Compressive Sensing Techniques for Detection and Estimation of Parameters in MIMO FDA Radars

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Dedicated

To my Late parents,

Wife, children, and teachers for

Their guidance and specially to

Dr Raja Asif and

Dr Naveed for their support

ABSTRACT

Different Radars technologies has been developed with each having their own advantage. Like Phased array radars are used to circumvent the limitation due to the mechanical radars. Using beam steering it has the additional advantage of array gain. However, it cannot remove range base interferences as well as clutters. Frequency diverse array (FDA) radars have been proposed not for a long time ago. It creates beam pattern which is range dependent. Similarly, Multiple Input Multiple Output (MIMO) radars have also been recently introduced where waveform diversity is exploited, and additional advantages have been achieved. However, the primary task is estimation of the parameters. In term of radars range and angle of a target is the utmost important parameter to be estimated.

The estimation of the parameter depends upon the number of samples as well as noise level. In this dissertation we exploit the application of compressive sensing technique for MIMO FDA radars. We establish a method to cast the problem of parameter estimation into compressive sensing framework. First the angle estimation using compressive sensing is studied. The grid mismatch problem is also addressed. An optimization method is developed for finding the optimal grid resolution for the estimation of the source angle. Different array configurations are considered for one dimension as well as two-dimension estimation. Compressive sensing using single and multiple snap shot is also considered. A preprocessing stage is considered to make the algorithm robust against Gaussian as well as Impulsive noise.

The technique developed is used in MIMO FDA radar for the estimation of the range and angle. Since range and angle information is coupled in MIMO FDA radar, double pulse method is considered to decouple them. Sub array structure is used in receive domain. Using compressive sensing the range and angle information is extracted.

List of Publications

Hamid Ali Mirza, Muhammad Asif Zahoor Raja, Naveed Ishtiaq Chaudhary, Ijaz Mansoor Qureshi, Aqdas Naveed Malik, " A Robust Multi Sample Compressive Sensing Technique for DOA Estimation Using Sparse Antenna Array", IEEE Access (Volume: 8), 140848 – 140861, 23 July 2020

Hamid Ali Mirza, Muhammad Asif Zahoor Raja, Naveed Ishtiaq Chaudhary, Ijaz Mansoor Qureshi, Aqdas Naveed Malik," A New Computing Paradigm for Off-Grid Direction of Arrival Estimation Using Compressive Sensing", Wireless Communications and Mobile Computing, Hindawi, 25 Aug 2020.

Hamid Ali Mirza, M.M.Ahmed "Interference rejection capability of cross dipole antenna array system" 2006 International Conference on Advances in Space Technologies, 20 February 2007

Hamid Ali Mirza, Ijteba-ul-Hasnain Shah, Fahim Ahmed Abbasi, Muhammad Amir Shafiq, "Hardware Implementation of Analog Beamformer for Phased Array Radar (PAR)", Proceedings of 2012 9th International Bhurban Conference on Applied Sciences & Technology (IBCAST), 03 April 2012.

Amina Khalid; Ijtebah Ul Hasnain Shah; **Hamid Ali Mirza**; Shahzad Amin Sheikh," Pattern Synthesis of Conformal Antenna Array to Achieve Optimized Amplitude Weights for Null Steering and PSLL Reduction", 14th International Bhurban Conference on Applied Sciences and Technology (IBCAST), 02 March 2017

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List of Abbreviation

AI Aircraft Interception

ATC Air Traffic Controller

AWGN Additive White Gaussian Noise

CW Continuous Wave

CR Cognitive radar

CRLB Cramer Rao lower bound

DOA Direction of arrival

DP Double Pulse

LPI Low probability of intercept

MIMO Multiple input multiple output

MIMO-FDA Multiple input multiple output- frequency diverse array

MSE Mean square error

MUSIC Multiple signal classification

MVDR Minimum variance distortion less response

PAR Phased array radar

RCS Radar cross section

SAR Synthetic aperture radar

SINR Signal to interference plus noise ratio

SLL Side lobe levels

SNR Signal to noise ratio

STAP Space-time adaptive processing

ULA Uniform linear array

CHAPTER 1

INTRODUCTION

The parameter estimation of the target consists of several parameters. However, detection and localization of the target is important and in certain applications it is of very high importance. Which is about estimation of the direction of the target known as direction of arrival estimation and estimation of the range of the target. Direction of arrival (DOA) estimation is an important parameter which is to be estimated in number of applications. It has applications in a number field. Where the aim is to accurately estimate the location of the sources. The problem of parameter estimation is dependent upon number of factors like the number of the sensors, the spacing between the sensors and the noise levels. The parameter estimation which includes range as well as direction of arrival of the source has a great importance in radars. It has application in different type of radars. Like phase array radar, MIMO radars, FDA radars etc. However, the data required for processing the direction of arrival is huge which requires a lot of processing power and memory. Compressive sensing is a technique where the data can be processed without satisfying the Nyquist criteria. Therefore, compressive sensing can be applied for parameter estimation for different radar types and array structure. Effect of different noise type on parameter estimation is considered and method to mitigate in compressive sensing is also considered.

1.1 Background

The range and angle of the source are an important parameter to be estimated. The aim in (DOA) problem is to accurately estimate the location of the sources and range also. However, the estimation of the source location and the range is treated separately [1-3]. There are number of algorithms that can be used for the estimation of the location of the sources [4-8]. Most used algorithms for source localization are "generalized cross correlation" (GCC), "minimum variance distortion less response" (MVDR) and sub-space technique like "Multiple signal classification" (MUSIC) algorithm, "estimation of signal parameter via a rotational variant technique" (ESPIRIT). However, these algorithms require that the sampling to be at Nyquist rate. Otherwise, the performance of the algorithms degrades. Also, the noise has an important role in estimation of parameters. The efforts are made to make the estimation algorithm as much robust possible.

Beamforming methods are also used for the estimation of the location of the sources. However, it relies on the prior knowledge, of the antenna pattern and depends upon the sampling at Nyquist rate and signal to noise ratio (SNR). In practical scenarios, it is possible that the antenna elements may malfunction, and this may lead to sparse antenna arrays. The performance of the algorithms mentioned above degrades using sparse antenna arrays.

Sparse antenna arrays can have also certain advantages. Using sparse antenna, larger aperture can be achieved [9]. Different techniques are used for designing sparse antenna array, some of them are using numerical methods. Similarly, sparse antenna array can be designed utilizing coprime antenna elements [4]. However, if the sparse array is random then there is not much control on the design due to the randomness. That is not desirable in operational scenarios. In certain cases, it is impossible to repair the antenna elements. Therefore, an alternative method is required to reconstruct the antenna pattern.

Compressive sensing (CS) techniques can be used for parameter estimation [10]. The CS technique is used for solving an underdetermined system and has gained lot popularity over the years and plenty of research has been done in this field. This exploits the sparsity of the system. It has application in several fields. According to the CS framework, if a signal is sparse in a certain domain, then it can be reconstructed using only a small number of measurements which are linear.

As most of the real time signals are sparse therefore the signal may be reconstructed using a linear equation with a smaller number of samples. In CS, we try to find the sparse vector that can be viewed as solving the inverse problem. In source localization problem the sources, can be sparse in the spatial domain, that they are not present at every angle, hence due to the sparsity of source in spatial domain the concept of compressive sensing can be applied. Super resolution can be achieved Using CS techniques for estimation of the sources. Which means that sources closer than Rayleigh resolution in the presence of noise can be resolved. Generally, source localization can be done using single or multiple measurements.

It is important to mention that noise plays an important role in estimation techniques. Generally, the distribution of noise is considered Gaussian. However, the signal can be corrupted with impulses in the time domain due to atmospheric noise or faulty receiver. The impulsive noise does not follow Gaussian distribution. For better result it is required that the algorithm to be robust against different noise distribution.

Radar is an active field of study. Several radar technologies have been developed like phased array radar (PAR), Multiple input multiple output (MIMO) radars and frequency diverse array (FDA) radar. These technologies have replaced the conventional mechanical system. One of such technology is FDA radar, which is based on the principle of implementing a small frequency increment across the array elements which is very small. This creates a pattern which is joint in range and angle. This technique can remove range dependent interferences to improve SNR. The beampattern of the mentioned radar is periodic in time as well as range. This has an effect on the localization parameter. Similarly, MIMO radar have been combined with FDA radar called "MIMO-FDA" radar. This can effectively locate targets in both which is range and angle dimensions.

Direction of arrival estimation is done in MIMO FDA radars. CS techniques are applied for the DOA estimation as well as range estimation using double pulse method and also the effect of noise is considered. Similarly, subarray structure is considered for more robustness and double pulse method using compressive sensing is applied.

1.2 Research Problem

Parameter estimation using compressive sensing is an interesting field of study especially DOA estimation and range estimation. However, parameter estimation using compressive sensing depends upon the resolution of the grid defined. It may happen such that the resolution of the sources does not coincides with the grid resolution. This leads to grid mismatch problem or off grid targets. A grid refinement scheme is developed to solve the problem. Similarly, a preprocessing stage is required to make the parameter estimation algorithm more robust against impulsive noise as well as Gaussian noise. To solve sparse solution l_1 regression is considered. The problem is formulated for single time sample as well as multiple time samples. To select the regularization term Generalized cross validation (GCV) method is used. DOA estimation problem for different array structure is considered like uniform linear sparse antenna array, L shaped sparse antenna array. Similarly, parameter estimation for MIMO FDA radar is done using compressive sensing. Application of compressed sensing for sub array antenna array is considered and applied to MIMO FDA radar also. Since the range and angle information is coupled in MIMO FDA radar. Double Pulse technique is used to decouple them.

1.3 Contribution

Here we apply compressive sensing framework for parameter estimation for MIMO FDA radar. The main challenge for the application of the compressive sensing is to define the grid resolution for the dictionary. To remove grid, mismatch a scheme is developed for the selection of the grid resolution for the dictionary. Similarly, the effect of impulsive noise as well as gaussian noise is studied on the parameter estimation and a preprocessing stage is suggested for multi sample compressive sensing to make the detection more robust. The main contribution of the dissertation is as follows.

- A novel framework DOA estimation problem using CS technique, taking into consideration off grid targets or grid mismatch.
- A new fitness function is calculated, which is based on the difference of source energy that is formed between adjacent grids due to grid mismatch in DOA estimation.
- An approach for finding the best discretization value using the designed objective function is presented for iterative grid refinement.
- The proposed scheme is viably tested for multiple sources with different energy and spatial resolution-based scenarios in DOA estimation.
- A new multi sample compressive sensing technique, is considered for source estimation using sparse antenna with geometry in ULA and L shaped array.
- A preprocessing stage based on MDCO and weighted moving average filter is introduced to make DOA estimation more robust against impulsive and Gaussian noises.
- The iterative grid refinement in CS is achieved through formulation of a fitness function.
- The proposed CS technique removes the ambiguity about the location of the sources due to grid mismatch and provides a discretization value.
- The simulation results verify the proposed method for scenarios of 1D/2D DOA estimation.
- Sub array strtucture for the receive side of MIMO FDA radar is suggested
- Compressive sensing model using double pulse method for MIMO FDA radar is designed.
- Angle and range is estimated using compressive sensing technique for MIMO FDA radar.

1.4 Organization

This dissertation is divided in to five chapters. The first chapter includes the introduction. The research problem and the main contribution of the dissertation. The second chapter includes necessary topics for understanding the research work. We introduce the direction of arrival schemes for source localization. The popular schemes like MUSIC, ML technique and beamforming technique is introduced. Then the idea of compressive sensing in the presence of the previous research is presented. The history of radar and modernization in the radar is also introduced in this chapter. The idea of PAR and FDA radar is also introduced here. Next in chapter three the source localization using compressive sensing is introduced. A source localization model using compressive sensing is developed for linear array. An optimization technique for grid refinement method is also developed. In chapter four the idea developed in the previous chapter in tailored for sparse linear as well as sparse L shaped antenna array for direction of arrival estimation in two dimensions. A system is developed for multi sample. The effect of different noise is also considered, and a method of preprocessing is introduced to remove Gaussian as well as impulsive noise. Finally in chapter 5 the technique developed in the previous chapter is applied to MIMO FDA radar having sub array structure in the receive domain. The compressive sensing technique using double pulse method is introduced. That decouple angle and range information,

CHAPTER 2

LITERATURE REVIEW

We introduce a brief review of the necessary topics before discussion of the main contributions. In this section we introduce different methods for source localization methods. In source localization we estimate the direction of the incoming sources. Each method has its advantage and disadvantage. In general, the algorithms for the estimation of the source localization depends upon the number of the sources, antenna elements and noise level. Later, we discuss the compressive sensing techniques. How the technique works with a smaller number of samples. Similarly, how to cast the parameter estimation problem into compressive sensing framework. The methods available to solve sparse solutions. The type of the antenna arrays also effects the estimation. In the end the section, we discuss different types of radars and their operation.

2.1 Direction of Arrival Schemes

The DOA estimation has a wide application. It is used in applications like radars, sonars, medical imaging and wireless communication. The aim is to estimate of the sources from the signal received by an antenna array or other sensor array. The structure of the sensor array can be linear or different like non uniform, circular, L shaped or rectangular. The sensor array is used for high signal gain and flexibility in term of beam shape and interference rejection. Hence sensor array is used for DOA estimation and other parameter estimation like range. We can say that for DOA estimation sensor array is required. The array can comprise of different structure. Using sensor array have certain benefits like improvement in signal to noise ratio (SNR), beam steering.

Before discussion on conventional source localization method, we consider the signal model. The same signal model will be used in the rest of the dissertation for uniform linear array. For simplification we consider a ULA with the spacing between the antenna elements to be constant. A uniform linear array (ULA) as shown in the Fig. 1, with N number of antenna elements. The inter element spacing between the antenna respective, elements is $\lambda/2$. Let us assume that there are P number sources at different angles, θ_l. We consider far field. The received signal at some m^{th} number antenna element is as

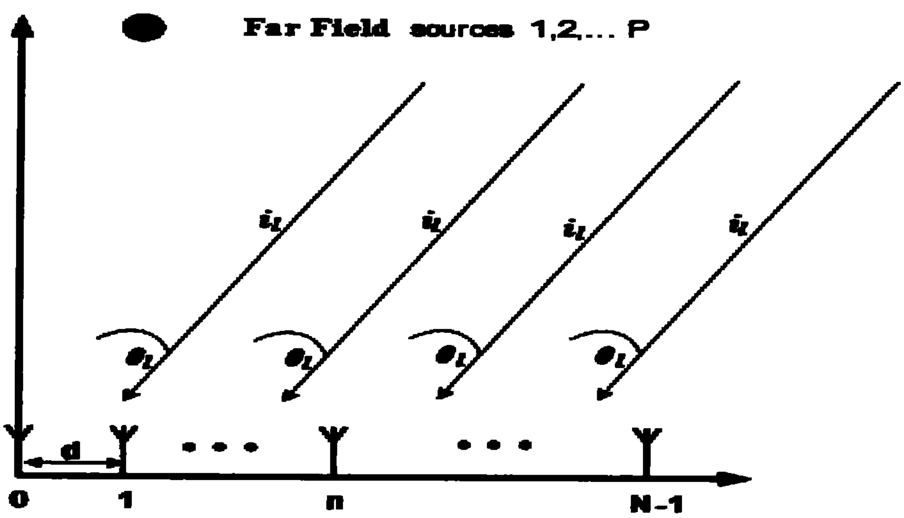


Fig2.1: Uniform Linear array

$$y_m(t) = \sum_{l=1}^{P} s_l(t) e^{j(m-1)kd\sin\theta_l} \tag{1}$$

where s_l in the above equation is the amplitude of the signal that is received. k and d, are wave number and distance between the antenna elements. Then, Equation (1) can be written as

$$y = As \tag{2}$$

The above equation is without noise. with noise, the received signal is given as

$$y = As + n \tag{3}$$

where n is the Gaussian noise, A is considered to be the steering matrix and y the received vector. Some of the methods for source localization are mentioned below.

2.1.1 Beamforming Technique

Beamforming is one of the methods for estimating the direction of arrival [11-13]. The classical beamforming method also known as sum and delay beamforming method operates by steering the main beam in the angle domain. If the return signal from the sources is above certain threshold level, then the location of the source is at that angle. The beam is steered using phase shifters. Where the phase among the adjacent antenna elements is changed such that the beam is steered in the direction using phase shifters. Let us consider a uniform linear array. The beamforming network consist of phase shifter for each antenna element as shown in figure below.

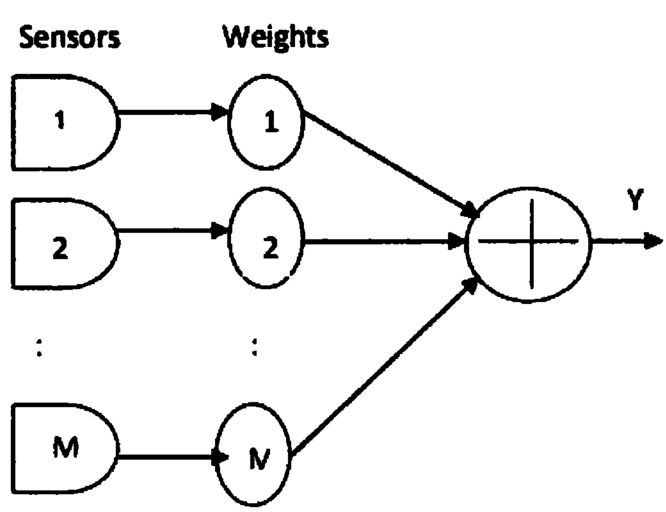


Fig2.2: Beamforming network

To create a beam in the broadside the phase difference between the antenna element is zero. To shift the beam to a specific direction then a progressive phase shift is added among the antenna elements. In localization the maximum power is achieved by steering the beam toward the sources.

A conventional beamforming method is shown in figure 2.2. Also known as delay and sum beam forming method. It consists of phase shifter. The phase due to path delay is compensated using the phase shifters. Consider a M sensors array, with a uniform spacing. We consider a signal $x(t)$ which is far field with an angle. The signal received by the M^{th} sensors

$$x(t) = [x(t-\tau_1), x(t-\tau_2), \cdots\cdots, x(t-\tau_M)] \tag{4}$$

Where τ's are the time delay due to the path difference for each signal received by the antenna array. For uniform linear array the delay is given as

$$\tau = (d\cos\theta_i)/c \tag{5}$$

If $[w_1, w_2, \cdots\cdots w_M]$ are the weights the output $y(t)$ generated by the linear combination of received signal and w can be represented as

$$\mathbf{y}(t) = \mathbf{w}^H \mathbf{x}(t) \tag{6}$$

The output power of the beamforming is given as

$$\mathbf{P}(y) = \sigma_y^{\,2} = E\{|\mathrm{y}|^2\} = \mathbf{w}^H \mathbf{R}_y \mathbf{w} \tag{7}$$

Where $w^H(\theta) = a(\theta)$, $a(\theta)$ is the steering vector of the main beam.

2.1.2 Maximum Likelihood Method

Maximum Likelihood (ML) is another method for estimating the direction of arrival [16,17]. In this method we try to maximize the likelihood that the signal is coming from a particular direction.

We consider the signal model that is previously stated for uniform linear array. The likelihood function of the different snap shot of the received signal y is $y(t_1), y(t_2), \cdots\cdots, y(t_{N_s})$ is given as

$$L = \prod_{t=1}^{N_s} \frac{1}{\pi \det[\sigma^2 \mathbf{I}_M]} . \exp(-\frac{1}{\sigma^2} | \mathbf{y}(t) - \mathbf{A}\mathbf{x}(t) |^2) \tag{8}$$

Therefore the function is given as

$$\ln L = -N_s \ln \pi - M N_s \ln \sigma^2 - \frac{1}{\sigma^2} \sum_{t=1}^{N_s} \left| \mathbf{y}(t) - \mathbf{A}\mathbf{x}(t) \right|^2 \tag{9}$$

Whereas σ^2, $x(t)$ and θ in A are the unknown parameters. Taking derivative with respect to σ^2 and keeping the other constant the above equation can be expressed as

$$\frac{\partial \ln L}{\partial \sigma^2} = -\frac{M N_s}{\sigma^2} + \frac{1}{\sigma^4} \sum_{t=1}^{N_s} \left| \mathbf{y}(t) - \mathbf{A}\mathbf{x}(t) \right|^2 \tag{10}$$

Which can be represented as

$$\sigma^2 = -\frac{1}{M N_s} \sum_{t=1}^{N_s} \left| \mathbf{y}(t) - \mathbf{A}\mathbf{x}(t) \right|^2 \tag{11}$$

Next taking derivative with respect to $x(t)$ and keeping the other constant

$$\frac{\partial \ln L}{\partial x(t)} = \frac{2}{\sigma^2} A^* [\mathbf{y}(t) - \mathbf{A}\mathbf{x}(t)] \tag{12}$$

Which can be simplified into

$$\mathbf{x}(t) = (\mathbf{A}^* \mathbf{A})^{-1} \mathbf{A}^* \mathbf{y}(t) \tag{13}$$

Finally by substituting the equations

$$\min_{\theta}\{\sum_{t=1}^{N_s}\left\|\left[\mathbf{I}_M - \mathbf{A}(\mathbf{A}^*\mathbf{A})^{-1}\mathbf{A}^*\right]\mathbf{y}(t)\right\|^2 \tag{14}$$

2.1.3 MUSIC

In the subspace method for the direction of arrival the MUSIC algorithm is one of the most prominent methods [18]. This algorithm is based upon the eigen decomposition of the covariance matrix. Let us consider the signal model presented in figure 2.3. The output correlation matrix from the sensor array is given as

$$\begin{aligned}
\mathbf{R}_y &= E\left[\mathbf{y}\mathbf{y}^H\right] \\
\mathbf{R}_y &= E\left[(\mathbf{A}\mathbf{s}+\mathbf{N})(\mathbf{A}\mathbf{s}+\mathbf{N})^H\right] \\
\mathbf{R}_y &= \mathbf{A}\mathbf{R}_{sig}\mathbf{A}^H + \mathbf{R}_{No}
\end{aligned} \tag{15}$$

Where $\mathbf{R}_{sig} = E[\mathbf{s}\mathbf{s}^H]$ is the correlation matrix of the signal and $\mathbf{R}_{No}$ is correlation matrix of the noise. The covariance matrix can be represented as

$$\mathbf{R}_y = \frac{1}{N}\sum_{i=1}^{N}\mathbf{y}(i)\mathbf{y}^H(i) \tag{16}$$

Then we apply decomposition on the covariance matrix. The eigen values and the eigen vectors corresponding the covariance matrix R_y consist of both the signal and the noise. The eigen values of R_y are sorted according to the size. The signal covariance matrix R_y have rank M, which are orthogonal to M steering vector. Therefore

$$\begin{aligned}
\mathbf{R}_y\mathbf{q}_m &= \mathbf{A}\mathbf{S}\mathbf{A}^H = 0 \\
\mathbf{q}_m^H\mathbf{A}\mathbf{s}\mathbf{A}^H\mathbf{q}_m &= 0 \\
\mathbf{A}^H\mathbf{q}_m &= 0
\end{aligned} \tag{17}$$

The plot of the pseudo spectrum is given as

$$P_{Music} = \frac{1}{a^H(\theta) Q_n Q_n^H a(\theta)} \tag{18}$$

The MUSIC exhibits sharp peaks at the source position.

Despite the resolution, MUSIC have a high sensitivity to model errors.

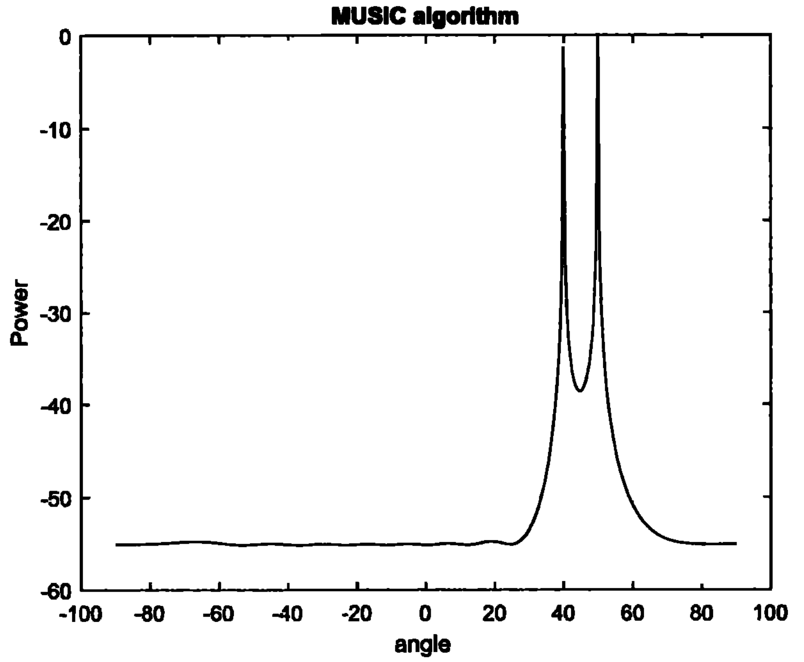


Fig 2.3: MUSIC algorithm

2.2 COMPRESSIVE SENSING

In this section we will look in to the concept of compressive sensing. Compressive sensing is a method of solving an underdetermined system [19-21]. As it can be applied to a number of fields like mathematics, computer science and communication where we try to solve an underdetermined matrix. It does not follow the traditional sampling theory that is celebrated Nyquist theorem.

The celebrated Nyquist theorem have is used in a number of applications. Where data can be exactly recovered from a set of uniformly samples [22,23]. According to the Nyquist theorem, "a signal can be reconstructed if sampling frequency to be twice the highest frequency present in the signal of interest".

The Fourier transform $\hat{f}$ of a function f is compactly supported in $\left[-\frac{B}{2},\frac{B}{2}\right]$. Then the function can be reconstructed from measurements $\left\{f(\frac{2\pi n}{b}), n \in Z\right\}$ via

$$f(t) = \sum_{n=-\alpha}^{\alpha} f(\frac{2\pi n}{B})\sin(\frac{Bt}{2} - \pi n)$$

$$\sin cx = \begin{cases} \frac{\sin(x)}{x} & if\ x \neq 0 \\ 1 & if\ x = 0 \end{cases} \tag{19}$$

Utilizing the Nyquist sampling theorem, the signal processing can be done in the digital domain. However, it poses certain challenges. As the highest frequency component increases so the requirement of the sampling frequency. It may happen so that sampling frequency at such a high rate may not be realizable practically. In many applications, the Nyquist rate is too high. Which is too costly. For this, we often depend on compression.

The aim of the compressive sensing is to reconstruct the signal with fewer measurements. We take in to consideration that the signal is sparse some domain. Many Signals such as real-world signals are sparse in some domain. Sparse signals have a lower dimensionality, they can be represented by few linear measurements. An optimal recovery, the algorithm recovers all sparse signals (with good probability).

2.2.1 Problem Formulation

The CS framework can be explained using figure 2.4 [24]. Some discrete signal x of length N, is sparse if some its coefficients are nonzero under some transformation. Then the information content of x is K dimensions. Therefore, for signal acquisition, K times. Which can be achieved by making some random, linear observations $y = Ax$ where A is an $M \times N$ *measurement matrix*. This is also known as dictionary. If A satisfies incoherency then x can be recovered. For practical applications, the condition of sparsity

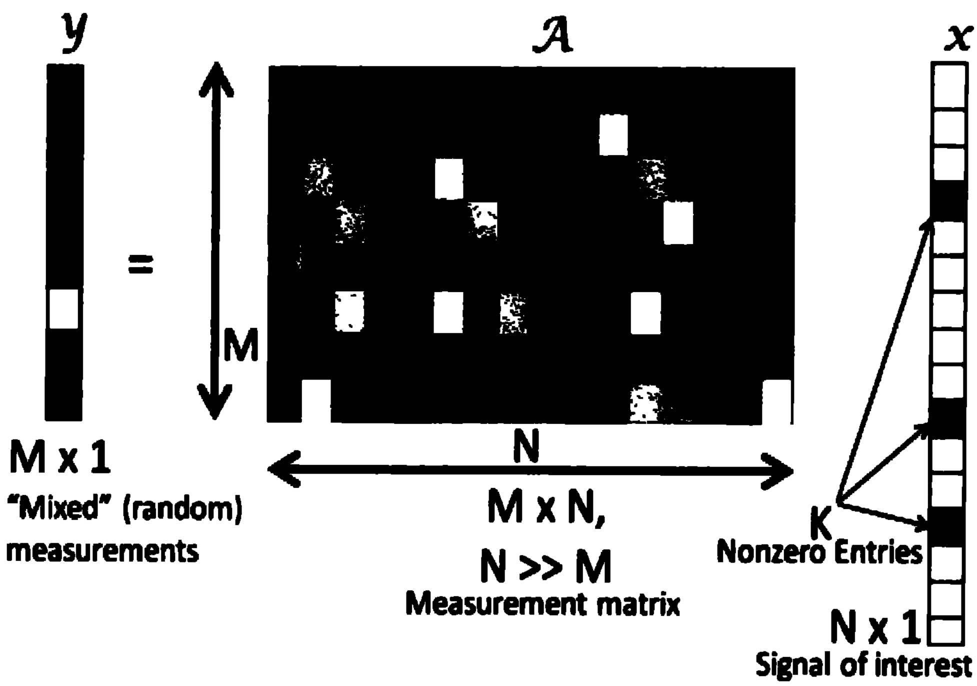


Fig 2.4: Recovery of compressed signal

The compressed sensing approach recovers the sparse signal x from y by finding the solution to the following problem [25-27].

$$\text{minimize } \|x\|_1 \text{ subject to } y = Ax \qquad (20)$$

2.3 RADAR

In this section we briefly discuss the concept of RADAR. We start with a brief introductory history of radar and some important details and then we discuss the latest research on PAR, FDA radar and MIMO radar. Study on radar like Phased MIMO radar and MIMO-FDA radar has been discussed.

2.3.1 History of RADAR

Radar stands for "*RAdio Detection And Ranging*". History of Radar extended dated to 1886, when Heinrich Hertz demonstrated that the radio waves can be reflected by a metallic object [1]. In 1903, Hülsmeyer successfully performed an experiment for detection of ships using reflected radio waves at a range of just over one mile. In next three decades, aircraft detection at a distance of 50 miles was achieved in 1932. This was followed by a patent on aircrafts detection in 1934 awarded to Taylor, Young, and Hyland [28-31].

In an anticipation of World War II, all countries especially Britain worked for development of radars that could detect from large distances. A pulsed radar with full capability of detection and ranging was presented by Robert Watson-Watt in 1935 [28]. Building on it, they developed an Aircraft-Interception (AI) radar in 1939, for the detection and interception of hostile aircraft. The system was mounted on an aircraft and had the capability to detect ships from air [31].

Earlier RADAR's had huge mechanical structures involving a lot of mechanical movement for detecting an aircraft. Due to mechanical system give rise to certain noises and limitation degrading the SNR. Therefore, a great urge was there to develop a radar system with the capability of electronic steering instead of mechanical steering. Dedicated research to produce a radar system with electronic steering resulted in development of most popular radar system in 1960 by the name of phased array radar (PAR) [32-34]. Antenna arrays using electronic beam steering techniques were explored and employed in military and civilian radars in the late 1970s. e.g., the PAVE PAWS radar [35]. This was followed by multimode programmable radar in 1980's and air borne electronically scanned antenna radar in 1990's [36]. Although the radar technology was basically flourished by military, several civilian applications also benefited from the technology. Most significant of these civilian applications include air traffic control (ATC) and marine navigation safety. TELEFUNKEN developed the first ATC in 1955. This ATC radar remained in use under the name Ground Radar System (GRS) between 1955-1957.

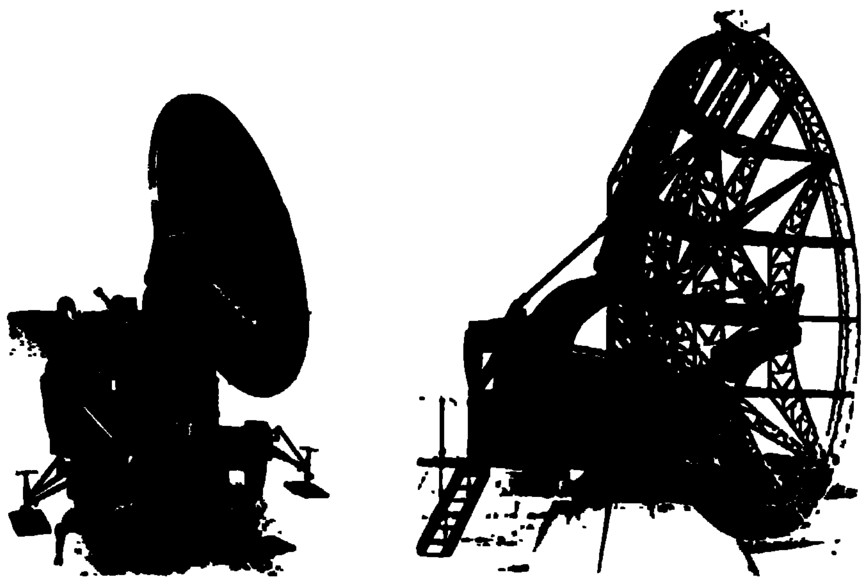


Fig 2.5: Early RADAR systems

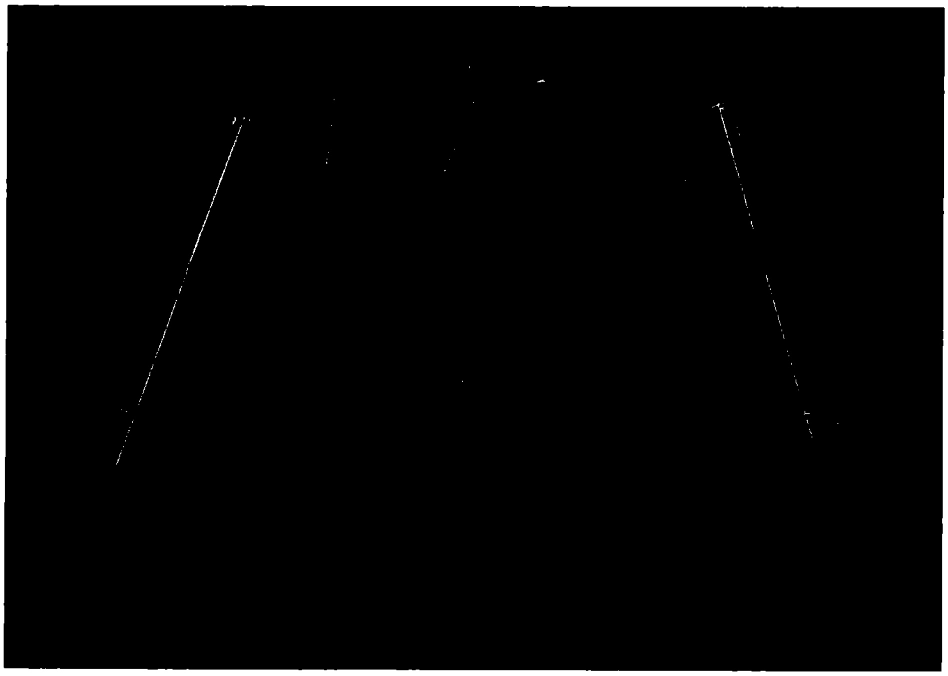


Fig 2.6: phased array Radar system

2.3.2 Classification of Radars

Radar systems can be classified in terms of antenna types, duration of signal, number of antennas used for transmission. On the basis of the distance between transmitter and receiver it can be classified [34]. In mono-static radar, the transmitter and receiver antenna elements are closely placed and only one antenna element performs both transmitting and receiving tasks by time multiplexing. Bistatic system, on the other hand, uses one transmitter and one receiver antenna that are separated by a significant distance. Multi-static radar uses two or more transmitting or receiving antennas with large distance between the antennas [37] .

In terms of signal duration, a radar system id classified into continuous waveform (CW) radar or pulse radar. CW radar transmits a steady frequency is transmitted and processes the received signal for Doppler shift in the received frequency. These radars, also called Doppler radars, have the inability to measure the range of target, however, applying a linear frequency modulation (LFM) to CW radar allows us to measure range as well as speed of target. Pulse radar on the other hand transmits a train of RF pulses with a low duty cycle. Direction of the target can be acquired from the angle of arrival and range can be measured from propagation time of the reflected signal. These radars, also known as pulse-Doppler radars, can also estimate the speed of target by using Doppler frequency [32].

In terms of number of antennas, radars can be categorized as single-antenna and multiple-antenna radar. For single-antenna based radar, a directional antenna is rotated on a mechanical pedestal to scan the whole region of interest. Multiple antennas radar uses an array of antenna to steer a beam towards target, as in PAR and FDA radar. Moreover, the electronic beam steering also eliminates the need of mechanical steering.

Finally, array radars use three different kinds of basic array structures. First is linear array in which antennas are placed close to each other in a straight line. In case of equal distance between antennas, it is called Uniform Linear array (ULA) [38]. Second array structure is planar array, which arranges the antenna elements as a grid of antenna in two dimensions [39]. Third type of array structure is circular array, in which antenna elements are placed in a circle [40].

2.4 FDA Radar

FDA stands for frequency diverse Array. Initially, FDA was investigated by Antonik [41]–[44]. The are different then Phased array radar. PAR have certain disadvantage also which is its inability to scan in range dimension. Beam steering in range dimension can be very useful in localizing a target in range dimension as well as rejecting the range dependent clutter. this inability of PAR in range dimension. In FDA radar a small frequency increment Δf at each element. This creates a beam pattern which is range dependent. This FDA radars have the ability to scan both in angle and range. FDA radar can be used for multiple mission simultaneously by utilizing the additional degree of freedom.

FDA radars work on the principle of small frequency increment, therefore there is no requirement of the phase shifter as in PAR. Since FDA scans without phase shifters, the radiation characteristics of FDA has been simulated in [45]. The FDA exhibit periodicity in time, angle, and distance. This is exploited in [46].

Another important research area synthetic aperture radar (SAR) [47], [48]. It has been shown that application of FDA to SAR has improved the high-resolution imaging. In addition to uniform linear array, FDA has also been investigated for planar arrays [49] and circular arrays [50]. Receiver architectures for different types of FDA arrays have been proposed in [51].

2.4.1 Signal Model

In this section we present the signal model for FDA. Let us consider a ULA as shown in the figure 2.7 given below. f_0 is the central operating frequency. As mentioned earlier small frequency increment is Δf. As shown in the figure each element has a increment of Δf. Thus the frequency at the pth antenna element is given as

$$f_p = f_0 + p.\Delta f \tag{21}$$

Where p = 0,1,2 …. (p-1). The signal transmitted is given as

$$s_p(t) = \exp(-j2\pi f_p t) \tag{22}$$

The array factor for the FDA radar can be given as

$$AF(t,\Delta f, r, \theta) = \sum_{p=0}^{p-1} \frac{1}{r_p} \exp\left\{-j2\pi\left(f_p t - \frac{r_p}{\lambda_p}\right)\right\} \tag{23}$$

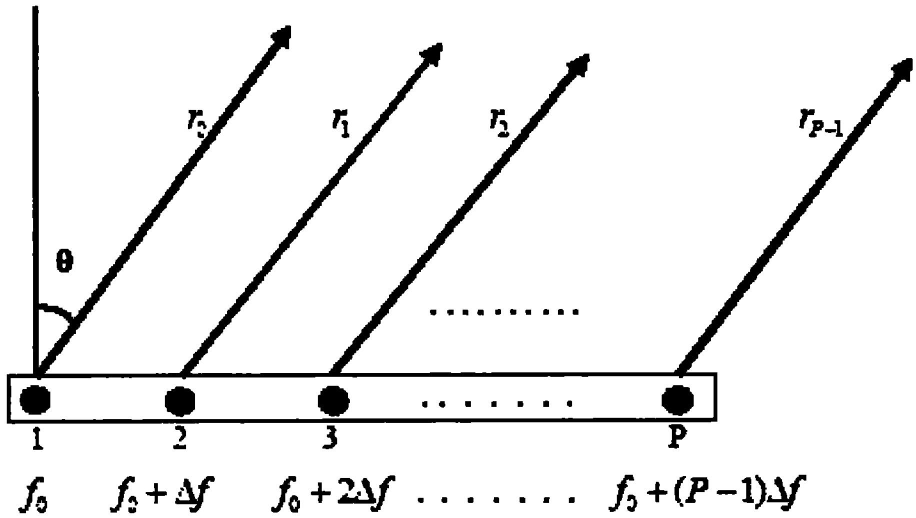


Fig 2.7: Frequency diverse array

In the above equation r_p is the range of the target from the pth element and λ_p is the wavelength of the antenna element. Assuming a target in space the beampattern with uniform weight can be given as

$$B_{FDA}(t,\Delta f,r,\theta)=\left|\sum_{p=0}^{P-1}\frac{1}{r_p}\exp\left\{-j2\pi(f_0+p\Delta f)\left(t-\frac{r_0}{c}+\frac{pd\sin\theta}{c}\right)\right\}\right| \tag{24}$$

Using the far field assumption i.e $r_n \approx r_0$ and $f_0 \gg \Delta f$. Then the beam pattern can be written as

$$B_{FDA}(t,\Delta f,r,\theta)\approx\left|\frac{\exp(j\gamma)}{r_0}\sum_{p=0}^{P-1}\exp(-jp\eta)\right|^2 \tag{25}$$

Where $\gamma=-2\pi f_0(t-\frac{r_0}{c})$ and $\eta=\left(2\pi\Delta ft+\frac{2\pi f_0 d\sin\theta}{c}-\frac{2\pi\Delta fr}{c}\right)$

The beam pattern can be simplified as

$$B_{FDA}(t,r,\Delta f,\theta)=\left|\frac{\sin\left(\frac{p}{2}\left(2\pi\Delta ft+\frac{2\pi f_0 d\sin\theta}{c}-\frac{2\pi\Delta fr}{c}\right)\right)}{\sin\left(\frac{1}{2}\left(2\pi\Delta ft+\frac{2\pi f_0 d\sin\theta}{c}-\frac{2\pi\Delta fr}{c}\right)\right)}\right|^2=\left|\frac{\sin\left(\frac{p}{2}\eta\right)}{\sin\left(\frac{1}{2}\eta\right)}\right|^2 \tag{26}$$

we can see that the beam pattern of the FDA radar is a function of time, range and frequency offset. Fixing any parameter the beampattern will be function of the other two. Since we are interested in range-angle analysis in this work, therefore, for a fixed frequency offset and time, FDA beampattern will vary in both angle and range dimension.

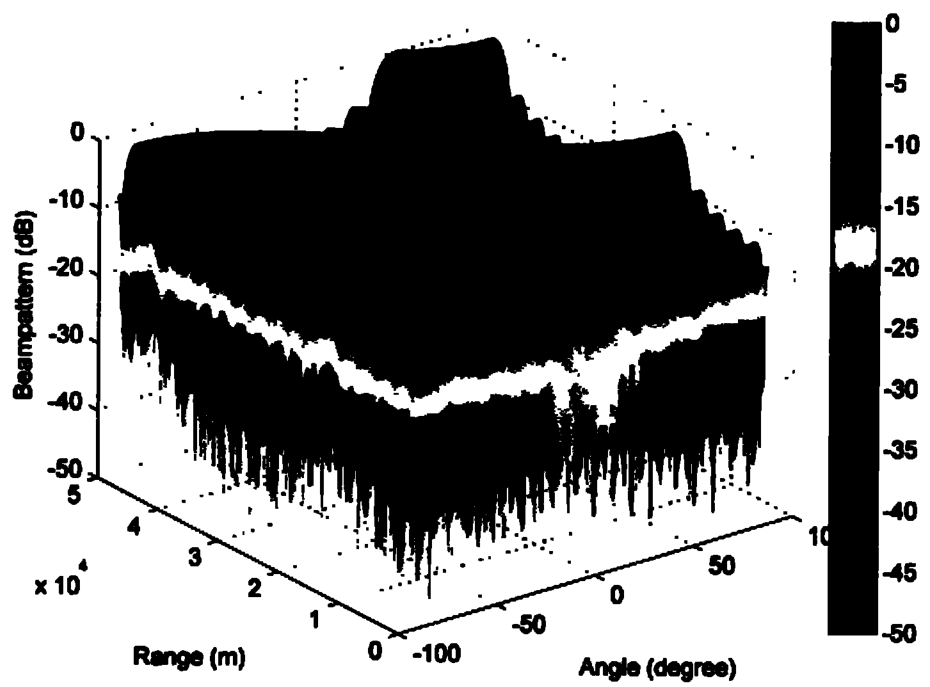


Fig 2.8: FDA pattern

It can be observed in fig 2.8 that the beampattern has more than one maxima at different ranges due to periodicity property of FDA. It can also observe that FDA radar produces multiple periodic maxima in range dimension. Moreover, it can be seen that number of maxima increases due to larger frequency offset. Multiple maxima property is not desirable due to its effects on target returns from a particular range-angle pair, resulting in low overall SINR.

2.5 MIMO Radar

MIMO stands for Multiple Input and Multiple Output system. It is primarily used in wireless communication systems to achieve higher data rates. In MIMO system multiple waveforms are transmitted in order to avoid fading channels. Due to path independence, different transmitted signals experience different level of fading, resulting in more or less constant average signal-to-noise ratio (SNR) levels. This was contrary to existing system at that time which transmit all of their energy over a single path. The concept of MIMO system was introduce in [52,53]. There is plenty research over advantages and disadvantages of application of MIMO in Radars. The basic application of MIMO in Radar is to achieve waveform diversity. Where different waveforms are transmitted. Concept of virtual length is introduced in [54-56]. In comparison with PAR, MIMO radars have the disadvantage of array gain. That is compensated using waveform diversity. Lot of research had been done for stealth detection using MIMO radars. The antenna array for the MIMO radars can be closely spaced or widely separated. Each having its advantages.

2.5.1 Phased MIMO Radar

Phased MIMO radar [57-60] has been introduced to combine the features of both PAR and MIMO radar in a single radar system. This radar has collocated antenna elements at the transmitter and receiver side just like PAR and Coherent MIMO radar. Transmit array can be divided into subarrays which can be non-overlapped or overlapped structure. Each subarray transmits a unique waveform and steers a beam of reasonable gain towards a target in the region of interest by designing proper weight vector. Furthermore, the waveforms transmitted by different subarray are orthogonal which resembles MIMO radar.

2.5.1.1 Signal Model

Radiating elements are placed in the vicinity of each other to make it a collocated radar. This phased MIMO radar divides the whole transmit array into subarrays which have full overlapping, as shown in Figure 2.9 below

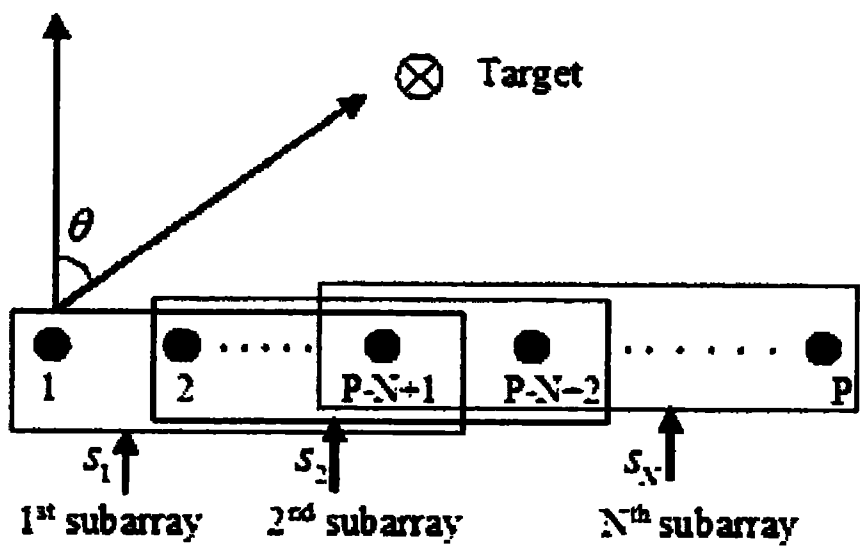


Fig 2.9: Subarray Phased MIMO radar

. In order to steer a beam in desired region, the antenna elements of subarray will transmit waveform, where is the number of samples of each pulse. At the output of N^{th} subarray, transmitted signal can

$$f_n(k) = \rho w_n^H u(\theta) s_n(k) \tag{27}$$

In the equation above s_n is the signal send by the n^{th} subarray. $u(\theta)$ is the steering vector and w_n is the weight for the corresponding subarray. Where as ρ represent the energy of the subarray. Which is given as

$$\rho = \frac{P}{N} \tag{28}$$

The combined effect of the beam pointing is given as

$$\begin{aligned} \mathbf{f}(k) &= \rho \sum_{n=1}^{N} \mathbf{w}_n^{H} \mathbf{u}(\theta) \mathbf{s}_n(k) \\ &= \rho \left[\mathbf{W}^{H} \mathbf{u}(q) \right] \mathbf{s}_N(k) \end{aligned} \tag{29}$$

W is PxN weight matrix and $s_N(k) = [s_1(k), s_2(k), \cdots \cdots, s_N(k)]$ is Nx1 waveform. Each entry of weight matrix will represent the association of an antenna element to a particular subarray. Each entry of the weight matrix will represent the association of the antenna element to the subarray. Moreover, all non-zero weights in a subarray scale same waveform to steer a beam towards region of interest. Transmitted signal by each antenna element can be represented as

$$\mathbf{g}(k) = r\mathbf{W}\,\mathbf{s}_n(k) \tag{30}$$

The antenna elements transmit a combination of waveforms which are orthogonal to each other. In literature, the phased MIMO radar has been presented in two different types of subarray arrangements i.e. overlapped subarrays and disjoint subarrays [61–66].

The disjoint subarrays have further applied to two different array architectures. One of them has used ULA as a transmitting array and divide it into disjoint subarrays [68–70]. The other architecture has simply divided a planar array into disjoint subarrays and used partially correlated waveforms in different subarrays [76–81].

Chapter 3

INTRODUCTION

In this chapter we will discuss the DOA estimation using compressive sensing and establish the basis at which the DOA estimation problem can be casted into a CS framework. We establish the basic for applying compressive sensing estimation to a uniform linear array. For range estimation similar methodology will be used in chapter 5. As mentioned earlier that Location of the sources are sparse, that are at few angles i.e., spatially sparse sources. This establishes the sparsity criteria for the application of the compressive sensing. Hence the location of the sources can be estimated using compressive sensing (CS) methods. The estimation using compressive sensing can achieve super resolution. However, one of the issues in CS is to define the resolution of the grid. There is always a possibility that the targets resolution may be in between the grid points, which means that the location of the target does not coincides with the grid defined. This creates ambiguity about the location of the source.

In order to remove the ambiguity about the location of the sources a grid refinement algorithm is also presented that iteratively calculated the best grid density in order to remove the ambiguity about the location of the sources. The CS model is developed for a single time stamp. For multiple time stamp the model is developed in the next chapter. The proposed technique is developed for uniform linear array.

3.1 CS Problem Formulation

We will formulate the estimation of the source location in to the CS framework. The method developed for casting the problem into CS framework can be generalized for estimation of the range of the sources. Which is shown in chapter 5 where the problem is formulized for MIMO FDA radar for estimation of the source angle as well as the range.

For the DOA estimation as mentioned we consider a uniform linear array (ULA) as shown in the Figure 2.1 in chapter 2. We consider N number of antenna elements at equal distance. Where the distance between the respective, antenna element is $\lambda/2$. There are P number sources at different angles, θ_l. The sources are at the far field. Then the received signal at the m^{th} antenna element is given as

$$y_m(t) = \sum_{i=1}^{P} s_i(t) e^{j(m-1)kd\sin\theta_i} \tag{1}$$

where s_i is the amplitude of the received signal. k and d are wave number and distance between the antenna elements. Then, the above equation can be represented as

$$\begin{bmatrix} y_0 \\ \cdot \\ \cdot \\ \cdot \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{jkd\sin\theta_1} & e^{jkd\sin\theta_2} & \cdots & e^{jkd\sin\theta_P} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ e^{j(N-1)kd\sin\theta_1} & e^{j(N-1)kd\sin\theta_2} & \cdots & e^{j(N-1)kd\sin\theta_P} \end{bmatrix} \begin{bmatrix} s_1 \\ \cdot \\ \cdot \\ \cdot \\ s_P \end{bmatrix} \tag{2}$$

The above equation in the vector form can be written as

$$\mathbf{y} = \mathbf{A}\mathbf{s} \tag{3}$$

With noise it can be represented as

$$\mathbf{y} = \mathbf{A}\mathbf{s} + \mathbf{n} \tag{4}$$

we see how the DOA problem can be explained in the CS domain. Let us consider the figure given below

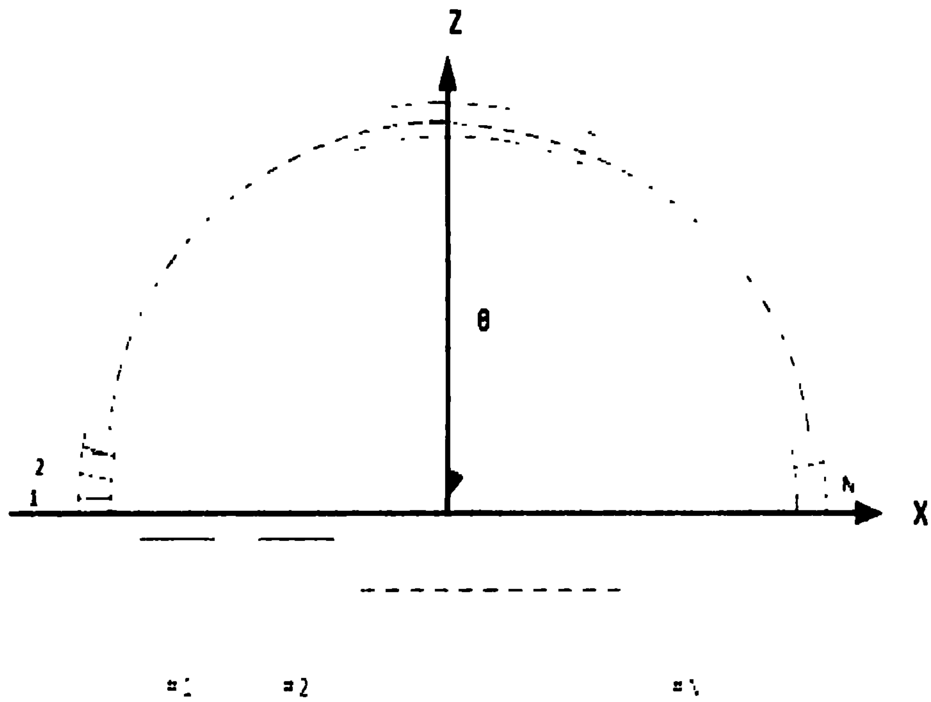


Fig 3.1: Sparse representation in angle domain

Let us consider a uniform linear array. The angle domain can be divided into small grid. As shown in the figure that the angle domain is divided into N grid points. We assume that the source is sparse. As it is shown in the figure that the source is at few angles. Hence the source is sparse in the angle domain as if consider the angle domain it is at few angles. The location of the source in the angle domain can be represented by the signal strength whereas those angles where the source is not present can be represented by noise or for simplification by zero. If the level of noise is less than the signal strength. Otherwise, a preprocessing stage is added which is discussed in next chapter. Initially to construct the basis we do not consider noise. Hence if we have a vector of signal strength in the angle domain it will be a sparse array. This satisfies the sparsity criteria of the CS framework.

Necessary background for the compressive sensing is presented in the previous chapter. A sparse representation of the signal can be reconstructed only with just few numbers of samples. Let us consider a signal s which is a discrete signal. It is a discrete signal which is sparse in certain domain $s \in C^N$. Let us consider that y is the received signal of dimension M such that $y \in C^M$. The simplest case is to consider without noise. The received signal is given as

$$\mathbf{y} = \mathbf{A}\mathbf{s} \tag{5}$$

A is a sensing matrix of dimension $A \in C^{MxN}$ where $M<<N$. The ideal assumption on the sparsity of s means that we count the number of non zeros element in s. Which is given by $\|s\|_0$ also known as l_0 - norm. This leads problem which is NP hard. To solve this many approximation methods have been developed. Like greedy approximation method. One of the methods is to use l_1 or l_p relaxation. The unknow signal s is considered sparse then the problem can be given as

$$\tilde{\mathbf{s}} = \min \|\mathbf{s}\|_p \;\; s.t \quad \mathbf{y} = \mathbf{A}\mathbf{s} \tag{6}$$

considering $p = 0$ then it will be a NP hard problem. So we consider $p = 1$. We can recast it as a l_1 - norm problem and solved following the given equation

$$\tilde{s} = \min \|y - As\|_2^2 + \lambda \|s\|_p \tag{7}$$

In practical scenario there is always a noise. Now, let us consider noise. If the received signal is contaminated with noise n which is

$$\mathbf{y} = \mathbf{A}\mathbf{s} + \mathbf{n} \tag{8}$$

The optimization problem is given as

$$\min \|\mathbf{s}\|_1 \;\; s.t \quad \|\mathbf{y} - \mathbf{A}\mathbf{s}\|_2^2 < \varepsilon \tag{9}$$

here ε is a parameter that specifies how much noise is allowed. In order to formulate the problem in the CS framework consider the figure given as in Figure 3.1 given in [95], where the spacing between the antenna elements is d which is $\lambda/2$. As the goal is to find the location of the sources, we consider a uniform linear array. We consider a narrow band signal for K number of sources arriving at the uniform linear array of M number of elements. The received signal is shown in equation 1. With m number of antenna elements and P number sources. As seen in the figure to cast this in the sparse representation problem an over complete dictionary of array steering vector A is introduced. where $A = [\theta_1, \theta_2, \ldots, \theta_N]$, N is the sampling of the grid. The N will be much higher than K. Therefore, the matrix A is given as

$$A = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{jkd\sin\theta_1} & e^{jk\sin\theta_2} & \cdots & e^{jk\sin\theta_N} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ e^{j(m-1)kd\sin\theta 1} & e^{j(m-1)kd\sin\theta_2} & \cdots & e^{j(m-1)kd\sin\theta_N} \end{bmatrix} \tag{10}$$

we consider the problem for a single time stamp T=1. Where s is Nx1 vector and non zero corresponding to N position of the angle at which the target is present. Therefore, the Equation 1 formulated for CS framework is given as in Equation 11.

$$\begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ y_M \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{jkd\sin\theta_1} & e^{kd\sin\theta_2} & \cdots & e^{jkd\sin\theta_N} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ e^{j(m-1)kd\sin\theta_1} & e^{j(m-1)kd\sin\theta_2} & \cdots & e^{j(m-1)kd\sin\theta_N} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \cdot \\ \cdot \\ s_M \end{bmatrix} \tag{11}$$

We estimate the source location through solving the l_1 normalization equation mentioned in the above equations.

First, we look into the effect of the number of the samples on the conventional DOA estimation algorithm in the figure given below. There are number of algorithms discussed in chapter 2. We consider MUSIC algorithm which is the most widely algorithm used for DOA estimation. As mentioned earlier the traditional algorithm depends upon the number of the samples. It is show in previous chapter that if there are sufficient samples then the location of the sources is estimating. However the performance degrades with the number of the samples.

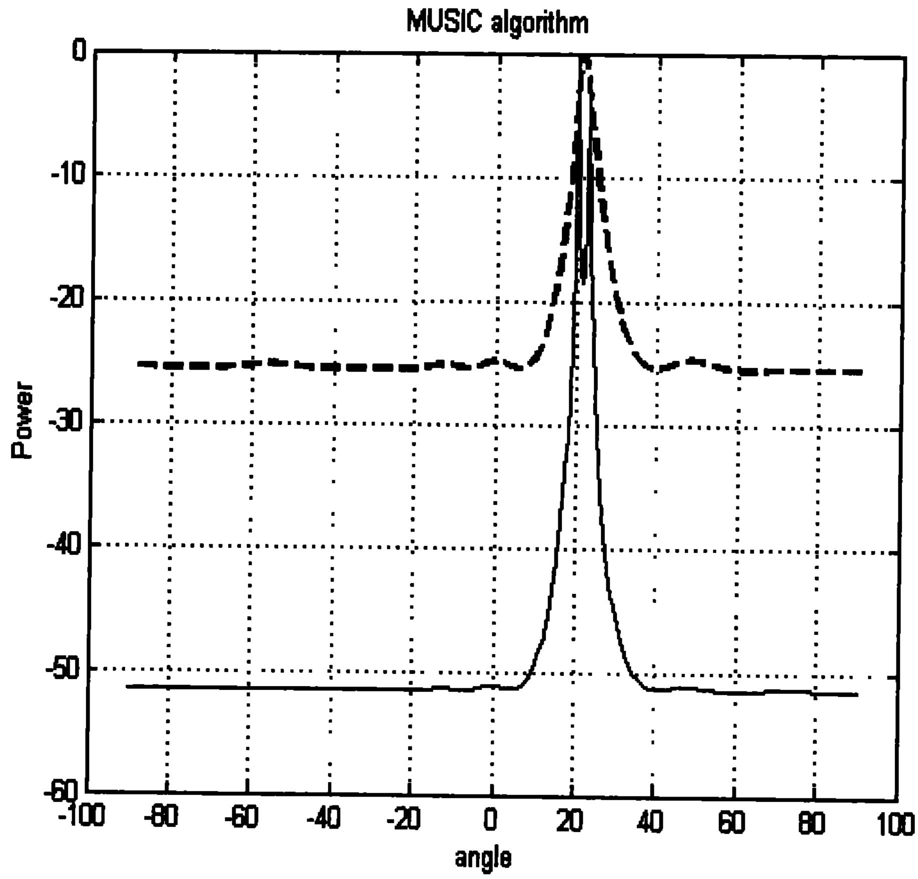


Fig 3.2: Effect of number of samples on MUSIC algorithm

In Figure 3.2, the results of two sources-based DOA estimation are presented that are at an angle of 20^0 and 23^0. The received signal SNR is 20dB and the number of antenna elements are 10.

As the number of the samples decreases then the estimation of the source localization also deteriorates.

Next, we consider CS techniques for solving the DOA problem. Considering a single sample at T=1 we consider a single snapshot problem. The method developed for single snapshot is generalized for multiple snapshot problem also keeping in mind the number of samples are still less than the Nyquist criteria. We create an over complete dictionary having a resolution of 1^0. The location of the targets can be resolved by solving equation (9) with the help of linear programming. We use convex optimization toolbox for solving this problem. For simplicity we consider a noiseless case. It is shown in Figure 3.3 those two targets with amplitude of 2 and 1 on normal scale at location 20^0 and 23^0 are resolved.

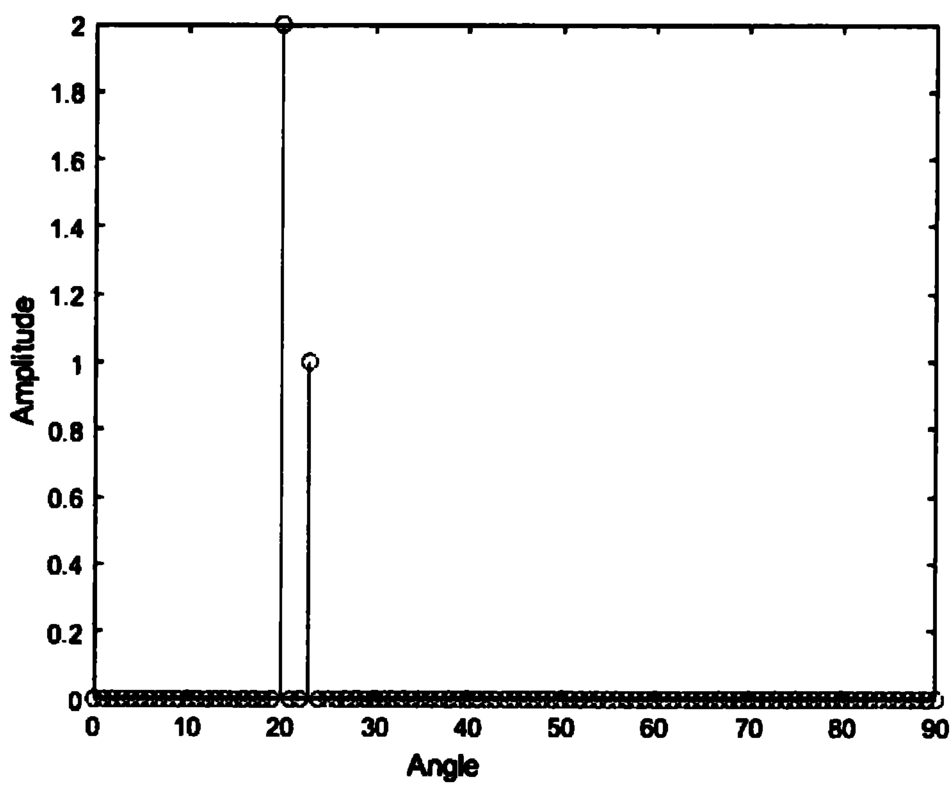


Fig 3.3: DOA estimation using CS

3.2 Grid Mismatch Problem

One of the main issues with application of CS in the DOA problem is definition of the grid resolution. The resolution depends upon the sampling grid formulation and the sampling grid is uniform. If the grid size is defined very fine, it increases the computational requirements. If the size of the sampling grid is large, then the resolution decreases and close targets cannot be detected.

Next, we consider a scenario in which the targets locations are not aligned with the grid resolution. Considering two targets; one is at 40.5^0 with amplitude 2 and the other is at 43.5^0 with amplitude 1

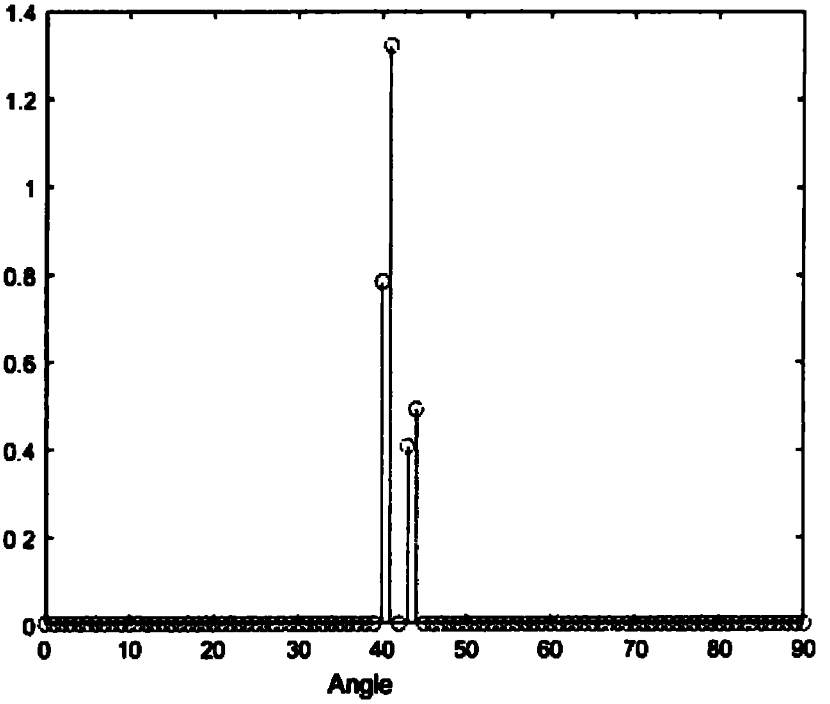


Fig 3.4: Grid mismatch of 2 sources

In Figure 3.4, it is shown that the four targets are detected. This creates ambiguity about the location of the targets and the number of the targets. However, it is observed that the amplitude of the received signal is distributed in the adjacent grid. Similarly, the figure 3.5 and 3.6 shows the grid mismatch problem for 3 and 4 sources.

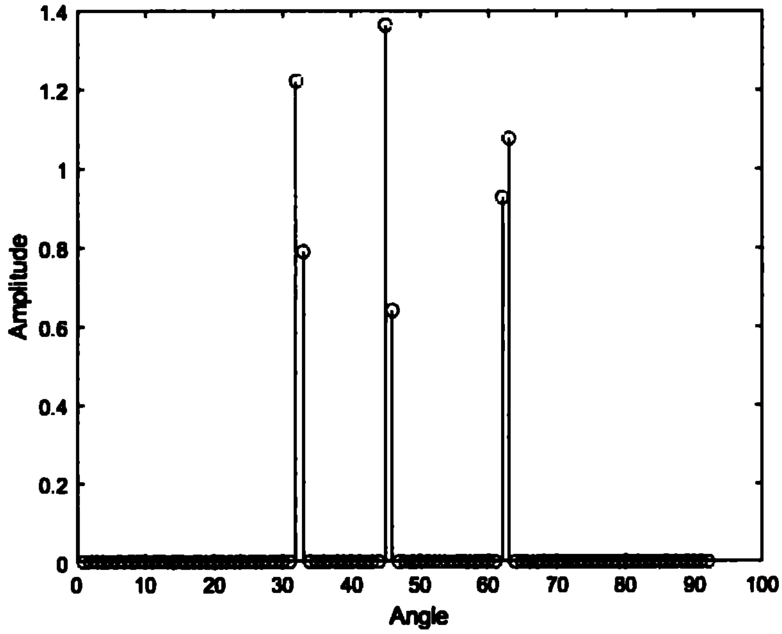


Fig 3.5: Grid mismatch for 3 sources

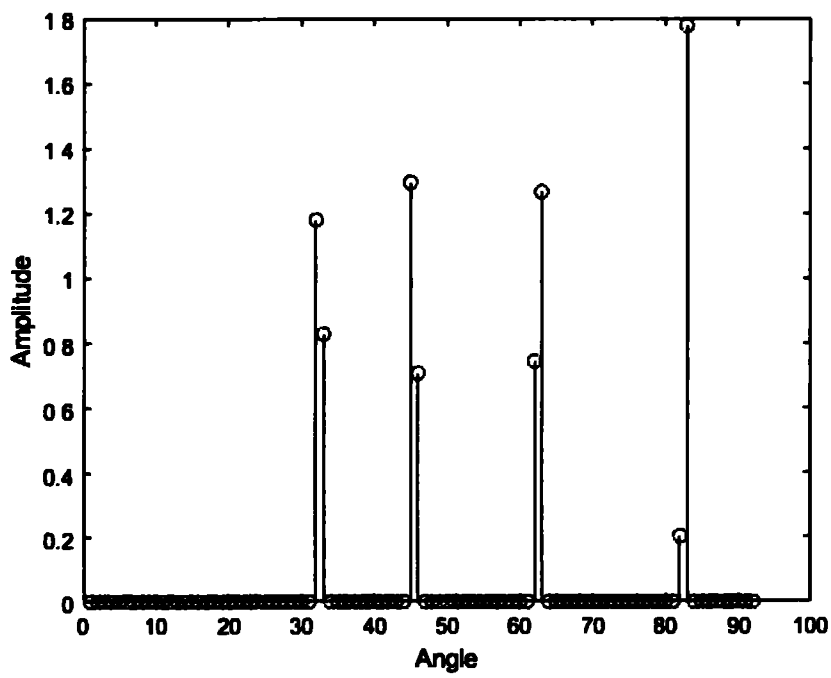


Fig 3.6: Grid mismatch for 4 sources

3.4 Proposed Solution

As shown in the previous section that due to grid mismatch multiple targets are created. This creates ambiguity about the location of the source in the angle domain. Therefore, we consider a grid refinement method depending on a fitness function. The fitness function governs the level of grid discretization.

The methodology we propose for the selection of discretization value for the grid is based on a fitness function. Considering Figure 3.7, two grids are shown. The upper grid represents the resolution of the grid with discretization value of $r = \theta_{n+1} - \theta_n$. Whereas in the lower grid θ_L represents the location of the source.

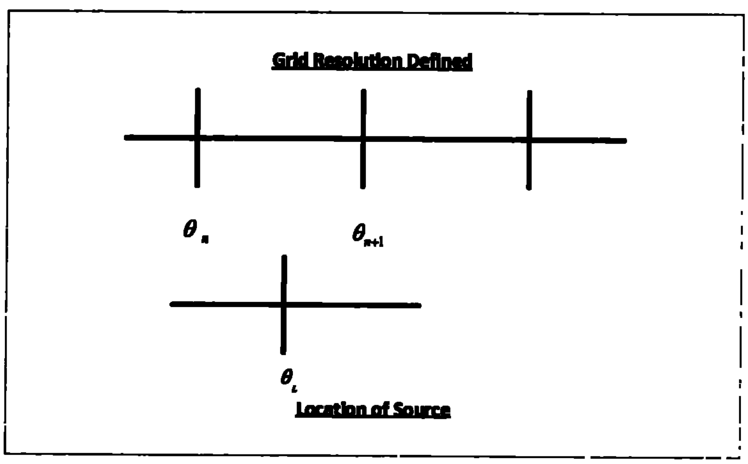


Fig 3.7: DOA grid mismatch

Using the l_1 regularization the first step is to estimate the vector s using the overcomplete dictionary defined for the iteration

$$\mathbf{s}' = \min \|\mathbf{y} - \mathbf{A}'\mathbf{s}\|_2^2 + \lambda \|\mathbf{s}\|_1 \tag{12}$$

Due to grid mismatch, the energy of the source is distributed among the adjacent grids as shown in figure 4. The discretised grid resolution is 1^0 defined and the location of the source is 60.4^0. The energy of the source distributed among the adjacent grids is mathematically presented as

$$E_i = \left| E_{\theta_K} - E_{\theta_{K+1}} \right| \tag{13}$$

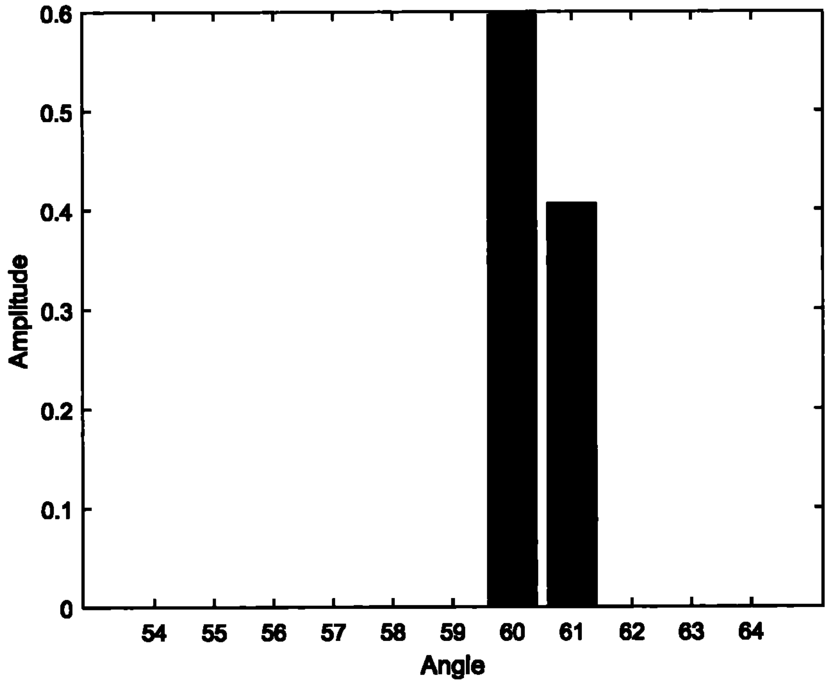


Fig 3.8: Ambiguity due to grid mismatch of single source

Let *i* be the iteration index in which the discretization value is r^i. The peaks in the vector *s* are detected, and the difference is taken as in above equation.

$$F_i = E_i \tag{14}$$

Then the process is repeated with $i+1^{th}$ iteration with finer grid discretised value and with dictionary defined with new discretization value. The fitness function is calculated as

$$F_{i+1} = E_{i+1} \tag{15}$$

A termination criterion is defined asset as

$$F_i \geq F_{i+1} \tag{16}$$

If it satisfies the termination criteria, then the discretization value in the i^{th} iteration is the best value for the grid to discretise at. For the case of multiple sources, the equation 15 can be generalized for sum of the difference of adjacent peaks and sum of individual peaks if there is no adjacent peak. As mentioned, i is the iteration number. In each iteration the discretization value is reduced. It is selected by the user. In our simulations we have selected a discretization value of 1, 0.5, 0.1, 0.01. The main steps involved in the proposed algorithm are given as follows.

Proposed Algorithm

Setup:

1- Define initial grid resolution of 1°

2- Calculate the Over Complete dictionary

3- Select the fitness function, i.e., equal to zero

4- Estimate the regularization term λ using GCV method

While ($F_i \geq F_{i+1}$)

1- Calculate s^i

2- Calculate F_i according

3- Change grid resolution

4- Calculate dictionary $\mathbf{A}^{i+1}$

5- Estimate $\mathbf{s}^{i+1}$

6- Calculate F_{i+1}

End while

Output: r^i

The regularization term, λ in (17) plays important role for the accuracy of the solution and must be estimated. It is a compromise between finding a solution that is sparse as possible and has lower error as possible. Two methods for estimating the regularization term are L curve and generalized cross validation (GCV). In GCV, it is more convenient as compared to L curve where we must find the corner [24]. It can be computed using the following relations.

$$GCV(\lambda) = \frac{\left\|\mathbf{A}\mathbf{s}_\lambda - \mathbf{y}\right\|^2}{trace(I - \mathbf{A}\mathbf{A}^{\#})^2} \quad (17)$$

$$\mathbf{A}^{\#} = (\mathbf{A}^T\mathbf{A} + \lambda\mathbf{I})^{-1}\mathbf{A}^T$$

$$\mathbf{s}_\lambda = \mathbf{A}^{\#}\mathbf{y}$$

The GCV estimate is variant of the above equation which is obtained by applying necessary calculation and results in GCV function [25]. This technique estimates λ by assuming that the optimum value of λ should be chosen to minimize GCV value.

Table 1: Grid mismatch case for two sources

Amplitude & Location		Grid Resolution 1^0		Grid Resolution 0.5^0		Grid Resolution 0.1^0	
Amplitude	Source # 1 =2	A_1	0.7830	A_1	1.999	A_1	1.998
		A_2	1.3208				
	Source # 2 =1	A_3	0.4061	A_2	0.999	A_2	0.998
		A_4	0.4905				
Location	40.5^0	θ_1	40^0	θ_1	40.5^0	θ_1	40.5^0
		θ_2	41^0				
	43.5^0	θ_3	43^0	θ_2	43.5^0	θ_2	43.5^0
		θ_4	44^0				
Fitness		$F_1 = 0.6221$		$F_2 = 2.999$		$F_3 = 2.998$	

Table 2 shows the iterations for solving the grid mismatch problem using the proposed algorithm. We consider a different case, where finer resolution is required to detect the sources. Two sources are considered with locations at 40.4^0 and 43.3^0 and amplitude of 2 and 1 shown in Table 3.

Table 2: Grid mismatch case for three sources

Amplitude & Location		Grid Resolution 1^0		Grid Resolution 0.5^0		Grid Resolution 0.1^0		Grid Resolution 0.01^0	
Amplitude	Source # 1 =2	A_1	0.9797	A_1	0.3381	A_1	1.9988	A_1	1.9916
		A_2	1.1367	A_2	1.6696				
	Source # 2 =1	A_3	0.5718	A_3	0.5718	A_2	0.9987	A_2	0.991
		A_4	0.3312	A_4	0.3122				
Location	40.4^0	θ_1	40^0	θ_1	40^0	θ_1	40.4^0	θ_1	40.4^0
		θ_2	41^0	θ_2	40.5^0				
	43.3^0	θ_3	43^0	θ_3	43^0	θ_2	43.3^0	θ_2	43.3^0
		θ_4	44^0	θ_4	43.5^0				
Fitness		$F_1 = 0.4167$		$F_2 = 1.4934$		$F_3 = 2.9975$		$F_4 = 2.9827$	

We consider the case of four sources at different location in table 3. Due to grid mismatch multiple targets are detected. However using the mention optimization algorithm the resolution of the grid is resolved for the best estimation of the targets.

Table 3: Grid mismatch case for four sources

Amplitude & Location		**Grid Resolution 1^0**		**Grid Resolution 0.5^0**		**Grid Resolution 0.1^0**		**Grid Resolution 0.01^0**	
Amplitude	Source # 1 =2	A_1	1.1805	A_1	0.3951	A_1	1.9998	A_1	1.9991
		A_2	0.8286	A_2	1.6061				
	Source # 2 =2	A_3	1.2950	A_3	0.7604	A_2	1.9997	A_2	1.9977
		A_4	0.7080	A_4	1.2409				
	Source # 3 =2	A_5	0.7446	A_5	1.9054	A_3	1.9990	A_3	1.9923
		A_6	1.2663						
	Source # 4 =2	A_7	0.2037	A_6	1.0556	A_4	1.9951	A_4	1.9631
		A_8	1.7768	A_7	0.9408				
Location	30.4^0	θ_1	30^0	θ_1	30^0	θ_1	30.4^0	θ_1	30.4^0
		θ_2	31^0	θ_2	30.5^0				
	43.3^0	θ_3	43^0	θ_3	43^0	θ_2	43.3^0	θ_2	43.3^0
		θ_4	44^0	θ_4	43.5^0				
	60.5^0	θ_5	60^0	θ_5	60.5^0	θ_3	60.5^0	θ_3	60.5^0
		θ_6	61^0						
	80.7^0	θ_7	80^0	θ_6	80.5^0	θ_4	80.7^0	θ_4	80.7^0
		θ_8	81^0	θ_7	81^0				
Fitness		$F_1 = 3.0338$		$F_2 = 3.7118$		$F_3 = 7.9938$		$F_4 = 7.9523$	

In the figure given below we consider two targets located which are at 30.5° and 60.5°. The regularization parameter for each SNR level is calculated using the GCV method mentioned. The mean square error (MSE) of the proposed method is compared with Cramer Rao lower bound (CRLB) for DOA estimation and off grid method with bias. For a greater SNR based scenario, the proposed algorithm is reasonable accurate. Additionally, the proposed approach is simpler and requires less computations.

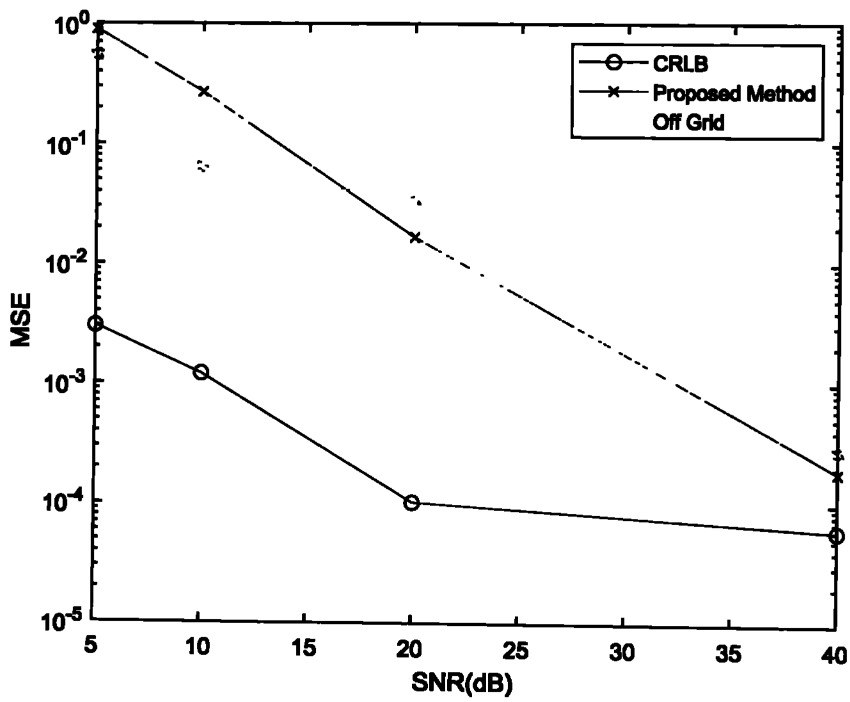


Fig 3.9: MSE comparison

CHAPTER 4

INTRODUCTION

In the previous section we discussed the technique for casting the DOA estimation problem in to the compressive sensing problem. Similarly, a grid refinement problem was considered for solving the target ambiguity due to grid mismatch. The problem was defined for ULA. However, the antenna arrays can be of different structure. Which can be of one dimension or two dimensions. Whereas planar array, circular array or L shaped array are used for DOA estimation in two dimensions. L shaped array consists of two orthogonal ULAs. It provides improved DOA estimation [98]. We have used CS using linear array. In this chapter we consider multiple snapshots and one dimension and two-dimension sparse antenna array. The number of snapshots is still less than the numbers of samples required by Nyquist criteria.

Noise have effect on the estimation of the above--mentioned technique. Generally, the distribution of noise is considered Gaussian. However, the signal can be corrupted with impulses in the time domain due to atmospheric noise or faulty receiver. This does not follow Gaussian distribution. The impulsive noise can be defined by alpha stable distribution [99]. To solve the impulsive noise median difference correntropy (MDCO) algorithm is suggested in [100]. The MDCO derives the weighting factor from the correntropy criterion, which suppresses impulsive noise. There are several methods to make the algorithm robust, like moving average filter. Weighted moving average filter is a derivative of moving average filter. Whereas the weighted moving average filter has better performance than moving average filter.

In practical scenarios, the antenna elements may malfunction which may lead to sparse antenna arrays. The traditional DOA estimation algorithms do not work for sparse antenna arrays. Whereas Sparse antenna arrays have certain advantages also. Using sparse antennas, larger aperture can be achieved [101]. For designing sparse antenna array different techniques have been developed. The idea of using numerical techniques for designing non-ULA was first utilized in minimum redundancy array which was introduced by Moffet. Similarly, sparse antenna array utilizing coprime antenna elements was introduced in [102]. However, if the sparse array is random then there is not much control over the design. In some papers different subspace decomposition methods are used, like in Yarris Wang who uses krylov subspace method then apply ESPIRIT algorithm for estimation of DOA using sparse antenna array.

4.1 Signal Model

Here we will develop the model for sparse antenna array for compressive sensing. Similarly different antenna structure is considered for one dimension and two-dimension search in the angle domain. Compressive sensing (CS) techniques are used for DOA estimation in sparse antenna array. As CS technique is used for solving an underdetermined system [103,104] and it has gained lot of popularity. According to the CS mathematical framework, if a signal is sparse in some domain, "then it can be reconstructed using only a small number of linear measurements" as mentioned in [105]. As most of the real time signals are sparse therefore the signal can be reconstructed using a linear equation. It is important that restricted isometric property (RIP) is satisfied. In CS, we try to find the sparse vector. It can be viewed as solving the inverse problem, which can be expressed in term of finding the l_0 norm. Mathematically it is given as

$$\min_{s} \|\mathbf{s}\|_0 \quad s.t \quad \mathbf{x} = \mathbf{A}\mathbf{s} \tag{1}$$

$\|s\|_0$ represents the number of nonzero in the vector. The $\mathbf{x}$ is the observed vector $\mathbf{x} \in \mathbb{C}^{M \times 1}$ and $\mathbf{A} \in \mathbb{C}^{M \times N}$ is the basis matrix called the dictionary. This is an NP hard problem. To find the sparse solution a number of methods have been developed, like greedy algorithm [106], basis pursuit [107], LASSO algorithm, In [108] a recursive weighted minimum norm with focul underdetermined system solver (FOCUSS) is applied. In [109] a hardware has been developed for spectral estimation using compressive sensing framework.

In most cases it is considered that the position of the sources is in the grid defined. That is not always possible. It may happen such that the location of the source does not coincide with the discretization of the grid. This creates grid mismatch or off grid targets. The problem can also be solved using off-grid sparse method [31]. The initial grid resolution is still defined. A bias is added to the signal model using first order approximation of the manifold matrix. This model may be non-convex and difficult to solve. Iterative grid refinement is one of the most popular methods which requires a procedure to select the discretization value for the grid.

We consider multi sample compressive sensing technique to sparse antenna array and evaluate the performance of the compressive sensing for nonuniform sparse antenna array. Two type of antenna array structures are considered. For one-dimension DOA estimation we consider non uniform linear sparse array and for DOA estimation in two dimension we consider non uniform sparse L shaped array. We consider that the sources are quasi-static. A multi sample CS scenario is considered.

Now we develop the signal model of the system. Initially we consider uniform linear array (ULA). Then we develop the signal model for random sparse linear array and L shaped array is developed. Then the application of the CS for DOA estimation for the developed array structure is considered.

First let us consider a uniform linear array. The signal received by the m^{th} element in the presence of noise not only Gaussian but also impulsive is given as

$$y_m(t) = \sum_{i=1}^{P} s_i(t) e^{-j(m-1)kd\sin\theta_i} + n_p(t) + n_g(t) \tag{2}$$

The distance between the antenna element is $\lambda/2$ with M number of antenna elements in the array. We consider that there are P number of sources at different angles θ_i and with amplitude of s_i, where as $i = 1,2 \ldots P$. in the far field, n_g is the Gaussian noise and n_i is the impulsive noise. The above equation can be expressed as

$$\mathbf{y} = \mathbf{A}\mathbf{s} + \mathbf{n}_p + \mathbf{n}_g \tag{3}$$

The above equation can also be written as

$$\mathbf{y} = \mathbf{A}\mathbf{s} + \mathbf{n}_T \tag{4}$$

$\mathbf{n}_T$ represents the total noise. Which consist of Gaussian as well as impulsive noise. In the above equation $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots \cdots, \mathbf{a}(\theta_P)]$ represents the steering matrix with $\mathbf{a}(\theta_p) = [1, e^{-jkd\sin\theta_p}, \cdots \cdots, e^{-j(M-1)kd\sin\theta_p}]^T$

After the model for the ULA, we develop the signal model for sparse antenna array. Using sparse antenna array can have certain advantages. In ULA it is possible that certain number of the antenna elements malfunction. This leads to a sparse antenna array such that the spacing between the antenna elements become non uniform and random. As shown in the figure given below.

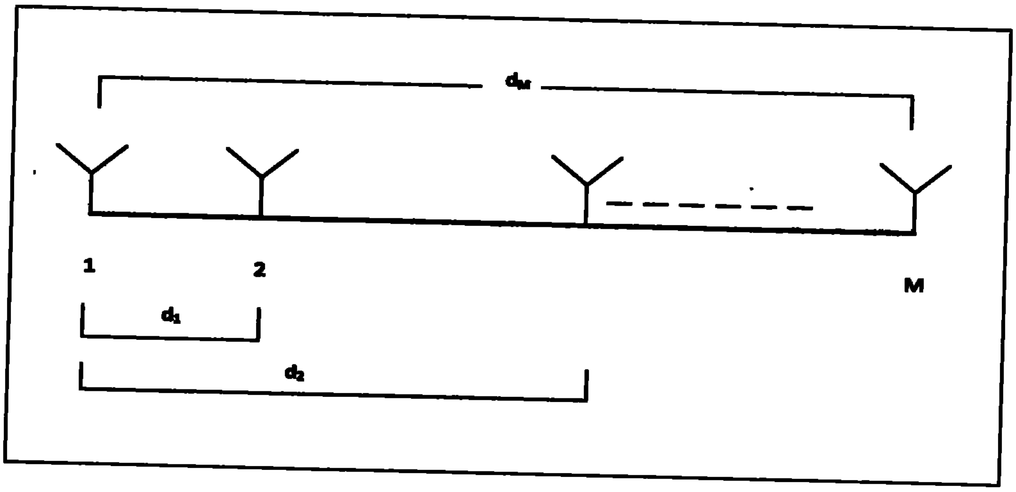


Fig 4.1: Sparse antenna Array

Let us consider an antenna array consist of M number elements. Some of the elements have malfunctioned and only few elements are working. We suppose that the first and the last antenna elements are functioning. Due to this the distance between the antenna element is not uniform, such that the distance d_2 is not twice the distance d_1 as shown in the figure given above. Hence the distance between the antenna element is not uniform. The received signal for such and antenna array is given as

$$\mathbf{x} = \bar{\mathbf{A}}\mathbf{s} + \mathbf{n}_T \tag{5}$$

$\mathbf{x}$ is the received vector from the sparse linear array where $\mathbf{x} \in \mathbb{C}^{M' \times 1}$. $\bar{\mathbf{A}} \in \mathbb{C}^{M' \times P}$ is steering matrix and $\mathbf{n}_T$ is combined noise defined earlier. M' = position of the active element in the array with respect to the first element. Using such an antenna array configuration, the traditional DOA estimation algorithm like MUSIC fails to estimate the location of the sources. For DOA estimation using sparse linear array CS technique will be used.

For estimation in 2 dimension a 2D array structure is required. We consider an L-shaped array for estimation of the source location in 2 dimensions. As the name suggests it consist of two ULA such that they are orthogonal to each other as shown in the figure 4.2 given below.

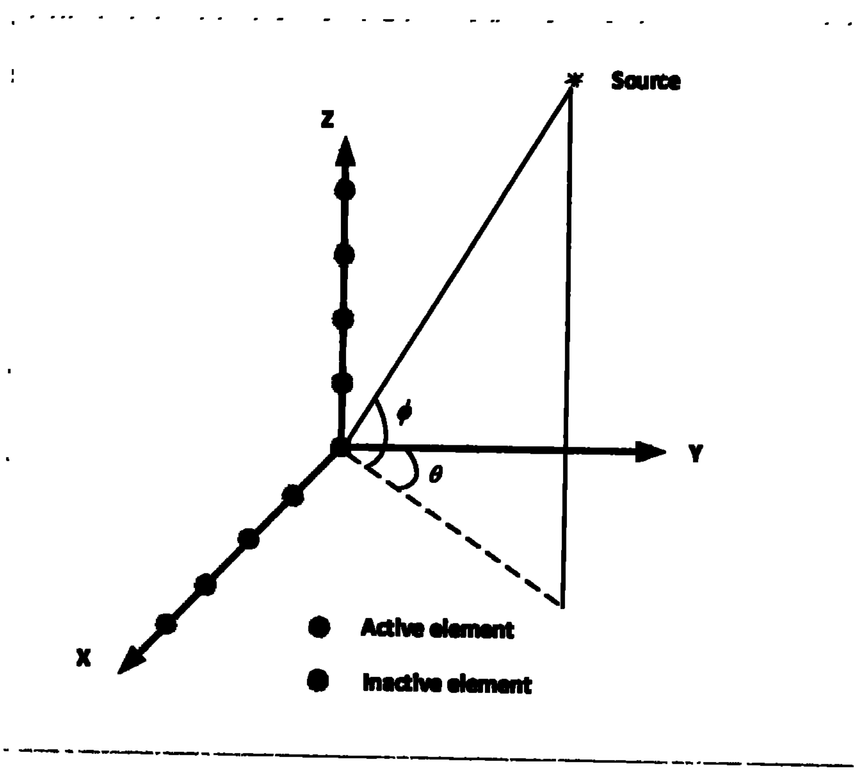


Fig 4.2: Sparse L shaped antenna array

In the above figure represent two linear sparse antenna arrays such that the two arrays are orthogonal to each other's. Since they are in two dimensions of the array configuration can be used for detection of the source in two dimensions also.

The L shaped antenna array have certain advantages when compared to other antennas regarding DOA estimation in two dimensions. The L-shaped array has a simpler configuration as compared to other configurations such as rectangular and triangular array, however it has a better accuracy for estimating 2-D DOA [110] the L-shaped arrays can provide the larger array aperture defined by the largest distance among the sensors. The cramer rao bound in DOA estimation for L shaped antenna is better than the antenna structure of the same category. In [101], Tayem and Kwon has shown that it is possible to decompose the 2-D problem into two independent 1-D problems by using the L-shaped . But, the two independent sets of angles have to be properly paired together using appropriate techniques [9].

We now develop the signal model for sparse L-shaped antenna array for estimation in two dimensions. Let us consider the structure shown in figure 4.2. Two random linear sparse arrays along the x axis and z axis are considered such that the element at the origin of the axis is common between them. Let M' and N' be the number of the active elements along the x and z axis. For simplicity we can assume that M' is equal to N'. We treat L shaped element configuration as two separate linear sparse elements array. Then the signal received along the z axis is given as

$$\mathbf{x}_z = \mathbf{A}_z\mathbf{s}_z + \mathbf{n}_T \tag{6}$$

In the above equation $\mathbf{A}_z$ is the steering matrix of dimension $\mathbf{A}_z \in \mathbb{C}^{M' \times P}$. M' represents the position of the active element in the array. $\mathbf{A}_z = [\mathbf{a}_z(\phi_1), \mathbf{a}_z(\phi_2), \cdots, \mathbf{a}_z(\phi_P)]$ whereas $a_z(\phi_p)$ is given as $\mathbf{a}_z(\phi_p) = [1, e^{-jkd\sin\phi_p}, \dots\dots, e^{-j(M'-1)kd\sin\phi_p}]^T$. The signal along the x axis is given as

With $\mathbf{A}_x = [\mathbf{a}_x(\theta_1,\phi_1), \mathbf{a}_x(\theta_2,\phi_2), \cdots, \mathbf{a}_x(\theta_P,\phi_P)]$ where $a_x(\theta_p,\phi_p)$ is given as

$\mathbf{a}_x(\theta_p,\phi_p) = [1, e^{-jkd\cos\theta_p\cos\phi_p}, \dots\dots, e^{-j(M'-1)kd\cos\theta_p\cos\phi_p}]^T$

In Compressive sensing we try to solve an underdetermined system [32]. Such that any sparse representation of the signal can be reconstructed. Cconsider a signal s which is sparse in certain domain $\mathbf{s} \in \mathbb{C}^{N \times 1}$ with $\mathbf{x}'$ as the received signal of dimension M such that $\mathbf{x}' \in \mathbb{C}^{M \times 1}$. The received signal is given as

$$\mathbf{x}' = \mathbf{A}'\mathbf{s} \tag{8}$$

$\mathbf{A}'$ is a sensing matrix of dimension $\mathbf{A}' \in \mathbb{C}^{M \times N}$ where $M << N$. The system is underdetermined and there is no unique solution for s. If the unknow signal s is considered sparse then the optimization problem, can be given as

$$\hat{\mathbf{s}} = \min \|\mathbf{s}\|_p \ s.t\ \mathbf{x}' = \mathbf{A}'\mathbf{s} \tag{9}$$

With $p = 0$ then the problem will be a NP hard problem. Therefore, we consider $p = 1$. This can be casted as a l_1 norm problem and solved following the given equation

$$\hat{s} = \min \left\| x' - A's \right\|_2^2 + \lambda \left\| s \right\|_p \tag{10}$$

In the above equation λ is the regularization term. It is a compromise between finding a solution that is sparse as possible and has lower error. If the received signal is contaminated with noise. The optimization problem without impulsive noise becomes

$$\min \left\| s \right\|_1 \quad s.t \quad \left\| x' - A's \right\|_2^2 < \varepsilon \tag{11}$$

where ε is a parameter that specifies how much noise is to be allowed? To formulate source localization problem in the compressed sensing framework, consider the figure given in [111]. For representing the estimation problem into a CS, we assume that the location of the sources is sparse in the angle domain. For representing the estimation problem into a CS framework, we assume that the location of the source is sparse in the angle domain. We define an overcomplete dictionary A' as shown in the equation 11.

$$A' = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{jkd\sin\theta_1} & e^{jk\sin\theta_2} & \cdots & e^{jk\sin\theta_N} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ e^{j(M-1)kd\sin\theta_1} & e^{j(M-1)kd\sin\theta_2} & \cdots & e^{j(M-1)kd\sin\theta_N} \end{bmatrix} \tag{12}$$

Where $A' = [\theta_1, \theta_2, \ldots, \theta_N]$, N is the sampling of the grid. The N will be much higher than M. However, considering a case for multi time samples let T be the duration for multi snap shot which is given as $T = 1,2,3,\ldots,T_p$. Then the received vector X is given as $X(t) = [x(t_1), x(t_2), \ldots, x(t_{T_p})]^T$. As mentioned earlier, we suppose that the location of the sources is stationary or very slow for the sampling period.

Some of the issues in applying compressive sensing is the presence of impulsive and Gaussian noise in the received signal. Also, application of compressive sensing for DOA estimation in two dimensions requires very high computation. To make the system more robust against the presence of noise, a preprocessing stage is added before applying compressive sensing technique. Similarly, due to grid mismatch it is possible multiple targets are detected. Therefore, a method is required for removal of ambiguity for DOA estimation in one and two dimensions by selecting the grid discretization value.

As mentioned earlier that noise plays an important role in the estimation of the sources. we take into consideration the effect of noise on the DOA estimation using single sample CS technique and our proposed scheme. Only Gaussian noise is considered We consider the same number of sparse antenna element as considered in the previous case. There are two sources located at 30^0 and 40^0. In this case the SNR is 2dB. As shown in the figure 4.3, that by directly applying CS technique using single time sample, the location of the sources is not much distinguishable from the noise.

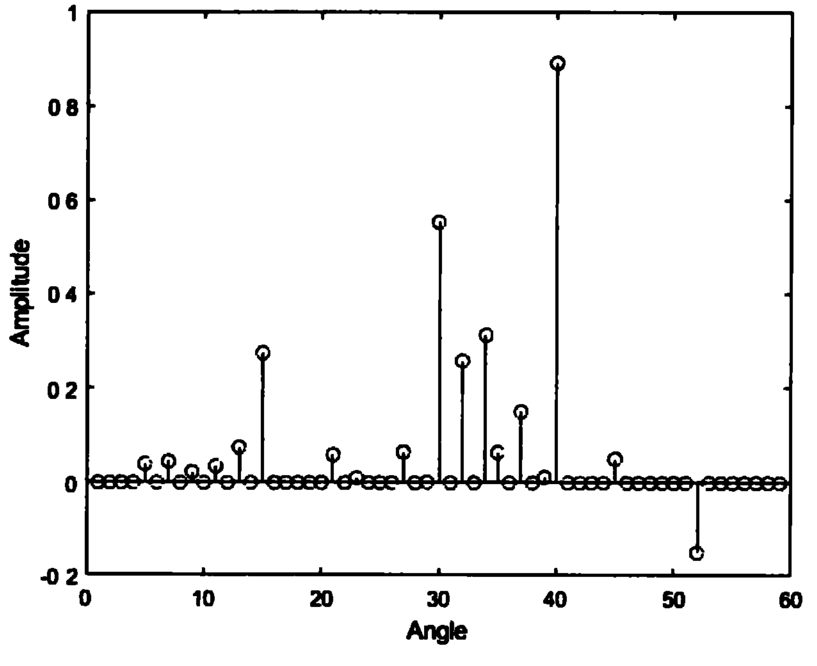


Fig 4.3: Two sources with SNR of 2dB

Similarly, the estimation criteria degrades in the presence of the impulsive noise. we consider a sparse linear array with two sources at -10° and 10° with SNR of 40dB. We consider an impulsive noise at one of the active elements. The effect of impulsive noise on estimation is shown in figure 4.4.

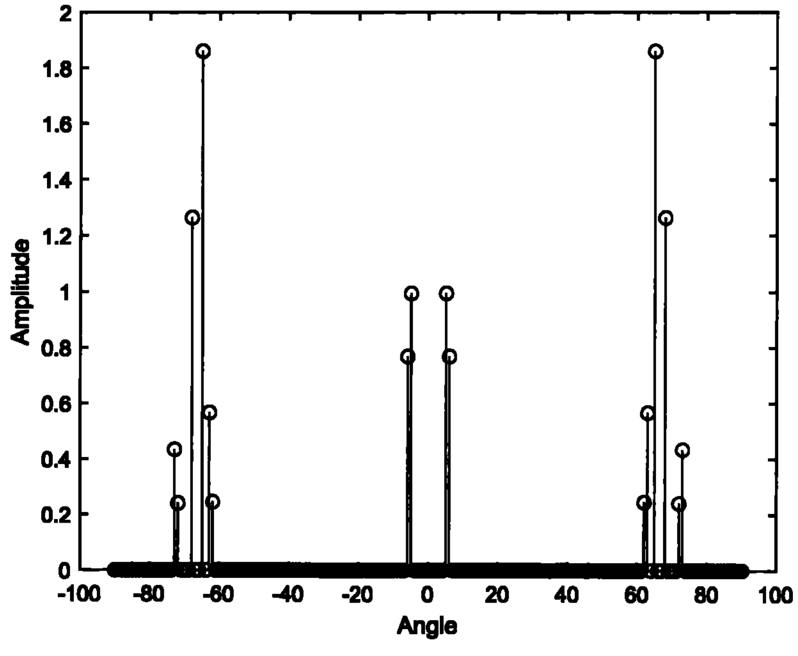


Fig 4.4: DOA estimation in the presence of Noise

4.2 PROPOSED METHODOLOGY

Here we consider a method for solving multi sample CS framework for sparse uniform linear array as well as L shaped array. As mentioned earlier we consider that two type of noise is present in the signal model. One is Gaussian noise, and the other is Impulsive noise. In order for the detection system to accurately estimate the location of the sources it is necessary to make it robust against noise by removing the effect of noise as much as possible. A pre processing stage before CS is suggested for removal of the impulsive noise and make the received signal more robust against

Gaussian noise. Also, a method for removing ambiguity about the position of the sources due to grid mismatch is proposed.

A preprocessing stage for the uniform linear sparse array is used to minimize the effect of noise. That can be extended for L shaped array. The pre processing stage consist of two parts. First part deal with the removal of impulsive noise using MDCO. The other part is the implementation of the weighted moving average filter. We assume such that the first antenna element is the reference element for a certain time. The energy of the signal received during the sampling period T_p is given as

$$E_{ref} = \frac{\sum_{i}^{T_p} x(t_i)}{T_p} \tag{13}$$

We set a threshold $\varepsilon_T = 6\sigma_{ref}^2 \cdot \sigma_{ref}^2$ is the variance of the signal at the reference antenna. Now we consider two variables P and Q such that $P = x_n(t)$ where $n = element\ number$ and $Q = x_{ref}(t)$. If $|P - Q| < \varepsilon_T$ it is considered that the signal is normal, otherwise it contains impulsive noise.

MDCO algorithm presented in [6] is used. According to which the correntropy is a method to deal with nonlinear and local optimal measurement of two random variables like P and Q. That is given as

$$V(P,Q) = E_{PQ}[\kappa(P,Q)] = \iint \kappa(p,q) P_{P,Q}(p,Q)\,dp\,dq \tag{14}$$

In the above equation $\kappa(\,.\,)$ is the translation. Generally Gaussian density function is used as a kernel function. The random variable P and Q follows alpha-distribution, then the MDCO is given as in equation 15 which is used to suppress the impulsive noise. σ is the size of the Gaussian kernel and B is the l_1 norm of the pre process data from each element, which represents the median of the received signal.

$$R_{MDCO} = E\left[\exp\left(-\frac{\|P| - B|^2}{2\sigma^2}\right)\exp\left(-\frac{\|Q| - B|^2}{2\sigma^2}\right)\exp\left(-\frac{\|P| - Q|^2}{2\sigma^2}\right)PQ\right] \tag{15}$$

Using the above equation the samples containing impulsive noise are removed. Next we consider a method for removing the Gaussian noise from the system. In general there are different methods for removing the Gaussian noise. One of the method is averaging when we have multiple samples. In general, a moving average filter is given as $x_t = (x_{t,t-k+1} + . \quad . \quad + x_{t,t})/K$. Where x is the received vector from each antenna element. We find out the variance of each received vector from antenna element.

$$w_p = \mathrm{var}(\mathbf{x}_p) \quad , p = element\,no \tag{16}$$

Once the w is calculated then we apply it to the filter defined in the equation below for smoothing.

$$\mathbf{x}_{t,p} = \frac{1}{|L - w_p|}\sum_{n=t+1}^{L}(x_{t,p} + x_{t-n,p}) \tag{17}$$

Here L is the number of the samples which is T_P and w_p is the variance of the samples received. The moving average filter is applied to samples of each antenna elements.

Now we apply the compressed sensing technique on linear sparse antenna array first. The flowgraph of the technique which includes compressive sensing with a preprocessing stage to make it robust is as follows.

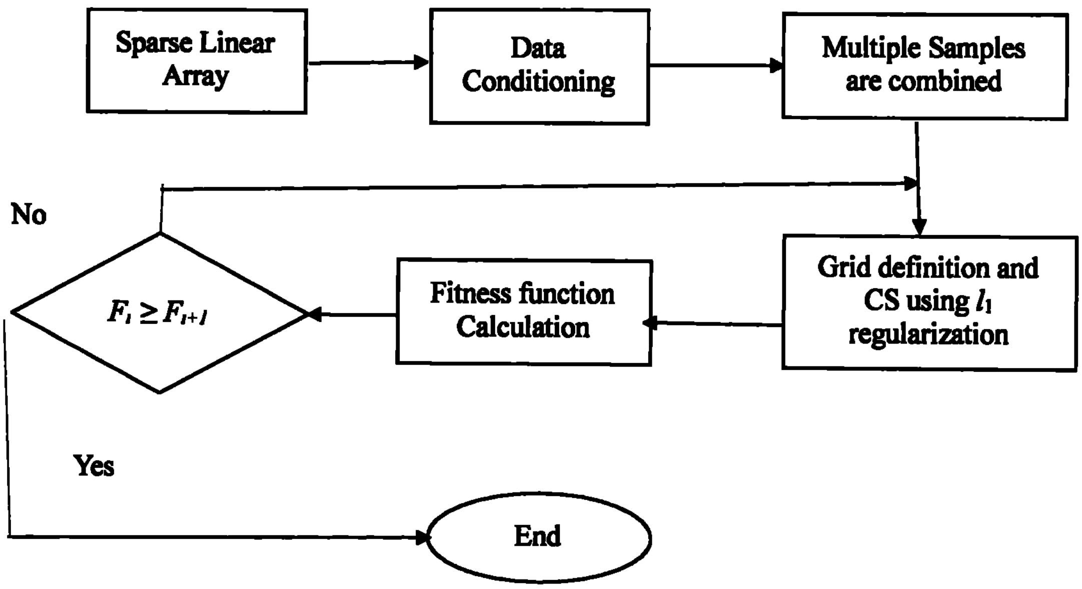


Fig 4.5: Flow graph. Cs for Linear sparse array

we study the DOA estimation technique using linear sparse array. For the current scenario impulsive noise is not considered only Gaussian noise is present. A uniform linear array of 60 element is selected. From these elements only 16 random elements are working. Keeping in view that the first and the last antenna elements are in working condition. Seven samples are taken from each active antenna elements. Two sources are at location of 51^0 and 71^0. Each with amplitude of 1 on the normal scale and with SNR of 20dB.

The proposed scheme is applied. The regularization term is selected to be 0.001. Figure 4.6 shows the result where two targets are detected.

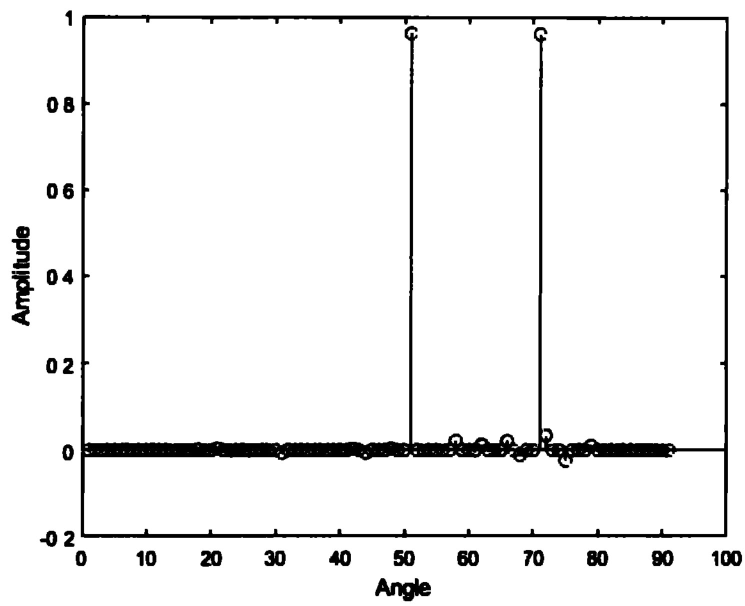


Fig 4.6: DOA estimation of two sources with SNR 20dB

Next in figure 4.7 and Figure 4.8 shows the same antenna array configuration with SNR of 15 and 9dB. The location of the sources in the angle domain is -30^0 and 30^0.

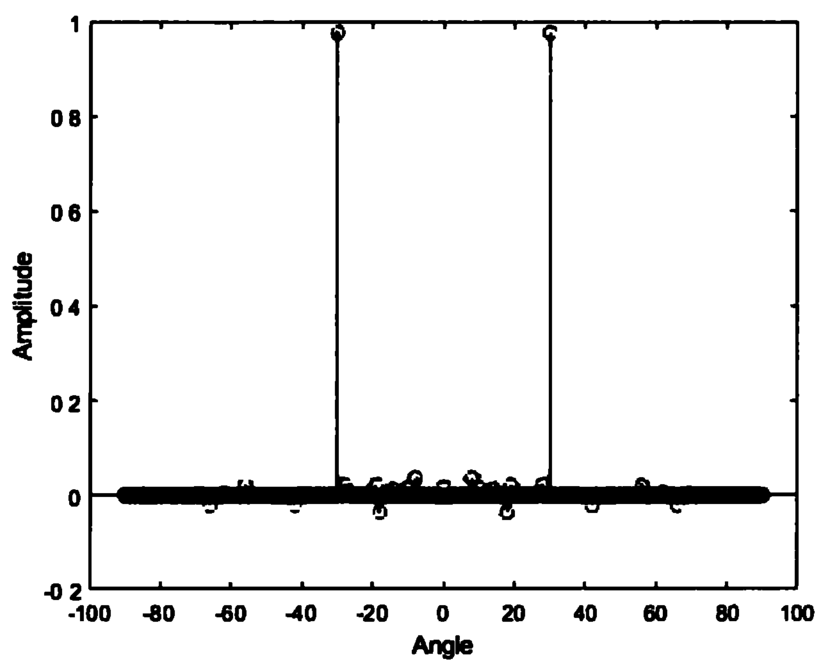


Fig 4.7: DOA estimation of two sources with SNR 15dB

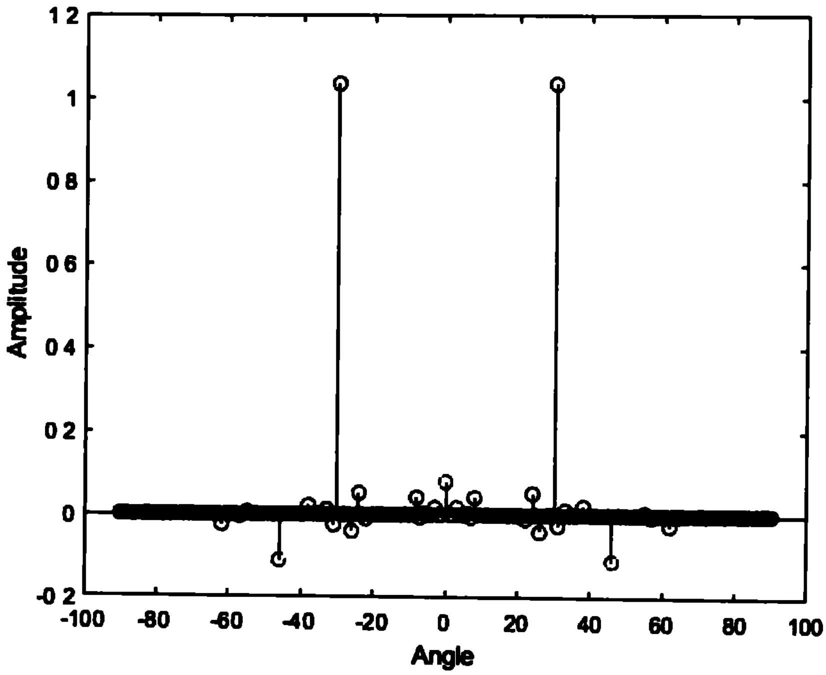


Fig 4.8: DOA estimation of two sources with SNR of 9dB

Next in figure 4.9 shows the same scenario for three sources at location of 30^0, 40^0 and 70^0 with SNR of 20dB.

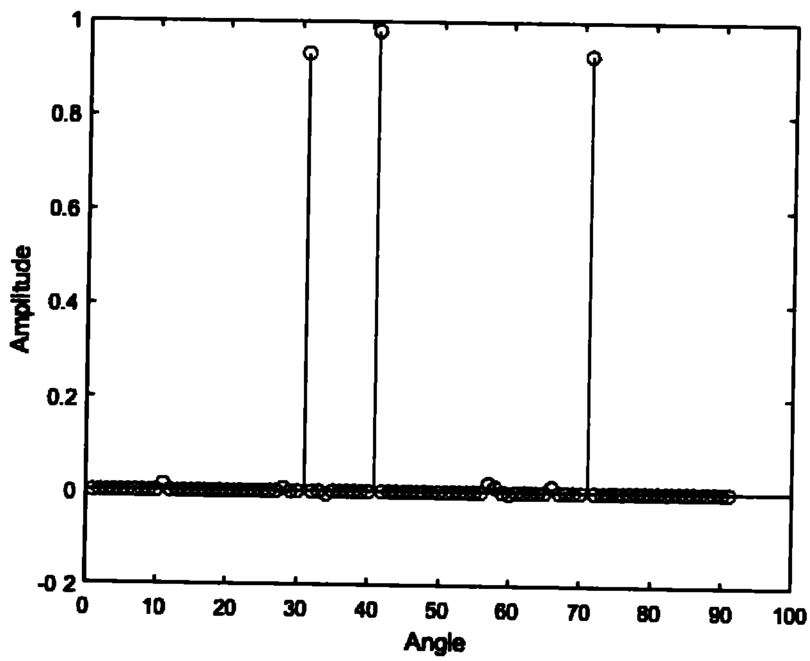


Fig 4.9: DOA estimation of three sources SNR 15dB

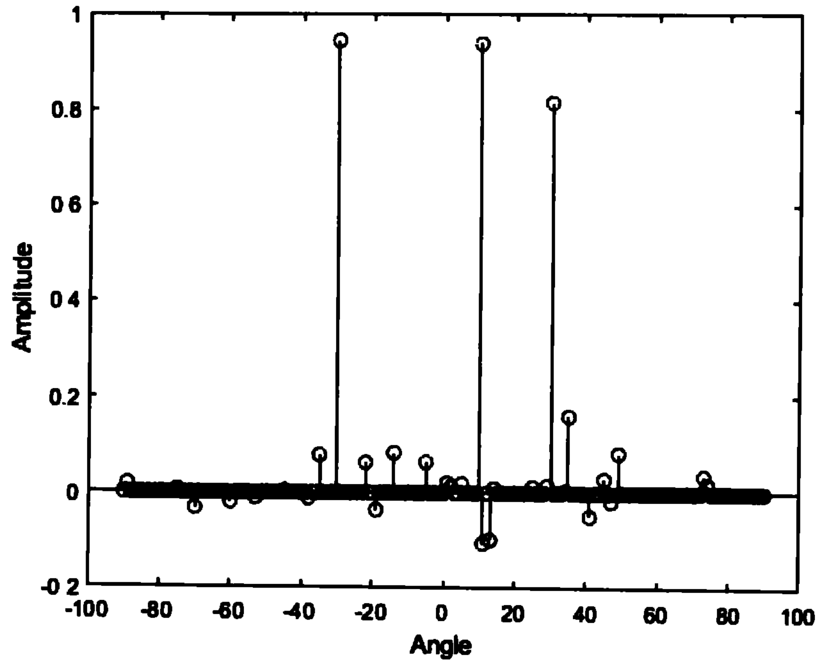


Fig 4.10: DOA estimation of three sources SNR 9dB

In figure 4.10 and figure 4.11 we estimate the DOA for three sources with same number of sparse antennas and at different SNR. Similarly, we consider a case for four sources. The location of the sources is -30^0, 10^0, 50^0 and 70^0. All the sources have the same power in figure 4.12.

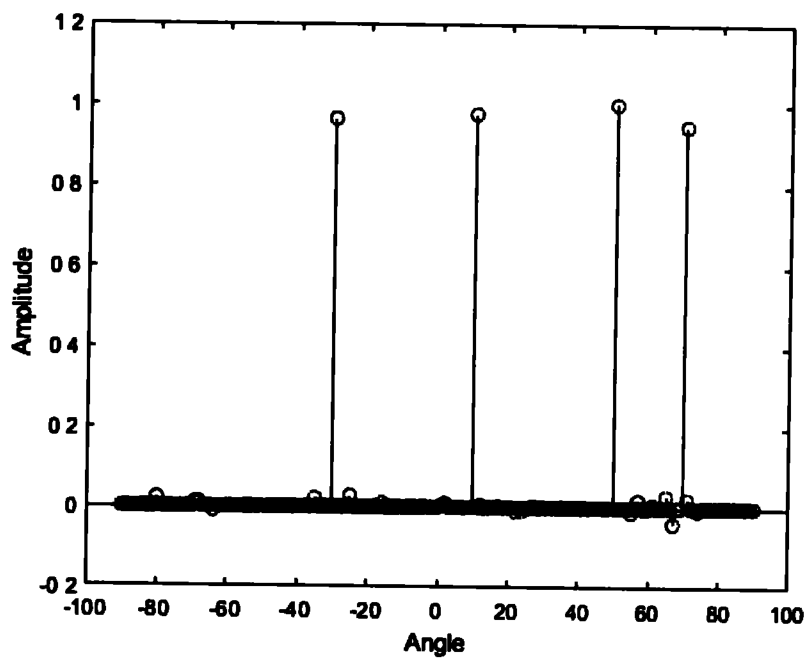


Fig 4.11: DOA estimation of four sources with SNR 20dB

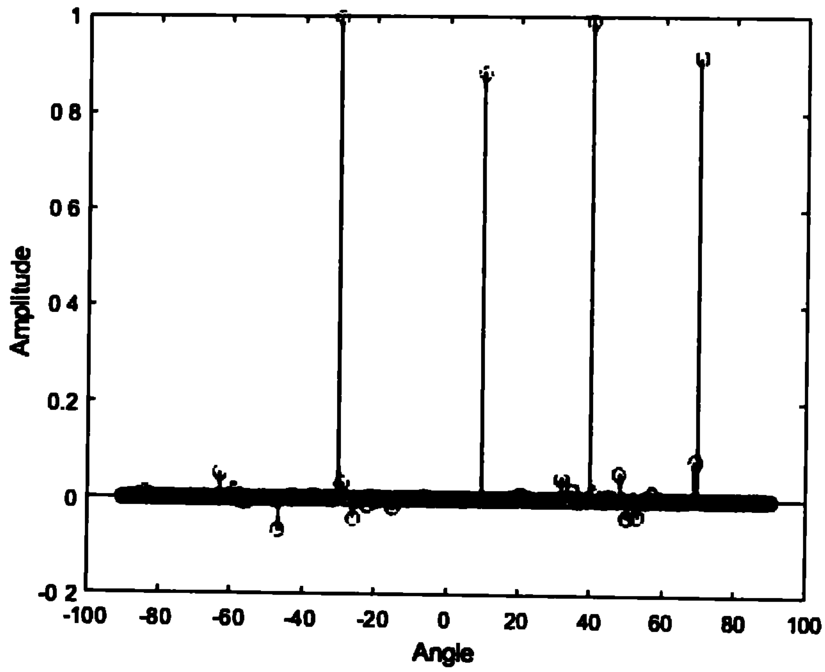


Fig 4.12: DOA estimation of four sources with SNR 15dB

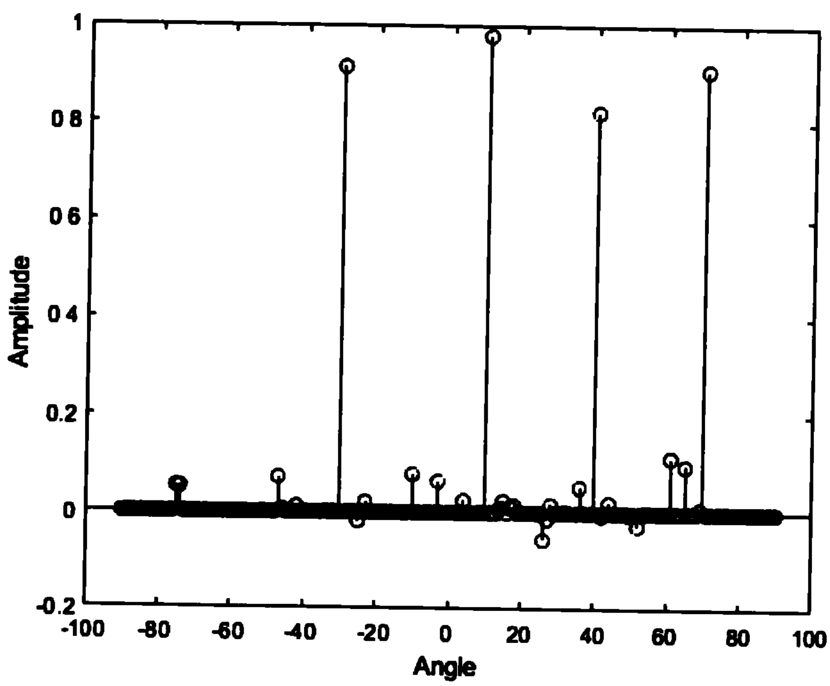


Fig 4.13: DOA estimation of four sources with SNR 9dB

The figure 4.13 and 4.14 shows detection of four sources at -30^0 ,10^0, 40^0 ,70^0 with SNR of 15dB and 9dB. It seen that multi sources can be detected using sparse antenna array.

In this case we take into consideration the effect of noise on the DOA estimation using single sample CS technique and our proposed scheme. Only Gaussian noise is considered We consider the same number of sparse antenna element as considered in the previous case. There are two sources located at 30^0 and 40^0. In this case the SNR is 2dB. As shown in the figure 4.15, that by directly applying CS technique using single time sample, the location of the sources is not much distinguishable from the noise.

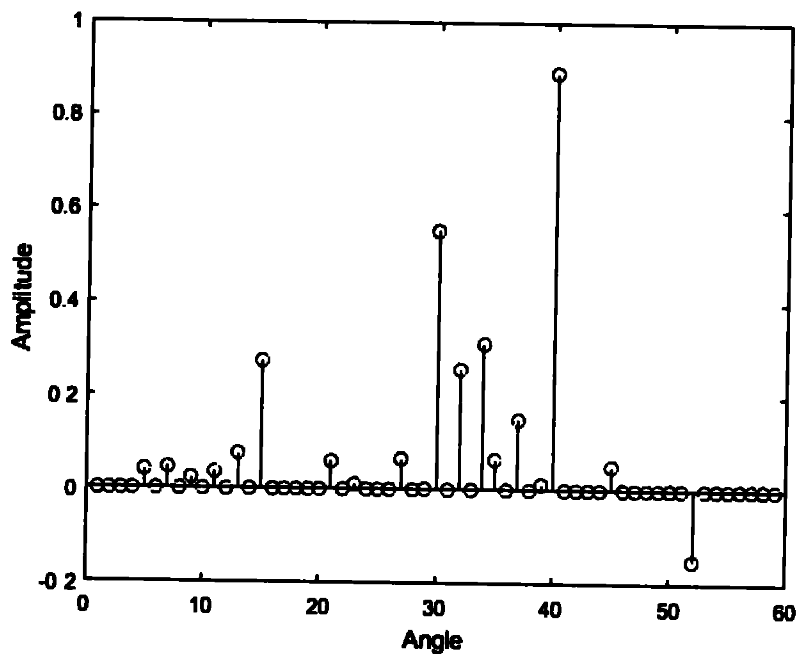


Fig 4.14: Two sources with SNR of 2dB

Next, we apply preprocessing stage before applying DOA estimation scheme. After combining multiple samples. Figure 4.16 shows the detection of the sources

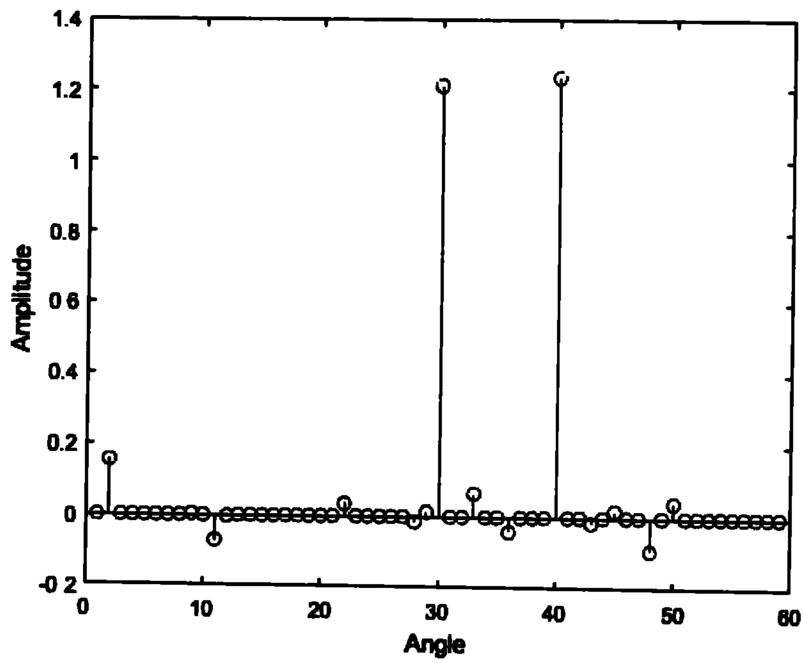


Fig 4.15: DOA estimation after data conditioning

Figure 4.14 shows the effect of calculating DOA using single sample in the presence of noise and figure 4.15 shows the effect of multiple samples on the DOA estimation. Which is more robust.

The same setup is analyzed for a signal with SNR of -6dB as shown in figure 4.17 and figure 4.18. In figure 16 preprocessing is not applied. The location of the sources cannot be identified.

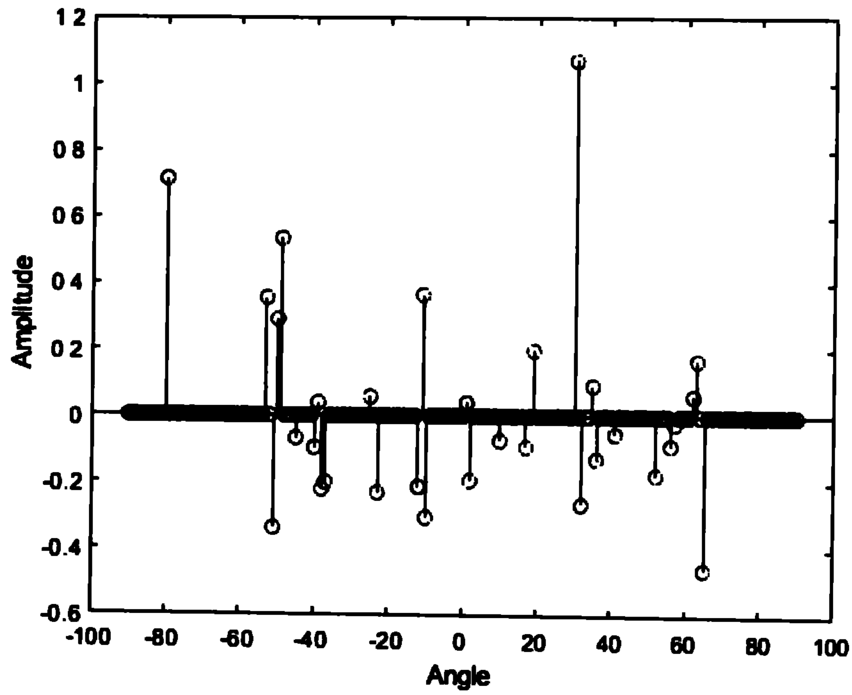


Fig 4.16: Two sources with SNR -6dB

After applying preprocessing stage before the estimation there is an improvement and the peaks in the angle domain are distinguished as in figure 4.17.

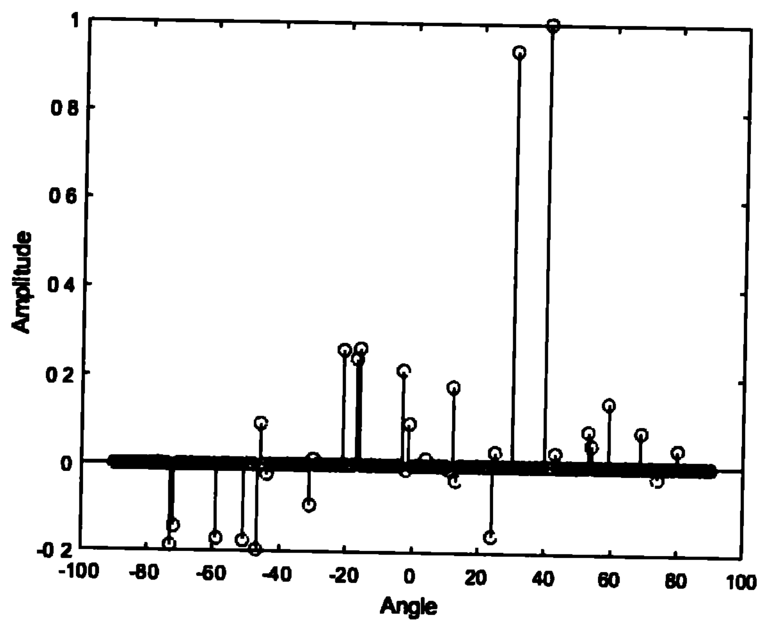


Fig 4.17: DOA estimation after preprocessing

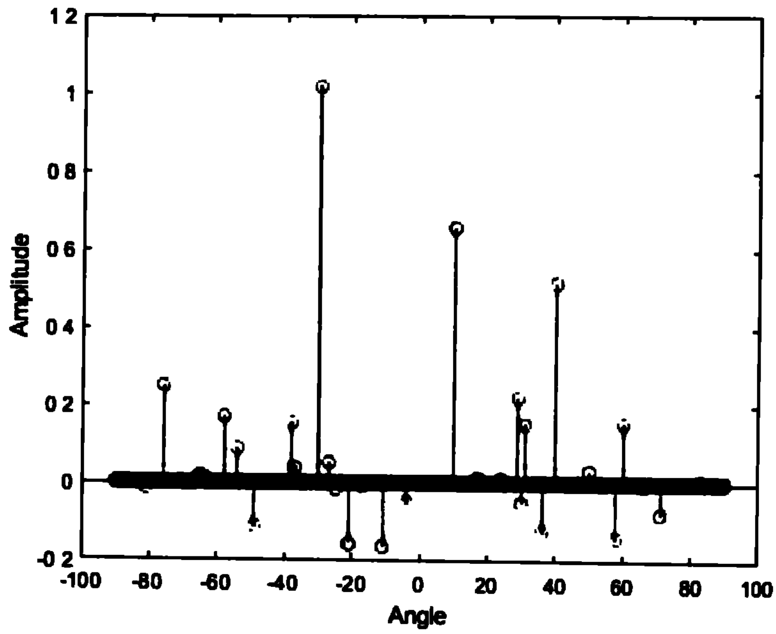


Fig 4.18: DOA estimation of three sources without preprocessing

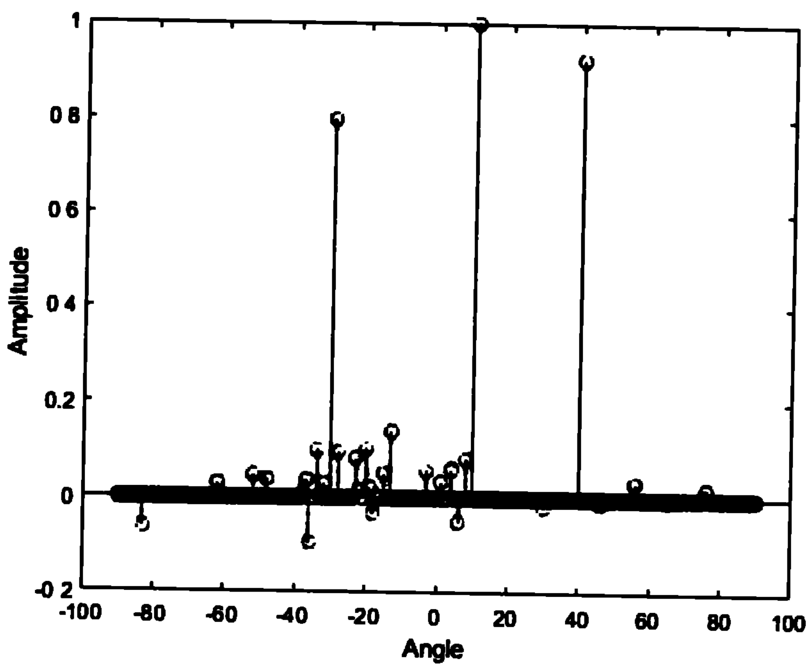


Fig 4.19: DOA estimation to three sources with preprocessing

Similarly, in figure 4.18 there are three sources at location of -30^0, 10^0 and 40^{0}with SNR of 2dB. In figure 4.19 we apply preprocessing on the data.

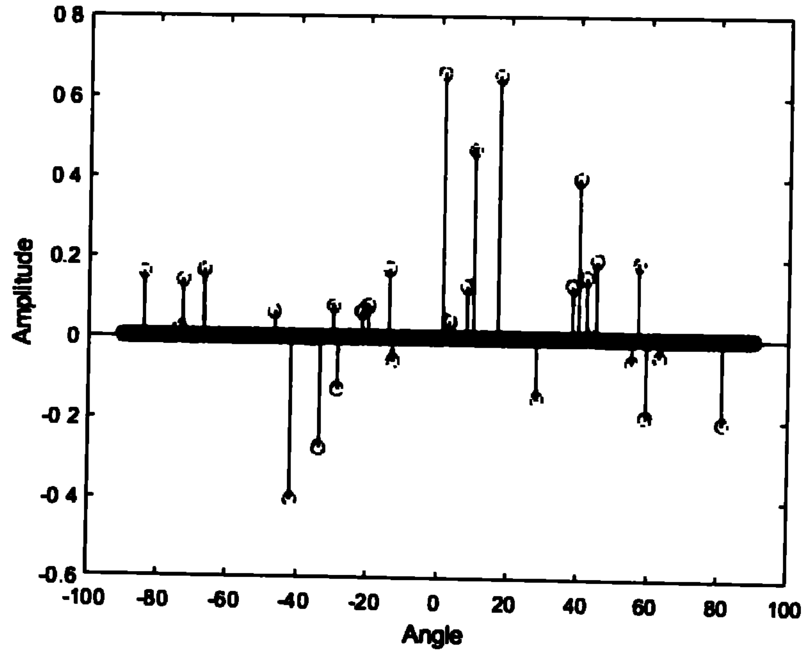


Fig 4.20: Three sources with SNR -2dB

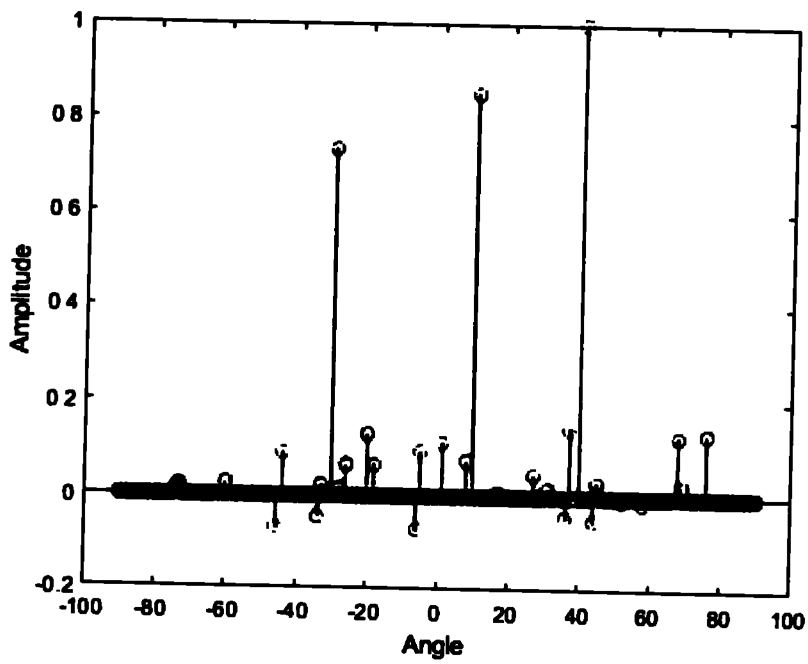


Fig 4.21: DOA estimation of three sources SNR -2dB a with preprocessing

In figure 4.20 we try to estimate the DOA of three sources at same location previously. Due to presence of noise the angle of the sources cannot be estimated properly. However, after preprocessing the location of the sources is estimated accurately as shown in figure 4.21.

Now we consider the effect of impulsive noise. we consider a sparse linear array with two sources at $-10°$ and $10°$ with SNR of 40dB. We consider an impulsive noise at one of the active elements.

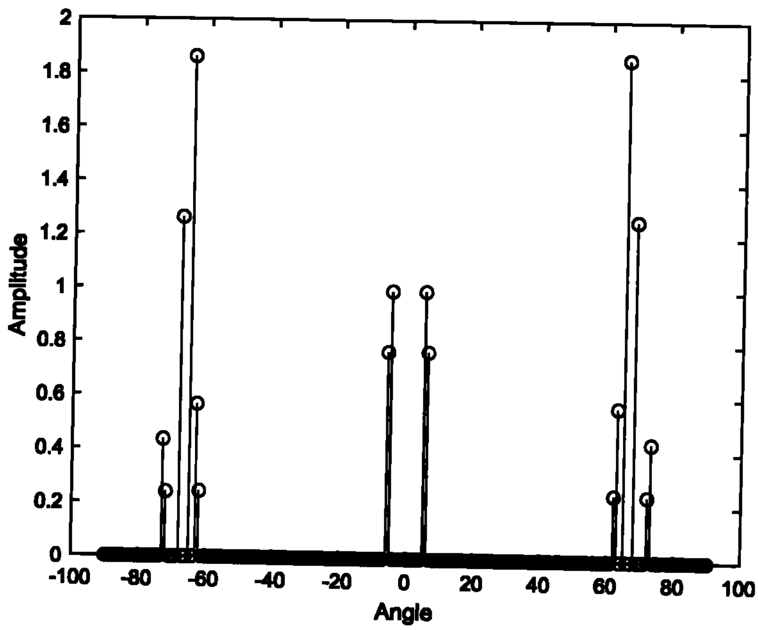


Fig 4.22: Effect of Impulsive noise on DOA estimation.

Applying the MDCO algorithm suggested in the preprocessing stage suppress the impulsive noise and the DOA of the sources is estimated accurately.

For solving the grid mismatch the same algorithm developed previously is applied for both sparse linear as well as L shaped antenna array. In this case, we consider the situation where the location of the sources does not coincide with the definition of the grid resolution. Initially it was supposed that the resolution of the grid is 1^0. The location of the sources does not coincide with the resolution of the grid. Then in such cases the energy of the source detected is distributed among the adjacent grids.

Let us consider two sources at 50.5^0 and 60.5^0. Initial grid resolution is selected to be 1^0 and SNR of 20dB.

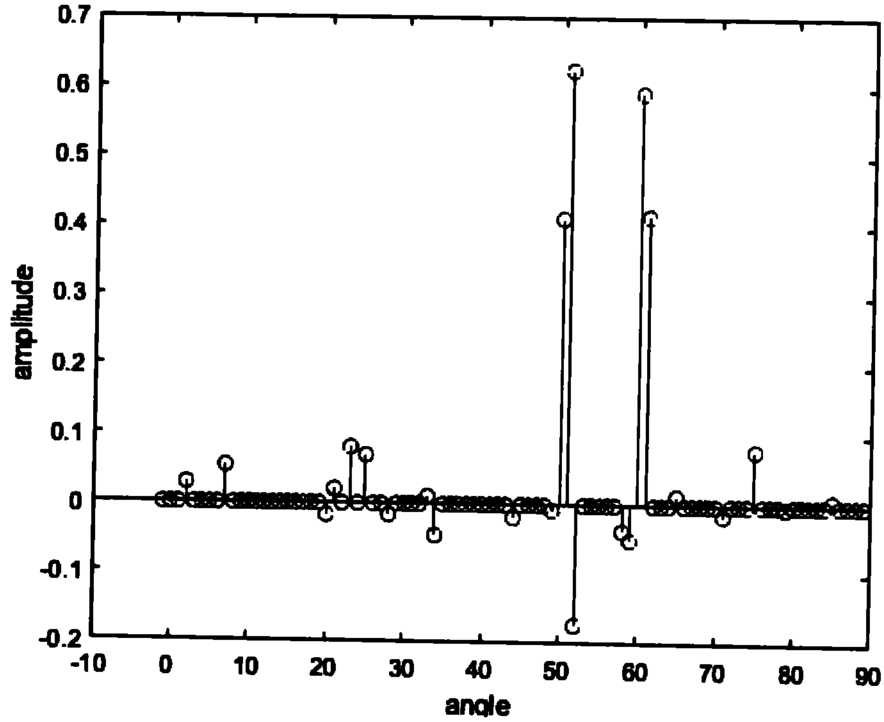


Fig 4.23: Ambiguity due to grid mismatch for two sources

As shown in figure 4.23 since the location of the sources does not coincide with the grid resolution four targets are detected at location of 50^0, 51^0, 60^0 and 61^0. This creates ambiguity about the position of the sources.

Next, we consider a scenario for three sources at location of location $\theta_1 = -30.4^0$, $\theta_2 = 50.5^0$, $\theta_3 = 60.5^0$ with same sparse antenna array as discussed earlier. We consider initial grid resolution of 1^0 and SNR of 20dB. It is shown in figure 4.24 that six targets are detected instead of three creating an ambiguity about the position of the sources.

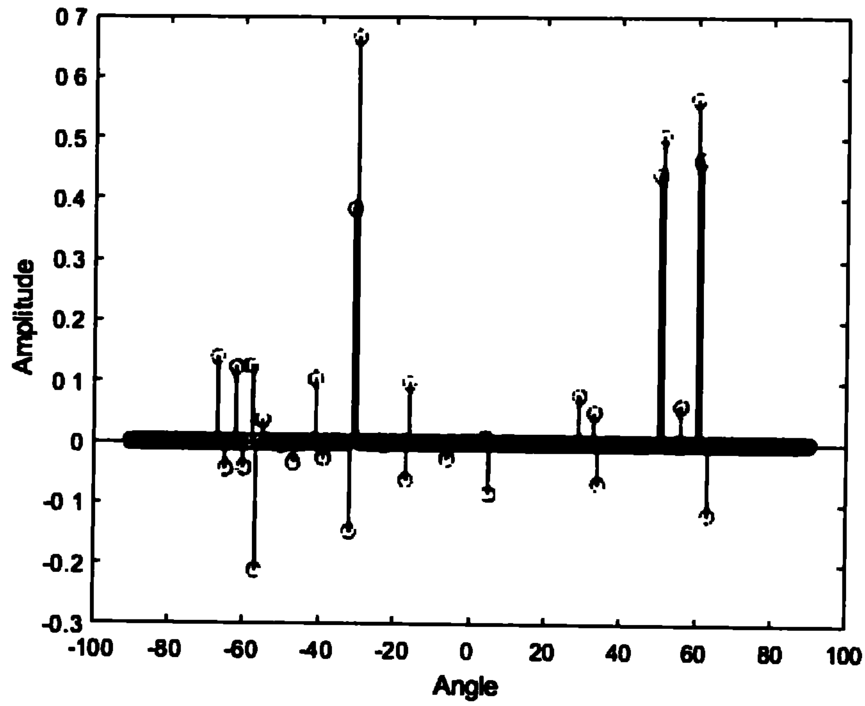


Fig 4.24: Ambiguity due to mismatch of Three sources

Figure 25 represents the target ambiguity due to four sources. The sources are at location of θ_1 = -30.5^0, θ_2 = -10.5^0, θ_3 = 50.5^0, θ_4 = 60.5^0 with SNR of 30dB. To remove the ambiguity a fitness function is used mentioned in proposed solution the grid is refined, and the new dictionary is created with slightly finer resolution.

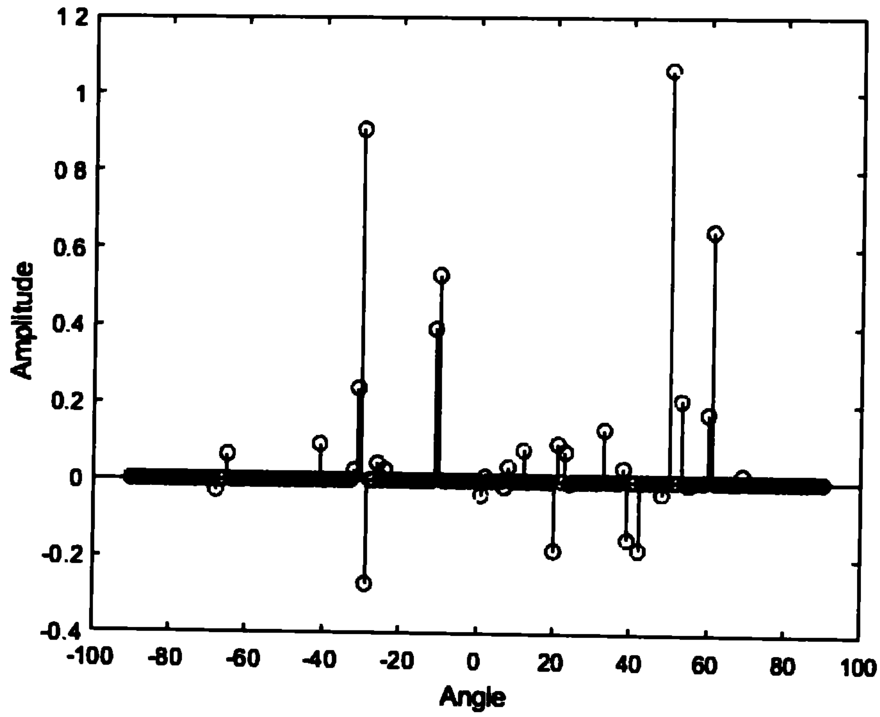


Fig 4.25: Ambiguity due to mismatch of four sources

The underdetermined system is solved and the amplitude of the peaks above certain limit is taken. The fitness function is calculated. The new fitness function is compared with the previous calculated value. If it satisfies the termination criteria the operation is terminated, and the peaks correspond to the true location of the sources. If the criteria are not satisfied the process is repeated with finer grid resolution.

The algorithm is applied to solve the ambiguity due to grid mismatch for two sources. Table 1 shows the application of the algorithm in tabular format. As mentioned, that there are two sources with equal amplitude at location of 30.5^0 and 50.5^0. Initial the grid resolution is taken to be 1^0. The mentioned method is applied, and the fitness function is calculated. The grid resolution in the second iteration is taken to be 0.5^0. The under determined system is solved using l_1 norm minimization and the new fitness function is calculated. The fitness function is increased therefore the process is repeated for a grid resolution of 0.1^0. The fitness function compared with the previous value is slightly decreased thus we conclude that the fitness function in the previous iteration is maximum and the best discretization value for the grid is 0.5^0.

Table 1: Grid Mismatch of two Sources

Amplitude & Location		**Grid Resolution 1^0**		**Grid Resolution 0.5^0**		**Grid Resolution 0.1^0**	
Amplitude	Source # 1 =1	A_1	0.4977	A_1	0.9527	A_1	0.9512
		A_2	0.5430				
	Source # 2 =1	A_3	0.6108	A_2	1.0106	A_2	0.9842
		A_4	0.3652				
Location	30.5^0	θ_1	30^0	θ_1	30.5^0	θ_1	30.5^0
		θ_2	31^0				
	50.5^0	θ_3	50^0	θ_2	50.5^0	θ_2	50.5^0
		θ_4	51^0				
Fitness		$F_1 = 0.2908$		$F_2 = 1.9363$		$F_3 = 1.9354$	

The CS technique can be applied on the signal after the preprocessing stage. The preprocessing stage for L shaped antenna array is similar, as we consider L shaped antenna as combination of two orthogonal sparse linear array. The preprocessing is applied independently on the multi sample signal

x_x and x_z.

Next, we apply CS technique on the received signal from sparse linear array and sparse L shaped array. First we apply the CS technique on the sparse linear array.

For the case of linear sparse array, the multi sample received signal is $\mathbf{X}'$ such that $\mathbf{X}' \in \mathbb{C}^{M' \times T_s}$ with impulsive noise suppressed and also applied the moving average filter for minimizing the effect of the Gaussian noise. We assume that the received signal is qausi-stationary. Qausi-stationary signals represents certain class of signals in which the statistics are locally static over a certain time. The underdetermined case of such class is used for DOA estimation in radars and sonars [33]. In [34] its utilization is mentioned to monitor birds in an airport for avoidance with aircraft. We combine all time samples of a spatial index of $\mathbf{X}'$ which can be represented by a vector $\mathbf{x}' \in \mathbb{C}^{M' \times 1}$. In order to cast the DOA problem in to CS framework we define an overcomplete dictionary $\mathbf{A}'$ such that $\mathbf{A}' \in \mathbb{C}^{M' \times N}$. We select initial discretization value of 1°. Then the dictionary is given as $\mathbf{A}' = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots \cdots, \mathbf{a}(\theta_N)]$ with $\mathbf{a}(\theta_N) = [1, e^{-jkd\sin\theta_N}, \cdots, e^{-jkd(M'-1)\sin\theta_N}]^T$. The sparse vector $\mathbf{s} \in \mathbb{C}^{N \times 1}$ is estimated by solving equation 12 with peaks corresponding to the estimated DOA.

Next, we cast the signal model developed for L shaped antenna array into the CS framework. As L shaped antenna array consist of two linear arrays. The received signal from each independent sparse linear array is $\mathbf{X}'_x, \mathbf{X}'_z \in \mathbb{C}^{M' \times T_s}$. The received signal is passed through the pre-processing stage independently then all-time samples of the spatial indexes are combined such they can be represented as $\mathbf{x}'_x, \mathbf{x}'_z \in \mathbb{C}^{M' \times 1}$. As shown in signal model the steering matrix $\mathbf{A}'_z$ consist of ϕ component only whereas, $\mathbf{A}'_x$ consist of θ and ϕ components. According to [34] array manifold matrix of x axis, a new parameter can be introduced defined as

$$\cos\omega_k = \cos\theta_k \cos\phi_k \tag{18}$$

Then the steering matrix $\mathbf{A}'_x = [\mathbf{a}(\omega_1), \mathbf{a}(\omega_2), \cdots, \mathbf{a}(\omega_P)]$ with $\mathbf{a}(\omega_P) = [1, e^{-jkd\cos\omega_P}, \cdots, e^{-jkd(M'-1)\cos\omega_P}]$. To apply CS technique an overcomplete dictionaries $\mathbf{A}'_z$ and $\mathbf{A}'_x$ are created, as for sparse linear array was created. Then the following optimization problem is solved.

$$\min \|\mathbf{s}_z\|_1 \quad s.t \quad \|\mathbf{x}'_z - \mathbf{A}'_z \mathbf{s}_z\|_2^2 < \varepsilon \tag{19}$$

$$\min \|\mathbf{s}_x\|_1 \quad s.t \quad \|\mathbf{x}'_x - \mathbf{A}'_x \mathbf{s}_x\|_2^2 < \varepsilon \tag{20}$$

The elevation angle can be estimated using equation 19. The azimuth angle for P sources is estimated from $\mathbf{s}_x$ using the equation given below.

$$\theta_k = \arccos\left[\cos(\varpi_k) \big/ \cos(\hat{\phi}_k)\right] \qquad k = 1, 2, \cdots, P \tag{21}$$

Now we consider the case of DOA estimation in two dimension using sparse L shaped antenna array. The proposed method is used to estimate the location of the target. Let us consider two targets with equal amplitude. With elevation angles of $\phi_1 = 40°, \phi_2 = 50°$ and elevation angle of $\theta_1 = 30°$, $\theta_2 = 40°$ with SNR of 40dB. The problem is solved using equation 19 for the elevation angles.

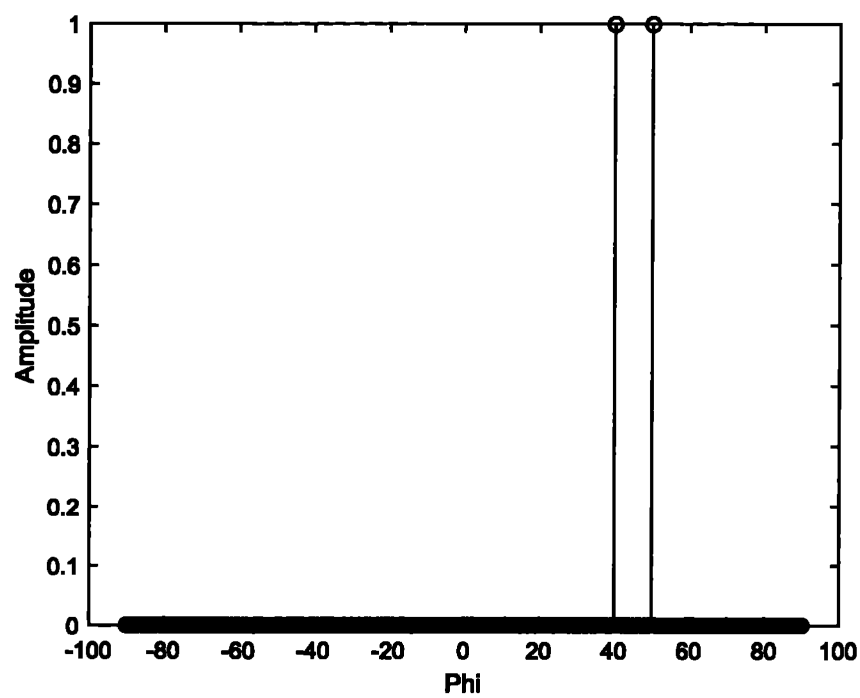


Fig 4.26: DOA estimation for the Elevation of two sources

As shown in figure 4.26 the elevation of the targets is resolved. Then equation 20 is solved for ω. It is observed that since the location of the targets aligned with the resolution of the grid but the value of the proposed ω does not. Using the suggested grid refinement method based on the distribution of the source energy the value of ω is estimated. Then with the help of equation 21 the azimuth angles are estimated which are $\theta_1 = 29.93^\circ$ and $\theta_2 = 40\ 02^\circ$.

CHAPTER 5

Introduction

In this chapter we accumulate the theory presented in the preceding chapters and apply it to MIMO FDA radars. We establish technique for the application of compressive sensing for MIMO FDA radars. In previous chapter we considered two types of noises which was Impulsive noise and Gaussian noise. As mentioned earlier a filtering stage is added for data conditioning to improve the detection in the presence of noise. Similarly, the optimization method for grid refinement is used for accurately estimating the parameters of the target.

The word MIMO stands for Multiple Input and Multiple Output. The concept behind it is waveform diversity. Where different antenna elements transmit different waveforms. In certain condition it provides superiority as compared to Phase array radars. This concept is employed in radars and have many applications. FDA radars have intrigued researchers for application in SAR (synthetic Aperture radar) and many more application where frequency diversity is applied at the antenna elements. The FDA radar have a range and angle dependent beam pattern. Several techniques have been presented to decouple the information. Similarly, FDA -MIMO radar have been suggested to accommodate the advantages of both the FDA and the MIMO radar. One of the methods for decoupling of the angle and the range is double pulse method suggested by [111]. In this method two pulses are transmitted. In the first pulse frequency diversity is not employed. It acts as a simple MIMO radar where the angle of the target is estimated. For the next pulse frequency diversity is employed. Since the information about the angle of the target is know the range is estimated.

We employ compressive sensing technique for MIMO FDA radar using sub arrays in the receive mode only. The parameters of the target are estimated accurately. In the next section we revisit the concept of MIMO-FDA radar and then we introduce the proposed architecture and the implementation of the compressive sensing for MIMO-FDA radar.

5.1 MIMO FDA Radar

Let us consider a collocated antenna array. Consist of M transmitting elements and N elements. We consider a ULA. The ULA can be divided in to transmitting and receiving array. Similarly it can be arranged in to subarray. Let us consider the signal transmitted by the m^{th} antenna element given in equation 1.

$$s_m(t) = \sqrt{\frac{E}{M}}\,\phi_m(t)e^{j2\pi f_m(t)} \tag{1}$$

Where f_m is the frequency component for the m^{th} antenna element which is

$$f_m = f_0 + (m-1)\Delta f \qquad m = 1,2,3\cdots,M \tag{2}$$

Similarly, E is energy and ϕ the waveform of the transmitted signal. The waveform of each transmitting antenna element is orthogonal. As shown in the equation given below

$$\int_T \phi(t)\phi^*(t-\tau)e^{j2\pi\Delta f(m-n)t}dt = 0 \quad m \neq n \tag{3}$$

$$otherwise$$

$$\int_T \phi(t)\phi^*(t)dt = 1$$

Let us consider free space loss. The echo signal received by the n^{th} receiving antenna element is given as

$$y_n(t) = \sum_{m=1}^{M} \sqrt{\frac{E}{M}} \xi \phi(t - \tau_{m,T} - \tau_{n,R}) \times e^{j2\pi(f_m + f_{d,m})(t - \tau_{m,T} - \tau_{n,R})} \tag{4}$$

In the above equation ξ is the reflection coefficient and $f_{d,m}$ is the doppler frequency. The doppler frequency can be estimated using $f_d = {2v_t}/{\lambda_0}$. Similarly, the transmit and the receive time delays are given as

$$\begin{aligned} \tau_{m,T} &= \frac{1}{c}[r - d_T(m-1)\sin\theta] \\ \tau_{n,R} &= \frac{1}{c}[r - d_R(n-1)\sin\theta] \end{aligned} \tag{5}$$

In the above equation d_T and d_R is the interelement spacing between in the transmitting and the receiving antenna elements. On the received side the signal is down converted passed through a match filter and converted in digital domain using an ADC (Analog to Digital Converter) for further processing. The signal is passed through N number of match filter which is equal to the number of the antenna elements. After match filtering the m^{th} output of the n^{th} antenna element is given as

$$\begin{aligned} y_{n,m} &\approx \sqrt{\frac{E}{M}} \xi e^{j2\pi f_d(t-\tau_0)} e^{-j2\pi \frac{f_m}{c}[d_T(m-1)\sin\theta + d_R(n-1)\sin\theta]} \\ &\approx \sqrt{\frac{E}{M}} \xi e^{j2\pi f_d(t-\tau_0)} e^{-j4\pi \frac{f_0}{c} r} e^{-j4\pi \frac{\Delta f}{c}(m-1)r} e^{j2\pi \frac{f_0}{c}[d_T(m-1)\sin\theta + d_R(n-1)\sin\theta]} \end{aligned} \tag{6}$$

The output of the above equation can be written as

$$y_n = \sqrt{\frac{E}{M}}\xi \begin{bmatrix} 1 \\ e^{-j4\pi\frac{\Delta f}{c}r + j2\pi\frac{d_T}{\lambda_0}\sin\theta} \\ \cdot \\ \cdot \\ \cdot \\ e^{-j4\pi(M-1)\frac{\Delta f}{c}r + j2\pi\frac{d_T}{\lambda_0}(M-1)\sin\theta} \end{bmatrix} \times e^{j2\pi\frac{d_R}{\lambda_0}(n-1)\sin\theta} \tag{7}$$

This can be expressed as

$$x_s = \left[y_1^T, y_2^T, \cdots\cdots, y_M^T\right]^T = \sqrt{\frac{E}{M}}\xi b(\theta) \otimes a(r,\theta) \tag{8}$$

Where $a(r,\theta)$ and $b(\theta)$ are represented as

$$a(r,\theta) = \begin{bmatrix} 1 \\ e^{-j4\pi\frac{\Delta f}{c}r + j2\pi\frac{d_T}{\lambda_0}\sin\theta} \\ \cdot \\ \cdot \\ \cdot \\ e^{-j4\pi(M-1)\frac{\Delta f}{c}r + j2\pi(M-1)\frac{d_T}{\lambda_0}\sin\theta} \end{bmatrix} \tag{9}$$

$$b(\theta) = [1, e^{j2\pi\frac{d_R}{\lambda_0}\sin\theta}, \cdots\cdots, e^{j2\pi(N-1)\frac{d_R}{\lambda_0}\sin\theta}]^T$$

Now let us consider the double pulse solution to the range angle ambiguity in FDA MIMO radar. As it is shown in the above equation that the range and the angle are coupled together. A number of methods have been developed to decouple the range and the angle information. One of the methods is double pulse method. In double pulse method two pulse are transmitted such that in the first pulse there is no frequency increment among the antenna elements while in the second pulse there is a frequency increment among the antenna elements.

5.1 Proposed Solution

We proposed a solution for the implementation of compressive sensing for FDA MIMO using double pulse method. The double pulse method is used to decouple the range and the angle information in FDA MIMO radars. We use sub array in the receive mode. Using subarray further improves the SNR and more robust against the noises. Different subarray architectures are present. A brief review of the subarrays used is as follows.

There is different configuration of subarrays used. However, the most common of them are contiguous and overlapped subarrays. As shown below. If we have a large number of elements in an array. Then applying phase shifter and attenuator at the element level is very expensive and difficult to control. Therefore, the array is divided into subarrays and the beamforming is applied at the subarray. However this scheme gives rise to grating lobes .

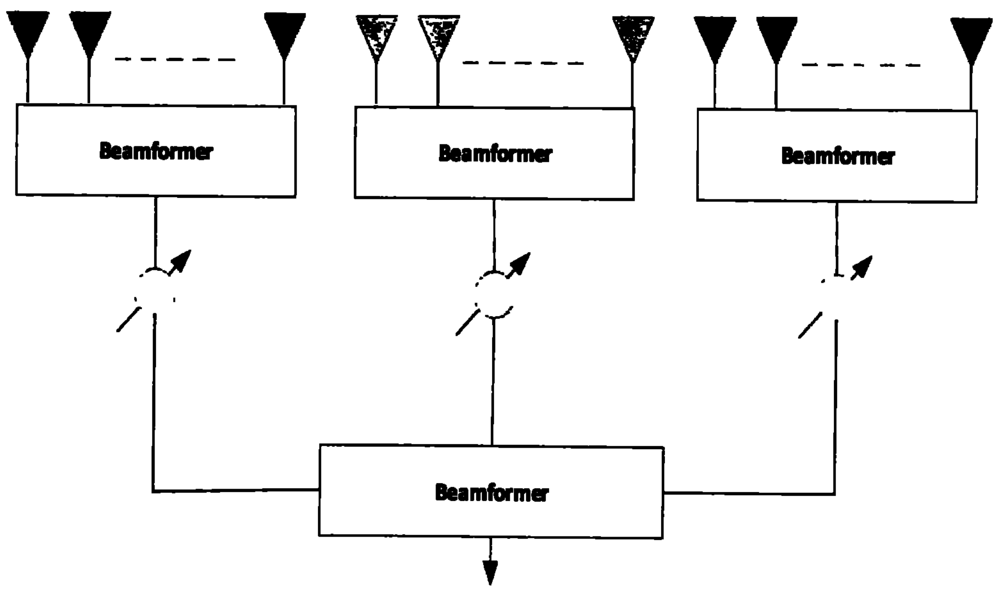


Fig 5.1: Subarray structure

Next in consideration is the overlapped subarray. It produces a flat-topped sector pattern with low sidelobes. It suppresses the grating lobes. It has applications, where it is required to minimize the number of controls to steer the beam. However, the architecture of an overlapped subarray antennas includes many splitting/combining ratios.

A typical design of overlapped subarray is given in [112] as shown in figure 5.2. Which shows a typical linear overlapped subarray with a 3:1 overlap ratio. With 4:1 combiner. The output combiner gets its signal from three different contiguous arrays. Taylor tapping is usually used to further suppress the side lobes. In general, some of the advantages of the contiguous subarrays is reduced number of phase shifters, Analog beamformer implementation but with high narrow band peak sidelobes and wideband grating lobes. Whereas in overlapped subarrays we have the advantage of low sidelobe performance, and it also suppresses the wideband grating lobes.

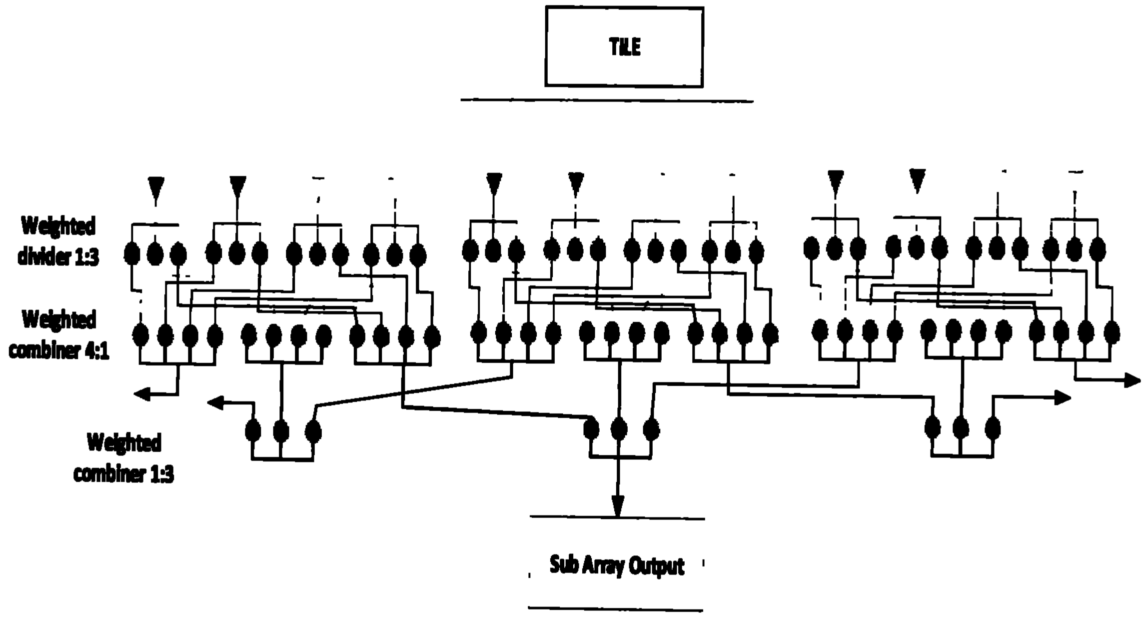
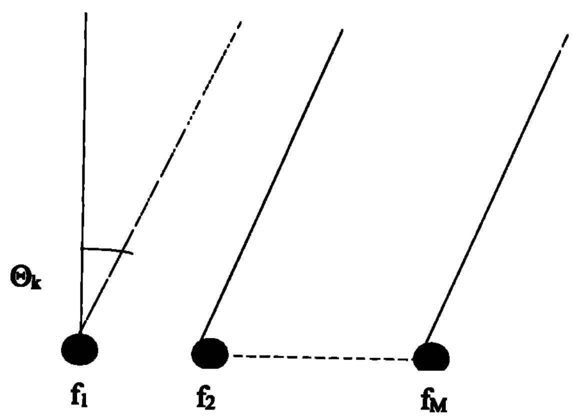


Fig 5.2: Subarray architecture

One of the main advantages of the FDA radar is that there is no need for the phase shifter. As digital phase shifter introduces quantization error, which creates beam pointing error and increases the sidelobe levels. Whereas control of the frequency is easier. The proposed architecture for the implementation of the compressive sensing for FDA MIMO is as follows



Transmit Array

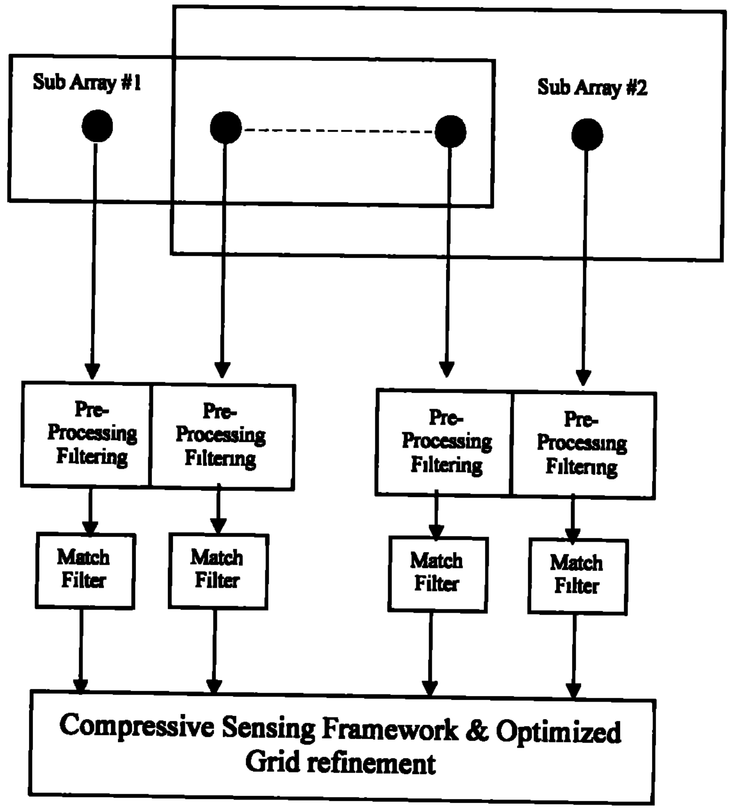


Fig5.3: CS architecture for FDA MIMO radar

Let us consider for simplicity that the receive antenna array is divided in to two subarrays. This can be generalized for more. Similarly, initially we will consider a single snap shot problem. This can be further extended for multiple snapshot case as shown in the previous chapters. The receive antenna array is divided in to two subarrays. The signal received by each subarray after match filtering is given as

$$x_1 = A_1 S + N$$
$$x_2 = A_2 S + N \tag{10}$$

Here $\mathbf{x}_1$ and $\mathbf{x}_2$ are the signal receive by the subarrays. $\mathbf{A}_1$ and $\mathbf{A}_2$ are the array factor of the corresponding array factor. The array factor in the previous section is derived which modified for the subarray antenna array as follows.

$$\mathbf{A}_1 = \mathbf{b}_1(\theta) \otimes \mathbf{a}(\theta, r) \tag{11}$$

$$\mathbf{b}_1(\theta) = \left[1, e^{j2\pi \frac{d_B}{\lambda_0}\sin\theta} \cdots\cdots, e^{j2\pi \frac{d_B}{\lambda_0}(M-2)\sin\theta}\right]^T$$

Similarly $\mathbf{A}_2$ is defined as

$$\mathbf{A}_2 = \mathbf{b}_2(\theta) \otimes \mathbf{a}(\theta, r) \tag{12}$$

$$\mathbf{b}_2 = \left[e^{j2\pi \frac{d_B}{\lambda_0}\sin\theta}, e^{j2\pi \frac{d_B}{\lambda_0}\sin\theta}, \cdots\cdots, e^{j2\pi \frac{d_B}{\lambda_0}(M-1)\sin\theta}\right]^T$$

In the above equation the total number of the antenna elements are M. The size of the subarray at the overlap ratio is up to the designer. In the above equation the size of the subarray is P.

Now as discussed that we have selected double pulse scenario. The first pulse is transmitted without any frequency increment. In this case the receive equation after match filtering by each sub array is given as

$$\begin{aligned} \mathbf{X}_{s1} &= \mathbf{A}_{s1}\mathbf{S}_{s1} + \mathbf{N}_{s1} \\ \mathbf{X}_{s2} &= \mathbf{A}_{s2}\mathbf{S}_{s2} + \mathbf{N}_{s2} \end{aligned} \tag{13}$$

In the above equation represent the received signal with respect to the subarray. $\mathbf{A}_{s1}$ and $\mathbf{A}_{s2}$ are the respective steering vector of the subarray which are given as

$$\begin{aligned} \mathbf{A}_{s1} &= \left[a_{s1}(\theta_1), a_{s1}(\theta_2), a_{s1}(\theta_3), \cdots\cdots, a_{s1}(\theta_k)\right]^T \\ \mathbf{A}_{s2} &= \left[a_{s2}(\theta_1), a_{s2}(\theta_2), a_{s2}(\theta_3), \cdots\cdots, a_{s2}(\theta_k)\right]^T \end{aligned} \tag{14}$$

Where $a_{s1} = a_{st} \otimes b_1$ and $a_{s2} = a_{st} \otimes b_2$. The new transmitting steering vector is given as

$$\mathbf{a}_{st} = \left[1, e^{j2\pi \frac{d}{\lambda}\sin\theta_1}, e^{j2\pi \frac{d}{\lambda}\sin\theta_2}, \cdots\cdots, e^{j2\pi(M-1)\frac{d}{\lambda}\sin\theta_k}\right] \tag{15}$$

N is the guassian noise $N \in (0,\sigma_n^2)$. A_1 and A_2 are the steering vector of the respective subarrays. The cross-covariance matrix between the data vectors of the two sub arrays.

$$R_{xx} = E\left[X_2 X_1^H\right] \tag{16}$$
$$R_{xx} = A_2 R_{ss} A_1^H + \sigma_n^2 I$$

Where as $R_{ss} = E[S\,S^H] = diag([\sigma_1^2, \sigma_2^2, \cdots, \sigma_k^2])$ .In the vector form it can be written as

$$\mathbf{z} = \mathbf{vec}(\mathbf{R}_x) = \bar{\mathbf{A}}\,\mathbf{R}_{ss} + \sigma_n^2 \mathbf{I} \tag{17}$$

As it is seen that the vector z consists of virtual array. Therefore, we take in to consideration the true position of the array the received vector after removing the virtual element position is given as

$$\tilde{z} = vec(\mathbf{R}_x) = \tilde{\mathbf{A}}\,\mathbf{R}_{ss} + \sigma_n^2 \mathbf{I} \tag{18}$$

Now we look in to the implementation of the compressive sensing for the received signal vector. As in the previous chapter we have mentioned that to solve a sparse solution we use l_1 regularization. However it is important to select the dictionary for the above problem. As mentioned earlier we select an over complete dictionary such that the received signal is sparse in that domain. Let us suppose that we select $\mathbf{B}$ as the dictionary such that $B \in \square^{M \Lambda x P}$. In the absence of noise the compressive sensing framework to the sparse solution is given as

$$\hat{\mathbf{s}} = \min \left\| \mathbf{z} - \mathbf{B}\mathbf{s} \right\|_2^2 + \lambda \left\| \mathbf{s} \right\|_p \tag{19}$$

However in the presence of noise the above equation is given as

$$\min \left\| \mathbf{s} \right\|_1 \;\; s.t \;\; \left\| \mathbf{z} - \mathbf{B}\mathbf{s} \right\|_2^2 < \varepsilon \tag{20}$$

Now let us consider a linear antenna array. It consist of total 10 antenna elements. As mentioned earlier we are using double pulse method. Where the first pulse does not contain frequency increment among the antenna elements. In the previous compressive sensing is applied for single snap shot and multiple snap shot. The signal in multiple snap shot is more robust against noise as a pre processing stage is added for Gaussian as well as Impulsive noise. The signal is pass through match filter first. For simplicity we consider that the array is split into to subarray. Each sub array consists of 5 antenna element and they are overlapped. Such the first and the last antenna element is not shared among them.

Next in figure 5.4 we estimate λ for a simple ULA with 10 elements and for a subarray defined above.

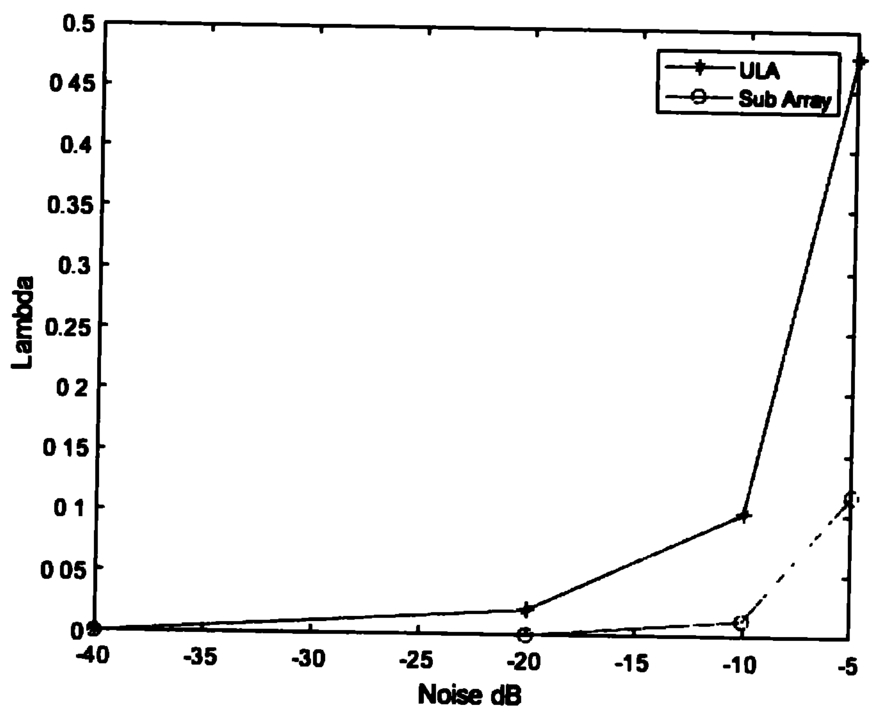


Fig 5.4: Comparison of λ calculated for ULA and Subarray structure

The scenario is tested to two sources at an angel of 30^0 and 40^0 degree. The horizontal line represents the values at different noise which is added to the signal from the sources which is at 0dB. It is clear from figure 5.4 that the proposed method for subarray is more robust against noise. Next, we compare the proposed algorithm with l_1 norm minimization method in figure 5.5.

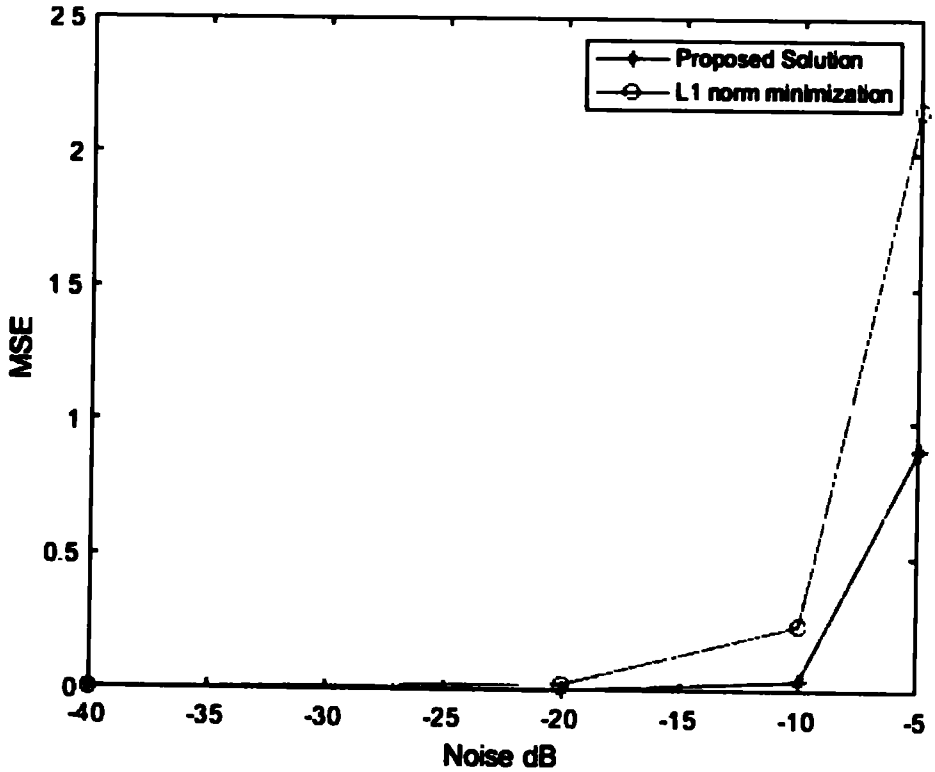


Fig 5.5: Comparison of proposed solution with l_1 norm minimization

In figure 5.5 we calculate the mean square error (MSE) of the DOA estimation between the proposed solution and l_1 norm minimization method. Both using single time sample. It is clear that the MSE of the Proposed method is far less than the traditional l_1 norm minimization method. Next, we consider a scenario of the detection of the proposed method for DOA estimation of multiple targets at different noise levels added to the signal.

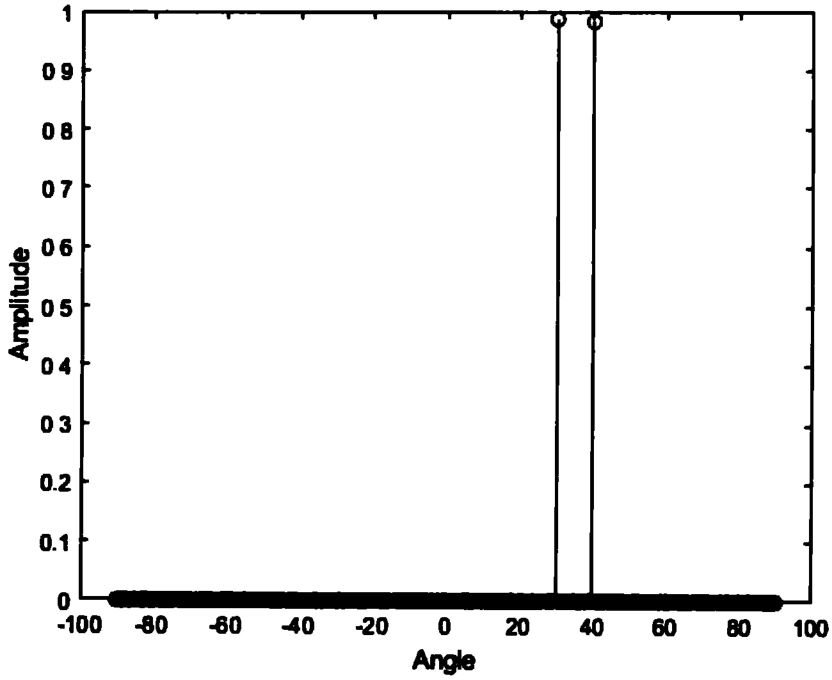


Fig 5.6: Detection of two sources with added noise of -40dB

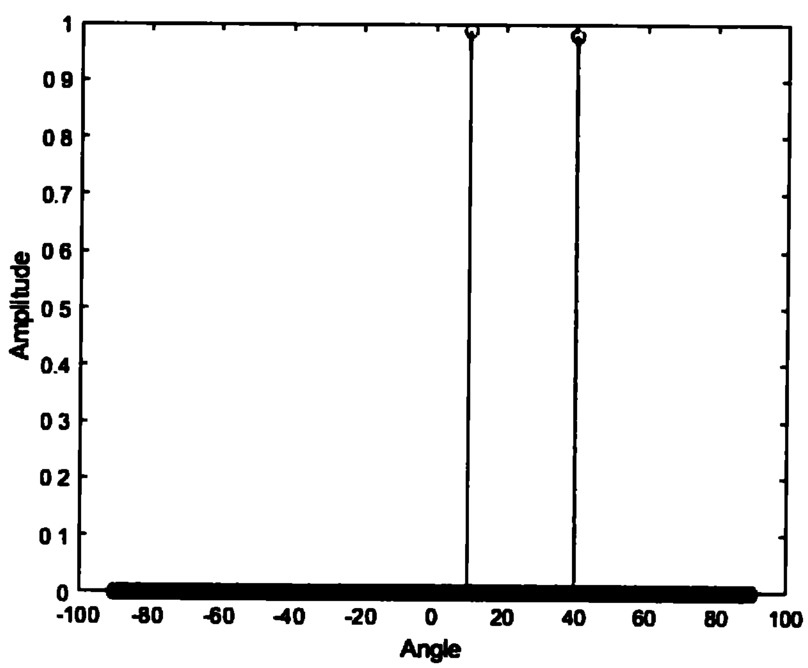


Fig 5.7: Detection of two sources with added noise of -20dB

Figure 5.6 and figure 5.7 shows the detection of two sources. The sources in figure 5.6 are at an angle of 30° and 40° with noise level of -40dB added to the source signal at 0dB. While in figure5.7 the sources are at 10° and 40° with noise level of -20dB added. The sources are estimated.

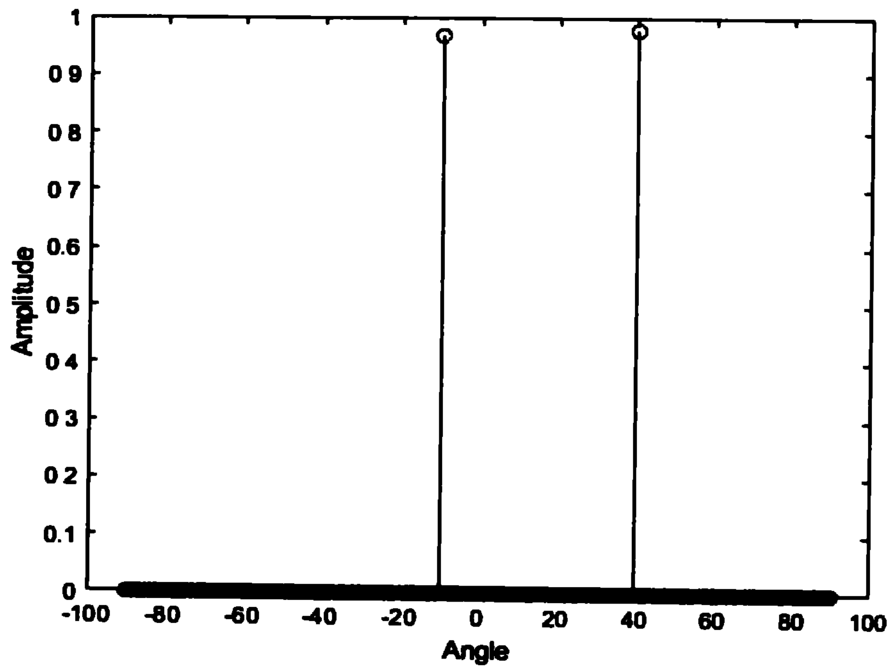


Fig 5.8: Detection of two sources with added noise of -10dB

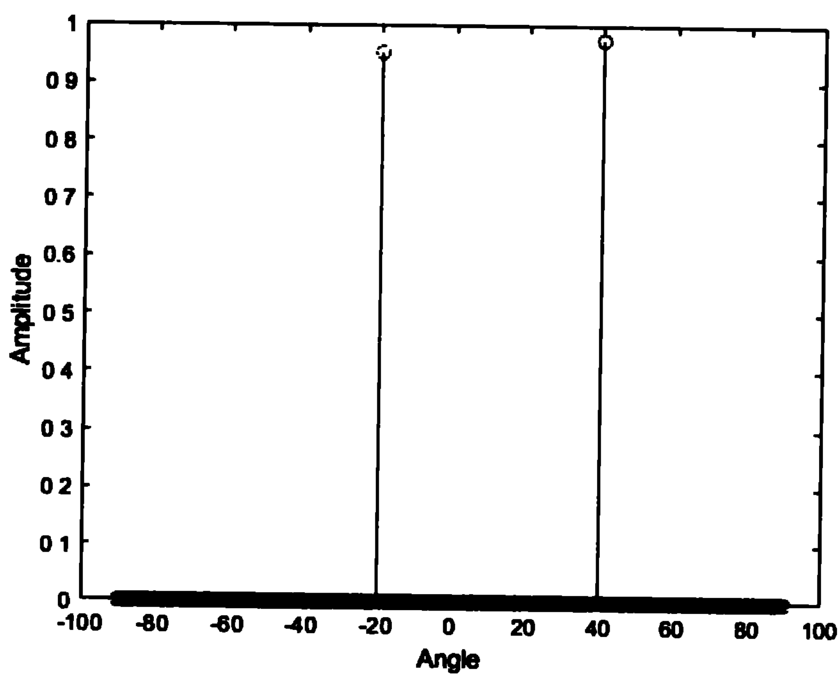


Fig 5.9: Detection of two sources with added noise of -5dB

The figure 5.8 and figure 5.9 shows the detection of two sources with the noise power of -10dB and -5dB.

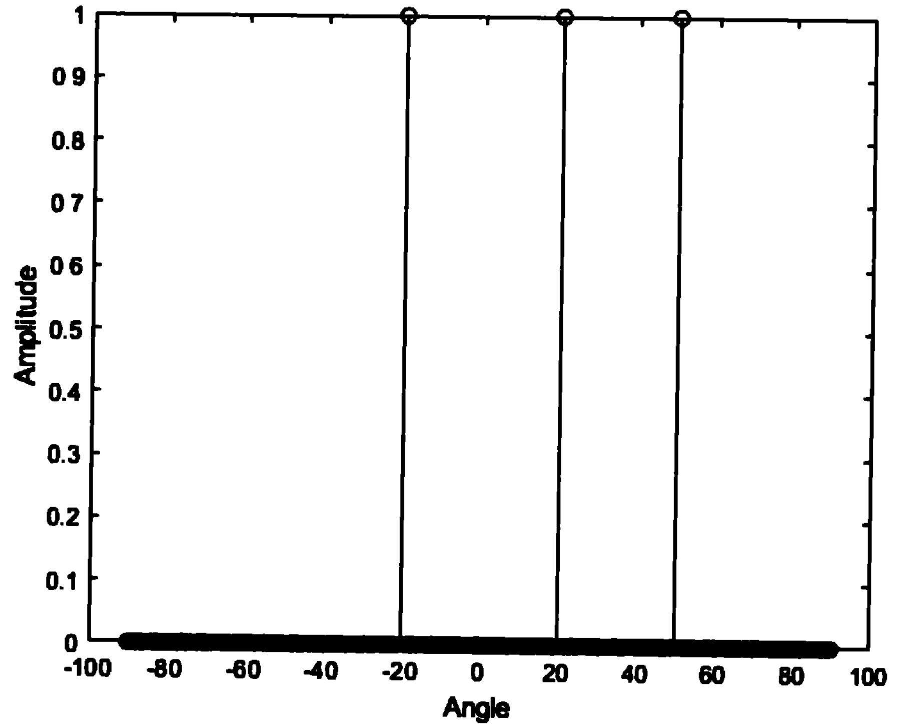


Fig 5.10: Detection of three sources with added noise of -40dB

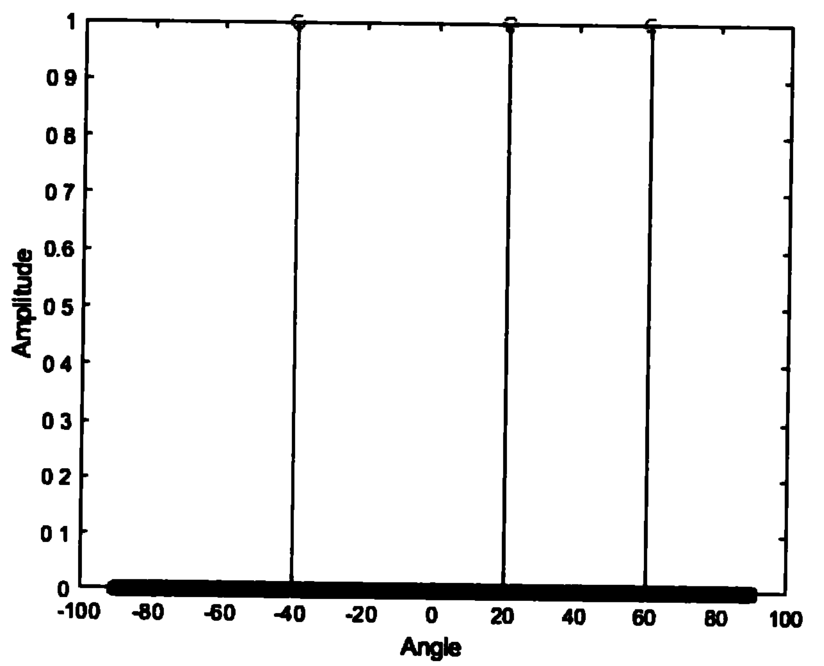


Fig 5.11: Detection of three sources with added noise of -20dB

In figure 5.10 and figure 5.11 shows the detection of three sources in the presence of noise of -40dB and -20dB. The location of the sources in the angle domain is resolved. Similarly in figure 5.12 shows the detection of three sources with noise of -5dB. The location of the sources is resolved however due to noise there are certain spikes, but they are low and can be ignored after proper thresholding.

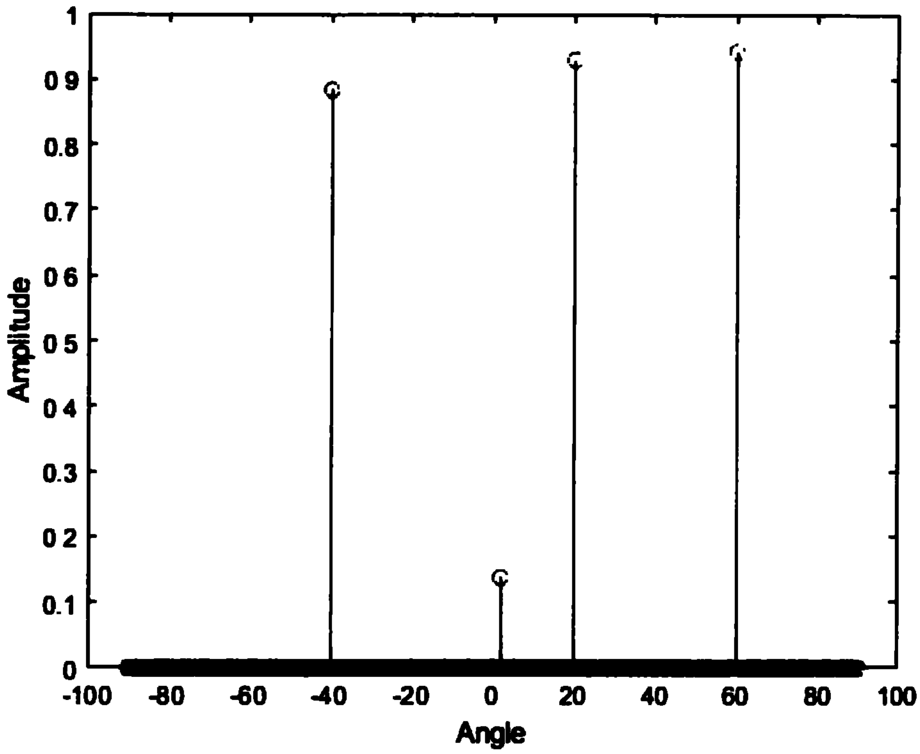


Fig 5.12: Detection of three sources with added noise of -5dB

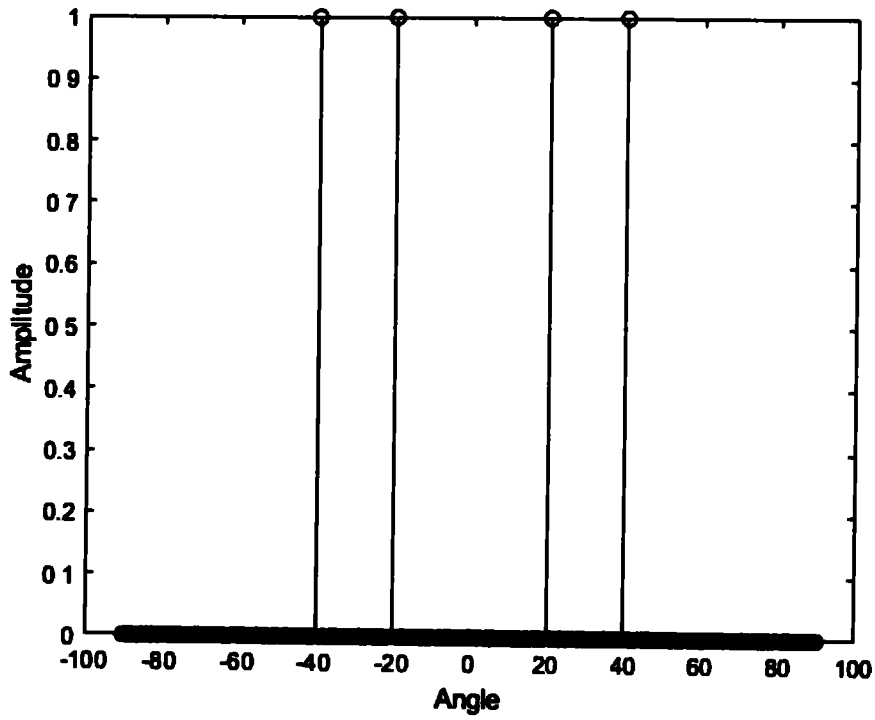


Fig5.13: Detection of four sources with added noise of -40dB

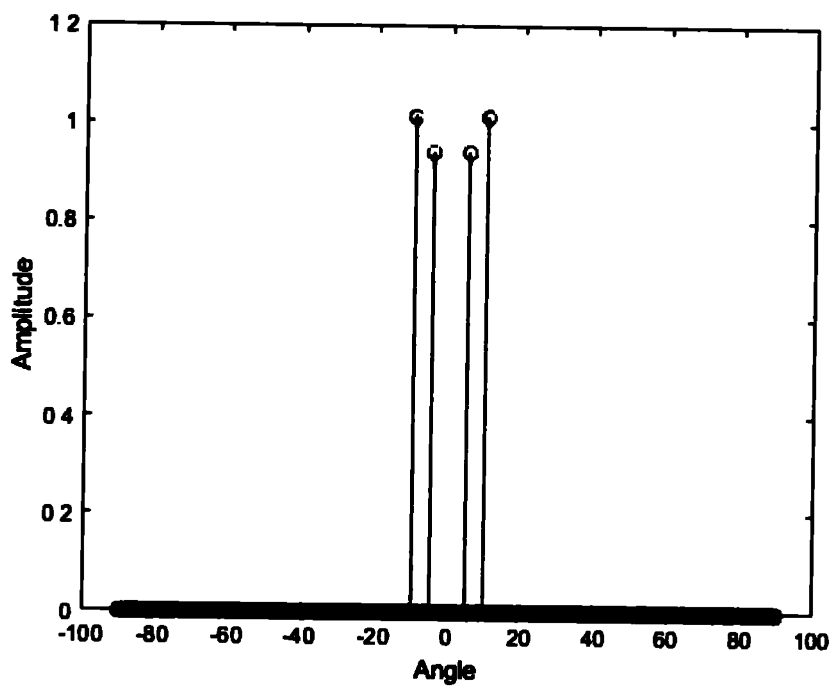


Fig 5.14: Detection of four sources with added noise of -20dB

We consider the situation where the location of the source does not coincide with the definition of the grid resolution. As initially we suppose the resolution of the grid is 1^0 and the location of the source is such that it is not in the grid. It is in between the grid then in such cases the amplitude of the source detected is distributed among the adjacent grid

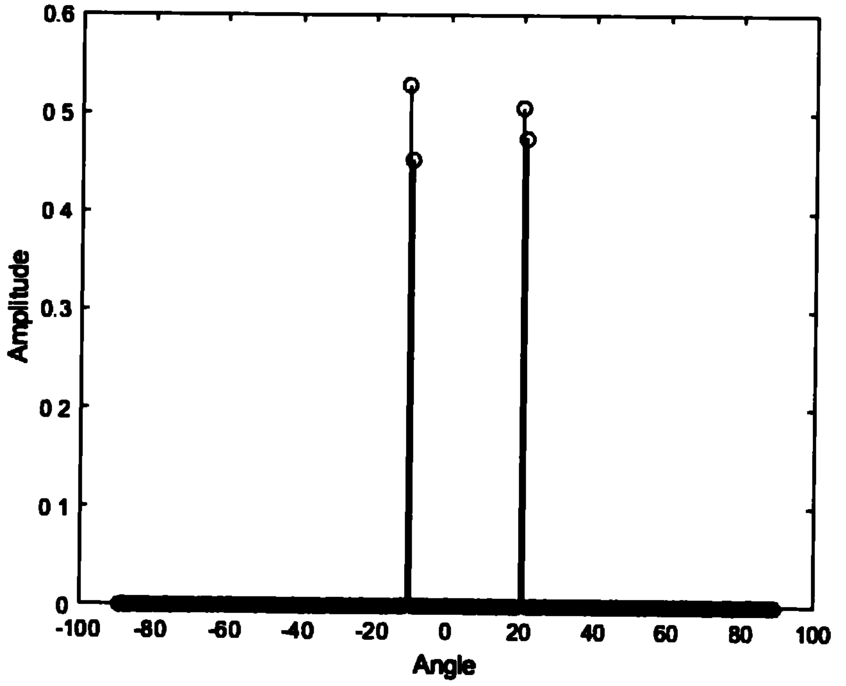


Fig 5.15: Ambiguity due to mismatch of two sources

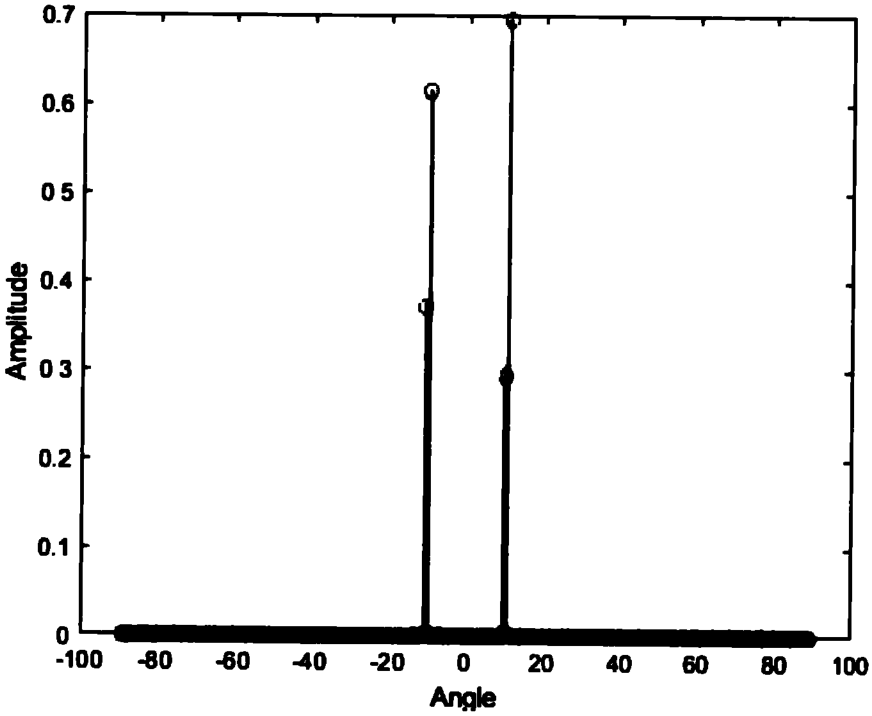


Fig 5.16: Ambiguity due to mismatch of two sources

As shown in figure 5.15 and figure 5.16. They are two sources at angles of -10.5° and 20.5°. However four targets are detected which are at angles of $-10^\circ, -9^\circ$ and at angle of $20^\circ, 21^\circ$ as shown in figure 19. Similarly for figure 20 sources at location of -10.5° and 10.5° are detected at angles of $-10^\circ, -9^\circ$ and $10^\circ, 11^\circ$. It is noted that since grid resolution defined is 1° and the location of the sources does not coincides with the grid resolution therefore this created ambiguity about the location of the sources and the power of the sources detected is distributed among the adjacent grid as shown in figure 5.16 and figure 5.17. Next in figure 21 we show the grid mismatch problem for three sources.

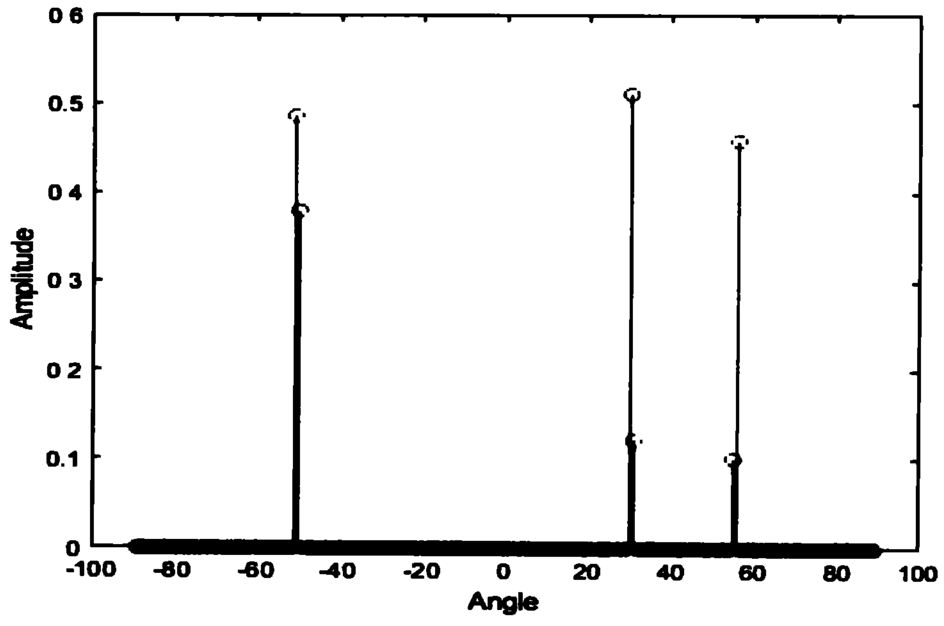


Fig 5.17 Ambiguity due to mismatch of three sources

As shown in figure 21 there are three sources at location of $-50.5^{\circ}, 30.5^{\circ}$ and 55.5°. The grid resolution of 1° is defined. Total of six targets are detected due to grid mismatch problem.

Grid Mismatch problem is solved using the algorithm defined before Using the optimization method defined in the previous chapter the true position of the target can be estimated as shown in table 1.

Table1:

Amplitude & Location			**Grid Resolution 1⁰**		**Grid Resolution 0.5⁰**		**Grid Resolution 0.1⁰**	
Amplitude	Source # 1 =1		A_1	0.5065	A_1	0.9967	A_1	0.9967
			A_2	0..4873				
	Source # 2 =1		A_3	0.4873	A_2	0.9967	A_2	0.9967
			A_4	0.5065				
Location	-10.5^0		θ_1	-10^0	θ_1	-10.5^0	θ_1	-10.5^0
			θ_2	-9^0				
	10.5^0		θ_3	10^0	θ_2	10.5^0	θ_2	10.5^0
			θ_4	11^0				
Fitness			$F_1 = 0.0384$		$F_2 = 1.9934$		$F_3 = 1.9934$	

Once the angle is estimated we come to the range estimation problem. As mentioned earlier that in double pulse problem two pulses are transmitted one without the frequency increment and the other with frequency increment. The range of the target is estimated using the second pulse which is periodic in range and angle.

The output of the array after match filtering can be represented using equation 8. Since the information about the angle of the target have been extracted then to estimate the range of the target then an overcomplete set of the ranges of the target is made which can be represented as $r = [r_1, r_2, \cdots \cdots, r_w]$ where w is the discretization point of the range domain. The range domain is discretized between $\left[0, {}^{c\Delta f}\!/_{2}\right]$. We stack the range set corresponding to the number K which is the number of the sources. That can be given as

$$\tilde{r} = \left[r_{(\theta_1,1)}, r_{(\theta 1,2)}, \cdots r_{(\theta_1,W)}, \cdots \cdots, r_{(\theta_k,1)}, r_{(\theta_k,2)}, \cdots \cdots r_{(\theta_k,W)}\right] \tag{21}$$

The sparse signal model can be given as

$$\begin{aligned} \mathbf{X}_1 &= \mathbf{A}_{r1}\mathbf{S}_{r1} + \mathbf{N} \\ \mathbf{X}_2 &= \mathbf{A}_{r2}\mathbf{S}_{r2} + \mathbf{N} \end{aligned} \tag{22}$$

Where as $\mathbf{A}_{r1}$ and $\mathbf{A}_{r2}$ is given as

$$\begin{aligned} \mathbf{A}_{r1} &= [\mathbf{a}_{r1}(\theta_1, r_1), \mathbf{a}_{r1}(\theta_1, r_2), \dots \dots, \mathbf{a}_{r1}(\theta_k, r_{w-1}), \mathbf{a}_{r1}(\theta_k, r_w)] \\ \mathbf{A}_{r2} &= [\mathbf{a}_{r2}(\theta_1, r_1), \mathbf{a}_{r2}(\theta_1, r_2), \dots \dots, \mathbf{a}_{r2}(\theta_k, r_{w-1}), \mathbf{a}_{r2}(\theta_k, r_w)] \end{aligned} \tag{23}$$

Which can be further given as $\mathbf{a}_{r1}(\theta_k, r_w) = \mathbf{a}_{rr1}(\theta_k) \otimes \mathbf{a}_{rt1}(\theta_k, r_w)$ similarly for $\mathbf{a}_{r2}$

(24)

$$\mathbf{a}_{rt1}(\theta_k, r_w) = \left[1, e^{-j4\pi(\Delta f/c)r_w}, \dots \dots, e^{-j4\pi(M-1)(\Delta f/c)r_w}\right].$$

$$\left[1, e^{j2\pi(d/\lambda)\sin\theta_k}, \dots \dots, e^{j2\pi(d/\lambda)(M-1)\sin\theta_k}\right]$$

$$\mathbf{a}_{rr1}(\theta_k) = \left[1, e^{j2\pi(d/\lambda)\sin\theta_k}, \dots \dots, e^{j2\pi(d/\lambda)(M-2)\sin\theta_k}\right]$$

$$\mathbf{a}_{rt2}(\theta_k, r_w) = \left[1, e^{-j4\pi(\Delta f/c)r_w}, \dots \dots, e^{-j4\pi(M-1)(\Delta f/c)r_w}\right].$$

$$\left[1, e^{j2\pi(d/\lambda)\sin\theta_k}, \dots \dots, e^{j2\pi(d/\lambda)(M-1)\sin\theta_k}\right]$$

$$\mathbf{a}_{rr2}(\theta_k) = \left[e^{j2\pi(d/\lambda)\sin\theta_k}, e^{j2\pi(d/\lambda)2\sin\theta_k} \dots \dots, e^{j2\pi(d/\lambda)(M-1)\sin\theta_k}\right]$$

Next we implement the above discussion for estimation of the range. We consider a single source at an angle of 30^0 and range of 3000 meters. Using double pulse MIMO FDA we first estimate the location of the source next using frequency increment among the antenna elements estimate the range of the source using the above discussed equation.

By implementing the equations and using grid refinement method the range of the source is estimated as shown in figure

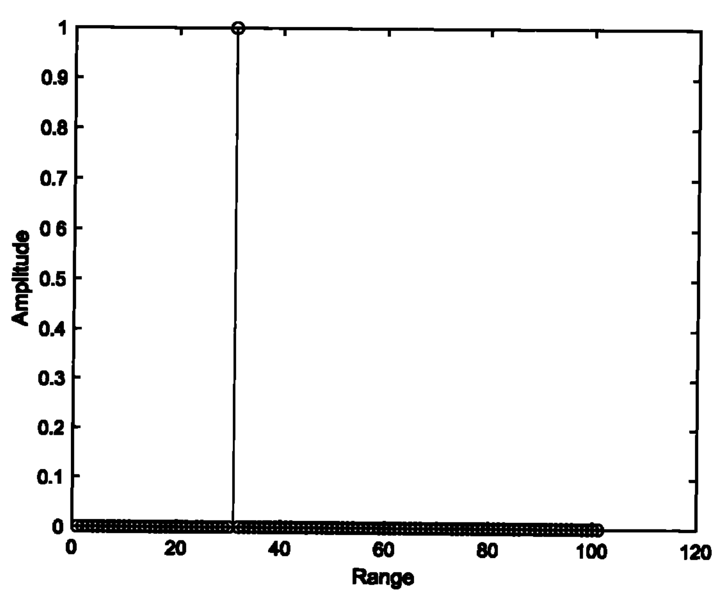


Fig 5.18: Range estimation of single source

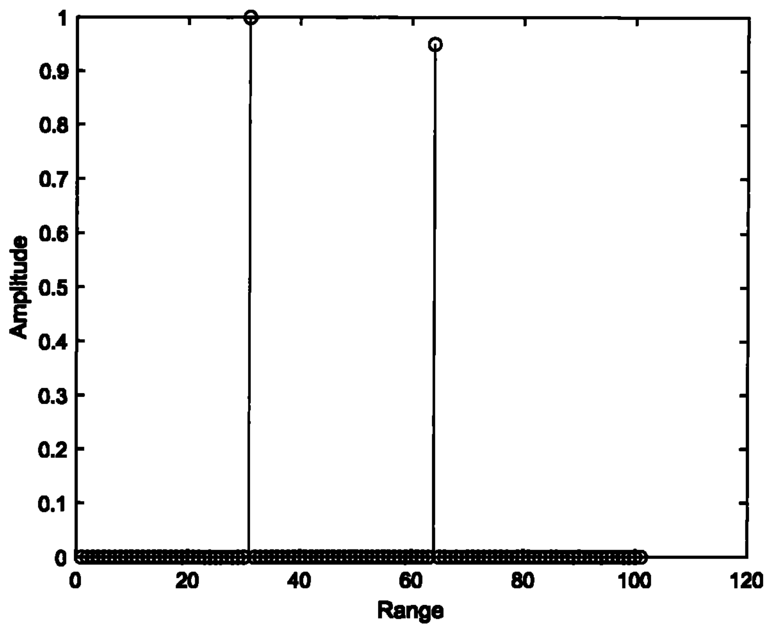


Fig 5.19: Range Detection for two sources

CHAPTER 6

Conclusion

In this dissertation we propose an application of compressive sensing for MIMO FDA radar. Where we estimate the parameters of a target. The parameters that are estimated are range and angle of the target. Compressive sensing using single snapshot as well as multi snap shot is considered. Initially we establish compressive sensing problem for estimating the angle of arrival of the target. Different array structure linear, sparse linear and L shaped for estimating the angle of arrival of the target in one dimension as well as two dimensions is studied. It is observed that the grid or resolution of the dictionary plays as important role. If the location of the target does not coincide with resolution of the grid, then the solution creates grid mismatch problem. Where an ambiguity is created due to the location of the target. To remove this ambiguity an optimization method is developed for estimating the exact grid resolution for the detection of the target. Similarly noise plays an important role in the estimation of the targets. Two types of noises are considered. One is Gaussian and the other is impulsive noise. A pre processing stage is included before the application of the compressive sensing. The pre processing stage removes the effect of the impulsive noise and also make the system more robust against the Gaussian noise.

It is observed in the MIMO FDA radar the range and the angle information is coupled. To decouple the range and the angle information double pulse method is employed. Where the first pulse is transmitted without the frequency increment among the antenna elements and the second pulse is transmitted with the frequency increment among the antenna elements. Sub array antenna structure in the receive domain is suggested. For simplicity two sub array are considered. The received signal from each subarray is passed through a pre processing stage and then a match filter. The compressive sensing is applied on each pulse separately. In order to make it multi sample then different pulses are transmitted and each pulses i.e with frequency increment and without frequency increment is staggered separately. First the angle information is extracted. The optimization algorithm developed is used for removing the ambiguity about the location of the sources. Once the angle information has been extracted the range information using compressive sensing from the second pulse which have frequency increment among the antenna element is extracted. In short we developed a CS framework for the DOA estimation. The problem of grid refinement is addressed, and a novel method is presented for solving it. The problem is explored for sparse antenna array using linear and L shaped antenna array, The grid refinement is applied.

The work is applied for subarray MIMO FDA radar. Linear case is considered only. Further array structure can be explored for future work.

Future Work

1- The compressive sensing problem for DOA estimation is developed for linear, sparse linear and sparse L shaped antenna array. The technique developed can be further extended for different antenna array like circular or rectangular.

2- To make the system robust a preprocessing stage based on weighted moving average filter and MDCO to remove Gaussian as well as Impulsive noise. Further techniques for making the system robust can be explored.

3- Multipath effect is not considered. Study of multipath and methods can be developed to remove them.

4- The multipath influence the estimation of the parameters. In order to remove multipath different algorithm are used with compressive sensing like ICA (Independent Component Analysis

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