Welfare Cost of Inflation in Pakistan

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Certificate

It is to certify that this thesis submitted by **Ms. Siffat Mushtaq,** is accepted in its present form by the Department of Economics (Women Campus), International Islamic University, Islamabad in satisfying the requirement for partial fulfillment for the Degree of Master in Philosophy in Economics.

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Abstract

This dissertation studies the welfare cost of inflation by allowing money to enter into aggregate utility function as an asset that provides liquidity services. The Money-in-Utility (MIU) model provides an estimable form of money demand function and a convenient framework for calculating the welfare cost of inflation. Welfare cost of inflation estimates are sensitive to the money demand specification, monetary aggregates, approaches to calculate the welfare cost and the benchmark inflation rate. The empirical analysis employs the autoregressive distributed lag model (ARDL) technique to estimate the coefficients of double-log (derived from MIU model) and semi-log money demand specifications. Inflation distorts the liquidity services of money by affecting the nominal interest rate. Welfare cost is measured at different nominal interest rates with benchmark taken as both zero inflation and the Friedman optimal deflation rate.

The welfare cost estimates for Pakistan's economy from Bailey's (1956) consumer's surplus and Lucas' (1994) compensating variation approaches imply that welfare gain of moving from positive to zero inflation is approximately same under both money demand specifications. However, moving further from zero inflation to deflation policy results in substantial gain under double-log model compared to trivial gain under constant semi-elasticity Cagan-type model. Welfare calculations are robust for the double-log specification in terms of similar costs across two approaches to compute welfare loss. Compensating variation approach and quadratic formula for the semi-log model gives higher welfare loss figures compared to Bailey's approach.

The welfare cost of inflation is sizable for Pakistan compared to the developed countries. Current sub-optimal policy of 12.5 percent nominal interest rate (10.5 percent inflation

rate) under double log specification and single monetary asset will result in a welfare loss of 0.63 percent of GDP (0.51 percent for the currency-deposit model) against zero inflation policy, while against deflation policy the cost is 0.87 percent of income (0.65 percent for currency-deposit model). The exact loss of deviating from price stability at current GDP of Rs. 5796.4 billion is Rs. 36.5 million (29.6 million for currency-deposit model). The study examines the welfare cost of inflation by distortions brought about by positive nominal interest rate on money demand. This direct cost of inflation gives lower bound of the actual cost of inflation, not addressing other channels through which inflation results in inefficient allocation of resources, by altering work-leisure choice and interacting with the tax structure of the economy.

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CHAPTER

1

INTRODUCTION

Inflation, generally defined as sustained increase in the general price level is viewed as having widespread implications for an economy on different accounts. It creates many economic distortions and stifles government's efforts to achieve macroeconomic objectives. More often than not price stability is considered a necessary condition for lessening income fidgets and disparities. In the long-run, price stability is not only considered a necessary but also a sufficient condition for sustained economic growth. Empirically it is found that growth declines sharply during a high inflation period, (Bruno & Easterly, 1996). High inflation creates uncertainty, grossly distorts investment plans and priorities, reduces the real return on financial assets and hence discourages savings, adversely affects economic efficiency and growth by distorting market signals. All these costs are associated with the unanticipated inflation and have received considerable attention in literature. Most of these costs involve transfer of resources from one group to another and the losses and gains tend to offset each other. It is widely agreed that most of the costs associated with unexpected inflation can be avoided if the inflation is correctly anticipated. But inflation even when fully anticipated results in a loss to the society via the net loss to the valuable services of real money balances.

Inflation is one of the major macroeconomic problems for Pakistan economy in the recent years. During the second half of 2000s inflation increased from previous levels of less than 5% annually to double digits by the end of the decade. The first significant acceleration occurred in 2005 when inflation rose to 9.3 percent, following excessive money flows towards the public and private sectors along with food inflation. To put a bar on the surge in demand and pressure on prices the State Bank of Pakistan (SBP) took action by using tight monetary policy. Consequently inflation was somewhat reduced to 7.8 percent for the next two years. In 2008 inflation again reached double digits figure of 12%,. This time the tight monetary policy (in the form of raising discount rate and cash requirement on demand and time deposits) was ineffective in reducing inflation due to excessive government borrowings from the SBP.

Governments of most of the developing countries are too weak to enact adequate tax programs and to administer them. Issuing money is an easy method of raising revenue to cover public expenditures or to fill the deficit. In Pakistan too, monetary policy does not seems to be formulated to achieve price stability; rather expansion in high powered money and bank credit is determined by the government's borrowing needs for budgetary support. This mode of financing the deficit is appealing and painless for the government. However, for the economy this method of financing has always been disastrous as resources move from the private sector to the public sector without imposing explicit tax. Sustained deficits not only retain the ongoing inflation rate but also cause it to accelerate. A monetary regime with such a built-in inflationary bias sends signals of high inflation in future.

Under inflationary environment people try to correctly anticipate inflation and accordingly adjust the ratio of real balances to income to the opportunity cost of holding money. Inflation resulting from this process imposes a tax on cash balances and a loss in terms of non-optimal holding of money. There is no close substitute for real balances and an unavoidable cost of holding money is its opportunity cost- the nominal interest rate.

Nominal interest rates reflect the expected inflation and according to Fisher hypothesis the cost of holding real balances increases with an increase in anticipated inflation. Beginning with Bailey (1956) welfare cost of inflationary finance is treated as the deadweight loss of inflation tax and is calculated by integrating the area under the money demand curve commonly known as Harberger Triangle. Traditional analysis of welfare cost of inflation has emphasized that these costs depend on the form of money demand function (Bailey 1956). Models based on Cagan-type semi-log and double-log money demand functions have been employed in estimating the welfare cost of inflation. However, different money demand specifications give very different estimates of welfare cost. This difference is mainly due to the behavior of the two demand curves towards low inflation (Lucas 2000).

Empirical literature on the welfare cost of inflation shows that money stock should be defined in the narrowest form representing the true liquidity services provided to the society. Money stock is taken in its narrow form as monetary base and M1. In some of the cases, M1 tends to overstate the welfare cost because when treated as a single aggregate, welfare integral runs from zero to the positive nominal interest rate. To accommodate for the interest bearing demand deposits component of M1, recent studies calculate welfare cost in currency-deposit framework[1].

Traditional studies on hyperinflation countries estimated the welfare cost of inflation against Friedman's deflation rate; as under hyperinflation real interest rate is zero and deflation rule implies zero inflation. But with the application of this analysis in the developed countries with stable prices and positive real interest rate, welfare cost of

[1] Distinct role of currency and deposits is emphasized in Marty (1999), Bali (2000), and Simonsen & Rubens (2001).

positive inflation is evaluated against both zero inflation and deflation policies[2]. All these issues regarding formulation of monetary model, definition of monetary aggregates and optimal inflation and interest rate policies are equally important areas of inquiry.

1.1 Objectives of the Study

Most of the empirical studies on inflation in Pakistan have primarily focussed on the determinants of inflation, and all of the studies reached the conclusion that the monetary factors have played dominant role in recent inflation [see for example, Qayyum 2006, Khan and Schimmelpfennin (2006), Kemal (2006), Khan et. al. (2007)]. Moreover, some of the studies have emphasized the role of State Bank of Pakistan to implement independent monetary policy with the objective of attaining price stability.[3] We do not find even a single study assessing the cost borne by the society due to positive inflation in Pakistan.

In this background, the present study attempts to comprehensively investigate the welfare cost of inflation in Pakistan so as to bridge the gap in empirical literature. We have used time-series data over the period 1960 to 2007 for monetary aggregates (monetary base, M1, currency and demand deposits), Gross Domestic Product (GDP) and nominal interest rates to estimate both semi-log and double log money demand functions. Moreover, the present study differs from the existing literature on money demand function with regard

[2] Using Fisher equation $i = r + \pi^e$ we can write the two benchmark inflation rates as

Zero-inflation/ Price Stability: $\pi^* = 0$ or equivalent to the condition that $i = r$

Friedman's optimal deflation rule: $\pi^* = -r$ implying $i = 0$ under hyperinflation with $r = 0$, zero-inflation is equivalent to Friedman's deflation rule.

[3] See for example Hussain (2005), Mubarik (2005), and Khan and Schimmelpfennig (2006) giving some threshold levels of inflation.

to estimation techniques.[4] We have employed different and novel estimation technique of Autoregressive Distributed Lag Model (ARDL) developed by Pesaran et. al. (2001). The ARDL modeling approach does not require any precise identification of the order of integration of the underlying data. In addition to that, this technique is applicable even if the explanatory variables are endogenous.

After estimating the parameters of the long run demand functions for narrow money we have quantitatively assessed the welfare losses associated with different rates of inflation. The policy issue of reducing inflation to zero and further reducing it to Friedman's deflation rule is addressed by computing and comparing welfare loss across different money demand specifications and monetary aggregates.

1.2 Plan of the Study

Rest of the research is organized as follows: Chapter 2 provides a brief review of existing literature on the welfare cost of inflation and followed by review of empirical work on money demand function in Pakistan. Chapter 3 explains the theoretical model specifications, the estimation technique and describes the data of variables used in empirical models. Chapter 4 reports the estimation results and the welfare cost calculations based on the estimated models. Chapter 5 contains the conclusions.

[4] See section 2.2 of Pakistan Specific Literature on Money Demand

CHAPTER

2

REVIEW OF LITERATURE

The objective of this chapter is to identify key issues involved in the computation of welfare cost and review the empirical literature on the welfare cost of inflation. Welfare cost analysis depends on the behavior of money demand function, in the second section we will provide exposition of the literature on money demand function in Pakistan.

2.1 Welfare Cost of Inflation

The issue of welfare cost of inflation is addressed under both partial equilibrium (traditional) and general equilibrium (neo-classical) frameworks. Bailey (1956) is the first to study the welfare implications of public sector inflationary finance; his analysis showed that open (anticipated) inflation costs members of the society more than the revenue which accrues to the government. The dead weight loss associated with this implicit tax is the difference between the cost to money holders and the transfer to the government. Inflation acts like an excise tax on money holding and the dead weight loss of anticipated (open) inflation is the welfare cost of inflation.

By its nature excise tax is levied on a specific commodity, where there is possibility to avoid this tax by switching over to the substitutes. In inflationary environment people economize on their holding of money, and welfare cost is the lost value of services of real money balances to all the households during the inflationary period. Marginal productivity of money at any nominal interest rate is shown by a point on the liquidity demand curve; welfare cost is measured as loss in consumer surplus or the loss in the valuable services of money. In welfare cost literature this framework is termed as partial equilibrium model and welfare cost is measured by integrating the area under the demand curve.

In addition to partial equilibrium model neoclassical general equilibrium models have been applied to quantify the costs of inflation. Neoclassical non-monetary models have been extended in three ways to allow for a role of money Walsh (2003): (i) Money-in-the-Utility Function model (MIU), money directly yields utility and is treated like a consumer good (Sidrauski 1967), (ii) Cash-in-Advance model (CIA), some transactions require cash and transactions or illiquidity costs create demand for money (Clower 1967; Kiyotaki and Wright 1989) and (iii) Overlapping Generation model where money is used for the intertemporal transfer of wealth (Samuelson 1958).

Welfare cost of inflation in its magnitude depends on the bench-mark inflation rate, that what should be the desirable or optimal rate of inflation. Optimal inflation rate in some of the studies is taken as zero-inflation or price stability and in others as the Friedman's deflation rate. Bailey (1956) measuring welfare cost of inflation for hyperinflation countries used zero-inflation rate as the benchmark, which was also equivalent to Friedman's deflation rule because in hyperinflation the real rate of interest is zero. But later studies showed that the welfare loss function is minimum when Friedman's optimal deflation rule is applied [Friedman (1969), Barro (1972), Lucas (2000)]. Friedman's deflation rule is based on Pareto optimality condition where socially efficient level of production of a commodity is the one where marginal cost is equal to marginal benefit (later being the price of the commodity). Marginal cost of producing money is nearly zero for the monetary authority but the social cost is the nominal interest rate, the opportunity cost of holding cash. To minimize the cost of holding money the nominal interest rate should be brought to zero which requires deflation equal to the real interest rate using Fisher equation $i = r + \pi^e$ we can write the two benchmark inflation rates as

Zero-inflation/ Price Stability: $\pi^* = 0$ or equivalent to the condition that $i = r$

Friedman's optimal deflation rule: $\pi^* = -r$ implying $i = 0$

The optimal inflation rate and gain of reducing inflation from positive inflation to price stability and further to the deflation rate depends on the shape of money demand function (Wolman 1997). Semi-log and double-log money demand functions have been estimated and used for the welfare cost analysis in most of the studies. The two specifications give different welfare cost estimates and the major difference is due to the behaviour of the two curves towards optimal deflation rate. The semi-log money demand function shows the presence of finite satiation level of real balances at zero nominal interest rate while for the double log model the real balances become arbitrarily large [Ireland (2007)]. For semilog money demand function larger gain is attained by moving from positive inflation rate to zero inflation policy whereas for the double-log bulk of gain comes from moving from zero inflation to deflation rate.

Bailey (1956) started the quantitative analysis of welfare cost of anticipated inflation. In this pioneering work inflation treated as excise tax on money holding and welfare cost is measured as the dead weight loss of this tax. Using Harberger welfare triangle, cost of inflation is measured as loss in consumer surplus moving from zero to positive inflation rate. To quantify this loss Cagan-type semi-log money demand function is estimated for seven hyperinflation countries, Austria, Germany, Greece, Hungary I(first Hungarian inflation, after first World War), Hungary II (after Second World War), Poland, and

Russia. Bailey has given theoretical explanation for this cost by identifying the fact that as people start expecting inflation they try to economize on their cash holdings thus raising the velocity of money and indulging in non-productive activities. Empirical estimates show that velocity did increased during hyperinflation episodes and that welfare cost was comparatively high in Hungary I (0.159) and Germany (0.0704) compared to other economies that experienced lower inflation.

Welfare cost measured by integrating the area under the money demand schedule from the stock of real money balances at zero inflation to that held at positive expected inflation rate is referred to as partial equilibrium measure of welfare cost in literature. This area represents the loss of productivity and convenience of forgone real balances. The partial equilibrium traditional model does not take into account the fact that the receipts from inflation tax can be used for the production of government capital and can contribute to economic growth. This aspect of inflationary finance was developed by Mundell (1965) and was later extended in Marty (1967) in welfare cost of inflation context. Marty (1967) using Cagan's and Mundell's money demand specifications for Hungary reaches the implication that the traditional measure of welfare is close to the measure of welfare cost in the model where inflation induces growth. Welfare cost of 10 percent inflation is 0.1 percent of income and 15.84 percent of government budget. Welfare cost estimates of Bailey (1956) and Marty (1967) are based on the average cost of revenue collection through money creation but Tower (1971) measures it as a marginal cost. For a hypothetical economy "Sylvania" the average and marginal costs are compared. The rate of inflation at which the average cost of inflationary finance is 7 percent corresponds to marginal cost of 15 percent.

Anticipated inflation raises the transaction costs as the individuals raise the frequency of transactions and results in increased velocity of money. This cost has been identified in Bailey (1956) but another cost of inflation arises when individuals, facing high inflation, employ alternative payments media with higher transaction costs. Barro (1972) is the first to identify the role of substitute transaction media. Using partial equilibrium model for Hungary the welfare costs of high, hyperinflation and unstable hyperinflation are calibrated. Welfare cost of 2-5 percent monthly inflation rate is between 3-7 percent. Welfare cost increases sharply for inflation rate above 5 percent per month. For hyperinflation where inflation is 25-50 percent per month, estimated cost is between 11-22 percent of income and for unstable hyperinflation where inflation is 100-150 percent per month, welfare cost of inflation is between 22-33 percent of income.

Fisher (1981) studied the distortionary costs of moderate inflation and applied the partial equilibrium analysis to the U.S. economy. Welfare loss is measured by consumer surplus measure that incorporates the production and taxation through portfolio choice decision. Using high-powered money as the monetary asset the welfare loss of 10 percent inflation is estimated to be about 0.3 percent of GNP. Using Bailey's (1956) consumer surplus formula Lucas (1981) calculated welfare cost of inflation for the U.S., defining money as M1. The welfare gain is estimated to be 0.45 percent of GNP as the economy move from 10 percent inflation to zero inflation.

Cooley and Hansen (1989) estimated the costs of anticipated inflation in real business model where money demand arises from cash-in-advance (CIA) constraint. In this model anticipated inflation operates as inflation tax on activities involving cash (consumption) and individuals tend to substitute the non-cash activities (leisure) for the cash activities. Welfare cost is measured as reduction in consumption as a percentage to GNP. Using quarterly data of U.S. from 1955:3 to 1984:1 for macroeconomic aggregates and using parameters of microeconomic data studies the model is calibrated. The simulation results show that the estimates of welfare loss are sensitive to the definition of money balances and to the length of time households are constrained to hold cash. For moderate annual inflation rate of 10 percent welfare loss is 0.387 percent of GNP where money is taken as M1 and the individual holds cash for one quarter. But this cost is substantially reduced to 0.1 percent for the monetary base and further the individual is constrained to hold cash for one month.

Extending Cooley and Hansen (1989) CIA model revenue and welfare implications of different taxes is analyzed in Cooley and Hansen (1991). Calibration and simulation techniques are used and the study reaches the conclusion that the presence of distortionary taxes (taxes on capital and labour) doubles the welfare cost of a given steady-state inflation policy. Permanent zero-inflation policy with other distortionary taxes held at their benchmark level improves welfare by 0.33 percent of GNP. Another type of zero-inflation policy that is assumed to be permanent, and where the lost revenue from inflation tax is replaced by raising distortionary taxes, the welfare cost is higher than the original policy with 5% inflation. Moreover, a temporary reduction of inflation rate to zero makes the economy worse-off due to intertemporal substitution.

Cooley and Hansen (1989) measured the welfare cost under the assumption of cash only economy and Cooley and Hansen (1991) allowed for the availability of costless credit. Further there is assumed predetermination of cash and credit goods in both the studies. Gillman (1993) introducing the Baumol (1952) exchange margin allowed the consumer to decide to purchase good from cash or credit with further assumption of costly credit. The consumer in making decision weighs the time cost of credit against the opportunity cost of cash. The interest rate elasticity and welfare loss from costly credit setup is compared with the cash-only and costless credit economies. Using U.S. average annual data from 1948 to 1988, the study shows that both interest elasticity and welfare cost in costly credit economy are greater than the cash-only and costless credit settings. The cost associated with 10 percent inflation is 2.19 percent of income compared to 0.582 percent and 0.098 percent for cash-only and costless credit economies respectively.

Eckstein and Leiderman (1992) in addition to Cagan semi-log model used Sidrauski-type money-in-uitility (MIU) model to study seigniorage implications and welfare cost of inflation for Israel. Parameters of the intertemporal MIU model are estimated by using Generalized Methods of Moments (GMM), on quarterly data from 1970:I to 1988:III. The simulation results show that inflation rate of 10 percent has welfare loss of about 1 percent of GNP. Degree of risk aversion is identified as an important determinant of welfare cost and loss of lower inflation rates predicted by the intertemporal model is higher than that calculated from the Cagan-type model. The welfare cost estimates from intertemporal model are more reliable as it produced national income ratios and seigniorage ratios much closer to the actual values.

López (2000) following Eckstein and Leiderman (1992) intertemporal model studied the seigniorage behavior and welfare consequences of different inflation rates in Columbia. For the period 1977:II to 1997:IV the parameters of the model are estimated using GMM. Welfare loss due to increase in inflation from 5 percent to 20 percent is 2.3 percent of GDP, and 1 percent of GDP when inflation increases from 10 percent to 20 percent. Eckstein and Leiderman (1992) model with some modification is employed in Samimi and Omran (2005) to study the consumption and money demand behavior from intertemporal choice. Welfare cost of inflation is calculated using annual data from 1970 to 2000 for Iran. Welfare cost is found to be positively related to the inflation rate. Welfare cost of 10 percent inflation is 2 percent of GDP and it is 4.37 percent of GDP for an inflation rate of 50 percent.

Bailey (1956), Wolman (1997), and Eckstein & Leiderman (1992) pointed out that the estimates of welfare cost depend largely on the money demand specification. Lucas (1994 & 2000) graphically plotting the annual figures of real balances and nominal interest rate for U.S prefers the double log money demand function in explaining the actual scatter plot than the semi-log functional form for the period 1900-1994. Bailey's consumers' surplus formulae are derived and used to compute the welfare cost of inflation for both semi-log and log-linear money demand functions. Based on double-log demand curve the welfare gain from moving from 3 percent to zero interest rate is about 0.009 percent of real GDP, while for semi-log estimates it is less than 0.001. To provide general equilibrium foundations for the money demand function, Sidruaski's MIU (1967) and McCallum & Goodfriend's (1987) transaction technology models are used. The study has used Friedman's deflation rule instead of zero inflation as a benchmark, and the difference of welfare gain for two specifications is due to their behavior towards moving from stable price policy to the deflation rule.

Benchmark inflation rate with which comparison of positive inflation is made to arrive at welfare cost has important implications depending on the money demand function. Lucas (1994 & 2000) using double-log money demand function calculates welfare gain as moving from positive inflation to optimal deflation rate, and found that there are sizable benefits to be accrued as the economy moves from zero inflation rate to the deflation rate. For the double-log specification as the nominal interest rate approaches zero the real balances increase towards infinity implying that individuals are not satiated with real money holdings at zero inflation rate. Wolman (1997) tested the non-satiation hypothesis using Nonlinear least squares (NLS) to the annual data of U.S from 1915 to 1992. Money demand explained in the transactions-time technology model shows that individuals are satiated with real balances at zero inflation rate. Larger gains of reducing inflation from 5 percent are achieved by price stability. The reduction of inflation from 5 percent to zero raises consumption that is equivalent of 0.6 percent of output.

Simonsen and Rubens (2001) theoretically extended Lucas (2000) transactions technology model to allow for the interest bearing assets. Lucas (2000) has emphasized that there is a need to formulate a model in which currency, deposits, and reserves be given distinct roles. The rationality for this reformulation is necessary as money when is defined as M1, includes the interest bearing assets that are partly indexed depending on the reserve requirement. Simonsen and Rubens (2001) reached the conclusion that with interest earning assets included, the upper bound lies between Bailey's consumer surplus measure and Lucas' measure of welfare cost. Bali (2000) using different monetary aggregates calculated welfare cost using two approaches, Bailey's welfare cost measure and compensating variation approach. It also estimated the costs of positive inflation to price stability and optimal deflation policy for both double-log and semi-log money demand specifications. Error Correction and Partial Adjustment models applied to find the long interest elasticities and semi-elasiticities. For the quarterly data ranging from 1957:I to 1997:II empirical results showed that constant elasticity demand function accurately fit the actual U.S data. Loss to welfare associated with 4 percent inflation turned out to be 0.29 percent of income (benchmark to be zero nominal interest rate) and the welfare gain in moving from 4 percent to zero inflation is 0.11 percent of income with currency-deposit specification, welfare cost is around 0.18 percent of GDP when monetary base is used whereas with M1 the loss is much higher than the earlier two cases and is approximately 0.55 percent of GDP.

Serletis and Yavari (2003) calculated and compared the welfare cost of inflation for two North American economies: Canada and the United States for the period 1948 to 2000. Following Lucas (2000) it is maintained that the proper money demand function has constant interest elasticity. Engle-Granger cointegration and Long-horizon regression approach are used to estimate the long run interest elasticities. Interest rate elasticity for Canada is 0.22 and for U.S it is 0.21, much lower than 0.5 assumed by Lucas (2000). Welfare cost is measured using traditional Bailey's approach and Lucas' compensating variation approach. Welfare gain of interest rate reduction from 14 percent to 3 percent (consistent with zero inflation) for U.S is equivalent to 0.45 percent increase in income. Reducing the nominal interest rate further to the optimal deflation rate yields an increase in income by 0.18 percent. For Canada the distortionary costs are marginally lower, reducing rate of interest from 14 percent to 3 percent increases real income by 0.35 percent, and further reducing to Friedman's zero nominal interest rate rule resulted in a gain of 0.15 percent of real income.

Serletis and Yavari (2005) estimated the money demand function and based on the estimated elasticity computed welfare cost of inflation for Italy. Long-horizon regression showed that interest elasticity for the annual data starting from 1861 to 1996 was 0.26. Using the same approaches of calculating welfare cost of inflation as in Serletis and Yavari (2003), the estimates showed that lowering the interest rate from 14 percent to 3 percent yield a benefit of about 0.4 percent of income. Same analysis was extended in Serletis and Yavari (2007) to calculate direct cost of inflation for seven European countries, Ireland, Australia, Italy, Netherlands, France, Germany, and Belgium. The welfare cost estimates of these countries showed that the cost is not homogeneous across these countries and is related to the size of the economy. Welfare cost was lower for Germany and France than the small economies.

Welfare costs of anticipated inflation are the distortions in the money demand brought about by the positive nominal interest rate so the major emphasis of studies after Lucas (2000) is first to check for the proper money demand specification. Ireland (2007) using Phillips-Ouliaris Cointegration and Johansen Cointegration tests found that Cagan-type semi-log money demand function is better description of post 1980 U.S data. For the quarterly data from 1980 to 2006 the semi-elasticity is estimated to be 1.79 and welfare cost of inflation is measured using consumer's surplus approach of calculating the area under the money demand curve. For 2 percent inflation rate the welfare cost is 0.04 percent of income and 0.22 percent of income for the 10 percent inflation rate. Price stability is taken as benchmark instead of Friedman's optimal deflation policy.

Gupta and Uwilingiye (2008) measured welfare cost of inflation for South Africa. Double log and semi-log money demand functions are estimated using Johansen cointegration method and long-horizon regression method. The study apart from estimating the proper money demand function analyzed whether time aggregation affects the long run nature of relationship or not. Interest elasticity and semi-elasticity estimates are used to measure welfare cost of inflation using Bailey's traditional approach and Lucas compensating variation approach. Estimation results show that for the period 1965:II to 2007:I, compared to cointegration technique, long-horizon approach gives more consistent long run relationship and welfare estimates under the two time aggregation sampling methods. Welfare cost of target inflation band of 3 percent to 6 percent lies between 0.15 percent to 0.41 percent of income.

Literature shows that the welfare cost of inflation has found its initial application in hyperinflation countries Bailey (1956), Marty (1967) and Barro (1972) and later welfare cost was measured for the developed countries with stable inflation like U.S and now it is extended to European countries and South Africa. This issue needs to be addressed for developing countries as inflation rate is primarily determined by money supply, for the conduct of monetary policy it is very much important to estimate the welfare cost of inflation based on a stable estimated money demand function. This is the first ever attempt to calculate the welfare cost of inflation in Pakistan.

Secondly there is also a transition from partial equilibrium analysis to general equilibrium analysis to calculate welfare cost of inflation. To provide general equilibrium rationale for holding money we will use the Money-in-the Utility Function model. Other general equilibrium models like Cash-in-Advance and transaction time technology models are relatively more sophisticated approaches but we cannot apply these due to two main reasons. First is the underlying assumption of the models regarding distinction among the cash and credit goods. Secondly the studies employing CIA constraint in Real Business Cycle (RBC) model use the calibration technique which makes use of the results of studies using microeconomic data. For most of the developing countries in general and specifically for Pakistan the data on non-durable (cash) goods and durable (credit) goods is not available. Similarly, the impact of inflation on the marginal decisions like working hours, capital accumulation and investment decisions at micro level have not been addressed for Pakistan.

The welfare cost analysis of present study is based on Money-in-utiltiy framework and we will apply Bailey's (1956) consumer surplus approach and Lucas (2000) compensating variation approach to calculate the inflation related welfare loss to the society.

2.2 Pakistan Specific Literature on Demand for Money

Welfare cost estimates are highly sensitive to the money demand specification; in this section we will provide review of some of the recent development on this issue in Pakistan. In theoretical literature the main determinants of money demand are the opportunity cost variables and the scale variable proxied by income. The attempt to empirically test this theory started with Mangla (1979) who estimated the real and nominal demand for M1. Scale variable is taken to be both GNP and permanent income. Oppurtunity cost for M1 is interest rate which is taken as both annual yield on government bonds and call money rate. The estimates of nominal money demand function for the study period from 1958 to 1971 showed that income elasticity is significantly greater than 1 and interest elasticity ranged from -0.04 to -0.16 for call money rate, while for the bonds yield it ranged from -0.31 to -0.96. Real money demand function has income elasticity greater than one, while the interest elasticity turned out to be low i.e., -0.02 to 0.02 for call money rate and positive for bonds yield.

Khan (1980) estimated the demand for money and real balances by defining money as M1 and M2 for the period 1960 to 1978. The main objective of the study was to identify the correct scale variable, current income or permanent income for the money demand function. Applying ordinary least squares income elasticity for both nominal and real money demand functions turned out to be significantly greater than one, implying diseconomies of scale. Both permanent and current income gave approximately similar results lending no superiority to one measure over the other. For nominal money demand functions for M1 and M2 the interest elasticity is insignificant but for the real money demand it has the expected negative sign. One of the issues regarding opportunity cost variable in developing countries is whether to take interest rate as opportunity cost or the inflation rate. This is due to the fact that the developing countries have controlled or limited financial markets (and interest rate) so substitution takes place between money and goods instead of between money and financial assets. In this study the effects of inflation and monetization on money demand function are measured by applying partial adjustment model. The coefficient of inflation is insignificant. Therefore, the demand for real money balances turned out to be related to interest rate and for the study period it is an appropriate opportunity cost variable.

Similar analysis of finding appropriate scale and opportunity cost variables for the money demand function was carried out in Khan (1982). The scale variables were taken to be permanent income and measured income, while opportunity cost variables were the interest rate (call money rate, interest on time deposits) and the expected and actual inflation rates. Using Cochrane-Orcutt technique the demand functions of M1 and M2 are estimated for the six Asian developing countries (Pakistan, India, Malaysia, Thailand, Sri Lanka, Korea) for the period 1960 to 1978. For Pakistan with M1 definition of money there is no difference between permanent and measured income elasticities. Income elasticity is greater than one representing diseconomies of scale. Money demand is significantly explained by interest on time deposits and interest elasticity ranged from -0.42 to -0.44. For broader money (M2) income elasticity is greater than for M1, and interest elasticity ranged from -0.37 to -0.39. For Pakistan inflation and expected inflation tend to affect money demand but the magnitude (-0.05) is much less than the coefficient of interest rate. This study also reaches the same conclusion as Khan (1980) that interest rate is the proper opportunity cost variable in money demand function.

Nisar and Aslam (1983) estimated the term structure of time deposits and the parameters were substituted in the money demand function for the period 1960 to 1979. The coefficient of term structure both for M1 and M2 monetary aggregates is negative and has a smaller magnitude for the M2 definition of money ranging from -0.51 to -0.73. Time deposits are positively related to interest rate (representing own rate of return), whereas interest rate has negative effect on currency so overall the magnitude of interest elasticity is low for M2 due to the inclusion of time deposits. The study reaches the same conclusion as Khan (1982) that the money demand is elastic with respect to the scale variable. While coefficient of inflation rate bears positive sign and is statistically not significant. Secondly the study compares the stability of estimated money demand function using term structure against the conventional money demand function with simple average interest rate (call money rate). The covariance analysis shows that the term structure money demand function remained stable while conventional function does not pass the stability test.

Developing countries like Pakistan lack sophisticated financial and banking sector. Currency constitutes a large proportion of total monetary assets. Qayyum (1994) using data from 1962:I to 1985:II estimated the long run demand for currency holding using Johansan Maximum likelihood method. Cointegration results show that currency demand is determined by interest rate defined as bond rate, rate of inflation and income. Coefficient of income is approximately unity, and money-income proportionality hypothesis is tested. Money-income proportionality holds and imposing this restriction, the steady state demand for currency turns out to be related to inflation and bonds rate.

The coefficients of inflation and interest are negative and significant showing that people can substitute between currency and real goods, and also between currency and financial assets.

Hossain (1994) estimated the money demand for both the real narrow (M1) and broad money (M2) balances using Johansen Cointegration test for the two sub-periods ranging from 1951 to 1991 and 1972 to 1991. Double log specification of money demand function is used with income, interest rate (govt. bond yield, call money rate) and inflation rate as the explanatory variables. The results for the sample period 1972 to 1991 are more encouraging where the income elasticity for broad money is around unity and about 0.86 for the narrow money. Interest elasticity in absolute terms is greater for narrow money (-0.54) than for M2 (-0.05). The results for both the sample periods show that real money balances are not cointegrated with inflation rate and that the narrow money demand is more stable than the broad money demand function.

Financial sector reforms of 1980s increased the interest in money demand function and two studies [Khan (1994) & Tariq and Mathew (1997)] investigated the impact of financial liberalization on money demand. First of these studies, Khan (1994) examined the effect of these reforms on stability of money demand. Engle-Granger two step method of cointegration is used to estimate the money demand function for the quarterly data starting from 1971:III to 1993:II. Cointegration results of double-log money (nominal M1 and M2) demand function show that demand for broader money is determined by real income, nominal interest rate of medium term maturity real interest rates, and the inflation rate. While for M1 definition the cointegral relationship holds for all the arguments except short term and medium term nominal interest rates.

The second study in the same line of addressing the effects of financial reforms is Tariq and Matthews (1997) that investigated the impact of deregulation on the definition of monetary aggregates. In this study divisia monetary aggregates are compared to the simple monetary aggregates in finding the stable money demand function. Using quarterly data from 1974IV to 1992IV and applying Johansen and Johansen & Juselius cointegration tests, the long run money demand function is estimated. Conventional money demand function is estimated with scale and opportunity cost variable, opportunity cost is taken as differential of interest on an alternative asset and own rate of return on the given monetary aggregate. Cointegration analysis showed that demand for all the four monetary aggregates, M1, M2, Divisia M1 and Divisia M2 is positively related to the scale variable and negatively to opportunity cost variable. Income elasticity is greater than unity implying that velocity has decreasing trend. Error correction model (ECM) is used to estimate the short run dynamic money demand function, which shows that all the four monetary aggregates are equally good in explaining the money demand function and there is no superiority of divisia aggregates over the simple-sum monetary aggregates.

There is difference between the money demand behavior of household and business sector and the sectoral money demand has been studied in developed countries. Its first application in Pakistan is Qayyum (2000) that studied the demand for money by business sector. Owing to the difference in the behavior of business sector, total sales is taken as the scale variable instead of income. The cointegration analysis for the period 1960:I to 1991:II shows that the long run demand for M1 is determined by sales and inflation rate. The sales/ transactions elasticity of business sector's demand for real balances is unitary. In the long run the demand for money is not determined by the interest rate but the short run dynamic ECM shows that money demand is determined by changes in the return on on saving deposits, changes in inflation rate, and movements in the previous money holding.

This sectoral money demand analysis is further extended in Qayyum (2001) that studies the money demand function at aggregate level and for both the household and business sectors using quarterly data from 1959:III to 1985:II. Johansen cointegration analysis identifies that all the three money demand functions are sensitive to income, inflation rate and interest rate. Monetary aggregate is defined as broad money M2, coefficient of call money rate representing own rate of interest is positive in all the specifications. Bond rate is the relevant opportunity cost variable in aggregate and household money demand functions. For the business sector the appropriate interest rate representing opportunity cost is the rate of interest on bank advances. Money-scale variable proportionality holds in all the money demand functions. Scale variable is defined as income/ real GDP for the aggregate and household money demand function while for the business sector it is real sales. Business sector demand for real balances is explained by own rate of return and inflation rate and further the money-sales proportionality holds in the long run. ECM shows that in the short run interest rate is an important variable determining the aggregate demand for real balances and liquidity demand of the business sector.

Qayyum (2005) estimated the demand for broader monetary aggregate M2 at aggregate level for the annual data frm 1960 to 1999. This study reaches similar conclusion as Qayyum (2001) that the major determinants of money demand are own rate of return (call money rate) and opportunity cost variables (inflation rate and the Govt. bond yield) and income. However, the magnitude of coefficients is high for both the interest rates, that is in accordance with the financial sector reforms during the decade of 1990s.

Using annual data from 1972 to 2005 Hussain et. al. (2006) estimated the demand for money; money is defined as monetary base, M1 and M2. The study finds that there is no cointegration and unit root in the data series so instead of ECM, ordinary least squares method is used. Demand for all the three monetary aggregates is explained by the real GDP, inflation rate, financial innovation and the interest rate on time deposits. The long run income elasticity ranges from 0.74 to 0.779 and interest elasticity ranges from -0.344 to -0.464. Of all the three definitions of money M2 is found to better explain the long run stable money demand function.

Ahmad et.al. (2007) estimated the long run money demand function using error correction model. The conventional money demand function with income and call money rate is estimated for the period 1953 to 2003.. The results show that both the arguments of money demand function have theoretically correct signs for M1 and interest semielasticity is -0.012. Interest rate coefficient is positive and insignificant for real M2. For both narrow money M1 and broad money M2 the money-income proportionality does not hold.

All the studies upto date estimated the long run money demand function using income as scale variable, and different types of interest rates as opportunity cost of holding money. Along with these variables inflation rate, monetization, financial innovation have also been found significant in explaining the money demand behaviour. M2 is the important instrument in the conduct of monetary policy. State bank of Pakistan formulates the monetary policy observing the trends in the demand and supply of M2. Most of the studies on money demand function have used M2 definition of money to determine the factors that explain the absorption of money in the economy and to analyze the effectiveness of monetary policy.

In present study we want to calculate the welfare cost of inflation based on the estimated parameters of a stable money demand function. The studies on welfare cost of inflation show that we have to define money in the narrowest form, like monetary base or M1 so that the interest rate is the opportunity cost of holding money[5]. Estimating the demand for broader monetary aggregate (M2) is not relevant for our analysis because it includes some interest bearing assets, interest coefficient in most of the studies turned out to be positive or insignificant showing that interest rate is own rate of return rather than an opportunity cost variable for M2.

Welfare cost is a steady state analysis for which the money-income proportionality is assumed to hold. Following the social welfare loss of inflation analysis we need to freshly estimate the money demand function taking the ratio of money balances to income (scale variable) as the dependent variable with a single argument; the nominal interest rate. We estimate demand functions defining money as Monetary Base, Narrow Money M1 and disintegrating M1 into its constituent components and estimate the demand functions of demand deposit and currency.

[5] Hussain (1994) narrow money demand in Pakistan is more stable than broad money demand function.

CHAPTER

3

METHODOLOGY

Theoretical and empirical studies on the welfare cost of inflation show that the magnitude of welfare cost depends on the choice of money, the selection of money demand function and the approaches of measuring welfare loss. In this chapter we will give an account of the general equilibrium Money-in-Utility framework to derive the demand for money, later we will discuss the welfare cost measures under different money demand specifications and monetary assets. Since the steady state analysis requires the estimation of long-run money demand function, the next section gives estimable form of money demand functions. This is followed by the discussion of econometric technique employed to get the parsimonious estimates of money demand function. Finally the last section describes the data to be used for the estimation of money demand function and computation of the welfare cost of inflation.

3.1 Money-in-Utility Function Approach to Money Demand

Money-in-utility model was formally developed by Sidrauski (1969) but the concept of introducing money in the utility function finds its first application in Friedman (1956) where money is treated like a consumer good and demand for money is determined by own price, price of substitute assets and the scale variable. MIU model has the advantage as it allows costs of inflation to be calculated within the money demand function that is consistent with optimizing behaviour of the economic agents [Walsh (2003)]. The money demand function derived from this model is directly estimable and it provides a convenient framework for calculating the welfare costs of inflation, because lower level of real money holdings due to higher rates of inflation has a direct effect on welfare when money enters the utility function. This model has been extended by Calvo and Leiderman (1992), Hansen and Singleton (1982), Lucas (2000), and Bali (2000), where all these studies used data for the U.S.

We will follow Bali's (2000) currency-deposit version of Sidrauski's MIU model to analyze the welfare cost of inflation in Pakistan. The rationale of employing currency-deposit model is that both currency and deposits have different opportunity costs. The implicit cost of holding currency is the nominal interest i, while that of demand deposit is the difference between the nominal interest rate, i and the interest on deposits, i_d. The studies that lump both the currency and demand deposits together as non-interest bearing assets are likely to overstate the true cost of inflation (Lucas 1994 & 2000). Another advantage of this disintegrated asset model is that the single monetary asset models are the nested versions of this broader model.

MIU model treats consumption and money demand behavior as jointly arising from a single optimizing framework of a representative agent. We will ignore uncertainty and labour-choice, focusing on the implications of the model for money demand and welfare cost of inflation. We assume that the representative household derives utility from a consumption good, c_t and flow of services from the real money balances that consist of currency m_t and demand deposits d_t.

$$\sum_{t=0}^{\infty}(1+\rho)^{-1}U(m_t,c_t,d_t) \qquad (3.1)$$

where ρ is the subjective rate of time preference. Utility function is assumed to be increasing in all the three arguments, strictly concave and continuously differentiable. Economy-wide budget constraint of household sector, in real units, is given by

$$(1+\pi_{t+1})m_{t+1}+(1+\pi_{t+1})d_{t+1}+k_{t+1}+(1+r_{t+1})^{-1}b_{t+1}=m_t+d_t(1+i_d(t))+k_t(1-\delta)+b_t+f(k_t)+c_t+h_t \tag{3.2}$$

The budget constraint indicates that the household can transfer resources from one period to the next by holding nominal currency m_t, demand deposits d_t, bonds b_t and physical capital k_t. Given the current income $f(k_t)$, the assets and any net transfer (h_t) from the government sector the household allocates its resources among current consumption (c_t) and savings (left side of equation 3.2). Real rate of return on bonds $(1+r_{t+1})$ is equal to $(1+i_{t+1})/(1+\pi_{t+1})$ where i_{t+1} denotes the nominal return on bonds held from t to $t+1$, whereas $(1+i_d)$ is the return on demand deposits.

Household maximizes (3.1) subject to budget constraint (3.2). Solving the optimization problem for two periods t and t-1 yields the following first-order Euler equations:

$$\frac{u_m(c_t,m_t,d_t)}{u_c(c_t,m_t,d_t)}=-1+(1+\rho)(1+\pi_t)\frac{(1+r_t)}{(1+\rho)}=i_t \tag{3.3}$$

$$\frac{u_d(c_t,m_t,d_t)}{u_c(c_t,m_t,d_t)}=-1(1+i_d)+(1+\rho)(1+\pi_t)\frac{(1+r_t)}{(1+\rho)}=i_t-i_d(t) \tag{3.4}$$

Euler equations (3.3) and (3.4) indicate that the marginal rate of substitution between money and consumption and between deposits and consumption is equal to the opportunity costs of respective assets. These first order Euler equations are the implicit form of asset demand functions which can be estimated by assuming some specific form of utility function.

In order to derive the implications of the model for welfare cost of inflation using Lucas compensation variation approach, the following CES isoelastic utility function is used:

$$u(c_t, m_t, d_t) = \frac{\left\{\left[\gamma_1^{1/\theta} c_t^{(\theta-1)/\theta} + \gamma_2^{1/\theta} m_t^{(\theta-1)/\theta} + \gamma_3^{1/\theta} d_t^{(\theta-1)/\theta}\right]^{\theta/(\theta-1)}\right\}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \tag{3.5}$$

where $\theta > 0$ is the elasticity of intertemporal substitution. Substituting the marginal utilities from (3.5) into Euler equations (3.3) and (3.4) gives the following real currency and real deposit demand functions:

$$m_t = \left(\frac{\gamma_2}{\gamma_1}\right) i_t^{-\theta} c_t \tag{3.6}$$

$$d_t = \left(\frac{\gamma_3}{\gamma_1}\right) (i_t - i_d(t))^{-\theta} c_t$$

Steady state analysis of welfare cost of inflation requires that the proportion of income held as cash, be independent of the growth in real income, which means that velocity remains constant.[6] Under steady state we write the money demand function as the ratio of real money balances to the real scale variable, which further requires that under both

[6] Velocity beocmes function of the interest rate and it is transformed to money demand function which is integrated under Bailey's approach to get welfare cost as a proportion of scale variable

$$u(c_t, m_t, d_t) = \left(\frac{1}{1-\omega}\right)\left[\left(\frac{1}{1-\gamma}\right)(c_t^{1-\gamma} - 1) + \left(\frac{\varphi}{1-\alpha}\right)(m_t^{1-\alpha} - 1) + \left(\frac{\phi}{1-\beta}\right)(d_t^{1-\beta} - 1)\right]^{1-\frac{1}{\omega}}$$

$$m_t = \varphi^{1/\alpha} i_t^{-1/\alpha} c_t^{\gamma/\alpha} \qquad d_t = \phi^{1/\beta}(i_t - i_d(t))^{-1/\beta} c_t^{\gamma/\beta}$$

where γ/α and γ/β are the scale elasticities of demand for real currency and real deposits and $1/\alpha$ and $1/\beta$ are elasticities of currency and deposits with respect to their respective opportunity costs. Unitary scale elasticities require that $\alpha = \beta = \gamma$ must hold, which implies that the assest demand functions have same opportunity cost elasticities.

currency and demand deposits have same interest elasticities (θ). Cost of holding money defined as demand deposits, is the differential between the yield on other assets i_t and interest on deposits i_d. If the banks are operating at zero profit condition then $i_d = (1-\mu)i_t$ (where μ is the reserve ratio). The zero profit condition implies that opportunity cost of holding deposits is $(i_t - i_d) = \mu i$. Equation (3.7) is the demand function for demand deposits.

$$d_t = \left[\frac{\gamma_3}{\gamma_1}\right](\mu i_t)^{-\theta} c_t \tag{3.7}$$

For the single monetary asset the utility function in MIU framework takes the form

$$\sum_{t=0}^{\infty}(1+\rho)^{-1}U(m_{t,}c_t,)$$

Solving the optimization problem with changing the budget constraint without distinct role of demand deposits gives the money demand function equivalent to the currency demand function 3.6.

3.2 Money Demand Specification

To compute the welfare cost function we will estimate both the double log and semi-log money demand functions.

3.2.1 Double-log Money Demand Function

To calculate the welfare cost of inflation we are interested specifically of the effect of opportunity cost (nominal interest rate) of money holding. The demand for real balances is given by

$$\left(\frac{M_t}{P_t}\right) = L(i_t, y_t)$$

The above equation states that the demand for real balances is function of nominal interest rate i_t, and the real income y_t. In the long run liquidity demand function takes the form

$$L(i\ ,y) = m(i)y \tag{3.8}$$

which shows that money demand is proportional to income. Estimates of the income elasticity of money (M1,M2 and currency) demand obtained for Pakistan tend to be around unity [Qayyum (1994), (2000), (2001) & (2005)]. Therefore, the unitary scale (income) elasticity restriction is imposed which enables us to estimate the money demand function *m(i)* defined as the ratio of real money balances to real income with single argument defined as the opportunity cost of holding money.

$$m / y = m(i) \tag{3.9}$$

Equations (3.6) and (3.7) are in the form of (3.8) and dividing by the scale variable lead to writing the final form of demand function suitable for the welfare cost analysis.

$$\left(\frac{m_t}{c_t}\right) = \left(\frac{\gamma_2}{\gamma_1}\right) i^{-\theta}$$

$$\left(\frac{d_t}{c_t}\right) = \left(\frac{\gamma_3}{\gamma_1}\right) (\mu i)^{-\theta}$$

These equations take the form of equation (3.9) and can be written in the following double-log form

$$m(i) = e^{\alpha_0} i^{-\alpha_1} \tag{3.10}$$

$$d(\mu i) = e^{\alpha_2} (\mu i)^{-\alpha_1} \tag{3.11}$$

where the dependent variables are taken as ratio to scale variable and welfare cost is expressed as the percentage of GDP.

3.2.2 Semilog Money Demand Function

Standard utility functions mostly yield double-log money demand function, but semilog models have gained wider application in money demand literature for their seigniorage implications. A number of studies like Lucas (2000), Bali (2000) and Gupta and Uwilingiye (2008) have estimated both the double-log and semi-log money demand functions and compared welfare costs associated with both the specifications. Keeping in view these studies we will also estimate the semi-log money demand function along-with the log-linear function to judge the sensitivity of the estimated welfare cost for the two models towards low interest rates.

For the single monetary asset the semilog money demand function is defined as

$$m(i) = e^{\alpha_0 - \alpha_1 i}$$

To compare the semi-log model with the derived double log currency-deposit model we restrict both currency and demand deposits to have same interest semi-elasticity. The demand functions for currency and deposits under semi-log specification are given as

$$m(i) = e^{\alpha_0 - \alpha_1 i} \quad (3.12)$$

$$d(i) = e^{\alpha_2 - \alpha_1 (\mu i)} \quad (3.13)$$

After estimating the steady state money demand functions the welfare cost will be computed using both Bailey's and Lucas' measures of welfare cost.

3.3 Welfare cost of inflation and money demand function

Following the literature on welfare cost of inflation we will apply the following approaches to compute the welfare cost of inflation:

(i) Bailey's (1956) consumer's surplus approach and

(ii) Lucas' (2000) compensation variation approach.

In this section we discuss the measure of welfare cost formulae under both approaches for all the monetary aggregates as well as for both money demand specifications.

3.3.1 Bailey's Consumer Surplus Approach

The first attempt to measure the welfare cost of anticipated inflation is credited to Bailey (1956) wherein the nominal interest rate was considered as the opportunity cost of holding money. Inflationary finance on part of the government is an excise tax on real cash holding; and welfare cost is the loss in consumer surplus, which is measured as the area under the money demand curve. Changes in inflation rate are related to changes in nominal interest rate through Fisher hypothesis that holds for Pakistan [Hassan (1999)] Welfare cost is measured as loss in consumer surplus not compensated by total revenue

$$w(r) = \int_{m(r)}^{m(0)} \psi(x)dx = \int_{0}^{r} m(x)dx - rm(r) \qquad (3.14)$$

where $m(r)$ is money demand function and $\psi(x)$ is the inverse demand function. m is defined as ratio of money to income, welfare function w is function of income therefore, welfare loss is defined as proportion of income.

Due to different behaviour of semi log and log linear money demand functions towards low inflation [Lucas (2000) & Wolman (1997)] and their implication on welfare cost, in the following sub-sections we give the formulae based on Bailey's approach under different money demand functions and monetary models.

3.3.1.1 Welfare Cost of Inflation for Semilog Money Demand function:

Bailey (1956), Marty (1967), Friedman (1969) and Tower (1971) have used Cagan semi-log money demand function. All these studies were based on hyperinflation economies, and welfare gain for this specification comes largely by moving from high interest to low interest rates, while for the interest rate approaching zero the solution is trivial.

a) Single Monetary Asset Model

When monetary stock is taken to be monetary base or M1 (single monetary asset model) the semilog money demand function is given as

$$m(r) = e^{\alpha_0 - \alpha_1 i}$$

Substituting the money demand function in (3.14) gives the following welfare cost measure (4.2)

$$\int_0^r e^{\alpha_0 - \alpha_1 x} dx - i\left(e^{\alpha_0 - \alpha_1 i}\right)$$

$$WC = \frac{e^{\alpha_0}}{\alpha_1}\left[1 - e^{-\alpha_1 i}(1 + i\alpha_1)\right] \qquad (3.15)$$

where α_0 is the intercept in money demand function and α_1 is the interest rate related semi elasticity of money demand.

b) Currency-Deposit Model

For the modified money-in-utility function which allows for the distinct role of currency and demand deposits, the welfare cost is given by:

$$WC = \int_0^i f(x)dx - if(i) + \int_0^{\mu i} g(x)dx - \mu i g(\mu i)$$

$$WC^{Semi-\log} = \frac{e^{\alpha_0}}{\alpha_1}\left[1 - e^{-\alpha_1 i}(1 + \alpha_1 i)\right] + \frac{e^{\beta_0}}{\beta_1}\left[1 - e^{-\beta_1 \mu i}(1 + \beta_1 \mu i)\right] \qquad (3.16)$$

where demand for currency is $f(x) = e^{\alpha_0 - \alpha_1 i}$ and semi-log demand function for deposits is $g(x) = e^{\beta_0 - \beta_1 \mu}$ First term in (3.16) represents the dead weight loss accruing from currency and second term is the dead weight loss measured for demand deposit. For currency the integral runs from zero to positive nominal interest i and for demand deposits it runs from zero to opportunity cost of holding demand deposits μi. Under restricted model where both currency and deposits are restricted to have the same semi elasticity $\alpha_1 = \beta_1$

3.3.1.2 Welfare Cost of Inflation for Double log Money Demand Function

Double log money demand function has been estimated and found to be the proper money demand specification in a number of studies like Lucas (2000) for the U.S., Serletis and Yavari (2003) for Canada and United States, the countries with relatively stable inflation.

a) Single Monetary Asset Model

The double log money demand specification for a single monetary asset i.e. monetary base or M1 is given as

$$m(i) = e^{\alpha_0} i^{-\alpha_1}$$

and welfare cost formula is derived by substituting the money demand function in (3.14)

$$WC = e^{\alpha_0} i^{1-\alpha_1} \left[\frac{\alpha_1}{1-\alpha_1} \right] \tag{3.17}$$

where α_0 and α_1 are the intercept and slope coefficient of double log money demand function respectively.

b) Currency-Deposit Model

For the double-log demand for currency and deposits (3.14) becomes

$$WC^{double-log} = \left(\frac{\alpha_1}{1-\alpha_1} \right) e^{\alpha_o} i^{1-\alpha_1} + \left(\frac{\beta_1}{1-\beta_1} \right) e^{\beta_o} (\mu i)^{1-\beta_1} \tag{3.18}$$

The welfare cost formula shows that the cost is entirely in terms of α_o, α_1, β_o and β_1, parameters of the estimated asset demand functions and their opportunity costs.

3.3.2 Lucas Compensating Variation Approach

In order to arrive at a welfare measure Lucas (1994) starts with the assumption that two economies have similar technology and preferences; the only difference is in the conduct of monetary policy. In one economy, Friedman's zero interest rate policy is adopted whereas in the second economy interest rate is positive. He defines welfare cost of inflation in terms of compensation in income required to leave the household (living in the second economy) indifferent to live in either of the two economies. Left hand side of equation (3.19) shows the welfare cost in second economy with positive interest rate and right hand side is the characterization of the first economy operating at deflation policy. *w(i)* is the measure of income compensation or the welfare cost of inflation.

$$u[1+w(i),\bar{m}(i),\bar{d}(\mu i)] = u[1,\bar{m}(0),\bar{d}(0)] \tag{3.19}$$

Lucas has proposed two measures of welfare cost for alternative specifications of long-run money demand function due to their different behavior at low interest rates; (a) Square-Root Formula and (b) Quadratic Approximation.

3.3.2.1 Welfare Cost of Inflation for Semilog Money Demand function and Quadratic Approximation

Semi-log money demand specification originally due to Cagan (1956) and Bailey (1956) leads to quadratic formula for welfare cost of inflation. Under this specification, there is the existence of satiation in money demand and the quadratic formula derived for this specification is sensitive to high interest rate. Wolman (1997) and Bakhshi (2001) have shown that for semi-log model there is satiation in asset holding *m(0)* and *d(0)* in (3.19), which represents maximum currency and demand deposits holdings at zero interest rate. The welfare gain of moving from positive inflation to zero inflation is higher as compared to the gains of moving further to Friedman's zero interest rule.

To derive the quadratic formula from equation (3.19), the second-order Taylor series expansion is applied to the welfare function around zero interest rate.

$$w(i) = w(i)\big|_{i=0} + w'(i)\big|_{i=0}(i-0) + \frac{1}{2}w''(i)\big|_{i=0}(i-0)^2 = \frac{1}{2}i^2\left[-\overline{m}'(0) - \mu^2\overline{d}'(0)\right] \qquad (3.20)$$

a) Single Monetary Asset Model

For the single-monetary-asset model, equation (3.19) takes the following form

$$u\left[1 + w(i), \overline{m}(i)\right] = u\left[1, m(0)\right]$$

and welfare cost of inflation is $w(i) = \frac{1}{2}\overline{m}(0)\eta i^2$ (3.21)

where η is the semi-elasticity of demand for M1 or monetary base with respect to interest rate.

b) Currency-Deposit Model:

Assuming that demand deposits and currency have same semi-elasticity (restricted case) welfare cost formula (3.20) is transformed as[7]

$$w(i) = \frac{1}{2}\eta i^2\left[\overline{m}(0) + \mu^2\overline{d}(0)\right] \qquad (3.22)$$

Given the semi-log demand functions $\overline{m}(i) = \overline{m}(0)e^{-\eta i}$, $\overline{d}(\mu i) = \overline{d}(0)e^{-\eta(\mu i)}$, $\overline{m}(0)$ and $\overline{d}(0)$ initial conditions are calculated by assuming $\overline{m}(i)$ and $\overline{d}(i)$ functions pass through the values of currency holdings, deposits, interest rates observed at the end of sample period. Semi-elasticity η is measured from long-run semi-log asset demand functions.

[7] For the unrestricted model that allows different semi-elasticities for currency and deposits the welfare cost formula is written as $w(i) = \frac{1}{2}\left[\eta\overline{m}(0) + \varepsilon\mu^2\overline{d}(0)\right]$ where η is the semi-elasticity of currency and ε is the semi-elasticity of demand deposits.

3.3.2.2 Welfare Cost of Inflation for Double log Money Demand Function and Square-Root Formula

Square-Root formula is applicable if double-log is the proper specification of money demand function. Under this specification, as nominal interest rate approaches zero, demand for real balances become arbitrarily large (Ireland 2007), and equation (3.19) takes the form: $u[1 + w(i), \overline{m}(i), \overline{d}(\mu i)] = u[1, \infty, \infty]$.

a) Single Monetary Asset Model

Welfare cost formula for a single monetary aggregate (Monetary base or M1) without assigning distinct roles to currency and deposits the welfare function is given as

$$w(i) = \left[1 - \left(e^{\alpha_0}\right) i_t^{1-\alpha_1}\right]^{\alpha_1/(\alpha_1 - 1)} - 1 \qquad (3.23)$$

Where $\boldsymbol{\alpha_1}$ is slope (interest elasticity) and $\boldsymbol{\alpha_0}$ is the intercept term in log-linear model with single monetary aggregate.

b) Currency-Deposit Model

For currency-deposit model, the welfare cost is calibrated by employing estimated parameters from log-log specification of the demand deposits and currency demand functions.

$$w(i) = \left[1 - \left(e^{\alpha_0}\right) i_t^{1-\alpha_1} - \left(e^{\alpha_2}\right) \mu_t^{1-\alpha_1} i_t^{1-\alpha_1}\right]^{\alpha_1/(\alpha_1 - 1)} - 1 \qquad (3.24)$$

This model is derived from CES utility function where α_1 is the interest elasticity for both the assets demand functions.

Welfare cost of inflation is measured by empirically estimating the money demand function parameters. Welfare cost is calculated as the value of welfare measures evaluated at different nominal interest rates.

3.4 Estimation Procedure and Empirical Technique

The main objective of the study is to estimate the stable money demand function for Pakistan and using the results to compute welfare cost of inflation. This investigation follows the methodological framework developed in section 3.3 and we are mainly concerned to estimate the long-run relationship between real money balances (defined as ratio to the scale variable) and the interest rate. First we discuss the stationarity of time series and Unit Root tests and then the co-integration and ARDL approach in detail.

3.4.1 Unit Root

The cointegration analysis determines the nature of long-run relationship between a set of various time series variables. Further it also determines the stability of relationship and sources of that stability. The cointegration technique is based on the idea that some non-stationary variables may deviate from each other in the short run but they tend to drift at roughly the same rate. Prior to estimating any relationship, it is essential to check the stationarity or order of integration of the time series involved, because if the concerned series is non-stationary then the conventional regression analysis gives rise to spurious regression leading to incorrect statistical inferences. To this end unit root tests are conducted to draw inference about the underlying stochastic process of the variables being studied.

3.4.1.1 Stationary and Non-Stationary

A time series variable is said to be stationary if its mean and variance are constant over time and covariance is finite and independent of time. Conversely, time series data will be non-stationary, if its mean and variance are changing overtime, in other words this variable has a unit root. We can examine the stationarity of the different time series by applying unti root test. To make the concept of stationarity clear, let us suppose that;

$$y_t = \rho y_{t-1} + \varepsilon \qquad -1 < \rho < 1 \qquad (3.25)$$

$$\Delta y_t = \delta y_{t-1} + \varepsilon_t \qquad (3.26)$$

In practice, instead of estimating equation (3.25) we estimate (3.26) and test the null hypothesis that $\delta = 0$ which implies $\rho = 1$. Null hypothesis is that the time series is non-stationary and has a unit root. If $\delta = 0$ then equation (3.26) reduces to $\Delta y_t = \varepsilon_t$ where ε_t is white noise error term, such a series is stationary, which means that first difference of a random walk variable is stationary.

A non-stationary time series can be transformed into stationary time series through successive differencing and the number of times a series needs to be differenced so that it becomes stationary is called the order of integration. A non-stationary time series X_t is said to be integrated of order "d" if it achieves stationary after being differenced "d" times and is denoted by $X_t \sim I(d)$.[8] Thus, any time if we have variable integrated of order one or greater then we have a non-stationary time series. By convention, if d = 0, the resulting I(0) process represents a stationary time series.

3.4.1.2 Test of Integration

Time series analysis requires pre-testing the order of integration of the underlying variables. There are several methods available for the unit root tests like Dickey-Fuller (DF) test (Dickey and Fuller 1976), Augmented Dickey-Fuller (ADF) test (Dickey and Fuller 1979), Phillips-Perron (PP) test (Phillips Perron 1988) etc. The most commonly used method to examine the stationarity of a time series is the ADF test.

[8] For details see Granger (1986) and Engle and Granger (1987).

(i) Dickey-Fuller (DF) Test

Dickey Fuller test is proposed by Dickey and Fuller (1976) to test for the existence of unit root in a first order autoregressive model AR(1).

$$y_t = \phi y_{t-1} + u_t \qquad (3.27)$$

To get the testable form, subtract y_{t-1} from both sides

$$\Delta y_{t-1} = \gamma y_{t-1} + u_t \qquad (3.28)$$

Where $\gamma = (\phi - 1)$, presence of unit root means that $\phi = 1$ in (4.18) or $\gamma = 0$ in (3.28), the null hypothesis of presence of unit root is tested against the alternative hypothesis that $\phi < 1$ or that the series is stationary. The generalized model taking into account the drift (α) and time trend (βt) factors is given by

$$\Delta y_{t-1} = \alpha + \beta t + \gamma y_{t-1} + u_t \qquad (3.29)$$

For all the cases the Dickey-Fuller test for stationarity is the 't' test on γ, the coefficient of y_{t-1}. The computed t values are compared to MacKinnon (1991) critical values defined for each of the three models mentioned above. Dickey-Fuller test is based on the assumption that the error term u_t is white noise.

(ii) Augmented Dickey-Fuller (ADF) Test

In practice the error term may not be white noise and may be serially correlated which means that Dickey-Fuller test will not detect unit root for such model. As a solution Dickey and Fuller (1979) have developed Augmented Dickey Fuller (ADF) test to detect unit root, in the presence of serial correlation. In this test the lagged values of dependent variable are introduced to correct for autocorrelation.

$$\Delta y_t = a_0 + \gamma y_{t-1} + a_2 t + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + u_t \qquad (3.30)$$

If γ = 0 the equation is entirely in first differences and has a unit root. Test for stationarity is the test on coefficient of the lagged dependent variable which is compared to the critical values given by MacKinnon (1991). If the computed absolute value of the t statistic exceeds the critical value, then we do not reject the null hypothesis that the given time series is stationary.

3.4.2 Cointegration and the Autoregressive Distributed Lag Model (ARDL)

Cointegration technique is used to determine the long run relationship between different time series. Different cointegration techniques have been employed in the literature to estimate the long run relationships like Engle-Granger two step method, Johansen and Juselius cointegration tests. We will use Autoregressive distributed lag (ARDL) model to estimate the long run interest elasticity and semi-elasticity of money demand function. This approach has an advantage that it provides long-run coefficients even for small data set and it does not require all the regressors to be integrated of same order that is I(1). It can be applied in case where the regressors have mixed order of integration. The only restriction is that none of the variable is I(2) or integrated of order greater than 1. Problem of endogneity does not affect the bounds test for cointegration.

To apply the bounds test for cointegration the Unrestricted Error Correction Model (UECM) representation of double log money demand function: $m(r) = e^{a_0} i^{-a_1}$ takes the following form

$$\Delta \log(m_t) = \beta_0 + \sum_{i=1}^{p} \beta_{1i} \Delta \log(m_{t-i}) + \sum_{i=0}^{p} \beta_{2i} \Delta \log(i_{t-i}) + \lambda u_{t-1} + \varepsilon_t$$

In this equation m_t is real money balances taken as ratio to real GDP, i_t is the interest rate, β_0 is the intercept, β_1 and β_2 are the slope coefficients and λ is the coefficient of error correction term u_{t-1}, which shows the correction of model towards the long-run equilibrium. If the error correction term is replaced with the lagged variables we get the ARDL model incorporating short run and long run information.[9]

$$\Delta \log(m_t) = \beta_0 + \sum_{i=1}^{p} \beta_{1i} \Delta \log(m_{t-i}) + \sum_{i=0}^{p} \beta_{2i} \Delta \log(i_{t-i}) + \beta_3 \log(m_{t-1}) + \beta_4 \log(i_{t-1}) + u_t \tag{3.31}$$

Similarly to estimate the interest semi-elasticity of money demand the ARDL model takes the following form

$$\Delta \log(m_t) = \beta_0 + \sum_{i=1}^{p} \beta_{1i} \Delta \log(m_{t-i}) + \sum_{i=0}^{p} \beta_{2i} (i_{t-i}) + \beta_3 \log(m_{t-1}) + \beta_4 (i_{t-1}) + u_t \tag{3.32}$$

We will employ ARDL two step method of Bahmani-Oskooee and Bohal (2000) and find the maximum lag length (p), the order of UECM and check the existence of long run relationship. The null hypothesis of no cointegration implies that coefficients of lagged level variables β_3 and β_4 are simultaneously zero. ARDL approach of Pesaran and Shin (1998) can be applied by OLS method and the test is based on comparing the F value (joint significance of lagged levels of variables) of the model with the critical bound values given in Pesaran (2001). It reports the two asymptotic critical bounds values under two conditions (i) lower bounds assuming all the regressors to be I(0) and (ii) upper bound taking all the regressors to be I(1). If the calculated F statistics is less than the lower bound it shows that there is no long run relationship. If F value falls between the lower and the upper bound it means we enter the indecisive region. It is only when F value is greater than the upper bound, there exists the cointegration relationship. After

[9] Long-run elasticity can be derived directly as $-(\beta_4/\beta_3)$.

identifying the existence of long-run relationship and maximum lag length, we proceed to the second step to find the optimal lag length based on Akaike Information Criterion (AIC), Schwarz Bayesian Criterion (SBC) and $\overline{R}^2$, and calculate the long-run coefficients of the model.[10] Finally diagnostic tests are applied to check that the model passes the functional form stability, existence of heteroscedasticity and serial correlation or otherwise.

This procedure can be repeated by taking each monetary asset as dependent variable and respective opportunity cost as the independent variable in equations (3.10), (3.11), (3.12) and (3.13) to estimate the parameters of the money demand functions.

3.5 Data Description

We estimate the parameters of money demand function for different monetary aggregates using annual data of Pakistan for the period 1960 to 2007. The aggregate time series data for income, used as scale variable is measured by GDP, monetary aggregates are taken to be M1, monetary base, currency and demand deposits. Data for the GDP, and monetary assets is divided by the Consumer price index to get real balances to real income ratio. For single monetary aggregates and currency the opportunity cost is the nominal interest rate which is taken as the call money rate. For the demand deposit model long term interest rate, the relevant opportunity cost variable is defined as the difference between interest rate offered on other assets (long term assets) minus own rate of return (rate of return on current deposits and other deposits). Data on deposits rates excluding current and other deposits are compiled by State Bank of Pakistan since 1990. Using State

[10] Computation of ARDL procedure in Microfit 4.0, optimal lag selected on the basis of maximum values of AIC and SBC.

Bank's definition we have calculated it for the period 1960 to 1990 as weighted average of the interest rates on the individual longer term components of time deposits, with weights being quantity shares of these deposits. Calculation of welfare cost of inflation requires the information of the average reserve ratio for the entire sample period. Reserve Ratio is measured as the reserves taken as ratio to deposits.[11] Data for nominal GDP, monetary stocks, CPI and call money rate are obtained from International Financial Statistics (IFS). Data for rate of return on long-term maturity deposits, however, is taken from State Bank of Pakistan Annual Reports.

[11] Following Agenor & Montiel (1996) reserve ratio is measured as (Reserve Money – Currency)/(M1 + Quasi Money – Currency).

CHAPTER 4

ESTIMATION RESULTS

As dicussed in chapter 3 we have calculated the welfare cost of inflation based on the long-run estimated demand functions for the four monetary aggregates. To estimate the interest semi-elasticity and elasticity of money demand functions ARDL test is conducted on each of the money stock. This chapter is divided into different sections. First section is related to determination of the order of integration for all the variables used in the analysis. The test statistics of the ARDL model for each money demand function and the welfare cost calculations are presented and interpreted in the subsequent sections.

4.1 Unit Root Test and the Order of Integration

To start the analysis we proceed by checking the stationarity characteristics of the data series using Augmented Dickey Fuller (ADF) test. ADF is applied to variables to test the presence of unit root in the final form in which they appear in the estimation model i.e. the natural logarithms of all the monetary aggregates taken as ratio to GDP, the interest and deposit rates along with their natural logarithms.

The number of lags ρ is selected to correct for serial correlation. For this purpose we started with zero lag and continued adding lags until the Breusch Godfrey LM test, applied to the residual of the ADF regression, showed no serial correlation. Whether the ADF regression has an intercept only or an intercept along with trend, ADF general to specific method was used Enders (2004). Starting with the general form which includes both the constant and deterministic trend, the significance of the trend coefficient a_2 based on the t-value is checked. If it is significant and hypothesis of unit root is not rejected we conclude that the test includes constant and trend.

Table 4.1 ADF Unit Root Test on Variables

	Levels			First Differences		
Variable	Lags (ρ)	Model	τ – value	Lags (ρ)	Model	τ- value
Log(M1/GDP)	1	Constant	-2.6890	0	Constant	-5.9508**
Log(Mo/GDP)	0	Constant	-2.4537	0	Constant	-6.7867**
Log (Currency/GDP)	2	Constant	-2.0391	0	Constant	-5.4555**
Log (Demand Deposits/GDP)	0	Constant & Trend	-2.6438	0	Constant	-7.1718**
Interest Rate	0	Constant	-2.4830	0	Constant	-6.7077**
Log (Interest Rate)	0	Constant	-2.6760	1	Constant	-5.7319**
Deposit Rate	0	Constant	-1.7544	0	Constant	-7.5469**
Log (Deposit Rate)	1	Constant	-2.3256	0	Constant	-5.3193**

Notes: ADF regression equation : $\Delta y_t = \alpha_0 + \gamma y_{t-1} + \alpha_2 t + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + u_t$

The null and alternative hypotheses for ADF test applies on the coefficient of the first lag of dependent variable γ. Under null hypothesis $\gamma = 0$ or the series is nonstationary and under alternative hypothesis of stationarity $\gamma < 0$. γ has non-standard distribution so τ-value is compared to McKinnon (1991) critical values. Critical values at 5% level of significance are -2.9266 and -3.51074 for the constant only and constant & trend models respectively. ** indicates that the series are stationary at 1% level of significance.

Table 4.1 shows the results of ADF test, except for log (demand deposits/GDP) all variables have insignificant trend coefficient, as shown in the third column. The null-hypothesis of non- stationarity is not rejected at 5% significance level for all the variables in their level while their first difference is stationary which means all the series are I(1). As none of the series is integrated of order greater than one, it means we can apply ARDL bounds test for cointegration.

4.2 Estimation of Money Demand Function and Calculation of Welfare Cost of Inflation for Monetary Base

4.2.1 Estimation of the Semi-Log Demand Function for the Monetary Base

As discussed in section 3.4 we apply two step ARDL approach to estimate the semi-log money demand function with money taken to be monetary base.

$$\Delta \log(m_{0_t}) = \beta_0 + \sum_{i=1}^{p} \beta_{1i} \Delta \log(m_{0_{t-i}}) + \sum_{i=0}^{p} \beta_{2i} \Delta i_{t-i} + \beta_3 \log(m_{0_{t-1}}) + \beta_4 i_{t-1} + u_t$$

In first step we are interested in the identifying the existence of long run relationship, the null hypothesis of no cointegration, Ho: $\beta_3 = \beta_4 = 0$ is tested against the alternative H_1: $\beta_3 \neq \beta_4 \neq 0$ by comparing the F-value for the joint significance of the coefficients of the first lag of the variables with the critical F-value bounds given in Pesaran et.al (2001).

Table 4.2 shows that the F statistics is sensitive to the no. of lags imposed on the first difference of the variables.[12] As unit root test showed that monetary base and interest rate series had drift only so the ARDL equation does not include trend term. F value is compared to the critical bounds of case iii in Pesaran et.al. (2001).

Table 4.2 Result of Bounds test for the existence of long-run relationship (Semi-log MB)

No. of Lag	AIC	SBC	CHSQsc(1)	$\overline{R}^2$	F-Statistics
1	50.9068	47.2065	0.046023[.830]	0.78119	(3,43) 55.7442[.000]
2	47.7316	42.2457	0.097278[.755]	0.76171	(5,40) 29.7689[.000]
3	46.4160	39.1893	.022695[.880]	0.76003	(7,37) 20.9076[.000]
4	42.9489	34.0280	.083171[.773]	0.72571	(9,34) 13.6410[.000]

Notes: Asymptotic critical value bounds are obtained from Pesaran et.al.(2001) Table F in appendix C, Case III: unrestricted intercept and no trend for k=1, at 1% level of significance lower bound = 6.84 and upper bound = 7.84, at 5% level of significance lower bound = 4.94 and upper bound = 5.73.

[12] This is identified in Bahmani-Oskooee and Bohl (2000) and Bahmani-Oskooee and Ng (2002).

For all the four lags the F- value exceeds the upper bound at 1% level of significance, representing that long run relationship exists between monetary base (taken as ratio to income) and the call money rate. Based on the Akaike Information Criterion (AIC) maximum lag is chosen to be 1.

Once the existence of long-run relationship is established we move to the second step to estimate the long-run coefficients (α_0 and α_1) of the money demand function.

$$\log[m_t(i)] = \alpha_0 - \alpha_1 i_t + \varepsilon_t$$

For this purpose we want to find the order of ARDL (p,q) for the two variables.

$$\log(m_{0t}) = \beta_0 + \sum_{i=1}^{p} \gamma_{1i} \log(m_{0t-i}) + \sum_{i=0}^{q} \gamma_{2i} i_{t-i} + u_t$$

Upper panel of table 4.3 shows the long-run estimates of the model, both the parameters of the model are significant at 5% level of significance. The interest rate semi-elasticity of monetary base (α_1) shows that a 1% increase in nominal interest rate lowers the demand for monetary base by 5.5%. Based on all the three criteria SBC, AIC and $\bar{R}^2$ the final ARDL (1,0) is selected.

Table 4.3 Long run Coefficients of the Semi log model for Monetary Base based on ARDL (1,0)

Regressor	Coefficient	Standard Errors	T value [Prob.]
C	-1.3668	0.13847	-9.8710 [0.000]
Interest Rate	-0.054999	0.017964	-3.0617 [0.004]
R^2 0.79490		DW-Stat 1.9974	
$\bar{R}^2$ 0.78558		F(2, 44) 85.2636[.000]	
Diagnostic Tests			
$\chi^2_{SC(1)}$ 0.00255[0.960]			
$\chi^2_{FF(1)}$ 1.3438[0.246]			
$\chi^2_{N(2)}$ 25.1256[0.000]			
$\chi^2_{Het(1)}$ 0.24024[0.624]			

Notes: For diagnostic test the Lagrange Multiplier statistics $\chi^2_{SC(1)}$, $\chi^2_{FF(1)}$, $\chi^2_{Het(1)}$ are the distributed with chi-square values for serial correlation, functional form mis-specification and Heteroscedasticity at 1 degree of freedom.

Lower panel shows the diagnostic tests and the overall goodness of fit of the model. High $R^2 = 0.79$ shows that the ARDL specification (1,0) is a quite good fit, and the model passes the diagnostic tests of functional form mis-specification, heteroscedasiticity and serial correlation.

4.2.2 Welfare Cost of Inflation for the Semi-log Model

In this subsection we derive the welfare cost of inflation formula based on (i) Lucas compensating variation measure and (ii) consumer's surplus measure.

(i) Compensating Variation Approach Following the procedure spelled out in section 3.3.2.1 for single monetary asset model the welfare cost function for semi-log specification is where $\overline{m}(i) = \overline{m}(0)e^{-\eta i}$ .To evaluate the welfare cost for Mo we substitute the 2007 observed value of (Mo/GDP) = $m(i)$ = 0.157106 , i = 0.093 and the estimated semi-elasticity with respect to nominal interest rate $\eta = 5.4999$ to find the intial condition $\overline{m}(0)$ and substituting in 3.21 gives us the following welfare function

$$w(i) = 0.7205 i^2. \qquad (4.1)$$

Welfare cost is measured both as moving from Friedman's optimal inflation rate to some positive inflation rate (from zero nominal interest rate to positive interest rate) and moving from zero inflation (stable price) to positive inflation rate. Real interest rate is approximately 2% for 2007 therefore, $i = 0.02$ is the benchmark value of nominal interest rate under zero inflation.[13] When $i = 0.08$ it means inflation rate is 6%, and for $i = 0.10$ inflation rate is 8%. Table 4.4 shows the welfare cost as percent of GDP associated with increasing interest rate from zero to a positive rate. Welfare cost entry against each interest rate is the loss in welfare for deviating from the Friedman's deflation rule.

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[13] Following Gillman (1993) $i = 0.093$ and $\pi = 0.0721$ giving the value of r approximately equal to 0.02

Table 4.4 Welfare cost of inflation for semi-log demand function for the Monetary Base

Interest rate	Compensating variation Approach $w(i) = 0.7205i^2$	Consumer's Surplus Approach $WC = \frac{e^{-1.3668}}{5.4999}\left[1 - e^{-5.4999i}(1 + 5.499i)\right]$
0	0.0000	0.0000
0.01	0.0072	0.0068
0.02	0.0288	0.0261
0.03	0.0648	0.0566
0.04	0.1153	0.0970
0.05	0.1801	0.1462
0.06	0.2594	0.2032
0.07	0.3530	0.2669
0.08	0.4611	0.3364
0.09	0.5836	0.4111
0.1	0.7205	0.4900
0.2	2.8820	1.3950
0.3	6.4845	2.2761
0.4	11.5280	2.9915
0.5	18.0125	3.5238
0.6	25.9380	3.8999
0.7	35.3045	4.1566
0.8	46.1120	4.3277
0.9	58.3605	4.4396
1	72.0500	4.5119

Second column shows the welfare cost using compensating variation approach, welfare cost of 5% nominal interest (3% inflation) is 0.15 percent of GDP against zero inflation, while comparing with zero nominal interest rate (optimal deflation rule) the cost is approximately 0.18 percent of GDP. Keeping in view the end of sample period inflation rate of 7% ($i = 0.09$) the welfare gain of moving towards zero inflation ($i = 0.02$) is 0.55 percent of GDP (the difference between the welfare costs at 9% and 2% nominal interest, 0.5836 and 0.0288 respectively) and further moving to the deflation rate results in an additional gain of 0.028 percent of GDP. The welfare cost estimates against zero inflation policy are shown in Appendix B.

(ii) Consumer's Surplus Approach

Plugging the values of the long-run parameters of semi-log monetary base demand function in equation 3.15 gives the following welfare cost formula

$$WC = \frac{e^{-1.3668}}{5.4999}\left[1 - e^{-5.4999i}(1 + 5.499i\ )\right] \qquad (4.2)$$

by substituting different values of nominal interest rate into the welfare function we get the welfare cost given in column 3 of table 4.4. Welfare cost of 5% nominal interest rate is 0.12 percent of GDP against price stability and slightly higher at 0.14 percent of GDP when compared to zero nominal interest rate. Similarly the welfare cost of 7 percent inflation is 0.41 percent of against the deflation rate, which is less than the 0.58 percent of GDP calculated under Lucas (2000) approach form. In table 4.4 we find that for all the nominal interest rates the welfare cost is higher under compensating approach than under Bailey's approach. The costs under two approaches are comparable for the single digit nominal interest rates and the difference widens for the higher interest rates. Deviating from Friedman's Deflation rule, cost of 20 percent nominal interest rate is 2.8 percent of income under Lucas' approach, while based on equation 4.2 for the Bailey's approach the cost is 1.4 percent. The difference between the calculated welfare loss from the two approaches is due to the quadratic nature of the compensating variation formula (4.1), in which the nominal interest rate appears in the quadratic form

Figure 4.1: Welfare cost of inflation for Monetary base Semi-log model

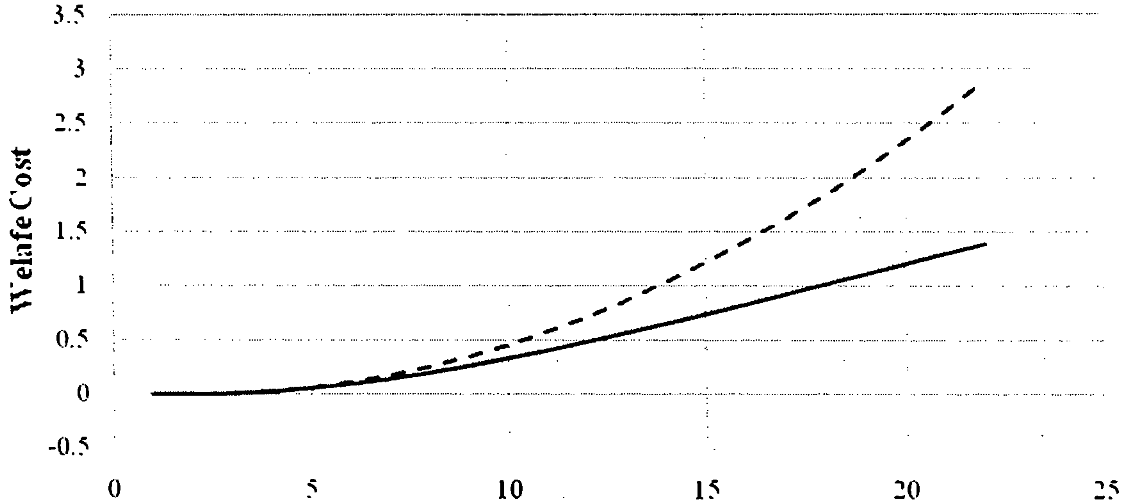


Figure 4.1 plots the welfare loss (relative to deflation rule), at different nominal interest rates. Nominal interest rate upto 20 percent is taken on the horizontal axis. Dotted curve shows the welfare loss based on compensating variation approach and the solid line under consumer surplus approach. The two approaches give almost same welfare loss calculations for low inflation/ interest rates but they tend to diverge for higher interest rates.

4.2.3 Estimation of the Double Log Demand Function for the Monetary Base

For the double log money demand function the ARDL approach takes the following form

$$\Delta \log(m_{0t}) = \beta_0 + \sum_{i=1}^{p} \beta_{1i} \Delta \log(m_{0t-i}) + \sum_{i=0}^{p} \beta_{2i} \Delta \log(i_{t-i}) + \beta_3 \log(m_{0t-1}) + \beta_4 \log(i_{t-1}) + u_t$$

We apply the OLS to the above equation to find the maximum lag of the first difference variables and apply the F test on the first lags levels of the variables exhibiting long run relationship. The F-stat doesn't follow standard distribution so its values are compared to the bounds values of Persaran et. al (2001). We check for the long run relationship taking 4 lags. The results are shown in table 4.5

Table 4.5 Results of Bounds test for the existence of long-run relationship (log-log MB)

No. of Lag	AIC	SBC	CHSQsc(1)	$\bar{R}^2$	F-Statistics
1	50.2872	46.5869	0.215E-3[.988]	0.77535	(3,43) 53.9208[.000]
2	47.2359	41.7499	1.0720[.301]	0.75652	(5,40) 28.9636[.000]
3	44.8887	37.6620	0.029734[.863]	0.74317	(7,37) 19.1886[.000]
4	41.6240	32.7031	0.43416[.510]	0.70868	(9,34) 12.6229[.000]

Notes: Asymptotic critical value bounds are obtained from Pesaran et.al.(2001) Table F in appendix C, Case III: intercept and no trend for k=1, at 1% level of significance lower bound = 6.84 and upper bound = 7.84, at 5% level of significance lower bound = 4.94 and upper bound = 5.73.

There exists long-run relationship for all the lags as the calculated F-value is greater than upper bound for $p=1$, at 5 and 1% levels of significance. Based on AIC, SBC and $\bar{R}^2$ the maximum lag is taken to be 1. Based on all the three criteria ARDL of order (1,0) is applied in the second step.

Table 4.6 Long run Coefficients of the double log model for MB based on ARDL (1,0)

Regressor	Coefficient	Standard Errors	T value [Prob.]
C	-2.6650	0.33587	-7.9347[.000]
log(Interest Rate)	-0.33186	0.12359	-2.6852[.010]
R^2 0.78891		DW-stat 1.9491	
$\bar{R}^2$ 0.77931		F (2,44) 82.2186[.000]	
Diagnostic Tests			
$\chi^2_{SC(1)}$ 0.017587[.894]			
$\chi^2_{FF(1)}$ 1.1021[.294]			
$\chi^2_{Het(1)}$ 0.15956[.690]			
$\chi^2_{N(2)}$ 22.7175[.000]			

Notes: For diagnostic test the Lagrange Multiplier statistics $\chi^2_{SC(1)}$, $\chi^2_{FF(1)}$, $\chi^2_{Het(1)}$ are the distributed with chi-square values for serial correlation, functional form mis-specification and Heteroscedasticity at 1 degree of freedom

The long run estimates and diagnostic test are reported in table 4.6. Long-run parameters have expected signs and are statistically significant. Interest elasticity of the demand for monetary base is 0.33 almost same (0.34) estimated in Hussain et.al.(2006). R^2 is 0.79 which shows the goodness of fit of the model. Diagnostic tests indicate that there is no problem of serial correlation, heteroscedasiticity and functional form mis-specification in the selected model.

4.2.4 Welfare Cost of Inflation for the Double-log Model

Welfare cost of inflation is calculated using both Lucas (2000) compensating variation approach and Bailey's (1956) consumer's surplus approaches.

(i) Compensating Variation Approach

For the log-linear money demand function $m(i) = e^{\alpha_0} i^{-\alpha_1}$ the welfare cost function under compensating variation approach is given by equation 3.23 substituting the values of the parameters the welfare function becomes

$$w(i) = \left[1 - 0.0696 i_t^{0.66814}\right]^{-0.4967} - 1. \qquad (4.3)$$

The welfare cost of 5 percent nominal interest rate is 0.47 percent of real income.

Welfare cost formula shows that after substituting the values of estimated parameters it is function of nominal interest only. Welfare cost of 5 percent inflation against the bench mark of zero inflation is calculated as $w(0.05) - w(0.02) = 0.4704 - 0..2542 = 0.21$ percent of income. Cost of 9 percent nominal interest rate the call money rate at the end of sample period costs about 0.7 percent of real output. Reducing nominal interest rate from 9 percent to 2 percent (under zero inflation) yields welfare gain equivalent to an increase in income by 0.44 percent.

Table 4.7 Welfare Cost of Inflation for Double log demand function for the Monetary Base

	Compensating Variation Approach	Consumer's Surplus Approach
Interest rate	$w(i) = \left[1 - 0.0696 i_t^{0.66814}\right]^{-0.4967} - 1$	$WC = 0.4967 e^{-2.665} i^{0.66814}$
0	0.0000	0.0000
0.01	0.1598	0.1594
0.02	0.2542	0.2533
0.03	0.3337	0.3320
0.04	0.4049	0.4024
0.05	0.4704	0.4671
0.06	0.5319	0.5276
0.07	0.5901	0.5849
0.08	0.6457	0.6395
0.09	0.6991	0.6918
0.1	0.7507	0.7423
0.2	1.2009	1.1795
0.3	1.5835	1.5465
0.4	1.9289	1.8742
0.5	2.2496	2.1756
0.6	2.5523	2.4574
0.7	2.8412	2.7240
0.8	3.1189	2.9782
0.9	3.3874	3.2220
1	3.6482	3.4570

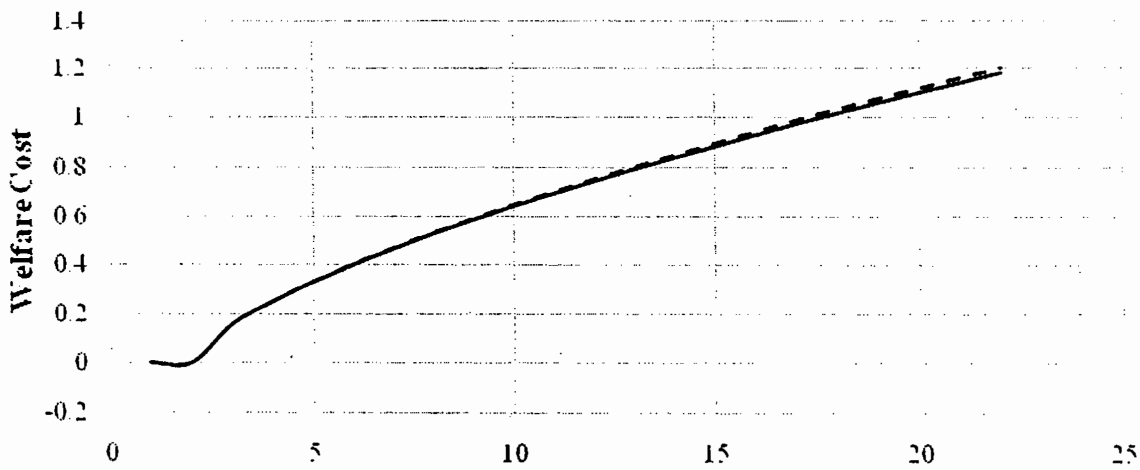
(ii) Consumer's Surplus Approach

Equation 3.17 gives the welfare cost function for the double log single asset model, with the estimated money demand function the it gets the form

$$WC = 0.4967 e^{-2.665} i^{0.66814} \qquad (4.4)$$

the welfare cost based on this formula for different nominal interest rates are shown in the third column of table 4.7.

Figure 4.2: Welfare Cost of Inflation for Monetary Base Log-Log Model



For all the nominal interest rates the cost of inflation under both Lucas approach (shown by dotted line in figure 2) and Bailey's approach (solid line) is almost same.

Comparing the table 4.6 and 4.7 (figures 4.1 and 4.2) we find that the welfare cost of inflation for moderate inflation under semi-log money demand function is relatively small compared to that under log-log model. Moving from zero inflation to deflation rule results in welfare gain of only 0.02 percent of GDP in semi-log model compared to a substantial gain of 0.25 percent of GDP for the double log model.[14]

[14] Wolman (1997) and Ireland (2007) show that towards low interest rate the semi-log model shows satiation in money holding but for the double log model the money holdings take asymptotic trend as nominal interest rate approaches zero.

4.3 Estimation of Money Demand Function and Calculation of Welfare Cost of Inflation for M1

4.3.1 Estimation of Semi-log demand function for M1

ARDL specification of the semi-log demand function for M1 is

$$\Delta \log(m_{1t}) = \beta_0 + \sum_{i=1}^{p} \beta_{1i} \Delta \log(m_{1t-i}) + \sum_{i=0}^{p} \beta_{2i} \Delta i_{t-i} + \beta_3 \log(m_{1t-1}) + \beta_4 i_{t-1} + u_t$$

Applying the two step ARDL procedure we first identify the maximum lag ρ and the existence of long-run relationship. Up to 4 lags are taken and the results for four key criteria for the selection of maximum lag are reported in the table 4.8.

Table 4.8 Result of Bounds test for the existence of long-run relationship

No. of Lag	AIC	SBC	CHSQsc(1)	$\bar{R}^2$	F-Statistics
1	48.8968	45.1965	5.0912[.024]	0.55948	(3, 43) 20.4738[.000]
2	52.3117	46.8258	10.3697[.001]	0.65777	(5, 40) 18.2982[.000]
3	60.3999	53.1732	.027268[.869]	0.78550	(7, 37) 24.0187[.000]
4	56.9606	48.0396	0.64497[.422]	0.77398	(9, 34) 17.3608[.000]

Notes: Asymptotic critical value bounds are obtained from Pesaran et.al.(2001) Table F in appendix C, Case III: intercept and no trend for k=1, at 1% level of significance lower bound = 6.84 and upper bound = 7.84, at 5% level of significance lower bound = 4.94 and upper bound = 5.73.

Based on all the criteria lag length 3 is selected. We further move to the second step and estimate the long-run parameters of the model with maximum lag set at 3. Based on SBC, AIC and $\bar{R}^2$ ARDL of order (3,3) is selected. The long-run parameters of M1 function and the diagnostic test results are summarized in table 4.9. The long run coefficients of money demand function have theoretically correct signs and are significant at 1% level of significance, the interest semi-elasticity of M1/GDP ratio is 3.1718. Semi-log money demand function for M1 satisfies all the diagnostic tests.

Table 4.9 Long run Coefficients of the model based on ARDL (3,3)

Regressor	Coefficient	Standard Errors	T value [Prob.]
C	-1.0131	0.031801	-31.8585[.000]
Interest Rate	-0.031718	0.0040270	-7.8763 [.000]
R^2 0.81963		DW-Stat 2.0111	
$\bar{R}^2$ 0.78550		F(7, 37) 24.0187[.000]	
Diagnostic Tests			
$\chi^2_{SC(1)}$ 0.02726 [0.869]			
$\chi^2_{FF(1)}$ 0.04996 [0.823]			
$\chi^2_{Het(1)}$ 1.0901 [0.296]			
$\chi^2_{N(2)}$ 0.00878[0.996]			

Notes: For diagnostic test the Lagrange Multiplier statistics $\chi^2_{SC(1)}$, $\chi^2_{FF(1)}$, $\chi^2_{Het(1)}$, and $\chi^2_{N(2)}$ are the distributed with chi-square values for serial correlation, functional form mis-specification, Heteroscedasticity, and Normality test with degrees of freedom in parentheses.

4.3.2 Welfare Cost of Inflation for the Semi-log Model

(i) Compensating Variation Approach

Substituting the end of period observed values of nominal interest rate and the M1 GDP ratio (0.093, 0.39091) in the quadratic welfare cost formula $w(i) = \frac{1}{2}\bar{m}(0)\eta i^2$ along with the semi-elasticity of M1 with respect to nominal interest rate as 3.1718, gives final form of formula as

$$w(i) = 0.8326i^2 \qquad (4.5)$$

Column 2 of table 4.10 gives the value of welfare loss against different nominal interest rates based on compensating variation approach. Welfare loss of 3 percent inflation corresponding to 5 percent nominal rate of interest is 0.21 percent of GDP against zero interest rate, while it reduces to 0.17 percent against price stability. Welfare loss associated with inflation rate of 7 percent is 0.64 percent of income compared to zero inflation rate, while reducing the inflation further to deflation rate results in additional gain of 0.03 percent of GDP or total gain of 0.67 percent of GDP. Semi-log money demand function under compensating variation gives quadratic formula for the welfare cost, column 2 of table 4.10 shows that the welfare cost associated with larger nominal interest rates/ inflation rates is substantially high compared to lower inflation rates.

Table 4.10 Welfare Cost of inflation for semi-log demand function for M1

	Compensating variation	**Consumer Surplus**
Interest rate	$w(i) = 0.8326i^2$	$WC = 0.1145\left[1 - e^{3.1718}(1+3.1718i)\right]$
0	0.0000	0.0000
0.01	0.0083	0.0056
0.02	0.0333	0.0221
0.03	0.0749	0.0487
0.04	0.1332	0.0847
0.05	0.2082	0.1296
0.06	0.2997	0.1828
0.07	0.4080	0.2437
0.08	0.5329	0.3117
0.09	0.6744	0.3865
0.1	0.8326	0.4674
0.2	3.3304	1.5264
0.3	7.4934	2.8209
0.4	13.3216	4.1446
0.5	20.8150	5.3861
0.6	29.9736	6.4922
0.7	40.7974	7.4448
0.8	53.2864	8.2456
0.9	67.4406	8.9068
1	83.2600	9.4452

(ii) Consumer's Surplus Approach

Substituting the semi-log money demand parameters of table 9 into equation (4.2) gives the welfare cost of inflation formula based on the Bailey's approach as

$$WC = 0.1145\left[1 - e^{3.1718}(1+3.1718i)\right] \qquad (4.6)$$

Welfare cost calculated of positive interest rates are reported in Column 3 of table 4.10. The welfare loss as a proportion of GDP rises from 0.02 percent when the rate of interest is 2 percent (inflation rate is zero) to over 0.38 percent at a rate of interest of 9 percent. Difference between these two welfare costs (0.38-0.02 = 0.36) gives the welfare loss of 7 percent inflation rate against zero inflation. Welfare cost of 3% inflation is 0.13 percent of income and moving to zero interest rate yields welfare gain of 0.02 percent of GDP.

4.3.3 Estimation of Double-log Demand Function for M1

Long run constant elasticity of the money demand function is estimated by applying two step ARDL approach, the results of the first step for the existence of long run relationship between nominal interest rate and M1 are reported in table 4.11. The maximum lag is chosen to be 3, based on the maximum value of SBC and AIC reported in Microfit 4.0. For all the lag lengths the F calculated value is greater than the upper bound critical value reported in Pesaran et. al.(2001).

Table 4.11 Result of Bounds test for the existence of long-run relationship

No. of Lag	AIC	SBC	CHSQsc(1)	$\bar{R}^2$	F-Statistics
1	48.5930	44.8928	4.0310[.045]	0.55375	(3,43) 20.0268[.000]
2	54.0301	48.5442	7.9827[.005]	0.68241	(5,40) 20.3382[.000]
3	56.4604	49.2337	0.081158[.776]	0.74446	(7,37) 19.3118[.000]
4	52.6491	43.7282	0.23365[.629]	0.72505	(9,34) 13.5989[.000]

Notes: Asymptotic critical value bounds are obtained from Pesaran et.al.(2001) Table F in appendix C, Case III: intercept and no trend for k=1, at 1% level of significance lower bound = 6.84 and upper bound= 7.84, at 5% level of significance lower bound = 4.94 and upper bound = 5.73

With maximum lag taken to be 3, the ARDL of order (3,3) is selected based on all three criteria of SBC, AIC and $\bar{R}^2$ the results for the long-run parameters are reported in table 4.12.

Table 4.12 Long run Coefficients of the model based on ARDL (3,3)

Regressor	Coefficient	Standard Errors	T value [Prob.]
C	-0.84401	.062659	-13.4699[.000]
log(Interest Rate)	-0.20890	0.031429	-6.6468[.000]
R^2 0.78511		DW-stat 2.0190	
$\bar{R}^2$ 0.74446		F (7,37) 19.3118[.000]	
Diagnostic Tests			
$\chi^2_{SC(1)}$ 0.081158[.776]			
$\chi^2_{FF(1)}$ 4.5980[.032]			
$\chi^2_{N(2)}$ 0.24570[.884]			
$\chi^2_{Het(1)}$ 1.1223[.289]			

Notes: For diagnostic test the Lagrange Multiplier statistics $\chi^2_{SC(1)}$, $\chi^2_{FF(1)}$, $\chi^2_{Het(1)}$, and $\chi^2_{N(2)}$ are the distributed with chi-square values for serial correlation, functional form mis-specification, Heteroscedasticity, and Normality test with degrees of freedom in parentheses.

Constant interest elasticity of money demand turns out to be 0.21 and the regression passes all the diagnostic tests except the functional form mis-specification test.

4.3.4 Welfare Cost of Inflation for Double-log Model

Based on the long run parameters of the log-log demand function for M1, we calculate welfare loss based on compensating variation and consumer surplus approaches.

(i) Compensating Variation approach

To evaluate the welfare cost of inflation for the log-linear demand function the values of the long run parameters are substituted into equation 3.23 the square-root welfare loss formula defined for single monetary stock and double-log money demand function. Column 2 of table 4.13 shows the welfare loss at different nominal interest rates based on the welfare formula

$$w(i) = \left[1 - 0.1643 i_t^{0.7911}\right]^{-0.2641} - 1 \qquad (4.7)$$

The welfare loss measured as percentage to GDP is about 0.65 percent for 9 percent interest rate relative to zero interest rate. While for stable prices the loss is slightly lower and its value is 0.45 percent of GDP. For 5 percent interest rate / 3 percent inflation the welfare loss is 0.21 percent of GDP compared to the zero inflation (reported in Appendix B) and raises by 0.21 percent of GDP when compared to zero nominal interest rate.

Table 4.13 Welfare Cost of inflation for Double –log model and M1 defined as the money

	Calibration	Consumer Surplus
Interest rate	$w(i) = \left[1 - 0.1643 i_t^{0.7911}\right]^{-0.2641} - 1$	$WC = 0.2640 e^{-1.8060} i^{0.7911}$
0	0.0000	0.0000
0.01	0.1139	0.1135
0.02	0.1974	0.1964
0.03	0.2726	0.2707
0.04	0.3428	0.3399
0.05	0.4096	0.4055
0.06	0.4739	0.4685
0.07	0.5362	0.5292
0.08	0.5968	0.5882
0.09	0.6560	0.6456
0.1	0.7140	0.7017
0.2	1.2512	1.2143
0.3	1.7445	1.6735
0.4	2.2143	2.1012
0.5	2.6698	2.5068
0.6	3.1158	2.8958
0.7	3.5553	3.2713
0.8	3.9906	3.6358
0.9	4.4232	3.9909
1	4.8544	4.3378

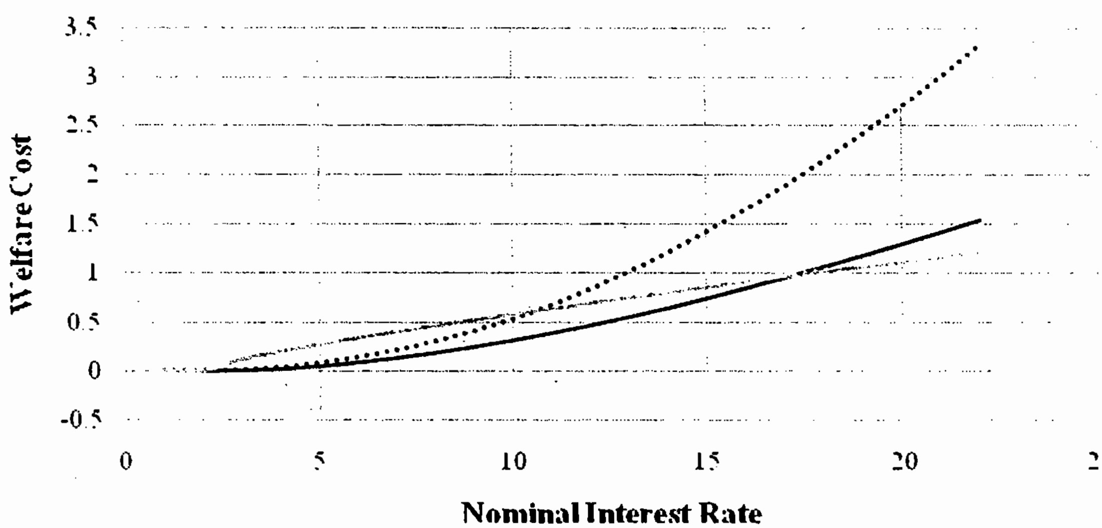
(ii) Consumer's Surplus Approach

Welfare loss formula derived by integrating the area under the log-linear money demand function given by equation (3.17) takes the following form

$$WC = 0.0434 i^{0.7911} \quad (4.8)$$

After substituting the long-run parameters of the money demand function we find that welfare cost is a function of nominal interest rate. Column 3 in table 4.13 shows the welfare loss as proportion to GDP at different interest rates. Each figure in this column is the cost to welfare against zero nominal interest rate/ Friedman's deflation rule. The welfare loss to the economy in terms of lost services of the real balances is 0.41 percent of GDP at 5 percent interest rate and it rises to 0.65 percent of GDP at 9 percent rate of interest. Welfare loss estimates for both the compensating variation and consumer's surplus approaches are approximately equal for low and moderate inflation rates.

Figure 4.3: Welfare cost functions for M1 relative to zero nominal interest rate

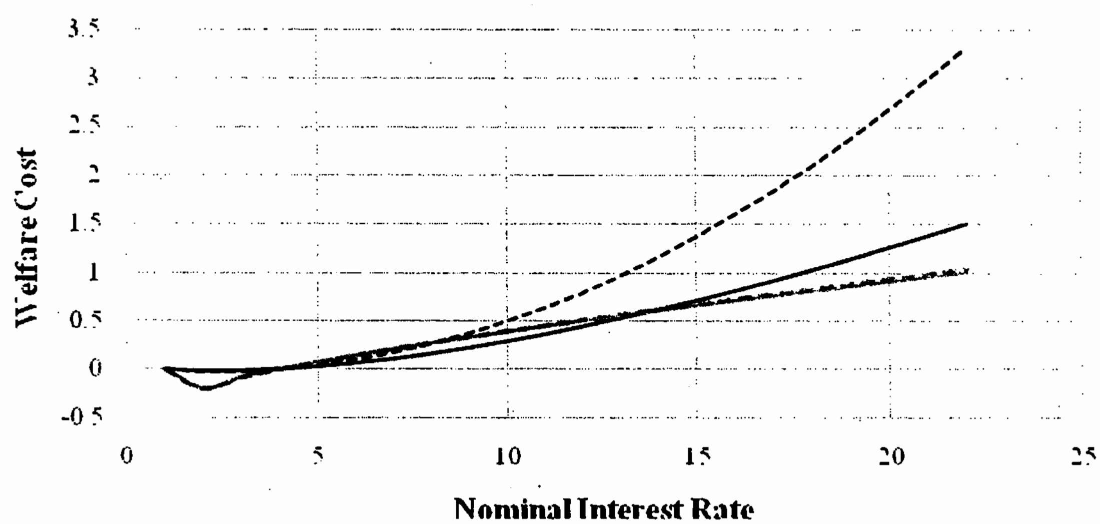


If we compare the welfare cost of inflation under both semi-log and log-log models for M1, we find that welfare cost from semi long money demand function (under both traditional and Lucas approaches) gives higher cost for higher interest rates shown by the darker lines in the figure 4.3 (dotted darker line for compensating variation approach and solid lines for the consumer surplus approach). Welfare cost based on the double-log money demand function shown by the two overlapping lighter coloured lines shows that both the approaches give almost same measure of welfare loss from deviating from the deflation rule. Welfare gain of moving from higher to lower nominal interest rate is almost same for the log-log model but for the semi-log model the welfare gain is associated with the rate of interest. Under the semi-log model one percent decrease in nominal interest rate for higher interest rate results in more benefit compared to one percent decrease in nominal interest rate at the lower end of the curve.

As in Wolman (1997) we are interested in the apportionment of the total gain of moving from positive interest rate to the deflation rate. This gain has two parts; first gain comes in moving from positive nominal interest rate to price stability and second from moving from zero inflation to the deflation policy. Owing to the sensitivity of the demand curves to low interest rates we find that for the semi-log model larger benefit accrues as the economy move towards zero inflation but further moving to deflation rate has very small gain. Figures 4.3 and 4.4 show that for semi-log money demand function under consumer surplus the welfare gain of moving from 12 percent interest rate to deflation rate is equal to 0.64 percent of GDP while for double log the gain is 0.81 percent of income. The proportion of gain from moving from zero inflation to deflation is only 5.15 percent of the total gain for the semi-log model and for the double log it is 24.2 percent.

The difference between the estimates of welfare loss is reduced when the cost is measured relative to zero inflation nominal interest rate. For the present study the end of period real interest rate is 2 percent which under price stability is equivalent to the nominal interest rate. As shown in figure 4.4 the welfare cost of non-optimal policy with positive inflation rate has same welfare loss under the three cases the semi-log model- the lighter coloured overlapping lines (drawn under compensating variation and traditional approaches) and the double-log model with consumer surplus approach shown by the dark solid line.[15]

Figure 4.4: Welfare Cost of Inflation under both specifications relative to 2 percent interest rate



The gain of moving from 12 percent interest rate to stable prices ranges from 0.61 to 0.63 percent of income. For both the money demand specifications for M1 the welfare loss is almost same for low and moderate inflation rates. The welfare loss line drawn for double log model under compensating variation approach diverges from the rest of the three cases as interest rate rises above 10 percent as shown in figures 4.3 (and figure 4.4).

[15] Lucas (2000) showed that the welfare gain of moving towards price stability is same for both the log-log and semi-log versions.

4.4 Estimation of Demand Function and Calculation of Welfare Cost of Inflation for Currency-Deposit Model

After estimating the demand function and the welfare costs for the single money stocks, Mo and M1 we estimate welfare loss for the Currency-Deposit Model. We disintegrate the two components of M1 for the reason that both currency and demand deposits don't have the same opportunity cost. Hand-to-hand used currency offers no return, its opportunity cost is the yield on other financial assets while banking system offers interest rate on the demand deposits, opportunity cost of holding demand deposits is the difference between the yield on alternative assets and the return on deposits. This difference requires that both currency and demand deposit demand functions be estimated separately with their own opportunity costs. This also required some modification in the welfare cost formulae as discussed in chapter 3. We apply the ARDL approach on bivariate models separately for currency and demand deposits.

4.4.1 Estimation of Semi-log Money demand function for Currency and Demand Deposits

ARDL specification for the currency demand function takes the following form

$$\Delta \log(m_t) = \beta_0 + \sum_{i=1}^{p} \beta_{1i} \Delta \log(m_{t-i}) + \sum_{i=0}^{p} \beta_{2i} \Delta i_{t-i} + \beta_3 \log(m_{t-1}) + \beta_4 i_{t-1} + u_t$$

Where m_t is the currency GDP ratio and i_t is the nominal interest rate. First step to estimate the long run parameters using ARDL approach is to select the maximum lag ρ and to apply F-test on the coefficients of lagged levels of the variables to check the presence of long-run relationship. We have taken annual data so to check the existence of long-run relationship we have taken upto 4 lags. Results of the first step reported in table

4.14 shows that for all the lag lengths there exists long-run relationship between currency (GDP ratio) and the nominal interest rate.

Table 4.14 Result of Bounds test for the existence of long-run relationship

No. of Lag	AIC	SBC	CHSQsc(1)	$\bar{R}^2$	F-Statistics
1	61.1356	57.4353	1.5636[0.211]	0.90632	(3,43) 149.3478[.000]
2	58.7107	53.2248	2.7919[0.095]	0.89831	(5,40) 80.5055[.000]
3	56.7558	49.5291	.00562[0.940]	0.89167	(7,37) 52.7386[.000]
4	53.0176	44.0966	.060046[0.806]	0.87084	(9,34) 33.2134[.000]

Notes: Asymptotic critical value bounds are obtained from Pesaran et.al.(2001) Table F in appendix C, Case III: intercept and no trend for k=1, at 1% level of significance lower bound = 6.84 and upper bound = 7.84, at 5% level of significance lower bound = 4.94 and upper bound = 5.73.

Based on SBC, AIC and $\bar{R}^2$ the maximum lag ρ is taken to be 1. For estimating the long-run parameters of the model, ARDL of order (1,1) is selected on the basis of SBC and AIC, the results are reported in table 4.15. The semi-elasticity of currency GDP ratio is 6.36 which is higher than for any other money stock.

Table 4.15 Long run Coefficients of the model based on ARDL (1,1)

Regressor	Coefficient	Standard Errors	T value [Prob.]
C	-1.5765	0.18165	-8.6790[.000]
Interest Rate	-0.063648	2.3260	-2.7363[.009]
R^2 0.91243		DW-stat 1.6405	
$\bar{R}^2$ 0.90632		F(3,43) 149.3478[.000]	
Diagnostic Tests			
$\chi^2_{SC(1)}$ 1.5636[0.211]			
$\chi^2_{FF(1)}$ 0.34365[0.558]			
$\chi^2_{N(2)}$ 3.8939[0.143]			
$\chi^2_{Het(1)}$ 0.26275[0.608]			

Notes: For diagnostic test the Lagrange Multiplier statistics $\chi^2_{SC(1)}$, $\chi^2_{FF(1)}$, $\chi^2_{Het(1)}$, and $\chi^2_{N(2)}$ are the distributed with chi-square values for serial correlation, functional form mis-specification, Heteroscedasticity, and Normality test with degrees of freedom in parentheses.

Lower panel of table 4.15 shows that model does not have serial correlation, heteroscedasticity and that the regression passes the functional form mis-specification and the normality tests.

After estimating the currency demand function we move to the estimation of the demand function for the demand deposits. Table 1 shows that demand deposits (taken as ratio to GDP) has deterministic trend and intercept therefore we apply the ARDL approach allowing for the trend. The ARDL specification for the demand deposits is written as:

$$\Delta \log(d_t) = \beta_0 + \sum_{i=1}^{p} \beta_{1i} \Delta \log(d_{t-i}) + \sum_{i=0}^{p} \beta_{2i} \Delta \mu i_{t-i} + \beta_3 \log(d_{t-1}) + \beta_4 \mu i_{t-1} + \beta_5 trend + u_t$$

where d_t are the real demand deposits taken ratio to real GDP, μi_t is the opportunity cost of demand deposits taken as difference between the interest paid on other assets and that paid on demand deposits. The results of first step of ARDL approach for the above equation are summarized in table 4.16.

Table 4.16 Result of Bounds test for the existence of long-run relationship

No. of Lag	AIC	SBC	CHSQsc(1)	$\overline{R}^2$	F-Statistics
1	33.4213	28.7960	0.051225[.821]	0.67416	(4,42)24.7935[.000]
2	34.2057	27.8055	1.8303[.176]	0.67804	(6,39)16.7951[.000]
3	32.1697	24.0397	0.12358[.725]	0.64879	(8,36)11.1602[.000]
4	29.0343	19.2213	0.11307[.737]	0.61146	(10,33) 7.7670[.000]

Notes: Asymptotic critical value bounds are obtained from Pesaran et.al.(2001) Table F in appendix C, Case V: intercept and trend for k=1, at 1% level of significance lower bound = 8.74 and upper bound = 9.63, at 5% level of significance lower bound = 6.56 and upper bound = 7.30.

The F statistics computed for the null hypothesis of no cointegrating relationship is compared to the critical value bounds of case V, the one that considers unrestricted intercept and trend. The upper bound at 95% for one regressor is equal to 7.3, therefore, F statistics for all the lags is greater than the this critical value which implies that we cannot accept the null hypothesis and there exists long run relationship. Maximum lag based on AIC and SBC is chosen to be 1. For the estimation of long run parameters of deposits demand function we proceed to the second step. ARDL (1,1) is selected on the basis of AIC.[16]

Table 4.17 Long run Coefficients of the model based on ARDL (1,1)

Regressor	Coefficient	Standard Errors	T value [Prob.]
C	-2.0872	.094660	-22.0495[.000]
Deposit Rate	-0.045215	.019470	-2.3223[.025]
Trend	0.015872	.0035846	4.4277[.000]
R^2 0.70249		DW-stat 1.8717	
$\bar{R}^2$ 0.67416		F (4, 42) 24.7935[.000]	
Diagnostic Tests			
$\chi^2_{SC(1)}$ 0.051225[.821]			
$\chi^2_{FF(1)}$ 0.43185[.511]			
$\chi^2_{N(2)}$ 0.31136[.856]			
$\chi^2_{Het(1)}$ 0.64700[.421]			

Notes: For diagnostic test the Lagrange Multiplier statistics $\chi^2_{SC(1)}$, $\chi^2_{FF(1)}$, $\chi^2_{Het(1)}$, and $\chi^2_{N(2)}$ are the distributed with chi-square vlaues for serial correlation, functional form mis-specification, Heteroscedasticity, and Normality test with degrees of freedom in parentheses

Semi-elasticity of demand deposits with respect to interest rate on long term deposits is 4.5 and the trend coefficient is significant as well. As shown in the lower panel of table 4.17, the regression passes all the diagnostic tests. The long run estimated currency and deposits demand function are given as:

$$\log(m_t) = -1.5765 - 0.0636 i_t \qquad (4.9)$$

$$\log(d_t) = -2.0872 - 0.0452 \mu i_t + 0.0159 trend \qquad (4.10)$$

Welfare cost of inflation under restricted currency-deposit model requires that both currency and deposits have the same semi-elasticity.

[16] On the basis of SBC and $\bar{R}^2$ orders of ARDL are (1,0) and (2,2) respectively which produce almost similar results as for ARDL (1,1), ARDL (1,1) is selected to enable us to apply the restriction of same semi-elasticity for currency and demand deposits.

Applying the same semi-elasticity restriction on equations (4.9) and (4.10) the currency deposit demand functions become[17]

$$\log(m_t) = -1.6653 - 0.0452 i_t \tag{4.11}$$

$$\log(d_t) = -2.0872 - 0.0452 \mu i_t + 0.0159 trend \tag{4.12}$$

4.4.2 Welfare Cost of Inflation for the Semi-log Model

Compensating Variation Approach

For the restricted model, welfare cost is measured by substituting the values of semi-elasticity η = 4.52 and the values of dependent variables projected at zero interest rate in equation 3.22. Currency holding at zero rate of interest is given by $\overline{m}(0) = \overline{m}(i)e^{\eta i}$ where end of sample values for currency holding and interest rate are taken as 0.1158 and 0.0930 respectively. Demand deposits at zero interest rate is given as $\overline{d}(0) = \overline{d}(i)e^{\eta(\mu i)}$ substituting the value for demand deposit to GDP ratio, semi elasticity of deposits (η = 4.52) and end period value of deposit rate μi (0.0351), $\overline{d}(0) = 0.2545e^{(4.52)(0.0351)}$ = 0.2983 and $\overline{m}(0) = 0.1763$ substituting these values and the average reserve ratio μ = 0.1225 the welfare function turns out to be

$$w(i) = 0.4086\ i \tag{4.13}$$

For welfare cost formula based for different long-run semi-elasticities of currency and deposits the welfare function takes the form $w(i) = \frac{1}{2} i^2 \left[\eta \overline{m}(0) + \varepsilon \mu^2 \overline{d}(0)\right]$. Equations (4.9) and (4.10) show that η = 6.36 is the semi-elasticity of currency and ε the interest

[17] We applied the Wald test on the slope coefficient and set the slope coefficient of currency model equal to semi-elasticity from the deposits model. The χ^2 of this model 1.4735 is less than the critical value of χ^2 3.84 for one restriction and 5% level of significance; therefore we cannot reject the null hypothesis. The slope coefficient in both the models is restricted to be 4.52.

semi-elasticity of demand deposits is 4.52. Currency holding at zero inflation is $\overline{m}(0) = \overline{m}(i)e^{\eta i} = 0.1158e^{(6.36)(0.093)} = 0.2092$ while deposit holding is given as $\overline{d}(0) = \overline{d}(i)e^{\varepsilon(\mu i)} = 0.2545e^{(4.552)(0.0351)} = 0.2986$ Substituting the values of all the parameters and asset demand at zero interest rate the welfare function becomes

$$w(i) = 0.6755i^2 \qquad (4.13)$$

Table 4.18 Welfare Cost of inflation for semi-log Currency-Deposit Model

	Compensating Variation Approach		**Consumer Surplus Approach**	
Interest rate	**Restricted Model**	**Unrestricted Model**	**Restricted Model**	**Unrestricted Model**
0	0.0000	0.0000	0.0000	0.0000
0.01	0.0041	0.0068	0.0042	0.0063
0.02	0.0163	0.0270	0.0163	0.0243
0.03	0.0368	0.0608	0.0355	0.0525
0.04	0.0654	0.1081	0.0613	0.0896
0.05	0.1022	0.1689	0.0931	0.1343
0.06	0.1471	0.2432	0.1302	0.1857
0.07	0.2002	0.3310	0.1721	0.2427
0.08	0.2615	0.4323	0.2183	0.3045
0.09	0.3310	0.5472	0.2685	0.3702
0.1	0.4086	0.6755	0.3221	0.4392
0.2	1.6344	2.7020	0.9737	1.1961
0.3	3.6774	6.0795	1.6777	1.8816
0.4	6.5376	10.8080	2.3155	2.4033
0.5	10.2150	16.8875	2.8482	2.7726
0.6	14.7096	24.3180	3.2742	3.0270
0.7	20.0214	33.0995	3.6074	3.2031
0.8	26.1504	43.2320	3.8659	3.3291
0.9	33.0966	54.7155	4.0669	3.4244
1	40.8600	67.5500	4.2254	3.5015

(ii) Consumer's Surplus Approach

Welfare cost of inflation measured for the currency deposit model is the sum of the area under the currency and deposits demand curves. The area under the curve is measured by integrating the asset demand functions where the integral runs from own rate of return to the positive interest rate. For the restricted model that requires same semi-elasticity for currency and deposits the welfare loss function 3.16 takes the form

$$WC = 0.04184\left[1 - e^{-4.52i}(1 + 4.52i)\right] + 0.0274\left[1 - e^{-0.5537i}(1 + 0.5537i)\right] \qquad (4.14)$$

Under the unrestricted model substituting different semi-elasticity of currency and deposits from equations (4.9) and (4.10), the welfare cost function becomes

$$WC = 0.0325\left[1 - e^{-6.36i}(1 + 6.36i)\right] + 0.0274\left[1 - e^{-0.5537i}(1 + 0.5537i)\right] \qquad (4.15)$$

Comparison of equations (4.14) and (4.15) shows that the welfare cost of inflation/ positive nominal interest rate is higher under the unrestricted model than under the restricted model.

Table 4.18 shows the welfare loss estimates at different nominal interest rates. The quadratic formula tends to give bigger values for the welfare cost compared to the area under the demand curve formula. Estimates for the unrestricted model are closer to the calculations of welfare loss measured for the monetary base model. But at all the nominal interest rates the cost based on currency-deposit model (under both restricted and the unrestricted cases) tends to be lower than the cost calculated for M1 money stock.

4.4.3 Double-log money demand function and the Currency-Deposit Model

Double-log currency and deposits demand functions are estimated using ARDL technique. The regression below is used to check the existence of long-run relationship between currency and the rate of interest. F- values based on the coefficients of the lagged levels of the variables show that for all the four lags there exists long run relationship as the F calculated value lies above the upper critical bounds. On the basis of all the lag selection criteria, maximum lag is chosen to be 1.

$$\Delta \log(m_t) = \beta_0 + \sum_{i=1}^{p} \beta_{1i} \Delta \log(m_{t-i}) + \sum_{i=0}^{p} \beta_{2i} \Delta \log i_{t-i} + \beta_3 \log(m_{t-1}) + \beta_4 \log i_{t-1} + u_t$$

Table 4.19: Result of Bounds test for the existence of long-run relationship

No. of Lag	AIC	SBC	CHSQsc(1)	$\bar{R}^2$	F-Statistics
1	50.2872	46.5869	0.2152E-3[.988]	0.77535	(3,43) 53.9208[.000]
2	47.2359	41.7499	1.0720[.301]	0.75652	(5,40) 28.9636[.000]
3	44.8887	37.6620	0.029734[.863]	0.74317	(7,37) 19.1886[.000]
4	41.6240	32.7031	0.43416[.510]	0.70868	(9,34) 12.6229[.000]

Notes: Asymptotic critical value bounds are obtained from Pesaran et.al.(2001) Table F in appendix C, Case III: intercept and no trend for k=1, at 1% level of significance lower bound = 6.84 and upper bound = 7.84, at 5% level of significance lower bound = 4.94 and upper bound = 5.73.

Based on the AIC to find the long run parameters of the model ARDL of order (1,1) is chosen. The results reported in table 4.20 show that the long run interest elasticity of currency demand is 0.372. The model does not violate any of the desirable assumptions which means that the application of OLS to autoregressive model is appropriate.

Table 4.20 Long run Coefficients of the model based on ARDL (1,1)

Regressor	Coefficient	Standard Errors	T value [Prob.]
C	-1.3402	.32624	-4.1078[.000]
log(Interest Rate)	-0.037175	.16587	-2.2412[.030]
R^2 .90996		DW-stat 1.5498	
$\bar{R}^2$ .90368		F (3,43) 144.8602[.000]	
Diagnostic Tests			
$\chi^2_{SC(1)}$ 2.4771[.116]			
$\chi^2_{FF(1)}$ .20659[.649]			
$\chi^2_{N(2)}$ 5.6950[.058]			
$\chi^2_{Het(1)}$ .069896[.791]			

Notes: For diagnostic test the Lagrange Multiplier statistics $\chi^2_{SC(1)}$, $\chi^2_{FF(1)}$, $\chi^2_{Het(1)}$, and $\chi^2_{N(2)}$ are the distributed with chi-square values for serial correlation, functional form mis-specification, Heteroscedasticity, and Normality test with degrees of freedom in parentheses.

After estimating the currency log-linear demand function we move to estimate double-log function for demand deposits. Results for the first step shows that the F statistics applied on the joint significance of the coefficients of first lags of the variables cannot be accepted at 5% level of significance. F calculated values are compared to the upper bounds reported as case V in Pesaran et. al. (2001), the case with constant and trend.

Table 4.21 Result of Bounds test for the existence of long-run relationship

No. of Lag	AIC	SBC	CHSQsc(1)	$\bar{R}^2$	F-Statistics
1	32.6334	28.0081	1.0012[.317]	0.66305	(4, 42) 23.6298[.000]
2	32.5631	26.1628	2.0556[.152]	0.65421	(6, 39) 15.1894[.000]
3	30.3029	22.1729	0.3005E-5[.999]	0.61841	(8, 36) 9.9134[.000]
4	27.7154	17.9024	2.6334[.105]	0.58745	(10, 33) 7.1230[.000]

Notes: Asymptotic critical value bounds are obtained from Pesaran et.al.(2001) Table F in appendix C, Case V: intercept and trend for k=1, at 1% level of significance lower bound = 8.74 and upper bound = 9.63, at 5% level of significance lower bound = 6.56 and upper bound = 7.30.

Optimal lag is chosen to be 1, where the Microfit calculated values of SBC and AIC are largest, there is no first order serial correlation and $\bar{R}^2$ is highest among different lags. After the existence of long-run relationship we proceed to second step and apply OLS on the regression with optimal ARDL lag order. ARDL of order (1,1) is selected on the basis of AIC and the results are summarized in table 4.22.

Table 4.22 Long run Coefficients of the model based on ARDL (1,1)

Regressor	Coefficient	Standard Errors	T value [Prob.]
C	-2.0881	.11168	-18.6981[.000]
log(Deposit Rate)	-0.15087	0.081483	-1.8516[.071]
trend	0.015402	0.0038030	4.0500[.000]
R^2 0.69235		DW-Stat 1.7370	
$\bar{R}^2$ 0.66305		F (4, 42) 23.6298[.000]	
Diagnostic Tests			
$\chi^2_{SC(1)}$ 1.0012[.317]			
$\chi^2_{FF(1)}$ 0.75474[.385]			
$\chi^2_{N(2)}$ 0.21768[.897]			
$\chi^2_{Het(1)}$ 0.55345[.457]			

Notes: For diagnostic test the Lagrange Multiplier statistics $\chi^2_{SC(1)}$, $\chi^2_{FF(1)}$, $\chi^2_{Het(1)}$, and $\chi^2_{N(2)}$ are the distributed with chi-square values for serial correlation, functional form mis-specification, Heteroscedasticity, and Normality test with degrees of freedom in parentheses.

Long-run interest elasticity of demand deposits is 0.15, trend is significant and the model does not posit any problem regarding the violation of the assumptions of Classical regression model. The double-log money demand function derived from CES utility function and the steady state welfare analysis requires that currency and deposits have the same interest elasticity. Applying this restriction we find that the currency and demand deposit models take the form

$$\log c_t = -1.71186 - 0.15087 \log i_t \qquad (4.17)$$

$$\log d_t = -2.0881 - 0.15087 \log \mu i_t + 0.0154 trend \qquad (4.18)$$

4.4.4 Welfare Cost of Inflation for Double log Model

For the currency-deposit model assuming that currency and deposits have same interest rate elasticity the welfare loss function is given by equation 3.24. Substituting the values of the estimated assets demand functions (4.17) and (4.18) and the mean value of the reserve ratio μ in the welfare function gives the final form of welfare function

$$w(i) = \left[1 - (0.15432) i_t^{0.84913}\right]^{-0.1777} - 1 \qquad (4.19)$$

Column (2) of table 4.23 presents the values of this function at different nominal rate of interest. Welfare gain of moving from 9% nominal interest rate to zero inflation (2% nominal interest rate) is 0.48 percent of GDP and further moving to deflation rate results in additional gain of 0.18 percent of GDP.

Based on the Bailey's approach welfare cost of inflation can be calculated by substituting the values of parameters from equations (4.17) and (4.18) into welfare formula 3.18

$$WC = 0.03614 i^{0.84193} \qquad (4.20)$$

Table 4.23 Welfare Cost of inflation for Double-log Currency-Deposit Model

	Calibration	Consumer Surplus
Interest rate	Double Log	Restricted model
0	0.0000	0.0000
0.01	0.0569	0.0748
0.02	0.1021	0.1341
0.03	0.1439	0.1887
0.04	0.1836	0.2404
0.05	0.2218	0.2901
0.06	0.2589	0.3383
0.07	0.2951	0.3852
0.08	0.3306	0.4310
0.09	0.3655	0.4759
0.1	0.3999	0.5201
0.2	0.7244	0.9322
0.3	1.0293	1.3115
0.4	1.3241	1.6709
0.5	1.6127	2.0162
0.6	1.8975	2.3508
0.7	2.1801	2.6765
0.8	2.4614	2.9950
0.9	2.7423	3.3072
1	3.0233	3.6140

Welfare calculations based on this formula are given in column 3 of table 4.23 which shows that 10 percent of inflation costs equivalent to a reduction of output by 0.38 percent. Under the log-log currency-deposit model the gain in moving from price stability to Friedman's optimal rule of deflation is 0.13 percent of GDP. The welfare estimates based on both the consumers' surplus and compensating variation approach tend to give similar costs of inflation.

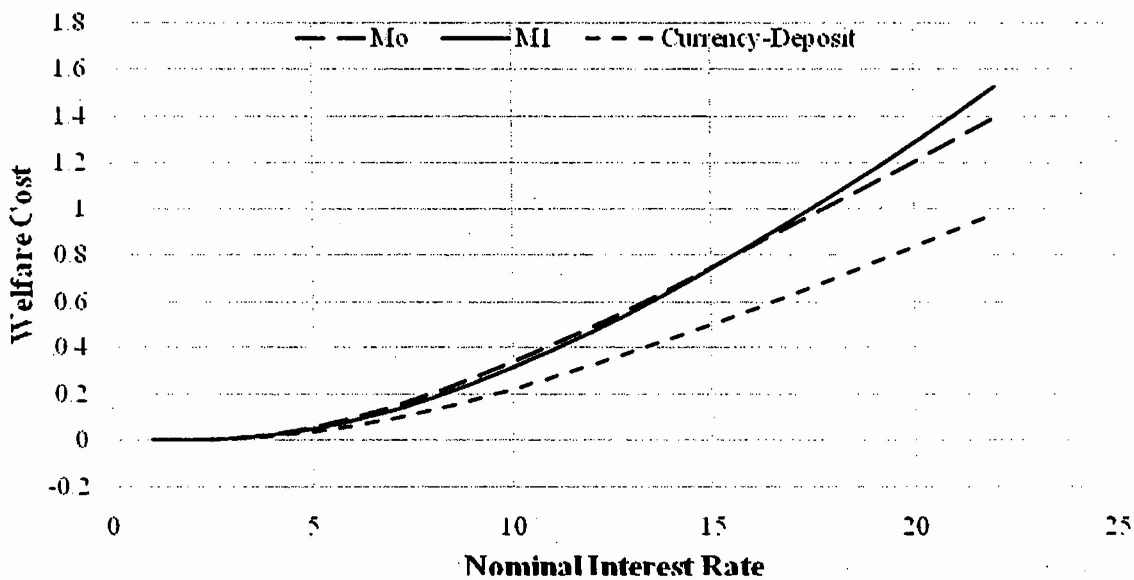
After estimating the money demand functions and calculating the welfare cost for the three models we draw the following conclusions regarding the welfare cost and its sensitivity to the selection of money demand function, approaches to calculate welfare loss and the definition of money.

(i) Comparing the two approaches to measure welfare loss we find that across all monetary assets under semi-log model, Lucas' quadratic formula gives bigger values of the loss function for higher interest rates. The difference between the loss estimates for the two approaches increases as interest rate rises above 3 percent. While for the double log model the two formulae give approximately the same loss in welfare.

(ii) Welfare cost of inflation is sensitive to the money demand specification. The welfare loss of deviating away from Friedman's Rule (zero nominal interest rate) is substantial under log-log specification than under semi-log model for low and moderate inflation rates. For all the monetary aggregates the welfare gain of moving from price stability to zero interest rate under double log model ranges from 0.10 percent to 0.25 percent of GDP, while for semi-log model the gain is trivial and ranges from 0.01 percent to 0.03 percent of GDP. This is due to the fact that the log log money demand function at low interest rates approaching zero takes asymptotic pattern showing high interest elasticity at low interest rate and higher gains of moving towards deflation rate.

(iii) Bailey and Lucas welfare cost formulae are based on the elasticity and semi-elasticity of money demand function. Long run estimates of both the semi log and double log models show that for all the four money stocks the elasticities and semi-elasticities are different. The interest elasticity and semi-elasticity are highest (6.36, 0.37) when money is defined as currency, followed by lower estimates for monetary base (5.4, 0.33) that is slightly countered by reserve component. M1, a combination of non-interest paying assets (currency) and demand deposits has lower elasticity and semi-elasticity, which is due to low interest rate elasticity of demand deposits (4.5, 0.15). The results show that for low and moderate inflation welfare cost is highest for monetary base than any other monetary asset except for the semi-log specification and compensating variation approach. Compensating variation formula is based on semi-elasticity as well as on the stock of money to calculate the initial condition for the welfare function, Monetary base is 40 percent of M1, therefore, welfare cost at each interest rate for the monetary base is less than that calculated for M1.

Figure 4.5: Welfare Cost for the three monetary assets Semi log model and Consumer Surplus Approach



The pattern of welfare cost for the three monetary models is shown in figure 4.5, welfare cost is evaluated for nominal interest rate upto 20 percent. Monetary base and M1 give approximately similar estimates of welfare loss. While the welfare loss for currency-deposit model lies much below the estimates for the single monetary aggregates. We observe similar behavior of welfare cost for the three monetary models for the double log money demand version and welfare cost measured by the compensating variation approach as shown in appendix B.

(iv) Comparing M1 and the Currency-Deposit model which calculates welfare loss based on different opportunity costs of the constituent components of M1, we find that the welfare cost for currency-deposit model is less than the loss measured using M1. These findings are in line with the empirical literature on welfare cost, that as currency and deposits are lumped together in M1 and cost evaluated at the same market rate of interest for both currency and demand deposits (treating deposits as non-interest bearing asset) exaggerates the true cost.[18]

(v) The welfare cost of inflation is sizable for Pakistan in comparison to the developed countries (shown in Appendix C). Welfare gain of moving from 14 percent to 3 percent nominal interest rate is 0.65 percent of GDP, which is greater than estimated gains for the U.S, Canada and the European countries (for double log specification using Lucas compensating variation approach).[19] Similarly the cost computed from semi-log model and using consumer surplus approach yields welfare loss of 0.06 percent and 0.62 percent of GDP as moving from 2 percent and 10 percent inflation rates to price stability. This cost is greater than computed for the U.S. which ranges from 0.04 to 0.21 percent of income under similar settings.[20]

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[18] Distinct role of currency and deposits is emphasized in Marty (1999), Bali (2000), and Simonsen & Rubens (2001).

[19] For details see Serletis & Yavari (2003, 2005, 2007).

[20] See Ireland (2007).

CHAPTER

5

CONCLUSION

In this dissertation we quantified the welfare cost of inflation from the estimated long run money demand functions for Pakistan for the period 1960-2007 using cointegration approach. Demand functions for four monetary aggregates; monetary base, narrow money (M1), currency and demand deposits taken as ratio to income against their respective opportunity costs are estimated. Welfare cost of inflation calculated for constant interest elasticity specification is compared to the constant semi-elasticity specification for two types of monetary asset models. For the single monetary asset model money stock is defined as monetary base and narrow money M1, while in the currency-deposit model M1 is disintegrated into currency and deposits based on the return on each of its constituent components. In calculating the welfare loss we have employed the traditional approach proposed by Bailey (1956) where loss due to inflation is measured by area under the money demand curve and Lucas (2000) compensating variation approach.

The empirical results show that all the monetary aggregates taken as ratio to the scale variable are negatively related to the interest rate, the single argument in money demand function. Increase in interest rate always result in reducing the holding of these monetary assets, and loss of services of liquid monetary assets. Therefore, for all the monetary asset models there is a loss associated with positive inflation and the nominal interest rate.

The welfare gain of moving from positive inflation to zero inflation is approximately same under both money demand specifications but the behavior of the two models is different towards low interest rates. Moving from zero inflation to zero nominal interest rate has substantial gain under log-log form compared to the semi-log function. Welfare calculations are robust for the double log specification in terms of similar costs across the two approaches to compute welfare loss. Compensating variation approach for the semi-log model gives higher welfare loss figures compared to the Bailey's approach due to the quadratic nature of nominal interest rate in the Lucas (2000) welfare measure.

Compared to the single monetary asset model, the currency-deposit model across all money demand specifications and welfare measures gives smaller welfare cost. For currency and deposit model the welfare gain of moving from 12 percent interest rate to 2 percent ranges from 0.37 percent to 0.57 percent of income. Reduction of interest rate from 2 percent to zero nominal interest rate yields a gain that ranges from 0.016 percent (semi-log form) to 0.13 percent (double-log version) of income. For the single monetary asset model, monetary base and narrow money give approximately the same welfare cost calculations. Welfare cost estimates for single monetary asset are almost same for the three cases: double-log model under both Bailey's and Lucas' approaches and semi-log model under Bailey's approach. Welfare gain of reducing the interest rate from 12 percent to 2 percent for M1 is 0.62 percent under the three cases while for the semi-log money demand function and the Lucas' approach the gain is 1.17 percent of income. Moving further to the zero interest rate, yields welfare gain of 0.03 percent of GDP for semi-log form and a substantial gain of 0.20 percent of income for the log-log form. For monetary base the benefit in terms of increase in income of pursuing price stability policy under similar settings ranges from 0.59 percent to 1 percent. While going further to the optimal deflation policy the gain ranges from 0.007 to 0.16 percent of income under semi-log and double-log specifications.

In recent years the State Bank of Pakistan has taken up hawkish stance in stabilizing the GDP in response to high inflation expectations. Applying "Taylor Principle" nominal interest rate is adjusted to changes in the expected inflation. During the last six months State Bank has lowered the policy rate by 250 basis points but for the last two months the inflationary pressure is gradually increasing. For December 2009 the increase in CPI stood at 10.5 percent and for the current fiscal year it is projected to remain in the range of 10-12 percent, SBP is likely to retain policy interest rate at 12.5 percent. Current sub-optimal policy of 12.5 percent nominal interest rate (10.5 percent inflation rate) under double log specification and single monetary asset will result in a welfare loss of 0.63 percent of GDP (0.51 percent for the currency-deposit model) against zero inflation policy, while against deflation policy the cost is 0.87 percent of income (0.65 percent for currency-deposit model). The exact loss of deviating from price stability at current GDP of Rs. 5796.4 billion is Rs. 36.5 million (29.6 million for currency-deposit model).

To conclude, the findings of this study suggest that the society bears a substantial loss due to inflation and positive nominal interest rate. This is the first attempt to break new grounds for measuring the welfare cost of inflation for Pakistan. However, limitation of this study is that the welfare cost analysis is based on the direct cost of inflation, not addressing other channels through which inflation results in inefficient allocation of resources. The direct cost of inflation under-states the actual cost of inflation, as inflation tends to distort marginal decisions by altering work-leisure choice and interact with the tax structure of the economy. The actual cost of inflation is much greater than estimated in this study, the State Bank of Pakistan should opt for an independent monetary policy.

For the last two years the government has financed its expenditures by borrowing heavily from the SBP, which contradicted the bank's tight monetary policy stance and passed on the signal of rising inflation in the economy. Furthermore, the Taylor Principle driven raising nominal interest rate contribute to inflation through cost side. Best policy contribution to sustain growth and welfare will be to maintain price stability.

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APPENDIX A

Each household maximizes the discounted utility

$$\sum_{t=0}^{\infty}(1+\rho)^{-1}U(m_t,c_t,d_t)$$

With respect to budget constraint

$$(1+\pi_{t+1})m_{t+1}+(1+\pi_{t+1})d_{t+1}+k_{t+1}+(1+r_{t+1})^{-1}b_{t+1}=m_t+d_t(1+i_d(t))+k_t(1-\delta)+b_t+f(k_t)+c_t+h_t$$

Substituting the budget constraint for the value of consumption in the utility function and differentiating with respect to b_t, m_t, and d_t and rearranging yields first order conditions for the maximization of utility.

For two periods (t and t-1) the utility function becomes

$$U(m_t,c_t,d_t)=(1+\rho)^{-1}U(m_{t-1},c_{t-1},d_{t-1})+(1+\rho)^{-1}U(m_t,c_t,d_t)$$

$$\begin{aligned}U(m_t,c_t,d_t)=(1+\rho)^{-1}U(m_{t-1},d_{t-1},(1+\pi_t)m_t+(1+\pi_t)d_t+k_t+(1+r_t)^{-1}b_t-m_{t-1}\\-d_{t-1}(1+i_d(t-1)-k_{t-1}(1-\delta)-b_{t-1}-f(k_{t-1})-h_{t-1}+(1+\rho)^{-1}U(m_t,d_t,(1+\pi_{t+1})m_{t+1}\\+(1+\pi_{t+1})d_{t+1}+k_{t+1}+(1+r_{t+1})^{-1}b_{t+1}-m_t-d_t(1+i_d(t))-k_t(1-\delta)-b_t-f(k_t)-c_t-h_t|\end{aligned}$$

Taking derivative with respect to b_t

$$(1+\rho)^{-1}\left[Uc(t-1)(1+r_t)^{-1}\right]+(1+\rho)^{-1}\left[Uc(t)(-1)\right]=0$$

Dividing the above equation by $(1+\rho)^{-1}Uc(t)$ we get

$$\frac{Uc(t-1)}{Uc(t)}=(1+r_t) \qquad (A.1)$$

Equation (A.1) is standard condition for optimally allocating consumption between periods *t-1* and *t*. It equates the marginal utility of giving up one unit of consumption in period *t* to the marginal gain from shifting resources to previous period consumption.

Taking derivative with respect to m_t

$$(1+\rho)^{-1}\left[Um(t)-Uc(t-1)\,(1+\pi_t)\right]+(1+\rho)^{-1}\left[Uc(t)(1)\right] = 0$$

Dividing the above equation by $(1+\rho)^{-1}Uc(t)$

$$\frac{Um(t)}{Uc(t)}-\frac{Uc(t-1)}{Uc(t)}(1+\pi_t)+1 = 0 \qquad \text{(A.2)}$$

Substituting equation (A.1) into (A.2) and further applying the condition that

$$(1+r_t) = (1+i_t)/(1+\pi_t)$$

$$\frac{Um(t)}{Uc(t)} = -1+(1+r_t)(1+\pi_t) = i_t$$

To find the Euler equation (3.4) we take the derivative of utility function with respect to

d_t

$$\frac{Ud(t)}{Uc(t)}-\frac{Uc(t-1)}{Uc(t)}(1+\pi_t)+(1+i_d(t))=0 \qquad \text{(A.3)}$$

$$\frac{Ud(t)}{Uc(t)} = -(1+i_d(t))+(1+r_t)(1+\pi_t) = i_t - i_d$$

APPENDIX B

(I) Welfare Cost of Inflation functions for Monetary Base (single monetary asset model) and Currency-Deposit model for semi-log and log-log money demand specifications and two approaches to measure welfare loss viz. Consumer Surplus (CS) Approach and Lucas' Compensating Variation (CV) Approach.

Figure 1: Welfare Cost Functions for Monetary Base relative to zero nominal interest rate

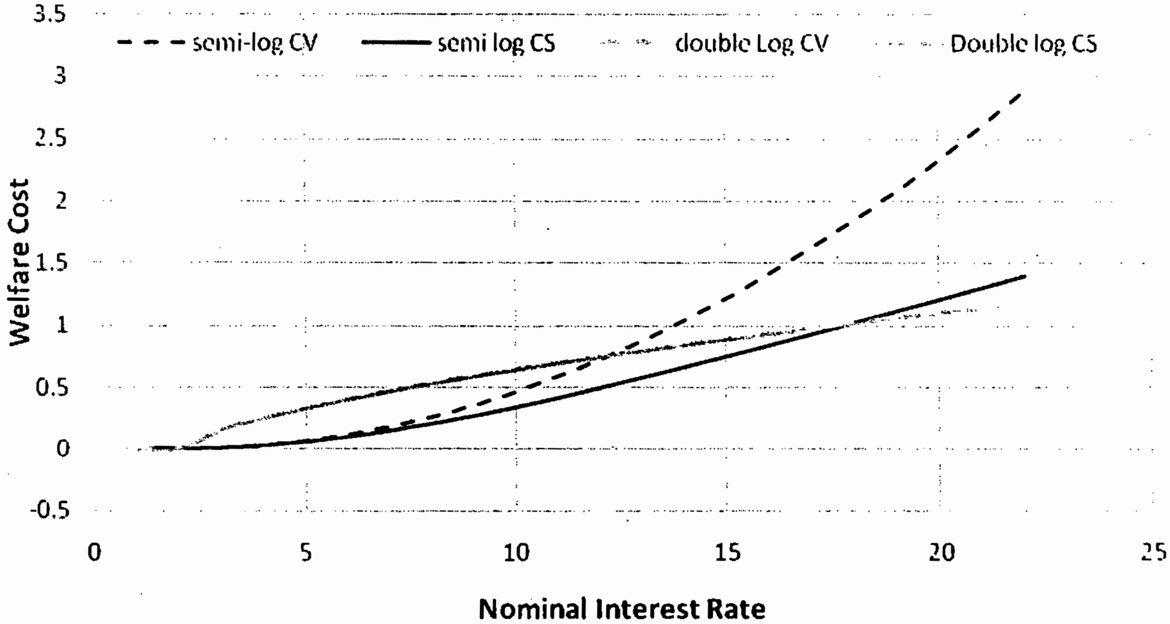


Figure 2: Welfare Cost Functions for Monetary Base relative to 2 percent nominal interest rate

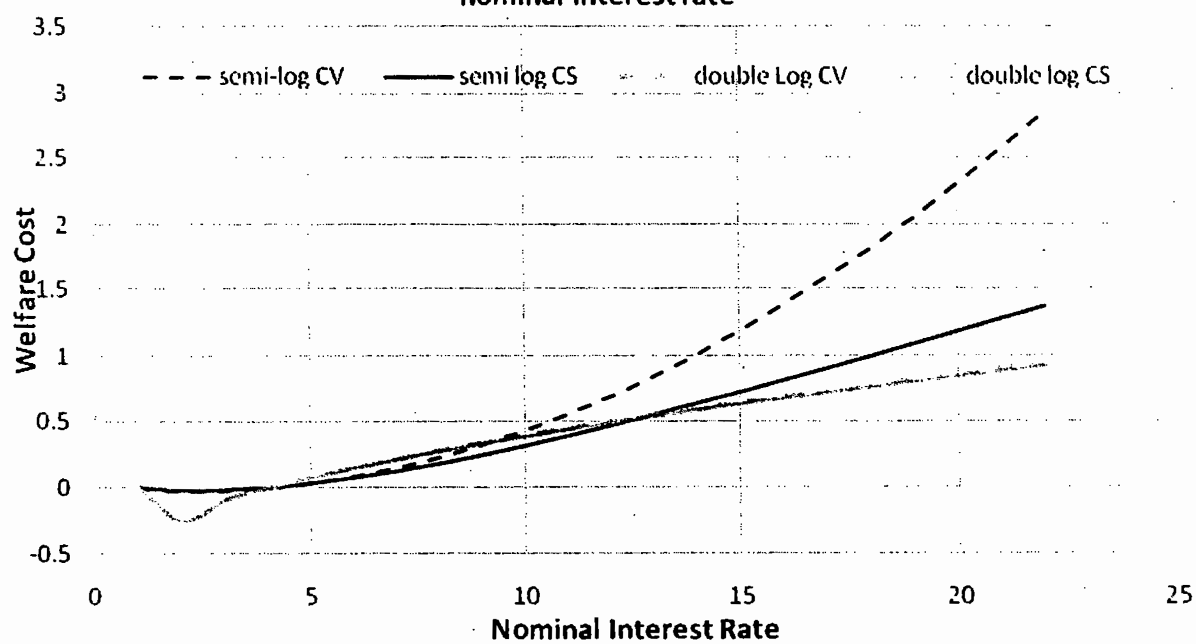


Figure 3: Welfare Cost Functions for Currency Deposit Model relative to zero nominal interest rate

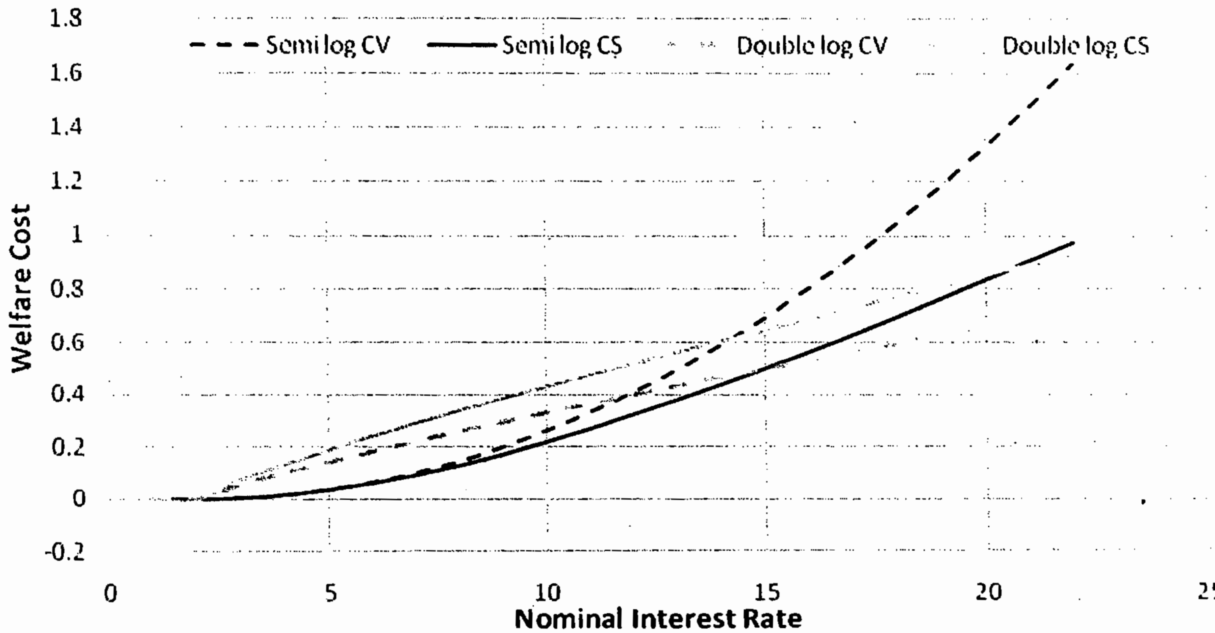
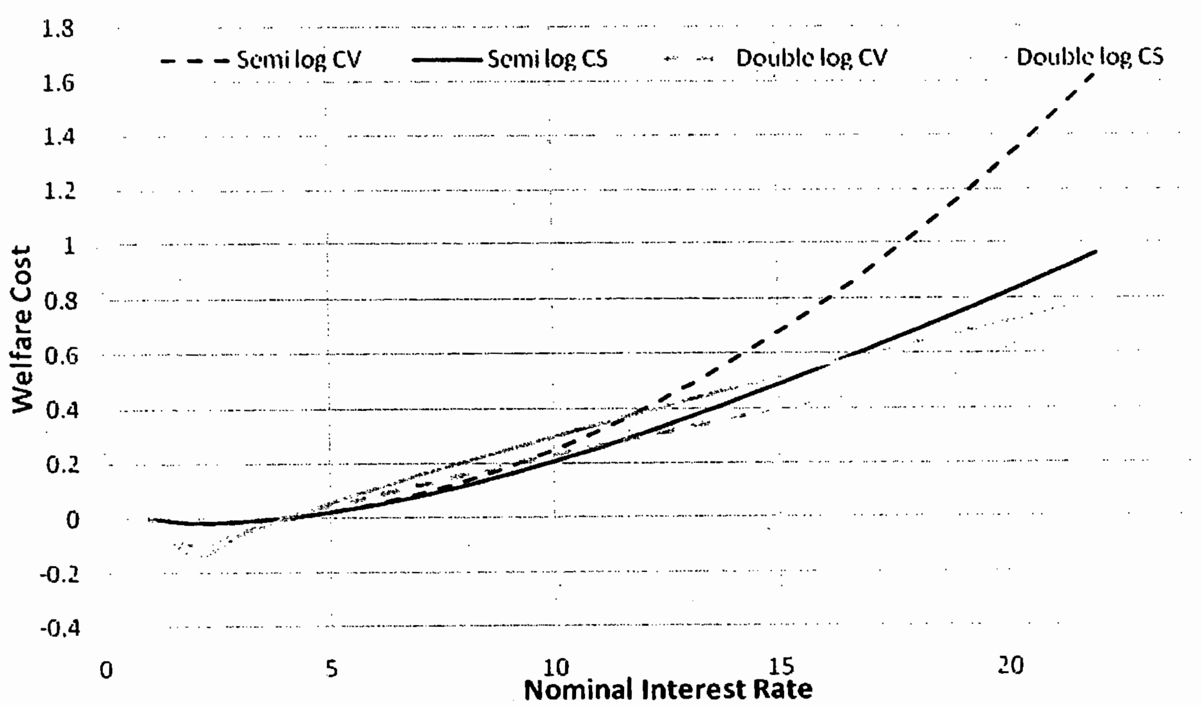


Figure 4: Welfare Cost Functions for Currency Deposit Model relative to zero nominal interest rate



(II) Comparison of Welfare Cost for the three monetary aggregates under different money demand specifications and welfare measurement approaches.

Figure 5: Semi-log Model and Compensating Variation Approach

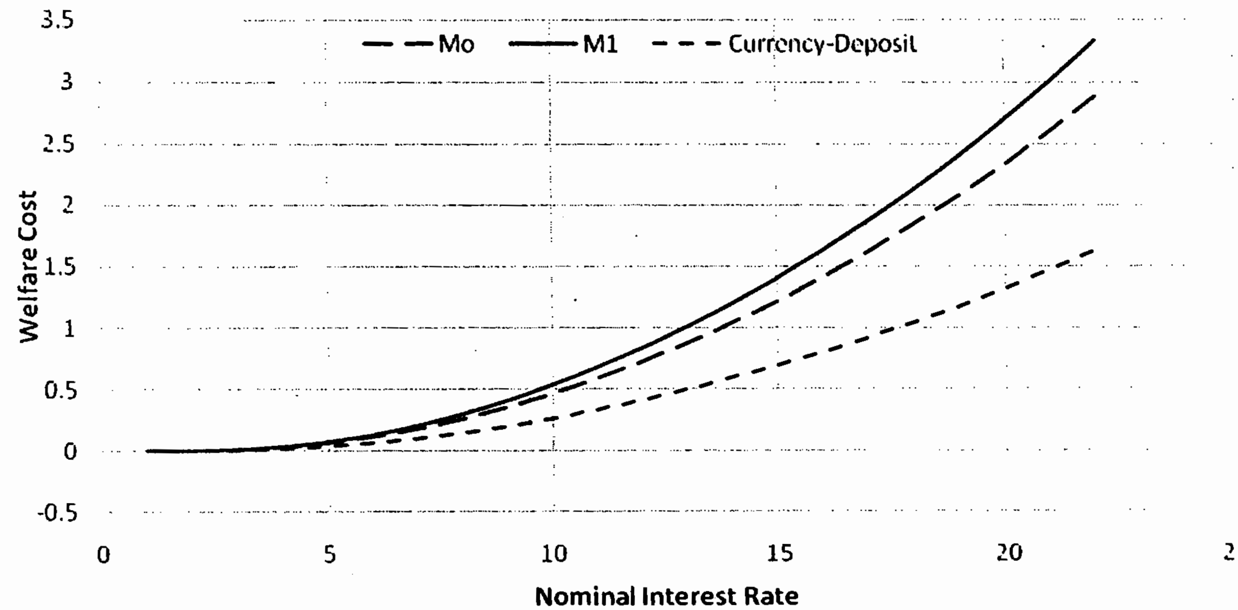


Figure 6: Double Log Model and Consumer Surplus Approach

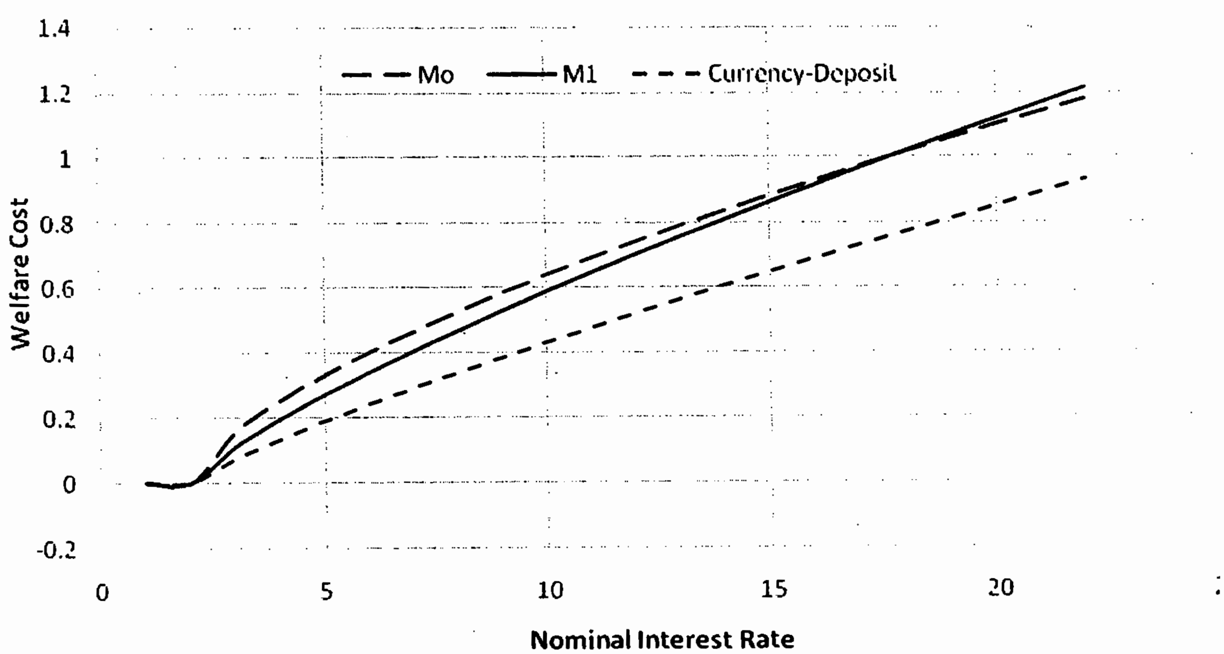
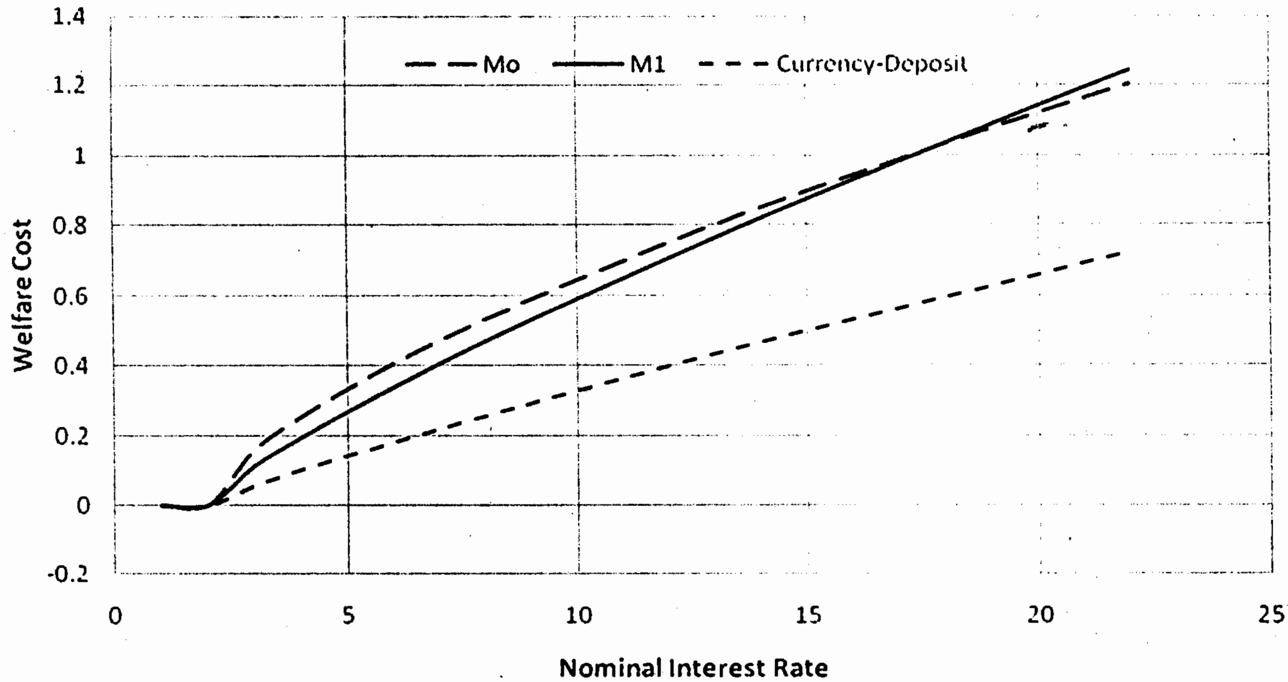


Figure 7: Double Log Model and Compensating Variation Approach



APPENDIX C

Table C.1 : Summary Table of the Empirical Studies of Welfare Cost of Inflation

Study	Features	Monetary Aggregate	Country	Inflation Comparisons	Welfare Cost (% of GDP)
Bailey (1956)	Partial Equilibrium Cagan money demand function	Monetary Base	Austria Germany Greece Hungary I Hungary II Polonia Russia	Highest inflation rate vs 0% equivalent to optimal where real interest rate is assumed to be zero.	0.0377 0.0704 0.0572 0.159 0.046 0.0346 0.0085
Marty (1967)	Partial Equilibrium Growth model Cagan and Mundell money demand specifications	M1	Seven Hyperinflation countries	10% vs 0%	0.310
Tower (1971)	Partial Equilibrium growth model with Cagan money demand function		Hypothetical Economy	20%	15.4
Barro (1972)	Partial Equilibrium Calibration		Hungary	2-5% vs 0% 25-50% 100-150% Monthly inflation rates	3-7% 11-22% 22-33%
Fisher (1981)	Partial Equilibrium	Monetary base	U.S	10 vs 0%	0.3
Lucas (1981)	Partial Equilibrium Cagan money demand function	M1	U.S	10% vs 0%	0.45

Study	Features	Monetary Aggregate	Country	Inflation Comparisons	Welfare Cost (% of GDP)
Cooley and Hansen (1989)	General Equilibrium model with CIA motive for holding money	M1 MB	U.S	10% vs 0%	0.387 0.1
Cooley and Hansen (1991)	General Equilibrium model with CIA motive for holding money	M1	U.S	5% vs 0%	0.33
Den Haan (1990)	General Equilibrium		U.S		4.6
Eckstein and Leiderman (1992)	General Equilibrium model with Money-in-Utility motive for money demand	Monetary Base M1	Israel	10% vs 0%	1
Imrohoroglu (1992)	Precautionary consumption smoothing motive for money demand			10%vs 0% 5% vs 0%	1.07 0.57
Gillman (1993)	General Equilibrium	M1	U.S	10% vs optimal	2.19
Lucas (1993)	Welfare triangle with General Equilibrium motivation	M1	U.S		1.64

Study	Features	Monetary Aggregate	Country	Inflation Comparisons	Welfare Cost (% of GDP)
Dotsey and Ireland (1996)	General Equilibrium, Endogenous growth with CIA and financial intermediation		U.S		1.73
Gillman (1995)	Partial Equilibrium	M1	U.S		0.349
Wolman (1997)	General Equilibrium model with Transaction-Time Technology	M1	U.S		0.6
Lucas (2000)	General Equilibrium	M1	U.S	3% vs optimal	1.0
Bali (2000)	General Equilibrium model with Money-in-Utility motive for money demand	Currency-deposits Monetary Base M1	U.S	4% vs optimal	0.11 0.18 0.55
Lopez(2001)	General Equilibrium model with Money-in-Utility motive for money demand	M1	Columbia	20%vs 5% 20% vs 10%	2.3 1

Study	Features	Monetary Aggregate	Country	Inflation Comparisons	Welfare Cost (% of GDP)
Serletis and Yavari (2003)	General Equilibrium Lucas (2000)	M1	U.S	11% vs 0% (nominal interest rate from 14% to 3%)	0.45
				0% vs optimal (nominal interest rate from 3% to 0%)	0.18
			Canada	11% vs 0% (nominal interest rate from 14% to 3%)	0.35
				0% vs optimal (nominal interest rate from 3% to 0%)	0.15
Samimi and Omran (2005)	General Equilibrium model with Money-in-Utility motive for money demand	Monetary Base	Iran	10% vs 0%	2
		M1		50% vs 0%	4.37
Serletis and Yavari (2005)	General Equilibrium Lucas (2000)	M1	Italy	Nominal interest rate from 14% to 3%	0.4%
Serletis and Yavari (2007)	General Equilibrium Lucas (2000)	M1	Australia	Nominal interest rate from 14% to 3%	0.45
			France		0.1
			Germany		0.2
			Netherlands		0.4
			Ireland		0.5
			Italy		0.4
			Belgium		0.3

Study	Features	Monetary Aggregate	Country	Inflation Comparisons	Welfare Cost (% of GDP)
Ireland (2007)	Partial Equilibrium	M1	U.S	2% vs 0% 10% vs 0%	0.04 0.22
Gupta and Uwilingiye (2008)	General Equilibrium Lucas (2000)	M3	South Africa	Band of 3% to 6% vs optimal	0.15 to 0.41