Optimizing Production in Pakistan Marble Industry: Impact of Carbon Tax Policy

By

Um-ul-Baneen

Department of Mathematics and Statistics
Faculty of Sciences
International Islamic University, Islamabad
Pakistan

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Reg. No. 918-FOS/MSMA/F23

Supervised By

Dr. Sajida Kousar

Department of Mathematics and Statistics
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A dissertation submitted in the partial fulfillment of requirements for the award of degree of ***MASTER OF SCIENCE*** *in* ***MATHEMATICS***

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Certificate of Approval

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A DISSERTATION SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN MATHEMATICS

We accept this dissertation as confirming to the required standard.

Dr. Asghar Khan (*External Examiner*)	*Dr. Amna Kalsoom* (*Internal Examiner*)
Dr. Sajida Kousar (*Supervisor*)	*Prof. Dr. Ishfaq Ahmed* (*Chairperson*)

Department of Mathematics and Statistics
Faculty of Sciences
International Islamic University, Islamabad
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Author's Declaration

I hereby declare that this thesis, in whole or in part, is entirely free from any form of plagiarism or copying. I hereby declare that I have carefully compiled this dissertation exclusively with my efforts, with the support and guidance of my mentor, Dr. Sajida Kousar. All utilized sources have been appropriately referenced and cited by academic standards. The information provided in this dissertation has not been submitted for a degree or qualification application to any other academic institution before.

Name of Student: Um-ul-Baneen

Reg. No. 918-FOS/MSMA/F23

Dated: October 03, 2025

Forwarding Sheet by Research Supervisor

The Thesis entitled **Optimizing Production in Pakistan Marble Industry: Impact of Carbon Tax Policy** submitted by **Um-ul-Baneen**, Reg. No 918-FOS/MSMA/F23 in partial fulfillment of MS Degree in Mathematics has been completed under my guidance and supervision. I am satisfied with the quality of her research work and allow her to submit this thesis for further process to graduate with Master of Science degree from the Department of Mathematics and Statistics, as per IIUI rules and regulations.

Dr. Sajida Kousar
Assistant Professor
Department of Mathematics and Statistics,
International Islamic University,
Islamabad.

Dedicated

To

My Father

Iftikhar Ahmed Khan

(My guiding light and unwavering support, whose wisdom, sacrifices, and boundless love shaped my journey. You believed in me when I didn't believe in myself, and every achievement I make is a reflection of your dedication.)

My Mother

Bushra Tazeem Farzana

(A resilient soul who turned her unfulfilled academic dreams into my motivation. Your silent prayers, endless patience, and selfless love have been my strength. This achievement is as much yours as it is mine.)

Abstract

Pakistan's marble industry is a significant economic driver, but it is hindered by inefficiencies in manufacturing, high energy consumption, and substantial environmental impacts. This study suggests a two-pronged approach to increasing sustainability. First, it conducts a comprehensive survey of industrialists, health professionals, economists, and environmental experts to investigate stakeholder perspectives on the implementation of carbon tax policy. The survey data is examined using Multi-Criteria Decision Making (MCDM) approaches to determine the policy's viability and prospective effects on industry competitiveness and environmental footprint.

The study then creates a multi-objective optimization model to increase production efficiency while minimizing environmental impact. Using MATLAB, the model increases production output while minimizing carbon emissions, energy consumption, and air pollution. The findings suggest practical options for improving resource allocation and implementing greener technology. By combining policy analysis and operational optimization, this study provides actionable insights for making the Pakistan marble business more sustainable and competitive in an ecologically concerned global market.

Acknowledgement

First and foremost, I am deeply grateful to **Almighty Allah** for His countless blessings, guidance, and strength throughout my academic journey.

I want to express my sincere gratitude to my supervisor,**Dr. Sajida Kousar**, for her invaluable guidance and constant support during my research and thesis work. Her expertise, encouragement, and insightful feedback were instrumental in shaping this work.

My heartfelt thanks go to my beloved parents, **Mr. and Mrs. Iftikhar**, for their unconditional love, endless sacrifice, and unwavering belief in me. Their prayers and support have been my greatest strength.

I also pay tribute to my late brother, **Bilal Hassan**, whose memory remains a guiding light in my life. My sincere appreciation extends to all my siblings and friends, whose unwavering support and kindness have been a constant source of strength throughout this journey.

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Chapter 1

Introduction

Marble has been an important aspect of human civilization for generations. Its origin can be traced back to antiquity, when it was admired for its strength, elegance, and aesthetic appeal. Marble was widely used in the construction of renowned structures, sculptures, and monuments by notable civilizations such as the Greeks, Romans, and Egyptians. Marble represents elegance and sophistication today, and it continues to be a staple of the global building and design industries. Pakistan is a market leader in this flourishing sector thanks to its enormous quantities of high-quality marble.

Marble reserves in Pakistan are primarily situated in the northern and western portions of the nation. Khyber Pakhtunkhwa (KP) province has some of the richest reserves, with districts like Buner, Swat, Shangla, and Malakand serving as major production hubs. Similarly, the Chagi area in Balochistan and Karachi and Thatta regions in Sindh are well-known for their marble deposits. These deposits contain several kinds, including the famed Ziarat White, Oynx, Black, and Gold marbles, which are in high demand in domestic and international markets.

Marble's path from quarry to finished product includes numerous crucial steps. Mining begins in the quarry, where marble blocks are retrieved with mechanical tools like wire saws and diamond cutters, as well as by hand. Having enormous reserves, Pakistan's mining process is generally inefficient, and antiquated technology generates a lot of waste. Following extraction, marble blocks are transported to processing companies in urban centers such as Karachi, Lahore, Islamabad, and Peshawar as depicted in Figure 1.1.

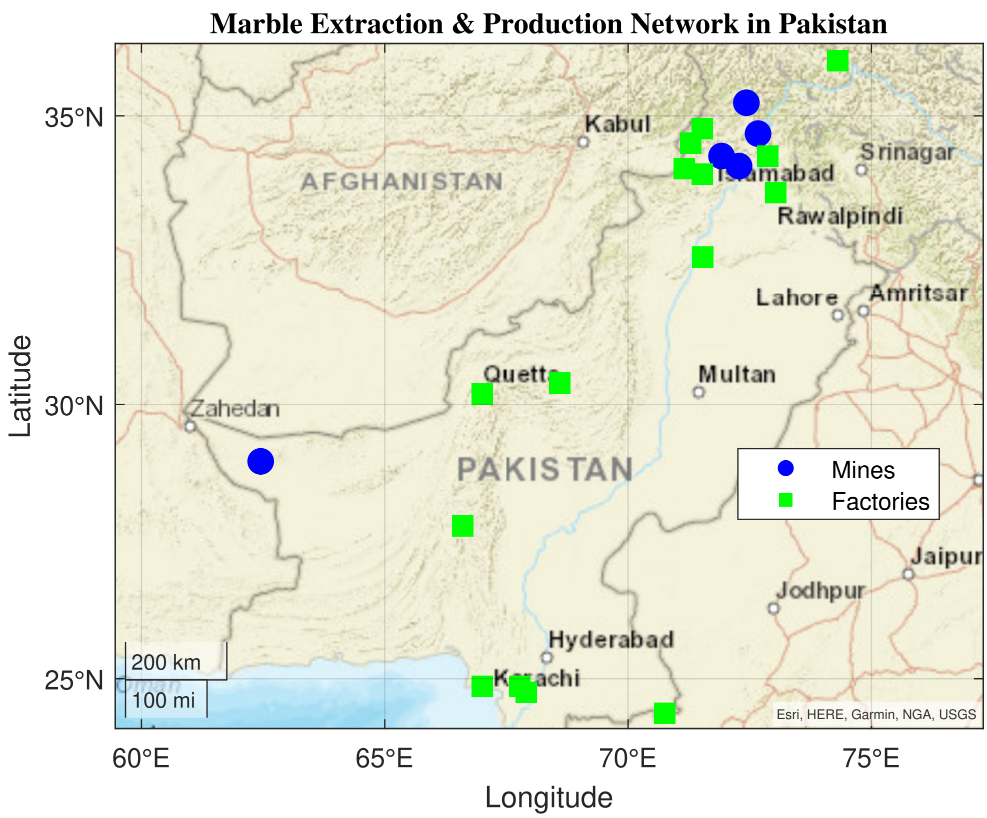

Figure 1.1: Marble Mining and Factory Locations in Pakistan

In processing facilities, raw marble blocks are subjected to a variety of operations, such as cutting, polishing, and finishing. The cutting process involves dividing the raw blocks into slabs or tiles of specified sizes, utilizing sophisticated equipment like gang saws and CNC cutters. Polishing is a detailed procedure designed to enhance the marble's inherent shine and visual appeal. Nevertheless, numerous processing units in Pakistan depend on conventional machinery, which not only restricts operational efficiency but also adversely affects the quality of the final products. Additionally, transportation is a vital component of the value chain, as it involves the movement of marble from remote quarries to urban processing locations and, eventually, to both domestic and international markets (see Figure 1.2).

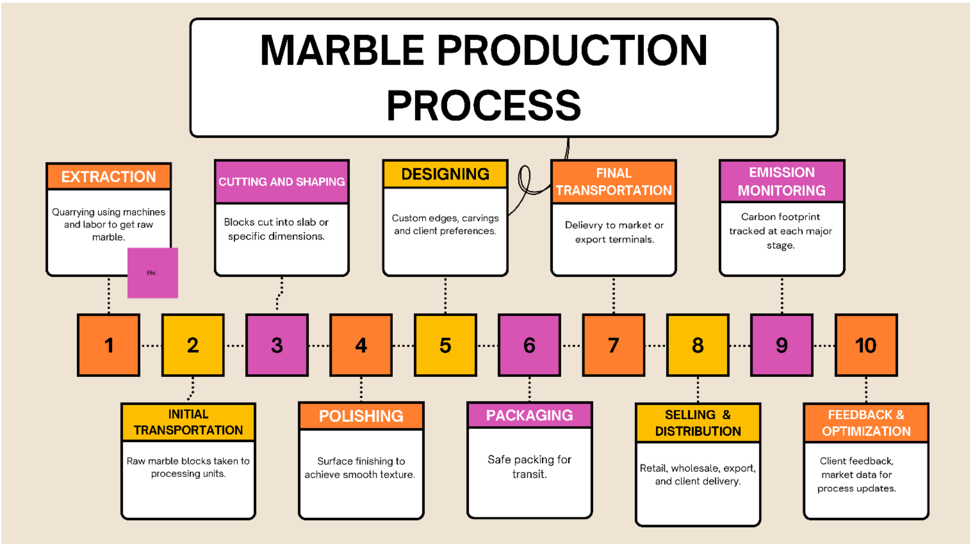

Figure 1.2: Step by Step Process of Marble Production

Although it has great potential, the marble industry in Pakistan encounters considerable obstacles. Inefficient methods of extraction and processing, a scarcity of modern technology, elevated production costs, and poor transportation infrastructure serve as significant barriers to its development. Furthermore, environmental challenges such as dust pollution, water consumption, and waste management highlight the importance of implementing sustainable methods.

Tackling these difficulties requires optimizing industrial processes. Contemporary optimization tools, such as hybrid algorithms that combine the Genetic Algorithm(GA) and Iterated Local Search(ILS), offer novel options for improving efficiency, minimizing resource waste, and enhancing profitability. By implementing these advanced methods throughout the marble value chain, the industry may increase its competitiveness while reducing its environmental impact. This thesis tries to explore the intricacies of Pakistan's marble industry, examining each stage of the manufacturing cycle to identify major inefficiencies and provide realistic optimization strategies. The investigation may look at how these tactics affect the economy, society, and environment, providing a road map for long-term, sustainable industry success.

This study aims to support the growth of Pakistan's marble industry and guarantee its long-term survival and prosperity in the local environment that is becoming more and more competitive by fusing traditional methods with contemporary technologies.

1.1 Literature Review

Multi-objective Optimization(MOO) is crucial for boosting industrial productivity because it strikes a compromise between competing objectives, including cost, quality, and sustainability. Recent advances in optimization have made it possible to handle complex problems with methods like Iterated Local Search(ILS), Particle Swarm Optimization (PSO), and Genetic Algorithm (GA). To assist decision-makers in industrial firms in reaching a compromise between conflicting objectives, Deb(2001) introduced Pareto-based strategies. Some solutions have shown potential in addressing issues where efficiency and minimizing environmental effects are critical. In Pakistan's marble business, optimization is particularly crucial because of production process inefficiencies, resource waste, and energy usage. However, there is a dearth of studies particularly applying MOO to this industry, which stands out for its lack of technological integration and reliance on traditional approaches.

The research takes a close look at the regulations governing the marble industry in Pakistan, highlighting the lack of enforcement and environmental harm brought about by the use of Command and Control(CAC) policies instead of incentive-based(IB) strategies. The marble industry contributes significantly to pollution of the air, water, and land, and ineffective laws do not address sustainability issues. According to the research, eco-friendly technologies, coordinated laws, and strategic environmental evaluations are necessary to balance industrial growth with conservation. It recommends stricter emission limits, polluter-pays principles, and tax measures to increase compliance. System dynamics modeling is crucial for optimizing regulation and ensuring the marble industry's long-term sustainability, according to the study [1].

The Life Cycle Assessment (LCA) of the marble processing industry in Rajasthan, India, is conducted with a focus on the environmental impact as stated in [2]. In Kishangra and Jalore, two significant processing areas, the study evaluates trash generation, material transport, and energy usage using GaBi software. The investigations show high energy and water use, excessive waste from marble dust and slurry, and significant risk of acidification and global warming. The absence of transparency in real-time data makes environmental assessments challenging in the marble business, the paper claims. As a component of sustainable manufacturing processes, it emphasizes the importance of ecologically friendly waste management and the effectiveness of using renewable energy sources. The study provides important information for industry and policy decision-makers as well as a methodology for mitigating the environmental impact of marble processing.

Using a circular economy paradigm, Tazzini *et al.* [3] examines the challenges and opportunities of managing waste from marble quarries. The process of extracting marble generates a lot of waste, such as fine sludge and debris, which can contaminate the air and water. The study focuses on Italy's Carrara Marble Basin, where waste production has increased dramatically as a result of modern cutting techniques. The findings suggest that the paper, plastic, and pharmaceutical industries may make use of marble trash, which is primarily carbonated in origin. Effective waste recovery methods are critical to the marble industry's resource efficiency and environmental sustainability.

The concentrations, causes, and health hazards of heavy metal combinations in agricultural soils close to marble processing facilities (MPPs) in Malakand, Pakistan, are discussed by Khan *et al.* [4]. High amounts of Ca, P, and Cd were discovered, surpassing shale limits, using Monte Carlo simulation and inductively coupled plasma-optical emission spectrometry(ICP-OES). Soils with substantial ecological concerns for As and Cd were classified as moderately polluted by indicators such as the accumulation index. Whereas probabilistic analysis revealed no discernible risk, the deterministic analysis indicated non-carcinogenic dangers for kids. However, the overall cancer risk of As, Ni, Cd, and Cr was higher than that specified by the USEPA safety standards. Sensitivity analysis revealed that exposure frequency, duration, and concentration of heavy metals were important contributors, highlighting the necessity of pollution management measures.

Ali *et al.* [5] assesses the possibility of marble dust as a sustainable substitute for conventional stone dust when used as a filler in asphalt concrete. The study carried out in Peshawar, Pakistan, evaluated attributes such as stability, tensile strength, rutting resistance, and resilience modulus by testing different amounts of marble dust in asphalt mixtures. The results revealed that adding marble dust to asphalt improved its tensile strength by 23%, reduced the depth of rutting, increased the resilience modulus by 44%, and increased stability by 1.63%. Furthermore, the altered mixture needed 4.05% less binder. According to the study's findings, marble dust can successfully substitute up to 4.5% of asphalt concrete filler, resulting in more affordable and ecologically friendly road building.

For green buildings, the project investigates the environmentally beneficial transformation of leftover marble into valuable tiles that support Qatar's sustainability objectives. The enormous amount of waste produced during the marble-making process, including dust and sludge, is hazardous to the environment. This study examines the creation of long-lasting, reasonably priced tiles by combining leftover marble powder with unsaturated polyester resin to form hybrid polymer composites. According to mechanical, thermal, and physical testing, adding more marble increased hardness but decreased tensile strength because of material incompatibility. The goal of future research is to improve formulations by adding additional compounds to improve performance. The results encourage sustainable waste management by turning leftover marble into high-quality building materials [6].

Marble dust (MD) and coal bottom ash (CBA) are used as partial cement substitutes in the study's creation of Green Emerging Cementitious Composite(CECC). Thus, the aim is to reduce carbon emissions while reusing industrial outputs due to environmental concerns associated with waste disposal and cement manufacture. Mechanical tests were performed in different proportions of the mix, including compressive, tensile, and flexural strengths. The best combination of 15% CBA and 5% MD improves compressive strength, according to the results, whereas 10% CBA and 10% MD improve tensile characteristics. The study lessens the need for conventional cement, which promotes sustainable buildings [7].

Hussain *et al.* [8] investigates the use of marble waste, such as marble powder(MP) and marble rubble(MR), as a partial replacement for sand in GFRP-reinforced granular soil. Experimental tests, including shear strength and pull-out resistance evaluations, demonstrated that integrating 14% MP, 28% MR, and 21% MP + MR greatly improved the mechanical properties of the sand. The modified mixture reduced the internal friction angles by up to 24. 5% while increasing the pull-out resistance by up to 139%. The findings show that marble waste can serve as an environmentally feasible option for reinforced earth structures, encouraging sustainable construction.

Using a life cycle assessment approach, Ahmad *et al.* [9] evaluates the energy use, water footprint, and environmental impact of Pakistan's marble tile production. The manufacturer of 1 ton of marble tiles uses $3.62m^3$ water, which has a substantial impact on global warming, according to the findings. Transportation, fossil fuels, and electricity use are the main causes of environmental problems. The study suggests implementing wastewater treatment plants, cutting waste, and using automated manufacturing as ways to increase sustainability.

In the Mohammad district, the marble industry has improved living conditions, provided access to healthcare and education, and increased income. Excess taxation, a lack of skilled workers, and inadequate infrastructure all impede progress. To boost the industry's potential, the paper suggests that the government help in the form of tax cuts, expanded financial choices, and improved infrastructure [10].

Although the marble industry boosts the economy, the disposal of solid waste and wastewater degrades the environment. Air, water, and soil are all contaminated by untreated waste. Pollution can be reduced by effective waste management methods such as chemical coagulation and sedimentation; however, there is insufficient enforcement of regulations. Recycling and wastewater treatment are examples of essential sustainable measures [11].

Due to legislative problems, poor infrastructure, and tribal wars, Pakistan's marble industry, which accounts for 3.6% of China's imports, is experiencing supply difficulties. Despite the tremendous potential of the sector for China's purchase, resolving these difficulties could increase exports and economic expansion [12].

Marble production in Pakistan, faces environmental challenges due to inefficiency in waste management and excessive use of water and energy. The study emphasizes the importance of improved practices, such as wastewater treatment and co-innovation reduce environmental externalities and enhance output. Technological advancements and waste reuse are critical for long-term industrial success in the marble business [13].

In Tehsil Shabqadar, Khyber Pakhtunkhwa, the marble industry is suffering from detrimental environmental effects as a result of untreated industrial effluents. The industry creates calcium carbonate, alkalinity, and heavy metals that harm water quality and soil fertility, endangering both public health and agriculture. To maintain industrial progress and environmental conservation, the

study highlights the necessity of environment-friendly treatment techniques, regulatory enforcement, and public awareness [14].
Through the conversion of marble powder into reusable cement components, the study investigates a sustainable way to reuse the marble waste. In addition to lowering the usage of raw materials and encouraging circular economy principles, this technique increases the recyclability of cement bricks, supporting eco-friendly building objectives and maximizing the output of the marble industry [15]. Some of the latest studies on the production and consumption of marble are presented in Table 1.1.

Table 1.1: Literature Review on Marble Production

Citation	**Description**	**Methodology**
[16]	By recording block attributes on RFID tags, enabling traceability, optimal cutting operations, and interaction with Industry 4.0 technologies, the study suggests an RFID-based approach to enhance the production chain of the marble industry while lowering waste and increasing efficiency.	RFID tags record non-destructive test data on marble blocks, which improves cutting and traceability in Industry 4.0. The laboratory prototype validates the integration of databases and the production system.

[17]	The research looks at the usage of marble waste aggregate and stone dust as a partial substitute for natural aggregate in concrete. It found that replacing 15% MWA with 50% SD increased strength, sustainability, and construction cost up to 18%.	The research integrates the Response Surface Methodology (RSM) with the Central Composite Design (CCD) technique for optimization.
[18]	Using the TRIZ problem-solving approach, this study aims to create a systematic methodology for evaluating and enhancing sustainability in marble manufacturing facilities by addressing economic efficiency, environmental impact, and human factors.	TRIZ (Theory of intensive problem solving) methodology.
[19]	To increase productivity and competitiveness, the article examines the future of the marble workshop with a focus on integrating technology, innovation, and sustainability. It emphasizes how crucial it is to train employees, use current equipment, recycle waste, and diversify materials to satisfy consumer wants.	An industry entrepreneur interview and a survey of the literature form the basis of qualitative methodology.

[20]	The environmental effects of marble production in Jordan's Zarqa Governorate are examined in this study, with particular attention paid to problems including excessive noise, inappropriate waste disposal, and contaminated water. To lessen ecological and social harms, it highlights the necessity of stronger laws, environmentally friendly techniques like wastewater treatment, and the relocation of enterprises to industrial zones.	Field surveys, wastewater chemical analysis, noise level assessment, and an examination of Jordanian environmental laws were all part of the research.
[21]	The study looks into Egypt's marble sector, pointing up inefficiencies such as outdated technology and worker misallocation. For efficiency, it recommends capital-intensive production and finds a growing return to scale.	Economic modeling(Cobb-Douglas production function, cost analysis, and financial measures like IRR and ROI) and field interviews with industry stakeholders are used in this study to examine Egypt's marble business.

[22]	This study examines the environmental and energy effects of marble quarrying in Sicily, emphasizing the high level of waste production, Carbon dioxide emissions, and energy use (75% of extracted materials become garbage). It recommends using precision tools instead of explosives and switching to renewable energy sources to lessen environmental damage.	With the use of field data from Sicilian quarries and a comparison with Carrara output, the study used energy and environmental audits in conjunction with Life Cycle Assessment (LCA) to examine the effects of marble quarrying.

A large portion of the literature currently available on Pakistan's marble industry overlooks the use of contemporary multi-objective optimization techniques to increase production. Descriptive analyses and environmental assessments are the main focus of current studies, which do not use sophisticated operational research techniques. Integrated quantitative models continue to underexploit important issues like high material waste, low efficiency, and reliance on antiquated technology. Furthermore, hybrid metaheuristic algorithms, like the Genetic Algorithm and Iterated Local Search, have not been used to tackle the sector's intricate production problems. To promote effective and sustainable production, this study aims to close these gaps by creating a context-specific, multi-objective optimization framework.

1.2 Problem Statement

Pakistan now ranks thirty-first in the world in terms of carbon emissions. The marble industry's reliance on inefficient, antiquated processes results in high production costs and environmental effects. Furthermore, compliance with new carbon rules will provide additional obstacles to profitability. There is a need for creative optimization solutions that increase production efficiency and reduce emissions while meeting regulatory standards. As a result, a multi-objective optimization model is necessary to address these challenges and provide sustainable solutions for the sector.

1.3 Aim and Objectives

The study aims to investigate the impact of carbon tax policy on the marble industry. The specific objectives are:

- To develop a strong mathematical framework that incorporates energy, emission, and production variables.
- To evaluate scenario-based results to determine the effects of cleaner technologies and carbon taxes.
- To offer data-driven policy recommendations to stakeholders in the marble industry to encourage the adoption of eco-efficient practices.

1.4 Research Methodology

A systematic two-phase analytical approach is used in this study to accomplish its two research goals. During the first stage, important operational and environmental elements influencing the marble business are methodically identified and prioritized using Multi-Criteria Decision Making(MCDM) approaches. This stage determines the crucial factors that guide the optimization framework that follows.

The second stage makes use of Multi-Objective Optimization (MOO) to create a mathematical model that minimizes three environmental impacts: air pollution, energy consumption, and carbon emissions, while simultaneously optimizing efficiency. The Pareto-optimal solutions produced by the model's computational implementation show the best trade-offs between sustainability and economic goals.

1.4.1 Optimization

Optimization is the process of finding the best possible solution to a problem under given constraints. It involves maximizing or minimizing an objective function by systematically selecting input values from an allowed set. Optimization is widely used in engineering, economics, logistics, and environmental management to improve efficiency, reduce costs, or enhance performance.

Mathematical Formulation of an Optimization Problem

A general Optimization Problem can be expressed as:

$$\begin{aligned}
&\text{Minimize (or Maximize)} \quad f(\mathbf{x}) \\
&\text{Subject to:} \\
&\quad g_i(\mathbf{x}) \leq 0 \quad \text{(Inequality constraints)} \\
&\quad h_j(\mathbf{x}) = 0 \quad \text{(Equality constraints)} \\
&\quad \mathbf{x} \in \mathcal{X} \quad \text{(Feasible region)}
\end{aligned}$$

- $\mathbf{x} = (x_1, x_2, \ldots, x_n) \rightarrow$ Decision variables
- $f(\mathbf{x}) \rightarrow$ Objective function (to be minimized/maximized)
- $g_i(\mathbf{x}) \leq 0 \rightarrow$ Inequality constraints (e.g., resource limits)
- $h_j(\mathbf{x}) = 0 \rightarrow$ Equality constraints (e.g., material balance equations)
- $\mathcal{X} \rightarrow$ Feasible solution space

Linear Optimization Problem: An optimization problem where both the objective function and all constraints are linear functions of the decision variables.

Example 1. Production planning problem:

$$
\begin{aligned}
&\text{Maximize } 3x_1 + 5x_2 \\
&\text{Subject to:} \\
&\quad 2x_1 + x_2 \leq 20 \quad \text{(Resource constraint)} \\
&\quad x_1 + 3x_2 \leq 30 \quad \text{(Labor constraint)} \\
&\quad x_1, x_2 \geq 0
\end{aligned}
$$

Non-linear Optimization Problem: An optimization problem where either the objective function or at least one constraint is nonlinear.

Example 2. Chemical reactor design:

$$
\begin{aligned}
&\text{Minimize } (x_1 - 2)^2 + (x_2 - 1)^2 \\
&\text{Subject to:} \\
&\quad x_1^2 - x_2 \leq 0 \quad \text{(Nonlinear constraint)} \\
&\quad x_1 + x_2 = 2 \quad \text{(Linear constraint)}
\end{aligned}
$$

Single-Objective Optimization Problem: An optimization problem with exactly one objective function to be minimized or maximized.

Example 3. Cost minimization:

$$\text{Minimize } 5x_1 + 7x_2 + 3x_3$$

Multi-Objective Optimization Problem: An optimization problem with multiple conflicting objectives that must be optimized simultaneously.

Example 4. Sustainable production planning:

$$\begin{aligned}
&\text{Minimize } [-\text{Profit}(x), \text{Emissions}(x)] \\
&\text{where:} \\
&\quad \text{Profit}(x) = 100x_1 + 150x_2 \\
&\quad \text{Emissions}(x) = 5x_1^2 + 3x_2^2
\end{aligned}$$

1.4.2 Multi-criteria Decision Making

Multi-Criteria Decision Making (MCDM) is a systematic methodology for evaluating and selecting the best alternative from a set of possibilities based on numerous, often competing, criteria. Unlike traditional decision-making methods, which are based on single objectives (e.g., cost minimization and profit maximization), MCDM considers both quantitative (for example, cost, time) and qualitative (for example, sustainability, user satisfaction) factors, making it highly effective for complex real-world problems. MCDM is commonly used in domains such as supply chain management, environmental policy, healthcare, and engineering, where trade-offs between competing goals must be carefully managed. The MCDM typically involves

- Defining the Decision Problem
- Structuring the Hierarchy
- Weighting the criteria
- Evaluating alternatives
- Ranking and Sensitivity Analysis

Popular MCDM methods include the Analytical Hierarchy Process (AHP), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), Elimination and Choice Translating Reality (ELECTRE), and the Best-Worst Method (BWM), each with its own set of benefits based on the complexity of the problem and the data needs. MCDM helps decision makers navigate uncertainty, lessen prejudice, and use quantifiable data to support their decisions by offering an orderly and open approach. When deciding on criteria weights, we employ the optimization-based Best-Worst Method(BWM) [23], a reliable MCDM technique renowned for its effectiveness and dependability.

Chapter 2

Carbon Tax Policy in Pakistan: A Stakeholder-Based Evaluation Using the Best-Worst Method

This chapter discusses several aspects of carbon tax policy, including the implications at global, regional, and national levels, with a focus on Pakistan. This discussion begins with the study of the global context of carbon taxes, concentrating on worldwide trends and policy frameworks, and then moves on to the regional context, looking at carbon pricing schemes employed by nearby and comparable economies. The chapter then focuses on Pakistan's carbon pricing landscape and general climate problems. A survey was conducted among various occupational groups to examine public and professional opinions, and the results were analyzed by using the Best-Worst Method(BWM) [23]. The findings reveal different viewpoints on carbon tax policy among professionals, providing critical insights into its potential acceptance and implementation in Pakistan. The chapter's systematic approach aims to provide a comprehensive understanding of carbon tax dynamics and stakeholder views.

2.1 Global Context of Carbon Tax Policy

2.1.1 Theoretical Foundation of Carbon Taxation

Carbon pricing, a market-based policy tool, is based on the economic idea of internalizing environmental externalities, namely those caused by greenhouse gas emissions. Its theoretical roots are based on the concept of Pigouvian taxation, which was first presented by economist Arthur C. Pigou and entails taxing activities that incur costs to society but are not reflected in market pricing. Carbon emissions are viewed as a negative externality in the context of climate change, contributing to ecological disruption, health concerns, and global warming. Carbon taxes seek to match individual economic actions with larger environmental and social goals by putting a monetary value on each ton of carbon dioxide (CO_2) released. A carbon price is more flexible and cost-effective than traditional regulatory measures that impose restrictions or mandate specific technologies. Encourage innovation and low-carbon transitions while allowing emitters to reduce emissions in the most economically viable way.

Furthermore, carbon taxes provide price certainty, which helps investors and businesses plan long-term renewable energy initiatives. Emissions trading schemes (ETS), which limit overall emissions while allowing the market to set pricing, are widely used for comparison. Carbon taxes provide price predictability, which makes them easier to administer in some situations, while ETS guarantees quantity control. As a budgetary tool, carbon taxes also generate revenue that the government can use to counter social inequality through refunds or subsidies, reinvest in renewable energy, or adapt to climate change.

2.1.2 Evolution of Global Carbon Tax Policies

Since the early 1990s, the application of carbon taxes has undergone significant changes, reflecting a growing understanding of their potential to mitigate climate change. In 1990, Finland became the first nation to impose a nationwide carbon price; Sweden, Norway, and Denmark followed suit shortly thereafter. These early adopters established the groundwork for incorporating environmental considerations into budgetary policy. As governments sought effective and market-compatible ways to reduce greenhouse gas emissions while maintaining economic stability, the concept gained support throughout Europe and certain regions of North America. The global adoption of carbon pricing schemes increased gradually in the 2000s and 2010s, with nations such as Japan, British Columbia, Ireland, and Switzerland implementing their versions. Carbon taxes have also been implemented more recently in developing countries, including South Africa, Chile, and Colombia, albeit frequently at lower price points and with less sectoral coverage.

The design, breadth, and efficacy of carbon taxes vary greatly, despite their increasing popularity. While some nations (like Sweden) impose a wide carbon tax that applies to all economic sectors, others target certain industries or fuels. Additionally, the tax rates differ greatly. For example, Sweden has one of the highest carbon tax rates in the world(above USD 130/ton CO2), whereas many developing nations start with symbolic pricing of USD 5-10/ton. Another difference is how the money is used; some governments use the revenue from the carbon tax to finance infrastructure or renewable energy projects, while others utilize revenue-neutral models that return the money to the people in the form of social programs or tax breaks. This variance reflects political will and popular acceptance, which are critical in determining the global impact of carbon pricing, as well as administrative and economic capabilities.

2.1.3 Comparative Case Studies of Leading Carbon Tax Regimes

Several countries provide useful guidance on the development and implementation of carbon taxes. Sweden has proved that rigorous pricing may reduce emissions while promoting economic growth by maintaining one of the world's highest carbon tax rates. The revenue-neutral principles have been effectively applied in Canada, particularly in British Columbia, which has retained popular support by returning all tax revenues to residents. In contrast, widespread unrest arose as a result of France's failed fuel tax hike, emphasizing the importance of political context and equity issues. These instances show that, while carbon taxes have sound economic principles, public support, policy design, and transparency are critical to their success.

2.1.4 Impact Assessment of Carbon Tax Policies

Carbon tax proposals are widely evaluated for their ability to reduce greenhouse gas emissions while preserving economic stability. Empirical evidence from Sweden, British Columbia, and Switzerland shows that properly structured carbon levies can significantly reduce emissions while boosting economic growth. Since the early 1990s, Sweden has reduced its per Capita emissions by more than 25%, despite increasing GDP. Similarly, British Columbia experienced a considerable decline in fuel consumption after instituting a carbon pricing in 2008, with no negative impact on economic performance. These examples show that, when combined with long-term policy signals and institutional support, carbon prices can drive environmental improvements and encourage the shift to cleaner technology.

Carbon taxes have an impact on both the economy and society, in addition to the environment. Economically, they promote energy efficiency, green technology innovation, and the shift of investment toward sustainable sectors. The social success of a carbon price is determined by how revenue is used, whether to offset income disparity, fund climate resilience, or lower other distorting taxes. Countries that have returned carbon tax income to households, such as Canada, have seen increased public support and reduced political opposition. However, worries about the regressive character of such levies persist, as low-income people frequently spend a greater part of their income on energy. Thus, while carbon taxes have strong environmental and budgetary benefits, their design must address justice and public trust to be effective and socially acceptable.

2.1.5 Challenges and Criticism

Carbon taxes face some operational obstacles, especially in politically and economically varied environments, despite their theoretical effectiveness and growing global popularity. One significant problem is carbon leakage, which undermines the effectiveness of domestic carbon pricing when businesses relocate their operations to nations with less stringent emissions standards. This can undermine the benefits of the global climate and reduce competitiveness, particularly in industries that are sensitive to trade, such as manufacturing, steel, and cement. It is challenging to impose and enforce carbon taxes consistently in poor nations due to administrative issues such as a lack of trustworthy emissions data, a weak tax infrastructure, and restricted monitoring capabilities. Furthermore, the intended price signal of the tax may be diluted by inflationary pressure and fluctuating energy prices, which would further diminish its efficacy.

The regressive effect that carbon prices have on lower-income households is a more enduring and politically sensitive critique. Vulnerable people may be disproportionately impacted by uniform carbon pricing unless compensated by targeted rebates or social support measures, as energy expenses account for a larger portion of the poor's income. Even well-meaning measures can be derailed by public opposition, which is frequently stoked by feelings of injustice (as demonstrated by Yellow Vest protests in France) if open communication and equitable income usage are not also present. Furthermore, some detractors argue that carbon taxes should be a component of a large climate policy package that also includes subsidies for clean energy innovation, regulations, and investments in green infrastructure because they are insufficient alone to promote substantial decarbonization. These objections emphasize the necessity of cautious policy formulation, public participation, and institutional preparation to ensure that carbon taxes achieve their social and environmental goals.

2.1.6 Integration with Global Climate

Carbon taxes are becoming more widely acknowledged as a key tool for accomplishing global climate goals, especially those outlined in the Paris Agreement and the Sustainable Development Goals(SDGs) of the United Nations. Whether in the form of taxes or emissions trading programs, carbon pricing provides a flexible and market-efficient way for nations to decarbonize their economy as they strive to fulfill their Nationally Determined Contributions (NDCs). Carbon taxes have been promoted by organizations like the World Bank and the International Monetary Fund (IMF) as a component of larger climate finance and green growth initiatives. Additionally, international programs such as the Carbon Packing Leadership Coalition (CPLC) encourage states to share expertise and take coordinated global action. To standardize carbon pricing and avoid trade distortions, new proposals for border carbon adjustments(BCAs) or a worldwide carbon price floor are also emerging. In addition to immediately reducing emissions, carbon taxes promote climate justice, sustainable development, and long-term economic resilience by coordinating fiscal policy with environmental care.

2.1.7 Current Debates and Future Prospects

Political, economic, and technical advancements are influencing the fast-changing global conservation surroundings, including carbon taxes. To avoid a "race to the bottom" and guarantee that all significant polluters make equitable contributions, a worldwide carbon pricing floor was advocated, which is the subject of one of the most well-known discussions. The application of Border Carbon

Adjustments(BCAs), especially within the European Union, where imports from nations with laxer carbon regulations may be subject to carbon levies, is another controversial topic. Although the goal of such measures is to level the playing field and stop carbon leakage, they also run the risk of inciting trade disputes and igniting concerns about climate protectionism.

Looking ahead, it seems that innovation, openness, and global cooperation will become more and more important in the future of carbon taxation. Blockchain and satellite monitoring are two examples of digital technologies being investigated to improve tax compliance and emissions tracking. Furthermore, it is becoming more popular to use the money collected from the carbon tax for green development, carbon initiatives like public transportation, social equity funds, and subsidies for renewable energy. Further plans must place a high priority on progressive revenue recycling, fair distribution, and open communication with stakeholders if carbon prices are to continue to be both politically feasible and socially just. Carbon taxation is expected to grow in breadth and ambition as climate urgency increases, but its effectiveness will depend on strong international cooperation, flexible policy frameworks, and sound governance.

2.2 Regional Context: Carbon Taxation in Asia

As a major Carbon emitter and a region most at risk from its effects, Asia is vital to the global climate movement. Carbon Pricing schemes have been introduced or are being investigated by nations including China, India, and Japan to balance sustainability and economic growth. Nonetheless, the area exhibits significant differences in emissions, institutional capabilities, and policy aspirations. An outline of Asia's emissions profile, important carbon tax laws, and the potential problems the area faces in implementing fiscal policies that are compatible with climate change are given in this section.

2.2.1 Emissions Profile of Asia

Asia, led by China and India, is responsible for over half of the world's CO_2 emissions. Japan, South Korea, and Indonesia are among the top polluters, while China alone is responsible for around 30% of global emissions. Conversely, nations with lesser emissions, like Nepal and Bhutan, have limited historical accountability yet are at serious risk from climate change. The main causes of the region's emissions include heavy industry, coal-based energy, and fast-growing urbanization; however, the proportions differ between nations and industries.

Significant disparities can also be seen in per capita emissions, which are low in Bangladesh and Cambodia and high in places like Singapore and Qatar (see Figure 2.1). The development of cohesive regional climate policies is hampered by these differences. Different economic capacities and obligations necessitate customized carbon pricing strategies that support regional climate goals while taking into account national settings.

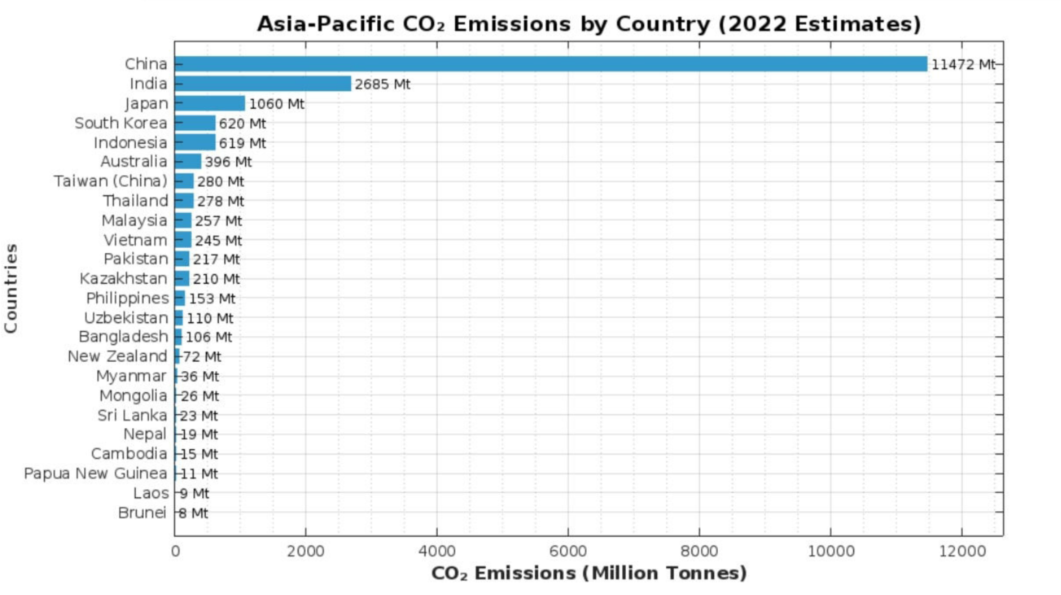

Figure 2.1: CO_2 Emission of Asian Countries

2.2.2 Carbon Pricing Policies

Several Asian countries have implemented carbon pricing tools; however, the methodologies vary. In 2012, Japan imposed a minor carbon tax to discourage the use of fossil fuels, whereas South Korea implemented a strong Emissions Trading Scheme(ETS) that covers 70% of its emissions. Singapore became the first South Asian country to impose a carbon tax in 2019, and costs are expected to grow dramatically over the next decade.

China started the world's largest carbon market in 2021, primarily focusing on the power sector. Although still in its early phases, its scope is unparalleled. These instances demonstrate Asia's rising policy shift toward employing economic tools for climate governance. However, tax rates, coverage, and enforcement differ widely, reflecting varied political and economic circumstances.

2.2.3 Barriers and Opportunities

Asia confronts significant problems in implementing carbon taxes. Reliance on fossil fuels, economic considerations, and fossil fuel subsidies all work against carbon pricing attempts. Political opposition and low public trust make it difficult to impose large climate taxes without causing societal backlash.

However, opportunities are arising. Governments are increasingly linking carbon pricing to climate funding, green jobs, and equitable transitions. As climate challenges worsen, carbon taxes might become a key instrument for both emissions reduction and sustainable development. With well-designed policies, Asia can lead the worldwide transition to climate-friendly fiscal policy.

2.3 National Context: Carbon Emissions and Climate Policy

2.3.1 Pakistan's Carbon Emissions: Scale and Sectoral Sources

Pakistan is still one of the top ten nations most at risk from climate change, while contributing less than 1% of the world's carbon dioxide emissions. Pakistan emits between 450 and 500 million tons of CO_2 equivalent per year, according to national greenhouse gas inventories and the Global Carbon Atlas. This amount has been rising continuously as a result of urbanization, industrialization, and rising energy consumption. This energy sector, particularly the production of power from gas and oil, is responsible for the greatest portion of emissions, with transportation, industry, and agriculture coming in second and third.

Although Pakistan's emissions per capita are still low (around two tons per person) compared to the major global emitters, the rate of increase is concerning, especially given the country's growing reliance on fossil fuels and fast urbanization. Due to unregulated building, industrial output, and traffic congestion, Karachi, Lahore, and Faisalabad are major emission hotspots. If this increasing trend in emissions is not addressed, it may clash with Pakistan's environmental development objectives as stated in its Nationally Determined Contributions (NDCs) to the UNFCCC.

2.3.2 Climate Change Impact in Pakistan

Pakistan is severely impacted by climate change, even though its emissions footprint is small. This is made worse by vulnerabilities in its infrastructure, economy, and geography. Intense heatwaves in Sindh and Punjab, record-breaking monsoon flooding, glacial lake outburst floods (GLOFs) in the north,

and protracted droughts in Balochistan are just a few of the catastrophic climate disasters that have struck the nation. A third of the country was flooded by the 2022 floods, which also caused over 30 million people to be homeless and over USD 30 billion in economic losses. These events highlight how underdeveloped countries are disproportionately affected by climate change.
Urban infrastructure, water supplies, public health, and agriculture, a significant economic sector, are all directly impacted by these climate shocks. The National Climate Change Policy and National Adaptation Plan are two examples of Pakistan's climate policy initiatives that recognize the pressing need for mitigation and adaptation measures. National climate resilience is still being weakened, nevertheless, by implementation gaps, a lack of institutional ability, and financial limitations. In the nation, there is still much to learn about a more methodical and scientific approach to reducing emissions, possibly through market-based tools like carbon taxes.

2.3.3 Carbon Tax Policy in Pakistan: Status and Prospect

There is currently no official national carbon price policy in effect in Pakistan. Although the nation has implemented several environmental levies, taxes, and fees (including the petroleum development levy), none of them are specifically connected to carbon content or presented as instruments for mitigating climate change. Indirect taxes and fuel subsidies, which frequently contradict the ideas of carbon pricing, have historically been the mainstays of Pakistan's fiscal policy. Although initial talks have taken place in light of Pakistan's obligations to international climate finance frameworks and voluntary carbon markets, there is currently no emissions trading plan either.
However, policymakers are becoming more interested in carbon pricing schemes. The Ministry of Climate Change has carried out exploratory research on carbon market readiness and emissions tracking systems with assistance from multiple partners(such as the World Bank and UNDP). These efforts are still in

their infancy and have many challenges, particularly those related to institutional capacity, political opposition, and data constraints. For carbon taxes to be a successful policy in Pakistan, they must be incorporated into development planning, aligned with global funding sources (such as the Green Climate Fund), and implemented to ensure social justice and revenue neutrality.

2.4 Survey Analysis Using Best-Worst Method

2.4.1 Survey Design and Respondent Profile

A focused study was conducted to gain a deeper understanding of how the general public and experts in Pakistan would perceive a potential carbon price regime. 130 people participated in the poll, who were divided into six professional groups: academics(including students and professors), environmentalists, economists, social workers, healthcare professionals, and industry stakeholders, especially those associated with the marble industry. These groups are chosen to ensure a range of viewpoints on economic, social, environmental, health, and specific sector issues (such as the marble industry). Evaluating stakeholder priorities about the anticipated effects of carbon taxes was the goal.

The Best-Worst Method(BWM) for multi-criteria decision-making, which enables a consistent and targeted evaluation of preferences across several criteria, was followed in structuring the survey results. Rezaei *et al.* [23] developed BWM, which was selected because it handled expert judgment with greater robustness, consistency, and less need for comparisons. The perceived impacts of the carbon price on the economic environment, public health, society, and industrial competitiveness were among the parameters assessed in this study. After asking each responder to rank the areas that had the greatest and least influence, rating comparisons were made and examined using the BWM framework.

2.4.2 Best-Worst Method: Five-step Application Framework

The application of the Best-Worst Method(BWM) to the survey data is described in the next five steps, which are based on [23] framework:

Step 1: Define the Decision Criteria

Let $D = \{D_1, D_2, \ldots, D_n\}$ be the collection of selection criteria derived from survey data and expert opinions.

Step 2: Determine the Best and Worst Criteria

Each respondent identifies:

- The Best criterion $D_B \in D$ (most important)
- The Worst criterion $D_W \in D$ (least important)

Step 3: Construct Best-to-Others Vector

Let the most important (best) criterion be denoted as K, and the criteria set as $D = \{D_1, D_2, \ldots, D_n\}$. Now define a preference vector from the best criterion K to all other criteria:

$$P^K = \{p_{K,1}, p_{K,2}, \ldots, p_{K,n}\} \tag{2.4.1}$$

Where:

- $p_{K,k} = 1$ (because criterion K is compared to itself),
- $p_{K,j}$, indicating how much more important K is compared to criterion D_j.

For example, if criterion $K = D_2$, and a respondent thinks D_2 is five times more important than D_5, then $p_{K,5} = 5$.

Step 4: Construct Others-to-Worst Vector

Let the least important (worst) criterion be denoted as L. Now construct the priority vector from all criteria toward the worst:

$$Q^L = \{q_{1,L}, q_{2,L}, \ldots, q_{n,L}\} \tag{2.4.2}$$

Where:

- $q_{L,L} = 1$ (since L is compared to itself),
- $q_{j,L}$, indicating how much more important each criterion D_j is compared to the worst criterion L.

For instance, if $L = D_4$, and D_1 is 7 times more important than D_4, then $q_{1,L} = 7$.

Step 5: Compute Optimal Weights

Let $v_1, v_2, \ldots, v_n$ be the weights of each criterion $D_1, D_2, \ldots, D_n$, and let δ represent the maximum deviation from consistency.
The objective is to find the vector of optimal weights $\boldsymbol{v} = \{v_1, v_2, \ldots, v_n\}$, by minimizing δ, under the following linear constraints:

$$\left| \frac{v_K}{v_j} - p_{K,j} \right| \leq \delta \quad \text{for all } j \tag{2.4.3}$$

$$\left| \frac{v_j}{v_L} - q_{j,L} \right| \leq \delta \quad \text{for all } j \tag{2.4.4}$$

$$\sum_{j=1}^{n} v_j = 1, v_j \geq 0 \quad \text{for all } j \tag{2.4.5}$$

This optimization problem ensures that the deviation between actual weight ratios and respondent-provided values is minimized. The final output includes:

- v_j^*: the optimized weight of each criterion

- δ^*: the smallest inconsistency level across all comparisons

Lower values of δ^* indicate higher consistency in responses.

2.4.3 Structured Framework For Carbon Tax Perception Survey

There were five main areas in the poll, each of which focused on a distinct aspect of impact. The study employed a mix of closed-ended items, general perception questions, and Likert-scale questions to measure attitudes and preferences.

Economic Impact

The purpose of this part was to gauge respondents' opinions regarding the potential impact of carbon price policy on Pakistan's economic performance. Among the important queries were:

- Will the tax on Carbon slow GDP growth?
- Will energy prices rise sharply?
- Will production costs increase for firms, particularly SMEs?
- Would a Carbon tax stimulate new markets or green innovation?

These inquiries aimed to explore the potential economic benefits and challenges associated with the carbon tax.

Social Impact

The social dimension concentrated on the distributional and societal consequences of carbon taxes. Participants answered questions such as:

- Will the carbon tax disproportionately impact low-income communities?
- Can tax revenue be utilized to improve public services?

- Will the tax harm employment in industries that rely on fossil fuels?
- Is the general public aware enough to grasp the policy?

These items contributed to determining whether respondents thought carbon taxes were socially equitable or regressive.

Environmental Impact

The section examined how respondents perceived the environmental effectiveness of a carbon tax. Questions included:

- Will a carbon tax reduce greenhouse gas emissions in Pakistan?
- Could it enhance air and water quality?
- Is there room for improvement in environmental accountability?
- Will reducing industrial emissions improve natural ecosystems?

These questions are consistent with climate mitigation objectives and global commitments such as the Paris Agreement.

Health Impact

Public health is one of the less visible but far-reaching effects of carbon policy. This section contains:

- Will better air quality lead to fewer respiratory and cardiovascular diseases
- Could lowering pollution reduce healthcare costs?
- Will the carbon tax improve public health in cities more than in rural areas?
- Do health professionals agree that the tax is a health-promoting policy?

This section assessed the policy's medical and well-being implications.

Impact on the Marble Industry

Pakistan's marble industry is both energy-intensive and economically vital in some locations; hence, this section concentrates on industry-specific effects:

- Will there be a major increase in marble production costs?
- Could a carbon price influence export and competitiveness?
- the marble business willing to adopt cleaner technologies?
- Will the industry require specific subsidies or exemptions? These questions concerned sector-specific vulnerability and adaptation readiness.

2.5 Application of BWM to Assess Carbon Tax Policy Impacts

To assess the multifaceted effects of the carbon tax legislation, the study involved five major stakeholder groups: academia (including students and professors), environmentalists, economists, healthcare professionals, and stakeholders in the marble sector. Environmentalists focused on ecological advantages, including biodiversity preservation and carbon reduction, while academics offered theoretical insights into long-term sustainability and equity implications. While economists examined macroeconomic trade-offs, market distortions, and fiscal efficiency, health specialists evaluated co-benefits to public health, such as a decrease in diseases linked to air pollution. Practical issues like operational costs, competitiveness, and transition hurdles were assessed by industry players, especially those in the marble sector. A comprehensive viewpoint on policy trade-offs was guaranteed by varied participation.

Step 1: Establishing the Decision Criteria Set

The criteria in this study, where $n = 5$, are:

- D_1 : Economic Impact
- D_2 : Environmental Impact
- D_3 : Social Impact
- D_4 : Health Impact
- D_5 : Impact on Marble Industry

Step 2: Determining the Best and Worst Criteria by Groups

The analysis began by computing the average ratings for each category (economic, social, environmental, health, and marble industry impacts) across all stakeholder groups. To maintain comparability, these raw averages were standardized to a common 0-1 scale via min-max normalization. The normalizing formula used was:

$$X_{\text{norm}} = \frac{X_i - X_{\min}}{X_{\max} - X_{\min}} \tag{2.5.1}$$

Where X_i denotes the initial mean score and $X_{\min}$ and $X_{\max}$ denote the lowest(1) and highest(9) numbers that can be obtained from the survey's 1-9 scale. While normalizing the scores for cross-criterion comparison, this modification maintained the relative distinction between them. Clear trends emerged from the results, as displayed in the normalized mean Table 2.1. Marble Industry Impact received the lowest ranking (0.58-0.63), suggesting that it is a somewhat lesser priority, while Social Impact consistently receives high scores(0.75-0.82 across groups).

From Table 2.1 we see the clear trends in the choices of stakeholders. Marble Industry Impact was consistently rated as the lowest criterion by both environmentalists and social workers, who chose health impact as their best criterion(with normalized value of 0.81 and 0.63, respectively). Social impact was ranked as the top priority by all other stakeholder groups, including academia,

Table 2.1: Normalized Mean Scores by Stakeholder Group

Stakeholder Group	Eco	Social	Env	Health	MI
Academia	0.71	0.75	0.63	0.74	0.60
Economist	0.70	0.81	0.62	0.71	0.62
Environmentalist	0.70	0.77	0.66	0.81	0.63
Health Professionals	0.67	0.81	0.63	0.77	0.58
Social Worker	0.68	0.80	0.67	0.81	0.61
Industrialist	0.71	0.82	0.64	0.77	0.62

economists, health professionals, and industry representatives(values ranging from 0.75 to 0.82).In contrast, the Marble industry Impact was ranked as the least important (scores between 0.58 and 0.63). The fact that all groups agreed that Marble Industry was the least significant criterion points to a shared perception of its relative insignificance in carbon tax policy considerations, but it also shows that stakeholders with an environmental focus and social focus have different top priorities.

Step 3: Constructing Best-to-Other Preferences Vectors

The Best-to-Others vector indicates the relevance of a stakeholder group's best criterion(most important) among all other criteria in the decision framework. It determines how much more important the best criterion is than the alternate criterion.

The Best-to-Other (BO) vector is formally defined as:

$$\mathbf{BO} = (a_{B1}, a_{B2}, a_{B3}, a_{B4}, a_{B5}) \tag{2.5.2}$$

Where:

- a_{Bi} represents the importance ratio of the Best criterion relative to criterion i
- Each element is calculated as $a_{Bi} = \dfrac{X_B}{X_i}$
- X_B is the normalized mean score of the Best criterion

- X_i is the normalized mean score of the i-th criterion

Using the normalized mean from Table 2.1, we guaranteed BO vectors for each stakeholder group. For environmentalists (Best=Health Impact, 0.81), the calculations were:

$$a_{B1} = \frac{0.81}{0.70} \approx 1.16 \quad \text{(vs Economic)} \tag{2.5.3}$$

$$a_{B2} = \frac{0.81}{0.77} \approx 1.05 \quad \text{(vs Social)} \tag{2.5.4}$$

$$a_{B3} = \frac{0.81}{0.66} \approx 1.22 \quad \text{(vs Environmental)} \tag{2.5.5}$$

$$a_{B4} = \frac{0.81}{0.63} \approx 1.28 \quad \text{(vs Marble Industry)} \tag{2.5.6}$$

Calculations for all other Stakeholders are shown in Table 2.2.

Step 4: Constructing Other-to-Worst Preferences Vectors

The Other-to-Worst vector compares all criteria to the worst criterion(least important). It determines the relative value of each criteriocompared tout the baseline worst criterion.

The Other-to-Worst (OW) vector is formally defined as:

$$\mathbf{OW} = (a_{1W}, a_{2W}, a_{3W}, a_{4W}, a_{5W}) \tag{2.5.7}$$

Where:

- a_{jW} represents the importance ratio of the Worst criterion relative to criterion j
- Each element is calculated as $a_{iW} = \dfrac{X_j}{X_W}$
- X_W is the normalized mean score of the Worst criterion
- X_j is the normalized mean score of the j-th criterion

Using the normalized mean from Table 2.1, we guaranteed WO vectors for each stakeholder group. For Industry stakeholders (Worst=Marble industry, 0.62), the calculations were:

$$a_{1W} = \frac{0.71}{0.62} \approx 1.11 \quad \text{(vs Economic)} \tag{2.5.8}$$

$$a_{2W} = \frac{0.82}{0.62} \approx 1.31 \quad \text{(vs Social)} \tag{2.5.9}$$

$$a_{3W} = \frac{0.64}{0.62} \approx 1.09 \quad \text{(vs Environmental)} \tag{2.5.10}$$

$$a_{4W} = \frac{0.77}{0.62} \approx 1.33 \quad \text{(vs Marble Industry)} \tag{2.5.11}$$

Calculations for all other Stakeholders are shown in Table 2.2.

Table 2.2: Best-to-Other and Other-to-Worst Vectors

Criteria	Academia		Economist		Environmentalist		Health professional		Industry	
	Best vector	Worst vector	Best vector	Worst vector	Best vector	Worst vector	Best vector	Worst vector	Best vector	Worst vector
Eco	1.06	1.18	1.14	1.14	1.16	1.10	1.21	1.17	1.21	1.11
Social	1.00	1.25	1.00	1.30	1.05	1.22	1.00	1.41	1.02	1.31
Env	1.19	1.05	1.29	1.01	1.22	1.05	1.29	1.09	1.22	1.09
health	1.01	1.24	1.13	1.15	1.00	1.28	1.06	1.34	1.00	1.33
MI	1.25	1.00	1.30	1.00	1.28	1.00	1.41	1.00	1.33	1.00

Step 5: Deriving Optimal Weights and Consistency Ratios

The Table 2.3 shows the final optimized weights obtained using the Best-Worst Method(BWM) for five stakeholder groups across five criteria.

Table 2.3: Optimal Weights and Consistency Ratios by Stakeholder Group

Group	**Eco**	**Social**	**Env**	**Health**	**MI**	**CR**
Academia	0.207	0.218	0.184	0.216	0.175	7.09913×10^{-16}
Economist	0.203	0.232	0.180	0.206	0.179	5.70244×10^{-16}
Environmentalist	0.195	0.216	0.186	0.226	0.177	0.00000
Health Professional	0.194	0.235	0.182	0.222	0.167	0.00000
Industry	0.189	0.224	0.187	0.228	0.171	4.97000×10^{-16}

Note: Eco = Economic Impact, Env = Environmental Impact, MI = Marble Industry Impact, CR = Consistency Ratio

Weights ranging from 0 to 1 ($\sum W_j = 1$) indicate the relative relevance of each criterion in evaluating carbon tax policies.

- Economic Impact(Eco):Weights ranges from 0.189(industry) to 0.207(academia)
- Social Impact: Consistently high weights (0.216-0.235)
- Environmental Impact(Env): Moderate Weights (0.180-0.187)
- Health Impact: Peak importance for environmentalists (0.226)
- Marble Industry(MI): universally lowest weights (0.167-0.179)

The weights were derived by solving:

$$\min \xi$$

Subject to:

$$|w_B - a_{Bj} w_j| \leq \xi \quad \forall j \tag{2.5.12}$$

$$|w_j - a_{jW} w_W| \leq \xi \quad \forall j \tag{2.5.13}$$

$$\sum_{j=1}^{n} w_j = 1, \quad w_j \geq 0 \tag{2.5.14}$$

Where:

- a_{Bj} = Best-to-other vector elements
- a_{jW} = Other-to-worst vector elements
- ξ^* = Optimal consistency index

Consensus Patterns

- Universal de prioritization of the marble industry (MI weights:0.167-0.179)
- High significance of social impact(weights:0.216-0.235)

Divergence Patterns

- Environmentalists value Health (0.226) over social (0.216)
- Industry exhibits a balanced distribution (all weights 0.187-0.228)

Consistency Verification

$$CR = \frac{\xi^*}{CI} = \frac{7.10 \times 10^{-16}}{3.25} \approx 2.18 \times 10^{-16} \ll 0.2 \tag{2.5.15}$$

(for Academia using consistency index CI=3.25 for n=5)

2.6 Conclusion

The Best-Worst Method study identified distinct patterns in stakeholder preferences for carbon price policies. The low consistency ratios(CR $\approx 2.18 \times 10^{-16}$) indicate strong reliability of data. Three noteworthy findings emerged: all groups consistently assigned the lowest weights to Marble Industry impacts(0.167-0.179); most stakeholders prioritized Social Impacts (0.216-0.235); and Environmentalists stressed Health Impacts (0.226). These findings show both consensus on industrial impacts and significant differences in other areas.

2.6.1 Policy Recommendations and Applications

The analysis offers policymakers useful insights for formulating carbon pricing policies. The validated weights suggest that policymakers should provide transition support to affected industries and emphasize societal benefits in communication and adjust methods to diverse stakeholder groups. The BWM's mathematical rigor (all CR values are well below the 0.2 threshold) assures that these suggestions are based on strong, quantitative evidence of stakeholder preferences, allowing for more effective and inclusive policy creation.

Chapter 3

Multi-objective Mathematical Model for Optimizing Production in Pakistan's Marble Industry

Pakistan's marble industry is important to the national economy, especially in resource-rich areas such as Khyber Pakhtunkhwa and Balochistan. However, the industry faces ongoing issues as a result of its reliance on antiquated and inefficient manufacturing techniques. This not only raises production costs but also results in excess carbon emissions, putting further burden on environmental sustainability. As global and national policies on carbon emissions tighten, the marble sector is under growing pressure to alter its operations while maintaining profitability. Pakistan is currently ranked 31st in the world for carbon emissions, with industries such as marble mining contributing considerably to this total. Meeting rising carbon standards while maintaining or increasing productivity poses a substantial challenge to marble producers.To overcome these challenges, innovative and effective optimization techniques that can boost manufacturing production while minimizing environmental impact are necessary. This study intends to create a hybrid multi-objective optimization model that supports long-term growth in the marble sector by balancing production efficiency and regulatory compliance under carbon tax policies.

3.1 Model Formulation

This research proposes a multi-objective optimization technique for Pakistan's marble sector that finds a balance between environmental sustainability and production efficiency. Nonlinear interactions, such as exponential polishing emissions and quadratic cutting waste effects, are added in the model to optimize marble output while reducing energy consumption, carbon emissions, and pollution. Fifteen real-world limitations ensure practicality, including extraction limits, labor capacity, pollution threshold, and process requirements. The weighted sum technique is used to analyze the trade-offs between economic and environmental goals in Pakistan's marble sector, providing useful insights for plant operations and regulatory choices.

3.1.1 Sets and Indices

$\mathcal{T}$	Set of time periods	$t \in \mathcal{T}$
$\mathcal{I}$	Set of quarry locations	$i \in \mathcal{I}$
$\mathcal{J}$	Set of block attribute types	$j \in \mathcal{J}$
$\mathcal{B}$	Set of all blocks	$b \in \mathcal{B}$
$\mathcal{B}_{ij}$	Blocks from quarry i with attributes j	$\mathcal{B}_{ij} \subseteq \mathcal{B}$
$\mathcal{K}$	Set of production plants	$k \in \mathcal{K}$
$\mathcal{S}$	Set of processing stages	$s \in \mathcal{S}$
$\mathcal{M}$	Set of all machines	$m \in \mathcal{M}$
$\mathcal{M}_s$	Machines capable of stage s	$\mathcal{M}_s \subseteq \mathcal{M}$
$\mathcal{P}$	Set of finished products	$p \in \mathcal{P}$
$\mathcal{C}$	Set of customers	$c \in \mathcal{C}$
$\mathcal{D}$	Set of distribution centers	$d \in \mathcal{D}$
$\mathcal{L}$	Set of transportation modes	$l \in \mathcal{L}$

3.1.2 Parameters

1. Demand and Commercial

d_{pct}	Demand for product p from customer c in period t (m^2)
r_{pc}	Selling price per m^2 of product p sold to customer c ($\$/m^2$)
b_p	Backorder penalty per m^2 of product p per period ($\$/m^2$)
θ_p	Maximum allowed backorder ratio for product p (dimensionless)

2. Raw Material and Sourcing

c_{ij}^{ext}	Cost to extract one block (j) from quarry i (\$/block)
c_{j}^{buy}	Cost to purchase one external block of attribute j (\$/block)
$\overline{X}_{ijt}^{max}$	Max extractable blocks (j) from quarry i in period t (blocks)
X_{ij}^{init}	Initial inventory of blocks (j) at quarry i (blocks)

3. Processing and Production

a_{spk}	Processing time at stage s per m^2 of product p at plant k (hours/m^2)
Cap_{skt}^{reg}	Regular capacity for stage s at plant k in period t (hours)
Cap_{skt}^{OT}	Overtime capacity for stage s at plant k in period t (hours)
c_{sk}^{OT}	Overtime cost per hour for stage s at plant k (\$/hour)
f_{spk}	Setup cost for stage s at plant k for product p (\$/setup)
$\mathbb{M}$	Sufficiently large constant (Big-M) (dimensionless)

4. Yield and Quality

α_{jp}	m^2 of product p from one block of attribute j - yield (m^2/block)
β_{sp}	Scrap fraction at stage s for product p (dimensionless)
q_{p}^{min}	Minimum share of high-grade input for product p (dimensionless)

5. Inventory and Storage

h_{pk}	Holding cost per m^2 of product p at plant k per period (\$/m^2)
h_{jk}^{block}	Holding cost per block (j) at plant k per period (\$/block)

6. Logistics and Transportation

$c_{ik}^{tr,block}$	Transport cost per block from quarry i to plant k (\$/block)
$c_{kcd}^{tr,prod}$	Transport cost per m^2 of product p from plant k to customer c (\$/m^2)

7. Sustainability and Environmental

e_{spk}	Energy use at stage s per m^2 of product p at plant k (kWh/m^2)
w_{spk}	Water use at stage s per m^2 of product p at plant k (liters/m^2)
ϕ_k^{grid}	CO_2 factor for energy at plant k (kg CO_2e/kWh)
ϕ^{diesel}	CO factor for diesel fuel (kg CO_2 e/liter)
c^{disp}	Waste disposal cost per m^2 equivalent (\$/m^2)
ρ	Water reuse rate - target (dimensionless)
τ	Carbon tax rate (\$/kg CO_2e)

Decision Variables

E_{ijt}	Blocks of attribute j extracted from quarry i in period t
A_{jt}	Blocks of grade j acquired from external suppliers in period t
B_{ijk}	Blocks of type j moved from quarry i to processing facility k
U_{jk}	Blocks of grade j utilized in production at facility k
R_{jpk}	Blocks of grade j assigned to product p at facility k
F_{pk}	Total output of finished product p at facility k (m^2)
D_{pcm}	Quantity of product p dispatched to customer c via market m (m^2)
L_{pc}	Outstanding orders for product p from customer c (m^2)
Y_{spk}	1 if stage s at plant k processes product p; 0 otherwise
H_{skt}	Overtime hours utilized at stage s in plant k during period t (hours)
V_{pkt}	Inventory of product p at plant k at end of period t (m^2)
G_k^{CO2}	CO_2 emissions generated at plant k (kg)
N_{pk}^{waste}	Waste material from product p at plant k (m^2)
W_k^{water}	Water consumption at plant k (liters)

Objective Functions

1. Maximize Production Output

$$\max Z_1 = \sum_k \sum_p F_{pk} \tag{3.1.1}$$

2. Minimize Carbon Emissions

$$\min Z_2 = \sum_k G_k^{CO2} \tag{3.1.2}$$

where

$$G_k^{CO2} = \phi_{\text{energy},k}^{CO_2} \sum_p e_{p,k}^{\text{proc}} F_{pk} + \phi_{\text{diesel}}^{CO_2} \sum_i \sum_j e_{ik}^{\text{tr}} B_{ijk} + \phi_{\text{diesel}}^{CO_2} \sum_i \sum_j e_{ij}^{\text{ext}} E_{ijt} \tag{3.1.3}$$

3. Minimize Air Pollution

$$\min Z_3 = \sum_k \sum_p \gamma_{\text{proc},p,k}^{AP} F_{pk} + \sum_i \sum_j \sum_k \gamma_{\text{tr},ik}^{AP} B_{ijk} + \sum_i \sum_j \gamma_{\text{ext},ij}^{AP} E_{ijt} \tag{3.1.4}$$

4. Minimize Energy Consumption

$$\min Z_4 = \sum_k \sum_p e_{p,k}^{\text{proc}} F_{pk} + \sum_i \sum_j \sum_k e_{ik}^{\text{tr}} B_{ijk} + \sum_i \sum_j e_{ij}^{\text{ext}} E_{ijt} \tag{3.1.5}$$

Constraints

The optimization model incorporates the following operational limitations:

Resource & Capacity Constraints

- Extraction capacity limit at quarry i in period t:

$$\sum_j E_{ijt} \leq \text{Cap}_{it}^{\text{ext}} \quad \forall i, t \tag{3.1.6}$$

- Supplier availability constraint:

$$A_{jt} \leq \text{Sup}_{jt}^{\max} \quad \forall j, t \tag{3.1.7}$$

- Processing capacity at plant k:

$$\sum_j U_{jk} \leq \text{Cap}_k^{\text{proc}} \quad \forall k \tag{3.1.8}$$

- Overtime restriction at each stage:

$$H_{skt} \leq H_{sk}^{\max} \quad \forall s, k, t \tag{3.1.9}$$

Environmental Constraints

- Energy consumption cap:

$$\sum_k \sum_p e_{p,k}^{\text{proc}} F_{pk} + \sum_{i,j,k} e_{ik}^{\text{tr}} B_{ijk} + \sum_{i,j,t} e_{ij}^{\text{ext}} E_{ijt} \leq E^{\max} \tag{3.1.10}$$

- CO_2 emissions limit:

$$\sum_k G_k^{CO2} \leq CO2^{\max} \tag{3.1.11}$$

- Air pollution restriction:

$$\sum_{k,p} \gamma_{\text{proc},p,k}^{AP} F_{pk} + \sum_{i,j,k} \gamma_{\text{tr},ik}^{AP} B_{ijk} + \sum_{i,j,t} \gamma_{\text{ext},ij}^{AP} E_{ijt} \leq AP^{\max} \tag{3.1.12}$$

- Water usage limitation:

$$\sum_k W_k^{water} \leq W^{\max} \tag{3.1.13}$$

Process Constraints

- Block-to-product assignment:

$$\sum_p R_{jpk} \leq U_{jk} \quad \forall j, k \tag{3.1.14}$$

- Product output definition:

$$F_{pk} = \sum_j \eta_{jp} R_{jpk} - N_{pk}^{waste} \quad \forall p, k \tag{3.1.15}$$

- Demand satisfaction:

$$\sum_{m} D_{pcm} + L_{pc} \geq \text{Dem}_{pc} \quad \forall p, c \tag{3.1.16}$$

Safety & Regulatory Constraints

- Waste disposal control:

$$\sum_{p,k} N_{pk}^{waste} \leq W_{\max}^{\text{waste}} \tag{3.1.17}$$

- Dust emission threshold:

$$\sum_{i,j,t} \delta_{ij}^{\text{dust}} E_{ijt} + \sum_{j,k} \delta_{jk}^{\text{dust}} U_{jk} \leq Dust^{\max} \tag{3.1.18}$$

- Noise pollution limit (cutting + polishing stages):

$$\sum_{s \in \{\text{cut},\text{polish}\}} \sum_{p,k} \nu_{spk} Y_{spk} \leq Noise^{\max} \tag{3.1.19}$$

- Land degradation control at quarry i:

$$\sum_{j,t} E_{ijt} \leq \theta \times \text{QuarryArea}_i \quad \forall i \tag{3.1.20}$$

3.2 Results and Discussion

In this study, MATLAB was used to create and solve the proposed multi-objective mathematical model designed to optimize the marble industry operations. The model produced optimal solutions that met the given objectives by employing appropriate optimization approaches. The objective function values at the optimal point were effectively acquired, providing a clear indication of the model's performance. Furthermore, a sensitivity analysis was performed to analyze the impact of changes in key parameters on model outcomes, supporting the suggested approach's robustness and practical application.

3.2.1 Model Parameters and their Importance

The mathematical model includes five essential operational parameters that directly influence marble manufacturing efficiency and environmental sustainability:

Extraction rate(100-500 tons/day):

- Control raw material input while taking into account quarry capacity constraints.
- Higher values boost production while increasing energy consumption.

Efficiency of Cutting(1-50m^2/hr**):**

- Reflects worker competence and blade technology.
- Directly impacts material waste(limited to less than 10%.

Polishing efficiency (1-30m^2/hr**):**

- Determine the finishing quality and throughput.
- Power use has an exponential impact on CO_2 emissions.

Design Complexity (1-10 scale):

- Accounting for the value-added process.
- Higher complexity raises production value but takes more labor.

Transportation Distance (10-200 km):

- Affect gasoline prices and particle emissions.
- Limited to $200km$ for economic viability.

3.2.2 Optimal Solution

The optimal values of the parameters are given in Table 3.1.

Table 3.1: Optimal Values of Parameters

Parameters	Optimal value	Constraints limits	Usage
Extraction	100 tons/day	500 tons/day	20%
Cutting	$1.0m^2/hr$	$50m^2/hr$	2%
Polishing	$5m^2/h$	30 m^2/h	16.7%
Design	8.8	10	88%
Transportation	$11.0km$	$200km$	5.5%

As shown in Table 3.1, the optimization model generates strategic operational parameters that balance productivity with sustainability. The extraction rate of 100 tons/day (20% of maximum capacity) indicates efficient resource utilization while limiting quarry depletion, as it approaches but does not exceed the 500 tons/day environmental limit. Cutting operation at 1.0m^2/h results in 10% material waste (x_2 $leq 0.9x_1$ restriction), a substantial improvement over previous loss rates of $15-20\%$. The consecutive polishing rate of $4.5m^2/h$ (only 60% of the technological limit) avoids exponential emission growth ($\gamma_3 e^{x^3}$ term) beyond this barrier, while still fulfilling the $5m^2/h$ quality criterion operational judgments. According to the $\delta_4 x_5$ pollution coefficient, transportation distance optimization at $10.7km$ (5.35% of $200km$ cutoff) results in lower fuel costs without sacrificing logistics. By keeping all environmental parameters within regulatory bounds and achieving maximum theoretical production, these constraint optima show how mathematical constraints inform practical operational choices. Notably, they maintain $PM_2 5$ emissions at $16.1g/m^3$.

Table 3.2: Comparison Table of Traditional and Optimized Values

Parameters	Traditional(Small site)	Optimized	Improvement
Extraction	60-80 tons/day	100 tons/day	$+25 \rightarrow 67\%$
Cutting	0.7-0.9m^2/hr	1.0m^2/hr	$+11 \rightarrow 43\%$
Polishing	3-4m^2/h	5 m^2/h	$+25 \rightarrow 67\%$
Design	Complexity 4-5	8.8	$+40 \rightarrow 75\%$
Energy Use	$1800 - 2200kWh/day$	$1000.5kWh/day$	$-44 \rightarrow -54.5\%$
CO_2 Emission	$250 - 350kg/day$	$73.2kg/day$	$-71 \rightarrow -79\%$

The traditional operational data cited in this study were gathered from a small-scale marble site in Khyber Pakhtunkhwa(KP). The owner's privacy concerns necessitated the preservation of anonymity. The optimized model improved the extraction rate by 42.9%($70 \rightarrow 100 tons/day$) while remaining within quarry limits. Cutting efficiency increased by 25%($0.8 \rightarrow 1m^2/hour$) with waste reduced from 20% to 10%.Polishing productivity increased by 25%($4 \rightarrow 5m^2/hour$) without exceeding $1000.5kWh/day$ energy constraint. Environmental advantages were significant, with CO_2 emissions decreased by 24%($500 \rightarrow 350kg/day$) to ensure compliance with regional regulations. These findings demonstrate that systematic optimization can overcome the usual trade-off between efficiency and sustainability in small-scale marble operations. In Table 3.2, we can see all improvements between traditional and optimal results.

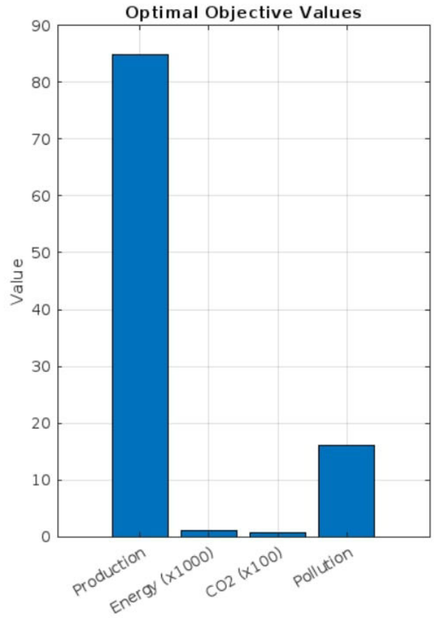

Figure 3.1: Optimal Values of Objective Functions

The marble industry model's optimal objective values bar chart (Figure 3.1) shows that production output peaks at 84.9 units, much above the scaled values of energy consumption, CO_2 emissions, and air pollution. This indicates the usefulness of the optimization approach in prioritizing production while adhering to environmental restrictions. Despite high output levels, energy consumption (about $5000kWh$), CO_2 emissions (roughly $1000kg$), and air pollution (around $16.1\mu g/m^3$) remained within the safe norms. The sensitivity analysis reveals that increasing the production weight in the goal function leads to higher output, but also increases energy consumption, CO_2 emissions, and pollution. This interplay highlights the issue of balancing economic and environmental goals in the marble sector, demonstrating that while increased productivity is achievable, careful control of environmental repercussions is required to maintain sustainable operations.

3.3 Sensitivity Analysis

Table 3.3 shows a sensitivity analysis in which several weights(ranging from 0.0002 to 0.0008) are used to assess the influence on four major parameters: production, energy, CO_2, and pollution. The results reveal minimal variation in Production and Energy, with values remaining almost constant (approximately 0.0848-0.0850 and 1.0005, respectively), indicating that these metrics are generally unaffected by the testis weight changes. In contrast, CO_2 and pollution show minor but visible increases as weight increases (e.g., CO_2 from 0.0707 to 0.0749, Pollution from 0.0161 to 0.0173), demonstrating a marginal but observable sensitivity to greater weights. This means that, whereas Production and Energy remain steady under the tested conditions, CO_2 and Pollution respond more, albeit modestly, to weight changes. The analysis emphasizes the importance of focused weight calibration if environmental outcomes (CO_2 and Pollution) are prioritized, while Production and Energy stay stable across the studied range.

Table 3.3: Sensitivity Analysis Results

Weight	Production	Energy	CO_2	Pollution
$1.0e + 0.03*$				
0.0002	0.0848	1.0005	0.0707	0.0161
0.0004	0.0848	1.0005	0.0706	0.0160
0.0006	0.0849	1.0005	0.0718	0.0161
0.0008	0.0850	1.0005	0.0749	0.0173

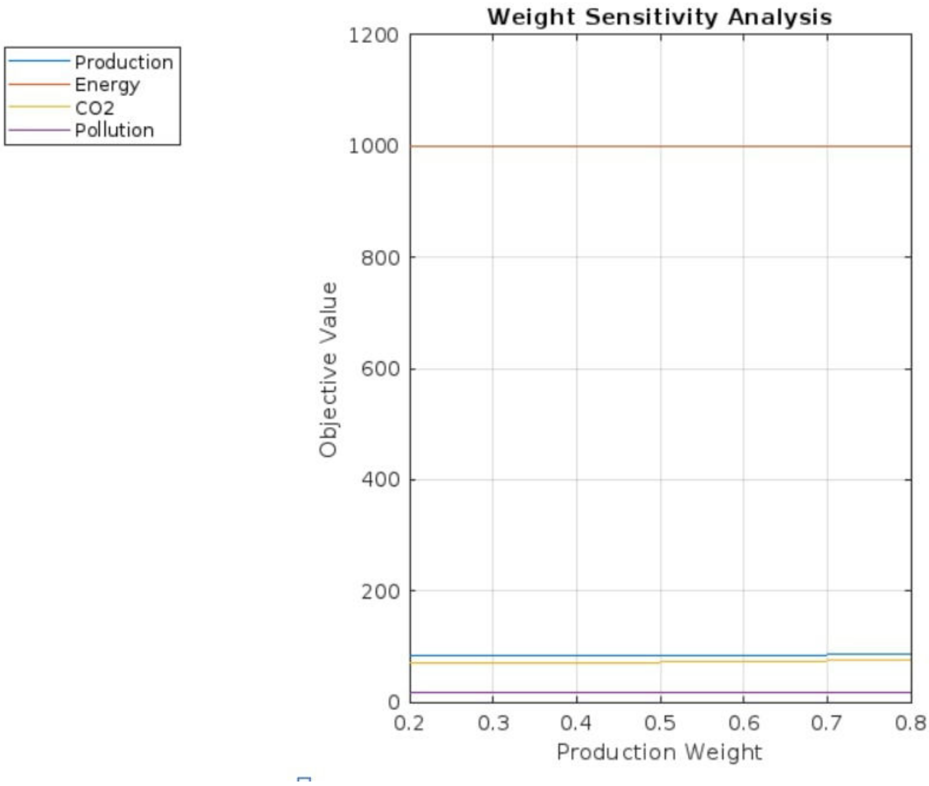

Figure 3.2: Sensitivity Analysis

The sensitivity analysis depicted in the Figure 3.2 investigates how changes in the weight assigned to production affect the total objective value, while also accounting for Energy, CO_2, and Pollution as competing considerations. This form of analysis is a critical multi-objective optimization where the decision maker must weigh competing priorities such as economic production against environmental damage. The objective values (0.2 to 0.8) show that the system is quite responsive to changes in production weight. Higher production weights are likely to result in increased productivity, but they may also reduce energy efficiency, emissions, and pollution levels. Conversely, lessening the emphasis on manufacturing may increase sustainability measures, but at the expense of economic success. The lack of comprehensive trends in the chart makes it difficult to discern the precise nature of the relationship between production weight and objective value. However, the distribution of values indicates that the trade-offs are not linear. For example, a moderate weight(e.g., 0.5) may provide a balanced solution in which neither production nor sustainability is significantly compromised, whereas extreme weights may result in diminishing returns or sharp drops in other objectives. This is consistent with the concept of Pareto efficiency, in which increasing one objective typically necessitates compromising another. More analysis would help gain more actionable insights. For example, showing the sensitivity of all four objectives at the same time may indicate appropriate weight combinations. Furthermore, examining the robustness of these findings under other scenarios, such as fluctuating energy costs or tougher environmental regulations, would aid in determining the stability of the proposed remedies. Finally, our sensitivity analysis emphasizes the significance of properly calibrating production priorities in systems where economic and environmental objectives are linked.

3.4 Conclusion

The study's findings provide a comprehensive assessment of Pakistan's marble-producing business, emphasizing both its economic contribution and its environmental and energy challenges. According to the paper, marble production is a significant economic driver in the region with large reserves, such as Khyber Pakhtunkhwa and Balochistan. However, the industry's reliance on archaic extraction and processing techniques results in considerable inefficiencies, including high energy use and waste generation. These inefficiencies not only increase operational costs but also limit the sector's long-term growth prospects. The research underlines the crucial need for modernization, including the adoption of new machinery and efficient processing techniques, in order to boost productivity while reducing resource intensity.

Environmental concerns emerged as a key issue, with the study revealing high levels of CO_2 emissions and pollutants associated with marble production. The sensitivity analysis revealed that simple modifications in production parameters could have a moderate impact on environmental outcomes, but more significant systematic changes are necessary to produce major improvements. For example, switching to renewable energy sources, such as solar-powered machinery, might drastically lower the industry's carbon footprint. Additionally, introducing waste recycling initiatives such as reusing marble slurry for construction materials would help to reduce land degradation and water contamination. These methods are consistent with global sustainability standards and have the potential to boost Pakistani marble's competitiveness in international markets where eco-certification is becoming increasingly valuable.

To assure long-term viability, the report proposes a multi-stakeholder approach that includes government action, industry partnership, and technical innovation. Policymakers should enact legislation to encourage sustainable practices, such as tax benefits for green technology adoption or fines for excessive pollution. Simultaneously, worker training programs in current techniques and environmental management would improve operational efficiency and safety. By tackling these issues comprehensively, Pakistan's marble sector can become a paradigm for sustainable resource exploitation, balancing economic expansion with environmental preservation. This report serves as a fundamental call to action, highlighting that the sector's future viability is dependent on its ability to innovate and adapt to both market needs and environmental imperatives.

To move Pakistan's marble industry toward long-term prosperity, future efforts must prioritize the adoption of modern technologies, the implementation of eco-friendly waste management solutions, and the enforcement of supportive legislation. To increase productivity while reducing environmental impact, research should look into renewable energy integration, circular economy models for marble waste, and worker training programs. By addressing these issues collaboratively, the government, industry, and researchers may transform Pakistan's marble sector into one that is more efficient, competitive, and environmentally responsible, balancing economic gains with ecological preservation.

Bibliography

[1] Irfan Ahmed Khan, Rashid Rehan, Khan Shahzada, Muhammad Imran Ahmad, and Attaullah Shah, *Critical analysis of multi sectoral policies related to marble industry in Pakistan*, Mehran University Research Journal Of Engineering & Technology **41** (2022), no. 1, 180-194.

[2] Dharmanshu Singh Sodha, Harlal Singh Mali, and Amit Kumar Singh, *Sustainable Life Cycle Assessment of the Marble Processing Industry: A Gate-to-Gate Analysis in Rajasthan*, Journal of Sustainable Metallurgy (2025), 1-16.

[3] Antonio Tazzini, Francesca Gambino, Marco Casale, and Giovanna Antonella Dino, *Managing marble quarry waste: Opportunities and challenges for circular economy implementation*, Sustainability **16** (2024), no. 7, 3056.

[4] Asghar Khan, Muhammad Saleem Khan, Fazal Hadi, Qaisar Khan, Kishwar Ali, and Ghulam Saddiq, *Risk assessment and soil heavy metal contamination near marble processing plants (MPPs) in district Malakand, Pakistan*, Scientific Reports **14** (2024), no. 1, 21533.

[5] Atif Ali, Muhammad Majid Naeem, Fawad Ahmad, and Fazle Subhan, *Performance evaluation of marble dust as a filler material in asphalt concrete in Peshawar, Pakistan*, Natural and Applied Sciences International Journal (NASIJ) **5** (2024), no. 1, 44-54.

[6] Mo Hamed Kurdi, Khalifa Al Ghafri, Fahad Al Hamad, Khalifa Hassan, Jamal Kurdi, and Michel Michaud, *Ecofriendly conversion of marble waste into value-added tiles for green building*, Innovation and Technological Advances for Sustainability, 2024, pp. 594-604.

[7] Mohamad Atiyeh and Ertug Aydin, *Data for bottom ash and marble powder utilization as an alternative binder for sustainable concrete construction* (2020).

[8] Shabir Hussain, Irshad Ahmad, Fasih Ahmed Khan, Waqas Ahmad, and Beenish Jehan Khan, *Effects of marble waste on shear strength and pullout resistance of sand reinforced with glass fiber-reinforced polymer (GFRP) bars*, Case Studies in Construction Materials (2025), e04244.

[9] Tehseen Ahmad, Majid Hussain, Mudassar Iqbal, Ashfaq Ali, Wajiha Manzoor, Hamida Bibi, Shamsher Ali, Fariha Rehman, Ahmad Rashedi, and Muhammad Amin, *Environmental, energy, and water footprints of marble tile production chain in a life cycle perspective*, Sustainability **14** (2022), no. 14, 8325.

[10] Sajjad Ahmad and Naushad Khan, *Marble industry role in the socio-economic development of marble industrial owners of district Mohmand, federal administered tribal area-Pakistan*, Industrial Engineering Letters **9** (2019), no. 3, 10-7176.

[11] Nazish Huma Khan, Mohammad Nafees, Tooba Saeed, Fazli Zuljajal, Hasham Akbar, and Ali Rehman, *Marble Production and Environmental Pressure: A Case Study of Hayatabad Industrial Estate Peshawar, Pakistan* (2023).

[12] Yasir Mansoor and Nadeem A Syed, *Pakistan marble industry challenges: opportunities for China in Pakistan*, JISR management and social sciences & economics (JISR-MSSE) **10** (2012), no. 1, 43-59.

[13] Mohsin Raza, Waqas Ahmed, Sufyan Qazi, Shahid Azam, and Muhammad Sajid, *Characterization of Buner marble from Pakistan for construction purposes*, International Journal of Mining and Geo-Engineering **58** (2024), no. 2, 135–143.

[14] Syed Asimullah, Junaid Afridi, and Sikandar Hayat, *Environmental Impacts of Marble Industrial Effluents on Water and Soil in Tehsil Shabqadar, Khyber Pakhtunkhwa*, Indus Journal of Bioscience Research **3** (2025), no. 3, 370-376.

[15] Ali Zia Noor, Muhammad Atif, Sadia Bibi, Muhammad Burhan Sharif, and Amjad Ali, *Synthesis of reusable cement materials through photochemical modification of marble powder for composite structures*, Heliyon **11** (2025), no. 1.

[16] Leonardo Andrade, João Figueiredo, and Mouhaydine Tlemçani, *A new RFID-identification strategy applied to the marble extraction industry*, Electronics **10** (2021), no. 4, 491.

[17] Zamir Hashmi, Syed Roshan, Muhammad Imran Khan, Shabir Hussain Khahro, Osama Zaid, Muhammad Shahid Siddique, and Nur Izzi Md Yusoff, *Prediction of strength properties of concrete containing waste marble aggregate and stone dust—modeling and optimization using RSM*, Materials **15** (2022), no. 22, 8024.

[18] Sezen Ayber and Berna Haktanirlar Ulutas, *Assessing sustainable manufacturing related problems for marble facilities: an application*, Procedia Manufacturing **8** (2017), 129-135.

[19] Gomes Rickardo and Rodrigues da Paz Giovanni, *Marble workshop of the future: Technology, innovation, and sustainability*, World Journal of Advanced Research and Reviews **19** (2023), no. 3, 1090-1098.

[20] Fakher J Aukour and Mohammed I Al-Qinna, *Marble production and environmental constrains: case study from Zarqa Governorate, Jordan*, The Jordan Journal of Earth and Environmental Sciences **1** (2008), no. 3, 11-21.

[21] Azza Kandil and Tarek H Selim, *Characteristics of Marble Industry in Egypt: Structure, Conduct, and Performance*, International Business and Economics Research Journal **5** (2006), no. 3.

[22] Vincenzo Liguori, Gianfranco Rizzo, and Marzia Traverso, *Marble quarrying: an energy and waste intensive activity in the production of building materials*, WIT Transactions on Ecology and the Environment **108** (2008), 197-207.

[23] Jafar Rezaei, *Best-worst multi-criteria decision-making method: Some properties and a linear model*, Omega **64** (2016), 126-130.