Hydromagnetic flow and heat transfer in converging/diverging channels

By

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Pakistan, 2011.

1 - Fluid mechanics

2. Fluid dynamics

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A Dissertation
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Dedicated to

*My **Father**, Mother and all of my family members.*

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Muhammad Muddassar Maskeen

DECLARTION

I hereby declare that this thesis, neither as a whole nor as a part thereof, has been copied out from any source. It is further declared that I have prepared this dissertation entirely on the basis of my personal efforts made under the sincere guidance of my supervisor. No portion of the work, presented in this dissertation, has been submitted in the support of any application for any degree or qualification of this or any other institute of learning.

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Preface

Flow of Newtonian and non-Newtonian fluids through converging/diverging channels is of great importance in many industrial applications [1-5]. One of the most important application is of the flow through nozzles, diffusers and reducers are encountered in polymers processing operation. Converging/diverging flows have also been studied successfully by Daripa et al [6] to stimulate the dilute polymer solution through porous media. Cases [7] discussed the cold drawing operations in polymer industry to improve different mechanical properties of products such as plastic sheets and rods. This important study also relate the flow through converging channels. Extrusions of molten polymers through converging dies [8-10] are another important application of this type of flow. Similarly different types of Newtonians and non-Newtonians fluids through converging and diverging channel have been discussed successfully by [1-19]. For Newtonian fluids, its importance is due to the fact that it renders itself to an exact (purely radial) solution known as Jeffery-Hamel flow [20]. This exact laminar solution is now using as a standard solution to locate the critical conditions for the occurrence of transition from laminar to turbulent flow [21]. Separation is indeed a common phenomenon with diffusers and may be witnessed at any Reynolds numbers depending on the length or channel half angle. It lowers the pressure recovery ratio of diffusers and increases its head loss at a time. Shina [22], discussed steady two dimensional, incompressible, laminar viscoelastic flow in converging/diverging channel with suction and injection. After that Khabazi [23] discussed the use of polymer additives to delay flow separation in diverging channel in his B.Sc thesis. Hartmann et al [24] discussed very successfully the effects of magnetic field on the laminar flow of viscoelastic fluids between two parallel plates. After this study, literature is in rich with many other geometries including converging/diverging channels [25-60]. Motivated by the above facts, the aim of this dissertation is to investigate the effects of heat transfer on viscoelastic flow through converging/diverging channels. This dissertation is arranged as follows:

Chapter 1 includes some basic definitions and concepts which will be used in subsequent chapters. Chapters 2 consist of revised study of magnetohydrodynamics (MHD) flows of viscoelastic fluids in converging/diverging channels by Sadeghy et al [61]. We extended this study in direction to investigate the effects of heat transfer on this flow and detailed discussions related to the heat transfer effects are given in chapter 3. (MHD) flows of viscoelastic fluids in converging/diverging channels by Sadeghy et al [61]. We extended this study in direction to investigate the effects of heat transfer on this flow and detailed discussions related to the heat transfer effects are given in chapter3.

Contents

Chapter 1

Fundamentals of fluid mechanics

In this chapter, some basic definitions and concepts [20] relating the problems illustrated in subsequent chapters are presented.

1.1 Fluid mechanics

The branch of applied mathematics in which we study the behavior of fluids in the states of rest and in motion is called fluid mechanics.

1.2 Fluid

Fluid is a material which deforms under the application of shear stress.

1.3 Deformation

It is a relative change in position or length of the fluid particles.

1.4 Flow

When different forces act on a material it goes under deformation. If this deformation increases continuously without limit then this phenomenon is known as flow.

1.5 Viscosity

Movement of the molecules of the fluid against each other produce some internal friction forces is called viscosity of the fluid, and it is mathematically defined as

$$\mu = \frac{\text{shear stress}}{\text{rate of shear strain}}. \tag{1.1}$$

It is denoted by μ and has dimension $[M/LT]$.

1.6 Kinematics viscosity

The ratio of the viscosity to density is referred as kinematic viscosity ν, defined as follows

$$\nu = \frac{\mu}{\rho}. \tag{1.2}$$

1.7 Density

Mass per unit volume of the fluid is called the density of the fluid.

In mathematical notations, we can write the density ρ at a point A as

$$\rho = \lim_{\delta v -> 0} \frac{\delta m}{\delta v}, \tag{1.3}$$

where δv is the total volume element around the point A and δm is the mass of the fluid within δv.

1.8 Pressure

The magnitude of the force per unit area is called pressure. It is denoted by p and can be defined as

$$p = \frac{F}{A}, \tag{1.4}$$

where A is the surface area of a fluid and F is the magnitude of force acting normal to the surface.

1.9 Types of fluid

1.9.1 Ideal fluids

Fluids of zero viscosity are called ideal fluids. In a particular fluid flow situation, it is sometimes useful to consider what happen to an ideal fluid. Actually an ideal flow do not exist.

1.9.2 Real fluids

Those fluids in which fluid friction has significant effects on the fluid motion are called real fluids. In other words, we can not neglect the viscosity effects of fluid on the motion. On the basis of *Newton's law of viscosity,* real fluids are further classified into two classes. According to this law"shear stress is directly proportional to the rate of deformation". For one dimensional flow it can be written as

$$\tau_{yx} = \mu \frac{du}{dy}, \tag{1.5}$$

where τ_{yx} is the shear stress and du/dy is the rate of deformation.

1.9.3 Newtonian fluids

Newtonian fluids are fluids which obeys *Newton's law of viscosity* Eq. (1.5) or one can say that is a type of fluid whose stress versus strain rate curve are linear and passes through the origin. Water and gasoline are common examples of Newtonian fluids.

1.9.4 Non-Newtonian fluids

The fluids for which shear stress is directly but non-linearly proportional to the rate of deformation are called non-Newtonian fluids. Mathematically it can be expressed as

$$\tau_{yx} = k \left(\frac{du}{dy} \right)^n, \qquad n \neq 1, \tag{1.6}$$

or

$$\tau_{yx} = \eta \left(\frac{du}{dy} \right). \tag{1.7}$$

where

$$\eta = \left(\frac{du}{dy}\right)^{n-1}. \tag{1.8}$$

is the apparent viscosity which is a function of the rate of deformation. Examples of non-Newtonian fluids are tooth paste, ketchup, gel, shampoo, blood and soaps etc.

1.9.5 Compressible fluids

A fluid for which the volume in unit mass changes with the change in pressure or temperature is called compressible fluid. or temperature will result in change in density is called compressible fluid. All the gases are treated as compressible fluids.

1.9.6 Incompressible fluids

A fluid for which the change in pressure or temperature produce negligible change in density is called an incompressible fluid. All the liquids are treated as incompressible fluids.

1.10 Types of Flow

1.10.1 Uniform flow

It is a flow in which the velocity of fluid particles is of the same magnitude and direction at every point in the domain.

1.10.2 Non-uniform flow

If the velocity of fluid particles is of different magnitude or direction at different points of the domain, such flow is called non-uniform flow.

1.10.3 Steady flow

It is a flow for which some properties of the fluid does not vary with time is called steady flow and mathematically, it is defined as

$$\frac{\partial \eta}{\partial t} = 0. \tag{1.9}$$

where η represent the specific fluid property and t is the time.

1.10.4 Unsteady flow

It is a flow in which properties of fluids get changes with time is called unsteady flow. It can be expressed as

$$\frac{\partial \eta}{\partial t} \neq 0. \tag{1.10}$$

1.10.5 Laminar flow

A flow in which every particle of fluid have a separate, definite path and do not intersect itself. Laminar flow is also known as stream line flow.

1.10.6 Turbulent flow

A flow in which every particle of fluid do not have a definite path and that every particle can intersect its path itself, is called turbulent flow.

1.10.7 Rotational flow

Flow of a fluid in which the curl of the fluid velocity is not zero. Mathematically, for rotational flow

$$\boldsymbol{\nabla} \times \mathbf{V} \neq 0, \tag{1.11}$$

where $\mathbf{V}$ is fluid velocity.

1.10.8 Irrotational flow

Flow of a fluid in which the curl of the fluid velocity is zero every where. In other words one can say that fluid particles does not rotate about any axis. Mathematically it can be expressed by

$$\boldsymbol{\nabla} \times \mathbf{V} = 0. \tag{1.12}$$

1.11 Thermodynamic properties

1.11.1 Heat

Heat is a form of energy flowing from an object at a higher temperature to an object at a lower temperature.

1.11.2 Heat flux

Heat flux is defined as rate of heat transfer per unit cross-sectional area.

1.11.3 Temperature

Measurement of the average kinetic energy of the molecules in a system is called temperature.

1.11.4 Heat transfer

Heat transfer is a discipline of thermal engineering that concerns the transfer of thermal energy from one physical system to another. Heat transfer is classified into various mechanisms, such as heat conduction, convection and thermal radiation.

1.11.5 Specific heat

An amount of heat or thermal energy required to raise the temperature of a body by one unit called specific heat, and it is denoted by c_p.

1.11.6 Thermal conductivity

The quantity k that relates the vector rate of heat flow per unit area $\mathbf{q}$ to the vector gradient of temperature ∇T is called thermal conductivity. Experimentally this proportionality observed for solid and fluids, known as Fourier's law of heat conduction, which is given by

$$\mathbf{q} = -k\nabla T. \tag{1.13}$$

Where minus sign satisfies the convention that heat flux is positive in the direction of decreasing temperature.

1.11.7 Enthalpy

Enthalpy is a measure of total energy of a thermodynamics system. It is state function and an extensive quantity

1.12 Dimensionless numbers

A dimensionless number is a number without any unit associated with it. It is the ratio of the quantities having same dimensions. They fully characterize the fluid under flow and processing. There are many dimensionless numbers. Which are used in fluid mechanics, few of them are presented as follows:

1.12.1 Reynolds number

The ratio which approximate the relationship between inertia force to the viscous force is called Reynold number. In mathematical notation, this number is denoted by Re and is defined by

$$\mathrm{Re} = \frac{UL}{\nu}, \tag{1.14}$$

where U is reference velocity and L is the characteristics length.

1.12.2 Hartman number

It is a dimensionless number which is expressed by the ratio of electromagnetic force to the viscous force. It is denoted by H and mathematically given by

$$H = BL\sqrt{\frac{\sigma}{\rho\nu}}, \tag{1.15}$$

where B is magnetic field and σ is electric conductivity.

1.12.3 Weissenberg number

It is a dimensionless number which is the ratio of relaxation time of the fluid to specific process time. It is used in the study of viscoelastic fluid and named after Karl Weissenberg. It is denoted by Wi and mathematically given by

$$Wi = \dot{\gamma}\lambda, \tag{1.16}$$

where $\dot{\gamma}$ is shear rate and λ is relaxation time.

1.12.4 Prandtl number

The approximate ratio of the product of dynamics viscosity and specific heat to the thermal conductivity gives dimensionless number called Prandtl number Pr. Mathematically, we can write it as

$$\mathrm{Pr} = \frac{c_p\mu}{k}, \tag{1.17}$$

where c_p is specific heat at constant pressure, μ is viscosity of the fluid, k is thermal conductivity.

1.12.5 Eckert number

The non-dimensional number which expresses the relationship between flow's kinetic energy and enthalpy is called the Eckert number Ec. Mathematically, it is expressed as

$$Ec = \frac{\nu^2}{c_p A}. \tag{1.18}$$

1.13 Fundamental equations of fluid mechanics

1.13.1 Equation of continuity

It is a natural fact that the matter cannot be created or destroyed in any closed system. The mathematical form of this law (conservation of mass) is known as equation of continuity.

It is defined as

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{V}) = 0. \tag{1.19}$$

For incompressible steady fluid, it can be written as

$$\nabla.\mathbf{V} = 0. \tag{1.20}$$

In polar coordinates, this equation is given by

$$\frac{1}{r}\frac{\partial(rv_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} = 0. \tag{1.21}$$

If the flow is purely radial, so above equation becomes

$$\frac{1}{r}\frac{\partial(rv_r)}{\partial r} = 0. \tag{1.22}$$

1.13.2 Equation of momentum

If there are no external forces then the total momentum of bodies consisting an isolated system act upon one another remain conserved. The mathematical form of this law (conservation of momentum) is known as equation of momentum. Mathematically, it can be expressed as

$$\rho\frac{d\mathbf{V}}{dt} = \mathbf{\nabla}.\mathbf{T} + \mathbf{f}_B, \tag{1.23}$$

where p is pressure, $\mathbf{V}$ is the velocity field, $\mathbf{T}$ is the Cauchy stress tensor, d/dt is the total material derivative and $\mathbf{f}_B$ is magnetic force. In polar coordinates $\mathbf{T}$ can be expressed as

$$\mathbf{T} = \begin{bmatrix} \tau_{rr} & \tau_{r\theta} & \tau_{rz} \\ \tau_{\theta r} & \tau_{\theta\theta} & \tau_{\theta z} \\ \tau_{zr} & \tau_{z\theta} & \tau_{zz} \end{bmatrix}, \tag{1.24}$$

where τ_{rr}, $\tau_{\theta\theta}$, τ_{zz} are the normal stresses and $\tau_{r\theta}$, $\tau_{\theta r}$, τ_{rz}, $\tau_{\theta z}$, τ_{zr}, $\tau_{z\theta}$ are shear stresses.

1.13.3 Equation of energy

The energy equation is described as

$$\rho c_p\frac{de}{dt} = \mathbf{T}.\mathbf{L} - \mathbf{\nabla}.\mathbf{q}, \tag{1.25}$$

in which

$$\mathbf{L} = \mathbf{\nabla}\mathbf{V}. \tag{1.26}$$

In polar coordinates, it is given a

$$\rho c_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} \right) = k \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} \right] + \mu \phi. \tag{1.27}$$

where ϕ is the viscous dissipation function, which is a quadratic function of spatial derivatives of components of fluid velocity which gives the rate at which mechanical energy is converted into heat in a viscous fluid per unit volume.

Chapter 2

Magnetohydrodynamics (MHD) flows of viscoelastic fluids in converging/diverging channels

In this chapter, we revise the study of Sadeghy et al. [61] in which they studied the applicability of magnetic field for controlling hydromagnetic separation in Jeffery-Hamel flows of viscoelastic fluids. Considered laminar, two-dimensional flow of viscoelastic fluids obeying second order fluid with the assumption that the flow is symmetric and purely radial. Similarity solution proposed by Sadeghy et al. [61] is used to reduce the given non-linear partial differential *equations to single non-linear ordinary differential equation. In order to obtain the numerical* solution of single equation governing the MHD flow through converging/diverging channel, with three physical boundary conditions, Chebyshev collocation point method is used. The physical parameters involved in the equations are Reynolds number, Weissenberg number, channel half-angle and the magnetic number. It is observed that all the parameters have a significant effect on the velocity profiles of Jeffery-Hamel flows are shown through graphs.

2.1 Problem formulation

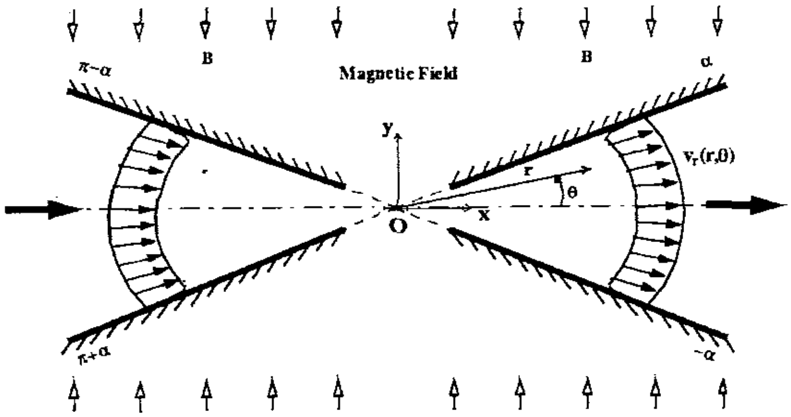

Fig. 2.1: Purely radial flow in a two dimensional converging/diverging channel subject to a uniform externally-imposed magnetic field.

Let us consider an incompressible, electrically-conducting, viscoelastic fluid obeying the second-order or the second-grade model is flowing through converging/diverging channel of subtended angle 2α as shown in Fig. 2.1. It is assumed that the channel walls are extend to infinity in the $z-$direction (i.e., perpendicular to the plane) and the flow is symmetric and purely radial. For convenience, the plane polar coordinates, with its origin at the apex of the channel, is used to formulate the problem. The fluid is assumed to be an incompressible, , as its constitutive equation. As shown in Fig. 2.1 , a uniform magnetic field of strength **B** is applied on the flowing fluid in the transverse direction. With all the assumption made on the flowing fluid, in polar coordinates, only single component v_r of velocity **V** survive, which under steady state condition is a function of r and θ only. We take into account the radial flow assumption, which is a restrictive condition can easily be ignored for Newtonian fluid only [39, 40]. In the absence of gravitational forces, the continuity equation and the equations of motion can be expressed, in vector form, as

$$\nabla.\mathbf{V} = 0, \tag{2.1}$$

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}.\nabla)\mathbf{V}\right) = -\nabla p + \nabla.\tau + \mathbf{f}_B, \tag{2.2}$$

where **V** is the velocity vector , $\boldsymbol{\tau}$ is the extra stress tensor, ρ is the fluid density and $\mathbf{f}_B$ is magnetic or Lorentz force [37]. The imposed Lorentz force $\mathbf{f}_B$ with constant electrical coductivity σ can be written as

$$\mathbf{f}_B{=}\sigma(\mathbf{E}+\mathbf{V}\times\mathbf{B})\times\mathbf{B}. \tag{2.3}$$

Here, induced magnetic and electrical field is neglected due to sufficiently small the Reynolds number. The Eq. (2.3)

$$\mathbf{f}_B=-\sigma\left[(v_r cos\theta)\,cos\theta B^2\hat{e}_r+(v_r cos\theta)\,sin\theta\hat{e}_\theta\right]. \tag{2.4}$$

Thus, in polar coordinates the governing equations of fluid (whether Newtonian or non-Newtonian) through a converging or diverging channel can be written as [23]:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r)=0, \tag{2.5}$$

$$\rho\left(v_r\frac{\partial v_r}{\partial r}\right)=-\frac{\partial p}{\partial r}+\left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr})+\frac{1}{r}\frac{\partial\tau_{\theta r}}{\partial\theta}-\frac{\tau_{\theta\theta}}{r}\right)-\sigma B^2 v_r(cos\theta)^2, \tag{2.6}$$

$$0=-\frac{1}{r}\frac{\partial p}{\partial\theta}+\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta})+\frac{1}{r}\frac{\partial}{\partial\theta}\tau_{\theta\theta}+\frac{\tau_{\theta r}-\tau_{r\theta}}{r}\right)-\sigma B^2 v_r(sin\theta.cos\theta). \tag{2.7}$$

From the continuity equation, Eq. (2.5), it can be concluded that the radial velocity $v_r(r,\theta)$ must be a separable function of the form $f(\theta)/r$. For a viscoelastic fluid obeying the second-order model, the extra stress tensor $\boldsymbol{\tau}$ can be related to the deformation field by [41]:

$$\boldsymbol{\tau}=\alpha_1\mathbf{D}+\alpha_2\overset{\nabla}{\mathbf{D}}+\alpha_3\mathbf{D}.\mathbf{D}, \tag{2.8}$$

where α_1, α_2, and α_3 are material properties, and $\mathbf{D}$, the velocity gradient tensor defined by

$$\mathbf{D}=\boldsymbol{\nabla}\mathbf{V}+(\boldsymbol{\nabla}\mathbf{V})^T. \tag{2.9}$$

Having determined all stress terms for second-order fluid in polar coordinate system [23], the purely radial-flow assumption can be invoked to simplify the stress tensor as [23]:

$$
\begin{aligned}
\tau \;=\; & \alpha_1 . \begin{bmatrix} 2\frac{\partial v_r}{\partial r} & \frac{1}{r}\frac{\partial v_r}{\partial \theta} \\ \frac{1}{r}\frac{\partial v_r}{\partial \theta} & 2\frac{v_r}{r} \end{bmatrix} \\
& +\alpha_2 . \left(\begin{bmatrix} 2v_r\frac{\partial^2 v_r}{\partial r^2} & -\frac{v_r}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{v_r}{r}\frac{\partial^2 v_r}{\partial r \partial \theta} \\ -\frac{v_r}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{v_r}{r}\frac{\partial^2 v_r}{\partial r \partial \theta} & -2\frac{v_r^2}{r^2} + 2\frac{v_r}{r}\frac{\partial v_r}{\partial r} \end{bmatrix} - \begin{bmatrix} 4(\frac{\partial v_r}{\partial r})^2 + \frac{2}{r^2}(\frac{\partial v_r}{\partial \theta})^2 & \frac{1}{r}\frac{\partial v_r}{\partial r}\frac{\partial v_r}{\partial \theta} + 3\frac{v_r}{r^2}\frac{\partial v_r}{\partial \theta} \\ \frac{1}{r}\frac{\partial v_r}{\partial r}\frac{\partial v_r}{\partial \theta} + 3\frac{v_r}{r^2}\frac{\partial v_r}{\partial \theta} & 4(\frac{v_r}{r})^2 \end{bmatrix} \right) \\
& +\alpha_3 . \begin{bmatrix} 4(\frac{\partial v_r}{\partial r})^2 + \frac{1}{r^2}(\frac{\partial v_r}{\partial \theta})^2 & \frac{2}{r}\frac{\partial v_r}{\partial r}\frac{\partial v_r}{\partial \theta} + 2\frac{v_r}{r^2}\frac{\partial v_r}{\partial \theta} \\ \frac{2}{r}\frac{\partial v_r}{\partial r}\frac{\partial v_r}{\partial \theta} + 2\frac{v_r}{r^2}\frac{\partial v_r}{\partial \theta} & \frac{1}{r^2}(\frac{\partial v_r}{\partial \theta})^2 + 4(\frac{v_r}{r})^2 \end{bmatrix} .
\end{aligned} \tag{2.10}
$$

Now, having calculated all stress terms for a second-order fluid (see Ref. [23]), Eqs. (2.6) and (2.7) can be combined into a single equation by cross differentiation and then eliminating the pressure p as follows

$$
\rho\left(-2\frac{ff'}{r^3}\right) = \alpha_1\left(\frac{f'''}{r^3} + 4\frac{f'}{r^3}\right) + \alpha_2\left(-4\frac{ff'''}{r^5} - 16\frac{ff'}{r^5}\right) - \frac{\sigma B^2}{r}\left(f'cos^2\theta - fsin(2\theta)\right). \tag{2.11}
$$

To non-dimensionalize this equation, we substitute

$$
F(\eta) = \frac{f(\theta)}{f_0}, \tag{2.12}
$$

where $\eta = \theta/\alpha$, F is a dimensionless function, and f_0 is a dimensional constant which can be related to the flow rate per unit length, Q, by means of the equation:

$$
Q = f_0 \int_{-1}^{1} \alpha F(\eta) d\eta. \tag{2.13}
$$

In order to assign a numerical value to f_0, we are at freedom to specify that the maximum value of F is equal to unity at the centerline and leave the constant f_0 to be determined by the flow rate Q. Obviously, by so doing at $F = 1$ we also have $F' = 0$. Furthermore, to satisfy the no-slip condition at the two walls, located at $\theta = \pm\alpha$ for the diverging case and $\theta = \pi \pm \alpha$ for the converging case, we require that $F(\pm 1) = 0$. After, substituting Eq. (2.11) into Eq. (2.12), we have

$$F''' + 2\,\mathrm{Re}\,\alpha F F' + 4\alpha^2 F' + Wi\left(8\alpha F F''' + 32\alpha^3 F F'\right) - H.\,\mathrm{Re}\left(F' cos^2(\alpha\eta) - \alpha F sin(2\alpha\eta)\right) = 0, \tag{2.14}$$

with the boundary conditions are

$$F(\pm 1) = 0,\ F(0) = 1. \tag{2.15}$$

Where Re the Reynolds number, Wi the Weissenberg(or elasticity) number, and H the Hartman(or magnetic) number, defined respectively as

$$\mathrm{Re} = \frac{\rho f_0 \alpha}{\mu_0}, \quad Wi = \frac{\psi_{1,0} f_0}{r^2}, \quad H = \frac{\sigma B^2 r^2 \alpha}{\rho f_0}. \tag{2.16}$$

It need to be mentioned that to keep the notation simple, in Eq. (2.15) we have allowed Re to be negative for the inward flow (converging case).

It is easy to check that Eq. (2.16) reduces to its Newtonian counterpart, as it should, by simply setting $Wi = 0$ and $H = 0$ [20]. Interestingly, the order of the differential equation remained three both for Newtonian and second-order/second-grade fluids. In order to solve the above Eq. (2.14) subject to the boundary conditions Eq. (2.15), Spectral collocation method is used.

2.2 Numerical solution

In a typical spectral method, the unknown function $F(\eta)$ is approximated by a sum of $N+1$ basis functions, $T_n(\eta)$ [52]

$$F(\eta) \approx F_N(\eta) = \sum_{n=o}^{N} a_n T_n(\eta). \tag{2.17}$$

For this purpose, Chebyshev polynomials [52] are used as a basis functions which are defined in the $-1 \leq \eta \leq 1$ as

$$T_N(\eta) = cos(N cos^{-1}\eta). \tag{2.18}$$

Since Eq. (2.17) is assumed approximate solution of Eq. (2.14), it is not an exact solution (for sure), after inserting Eq. (2.17) into Eq. (2.14), it will not satisfy the equation, residue will arise in this ease. The problem of finding the solution is now reduce to minimize the residue. There are several methods available for this purpose [52] collocation point method is used in which the residual is forced to become exactly equal to zero at a set of N predefined (collocation) points. The points in common use are the Gauss-Labatto points defined as [52].

$$\eta_j = cos\left(\frac{j\pi}{N}\right) \qquad j = 1, 2, ..., N. \tag{2.19}$$

Obtained set of N algebraic equations are solved by using Newton's iteration method [53]. The solution of the equations are infect the N unknown values of a_n at N collocation points. For the results to become independent of collocation points N, the number of N was increased until the converged results were found. It is observed that for the converged solution (independent of N), N is taken around 50.

2.3 Result and discussion

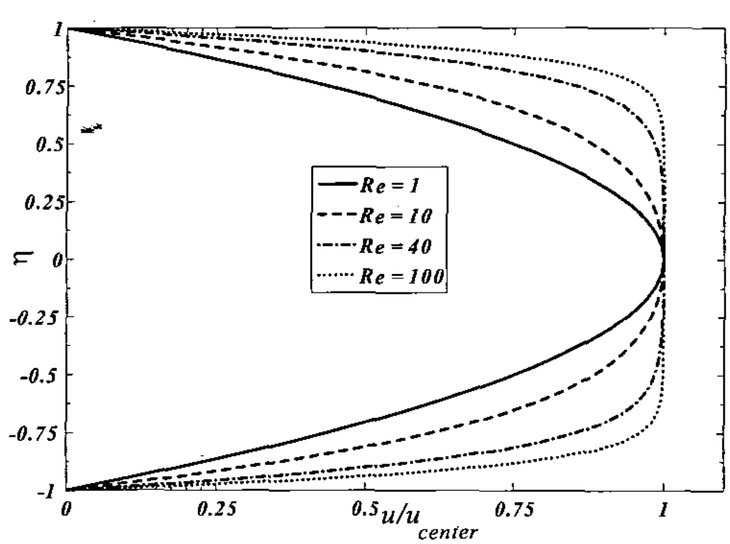

Fig. 2.2: The effect of the Reynolds number on the velocity profiles in a converging channel with $\alpha = \pi/8$, $Wi = 0.2$ and $H = 0$.

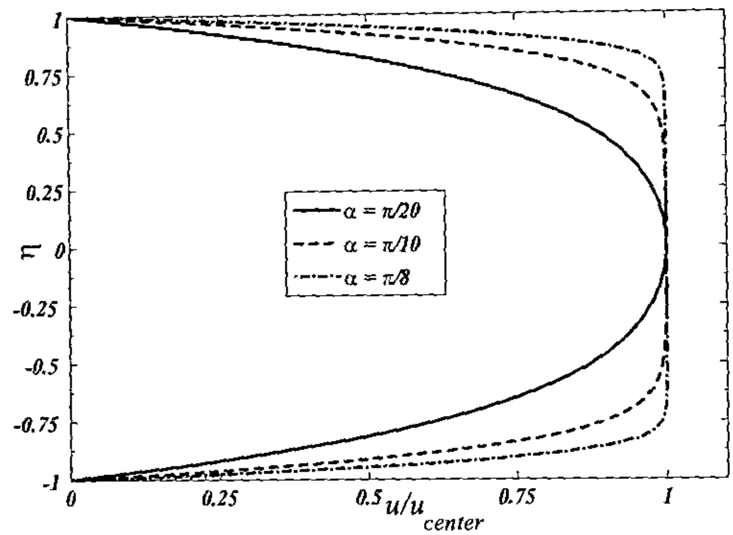

Fig. 2.3: The effect of the channel half-angle on the velocity profiles in a converging channel with $Re = 50$, $Wi = 0.3$ and $H = 0$.

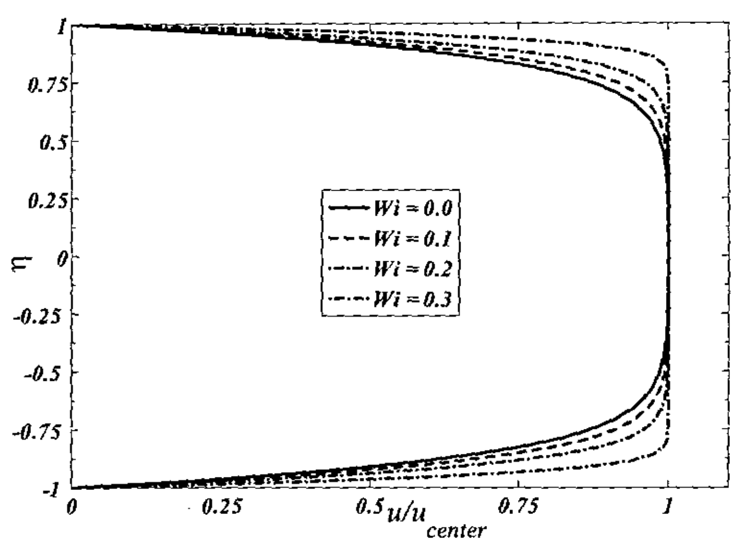

Fig. 2.4: The effect of the Weissenberg number on the velocity profiles in converging channel with $\alpha = \pi/8$, Re = 100 and $H = 0$.

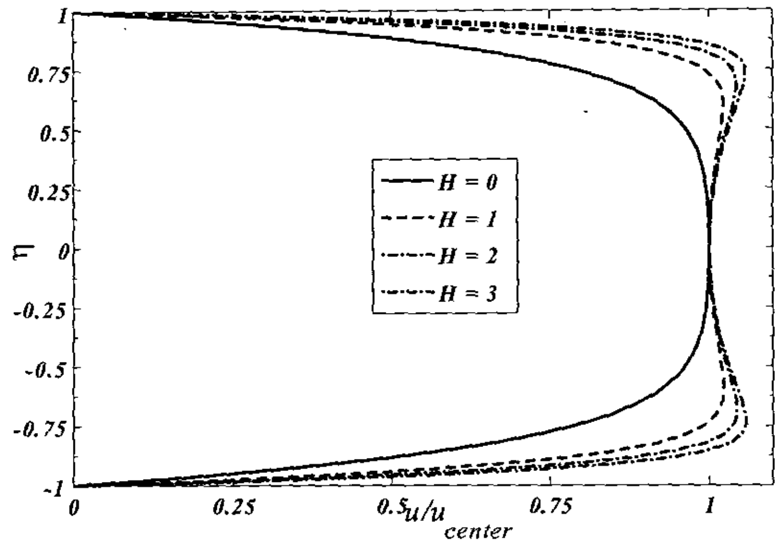

Fig. 2.5: The effect of the magnetic number on the velocity profiles in converging channel with $\alpha = \pi/8$, $\mathrm{Re} = 60$ and $Wi = 0.1$.

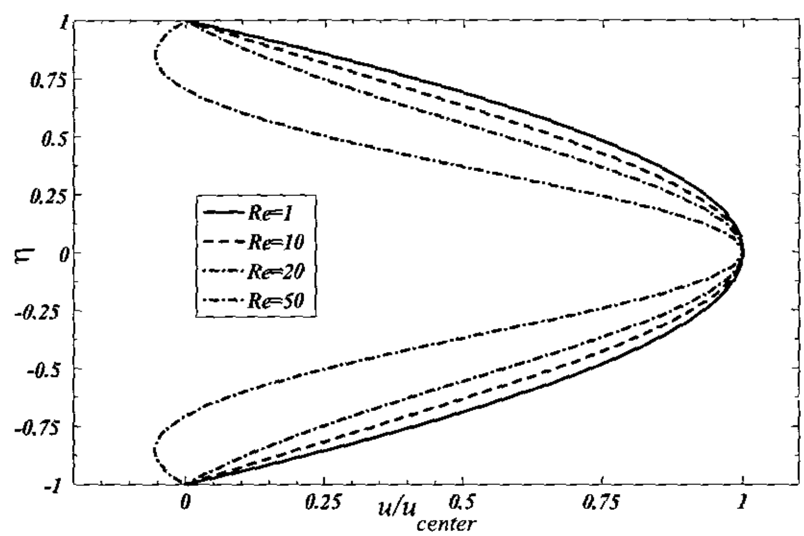

Fig. 2.6: The effect of the Reynolds number on the velocity profiles in diverging channel with $\alpha = \pi/8$, $Wi = 0.2$ and $H = 0$.

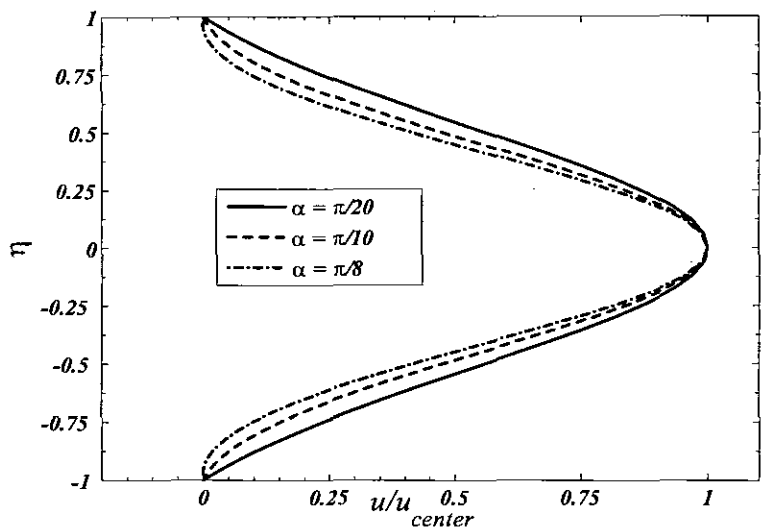

Fig. 2.7: The effect of the channel half-angle on the velocity profiles in diverging channel with Re = 40, $Wi = 0.3$ and $H = 0$.

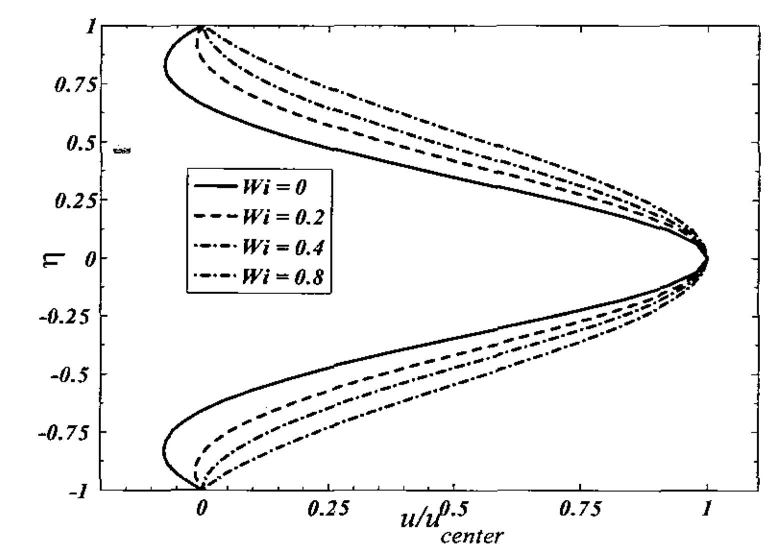

Fig. 2.8: The effect of Weissenberg number on the velocity profiles in diverging channel with $\alpha = \pi/8$, Re = 40 and $H = 0$.

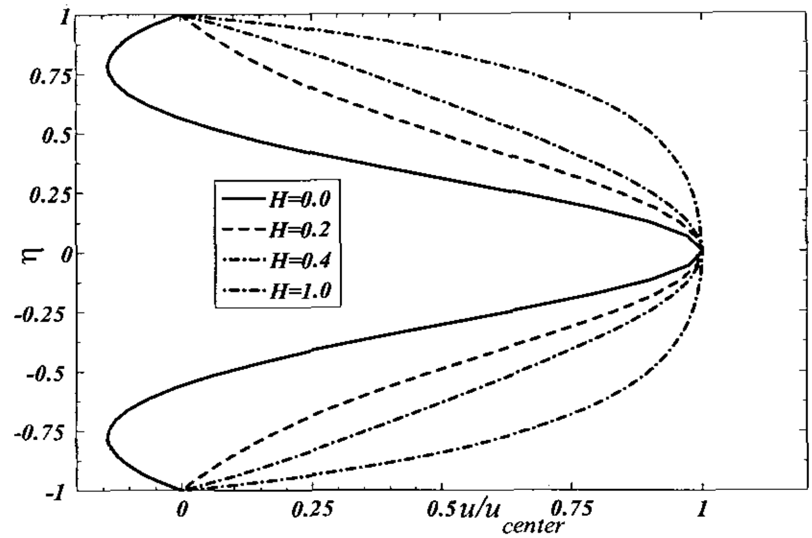

Fig. 2.9: The effect of the magnetic number on the velocity profiles in diverging channel with $\alpha = \pi/8$, Re = 50 and $Wi = 0.1$.

The numerical technique described in the last section is translated into a Matlab program and obtained the solution for some values of all the physical parameters involved. But before doing so, the numerical solution obtained is compared for Newtonian fluids [20] for a channel having $\alpha = 22.5^o$, and is found in excellent agreement.

Fig. 2.2 – 2.5 are drawn to present the effects of the Reynolds number Re, channel half-angel α, Weissenberg number Wi, and magnetic number H, on the velocity profiles inside a converging channel for a second-order fluid. Fig. 2.6 – 2.9 are drawn to elaborate the effects of the parameters for diverging channel for the same fluid. These all figure clearly exhibit the strong influence of all these parameters on the for inside the converging/diverging channel. It is seen from Fig. 2.2 that increase in the Reynold number Re, is responsible to accelerate the fluid elements near the channel wall. In contrast, an increase in Reynolds number may lead to a flow reversal in diverging channels, (see Fig. 2.6). Since the increase in channel half angle creates the favourable pressure gradient in converging channel, and adverse pressure gradient for diverging channel. Therefor by increasing the channel half angle α, the fluid element accelerated near the channel wall in converging channel, as shown in Fig. 2.3. However, increasing in Reynolds number Re, in diverging channel, leads to the reversal of flow near the wall, as shown in Fig. 2.7.

Interestingly, it is seen from Fig. 2.6 and 2.7 that, for a very high value of Re and channel half angle α, for a diverging channel and second order fluids the flow may easily become susceptible to hydrodynamic stability [54, 55]. To observe the effects of the Weissenberg number on the velocity profiles, Figs 2.4 and 2.8 suggest that by an increase in this number (say, by increasing the concentration of the polymeric additive), fluid elements near the wall are more intensely accelerated in converging channel. However, for diverging channel, comparitively high values are required to over from the flow separation near the wall for second order fluid. Finally, Figs 2.5 and 2.9 show the strong effect of magnetic number on the velocity profiles inside converging/diverging channels. The effect is quite striking in that in converging channels, fluid elements near the walls are seen to accelerate to velocities higher than the centerline velocity. More importantly, the acceleration as caused by the magnetic forces may completely suppress flow separation in diverging channels (see Fig. 2.9). Fig. 2.10 and 2.11 are presented to show the effects of magnetic number on the second grade fluid (fluid having negative $\psi_{1,0}$) for converging/diverging channels. It is observed that these effects are almost same qualitatively with the second-order fluid.

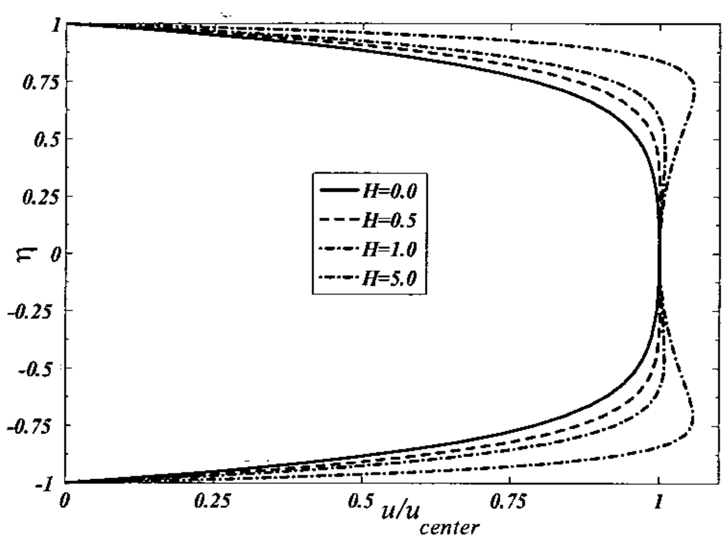

Fig. 2.10: The effect of the magnetic number on the velocity profiles for second-grade fluid in a converging channel with $\alpha = \pi/8$, Re $= 60$ and $Wi = -0.1$.

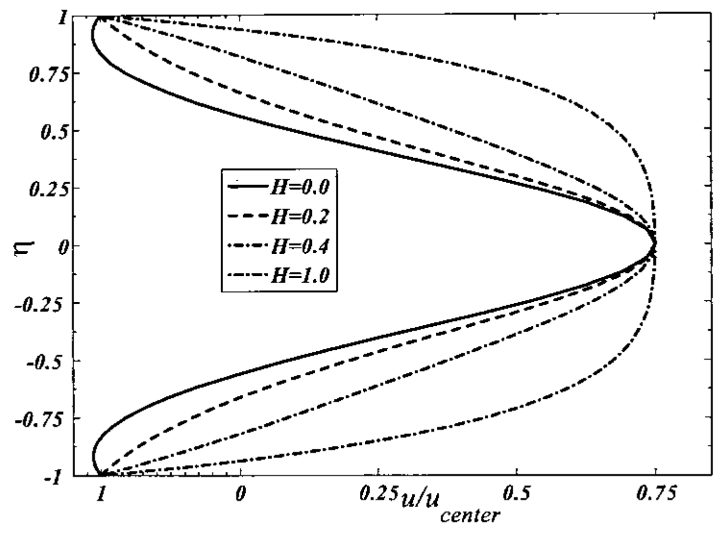

Fig. 2.11: The effect of the Magnetic number on the velocity profiles for second-grade fluid in a diverging channel with $\alpha = \pi/8$, Re = 30 and $Wi = -0.1$.

Chapter 3

Hydromagnetic flow and Heat transfer in converging/diverging channels

In this chapter, we study the effect of heat transfer on the hydromagnetic flow through converging/diverging channel. To do this, a local similarity solution is found to convert the modelled partial differential equation for heat transfer analysis to an ordinary differential equation. We further assume that there is no temperature at the wall and the phenomenon of heat transfer is occurring due to the viscous dissipation. The governing linear ordinary differential equation is then solved by the same Spectral collocation method. The effects of various parameters appearing in the problem on temperature profile discussed through the graphs. This chapter is an extension of the work made by Sadeghy et al. [61].

3.1 Problem formulation

Consider, steady two-dimensional flow of an incompressible , electrically-conducting viscoelastic fluid obeying second-order model flowing through converging/diverging channel of substended angel 2α as discussed in chapter 2. It is also assumed that flow is symmetric and purely radial. Due to these assumption we have seen that there is only one non-zero velocity component, v_r, which under steady state conditions is a function of r and θ only. The temperature is also assumed to be a function of r and θ. The equation that govern the heat transfer in cylindrical coordinates is given by

$$\rho c_p v_r \frac{\partial T}{\partial r} = k\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right) + \alpha_1\left(4\left(\frac{\partial v_r}{\partial r}\right)^2 + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}\right), \tag{3.1}$$

in which c_p is the specific heat at constant pressure, T is the temperature and k is the thermal conductivity. We assumed that there is no temperature at the wall and heat transfer occur due to viscous dissipation term in Eq. (3.1) i.e.

$$T = 0 \quad \text{at} \quad \theta = \pm\alpha.$$

Defining

$$v_r = \frac{f(\theta)}{r}, \quad T = \frac{g(\theta)}{r^2}. \tag{3.2}$$

Eq. (3.1) reduced to an ordinary differential equation in the following form

$$-2\rho c_p f g = k\left(g^{''} + 4g\right) + \alpha_1\left(4f^2 + f^{'2}\right). \tag{3.3}$$

and the boundary conditions reduces to

$$\frac{\partial T}{\partial \theta} = \frac{g^{'}(0)}{r^2} = 0 \quad \Rightarrow \quad g^{'}(0) = 0. \tag{3.4}$$

$$T(\alpha) = T_w \quad \Rightarrow \quad \frac{g(\alpha)}{r^2} = T_w \quad \Rightarrow \quad g(\alpha) = r^2 T_w. \tag{3.5}$$

Also

$$v_r = U \quad \text{at} \quad \theta = 0. \tag{3.6}$$

$$f(0) = rU. \tag{3.7}$$

For making the governing equation dimensionless we use

$$F(\eta) = \frac{f}{\upsilon}, \quad \phi(\eta) = \frac{g}{A}. \tag{3.8}$$

where $\eta = \theta/\alpha$, ϕ is dimensionless function. Eq. (3.3) reduces to the form

$$\phi'' + 4\alpha^2\phi + 2\,\mathrm{Pr}\,F\phi + \mathrm{Pr}\,Ec\left(F'^2 + 4\alpha^2F^2\right) = 0, \tag{3.9}$$

with the boundary conditions given by:

$$\phi(+1) = 0; \quad \phi(-1) = 0. \tag{3.10}$$

Where the Prandtl number Pr and the Eckret number Ec are defined as

$$\mathrm{Pr} = \frac{\mu c_p}{k}. \tag{3.11}$$

$$Ec = \frac{\nu^2}{c_p A}. \tag{3.12}$$

Now the problem here is to find the solution of Eq. (2.14) subject to the boundary conditions Eq. (2.15) and Eq. (3.9) subject to the boundary condition Eq. (3.10). Since the given equations are highly non-linear, although current practice is to find such type of boundary layer problem by using Runge-Kutta fourth order scheme, but due to the highly nonlinearity of the Eq. (2.14), it is decided to solve through Spectral collocation method.

3.2 Numerical solution

Since the Eq. (2.14) does not involve any term of temperature profile ϕ. Therefore, that equation can be solved separately by using Spectral method. Once the solution of Eq. (2.14) is obtained, the problem Eq. (3.9) become linear in ϕ, as the nonlinearity in F and $F'$is known to us at this stage. Now, for the solution of Eq. (3.9) subject to the boundary conditions Eq. (3.10), the unknown function $\phi(\eta)$ is approximated by a sum of $N+1$ basis functions, $T_n(\eta)$ as

$$\phi(\eta) \approx \phi_N(\eta) = \sum_{n=o}^{N} e_n T_n(\eta). \tag{3.13}$$

The basis function $T_n(\eta)$ here are the chebyshev polynomials which are used in chapter 2. Since the solution of Eq. (2.14) with boundary conditions Eq. (2.15) is already calculated separately, after putting the values of $F(\eta)$ and $F^{'}(\eta)$ for $-1 \leq \eta \leq 1$ and putting Eq. (3.13) into Eq. (3.9), we get the residue of the equation, as approximated solution Eq. (3.13) is not the exact solution of Eq. (3.9). In order to minimize the residue of this equation we used Newton iteration method as discussed in chapter 2. For this convergent solution, we observed that the value of N is found to be around 50. In this way, after ensuring the convergence of the solution we draw the effects of Eckert number Ec, Prandtl number Pr, channel half angle α, Reynold number Re, magnetic number(Hartman number) H and Weissenberg number Wi on the temperature profile are shown in Fig. 3.1 – 3.12.

3.3 Result and discussion

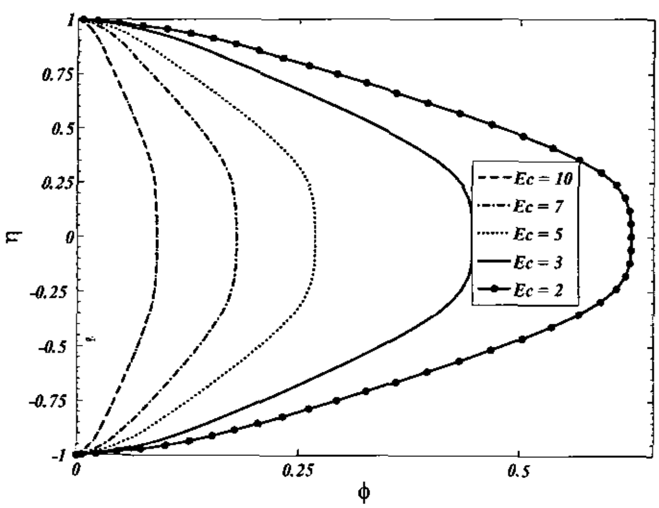

Fig. 3.1: The effect of Eckret number on the temperature profiles for a converging channel with $\alpha = \pi/20$, Re $= 60$, $H = 0$, $Wi = 0$ and Pr $= 0.7$.

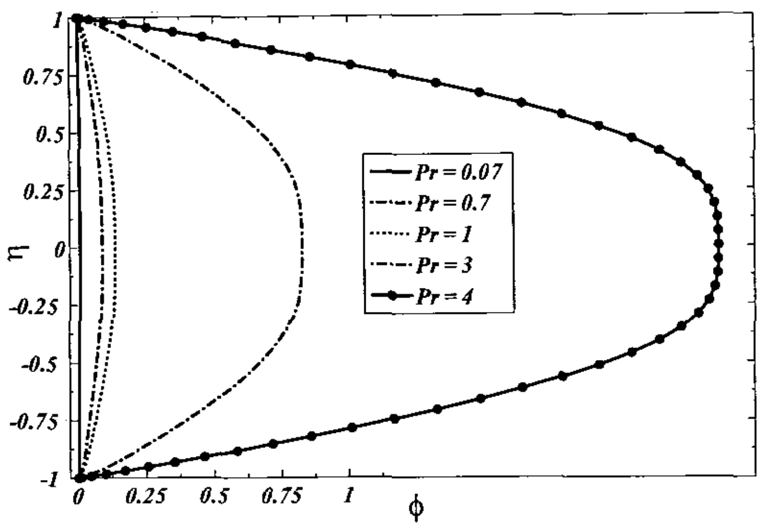

Fig. 3.2: The effect of Prandtl number on the temperature profiles for a converging channel with $\alpha = \pi/20$, $\mathrm{Re} = 60$, $H = 0$, $Wi = 0$ and $Ec = 1$.

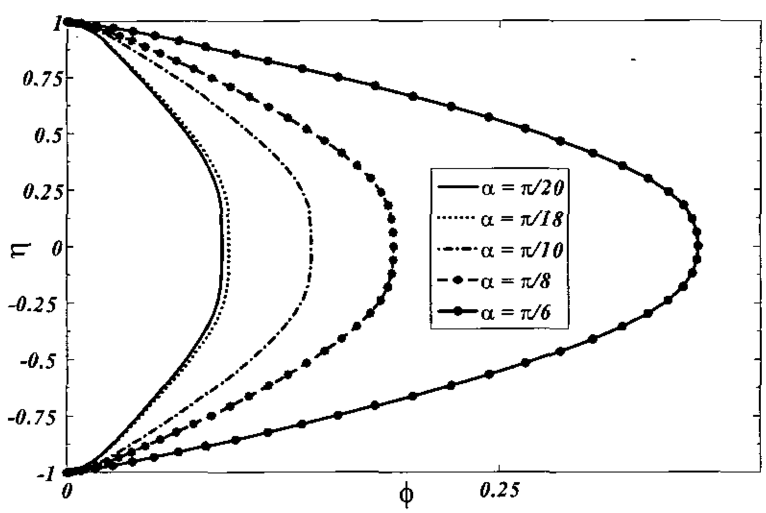

Fig. 3.3: The effect of channel half angle on the temperature profiles for a converging channel with $\mathrm{Re} = 60$, $H = 0$, $Wi = 0$, $Ec = 1$ and $\mathrm{Pr} = 0.7$.

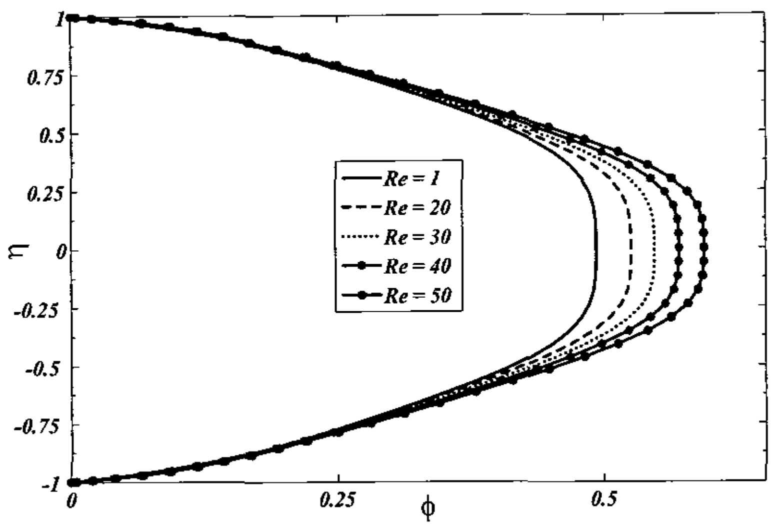

Fig. 3.4: The effect of Reynolds number on the temperature profiles for a converging channel with $\alpha = \pi/20$, $H = 0$, $Wi = 0.1$, $Ec = 5$ and $\Pr = 0.9$.

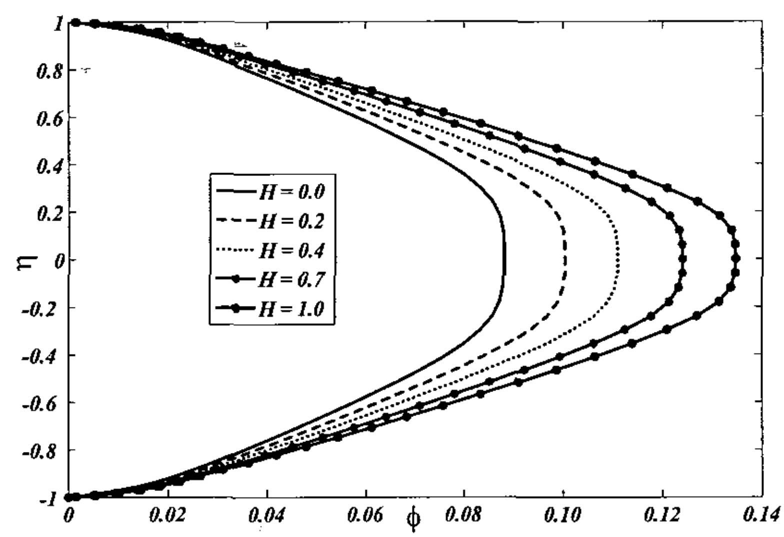

Fig. 3.5: The effect of magnetic number on the temperature profiles for a converging channel with $\alpha = \pi/20$, $\text{Re} = 50$, $Wi = 0.1$, $Ec = 1$ and $\Pr = 0.7$.

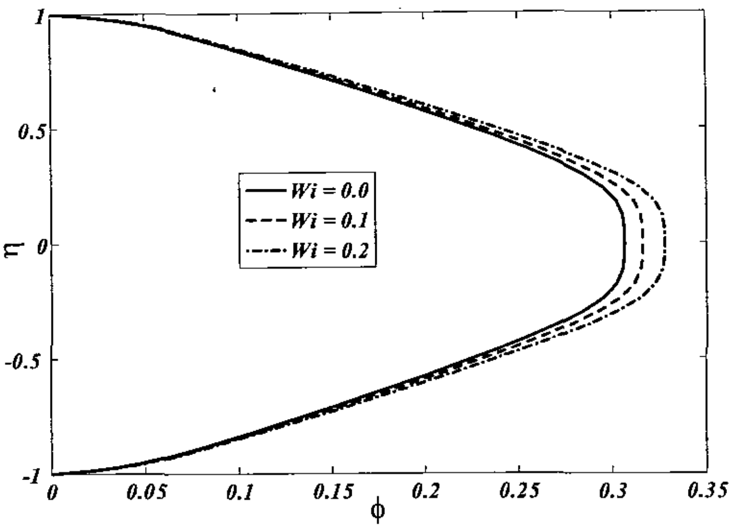

Fig. 3.6: The effect of Weissenberg number on the temperature profiles for a converging channel with $\alpha = \pi/20$, Re $= 60$, $H = 0.2$, $E_c = 3$ and Pr $= 0.7$.

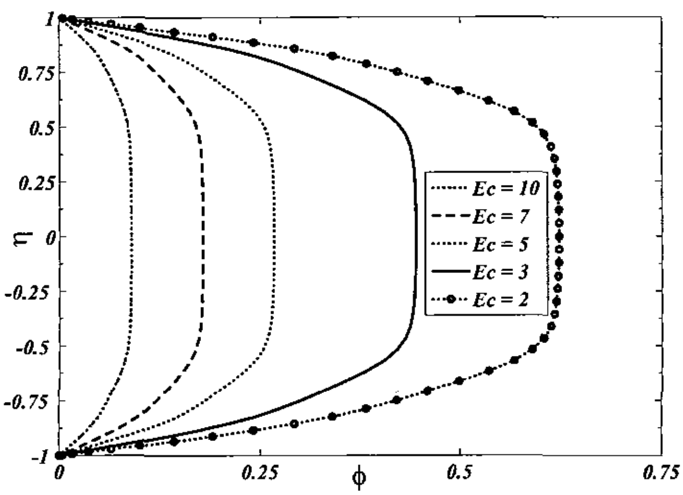

Fig. 3.7: The effect of Eckret number on the temperature profiles for a diverging channel with $\alpha = \pi/20$, Re $= 60$, $H = 0$, $Wi = 0$ and Pr $= 0.7$.

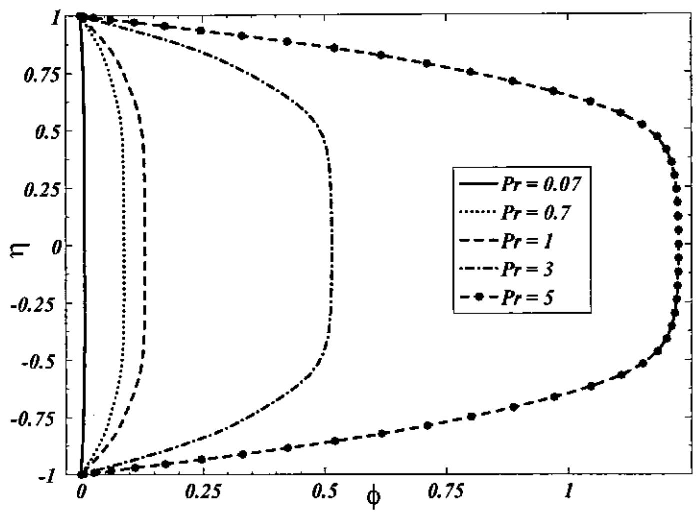

Fig. 3.8: The effect of Prandtl number on the temperature profiles for a diverging channel with $\alpha = \pi/20$, $\mathrm{Re} = 60, H = 0$, $Wi = 0$ and $Ec = 1$.

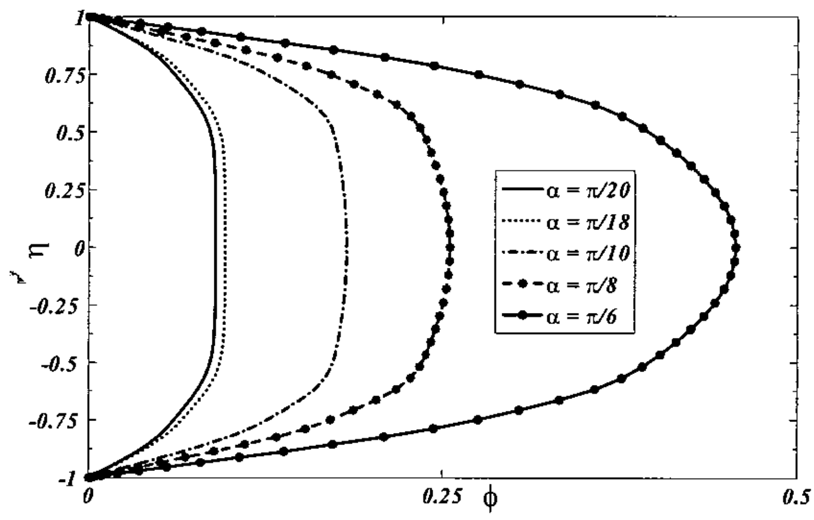

Fig. 3.9: The effect of channel half angle on the temperature profiles for a diverging channel with $\mathrm{Re} = 60$, $H = 0$, $Wi = 0$, $Ec = 1$ and $\mathrm{Pr} = 0.7$.

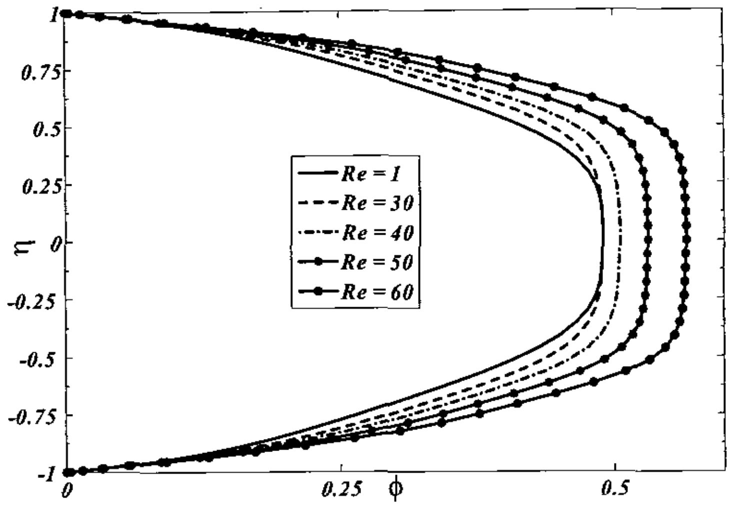

Fig. 3.10: The effect of Reynolds number on the temperature profiles for a diverging channel with $\alpha = \pi/20$, $H = 0$, $Wi = 0.1$, $Ec = 5$ and $\Pr = 0.9$.

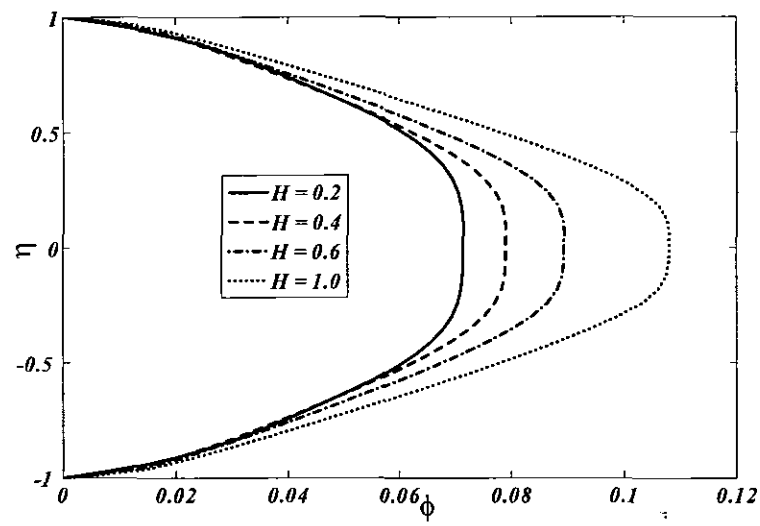

Fig. 3.11: The effect of magnetic number on the temperature profiles for a diverging channel with $\alpha = \pi/20$, $\mathrm{Re} = 50$, $Wi = 0.1$, $Ec = 1$ and $\Pr = 0.7$.

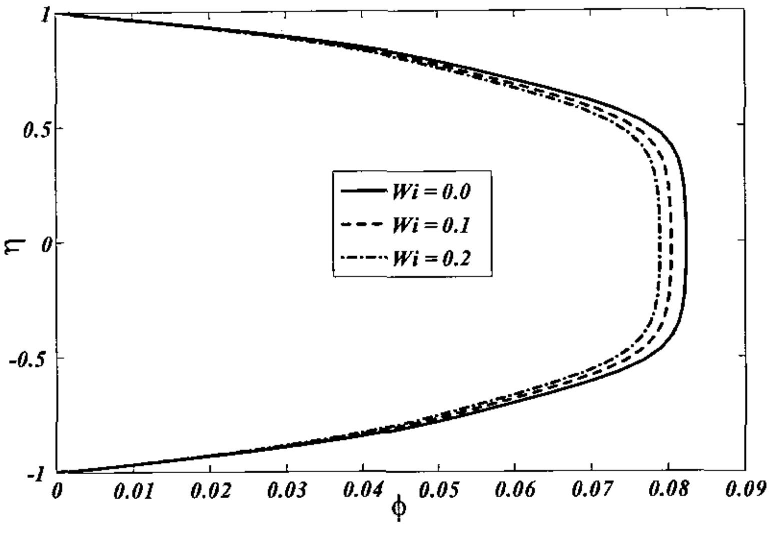

Fig. 3.12: The effect of Weissenberg number on the temperature profiles for a converging channel with $\alpha = \pi/20$, $\mathrm{Re} = 50$, $H = 0.0$, $Ec = 1$ and $\mathrm{Pr} = 0.7$.

For the analysis, Figures 3.1 – 3.6 present the effects of the Eckert number Ec, Prandtl number Pr, channel half angle α, Reynold number Re, Weissenberg number Wi and magnetic number H on the temperature profile inside a converging channel for a second-order fluid. Figures 3.7 – 3.12 show the variation in temperature profile due to the same parameters in a diverging channel. Since, it was assumed that there is no temperature at the walls of the channel, phenomenon of heat transfer occur only due to viscous dissipation term in temperature field Eq. (3.9). For $\mathrm{Pr} = 0.7$, the effects of the Eckert number Ec on temperature field are shown in Fig. 3.1. It is observed that, by increasing the value of Ec, heat dissipation increases within the converging channel and approach to its maximum value only at the center of the channel. However, in comparison with Fig. 3.7, for diverging channel, the phenomenon of heat transfer occurs only adjacent to the walls of the channel. It remain constant for very wide range within the channel around $\eta = 0$. It is seen that, qualitatively the effects of Eckert number Ec on the temperature profile for converging and diverging cases are same. For $Ec = 1$ and $\mathrm{Pr} = 0.07$, there is almost negligible heat transfer appear in the converging/diverging channel as shown in Fig. 3.2 and 3.8. It is due to the fact that for these values of the parameters, coefficients of the viscous dissipation term are very small numerically. However for large value of Pr, heat dissipation increases both for the converging and diverging channels. The effects of variations in the channel half angle α on the temperature profile for converging and diverging cases are shown in Figs. 3.3 and 3.9 respectively. It is observed that by increasing α, heat dissipation increases in the channel. In contrast to the effects of the Eckert number Ec and Prandtl number Pr in diverging channel, for large value of channel half angle, the heat dissipation occurs throughout the channel as shown in Fig. 3.9. However, for small angle, heat dissipation occurs only adjacent to the wall for the diverging channel. It is quite similar as the effects of Ec and Pr on temperature profile for diverging case. Variation in the values of Reynold number Re, produces no change in the phenomenon of the heat transfer near the wall for converging case as shown in Fig. 3.4. Variation in temperature profile is observed only near the center of the channel due to change in Re . The effects of Re on temperature profile for diverging channel are shown in Fig. 3.10. Its effects are qualitatively and quantitatively same as of Pr, Re and channel half angle α. It is surprising to observe the effects of magnetic number H on the temperature profiles as its effects are same both for converging and diverging channels as shown in Fig. 3.5 and 3.11 respectively. By increasing the value of Weissenberg number W_i, temperature profiles increases for converging channel and decreases for diverging channel as shown in Figs. 3.6 and 3.12 respectively. It is noted that our solution holds for small values of Weissenberg number only.

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