QUANTUM MHD WAVES WITH EFFECT OF SPIN MAGNETIZATION AND TEMPERATURE DEGENERACY

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Dedicated to

My Father, my late mother, my wife, my children and my brothers like friends for their ***Love, Prayers*** *and* ***Support*** *to complete my* ***PhD***

DECLARATION

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ABSTRACT

In this thesis, we have studied the linear properties of magnetohydrodynamic (MHD) waves in degenerate plasma composed of three species; electron-positron-ion (e-p-i) by considering Bohm potential, temperature degeneracy and spin magnetization. A generalized dispersion relation is derived using quantum magnetohydrodynamic fluid model. Perpendicular and oblique propagation modes of magnetoacoustic waves are investigated. It is noted that spin magnetization and degree of arbitrary temperature modify the dispersive properties of the waves. The results of this investigation are beneficial for understanding the collective phenomena of space and astrophysical plasma.

Similarly, MHD waves are also studied in quantum degenerate electron- ion (e-i) plasma with strong magnetic field in the presence of degenerate pressure due to Landau diamagnetic levels and Pauli spin magnetization. A linear dispersion relation of low frequency propagation wave in the direction of magnetic field is derived that strongly depends on the magnetic field while in classical regime this field has no such a role. Quantum effects are incorporated through the Landau pressure due to Landau quantization of magnetic field, magnetization energy linked to spin motion and Bohm potential. New modes of wave propagation associated with quantization of orbital motion and spin magnetization in quantum plasmas are analyzed. It is found that spin magnetization energy and quantum acoustic velocity affect the Alfven mode propagation. The current study in the context of spin magnetization along with Landau diamagnetic pressure is sufficient for investigating the compact astrophysical systems such as neutron stars and white dwarfs.

Moreover, the generalized dispersion relation for the propagation of MHD waves in Cd^{+} ion trapped semiconductor electron-hole-ion plasmas is studied with effect of quantum corrections. The important ingredients of these corrections occurred due to Bohm potential, relativistic degeneracy, exchange-correlation potential and spin magnetization and have significant impact on the dispersion properties of perpendicular and oblique modes of MHD wave. The derived results are numerically analyzed by using the numerical parameters of GaAs, GaSb, GaN, and InP semiconductors plasmas. From the numerical analysis, it is observed that for higher number density, the phase speed of magnetosonic wave is larger for the InP semiconductor, while for low number density plasma region, it gives lower values for GaAs semiconductor. Similarly the phase speed of magnetosonic wave in GaAs decreases with applied magnetic field for different regime of number density. Due to exchange-correlation potential it is found that the frequencies of magnetosonic waves are blue-shifted means that it has magnified the phase speed. It is also shown that frequency of oblique MHD wave for GaAs semiconductor plasmas increases (decreases) with number density of electrons (holes). The relativistic degeneracy term (γ) for given number density is numerically calculated (1.00011~1.0058) for all the above-mentioned semiconductors. It is observed that due to its mild numerical value it has not significant impact on graphical manipulation. The Alfven

speed for above compound semiconductors with $B_0 \leq 10^4 G$ is also calculated which exits in a permissible range of order 10^4 cm/s to 10^7 cm/s. The results are helpful to understand the energy transport in semiconductor plasma in the presence of magnetic field.

LIST OF PUBLICATIONS

This thesis is based on the following four publications:

1. **Safdar, A., et al.** "Magnetosonic waves in ion trapped semiconductor chip plasma with effect of exchange correlation potential and relativistic degeneracy." *Physica Scripta* **97** 025603 (2022).

2. **Safdar, A., et al.** "Effect of arbitrary temperature degeneracy and ion magnetization on shear Alfven waves in electron-positron-ion plasmas." *Contributions to Plasma Physics* **61**.2 (2021).

3. **Safdar, A., et al.** "MHD waves with Landau diamagnetic pressure and Pauli paramagnetizim in degenerate plasmas." *Physica Scripta* **96**.1 (2020).

4. **Safdar, A., et al.** "Magnetoacoustic waves with effect of arbitrary degree of temperature and spin degeneracy in electron-positron-ion plasmas." *Contributions to Plasma Physics* **60**.2 (2020).

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Chapter 1

Introduction

1.1 Plasma Physics

The word "plasma" comes from the Greek which means something formed or molded. It was introduced for the first time in 1929 by Tonks and Langumir to describe ionized gases. Ionized means that when an electron is stripped off from a significant fraction of a molecule. The difference between a gas and plasma is the presence of electrostatic force among the charged particles that cannot be ignored in the study of dynamics of plasma. By applying of magnetic field a Lorentz force is also developed which leads to many novels and spectacular behavior of plasma. Plasma is considered as the fourth state of matter. The basic distinction among the three states of matter (solids, liquids and gases) lies in the difference between the binding forces that hold together their constituent particles i.e., atoms and molecules. When a solid or liquid is heated sufficiently the atoms or molecules gain more thermal kinetic energy that overcomes their potential binding energy and phase transition occurs. Therefore, at high elevated temperatures an ionized gas or plasma occurs

Three fundamental parameters of plasma are given as below

i. the plasma density n (that's measured in particles per cubic meter),

ii. the temperature of particle (that's measured in eV, $1eV = 11605\ \mathring{K}$),

iii. the static magnetic field $\mathbf{B}_0$ (that's measured in $Tesla$).

Other parameters, e.g., Larmor radius, Debye length, thermal velocity, Cyclotron frequency and Plasma frequency are derived from the above parameters. One of the main properties of plasma is collective behavior that differentiates it from a neutral gas. Particles in neutral gas interact only through collisions, which involve short range Vander Walls forces. An independent straight path is followed by these particles from the other neighborhood particles. On the other hand in plasma, situation is quite different where interaction between charged particles involves the long range Columb force and at the same time a charged particle interacts with several other particles. Consequently, plasma shows an immediate reaction to an external perturbation which is recognized as collective behavior. Debye shielding is main feature of plasma which is the ability of plasma to reduce electric fields effectively. When a test charge (positive charge) enteres in to plasma, the particles of plasma will try to neutralize the effect of this test charge by confining its electric field to a specified distance which is known as Debye length represented by λ_D. In plasma, the test charge is surrounded by electrons while ions feel repulsive force. Debye length (λ_{De}) is an important parameter of plasma which is given by

$$\lambda_{De} = \sqrt{\frac{\epsilon_0 K_B T_e}{e^2 n_{e0}}}, \tag{1.1}$$

where n_{e0} is equilibrium number density, T_e is the electron temperature, ϵ_0 is permittivity of free space, e is the elementary electronic charge and K_B is the Boltzmann constant. This shielding process (non-static) depends on the thermal energy of electrons (and ions).

Another property of plasma is the quasineutrality which is mixture of electrons and ions that is not strictly neutral but like Debye length its deviation from neutrality can occur at short distance. However, at lengths larger than Debye length, the plasma must be quasineutral i.e., $n_{i0} \approx n_{e0}$ (the number of positive charges is almost equal to negative charges). This quasineutrality occurs due to collective behavior of plasma and is a defining property of classical plasma. For small perturbations, the electrons must attain the new equilibrium by getting its new position at Debye length λ_{De}. The probable time taken by electrons is $\tau \approx \lambda_{De}/v_{the}$ with the thermal velocity v_{the}

($=\sqrt{K_BT_e/m_e}$). The reciprocal of response time is termed plasma frequency ω_{pe} and is the basic plasma parameter

$$\omega_{pe} = \sqrt{\frac{n_{e0}e^2}{\epsilon_0 m_e}}, \tag{1.2}$$

that shows the typical electron oscillations with a fixed background of ions. On the basis of above-defined plasma parameters, we have three basic conditions for a gas to be in plasma state.

(*i*) $L >> \lambda_{De}$ (size of plasma L must be larger than the Debye length) in order to satisfy the quasineutrality condition.

(*ii*) In Debye shielding, number of charged particles must be large $N_D >> 1$ so that plasma behave in a collective manner.

(*iii*) $\omega_{pe} >> v_{ce}$ i.e., the plasma frequency ω_{pe} must be greater than the collisional frequency v_{ce} so that the ionization rate is greater than the combination rate.

1.2 Multi-Component Plasma

Mostly in plasma, we focus at the contribution of negatively charged electrons and positively charged ions. However there are also some other charged species that exit in it, e.g., positrons, negative ions, negative dust, positive dust etc. Such type of plasma is known as multi-component plasma. Details of some types of multi-component plasmas, studied in our research work are given below:

1.2.1 Electron-Positron-Ion Plasma

The plasma state has received a remarkable attention in the field of physics due to the fact that most observable state of matter in the universe exists in ionized form. Due to this attention, the plasma physics has opened up various new scientific and industrial applications. Among the classification of plasma, the pair-plasma containing charged particles in the form of electrons and positrons is unique in its nature and existence. It is believed that early universe existed in pair plasma state [ ]. It is also obvious that such a pair plasma consists of some nuclei (considered to be ions) in the background by making it three component plasma known as electron-positron-ion plasma. The addition of ions makes the three component plasma different from the pair plasma and introduces some new modes of propagation. This three component of plasmas occur in solar atmospheres [ ], galactic nuclei [ ], pulsars and neutron star's environment [ ] etc. Electron-positron-ion plasma can also be found in astrophysical jets of quasars and blazars [ ]. The formation of this type of plasma is also detected in the magneto-tail of the earth where electrons and positrons from the pulsars arrive and interact with the electron-ion plasma in this region [ ]. Several experimental [ ] ideas have been introduced to produce the electron-positron-ion plasma at laboratory level through strong and high frequency lasers, in Tokomaks with the application of beam-plasma system [ ]. The electron-positron-ion plasma has diverse applications and especially in the understanding of astrophysical objects. The study of pulsars is the fundamental application of electron-positron-ion plasma, which provides the basis for understanding the phenomena occurring in distant astrophysical objects. The study of neutron stars consisting of this three component plasma can provide diverse applications in the field of solid state physics, where matter with high densities and strong fields are involved. It is thought that the deep study and understanding of pair plasma or electron-positron-ion plasma may provide ways to realize the concepts about the earlier universe. While producing the electron-positron-ion plasma experimentally may help out to achieve astrophysical like conditions at laboratory environments.

1.2.2 Electron-Ion-Hole Plasma

The most significant example of plasma is electron gas in an ordinary metal, where valence electrons cannot be bound to any specific nucleus, but rather they behave like a gas particles that's why a metal is a good conductor. Moreover, at ambient temperature and typical density, quantum effects can no longer be ignored. Thus electron gas along with lattice ions in a metal makes a real quantum plasma. Similarly, semiconductor physics contributes another possible application of quantum plasmas [, , ,]. Though electron density in semiconductor materials is comparatively less than in metals, the quantum effects are investigated due to the emerging of miniaturization of electronic devices. The electron-hole (e-h) plasmas can be produced by interaction of short laser pulses with matter where electrons gained energy during interaction from valence band (V.B) and jump to conduction band (C.B) via absorption of single (or multi) photon depending upon band gap energy $E_g(T)$ and that of photon energy. As a result of electrons transition holes are created in V.B [ ]. The electrons and holes fluids in semiconductor behave as quantum plasma under the condition $T_{Fe,h} \geq T_{e,h}$ and they obey F.D statistics. Likewise, quantum effects become more essential in semiconductors at such a small space scales i.e., the de Broglie wavelengths associated with the electrons (and holes) are comparable to their average inter-particle distances. For further manufacturing of modified semiconductor devices, ion-implantation techniques are generally employed in which ions in host materials alter their characteristics through introduction of metal ions (such as Fe^+, Cu^+, Ag^+ etc.). In the recent past, ion-implanted semiconductor (IIS) plasmas have been explored [, , ,], where quantum effects were studied. Also, Cd^+ ion trapped in semiconductor gallium-arsenide (GaAs) heterostructure chip has been fabricated [ ] which gives an interesting opportunity of three species electron-hole-ion (e-h-i) semiconductor magnetoquantum plasmas to determine their quantum collective effects on dispersive properties of magnetosonic propagation waves. In the last chapter, we have studied the linear dispersive properties of low frequency magnetoacoustic waves in spin-1/2 semiconductor quantum magnetoplasmas (GaAs, GaSb, GaN, and InP) taking into account the degenerate relativistic and non relativistic pressure with Bohm potential as well as exchange-correlation potential.

1.3 Premier of Quantum Plasma

Generally, plasma is divided into two types, classical plasma and quantum plasma. Classical plasma is defined on the basis of low density and high temperature while on another hand, quantum plasma is characterized by high density and low temperature. The quantum mechanical effects can be observed in astrophysical objects (e.g., white dwarf, neutron stars, pulsars, magnetars, black holes etc.) [, ,] as well as in solid-state objects (e.g., semiconductors, metals, nano-structures materials etc.) [,]. The quantum effects play an important role when the average interparticle distance ($n^{-1/3}$) is comparable to the de Broglie's wavelength, (λ_B) i.e.,

$$n\lambda_B^3 \geq 1, \tag{1.3}$$

where

$$\lambda_B = \frac{\hbar}{mV_{th}}. \tag{1.4}$$

$V_{th} = \sqrt{K_B T/m}$ is the thermal speed with T (temperature), K_B (Boltzmann constant) and m (mass of the quantum particle). The quantum effects may appear when the Fermi temperature T_F becomes equal or greater than thermal temperature T, we then have a relation

$$E_F(\equiv K_B T_F) = \frac{\hbar^2}{2m}(3\pi^2)^{2/3}n^{2/3}, \tag{1.5}$$

of Fermi energy, E_F in terms of T_F. For temperature condition of $T_F \geq T$ the Maxwell-Boltzmann (MB) distribution changes to Fermi-Dirac (FD) distribution. Therefore another dimensionless quantum degeneracy parameter σ $(= \frac{T_F}{T})$ may be described as

$$\sigma = \frac{T_F}{T} = \frac{1}{2}(3\pi^2)^{2/3}(n\lambda_B^3)^{2/3}. \tag{1.6}$$

This parameter quantifies the quantum regime in the limit range $\sigma \geq 1$ and its more detail is given in chapter (3). In other words at very low system temperature as compared with Fermi Temperature i.e., $T \ll T_F$ the thermal energy may be ignored and in this situation, typical velocity is called Fermi velocity which can be defined, by

$$v_F = \sqrt{\frac{2E_F}{m}} = \frac{\hbar}{m}(3\pi^2 n)^{1/3}. \tag{1.7}$$

In term of plasma frequency and Fermi velocity, we obtain typical scale length (length of electrostatic screening) λ_{Fe} as

$$\lambda_{Fe} = \frac{v_F}{\omega_{pe}} = \left(\frac{2\epsilon_0 E_F}{ne^2}\right)^{1/2}. \tag{1.8}$$

The quantum coupling parameter g_Q is the ratio of interaction energy E_i $(= E_F)$ to the average kinetic energy E_k

$$g_Q = \frac{E_i}{E_k} \sim \left(\frac{1}{n\lambda_{Fe}^3}\right)^{\frac{2}{3}}. \tag{1.9}$$

Plasma may be treated as quantum in nature when its particles significantly affect its macroscopic properties. For example, miniaturization of semiconductor and metallic structures to nano scales (thin metallic films, nanowires, quantum dots) has reached a stage where the quantum tunneling effect of charge carriers has a substantial influence on collective plasma processes in such structures.

1.4 Application of Quantum Plasma

Quantum plasma physics is a new research field which is rapidly grown in recent years because of its potential applications in different areas of research. Quantum plasma is usually used in quantum semiconductor (diodes), quantum dots [ ], nano wires quantum computers [ ], micro plasma, laser produced plasma, ultra cold plasma, ultra-small electronic components, bio photonics [ ] and in compact astrophysical objects (supernova, neutron stars, black hole etc.) [ ]. Similarly quantum plasma composed of electron gas along with ions that exists in ordinary metals and metallic nanostructures e.g., nanoparticles, metal clusters and thin metal films. Moreover, quantum plasma exists in quantum semiconductors and Laser produced plasma which is briefly discussed application wise as follow:

1.4.1 Semiconductor Plasma

The field of semiconductor materials provides another probable application of quantum plasma. Recently, the quantum electron-ion (e-i) semiconductor has been studied by viewing the quantum effects i.e., quantum tunneling and quantum degeneracy pressure, which arise due to the Pauli's exclusion principle and the Heisenberg's uncertainty principle. These effects play an important role in the behavior of electronic devices. Quantum mechanical effects become prominent when the de Broglie's wavelength associated with charge carriers (electrons and holes) become comparable to the spatial variation of the doping profiles [.]. The de Broglie´s wavelength is given by the relation, $\lambda_B = \frac{\hbar}{\sqrt{mK_BT}}$). Quantum effects start playing a significant role when the de Broglie´s wavelength is comparable to the interparticle distance of plasma $\lambda_B \sim d$ and the plasma will be considered as quantum plasma which means that the plasma is high dense. Otherwise the plasma will be treated as in classical nature. In other words we can also say that if $T < T_F$ then the plasma behaves as quantum nature. Accordingly for $T << T_F$, the plasma is so in dense form that all the particles of plasma are considered dimensionally like as Broglie´s wavelength.

In quantum semiconductor plasma, the charge carriers due to the Pauli exclusion principle obey the F.D distribution rather MB distribution. (The F.D distribution is used for fermion particles e.g; electrons, protons, neutrons etc. follow both Heisenberg uncertainty and Pauli Exclusion Principle. In this distribution the particles have fermi energy E_F i.e., the energy of the highest occupied state at absolute zero temperature. Therefore at absolute zero temperature, fermions will fill up all available energy states below a level E_F with one (and only one) particle according to Pauli Exclusion Principle. At higher temperature, some particles are elevated to level above the Fermi level. On the other hand MB distribution can be used for bosons e.g; photons, bosons, gluons, graviton etc. These particles have thermal energy E_T. They follow Heisenberg uncertainty and not follow Pauli Exclusion Principle). These quantum semiconductor devices are like resonant tunneling diodes, high-electron-mobility transistors, or super lattices [ ]. Though electron density in ordinary semiconductor materials is comparatively less than in metals, the quantum effects are investigated due to the emerging of miniaturization of electronic components. To detect frequency shifts due to quantum effects, X-ray Thomson scattering in high energy density plasmas provide experimental techniques for accessing narrow band width spectral lines [ ].

1.4.2 Laser Produced Plasma

The rapid development of laser technology provides an excellent opportunity to construct a high power laser source with high intense beam. This high power laser pulses will open a new window dealing with interaction of laser beam with plasma in the relativistic and quantum regime. There are a lot of physical linear and non linear phenomena which are the subject interest in the theoretical investigations. These phenomena contain self foucusing laser beam in quantum plasma [ ], laser filamentation [ ], linear and non linear electrostatic waves [ ] and electromagnetic waves [ ], Cherenkov beam instabilities in quantum plasma [ ] and modulation instabilities in the interaction of laser beam with relativistic quantum plasma [ ]. Semiconductor provides a less expensive medium to model the phenomena in which laser produced plasma has been found. When a beam of laser is incident on a semiconductor material, electrons will excite from valance band to conduction band with the absorption of photon energy. This inter band transition of the electrons creates holes in the valence band, and this state may satisfy the plasma condition to produce electron-hole plasma.

1.4.3 Dense Astrophysical Plasma

Quantum plasma has a verity of applications in dense astrophysical objects, composed of degenerate particles as well as non degenerate particles. In quantum plasmas, the electron and positrons/holes are assumed to be degenerate while the ions are preserved as non degenerate particles due to their much smaller de-Broglie wavelength in correlation with thermal electrons. The dense astrophysical objects are discussed below in detail.

White Dwarf Star

A white dwarf (WD) star is basically a long-dead high dense star that is formed through the evolution process of intermediate mass stars like Sun. During the whole evolution process period at about of several billion years, the star loses its original mass through stellar winds and becomes in cool and dense form. However such a star at the end of burning stage remains hot core ($T > 10^5 \mathring{K}$) called young WD. The mass of WD is not as high enough as that of neutron star and black hole. Generally its mass is comparable to that of sun while its volume is considered the same as that of earth. A WD undergoes gravitational collapse when its mass exceeded to $M_{WD} = 1.4M_{sun}$ (known as Chandrasekhar limit). WD star is so called because of its white colour that has low luminosity. This type of star is in hydrostatic equilibrium which is due to the electron degeneracy of pressure. Recently, strongly magnetized WD and its instability due to nuclear processes have been studied by Otoniel et. al [ ]. Mostly WDs made of oxygen, carbon, or helium which possesses central densities up to $\sim 10^{11} g\ cm^{-3}$. The observed surface magnetic fields are from $10^6 G$ to $10^9 G$. On the other hand, in the framework of both Newtonian gravity and general relativity, analytical and numerical calculations, show that WDs may have internal magnetic fields as large as $10^{12-16} G$ [,].

Due to quantum mechanical properties, the structure of WD star has a fascinating feature inwhich electron degeneracy pressure (a phenomenon described by the Pauli exclusion principle) against of its collapse [ ]. Other attractive property is that the electrons are forced to squeeze with increasing mass of star and as a result the radius of star decreases. The equation of state is modified through the incorporation of additional factors associated with time period of rotation. In this model the restoring force is applied due to the thermal electrons while ions are treated as inactive particles. Different modes such as p-mode (propagating acoustic mode) and g-mode (gravity oscillation modes) have been theoretical studied [,].

Neutron Star

A neutron star (NS) is a collapsed core of very high dense giant star which has the total mass in the range 10 - 25 of solar mass. In fact this type of star has a massive amount of neutrons in its core that's why it is called NS (or also known as giant nucleus). It collapses so much that electrons and protons combine to form neutrons. Therefore this system object has a high amount of neutrons but very slight variations of protons as well as electrons which are a clear evident of its neutralized matter. These stars contain

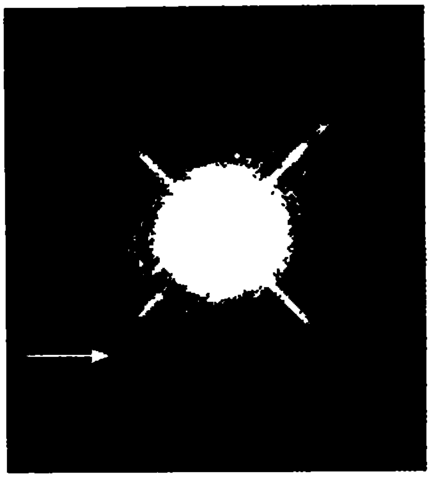

Figure 1.1: White Dwarf

iron element as a main component, but the outer layers above of the core of iron layer are composed of lighter elements [ ]. A NS is partially supported against further collapse by neutron degeneracy pressure, just as WD star is supported against collapse by electron degeneracy pressure. Moreover, due to high density of NS, it has intense gravitational and magnetic fields. The magnetic field of NS can be a billion times the magnetic field on the surface of Earth. Usually, a huge dense NS may collapse into a black hole and disappeared itself.

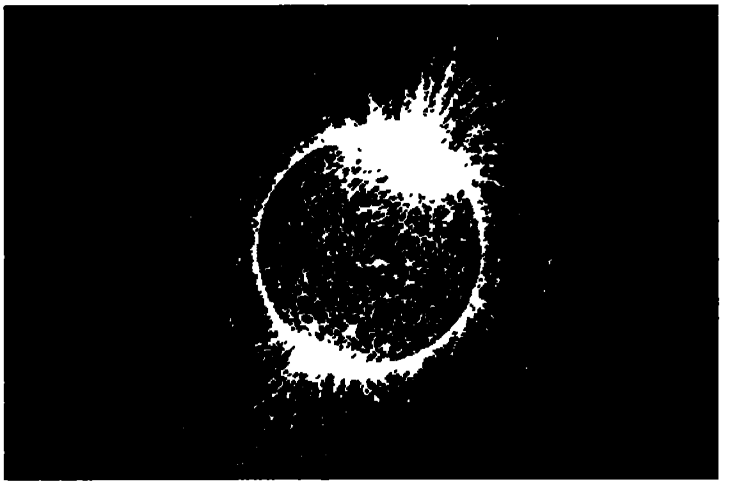

Figure 1.2: Neutron star

The temperature of a newly formed NS is in the range of $10^{11} - 10^{12}\mathring{K}$. However, after a few years its temperature falls to $10^{6}\mathring{K}$ due to the emission of huge number of

Figure 1.3: Magnetar

neutrinos [ ] and at this lower range of temperature, most of the light generated by a NS is in X-rays. The magnetic field strength on the surface of NS ranges from 10^8 to $10^{15}G$ [ ].

Pulsars and Magnetars

Generally NS is about $20km$ ($12.5miles$) in diameter and one sugar cube of its material has weigh about $1billion\ tons$ (or 1 $trillion\ Kg$) on our Earth. Due to its small size and high density NS possesses gravitational field about 2×10^{11} times that of Earth. NS can also carry magnetic field a million times stronger than the strongest magnetic field produced on Earth. On the base of spinning effect and strong magnetic field these stars can be further divided into two types: Pulsars and Magnetars.

Pulsars are spinning NSs which were first discovered by graduate student Jocelyn Bell Burnett in 1967. Pulsars have strong magnetic field which have jets of particles out moving often at the speed of light along their magnetic poles. Since the magnetic field is not aligned with the spinning axis of star, therefore the beam of particles as a light is swept around as it rotates. This light seems as the spotlight in a lighthouse.

Another type of NS is Magnetar which has magnetic field in the range $10^{13} - 10^{15}G$ [ ]. In Magnetars their crust is tightly locked with strong magnetic field so the change occurred in one will cause to change the other. Therefore due to a huge magnetic field the moment of star will release a tremendous amount of energy in the form of EM

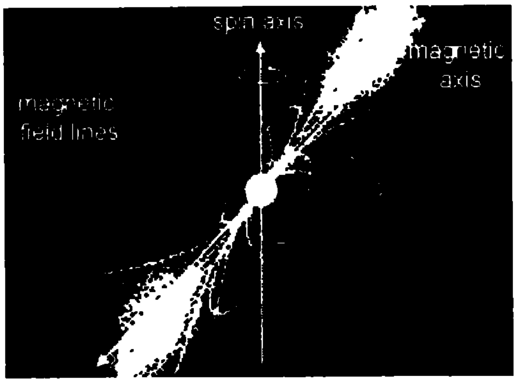

Figure 1.4: Pulsar

radiation. The SGR 1806-20 is a Magnetar which has been discovered in 1979 and identified as soft gamma repeater. This star releases energy in 1/10 second which is more than the energy released by Sun in the last 100,000 years.

1.5 Physics of MHD Waves

1.5.1 Alfven wave

Since low frequency EM waves can propagate in conducting fluid like plasma which is not possible in rigid conducting medium. The two basic propagation modes are Alfven waves and magnetosonic waves which are experimentally and theoretically studied. Hannes Alfven in 1942 first predicted Alfven waves and assumed the plasma that immersed in magnetic field as a conducting and incompressible medium. Alfven waves are EM waves which have propagation vector $\mathbf{K}$ parallel to magnetic field $\mathbf{B}_0$ (i.e., $\mathbf{K} \parallel \mathbf{B}_0$) with $\omega < \omega_{ci}$, as $\omega_{ci}(= \frac{eB_0}{m_i})$ is the ion cyclotron frequency. The dispersion relation for such a type of EM wave is given by

$$\omega = V_A k. \tag{1.10}$$

In this equation $V_A(= \frac{1}{\sqrt{\mu_0 n_0 m_i}})$ is the Alfven speed and $V_A < c$. Further Alfven waves can be categorized into two main types. First is the torsional Alfven wave and second is the compressional Alfven wave. Torsional Alfven wave is also known as shear Alfven wave and at frequency near to the ion cyclotron frequency is called ion cyclotron wave. The word shear is appropriate in cartesian coordinate while the word torsional is in cylindrical geometry. The propagation of the wave is angle dependent. Moreover, this type of wave is slow and compressionaless which means that it is independent of the change in density of fluid and related to low temperature. When we add the kinetic effects then it said to be kinetic Alfven waves. On the other hand compressional waves are known as magnetoacoustic or magnetosonic waves if magnetic field pressure is incorporated. In this compressional wave the compression plasma is transverse to $\mathbf{B}_0$. Here dispersion relation is given by

$$\omega = k\sqrt{V_A^2 + V_s^2}. \tag{1.11}$$

For compressional waves $V_A > V_s$ as $V_s = \sqrt{\frac{P}{\rho}}$ denotes the ion acoustic speed which describes that it is partially electrostatic and partially magnetic. If the magnetic fluctuations increase as compare to density flux then electromagnetic behavior is dominante likewise, if magnetic component is less than density fluctuation then electrostatic behavior will dominante. This wave is non-dispersive as phase and group velocities are the same [ ]. Compressional waves can also be designated as fast waves comparing to torsional waves while at frequency above of the cyclotron frequency is known as whistler wave or helicon wave [ ].

The Alfven waves have an important role in the development of controlled nuclear fusion plasma which contributes to heat plasma at fusion temperature and carrying current so that to generate magnetic field for the use of confining charged particles. These waves may be helpful to understand the different instabilities and turbulance arising in such controlled nuclear fusion plasma [ ]. The Alfven waves play a crucial role in Tokamaks and Stellerators. These waves are also observed in space plasma such as earth's magnetosphere, solar atmosphere and the interplanetary plasma. Similarly, the coupling of energy between the ionosphere and magnetosphere at their sharp boundary of the two regions may be explained through reflection of Alfven waves [ ]. In solar wind low frequency EM waves were examined through spacecraft measurements which are now called Alfven waves [ ]. Likewise, the Alfven waves play a significant role in astrophysical plasma under the influence of strong magnetic field. Recently, the effect of Alfven wave on the thermal instability of the Interstellar medium is investigated both numerically and analytically [ ]. The temperature difference between the sun's corona and its surface may be described on the base of energy transportation property which is the basic idea given by Hannes Alfven. Thus the Alfven wave as a fundamental EM mode involved in many astrophysical phenomena e.g., energy transportation, heating, damping, turbulence, mass loss in stars with cold corona etc.

1.5.2 Fast and slow Magnetosonic waves

A magnetosonic wave is a linear MHD wave which is propagated due to thermal and magnetic pressures. Magnetosonic wave is categorized into two types, fast and the slow magnetosonic modes. Both these modes have been recently discovered in the sun corona [ ].

Let ς is the displacement vector which gives the distance of a parcel plasma displaced from equilibrium position in a specific direction. While $F(\varsigma)$ is the force applied by a parcel plasma displaced through displacement ς. In cartesian axes $K = k_\perp \hat{y} + k_\parallel \hat{z}$ we can derive the dispersion relation for MHD waves by using the following equations;

$$\left(\omega^2 - V_A^2 k_\parallel^2\right) \varsigma_x = 0, \tag{1.12}$$

$$\left(\omega^2 - V_S^2 k_\perp^2 - V_A^2 k_\parallel^2\right) \varsigma_y - V_S^2 k_\perp k_\parallel \varsigma_z = 0, \tag{1.13}$$

$$- V_S^2 k_\perp k_\parallel \varsigma_y + \left(\omega^2 - V_A^2 k_\parallel\right) \varsigma_z = 0. \tag{1.14}$$

For a non-trivial solution (i.e., ς is not equal to zero), we need

$$\det \begin{bmatrix} \omega^2 - V_A^2 k_\parallel^2 & 0 & 0 \\ 0 & \omega^2 - V_S^2 k_\perp^2 - V_A^2 k_\parallel^2 & -V_S^2 k_\perp k_\parallel \\ 0 & -V_S^2 k_\perp k_\parallel & \omega^2 - V_A^2 k_\parallel \end{bmatrix} = 0. \tag{1.15}$$

Equation (1.15) reduces the dispersion relation of MHD waves

$$\left(\omega^2 - V_A^2 k_\parallel^2\right)\left[\omega^4 - k^2\left(V_A^2 + V_S^2\right)\omega^2 + k_\parallel k^2 V_A^2 V_S^2\right] = 0. \tag{1.16}$$

The solution for shear Alfven waves is given by

$$\omega^2 = V_A^2 k_\parallel^2. \tag{1.17}$$

A generalized form of magnetosonic wave in homogenous plasma is given by

$$\omega^4 - k^2\left(V_A^2 + V_S^2\right)\omega^2 + k_\parallel k^2 V_A^2 V_S^2 = 0. \tag{1.18}$$

For fast and slow magnetosonic waves, the above equation can be written in simplified form as

$$\omega_{f,sl}^2 = \frac{k^2\left(V_A^2 + V_S^2\right)}{2}\left(1 \pm \sqrt{1 - \frac{4k_\parallel^2 V_A^2 V_S^2}{k^2\left(V_A^2 + V_S^2\right)^2}}\right), \tag{1.19}$$

here the subscripts f and sl stand for fast and slow respectively. It can be shown that $\omega_{sl} \leq \omega \leq \omega_f$, where ω $(= \omega_A = V_A k)$ is Alfven frequency. If the external magnetic field is zero then $V_A = 0$ and $\omega_{sl} = 0$, implies $\omega_f = kV_S$. It means that slow mode is disappeared while only fast mode remained in the system which is just as a sound wave that propagates isotropically. Similarly, under the assumption of cold plasma $(T_0 = 0)$, it follows that thermal pressure $P_0 = 0$ and therefore $V_S = 0$. Here $\omega_{sl}^2 = 0$ and $\omega_f^2 = k^2 V_A^2$. Hence there is no slow wave, and the fast wave propagate in the system isotropically with the Alfvén speed.

1.6 Quantum Magnetohydrodynamics Model

Magnetohydrodynamics (MHD) is the study which deals with the interaction of magnetic fields and moving conducting fluids. Here "Magneto" stands for electromagnetic fields, "hydro" for fluid and "dynamics" for forces or moments. Three process are important in MHD

- According to Faraday's law of induction an induced e.m.f. is produced due to the relative moment of a conducting fluid and a magnetic field.

• The induced current will give a second, induced magnetic field according to Ampere's law. This induced magnetic field will add to the applied (original) magnetic field and the change is usually such that the fluid appears to drag the magnetic field lines along with it and this is called frozen-in-flux condition.

• The combined magnetic field i.e., origional magnetic field plus induced magnetic field interacts with induced current density, to give rise a Lorentz force [ ].

Some salient features of MHD are described as below:

• MHD model is used for low frequency plasma interaction i.e., $\omega < \omega_{ci}$. The main propagation source of these MHD waves in plasma is ions particles in which ions oscillate in response of a restoring force provided by an effective tension on the magnetic field lines. Therefore, these waves are considered as low frequency waves.

• Ions and electrons are strongly magnetized which means that they are tied to the magnetic field lines.

• Plasma will act like a single fluid.

• Scale may be considered as a bulk i.e., particle identity is lost.

• MHD describes the macroscopic behavior of plasma.

• Each term in the MHD equations represents a different physical effect.

Quantum hydrodynamic (QHD) model is one of the possible approach to investigate charged particle system with relevant quantum effects. Quantum MHD is the macroscopic fluid model that is used to focus the global properties of quantum plasma. The QMHD equations in a simplified form are given below [ ]:

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \mathbf{v}) = 0, \tag{1.20}$$

$$\rho_m \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{v} = \mathbf{J} \times \mathbf{B} - \nabla P + \frac{\rho_m \hbar^2}{2 m_e m_i} \nabla \left(\frac{\nabla^2 \sqrt{\rho_m}}{\sqrt{\rho_m}} \right), \tag{1.21}$$

$$\nabla P = V_s^2 \nabla \rho_m, \tag{1.22}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J}, \tag{1.23}$$

and

$$\mathbf{J} = y_\sigma \left[\mathbf{E} + \mathbf{v} \times \mathbf{B} - \frac{m_i}{e\rho_m} (\mathbf{J} \times \mathbf{B}) - \frac{\hbar^2}{2em_e} \nabla \left(\frac{\nabla^2 \sqrt{\rho_m}}{\sqrt{\rho_m}} \right) \right]. \tag{1.24}$$

Here the plasma centre of mass density with $m_i >> m_e$ is given as $\rho_m (= n_e m_e + n_i m_i \approx nm_i)$, the modified pressure as $P(= P_e + P_i)$, current density as $J(= (v_i - v_e)ne)$ and $y_\sigma (= \rho_m e^2 / m_e m_i \nu_{ei})$ with ν_{ei} being the collisional frequency for momentum transfer between electrons and ions. The adiabatic sound speed is represented by V_s in the above Equation (1.22). To use MHD model quasineutrality condition is considered. Quasineutrality means neutral enough so that one can take $n_e = n_i \approx n$ where n is the common density called plasma density, but not so neutral that all the electromagnetic forces vanish. Accordingly plasma has also collective behavior. Therefore to study MHD waves in quantum plasma quasineutrality condition must be applied. Usually an infinite conductivity is considered in an ideal MHD model by neglecting the Hall force term in Equation (1.24) which gives

$$E = -\mathbf{v} \times \mathbf{B} + \frac{\hbar^2}{2em_e} \nabla \left(\frac{\nabla^2 \sqrt{\rho_m}}{\sqrt{\rho_m}} \right), \tag{1.25}$$

$$\rho_m \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{v} = \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla P + \frac{\rho_m \hbar^2}{2m_i m_e} \nabla \left(\frac{\nabla^2 \sqrt{\rho_m}}{\sqrt{\rho_m}} \right), \tag{1.26}$$

and

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}). \tag{1.27}$$

The term that makes the quantum MHD different from the classical one is the quantum correction term which appears as last term in Equation (1.26). The quantum correction term in non-ideal QMHD vanishes if the curl of Equation (1.25) is operated and leads to a dynamo Equation (1.27) similar to that of classical MHD. Therefore, with the assumption of infinite conductivity, still the condition of magnetic field lines frozen in fluid is satisfied. Now we introduce the following some dimensionless parameters as

$$\bar{\rho}_m = \frac{\rho_m}{\rho_0},\ \bar{\mathbf{v}} = \frac{\mathbf{v}}{V_A},\ \bar{\mathbf{B}} = \frac{\mathbf{B}}{B_0},\ \bar{\mathbf{r}} = \frac{\omega_{ci}\mathbf{r}}{V_A} \text{ and } \bar{t} = \omega_{ci}t, \tag{1.28}$$

where ρ_0 and B_0 are the unperturbed mass density and magnetic field, respectively, $\omega_{ci}(= eB_0/m_i c)$ is the ion cyclotron frequency and the typical Alfvén velocity $V_A(= \sqrt{\frac{B_0^2}{\rho_0\mu_0}})$ which provides a natural velocity scale in MHD. Rescaling the QMHD equations with the dimensionless parameters provided in Equation (1.28), these equations in normalized forms may be expressed as under

$$\frac{\partial\bar{\rho}_m}{\partial t} + \nabla\cdot(\bar{\rho}_m\bar{\mathbf{v}}) = 0, \tag{1.29}$$

$$\left(\frac{\partial\bar{\mathbf{v}}}{\partial t} + \bar{\mathbf{v}}\cdot\nabla\bar{\mathbf{v}}\right)\bar{\rho}_m = \left(\nabla\times\bar{\mathbf{B}}\right)\times\bar{\mathbf{B}} - \frac{v_S^2}{V_A^2}\nabla\bar{\rho}_m + \frac{\bar{\rho}_m H^2}{2}\nabla\left(\frac{\nabla^2\sqrt{\bar{\rho}_m}}{\sqrt{\bar{\rho}_m}}\right), \tag{1.30}$$

$$\frac{\partial\bar{\mathbf{B}}}{\partial t} = \nabla\times\left(\bar{\mathbf{v}}\times\bar{\mathbf{B}}\right), \tag{1.31}$$

where H_e is dimensionless parameter and is defined as

$$H_e = \frac{\hbar\omega_{ci}}{V_A^2\sqrt{m_e m_i}}. \tag{1.32}$$

This parameter H_e measures the quantum effects which in MKS system can be written as

$$H_e = 3.42\times10^{-30}\frac{n_0}{B_0}, \tag{1.33}$$

here n_0 (and B_0) is the ambient particle density (and the externally applied magnetic field). It can be seen from Equation (1.33), that H_e can be neglected in the case of classical plasma systems where the number densities are low. However, for astrophysical plasmas such as WD or NS, where the number densities lie in the range $10^{29} - 10^{34} m^{-3}$, the quantum parameter H_e can take values near to unity or greater and, thus, quantum effects become important.

1.7 Temperature Degeneracy and Fermi-Dirac Statistic

When the number density of quantum plasma increases and the system temperature decreases, the interaction among the particles may play a vital role. As these particles become closer, the overlapping of their associated waves may occur and consequently energy levels split and therefore the effect of degeneracy observes. In case of degenerate plasma equilibrium distribution changes from Maxwellian (classical) to F.D (degenerate) [ ]. In quantum plasma, the two effects i.e., Pauli's exclusion principle and Heisenberg uncertainty principle are combined then as a result quantum pressure is produced. The Pauli's exclusion principle is due to fermions (particles with half-integer spin) which obey F.D distribution while Heisenberg's uncertainty principle is linked with wave nature of particles. The electron degeneracy pressure may be generated when electron concentration is confined in a small volume [ ]. The $\xi (= e^{\frac{\mu}{K_B T_e}})$ (known as Fugacity) in F.D distribution which describes the degeneracy in chemical potential (μ) and thermodynamic electron particle temperature (T_e). To show that the regime is either degenerate or non degenerate depends on the numerical value of ξ. If $\xi \ll 1$ i.e., $\frac{\mu}{K_B T_e}$ is large and negative, the regime is a non degenerate implying $\xi \rightarrow 0$, and similarly on the other hand, if $\xi \gg 1$ i.e., $\frac{\mu}{K_B T_e}$ is large and positive, the regime is completely degenerate implying $\xi \rightarrow \infty$ with $\mu \rightarrow T_{Fe} = 1/2 m v_{Fe}^2$, here $v_{Fe} = (K_B T_{Fe}/m)^{1/2}$ is the Fermi speed of electron. Alternatively, we can also define the above system of regimes as (a) if $T_{Fe} \ll T_e$, the regime is non degenerate and (b) if $T_{Fe} \gg T_e$ the regime is considered as degenerate [ ]. In the theory of F.D system we take the F.D distribution function which for electrons can be written as

$$f(v,r,t) = \frac{A}{1+e^{\frac{(\varepsilon_e+\mu)}{K_B T_e}}}, \tag{1.34}$$

here $\varepsilon_e = 1/2 m_e v_e^2$, v_e is the velocity of electron and μ is the chemical potential of electron which is a function of time and space. The chemical potential exists in the limit range $-\infty \leftarrow \mu \rightarrow +\infty$ and is an arbitrary quantity. In above equation A is the normalization constant and $\int f d^3v = n_e$, so that $A = -\frac{n_e}{Li_{3/2}(-\xi)}\left(\frac{m_e}{2\pi K_B T_e}\right)^{3/2} = 2\left(\frac{m_e}{2\pi\hbar}\right)^3$, here $Li_v(\xi)$ of index v is the polylogarithm function given by [ ]

$$Li_v(\xi) = \frac{1}{\Gamma(v)}\int_0^\infty \frac{\chi^{v-1}}{\frac{e^\chi}{\xi}-1}d\chi, \tag{1.35}$$

where $\chi = \frac{\varepsilon_e}{K_B T_e}$ and $\Gamma(v)$ is the gamma function. Now the thermal equation of the state of Fermi gase can be found from the following system of equations

$$P = -\left[\frac{\partial\Omega(V,T,\mu)}{\partial V}\right]_{T,\mu}, \tag{1.36}$$

$$n = -\left[\frac{\partial\Omega(V,T,\mu)}{\partial \mu}\right]_{V,T}. \tag{1.37}$$

Here $\Omega(V,T,\mu)$ is the grand thermodynamic potential of the electron particle (i.e., it depends upon volume,teperature and chemical potential) which is given by the expression as

$$\Omega_e = -K_B T_e \sum_k \ln\left(1+e^{(\mu-\varepsilon)/K_B T_e}\right). \tag{1.38}$$

This equation implies

$$\Omega_e = -\frac{K_B T_e V g_0}{(2\pi)^2}\frac{(2m_e)^{3/2}}{\hbar^3}\int_0^\infty \ln\left(1+e^{(\xi-\varkappa)/K_B T_e}\right)\varkappa^{1/2}d\varkappa, \tag{1.39}$$

where $g_0 = (2s+1)$ is the degeneracy with respect to the spin, $\varkappa = \varepsilon/K_B T_e$, $\xi = \mu/K_B T_e$ and the chemical potential changes in the range $-\infty < \mu < +\infty$,The system of thermal equations (1.36-1.37) of state are given by

$$P(\mu, T) = \frac{2}{3}\frac{g_0\,(2m)^{\frac{3}{2}}}{(2\pi)^2\hbar^3}\int_0^{\infty}\frac{\varkappa^{\frac{3}{2}}d\varkappa}{e^{(\varkappa-\xi)}+1}, \tag{1.40}$$

$$n(\mu, T) = \frac{V g_0\,(2m)^{\frac{3}{2}}}{(2\pi)^2\hbar^3}\int_0^{\infty}\frac{\varkappa^{\frac{3}{2}}d\varkappa}{e^{(\varkappa-\xi)}+1}. \tag{1.41}$$

F.D integrals in the above two equations using $g_0 = 0$ with other parameters are given below

$$P(\mu, T) = \frac{2}{3}\frac{(2mk_BT_e)^{\frac{5}{2}}}{4m\pi^2\hbar^3}\int_0^{\infty}\frac{\varkappa^{\frac{3}{2}}d\varkappa}{e^{(\varkappa-\xi)}+1}, \tag{1.42}$$

$$n(\mu, T) = \frac{(2mk_BT_e)^{\frac{3}{2}}}{2\pi^2\hbar^3}\int_0^{\infty}\frac{\varkappa^{\frac{1}{2}}d\varkappa}{e^{(\varkappa-\xi)}+1}. \tag{1.43}$$

In terms of polylogarithmic function Equations (1.42) and (1.43) can be written as [ ]

$$P(\mu, T) = -\frac{2}{3}\frac{(2mk_BT_e)^{\frac{5}{2}}}{4m\pi^2\hbar^3}\Gamma\left(\frac{5}{2}\right)Li_{\frac{5}{2}}\left(-e^{\frac{\mu}{k_BT_e}}\right), \tag{1.44}$$

$$n(\mu, T) = -\frac{(2mk_BT_e)^{\frac{3}{2}}}{2\pi^2\hbar^3}\Gamma\left(\frac{3}{2}\right)Li_{\frac{3}{2}}\left(-e^{\frac{\mu}{k_BT_e}}\right). \tag{1.45}$$

Solving Equations (1.44) and (1.45) we can obtain the generalized form of pressure in terms of polylogarithm as

$$P = Gnk_BT_e, \tag{1.46}$$

which is the barotropic equation of state with $G = \frac{Li_{\frac{5}{2}}(-\xi)}{Li_{\frac{3}{2}}(-\xi)}$ being defined as arbitrary temperature degeneracy parameter. For $\xi \ll 1$ we have $G = 1 + \frac{\xi}{2^{5/2}}$. Therefore Equation (1.46) becomes as

$$P_{NND} = (1 + \frac{\xi}{2^{5/2}})nk_BT_e, \tag{1.47}$$

which gives the expression of pressure for in case of nearly non degenerate (NND) plasma. Accordingly, for $\xi \gg 1$ polylogarithm function can be expanded as $G = \frac{2T_{Fe}}{5T_e}(1 - \frac{\pi^2}{12}(\frac{T_e}{T_{Fe}})^2)$. Hence Equation (1.46) implies as under

$$P_{ND} = \frac{2}{5}(1 - \frac{\pi^2}{12}(\frac{k_B T_e}{\varepsilon_F})^2)n\varepsilon_F. \tag{1.48}$$

This equation shows pressure for in case of nearly degenerate plasma with Fermi energy $\varepsilon_F = k_B T_{Fe}$.

1.8 Spin Magnetization

Brodin et al. [ ] investigated novel collective EM wave phenomina including electron spin-1/2 effect. The electron tends to align its spin effect with external magnetic field. Consequently, there is emerged a plasma magnetization in the direction of this field. The spin effect modifies the plasma current density and produces a magnetic moment force on electron. The Maxwell's equations are written as

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{1.49}$$

and

$$\nabla \times \mathbf{B} = \mu_0 (\mathbf{J}_m + \mathbf{J}_p). \tag{1.50}$$

Here $\mathbf{J}_p \, (= \sum qnV)$ is the plasma current density due to free electrons and similarly, the electron magnetization spin curret density ($\mathbf{J}_m$) is as under;

$$\mathbf{J}_m = \nabla \times \mathbf{M}, \tag{1.51}$$

where M is spin magnetization (Pauli magnetization) per unit volume and its mathematical expression can be derived in following manner.

As the energy of free electron in presence of external magnetic field in simple form is given by

$$E = (N + 1/2)\hbar\omega + (\hbar^2/2m)k_z z \pm \mu_B B. \tag{1.52}$$

Here N denotes the Landau quantum number ($N = 0, 1, 2, ...$). The first two terms in right hand side of the above equation are related to the orbital motion of the electron. While the last term results from the to the electron spin (s) coupled magnetic moment ($\mu_s = -\mu_B mg = \mp\mu_B$, as $m = \pm 1/2, g = 2$). A simple relation of spin magnetization (pauli magnetization) is as under,

$$\mathbf{M} = (n_+ - n_-)\mu_B, \tag{1.53}$$

here n_+ is the number density of electrons with magnetic moment (μ_s) along the magnetic field. Similarly, n_- is the number density of electrons with μ_s against the field direction.

Under thermal equilibrium, the condition for high temperature is $\mu_s B \ll K_B T$ therefore we have the relation for magnetization

$$\mathbf{M} = \frac{n\mu_B^2 B}{K_B T}, \tag{1.54}$$

It means that magnetization in this case is temperature dependent i.e.,

$$\mathbf{M} \propto \frac{1}{T}, \tag{1.55}$$

Similarly, for low temperature new magnetization will become as

$$\mathbf{M} = (n_+ - n_-)\mu_B = -\frac{B\mu_B^2}{V}\int_0^\infty D(E)\frac{df}{dE}dE, \tag{1.56}$$

In the above equation $-\frac{df}{dE}$ can be replaced by $\delta(E - E_f)$ for low temperature condition. The density of states for the free electron gas,

$$\mathbf{D}(E) = \frac{V}{2\pi^2}(\frac{2m}{\hbar^2})^{3/2}(K_B T_F)^2, \tag{1.57}$$

Therefore,

$$\mathbf{M} = \frac{n\mu_B^2 B}{K_B T_F}, \tag{1.58}$$

This equation gives the expression for spin magnetization (Pauli magnetization) per unit volume. The magnetization arises due to the spin of unpaired electrons and is proportional to the difference of densities of electrons spin up and spin down ($n_{up} - n_{down}$). At low temperature this difference vanishes due to pauli blocking. Therfore the spin magnetization becomes zero for an ideal Fermi gas at ground state [ ]. However the spin magnetization may play a significant role for those electrons which are closed around to Fermi level. For spin quantum plasma the evoluation equation can be written as

$$\frac{d\mathbf{s}}{dt} = \frac{2\mu_B}{\hbar}(\mathbf{s} \times \mathbf{B}). \tag{1.59}$$

In MHD, spin inertia can be neglected under the the assumption $\omega < \omega_{ci} < \omega_{ce}$, which gives the spin equation of motion as $\mathbf{s} \times B = 0$, and the solution is

$$\mathbf{s} = -\frac{1}{2}\hbar\eta(\alpha_G)\hat{\mathbf{B}}, \tag{1.60}$$

where $\eta(\alpha_G)(= \tanh(\alpha_G))$ is the Langevin parameter due to the magnetization of a spin distribution in thermodynamic equilibrium and α_G is magnetic energy due to thermal energy ratio i.e., $\alpha_G = \frac{\mu_B \mathbf{B}_0}{GK_BT}$ as G is temperature degeneracy parameter and $\mu_B(|\frac{e\hbar}{2mc}|)$ is the Bohr magneton. The spinning effect is as one of the most important property of quantum plasma in the presence of high magnetic field. In quantum plasma, spin effect urges the continuation of different types of waves. By employing separate spin (spin-up and spin-down) evolution model, new longitudinal waves are obtained and these types of waves are propagated along perpendicular and parallel direction of external magnetic field.

1.9 Quantization of Magnetic Field and Landau Pressure

Landau quantization is a mechanism by which the quantization of energy levels occurred due to the presence of magnetic field. It is known that the moving electron (either spinning or orbital motion) produces a magnetic field which has a magnetic moment along the axis of gyration. The orientation of magnetic moments generates magnetism in medium. The spinning or orbital motion of electron is altered by applying the external magnetic field. In the presence of strong magnetic field the quantum plasma has two magnetic effects: one is the Pauli magnetism due to the spin effect of electron and the second is the Landau diamagnetism (Landau quantization) due to the quantization of the orbital motion of electron. Landau quantization effect is a pure quantum nature which has no classical analogy and can only be explained in quantum mechanics [ ]. By applying external magnetic field, the energy of the system in terms of quantized energy levels is enhanced due to diamagnetism. The fixed electron shows Pauli paramagnetism that higher than Landau diamagnetism while the free electron experiences Landau diamagnetism at $T_{Fe} > T$, although, the Pauli paramagnetism becomes too small. The electrons rotate in circular orbit in a plane perpendicular to the magnetic field in the direction of $z-axis$ $(H\hat{z})$. The motion of electron can be resolved into two parts : one is parallel to magnetic field inwhich the longitudinal energy is not quantized that can be given as $\varepsilon_{\shortparallel} = \frac{p_z^2}{2m}$ and the second, is quantized transverse energy which can be written as

$$\varepsilon_{\perp} = \varepsilon_N = (2N_L + 1 + s)\,\hbar\omega_{ce}, \tag{1.61}$$

here N_L denotes the Landau quantum number ($N_L = 0, 1, 2, ...$) and the cyclotron frequency is $\omega_{ce} = \frac{eH_0}{m_e}$, while s defines the spin orientation $\vec{s} = \frac{1}{2}\vec{\sigma}_s$ with $\sigma_s(\pm 1)$ is the operator. Relation (1.61) manifests that the energy spectrum of electrons consist of the lowest Landau level, $N_L = 0$, $s = -1$, while 2 degenerate levels give us polarization $s = +1$. Thus each value with $\ell \neq 0$ occurs twice, and that with $\ell = 0$ once. In terms of Bohr magneton, $\mu_B(= \frac{e\hbar}{2m_e})$, Equation (1.61) can be written as

$$\varepsilon_N \equiv (2N_L + 1 + s)\,\mu_B H_0. \tag{1.62}$$

Thus the net energy of electron in the presence of magnetic field ($\varepsilon = \varepsilon_{\perp} + \varepsilon_{\shortparallel}$) is

$$\varepsilon \equiv \varepsilon\left(N_L, k_z\right) = \left(2N_L + 1 + s\right)\mu_B H_0 + \frac{\hbar^2 k_z^2}{2m_e}. \qquad (1.63)$$

The first part of energy in right hand side of Equation (1.63) is discrete in nature corresponding to the quantization of the circular motion perpendicular to the magnetic field, it is to be noted that the spectrum discrete levels differentiated from each other by the energy gap, $2\mu_B H_0$. Accordingly, the second part of energy is not quantized and depends on k_z, corresponding to the motion of an electron along the magnetic field.

In order to empathize thermodynamic properties of electron gas embeded in quantized magnetic field, the pressure can be written in the following way [ ]. As the thermodynamic function (also called grand thermodynamic potential) in explicit form can be expressed in the form $\Omega_e = \Omega_e(T, V, \mu, H)$. Therefore pressure is defined in terms of grand thermodynamic potential as

$$P = -\left[\frac{\partial \Omega_e}{\partial V}\right]_{\mu,H,T}. \qquad (1.64)$$

For F.D distribution of electron we have

$$\Omega_e = \sum_k \Omega_k = -K_B T_e \sum_{N_L} \ln\left(1 + e^{(\mu-\varepsilon)/K_B T_e}\right), \qquad (1.65)$$

where ε is the energy of non-interacting particles i.e., Fermions or bosons in quantum state N_L at temperature T_e and μ is the chemical potential that changes in the limit from $-\infty$ to $+\infty$ in F.D distribution function. For electrons in e-i plasma the thermodynamic function can be written as

$$\Omega_e = -2K_B T \sum_{N_L} \ln\left(1 + e^{\frac{\mu-\varepsilon}{K_B T_e}}\right). \qquad (1.66)$$

Here $k \rightarrow (N_L, k_y, k_z)$ and $\varepsilon\left(N_L, k_z\right)$ is given Equation (1.63) while the factor 2 takes into account degeneracy with respect to spin. Let the electron gas is occupied in volume $V(= L_1 L_2 L_3)$, after further simplification we get the following relation

$$\Omega_e = -\frac{4K_BT_eV}{(2\pi R)^2}\sum_{N_L}\int\limits_{\varepsilon_{N_L}}^{\infty}\frac{dk_z}{d\varepsilon}\ln\left(1+e^{\dfrac{\mu-\varepsilon}{K_BT_e}}\right)d\varepsilon, \tag{1.67}$$

where $\varepsilon_{N_L} = (N_L + 1/2)\,\hbar\omega_{ce}$ is the root of Equation (1.63) at $k_z = 0$. Integrate up this equation w.r.t ε_{N_L} one can obtain

$$I = \frac{1}{K_BT_e}\int\limits_{\varepsilon_N}^{\infty} k_z f(\varepsilon)\, d\varepsilon,$$

here $k_z(\varepsilon, N_L) = \frac{\sqrt{2m_e}}{\hbar}(\varepsilon-\varepsilon_{N_L})^{1/2}$ and $f(\varepsilon) = \left(1+e^{\dfrac{\varepsilon-\mu}{K_BT_e}}\right)^{-1}$ is the F.D distribution. It then modifies Equation (1.67) as

$$\Omega_e = -\frac{4V}{(2\pi R)^2}\sum_{N_L}\int\limits_{\varepsilon_{N_L}}^{\infty} k_z(\varepsilon, N_L)\, f(\varepsilon)\, d\varepsilon. \tag{1.68}$$

In the above equation $R(=\sqrt{\frac{\hbar}{eH}})$ is the magnetic-length. Now the pressure of the electron gas can be written in the form as

$$\begin{aligned} P_e &= \frac{4}{(2\pi R)^2}\sum_{N_L}\int\limits_{\varepsilon_{N_L}}^{\infty} k_z(\varepsilon, N_L)\, f(\varepsilon_N)\, d\varepsilon \\ &= \frac{4\sqrt{2m_e}}{(2\pi R)^2\hbar}\sum_{N_L}\int\limits_{\varepsilon_{N_L}}^{\infty}(\varepsilon-\varepsilon_{N_L})^{1/2} f(\varepsilon)\, d\varepsilon. \end{aligned} \tag{1.69}$$

Integration of the equation can be modified in the following form as

$$\begin{aligned} I_1 &= \int\limits_{\varepsilon_{N_L}}^{\infty}(\varepsilon-\varepsilon_{N_L})^{1/2} f(\varepsilon)\, d\varepsilon \\ &= \frac{2}{3}\int\limits_{\varepsilon_{N_L}}^{\infty}(\varepsilon-\varepsilon_{N_L})^{3/2}\left(-\frac{\partial f(\varepsilon)}{\partial\varepsilon}\right)d\varepsilon, \end{aligned}$$

substituting the above relation in Equation (1.69) the pressure for arbitrary degenerate plasma becomes as

$$P_e = \frac{8\sqrt{2m_e}}{3\left(2\pi R\right)^2 \hbar} \sum_{N_L} \int_{N_L}^{\infty} \left(\varepsilon - \varepsilon_{N_L}\right)^{3/2} \left(-\frac{\partial f}{\partial \varepsilon}\right) d\varepsilon. \tag{1.70}$$

1.9.1 Pressure for Non Degenerate Plasma

Let's solve the Equation (1.70) for non degenerate electron gas and that can be written as

$$I_2 = \int_{\varepsilon_{N_L}}^{\infty} \left(\varepsilon - \varepsilon_{N_L}\right)^{3/2} \left(-\frac{\partial f\left(\varepsilon\right)}{\partial \varepsilon}\right) d\varepsilon, \tag{1.71}$$

in this case the distribution $f\left(\varepsilon, \mu\right) = \left(1 + e^{\frac{\mu - \varepsilon}{K_B T_e}}\right)$ for large negative value of $\frac{\mu}{K_B T_e}$ will become as $f\left(\varepsilon\right) = e^{\frac{\mu - \varepsilon}{K_B T_e}}$ thus $-\frac{\partial f\left(\varepsilon\right)}{\partial \varepsilon} = \frac{e^{\frac{\mu}{K_B T_e}}}{K_B T_e} e^{\frac{-\varepsilon}{K_B T_e}}$. It makes Equation (1.71) as

$$I_2 = \frac{e^{\frac{\mu}{K_B T_e}}}{K_B T_e} \int_{\varepsilon_{N_L}}^{\infty} \left(\varepsilon - \varepsilon_{N_L}\right)^{3/2} e^{\frac{-\varepsilon}{K_B T_e}} d\varepsilon. \tag{1.72}$$

let $\varepsilon' = \varepsilon - \varepsilon_{N_L}$ and for $\varepsilon = \varepsilon_{N_L}$ then $\varepsilon' = 0$ and for $\varepsilon = \infty$ it implies that $\varepsilon' = \infty$. Using these values in Equation (1.72) We get

$$\begin{aligned}
I_2 &= \frac{e^{\frac{\mu - \varepsilon_{N_L}}{K_B T_e}}}{K_B T_e} \int_0^{\infty} \left(\varepsilon'\right)^{\frac{3}{2}} e^{\frac{-\varepsilon'}{K_B T_e}} d\,\varepsilon' \\
&= \frac{e^{\frac{\mu - \varepsilon_{N_L}}{K_B T_e}}}{K_B T_e} \left[0 - \int_0^{\infty} \frac{d}{d\varepsilon'} (\varepsilon')^{\frac{3}{2}} \left(\int e^{\frac{-\varepsilon'}{K_B T_e}} d\,\varepsilon' \right) d\,\varepsilon' \right] \\
&= \frac{e^{\frac{\mu - \varepsilon_{N_L}}{K_B T_e}}}{K_B T_e} \left[\frac{3}{2} K_B T_e \int_0^{\infty} (\varepsilon')^{\frac{1}{2}} e^{\frac{-\varepsilon'}{K_B T_e}} d\,\varepsilon' \right] \\
&= \frac{3}{2} e_e^{\frac{\mu - \varepsilon_{N_L}}{K_B T_e}} \int_0^{\infty} (\varepsilon')^{\frac{1}{2}} e^{\frac{-\varepsilon'}{K_B T_e}} d\,\varepsilon'.
\end{aligned} \tag{1.73}$$

Again subtitute $x = \frac{\varepsilon'}{K_B T_e}$ that implies $\varepsilon' = x K_B T_e$ and $d\varepsilon' = K_B T_e dx$

$$I_3 = \int_0^\infty (\varepsilon')^{\frac{1}{2}} e^{-\frac{\varepsilon'}{k_0 T}} d\,\varepsilon' = (K_B T_e)^{\frac{3}{2}} \int_0^\infty x^{1/2} e^{-x} dx. \tag{1.74}$$

Now using the the Gamma function definition $\Gamma(z+1) = \int_0^\infty x^z e^{-x} dx$ and it shows in our case that for $z = 1/2$ we have $\Gamma(1/2+1)$ that is equal to $\Gamma\left(\frac{3}{2}\right) = \int_0^\infty x^{1/2} e^{-x} dx = \frac{\sqrt{\pi}}{2}$. Using these manipulations we get

$$I_3 = (K_B T_e)^{\frac{3}{2}} \int_0^\infty x^{1/2} e^{-x} dx = \frac{\sqrt{\pi}}{2} (K_B T_e)^{\frac{3}{2}}, \tag{1.75}$$

and hence

$$I_2 = \frac{3\sqrt{\pi}(K_B T_e)^{\frac{3}{2}}}{4} e^{\frac{\mu - \varepsilon_{N_L}}{K_B T_e}}, \tag{1.76}$$

which makes Equation (1.70) as

$$P_e = \frac{8(2m_e)^{\frac{1}{2}}}{3\hbar(2\pi R)^2} \frac{3\sqrt{\pi}(K_B T_e)^{\frac{3}{2}} e^{\frac{\mu}{K_B T_e}}}{4} \sum_{N_L} e^{-\frac{\varepsilon_{N_L}}{K_B T_e}}. \tag{1.77}$$

by putting the value of ε_{N_L} from Equation (1.62) we get

$$P_e = \frac{8(2m_e)^{\frac{1}{2}}}{3\hbar(2\pi R)^2} \frac{3\sqrt{\pi}(K_B T_e)^{\frac{3}{2}} e^{\frac{\mu}{K_B T_e}}}{4} \sum_{N_L} e^{-(2N_L+1)\mu_B H_0 / K_B T_e}. \tag{1.78}$$

The summation w.r.t N_L in Equation (1.78) gives

$$\sum_{N_L=0} e^{\frac{-(2N_L+1)\mu_B H}{k_0 T}} = \left[2 \sin h(\mu_B H_0 / K_B T_e)\right]^{-1}, \tag{1.79}$$

by putting in Equation (1.78) We have

$$P_e = \frac{(2\pi m_e)^{\frac{1}{2}}(K_BT_e)^{\frac{3}{2}}}{\hbar(2\pi R)^2}\frac{e^{\frac{\mu}{K_BT_e}}}{\sin h(\mu_B H_0/K_BT_e)}. \tag{1.80}$$

The expression $e^{\frac{\mu}{K_BT_e}}(=\xi_e)$ for non-degenerate gas, using Equation (8.42) of Ref. [ ] thats holds true for any value of a strong field, including a quantizing magnetic field has the value

$$\xi_e = e^{\frac{\mu}{K_BT_e}} = \frac{4n_e(\pi)^{\frac{3}{2}}\hbar^3}{(2m_eK_BT_e)^{\frac{3}{2}}}\frac{\sin h(\mu_B H_0/K_BT_e)}{(\mu_B H_0/K_BT_e)}.$$

It then makes the expression Equation (1.80) as

$$P_e = n_e K_B T_e. \tag{1.81}$$

It is observed that in case of non-degenerate and classical plasma pressure is independent of magnetic field.

1.9.2 Pressure for Degenerate Plasma

The problem of distribution of an ideal gas (Fermions or Bosons) can be solved by the proposed method of Landau [ ]. At $T \to 0$, the F.D distribution becomes Heaviside step function. In this limit the chemical potential is equal to the Fermi energy, and all states of energy below the Fermi energy is occupied, while all states above are empty and such a state of plasma is called complete degenerate plasma. Following this argument, $\left(-\frac{\partial f}{\partial \varepsilon}\right)$ tends to a step-function with argument $\varepsilon - \mu_F$, such that

$$\left(-\frac{\partial f}{\partial \varepsilon}\right) = \delta\left(\varepsilon - \xi_F\right)$$

where δ is the Dirac-Delta function which is one for $\varepsilon << \xi_F$ (degenerate state of plasma) and zero for $\varepsilon >> \xi_F$. Using this idea Equation (1.70) becomes

$$P_e = \frac{8\sqrt{2m_e}}{3\hbar\left(2\pi R\right)^2}\sum_{N_L}\int_{\varepsilon_{N_L}}^{\infty}\left(\varepsilon - \varepsilon_{N_L}\right)^{3/2}\delta\left(\varepsilon - \xi_F\right)d\varepsilon. \tag{1.82}$$

Using the Dirac-Delta function property of $\int_0^\infty f(x)\delta(x-a)\,dx = f(a)$. Therefore,

$$\int_{\varepsilon_N}^{\infty} (\varepsilon - \varepsilon_{N_L})^{3/2}\,\delta(\varepsilon - \mu_F)\,d\varepsilon = (\varepsilon - \varepsilon_{N_L})^{3/2},$$

where $\varepsilon_{N_L} = (2N_L + 1)\,\mu_B H_0$ and therefore the integral under the above condition of degenerate state of plasma $\varepsilon << \xi_F$ becomes as

$$\int_{\varepsilon_N}^{\infty} (\varepsilon - \varepsilon_{N_L})^{3/2}\,\delta(\varepsilon - \mu_F)\,d\varepsilon = (\xi_F - (2N_L + 1)\,\mu_B H_0)^{3/2}.$$

Equation (1.82) in simplified form is as under

$$P_e = \frac{8(2m_e)^{\frac{1}{2}}}{3\hbar(2\pi R)^2}[\sum_{N_L=0}^{N_0} \{\xi_F - (2N_L + 1)\,\mu_B H_0\}]^{3/2} \tag{1.83}$$

Here $N_0 = \dfrac{\mu_F - \mu_B H_0}{2\mu_B H}$. Using the quantum limit condition (when all electrons are found at the zero Landau level i.e., $\mu_B H_0 < \mu_F < 3\mu_B H_0$), the Equation (1.83) can be written as $P_e = \frac{8(2m_e)^{\frac{1}{2}}}{3\hbar(2\pi R)^2}\,[\xi_F - \mu_B\mu_0 H_0]^{3/2}$,

The concentration of electrons is expressed in the following matheimatical relation given in Ref. [ ]

$$n = \frac{4V}{(2\pi R)^2}\sum_{N_L=0}^{N_0}\int_{\varepsilon_N}^{\infty}\left(-\frac{\partial f}{\partial \varepsilon}\right) k_z(\varepsilon,_{N_L})d\varepsilon,$$

From Equation (1.63)

$$k_z(\varepsilon,_{N_L}) = \frac{\sqrt{2m}}{\hbar}\,(\varepsilon - \varepsilon_{N_L})^{1/2},$$

Therefore we have

$$n = \frac{4(2m_e)^{\frac{1}{2}}}{\hbar(2\pi R)^2}\sum_{N_L=0}^{N_0}\int_{\varepsilon_N}^{\infty}(\varepsilon - \varepsilon_{N_L})^{1/2}\left(-\frac{\partial f(\varepsilon)}{\partial \varepsilon}\right)d\varepsilon,$$

Further after the derivation one can get

$$n = \frac{(2m_e)^{\frac{3}{2}}}{\pi^2\hbar^3}\mu_B H_0 \sum_{N_L=0}^{N_0}[\xi_F - (2N_L + 1)\,\mu_B H_0]^{1/2},$$

which implies that

$$[\sum_{N_L=0}^{N_0}\{\xi_F-(2N_L+1)\,\mu_B H_0\}]^{3/2}=\frac{n^3}{\left(\frac{(2m_e)^{\frac{3}{2}}}{\pi^2\hbar^3}\right)^3}\frac{1}{\mu_B^3 H_0^3} \tag{1.84}$$

Using Equation (1.84) in Equation (1.83) we get

$$P_e=\frac{8(2m_e)^{\frac{1}{2}}}{3\hbar(2\pi R)^2}\frac{n^3}{\left(\frac{(2m_e)^{\frac{3}{2}}}{\pi^2\hbar^3}\right)^3}\frac{1}{\mu_B^3 H_0^3}$$

By using the values of $R=\left(\frac{\hbar}{e\mu_0 H_0}\right)^{\frac{1}{2}}$ and $\mu_B(=\frac{e\hbar}{2mc})$ we arrive at

$$P_e=\left(\frac{\pi^4\hbar^4}{3e^2 m_e}\right)\frac{n_e^3}{H_0^2}\approx\frac{n_e^3}{H_0^2}. \tag{1.85}$$

The above equation describes that the pressure of degenerate electron gas in the quantum limit has strong dependency on both density as well as magnetic field. This equation is valid for the degenerate plasma in the presence of quantized magnetic field.

1.10 Thesis Layout

The thesis is based on theoretical analysis of quantum MHD waves with effect of spin magnetization and temperature degeneracy and is divided into four chapters as follow.

In Chapter (1), we are discussing the basics of plasma physics, multi-component plasma, application of quantum plasma, physics of MHD waves, quantum MHD model, F.D distribution, arbitrary temperature degeneracy, spin Effect, Landau quantization and Landau Pressure.

Chapter (2), is about our first research problem related to the study of MHD waves in quantized magnetic field. This chapter consists of five sections. Section (I) gives a literature review concerned to the problem. In Section (II) dispersion relation with effect of quantizing magnetic field is derived and further different modes of MHD waves are numerically and graphically analyzed. Similarly, we discuss dispersion relation with effect of both quantizing field and spin magnetization for different MHD wave modes in Section (III). Section (IV) is delegated to results and discussion . The conclusion of the whole chapter is then given in Section (V).

In Chapter (3), we study our next research problem related to the linear properties of MHD waves in nearly non degenerate and nearly degenerate quantum plasma composed of three species (electrons, positrons and ions) in the presence of spin- 1/2 effect. This chapter consists of four sections. Section (I) gives a literature review concerned to the problem. In Section (II) a generalized dispersion relation is derived with the help of equation of motion and continuity equation along with Maxwell equations using MHD model and MHD wave propagation modes are analyzed. Results and discussion are presented in Section (III). The conclusion is then described in Section (IV).

In Chapter (4), we describe the linear properties of magnetosonic waves in ion trapped semiconductor electron-hole-ion (e-h-i) plasma with effects of density correlation via Bohm potential, spin magnetization, exchange correlation potential and relativistic degeneracy. This chapter consists of five sections. Section (I) is related to literature review. A modified and generalized dispersion relation for oblique propagation of waves is derived by using the quantum MHD model in section (II). Propagation modes are described in section (III). Results and discussion for in case of some typical parametric values of Cd^{+} ion trapped in Gallium Arsenide (GaAs) and Gallium Nitride (GaN) semiconductor compounds at room temperature are then given in section (IV). The conclusion is then given in section (V).

Chapter 2

MHD Waves with Quantized Magnetic Field

2.1 Introduction

Degenerate plasmas have gained a lot of interest owing to their important potential applications in modern technology such as metallic and semiconductor nanostructures (e.g., quantum dots, quantum wells, quantum wires, spintronics, quantum free electron lasers, nanoplasmonic devices, nanotubes, metallic nanoparticles, thin metal films, metal clusters, etc.). Likewise, quantum plasmas commonly exist in dense astrophysical systems particularly, WD, NS and superdense interior core of Jupiter [,]. It is to be noted that the quantum plasma may play an important role when a hydrogen pellet is compressed many times to that of its solid density state. In quantum plasmas, the spin effect as well as quantization of the orbital motion of electrons are very significant however they may not survive on classical scales. Quantum mechanical effects appear when the average interparticle distance (or Wigner-Seitz radius) $a = (3/4\pi n)^{1/3}$ is almost equal to or smaller than the thermal de Broglie wavelength i.e., $n\lambda_B^3 \geq 1$.

It is well-established that Fermion gas under a constant external magnetic field gives rise to two phenomena. First, is known as pauli paramagnetism and the second is called Landau diamagnetism. The former phenomenon is occured due to the spin of electrons, whereas in the latter case the orbital motion is quantized. The quantization (discreteness) of the orbital motion of electron gas was explained by Landau [ ] under the external magnetic field while experimenting on metals. The electron gas in the presence of a constant external magnetic field performs dual motions (i.e., helical motions) simultaneously; One is a perpendicular motion which is purely rotational with radius $r = \frac{v_\perp}{\omega_{ce}}$ (here $v_\perp$ is perpendicular component of velocity and ω_{ce} is the cyclotron frequency) and the other is the parallel (along the direction of external magnetic field) which is linear. The former motion is purely quantum in nature, whereas latter motion is purely classical in nature so that $k_B T \gg \hbar\omega_{ce}$ [,]. Lundin et al. investigated the high frequency EM waves in the presence of extreme magnetic fields $H \simeq 10^{14} - 10^{15}G$ (in the vicinity of magnetars and pulsars) [ ]. The equation of state (EOS) has been calculated for laser-plasma interaction with the effect of quantizing magnetic field in laboratory level by Eliezer et al. [ ]. They further stressed to study the research on laser-plasma interaction now becomes a domain of physics equivalent to astrophysical plasmas such as WD and NS. Later on, Tsintsadze and his co-authors in their number of papers have studied the relativistic thermodynamical properties of a Fermi gas in a strong magnetic field [ ]. It was observed that the dispersion relation of longitudinal electric waves is strongly dependent on the magnetic field strength. The impact of such a strong magnetic field on the wave propagation and thermodynamical properties of a medium is a vital issue in supernovae and NS. The prestellar of the evolution of the universe and the convective zone of the sun as well as in laboratory plasma (e.g., the laser-matter interaction) are also characterized by strong magnetic fields. On the basis of measured data of astrophysical system, the magnetic field on this surface NS is reported, as $10^{11} - 10^{13}G$ and while the field of interior core of NS can reach to $10^{15}G$ or even may be higher values [, ,]. However, the rotational movement of stars may cause to increase this field by a factor $10^3 \sim 10^4$ shown by Bisnovati-Kogan [ ]. Here in this situation the characteristic energy of electrons on a Landau level reaches the non-relativistic limit of the electrons chemical potential $\mu = E_F = \frac{e\hbar H_0}{2m_e c}$, where $H_0 = H_s\left(\frac{v_F^2}{c^2}\right)$ i.e., $H_s(= \frac{m_e^2 c^3}{e\hbar} \simeq 4.4 \times 10^{13}G)$ is Schwinger's magnetic field and $v_F(= \sqrt{\frac{K_B T_F}{m_e}})$ is the speed of electrons at Fermi surface with $T_F = \frac{(3n\pi^2)^{2/3}\hbar^2}{2m_e K_B}$ (Fermi temperature of electron), m_e(mass of electron), and $\hbar = \frac{h}{2\pi}$(reduced Plank's constant).

Alternatively, in terms of number density of plasma (n) along with magnetic field (H_0), Fermi temperature may also be defined as $T_F = \frac{c^2\pi^4\hbar^4}{2m_e e^2}(\frac{n}{H_0})^2$. Therefore, in the presence of extreme magnetic field the wave dynamics and thermodynamical properties would be quite different in quantum plasma governed by the quantum effects. In classical plasma physics, fluid (or hydrodynamic) models are often derived by taking moments of the appropriate kinetic equation (e.g., Vlasov's equation) in velocity space. In the magnetohydrodynamic (MHD) description of the plasma, the latter is considered as a single charged fluid in which the plasma particles (viz, electrons and ions) move with the same velocity. The quantum hydrodynamic model (QHD) and quantum MHD model are the extensions of the classical hydrodynamic model. The QHD model for charged particle systems is extended to the cases of nonzero magnetic fields. These two models focus on some microscopic variables of quantum plasmas such as charge, momentum and energy transport. Therefore, these two models are widely used to study the propagation properties of linear and nonlinear waves in quantum plasmas.

Usually, in quantum plasma, high density and low temperature are considered as typical plasma environment in which quantum effects arise. Criteria of the electron pumping in electron-hole (e-h) quantum plasma has been discussed by Afify [ ]. The linear and nonlinear propagation of kinetic Alfven waves (KAWs) in quantum plasma have been studied with the inclusion of spin magnetization effects. Using the numerical values of physical parameters, it is found that the dynamics of linear and nonlinear structures are significantly modified due to the change in spin magnetization effects [ ]. The nonlinear propagation characteristics of inertial Alfven waves (IAWs) are investigated by taking account into spin magnetization effects along with exchange correlation effect [ ]. The features of shock waves in Landau quantized plasma are also analyzed with higher (lower) electron density and magnetic field stength by Deka and dev [ ]. Similarly, different spectra of spin magnetosonic waves are studied in a nonrelativistic quantum degenerate plasma, accounting for spin-up and spin-down electrons (assumed these electrons as two different fluids). It is found that the energy of the degenerate electrons in the perpendicular direction becomes quantized into Landau levels due to very strong magnetic field. It is also shown that the spin polarization may cause to give birth to a new wave [ ]. Zedan et. al., investigated theoretically the multi-dimensional instability of ion acoustic solitary waves in a magnetized, degenerate, collisionless multi-ion plasma with trapping distributed electrons under the influence of quantizing magnetic field and polarization effect. The stability of oblique propagating wave with a polarization force effect is studied. Moreover, the growth rate of the wave instability has also been calculated [ ]. The degenerate characteristics of the weakly ionized NS are explained in the presence of quantized magnetic field. The quantized magnetic pressure is taken along with Fermi pressure and therefore the sound wave in the dispersive ionized neutrino stars is described. It has been shown that solitary wave may exist for a NS (weakly ionized, degenerate and magnetized), where the density of neutrons is much larger than the unperturbed density of charged electron and ion species [ ].

The spin effects on linearly EM wave propagation in arbitrary direction have been examined in dense magnetized plasmas by Hu et al., [ ], where it was shown that spin phenomena to be more significant for low frequency modes as compare to high frequency modes. Later on, the magnetoacoustic waves were studied in degenerate e-p-i plasmas with effects of arbitrary temperature and spin magnetization. Here it has been observed that the temperature degeneracy and spin magnetization modify the dispersion modes of propagation waves [ ]. Similarly, the waves and instabilities in relativistic anisotropic MHD plasma were studied and the dispersion relation was derived for two modes of propagation, i.e., modified magnetosonic mode and shear Alfven mode [ ]. The low frequency quantum MHD wave has been analyzed in dense degenerate plasmas [ ] to account for Bohm potential (along with spin effect) and derived dispersion relation for perpendicular, oblique and parallel propagation modes.

In this Chapter, we investigate the radical changes in the propagation of MHD waves when the effects of pressure caused by quantizing magnetic field and pauli paramagnetism are incorporated. Here the main objective of our work is to observe the quantum corrections (Landau quantization, Spin effect, and Bohm potential) on the dispersion modes of the MHD waves. The layout of the manuscript is given as follows: Dispersion relation including the effect of quantizing magnetic field is derived in Sec.

II. MHD waves with effect of combined orbital quantizition and spin magnetization is given in Sec. III. Results and discussion are described in Sec. IV. The conclusion is then given in the final section.

2.2 Dispersion relation with quantizing magnetic field effect

Let's consider a two fluid plasma model which is composed of electron and ion species. The amplitudes of oscillations are assumed to be very small so that by using linearized equations one can easily solve the system problem. At equilibrium the plasma is considered to have null zeroth order velocities of electrons and ions i.e., $\mathbf{v}_{e0} = \mathbf{v}_{i0} = 0$. Therefore for both species, at first order, the equations of continuity can be given as:

$$\frac{\partial n_e}{\partial t} + n_0(\nabla \cdot \mathbf{v}_e) = 0, \tag{2.1}$$

and

$$\frac{\partial n_i}{\partial t} + n_0(\nabla \cdot \mathbf{v}_i) = 0, \tag{2.2}$$

where n_e and n_i are the first order of linearly perturbed number densities of electrons and ions, respectively. Here n_0 is taken as unperturbed number density of particles (electrons/ions). According to quasi- neutrality condition, we have $n_{e0} \approx n_{i0} \approx n_0$. Similarly, $\mathbf{v}_e$ (and $\mathbf{v}_i$) is the electron fluid (and ion fluid) perturbed velocity.

In quantum fluid model, the DFT is a general mathematical tool that is used to study the electron exchange correlation potentials, where exchange potential comes up from the Pauli exclusion principle, while the correlation potential happens when the energy is lowered due to the wave function described by the Hartree-Fock hypothesis. Brey et al. [ ] introduced the exchange-correlation potential in the field of solid state physics. Later, in plasma physics, it became notable by incorporating the term of the exchange-correlation potential by Crouseilles et al. [ ] in the quantum fluid model and compared their results with DFT for thin films. This effect represents a short range electric potential which only depends on the number density of the Fermi particles. It is observed that on a large time scale, the exchange-correlation potential effect becomes weak as compared to other quantum corrections [, ,]. That's why we have ignored this effect in our current problem.

Let us suppose that an e-i plasma is immersed in the external uniform magnetic field in the direction of z-direction **H** (0, 0, H). The momentum equations for both fluids of electrons and ions are given by

$$\frac{\partial \mathbf{v}_e}{\partial t} = -\frac{e\mathbf{E}}{m_e} - \frac{e}{m_e}(\mathbf{v}_e \times \mathbf{H}) - \frac{\boldsymbol{\nabla} P_e}{m_e n_e} + \frac{\hbar^2}{2m_e^2}\boldsymbol{\nabla}\left[\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right], \tag{2.3}$$

and

$$\frac{\partial \mathbf{v}_i}{\partial t} = \frac{e\mathbf{E}}{m_i} + \frac{e}{m_i}(\mathbf{v}_i \times \mathbf{H}) - \frac{\boldsymbol{\nabla} P_i}{m_i n_i} + \frac{\hbar^2}{2m_i^2}\boldsymbol{\nabla}\left[\frac{\nabla^2\sqrt{n_i}}{\sqrt{n_i}}\right], \tag{2.4}$$

where m_e (m_i) is the mass of electrons (ions) and e is the charge of electrons. In Equation (2.4), P_i is perturbed arbitrary pressure given by $P_i = n_i K_B T_i$, which in turn leads to $\boldsymbol{\nabla} P_i = K_B T_i \boldsymbol{\nabla} n_i$ with T_i (ion fluid temperature). The last term is the Bohm potential at right hand side of Equation (2.3) (and in Equation (2.4)) which arises due to density fluctuations. Similarly the parameter P_e in Equation (2.3) is the Landau diamagnetic pressure (detailed explanation is given in chapter one), can be expressed, as

$$P_e = \left(\frac{n_e^3}{H_0^2}\right)\frac{\pi^4\hbar^4}{3m_e e^2} \approx \frac{n_e^3}{H_0^2}. \tag{2.5}$$

This describes that the pressure has dependency on both number density as well as magnetic field. By combining Equations (2.3) and (2.4), we get

$$\rho_m\frac{\partial \mathbf{v}}{\partial t} = \mathbf{J} \times \mathbf{H} - \boldsymbol{\nabla} P + \frac{\hbar^2\rho_m}{2m_i m_e}\boldsymbol{\nabla}\left[\frac{\nabla^2\sqrt{\rho_m}}{\sqrt{\rho_m}}\right]. \tag{2.6}$$

Equations (2.1) and (2.2) can be combined to obtain

$$\frac{\partial \rho_m}{\partial t} + \rho_m(\boldsymbol{\nabla} \cdot \mathbf{v}) = 0, \tag{2.7}$$

In Equations (2.6) and (2.7), we used $\mathbf{v}(\simeq \dfrac{m_i\mathbf{v}_i + m_e\mathbf{v}_e}{m_i})$ which shows the global fluid velocity, and $\rho_m(= n_i m_i + n_e m_e \simeq n m_i)$ is the plasma centre of mass density with $n_e \simeq n_i = n$. Here P is the global pressure and is defined as $P = P_i + P_e$, where $P_e(\approx \frac{n^3}{H_0^2})$ is the pressure for degenerate electrons. The system is closed with an EOS for the pressure, therefore in terms of mass density, we can write $\nabla P = C_{ie}^2 \nabla \rho_m$, where $C_{ie} = \sqrt{\frac{dP}{d\rho_m}} = \sqrt{V_i^2 + V_e^2}$ is the sound velocity duly modified by Landau diamagnetic levels with $V_i = \left(\frac{K_B T_i}{m_i}\right)^{1/2}$ and $V_e = \left(\dfrac{\pi^4 \hbar^4 n_0^2}{m_e m_i e^2 H_0^2}\right)^{1/2}$, thermal speed of ions and modified Fermi speed of electrons respectively. The term $\mathbf{J}$ is the current density and is given by $\mathbf{J} = (n_i V_i - n_e V_e)\, e = nVe$. Further $\mathbf{J}$ can be deduced by using the Maxwell's equation

$$\mathbf{J} = \frac{1}{\mu_0}(\nabla \times \mathbf{H}), \tag{2.8}$$

which gives as Ampere's law. Using other Maxwell's equation, i.e., Faraday's law is

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{H}}{\partial t}, \tag{2.9}$$

where μ_o is the permeability of free space and the Ohm's law is

$$\mathbf{E} = -\mathbf{v} \times \mathbf{H}. \tag{2.10}$$

The result is obtained after simplification, as

$$\mathbf{J} \times \mathbf{H} = \frac{1}{\mu_0}(\nabla \times \mathbf{H}) \times \mathbf{H}. \tag{2.11}$$

Now substituting the above result in Equation (2.6), we have the following relation

$$\rho_m \frac{\partial \mathbf{v}}{\partial t} = \frac{1}{\mu_0}(\nabla \times \mathbf{H}) \times \mathbf{H} - C_{ie}^2 \nabla \rho_m + \frac{\rho_m \hbar^2}{2 m_i m_e} \nabla \left[\frac{\nabla^2 \sqrt{\rho_m}}{\sqrt{\rho_m}}\right]. \tag{2.12}$$

In order to write the above equations in linearized forms, lets suppose the small amplitude variations in each physical quantities with their equilibrium values i.e., $\rho_m(r,\ t) = \rho_{m0} + \rho_{m1}(r,\ t)$, $\mathbf{H}(r,\ t) = \mathbf{H}_0 + \mathbf{H}_1(r,\ t)$ and $\mathbf{v}(r,\ t) = \mathbf{v}_1(r,\ t)$. Therefore, Equation (2.7) and Equation (2.12) will become

$$\frac{\partial \rho_{m1}}{\partial t} + \rho_{m0}(\nabla \cdot \mathbf{v}_1) = 0, \tag{2.13}$$

and

$$\rho_{m0}\frac{\partial \mathbf{v}_1}{\partial t} = \frac{1}{\mu_0}\mathbf{H}_0 \times (\nabla \times \mathbf{H}_1) - C_{ie}^2 \nabla \rho_{m1} + \frac{\rho_{m0}\hbar^2}{2m_i m_e}\nabla\left[\nabla^2\sqrt{\rho_{m1}}\right]. \tag{2.14}$$

Differentiating Equation (2.14) w.r.t time t and after rearranging, we have

$$\frac{\partial^2 \mathbf{v}_1}{\partial t^2} - C_{ie}^2 \nabla(\nabla \cdot \mathbf{v}_1) + \mathbf{V}_A \times [\nabla \times \{\nabla \times (\mathbf{v}_1 \times \mathbf{V}_A)\}] - \frac{\hbar^2}{4m_i m_e}\frac{\partial}{\partial t}\left[\nabla\left(\nabla^2\sqrt{\rho_{m1}}\right)\right] = 0, \tag{2.15}$$

where $\mathbf{V}_A \left(= \frac{\mathbf{H}_0}{\sqrt{\mu_0 \rho_m}}\right)$ is the Alfven velocity and $\mathbf{v}_1(r,\ t) \propto e^{(i\mathbf{k}\cdot\mathbf{X} - i\omega t)}$ with ω is the plasma mode frequency. Using $\frac{\partial}{\partial t} \rightarrow -i\omega$, $\nabla \rightarrow i\mathbf{k}$ and $\rho_{m1} = \frac{\rho_{m0}(\mathbf{k}.\mathbf{v}_1)}{\omega}$ in Equation (2.15). By simplification the following relation is obtained (in appendix A the detail derivation is given)

$$\omega^2 \mathbf{v}_1 - \mathbf{k}\left(C_{ie}^2 + V_A^2\right)(\mathbf{k}\cdot\mathbf{v}_1) + \left[(\mathbf{V}_A\cdot\mathbf{v}_1)\mathbf{k} - (\mathbf{V}_A\cdot\mathbf{k})\mathbf{v}_1 + (\mathbf{k}\cdot\mathbf{v}_1)\mathbf{V}_A\right](\mathbf{k}\cdot\mathbf{V}_A) - \mathbf{k}\frac{\hbar^2 k^2}{4m_i m_e}(\mathbf{k}\cdot\mathbf{v}_1) = 0. \tag{2.16}$$

Consider the following dimensionless variables

$$\omega \rightarrow \frac{\omega}{\omega_{ci}},\ n \rightarrow \frac{n}{n_0},\ \mathbf{k} \rightarrow \mathbf{k}\frac{\mathbf{v}_A}{\omega_{ci}},\ \mathbf{v}_1 \rightarrow \frac{\mathbf{v}_1}{\mathbf{v}_A}$$

The dispersion relation (2.16) can be written in dimensionless form as

$$\omega^2 \mathbf{v}_1 - \mathbf{k}(1 + \beta_L)(\mathbf{k}\cdot\mathbf{v}_1) + \left[(\hat{\mathbf{z}}\cdot\mathbf{v}_1)\mathbf{k} - (\hat{\mathbf{z}}\cdot\mathbf{k})\mathbf{v}_1 + (\mathbf{k}\cdot\mathbf{v}_1)\hat{\mathbf{z}}\right](\mathbf{k}\cdot\hat{\mathbf{z}}) - \mathbf{k}(\mathbf{k}\cdot\mathbf{v}_1)H_e^2 = 0. \tag{2.17}$$

In the above expression, $\beta_L = \frac{C_M^2}{V_A^2} = \frac{n_0^3}{H_0^4}\left(\frac{\mu_0\pi^4\hbar^4}{m_e e^2}\right)$ is the Landau diamagnetic pressure modified plasma beta, while $H_e(=\frac{\hbar\omega_{ci}}{V_A^2\sqrt{m_e m_i}})$ is a dimensionless parameter that arises due to the collective electron tunnelling effect through the so-called Bohm potential, with $\omega_{ci} = \frac{eH_0}{m_i}$ being the ion cyclotron frequency. Equation (2.17) gives the modified dispersion relation for e-i plasma with quantum corrections including Bohm potential and modified quantum sound velocity due to Landau diamagnetic effect through the correction of quantizing magnetic field. By ignoring the quantum corrections, one can easily obtain the classical standard definition of dispersion. The above modified dispersion equation can be used to investigate different modes of propagation of MHD wave however in the following manner we only discuss the perpendicular propagation and parallel propagation.

2.2.1 Perpendicular Propagation

This mode of propagation wave is the magnetosonic mode and for $\mathbf{k} \perp \mathbf{H_0}$, i.e., $\mathbf{k}$ perpendicular to $\mathbf{H_0}$, we have $\mathbf{k}\cdot\hat{\mathbf{z}} = 0$, as the pure Alfven wave always propagates in the direction of magnetic field. Therefore, the dispersion Equation (2.17) is reduced to in the following form:

$$\omega^2\mathbf{v}_1 - \mathbf{k}\left(1+\beta_L+H_e^2\right)(\mathbf{k}\cdot\mathbf{v}_1) = 0. \tag{2.18}$$

The vector nature of equation requires that $\mathbf{k}$ will be parallel to the perturbed fluid velocity $\mathbf{v}_1$, i.e., $\mathbf{k}\cdot\mathbf{v}_1 = kv_1$. The wave is longitudinal in nature and its normalized dispersion relation is given by

$$\omega = k\sqrt{1+\beta_L+H_e^2}, \tag{2.19}$$

we can also write in dimensional form as

$$\omega = k\sqrt{C_{ie}^2 + V_A^2 + \frac{\hbar^2 k^2}{4m_i m_e}}. \tag{2.20}$$

It is clear from the above equation that the dispersion relation of magnetosonic wave is modified by quantum speed due to Landau diamagnetic effect through pressure term, Alfven propagation mode and Bohm potential.

2.2.2 Parallel Propagation

The wave propagates parallel to $\mathbf{H}_0$, i.e., $\mathbf{k} \parallel \mathbf{H}_0$ which implies that $\mathbf{V}_A \parallel \mathbf{H}_0$ means $\mathbf{k} \cdot \hat{\mathbf{z}} = k$. In the mode of this type of propagation, the two dispersion relations i.e., longitudinal propagation wave and transverse propagation wave are obtained. In longitudinal wave propagation, $\mathbf{v}_1$ is parallel to $\mathbf{H}_0$ (i.e., $\mathbf{v}_1 \parallel \mathbf{V}_A$, along with $\mathbf{v}_1 \parallel \mathbf{k}$ means $\hat{\mathbf{z}} \cdot \mathbf{v}_1 = v_1$ and $\mathbf{k} \cdot \mathbf{v}_1 = kv_1$) the dispersion Equation (2.17) is then further reduced to in the following way:

$$\omega = k\sqrt{\beta_L + H_e^2}. \tag{2.21}$$

In dimensional form, we have

$$\omega = k\sqrt{C_{ie}^2 + \frac{\hbar^2 k^2}{4m_i m_e}}. \tag{2.22}$$

This only depends on Bohm potential. Whereas in transverse wave mode (i.e., $\mathbf{v}_1 \perp \mathbf{V}_A$, along with $\mathbf{v}_1 \perp \mathbf{k}$ means $\hat{\mathbf{z}} \cdot \mathbf{v}_1 = 0$ and $\mathbf{k} \cdot \mathbf{v}_1 = 0$) then $\omega^2 - V_A^2 k^2 = 0$ and the same phase velocity is obtained as for in case of pure Alfven mode $\omega = V_A k$.

2.3 MHD waves with Effect of Combined Orbital Quantizition and Spin Magnetization

The magnetic moment associated with a quantum plasma is a new collective dynamical property that arises due to indirect spin interaction through external effective magnetic field and spin velocity coupling. Let us suppose a collisionless, quasineutral and spin magnetized plasma which is assumed as perfectly dense degenerate. The dynamics of electron may be modified due to the contribution of its spin force F_{spin}. The continuity equation remains the same as Equation (2.7), while only the momentum Equation (2.6) is modified and can be written, as

$$\rho_m \frac{\partial \mathbf{v}}{\partial t} = \mathbf{J} \times \mathbf{H} - \nabla P + \frac{\rho_m \hbar^2}{2m_i m_e} \nabla \left[\frac{\nabla^2 \sqrt{\rho_m}}{\sqrt{\rho_m}} \right] + \mathbf{F}_{spin}, \tag{2.23}$$

where $\mathbf{F}_{spin}$ is the magnetization energy which arises due to spin effect of electrons and its mathematical expression is given by

$$\mathbf{F}_{spin} = \mu_0 \nabla (\mathbf{M} \cdot \mathbf{H}). \tag{2.24}$$

The Pauli magnetization (spin magnetization) per unit volume is given by $\mathbf{M}_p = \left(\frac{3n\mu_o \mu_B^2}{2T_{Fe} K_B} \right) \mathbf{H}$ and is related to magnetization current. For a magnetized medium with magnetization $\mathbf{M}$, the Ampere's law in general form for the free current in a finite volume is defined by $\mathbf{J} = \frac{1}{\mu_0} \nabla \times \mathbf{H} - \nabla \times \mathbf{M}$. It is known that in a complete degenerated plasma the Pauli magnetization dominates the Langevin-type susceptibility which is given as $M_{lang} = n\mu_B \tanh\left(\frac{\mu_B H_0}{K_B T_e} \right)$. The Landau demagnetization susceptibility, $\mathbf{M}_L (= -\frac{1}{3} \mathbf{M}_p)$ is added to the effect due to electron orbital (spatial) contribution implying that $\mathbf{M} = \mathbf{M}_L + \mathbf{M}_p = \left(\frac{n\mu_o \mu_B^2}{E_F} \right) \mathbf{H}$. In a complete degenerated quantum magnetized plasma, $T_e \sim 0$ i.e., $T_{Fe} \gg T_e$ is assumed. Thus, Equation (2.24) leads to the following simplified form

$$\mathbf{F}_{spin} = n\beta\mu_B (\nabla H). \tag{2.25}$$

The effect of the spin magnetization along with orbital motion of electron appears via parameter $\beta (= \frac{\mu_B \mu_o^2 H_0}{E_F})$ and the condition $\beta > 1$ should be satisfied for spin magnetization. Using Equation (2.25) in Equation (2.23), we differentiate w.r.t time (t) to obtain

$$\begin{aligned} &\frac{\partial^2 \mathbf{v}_1}{\partial t^2} - C_{ie}^2 \nabla (\nabla \cdot \mathbf{v}_1) + \mathbf{V}_A \times [\nabla \times \{\nabla \times (\mathbf{v}_1 \times \mathbf{V}_A)\}] \\ &- \frac{\hbar^2}{4m_i m_e} \frac{\partial}{\partial t} [\nabla (\nabla^2 \sqrt{\rho_{m1}})] + \beta (\frac{\mu_B}{m_i}) \frac{\partial}{\partial t} (\nabla H) = 0 . \end{aligned} \tag{2.26}$$

Note that one can obtain $\omega H = H_0(\mathbf{k}\cdot\mathbf{v}_1)$ from Equations (2.9) and (2.10). Further simplificating Equation (2.26) (in the appendix A the derivation detail is given) we eventually get the following relation:

$$\omega^2\mathbf{v}_1 - \mathbf{k}\left(C_{ie}^2 + V_A^2\right)(\mathbf{k}\cdot\mathbf{v}_1) + [(\mathbf{V}_A\cdot\mathbf{v}_1)\mathbf{k} - (\mathbf{V}_A\cdot\mathbf{k})\mathbf{v}_1 + (\mathbf{k}\cdot\mathbf{v}_1)\mathbf{V}_A](\mathbf{k}\cdot\mathbf{V}_A)$$
$$-\mathbf{k}\frac{\hbar^2k^2}{4m_em_i}(\mathbf{k}\cdot\mathbf{v}_1) + \beta H_0\left(\frac{\mu_B}{m_i}\right)\mathbf{k}(\mathbf{k}\cdot\mathbf{v}_1) = 0\,, \tag{2.27}$$

Equation (2.27) can be expressed in a dimensionless form, as

$$\omega^2\mathbf{v}_1 - \mathbf{k}(1+\beta_L)(\mathbf{k}\cdot\mathbf{v}_1) + [(\hat{\mathbf{z}}\cdot\mathbf{v}_1)\mathbf{k} - (\hat{\mathbf{z}}\cdot\mathbf{k})\mathbf{v}_1 + (\mathbf{k}\cdot\mathbf{v}_1)\hat{\mathbf{z}}](\mathbf{k}\cdot\hat{\mathbf{z}})$$
$$-\mathbf{k}(\mathbf{k}\cdot\mathbf{v}_1)\left(H_e^2 - \varepsilon_0^2\right) = 0. \tag{2.28}$$

In Equation (2.28), $\varepsilon_0 = \dfrac{\mu_0\mu_B H_0}{\sqrt{m_i E_F V_A^2}}$ is the dimensionless parameter, showing the normalized Zeeman energy which is due to spin magnetization. The above expression gives the dispersion relation including quantum correction, i.e., spin magnetization energy due to spin effect, Landau pressure due to quantizing magnetic field and Bohm potential.

2.3.1 Perpendicular Propagation

In case of perpendicular propagation mode, we consider $\mathbf{k}\cdot\hat{\mathbf{z}} = 0$ and obtain the dispersion relation from (2.28),

$$\omega^2\mathbf{v}_1 - \mathbf{k}(1+\beta_L)(\mathbf{k}\cdot\mathbf{v}_1) - \mathbf{k}(\mathbf{k}\cdot\mathbf{v}_1)\left(H_e^2 - \varepsilon_0^2\right) = 0, \tag{2.29}$$

further by simplification, i.e., $\mathbf{k}\cdot\mathbf{v}_1 = kv_1$, one can easily arrive at the following result

$$\omega = k\sqrt{1+\beta_L + H_e^2 - \varepsilon_0^2}. \tag{2.30}$$

Under the condition; $\varepsilon_0^2 > (1 + \beta_L + H_e^2)$, the magnetosonic waves become unstable. In the presence of very strong magnetic field like due to spin magnetization if it overcomes all other quantum corrections then the growth rate ω_{gr} at which magnetosonic waves become unstable can be written as

$$\omega_{gr} = k\sqrt{\varepsilon_0^2 - (1 + \beta_L + H_e^2)}. \tag{2.31}$$

2.3.2 Parallel Propagation

Similarly, in parallel propagation when $\mathbf{k}.\mathbf{V}_A = kV_A$, we obtain the relation for longitudinal wave propagation, as

$$\omega = k\sqrt{\beta_L + H_e^2 - \varepsilon_0^2}. \tag{2.32}$$

From Equations (2.30 and 2.32), it is clear that the unstable modes of waves in case of dominant spin effect over other quantum effects could be affected and if the spin effect is ignored then one can obtain again the same general MHD wave propagation for quantum plasma as in Equations (2.19 and 2.20), respectively.

2.4 Results and Discussion

The study of dense astrophysical systems delivers basic information about the evolution and structure of stars and galaxies. The exact description of mechanical as well as thermal properties of dense astrophysical objects need more specific knowledge of EOS in dense astrophysical plasmas. It is noted that in the evolution process, usually the end product of the majority of stars are converted into the WD whose size is approximately equal to the earth and mass equal to the NS. Typically, the WD has the strength of magnetic field $\sim (10^6 - 10^9)\ G$, interior density of the order of $\sim 10^{29} cm^{-3}$ with an average electron temperature $\sim 10^4 \mathring{K}$ [ ]. The discovery of pulsar has shown the presence of NS in universe. While magnetic field strength has been determined in the NS in the range $\sim (10^{11} - 10^{13})\,G$. Furthermore, depending upon the mass and age of NS, the average value of temperature ($10^6\mathring{K}$) indicates its photospheric properties. Moreover, its number density of the order of $\sim 10^{28}cm^{-3}$ depends upon the value of the strength of magnetic field and thermal temperature [,]. Generally, the plasma is assumed as non-isothermal $T_e \gg T_i$. For $T_e = 100T_i$ that is for dense astrophysical systems if $n = 10^{26}cm^{-3}$, $T_e(= 10^4\mathring{K}) \ll T_{Fe}(= 10^7\mathring{K})$ and $T_i = 10^2\mathring{K}$ then $V_e = 1.7 \times 10^6\ cm/\sec$, and $V_i = 9.09 \times 10^4\ cm/\sec$ that means $V_i \ll V_e$.

The following assumptions have been taken which are valid for the system under consideration such as WD and NS (a) $n_0\lambda_B^3 \gtrsim 1$, here $\lambda_B = \frac{\hbar}{\sqrt{m_e k_B T_e}}$ is the de Broglie wavelength; For number density of plasma $n_0 \approx 1.7\times10^{26}cm^{-3}$, one can obtain $n_0\lambda_B^3 \approx 3.02$, which will confirm the validity of using the QMHD model. (b) $T_e < T_F$; as the Fermi temperature of electron is proportional to the number density, i.e., $T_F \propto n_0^{\frac{2}{3}}$. For $n_0 \approx 10^{26}cm^{-3}$, we have $T_F \approx 1.3 \times 10^7\mathring{K}$, and this value is greater than system temperature. Therefore, the condition $T_e < T_F$ is valid for degenerate electrons in quantum plasma. (c) $E_0 \gg E_F$, where $E_0 = m_ec^2$ is the rest mass energy of electrons, and degenerate electrons are assumed to be in the non relativistic regime. Let for $T_F \approx 1.3\times10^7\mathring{K}$, we derive $E_F \approx 1.78\times10^{-16}ergs$ and this value is much smaller than the electron rest mass energy $E_0 \approx 8.19 \times 10^{-14}ergs$. Therefore, in our current study the condition $E_0 \gg E_F$ holds.

It is convenient to describe that the thermodynamic properties of degenerate plasma in the presence of strong magnetic fields is significantly modified. We know that Landau levels may be analyzed in the presence of the magnetic field. The ratio of cyclotron energy of electron ($\hbar\omega_{ce}$) to rest mass-energy of electron (m_ec^2) gives the term $b \sim 0.0016$, as $b = \frac{\hbar\omega_{ce}}{m_ec^2} = \frac{H_{12}}{44.12}$ which verifies the condition of non relativistic plasma. Here, $H_{12}(\approx \frac{H}{8\times10^{10}G})$ denotes the value of the magnetic field scale for the system. Also, the atomic unit of the strength magnetic field is calculated as $H_0 = 2.35 \times 10^9G$ from $\hbar\omega_{ce} = e^2/a_0$, (as a_0 is Bohr radius). Now dimensionless magnetic field strength can be defined as $\sigma_1 = \frac{H}{H_0} = \frac{b}{\alpha_f^2}$, where α_f is the fine structure constant and has a value ~ 0.0072. In our current study $\sigma_1 (\simeq 30) \gg 1$, the electron cyclotron energy ($\hbar\omega_{ce}$) becomes greater than electron thermal energy (k_BT_e) as well as Fermi energy (k_BT_F) and hence majority of electrons adjust themselves at Landau ground state. In case of

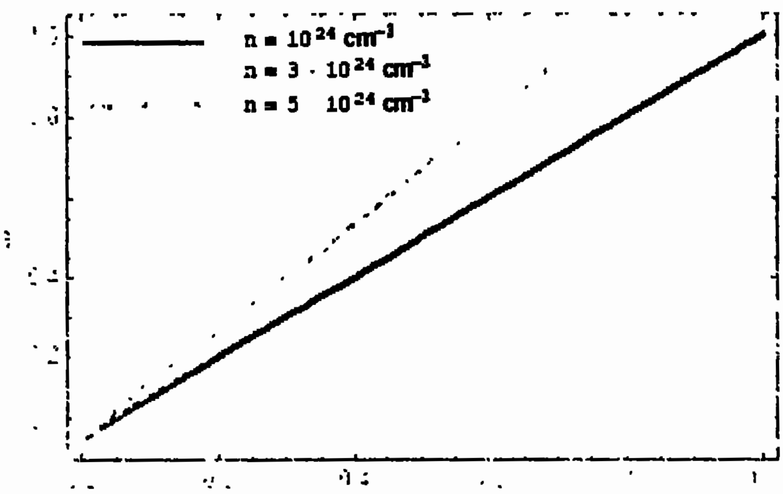

Figure 2.1: Numerical diagram of normalized dispersion relation given by Equation (2.19) of QMHD wave is shown for different plasma number density n such that $n = 10^{24}cm^{-3}$(Thick, Blue), $3 \times 10^{24}cm^{-3}$(Dotdashed, Green) and $5 \times 10^{24}cm^{-3}$(Dashed, Red) with $T_i = 10^2\ \mathring{K}$ and $H_0 = 10^9 G$.

$\sigma_1 \gg 1$, the electron spins are aligned antiparallel to H_0. It is known that the orbital motion of ions is also quantized but the cyclotron frequency is much smaller and hence ignored here i.e., $\hbar\omega_{ci} = \hbar\omega_{ce}(m_e/m_i)$. Similarly, for $\sigma_1 \gg 1$, the magnetic fields affect the interaction among different particles such as molecules, atoms and protons due to high cyclotron energy over typical electrostatic energy [ ]. In the presence of a strong magnetic field the binding energy of atoms and formation of molecules is altered and therefore, the quantum mechanical calculation of binding energy can be found in Refs. [, ,].

Now we graphically represent (ω versus k) the already obtained theoretical results so that to visualize the complete picture of quantum effects on MHD waves. Using the above numerical data in Equation (2.17) and Equation (2.28), the modification in dispersive properties of low frequency waves have been observed by varying the number density (n) and magnetic field (H_0).

Figure 2.1 represents the effect of number density of plasma on the perpendicular MHD wave mode (using linear dispersion relation 2.19) including the Landau diamagnetic effect and such a type of wave is longitudinal in nature. It is examined that the increase in number density ($10^{24} - 5 \times 10^{24} cm^{-3}$) at constant magnetic field ($10^9 G$) will

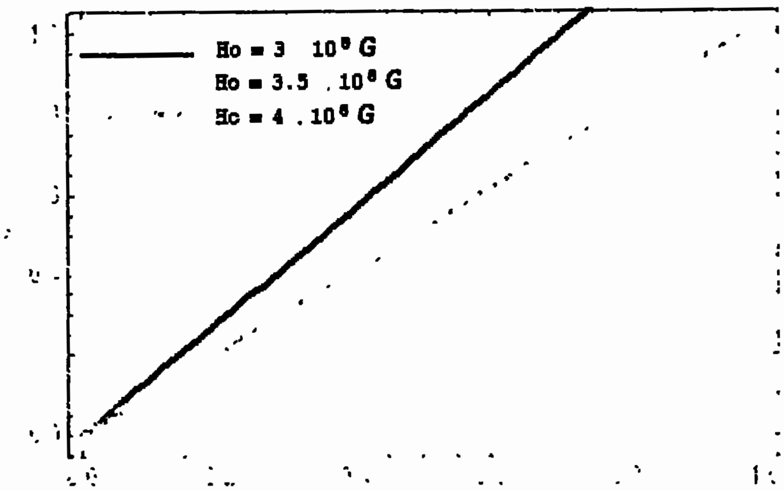

Figure 2.2: Numerical diagram of dispersion relation given by Equation (2.21) of QMHD wave is shown for different magnetic fields such that $H_0 = 3.0 \times 10^8 G$ (Thick, Blue), $3.5 \times 10^8 G$ (Dotdashed, Green) and $4.0 \times 10^8 G$ (Dashed, Red) with $T_i = 10^2 \mathring{K}$ and $n = 10^{26} cm^{-3}$, where the (normalized) wave frequency and (normalized) wave number are presented by vertical and horizontal axes respectively.

cause to increase the phase velocity. Physically, it can be interpreted that the number density of plasma is inversely proportional to Fermi screening therefore by increasing n, decreases the Fermi screening length and so the dispersion of wave increases.

Similarly, **Figure 2.2** shows the effect of magnetic field on the parallel MHD wave mode (using linear dispersion relation 2.21) including Landau diamagnetic effect. It has been shown that the wave frequency decreases with the increasing of magnetic field ($3 \times 10^8 \sim 4 \times 10^8 G$) at constant number density ($10^{24} cm^{-3}$). It means that with increasing H_0 bound state increases and hence less numbers are then involved with dispersion as a result the phase velocity decreases.

Figure 2.3 shows the effect of number density of plasma on the perpendicular MHD wave mode (using linear dispersion relation 2.30) including Landau diamagnetic effect along with spin magnetization and this wave is longitudinal in nature. It is observed that increase in number density from $10^{25} cm^{-3}$ to $9 \times 10^{25} cm^{-3}$ at fixed magnetic field ($10^{10} G$) will cause to increase in phase velocity. Both the spin Pauli magnetism and Landau diamagnetism depend on the magnetic field.

Accordingly, quantitatively the effect of H_0 ($7.5 \times 10^9 - 8.8 \times 10^9 G$) at constant

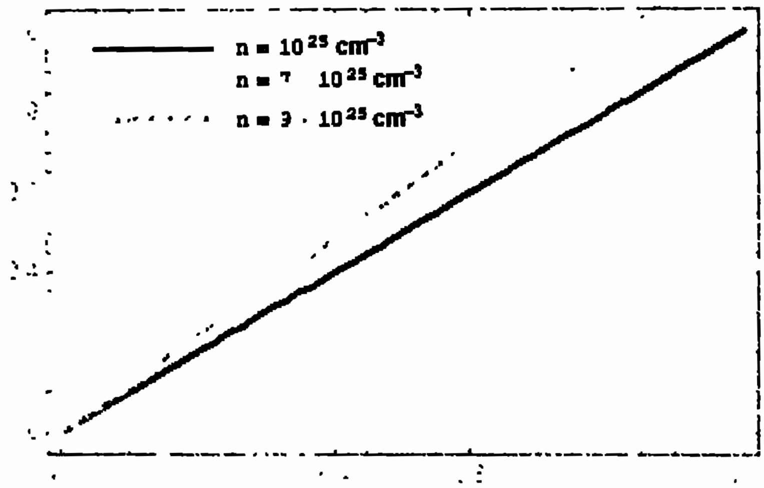

Figure 2.3: Normalized wave frequency as a function of normalized wave number (Equation (2.30)) of QMHD wave is shown for plasma number density i.e., $n = 10^{25}cm^{-3}$(Thick, Blue), $7 \times 10^{25}cm^{-3}$(Dotdashed, Green) and $9 \times 10^{25}cm^{-3}$(Dashed, Red) with $T_s = 10^2\mathring{K}$ and $H_0 = 10^5G$.

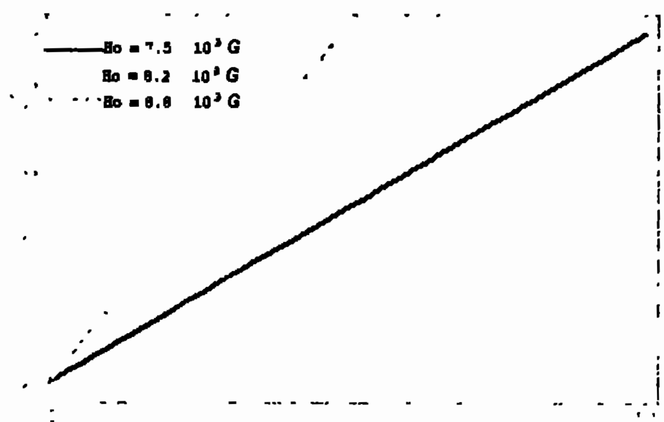

Figure 2.4: Numerical plots of dispersion relation given by Equation (2.32) of QMHD wave are shown for different magnetic fields i.e., $H_0 = 7.5 \times 10^9G$ (Thick, Blue), 8.2×10^9G (Dotdashed, Green) and 8.8×10^9G (Dashed, Red) with $T_s = 10^2\mathring{K}$ and $n = 10^{26}cm^{-3}$, where the (normalized) wave frequency and (normalized) wave number are presented by vertical and horizontal axes respectively. Other Parameters are considered as in Figure (3.3).

number density ($10^{26}cm^{-3}$) is shown in **Figure (2.4)**. This describes the effects of H_0 on the dispersive properties of parallel MHD wave mode including Landau diamagnetic effect along with spin effect (Equation 2.32). The same trend has been observed as that of **Figure (2.2)**.

2.5 Conclusion

Impact of quantizing magnetic field and spin magnetization on the propagation of low frequency MHD waves has been theoretically investigated in quantum magnetoplasmas. Using QMHD model a linear dispersion relation has analytically derived. The dispersion relation has elaborated both qualitatively and quantitatively for dense plasma in the presence of quantizing magnetic field and spin magnetization. The impact of other plasma parameters such as number density (n), temperature (T), and magnetic field (H_0), on the dispersion relation of magnetoacoustic waves, has studied. The perpendicular and parallel modes of wave propagation are deduced. Some novel branches of magnetoacoustic waves were found, which have no analogies without the Landau quantization of magnetic field and spin effect. It is noted that the longitudinal wave propagation in the direction of magnetic field strongly depends on the magnetic field while in the classical limit the magnetic field has no such a role. We have found that the contribution of spin of electrons affects the Alfven propagation mode and the condition $\beta_L > 1$ should be satisfied for spin Pauli magnetization. The variation of the dispersion features for different number density of plasma associated with quantum correction (i.e., Bohm potential, Landau quantization of magnetic field and spin effect) were also explored. Mostly, in plasmas these quantum corrections are very small; however, they may be relevant for dense astrophysical systems in the presence of strong magnetic field. It is found that the effects of the quantization of electron orbital motion and that of electron spin have a substantial influence on the dynamics of magnetosonic waves. The current model in the context of Landau diamagnetic pressure along with spin magnetization is sufficient to study the astrophysical plasma environment existing in the compact systems e.g., white dwarfs, neutron stars and magnetars. Also at laboratory level, it suffices to study the perturbation in solid state objects (i.e. metals and their nanostructures) and in inertial confinement processes as well.

Chapter 3

Effect of Temperature Degeneracy on Spin Magnetosonic Waves

3.1 Introduction

Recently, the quantum plasma have gained much interest owing to its potential applications in miniaturized electronic devices [ ], laser induced plasma [ ], astrophysical compact systems like WD, NS, magnetar and pulsar [ ], ultra-cold plasma [ ] and so on. Quantum plasma is usually characterized by high density and relatively low temperature parameters. Fermions (i.e., particles with spin of one half, e.g., electrons) in such a system obey Pauli's exclusion principle and that's why follow the familiar F.D statistics. In quantum plasma, spin effects play an important role at a scale of high density with low temperature and strong magnetic field. There has been a great deal of interest in excitations of collective modes in spin system, such as spin waves [ ]. The coupling of spin waves to the fermionic degrees of freedom produces the damping of spin waves which may be experimentally measured. Using quantum theory, the study of plasma along with charged particles has gained a lot of attention in astrophysical objects e.g., strongly magnetized plasma [ ]. Cheng and Wu [ ] investigated the spin 1/2 effects associated with electrons in quantum plasma and derived the basic equations by assuming spin-orbit coupling effect. Further, it has been found that spin alignment may be induced in such a plasma through spin-orbit coupling by the interaction of Langmuir waves with EM waves. Similarly, on the basis of MHD model using the non relativistic Pauli spin equations, Wang et al. [ ] investigated the spin effects associated to electrons due to the random orientation of the plasma particles in a non uniform magnetic field with one-body particle-antiparticle Dirac theory of electrons. The physical properties of the magnetosonic solitary waves (MSWs) induced by head-on collision have been reported in quantum magnetoplasma with spin-1/2 non relativistic degnerate electron [ ]. Iqbal et al. employed spin quantum kinetic theory to study the spatial and temporal damping mechanism of the right handed circularly polarized (RCP) waves for both non degenerate and degenerate magnetized quantum plasma. Accordingly, a spin-modified dispersion relation is derived for such RCP-waves in electron-ion (e-i) quantum plasma [ ]. Sharma and Chhajlani [ ] studied the spin effect due to induced magnetization, electrical resistivity and viscosity of medium on the Jeans instability of quantum plasma embedded in the uniform magnetic field by using the quantum MHD model. Shahid et al. [ ] studied the EM waves in e-p plasma using non relativistic spin quantum fluid theory, they predicted that Faraday rotations appear with the inclusion of spin effects of plasma species which is absent in normal e-p plasma, i.e., without the presence of spin effects. The influence of spin effects to linear EM wave modes in dense magnetized plasma have been found by Hu et al. [ ] in arbitrarily direction, where it is shown that spin effect has more significant role for low frequency mode as compared to high frequency mode. Recently, Andreev has separately described spin-up and spin-down effects for electron particles using quantum hydrodynamic (QHD) model and introduced a new general mathematical form for wave solution. It may be appeared as a sound-like solution, which is called as spin-electron acoustic wave [ ]. The effects of quantum corrections, i.e., the spin magnetization energy, and arbitrary temperature degeneracy along with Bohm potential on the dispersive properties of spin MHD wave in magnetized e-p-i plasma have been analyzed by Mushtaq et al. [ ]. Haas and Shahzad [,] have also studied linear and non linear properties of ion-acoustic waves and magnetosonic waves with effect of arbitrary temperature degeneracy.

Degenerate plasma exits at high number density and relatively low temperature. For such high density plasma, the mean inter-particle distance smaller (or of the same order as) the de-Broglie wave length and are following F.D statistics and linked with Fermi velocity, $v_F(=\frac{\hbar}{m}(3\pi^2 n_0)^{1/3})$. In dense astrophysical environments degenerate quantum plasma can be expected to occur naturally. On other hand, non degenerate (classical) plasma exists at low number density and relatively high temperature. In classical mechanics, even though the properties of all particles are identical, they do not lose their individuality and are easily distinguishable from each other. Therefore such a regime of classical plasma obey Maxwell–Boltzmann statistics. Absolute non degenerate (and degenerate) plasmas cannot exist in universe. However, at low temperature and high density the equilibrium distribution of particles varies from non degenerate (classical) plasma to F.D distribution (degenerate plasma). The pressure in quantum degenerate plasma arises from the combined effects of Pauli's exclusion principle (obeying F.D distribution) and Heisenberg's uncertainty principle due to wave-like nature of the particles shown by Schrodinger's wave equation, which is dependent on the number density of plasma but is independent of their own temperature (thermal). The ions can be assumed as non degenerate because of their heavy mass as compared to the electrons (and positrons).

In this Chapter, we study the dispersive features of low frequency MHD waves in magnetized e-p-i plasma with quantum spin-1/2 effects. Here the main objective of this work is to study the quantum corrections, i.e., the degree of temperature degeneracy and spin magnetization of electrons (and positrons) along with Bohm quantum potential on dispersion modes of MHD waves.

3.2 Basic Formulation and dispersion relatioin

Let us consider an e-p-i quantum plasma in which electrons and positrons are treated as partially degenerate particles. Both the electrons and positrons follow the arbitrary degenerate pressure's law. Moreover, these particles have spin magnetization energy due to spin effect and quantum Bohm potential associated with density fluctuation. On the other hand, ion particles are assumed as non degenerate (classical) due to their large inertia. The plasma is taken in the presence of external magnetic field having direction along the yz-plane i.e., $\mathbf{B}_0 = B_0(\cos\theta\ \hat{y} + \sin\theta\ \hat{z})$ with $\hat{y}$ and $\hat{z}$ being the unit vectors, where θ is the angle that lies between magnetic field and propagation vector $\mathbf{k}(= k\hat{y})$. In current situation, the neutrality condition at equilibrium can be defined as $n_{e0} = n_{i0} + n_{p0}$, where n_{e0}, n_{p0} and n_{i0} are the unperturbed number densities of electrons, positrons and ions, respectively. The amplitude of oscillations is considered so small that we can easily solve the system by using linearized equations. The linearized equations of momentum and continuity of the given dynamical system are given as under

$$n_{j0}m_j\frac{\partial \mathbf{V}_{j1}}{\partial t} = n_{j0}q_j(\mathbf{E}_1 + \frac{\mathbf{V}_{j1}\times\mathbf{B}_0}{c}) - \nabla P_j + \frac{\hbar^2}{4m_j}\nabla(\nabla^2 n_j) + \frac{2n_{j0}\mu_j}{n_{j0}m_j\hbar}\nabla(\mathbf{s}\cdot\mathbf{B}_1), \quad (3.1)$$

and

$$\frac{\partial n_j}{\partial t} + n_{j0}(\nabla\cdot\mathbf{V}_j) = 0, \quad (3.2)$$

where $n_{j0}(n_j)$ represents the unperturbed number density (first order perturbed number density). Here the subscript j stands for particles e, p and i (electrons, positrons and ions, respectively). While $\mathbf{V}_j$ is the fluid velocity, m_j is the mass and q_j is the charge of j^{th} particles, c is the speed of light and $\hbar(= \frac{h}{2\pi})$ is the reduced Plank's constant. The geometry of EM field is considered in such a way that the perturbed electric and magnetic fields can be expressed, as $\mathbf{E}_1 = E_1\hat{x}$ and $\mathbf{B}_1 = -(ckE_1/\omega)\hat{z}$, respectively. The third term in right side of momentum Equation (3.1) is the quantum Bohm Potential that arises due to density fluctuation (overlapping of wave function) and accordingly the last term is the magnetization energy due to spin effect of electrons/positrons. Also in the above corresponding equations the parameter $\mu_j = \frac{q_j\hbar}{2m_jc}$ shows the magnetic moment and while $\mathbf{s}$ being the spin vector. Therefore, in terms of Bohr magneton $\mu_B = |\frac{e\hbar}{2mc}|$ the magnetic moments for electrons (and positrons) can be written by relations, $\mu_e = -\mu_B$ (and $\mu_p = \mu_B$) with $m_e = m_p = m$.

Regarding to pressure, it is supposed that ion follows the Maxwellian (non degenerate) pressure law, i.e., $P_i = n_i K_B T_i$ implying that the pressure gradient force is $\nabla P_i = \gamma_i \nabla n_i k_B T_i$, with γ_i being the polytropic index. A quantum thermal electron/positron gas corresponds to a F.D distribution. A local quasi-equilibrium F.D distribution function for electrons/positrons is given by $f(v, r, t) = \dfrac{A}{1+\exp\left(\dfrac{\varepsilon_e - \mu}{k_B T}\right)}$, here $\varepsilon_e (= mV^2/2)$ is the K.E with V is the microscopic velocity, μ is the chemical potential term which can vary in the range from $-\infty$ to $+\infty$ and T is the thermodynamic temperature. The EOS for a degenerate gas is described by F.D probability distribution has two independent parameters, i.e., one is T and the other is μ. The thermodynamic temperature T is the main parameter for near-to-equilibrium Maxwellian particles while on other hand, the chemical potential μ is a unique parameter which has prominent role in the energy spectrum of degenerate gas. Therefore it is interesting to examine the different modes of wave propagation with EOS in which these two parameters, i.e., μ and T have equal relevance. This situation is more relevant to systems which are neither complete degenerate nor close to classical (non degenerate) statistics. Alternatively, one can also say that it is more relevant to systems where thermal and Fermi temperatures are of almost same order of magnitude i.e., $T \approx T_F$. The expression for electron (and positron) pressure in terms of polylogarithmic function can be written in the following generalized form:

$$P = Gnk_B T, \tag{3.3}$$

with $G(= \frac{Li_{\frac{5}{2}}(-\xi)}{Li_{\frac{3}{2}}(-\xi)})$ being defined as arbitrary temperature degeneracy with thermal temperature T. One can obtain a generalized relation for Maxwellian pressure as $P = nk_B T$ in the limit of complete non degenerate case i.e.; $G = 1$. Introducing the Sommerfeld lemma [,], $\mu = k_B T_F \left[1 - \sigma^{-2}\frac{\pi^2}{12}\right]$ and therefore $\ln \xi = \sigma\left(1 - \sigma^{-2}\frac{\pi^2}{12}\right)$ that implicates $G_{ND} = \sigma(1 - \sigma^{-2}\frac{\pi^2}{12})$. We obtain the pressure in case of nearly non degenerate and nearly degenerate respectively, as

$$P_{NND} = nk_B T(1 + \frac{\xi}{2^{\frac{5}{2}}}), \tag{3.4}$$

and

$$P_{ND} = \frac{2}{5}nk_BT_F(1 - \sigma^{-2}\frac{\pi^2}{12}). \tag{3.5}$$

The derivation detail of above three equations is given in Chapter one. Now the Maxwell equations for the considered system are given, as

$$\nabla \times \mathbf{E}_1 = -\frac{1}{c}\frac{\partial \mathbf{B}_1}{\partial t}, \tag{3.6}$$

$$\nabla \times \mathbf{B}_1 = \frac{1}{c}\frac{\partial \mathbf{E}_1}{\partial t} + \frac{4\pi}{c}\mathbf{J}_1. \tag{3.7}$$

Taking the curl of Equation (3.6) and then using it in Equation (3.7), one can obtain the following relation

$$\omega^2\mathbf{E}_1 + i4\pi\omega\mathbf{J}_1 = c^2k^2\mathbf{E}_1, \tag{3.8}$$

where $\mathbf{J}_1$ is the total current density which is given by

$$\mathbf{J}_1 = \sum_j q_j n_j \mathbf{v}_{j1} + c\mathbf{J}_{Me} + c\mathbf{J}_{Mp}. \tag{3.9}$$

Here $\mathbf{J}_{Me}$ and $\mathbf{J}_{Mp}$ are are the magnetization spin currents of electrons and positrons respectively, which are defined by the following relations:

$$\mathbf{J}_{Me} = -\nabla \times (2n_e\mu_B\mathbf{s}/\hbar), \tag{3.10}$$

$$\mathbf{J}_{Mp} = -\nabla \times (2n_p\mu_B\mathbf{s}/\hbar). \tag{3.11}$$

In spin e-p-i quantum plasma, the spin evolution equation is defined as $\frac{d\mathbf{s}_j}{dt} = \frac{2\mu_j}{\hbar}(\mathbf{s}_j \times \mathbf{B})$, where the spin-thermal coupling terms are ignored. The exact term of spin inertia is $\frac{d\mathbf{s}_j}{dt}$ which is also called spin angular frequency. The spin angular frequency of ion is low as compared to electrons and positrons because of heavy mass. The gyration radius and time taken for revolution of ions is comparatively smaller than for considering electrons and positrons. Under the assumption ($\omega < \omega_{ci} < \omega_{ce}, \omega_{cp}$), the spin inertia can be neglected and we obtain the spin equation of motion as $\mathbf{s}_j \times B = 0$, which has the solution $\mathbf{s}_j = -\frac{1}{2}\hbar\eta(\alpha_G)\hat{\mathbf{B}}$. Under the assumption of considered geometry the solution for spin particles (electrons and positrons) can be written as

$$\mathbf{s}_e = -\frac{\hbar}{2}\eta_e(\alpha_{Ge})(\cos\theta\hat{y} + \sin\theta\hat{z}), \tag{3.12}$$

and

$$\mathbf{s}_p = -\frac{\hbar}{2}\eta_p(\alpha_{Gp})(\cos\theta\hat{y} + \sin\theta\hat{z}), \tag{3.13}$$

where Langevin parameter $\eta(\alpha_G) = \tanh(\alpha_G)$ is due to the magnetization of a spin distribution in thermodynamic equilibrium. Here, the term $\alpha_G(= \frac{\mu_B \mathbf{B}_0}{GK_BT})$ is the ratio between magnetic energy and thermal energy. This term is modified by temperature degeneracy and therefore for NND regime, we have

$$\alpha_{NND} = \frac{\mu_B \mathbf{B}_0}{(1 + \frac{\xi}{2^{\frac{5}{2}}})K_BT}, \tag{3.14}$$

and for ND regime, the term can be written in the forms as

$$\alpha_{ND} = \frac{5}{2}\frac{\mu_B \mathbf{B}_0}{\ln(\xi)K_BT_F}. \tag{3.15}$$

To study the linear perturbations, let us assume that all the perturbed quantities in the plasma system are proportional to $e^{i(ky-\omega t)}$, then Equation (3.1) may be expressed for ions, electrons and positrons, using the concept of linearization, as

$$-im_in_{i0}\omega\mathbf{V}_{i1} = n_{i0}e\left(\mathbf{E}_1 + \frac{\mathbf{V}_{i1} \times \mathbf{B}_0}{c}\right) - i\gamma_in_{i1}K_BT_ik\hat{y}, \tag{3.16}$$

and

$$-im_en_{e0}\omega\mathbf{V}_{e1} = -n_{e0}e\left(\mathbf{E}_1 + \frac{\mathbf{V}_{e1} \times \mathbf{B}_0}{c}\right) - iG_en_{e1}k_BT_ek\hat{y} - \frac{i\hbar^2k^3n_{e1}}{4m_e}\hat{y} + i\eta_e(\alpha_{Ge})n_{e0}\mu_BB_1k\sin\theta\hat{y}, \tag{3.17}$$

$$-im_p n_{p0}\omega \mathbf{V}_{p1} = n_{p0}e\left(\mathbf{E}_1 + \frac{\mathbf{V}_{p1}\times\mathbf{B}_0}{c}\right) - iG_p n_{p1}k_B T_p k\hat{y} - \frac{i\hbar^2 k^3 n_{p1}}{4m_p}\hat{y} - i\eta_p(\alpha_{Gp})n_{p0}\mu_B B_1 k\sin\theta\hat{y}. \tag{3.18}$$

Similarly, the linearized continuity Equation (3.2) becomes as

$$n_{j1} = \frac{n_{j0}k}{\omega}\hat{y}\cdot\mathbf{V}_{j1}. \tag{3.19}$$

In magnitude form, the magnetization current density for electrons and positrons can also be written as

$$J_{me1} = i\eta_e(\alpha_{Ge})n_{e0}\mu_B k^2 V_{ey1}\sin\theta/\omega, \tag{3.20}$$

and

$$J_{mp1} = -i\eta_p(\alpha_{Gp})n_{p0}\mu_B k^2 V_{py1}\sin\theta/\omega. \tag{3.21}$$

To study the dispersive features of low frequency magnetoacoustic waves in a magnetized quantum e-p-i plasma with spin effects in arbitrary degenerate state, we solve Equations (3.16)-(3.21) along with Equation (3.8) to get a generalized dispersion relation in the following form

$$\begin{aligned}\omega^2 - c^2k^2 = {} & \Omega_{pi}^2\left[\frac{(1-A_i)\omega^2}{\omega^2(1-A_i) - \omega_{ci}^2 + \omega_{ci}^2\cos^2\theta A_i}\right] + \Omega_{pe}^2\left[\begin{array}{c} 1 + \frac{2\eta_e(\alpha_{Ge})\omega_{ce}\mu_B ck^2\sin^2\theta}{(1-A_e)e\omega^2} \\ + \frac{\eta_e^2(\alpha_{Ge})\mu_B^2 c^2k^4\sin^2\theta}{(1-A_e)e^2\omega^2}\left(1 - \frac{\omega_{ce}^2}{\omega^2}\cos^2\theta\right)\end{array}\right] \\ & \times\left[\frac{(1-A_e)\omega^2}{\omega^2(1-A_e) - \omega_{ce}^2 + \omega_{ce}^2\cos^2\theta A_e}\right] + \Omega_{pp}^2\left[\begin{array}{c} 1 - \dfrac{2\eta_p(\alpha_{Gp})\omega_{cp}\mu_B ck^2\sin^2\theta}{(1-A_p)e\omega^2} \\ + \frac{\eta_p^2(\alpha_{Gp})\mu_B^2 c^2k^4\sin^2\theta}{(1-A_p)e^2\omega^2}(1 - \frac{\omega_{cp}^2}{\omega^2}\cos^2\theta)\end{array}\right] \\ & \times\left[\frac{(1-A_p)\omega^2}{\omega^2(1-A_p) - \omega_{cp}^2 + \omega_{cp}^2\cos^2\theta A_p}\right],\end{aligned} \tag{3.22}$$

where $\Omega_{pj}(=\sqrt{\frac{4\pi e^2 n_{j0}}{m_j}})$ and $\omega_{cj}(=\frac{eB_0}{m_j c})$ are the plasma oscillation frequency and cyclotron frequency of the j^{th} species, respectively. In dispersion Equation (3.22), the spin effects with arbitrary degree of temperature degeneracy appear in $\eta_e(\alpha_{Ge})$ and $\eta_p(\alpha_{Gp})$. Here $A_i = \frac{V_{Ti}^2 k^2}{\omega^2}$, $A_e = \left[\frac{G_e K_B T_e}{m} + \frac{\hbar^2 k^2}{4m^2}\right]\frac{k^2}{\omega^2}$ and $A_p = \left[\frac{G_p K_B T_p}{m} + \frac{\hbar^2 k^2}{4m^2}\right]\frac{k^2}{\omega^2}$ are the dimensionless quantities, while $V_{Ti}(=\sqrt{\frac{\gamma_i k_B T_i}{m_i}})$ is the ion thermal velocity. For the case of NND, these dimensionless parameters for electrons and positrons; A_e and A_p, respectively, can be written as

$$A_{eNND} = \left[\frac{K_B T_e}{m}\left(1 + \frac{\xi_e}{2^{\frac{5}{2}}}\right) + \frac{\hbar^2 k^2}{4m^2}\right]\frac{k^2}{\omega^2}, \tag{3.23}$$

and

$$A_{pNND} = \left[\frac{K_B T_p}{m}\left(1 + \frac{\xi_p}{2^{\frac{5}{2}}}\right) + \frac{\hbar^2 k^2}{4m^2}\right]\frac{k^2}{\omega^2}. \tag{3.24}$$

Similarly, in terms of ND, it can be expressed as

$$A_{eND} = \left[\frac{2k_B T_e}{5m}\ln(\xi_e) + \frac{\hbar^2 k^2}{4m^2}\right]\frac{k^2}{\omega^2}, \tag{3.25}$$

and

$$A_{pND} = \left[\frac{2k_B T_p}{5m}\ln(\xi_p) + \frac{\hbar^2 k^2}{4m^2}\right]\frac{k^2}{\omega^2}. \tag{3.26}$$

In Equations (3.23)–(3.26), the first term show that the temperature degeneracy effect (because of F.D particle distribution) which is coming through the definition of $\xi = e^{\frac{\mu}{k_B T}}$. While the second term in the same equations indicates the contribution of the respective quantum Bohm potential effect that arises due to density correlation and is proportional to the order of $\hbar^2$. The generalized dispersion relation (3.22) in the limiting cases are discussed below for investigating, the spin magnetization and arbitrary temperature degeneracy effects on the propagation of MHD wave with respect to different angles.

3.2.1 Perpendicular Propagation ($\theta = \pi/2$)

In this case, we consider a compressional magnetoacoustic wave with low frequency mode. The inertia associated with the electrons and positrons is taken to be small such that $\omega^2 \ll \omega_{ce}^2, \omega_{cp}^2$. Quantum effects, appearing in A_e (and A_p), become important for large wave number and therefore the limit can be assumed as $\omega^2 \ll \omega_{ce}^2 A_e, \omega_{cp}^2 A_p$. Further, by ignoring the Hall effect, it is considered that $\omega^2 \ll \omega_{ci}^2$ and so $\omega^2 \ll \omega_{ce}^2$, ω_{cp}^2. Using these assumptions Equation (3.22) can be reduced to the following form

$$\omega^2 = \frac{c^2k^2}{V_A^2 + c^2}\left[\tilde{V}_A^2 + V_S^2 + \frac{\hbar^2 k^2}{4m_i m}\{\delta_e[1-\eta_e^2(\alpha_{Ge})] + \delta_p[1-\eta_p^2(\alpha_{Gp})]\}\right]. \tag{3.27}$$

Equation (3.27) shows the compressional magnetoacoustic or fast MHD modes in quantum e-p-i plasma. Here the modified acoustic speed is $V_S(=\sqrt{V_{Ti}^2 + \beta_p V_{Gp}^2 + \beta_e V_{Ge}^2})$ with $V_{Gp,e}(=\sqrt{\frac{GK_BT_F}{m}})$ being the thermal speed for electrons and positrons with arbitrary temperature degeneracy. Also $V_A(=\frac{B_0}{\sqrt{4\pi n_{i0} m_i}})$ is the Alfven velocity and $\tilde{V}_A = V_A\sqrt{1 + \frac{8\pi\eta_p(\alpha_{Gp})\mu_B n_{p0}}{B_0} - \frac{8\pi\eta_e(\alpha_{Ge})\mu_B n_{e0}}{B_0}}$ is the spin modified Alfvenic velocity. Other parameters in Equation (3.27) are defined as $\beta_e = (\frac{m_e}{m_i})\delta_e$, $\beta_p = (\frac{m_p}{m_i})\delta_p$ with $\delta_e = \frac{n_{eo}}{n_{io}}$ and $\delta_p = \frac{n_{po}}{n_{io}}$. Therefore the spin modified version of the Alfven velocity has an important role in propagation of waves which depends on number density of plasma and temperature degeneracy. This constant quantum correction produces a contribution to the linear portion of magnetosonic wave. The other correction that is not constant which is due to Bohm potential of order k^4 and depends on the number density of electrons and positrons. The degree of temperature degeneracy by using F.D particle distribution is considered in the third correction. Equation (3.27) can be written for NND and ND systems with definitions of G_{NND} and G_{ND} in the following forms

$$\omega_{NND} = \frac{ck}{\sqrt{V_A^2 + c^2}}\left[\begin{array}{l}(1 + \frac{8\pi\eta_p(\alpha_{NND})\mu_B n_{p0}}{B_0} - \frac{8\pi\eta_e(\alpha_{NND})\mu_B n_{e0}}{B_0})V_A^2 + V_{Ti}^2 + \frac{\sigma_p^{-1}K_BT_{Fp}}{m}(1+\frac{\xi_p}{2^{\frac{5}{2}}})\beta_p \\ +\frac{\sigma_e^{-1}K_BT_{Fe}}{m}(1+\frac{\xi_e}{2^{\frac{5}{2}}})\beta_e + \frac{\hbar^2k^2}{4m_i m}\{\delta_p[1-\eta_p^2(\alpha_{NND})] + \delta_e[1-\eta_e^2(\alpha_{NND})]\}\end{array}\right]^{1/2}, \tag{3.28}$$

$$\omega_{ND} = \frac{ck}{\sqrt{V_A^2 + c^2}} \left[\begin{array}{c} (1 + \frac{8\pi\eta_p(\alpha_{ND})\mu_B n_{p0}}{B_0} - \frac{8\pi\eta_e(\alpha_{ND})\mu_B n_{e0}}{B_0})V_A^2 + V_{Ti}^2 + \frac{2}{5}\frac{k_B T_{Fp}}{m}(1 - \frac{\pi^2}{12}\sigma_p^{-2})\beta_p \\ +\frac{2}{5}\frac{k_B T_{Fe}}{m}(1 - \frac{\pi^2}{12}\sigma_e^{-2})\beta_e + \frac{\hbar^2 k^2}{4m_i m}\{\delta_p[1 - \eta_p^2(\alpha_{ND})] + \delta_e[1 - \eta_e^2(\alpha_{ND})]\} \end{array} \right]^{1/2}. \tag{3.29}$$

Equation (3.28) relates the arbitrary temperature degeneracy parameter through $\xi_{e,p}$ which has been modified both the spin and pressure terms for NND plasma. For complete non degenerate (CND) quantum plasma, i.e., $\xi_{e,p} \to 0$ one can get relation (11) of [ ] for zero positron concentration. On the other hand for the case of complete degenerate (CD) plasma ($T_{e,\,p} \to 0$), using Equation (3.29), one can easily retrieve relation (16) of Ref. [ ]. It is to be noted that wave propagation mode has been greatly impacted for both NND and ND systems by the degree of temperature degeneracy.

Using the following dimensionless variables

$$\omega \to \frac{\omega}{\omega_{ci}}, \quad k{\to}k\lambda_{Fe}.$$

The dimensionless form of the dispersion relation (3.28) can be written as

$$\omega_{NND}^2{=}k^2\left(V_{AnNND}^2 + V_{SnNND}^2 + k^2 H_e^2 \Psi_{nNND}\right), \tag{3.30}$$

here

$$V_{AnNND} = \frac{\tilde{V}_{AnNND}}{V_F} = V_{AFn}\sqrt{1 + \gamma_{Pnp}\eta_p(\alpha_{NNDp}) - \gamma_{Pne}\eta_e(\alpha_{NNDe})}, \tag{3.31}$$

where $\gamma_{Pnp} = \frac{8\pi\mu_B n_{p0}}{B_0}$, $\gamma_{Pne} = \frac{8\pi\mu_B n_{e0}}{B_0}$ and $V_{AFn} = \frac{V_A}{V_F}$.

$$V_{SnNND} = \frac{V_{SNND}}{V_F}, \quad V_{SNND}(= \sqrt{V_{TiN}^2 + \beta_p V_{GpN}^2 + \beta_e V_{GeN}^2}). \tag{3.32}$$

Here $V_{TiN} = \frac{V_{Ti}}{V_F}$, $V_{GpN} = \frac{V_{Gp}}{V_{Fp}}$ and $V_{GeN} = \frac{V_{Ge}}{V_{Fe}}$. In the above representation (3.30), $H_e = \dfrac{\hbar^2}{\sqrt{4mm_i\lambda_F^2 V_F^2}}$ is a dimensionless parameter which arises due to the collective electron tunnelling effect through the so-called Bohm potential and $\Psi_{nNND} = \delta_p[1 -$

$\eta_p^2(\alpha_{NNDp})] + \delta_e[1 - \eta_e^2(\alpha_{NNDe})]$.

Similarly, the dimensionless form of the dispersion relation (3.29) can also be written, as

$$\omega_{ND}^2 = k^2\left(V_{AnND}^2 + V_{SnND}^2 + k^2 H_e^2 \Psi_{nND}\right), \tag{3.33}$$

where $k \to k\lambda_{Fe}$ and dimensionless parameters are

$$V_{AnND} = \frac{\tilde{V}_{AnND}}{V_A} = \sqrt{1 + \gamma_{Pnp}\eta_p(\alpha_{NDp}) - \gamma_{Pne}\eta_e(\alpha_{NDe})}, \tag{3.34}$$

$$V_{SnND} = \frac{V_{SND}}{V_A}, \quad V_{SND}(= \sqrt{V_{TiN}^2 + \beta_p V_{GpN}^2 + \beta_e V_{GeN}^2}). \tag{3.35}$$

and

$$\Psi_{nNND} = \delta_p[1 - \eta_p^2(\alpha_{NND})] + \delta_e[1 - \eta_e^2(\alpha_{NND})]. \tag{3.36}$$

3.2.2 Parallel Propagation ($\theta = 0$)

Let us assume that wave propagates in the direction of magnetic field (along the z−axis). Therefore Equation (3.22) in this case, yields

$$\omega^2 - c^2k^2 = \frac{\Omega_{pi}^2}{1 - \frac{\omega_{ci}^2}{\omega^2}} + \frac{\Omega_{pp}^2}{1 - \frac{\omega_{cp}^2}{\omega^2}} + \frac{\Omega_{pe}^2}{1 - \frac{\omega_{ce}^2}{\omega^2}}. \tag{3.37}$$

Under the assumption of low frequency limit i.e., $\omega^2 \ll \omega_{ci}^2$, $\omega^2 \ll \omega_{cp}^2$, $\omega^2 \ll \omega_{ce}^2$, $\omega^2 \ll c^2k^2$, and $m \ll m_i$, Equation (3.37) gives us $\omega = kV_A$, means that the spin and other quantum corrections are vanished for Alfven propagation mode, because the spin (along the z-axis) will be aligned parallel to the background magnetic field ($\mathbf{B}_0$) and therefore does not couple with the perturbed magnetic field.

3.2.3 Oblique Propagation ($\theta = \pi/4$)

Here in this case we use in Equation (3.22), the MHD assumptions i.e., $\omega^2 \ll \omega_{ci}^2 \ll \omega_{cp}^2$, ω_{ce}^2; $\omega^2 \ll c^2k^2$; $\omega^2 \ll \omega^2 A_p$; $\omega^2 \ll \omega^2 A_e$ and after some steps of algebra, the following relation is obtained

$$\frac{c^2k^2}{\omega_{NND}^2}\left[1-\frac{2\pi n_{p0}\mu_B^2\eta_p^2(\alpha_{NNDp})}{k_BT_p\left(1+2^{-\frac{5}{2}}\xi_p\right)+\frac{\hbar^2k^2}{4m}}+\frac{2\pi n_{e0}\mu_B^2\eta_e^2(\alpha_{NNDe})}{k_BT_e\left(1+\frac{\xi_e}{2^{\frac{5}{2}}}\right)+\frac{\hbar^2k^2}{4m}}\right]=\left(\frac{\omega_{NND}^2-V_{Ti}^2k^2}{\omega_{NND}^2-V_{Ti}^2k^2/2}\right)\frac{c^2}{V_A^2}$$
$$+\frac{8\pi n_{p0}\mu_B\eta_p(\alpha_{NNDp})c^2}{B_0\left[\frac{k_BT_p}{m}\left(1+\frac{\xi_p}{2^{\frac{5}{2}}}\right)+\frac{\hbar^2k^2}{4m^2}\right]}-\frac{8\pi n_{e0}\mu_B\eta_e(\alpha_{NNDe})c^2}{B_0\left[\frac{k_BT_e}{m}\left(1+\frac{\xi_e}{2^{\frac{5}{2}}}\right)+\frac{\hbar^2k^2}{4m^2}\right]}, \tag{3.38}$$

for oblique MHD propagation mode, in NND plasma. While for ND plasma we have

$$\frac{c^2k^2}{\omega_{ND}^2}\left[1-\frac{2\pi n_{p0}\mu_B^2\eta_p^2(\alpha_{NDp})}{\frac{2}{5}k_BT_p\ln(\xi_p)+\frac{\hbar^2k^2}{4m}}+\frac{2\pi n_{e0}\mu_B^2\eta_e^2(\alpha_{NDe})}{\frac{2}{5}k_BT_e\ln(\xi_e)+\frac{\hbar^2k^2}{4m}}\right]=\left(\frac{\omega_{ND}^2-V_{Ti}^2k^2}{\omega_{ND}^2-V_{Ti}^2k^2/2}\right)\frac{c^2}{V_A^2}$$
$$+\frac{8\pi n_{p0}\mu_B\eta_p(\alpha_{NDp})c^2}{B_0\left[\frac{2}{5}\frac{k_BT_p}{m}\ln(\xi_p)+\frac{\hbar^2k^2}{4m^2}\right]}-\frac{8\pi n_{e0}\mu_B\eta_e(\alpha_{NDe})c^2}{B_0\left[\frac{2}{5}\frac{k_BT_e}{m}\ln(\xi_e)+\frac{\hbar^2k^2}{4m^2}\right]}. \tag{3.39}$$

Equations (3.38) and (3.39) can be expressed in simplified form as

$$a_{NND}\omega_{NND}^4-b_{NND}\omega_{NND}^2+c_{NND}=0, \tag{3.40}$$

and

$$a_{ND}\omega_{ND}^4-b_{ND}\omega_{ND}^2+c_{ND}=0. \tag{3.41}$$

The coefficients in Equation (3.40) for NND limit are defined as

$$a_{NND}=\frac{c^2}{V_A^2}+\frac{8\pi n_{p0}\mu_B\eta_p(\alpha_{NNDp})c^2}{\left[\frac{\sigma_p^{-1}K_BT_{Fp}}{m}\left(1+\frac{\xi_p}{2^{\frac{5}{2}}}\right)+\frac{\hbar^2k^2}{4m^2}\right]B_0}-\frac{8\pi n_{e0}\mu_B\eta_e(\alpha_{NNDe})c^2}{\left[\frac{\sigma_e^{-1}K_BT_{Fe}}{m}\left(1+\frac{\xi_e}{2^{\frac{5}{2}}}\right)+\frac{\hbar^2k^2}{4m^2}\right]B_0}, \tag{3.42}$$

$$b_{NND} = \frac{V_{Ti}^2 k^2}{2}\left(\frac{8\pi n_{p0}\mu_B \eta_p(\alpha_{NNDp})c^2}{\left[\frac{\sigma_p^{-1}K_B T_{Fp}}{m}\left(1+\frac{\xi_p}{2^{\frac{5}{2}}}\right)+\frac{\hbar^2 k^2}{4m^2}\right]B_0} - \frac{8\pi n_{e0}\mu_B \eta_e(\alpha_{NNDe})c^2}{\left[\frac{\sigma_e^{-1}K_B T_{Fe}}{m}\left(1+\frac{\xi_e}{2^{\frac{5}{2}}}\right)+\frac{\hbar^2 k^2}{4m^2}\right]B_0}\right) + \frac{c^2 k^2 V_{Ti}^2}{V_A^2}$$

$$+ c^2 k^2 \left(1 - \frac{2\pi n_{p0}\mu_B^2 \eta_p^2(\alpha_{NND})}{\sigma_p^{-1}K_B T_{Fp}\left(1+\frac{\xi_p}{2^{\frac{5}{2}}}\right)+\frac{\hbar^2 k^2}{4m}} + \frac{2\pi n_{e0}\mu_B^2 \eta_e^2(\alpha_{NND})}{\sigma_e^{-1}K_B T_{Fe}\left(1+\frac{\xi_e}{2^{\frac{5}{2}}}\right)+\frac{\hbar^2 k^2}{4m}}\right), \tag{3.43}$$

and

$$c_{NND} = \frac{c^2 k^4 V_{Ti}^2}{2}\left(1 - \frac{2\pi n_{p0}\mu_B^2 \eta_p^2(\alpha_{NNDp})}{\sigma_p^{-1}K_B T_{Fp}\left(1+\frac{\xi_p}{2^{\frac{5}{2}}}\right)+\frac{\hbar^2 k^2}{4m}} + \frac{2\pi n_{e0}\mu_B^2 \eta_e^2(\alpha_{NNDe})}{\sigma_e K_B T_{Fe}\left(1+\frac{\xi_e}{2^{\frac{5}{2}}}\right)+\frac{\hbar^2 k^2}{4m}}\right). \tag{3.44}$$

The dimensionless form of the dispersion relation (3.40) can be written as

$$\acute{a}_{NND}\omega_{NND}^4 - \acute{b}_{NND}\omega_{NND}^2 + \acute{c}_{NND} = 0, \tag{3.45}$$

where

$$\acute{a}_{NND} = V_{AFn}^{-2} + \frac{\gamma_{np}\eta_p(\alpha_{NNDp})}{\sigma_p^{-1}\left(1+\frac{\xi_p}{2^{\frac{5}{2}}}\right)V_{FpN}^2 + H_e^2} - \frac{\gamma_{ne}\eta_e(\alpha_{NNDe})}{\sigma_e^{-1}\left(1+\frac{\xi_e}{2^{\frac{5}{2}}}\right)V_{FeN}^2 + H_e^2}, \tag{3.46}$$

$$\acute{b}_{NND} = \frac{V_{TiN}^2}{2}\left(\frac{\gamma_{np}\eta_p(\alpha_{NNDp})}{\sigma_p^{-1}\left(1+\frac{\xi_p}{2^{\frac{5}{2}}}\right)V_{FpN}^2 + H_e^2} - \frac{\gamma_{ne}\eta_e(\alpha_{NNDe})}{\sigma_e^{-1}\left(1+\frac{\xi_e}{2^{\frac{5}{2}}}\right)V_{FeN}^2 + H_e^2}\right) + k^2 V_{TiN}^2$$

$$+k^2\left(1 - \frac{\acute{\gamma}_{np}\eta_p(\alpha_{NNDp})}{\sigma_p^{-1}\left(1+\frac{\xi_p}{2^{\frac{5}{2}}}\right)V_{FpN}^2 + H_e^2} - \frac{\acute{\gamma}_{ne}\eta_e(\alpha_{NNDe})}{\sigma_e^{-1}\left(1+\frac{\xi_e}{2^{\frac{5}{2}}}\right)V_{FeN}^2 + H_e^2}\right), \tag{3.47}$$

and

$$\acute{c}_{NND} = \frac{k^4 V_{TiN}^2}{2}\left(1 - \frac{\acute{\gamma}_{np}\eta_p(\alpha_{NNDp})}{\sigma_p^{-1}\left(1+\frac{\xi_p}{2^{\frac{5}{3}}}\right)V_{FpN}^2 + H_e^2} - \frac{\acute{\gamma}_{ne}\eta_e(\alpha_{NNDe})}{\sigma_e^{-1}\left(1+\frac{\xi_e}{2^{\frac{5}{3}}}\right)V_{FeN}^2 + H_e^2}\right). \quad (3.48)$$

Here $\gamma_{np} = \frac{8\pi n_{p0}\mu_B}{B_0}$, $\gamma_{ne} = \frac{8\pi n_{e0}\mu_B}{B_0}$, $\acute{\gamma}_{np} = \frac{2\pi n_{p0}\mu_B^2}{mV_A^2}$, and $\acute{\gamma}_{pe} = \frac{2\pi n_{e0}\mu_B^2}{mV_A^2}$.

Similarly, the dimensionless form of the dispersion relation (3.41) can be written as

$$\acute{a}_{ND}\omega_{ND}^4 - \acute{b}_{ND}\omega_{ND}^2 + \acute{c}_{ND} = 0, \quad (3.49)$$

$$\acute{a}_{ND} = V_{AFn}^{-2} + \frac{\gamma_{np}\eta_p(\alpha_{NDp})}{\frac{2}{5}\left(1-\frac{\pi^2}{12}\sigma_p^{-2}\right)V_{FpN}^2 + H_e^2} - \frac{\gamma_{ne}\eta_e(\alpha_{NDe})}{\frac{2}{5}\left(1-\frac{\pi^2}{12}\sigma_e^{-2}\right)V_{FeN}^2 + H_e^2}, \quad (3.50)$$

$$\begin{aligned}\acute{b}_{ND} &= \frac{k^2 V_{TiN}^2}{2}\left(\frac{\gamma_{np}\eta_p(\alpha_{NDp})}{\frac{2}{5}\left(1-\frac{\pi^2}{12}\sigma_p^{-2}\right)V_{FpN}^2 + H_e^2} - \frac{\gamma_{ne}\eta_e(\alpha_{NDe})}{\frac{2}{5}\left(1-\frac{\pi^2}{12}\sigma_e^{-2}\right)V_{FeN}^2 + H_e^2}\right) + k^2 V_{TiN}^2 \\ &+k^2\left(1 - \frac{\acute{\gamma}_{np}\eta_p(\alpha_{NDp})}{\frac{2}{5}\left(1-\frac{\pi^2}{12}\sigma_p^{-2}\right)V_{FpN}^2 + H_e^2} - \frac{\acute{\gamma}_{ne}\eta_e(\alpha_{NDe})}{\frac{2}{5}\left(1-\frac{\pi^2}{12}\sigma_e^{-2}\right)V_{FeN}^2 + H_e^2}\right), \quad (3.51)\end{aligned}$$

and

$$\acute{c}_{ND} = \frac{k^4 V_{TiN}^2}{2}\left(1 - \frac{\acute{\gamma}_{np}\eta_p(\alpha_{NDp})}{\frac{2}{5}\left(1-\frac{\pi^2}{12}\sigma_p^{-2}\right)V_{FpN}^2 + H_e^2} - \frac{\acute{\gamma}_{ne}\eta_e(\alpha_{NDe})}{\frac{2}{5}\left(1-\frac{\pi^2}{12}\sigma_e^{-2}\right)V_{FeN}^2 + H_e^2}\right). \quad (3.52)$$

Equations (3.45) and (3.49) are in the forms of biquadratic equations and can be solved as

$$\omega_{NND}^2 = \frac{\acute{b}_{NND} \pm \sqrt{\acute{b}_{NND}^2 - 4\acute{a}_{NND}\acute{c}_{NND}}}{2\acute{a}_{NND}}, \quad (3.53)$$

and

$$\omega_{ND}^2 = \frac{\acute{b}_{ND} \pm \sqrt{\acute{b}_{ND}^2 - 4\acute{a}_{ND}\acute{c}_{ND}}}{2\acute{a}_{ND}}. \quad (3.54)$$

From Equations (3.53) and (3.54), it is concluded that oblique MHD mode is substantially modified by incorporating the quantum correction terms (Bohm Potential, degree of temperature degeneracy in pressure term and in spin magnetization for e-p particles), highlightly important and interesting features in the propagation of such a type of waves.

3.3 Results and Discussion

For numerical analysis, the dispersion relation (3.22) is solved to investigate MHD wave in three species (e-p-i) plasma by varying different values of angle θ, spin magnetization energy and degree of temperature degeneracy. This situation can be plotted with the help of some numerical data of quantum plasma taken from dense astrophysical systems. A compact dense astrophysical system forms the endpoint of stellar evolution. Luminous stars lose their nuclear energy source in a finite time. Finally, all the available energy of stars is consumed (stellar death) and the stars collapse to form compact dense stars. It has been observed that the compact systems like WD and NS, as a solid state are opposed to the gaseous interior of all other stars. Corresponding, in these compact dense astrophysical systems, parameters such as number density, magnetic field and arbitrary temperature degeneracy vary over a wide range of values. The number densities for such a dense plasma varies about $n = (10^{30} - 10^{36})\, m^{-3}$ and their temperatures are $T \leq 10^{9} \mathring{K}$ [ ]. Similarly, the magnetic field in the WD and the NS is predicted upto $B_0 \leq 10^{10} Tesla$ [,]. The Fermi temperature of degenerate plasma depends on number density i.e., $T_F \propto n^{2/3}$ has numerical values for NND plasma to ND plasma $\sim 10^4$ to $10^8 \mathring{K}$, respectively. Other parameters are taken as $m = 9.1 \times 10^{-31} kg$, $m_i = 1.67 \times 10^{-27} kg$, $c = 3 \times 10^8 m/sec$, $k_B = 1.38 \times 10^{-23}\ J/\mathring{K}$, $e = 1.602 \times 10^{-19} c$ and $\hbar = 1.05 \times 10^{-31} J - \text{sec}$. The change in magnitude of parameters, such as, temperature degenerate fugacity (ξ), temperature ratio σ, magnetic field and density will modify the dispersive properties of QMHD wave. The present work is related with borderline system ($T_F \approx T$) i.e., neither complete non degenerate regime nor complete degenerate regime. However, NND is slightly differentiated from ND by intermediate regime.

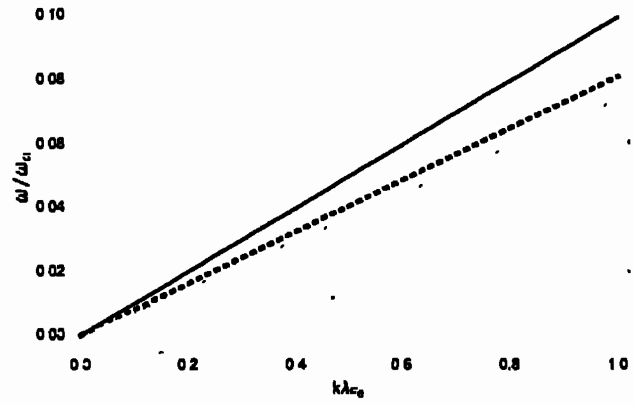

Figure 3.1: Dispersion diagram given by Equation (3.33) for perpendicular propagation mode of MHD wave in ND plasma with different electron's concentration effect such that $n_{e0} = 1 \times 10^{33} m^{-3}$(solid black line), $5 \times 10^{33} m^{-3}$ (dashed black line), $10^{34} m^{-3}$ (solid red line) with $n_{p0} = 10^{32} m^{-3}$ as a fixed value. Other parameters are $\sigma_e = 2$, $\sigma_p = 2$ and $B_0 = 5 \times 10^6 Tesla$, where $\omega (= \frac{\omega}{\omega_{cs}}$, along horizontal axis) and $k (= k\lambda_{Fe}$, along vertical axis).

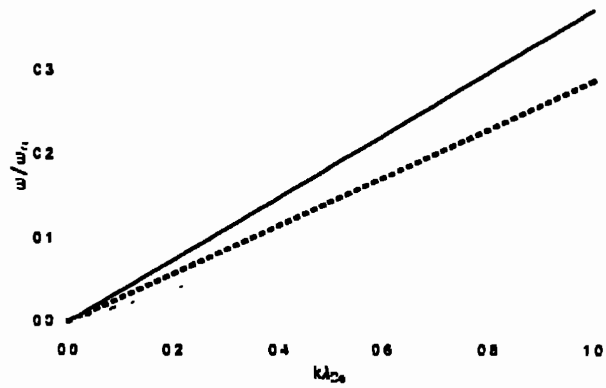

Figure 3.2: Dispersion diagram given by Equation (3 30) for perpendicular propagation mode of MHD wave in NND plasma with different electron's concentration effect such that $n_{e0} = 1 \times 10^{25} m^{-3}$(solid black line), $2 \times 10^{25} m^{-3}$ (dashed black line), $6 \times 10^{25} m^{-3}$ (solid red line) with $n_{p0} = 10^{24} m^{-3}$ as a fixed value. Other parameters are $\sigma_e = 0.02$, $\sigma_p = 0.02$ and $B_0 = 5 \times 10^4 Tesla$, where $\omega (= \frac{\omega}{\omega_{cs}}$, along horizontal axis) and $k (= k\lambda_{De}$, along vertical axis).

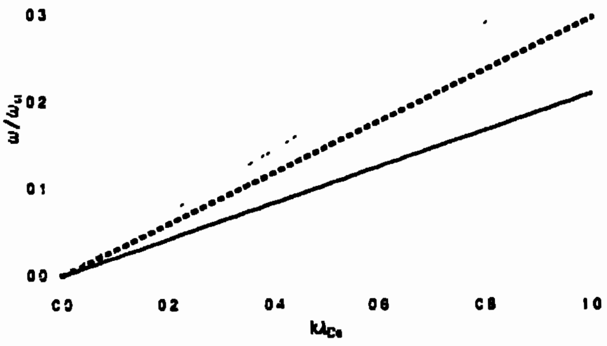

Figure 3.3: Dispersion diagram given by Equation (3.30) for perpendicular propagation mode of MHD wave in NND plasma with different values of quantum temperature degeneracy factor such that σ_e, $\sigma_p = 0.02, 0.02$ (solid black line), $0.04, 0.04$ (dashed black line), $0.06, 0.06$ (solid red line). Other parameters are $n_{e0} = 10^{25}m^{-3}$, $n_{p0} = 10^{25}m^{-3}$ and $B_0 = 5 \times 10^4 Tesla$.

The influence of electrons concentration on the dispersion of magnetosonic wave (using Equation 3.29) for ND (and NND) plasma is represented in **Figure 3.1 (and Figure 3.2)**. In **Figure 3.1**, the normalized wave frequency ω (i.e., $\omega = \frac{\omega}{\omega_{ci}}$) as a function of normalized wavenumber k (i.e., $k = k\lambda_{Fe}$) decreases by increasing the electrons number density of ND plasma n_{e0} ($10^{33} - 10^{34}m^{-3}$) at a fixed number density of positrons ($n_{p0} = 10^{32}m^{-3}$). As increase in number density of electrons will cause to decrease the wave frequency with more degeneracy factor and therefore it will enlarge the Fermi screening length. The same trend is also shown in **Figure 3.2** for in case of NND plasma with electrons number density ($1 \times 10^{25} - 6 \times 10^{25}m^{-3}$) at a fixed number density of positrons ($n_{p0} = 10^{24}m^{-3}$), where the vertical axis (horizontal axis) shows the normalized wave frequency, $\omega = \frac{\omega}{\omega_{ci}}$ (normalized wave number, $k = k\lambda_{De}$).

The effect of degeneracy parameter $\sigma(= \frac{T_F}{T})$ on the dispersion of perpendicular MHD wave for NND plasma i.e., $\sigma \leq 1$ is graphically shown in **Figure 3.3**. It has been observed that increase in the values of σ_e, σ_p from 0.02, 0.02 to 0.06, 0.06 will cause to increase the frequency of wave. **Figure 3.4** shows the impact of magnetic field on MHD wave in NND plasma with number density ($n_{e0} = 5 \times 10^{28}m^{-3}$, $n_{p0} = 10^{27}m^{-3}$). It is observed that the phase speed of wave decreases by increasing the value of $\mathbf{B}_0$ from 10^4T to 10^8T. It means that due to increase in $\mathbf{B}_o$, the frequency increases. Similarly, **Figure 3.5** shows the effect of magnetic field on MHD wave in ND plasma

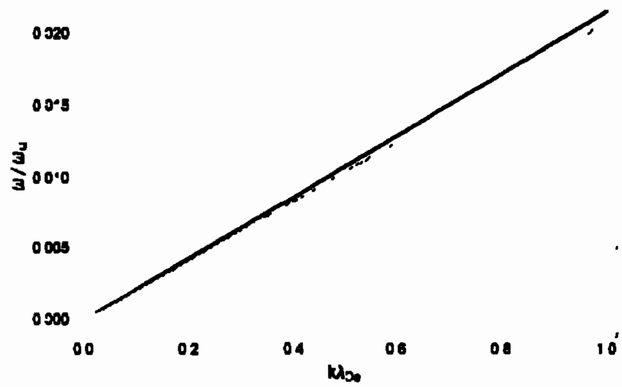

Figure 3.4: Dispersion diagram given by Equation (3.30) for perpendicular propagation mode of MHD wave in NND plasma with different magnetic fields, i.e., $B_0 = 10^4 Tesla$ (solid black line), $10^8 Tesla$ (solid red line). Other parameters are $n_{e0} = 5 \times 10^{28} m^{-3}$, $n_{p0} = 10^{27} m^{-3}$, $\sigma_e = 0.02$ and $\sigma_p = 0.02$.

with number density ($n_{e0} = 5 \times 10^{30} m^{-3}$, $n_{p0} = 10^{30} m^{-3}$) in which the phase speed of wave increases. At lower density the effect of spin magnetization due to magnetic field on the fast MHD mode in NND plasma is dominated and the mode is not altering by the magnetic field upto number density $10^{27} m^{-3}$ and with values higher than this upto $10^{29} m^{-3}$ it is increasing.

It is to be noted that the plasma density is considered low in NND regime as compared to ND regime. The Alfven velocity is modified by the spin effect that affects the dispersion features of the wave and corresponding the temperature degeneracy associated with the NND and ND electrons (and positrons), can also play a vital role in dynamics of magnetoacoustic waves.

In order to study oblique MHD wave in NND and ND quantum plasma we solve the Equations 3.53 and 3.54. All the parameters considered here are the same as taken in **Figures 3.1** and **3.2**. **Figure 3.6** shows the effect of degenerate fugacity related to electrons and positrons, i.e., $\xi_e, \xi_p = 0.1, 0.1 - 0.9, 0.9$ with number density $n_{e0} = 10^{25}$, $n_{p0} = 10^{24} m^{-3}$ in NND magnetoplasma ($\xi \leq 1$) on dispersion of oblique propagation of MHD wave. The increase in the magnitude of frequency is observed for the degree of degeneracy in terms of fugacity. In **Figure 3.7**, the same trend of magnetic field effect on oblique MHD wave in NND plasma is investigated as in **Figure 3.4**.

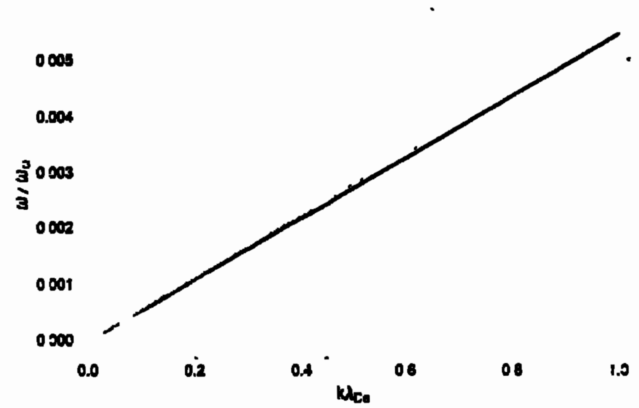

Figure 3.5: Dispersion diagram given by Equation (3.30) for perpendicular propagation mode of MHD wave in NND plasma with different magnetic fields, i.e., $B_0 = 10^4 Tesla$ (solid black line), $10^8 Tesla$ (solid red line). Other parameters are $n_{e0} = 5 \times 10^{30} m^{-3}$, $n_{p0} = 10^{30} m^{-3}$, $\sigma_e = 0.02$ and $\sigma_p = 0.02$.

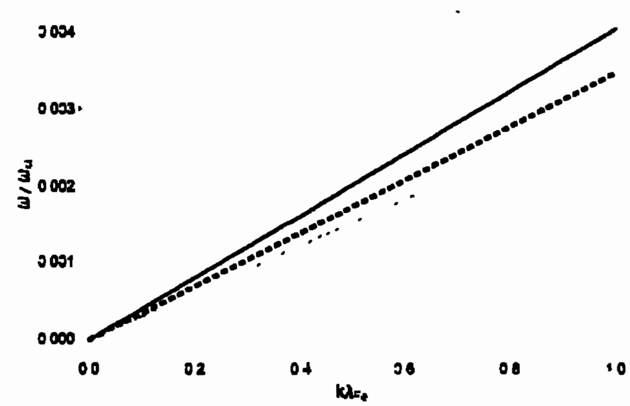

Figure 3.6: Dispersion diagram given by Equation (3.54) for oblique propagation mode of MHD wave in ND plasma with different magnetic fields, i.e., $B_0 = 10^5 Tesla$ (solid black line), $2 \times 10^5 Tesla$ (dashed black line), $3 \times 10^5 Tesla$ (solid red line). Other parameters are $n_{e0} = 5 \times 10^{35} m^{-3}$, $n_{p0} = 10^{34} m^{-3}$, $\sigma_e = 200$ and $\sigma_p = 200$.

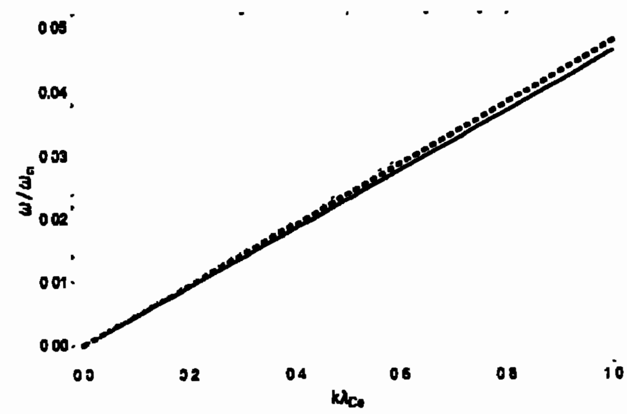

Figure 3.7: Dispersion diagram given by Equation (3.53) for NND plasma of oblique propagation mode of MHD wave for varying quantum degenerate fugacity factor of species, i.e., ξ_e, ξ_p such that ξ_e, ξ_p $= 0.1, 0.1$ (solid black line), $0.5, 0.5$ (dashed black line), $0.9, 0.9$ (solid red line). Other parameters are $\sigma_e = 0.2$, $\sigma_p = 0.2$, $n_{e0} = 10^{25}m^{-3}$, $n_{p0} = 10^{25}m^{-3}$ and $B_0 = 5 \times 10^2 Tesla$.

3.4 Conclusion

In this chapter, we have studied the propagation of low frequency MHD waves with arbitrary degree of temperature and spin degeneracy in e-p-i quantum plasma. Temperature degenerate parameter has been incorporated by means of the pressure term using the F. D Distribution. Starting from quantum hydrodynamics equations, including the effects of temperature degeneracy and spin magnetization of electrons and positrons, a generalized dispersion relation is derived. The electrons (and positrons) are treated as quantum degenerate particles because of their smaller masses, while ions are taken as non degenerate (classical). Further, the generalized dispersion relation has been reduced to get three different modes in case of NND as well as ND plasma with respect to angle of propagation under the low frequency MHD assumptions. The quantum effects cannot affect the parallel propagation ($\theta = 0$) at low-frequency. Three quantum corrections, i.e., temperature degeneracy due to the quantum statistics, Bohm potential arises due to the density fluctuation, and spin magnetization due to spin effects have been investigated. It is observed that number density of positron leads to new modifications in the wave dispersion of MHD modes both in the NND and ND regimes and also the background magnetic field enhances the dispersion mode. It is also noted that the effect of temperature degeneracy in the presence of $1/2-$spin magnetization and Bohm potential modifies the dispersive property of magnetosonic waves in quantum astrophysical dense plasmas. The modified Alfven velocity by spin magnetization affects, the dispersion features of the wave is comparatively less than that of temperature degeneracy. It is found that the arbitrary degree of temperature and spin degeneracy associated with electrons and positrons has a substantial influence on the dynamics of magnetoacoustic waves.

Chapter 4

Magnetosonic Waves in Ion Trapped Semiconductor Chip Plasma

4.1 Introduction

Recently, the quantum plasma has gained much interest due to its potential applications in astrophysical systems (e.g., WD, NS, magnetars, black holes, etc.) at low temperature and high density. As normal plasmas are characterized by regimes of low density at high temperature with negligible quantum effects [ ] but physical systems where plasmas and quantum effects coexist is the noteworthy example of electron gas in an ordinary metal. In case of metal, valence electrons cannot be bound to any specific nucleus, but rather they behave like a gas particles that's why a metal is a good conductor. At ambient temperature and typical density, quantum effects can no longer be ignored. Thus, the electron gas along with lattice ions in a metal makes a real quantum plasma. Similarly, semiconductor physics contributes another possible application of quantum plasmas. Though the electron density in semiconductor materials is comparatively less than in metals, even then quantum effects are investigated due to the emerging of miniaturization of electronic devices. The electron-hole (e-h) plasmas can be produced by interaction of short laser pulses with matter where electrons gained energy during interaction from valence band (V.B) and are excited to conduction band (C.B) via absorption of single (or multi) photon depending upon the band gap energy $E_g(T)$ and that of photon energy. As a result of electrons transition holes are created in V.B. The electrons and holes fluids in semiconductor behave as quantum plasma under the condition $T_{Fe,h} > T_{e,h}$ and obey FD statistics. Moreover, quantum effects become more essential in semiconductors at such a small space scales when the de Broglie wavelengths associated with the electrons (and holes) are comparable to their average inter-particle distances, viz., $d \sim n_0^{-\frac{1}{3}}$ as n_0 is the equilibrium number density. For further manufacturing of modified semiconductor devices, ion-implantation techniques are generally employed in which ions in host materials alter their characteristics through introduction of metal ions such as Fe^+, Cu^+, Ag^+ etc. In the recent past, ion-implanted semiconductor (IIS) plasmas have been explored [, , ,], where quantum effects were studied. Also, Cd^+ ion trapped in semiconductor gallium-arsenide (GaAs) heterostructure chip has been fabricated [ ] which gives an interesting opportunity of three species electron-hole-ion (e-h-i) semiconductor magnetoquantum plasmas to determine their quantum collective effects on dispersive properties of magnetosonic propagation waves. Non-linear property of the wave propagation in the medium can be resulted in the formation of solitary pulses called solitons. The experimental investigations of the acoustic solitons are carried out for a number of systems like Si, Sapphire, MgO, Al_2O_3, GaAs and alpha-quartz materials. Later, Moslem *et al.* [ ] have also applied their theoretical results to quantum semiconductors plasmas like GaAs, GaN, GaSb, and InP.

Several new quantum plasma models [, , ,] have been successfully studied and made progress in investigation of spin magnetization of electrons [ ], kinetic quantum effects due to Fermi particles [ ], non relativistic effects in dense quantum plasmas [ ] immersed in high magnetic field, and spin dynamics in semi-relativistic plasmas [ ]. Similarly, Asenjo [ ] selected the quantum MHD model to investigate the propagation of low frequency magnetosonic waves of two species (electrons and ions) in quantum magnetoplasmas with Bohm potential and spin magnetization energy and discussed the effects of quantum corrections on the dispersion of oblique, perpendicular and parallel modes of propagation. Accordingly it has been examined that temperature degeneracy along with spin-1/2 effect associated with electrons and positrons, has a substantial influence on the dynamics of magnetosonic waves in e-p-i plasmas [ ]. Typically, whenever quantum effects in plasmas are incorporated, the exchange and correlation potential (V_{xc}) effects can no longer be ignored. Actually, this effect was effectively described by density function theory (DFT). A lot of theoretical work has been done on an electron gas with the exchange and correlation potential (V_{xc}) effect (especially in quantum wells) [, , , , , ,]. Therefore, the contribution of this effect along with Fermi degenerate pressure and Bohm potential will reshape the dispersive properties of magnetoacoustic waves in quantum plasmas. The Fermi degenerate pressure at high density dominates over the thermal pressure which supports the high dense objects against the gravitational burst. In such a situation, thermal energy is always less than Fermi energy. According to degeneracy pressure's law, the degenerate pressure is directly related to 5/3 power of density in case of non relativistic and is also related to 4/3 power of density in case of ultrarelativistic [,]. Moreover, Shahid *et al.* [ ] have studied the parallel propagating electromagnetic waves with the help of non relativistic spin quantum fluid theory in e-p plasma to examine spin effect.

In this Chapter, we investigate linear dispersive properties of the low frequency magnetoacoustic waves in spin-1/2 semiconductor quantum magnetoplasmas (GaAs, GaSb, GaN, and InP) taking into account the degenerate relativistic and non relativistic pressure with Bohm potential as well as exchange-correlation potential. A generalized dispersion relation is derived by using the QMHD model including the quantum effects. The layout of the chapter is given as follows: Mathematical formulation is described in Sec.II. Propagation modes are given in Sec.III. Numerical analysis is discussed in its final portion.

4.2 Mathematical Formulation

Consider an e-h-i quantum semiconductor plasma in the presence of applied magnetic field $\mathbf{B}_0(= B_0 \cos\theta\hat{y} + B_0 \sin\theta\hat{z})$ with $\hat{y}$ and $\hat{z}$ being the unit vectors, where θ is the angle subtended by magnetic field strength with wave propagation direction (or wave number) $\mathbf{k}(= k\hat{y})$. The geometry of perturbed EM field is sketched in such a way that electric field ($\mathbf{E}_1 = E\hat{x}$) is considered along X-axis and magnetic field ($\mathbf{B}_1 = -\frac{ckE_1}{\omega}\hat{z}$) is along Z-axis. The electrons (and holes) obey the relativistic degeneracy pressure's law, while ions as ND particles follow the classical's law. The quantum mechanical effect of ion is ignored due to its heavier mass. At equilibrium, background charge neutrality condition can be taken by the relation $n_{i0} + n_{h0} = n_{e0}$ as n_{r0} (in this Chapter) is the unperturbed number density of r^{th} species, $r = i$ (ions), $r = e$ (electrons) and $r = h$ (holes). The amplitude of the oscillations is assumed to be so small that this problem may be solved by employing the linearized equations. In order to investigate the MHD waves (or magnetosonic waves) in semiconductor plasma along Y-axis, we start with that of balance equations of continuity and momentum as

$$\frac{\partial n_r}{\partial t} + n_{r0}(\nabla \cdot \mathbf{V}_r) = 0 \qquad r = i,\ e,\ h \tag{4.1}$$

$$\frac{\partial \mathbf{V}_i}{\partial t} = \frac{q_i}{m_i}\left(\mathbf{E}_1 + \frac{\mathbf{V}_i \times \mathbf{B}_0}{c}\right) - \frac{\nabla P_i}{n_{i0} m_i}, \tag{4.2}$$

$$\frac{\partial \mathbf{V}_e}{\partial t} = \frac{q_e}{m_e^*}\left(\mathbf{E}_1 + \frac{\mathbf{V}_e \times \mathbf{B}_0}{c}\right) - \frac{\nabla P_{Re}}{n_{e0} m_e^*} - \frac{\nabla V_{xce}}{m_e^*} + F_{Qe}, \tag{4.3}$$

and

$$\frac{\partial \mathbf{V}_h}{\partial t} = \frac{q_h}{m_h^*}\left(\mathbf{E}_1 + \frac{\mathbf{V}_h \times \mathbf{B}_0}{c}\right) - \frac{\nabla P_{Rh}}{n_{h0} m_h^*} - \frac{\nabla V_{xch}}{m_h^*} + F_{Qh}. \tag{4.4}$$

In these equations, $\mathbf{V}_r$, n_r and q_r (for subscript, $r = e,\ h,\ i$) are the fluid velocity, perturbed number density and charge of electrons, holes, and ions, respectively. In such a situation the perturbed number is very small as compared to unperturbed number density i.e., $n_r \ll n_{r0}$. Here the effective mass of electrons (holes) is denoted by m_e^* (m_h^*) and the mass of ions is m_i. Regarding to the pressure term, as ions are considered to be non degenerate (classical) so that the pressure gradient for ions in Equation (4.2) becomes as $\nabla P_i = \gamma_i E_{Ti} \nabla n_i$, with γ_i being the polytropic index and $E_{Ti} = k_B T_i$ is the thermal energy of ions. All the quantum corrections are ignored for ions due to its large inertia as compared to electrons (and holes). The electron pressures in the non relativistic and ultrarelativistic limit [, , ,], are

$$P_{NR,e} = \frac{(\pi)^{2/3}\hbar^2}{5m} n_e^{5/3}, \tag{4.5}$$

and

$$P_{UR,e} = \frac{3\hbar c}{4} n_e^{4/3}. \tag{4.6}$$

The relativistic pressures for electrons and holes can be written in a generalized form as

$$P_{R,e,h} \simeq P_{e0,h0} + \frac{2E_{Fe,h}}{3\gamma_{e0,h0}} n_{e,h}. \tag{4.7}$$

Here $E_{Fe,h} \left(= \frac{(3\pi^2 n_{e0,h0})^{2/3}\hbar^2}{2m^*_{e,h}}\right)$ is the Fermi energy and $\gamma_{e0,h0} = \sqrt{1+\zeta^2_{R(e0,h0)}}$ is the relativistic factor due to electrons (and holes) with $\zeta_R = \frac{(3h^3 n_{e0,h0}/8\pi)^{2/3}}{m^*_{e,h}c}$. The fourth term in right side of Equation (4.3) (and Equation (4.4)) denotes the exchange and correlation force ($V_{xc\ e,h}$) due to identical particles i.e., electrons (and holes) in quantum semiconductor with parallel and antiparallel spin-1/2. The contribution of this effect is usually very weak, and is normally neglected. However, it is expected that exchange-correlation effect plays an essential role due to the recent miniaturization of electronic components. The exchange-correlation is a function of electron (and hole) density which can be obtained via the adiabatic local density approximation(ALDA) [, ,] as

$$V_{xc\ e,h} = -0.985 \frac{e^2 n_{e,h}^{1/3}}{\epsilon} [1 + \frac{0.034}{a_B^* n_{e,h}^{1/3}} \ln(1 + 18.37 a_B^* n_{e,h}^{1/3})],$$

where $a_B^* (= \frac{\hbar^2 \epsilon}{m^*_{e,h} e^2})$ is the effect of Bohr atomic radius. The parameter ϵ is the effective dielectric constant of semiconductor. The dielectric constant is a measure of the amount of electric potential energy, in the form of induced polarization that is stored in a given volume of material under the action of an electric field. It is expressed as the ratio of the dielectric permittivity of material to that of a vacuum or dry air. Practically, it is determined as the ratio of the capacity of a capacitor with wood as the dielectric between the plates to that with dry air. Using the Taylor's expansion under the condition $18.37a_B^* n_{e,h}^{1/3} \ll 1$ (i.e., $n_{e,h} = \frac{m_{e,h}^* e^2}{18.37\hbar^2\epsilon} \ll 10^{37}cm^{-3}$), one gets the expression given by

$$V_{xc\,e,h} = -1.6\frac{e^2 n_{e,h}^{1/3}}{\epsilon} + 5.65\frac{\hbar^2}{m_{e,h}^*} n_{e,h}^{2/3}.$$

We have considered the number density $n_{e,h} \leq 10^{20}cm^{-3}$ in our calculations according to the above approximation. The last term in the right side of Equation (4.3) (and Equation (4.4)) can be expressed, as

$$F_{Qe,h} = \frac{\hbar^2}{4n_{e,h0}(m_{e,h}^*)^2}\nabla\left(\nabla^2 n_{e,h}\right) + \frac{2n_{e0,h0}\mu_{e,h}}{\hbar}\nabla(\mathbf{s}\cdot\mathbf{B}_1), \tag{4.8}$$

where the first term in the right side of Equation (4.8) shows the Bohm quantum potential associated with density fluctuation and the next second term corresponds to magnetization energy due to particle spining effect. The parameter $\mu_{e,h}(= \left|\frac{q_{e,h}\hbar}{2m_{e,h}^* c}\right| = \mp\mu_{Be,h})$ describes the magnetic moment for electrons and holes i.e., $\mu_e = -\mu_{Be}$ (for electrons) and $\mu_h = \mu_{Bh}$ (for holes) while s is the spin vector. Here the spin evolution equation may be written, as

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_{e,h}\cdot\nabla\right)\mathbf{s} = \frac{2\mu_{e,h}}{\hbar}(\mathbf{s}\times\mathbf{B}). \tag{4.9}$$

Under the MHD assumption ($\omega < \omega_{ci} < \omega_{ce}, \omega_{ch}$), the spin inertia can be neglected that gives $s \times B = 0$, which has the solution $s = -\frac{1}{2}\hbar\eta(\alpha_{e,h})\hat{B}$. For electrons and holes under the above approximation with assumed geometry, the solution can be more simplified in the following form as

$$\mathbf{s}_{e,h} = -\frac{1}{2}\hbar\eta_{e,h}(\alpha_{e,h})(\cos\theta\hat{y} + \sin\theta\hat{z}), \tag{4.10}$$

In Equation (4.10) $\eta(\alpha_{e,h}) = \tanh(\alpha_{e,h})$ is the Langevin parameter due to the magnetization of a spin distribution in thermodynamic equilibrium, where $\alpha_{e,h} = \frac{\mu_{Be,h}B_0}{K_B T_{Fe,h}}$ and

$T_{Fe,h} \cong \frac{E_{Fe,h}}{K_B} = \frac{(3\pi^2 n_{e,h})^{2/3}\hbar^2}{2K_B m^*_{e,h}}$ is Fermi temperature of degenerate electrons and holes. In case of non degenerate plasma, $T_{Fe,h}$ is replaced by Maxwellian temperature $T_{e,h}$. Using the Maxwell's EM equations $\nabla \times E_1 = -\frac{1}{c}\frac{\partial \mathbf{B}_1}{\partial t}$ and $\nabla \times B_1 = \frac{1}{c}\frac{\partial \mathbf{E}_1}{\partial t} + \frac{4\pi}{c}J_1$ we get the following relation

$$\left(\omega^2 - c^2k^2\right)\mathbf{E}_1 + i4\pi\omega\mathbf{J}_1 = 0. \tag{4.11}$$

Here $\mathbf{J}_1$ is the total current density which can be given by relation $\mathbf{J}_1 = \sum_r q_r n_r \mathbf{v}_r + c\mathbf{J}_{eM} + c\mathbf{J}_{hM}$ where $\mathbf{J}_{eM} = -\nabla \times (2n_e\mu_{Be}\mathbf{s}/\hbar)$ (and $\mathbf{J}_{hM} = \nabla \times (2n_h\mu_{Bh}\mathbf{s}/\hbar)$) is the magnetization spin current of electrons (and holes).

To analyze the linear perturbation in semiconductor quantum plasmas, we assume that all the first order perturbed quantities are proportional to $e^{i(ky-\omega t)}$, here k is the wave number or direction of propagation and ω is taken as the wave frequency. Using the linearization, procedure Equations (4.1-4.4) can be written for ions, electrons and holes as

$$n_r = \frac{n_{r0}k}{\omega}\hat{y}.\mathbf{V}_r, \tag{4.12}$$

$$-i\omega m_i n_{i0}\mathbf{V}_i = en_{i0}\left(\mathbf{E}_1 + \frac{\mathbf{V}_i \times \mathbf{B}_0}{c}\right) - i\left(\gamma_i E_{Ti} k n_i\right)\hat{y}, \tag{4.13}$$

$$\begin{aligned}-i\omega m_e^* n_{e0}\mathbf{V}_e = {} & -en_{e0}\left(\mathbf{E}_1 + \frac{\mathbf{V}_e \times \mathbf{B}_0}{c}\right) - i\frac{2}{3}\left(\frac{kE_{Fe}}{n_{e0}\gamma_{e0}}n_e\right)\hat{y} - i(2E_{Fe}kn_{e0}n_e)V_{xcR_e}\hat{y} \\ & -i\left(\frac{\hbar^2k^3}{4m_e^*}n_e\right)\hat{y} + i\eta(\alpha_e)\mu_{Be}B_1kn_{e0}\sin\theta\hat{y}.\end{aligned} \tag{4.14}$$

$$\begin{aligned}-i\omega m_h^* n_{h0}\mathbf{V}_h = {} & en_{h0}\left(\mathbf{E}_1 + \frac{\mathbf{V}_h \times \mathbf{B}_0}{c}\right) - i\frac{2}{3}\left(\frac{kE_{Fh}}{n_{h0}\gamma_{h0}}n_h\right)\hat{y} - i(2E_{Fh}kn_{h0}n_h)V_{xcR_h}\hat{y} \\ & -i\left(\frac{\hbar^2k^3}{4m_h^*}n_h\right)\hat{y} - i\eta(\alpha_h)\mu_{Bh}B_1kn_{h0}\sin\theta\hat{y}.\end{aligned} \tag{4.15}$$

Here $V_{xcR_{e,h}} = \frac{1}{3}\rho_{e,h} - \frac{2}{3}\delta_{e,h}$ with $\rho_{e,h} = \dfrac{1.6e^2 n_{e0,h0}^{1/3}}{2\epsilon E_{Fe,h}}$ and $\delta_{e,h} = \dfrac{5.65\hbar^2 n_{e0,h0}^{2/3}}{2m_{e,h}^* E_{Fe,h}}$. Solving Equations (4.12-4.15) along with Equation (4.11) we obtain the following generalized dispersion relation for MHD waves.

$$\omega^2 - c^2k^2 = \left(\frac{4\pi e^2 n_{i0}}{m_i}\right)^{1/2}\beta_i + \left(\frac{4\pi e^2 n_{e0}}{m_e^*}\right)^{1/2}\beta_e + \left(\frac{4\pi e^2 n_{h0}}{m_h^*}\right)^{1/2}\beta_h, \tag{4.16}$$

where

$$\beta_i = \frac{\omega^2(1-A_i)}{\omega^2(1-A_i) - \omega_{ci}^2 + \omega_{ci}^2\cos^2\theta A_i}, \tag{4.17}$$

$$\begin{aligned}\beta_e = &\left[1 + \frac{2\eta_e(\alpha_e)\omega_{ce}\mu_B ck^2\sin^2\theta}{e\omega^2(1-A_e)} + \frac{\eta_e^2(\alpha_e)\mu_B^2 c^2k^4\sin^2\theta}{e^2\omega^2(1-A_e)}\left(1 - \frac{\omega_{ce}^2}{\omega^2}\cos^2\theta\right)\right]\\ &\times\left[\frac{\omega^2(1-A_e)}{\omega^2(1-A_e) - \omega_{ce}^2 + \omega_{ce}^2\cos^2\theta A_e}\right],\end{aligned} \tag{4.18}$$

and

$$\begin{aligned}\beta_h = &\left[1 - \frac{2\eta_h(\alpha_h)\omega_{ch}\mu_{Bh} ck^2\sin^2\theta}{e\omega^2(1-A_h)} + \frac{\eta_h^2(\alpha_h)\mu_{Bh}^2 c^2k^4\sin^2\theta}{e^2\omega^2(1-A_h)}\left(1 - \frac{\omega_{ch}^2}{\omega^2}\cos^2\theta\right)\right]\\ &\times\left[\frac{\omega^2(1-A_h)}{\omega^2(1-A_h) - \omega_{ch}^2 + \omega_{ch}^2\cos^2\theta A_h}\right].\end{aligned} \tag{4.19}$$

In the above Equations (4.16-4.19), $\omega_{ci} = \frac{eB_0}{m_i c}$, $\omega_{ce} = \frac{eB_0}{m_e^* c}$ and $\omega_{ch} = \frac{eB_0}{m_h^* c}$ respectively represent the cyclotron frequency of ions, electrons and holes. The dimensionless quantities are given as $A_i = \left(\frac{V_i k}{\omega}\right)^2$ with $V_i = \left(\frac{\gamma_i E_{T_i}}{m_i}\right)^{1/2}$, $A_e = \left[\frac{V_{Fe}^2}{3}\left(\frac{1}{\gamma_{e0}} + V_{xcR_e}\right) + \frac{\hbar^2 k^2}{4m_e^*}\right]\frac{k^2}{\omega^2}$ and $A_h = \left[\frac{V_{Fh}^2}{3}\left(\frac{1}{\gamma_{h0}} + V_{xcR_h}\right) + \frac{\hbar^2k^2}{4m_h^*}\right]\frac{k^2}{\omega^2}$. The relativistic factor ($\gamma_{e0,h0}$), Bohm potential (proportional to $\hbar^2$ order), and exchange correlation parameter ($V_{xcR_{e,h}}$) appear in modified dimensionless quantities A_e (and A_h) while the spin effects appear in $\eta(\alpha_e)$ and $\eta(\alpha_h)$. Thus, dispersion relation (4.16) is significantly modified by relativistic pressure through electron-hole Fermi energies $E_{Fe,h}(= \frac{3\pi^2 n_{e0,h0}\hbar^2}{2m_{e,h}^*})$ along with relativistic factor ($\gamma_{e0,h0}$) and that of contribution of exchange correlation parameter ($V_{xcR_{e,h}}$). Also $V_{Fe,h}$ is the Fermi velocity of electrons and holes given by $V_{Fe,h} = \sqrt{\frac{2E_{Fe,h}}{m^*_{e,h}}}$. In the following, we will discuss the above dispersion relation of MHD waves with different limiting modes in quantum semiconductor plasmas for the above mentioned quantum effects.

4.3 Propagation Modes

In order to consider the wave propagation perpendicular to magnetic field in a relativistically degenerate e-h-i quantum semiconductor plasma, we assume $B_{0y} = 0$ and only incorporate the perpendicular background magnetic field (viz., $B_0 = B_0\hat{z}$), which makes an angle $\theta(= \pi/2)$ with the wave propagation in the y-direction. This would lead to the perpendicular EM waves and one can get a compressional magnetosonic waves. Due to small inertia of electrons (and holes) we may consider the low frequency limit ($\omega^2 \ll \omega_{ce}^2, \omega_{ch}^2$). The quantum corrections (like, relativistic degeneracy, exchange-correlation potential and Bohm quantum potential appear in A_e and A_h) become significant for large value of wave number and therefore the limit used as, $\omega^2 \ll A_e\omega_{ce}^2, A_h\omega_{ch}^2$. Moreover, neglecting the Hall effects, it is considered, $\omega^2 \ll \omega_{ci}^2$ in such away that $(1 - A_i) \ll \omega_{ci}^2/\ \omega^2$. By incorporating the above mentioned assumptions in Equation (4.16) the following reduced form is obtained as

$$\omega^2 = \frac{c^2k^2}{V_A^2 + c^2}\left[\tilde{V}_A^2 + V_S^2 + \frac{\hbar^2k^2}{4m_i}\left\{\frac{n_{eo}}{m_e^* n_{io}}\left[1 - \eta^2(\alpha_e)\right] + \frac{n_{ho}}{m_h^* n_{io}}\left[1 - \eta^2(\alpha_h)\right]\right\}\right]. \tag{4.20}$$

Equation (4.20) is the dispersion relation of compressional (or fast) MHD modes in e-h-i semiconductor quantum plasmas. Here

$$\tilde{V}_A = V_A\left(1 + \frac{8\pi\eta(\alpha_h)\mu_{Bh}n_{h0}}{B_0} - \frac{8\pi\eta(\alpha_e)\mu_{Be}n_{e0}}{B_0}\right)^{1/2}, \tag{4.21}$$

is the modified Alfven velocity with spin magnetization effects of electron and hole ($\eta_{e,h}$), where V_A is the standard Alfven velocity. Moreover, in Equation (4.20), V_S is the modified acoustic speed including the relativistic degeneracy factor (γ) and exchange correlation parameter (V_{xcR}) given by

$$V_S = \sqrt{V_i^2 + \left(\frac{n_{eo}}{n_{io}}\right)\left(\frac{m_e^*}{m_i}\right)\frac{V_{Fe}^2}{3}(\frac{1}{\gamma_{e0}} + V_{xcR_e}) + \left(\frac{n_{ho}}{n_{io}}\right)\left(\frac{m_h^*}{m_i}\right)\frac{V_{Fh}^2}{3}(\frac{1}{\gamma_{h0}} + V_{xcR_h})}. \tag{4.22}$$

Using Equations (4.21),(4.22) with the value of $V_{Fe,h}$ in Equation (4.20) we get

$$\omega^2 = k^2\left[V_A^2(1 + \frac{8\pi\eta(\alpha_h)\mu_{Bh}n_{h0}}{B_0} - \frac{8\pi\eta(\alpha_e)\mu_{Bh}n_{e0}}{B_0}) + V_i^2 + \frac{2}{3}\frac{E_{Fe}}{m_i}\left(\frac{n_{eo}}{n_{io}}\right)(\frac{1}{\gamma_{e0}} + V_{xcR_e})\right.$$
$$+\frac{2}{3}\frac{E_{Fh}}{m_i}\left(\frac{n_{ho}}{n_{io}}\right)(\frac{1}{\gamma_{h0}} + V_{xcR_h}) + \frac{2}{3}\frac{E_{Fe}}{m_i}\left(\frac{n_{eo}}{n_{io}}\right)(\frac{1}{\gamma_{e0}} + V_{xcR_e})$$
$$\left.+\frac{2}{3}\frac{E_{Fh}}{m_i}\left(\frac{n_{ho}}{n_{io}}\right)(\frac{1}{\gamma_{h0}} + V_{xcR_h}) + \frac{\hbar^2k^2}{4m_i}\left\{\frac{n_{eo}}{m_e^*n_{io}}[1-\eta^2(\alpha_e)] + \frac{n_{ho}}{m_h^*n_{io}}[1-\eta^2(\alpha_h)]\right\}\right]. \tag{4.23}$$

Equation 4.20 (and 4.23) is the dispersion relation of magnetosonic waves in quantum semiconductor plasma including three species (e-h-i). This relation contains $\eta(\alpha_{e,h})$ (the spin magnetization effect due to spin-$\frac{1}{2}$ effect). From mathematical relation (4.21) it is obvious that Alfven velocity increases due to spin magnetization effect of holes but the converse effect is observed for electrons showing that there is a constant correction to Alfven velocity and as a result it produces a contribution to linear portion of the propagation wave mode. The other correction is due to the Bohm quantum potential that produces a contribution (of order k^4) to the linear part of the wave. Third correction may be added because of relativity factor ($\gamma_{e0,h0}$) of quantum plasmas that causes to modify the acoustic speed and the fourth correction comes from the exchange correlation parameter as $V_{xcR_{e,h}}$ bringing a linearly change in dispersive properties of magnetoacoustic waves. The above quantum corrections have more substantial influence in semiconductor quantum plasmas with high number density and low temperature and parametrically are shown in the section of numerical analysis and discussion.

Using the following dimensionless variables

$$\omega \to \frac{\omega}{\omega_{ci}}, \quad k \to k\lambda_{Fe}.$$

The dimensionless form of the dispersion relation (4.23) can be written as

$$\omega^2 = k^2\lambda_{Fe,h}^2\left(V_{An}^2 + V_{Sn}^2 + k^2 H_{e,h}^2\Psi_n\right). \tag{4.24}$$

Here

$$V_{An} = \frac{\tilde{V}_{An}}{V_F} = V_{AFn}\sqrt{1 + \gamma_{onh}\eta_h(\alpha_h) - \gamma_{Pne}\eta_e(\alpha_e)}, \tag{4.25}$$

where $\gamma_{onh} = \frac{8\pi\mu_B n_{h0}}{B_0}$ and $V_{AFn} = \frac{V_A}{V_F}$.

$$V_{Sn} = \frac{V_{Sn}}{V_F}, \quad V_S (= \sqrt{V_{in}^2 + (\frac{1}{\gamma_{h0}} + V_{xcR_h})\beta_{h1} + (\frac{1}{\gamma_{e0}} + V_{xcR_e})\beta_{e1}}). \tag{4.26}$$

Here $V_{in} = \frac{V_i}{V_F}$, $\beta_{e1} = \frac{1}{3}\left(\frac{n_{e0}}{n_{io}}\right)\left(\frac{m_e^*}{m_i}\right)$ and $\beta_{h1} = \frac{1}{3}\left(\frac{n_{h0}}{n_{io}}\right)\left(\frac{m_h^*}{m_i}\right)$.

In the above representation (4.24), $H_{e,h} = \dfrac{\hbar^2}{\sqrt{4m_{e,h}^* m_i \lambda_F^2 V_F^2}}$ is a dimensionless parameter which arises due to the collective electron (and hole) tunnelling effect through the so-called Bohm potential and $\Psi_n = \delta_h[1 - \eta_h^2(\alpha_h)] + \delta_e[1 - \eta_e^2(\alpha_e)]$.

For oblique MHD propagation mode say for example; $\theta = \frac{\pi}{4}$ and by using the above-mentioned low frequency assumptions we get the following relation after some steps of algebraic manipulation:

$$\frac{c^2k^2}{\omega^2}\left[1 - \frac{2\pi n_{h0}\mu_{Bh}^2\eta^2(\alpha_h)}{\frac{2E_{Fh}}{3}(\frac{1}{\gamma_{h0}} + V_{xcR_h}) + \frac{\hbar^2k^2}{4m_h^*}} + \frac{2\pi n_{e0}\mu_{Be}^2\eta^2(\alpha_e)}{\frac{2E_{Fe}}{3}(\frac{1}{\gamma_{e0}} + V_{xcR_e}) + \frac{\hbar^2k^2}{4m_e^*}}\right] = \frac{c^2}{V_A^2}\left(\frac{\omega^2 - V_{Ti}^2k^2}{\omega^2 - V_{Ti}^2k^2/2}\right)$$
$$+\frac{8\pi n_{h0}\mu_{Bh}\eta(\alpha_h)c^2}{B_0\{\frac{2E_{Fh}}{3m_h^*}(\frac{1}{\gamma_{h0}} + V_{xcR_h}) + \frac{\hbar^2k^2}{4m_h^{*2}}\}} - \frac{8\pi n_{e0}\mu_{Be}\eta(\alpha_e)c^2}{B_0\{\frac{2E_{Fe}}{3m_e^*}(\frac{1}{\gamma_{e0}} + V_{xcR_e}) + \frac{\hbar^2k^2}{4m_e^{*2}}\}}, \tag{4.27}$$

Equation (4.27) shows the oblique propagation mode that depends on all the four quantum corrections. In a more simplified form this equation can be written as

$$a\omega^4 - b\omega^2 + d = 0. \tag{4.28}$$

The coefficients in equation (4.28) are given as

$$a = \frac{c^2}{V_A^2} + \frac{8\pi n_{h0}\mu_{Bh}\eta(\alpha_h)c^2}{B_0\left\{\frac{2}{3}\frac{E_{Fh}}{m_h^*}(\frac{1}{\gamma_{h0}} + V_{xcR_h}) + \frac{\hbar^2k^2}{4m_h^{*2}}\right\}} - \frac{8\pi n_{e0}\mu_{Be}\eta(\alpha_e)c^2}{B_0\left\{\frac{2}{3}\frac{E_{Fe}}{m_e^*}(\frac{1}{\gamma_{e0}} + V_{xcR_e}) + \frac{\hbar^2k^2}{4m_e^{*2}}\right\}}, \tag{4.29}$$

$$b = \frac{V_i^2k^2}{2}\left[\frac{8\pi n_{h0}\mu_{Bh}\eta(\alpha_h)c^2}{B_0\left\{\frac{2}{3}\frac{E_{Fh}}{m_h^*}\left(\frac{1}{\gamma_{h0}} + V_{xcR_h}\right) + \frac{\hbar^2k^2}{4m_h^{*2}}\right\}} - \frac{8\pi n_{e0}\mu_{Be}\eta(\alpha_e)c^2}{B_0\left\{\frac{2}{3}\frac{E_{Fe}}{m_e^*}\left(\frac{1}{\gamma_{e0}} + V_{xcR_e}\right) + \frac{\hbar^2k^2}{4m_e^{*2}}\right\}}\right] + \frac{c^2k^2V_i^2}{V_A^2}$$
$$+c^2k^2\left[1 - \frac{2\pi n_{h0}\mu_{Bh}^2\eta^2(\alpha_h)}{\frac{2}{3}E_{Fh}\left(\frac{1}{\gamma_{h0}} + V_{xcR_h}\right) + \frac{\hbar^2k^2}{4m_h^*}} + \frac{2\pi n_{e0}\mu_{Be}^2\eta^2(\alpha_e)}{\frac{2}{3}E_{Fe}\left(\frac{1}{\gamma_{e0}} + V_{xcR_e}\right) + \frac{\hbar^2k^2}{4m_e^*}}\right], \tag{4.30}$$

and

$$d = \frac{c^2k^4V_i^2}{2}\left(1 - \frac{2\pi n_{h0}\mu_{Bh}^2\eta^2(\alpha_h)}{\frac{2}{3}E_{Fh}(\frac{1}{\gamma_{h0}} + V_{xcR_h}) + \frac{\hbar^2k^2}{4m_h^*}} + \frac{2\pi n_{e0}\mu_{Bh}^2\eta^2(\alpha_e)}{\frac{2}{3}E_{Fe}(\frac{1}{\gamma_{e0}} + V_{xcR_e}) + \frac{\hbar^2k^2}{4m_e^*}}\right). \tag{4.31}$$

Equation (4.28) is in the form of biquadratic equation and one can obtain the following equation

$$\omega = \sqrt{\frac{b \pm (b^2 - 4ad)^{1/2}}{2a}}, \tag{4.32}$$

From the above equation (4.32), it is obvious that dispersion relation of oblique MHD wave in semiconductor quantum plasmas depends on the spin effect, relativistic factor, Bohm potential and exchange correlation, which cause to bring interesting features in dispersive property of such a type of wave mode.

Similarly,the dimensionless form of the dispersion relation (4.27) with $k(=k\lambda_{Fe})$ can be written as

$$\acute{a}\omega^4 - \acute{b}\omega^2 + \acute{c} = 0, \tag{4.33}$$

$$\acute{a} = V_{AFn}^{-2} + \frac{\gamma_{onh}\eta_h(\alpha_h)}{\frac{2}{3}(\frac{1}{\gamma_{h0}} + V_{xcR_h}) + H_h^2} - \frac{\gamma_{pne}\eta_e(\alpha_e)}{\frac{2}{3}(\frac{1}{\gamma_{e0}} + V_{xcR_e}) + H_e^2}, \tag{4.34}$$

$$\begin{aligned} \acute{b} &= \frac{k^2 V_{Tin}^2}{2}\left(\frac{\gamma_{onh}\eta_h(\alpha_h)}{\frac{2}{3}(\frac{1}{\gamma_{h0}} + V_{xcR_h}) + H_h^2} - \frac{\gamma_{pne}\eta_e(\alpha_e)}{\frac{2}{3}(\frac{1}{\gamma_{e0}} + V_{xcR_e}) + H_e^2}\right) + k^2 V_{Tin}^2 \\ &+k^2\left(1 - \frac{\acute{\gamma}_{onh}\eta_h(\alpha_h)}{\frac{2}{3}(\frac{1}{\gamma_{h0}} + V_{xcR_h}) + H_h^2} - \frac{\acute{\gamma}_{one}\eta_e(\alpha_e)}{\frac{2}{3}(\frac{1}{\gamma_{e0}} + V_{xcR_e}) + H_e^2}\right), \end{aligned} \tag{4.35}$$

and

$$\acute{c} = \frac{k^4 V_{Tin}^2}{2}\left(1 - \frac{\acute{\gamma}_{onh}\eta_h(\alpha_h)}{\frac{2}{3}(\frac{1}{\gamma_{h0}} + V_{xcR_h}) + H_h^2} - \frac{\acute{\gamma}_{one}\eta_e(\alpha_e)}{\frac{2}{3}(\frac{1}{\gamma_{e0}} + V_{xcR_e}) + H_e^2}\right), \tag{4.36}$$

where $\acute{\gamma}_{onh} = \frac{2\pi n_{h0}\mu_B^2}{m_h^* V_A^2}$, and $\acute{\gamma}_{one} = \frac{2\pi n_{e0}\mu_B^2}{m_e^* V_A^2}$. Equation (4.33) is in the form of biquadratic equation and can be solved as

$$\omega^2 = \frac{\acute{b} \pm \sqrt{\acute{b}^2 - 4\acute{a}\acute{c}}}{2\acute{a}}. \tag{4.37}$$

4.4 Results and Discussion

For a quantitative analysis, we have solved dispersion relation (4.16) for perpendicular and oblique modes of MHD waves with respect to external uniform magnetic field in quantum semiconductor plasmas. This type of plasma is considered to be composed of three species (ions, electrons and holes). In order to further study the complete picture of quantum effects (including spin magnetization, Bohm quantum potential and exchange correlation along with relativistic degeneracy) on dispersive properties, Equations (4.23) and (4.32) are graphically represented between ω and k with the help of numerical values of magnetized quantum plasmas system. Here we have to apply our results to four cases of compound semiconductors at room temperature $T = 300\mathring{K}$ with typical parameters [, ,] given as:

(a) GaAs with $n_0 = 4.7 \times 10^{16} cm^{-3}$, $\epsilon = 12.80$, $m_e^* = 0.067 m_e$, and $m_h^* = 0.5 m_e$, (b) GaSb with $n_0 = 1.6 \times 10^{17} cm^{-3}$, $\epsilon = 15.69$, $m_e^* = 0.047 m_e$, and $m_h^* = 0.4 m_e$, (c)

GaN with $n_0 = 5.0 \times 10^{19} cm^{-3}$, $\epsilon = 11.30$, $m_e^* = 0.13 m_e$, and $m_h^* = 1.3 m_e$, (d) InP with $n_0 = 5.7 \times 10^{17} cm^{-3}$, $\epsilon = 12.6$, $m_e^* = 0.077 m_e$, and $m_h^* = 0.6 m_e$.

Consider ion concentration in above electron-hole compound semiconductors (according to charge neutrality condition) in the presence of applied magnetic field $[10^2 - 10^8 G]$. Using the typical values, we can find out other parameters in case of GaAs such as the electron Fermi energy $\varepsilon_{Fe} = 1.13 \times 10^{-14} ergs$, electron thermal energy $\varepsilon_{Te} = 4.14 \times 10^{-14} ergs$, the rest effective mass energy $m_e^* c^2 = 5.49 \times 10^{-8} ergs$ and the magnetization energy (at $10^2 G$) $\mu_{Be} B_0 = 9.27 \times 10^{-19} ergs$ and similarly the same energies calculated are given for case of holes in InP such as the Fermi energy $\varepsilon_{Fh} = 4.31 \times 10^{-14} ergs$, the thermal energy $\varepsilon_{Th} = 4.14 \times 10^{-14} ergs$, the rest effective mass energy $m_h^* c^2 = 1.06 \times 10^{-6} ergs$ and the magnetization energy (at $10^2 G$) $\mu_{Bh} B_0 = 9.27 \times 10^{-19} ergs$. These results show that $\mu_{Be,h} B_0 \ll \varepsilon_{Fe,h}, \varepsilon_{Te,h} \ll m_{e,h}^* c^2$. It may be noted that quantum spin effects can be minimized through this condition but magnetization due to higher values of magnetic fields can even have influence on dispersive characteristics of magnetoacoustic waves. In the following, we will numerically show the MHD waves in magnetized compound semiconductor quantum plasmas. Other parameters in *cgs* system are $c = 3 \times 10^{10} cm/sec$, $m_i = 1.67 \times 10^{-24} g$, $m_e = 9.1 \times 10^{-28} g$, $\hbar = 1.05 \times 10^{-27} erg - \text{sec}$, $k_B = 1.38 \times 10^{-16} erg/\mathring{K}$, and $e = 4.8 \times 10^{-10} statc$.

Here we have considered the compound semiconductors (GaAs, GaSb, GaN and InP). Compound semiconductors are made from two or more elements. Most compound semiconductors are from combinations of elements from Group III and GroupV of the Periodic Table of the Elements (GaAs, GaP, InP, etc.). Other compound semiconductors are made from Groups II and VI (CdTe, ZnSe, etc.). It is also possible to use different elements from within the same group (IV), to make compound semiconductors such as SiC. In recent years, however, the cost of manufacturing compound semiconductors has come down. It is still much higher than silicon, but at the same time, the special properties of these crystals have become more important for certain applications. Because of their fundamental material properties, compound semiconductors can do things that simply aren't possible with silicon. During manufacturing of compound semiconductor, the number density can be increased comparatively to

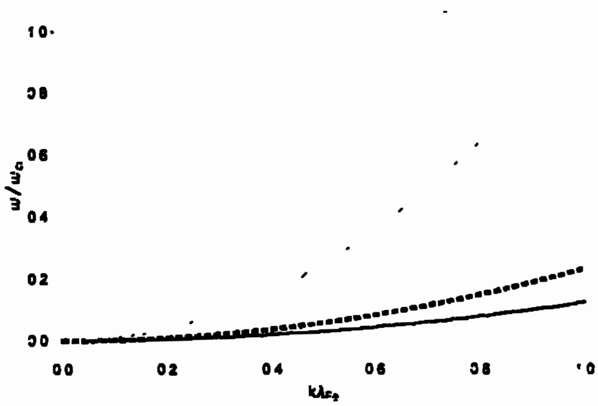

Figure 4.1: Dispersion diagram of relation (4.23) for perpendicular propagation mode of MHD waves in magnetized semicoductor plasmas (GaAs, GaSb and InP) at $B_0 = 10^2G$ is plotted between wave frequency and wavenumber. The number density of electrons (and holes) for these semiconductors are taken as ($n_{e0} = 4.7 \times 10^{16}cm^{-3} \rightarrow 5.7 \times 10^{17}cm^{-3}$), (and $n_{h0} = 5 \times 10^{15}cm^{-3} \rightarrow 3.7 \times 10^{17}cm^{-3}$) and accordingly GaAs (solid black curve), GaSb (dashed black curve), and InP (solid red curve).

silicon according to their properties.

The numerical values obtained for relativistic factor of electron (γ_{e0}) in GaAs, GaSb, GaN and InP are 1.00011, 1.00023, 1.0058 and 1.00022, respectively, which show that plasma with number density below than $n = 10^{16}cm^{-3}$ is almost completely nonrelativistic while relativistic degeneracy starts in plasma with number density above of this value as relativistic factor is density dependent function. The Alfven speed V_A for the above semiconductors immersed in constant applied magnetic field (10^4G) are calculated as in the range of $4.88 \times 10^5cm/s - 1.06 \times 10^7cm/s$. Similarly, Alfven Speed for different applied magnetic fields 10G-10^4G with fixed number density of GaAs ($n = 4.7 \times 10^{16}cm^{-3}$) are calculated as $1.33 \times 10^4cm/s - 1.33 \times 10^7cm/s$.

Figure (4.1) shows the graphical representation between normalized wave frequency ω (i.e., $\omega = \frac{\omega}{\omega_{ci}}$) and normalized wavenumber k (i.e., $k = k\lambda_{Fe}$) for different compound semiconductors plasmas (GaAs, GaSb and InP) to study perpendicular MHD waves by using equation (4.23). It is observed that phase velocity increases with increasing number density of plasmas and therefore it is concluded that dispersive properties of magnetosonic waves enhanced with respect to number density.

Figure (4.2) represents the normalized wave frequency ω (i.e., $\omega = \frac{\omega}{\omega_{ci}}$) and normalized wavenumber k (i.e., $k = k\lambda_{Fe}$) for a semiconductor (GaAs) with external applied

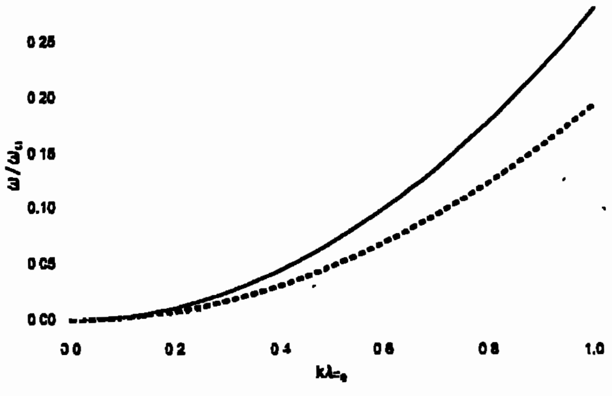

Figure 4.2: Dispersion diagram given by Equation (3 30) for perpendicular propagation mode of MHD wave in NND plasma with different values of quantum temperature degeneracy factor such that σ_e, $\sigma_p = 0.02, 0.02$ (solid black line), $0.04, 0.04$ (dashed black line), $0.06, 0.06$ (solid red line). Other parameters are $n_{e0} = 10^{25}m^{-3}$, $n_{p0} = 10^{25}m^{-3}$ and $B_0 = 5 \times 10^4 Tesla$.

magnetic fields ($10^5 - 1.4 \times 10^5 G$). It is observed that wave frequency is decreased with increasing magnetic field strength.

Figure (4.3) describes the significance of exchange correlation potential on dispersive properties of perpendicular MHD wave by plotting the wave frequency of wave mode against wavenumber. The same parameters in **Figure (4.1)** and **Figure (4.2)** are considered here. Graphically, the exchange correlation effect is more prominent for $B_0 \leq 10^8 G$, which concludes that spin magnetization is independent of this effect because exchange correlations depend only on number densities of electrons/holes in semiconductor plasmas and that of their effective masses. Hence, both the exchange correlation (i.e., $\rho_{e,h}$, $\delta_{e,h}$) and spin magnetization (i.e., $\eta_{e,h}$) effects modify the dispersive properties of MHD waves.

In order to investigate numerically the oblique MHD waves in semiconductor quantum plasmas, such as GaAs and GaN, we solve Equation (4.32). Here all the plasma parameters (in the above Perpendicular propagation case) are taken as the same in the previous figures. The oblique propagation modes are given below.

Figure (4.4) displays the graphical representation of wave frequency ($\frac{\omega}{\omega_{ci}}$) versus the wavenumber ($k\lambda_{Fe}$) for oblique waves in semiconductor (GaAs) with different values of number densities i.e., ($n_{e0} = 4.7 \times 10^{16} cm^{-3} \sim 6.7 \times 10^{16} cm^{-3}$) and with fixed value of $n_{h0} = 2 \times 10^{16} cm^{-3}$ as shown in 4.4 (a). It is clear that the phase velocity decreases

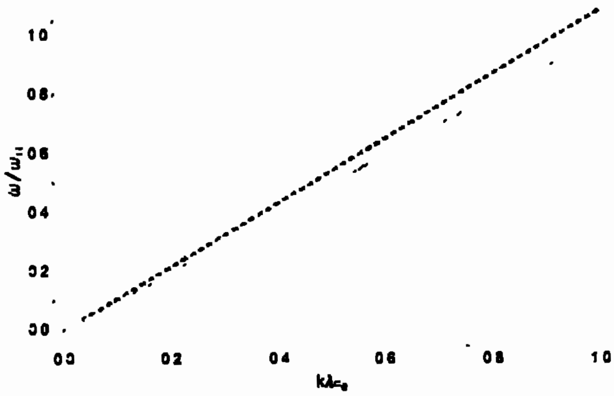

Figure 4.3: Diagram of Dispersion relation (4.23) for perpendicular propagation mode of MHD waves in magnetized semiconductor plasma (GaSb) with $n_{e0} = 1.6 \times 10^{17} cm^{-3}$, $n_{h0} = 7 \times 10^{16} cm^{-3}$ is plotted with exchange correlation (dashed blue line) and without exchange correlation (solid red line).

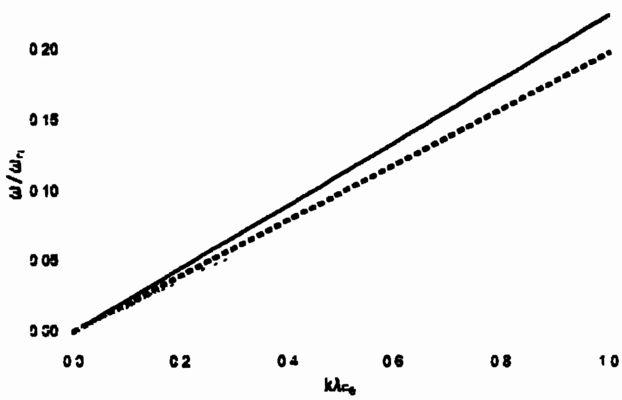

Figure 4.4: Dispersion Diagram (4.32) of oblique propagation mode of magnetoacoustic waves in semiconductor plasma (GaAs) at $B_0 = 10^2 G$ is shown for different number densities of electrons; $n_{e0} = 4.7 \times 10^{16} cm^{-3}$(solid black line), $n_{e0} = 5.7 \times 10^{16} cm^{-3}$(dashed black line), $n_{e0} = 6.7 \times 10^{16} cm^{-3}$ (solid red line) with fixed value $n_{h0} = 2 \times 10^{16} cm^{-3}$.

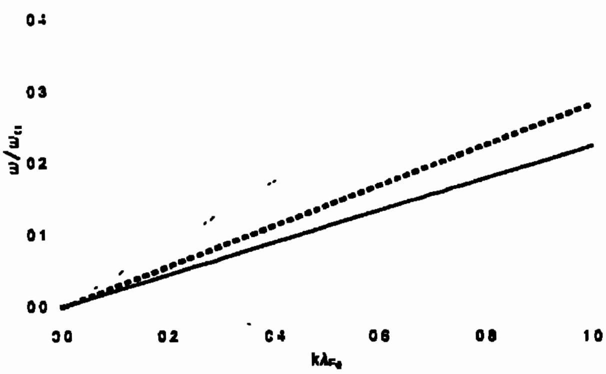

Figure 4.5: Dispersion diagram (4.32) of oblique propagation mode of magnetoacoustic waves in semiconductor plasma (GaAs) at $B_0 = 10^2 G$ is shown for different number densities of holes; $n_{h0} = 2 \times 10^{16} cm^{-3}$ (solid black line), $n_{h0} = 3 \times 10^{16} cm^{-3}$ (dashed black line), $n_{h0} = 4 \times 10^{16} cm^{-3}$ (solid red line) with fixed value $n_{e0} = 4.7 \times 10^{16} cm^{-3}$ and other parameters are taken the same as in Figure (4.1).

with increasing number density of electrons while the trend of graph for different values of holes density n_{h0} is the same but opposite to that of n_{e0} as shown in 4.4 (b). It is revealed from **Figure** (4.5) that by increasing the magnetic field from 1×10^2 G to $3 \times 10^2 G$, the dispersive properties of the oblique waves enhance.

4.5 Conclusion

To summarize, we have investigated the low frequency MHD waves in quantum semiconductor plasmas with quantum effects (Bohm potential, spin magnetization energy, exchange correlation potential and relativistic degeneracy). A modified dispersion relation is derived by using the QMHD model and further its reduced forms are also obtained for propagation modes of magnetosonic waves in the presence of external magnetic field. Relativistic degeneracy term (γ_{e0}) is numerically calculated for some compound semiconductors plasmas as it is related to density but due to its smaller numerical value, it could not be shown graphically. However, it is hoped that semiconductor quantum plasmas with relativistic degeneracy factor due to high degree density miniaturization in electronic components will have a crucial role in future. It is found that the spin magnetization degeneracy and exchange correlation potential along with

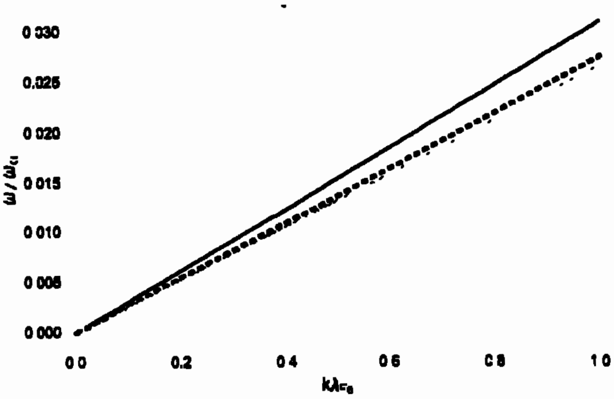

Figure 4.6: Dispersion diagram (4.32) of oblique propagation mode of magnetoacoustic waves in semiconductor plasma (GaN) is shown for different magnetic fields i.e., $B_0 = 1 \times 10^2 G$ (solid black line), $B_0 = 2 \times 10^2 G$ (dashed black line) and $B_0 = 3 \times 10^2 G$ (solid red line), where $n_{e0} = 5.0 \times 10^{19} cm^{-3}$, $n_{h0} = 3.0 \times 10^{19} cm^{-3}$ and other parameters are taken the same as in Figure (4.1).

relativistic degeneracy associated with electrons (and holes) have a substantial influence on the dynamics of magnetosonic waves. We have applied our theoretical results to four kinds of semiconductors plasmas; namely, GaAs, GaSb, GaN, and InP. The present investigation reveals that the characteristics of the linear MHD waves in each semiconductor are different depending upon the typical plasma parameters.

Actually, the current model is beneficial in the relevance and interest of scientific community in context of practical and simulation like studies. When the number density is increased upto $10^{16} cm^{-3}$ then the quantum diffraction and exchange correlation like terms which are number density dependent are increasing and overall affecting the fast mode significantly. At this particular (and above it) value the magnetic field effect upto 10^5 G is constant and does not make any further variation in this mode. Beyond 10^5 G, i.e., upto 10^8 G we then observe the variation in this mode, which is signeficant for simulation like studies, but practically upto this time at given semiconductor magnetic field, it is not useful. On the other hand when we consider the number density below of order $10^{16} cm^{-3}$ (e.g., at $10^{12} cm^{-3}$) then the quantum diffraction and exchange correlation like terms may be ignored and we can easily observe graphically the variation in low magnetic field i.e., even at below of 10^3 -10^5 G. We can also say that the quantum effects (including of the exchange-correlation effects and the quantum Bohm potential) was seen to cause an increase in the phase speed of MHD waves.

Moreover the results are also beneficial to understand the energy transport in semiconductor plasma in the presence of perpendicular magnetic field. Although they may also be applicable for understanding the dynamics of semiconductor plasma to produce high power high band-width devices in analogy, which are contrast to the existing gas plasma devices.

Appendix A

Derivations

A.1 Derivation of Equations (2.17 and 2.28)

As we know

$$\nabla P = C_{ie}^2 \nabla \rho_m, \tag{A.1}$$

here $C_{ie}^2 = (\frac{dP}{d\rho_m}) = (V_i^2 + V_e^2)$ is the modified quantum acoustic velocity which is basically a new type of sound velocity and is duly modified by Landau diamagnetic levels with ion thermal velocity $V_i(=\sqrt{\frac{K_B T_i}{m_i}})$ and modified Fermi velocity of electron $V_e(=\sqrt{\frac{n_0^2 \pi^4 \hbar^4}{m_i m_e e^2 H_0^2}})$. Linearize Equation (A.1) and then differentiate w.r.t time (t) we get

$$\nabla \frac{\partial P}{\partial t} = C_{ie}^2 \nabla (\frac{\partial}{\partial t} \rho_{m1}) = \rho_{m0} C_{ie}^2 \mathbf{k}(\mathbf{k} \cdot \mathbf{v}_1) \tag{A.2}$$

From linearized continuity equation with $\rho_m = \rho_{m0} + \rho_{m1}$ we have $\frac{\partial}{\partial t}\rho_{m1} = -\rho_{m0}\,(\nabla \cdot \mathbf{v}_1)$ which makes Equation (A.2)

$$\nabla \frac{\partial P}{\partial t} = -\rho_{m0} C_{ie}^2 \nabla\,(\nabla \cdot \mathbf{v}_1) = \rho_{m0} C_{ie}^2 \mathbf{k}(\mathbf{k} \cdot \mathbf{v}_1). \tag{A.3}$$

Now Consider the Bohm potential term as

$$T = \frac{\hbar^2}{2 m_i m_e} \nabla \left[\frac{\nabla^2 \sqrt{\rho_m}}{\sqrt{\rho_m}} \right].$$

By using linearized continuity equation and differentiate the above equation w.r.t time (t) one can obtain

$$\frac{\partial T}{\partial t} = -\mathbf{k}\frac{\pi^4 k^2}{4m_i m_e}(\mathbf{k}\cdot\mathbf{v}_1). \tag{A.4}$$

Also,

$$\mathbf{J}\times\mathbf{H} = (\nabla\times\mathbf{H})\times\mathbf{H}.$$

Differentiating again the last one eaquation w.r.t time (t) we can get the following relation

$$\frac{\partial}{\partial t}(\mathbf{J}\times\mathbf{H}) = \mathbf{V}_A\times[\nabla\times\{\nabla\times(\mathbf{v}_1\times\mathbf{V}_A)\}].$$

The above equation can be written in simplest form by using the vector identity rule and plane wave solution as

$$\frac{1}{\rho_{m0}}\frac{\partial}{\partial t}(\mathbf{J}\times\mathbf{H}) = [(\mathbf{V}_A\cdot\mathbf{v}_1)\mathbf{k}-(\mathbf{V}_A\cdot\mathbf{k})\mathbf{v}_1+(\mathbf{k}\cdot\mathbf{v}_1)\mathbf{V}_A](\mathbf{k}\cdot\mathbf{V}_A). \tag{A.5}$$

The spin density force in linearized form is given as under

$$\mathbf{F}_{spin} = \mu_0 M(\nabla\mathbf{H}) = n\mu_B\mu_0\beta(\nabla H),$$

Differentiate w.r.t time (t) and then dividing the above equation by ρ_{m0} we get

$$\frac{1}{\rho_{m0}}\frac{\partial}{\partial t}(\mathbf{F}_{spin}) = \beta(\frac{\mu_B}{m_i})\frac{\partial}{\partial t}(\nabla H) = \beta H_0(\frac{\mu_B}{m_i})\mathbf{k}(\mathbf{k}\cdot\mathbf{v}_1). \tag{A.6}$$

Using the values from Equations (A.3-A.5) in Equation (2.14) one can obtain

$$\omega^2\mathbf{v}_1 - \mathbf{k}\left(C_{ie}^2+V_A^2\right)(\mathbf{k}\cdot\mathbf{v}_1) + [(\mathbf{V}_A\cdot\mathbf{v}_1)\mathbf{k}-(\mathbf{V}_A\cdot\mathbf{k})\mathbf{v}_1+(\mathbf{k}\cdot\mathbf{v}_1)\mathbf{V}_A](\mathbf{k}\cdot\mathbf{V}_A)$$
$$-\frac{\hbar^2 k^2}{4m_i m_e}\mathbf{k}(\mathbf{k}\cdot\mathbf{v}_1) = 0.$$

By Substituting Equations (A.3-A.6) in Equation (2.23) we can get

$$\omega^2\mathbf{v}_1 - \mathbf{k}\left(C_{ie}^2 + V_A^2\right)(\mathbf{k}\cdot\mathbf{v}_1) + \left[(\mathbf{V}_A\cdot\mathbf{v}_1)\,\mathbf{k} - (\mathbf{V}_A\cdot\mathbf{k})\,\mathbf{v}_1 + (\mathbf{k}\cdot\mathbf{v}_1)\,\mathbf{V}_A\right](\mathbf{k}\cdot\mathbf{V}_A)$$
$$-\frac{\hbar^2k^2}{4m_im_e}\mathbf{k}\,(\mathbf{k}\cdot\mathbf{v}_1) + \beta H_0(\frac{\mu_B}{m_i})\mathbf{k}\,(\mathbf{k}\cdot\mathbf{v}_1) = 0.$$

Using the non dimension parameters one can easily derive Equations (2.17 and 2.28).

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