DOA Estimation Algorithms For Smart Antenna Systems

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Certificate of Approval

It is certified that we have read the project report submitted by Ishtiaq **Akbar** Registration No: **[69-FET/MSEE/F-07]**. It is our judgment that this report is of sufficient standard to warrant its acceptance by the International Islamic University, Islamabad for degree of MS Electronic Engineering (MSEE).

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To My Parents and Teachers

Abstract:

In many practical signal processing problems, the objective is to estimate from a collection of noise "contaminated "measurements a set of constant parameters upon which the underlying true signals depend . Moreover, the accurate estimation of the direction of arrival of all signals transmitted to the adaptive array antenna contributes to the maximization of its performance with respect to recovering the signal of interest and suppressing any present interfering signals. The same problem of detennining the direction of arrival (DOA) of impinging wave fronts, given the set of signals received at an antenna array from multiple emitters, arises also in a number of radar, sonar, electronic surveillance, and seismic exploration applications. In general, the DOA estimation algorithms can be categorized into two groups; the conventional algorithms and the subspace algorithms.

In this thesis report different Direction of Arrival (DOA) techniques like Pisarenko Harmonic Decomposition (PHD), The Minimum Norm (MN) Algorithm, The Minimum Variance Distortion less Method (MVD), Multiple Signal Classifier (MUSIC) and ESPRIT Algoritlun have been discussed.

Computer simulations have been carried out for the following techniques

1-spectral estimation

2-MVD(capon) method

3-Music Algorithm

Simulations were carried for two uncorrelated signals impinging on uniform linear array (ULA) having 10 antenna elements, which were equally spaced by half the wave length. Number of snapshots was 100.

Computer simulation shows that subspace based DOA estimation technique Music Algorithm outperformed the other two technique and give better resolution for two uncorrelated signal coming from different directions.

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Table 1: Symbols and Description

Terms	Descriptions
p	Number of Sources
N	Number of Sensors (Array Elements)
$\mathbf{s}(t)$	P x 1 Original Signal Vector
$\mathbf{a}(\theta)$	N x 1 steering vector
$\mathbf{n}(t)$	N x 1 Noise Vector
$\mathbf{x}(t)$	N x 1 Vector of Array Outputs
$\mathbf{w}$	N x 1 Weight vector
y(t)	Beam former Output
$d(t)$	Reference signal
$e(t)$	Error Signal
σ^2	Noise Variance
$A(\theta)$	Nxp Steering Matrix
$\mathbf{R}_{ss}$	pxp Autocorelation Matrix defined by $\mathbf{R}_{SS}=E[\mathbf{s}(t)\mathbf{s}^H(t)]$
$\mathbf{R}_x$	NxN Signal autocorrelation Matrix defined by $R_x = E\{\mathbf{x}(t)\mathbf{x}^H(t)\}$
$\mathbf{R}_s$	NxN Matrix defined by $R_s = \mathbf{A}(\theta)\mathbf{R}_{SS}$ (8)
$\mathbf{R}_n$	NxN Noise Autocorrelation Matrix defined by $R_n = \sigma^2 I$
$\mathbf{R}_x$	An Estimate of R_x
q_i^s	Eigenvalues of R_s
q_i^x	Eigenvalues of R_x
$\mathbf{e}_i$	Eigenvectors of R_x
$\hat{\mathbf{e}}_i$	Eigenvectors of $\hat{\mathbf{R}}_x$
N-p	Noise Subspace Dimension
M	Number of Snapshots
H	Conjugate Transpose Operation

Chapter 1

INTRODUCTION

Many refer to smart antenna systems as smart antennas, but in reality antennas by themselves are not smart. It is the digital signal processing capability, along with the antennas, which make the system smart. Although it may seem that smart antenna systems are a new technology, the fundamental principles upon which they are based are not new. In fact, in the 1970s and 1980s two special issues of the IEEE Transactions on Antennas and Propagation were devoted to adaptive antenna arrays and associated signal-processing techniques [1,2]. The use of adaptive antennas in communication systems initially attracted interest in military applications. Particularly, the techniques have been used for many years in electronic warfare (EWF) as countermeasures to electronic jamming. In military radar systems, similar techniques were already used during World War II [3]. However, it is only because of today's advancement in powerful low-cost digital signal processors, general-purpose processors and ASICs (Application Specific Integrated Circuits), as well as innovative software-based signal processing techniques (algorithms), that smart antenna systems are gradually becoming commercially available.

1.1 NEED FOR SMART ANTENNAS

Wireless communication systems, as opposed to their wire line counterparts, pose some unique challenges.

i- The limited allocated spectrum results in a limit on capacity.

ii- The radio propagation environment and the mobility of users give rise to signal fading and spreading in time, space and frequency

iii- The limited battery life at the mobile device poses power constraints.

In addition, cellular wireless communication systems have to cope with interference due to frequency reuse. Research efforts investigating effective technologies to mitigate such effects have been going on for the past twenty five years, as wireless communications are been going on for the past twenty five years, as wireless communications are experiencing rapid growth . Among these methods are multiple access schemes, channel coding and

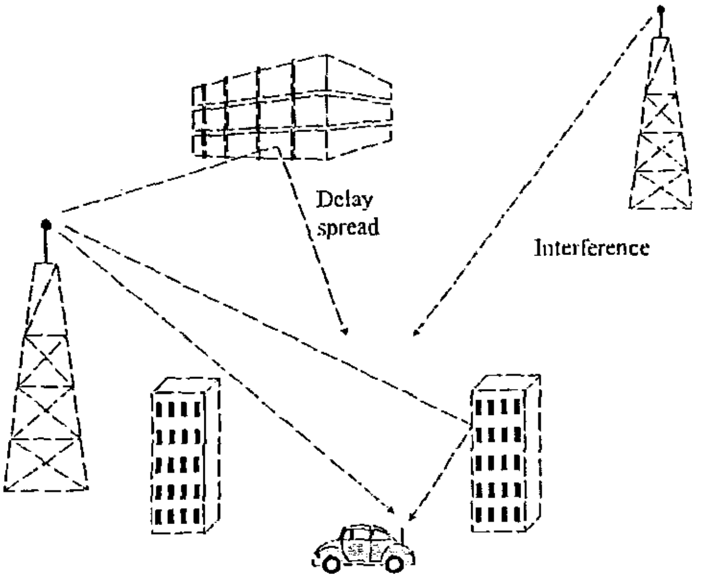

FIGURE 1.1: Wireless systems impairments[5].

equalization and smart antenna employment. Fig. 1.1 summarizes the wireless communication systems impairments that smart antennas are challenged to combat.

An antenna in a telecommunications system is the port through which radiofrequency (RF) energy is coupled from the transmitter to the outside world for transmission puposes,and in reverse, to the receiver from the outside world for reception purposes.

To date,antennas have been the most neglected of all the components in personal communications systems. Yet, the manner in which radio frequency energy is distributed into and collected from space has a profound influence upon the efficient use of spectrum, the cost of establishing new personal communications networks and the service quality provided by those networks . The commercial adoption of smart antenna techniques is a great promise to the solution of the aforementioned wireless communications' impairments.

OVERVIEW

The basic idea on which smart antenna systems were developed is most often introduced with a simple intuitive example that correlates their operation with that of the human auditory system. **A** person is able to determine the Direction of Arrival (DOA) of a sound by utilizing a three-stage process:

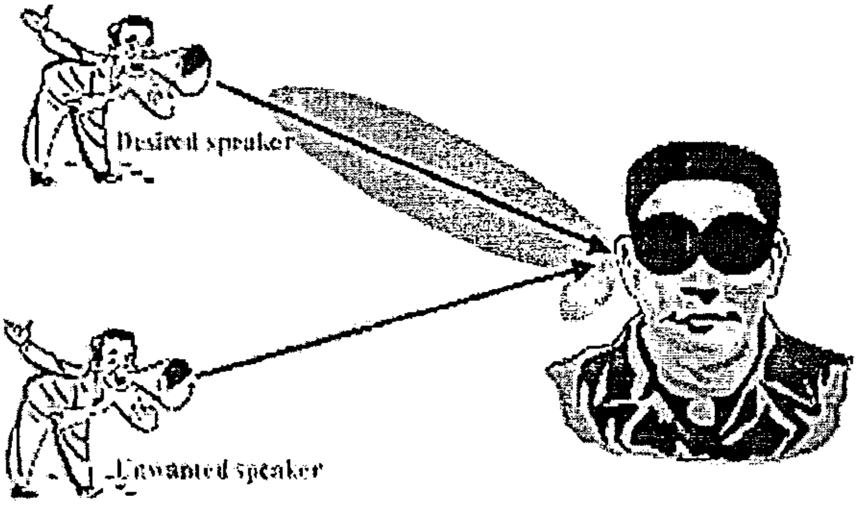

FIGURE. 1.2: Human auditory function [4]

1- One's ears act as acoustic sensors and receive the signal.

2 -Because of the separation between the ears, each ear receives the signal with a different time delay.

3 -The human brain, a specialized signal processor, does a large number of calculations to correlate information and compute the location of the received sound.

To better provide an insight of how a smart antenna system works, let us imagine two Persons carrying on a conversation inside an isolated room as illustrated in Figl.2. The listener between the two persons is capable of determining the location of the speaker as he moves about the room because the voice of the speaker amves at each acoustic sensor, the ear, at a different time. The human "signal processor," the brain, computes the direction of the speaker from the time differences or delays received by the two ears. Afterward, the brain adds the strength of the signals from each ear so as to focus on the sound of the computed direction utilizing a similar process, the human brain is capable of distinguishing between multiple signals that have different directions of arrival **(DOA).** Thus, if additional speakers join the conversation, the brain is able to enhance the received signal from the speaker of interest and tune out unwanted interferers. Therefore, the listener has the ability to distinguish one person's voice, from among many people talking simultaneously, and concentrate on one conversation at a time. In this way, any unwanted interference is attenuated. Conversely, the listener can respond back to the same direction of the desired speaker by orienting his/her transmitter, his/her mouth, toward the speaker.

Electrical smart antenna systems work the same way using two antennas instead of two ears, and a digital signal processor instead of the brain . Thus, based on the time delays due to the impinging signals onto the antenna elements, the digital signal processor computes the direction-of-arrival **(DOA)** of the signal-of-interest (SOI), and then it adjusts the excitations (gains and phases of the signals) to produce a radiation pattern that focuses on the SOI while tuning out any interferers or signals-not-of-interest **(SNOI).**

Transfemng the same idea to mobile communication systems, the base station plays the role of the listener, and the active cellular telephones simulate the role of the several sounds heard by human ears. In mobile communication system digital signal processor located at the base station works in conjunction with the antenna array and is responsible for adjusting various system parameters to filter out any interferers or *signals-not-of-interest* (SNOI) while enhancing desired communication or ***signals-ofinterest*** **(SOI)**. Thus, the system forms the radiation pattern in an adaptive manner, responding dynamically to the signal environment and its alterations. The principle of beam forming is essentially to weight the transmit signals in such a way that obtains a constructive superposition of different signal parts. Note that some knowledge of the transmission channel at the transmitter is necessary in order for beam fonning to transmission be feasible, such that maximum radiated power is produced in the directions of desired mobile users and deep nulls are generated in the directions of undesired signals representing co-channel interference from mobile users in adjacent cells. Prior to adaptive beamforming, the directions of users and interferes must be obtained using a direction of arrival algorithm.

1.2 Overview of Direction of Arrival (DOA) Algorithms

After the signal from all directions has been anticipated by antenna array, The DOA algorithm determines the direction of all incoming signals based on the time delays. There are number of DOA estimation techniques. These techniques can be categorized on the basis of data analysis and implementation into four different area : conventional methods, subspace based methods, maximum likelihood methods, and integrated methods.

Conventional methods for DOA estimation are based on the concepts of beam forming and null steering and do not exploit the statistics of the received signal [4]. In this technique, the DOA of all the signals is determined from the peaks of the output power spectrum obtained from steering the beam in all possible directions. Examples of conventional methods are the delay-and-sum method (classical beam former method or Fourier method) and Capon's minimum variance method. One major disadvantage of the delay-and-sum method is its poor resolution; that is, the width of the main beam and the height of the side lobes limit its ability to separate closely spaced signals .On the other hand, Capon's minimum variance technique tries to overcome the poor resolution problem associated with the delay-and-sun1 method, and in fact, it gives a significant improvement. Although it provides better resolution, Capon's method fails when the SNOIs are correlated with the SOI. unlike conventional methods, *subspace methods* exploit the structure of the received data, resulting in a dramatic improvement in resolution. Two main algorithms that fall into this category are the Multiple Signal Classification (MUSIC) algorithm and the Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT). In 1979, Schmidt proposed the conventional MUSIC algorithm that exploited the eigen structure of the input covariance matrix [4]. This algorithm provides information about the number of incident signals, DOA of each signal, strengths and cross correlations between incident signals, and noise powers. Like many algorithms, the conventional MUSIC possesses drawbacks. One of the drawbacks is that it requires very precise and accurate array calibration. Another drawback is that, if the impinging signals are highly correlated, it fails because the covariance matrix of the received signals becomes singular. And lastly, it is computationally intensive. To improve the conventional MUSIC algorithm further, several attempts were made to increase its resolution performance and decrease its computational complexity. In 1983, Barbell developed the Root-MUSIC algorithm based on polynomial rooting and provided higher resolution; its drawback was that it was applicable only to uniformly spaced linear arrays [5] . In 1989, Schmidt proposed the Cyclic MUSIC, a selective direction finding algorithm, which exploited the spectral coherence properties of the received signal and made it possible to resolve signals

Chapter 2

Signal Model and Problem Formulation

2.1 Signal Model for Array Processing

The essential goal of sensor array signal processing is to estimate signal parameters by combining temporal and spatial information, captured via sampling a wave field with a set of judiciously placed antenna sensors. The wave field is assumed to be generated by a finite number of emitters, and contains information about the signal parameters characterizing the emitters. In this section, we will provide the mathematical model that has been widely used for array signal processing through the recent decades.

2.2 Antenna Array Response

Consider a uniform linear array (ULA) consisting of N identical antenna elements illustrated in Fig. 2.1 Suppose there is only one narrow-band point source $s_o(t)$ with carrier frequency w_o present in the far field. If the distance between the array and the source is large enough compared to the aperture of the array, the wave front impinging on the array can be approximately considered as planar. Assuming that antenna array is composed of identical isotropic elements, each element receives a time-delayed version of the same plane wave with wavelength 1. In other words, each element receives a phase-shifted version of the signal[4]. For example, with a *uniform linear array* (ULA), as shown in Fig.2.1, the relative phases are also uniformly spaced, with $\Psi = \frac{2\pi}{a} d \sin(\theta)$ being the relative phase difference between adjacent elements. The vector of relative phases is referred to as the *steering vector* (SV) $\mathbf{a}(\theta)$. A more general concept is the *array response vector* (ARV) which is the response of an array to an incident plane wave. It is a combination of the steering vector and the response of each individual element to the incident wave.

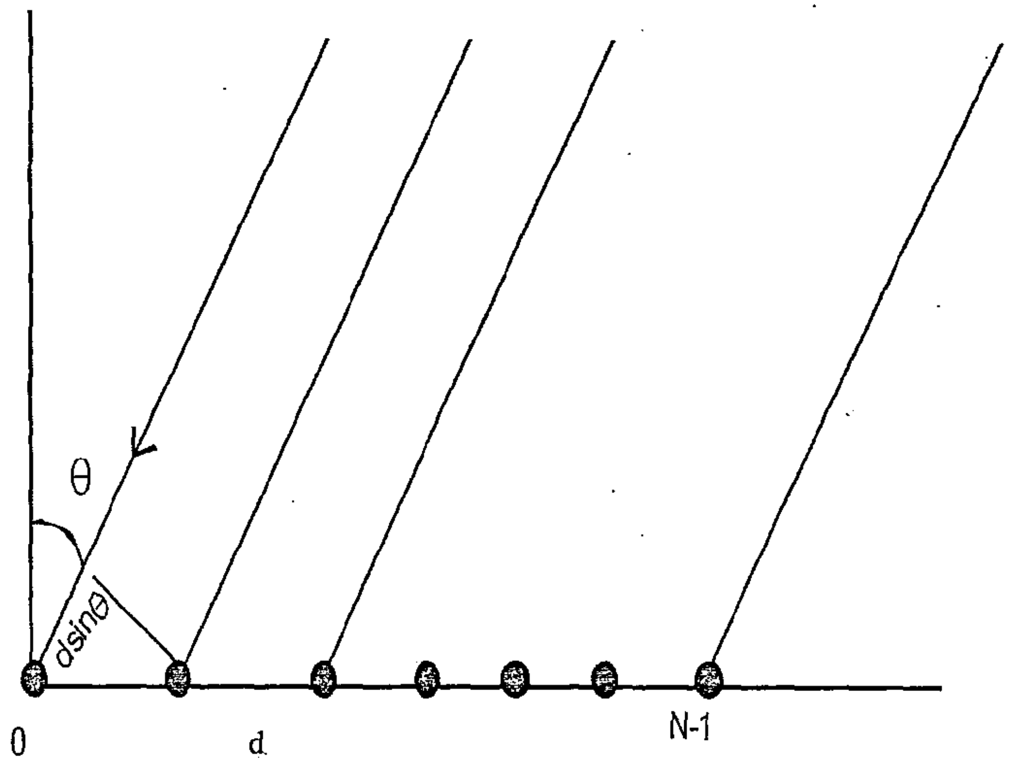

FIGURE. 2.1: A uniform linear array (ULA) with N sensor elements along with an impinging uniform plane EM wave[4].

the array response vector(ARV)f or uniform linear array(ULA) under discussion is given as

$$[e^{0} \quad e^{-j\psi} \quad e^{-j2\psi} \ldots\ldots\ldots\ldots\ldots\ldots \; e^{-j(N-1)\psi}]^{T} \qquad (2.1)$$

inserting value of $\Psi = \frac{2\pi}{a} d \sin(\theta)$ we get

$$a(\theta) = \begin{bmatrix} 1 \\ \bullet \\ \bullet \\ e^{-j\frac{2\pi}{\lambda}(N-1)\sin(\theta)} \end{bmatrix} \tag{2.2}$$

Where N x 1 array response vector $a(\theta)$ is referred to as the steeling vector of the array to a planar wave arriving from the direction θ. It is known that N different steering vectors randomly selected from the manifold are generally linearly independent.

2.3 Direction of Arrival (DOA) Estimation algorithms

For the Beamformer to steer the radiation in a particular direction and to'place the nulls in the interfering directions the direction of arrival(DOA) has.to be known beforehand. The DOA algorithms does exactly the same, they work on the signal received at the output of the array and computes the direction of arrivals of all the incoming signals. Once the angle information is known it is fed into the beamforming network to compute the complex weight vectors required for beam steering. It is shown in the following figure.

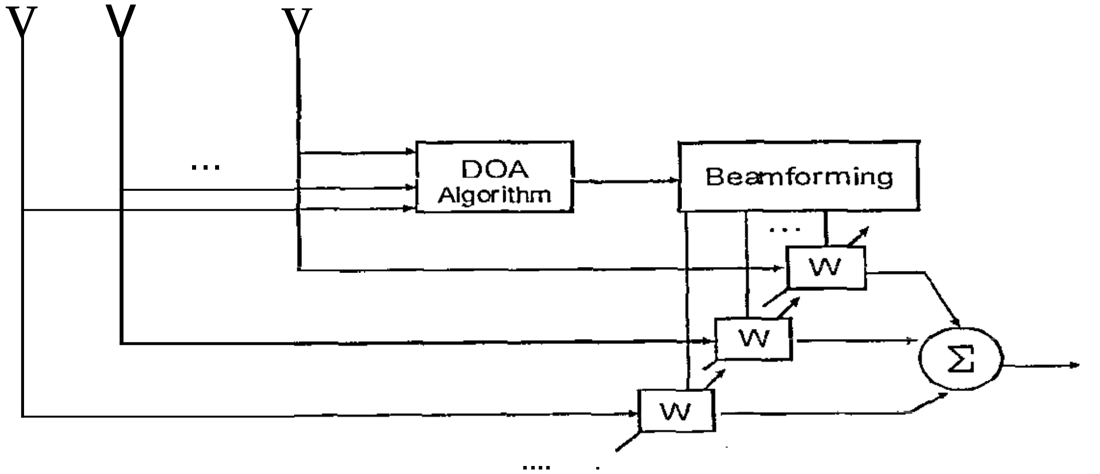

Fig. 2.2: Beamforming Setup with Direction Of Arrival Estimation

2.4 Problem formulation

Consider a beam forming setup shown in figure 2.3, consisting of an array of N sensors with arbitrary locations and arbitrary directional characteristics, which receives signals generated by p narrowband sources with known center frequency w_o and locations θ_1, θ_2, θ_3 …… θ_p . The output $y(t)$ of the array with variable element weights w_m is the weighted sum of the received signals $S_k(t)$ at the array elements and the noise $n(t)$ at the receivers connected to each element. The weights are iteratively computed based on the output $y(t)$, a reference signal $d(t)$ that approximates the desired signal, and previous weights.

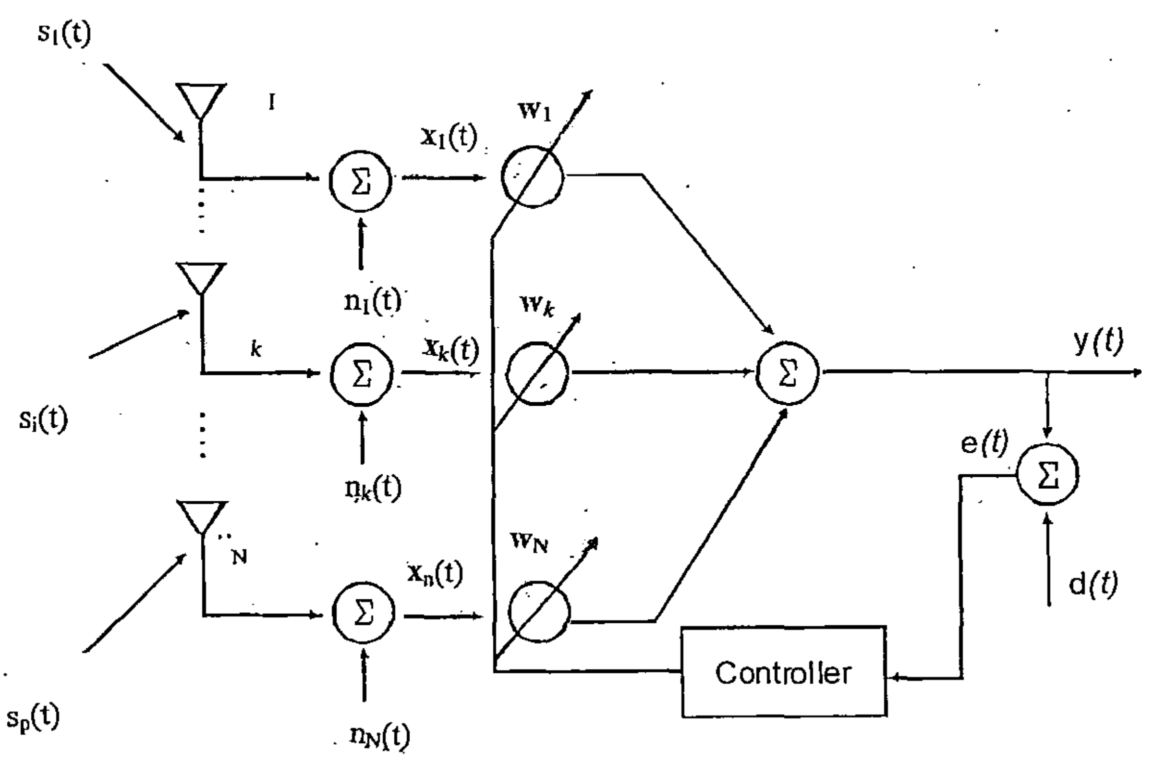

Fig. 2.3: An Addaptive Beamforming System.

The beam former output is given by

Where $\mathbf{w}^H$ denotes the complex conjugate transpose of the weight vector $\mathbf{w}$. In order to compute the optimum weights, the array response vector from the sampled data of the array output has to be known. The array response vector is a function of the incident angle as well as the frequency. Since the signals are narrowband, the propagation delay across the array is much smaller then the reciprocal of the signal bandwidth, and it follows that by using a complex envelop representation, the output of the k_{th} element of the array, $x_k(t)$ (see figure 2.3), can be expressed as a sum of phase-shifted and attenuated versions of the original signal $si(t)$ and is given in the following equation [8].

$$\mathbf{x}_k(t) = \sum_{i=1}^{p} a_k(\theta_i) s_i(t) e^{-j\omega_0 \tau_k(\theta_i)} + n_k(t) \qquad (2.4)$$

Where $\tau_k(\theta i)$ is the propagation delay between a reference point and the k_{th} sensor for the i_{th} wave front impinging on the array from direction θ_i, $a_k(\theta_i)$ is the corresponding sensor element complex response (gain and phase) at frequency w_0 and $n_k(t)$ is the noise present at k_{th} sensor. Employing vector notation for the outputs of the N sensors, the data model becomes.

$$\mathbf{x}(t) = \sum_{i=1}^{p} \mathbf{a}(\theta i) si(t) + \mathbf{n}(t) \qquad (2.5)$$

Where we have the following

- $\mathbf{x}(t) = [x_1(t), x_2(t), \ldots\ldots x_N(t)]^T$ is $Nx1$ vector of signals representing sensor outputs.
- $s_i(t)$ is the signal emitted by the i_{th} source as received at the reference sensor 1 of the array.

- $\mathbf{a}(\theta i) = [a_1(\theta_i)e^{-j\omega_0\tau_1(\theta_i)}, \ldots\ldots\ldots, a_N(\theta_i)e^{-j\omega_0\tau_N(\theta_i)}]^T$ is a Nxl vector often termed as array response or array steering vector towards direction θ_i.
- $\mathbf{n}(t) = [n_1(t), n_2(t), \ldots\ n_N(t)]^T$ is a ***Nxl*** noise vector.

Equation (2.7) can be written in a more compact form using matrix notation as follows

$$\mathbf{x}(t) = \mathbf{A}(\theta_i)\mathbf{s}(t) + \mathbf{n}(t) \qquad (2.6)$$

where $\mathbf{A}(\theta)$ is the ***Nxp*** matrix of the array steering vectors, i.e

$$\mathbf{A}(\theta) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots\ldots \mathbf{a}(\theta_p)]$$

and $s(t)$ is *px1* vector of the signals generated by p narrowband sources, i.e

$$\mathbf{s}(t) = [s_1(t), s_2(t), \ldots\ s_p(t)]^T$$

Our objective is to determine the Direction of Arrivals (DOA's) $\theta_1, \theta_2, \theta_3, \ldots \theta_p$, of the sources from the information given by Eq (2.6). To achieve the goal ***let's*** make the following assumptions

A.1 The number of sources generating the signals is known and is smaller than the number of sensors i.e $p < N$.

A.2 The set of any p steering vectors is linearly independent.

A.3 The signal waveforms are non-coherent Gaussian processes.

A.4 The noise samples $n_k(t)$ are zero mean Gaussian random pmcess with variance and are statistically independent from each other.

Assumptions **A.l** and A.2 guarantee the uniqueness of the solution and assumptions A.3 and **A.4** have been made to make the mathematical treatment an easier one.

2.5 SIGNAL AUTOCOVARIANCE MATRICES

Let us consider the problem in the context of eigen space, a space which is spanned by the eign vectors of a matrix, that corresponds to some scalar constant called eign value of the matrix . Consider the array output $x(t)$ which is given by

$$\mathbf{X} = A(\theta)\, S + N.$$

The N x N correlation matrix R_x of the vector $x(t)$ is given by

$$\mathbf{R}_x = E\{ \mathbf{X}\,\mathbf{X}^H \} \tag{2.7}$$

Now

$$\mathbf{X}\,\mathbf{X}^H = (A(\theta)S + N)\,(A(\theta)\,S + N)^H$$

$$= (A(\theta)\,S + N))\,(S^H A^H(\theta) + N^H)$$

$$=(A(\theta)\,S\,S^H A^H(\theta) + (A(\theta)\,S\;N^H) + (NS^H A^H(\theta) + (N\,N^H)$$

Therefore from Eq ***(2.7)*** we have

$$\mathbf{R}_x = E\{(A(\theta)\,S\,S^H A(\theta) + (A(\theta)\,S\;N^H) + (N\,S^H A^H(\theta) + (N\,N^H)$$

$$= E\{A(\theta)\,S\,S^H A(\theta\} + E\{A(\theta)\,S\;N^H) + E\{N\,S^H A^H(\theta\;\} + E\{N\,N^H\}$$

$$=A(\theta)E\{S\,S^H\}A(\theta\} + A(\theta)E\{S\,N^H\} + E\{N\,S^H\}\,A^H(\theta + E\{N\,N^H\}$$

Assuming that the signal and the noise are uncorrelated Gaussian random process then the following holds.

$$E\{S\,N^H\} = E\{N\,S^H\} = 0$$

Hence we are left with

$$R_x = A(\theta)R_{SS}A^H(\theta) + \sigma^2 I \qquad (2.8)$$

Where $R_{SS} = E\{SS^H\}$ $k \times k$ autocorelation matrix of the original signals $s_i(t)$ and $\sigma^2 1 = E\{n(t)n^H(t)\}$. Let us define $\mathbf{R}_s$ as signal correlation matrix and R,, as noise correlation matrix i.e.

$$R_s = A(8)R_{SS}A^H(8) \qquad (2.9)$$

And

$$\mathbf{R}_n = \sigma^2 \mathbf{I} \qquad (2.10)$$

Then Eq (2.7) becomes

$$\mathbf{R}_x = \mathbf{R}_s + \mathbf{R}_n \qquad \mathbf{(2.11)}$$

A class of spatial spectral estimation techniques is based on the eigenvalue decomposition (EVD) of the spatial correlation matrix R,. The rationale behind this approach is that one wants to emphasize the choices for the steering vector $a(\theta)$, which correspond to signal directions. The method exploits the property that the directions of arrival determine the eigen Structure of the matrix. To see this let us suppose that $q_1^x \geq q_2^x \geq \ldots\ldots \geq q_N^x$ denote the eigenvalues of the matrix R_x and $q_1^s \geq q_2^s \geq \ldots\ldots \geq q_N^s$ denote the eigenvalues of the matrix $\mathbf{R}_s$. From assumption ***A.2*** it follows that the steering matrix $\mathbf{A}(\theta)$ is of full column rank p. Also the non-coherence of the p incoming plane waves implies that the p x p matrix $\mathbf{R}_{ss}$ given by $\mathbf{R}_{SS} = E\{\mathbf{s}(t)\mathbf{s}^H(t)\}$ is a full rank matrix. Therefore the R_s matrix given by Eq (2.9) will also have a rank equal to p. It means that R_s will have p nonzero and N-p zero eigenvalues. Since q_k^s have been arranged in a descending order, then the smallest N-P eigenvalues of R_s will be equal to zero. Thus we have q_i^x

$$q_k^s = \begin{cases} \text{Non zero} & k=1,2,\ldots\ldots\ldots p \\ \text{zero} & k=p+1,p+2,\ldots\ldots N \end{cases} \qquad (2.12)$$

$$q_k^x = q_k^s + \sigma^2 \qquad \text{from eq} \qquad (2.11)$$

Using Eq (2.11) we see that eigenvalues of R_x can be written as

$$q_k^x = \begin{cases} q_k^s + \sigma^2 & k=1,2,\ldots\ldots\ldots p \\ \sigma^2 & k=p+1,p+2,\ldots.N \end{cases} \qquad (2.13)$$

Hence the EVD of the correlation matrix R_x can be written as

$$\mathbf{R}_x = \sum_{k=1}^{p} [q_k^s + \sigma^2]\, \mathbf{e}_k \mathbf{e}_k^H + \sum_{k=p+1}^{N} \sigma^2\, \mathbf{e}_k \mathbf{e}_k^H \qquad (2.14)$$

Where

$$\mathbf{e}_k \mathbf{e}_l^H = \delta_{kl} = \begin{cases} 1 & k=l \\ \text{zero} & k \neq l \end{cases} \qquad (2.15)$$

are the orthogonal eigenvectors of the matrix R_x and Satisfy the eigenvalue equation given by

$$\mathbf{R}_x\, \mathbf{e}_k = q_k^x\, \mathbf{e}_k \qquad k = 1,2, \ldots\ldots N \qquad \textit{(2.16)}$$

Using Eq (2.11), Eq (2.14) can be written as

$$\mathbf{R}_x\, \mathbf{e}_k = \sigma^2\, \mathbf{e}_k \qquad k = p + 1, \ldots.. N \qquad \textit{(2.17)}$$

Or equivalently,

$$(\mathbf{R}_x - \sigma^2 \mathbf{I})\, \mathbf{e}_k = 0 \qquad k = p + 1, \ldots\ldots, N \qquad (2.18)$$

Using Eq (2.7) above equation can be written as

$$\mathbf{A}(\theta)\mathbf{R}_{SS}\mathbf{A}^H(\theta)\mathbf{e}_k = 0 \qquad k = p + 1, \ldots\ldots, N \qquad (2.19)$$

From which it follows that

$$\mathbf{A}^H(\theta)\mathbf{e}_k = 0 \qquad k = p + 1, \ldots\ldots, N \qquad (2.20)$$

Or $$\mathbf{a}^H(\theta_i)\,\mathbf{e}_k = 0 \qquad i=1,2,\ldots,p \qquad k = p+1, p+2, \ldots, N \tag{2.21}$$

Equation (2.23) readily implies that the subspace spanned by the eigenvectors $\{\mathbf{e}_{p+1}, \mathbf{e}_{p+2}, \ldots\ldots \mathbf{e}_N\}$ is the orthogonal complement of the subspace spanned by the steering vectors $\{\mathbf{a}\{(\theta_1), \mathbf{a}(\theta_2), \ldots\ldots\ \mathbf{a}(\theta_p)\}$. Since the eigenvectors of the correlation matrix $\mathbf{R}_x$ are orthogonal to each other we can also conclude that the subspace spanned by the eigenvectors $\{\mathbf{e}_1, \mathbf{e}_2, \ldots\ldots, \mathbf{e}_p\}$ is exactly the same as the subspace spanned by the vectors $\{\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots\ldots \mathbf{a}(\theta_p)\}$.

The previous analysis leads to the following observations. If the propagation field contains p distinct non-coherent propagating signals in a spatially white noise environment, then the eigenvalue decomposition of the spatial correlation matrix $\mathbf{R}_x$ results in the formation of two disjoint subspaces that are the orthogonal complement of each other. The first one, called the ***Signal plus noise subspace*** is spanned by the eigenvectors corresponding to the p largest eigenvalues of $\mathbf{R}_x$. The second, called the ***noise subspace*** is spanned by the eigenvectors corresponding to the N-$p$smallest eigenvalues of $\mathbf{R}_x$. Thus given the eigenvectors of the matrix $\mathbf{R}_x$, we may determine the signal directions of arrival by searching for those steering vectors $\mathbf{a}(\theta)$ that are orthogonal to the noise subspace which is spanned by the eigenvectors $\{\mathbf{e}_{p+1}, \mathbf{e}_{p+2}, \ldots\ldots, \mathbf{e}_N\}$ corresponding to the N-p smallest eigenvalues of R,. In practice, $\mathbf{R}_x$ is unknown but can be consistently estimated from the available data as

$$\mathbf{R}_x = \frac{1}{M}\sum_{t=1}^{M} \mathbf{x}(t)\mathbf{x}^H(t) \tag{2.22}$$

Where M represents number of snapshots of $\mathbf{x}(t)$ taken at time instants $t_1, t_2, \ldots\ldots t_M$. Because of the uncertainty in the eigenvector estimates$\{\mathbf{e}_{p+1}, \mathbf{e}_{p+2}, \ldots\ldots, \mathbf{e}_N\}$ introduced by the way we estimate the matrix $\mathbf{R}_x$, we can only search for those steering vectors $\mathbf{a}(\theta)$ that are most closely orthogonal to the noise subspace.

Chapter 3

DOA Estimation Techniques

3.1 Pisarenko Harmonic Decomposition (PHD)

In this method it is assumed that $x(t)$ is a sun of p narrow band non-coherent plane wave signals in white noise and that the number of sources, p generating these signals is known. A (p+1)x(p+1) autocorrelation $\hat{R}_x$ is estimated and its EVD is performed. With a (p+1)x(p+1) autocorrelation matrix R_x the dimension of the noise subspace becomes equal to one and is spanned by the eigenvector corresponding to the minimum eigenvalue $q_{min} = \sigma^2$. Denoting this eigenvector by $\hat{e}_{min}$, it follows that will $\hat{e}_{min}$ be approximately orthogonal to each steering vector $a(\theta_i)$, i.e $a^H(\theta_i)\,\hat{e}_{min} = 0$ for i=1,2,................,P

Therefore if we form a Direction Estimation Function (DEF), also called the **Pseudo-Spectrum**, like the one given below.

$$\hat{P}_{PHD} = \frac{1}{\left| a^H(\theta)\hat{e}_{min} \right|^2} \tag{3.1}$$

Then $P_{PHD}(\theta)$ will be infinite (theoretical) at Locations where $\theta = \theta_i$ for i = 1, .., p. In practice however, a plot of $P_{PHD}(\theta)$ will contain p peaks. The locations of these peaks in the plot of DEF may be used to estimate the Directions of Arrival (DOA) of the original signals $s_i(t)$.

The major disadvantage with this method is that if an eight element array is used, seven signal sources are assumed to exist and the method will locate seven DOA's. If only one signal source is known to exist, only signals from the first two antenna elements should be taken. However the method being of theoretical interest, has led to the important insights into the DOA estimation problems, and has provided the stimulus for the development of other EVD methods that are more robust.

3.2 Multiple Signal Classifier (MUSIC)

Music Algorithm is an extension of PHD method and was presented by Schmitt in 1979. to see how it works, let us assume that $x(t)$ is a sum of p narrow band non-coherent plane wave signals in white noise and that the number of sources, p generating these signals is known. Let an N x N estimate of the autocorrelation matrix of $x(t)$ with $N > p + 1$ (In PHD method $N = p + 1$). If the eigenvalues of R_x are arranged in descending order, $q_1, \geq q_2, \geq q_3, \geq \ldots, q_N$ and if $\{\hat{e}_1, \hat{e}_2, \ldots, \hat{e}_p, \hat{e}_{p+1}, \ldots, \hat{e}_N\}$ are the corresponding eigenvectors, then these eigenvectors may be divided into two groups: the p signal plus noise eigenvectors correspoinding to p largest eigenvalues and the $N - p$ noise eigenvectors corresponding to the $N - p$ smallest eigenvalues that ideally are equal to σ^2. These N-p noise eigenvectors will be approximately orthogonal to the p steering vectors $a(\theta)$, i.e.

$$a^H(\theta_i)\,\hat{e}_k = 0 \qquad \text{for } i = 1, 2, \ldots, p \quad k = p + 1, \ldots, N \qquad (3.2)$$

or simply

$$a^H(\theta_i)\,\hat{e}_k = 0 \qquad \text{for } k = p + 1, \ldots, N \qquad (3.3)$$

where $\hat{e}_k$ represents the k_{th} noise eigenvector.

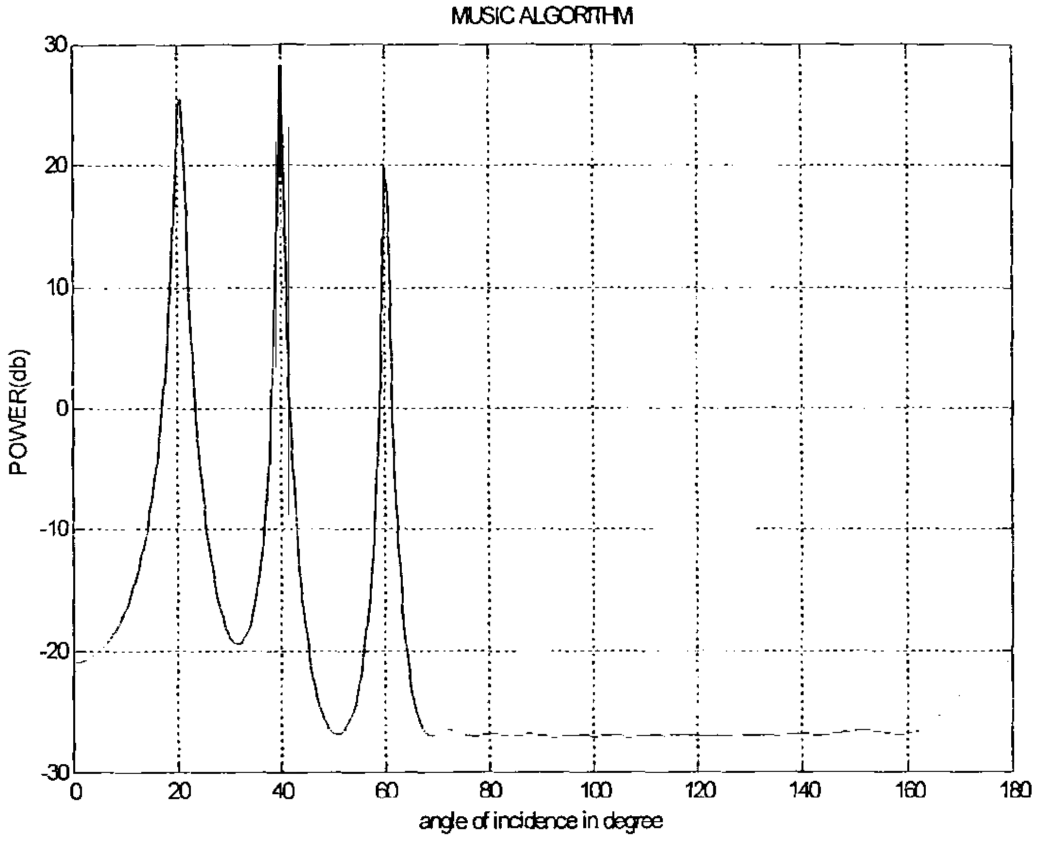

Fig. 3.1: Music pseudospectrum for $\theta 1 = 20'$ $\theta 2 = 40'$ $\theta 3 = 60^{\circ}$

All the N – p noise eigenvectors will share the same p roots. However, because each noise eigenvector is a length N vector, there are an additional N – p roots that are due entirely to the noise i.e each of the noise eigenvector will have these roots due to noise only and that also at random frequencies. This may given rise to the spurious peaks in the pseudo spectrum of single noise eigenvector (as in the case of PHD method.) Therefore when only one noise eign vector is used to estimate DOA's, there may be some ambiguity in distinguishing the desired peaks from the spurious ones. In MUSIC algorithm the effects of these spurious peaks are reduced by averaging the pseudo spectra obtained for each of the noise eigenvectors i.e. MUSIC algorithm assumes the following form of the DEF or pseudo spectrum.

$$P_{MUSIC}(\theta) = \frac{1}{\sum_{k=p+1}^{N} \left| a^H(\theta)\hat{e}_k \right|^2} \quad (3.4)$$

Above equation can be written as

$$P_{MUSIC}(\theta) = \frac{1}{\sum_{k=p+1}^{N} a^H(\theta)\hat{e}_k) (\hat{e}_k^H a(\theta))} \quad (3.5)$$

Let us define

$$\hat{U}_N = \sum_{k=p+1}^{N} \hat{e}_k \hat{e}_k^H \quad (3.6)$$

Then Eq (3.5) can be written in amore compact form as follows.

$$\hat{P}_{MUSIC} = \frac{1}{a^H(\theta)\hat{U}_N a(\theta)} \quad (3.7)$$

This is called MUSIC pseudo-spectrum. One note able thing about this spectrum is that, it is based on single realization of the stochastic process represented by the snap shots $x(t)$ for t = 1,2, ..., M. Music estimates are consistent and converge to true source bearings as the number of snapshots grows to infinity. However this is true only for the case of uncorrelated signal. For strongly correlated signals, the estimates provided by the MUSIC pseudo spectrum are extremely poor compared with the situation where uncorrelated signals are used. This is the major problem with the music algorithm.

3.3 The Minimum Norm (MN) Algorithm

The Minimum Norm (MN) method is also based on the EVD of the spatial correlation marix R_x. In this method, two sets of eigenvectors are formed as follows.

$$E_s = [\, e_1, e_2, \ldots.., e_p] = \begin{bmatrix} g_s^T \\ G_s \end{bmatrix} \tag{3.8}$$

And

$$E_n = [\, e_{p+1}, \ldots..., e_N] = \begin{bmatrix} g_n^T \\ G_n \end{bmatrix} \tag{3.9}$$

where g_s^T and g_s^T are 1 x p and 1 x $(N - p)$ row vectors respectively. The matrices G_s and G_n have dimensions (N − 1) x p and $(N$ -1) x $(N - p)$, respectively. Instead of forming pseudo spectrum that uses all of the noise eigenvectors as in the MUSIC, the MN algorithm aims to find a single N x 1 vector Q that satisfies the following conditions;

- The vector Q lies in the noise subspace and therefore it is orthogonal to the signal plus noise subspace i.e.

$$E_n^H Q = 0 \tag{3.10}$$

- The first element of Q is equal to unity i.e.

$$Q = \begin{bmatrix} 1 \\ -q \end{bmatrix} \tag{3.11}$$

- The vector Q has minimum norm.

In order to ensure that the minimum norm solution is not a null vector, the first element of Q has taken to be unity. The aim is now to find a $(N$ -1) x 1 vector q that has minimum norm. By putting Eq *(3.8)* and Eq *(3.11)* into Eq *(3.10)* we get the following equation.

$$G_s^H q = g_s \qquad (3.12)$$

This is a linear system of p equations in N-1 unknowns. Since we assumed that N > p, Eq (3.12) represents an undetermined system of linear equations with no unique solution. One approach that is often used to define a unique solution is to find a vector satisfying the equations that has the minimum norm i.e.

$$\|q\| = g_s \text{ subject to } G_s^H q = g_s \qquad (3.13)$$

If G_s has rank equal to N – l then the (N – l) x (N – I) matrix $G_s G_s^H$ is invertible and the minimum norm solution is given by;

$$q = (G_s G_s^H)^{-1} G_s g_s \qquad \textbf{(3.14)}$$

The matrix

$$G' = (G_s G_s^H)^{-1} G_s \qquad (3.15)$$

is known as the pseudo-inverse of the matrix G_s^H for the underdetermined problem. Correspondingly the minimum norm solution for the vector Q is given by

$$Q = \begin{bmatrix} 1 \\ -(G_s G_s^H)^{-1} G_s g_s \end{bmatrix} \qquad (3.16)$$

With the knowledge of minimum norm vector Q, the MN algorithm forms the following form of the DEF

$$P_{MN} = \left| \frac{1}{a^H(\theta) Q} \right| \qquad (3.17)$$

Again as with the MUSIC, MN pseudo spectrum is also based on a single realization of the underlying stochastic process.

3.4 The Minimum Variance Distortionless Method (MVD)

The methods we discussed so far are also referred to as noise subspace methods, because the DOA's are estimated using the fact that all the noise eigenvectors are orthogonal to the steering vectors (that contain DOA information). In this section we shall discus a method called MVD , which does not use the noise eigenvectors to determine the pseudo spectrum; instead it uses signal plus noise eigenvectors to obtain the pseudo spectrum and hence this method is also referred to as signal plus noise subspace method.

MVD method is developed as a constrained optimization problem. Recall that the sensors outputs $x(t)$ are weighted by a vector $\mathbf{w}$ to produce the beamformer output $y(t)$ i.e.

$$y(t) = \mathbf{w}^H \mathbf{x}(t)$$

The spectral estimates are derived by finding a weight vector $\mathbf{w}$, which minimizes the output noise variance. In order to ensure that the desired signal from some direction θ is passed to the output with a specific gain and phase (i.e without any distortion), a constraint may be used so that the response of the beamformer to the desired signal is

$$\mathbf{w}^H \mathbf{a}(\theta) = 1$$

where $\mathbf{a}(\theta)$ represents an ideal plane wave corresponding to the direction of interest 8. Minimization of contributions to the output due to noise is accomplished by choosing the weights to minimize the variance of the output power, which is the mean square value of $y(t)$ i.e.

$$Var\{y(t)\} = E\{|y(t)|^2\} \tag{3.18}$$

$$= E\{(\mathbf{w}^H \mathbf{x}(t))(\mathbf{w}^H \mathbf{x}(t))^H\}$$

$$= \mathbf{w}^H E\{\mathbf{x}(t)\mathbf{x}^H(t)\}\mathbf{w}$$

or

$$\text{Var}\{y(t)\} = \mathbf{w}^H \mathbf{R}_x \mathbf{w}$$

$$\text{Var}\{y(t)\} = \mathbf{w}^H \mathbf{R}_s \mathbf{w} + \mathbf{w}^H \mathbf{R}_n \mathbf{w} \tag{3.19}$$

Where we have made use of Eq $R_x = R_s + R_n$. thus we have a complete constrained optimization problem stated as follows

Minimize

$$Var\{y(t)\} = \mathbf{w}^H \mathbf{R}_x \mathbf{w} \qquad (3.20)$$

$$= \mathbf{w}^H \mathbf{R}_S \mathbf{w} + w^H \mathbf{R}_n \mathbf{w}$$

Subject to

$$\mathbf{w}^H \mathbf{a}(\theta) = 1$$

This is equivalent to minimizing the output noise power w^H R,, w alone. Now any constrained optimization method can be used to solve this problem, however the most simplest is the Lagrange multiplier method. If we follow this method, the optimum weight vector is given by

$$w_{opt} = \frac{\mathbf{R}_x^{-1} \mathbf{a}(\theta)}{\mathbf{a}^H(\theta) \mathbf{R}_x^{-1} \mathbf{a}(\theta)} \qquad (3.21)$$

The power in the beam when steered in the direction of interest determined by $\boldsymbol{a}(\theta)$ becomes

$$P_{MVD}(\theta) = \frac{1}{\mathbf{a}^H(\theta) \mathbf{R}_x^{-1} \mathbf{a}(\theta)} \qquad (3.22)$$

Fig 3.2 shows capon (MVD) pseudospectrum for three uncorelated signals coming from different directions

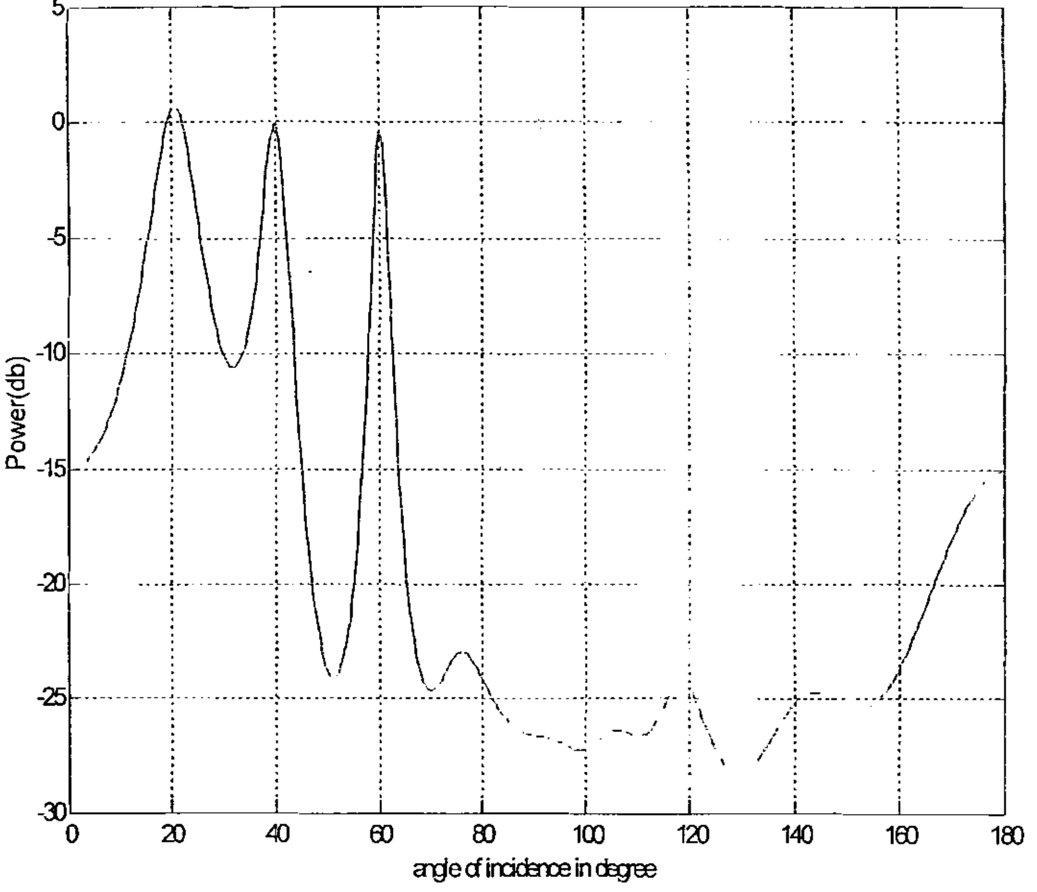

Fig.3.2:Capon(MVD) pseudo spectrum for $\theta 1= 20^{\circ}$ $\theta 2= 40^{\circ}$ $\theta 3= 60^{\circ}$

this is the MVD direction estimation function (DEF) or MVD pseudo spectrum. Now let us perform EVD of the spatial correlation matrix i.e.

$$\mathbf{R}_x = \sum_{k=1}^{N} q_k^{x} \mathbf{e}_k \mathbf{e}_k^{H} \qquad (3.23)$$

The inverse R_x^{-1} has the same eigenvectors as that of R_x, but its eigen values are reciprocal of those of R_x. Therefore

$$\mathbf{R}_x^{-1} = \sum_{k=1}^{N} \frac{1}{q_k^{x}} \mathbf{e}_k \mathbf{e}_k^{H} \qquad (3.24)$$

$$\mathbf{R}_x^{-1} = \sum_{k=1}^{p} \frac{1}{q_k^S + \sigma^2} \mathbf{e}_k \mathbf{e}_k^H + \sum_{k=p+1}^{N} \frac{1}{\sigma^2} \mathbf{e}_k \mathbf{e}_k^H \qquad (3.25)$$

Since MVD is a signal plus noise subspace method, The EVD of Rx^{-1} is truncated to include only the terms that correspond to signal plus noise subspace i.e.

$$\mathbf{R}_x^{-1} = \sum_{k=1}^{p} \frac{1}{q_k^S + \sigma^2} \mathbf{e}_k \mathbf{e}_k^H \qquad (3.26)$$

So equation (3.27) becomes

$$P_{MVD}(\theta) = \frac{1}{\mathbf{a}^H(\theta) \left[\sum_{k=1}^{P} \frac{1}{q_k^S + \sigma^2} \mathbf{e}_k \mathbf{e}_k^H \right] \mathbf{a}(\theta)} \qquad (3.27)$$

$$= \frac{1}{\left[\sum_{k=1}^{p} \frac{1}{q_k^S + \sigma^2} \left(\mathbf{a}^H(\theta) \mathbf{e}_k \mathbf{e}_k^H \mathbf{a}(\theta) \right) \right]}$$

$$= \frac{1}{\left[\sum_{k=1}^{p} \frac{1}{q_k^S + \sigma^2} \left(\mathbf{a}^H(\theta) \mathbf{e}_k \right) \left(\mathbf{a}^H(\theta) \mathbf{e}_k \right)^H \right]}$$

$$P_{MVD} = \frac{1}{\left[\sum_{k=1}^{P} \frac{1}{q_k^S + \sigma^2} \left| a^H(\theta) e_k \right|^2 \right]}$$

Equation gives another form of MVD pseudo spectrum

3.5 The ESPRIT Algorithm

ESPRIT stands for *Estimation of Signal Parameters via Rotational Invariance Techniques* and was first proposed by Roy and Kailath in 1989. The goal of the ESPRIT techniques to exploit the rotational invariance in the signal subspace which created by two arrays with a translational invariance structure. ESPRIT inherently assumes narrowband signals so that one knows the translational phase relationship between the multiple arrays to be used[7]. As with MUSIC, ESPRIT assumes that there are p $<N$ narrow-band sources centered at the center frequency $f0$. These signal sources are assumed to be of a sufficient range so that the incident propagating field is approximately planar. The sources can be either random or deterministic and the noise is assumed to be random with zero-mean. ESPRIT assumes multiple identical arrays called *doublets*. These can be separate arrays or can be composed of sub arrays of one larger array. It is important that these arrays are displaced translationally but not rotationally. An example is shown in Fig. 3.3 where a four element linear array is composed of two identical three-element sub arrays or two doublets. These two sub arrays are translationally displaced by the distance d. Let us label these arrays as array 1 and array 2

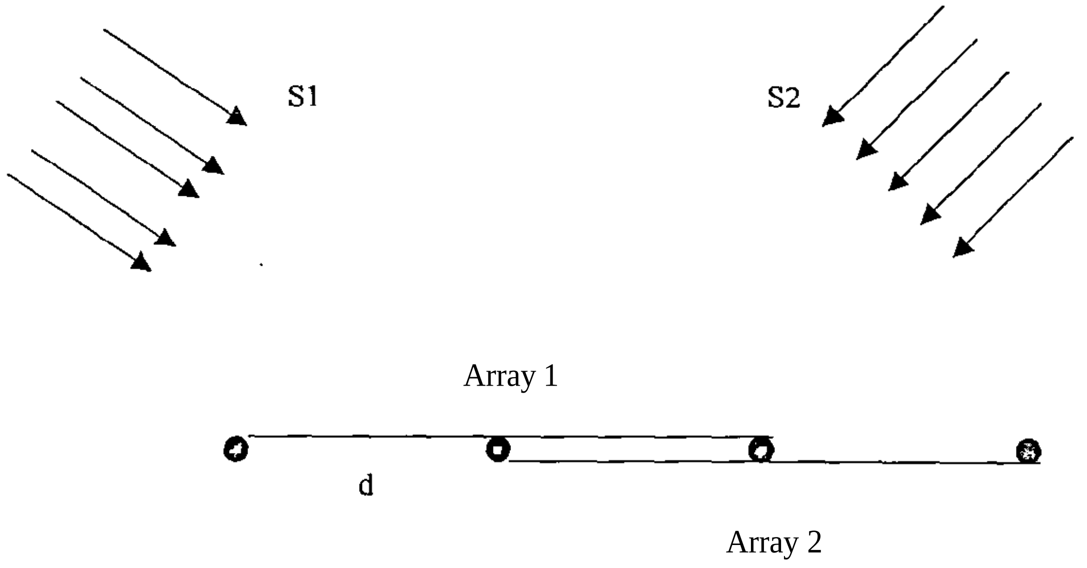

Fig. 3.3:Doublet composed of two identical displaced arrays[6]

$$\mathbf{x}_1(k)=[\mathbf{a}_1(\theta 1)\ \ \mathbf{a}_1(\theta 2),\ldots\ldots\ldots\ldots\ \mathbf{a}_1(\theta p)].\begin{bmatrix} s_1(k) \\ s_2(k) \\ . \\ . \\ s_p(k) \end{bmatrix}+\mathbf{n}_1(k) \qquad (3.28)$$

$$=\mathbf{A}_1.s(k)+\mathbf{n}_1(k)$$

and

$$\mathbf{x}_2(k)=\mathbf{A}_2.\mathrm{s}(k)+\mathbf{n}_2(k) \qquad (3.29)$$

$$=\mathbf{A}_1.\mathbf{\Phi}.s(k)+\mathbf{n}_2(\mathrm{k})$$

where

$$\Phi = diag\left\{e^{jkd\sin\theta 1}, e^{jkd\sin\theta 2}\ldots\ldots\ldots\ldots, e^{jkd\sin\theta p}\right\}$$

= a *pxp* diagonal unitary matrix with phase shifts between the doublets for each AOA

Ai = Vandermonde matrix of steering vectors

for subarrays i = 1 , 2

The complete received signal considering the contributions of both sub arrays is given as

$$\mathbf{x}(k) \;=\; \begin{bmatrix} \mathbf{x}_1(k) \\ \mathbf{x}_2(k) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_1.\varphi \end{bmatrix} .s(k) + \begin{bmatrix} \mathbf{n}_1(k) \\ \mathbf{n}_2(k) \end{bmatrix} \qquad (3.30)$$

We can now calculate the correlation matrix for either the completearray or for the two subarrays. The correlation matrix for the complete array is given by

$$\mathbf{R}_{xx} = \mathrm{E}[\mathbf{x}\,.\mathbf{x}^{H}] = \mathbf{A}\mathbf{R}_{ss}\,\mathbf{A}^{H} + \sigma^2\,\mathbf{I} \qquad (3.31)$$

whereas the correlation matrices for the two subarrays are given by

$$\mathbf{R}_{11} = \mathrm{E}[\mathbf{x}_1.\mathbf{x}_1^{H}] = \mathbf{A}\mathbf{R}ss\,\mathbf{A}^{H} + \sigma^2\,\mathbf{I} \qquad (3.32)$$

and

$$\mathbf{R}_{22} = \mathrm{E}[\mathbf{x}_2.\mathbf{x}_2^{H}] = \mathbf{A}\,\Phi\,\mathbf{R}_{ss}\,\mathbf{A}^{H} + \sigma^2\,\mathbf{I} \qquad (3.33)$$

Each of the full rank correlation matrices given in Eq. (3.32) and (3.33) has a set of eigenvectors corresponding to the p signals present. Creating the signal subspace for the two subarrays results in the two matrices $\mathbf{E}_1$ and E_2. Creating the signal subspace for the entire array results in one signal subspace given by Ex. Because of the invariance structure of the array, ***Ex*** can be decomposed into the subspaces $\mathbf{E}_1$ and $\mathbf{E}_2$.Both $\mathbf{E}_1$ and $\mathbf{E}_2$ are Nx p matrices whose columns are composed of the P eigenvectors corresponding to the largest eigenvalues of $\mathbf{R}_{11}$ and $\mathbf{R}_{22}$. Since the arrays are translationally related, the subspaces of eigenvectors are related by a unique non-singular transformation matrix such that

$$E_1\Psi = E_2 \qquad (3.34)$$

There must also exist a unique non-singular transformation matrix T

Such that

$$E_1 = A\,T \qquad (3.35)$$

$$E_2 = A\Phi T \qquad (3.36)$$

and

By substituting Eqs. **(3.39)** and **(3.40)** into Eq. **(3.41)** and assuming that *A* is of full-rank, we can derive the relationship

$$T\,\Psi\,T^{-1} = \Phi$$

Thus, the eigenvalues of Ψ must be equal to the diagonal elements of Φ such that $\lambda_1 = e^{jkd\sin\theta 1}$, $\lambda_2 = e^{jkd\sin\theta 2}$, . . . , $\lambda p = e^{jkd\sin\theta p}$ and the columns of T must be the eigenvectors of Ψ.Ψ is a rotation operator that maps the signal subspace E_1 into the signal subspace E_2. One is now left with the problem of estimating the subspace rotation operator Ψ and consequently finding the eigenvalues of Ψ.If we are restricted to a finite number of measurements and we also assume that the subspaces E_1 and E_2 are equally noisy, we can estimate the rotation operator Ψ using the *total least-squares* (TLS) criterion.Details of the TLS criterion can be found in van Huffel and Vandewalle . This procedure is outlined as follows.

- Estimate the array correlation matrices R_{11}, R_{22} from the data samples.

Knowing the array correlation matrices for both subarrays, one can estimate the total number of sources by the number of large eigenvalues in either R_{11} or R_{22}.

Calculate the signal subspaces E_1 and E_2 based upon the signal eigenvectors of R_{11} and R_{22}. For ULA, one can alternatively construct the signal subspaces from the entire array signal subspace E_x.E_x is an **Nxp** matrix composed of the signal eigenvectors. E_1 can be constructed by selecting the first N/2 + 1 rows ((N + 1)/2 + 1 for odd mays) of $\mathbf{E}_x$. E_2 can be constructed by selecting the last N/2+1 rows ((N+ 1)/2 + 1 for odd arrays) of $\mathbf{E}_x$.

Next form a 2P x 2P matrix using the signal subspaces such that

$$\mathbf{C} = \begin{bmatrix} \mathbf{E}_1 \\ \mathbf{E}_2 \end{bmatrix} [\mathbf{E}_1 \quad \mathbf{E}_2] = \mathbf{E}_C \Lambda \mathbf{E}_C^H$$

where the matrix E_C is from the ***eigenvalue decomposition*** (EVD) of C such that $\lambda_1 \geq \lambda_2 \geq \ldots\ldots \lambda_{2p}$ and $\Lambda = diag\{\lambda_1, \lambda_2, \ldots, \lambda_{2p}\}$

.Partition E_C into four pxp submatrices such that

$$\mathbf{E}_C = \begin{bmatrix} \mathbf{E}_{11} & \mathbf{E}_{12} \\ \mathbf{E}_{21} & \mathbf{E}_{22} \end{bmatrix}$$

.Estimate the rotation operator

$$\Psi = -\mathbf{E}_{12}\mathbf{E}_{22}^{-1}$$

.Calculate the eigenvalues of $\lambda_1, \lambda_2, \ldots, \lambda p$

▪Now estimate the angles of arrival, given that $\lambda i = |\lambda i| e\, j \arg(\lambda i)$

$$\theta_i = \sin^{-1} \arg(\lambda i)/kd \qquad i = 1, 2, \ldots, p$$

If so desired, one can estimate the matrix of steering vectors from the signal subspace Es and the eigenvectors of Ψ given by E_Ψ such that

$$\hat{\mathbf{A}} = \mathbf{E}_S\mathbf{E}_\Psi$$

Chapter 4

Computer simulations

Matlab programme is developed to compute direction of arrival (DOA) for following DOA estimation algorithms

1-spectral estimation

2-MVD method

3-Music algorithm

In this simulation we used the following common parameters

1- Number of sources p=2 uncorrelated signals with the same *S/R* of 20 dB are corning from 30' and 35° .
2- Number of snapshots k=100
3- Number of antenna elements N=10

The noise at each antenna element is assumed to be additive white Gaussian with zero mean and variance equal to 0.5.

Fig (1) shows simulation result for spectral estimator, this technique is based on signal and noise power consideration, this technique is not showing good resolution .

Fig (2) shows simulation result for capon method based on following function

$$P_{MVD}(\theta) = \frac{1}{a^H(\theta)\, R_x^{-1}\, a(\theta)}$$

showing better resolution and giving peaks at 30° and 35"

Fig(3) shows simulation result for Music algorithm based on following function-

$$P_{MUSIC}(\theta) = \frac{1}{\sum_{k=p+1}^{N} \mathbf{a}^H(\theta)(\hat{\mathbf{e}}_k \hat{\mathbf{e}}_k^H)\,\mathbf{a}(\theta)}$$

Music algorithm outperformed the above two algorithms and gave sharp peaks at 30" and 35'

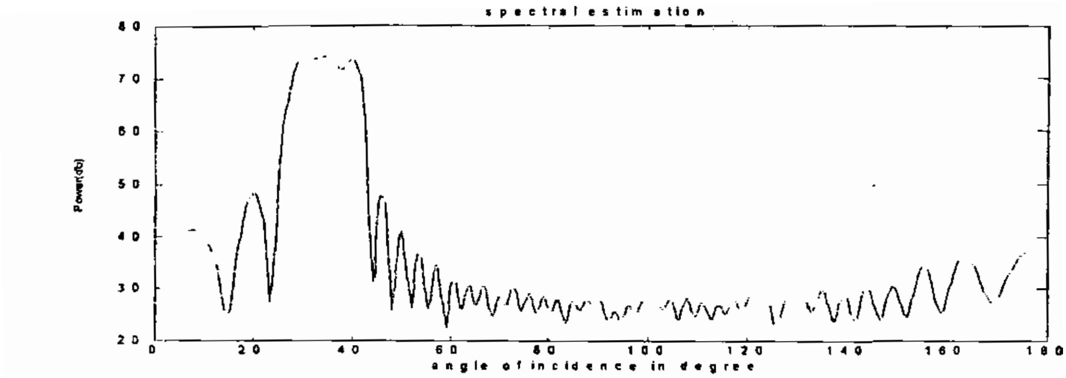

fig. 4.1: spectral estimation for two uncorrelated signals coming from directions θ1=30° and θ1=35°

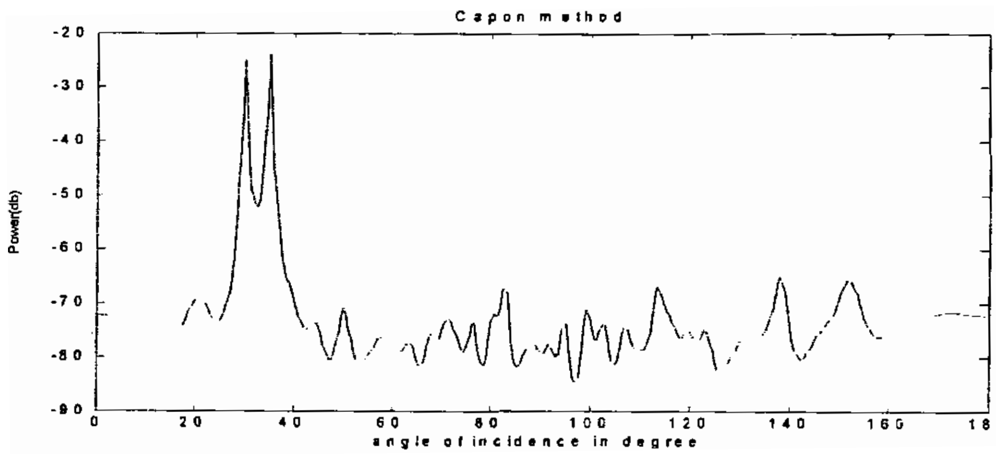

fig. 4.2: MVDR(capon method)pseudo spectrum for two uncorrelated signals coming from directions θ1=30° and θ1=35°

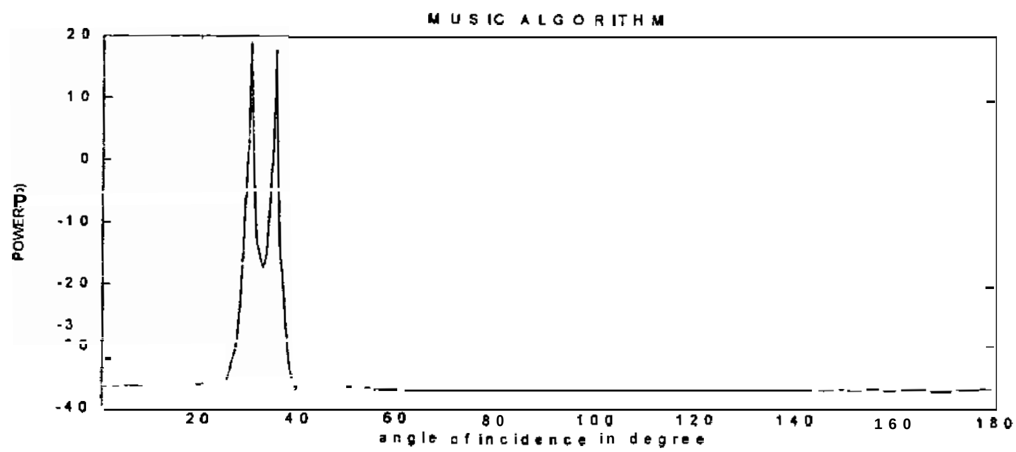

fig.: 4.3 MUSIC pseudo spectrum for two uncorrelated signals coming from directions θ1=30° and θ1=35°

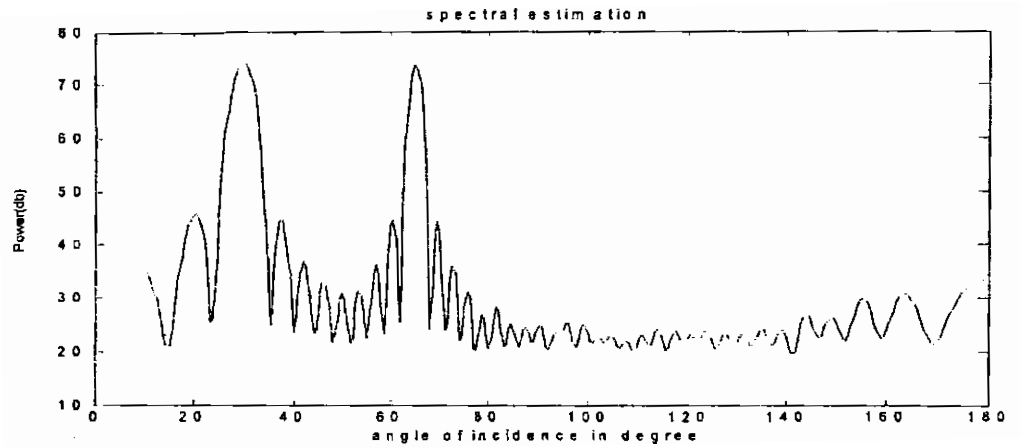

fig 4.4 spectral estimation for two uncorrelated signals coming from directions $\theta1=30^o$ and $\theta1=65^o$

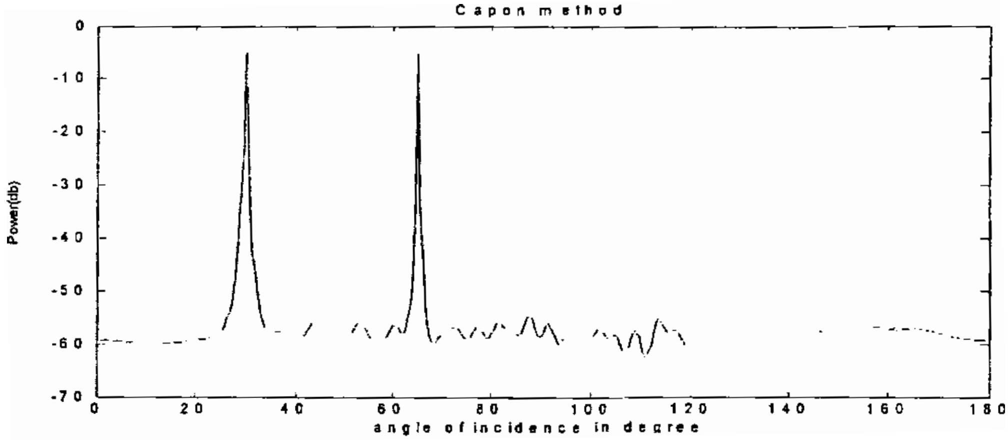

fig 4.5 MVDR(capon method) for two uncorrelated signals coming from directions $\theta1=30^o$ and $\theta1=65^o$

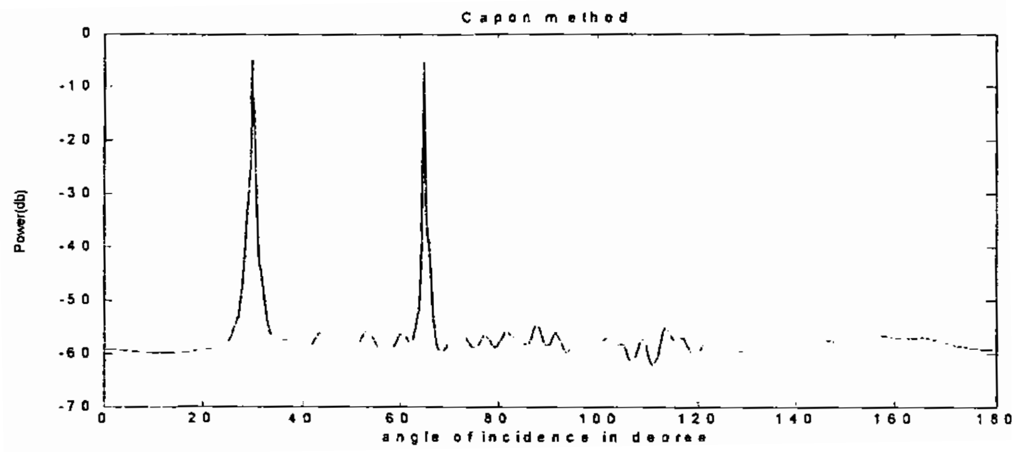

fig 4.6 MUSIC Pseudo spectrum for two uncorrelated signals coming from directions $\theta1=30^o$ and $\theta1=65^o$

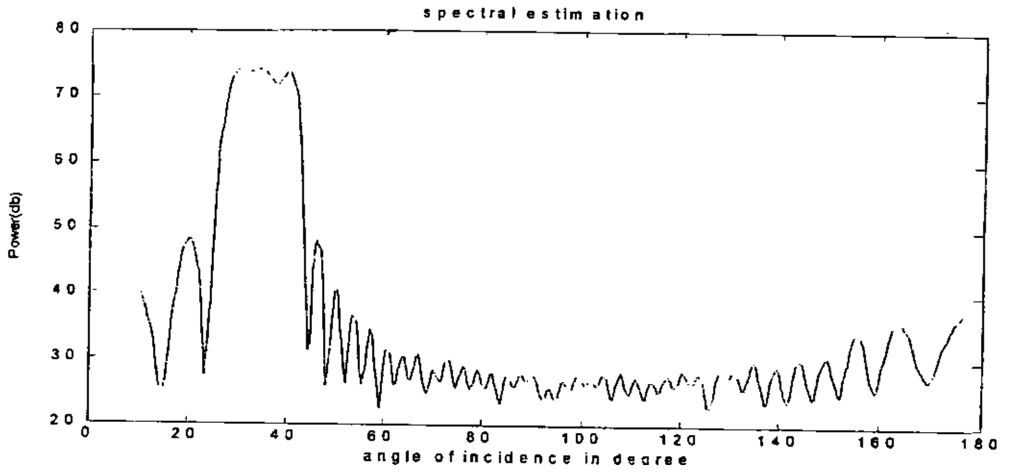

fig 4.7 spectral estimation for three uncorrelated signals coming from directions $\theta1=30^{o}$ $\theta2=35^{o}$ and $\theta3=40^{o}$

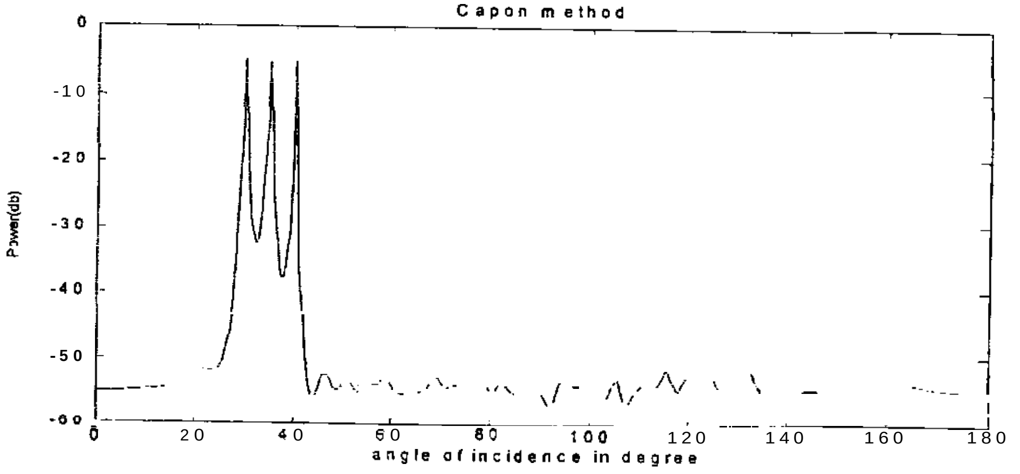

fig 4.8 MVDR (capon method) for three uncorrelated signals coming from directions $\theta1=30^{o}$ and $\theta2=35^{o}$ and $\theta3=40^{o}$

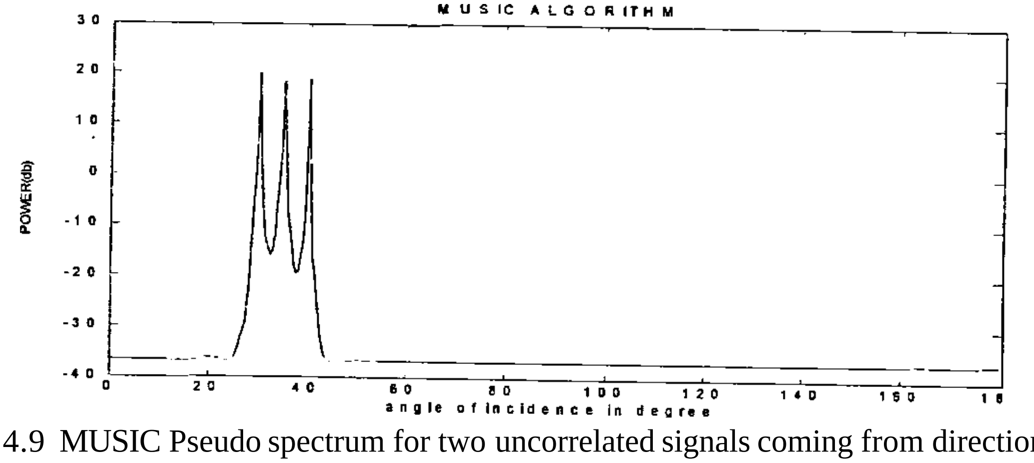

fig 4.9 MUSIC Pseudo spectrum for two uncorrelated signals coming from directions $\theta1=30^{o}$ $\theta2=35^{o}$ and $\theta3=40^{o}$

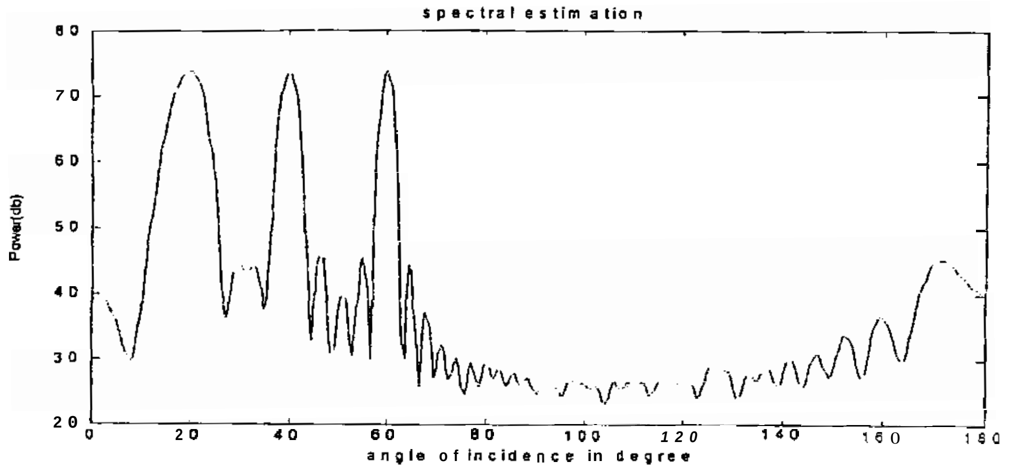

fig 4.10 spectral estimation for three uncorrelated signals coming from directions $\theta 1=20^{o}$ $\theta 2=40^{o}$ and $\theta 3=60^{o}$

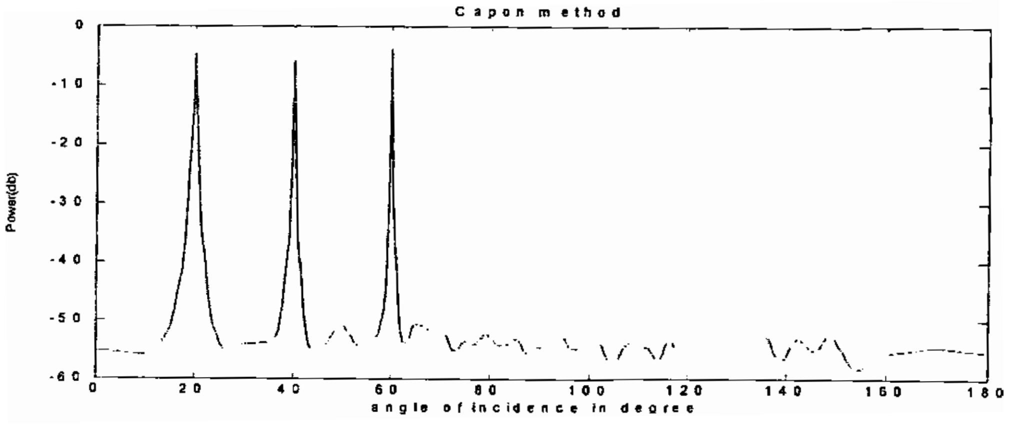

fig 4.11 MVDR (capon method) for three uncorrelated signals coming from directions $\theta 1=20^{o}$ and $\theta 2=40^{o}$ and $\theta 3=60^{o}$

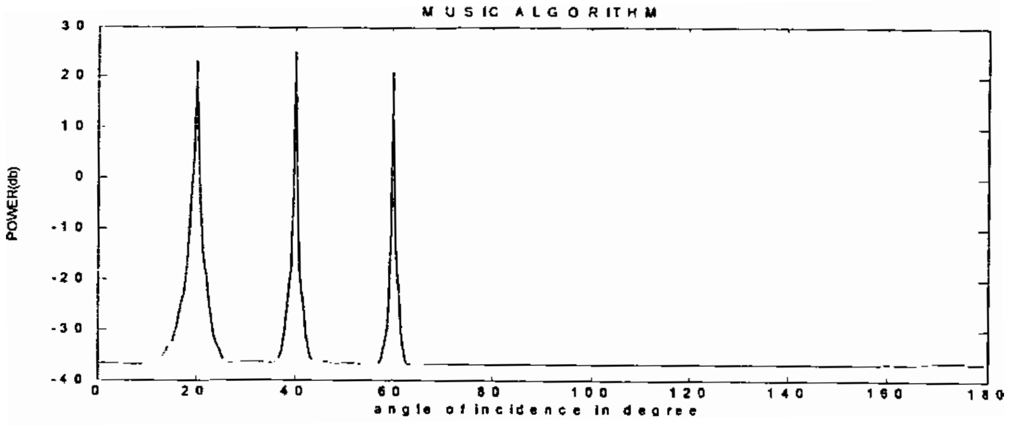

fig 4.12 MUSIC Pseudo spectrum for two uncorrelated signals coming from directions $\theta 1=20^{o}$ $\theta 2=40^{o}$ and $\theta 3=60^{o}$

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