Entropy Analysis of a Chemically Reactive Flow of Nanofluid Past a Stretching Cylinder

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A Thesis

Submitted in the Partial Fulfillment of the Requirements for Degree of

MASTER OF SCIENCE

IN

MATHEMATICS

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Certificate

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Muhammad Zeeshan

A dissertation submitted in the partial fulfillment of the requirements for the degree of the **Master of Science in Mathematics**

We accept this dissertation as conforming to the required standard.

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2022

Declaration

I hereby declare and affirm that this research work neither as a whole nor as a part has been copied out from any source. It is further declared that I have developed this research work entirely on the basis of my personal efforts made under the sincere guidance of my supervisor. No portion of the work presented in this thesis has been submitted in the support of any application for any degree or qualification of this or any other institute of learning.

Signature:____________________

MUHAMMAD ZEESHAN

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The Holy Prophet
Hazrat Muhammad PBUP

&

My Parents

Acknowledgement

I feel proud to thank the Almighty **ALLAH** who created us as human beings with the power of reasoning and has showered countless blessings upon us. I praise the **Holy Prophet Hazrat Muhammad (PBUH)** who guided us to the way of knowledge and research and whose life is the best role model for the whole of humanity.

I would like to express my most deep and sincere gratitude to my respected supervisor **Dr. Ahmer Mehmood** for his support, encouragement and guidance through every step of this research. Working with him has been a true privilege and a great experience for me. His kind efforts contributed greatly to my knowledge, understanding and enthusiasm for this research. Without his guidance and support this thesis would never have been accomplished. It has been a real honor for me to work under his supervision and again thanks go to **Dr. Ahmer Mehmood** for providing me this opportunity.

I want to appreciate to all the teachers of international Islamic university, who trained me during course and research work, by delivering his precious and valuable thoughts, in building my concept. I owe my deep gratitude and heartfelt thanks to my parents and to my all family members for their endless prayers support and love. Also I am specially appreciated Dr. Iqrar sahib, Dr. Usman sahib and Dr. Awais sahib for his support in the every step of research. At last I am also thankful to Shakir Ullah Khan, Zeeshan Marwat, Junaid Ahmad and Asmat Ullah Khan for his support, guidance and encouragement.

MUHAMMAD ZEESHAN

Preface

Nanofluid is formed by a combination of liquid suspension, comprises of very small particles of size range 1-100 nm in a base fluid (water, oil, etc,). These small particles are named as nanoparticles and can be made from different materials such as metals (*Cu*, *Ag*, *Au*), nitride ceramics (AlN, SiN), and semiconductor (SiC). The suspension of nanoparticles into the base fluid enhances the proficiency of heat and mass transfer process in liquid. The notion of nanofluid has made significant development in thermal engineering and material science in recent times. The applications of nanofluid include power generation, chemical and metallurgical sectors, manufacturing, and aerospace, etc. There are two types of models which have been commonly employed to nanofluid flows, namely Tiwari-Das model and Buongiorno model. Tiwari-Das model [1] emphasis on the solid volume friction of nanoparticles. However, Buongiorno model [2] states that Brownian diffusion and thermophoresis are the most significant parameter in the nanofluid flow. Moreover, nanofluid velocity is assumed to be the sum of the slip velocity and the base fluid velocity. Choi [3] gave the notion of nanofluid flow by cooling nanoparticles in the base fluid.

Investigations on fluid flow due to stretching surfaces are of great significance in manufacturing processes such as glass blowing, paper production, aerodynamic extraction of plastic sheets, etc. Hayat et al [4] studied the nanofluid flow over a stretching slim cylinder subjected to variable heat flux. Waini et al [5] examined the heat and mass transfer in hybrid nanofluid flow along a vertical slim cylinder by considering the surface heat flux. Waini et al [6] investigated the hybrid nano fluid flow past a porous slim cylinder in order to examine the effects of Brownian diffusion and thermophoresis parameters. Narain and Uberoi [7] studied the free and forced convection nanofluid over a vertical slim cylinder. Khan et al [8] employed the Buongiorno model to analyze the fluid flow over a horizontal slim cylinder.

Rudolf Clawsias [9] proposed the concept of entropy to examine the theory of heat and its uses associated with engine. Carrington and San [10] determined the formation of entropy generation by stating that entropy is generated when both, the mass transfer process and heat transfer process, are present. In their another study [11], they used control volume method to observe the creation of entropy brought on by the movement of heat and mass process for both internal and external fluxes. In other words, entropy is a quantity which measure the energy loses in a system which cannot be reversed. In this way, they described the second law of thermodynamics in terms of increase of entropy. Later on, Boltzman [12] and Gibbs [13] contributed to explain the notion of entropy and its concentration with irreversibility. Bejan [14−15] used the idea of entropy to discuss the efficiency of the thermo-mechanical devices. He pointed out the various source of entropy generation such as fluid friction, temperature difference, pressure of magnetic field, and thermal radiation.

This thesis consist of three Chapters. First Chapter contains some basic definitions, concept, and the governing equations associated with the current study. Chapter 2 covers the investigation of chemically reactive hybrid nanofluid flow over stretching slim cylinder subjected to viscous dissipation. Chapter 3 contains the analysis of entropy generation for chemically reactive nanofluid flow over a stretching slim cylinder. The impact of dimensionless parameters on entropy generation and Bejan number has been reported through various graphs.

Contents

CHAPTER 1

Introduction and basic definitions

This section contains introduction, basic definitions, and fundamental equations for the subsequent Chapters.

1. 1 Introduction

Fluid mechanics is the branch of mechanics that deals about fluid and the forces on them. It is an important subject of Civil, Mechanical and Chemical Engineering, etc. The two subsystem static and dynamic of fluid mechanics are used to examine fluid. One can observe the major role of fluid mechanics in the life of creatures. Our body is composed of 65% of water. Blood carries nutrients and energy to tissues so they can continue to function normally. Fluids include all gaseous and liquid substances. For instance water, oil, and other materials are used for a variety of purposes and are important to our daily lives. Hydroelectric power plants use water to generate energy. Water is also utilized as the coolant in nuclear power plants. Automobiles are lubricated with oil. Many fluids can be employed for a variety of purposes since they burn quickly and produce a lot of heat such as petrol and diesel for vehicles. Some liquids, such as oil, tend to exert a lot of pressure or force. Various heavy loads can be lifted using these fluids. The fluids used in hydraulic machines and hydraulic lifters are examples of such fluid.

By the end of the 19th century fluid mechanics had been separated into branches, namely, hydrodynamic and hydraulics. At the start, the viscosity of the fluid was ignored by hydrodynamic and consequently contradicting with experimental results. This led to the development of hydraulics, which had a significant impact on practical work. In 1904, Ludwig Prandtl [16] proposed the notion of boundary layer and described how to merge the two branches (hydrodynamic and hydraulics) of fluid mechanics. It simplifies the equations of fluid flow by dividing the flow field into two areas: one inside the boundary layer, dominated by viscosity and creating the majority of drag experienced by the boundary body and other outside the boundary layer, where viscosity can be neglected without significant effects on the solution. Blasius [17], the student of Prandtl, was the first who investigated the boundary layer flow over a flat sheet at rest. Later on, there had been a number of studies about boundary layer flow on static surface. After five decades later, Sakiadas [18 – 19] performed flow analysis of boundary layer on moving solid surface.

Cooling and heating are very essential in all industrial fields such as manufacturing, transportation, power station and highly developed electronic equipment's etc. Ethylene glycol, engine oil and water are most common fluids that are used for energy transportation because of their fluidity. Such fluids have limited heat transfer capabilities. On the other hand metallic materials dissipate a lot of heat conduction than the conventional liquids. For this nanofluid possesses greater thermal conductivity than that of ordinary fluid. This is because of proper suspension of metallic nano-sized particles in the base fluid. The fluid was first prepared by choi and Estman in 1995 [3]. Suspension or dispersion phase model [20] and Buongiornos model [2] was established to investigate nanofluid flow. Lin and Shah [21 – 22] examined the heat transfer and boundary layer flow on a stretching cylinder. Maskeen et al [23] examined the nanofluid flow characteristics and the performance of heat transport of hybrid nano liquid past a stretching cylinder.

The 2nd law of thermodynamics, basically deals with the ideology of entropy or irreversibility. It focuses on estimating the degree and average energy loss in all heat transfer processes. Generation of entropy will lead to shortening of the available energy in any system. Bejan [14] was the first man who open up a new line of research disclosing the concept of entropy in a convective heat transport flow problem. Bejan concluded the main reason for entropy production is the velocity gradient and temperature differences. San and Laven [24] examined the entropy generation due to heat and mass transfer through an isothermal channel.

1.2 Fundamental definitions

1.2.1 Fluid

Fluid is a substance that continuously deforms (flows) under an applied shear stress, or external force. Gasses (oxygen, hydrogen), liquid (water, petrol) are the examples of fluid.

1.2.2 Density

The measurement of quantity or mass per unit volume in a given substance is called density. Mathematically,

$$\rho = \frac{m}{v}, \tag{1.1}$$

where ρ represents density, m represents mass of an object and v represents volume of an object.

1.2.3 Pressure

The force per unit area that is applied perpendicular to an object's surface. Mathematically,

$$P = \frac{|F|}{A}, \tag{1.2}$$

where P represents pressure, $\boldsymbol{F}$ represents force and A represents area.

1.3 Types of fluid

1.3.1 Ideal fluid

Fluid that is incompressible and has no internal flow resistance (zero viscosity) is called ideal fluid.

It is a hypothetical fluid that does not exist in actual.

1.3.2 Real fluid

Fluid which possesses at least some viscosity is known as real fluid. Every fluid that we see around us like water, diesel, petrol, and so on are examples of real fluids.

1.3.3 Newtonian fluid

Newtonian fluid obeys Newton's law of viscosity. Newtonian fluids include things like water, organic solvents, and honey.

1.3.4 Non-newtonian fluid

Non-Newtonian fluids do not adhere to Newton's law, their viscosity (the ratio of shear stress to shear rate) is not constant and is depending on the shear rate. Soap solutions, toothpaste, butter, cheese, jam, soup, and yogurt are the some examples of Non-Newtonian fluid.

1.4 Classification of flow

1.4.1 Uniform flow

Flow of a fluid where each particle follows its line of flow at a constant speed and where the cross section of each stream tube stays constant. Mathematically,

$$\frac{\partial(,)}{\partial s} = 0. \tag{1.3}$$

1.4.2 Non-uniform flow

When a flow's velocity fluctuates at a specific moment, it is said to be non-uniform flow. Mathematically,

$$\frac{\partial(,)}{\partial s} \neq 0. \tag{1.4}$$

1.4.3 Steady flow

A steady flow is one in which all conditions at any point in a stream remain constant with respect to time. Mathematically,

$$\frac{\partial()}{\partial t} = 0. \tag{1.5}$$

1.4.4 Unsteady flow

Unsteady flow is defined as a flow in which the amount of liquid flowing per second is not constant. Mathematically,

$$\frac{\partial()}{\partial t} \neq 0 \tag{1.6}$$

1.4.5 Compressible flow

Compressible flow is the flow of fluids whose density considerably changes in response to a change in pressure. Mathematically,

$$\rho(x, y, z, t) \neq \text{Constant}. \tag{1.7}$$

1.4.6 Incompressible flow

A flow is considered to be incompressible if every fluid parcel has the same density. Mathematically,

$$\rho(x, y, z, t) = \text{Constant}. \tag{1.8}$$

1.4.7 Laminar flow

Laminar flow is the one in which the fluid travels smoothly or in regular paths. Oil flow through a thin tube, blood flow through capillaries are the example of laminar flow.

1.4.8 Turbulent flow

Turbulent flow is the one in which fluid moves with irregular fluctuations or mixing. Blood flow in arteries, oil transport in pipelines are the example of turbulent fluid.

1.4.9 Rotational flow

Rotational flow is a flow in which the movement of fluid is in a circular pattern across a space between spinning cylinders, a space between two spherical surfaces, a space between a cone and a plate, or other arrangements of circular bodies. Mathematically,

$$\boldsymbol{\nabla} \times \boldsymbol{V} \neq 0. \tag{1.9}$$

1.4.10 Irrotational flow

Irrotational flow can be defined as flow where no component of the moving fluid rotates at all from one instant to the next in relation to a specific frame of reference. Mathematically,

$$\boldsymbol{\nabla} \times \boldsymbol{V} = 0. \tag{1.10}$$

1.4.11 Axially-symmetric flow

A flow whose every flow quantity is independent of angle when cylindrical polar coordinates $r, z,$ are properly chosen, is known as axially symmetric flow. For example a flow past bodies of revolution.

1.4.12 One-dimensional flow

One-dimensional flow is a type of flow in which variables like velocity, pressure, density, temperature, etc. depend solely on one space coordinate and one time dimension. Mathematically,

$$\boldsymbol{V} = \boldsymbol{V}(x, t). \tag{1.11}$$

1.4.13 Two-dimensional flow

When a flow is two dimensional, the flow characteristics depend on both time and two rectangular space coordinates. Mathematically,

$$V = V(x, y, t). \tag{1.12}$$

1.4.14 Three-dimensional flow

Three dimensional flow is a type of flow in which the flow characteristics are the function of time and three mutually perpendicular directions. Mathematically,

$$V = V(x, y, z, t). \tag{1.13}$$

1.4.15 Internal flow

Flows completely bounded by solid surfaces are called internal flows. Example of such flow is flow in a pipe.

1.4.16 External flow

External flow is a type of flow in fluid dynamics where boundary layer can grow freely without being constrained by nearby surfaces. The flow over air foils, ship hulls and over turbine blades are some examples of external flows.

1.5 Properties of fluid

1.5.1 Viscosity

Viscosity is the measure of resistance of a fluid to deformation under shear stress. It is commonly perceived as thickness, or resistance to pouring. Viscosity describes a fluids internal resistance to flow and may be thought of as a measure of fluid friction. Thus, methanol is thin, having a low viscosity, while vegetable oil is thick having a high viscosity.

1. 5. 2 Dynamic viscosity

Dynamic viscosity is also known as absolute viscosity which is the ratio of shear stress τ to the deformation rate $\frac{\partial u}{\partial y}$. It is denoted by μ. Mathematically,

$$\mu = \frac{\tau}{\frac{\partial u}{\partial y}}. \tag{1.14}$$

1. 5. 3 Kinematic viscosity

The ratio of absolute viscosity μ to the density ρ is called kinematic viscosity. It is denoted by ν. Mathematically,

$$\nu = \frac{\mu}{\rho}. \tag{1.15}$$

1. 6 Temperature

Temperature is a unit used to represent hotness or coolness using any of a number of scales, including Fahrenheit and Celsius. Temperature specifies the direction in which heat energy will spontaneously flow, i.e., from a hotter body (one with a higher temperature) to a colder body (one at a lower temperature).

1. 7 Heat

Heat is the transfer of kinetic energy from one medium or object to another, or from an energy source to a medium or object. Such energy transfer can occur in three ways: radiation, conduction, and convection.

1. 8 Heat transfer

Heat transfer is defined as the process in which the molecules are moved from the region of higher temperature to lower temperature. There are three modes of heat transfer, which are given below

(i) Conductive heat transfer.

(ii) Convective heat transfer.

(iii) Thermal radiation.

1.8.1 Conductive heat transfer

Conduction is the process by which energy is transferred from a substance's more energetic particles to its less energetic particles through particle contact.

1.8.2 Convective heat transfer

The transfer of heat from one location to another through the flow of fluid is known as convection (or convective heat transfer). Convective heat transfer involves the combined processes of conduction (heat diffusion) and advection (heat transfer by bulk fluid flow).

1.8.3 Thermal radiation

Thermal radiation is electromagnetic radiation emitted from all matter due to its possessing thermal energy which is measured by the temperature of the matter. Examples of thermal radiation are an incandescent light bulb emitting visible light, infrared radiation emitted by a common household radiator or electric heater, as well as radiation from hot gas in outer space.

1.9 Governing equations

1.9.1 Continuity equation

According to the law of conservation of mass the rate at which the mass enters into the system is equal to the rate at which the mass leaves the system.

The law of conservation of mass for compressible fluid flow is given by

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\boldsymbol{V})=0\,, \tag{1.16}$$

Let u, v, w denote the velocity components in Cartesian co-ordinates, then Eq.(1.16) read as

$$\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial x}+v\frac{\partial\rho}{\partial y}+w\frac{\partial\rho}{\partial z}+\rho\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right]=0. \tag{1.17}$$

Suppose $v_{r,}$ $v_{\Theta,}$ and v_z are velocity components in directions of $r, \theta,$ and z respectively, then in cylindrical polar coordinates, the equation of continuity observe a form

$$\frac{\partial\rho}{\partial t}+v_r\frac{\partial\rho}{\partial r}+\frac{v_\theta}{r}\frac{\partial\rho}{\partial\theta}+v_z\frac{\partial\rho}{\partial z}+\rho\left[\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{1}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{\partial v_z}{\partial z}\right]=0. \tag{1.18}$$

For the case of incompressible steady flow, equation (1.18) becomes

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{1}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{\partial v_z}{\partial z}=0 \tag{1.19}$$

1.9.2 Momentum equation

Every fluid motion is actually governed by the Newton's second law of motion. In vector form, it is written as

$$\rho\frac{DV}{Dt}=div\boldsymbol{T}+\rho\boldsymbol{b}, \tag{1.20}$$

where $\frac{D}{Dt}$ is material derivative, $\boldsymbol{b}$ is the body force per unit mass, $\boldsymbol{T}$ is the Cauchy stress tensor, and $\boldsymbol{V}$ is velocity vector. Here in the above equation left hand side represent the sum of inertial forces whereas first and second terms on the right hand side are the sum of surface and body forces, respectively. The momentum equation (1.22) is also known as Navier Stokes equation (momentum equation).

The momentum equation in cylindrical coordinates is given as

Radial component:

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{{v_\theta}^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)=$$

$$-\frac{\partial p}{\partial r}+\mu\left(\frac{\partial^2 v_r}{\partial r^2}+\frac{1}{r}\frac{\partial v_r}{\partial r}-v_r\frac{v_r v_\theta}{r^2}+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial^2 v_r}{\partial z^2}\right)+\rho b_r. \tag{1.21}$$

Circumferential component:

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_\theta\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}-\frac{{v_\theta}^2}{r}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=$$

$$-\frac{\partial p}{\partial \theta}+\mu\left(\frac{\partial^2 v_\theta}{\partial r^2}+\frac{1}{r}\frac{\partial v_\theta}{\partial r}-\frac{v_\theta}{r^2}+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right)+\rho b_\theta. \tag{1.22}$$

Axial component:

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 v_z}{\partial r^2}+\frac{1}{r}\frac{\partial v_z}{\partial r}+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho b_z. \tag{1.23}$$

For an incompressible, steady, two-dimensional, axis-symmetric flow in the absence of body force the velocity vector is chosen:

$$\boldsymbol{V}=[v_r(r,z),0,v_z(r,z)].$$

In view of this velocity vector the above equations are reduced to the form

Radial component:

$$v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\left(\frac{\partial^2 v_r}{\partial r^2}+\frac{1}{r}\frac{\partial v_r}{\partial r}+\frac{\partial^2 v_r}{\partial z^2}\right), \tag{1.24}$$

Axial component:

$$v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left(\frac{\partial^2 v_z}{\partial r^2}+\frac{1}{r}\frac{\partial v_z}{\partial r}+\frac{\partial^2 v_z}{\partial z^2}\right), \tag{1.25}$$

Circumferential component:

$$-\frac{1}{\rho}\frac{\partial p}{\partial \theta} = 0. \tag{1.26}$$

1.9.3 Energy equation

In fluid dynamics, the study of laws of conservation of mass and energy is important. The law of conservation of energy for fluid flow is given by

$$\rho C_p\left(\frac{\partial T}{\partial t} + \boldsymbol{V}\cdot\boldsymbol{\nabla} T\right) = K\boldsymbol{\nabla}^2 T + \mu\Phi, \tag{1.27}$$

where Φ, K, and C_P are dissipation function, thermal conductivity, and specific heat at constant pressure, respectively. The energy equation in cylindrical coordinates is given by,

$$\rho C_p\left(\frac{\partial T}{\partial t} + v_z\frac{\partial T}{\partial z} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta}\right) = K\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right] + \mu\Phi, \tag{1.28}$$

where dissipation function Φ is given by

$$\Phi = 2\left(\frac{\partial v_r}{\partial r}\right)^2 + 2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)^2 + 2\left(\frac{\partial v_z}{\partial z}\right)^2 + \left(\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)^2 + \left(\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right)^2 +$$

$$\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right)^2. \tag{1.29}$$

For a steady, two-dimensional, and axi-symmetric flow the energy equation in the absence of viscous dissipation and thermal radiation reads as

$$v_z\frac{\partial T}{\partial z} + v_r\frac{\partial T}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(\alpha r\frac{\partial T}{\partial r}\right). \tag{1.30}$$

1.10 Boundary layer

The concept of boundary layer was first introduced by Ludwig Prandtl in 1904. The thin layer of a flowing gas or liquid in contact with a surface such as that of an airplane wing or of the inside of a pipe is called boundary layer.

1.11 Boundary layer equations

Prandtl observed that inside the boundary layer u is dominant velocity component, for a two-dimensional flow situation. Moreover, the variations in velocity, as a consequence of fluid viscosity, are significant in y-direction and less significant in x-direction. On this basis he introduced an order of magnitude analysis of the Navier-stokes equations which resulted in a significant simplification to the Navier-stokes equations. The resulting simplified equations were then named as the boundary layer equations.

Corresponding to the governing equations in cylindrical coordinates, the boundary layer equation for a steady, two-dimensional flow of an incompressible fluid are given as:

Continuity equation:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{\partial v_z}{\partial z} = 0. \tag{1.31}$$

Momentum equations:

$$v_r\frac{\partial v_z}{\partial r} + v_z\frac{\partial v_z}{\partial z} = -\frac{1}{\rho}\frac{\partial \rho}{\partial z} + v\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v_r}{\partial r}), \tag{1.32}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial r}, \tag{1.33}$$

Energy equation:

$$v_r\frac{\partial T}{\partial r} + v_z\frac{\partial T}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(\alpha r\frac{\partial T}{\partial r}\right). \tag{1.34}$$

1.12 Laws of thermodynamics

1.12.1 First law of thermodynamics

The law of conservation of energy is the first law of thermodynamics. Energy, according to this statement, can take on several forms, but the overall amount of energy is conserved, i.e, it cannot be created or destroyed. The equivalence of various kinds of energy is indicated by the first law of thermodynamics. It gives a link between work and absorbed energy and characterizes the system's internal energy as a state function. However, it is unable to predict whether or not the energy will change and how much the change will be. Additionally, it offers no information regarding the reversibility of thermodynamic processes, or the direction of the change during the spontaneous process.

1.12.2 Second law of thermodynamics

The first law of thermodynamics asserts that energy is conserved, i.e, the amount of energy remains constant. However, energy has qualitative aspect as well. When energy is transferred from one form to another, some of it loses its usefulness and cannot be recovered. This useless energy is measured by using a standard metric known as entropy. The second law of thermodynamics asserts that the entropy of the universe (a system and its surrounds) either stays constant or rises. The effectiveness and performance of engineering systems are assessed using the second law of thermodynamics.

1.13 Entropy generation

The thermodynamic reversibility of the flow and heat transfer systems is related to entropy. It is a systemic measure of disordered randomness. The amount of energy that the system can use decreases as its entropy rises. As a result, the energy that is accessible for use in work is destroyed by entropy. The investigation of entropy generation effects is important because it can help engineering systems to operate more effectively and efficiently.

1.14 Some useful dimensionless numbers

1.14.1 Reynolds number

The Reynolds number Re is defined as the ratio between inertial forces and viscous forces

$$Re=\frac{UL}{\nu}, \tag{1.35}$$

where L represent characteristics length and U represent reference velocity.

1.14.2 Prandtl number

The ratio of momentum diffusivity (also known as kinematic viscosity) and thermal diffusivity of the fluid is known as the Prandtl number

$$Pr=\frac{\nu}{\alpha}, \tag{1.36}$$

where α is thermal diffusivity and ν is kinematic viscosity.

1.14.3 Nusselt number

Nusselt number is expressed as the ratio between convection and conduction. Mathematically,

$$Nu=\frac{hL}{K} \tag{1.37}$$

where L is the characteristic length, K represents the fluid thermal conductivity, and h is the convective heat transfer coefficient.

1.14.4 Skin-friction coefficient

Skin friction is the unit of measurement for the resistance created by the interaction of a solid surface with a fluid that is moving relative to it. A dimensionless quantity known as the skin-friction coefficient is defined as

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2}, \tag{1.38}$$

where τ_w represents the shear stress at wall, ρ is density, and U is the reference velocity.

1.14.5 Bejan number

Bejan number provides the detail whether the effects of viscous dissipation on entropy generation outweigh those of heat transport, it is defined as

$$\text{Be} = \frac{Entropy\ due\ to\ heat\ transfer}{total\ entropy\ genration}.$$

1.14.6 Eckert number

The Eckert number is a dimensionless number used in continuum mechanics. It expresses the relationship between a flow's kinetic energy and the boundary layer enthalpy difference, and is used to characterize heat dissipation. It is named after Ernst R. G. Eckert. Mathematically it is given as:

$$Ec = \frac{u^2}{C_p \Delta T}, \tag{1.39}$$

where, u is the local flow velocity of the continuum, C_p is the specific heat of the continuum and ΔT is the difference between wall temperature and local temperature.

1.15 Nanofluid

Fluid having a very small quantity of solid particle whose diameter is $1 - 100$ nm that are constantly dispersed into base fluid, such as water etc. These types of mixtures are called nanofluids.

1.15.1 Hybrid nanofluid

Hybrid nanofluids are nanofluids which has different nano-sized particles in a particular ratio. The Hybrid nanofluids gain higher heat transfer rate as compared to pure fluid as it is proved experimentally and numerically by numerous researchers. Since hybrid nanofluids show favorable characteristics and performance, authors are inspired to review different aspects of hybrid nanofluids which include thermophysical properties, heat transfer performance, preparation method and stability investigation method.

The hybrid nanofluid parameters bear the following forms.

$$\rho_{nf} = \left\{(1-\varphi_2)\left((1-\varphi_1)+\varphi_1\frac{\rho_{s1}}{\rho_f}\right)+\varphi_2\frac{\rho_{s2}}{\rho_f}\right\}\rho_f, \qquad \mu_{nf} = \frac{\mu_f}{(1-\phi_1)^{2.5}}$$

$$(\rho C_p)_{nf} = (1-\phi_1)(\rho C_p)_f + \phi_1(\rho C_p)_{s1}, \quad K_{nf} = \frac{K_{s2}+2K_f-2\varphi_2(K_f-K_{s2})}{K_{s2}+2K_f+\varphi_2(K_f-K_{s2})}K_{nf}.$$

$$\rho_{hnf} = \{(1-\varphi_2)\left((1-\varphi_1)+\varphi_1\frac{\rho_{s1}}{\rho_f}\right)+\varphi_2\frac{\rho_{s2}}{\rho_f}\}\rho_f, \quad \mu_{hnf} = \frac{\mu_f}{(1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}},$$

$$(\rho C_p)_{hnf} = \left\{(1-\varphi_2)\left((1-\varphi_1)+\varphi_1\frac{(\rho C_p)_{s1}}{(\rho C_p)_f}\right)+\varphi_2\frac{(\rho_{s2})}{\rho_f}\right\}(\rho C_p)_f.$$

Here ρ_{nf} and ρ_{hnf} represents the nanofluids and hybrid nanofluid densities respectively, μ_{nf} and μ_{hnf} are its viscosities, $(\rho C_p)_{nf}$ and $(\rho C_p)_{hnf}$ are the specific heat of nanofluid and hybrid nanofluid respectively.

Numerical values of thermophysical quantities for a base fluid (water) and nanoparticles are given in table 01.

Table 01: Numerical estimates of the base fluid's thermophysical characteristics in relation to nanoparticles.

	(Silver – Ag)	(Copper oxide-CuO)	(Water - H_2O)
Density	10500	6320	997.1
Specific heat	235	531.8	4179
Thermal conductivity	429	76.5	0.613

1.16 Buongiorno model

There exist several empirical and theoretical nanofluid models. In literature among them all, everyone has its own merits and demerits. In most of the theoretical studies, Tiwari and Das model [1] and Buongiorno model [2] are preferred over other famous models. In addition to the volume fraction of the nanoparticles, the Buongiorno model also considers the thermophoretic and Brownian motion effects of the nanoparticles. The Buongiorno nanofluid model helps to examine the Brownian motion and thermophoresis impact on the thermal transport phenomenon. This model has the following advantages and disadvantages:

Advantages

- Higher single-phase heat transfer coefficient, especially for laminar flow, due to increased thermal conductivity
- This model is used to investigate the effect of Rayleigh lewis thermophoresis, Brownian motion and heat transfer rate of natural convection inside a cavity

Disadvantage

- Increase axial rise in wall temperature due to degraded specific heat
- Increase pumping power due to greater pressure drops

CHAPTER 2

Chemically reactive nanofluid flow past a slim moving cylinder with viscous dissipation

2.1 Introduction

In this Chapter, heat transmission and hybrid nanofluid flow through a slim, heated cylinder are studied in the presence of viscous dissipation. The nanoparticles, Copper Oxide (CuO) and Silver (Ag), are considered with water as base fluid. The impact of thermophoretic and Brownian force is investigated using the Buongiorno model [2]. By employing similarity transformations, the governing equations of the flow problem are converted into a set of ordinary differential equations. The transformed equation are solved numerically by using $bvp4c$ method on MATLAB. The results show that increasing the volume fraction of nanoparticles reduces fluid flow. Thermal characteristics increase with the expansion of the Eckert number, Brownian motion parameter, and the volume percentage of nanoparticles, whereas they decrease with the growth of the Prandtl number. Moreover, nanoparticles concentration decreases by increasing chemical reaction parameter, thermophoresis and Lewis number whereas it increases by increasing Brownian motion parameter.

.2.2 Problem formulation

Consider the flow of a viscous, incompressible nanofluid around a slim, heated cylinder. In the base fluid (water), copper oxide (CuO) and silver (Ag) are considered to be nanoparticles. The velocity components u and v are assumed along x and r directions respectively. The cylinder moves with constant velocity u_w in the same or opposite direction of the flow system $U = u_w + u_\infty$.

The cylinder surface temperature is assumed to be T_w (constant) whereas the concentration of species at the surface is C_w (constant). The corresponding temperature and concentration of ambient fluid are $T_\infty (T_w > T_\infty)$ and C_∞ $(C_w > C_\infty)$, respectively. The radius of slim cylinder is expressed as $R(x) = (\upsilon_f cx/U)^{\frac{1}{2}}$. In addition, it is considered that the size of cylinder is slim enough so that the surface transverse curvature has a considerable impact.

In view of given assumption along with the utilization of described Buongiorno model, the governing equations read as:

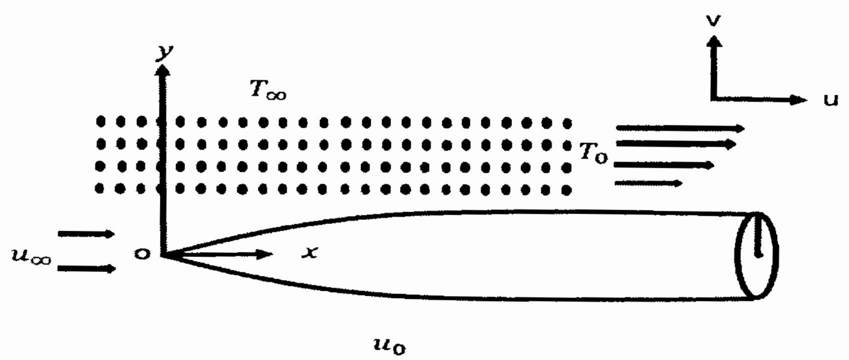

Figure 2.1: schematic diagram of the problem.

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0\ , \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \frac{\mu_{hnf}}{\rho_{hnf}}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right), \tag{2.2}$$

$$\left(\rho C_{P\,hnf}\right)\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r}\right) = \tau\left(D_B\ \frac{\partial T}{\partial r}\frac{\partial C}{\partial x} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial r}\right)^2\right) + K_{hnf}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \mu_{hnf}\left(\frac{\partial u}{\partial r}\right)^2 \tag{2.3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial r} = \frac{D_T}{T_\infty}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{D_B}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right) - K^*(C - C_\infty)\ , \tag{2.4}$$

In above equations $(2.1) - (2.4)$, u and v represent velocity components in axial and radial directions. T and C represent temperature and concentration of the fluid, respectively. K_{hnf}, μ_{hnf}

and ρ_{hnf} represent the thermal conductivity, viscosity and density of hybrid nanofluid, respectively. C_p represents heat capacity where as T_∞ and C_∞ respectively denotes the temperature and concentration of ambient fluid. D_B and D_T are the Brownian and thermophoretic parameters where as $K^* = \frac{K_\circ}{x}$ represents the rate of dimensionless reaction.

Boundary condition for the above equations are

$$u(x\,,\,r) = U_w,\quad v(x\,,\,r) = 0,\; T(x\,,\,r) = T_w\,,\; C(x,r) = C_w \text{ at } r = R(x),$$

$$u(x\,,\,r) \to U_\infty\,,\; T(x\,,\,r) \to T_\infty,\; C(x\,,\,r) \to C_\infty \text{ when } r \to \infty. \tag{2.5}$$

To change the system of equations, (2.1) – (2.4) a set of similarity variables is introduced as

$$\psi = v_f x f(\eta),\; \eta = \frac{Ur^2}{v_f x}\,, \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}\,, \phi(\eta) = \frac{C-C_\infty}{C_w-C_\infty}\,, \tag{2.6}$$

with $U = u_w + u_\infty$ as a composite velocity of the system.

The components of flow velocity for the assumed stream function are expressed as

$$u = \frac{1}{r}\frac{\partial\psi}{\partial r},\quad v = -\frac{1}{r}\frac{\partial\psi}{\partial x}\,. \tag{2.7}$$

In this study the hybrid nanoparticles are made up of copper oxide (CuO) and silver (Ag) suspended in water (H_2O) in order to achieve hybrid nanofluids $Ag - CuO/H_2O$. At first nanoparticles CuO with a volume fraction denoted by φ_1 are suspended in water and in turns, CuO/H_2O nanofluid is obtained. Secondly, nanoparticles Ag with volume fraction denoted by φ_2 is suspended in nanofluid CuO/H_2O and in turns, $Ag - CuO/H_2o$ is obtained.

In view of equation (2.7), continuity equation (2.1) satisfies identically. However, insertion of equations (2.6) and (2.7) in equations (2.2) – (2.4) give the following system:

$$\frac{2}{(1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}}(\eta f''' + f'') + \left\{(1-\varphi_2)\left((1-\varphi_1)+\varphi_1\frac{\rho_{s1}}{\rho_f}\right)+\varphi_2\frac{\rho_{s2}}{\rho_f}\right\}ff'' = 0, \qquad (2.8)$$

$$2\frac{K_{hnf}}{K_f}(\theta' + \eta\theta'') + \left\{(1-\varphi_2)\left((1-\varphi_1)+\varphi_1\frac{(\rho C_p)_{s1}}{(\rho C_p)_f}\right)+\varphi_2\frac{(\rho C_p)_{s2}}{(\rho C_p)_f}\right\}Prf\theta' +$$

$$2\eta \Pr(N_b\theta'\phi' + N_t(\theta')^2) + 8PrEc\left(\frac{\eta}{(1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}}(f'')^2 = 0, \qquad (2.9)$$

$$2(\phi' + \eta\phi'') + 2\frac{N_t}{N_b}(\theta' + \eta\theta'') + Le\phi' f - \frac{1}{2}LeK\phi = 0. \qquad (2.10)$$

The subjected boundary condition in dimensionless form are given as

$$f(c) = \frac{\varepsilon}{2}c, \quad f'(c) = \frac{\varepsilon}{2}, \quad \theta(c) = 1, \quad \phi(c) = 1,$$

$$f(\infty) = \frac{1}{2}(1-\varepsilon), \ \theta(\infty) = 0, \ \phi(\infty) = 0, \qquad (2.11)$$

where $\varepsilon = \frac{u_w}{U}$.

In equations (2.8) to (2.10), the physical quantities denoted by subscript 1 correspond to CuO-nanoparticles whereas those denoted by subscript 2 correspond to Ag-nanoparticles. Further, there are some parameters which are described in Table 1 along with their mathematical and physical descriptions. In addition, there arise three cases for the parameter $\varepsilon = \frac{u_w}{U}$.

(i) $\varepsilon = 1$, indicates that cylinder is moving and fluid is it rest.

(ii) $\varepsilon = 0$, indicates that fluid is moving and cylinder is it rest.

(iii) $0 < \varepsilon < 1$, indicates that both fluid and cylinder are moving in the same direction.

There are also two cases for chemical reaction parameter K such that $K > 0$ depicts a destructive reaction whereas $K < 0$ depicts a generation reaction. The thermophoresis properties of nanofluid and hybrid nanofluid are given in Tables $2.2 - 2.3$.

According to our flow system, the local Nusselt number, Sherwood number, and the coefficient of skin friction are defined as

$$\mathrm{Nu}_x = -\frac{x k_{hnf}}{k_f (T_w - T_\infty)} \left(\frac{\partial T}{\partial r}\right)_{r=R}, \; Sh_x = \frac{-x}{(C_w - C_\infty)} \left(\frac{\partial C}{\partial r}\right)_{r=R}, \; C_f = \frac{\mu_{hnf}}{\rho_f u_w^2} \left(\frac{\partial u}{\partial r}\right)_{r=R}, \tag{2.12}$$

After incorporating (2.6) in (2.12) the dimensionless form of these quantities read as.

$$\mathrm{Nu}_x Re_x^{\frac{-1}{2}} = -2 \frac{K_{hnf}}{K_f} c^{\frac{1}{2}} \theta'(c), \; Sh_x Re_x^{\frac{1}{2}} = -2 c^{\frac{1}{2}} \phi'(c), \; C_f Re_x^{\frac{1}{2}} 4 c^{\frac{1}{2}} \frac{1}{(1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}} f''(c), \tag{2.13}$$

In equation (2.15) $Re_x = \frac{Ux}{\rho_f}$ represents local Reynolds number.

Table 2.1: Details about the emerging characteristics.

Symbolic form	Mathematical form	Physical mean
Pr	$\frac{V_f (\rho C_p)_{hnf}}{k_f}$	Prandtl number
N_b	$\frac{\tau D_b (C_w - C_\infty)}{v_f}$	Brownian motion parameter
N_t	$\frac{\tau D_T (T_w - T_\infty)}{T_\infty v_f}$	Thermophoretic parameter
Ec	$\frac{U^2}{(T_w - T_\infty) C_p}$	Eckert number
Le	$\frac{v_f}{D_B}$	Lewis number
K	$\frac{K^*}{U}$	Chemical reaction parameter
ε	$\frac{u_w}{U}$	Velocity ratio parameter

Table 2.2: Thermophoresis characteristics of a Nano fluid

Characteristics	**Nano fluid (CuO)**
Density	$\rho_{nf}=(1-\varphi_1)\rho_f+\varphi_1\rho_{s_1}$
Heat capacity	$(\rho C_p)_{nf}=(1-\varphi_1)(\rho C_p)_f+\varphi_1(\rho C_p)_{s_1}$
Viscosity	$\mu_{nf}=\frac{\mu_f}{(1-\varphi_1)^{2.5}}$
Thermal-conductivity	$K_{nf}=\frac{K_{s_2}+2K_f-2\varphi_2(K_f-K_{s_2})}{K_{s_2}+2K_f+\varphi_2(K_f-K_{s_2})}K_{nf}$

Table 2.3: Thermophoresis characteristics of a Nano fluid

Characteristics	**Hybrid nano fluid ($CuO-Ag$)**
Density	$\rho_{hnf}=\{(1-\varphi_2)\left((1-\varphi_1)+\varphi_1\frac{\rho_{s_1}}{\rho_f}\right)+\varphi_2\frac{\rho_{s_2}}{\rho_f}\}\rho_f$
Heat capacity	$(\rho C_p)_{hnf}=\left\{(1-\varphi_2)\left((1-\varphi_1)+\varphi_1\frac{(\rho C_p)_{s_1}}{(\rho C_p)_f}\right)+\varphi_2\frac{(\rho_{s_2})}{\rho_f}\right\}(\rho C_p)_f$
Viscosity	$\mu_{hnf}=\frac{\mu_f}{(1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}}$
Thermal-conductivity	$K_{nf}=\frac{K_{s_2}+2K_f-2\varphi_1(K_f-K_{s_2})}{K_{s_2}+2K_f+\varphi_2(K_f-K_{s_2})}K_{nf}$

2.3 Numerical solution

The set of equations (2.8) – (2.10) along with boundary conditions (2.11) has been solved with the aid of bvp4c method on MATLAB. For this, equations (2.8) – (2.10) are transformed into a system of first–order differential equations in the following way.

Let

$$f = y(1),$$

$$f' = y(2),$$

$$f'' = y(3),$$

$$f''' = -\frac{y(3)}{\eta} - \frac{(1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}}{2\eta}\left\{(1-\varphi_2)\left((1-\varphi_1)+\varphi_1\frac{\rho_{s_1}}{\rho_f}\right)+\varphi_2\frac{\rho_{s_2}}{\rho_f}\right\}y(1)y(3),$$

and

$$\theta = y(4),$$

$$\theta' = y(5),$$

$$\theta'' = -\frac{y(5)}{\eta} - \frac{K_f}{2\eta K_{hnf}}\left\{(1-\varphi_2)\left((1-\varphi_1)+\varphi_1\frac{(\rho C_p)_{s_1}}{(\rho C_p)_f}\right)+\varphi_2\frac{(\rho C_p)_{s_2}}{(\rho C_p)_f}\right\}Pry(1)y(5) -$$

$$\frac{K_f}{K_{hnf}}Pr(N_b y(5)y(8) + N_t(y(5))^2 - 4PrEc\frac{K_f}{K_{hnf}}\left(\frac{1}{(1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}}\right)(y(3))^2,$$

we further more let

$$\phi = y(6),$$

$$\phi' = y(7),$$

$$\phi'' = -\frac{2y(8)}{\eta} - \frac{2N_t}{\eta N_b}\big(y(5) + \eta y(6)\big) - Ley(8)y(1) + 0.5LeKy(7),$$

The corresponding boundary conditions take the form as follows

$$y_a(1) - \frac{\varepsilon}{2}c,$$

$$y_a(2) - \frac{\varepsilon}{2},$$

$$y_a(4) - 1,$$

$$y_a(6) - 1,$$

$$y_b(2) \rightarrow \frac{1}{2}(1-\varepsilon),$$

$$y_b(4) \rightarrow 0\,,$$

$$y_b(6) \rightarrow 0.$$

2.4 Results and discussion

In this study, heat and mass transfer in a hybrid nanofluid comprising Copper Oxide (CuO) and Silver (Ag) over a stretched slim cylinder. The impact of nanoparticles CuO and Ag volume fraction $\varphi_1$and φ_2, Eckert number Ec, Prandtl number Pr, Lewis number Le, Brownian motion parameter N_b, thermophoresis parameter N_t and chemical reaction parameter K on temperature, velocity, and concentration characteristics has been discussed. The influence of nanoparticles φ_1 and φ_2 on velocity distribution is depicted by Figures (2.2 –2.3) These figures show that velocity profiles reduce upon increasing φ_1 and φ_2. Figures (2.4 – 2.9) show that variation in temperature distribution influenced by involved parameters. Increment in Eckert number Ec causes to increase the temperature profile and is shown in Figure 2.4. Figures $(2.5-2.6)$ depict the effects of nanoparticles volume friction φ_1 and φ_2 on temperature distributions. It is noted that temperature distribution raises upon increasing φ_1 and φ_2. Figure 2.7 describes the influence of Brownian motion parameter N_b on the temperature distribution. It is to be noticed that temperature profile grows as Nb increases. Figure 2.8 depicts that temperature distribution enhances upon changing the thermophoretic parameter N_t. However, temperature distribution decreases as Prandtl number Pr increases that is described in Figure 2.9. Figures $2.10-2.13$ depict the impacts of involved parameters on concentration distribution. Figure 2.10 shows that concentration profile reduces upon increasing Lewis number Le. The influence of Brownian motion parameter N_b on the concentration distribution is described in Figure 2.11 in which concentration profile grows as N_b increases. Increment in thermophoretic parameter N_t causes to decline in concentration profile as shown in Figure 2.12. Figure 2.13 depict that concentration distribution falls upon increasing the chemical reaction parameter K. The impact of various parameters on the Nusselt number, Sherwood number and coefficient of skin-friction are described in Tables $2.4-2.6$. Table 2.4 indicates that skin-friction coefficient enhances as the size of cylinder enhance whereas it decreases upon increasing the velocity ratio parameter ε. Table 2.5 describe that Nusselt number decreases by increasing the size of the slim cylinder c and velocity ratio parameter ε and in contrast, it increases the greater values of nanoparticles, concentration, Eckert number, Brownian motion parameter, and Thermophoretic parameter. Table 2.6 shows that Sherwood number enhances by increasing the size of the slim cylinder c, velocity ratio parameter ε ,Brownian parameter Nb and Thermophoretic parameter Nt.

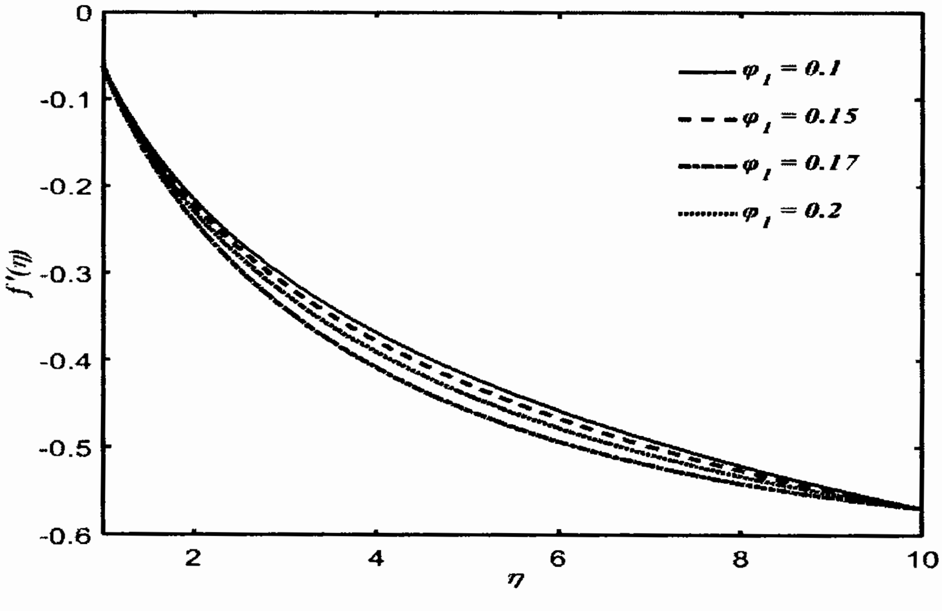

Figure 2. 2: Velocity profile for different values of CuO volume fraction.

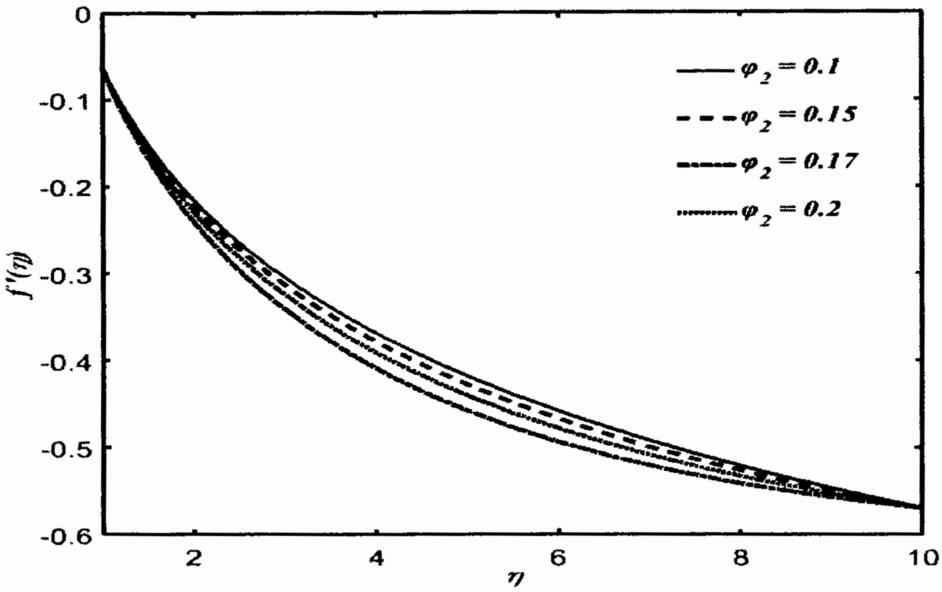

Figure2. 3: Velocity profile for different values of CuO volume fraction.

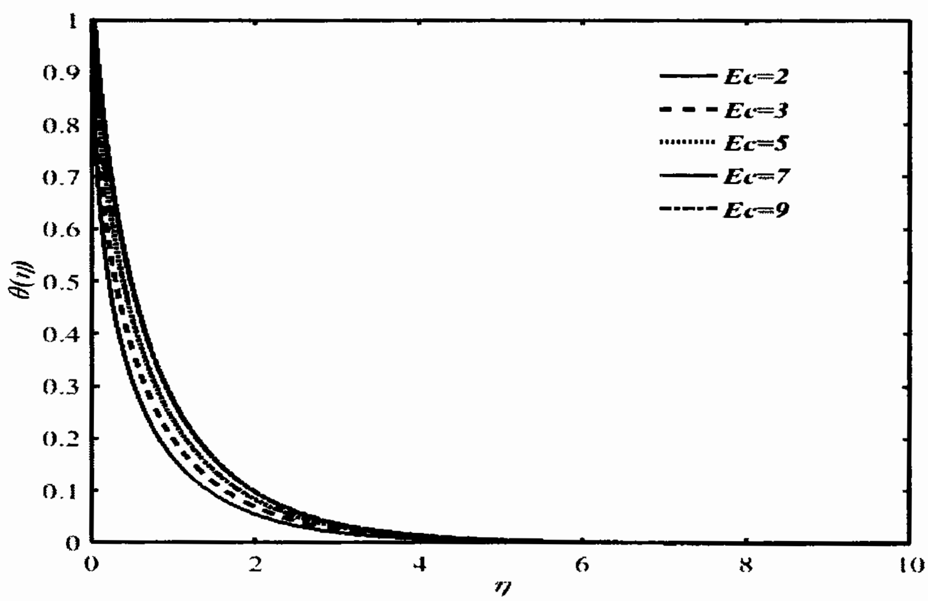

Figure 2.4: Thermal feature for different values of Eckert number.

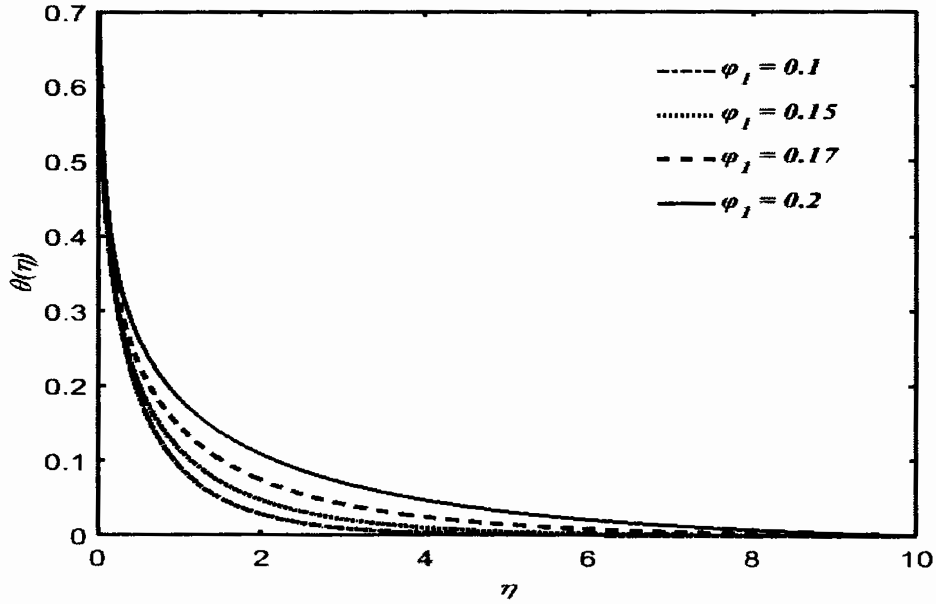

Figure 2. 5: Thermal features for different values of CuO volume fraction.

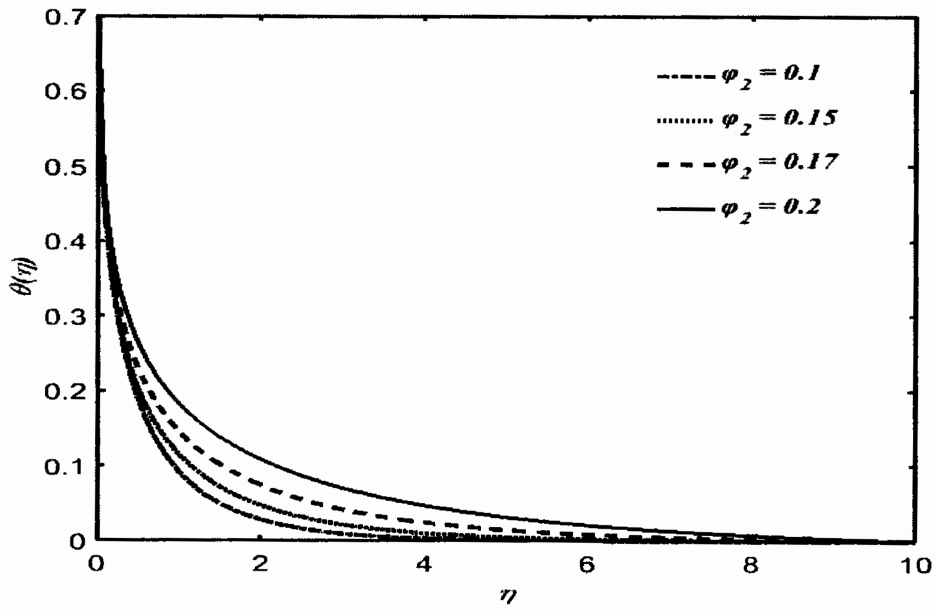

Figure 2. 6: Thermal characteristic for different number of Ag volume fraction.

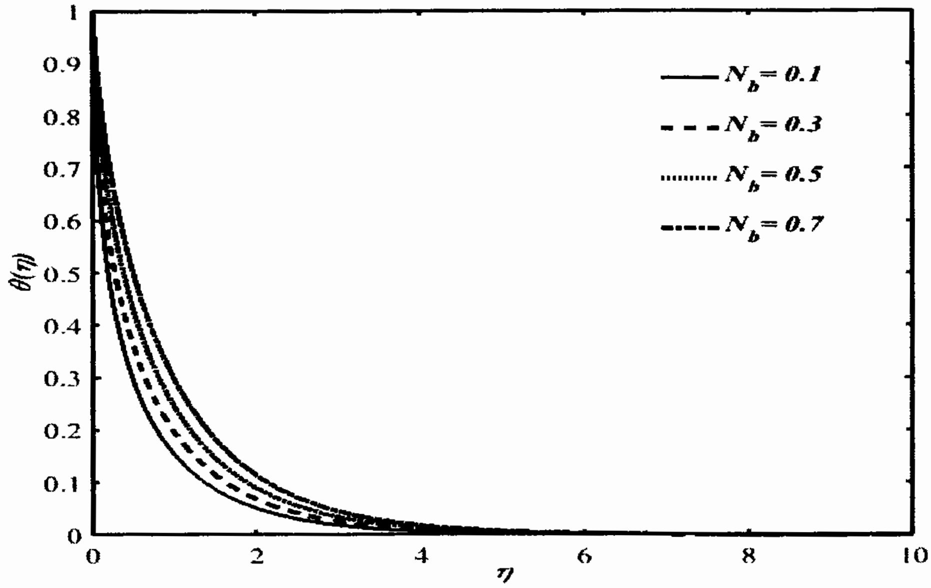

Figure 2. 7: Thermal characteristics for different values of Brownian motion parameter.

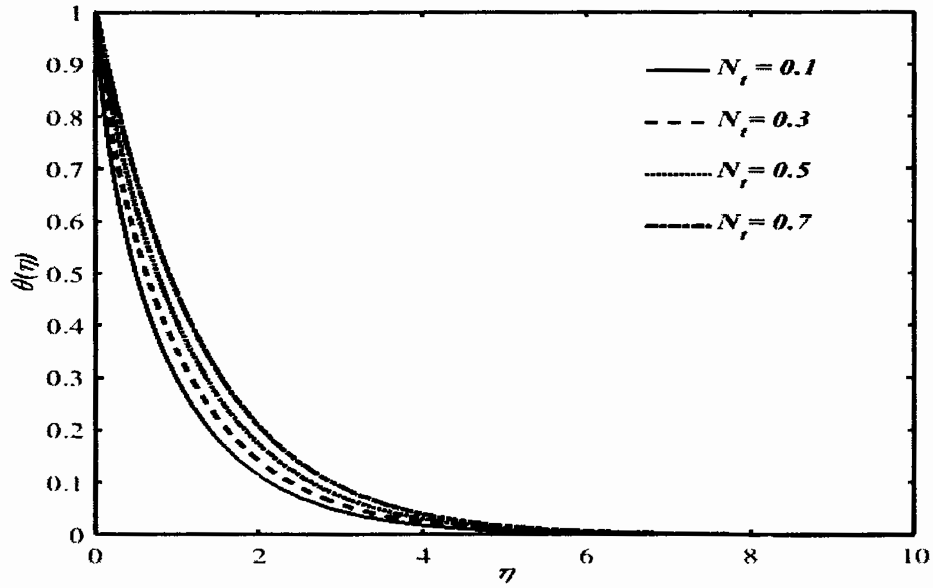

Figure 2. 8: Thermal features for different values of the thermophoresis parameter.

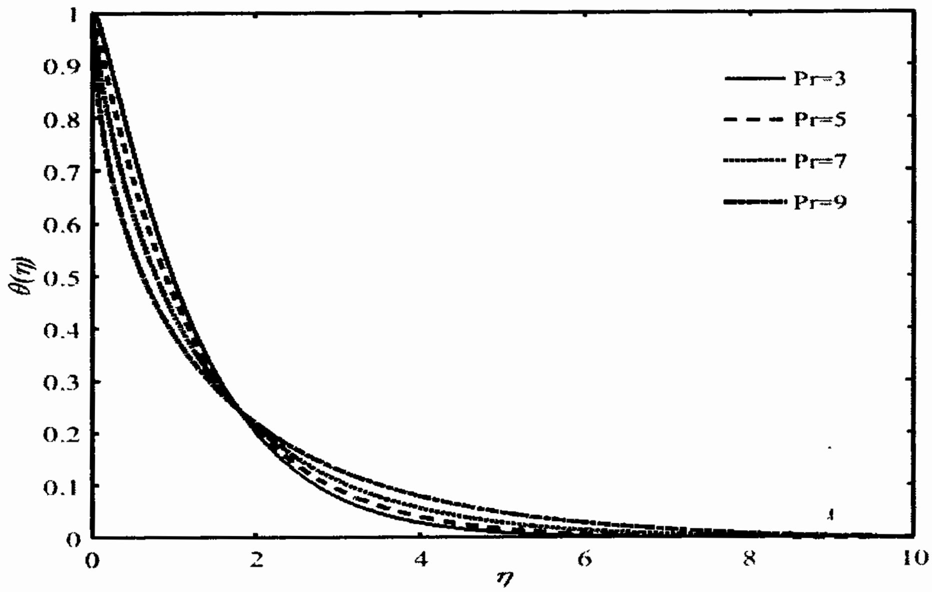

Figure 2. 9: Thermal features for different values of Prandtl number.

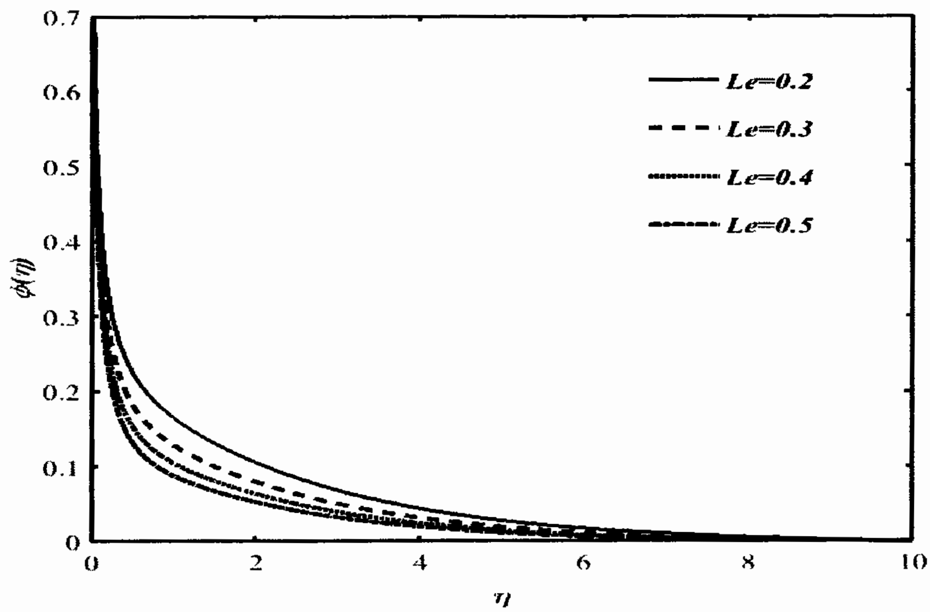

Figure 2. 10: Concentration features for various number of Lewis number.

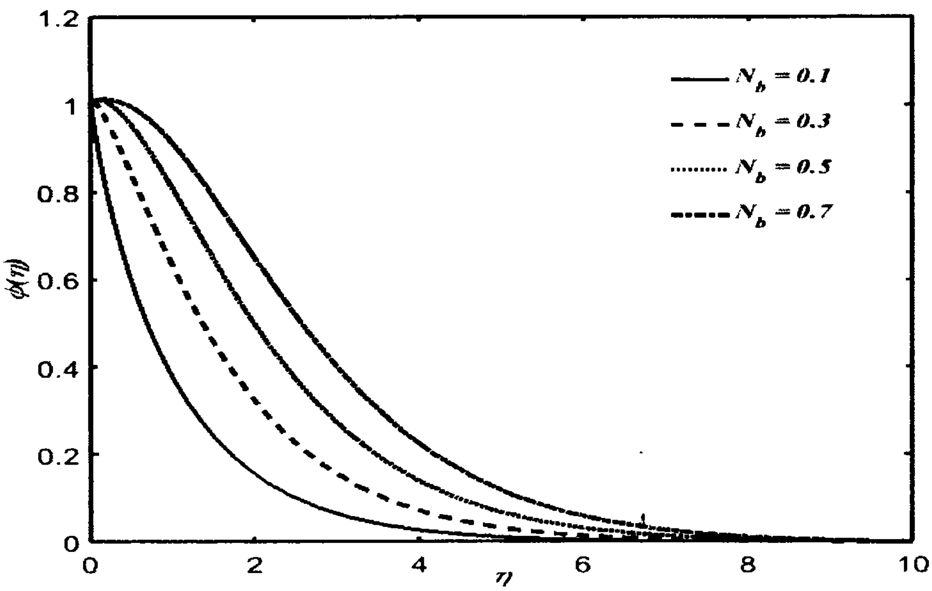

Figure 2.11: Concentration features for different values of Brownian motion parameter.

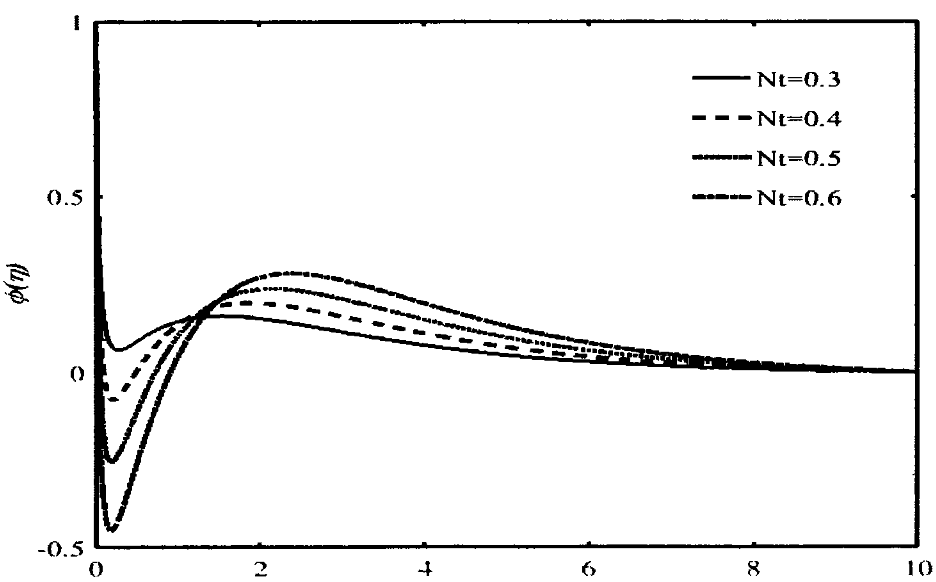

Figure 2.12: Concentration characteristics for different values of thermophoresis parameter.

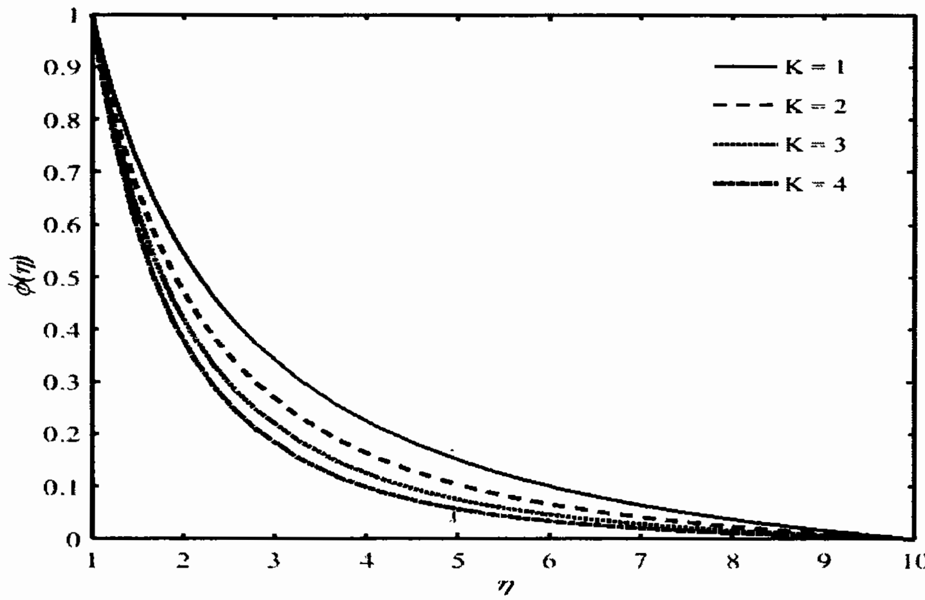

Figure2. 13: Concentration features for non-identical numbers of chemical reaction parameter.

Table 2.4: Impact upon skin friction $f''(c)$ when $Pr = 7.0$ and $\phi_1 = \phi_2 = 0.02$.

c	ε	$f''(0)$
0.002	0.1	0.1935
0.02	0.2	0.1936
0.1	0.2	0.1942
0.002	0.2	0.1471
0.002	0.3	0.0992

Table 2.5: Impact of different parameters on heat transfer coefficient $\theta'(c)$.

c	ε	$\phi_1 = \phi_2$	E_c	Nb	Nt	$\theta'(c)$
0.002	0:1	0.01	2	0.1	0.1	−0.3860

0.02	0.1	0.01	2	0.1	0.1	−0.3872
0.1	0.1	0.01	2	0.1	0.1	−0.3923
0.002	0.2	0.01	2	0.1	0.1	−0.5024
0.002	0.3	0.01	2	0.1	0.1	−0.5946
0.002	0.2	0.02	2	0.1	0.1	−0.5915
0.002	0.2	0.03	2	0.1	0.1	−0.5882
0.002	0.2	0.02	5	0.1	0.1	−0.5326
0.002	0.2	0.02	7	0.1	0.1	−0.4955
0.002	0.2	0.02	2	0.2	0.1	−0.4070
0.002	0.2	0.02	2	0.3	0.1	−0.3287
0.002	0.2	0.02	2	0.1	0.2	−0.2655
0.002	0.2	0.02	2	0.1	0.3	−0.2107

Table 2.6: Influence of various parameters on coefficient of Sherwood $\phi'(c)$ when $Pr = 7$.

c	ε	Nt	Nb	$\phi^{\wedge\prime}(c)$
0.002	0.1	0.1	0.2	0.0118
0.02	0.1	0.1	0.2	0.0113
0.2	0.1	0.1	0.2	0.0068
0.002	0.2	0.1	0.2	−0.2342
0.002	0.3	0.1	0.2	−0.4240
0.002	0.2	0.2	0.2	−0.3493
0.002	0.2	0.3	0.2	−0.2838

0.002	0.2	0.3	0.3	−0.2219
0.002	0.2	0.3	0.4	−0.1689

2.5 Conclusions

Investigation of hybrid nanofluid flow over a stretching cylinder in the vicinity of viscous dissipation has performed in this Chapter. Nanofluid consist of nanoparticles (Copper Oxide-CuO) and Silver (Ag) along with water as a pure fluid. After a detailed analysis, following observations are highlighted below.

- Velocity distribution reduces upon increasing the nanoparticle volume friction
- Temperature distribution enhances as nanoparticle volume friction, Eckert number, Brownian motion parameter, and thermophoretic parameter increases and in contrast, it reduces upon increasing Prandtl number
- Concentration distribution grows upon increasing Brownian motion parameter, whereas it decreases upon increasing Lewis number, thermophoretic parameter, and chemical reaction parameter

Chapter 3

Entropy generation of chemically reactive nanofluid flow past a slim moving cylinder with viscous dissipation

3.1 Introduction

This chapter consists of the investigation of entropy generation effects in a chemically reactive hybrid nanofluid flow over a stretched slim cylinder in the presence of viscous dissipation. Nanofluid contains Copper Oxide (CuO) and Silver (Ag) as nano particles along with water as a base fluid. The entropy generation number is obtained from entropy production dividing by characteristic entropy. Bejan number is formed with the ratio of entropy generation due to the heat and mass transfer to the total entropy generation. A group of similarity variables is used to transform the governing partial differential equations of the flow problem into a system of ordinary differential equations (ODEs). The transformed system is then solved numerically using bvp4c method in MATLAB. The impact of temperature parameter, concentration parameter, diffusion parameter and Eckert number on entropy generation number and Bejan number has been examined through graphs. Results reveal that the entropy generation number rises as increasing the temperature parameter (α_1), concentration parameter (α_2), diffusion parameter (L), and Eckert number (Ec). Moreover, Bejan number also increases on these parameters except the Eckert number (Ec).

3.2 Problem formulation

Consider the flow of a hybrid nanofluid (mixture of Copper Oxide and Silver with a pure fluid) over a horizontal stretched slim cylinder. The cylinder is assumed to be heated, placed in the axial direction of flow and moves with the wall velocity u_w. In contrast, fluid is also moving with free stream velocity u_∞. The velocity of fluid in entire flow system is $U = u_w + u_\infty$. Temperature and concentration at the cylinder surface are T_w and C_w which are constants whereas the corresponding values for the ambient fluid are T_∞ ($T_w > T_\infty$) and C_∞ ($C_w > C_\infty$), respectively.

Let u and v be the velocity components in the axial and radial directions. Keeping in view the above assumptions, the governing equations of the flow system in the presence of viscous dissipation are.

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0, \tag{3.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \frac{\mu_{hnf}}{\rho_{hnf}}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right), \tag{3.2}$$

$$\left(\rho C_{P\,hnf}\right)\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r}\right) = \tau\left(D_B\ \frac{\partial T}{\partial r}\frac{\partial C}{\partial x} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial r}\right)^2\right) + K_{hnf}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \mu_{hnf}\left(\frac{\partial u}{\partial r}\right)^2, \tag{3.3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial r} = \frac{D_T}{T_\infty}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{D_B}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right) - K^*(C - C_\infty)\ , \tag{3.4}$$

The relevant quantities in the aforementioned equations have the same meaning as usual and are described in Chapter 2.

Boundary condition for the above equations are

$$u(x,r) = U_w,\quad v(x,r) = 0,\ T(x,r) = T_w,\ C(x,r) = C_w \text{ at } r = R(x),$$

$$u(x,r) \to U_\infty,\ T(x,r) \to T_\infty,\ C(x,r) \to C_\infty \text{ at } r \to \infty\ , \tag{3.5}$$

Let us introduce the following similarity variables

$$\psi = \nu_f x f(\eta),\ \eta = \frac{Ur^2}{\nu_f x}\ ,\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}\ ,\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}\ , \tag{3.6}$$

where ψ is a stream function. The velocity components u and v can be expressed in terms of stream function as

$$u = \frac{1}{r}\frac{\partial \psi}{\partial r}, \quad v = -\frac{1}{r}\frac{\partial \psi}{\partial x} \tag{3.7}$$

In view of equation (3.7), the continuity equation (3.1) satisfies identically. Rest of the governing equations (3.2) – (3.4) are converted into set of ODEs after the utilization of equation (3.6) and (3.7), and are given below

$$\frac{2}{(1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}}(\eta f''' + f'') + \left\{(1-\varphi_2)\left((1-\varphi_1) + \varphi_1\frac{\rho_{s_1}}{\rho_f}\right) + \varphi_2\frac{\rho_{s_2}}{\rho_f}\right\} ff'' = 0 \ , \tag{3.8}$$

$$2\frac{K_{hnf}}{K_f}(\theta' + \eta\theta'') + \left\{(1-\varphi_2)\left((1-\varphi_1) + \varphi_1\frac{(\rho C_p)_{s_1}}{(\rho C_p)_f}\right) + \varphi_2\frac{(\rho C_p)_{s_2}}{(\rho C_p)_f}\right\} Prf\theta' +$$

$$2\eta \Pr(N_b\theta'\phi' + N_t(\theta')^2) + 8PrEc(\frac{\eta}{(1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}}(f'')^2 = 0 \ , \tag{3.9}$$

$$2(\phi' + \eta\phi'') + 2\frac{N_t}{N_b}(\theta' + \eta\theta'') + Le\phi' f - \frac{1}{2}LeK\phi = 0 \ , \tag{3.10}$$

where the subscript 1 denotes the quantities associated with CuO-nanoparticles whereas the subscript 2 denotes the quantities associated with Ag-nanoparticles. Rest of the involved parameters are described in previous chapter with their usual meanings.

Boundary conditions in the dimensionless form are given as

$$f(c) = \frac{\varepsilon}{2}c, \ f'(c) = \frac{\varepsilon}{2}, \ \theta(c) = 1, \ \phi(c) = 1 \ ,$$

$$f(\infty) = \frac{1}{2}(1-\varepsilon), \ \theta(\infty) = 0, \ \phi(\infty) = 0 \ . \tag{3.11}$$

3.3 Entropy generation

The volumetric entropy generation in dimensional form is given as

$$S_G = \frac{K_{hnf}}{T_\infty{}^2}\left(\frac{\partial T}{\partial r}\right)^2 + \frac{\mu_{hnf}}{T_\infty}\left(\frac{\partial u}{\partial r}\right)^2 + \frac{RD_B}{T_\infty}\left(\frac{\partial T}{\partial r}\frac{\partial C}{\partial r}\right) + \frac{RD_B}{T_\infty}\left(\frac{\partial C}{\partial r}\right)^2. \tag{3.12}$$

There are four entropy generating sources in Equation (3.12) as contributing. The first term on right hand side is local entropy generation due to heat transfer, the second term is the local entropy generation due to viscous dissipation and the last term represent the local entropy generation due to mass transfer. In addition R represents fluid resistance and D_B represents Brownian motion parameter. For entropy generation rate, it is appropriate to specify a dimensionless number Ns. This number is defined by dividing the local volumetric entropy generation rate S_G to a characteristic entropy generation rate S_{G_0}. For prescribed boundary condition the characteristic entropy generation rate is given by

$$S_{G_0} = \frac{4K_f U}{x\rho_f}. \tag{3.13}$$

Therefore, the entropy generation number becomes

$$Ns = \frac{S_G}{S_{G_0}}. \tag{3.14}$$

Using the expressions of dimensionless velocity, temperature and concentration, the entropy generation number can be transformed as

$$Ns = \alpha_1\left(\alpha_1 \eta \theta'^2 + \frac{4PrEc}{(1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}}\eta f''^2\right) + \alpha_1 L\, \eta\theta'\phi' + \alpha_2 L\eta\phi'^2 \ , \tag{3.15}$$

where Pr and Ec represent the Prandtl number and Eckert number, and α_1, α_2 and L represent the dimensionless temperature difference, the dimensionless concentration difference and diffusion parameters. These parameters are given by the following relationships,

$$Ec = \frac{U^2}{(T_w - T_\infty)C_p}\ , Pr = \frac{V_f(\rho C_p)_{hnf}}{k_f},\ \alpha_1 = \frac{T_w - T_\infty}{T_\infty}, \alpha_2 = \frac{C_w - C_\infty}{C_\infty}\ , L = RD_B\ \frac{C_w - C_\infty}{k_f}.$$

Now we define Bejan number to exhibit the supremacy between the contribution of entropy generation in view of heat transfer, concentration of mass and fluid friction as

$$\mathrm{Be} = \frac{Entropy\ due\ to\ heat\ and\ mass\ transfer}{Total\ Entropy\ genration},$$

$$Be = \frac{\alpha_1{}^2\eta\theta'^2+\alpha_1 L\,\eta\theta'\phi'+\alpha_2 L\eta\phi'^2}{\alpha_1\left(\alpha_1\eta\theta'^2+\frac{4PrEc}{(1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}}\eta f''^2\right)+\alpha_1 L\,\eta\theta'\phi'+\alpha_2 L\eta\phi'^2}. \tag{3.16}$$

3.4 Results and discussion

In the present work, chemically reactive nanofluid flow past a moving slim cylinder with viscous dissipation is considered to investigate the effects of entropy generation. Similarity transformation are used to reduce the governing equations (3.1 − 3.4) and then solved numerically using bvp4c method on MATLAB. The results are discussed through several graphs. A comparison of entropy generation due to heat transfer and entropy effects due to fluid friction and viscous dissipation is made with the help of Bejan number. The influence of involved quantities such as nanoparticles volume fraction $\varphi_1$and φ_2, temperature parameter α_1, concentration parameter α_2, Eckert number Ec and diffusion parameter L on entropy generation Ns and Bejan number Be is investigated in Figures 3.1 − 3.12. Figure 3.1 shows the impact of α_1 on entropy generation profile. It is noted that entropy generation profile grows as α_1 increases. The influence of α_2 on entropy generation is depicted by Figure 3.2 and it shows that entropy generation profile enhances upon increasing α_2 which due to the direct relation between the concentrations of the nanoparticles with heat transfer. Figure 3.3 described that entropy generation profile rises as Ec increases. This is because heat is a form of disorganized energy. Therefore, more heat transfer to the system is generated due to entropy. That is why kinetic energy of liquid particles converted low grade energy (heat energy) and consequently (*Ns*) enhances. Increment in diffusion parameter L also cases to enhance the entropy generation profile as shown in Figure 3.4. This is because for higher L more disturbance is created in the system and thus, entropy of the system is increased. Figures 3.5 and 3.6 show that entropy generation number increase upon increasing the nanoparticles volume friction φ_1 and φ_2. Figure.3.7 depicts the impact α_1 on the Bejan number Be. It can be observed from this figure that Bejan number profile raises as α_1 increases. Figure 3.8 described the influence of α_2 on Bejan number profile enhance it increases upon increasing α_2.for different values of α_2. We see that increment in α_2 results in increasing Be. Figure 3.9 depicts the influence of diffusion parameter L on Bejan number profile. It is to be noticed that Bejan number profile enhances upon increasing L. Figure 3.10 shows the impacts of Eckert number Ec on Bejan number profile. It can be observed from figure that Be number profile reduces as Eckert number increases. Figures 3.11 and 3.12 depict that Bejan number declines as the nanoparticles volume friction φ_1 and φ_2 grow.

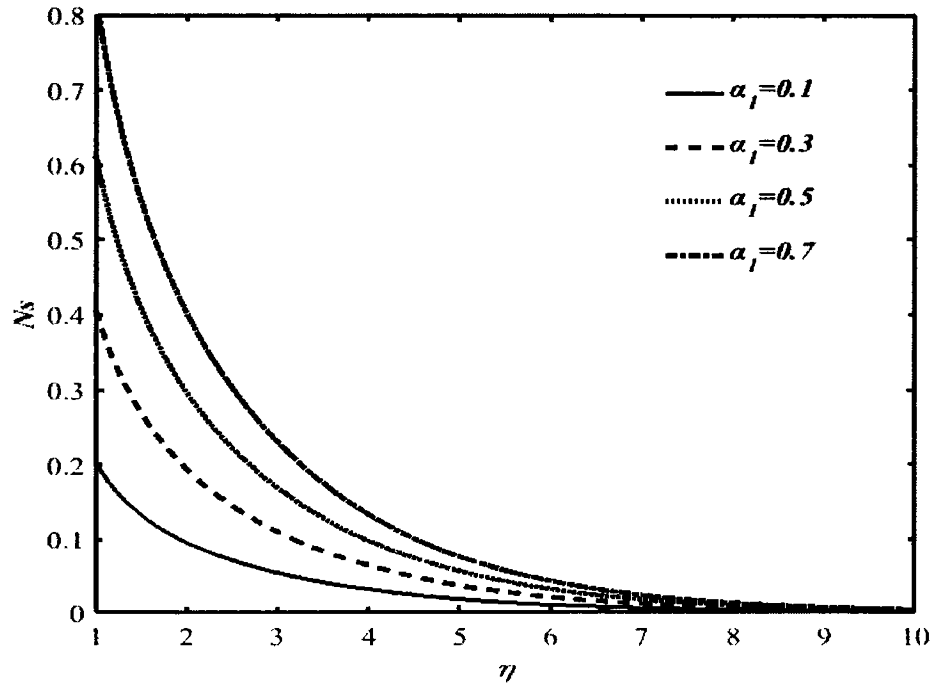

Figure 3.1: Effect of temperature parameter α_1 on entropy generation number Ns.

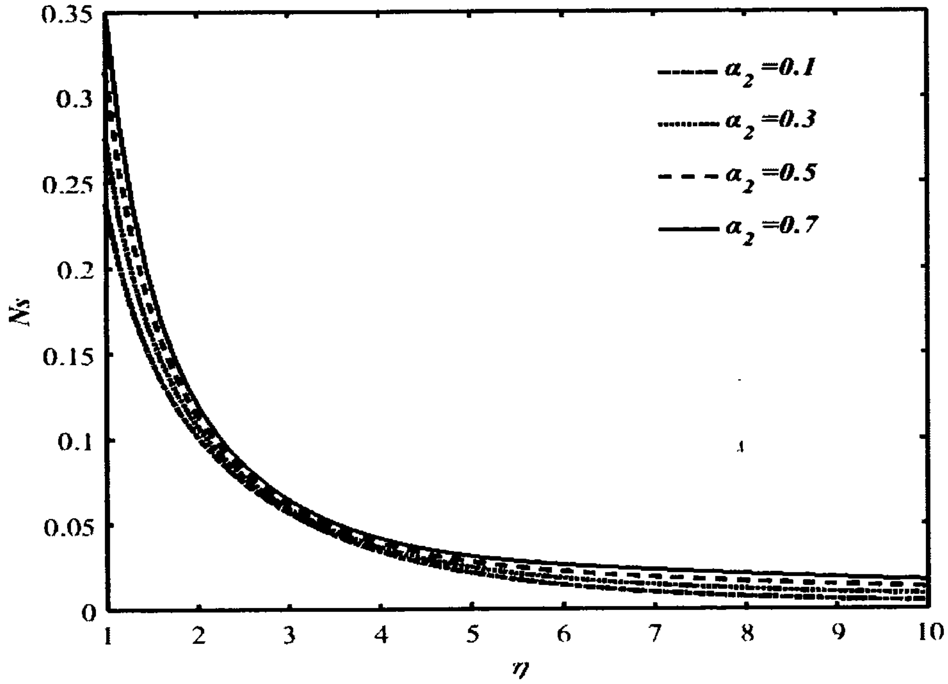

Figure 3.2: Effect of concentration parameter α_2 on entropy generation number Ns.

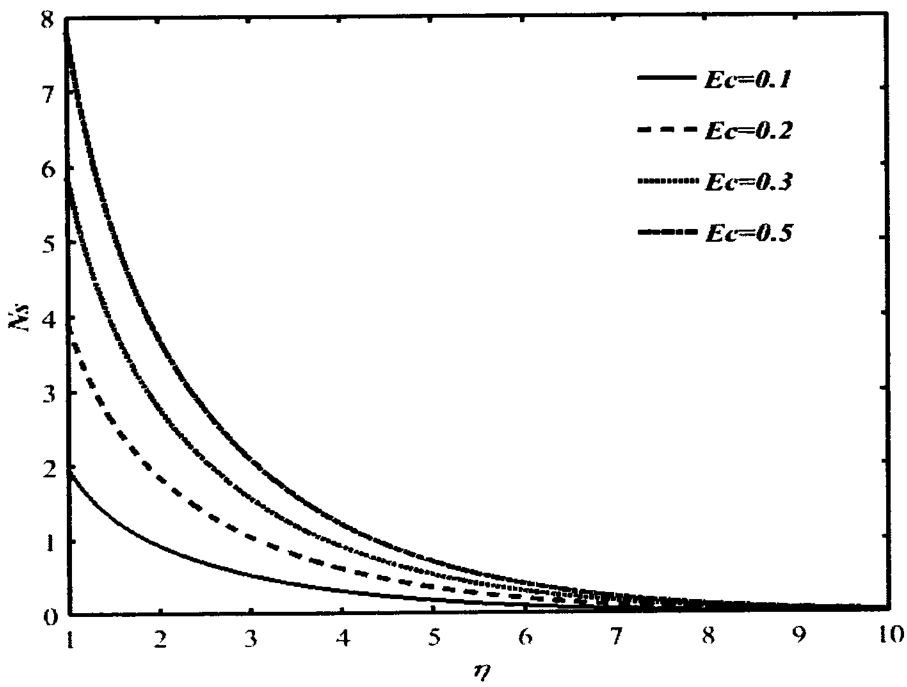

Figure 3.3: Effect of Eckert number Ec on entropy generation number Ns.

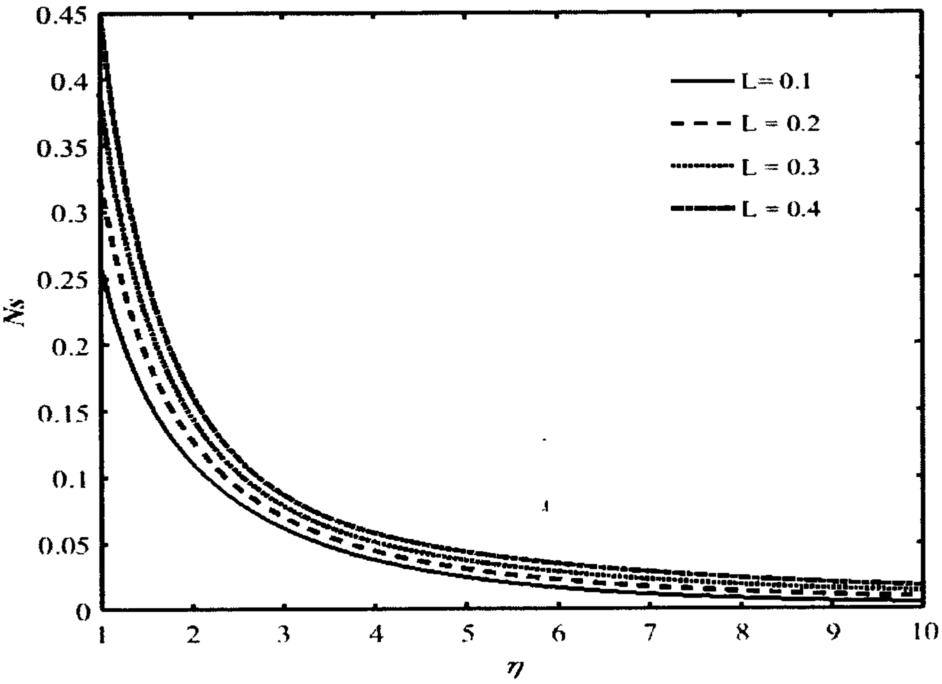

Figure 3.4: Effect of diffusion parameter L on entropy generation number Ns.

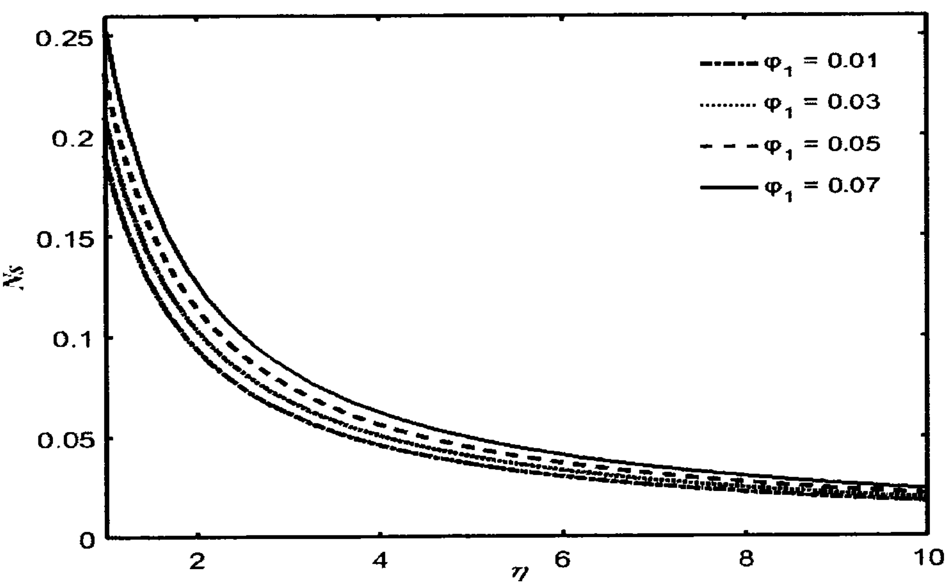

Figure.3.5: Effect of different values of φ_1 (CuO) on entropy generation Ns

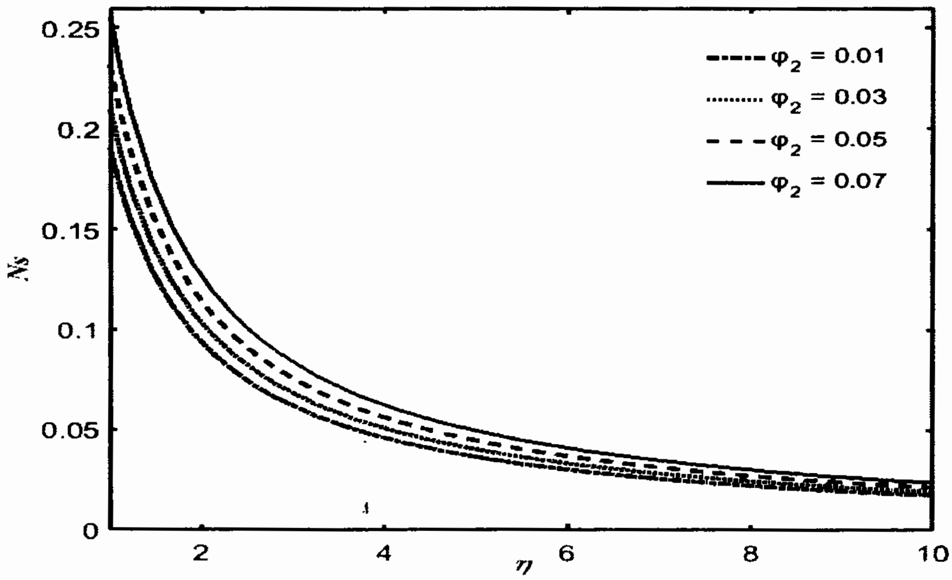

Figure 3.6: Effect of different values of φ_2 (Ag) on entropy generation Ns.

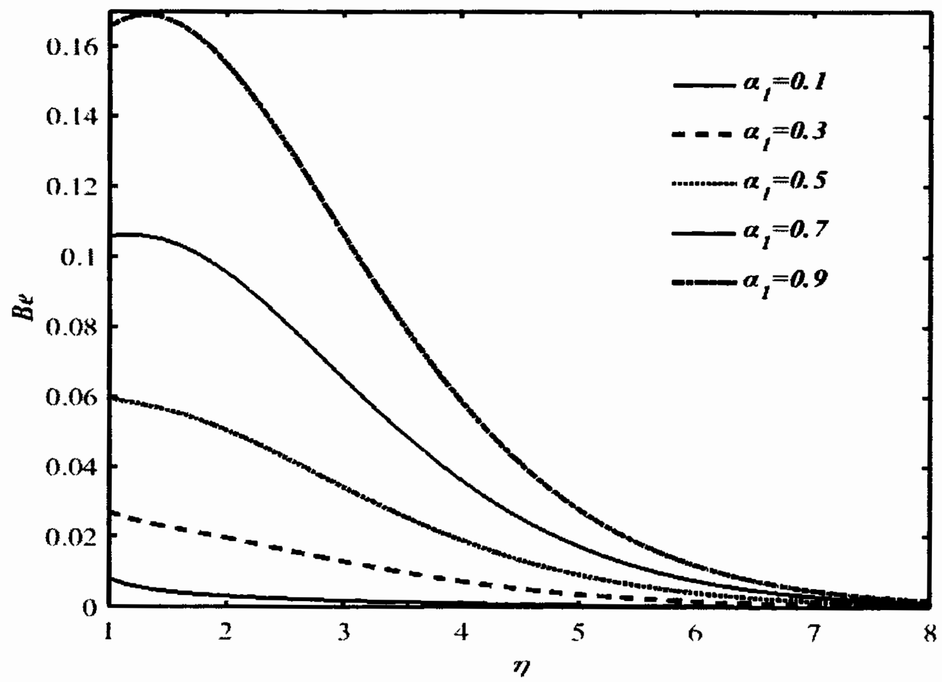

Figure 3.7: Effect of temperature parameter α_1 on Bejan number Be.

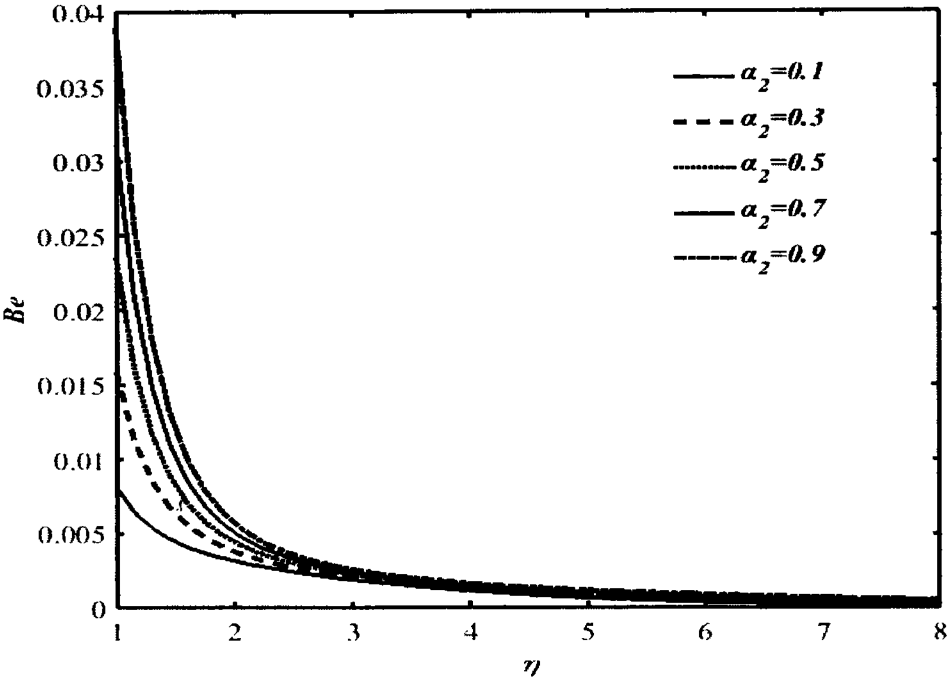

Figure 3.8: Effect of concentration parameter α_2 on Bejan number Be.

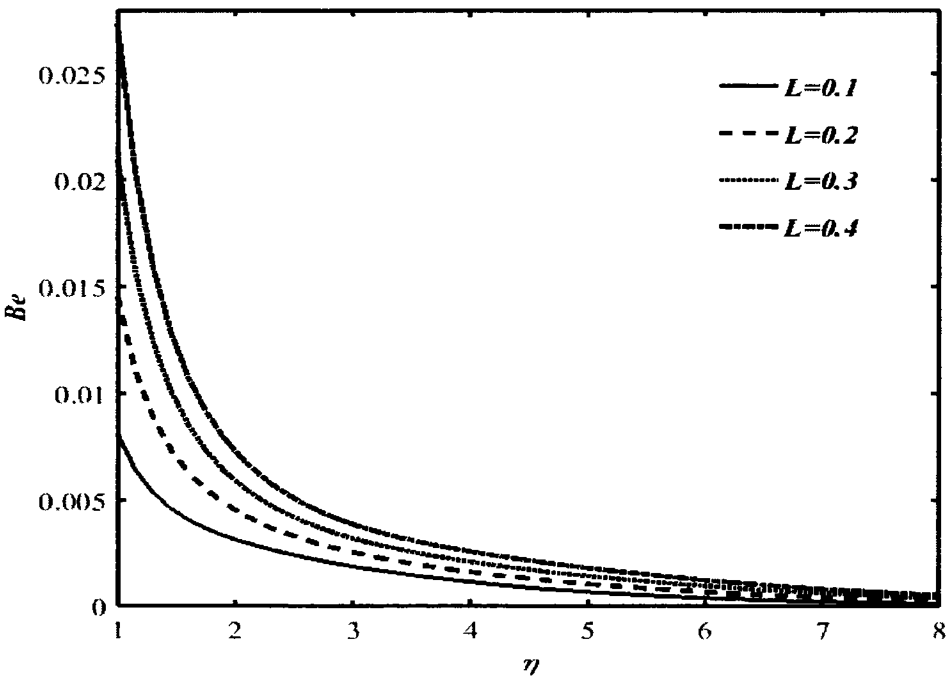

Figure 3.9: Effect of diffusion parameter L on Bejan number Be.

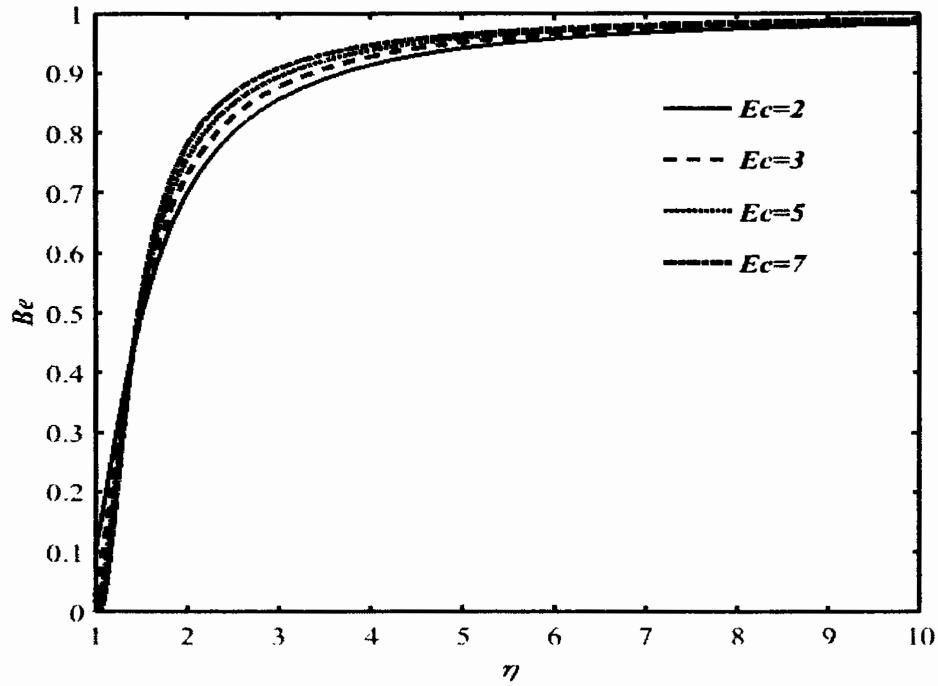

Figure 3.10: Effect of Eckert number Ec on Bejan number Be.

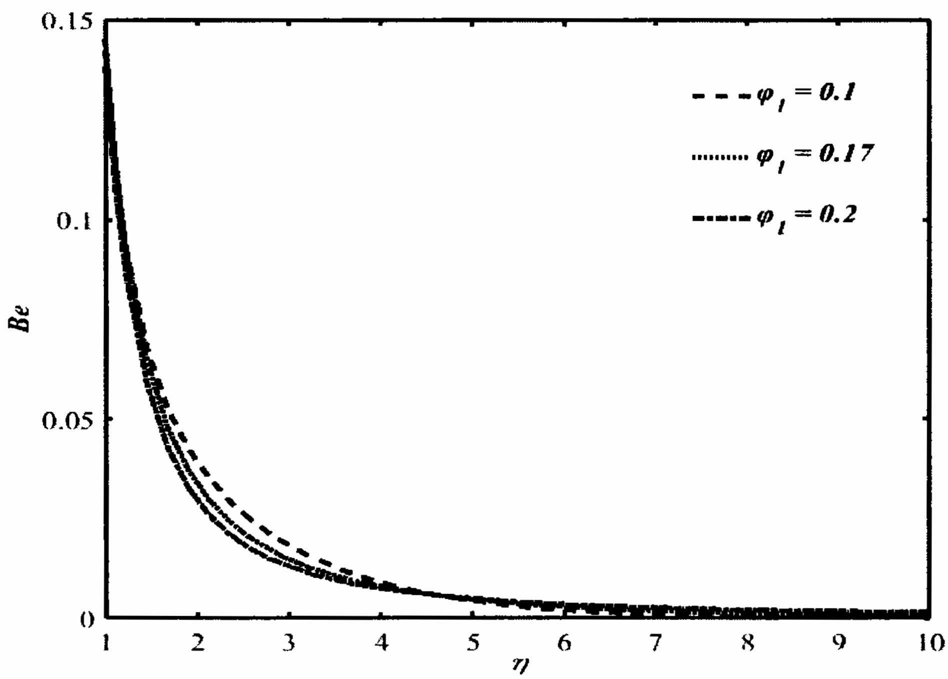

Figure. 3.11: Effect of different values of φ_1 (CuO) on Bejan number Be.

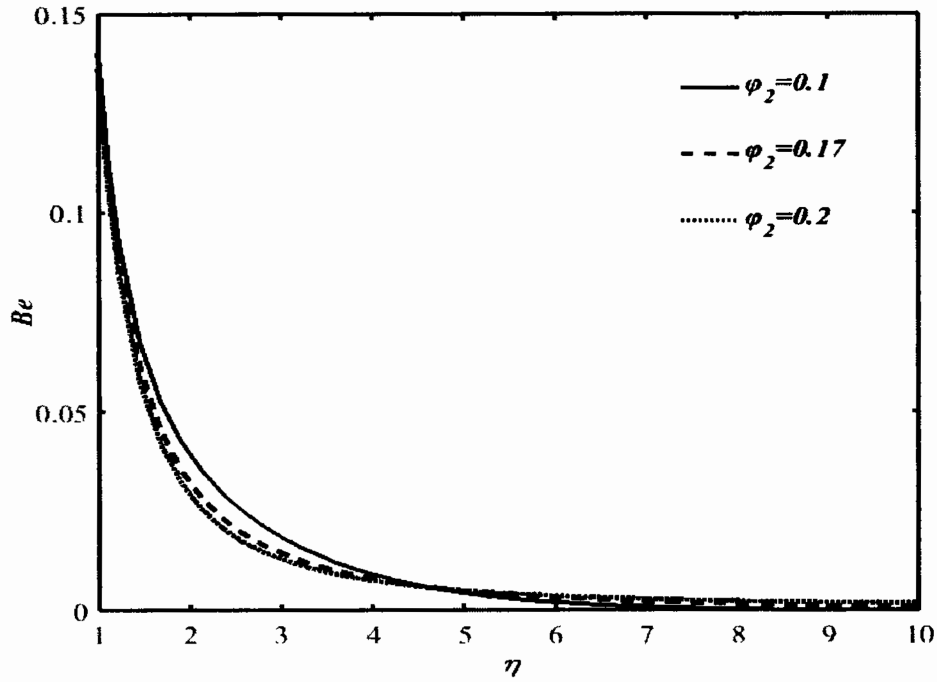

Figure 3.12: Effect of different values of φ_2 (Ag) on Bejan number Be.

3.5 Conclusion

In the present work chemically reactive nanofluid flow past a slim moving cylinder with viscous dissipation is considered to investigate the effects of entropy generation. The following conclusions are depicted from results:

- Entropy generation number Ns increases by increasing Eckert number Ec and diffusion parameter L
- Entropy generation number Ns increases due to the concentration of fluid α_2 and temperature parameter α_1
- Entropy generation number Ns rises upon rising the nanoparticles volume friction φ_1 and φ_2
- Bejan number Be increases due to the concentration of fluid α_2, temperature parameter α_1, and diffusion parameter L

- The Bejan number Be decreases by increasing the nanoparticle volume friction φ_1 and φ_2

References

[1] R. K. Tiwari and M. K. Das, Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids Int. J. Heat and Mass Transf 50, 2002-2018 (2007).

[2] J. Buongiorno, Convective transport in nanofluids ASME J. Heat Transf 128, 240–250 (2006).

[3] Choi SUS, (1995) Enhancing thermal conductivity of fluids with nanoparticles. ASME Int Mech Eng 66: 99–105.

[4] Hayat T. et al, (2016) investigated variable heat flux for nanofluid at stagnation point of horizontal stretching slim cylinder. Journal of Molecular Liquids, 224, 786–791.

[5] Waini et al, (2019) Hybrid nanofluid flow and heat transfer past a vertical thin needle with prescribed surface heat flux. International Journal of Numerical Methods for Heat & Fluid Flow 29 (12), 4875–4894.

[6] Waini et al, (2020) have examined transfer of heat over a porous needle using Brownian motion parameter and thermophoretic effect upon hybrid nanofluid.

[7] Narain JP, Uberoi MS (1972) Free-convection heat transfer from a thin vertical needle, Phys. Fluids 15, 928–929.

[8] Khan et al, (2016) has investigated the boundary layer flow around a needle that is positioned horizontally while taking into account incompressible viscous fluid by employing famous Buongiorno model.

[9] Clausius. R, (1872) the mechanical theory of heat - with its application to the steam engine and to physical properties of body, John Van Voorst, London,

[10] Carrington .C.G, Sun. Z.F, (1991) Second law analysis of combined heat and mass transfer phenomena, Int. J. Hear Mass Transf., 34 (11) 2767-2113.

[11] Carrington. C.G, Sun Z.F. (1992) , Second law analysis of combined heat and mass transfer in internal and external flows, Int. J. Heat Fluid Flow, 13 (1) 65-70.

[12] Boltzmann's, et al (2004) Work in Statistical Physics, Stanford Encyclopedia of Philosophy,

[13] Gibbs J. W, (1960) Elementary Principles in Statistical Mechanics, Dover Publication, New York.

[14] Bejan A, (1979) A study of entropy generation in fundamental convective heat transfer, J. Heat Transf.,101 718-725.

[15] Bejan A, (1982) Entropy generation through heat and fluid flow, John Wiley and Sons, New York,

[16] Prandtl L, (Heidelberg, 1904). Uber Flussigkeits bewegung bei sehr kleiner Reibung, in Proceedings of 3rd international Mathematical Congress.

[17] Blasius, (1908) was the first who analyzed a boundary layer flow on a thin sheet at rest.

[18] Sakiadis B.C, (1961) Boundary layer behavior on continuous solid surface: I. Boundary-layer equation for two-dimensional and axis-symmetric flow. AIChE 7(1) 26–28.

[19] Sakiadis B.C, (1961) Boundary layer behavior on continuous solid surface: II. The Boundary layer on a continuous flat surface. AIChE 7(2) 221–225.

[20] Xuan Y, Roetzel W. Conceptions for heat transfer correlation of nanofluid. Int J Heat Mass Transf. 2000; 43(19):3701-7.

[21] Lin H.T, Shah Y.P, Laminar Boundary Layer Heat Transfer along Static and Moving cylinder, J. Chinese Institute of Engineers, V 3, No. 1 (1980) 73-79.

[22] H.T. Lin, Y.P. Shah, examined Heat transfer and boundary Layer flow on stretching Cylinders, J. Chinese Institute of Engineers, V 4, No. 1 (1981) 47-57.

[23] Maskeen MM, Zeeshan A, Mehmood OU, Hassan M. Heat transfer enhancement in hydromagnetic aluminacopper/water hybrid nanofluid flow over a stretching cylinder. J Therm Anal Calorim. 2019; 138(2):112736.

[24] San JY, Laven Z. Entropy generation in convective heat transfer and isothermal mass transfer. J Heat Transfer 1987; 109:647–52.