right) + \frac{h^5k}{2}(-k+q) + \frac{hk^5}{2}(k+q) + \frac{k^6q}{4}\right\}\ln[k+h]
$$

$$
+\Bigg(-\frac{h^7}{2} - \frac{hk^5}{2}(k+q) + \frac{h^5k}{2}(k-q) - h^6\left(k+\frac{q}{4}\right) + h^2k^4\left(\frac{q}{4}-k\right) - h^4k^2\left(2k+\frac{q}{4}\right)
$$

$$
-\frac{k^6q}{4} + h^3k^3\left(\frac{k}{2}+q\right)\Bigg)\ln[k+h]^2\Big\}\Big)\Big] \Big/ \Big[(k-h)(k+h)^2\left(-\frac{1}{2}+\ln[k+h]\right)
$$

$$
+\left(\frac{h^2-k^2}{4}\right)\ln\left[\frac{k-h}{k+h}\right]\Bigg(\frac{h^2k^2}{4}(k+h) - \frac{h^2k^2}{2}(k+h)\ln[k+h]
$$

$$
+\left(-\frac{h^5}{16} - \frac{h^4k}{16} + \frac{h^3k^2}{8} + \frac{h^2k^3}{8} - \frac{hk^4}{16} - \frac{k^5}{16}\right)\ln[k+h]^2
$$

$$
+\left(\frac{h^5}{8} + \frac{h^4k}{8} - \frac{h^3k^2}{48} - \frac{h^2k^3}{4} + \frac{hk^4}{8} + \frac{k^5}{8}\right)\ln[k+h]^3 + \ln[k-h]\ln[k+h]\Bigg)
$$

$$
\Bigg\{\frac{h^5}{8} + \frac{h^4k}{8} - \frac{h^3k^2}{48} - \frac{h^2k^3}{4} + \frac{hk^4}{8} + \frac{k^5}{8} +
$$

$$
\left(-\frac{h^5}{4} - \frac{h^4k}{4} + \frac{h^3k^2}{2} + \frac{h^2k^3}{2} - \frac{hk^4}{4} - \frac{k^5}{4}\right)\ln[k+h] +
$$

$$
\ln[k+h]^2\Bigg(-\frac{h^5}{16} - \frac{h^4k}{16} + \frac{h^3k^2}{8} + \frac{h^2k^3}{8} - \frac{hk^4}{16} - \frac{k^5}{16} +
$$

$$
\left(\frac{h^5}{16} + \frac{h^4k}{16} - \frac{h^3k^2}{8} - \frac{h^2k^3}{8} + \frac{hk^4}{16} + \frac{k^5}{16}\right)\ln[k+h]\Bigg)\Bigg\}.
$$

$$
\begin{aligned}
c_2 \;=\; & \left[-\frac{k}{2}\left\{\frac{(k-h)\left(-\frac{1}{2}+\ln[k-h]\right)}{k+h}-\frac{(k+h)\left(-\frac{1}{2}+\ln[k+h]\right)}{k-h}\right\}\right. \\
& +\left(\frac{(k-h)}{2}\left\{-\frac{(k-h)\left(-\frac{1}{2}+\ln[k-h]\right)}{k+h}+\frac{(k+h)\left(-\frac{1}{2}+\ln[k+h]\right)}{k-h}\right\}\right. \\
& \left[-\frac{(k-h)\,k}{2}\left\{\frac{(k-h)\left(-\frac{1}{2}+\ln[k-h]\right)}{k-h}-\frac{(k+h)\left(-\frac{1}{2}+\ln[k-h]\right)}{k-h}\right\}\right. \\
& \left(\frac{kh}{4}+\frac{(k-h)^2}{16}\ln\left[\frac{k-h}{k+h}\right]\right)+\frac{(k^2-h^2)}{4}\ln\left[\frac{k-h}{k-h}\right]\Bigg) \\
& \left(\frac{k\,(k-h)^2\left(-\frac{1}{2}+\ln[k-h]\right)}{4\,(k+h)}+\frac{k\,(k+h)^2\left(-\frac{1}{2}+\ln[k+h]\right)}{4\,(k+h)}-\right. \\
& \left.\left.\left.\frac{(k+h)\left(-\frac{1}{2}+\ln[k-h]\right)}{2}\left(2h+q+k\ln\left[\frac{k-h}{k+h}\right]\right)\right)\right]\right]\Bigg/\left[\frac{h^2k^2}{4}-\frac{h^2k^2}{2}\ln[k-h]-\right. \\
& \frac{(k^2-h^2)^2}{16}\ln[k+h]^2+\frac{(k^2-h^2)^2}{16}\ln[k+h]^3+ \\
& \left.\ln[k-h]\ln[k+h]\left(-\frac{(k-h)^2}{16}+\frac{(k-h)^2}{8}\ln[k+h]\right)\right]\Bigg/ \\
& \left[\frac{(k^2-h^2)}{4}\ln\left[\frac{k+h}{k-h}\right]\right].
\end{aligned}
$$

$$
\begin{aligned}
c_3 \;=\; & -\left[(k-h)\left\{\left[-\frac{(k-h)\,k}{2}\left\{\frac{(k-h)\left(-\frac{1}{2}+\ln[k-h]\right)}{k+h}-\frac{(k+h)\left(-\frac{1}{2}+\ln[k+h]\right)}{k-h}\right\}\right.\right.\right. \\
& \left(\frac{kh}{4}+\frac{(k-h)^2}{16}\ln\left[\frac{k-h}{k+h}\right]\right)+\frac{(k^2-h^2)}{4}\ln\left[\frac{k+h}{k-h}\right]\left(-\frac{k\,(k-h)^2\,(-1+\ln[k-h])}{4\,(k-h)}+\right. \\
& \left.\left.\left.\frac{k\,(k+h)^2\,(-1+\ln[k+h])}{4\,(k+h)}-\frac{(k+h)\left(-\frac{1}{2}+\ln[k+h]\right)}{2}\left(2h+q+k\ln\left[\frac{k-h}{k+h}\right]\right)\right)\right\}\right] \\
& \left[\frac{h^2k^2}{4}-\frac{h^2k^2}{2}\ln[k+h]-\frac{(k^2-h^2)^2}{16}\ln[k-h]^2+\frac{(k^2-h^2)^2}{16}\ln[k+h]^3+\right. \\
& \ln[k-h]\ln[k+h]\left(\frac{(k-h)^2}{8}-\frac{(k-h)^2}{4}\ln[k+h]\right)+ \\
& \left.\ln[k-h]^2\left(-\frac{(k^2-h^2)^2}{16}+\frac{(k^2-h^2)^2}{8}\ln[k+h]\right)\right].
\end{aligned}
$$

$$
\begin{aligned}
c_4 = {} & h + \frac{q}{2} + k\ln[k-h] + \Bigg[\frac{kh\,(k-h)\,(-1+\ln[k-h])}{4} \ln[k+h] \Bigg\{ \frac{h^7}{8} + h^3k^3\left(-\frac{k}{8} - \frac{q}{4}\right) \\
& -h^4k^2\left(\frac{k}{2} + \frac{q}{16}\right) + h^2k^4\left(\frac{k}{4} - \frac{q}{16}\right) + h^6\left(\frac{k}{4} + \frac{q}{16}\right) + \frac{h^5k}{8}(q-k) + \frac{hk^5}{8}(q+k) + \frac{k^6q}{16} \\
& \left(-\frac{h^7}{2} - \frac{hk^5}{2}(k+q) + \frac{h^5k}{2}(k-q) - h^6\left(k + \frac{q}{4}\right) + h^2k^4\left(\frac{q}{4} - k\right) + h^4k^2\left(2k + \frac{q}{4}\right) - \right. \\
& \left. \frac{k^6q}{4} + h^3k^3\left(\frac{k}{2} + q\right)\right) \ln[k+h] + \Bigg\{ \frac{h^7}{2} - h^3k^3\left(\frac{k}{2} + q\right) - h^4k^2\left(2k + \frac{q}{4}\right) - h^2k^4\left(\frac{q}{4} - k\right) \\
& +h^6\left(\frac{q}{4} + k\right) + \frac{h^5k}{2}(-k+q) + \frac{hk^5}{2}(k+q) + \frac{k^6q}{4} \Bigg\} \ln[k+h]^2 \Bigg\} + \\
& \ln[k-h] \Bigg\{ -\frac{h^7}{8} + h^3k^3\left(\frac{k}{8} + \frac{q}{4}\right) + h^4k^2\left(\frac{k}{2} + \frac{q}{16}\right) - h^2k^4\left(\frac{k}{4} - \frac{q}{16}\right) - h^6\left(\frac{k}{4} + \frac{q}{16}\right) \\
& -\frac{h^5k}{8}(q-k) - \frac{hk^5}{8}(q+k) - \frac{k^6q}{16} + \left(-\frac{h^7}{2} - \frac{hk^5}{2}(k+q) + \frac{h^5k}{2}(k-q) - h^6\left(k + \frac{q}{4}\right) \right. \\
& \left. +h^2k^4\left(\frac{q}{4} - k\right) + h^4k^2\left(2k + \frac{q}{4}\right) - \frac{k^6q}{4} + h^3k^3\left(\frac{k}{2} + q\right)\right) \ln[k+h] \Bigg\{ \frac{h^7}{2} - h^3k^3\left(\frac{k}{2} + q\right) \\
& -h^4k^2\left(2k + \frac{q}{4}\right) - h^2k^4\left(\frac{q}{4} - k\right) + h^6\left(\frac{q}{4} + k\right) + \frac{h^5k}{2}(-k+q) + \frac{hk^5}{2}(k+q) + \frac{k^6q}{4} \Bigg\} \ln[k-h]^2 \Bigg\} \\
& + \ln[k-h] \Bigg\{ -\frac{h^7}{8} + h^3k^3\left(\frac{k}{8} + \frac{q}{4}\right) + h^4k^2\left(\frac{k}{2} + \frac{q}{16}\right) - h^2k^4\left(\frac{k}{4} - \frac{q}{16}\right) - h^6\left(\frac{k}{4} + \frac{q}{16}\right) \\
& -\frac{h^5k}{8}(q-k) - \frac{hk^5}{8}(q+k) - \frac{k^6q}{16} \\
& + \left[\left(\begin{array}{c} \frac{h^7}{2} - h^3k^3\left(\frac{k}{2} + q\right) - h^4k^2\left(2k + \frac{q}{4}\right) - h^2k^4\left(\frac{q}{4} - k\right) \\ +h^6\left(\frac{q}{4} + k\right) + \frac{h^5k}{2}(-k+q) + \frac{hk^5}{2}(k+q) + \frac{k^6q}{4} \end{array} \right) \ln[k+h] \right) \\
& + \left(-\frac{h^7}{2} - \frac{hk^5}{2} - h^6\left((k+q) + \frac{h^5k}{2}(k-q)\,k + \frac{q}{4}\right) + \right. \\
& \left. h^2k^4\left(\frac{q}{4} - k\right) + h^4k^2\left(2k + \frac{q}{4}\right) - \frac{k^6q}{4} + h^3k^3\left(\frac{k}{2} + q\right)\right) \ln[k+h]^2 \Bigg]\Bigg].
\end{aligned}
$$

From Eq. (2.28), the pressure gradient turns out to be

$$
\frac{\partial p}{\partial s} = -\frac{1}{k} c_1. \tag{2.45}
$$

The dimensionless pressure rises over one wave length is defined by

$$
\Delta P_\lambda = \int_0^{2\pi} \frac{\partial p}{\partial s} ds. \tag{2.46}
$$

2.3 Results and Discussion

This section is divided into three subsections. Flow characteristics are described in first subsection. Second subsection is devoted to the discussion of pumping characteristics. In the last subsection, trapping phenomenon is illustrated. The analytical expressions of Ψ, $u(\eta)$ and dp/ds given in the previous section are used in this section to discuss these features of peristaltic motion. In the present analysis, the extra parameter that comes into play in contrast with previous attempts on peristalsis is the radius of curvature of the channel, i.e., k.

2.3.1 Flow Characteristics

The expression of u given by Eq. (2.43) can be used to discuss the flow characteristics. Therefore, its variation with η for different values of k is plotted in Fig. (2.2).

It is noticed that for large values of k (i.e. for straight channel), the velocity profile is symmetric about the axis of channel and the maxima occurs at $\eta = 0$. However, for small values of k (i.e. for curved channel), the profiles are not symmetric about $\eta = 0$ and maxima shifts towards the negative values of η. Furthermore it is observed from the computations that in the narrow part of the channel, the effects of the curvature are not pronounced.

2.3.2 Pumping Characteristics

The Pumping characteristics can be well described by studying the axial pressure gradient dp/ds given by expression Eq. (2.45) and the pressure difference over one wave length calculated from expression Eq. (2.46).

The variation of dp/ds per wavelength for different values of k is seen in Fig. (2.3). This figure depicts that the magnitude of dp/ds decreases in going from curved to straight channel.

An interesting feature of peristalsis is pumping against the pressure rise. For such characteristics, we have plotted pressure rise per wavelength ΔP_λ against dimensionless time mean flow rate Θ (Fig. (2.4)) for different values of k. The maximum pressure rise against which peristalsis works as a pump, i.e., ΔP_λ for $\Theta = 0$, is denoted by P_0. When $\Delta P_\lambda > P_0$, then flux is negative, i.e., against the peristaltic wave direction. The value of Θ corresponding to $\Delta P = 0$ (which is known as free pumping) is denoted by Θ_0. When $\Delta P_\lambda < 0$, the pressure assists the flow and this is known as co-pumping. The following information can be extracted from Fig. (2.4).

⋆ P_0 increases as one moves from straight to curved channel. This means that the peristalsis has to work against greater pressure rise in curved channel as compared to flow in straight channel.

⋆ The free pumping flux Θ_0 increases in going from curved to straight channel.

⋆ In co-pumping similar to free pumping, the pumping rate for straight channel is greater in magnitude as compared to curved channel.

2.3.3 Trapping

The analytical expression of Ψ due to Eq. (2.44) is plotted in Figs. (2.5 a) – (2.5 d) to discuss the trapping phenomenon for various values of k. In general, the shape of streamlines is similar to that of the boundary wall in the wave frame. However, under certain conditions, some of the streamlines split and enclose a bolus, which moves as a whole with wave. We observe from Figs. (2.5 a) – (2.5 d) that for small value of k, the bolus is not symmetric about $\eta = 0$ and is pushed towards the lower wall. However, as k increases, the results of straight channel can be recovered.

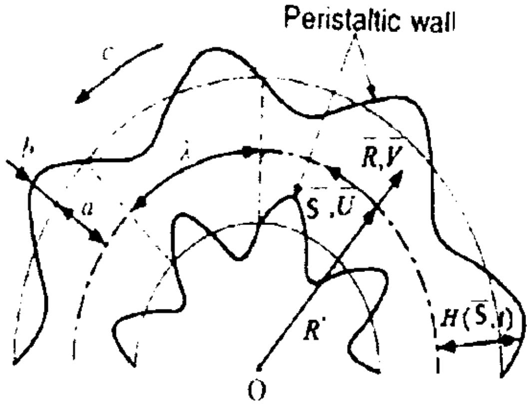

Fig. (2.1). Schematic diagram of the problem.

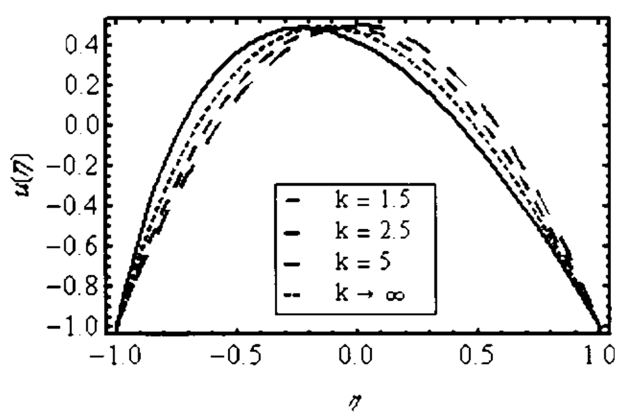

Fig. (2.2). Variation of $u(\eta)$ for different values of k with

$\Phi = 0.8$ and $\Theta = 2$.

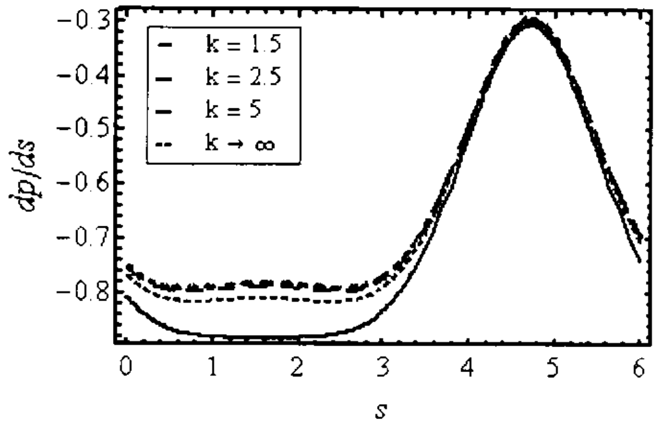

Fig. (2.3). Variation of dp/ds for different values of k with $\Phi = 0.2$ and $\Theta = 0.5$.

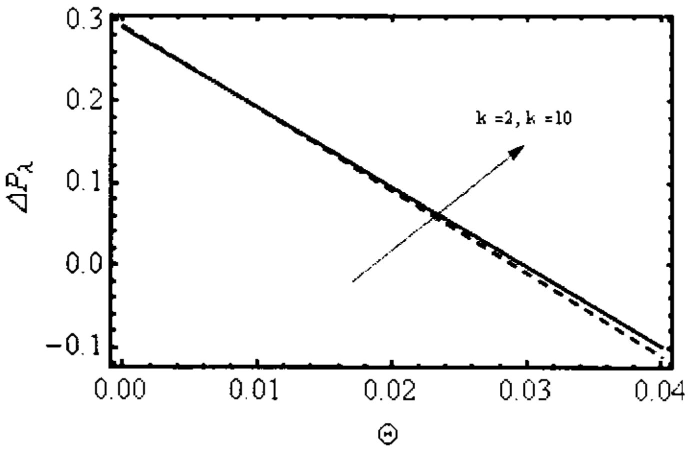

Fig. (2.4). Variation of ΔP_λ for different values of k with $\Phi = 0.1$.

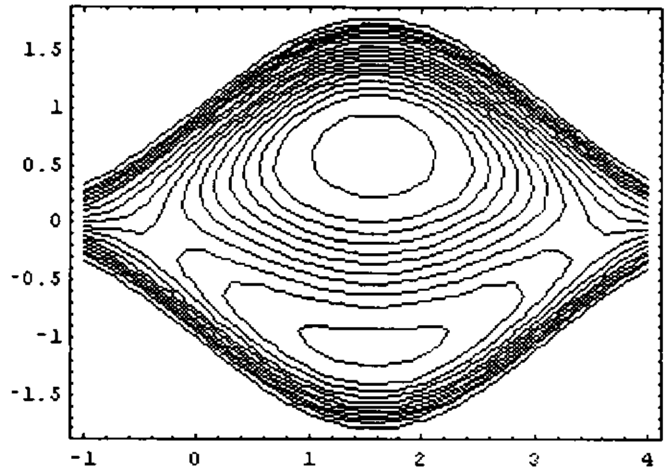

Fig. (2.5 a). Streamlines for $k = 3.5$ with parameters

$\Phi = 0.8$ and $\Theta = 1.5$.

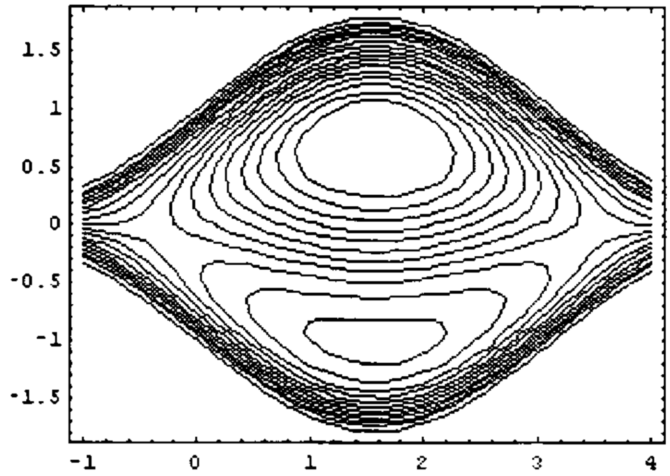

Fig. (2.5 b). Streamlines for $k = 5$ with parameters

$\Phi = 0.8$ and $\Theta = 1.5$.

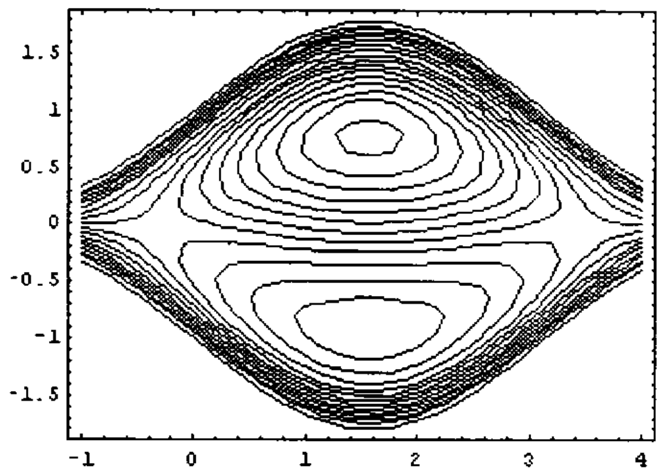

Fig. (2.5 c). Streamlines for $k = 10$ with parameters

$\Phi = 0.8$ and $\Theta = 1.5$.

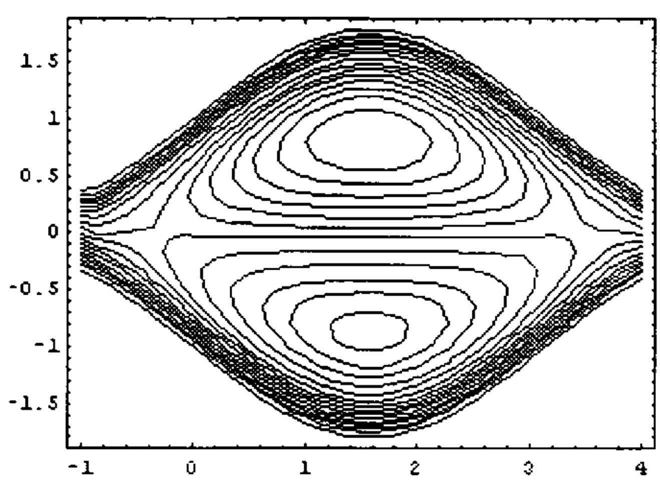

Fig. (2.5 d). Streamlines for $k \to \infty$ with parameters

$\Phi = 0.8$ and $\Theta = 1.5$.

Chapter 3

Slip effects on peristaltic flow in a curved channel

3.1 Introduction

The purpose of this chapter is to extend the analysis of chapter 2 by considering the slip at the walls of the channel. The expressions of stream function, pressure gradient and velocity are obtained analytically in terms of dimensionless slip parameter. A graphical study is performed to analyze the effects of slip parameter on velocity, pressure rise per wavelength and trapping phenomenon.

3.2 Problem Formulation

The geometry and governing equations of the problem which are to be attempted in this chapter are same as described in chapter 2. The only change comes through the boundary conditions at the walls of the channel.

In chapter 2, no-slip boundary condition was used at the upper and lower walls of the channel. Here the no-slip condition is replaced by the slip condition, i.e., Eqs. (2.38 b) and (2.39 b) now become

$$-\beta\frac{\partial^2\Psi}{\partial\eta^2}+\left[1+\frac{\beta}{k+\eta}\right]\frac{\partial\Psi}{\partial\eta} = \frac{\beta}{k+\eta}+1 \text{ at } \eta=-h=-1-\Phi\,\mathrm{Sin}\,(s). \qquad (3.1)$$

$$\beta\frac{\partial^2\Psi}{\partial\eta^2}+\left[1-\frac{\beta}{k+\eta}\right]\frac{\partial\Psi}{\partial\eta} = \frac{-\beta}{k+\eta}+1 \text{ at } \eta=h=1+\Phi\,\mathrm{Sin}\,(s). \qquad (3.2)$$

where $\beta = b/a$ is the dimensionless slip parameter.

It is interesting to note that due to the slip at the walls, the dimensionless curvature radius parameter k comes in the boundary condition and thus for the fixed values of β, the velocity will take the different values at the walls for different values of k.

Employing the same methodology as used in chapter 2, the solution of the boundary value problem consisting of differential equation (2.29) and the boundary conditions Eqs. (2.38 a), (2.39 a), (3.1) and (3.2) is

$$\Psi = \eta - k\ln(k+\eta) + c_1^*\frac{(k+\eta)^2}{4}\{\ln(k+\eta)-1\} + \frac{(k+\eta)^2}{4}c_2^* + c_3^*\ln(k+\eta) + c_4^*. \qquad (3.3)$$

where c_1^*, c_2^*, c_3^* and c_4^* are the integration constants and are given as below.

$$\begin{aligned} c_1^* = {} & \left(8k(2h+q)\left(-h^3+hk^2+3h^2\beta+k^2\beta\right)\right) / \left(4hk^2\left(-h^3-2h^2\beta+2k^2\beta+h\left(k^2+4\beta^2\right)\right)\right. \\ & +\left(h^2-k^2\right)^2\ln[k-h]^3-4k\left(h^2-k^2\right)^2\beta\ln[k+h]+\left(h^2-k^2\right)^3\ln[k+h]^2 \\ & \left.-\left(h^2-k^2\right)^2\ln[k-h]\left(-2k\beta+\left(h^2-k^2\right)\ln[k+h]\right)\right). \end{aligned}$$

$$\begin{aligned} c_2^* = {} & -\Big(2(2h+q)\Big((h-k)^3(k+h-2\beta)\ln[k-h]-(-k+h-2\beta). \\ & \Big(-2kh(k+h-2\beta)+(h+k)^3\ln[k+h]\Big)\Big)\Big) / \left(4hk^2\left(-h^3+2h^2\beta-2k^2\beta+h\left(k^2+4\beta^2\right)\right)\right. \\ & +\left(h^2-k^2\right)^2\ln[k-h]^3-4k\left(h^2-k^2\right)^2\beta\ln[k+h]+\left(h^2-k^2\right)^3\ln[k+h]^2 \\ & \left.-\left(h^2-k^2\right)^2\ln[k-h]\left(-2k\beta+\left(h^2-k^2\right)\ln[k+h]\right)\right). \end{aligned}$$

$$
\begin{aligned}
c_3^* = {} & 2k\left(2h^5\beta + 6hk^4\beta + qk^4\beta + h^4\right. \\
& \left(-2k^2 + q\beta + 2h^2k^2\left(k^2 + \beta\left(-q + 4\beta\right)\right) + k\left(h^2 - k^2\right)^3 \ln\left[k - h\right]^2 - \right. \\
& - \left(h^2 - k^2\right)^2 \left(2h^3 - 2hk^2 + h^2q - k^2\left(q - 4\beta\right) \ln\left[k + h\right] + k\left(h^2 - k^2\right)^3 \ln\left[k - h\right]^2\right. \\
& \left.\left. + \left(h^2 - k^2\right)^2 \ln\left[k - h\right]\left(2h^3 - 2hk^2 + h^2q - k^2\left(q - 4\beta\right) + \left(2k^3 - 2kh^2\right)\ln\left[k + h\right]\right)\right) / \\
& \left(4hk^2\left(-h^3 + 2h^2\beta + 2k^2\beta + h\left(k^2 + 4\beta^2\right)\right) + \left(h^2 - k^2\right)^2 \ln\left[k - h\right]^3 - 4k\left(h^2 - k^2\right)^2 \beta \ln\left[k + h\right]\right. \\
& \left. + \left(h^2 - k^2\right)^3 \ln\left[k + h\right]^2 - \left(h^2 - k^2\right)^2 \ln\left[k - h\right]\left(-2k\beta + \left(h^2 - k^2\right)\ln\left[k + h\right]\right)\right)\Big).
\end{aligned}
$$

$$
\begin{aligned}
c_4^* = {} & \left(2h + q\right)\left(-2k + \left(h^2 + k^2\right)\left(-h^3 + 2h^2\beta + 2k^2\beta + h\left(k^2 + 4\beta^2\right)\right)\right. \\
& - \left(h - k\right)^2\left(h + k\right)^3\left(-k - h - 2\beta\right)\ln\left[k - h\right] + \left(h^2 - k^2\right)^2 \ln\left[k - h\right]^3 \\
& \left. + \left(h - k\right)^3\left(h + k\right)^2\left(k + h - 2\beta\right)\ln\left[k + h\right] + \left(h^2 - k^2\right)^3 \ln\left[k + h\right]^2\right) \\
& \left(2\left(4hk^2\left(-h^3 + 2h^2\beta + 2k^2\beta + h\left(k^2 + 4\beta^2\right)\right) + \left(h^2 - k^2\right)^2 \ln\left[k - h\right]^3 - 4k\left(h^2 - k^2\right)^2 \beta \ln\left[k + h\right]\right.\right. \\
& \left.\left. + \left(h^2 - k^2\right)^3 \ln\left[k + h\right]^2 - \left(h^2 - k^2\right)^2 \ln\left[k - h\right]\left(-2k\beta - \left(h^2 - k^2\right)\ln\left[k + h\right]\right)\right)\right).
\end{aligned}
$$

The dimensionless pressure gradient can be obtained with the help of Eq. (2.45) and is given as:

$$
\frac{\partial p}{\partial s} = -\frac{1}{k} c_1^*. \tag{3.4}
$$

Using the above expression in Eq. (2.46), one can discuss the effect of slip parameter β on the pumping characteristics.

3.3 Discussion

In this section, the intention is to analyze the effects of emerging parameters such as β and k on flow velocity, pressure gradient, pressure rise per wavelength and trapping phenomenon through Figs. (3.1) – (3.15). Now the brief description of each figure and the results obtained from it. Fig. (3.1) is prepared to see the effects of dimensionless curvature radius k on longitudinal velocity component u. It is seen that the flow is symmetric about the centre-line of the channel for large value of k. However as k increases, the profiles of u are no more symmetric about the centre-line. This observation is in accordance with the one made in chapter (2). Interestingly, due to the imposition of slip condition at the walls of the channel, the longitudinal velocity u at the walls becomes a function of k. Therefore, as k increases, u decreases at the lower wall and increases at the upper one.

In Fig. (3.2), $u(\eta)$ is plotted against η for various values of β by keeping k fixed. This figure reveals that an increase in β increases the longitudinal velocity at both the walls.

The profiles of dp/ds for various values of k and for non-zero value of β are shown in Fig. (3.3). It can be inferred from this figure that the magnitude of gradient decreases by increasing k.

The effects of β on dp/ds is similar to that of k (Fig. 3.4). Figs. (3.5) and (3.6) are plotted to analyze the behaviour of pressure rise per wavelength for different values of β and k. It is also important to mention that in peristaltic flow, the peristaltic wave works against the pressure rise to propel the fluid. Thus the agents which reduces the pressure rise can significantly affects the performance of machinery which works on the principles of peristalsis. It is interesting to note that the maximum values of ΔP_λ (i.e. ΔP_λ for $\Theta = 0$) decreases for large values of β and k. Thus the slip parameter β has significant effects on ΔP_λ and cannot be ignored in such studies.

In order to see the effects of β and k on trapping phenomenon, Figs. (3.7) – (3.15) are plotted. These figures demonstrate that on one hand the bolus is symmetric about the centre-line for a straight channel. However, it becomes unsymmetrical for small values of k (i.e. for a curved channel) and splits into two halves. Further, the upper half pushes the lower one towards the lower wall. It is also to be noted that the circulation of fluid in the upper half is faster than the circulation of fluid in the lower one. On the other hand, the size and circulation of bolus reduces for large value of β.

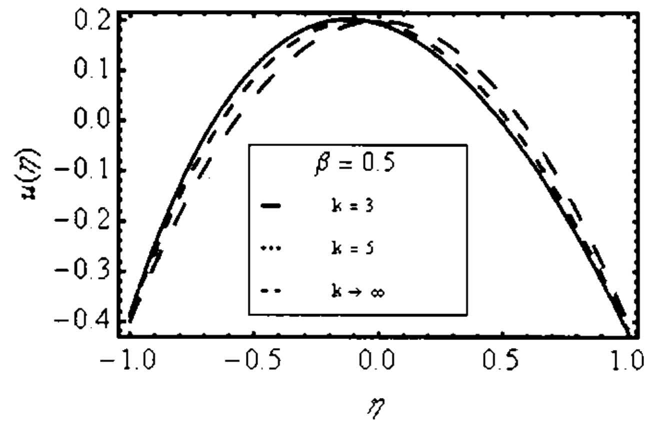

Fig. 3.1. Variation of $u(\eta)$ for different values of k for $\beta = 0.5$ with $\Phi = 0.8$ and $\Theta = 2$.

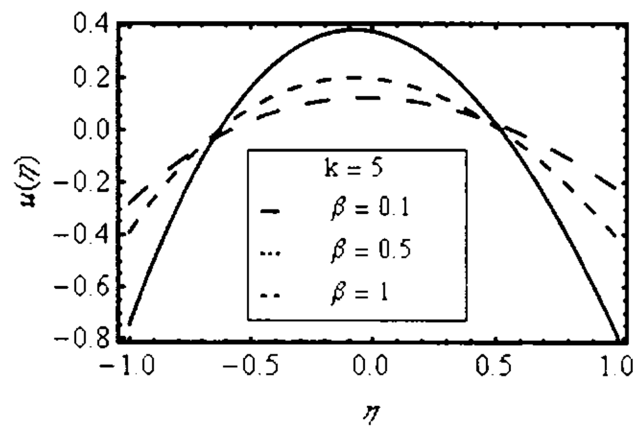

Fig. 3.2. Variation of $u(\eta)$ for different values of β for $k = 5$ with $\Phi = 0.8$ and $\Theta = 2$.

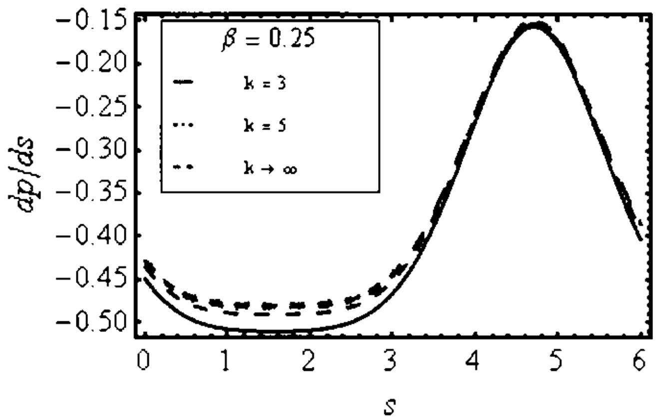

Fig. 3.3. Variation of dp/ds for different values of k for $\beta = 0.25$ with $\Phi = 0.5$ and $\Theta = 0.2$.

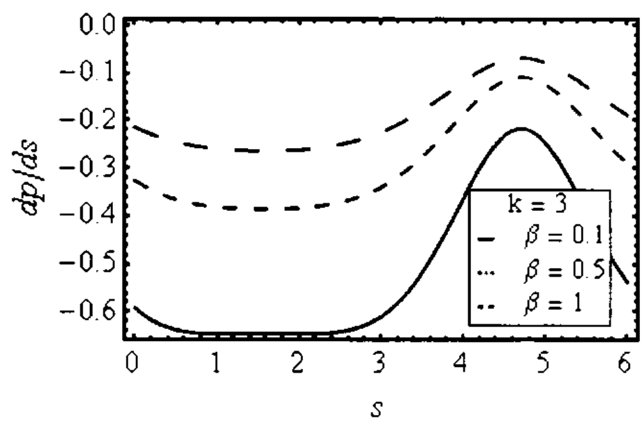

Fig. 3.4. Variation of dp/ds for different values of β for $k = 3$ with $\Phi = 0.5$ and $\Theta = 0.2$.

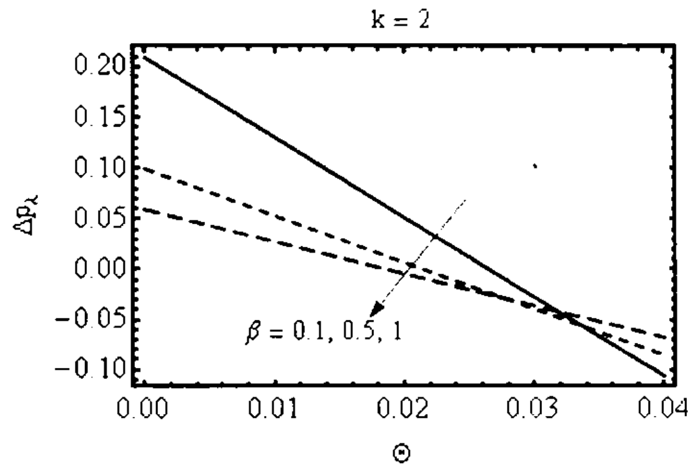

Fig. 3.5. Variation of ΔP_λ for different values of β for $k = 2$ with $\Phi = 0.1$.

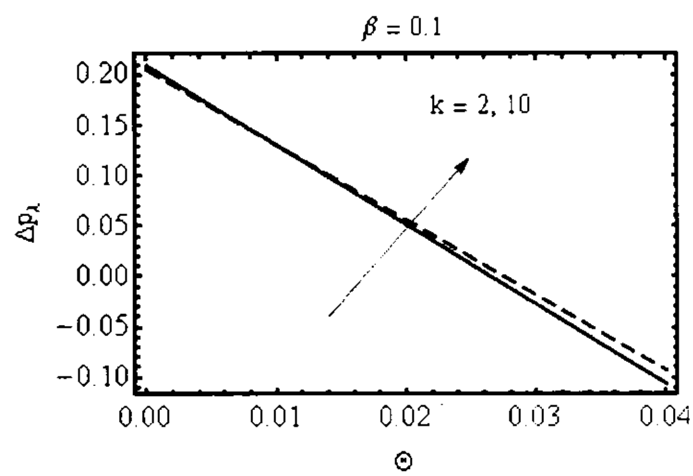

Fig. 3.6. Variation of ΔP_λ for different values of k for $\beta = 0.1$ with $\Phi = 0.1$.

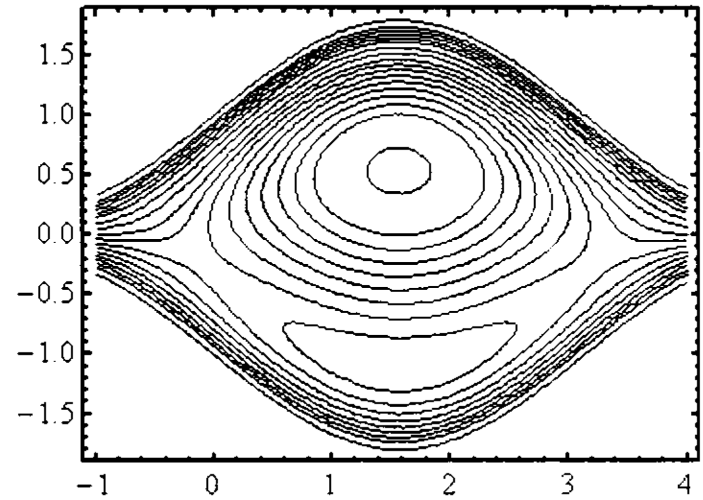

Fig. 3.7. Streamlines for $\beta = 0.1$, $k = 3.5$ with $\Phi = 0.8$ and $\Theta = 1.5$.

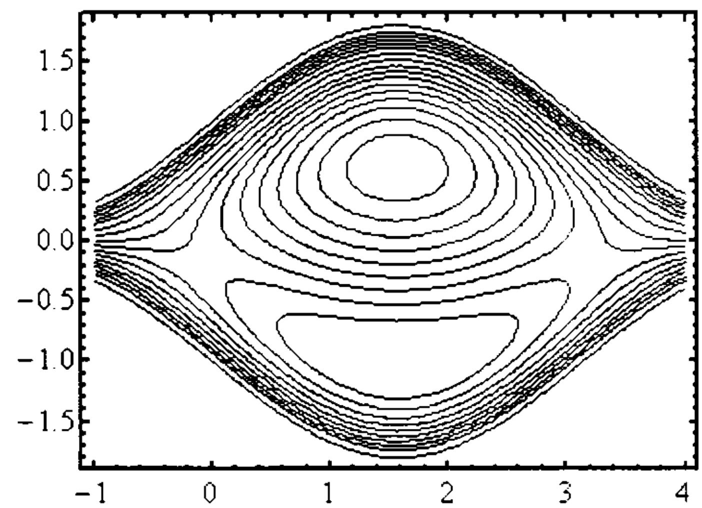

Fig. 3.8. Streamlines for $\beta = 0.1$, $k = 5$ with $\Phi = 0.8$ and $\Theta = 1.5$.

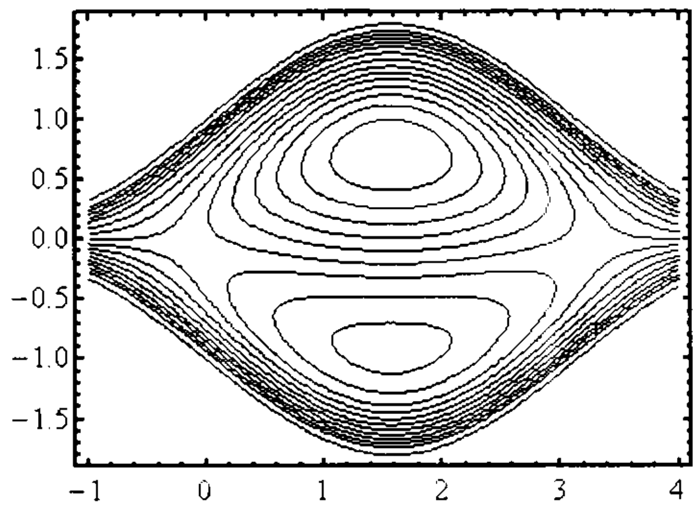

Fig. 3.9. Streamlines for $\beta = 0.1$, $k = 10$ with $\Phi = 0.8$ and $\Theta = 1.5$.

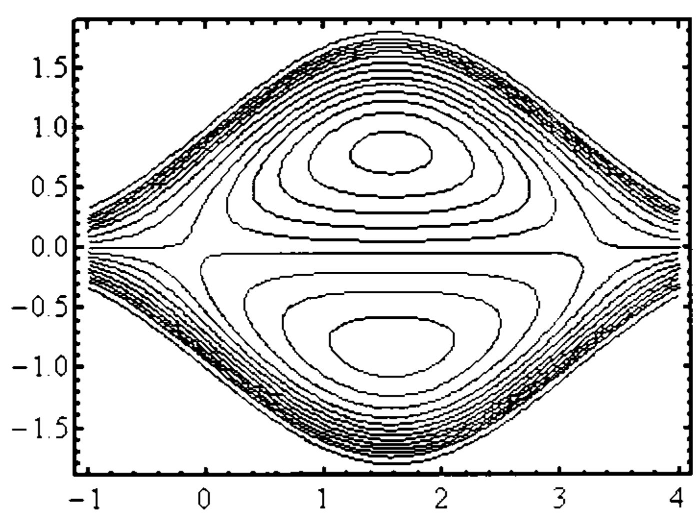

Fig. 3.10. Streamlines for $\beta = 0.1$, $k \to \infty$ with $\Phi = 0.8$ and $\Theta = 1.5$.

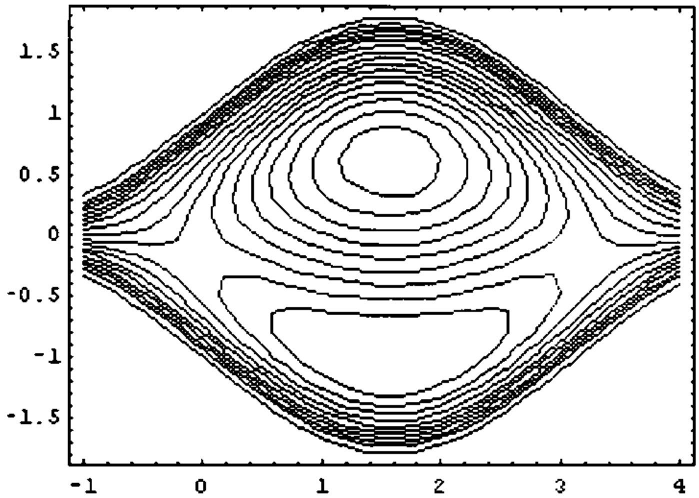

Fig. 3.11. Streamlines for $k = 5$, $\beta = 0.1$ with $\Phi = 0.8$ and $\Theta = 1.5$.

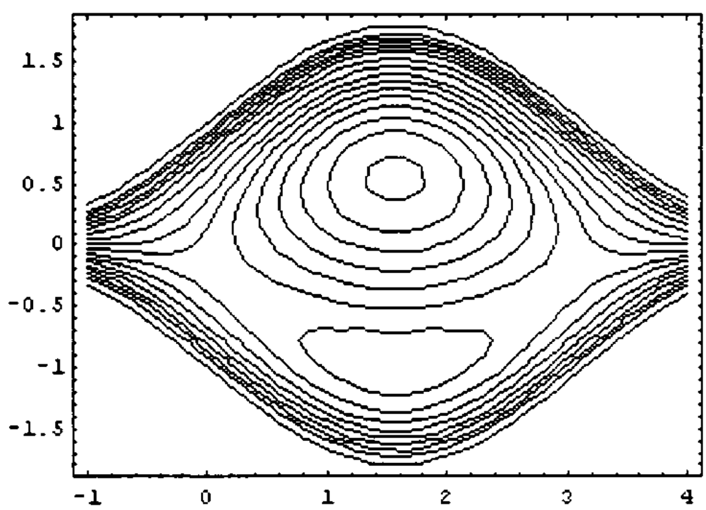

Fig. 3.12. Streamlines for $k = 5$, $\beta = 0.25$ with $\Phi = 0.8$ and $\Theta = 1.5$.

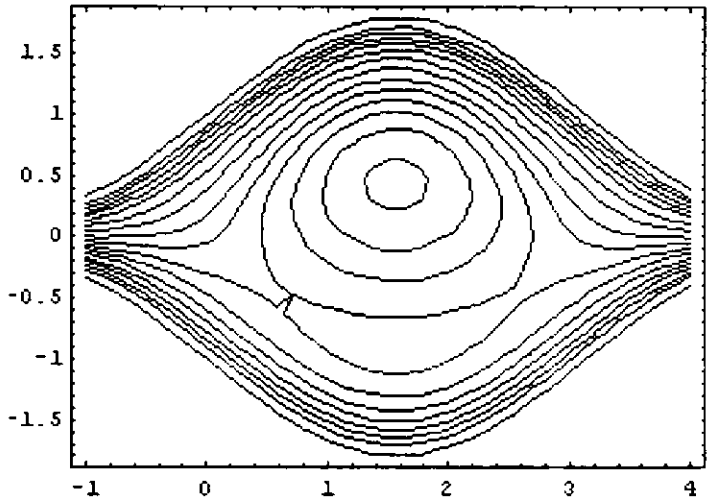

Fig. 3.13. Streamlines for $k = 5$, $\beta = 0.5$ with $\Phi = 0.8$ and $\Theta = 1.5$.

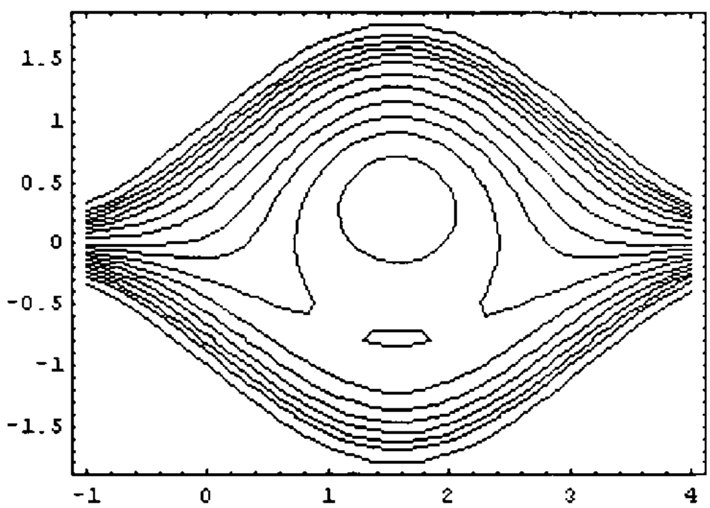

Fig. 3.14. Streamlines for $k = 5$, $\beta = 0.75$ with $\Phi = 0.8$ and $\Theta = 1.5$.

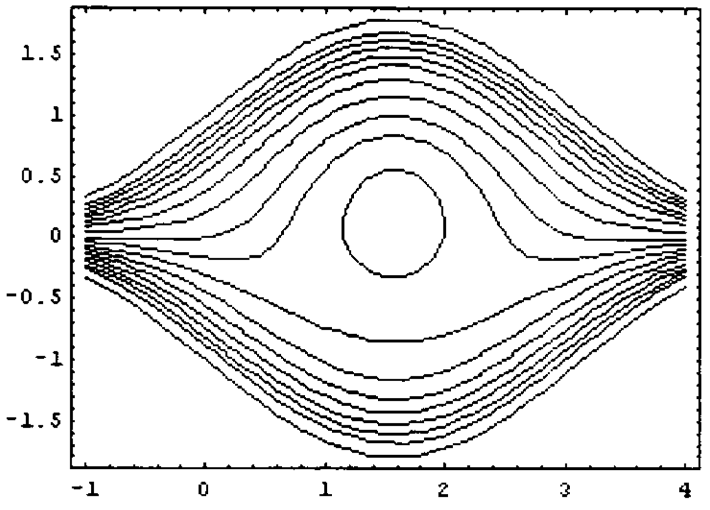

Fig. 3.15. Streamlines for $k = 5$, $\beta = 1$ with $\Phi = 0.8$ and $\Theta = 1.5$.

Bibliography

[1] T. W. Latham. Fluid motion in peristaltic pump. MS Thesis. MIT, Cambridge. MA. (1966).

[2] A. H. Shapiro, M. Y. Jaffrin and S. L. Weinberg. Peristaltic pumping with long wavelength at low Reynolds number. J. Fluid Mech. **37** (1969) 799 – 825.

[3] M. Y. Jaffrin and A. H. Shapiro. Peristaltic pumping, Annual Review of fluid mechanics **3** (1971) 13 – 36.

[4] Y. C. Fung and C. S. Yih. peristaltic transport. AMSME J. Appl. Mech. **33** (1968) 669 – 675.

[5] F. C. P. Yin and Y. C. Fung. Comparison of theory and experiment in peristaltic transport. J. Fluid Mech. **47** (1971) 93 – 112.

[6] K. K. Raju and R. Devanathan. Peristaltic motion of a non-Newtonian. Part-1. Rheol. Acta **11** (1972) 170 – 178.

[7] K. K. Raju and R. Devanathan. Peristaltic motion of a non-Newtonian. Part-2. Viscoelastic. Rheol. Acta **13** (1974) 944 – 948.

[8] Kh. S. Mekheimer, Peristaltic transport of couple-stress fluid in uniform and non-uniform channels. Biorheology **39** (2002) 755 – 765.

[9] E. F. El Shehawey and Kh. S. Mekheimer. couple-stresses in peristaltic transport of fluids. J. Phys. D, **27** (1994) 1163 – 1170.

[10] Kh. S. Mekheimer, E. F. El-Shehawey and A. M. Alaw. Peristaltic motion of a particle fluid suspension in a planar channel. Int. J. Theor. Phys., **37** (1998) 2895 – 2920.

[11] M. H. Haroun, Non-linear peristaltic flow of a fourth grade fluid in an inclined asymmetric channel. Comput Mater. Sci. **39** (2007) 324 – 333.

[12] M. H. Haroun, Effect of Deborah number and phase difference on peristaltic transport of third-order fluid in an asymmetric channel, Commun. Nonlinear Sci. Numer. Simul., **12** (2007) 1464 – 1480.

[13] A. M. Siddiqui and W. H. Schwarz, Peristaltic flow of second order fluid in tubes. J. Non-Newtonian Fluid Mech. **53** (1994) 257 – 284.

[14] T. Hayat, Y. Wang, A. M. Siddiqui, K. Hutter and S. Asghar, Peristaltic transport of third-order fluid in a circular cylindrical tube. Math. Models Meth. Appl. Sci., **12** (2002) 1691 – 1706.

[15] T. Hayat, F. M. Mahomed and S. Asghar, Peristaltic flow of magnetohydrodynamic Johnson–Segalman fluid, Nonlinear Dyn., **40** (2005) 375 – 385.

[16] T. Hayat and N. Ali, On mechanism of peristaltic flows for power-law fluids. Physica A. **371** (2006) 188 – 194.

[17] Y. Wang, T. Hayat and K. Hutter, Peristaltic flow of a Johnson-Segalma fluid through a deformable tube. Theor. Comput. Fluid Dyn.. **21** (2007) 369 – 380.

[18] Y. Wang, T. Hayat, N. Ali and M. Oberlack, Magnetohydrodynamic peristaltic motion of a Sisko fluid in a symmetric or asymmetric channel, Physica A **387** (2008) 347 – 362.

[19] H. Sato, T. Kawai, T. Fujita and M. Okabe, Two dimensional peristaltic flow in curved channels. Trans. The Japan Soc. Mech. Eng. B **66** (2000) 679 – 685.

[20] N. Ali, M. Sajid and T. Hayat, Long wavelength flow analysis in a curved channels, Z. Naturforsch. **65*a*** (2010) 191 – 196.

[21] W. K.H. Chu and J. Fang, J. On peristaltic transport in small Knudsen number flow, Meccanica 35 (2000) 69 – 74.

[22] E. F. El-Shehawy, N. T. El-Dabe and I. M. El-Desoky, Slip effects on the peristaltic flow of a non-Newtonian Maxwellian fluid, Acta Mech. **186** (2006) 141 – 159.

[23] T. Hayat, Q. Hussain and N. Ali. Influence of partial slip on the peristaltic flow in a porous medium. Physica A, **387** (2008) 3399 – 3409.

[24] T. Hayat, Umer and N. Ali. The influence of slip on the peristaltic motion of a third grade fluid in an asymmetric channel. Phys. Letts A, **372** (2008) 2653 – 2664.

[25] T. Hayat, N. Ali, Q. Hussain and S. Asghar. Slip effects on the peristaltic transport of MHD fluid with variable viscosity. Phys. Letts A, **372** (2008) 1477 – 1489.