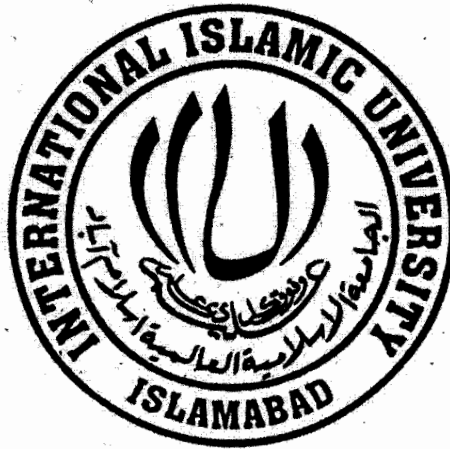


# Analysis of Alamouti Scheme and it's Extension

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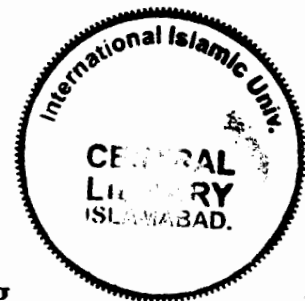


By

**Umair Saeed**

A dissertation submitted to the Faculty of Engineering & Technology, IIU,  
As a partial fulfillment of the requirements for the award of the degree of

**MS Electronic Engineering**



**Department of Electronic Engineering  
Faculty of Engineering and Technology  
International Islamic University, Islamabad**

## Certificate of Approval

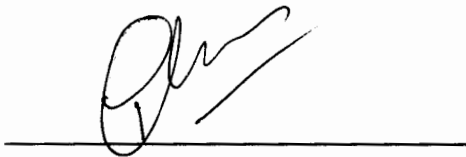
It is certified that we have read the thesis submitted by Umair Saeed [134-FET/MSEE/F07]. It is our judgment that this project is of sufficient standard to warrant its acceptance by the International Islamic University, Islamabad for MSEE degree in Electronic Engineering (Telecommunication Engineering).



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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

**Dedicated To,**

THE HOLY PROPHET HAZRAT MUHAMMAD (S.A.W.),  
HIS SAHABA, MY PARENTS, TEACHERS & UMMAH.

## DECLARATION

I, hereby declare that this research work, neither as a whole nor as a part thereof has been copied from any source. It is further declared that I have developed this research and the accompanied report entirely on the basis of my personal effort made under the guidance of my supervisor.

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A handwritten signature in black ink, appearing to read 'Saeed', is written over a horizontal line. The signature is stylized and includes a large loop on the left side.

**Umair Saeed**  
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***Umair Saeed***

## **Abstract**

The demand for mobile communication systems with high data rates and improved link quality for a variety of applications has dramatically increased in recent years. New concepts and methods are necessary in order to cover this huge demand, which counteract or take advantage of the impairments of the mobile communication channel and optimally exploit the limited resources such as bandwidth and power. Multiple antenna systems are an efficient means for increasing the performance. In order to utilize the huge potential of multiple antenna concepts, it is necessary to resort to new transmit strategies, referred to as Space-Time Codes, which, in addition to the time and spectral domain, also use the spatial domain. The performance of such Space-Time Codes is analyzed in this thesis. In this thesis, the focus is on diversity oriented Space Time Codes. Starting with Alamouti scheme then Extended Alamouti and after that I have analyzed the performance of some other schemes with these two and plot their graphs.

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## List of Symbols

$U$	Transmission matrix
$B$	Number of receive antennas
$C$	Number of receive antennas
$Y$	Signal constellation
$R$	Data rate
$S$	Transmitted symbol
$E$	Energy of the transmitted symbol
$W$	Column vector in transmission matrix
$P$	Probability
$Q$	Transmitted symbol in one time slot
$d$	Number of bits per symbol
$Z$	Resulting matrix
$i$	Indeterminate of transmission matrix $U$
$r$	Received vector
$h$	Channel coefficient
$n$	Noise term
$\beta$	Path gain
$A$	Unitary matrix
$\sigma$	Variance
$\rho$	Transmit power

# CHAPTER 1

## Introduction

### 1.1 Introduction

The demands for the high data rates increase a lot in the wireless communications systems. The main problem for service provider in the wireless communication is the unavailability of enough bandwidth and the process of allotment of new spectrum by the government is very slow. The devices used in the communication system uses a lot of power and the current focus are also towards the power consumption by the devices. Efforts have been put to design such devices in such a way that they use little power as possible to increase battery life and keep the products small. Thus, the major challenge faced by the designer are increase data rates and improve the performance keeping in view of the fact of smaller bandwidth [1]. The wireless channel is unpredictable by its nature and in general the channel error rates are very poor over the wireless channel as compared to a wired channel.

The major problem in the wireless communication channel is the multipath fading, which results in attenuation in the received signal. This fading occurs due to delays, reflections, scattering and diffracted signal components [2]. The other problem is the variation in the time, which is due to the movements of the mobile units and the objects in the environment. This results in severe attenuation of the signal, which decreases the signal to noise ratio (SNR), which degrades the performance significantly.

The only solution for the reliable communication is to use the diversity techniques, through which the receiver antenna receives the multiple replicas of the transmitted signal under varying fading condition. By using the diversity technique, it reduces the probability that all the replicas are simultaneously affected by a severe attenuation. There are different methods to achieve the diversity, which are Antenna/Spatial diversity, Polarization diversity, Frequency diversity and temporal diversity. By using the combination of these methods produces much better results.

There is also another technique to address the problem of multipath fading which is the use of multiple transmitting antennas and multiple receive antennas. That technique is known as Space Time Coding (STC) [3]. The introduction of this technique provides high data rates and reliable communication over fading channels, this concept combines coding, modulation and spatial diversity into a two-dimensional coded modulation technique. STC provide simple decoding and full diversity, but no coding gain. STC use the concept of space (different antennas) and the time while encoding and decoding information symbols. I use the terms multiple-input multiple-output (MIMO) and space-time coding interchangeably in this thesis.

In MIMO system, antennas at the transmitter and the receiver are placed far enough (spatially) such that the effect of the channel at a particular antenna element is different from the effect at all other antenna elements. This implies independent or spatially uncorrelated fading. This holds true only if spacing between transmits antennas or receive antennas is of the order of several wavelengths. However, if antenna spacing is not enough, the fading channel from multiple antennas might be correlated, and the performance will be degraded.

## **1.2 Contribution**

- I have analyzed the Alamouti STC scheme for two transmit antenna.
- I have also analyzed the Alamouti STC scheme extension and investigate a generic matrix by which I can design space time block code with  $N_T$  transmit antennas ( $N_T > 2^n$ ) where  $n$  is the constellation symbol.
- With help of this formula I can extend this scheme to affordable number of transmit antennas and accordingly can achieve the transmit diversity by a sophisticated maximum ratio combiner (MRC).
- Demonstration of given scheme for different SNR is obtained and discussed that how this scheme combats with noise and BER.

## **1.3 Organization of thesis**

- Chapter 2 contains an introduction to Wireless Communication System
- Chapter 3 contains a brief description of Space-Time Codes
- Chapter 4 contains the Analysis of Alamouti Space-Time Block Code with its Extension
- Chapter 5 contains the conclusions and future enhancements of the scheme.

## **Chapter 2**

### **Wireless Communication Systems**

#### **2.1 Introduction**

In the last ten years remarkable evolution occurs in the communication field like introduction of cellular system and the introducing of AM and FM modulation techniques [4]. Also in this period WC system reached to its highest point and introduced new and advance equipments for handling the data. We are using so much advance devices and different WL networks some are cellular phone, walkie-talkie, handheld PDA and especially wireless internet, etc .As every one want to talk with any one so to achieve this goal the major problem which creates much difficulty in achieving the goal is the capacity of the WC system. In order to achieve this goal with in limited resources especially using devices that consumes less power and complexity.

As the signal is transmitted from only one transmit antenna and there are lot of moving things present in the signal path so, only by increasing the power of the transmitted signal we can't achieve the required capacity. Lot of researchers are working to find the simplest and most efficient way to solve this problem and they have found some efficient techniques, like frequency reuse [5], OFDM [6], turbo codes [7] and low-density parity check codes [8].

The major disadvantage of single antenna system is its high error rate. In an additive white Gaussian noise (AWGN) channel, the bit error probability BEP decreases linearly with increasing SNR in a multipath-fading channel. Therefore, in order to obtain the required SNR in a single antenna communication system a much high power and

some special codes are needed. If we want to use the single antenna system, which have many disadvantages, we should have to adopt some new communication techniques to provide high capacity and less BER for future WC. One most convenient system is called multiple-input-multiple-output (MIMO) wireless system in which there is multiple antennas are used at both side. The specialty of the MIMO system is that it turns multiple-path propagated signals into a single and strong signal, which is linear in the number of, transmit antennas [9].

Other thinking is about channel knowledge and it is considered that the information about channel is partial or unknown at the receiver side but still new scheme have higher Shannon capacity then the old system that have only one antenna at both ends. Although Shannon capacity is achieved only by using codes with unbounded complexity but the above scheme is not useable for real transmission systems. Let there are two antennas at transmitter side and transmit same signal from both antennas then PEP is obtained although it is inversely proportional to SNR but only the coding gain is improved.

At this point we need some algorithm that utilized all benefits of spatial diversity obtained from multiple antennas. Lot of codes, algorithm with affordable complexity and performance has been proposed, like diversity techniques and diversity combining methods. The most powerful and successful one is space-time code. The difference in STC system from the old single antenna system is that here the signal processing at the transmitter and receiver side is done in two dimensions one in time and second in spatial dimensions. This dual signal processing improves the performance of data rate by many



orders of magnitude without using more bandwidth. The saving of bandwidth makes this scheme so much attractive for the researchers and engineers.

It is proved in [10] that for STC the PEP is inversely proportional to  $\text{SNR}^{BC}$ , B represents number of antennas and C are the number of receive antennas. The product BC is called the diversity of the selected STC. If we compare the PEP of a single antenna system, which is inversely proportional to the SNR, then in the above coding scheme the BER decreases so fast. Alamouti in [11] is the one who introduced this STC scheme but this scheme applicable only for two transmit antennas. There is lot of experiments performed on different codes and got better results but in all experiments the channel knowledge is assumed to be known, which not possible approach in very fast changing channels is. Hochwald studies the real coding scheme where channel is unknown first time and Meir Feder [12] also introduced a unitary space-time modulation, which also has the same diversity, and in this scheme the channel at transmitter and receiver is unknown which is introduced by I. Bahceci and T. M. Duman [13].

As the need of communication systems increases day by day so lot of new schemes are investigating by using latest technology of multiple antennas which make space-time coding successful. Lot of researchers has been written many papers on differential and non-differential unitary space-time codes. There is so much work have been done in improving the coding gain by using different techniques but still this area need much study.

## 2.2 Communication System Model using Multiple-Antenna

In Figure 2.1 there is a WC system that have B transmit and C receive antennas. In real case there will be a wireless channel for linking of each transmitter and receiver. Here  $h_{mn}$  is the propagation coefficients of the channel between the  $b_{th}$  transmit and the  $c_{th}$  receive antenna. In each time slot  $s_1, s_2, s_3, \dots, s_B$  information is transmitted from

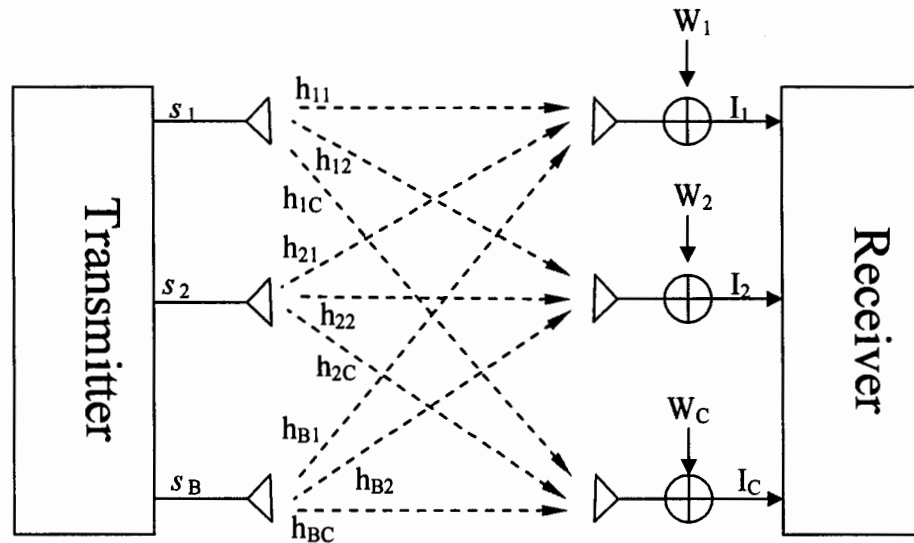


Figure 2.1 Multiple-antenna communication systems

all B transmit antennas. Now at the receiver end there is B signals arrived from all transmitters after passing some fading coefficient. Passing through the channel information got some noise signal and  $n_c$  is the noise present at  $c_{th}$  receive antenna. Now after collecting all signals from all transmitters the receiver will get

$$i_c = \sum_{b=1}^B h_{bc} s_b + n_c \quad (2.1)$$

These are the vector notation of noise  $n=[1, 2, \dots, N]$ , transmitted  $s=[s_1, s_2, s_3, \dots, s_B]$ , received  $i=[i_1, i_2, i_3, \dots, i_M]$ , signal and the channel matrix as

$$\mathbf{H} = \begin{bmatrix} h_{11} & \dots & \dots & \dots & h_{1C} \\ h_{21} & \dots & \dots & \dots & h_{2C} \\ \vdots & \dots & \dots & \dots & \vdots \\ \vdots & \dots & \dots & \dots & \vdots \\ h_{B1} & \dots & \dots & \dots & h_{BC} \end{bmatrix}, \quad (2.2)$$

This equation can be written in this way

$$I = s\mathbf{H} + n \quad (2.3)$$

### 2.3 Rayleigh Flat-Fading Channel

As there are lots of moving things especially transmitter and receiver in our surrounding so these all produce interference in the transmitted signal this make the analysis of signal to much difficult. On other side if we see the wired channels then we come to know that there are not so much problems due to stationary environment. Electromagnetic waves are transmitted in wireless system the transmitted signal struck with the surrounding moving things this change the waves path, amplitude and their phases. This change in wave path creates multiple fading in the signal and the signal strength decreases at the receiver side. Propagation models those observe signal variations from long distances are called large-scale models and those observe variation from small distance are called small scale or fading models.

Small-scale fading is more serious because this comes more in daily life environments due to the multiple-path propagation, moving transmitter, receiver, and surroundings and especially from the transmission bandwidth. In this case the bandwidth of the transmitted signal is much lesser then the frequency range over which the channel fading process is correlated and this is known as frequency nonselective fading. The flat-fading signal envelope at the receiver the Rayleigh distribution is commonly used. In

real wireless system the mobile receives scattered waves because the surrounding objects reflect all line-of-sight waves. As the number of reflected waves increases when they reach equal to central limit theory then we have two quadrature components at the receiver with mean zero and variance  $\sigma^2$ . So the received envelop for any time has Rayleigh distribution having phase between  $-\pi$  and  $\pi$ . The Rayleigh distribution has probability density function

$$p(r) = \begin{cases} \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} & r \geq 0 \\ 0 & r \leq 0 \end{cases} \quad (2.4)$$

Normalized form of the equation (2.3) is

$$\sum_{b=1}^B |h_{bc}^2| = B, \text{ for } i=1,2,\dots,C, \quad (2.5)$$

I have  $\sigma^2 = \frac{1}{2}$  so the channel coefficient  $h_{bc}$  has a complex Gaussian distribution with mean zero and variance equal to one. The imaginary and real part of  $h_{bc}$ , is independent Gaussians with mean zero and variance  $\frac{1}{2}$ . Using equation 2.5, and make in normalized form shows that the total transmitted energy is equal to the sum of all energies present on all receive antennas.

$$E \left| \sum_{b=1}^B h_{bc} s_c \right|^2 = \sum_{b=1}^B E |h_{bc}|^2 |s_c|^2 = \sum_{b=1}^B E |s_c|^2 = P, \quad (2.6)$$

If there is a stationary environment and we have line-of-sight propagation path then we have an other model known as Ricean model and this model have density function,

$$p(r) = \begin{cases} \frac{r}{\sigma^2} e^{-\frac{r^2 + A^2}{2\sigma^2}} I_0\left(\frac{Ar}{\sigma^2}\right) & r \geq 0 \\ 0 & r \leq 0 \end{cases} \quad (2.7)$$

Here  $A$  represents the amplitude of the strongest received signal which remains positive and  $I_0(\cdot)$  is the 0<sup>th</sup>-order modified Bessel function.

## 2.4 Capacity Results

As in communication world capacity is the major problem and using multiple antennas at transmitter and receiver solves this problem. Here I shall discuss three cases and see how much capacity will increase in different cases by assuming the Rayleigh fading channel.

These three cases are as below.

- 1 Transmitter and the receiver know the channel behavior
- 2 Only the receiver knows the channel behavior
- 3 Transmitter and receiver both don't know about the channel behavior

Let us start with first case where I assume that the channel matrix  $\mathbf{H}$  is known by transmitter and receiver also  $\mathbf{H}$  is deterministic. If we decompose the matrix  $\mathbf{H}$  then shall get the  $\mathbf{H} = \mathbf{ABC}^*$  where  $\mathbf{A}$  is a  $B \times C$  unitary matrix,  $\mathbf{C}$  is an  $I \times I$  unitary matrix, and  $\mathbf{B}$  is also  $B \times C$  diagonal matrix and there is not any negative entry at diagonal of the matrix  $\mathbf{B}$ . By defining  $\tilde{i} = \mathbf{C}i$ ,  $\tilde{s} = \mathbf{sA}$ , and  $\tilde{c} = \mathbf{C}c$ , the system equation (2.3) is equivalent to  $\tilde{i} = \mathbf{B}\tilde{s} + \tilde{c}$

As  $c$  is symmetric complex Gaussian that have mean zero and variance  $\sigma_c$  then  $\tilde{c}$  will also be the same like  $c$ . The matrix  $\mathbf{H}$  has non-zero singular values so its rank will be at

most  $\{B, C\}$ . By putting the non-zero singular values of  $\mathbf{H}$  as  $\sqrt{\lambda_i}$ . Now the system equation is equivalent to

$$\tilde{i}_i = \sqrt{\lambda_i} \tilde{s}_i + \tilde{n}_i \quad \text{for } 1 \leq i \leq \min\{B, C\} \quad (2.8)$$

The distribution of  $\tilde{s}_i$  is similarly circularly symmetric Gaussian and we know the capacity for the k-th independent channel is  $\log(1 + \lambda_k p_k)$  and  $p_k = E \tilde{s}_k \tilde{s}_k^*$  is the consumed power in the k-th channel. In order to get the maximum information,  $\tilde{s}_k$  should be circularly symmetric Gaussian distribution and this power is allotted to an independent channel. Now the power given to the k-th channel would be  $E \tilde{s}_k^* \tilde{s}_k = (\tilde{\mu} - \lambda_k^{-1})^+$ , and  $\mu$  is equal to  $\sum_{k=1}^{\min\{B, C\}} (\mu - \lambda_k^{-1})^+ = p^3$  and now by solving all equations we get the channel capacity

$$C = \sum_{k=1}^{\min\{B, C\}} \log(\mu \lambda_k) \quad (2.9)$$

It is clear from above Eq that capacity increases linearly with  $\min\{B, C\}$ .

Let us start work on second case where only the receiver knows about channel behavior. If  $p$  is circularly symmetric complex Gaussian having mean equal to zero and with variance  $(P/B)I_B$  then the channel capacity is given by  $C = \log \det(I_C + (P/B)\mathbf{H}^* \mathbf{H})$ . If the channel matrix  $\mathbf{H}$  is random Rayleigh distribution then the channel expected capacity would be

$$C = E \log \det(I_C + (P/B)\mathbf{H}^* \mathbf{H}) \quad (2.10)$$

If the number of receive antennas is fixed to  $C$  then from the “large number” law we have  $\lim_{B \rightarrow \infty} \frac{1}{B} \mathbf{H}^* \mathbf{H} = I_C$  with a probability equal to 1. Now the capacity will behave,

with probability 1, as  $C \log(1+P)$ , here the capacity increases linearly as number of receives antennas  $C$  increases. Similarly with receive antennas if transmit antennas are fixed then from the law of large number we have  $I_B$  with a probability 1 is

$$\lim_{C \rightarrow \infty} \frac{1}{C} \mathbf{H}^* \mathbf{H} = I_B \quad (2.11)$$

Since we have  $\det(I_C + (P/B)\mathbf{H}^* \mathbf{H}) = \det(I_B + (P/B)\mathbf{H}\mathbf{H}^*)$ , by solving above equations the capacity with probability 1 is  $B \log(1 + \frac{PC}{B})$ . It is clear from the capacity equation that it will increase linearly as number of transmit antenna  $B$  increase. Now if we see on the capacity of a single antenna system that is equal to  $C = \log(1 + P)$ . After looking both capacity equations we come to know that multiple antenna system have much better results.

## 2.5 Diversity Techniques

I can get better capacity and better transmission rate with out increasing the power and bandwidth of the transmitter side by using multiple antennas. If we send two same signals from two different antennas at the same time and both have this information then the chance of getting fading effect at both signals is much lower then that of only one signal. So at receiver all coming signals are combined to get a strongest signal it improves the reliability of transmission. I gave a very simple analysis below.

A communication system with a single antenna has this equation  $i = \sqrt{\rho}sh + n$  where  $h$  represents Rayleigh flat-fading channel coefficient,  $n$  is the noise term with mean equal to zero and variance is one,  $\rho$  is the transmit power. If we see on  $s$  it

satisfies the power constraint  $E|s|^2 = 1$ . Now the receiver has the SNR equal to  $\rho|h|^2$ . As

$h$  is Rayleigh distributed so the probability density function of  $|h|^2$  is

$$p(i) = e^{-i}, \quad i > 0 \quad (2.12)$$

As the signal at the receiver will be less than the transmitted one so here we have the probability equation for SNR is less than a level  $\varepsilon$  is,

$$P(\rho|h|^2 < \varepsilon) = P(|h|^2 < \frac{\varepsilon}{\rho}) = \int_0^{\frac{\varepsilon}{\rho}} e^{-i} di = 1 - e^{-\frac{\varepsilon}{\rho}} \quad (2.13)$$

When the transmitted signal power is high ( $\rho \gg 1$ ),

$$P(\rho|h|^2 < \varepsilon) \approx \frac{\varepsilon}{\rho} \quad (2.14)$$

From above equation it is clear that transmitted power is inversely proportional. Now with this power the system equation for multiple-antenna system is

$$i = \sqrt{\rho} s \mathbf{H} + n \quad (2.15)$$

Where  $E s s^* = 1$ . Here we assume that the transmitted symbol  $s$  are i.i.d and

$$E|s_k|^2 = \frac{1}{B} \quad (2.16)$$

Since  $h_{kj}$  both are independent so the SNR at the receiver is

$$\rho E s \mathbf{H} \mathbf{H}^* s^* = \rho \sum_{k=1}^B \sum_{j=1}^B \sum_{i=1}^B E s_k \overline{s_j} \overline{h_{ik}} h_{jk} = \rho \sum_{k=1}^B E|s_k|^2 \sum_{k=1}^B |h_{ik}|^2 = \frac{\rho}{B} \sum_{i=1}^B \sum_{k=1}^B |h_{ik}|^2 \quad (2.17)$$

Now the probability that how much SNR at the receiver is less than a specific level  $\varepsilon$

$$P\left(\frac{\rho}{B} \sum_{i=1}^B \sum_{k=1}^C |h_{ik}|^2 < \varepsilon\right) = P\left(\sum_{i=1}^B \sum_{k=1}^C |h_{ik}|^2 < \varepsilon \frac{B}{\rho}\right) \quad (2.18)$$



$$P(|h_{11}|^2 < \frac{cB}{\rho}, \dots, |h_{BC}|^2 < \frac{\varepsilon B}{\rho}) \quad (2.19)$$

$$= \prod_{i=1, k=1}^{B, C} P(|h_{ik}|^2 < \frac{\varepsilon B}{\rho}) \quad (2.20)$$

$$= (1 - e^{-\frac{\varepsilon B}{\rho}})^{BC} \quad (2.21)$$

In an other case where the transmitted power is high ( $\rho \gg 1$ ),

$$P(\frac{\rho}{B} \sum_{i=1}^B \sum_{k=1}^C |h_{ik}|^2 < \varepsilon) < (\frac{\varepsilon B}{\rho})^{BC} \quad (2.22)$$

If we see on above equation then we come to know that the probability of error is inversely proportional to  $\rho^{BC}$  so if the number of antennas increases at transmitter side then error will be less than a single antenna system.

Now we have three types of diversity time, frequency and antenna. In time diversity the same information is transmitted after same time slots. In frequency diversity the same information is transmitted in same time but at different frequencies. In last type where we have space diversity in this scheme the antennas are placed physically at different places and the distance between two antennas is equal to equal to some wavelengths. In space diversity there are two schemes in first there are multiple antennas at transmitter side and in second type the number of antennas increased at receiver side at receiver the multiple signals received from all antennas is combined to increased the SNR and eliminate the error chances. At receiver there are some methods to get a strong signal like switching combining, selection combining, equal gain combining, maximum ratio combining and switching combining.

## CHAPTER 3

### Space-Time Block Code

#### 3.1 Introduction

The requirement of better quality wireless systems is going to increase day by day and every one need less error rate and high data rate by using small and light weight mobile sets at different types of environments. It is assumed that the latest system is more efficient and more power full against the hurdles present for wireless systems. The size of mobile is much small to keep in pocket and also the services are in affordable ranges. There are lots of communication systems like line-of-sight microwave, coaxial cable, fiber and satellite transmissions but in all only a wireless system is one that has lot of problems due to time-varying multipath fading [13].

In real life the reduction of BER is much difficult in a multipath-fading channel. If we use some special scheme then to remove the BER from 10 to 20 may need only 1 or 2 dB SNR but in a multipath-fading channel with additive white Gaussian noise (AWGN) the reduction of same BER need more then 10dB SNR. In order to achieve more SNR there is need of more power and bandwidth and it is much difficult to achieve this by using some coding techniques so with out sacrificing some power or some bandwidth we can't achieve required SNR.

Theoretically, there are two major problems in wireless communication system. First, if the behavior of the channel and the signal received at receiver side is known then by using some techniques at transmitter the SNR can be increased by only increasing transmitted power. There is a problem of varying distances between transmitter and

receive and it can't be calculated so the transmitted power can't be so much increased due to radiation power limitations, size and cost of the power amplifier. Second, the transmitters don't have any knowledge about the channel behavior after the reception of the signal only in the systems where the uplink (remote to base) and downlink (base to remote) transmissions are done by using the same frequency for both links. Here the channel behavior is fed back to the transmitter from the receiver but this increases the circuit complexity so much at both ends. As there is not any special link to feed back the required information.

Other techniques are time and frequency, time interleaving also with some corrective codes can improve diversity gain. The same results are obtained from the spread spectrum. Although the time interleaving produced large delays if the channel is slow changing. Similarly spread techniques don't have much effect when the coherence bandwidth of the channel is much faster than the spreading bandwidth and also for the channels, which have small delay spread.

The areas where there is lot of obstacles are present in the way of electromagnetic waves the antenna diversity is the best approach this reduces the effect of multipath fading [14]. In old technology the number of received antennas is increased and by using combining, selection and switching techniques the quality of the signal is improved. But doing this the size, power and cost of the remote unit is much increased. That's why the use of more antennas is only applicable at the transmitter side as one base station is capable to control thousands of remote units so better to have many antennas at base station not at the remote unit. Firstly only one antenna is installed at the base station and found better results so it is preferable to install more antennas.

There are two major techniques to obtain the information from the transmitted signal, minimum mean squared error (MMSE) or a maximum likelihood sequence estimator (MLSE). Second approach is space-time trellis coding [15], in these techniques the information is encoded exactly similar to the transmitter antennas and then decoded by maximum likelihood decoder at receiver in the same manner. This scheme utilizes two techniques first forward error correction (FEC) coding technique and second diversity technique and obtains so much gain. The cost of this technique is much more and it increases exponentially that's why this is not most applicable approach.

In this thesis, I have discussed a simple transmit diversity scheme where the transmitter have two antennas but receiver has only one antenna. There is used a maximal-ratio receiver combining (MRRC) technique in order to obtained diversity at the receiver. For the same way we can construct a system with two transmit and  $C$  receive antenna to obtain  $2C$  diversity by applying some algorithms and some complexity in circuit. There is not any requirement for more bandwidth because we did not do any change in time or frequency domain. With the help of new scheme we can obtain better results in capacity and data rate. As the sensitivity of the system from the fading is decreased so we can use higher level modulation schemes to maintain higher data rates and also makes smaller reuse factor in a cluster to improve the capacity and similarly the coverage area is increased. This diversity technique is much effective in all aspects like to obtain high data rate better quality communication in the presence of multipath fading problem but there is some increase in cost and more signal processing

is done at the base stations. After looking on the feed back improvement in wireless systems in the presence of all fading effects the described scheme is best.

Here I have discussed the classical maximal ratio receive combining scheme then at second stage there is a two branch transmit diversity with having one receive antenna then having two receive antenna and then compare all three. In this chapter, I have also discussed the comparison of BER of new scheme with the old binary phase-shift keying (BPSK) modulation and also compare with MRRC. We come to know from the simulation results that new scheme is much better but the cost is high then the old technology, here quality is most important for the day by day increasing demand if wireless communication systems.

### 3.2 Classical MRRC

Here I have a classical two-branch MRRC system shown in Figure 3.1 the signal  $s_0$  is transmitted from the single antenna system at one time. There are lot of effects will come in this signal like the air-link, fading, addition of noise term some complex multiplicative distortion in shape of magnitude and phase response. There is a channel between the transmit antenna and the zero receive antenna is  $h_0$  and similarly  $h_1$  for the first receive antenna.

$$h_0 = \beta_0 e^{j\theta_0} \quad (3.1)$$

$$h_1 = \beta_1 e^{j\theta_1} \quad (3.2)$$

After passing through the channel some noise term will add and at the receiver side received base-band signals are

$$r_0 = h_0 s_0 + n_0 \quad (3.3)$$

$$r_1 = h_1 s_1 + n_1 \quad (3.4)$$

Where  $n_0$  and  $n_1$  are the noise terms.

Let suppose that  $n_0$  and  $n_1$  are Gaussian then the at receiver we use maximum likelihood decision rule to check which  $s_k$  is received and this is possible only if (iff)

$$d^2(r_0, h_0 s_k) + d^2(r_1, h_1 s_k) \leq d^2(r_0, h_0 s_i) + d^2(r_1, h_1 s_i), \forall i \neq k \quad (3.5)$$

Here  $d^2(u, v)$  represents the squared Euclidean distance between two received signal symbols and these are calculated as:

$$d^2(u, v) = (u - v)(u^* - v^*) \quad (3.6)$$

At receiver the classical two-branch MRRC will give this:

$$\begin{aligned} s_0 &= h_0^* r_0 + h_1^* r_1 \\ &= h_0^* (h_0 s_0 + n_0) + h_1^* (h_1 s_0 + n_1) \\ &= (\beta_0^2 + \beta_1^2) s_0 + h_0^* n_0 + h_1^* n_1 \end{aligned} \quad (3.7)$$

By putting (3.6) and (3.7) in (3.5) I get this expression

$$\begin{aligned} &(\beta_0^2 + \beta_1^2) |s_k|^2 - s_0 s_k^* - s_0^* s_k \\ &\leq (\beta_0^2 + \beta_1^2) |s_i|^2 - s_0 s_i^* - s_0^* s_i, \quad \forall i \neq k \end{aligned} \quad (3.8)$$

By solving and simplifying

$$\begin{aligned} &(\beta_0^2 + \beta_1^2 - 1) |s_k|^2 - d^2(\bar{s}_0, s_k) \\ &\leq (\beta_0^2 + \beta_1^2 - 1) |s_i|^2 - d^2(\bar{s}_0, s_i), \quad \forall i \neq k \end{aligned} \quad (3.9)$$

If I have a PSK signals with same energy symbols

$$|s_k|^2 = |s_i|^2 = E_s, \quad \forall i \neq k \quad (3.10)$$

$E_s$  is the transmitted energy. So by solving equation (3.9) for a PSK signal that will give

$$d^2(\bar{s}_0, s_k) \leq d^2(\bar{s}_0, s_i) \quad \forall i \neq k \quad (3.11)$$

It is clearly explain in fig below that how a MRRC scheme will produce the signal  $\bar{s}_0$ , which is exactly close to  $\tilde{s}_0$  or we can say it is the maximum likelihood estimate is of  $s_0$ .

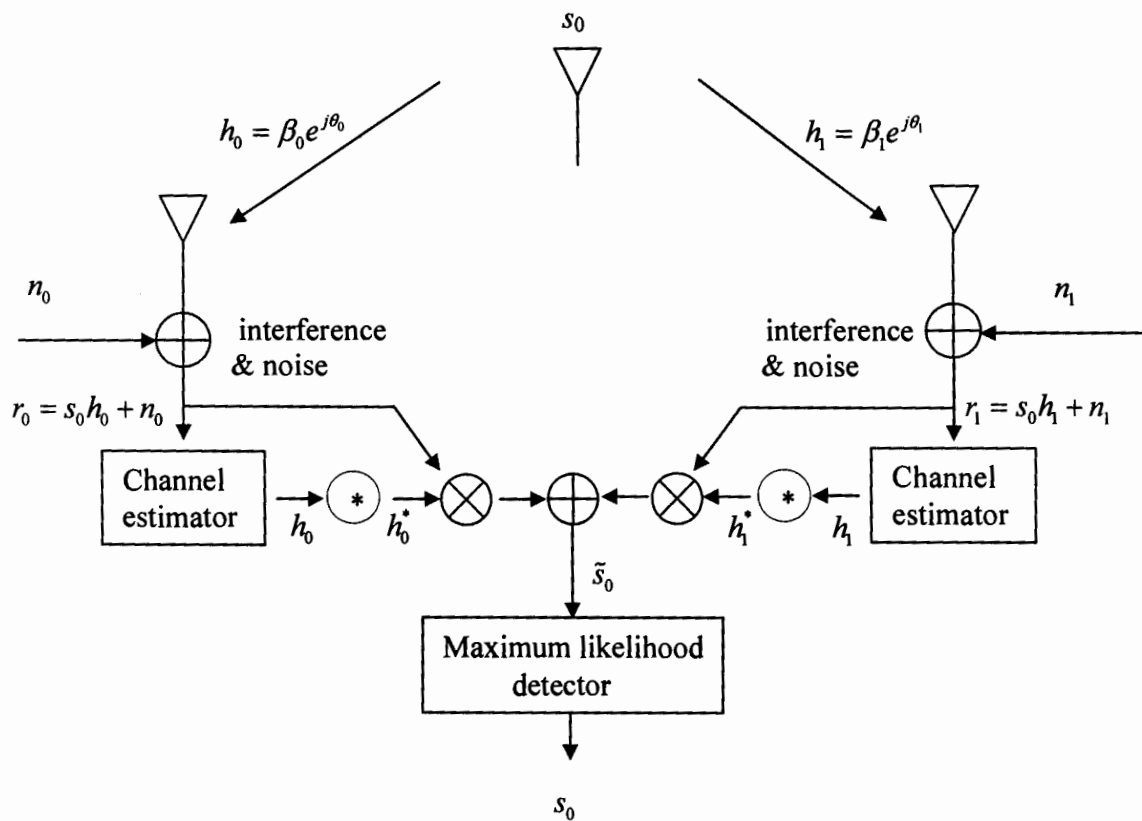


Figure 3.1 Two-branch MRRC

### 3.3 New transmit diversity scheme

In Figure 3.2 there is presented Two-Branch Transmit Diversity with a single antenna at the receiver side. The scheme performing three functions collectively at transmitter and receiver these are:

- At the transmitter first the signal is encoded then symbols are transmitted from transmitter.
- Only combining is performed at the receiver side.
- A maximum likelihood criterion is used to decide the constructed symbol.

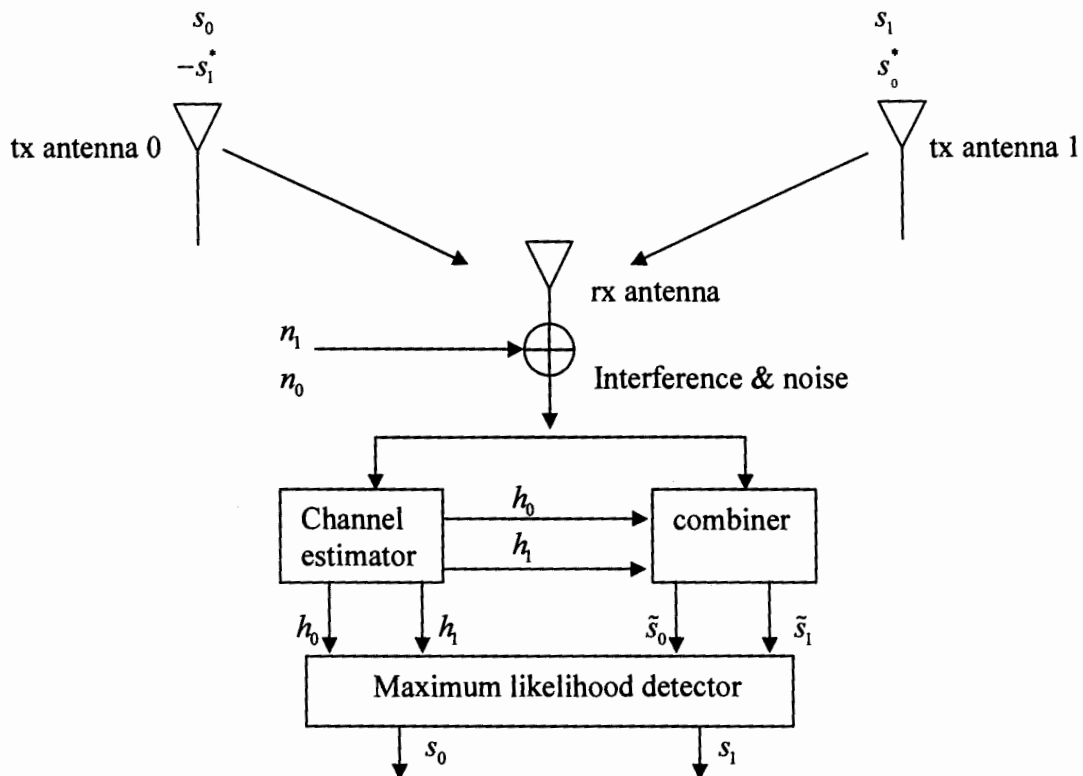


Figure 3.2 Two-branch transmit diversity scheme with one receiver



Here I have two transmit antennas so at the same time two symbols are transmitted,  $s_0$  from zero and  $s_1$  from first antenna. In second time slot ( $-s_1^*$ ) for zero antenna and  $s_0$  is transmitted from first antenna. The encoding is done in time and space. We can do this by using different frequency instead of time then this will called the space and frequency encoding. The time and space encoding is shown in Table 3.1 below.

Table 3.1 Encoding and Transmission Sequence

Antenna \ Time	Antenna0	Antenna1
Time t	$s_0$	$s_1$
Time t+T	$-s_1^*$	$s_0^*$

The channel coefficient for transmit antenna zero is  $h_0(t)$  and for first antenna  $h_1(t)$  here t represents the time at which the signal is transmitted. Let us consider that the fading is constant for the both time slots then

$$\begin{aligned} h_0(t) &= h_0(t+T) = h_0 = \beta_0 e^{j\theta_0} \\ h_1(t) &= h_1(t+T) = h_1 = \beta_1 e^{j\theta_1} \end{aligned} \quad (3.12)$$

T represents the total symbol duration. Now the received signal at time t and after T is expressed as

$$\begin{aligned} r_0 &= r(t) = h_0 s_0 + h_1 s_1 + n_0 \\ r_1 &= r(t+T) = -h_0 s_1^* + h_1 s_0^* + n_1 \end{aligned} \quad (3.13)$$

There are two signals  $r_0$  and  $r_1$  with two noise terms  $n_0$  and  $n_1$  at the receiver one signal reaches at time t and second at t+T. The output of combiner is fed to maximum

likelihood detector and the output of the combiner is given below and the steps are shown in Figure 2.

$$\begin{aligned}\tilde{s}_0 &= h_0^* r_0 + h_1^* r_1 \\ \tilde{s}_1 &= h_1^* r_0 - h_0^* r_1\end{aligned}\tag{3.14}$$

By looking on above equation I come to know that the above results are so much different from the old MRRC results present in equation (3.7) now by putting (3.12) and (3.13) in (3.14),

$$\begin{aligned}\tilde{s}_0 &= (\beta_0^2 + \beta_1^2) s_0 + h_0^* n_0 + h_1^* n_1 \\ \tilde{s}_1 &= (\beta_0^2 + \beta_1^2) s_1 - h_0^* n_1 + h_1^* n_0\end{aligned}\tag{3.15}$$

Maximum likelihood detector received these two equations and then apply decision rule expressed in (3.7) or (3.9) on above equation to get the transmitted signal  $s_0$  and  $s_1$  as I use PSK signals. If we see the results obtained from the two-branch MRRC then we conclude that the above results are similar to that one except there is a phase difference in noise terms, which don't have any part in SNR. So, I conclude that the new scheme have the same results like the two-branch MRRC scheme. In future there could be required to obtain a higher order diversity of  $2C$  where  $C$  is the number of received antennas, which can be possible. To understand this approach I have discussed the case where I have used two transmit and also two receive antennas. This will give a generalized overview to use any number of antennas. The channel coefficient for the two antennas system is given below in Table 3.2.

Table 3.2 Channel coefficients for the two antennas system

Channel \ Tx antenna	Rx Antenna0	Rx Antenna1
Tx antenna 0	$h_0$	$h_2$
Tx antenna 1	$h_1$	$h_3$

In Figure 3.3 there is a new scheme implementation with two transmitter and two receiver antennas. If we see in depth then we come to know that it is similar to that of a single antenna system like encoding and decoding both are similar. Here Table 3.2 represents the channel coefficients present between the transmitter and receiver antennas and the last Table 3.3 has the received signal at two receive antennas.

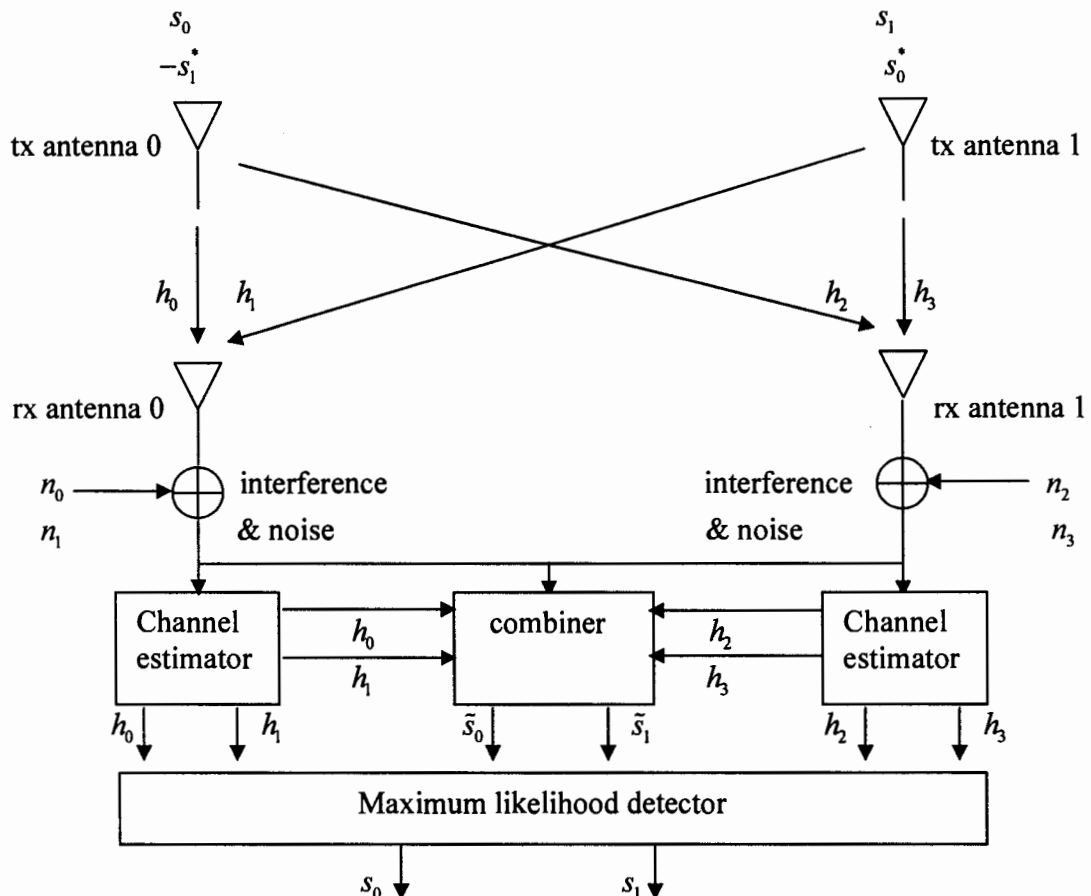


Figure 3.3 Two-branch transmit diversity scheme with two receiver

The received signal at two receive antennas is explained in Table 3.3.

Table 3.3 Received signal at two antennas

Receiver \ Time	Rx Antenna0	Rx Antenna1
Time t	$r_0$	$r_2$
Time t+T	$r_1$	$r_3$

At receiver I have these equations;

$$\begin{aligned}
 r_0 &= h_0 s_0 + h_1 s_1 + n_0 \\
 r_1 &= -h_0 s_1 + h_1 s_0 + n_1 \\
 r_2 &= h_2 s_0 + h_3 s_1 + n_2 \\
 r_3 &= -h_2 s_1 + h_3 s_0 + n_3
 \end{aligned} \tag{3.16}$$

$n_0, n_1, n_2$  and  $n_3$  all are the noise terms added from the channel in each received signal.

Two signals are received at maximum likelihood detector from the above-discussed combiner and the combiner will produce

$$\begin{aligned}
 \tilde{s}_0 &= h_0^* r_0 + h_1^* r_1 + h_2^* r_2 + h_3^* r_3 \\
 \tilde{s}_1 &= h_1^* r_0 - h_0^* r_1 + h_3^* r_2 - h_2^* r_3
 \end{aligned} \tag{3.17}$$

By solving and arranging all equations I get

$$\begin{aligned}
 \tilde{s}_0 &= (\beta_0^2 + \beta_1^2 + \beta_2^2 + \beta_3^2) s_0 + h_0^* n_0 \\
 &\quad + h_1^* n_1 + h_2^* n_2 + h_3^* n_3 \\
 \tilde{s}_1 &= (\beta_0^2 + \beta_1^2 + \beta_2^2 + \beta_3^2) s_1 - h_0^* n_1 \\
 &\quad + h_1^* n_0 - h_2^* n_3 + h_3^* n_2
 \end{aligned} \tag{3.18}$$

Now after all necessary calculations the combiner output for maximum likelihood decoder is equation (3.19) here decision criteria rule is applied to retrieve back the two transmitted signals  $s_0$  and  $s_1$  as I have used PSK signaling.

$$\begin{aligned}
& (\beta_0^2 + \beta_1^2 + \beta_2^2 + \beta_3^2 - 1)|s_k|^2 + d^2(\bar{s}_0, s_k) \\
& \leq (\beta_0^2 + \beta_1^2 + \beta_2^2 + \beta_3^2 - 1)|s_i|^2 + d^2(\bar{s}_0, s_i)
\end{aligned} \tag{3.19}$$

This above equation will produce  $s_0$  if and only iff

$$d^2(\bar{s}_0, s_k) \leq d^2(\bar{s}_0, s_i) \quad \forall i \neq k \tag{3.20}$$

In second step the maximum likelihood decoder will produce this equation.

$$\begin{aligned}
& (\beta_0^2 + \beta_1^2 + \beta_2^2 + \beta_3^2 - 1)|s_k|^2 + d^2(\bar{s}_1, s_k) \\
& \leq (\beta_0^2 + \beta_1^2 + \beta_2^2 + \beta_3^2 - 1)|s_i|^2 + d^2(\bar{s}_1, s_i)
\end{aligned} \tag{3.21}$$

Now apply decision rule on above equation to get  $s_1$  as PSK modulation is used. It will retrieve if and only iff

$$d^2(\bar{s}_1, s_k) \leq d^2(\bar{s}_1, s_i) \quad \forall i \neq k \tag{3.22}$$

After looking on equation (3.18) I fund that it is the same result that I can get from a four branches MRRC. The output of the new transmit diversity scheme for two transmit and two receive antennas is similar to that one which is much interesting as in order to obtain the same result from old technology I have to use fore branch MRRC.

From all above calculation I have also seen that the result of a two combining receives antenna is just the addition of two isolated antennas combiner output. So here I can conclude that in order to get a diversity of 2C with two transmit and C receives antennas there is needed to just add the combiner output of each receive antenna. It will give the same diversity order as 2C branch MRRC. We can say that this new scheme will double the diversity order if we only use two transmit antenna instead of one and the diversity will be the product of these two transmit and C receive antennas.

## CHAPTER 4

### Analysis of Alamouti Space-Time

#### Block Code with its extension

##### 4. I Introduction

The diversity techniques are used extensively for the improvement in the wireless fading channels. This is only possible due to easy and cheap implementation of multiple antennas at the base station. The STBC [16], [17] is more attractive towards transmit diversity than receive diversity. A simple decoding is used in maximum likelihood decoding algorithm at the decoding end to obtain full diversity. Orthogonal property is used for designing new space-time block codes. Using orthogonal property I have analyzed the code, which achieves half of the full transmission rate at any number of transmission antennas. I have also analyzed code, which provides  $3/4$  of the full transmission rate for the special cases of three, and four transmit antennas.

In this work, the receiver decodes pairs of transmitted symbol in analyzing STBC that have rate one and provide half of the maximum possible diversity. The codes designed in [17] utilize the orthogonal property. It means that the transmission code matrix columns are divided into groups and these groups are orthogonal to each other. Using orthogonal property the recovery of transmitted symbol is effectively so simple. I have analyzed, the structures having normal code matrix, and seen that the separation of transmitted symbols from each other is much difficult.

In a transmission matrix only groups are orthogonal to each other instead of columns. Such a structure is called Extended Alamouti design. It is seen that by using an Extended Alamouti design pairs of transmitted symbols can be decoded separately. By using these structures the higher transmission rate is possible while sacrificing the full diversity. Simulation shows that full rate is more important for very low SNRs and high bit error rates (BERs); however, full diversity is important for high SNRs and low BERs. In fact a code having full diversity is more important than increasing the SNR.

## 4.2 Space Time Block Codes

In wireless communication system the space-time block coding [18] provides maximum achievable diversity for multiple transmit antennas. Here  $B \times C$  is a complex transmission matrix  $\mathbf{U}$  where  $B$  is the number of time slots and  $C$  represents the number of transmit antennas. The matrix is linear combinations of indeterminate  $i_1, i_2, \dots, i_k$  and their conjugates. So

$$\mathbf{U}^* \mathbf{U} = (|i_1|^2 + |i_2|^2 + \dots + |i_k|^2) \mathbf{I} \quad (4.1)$$

Where  $\mathbf{U}^*$  is the Hermitian of  $\mathbf{U}$  and  $\mathbf{I}$  is the  $V \times V$  identity matrix. Where  $R$  is the rate of  $\mathbf{U}$  which is  $R = Q/B$ .  $Q$  constellation symbols are transmitted in  $B$  time slots. It has been proved in [19] that the rate of a full-diversity code is  $R \leq 1$ .

Let  $Y$  is a signal constellation of size  $2^d$ .  $Qd$  bits arrive at the encoder at same time, where  $Q$  is the variables present in the transmission matrix. These  $Qd$  bits choose  $Q$  constellation symbols  $s_1, s_2, \dots, s_k$ .

The encoder changes all  $s_k$  with  $i_k$  in the transmission matrix  $\mathbf{U}$  for all  $1 \leq k \leq K$ . The resulting matrix is denoted by  $\mathbf{Z}$ . Now at time  $b, b = 1, 2, \dots, B$  the  $q$ -

th element of the r-th row of  $\mathbf{Z}$ ,  $\mathbf{Z}_{r,q}$ , is transmitted from  $c = 1, 2, \dots, C$  antennas. I insist that all transmissions are continuous and all signals have the same time duration. Since all elements of  $\mathbf{U}$  are linear combinations of  $i_1, i_2, \dots, i_K$  and their conjugates.

I have considered a system with  $C$  transmit antenna and  $R$  receive antennas at the remote. Channel is considered to be flat fading having path gain, from  $n$  transmit to  $r$  receive antenna,  $\beta_{n,r}$ . The variance of path gain of the transmission channel is 0.5 and channel properties are considered to be quasi-static it means the channel path gains are constant for the same time slot  $B$  and vary for next time slot. The received signal at time  $b$  on  $r$ -th receive antenna from  $C$  transmit antennas is

$$r_{b,r} = \sum_{n=1}^C \beta_{n,r} \mathbf{Z}_{n,r} + \eta_{b,r} \quad (4.2)$$

Where noise term  $\eta_{b,r}$  is zero-mean complex Gaussian random variable.

The symbols are transmitted from each antenna with normalized energy which is equal to be 1, which make the received signal power on each receive antenna is  $C$  and the signal to noise ratio is SNR. If channel behavior is known at receiver then the received decision metric is

$$\sum_{r=1}^R \sum_{b=1}^B \left| r_{b,r} - \sum_{c=1}^C \beta_{c,r} \mathbf{U}_{bc} \right|^2 \quad (4.3)$$

Give decision in favor of the constellation symbols  $s_1, s_2, \dots, s_K$  that will give the minimum answer for the above equation, which results in a maximum likelihood decoding. If the above equation (4.3) is expanded, all cross terms having  $\beta_{p,r} \beta_{q,r}^*$ ,  $1 \leq p \neq q \leq C$  will cancel each other and the  $p$ -th and  $q$ -th columns of  $\mathbf{U}$  are



orthogonal. If we analyze the sum term then we come to know that it has  $Q$  components in the form  $i_k$ ,  $q = 1, 2, \dots, Q$ .

### 4.3 Extended Alamouti Space Time Block Codes

Here we have some examples of STBC having full-rate and full-diversity, which is

$$Y_{12} = \begin{pmatrix} i_1 & i_2 \\ -i_2^* & i_1^* \end{pmatrix} \quad (4.4)$$

$$Y = \begin{pmatrix} Y_{12} & Y_{34} \\ -Y_{34}^* & Y_{12} \end{pmatrix} \quad (4.5)$$

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ -i_2^* & i_1^* & -i_4^* & i_3^* \\ -i_3^* & -i_4^* & i_1^* & i_2^* \\ i_4 & -i_3 & -i_2 & i_1 \end{pmatrix} \quad (4.6)$$

constructed by using Alamouti scheme. Here the subscript 12 used with Y represent the indeterminate  $i_1$  and  $i_2$  which are present in the transmission matrix. Now, there is a special case of Alamauti scheme in which  $C = B = Q = 4$ .

If we find the rank of matrix  $Y(s_1 - \bar{s}_1, s_2 - \bar{s}_2, s_3 - \bar{s}_3, s_4 - \bar{s}_4)$  this matrix is built by putting  $s_i - \bar{s}_i$  in Y instead of  $i_i$ , are 2. As the rank is two and there are R receivers so achieved diversity is 2R with rate one. If  $W_i$  represents a column of matrix Y, where  $i = 1, 2, 3, 4$  then from the orthogonal behavior the result will be zero.

$$\langle W_1, W_2 \rangle = \langle W_1, W_3 \rangle = \langle W_2, W_4 \rangle = \langle W_3, W_4 \rangle = 0 \quad (4.7)$$

The dot product of two vectors  $W_i$  and  $W_j$  is

$$\langle W_i, W_j \rangle = \sum_{l=1}^4 (W_i)_l (W_j)_l^* \quad (4.8)$$

As two vectors are orthogonal, so their subspaces will also be orthogonal to each other are orthogonal. The maximum likelihood decision matrix can also be calculated by using the vectors orthogonal property  $f_{14}(i_1, i_4) + f_{23}(i_2, i_3)$ , where first term  $f_{14}$  is independent from  $i_2, i_3$  and second term  $f_{23}$  is independent from  $i_1, i_4$ . In order to reduce the complexity and to obtain the similar results like equation (4.3) we need to solve above two terms independently and the performance will remain the same. By doing some mathematical calculation on equation (4.3) we get the two formulas for both terms  $f_{14}(\cdot)$  and  $f_{23}(\cdot)$ .

$$\begin{aligned} f_{23}(i_2, i_3) = & \sum_{r=1}^R \left[ \left( \sum_{c=1}^4 |\beta_{c,m}|^2 \right) (|i_2|^2 + |i_3|^2) + \right. \\ & 2 \operatorname{Re} \{ (-\beta_{2,r} r_{1,r}^* + \beta_{1,r} r_{2,r}^* - \beta_{4,r} r_{3,r}^* + \beta_{3,r} r_{4,r}^*) i_2 + \\ & \quad (-\beta_{3,r} r_{1,r}^* - \beta_{4,r} r_{2,r}^* + \beta_{1,r} r_{3,r}^* + \beta_{2,r} r_{4,r}^*) i_3 + \\ & \quad \left. (\beta_{2,r} \beta_{3,r}^* - \beta_{1,r} \beta_{4,r}^* - \beta_{1,r} \beta_{4,r}^* + \beta_{2,r} \beta_{3,r}^*) i_2 i_3^* \} \right] \quad (4.9) \end{aligned}$$

$$\begin{aligned} f_{14}(i_1, i_4) = & \sum_{r=1}^R \left[ \left( \sum_{c=1}^4 |\beta_{c,r}|^2 \right) (|i_1|^2 + |i_4|^2) + \right. \\ & 2 \operatorname{Re} \{ (-\beta_{1,r} r_{1,r}^* - \beta_{2,r} r_{2,r}^* - \beta_{3,r} r_{3,r}^* - \beta_{4,r} r_{4,r}^*) i_1 + \\ & \quad (-\beta_{4,r} r_{1,r}^* + \beta_{3,r} r_{2,r}^* + \beta_{2,r} r_{3,r}^* - \beta_{1,r} r_{4,r}^*) i_4 + \\ & \quad \left. (\beta_{1,r} \beta_{4,r}^* - \beta_{2,r} \beta_{3,r}^* - \beta_{2,r} \beta_{3,r}^* + \beta_{1,r} \beta_{4,r}^*) i_1 i_4^* \} \right] \quad (4.10) \end{aligned}$$

Where  $\operatorname{Re}\{\beta\}$  represents real part of  $\beta$ . It is much difficult to decode the new received signal because this code is in pair form of symbol [20]. Here are some examples having similar behaviors are given below:

$$\begin{pmatrix} Y_{12} & Y_{34} \\ -Y_{34} & Y_{12} \end{pmatrix}, \begin{pmatrix} Y_{12} & Y_{34} \\ Y_{34} & -Y_{12} \end{pmatrix}, \begin{pmatrix} Y_{12} & Y_{34} \\ Y_{34} & -Y_{12}^* \end{pmatrix} \quad (4.11)$$

The main idea behind the matrix  $Y$  in equation (4.6) is to construct a  $4 \times 4$  matrix by using  $2 \times 2$  matrices but with out disturbing the rate and similarly the same approach is used to build an  $(8 \times 8)$  matrix by using the  $(4 \times 4)$  matrix and keeping the rate fixed.

$$\begin{pmatrix} i_1 & i_2 & i_3 & 0 & i_4 & i_5 & i_6 & 0 \\ -i_2^* & i_1^* & 0 & -i_3 & i_5^* & -i_4^* & 0 & i_6 \\ i_3^* & 0 & -i_1^* & -i_2 & -i_6^* & 0 & i_4^* & i_5 \\ 0 & -i_3^* & i_2^* & -i_1 & 0 & i_6^* & -i_5^* & i_4 \\ -i_4 & -i_5 & -i_6 & 0 & i_1 & i_2 & i_3 & 0 \\ -i_5^* & i_4^* & 0 & i_6 & -i_2^* & i_1^* & 0 & i_3 \\ i_6^* & 0 & -i_4^* & i_5 & i_3^* & 0 & -i_1^* & i_2 \\ 0 & i_6^* & -i_5^* & -i_4 & 0 & i_3^* & -i_2^* & -i_1 \end{pmatrix} \quad (4.12)$$

For the above  $(8 \times 8)$  code we have some orthogonal columns like  $W_i$ ,  $i = 1, 2, \dots, 8$ , as

$$\left[ \begin{array}{l} \langle W_1, W_i \rangle = 0, i \neq 5, \quad \langle W_2, W_i \rangle = 0, i \neq 6, \\ \langle W_3, W_i \rangle = 0, i \neq 7, \quad \langle W_4, W_i \rangle = 0, i \neq 8, \\ \langle W_5, W_i \rangle = 0, i \neq 1, \quad \langle W_6, W_i \rangle = 0, i \neq 2, \\ \langle W_7, W_i \rangle = 0, i \neq 3, \quad \langle W_8, W_i \rangle = 0, i \neq 4, \end{array} \right] \quad (4.13)$$

The similar results like equation (4.3) can be calculated by solving the sum of three independent terms  $f_{14}(i_1, i_4) + f_{25}(i_2, i_5) + f_{36}(i_3, i_6)$  and but the decoding is little bit difficult due to pairs of constellation symbols.

#### 4.4 Generalized Alamouti STBC

To reach at a point where we can construct any number of transmission matrix we start from well-known Alamouti scheme which is for  $c_T = 2$  transmit antennas

$$\mathbf{U}_2(i_1, i_2) = \begin{pmatrix} i_1 & i_2 \\ i_2^* & -i_1^* \end{pmatrix} \quad (4.14)$$

Here is the generalized formula for construction of  $c_T = 2^c$  ( $c_T \geq 4$ )

$$\mathbf{U}_{c_T}(\{i_j\}_{j=1}^{c_T}) = \begin{pmatrix} \mathbf{U}_{\frac{c_T}{2}}(\{i_j\}_{j=1}^{\frac{c_T}{2}}) & \mathbf{U}_{\frac{c_T}{2}}(\{i_j\}_{j=\frac{c_T}{2}+1}^{c_T}) \\ \mathbf{U}_{\frac{c_T}{2}}(\{i_j\}_{j=\frac{c_T}{2}+1}^{c_T}) \Theta_{c_T} & -\mathbf{U}_{\frac{c_T}{2}}(\{i_j\}_{j=1}^{\frac{c_T}{2}}) \Theta_{c_T} \end{pmatrix} \quad (4.15)$$

Here  $\{i_j\}_{j=1}^{c_T} = i_1, i_2, \dots, i_{c_T}$  and the diagonal  $\left[ \frac{c_T}{2} \times \frac{c_T}{2} \right]$  matrix  $\Theta_{c_T}$  is given by

$$\Theta_{c_T} = \text{diag} \left( \left\{ (-1)^{j-1} \right\}_{j=1}^{\frac{c_T}{2}} \right) \quad (4.16)$$

To analyzed the above generalized formula here is an example having  $c_T = 4$  transmit antennas I have

$$\mathbf{U}_4(\{i_j\}_{j=1}^4) = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ i_2^* & -i_1^* & i_4^* & -i_3^* \\ i_3 & -i_4 & -i_1 & i_2 \\ i_4^* & i_3^* & -i_2^* & -i_1^* \end{pmatrix} \quad (4.17)$$

In this work, I have use the Alamouti scheme as the building block in order to construct the rate one Extended Alamouti code. Similarly, I can build a matrix with dimension  $8 \times 8$ . However, it is also possible to construct Extended Alamouti with rates lower than one.

## 4.5 Simulation results and comparison table

In this section, I have provided simulation results of Extended Alamouti code and compare them with the results of Alamouti scheme and a NON-linear STBC [21]. In order to simulate my all results I have used different number of transmit and receive antennas. Since the code having full-rate, full diversity exists only for real signal constellations [22], there is advantage in using the Extended Alamouti code of BPSK which results in the transmission of 1 bit/s/Hz. Figure 4.1 provides simulation results for the transmission of 2 bits/s/Hz using two and four transmit antennas but used only one receive antenna for NON-linear STBC, Alamouti and Extended Alamouti. I have used the two modulation schemes, 4-PSK scheme give rate one and 8-PSK-scheme give half rate. Figure 4.2 provide simulation results with same number of transmit antennas but it has two antennas at receiver side. In Figure 4.2, the rate one code and the un-coded system use 8-PSK. Figure 4.3 provides simulation results with two and four transmit antennas with four receive antenna. In Figure 4.4, 4.5 and 4.6 I have used four and eight transmit antennas and one, two and four receive antennas.

After getting simulation curves we come to know that full rate transmission is better choice at low SNRs and high BERs while full diversity is better when we have high SNRs and low BER. The reason behind this is that the degree of diversity dictates the slope of the BER-SNR curve. Therefore, although a rate one extended Alamouti code starts from a better point in the BER-SNR plane, a code with full-diversity benefits more from increasing the SNR. Therefore, the BER-SNR curve of the full-diversity scheme passes the curve for the new code at some moderate SNR.

Also, note that the receiver of the full-diversity codes can decode the symbols one by one while the decoding for the new rate one Extended Alamouti code is done for pairs of symbols. It seems very clear that decoding for new scheme is less than the old one and the encoding techniques are similar for both codes. Extended Alamouti code for 8 transmit antenna is an order different from that of 4 transmit antennas. Similarly, 4 transmit antennas are better than that of 2 in almost same fashion. But the difference is more significant in higher SNR regions. The computational comparison Table below provides the detail of additions, subtractions and the errors in all the codes.

Table 4.1 Computational Comparison of Non Linear, Alamouti and Extended Alamouti Schemes

Codes	Addition	Multiplication	Error
Alamouti 2x1	14	18	2
Alamouti 2x2	14	18	1
Extended Alamouti 4x1	30	62	6
Extended Alamouti 4x2	30	62	4
Extended Alamouti 4x4	30	62	2
Non Linear 2x1	08	26	8
Non Linear 2x2	08	26	4
Non Linear 4x1	17	35	12
Non Linear 4x2	17	35	7
Non Linear 4x4	17	35	5
Non Linear 8x1	36	84	26
Non Linear 8x4	36	84	20
Extended Alamouti 8x1	48	76	13
Extended Alamouti 8x4	48	76	7

In above table  $A \times B$  represents the number of antennas. A represents number of transmit antennas and B represents the number of receive antennas.

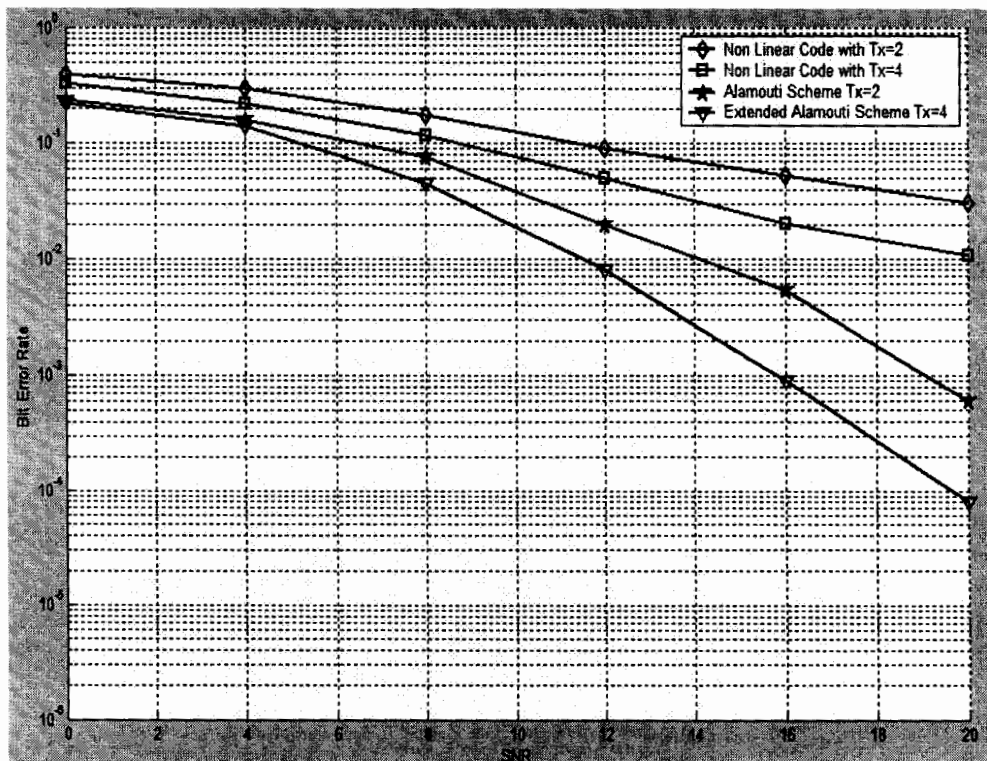


Figure 4.1 Bit error probabilities vs. SNR for two, four transmit and only one receive antenna

In Figure 4.1 I have used four transmit antennas for Extended Alamouti, Non Linear STBC scheme and two transmit antenna for Alamouti, Non Linear STBC scheme but there is only one antenna at receiver side for all cases. In the low SNR region, all schemes perform same for two and four transmit antennas as SNR increases there comes little improvement in data rate and at 8db SNR the Extended Alamouti scheme shows much better results than all the other schemes with only one receive antenna. There is a significant improvement in Extended and simple Alamouti schemes after 16db SNR.

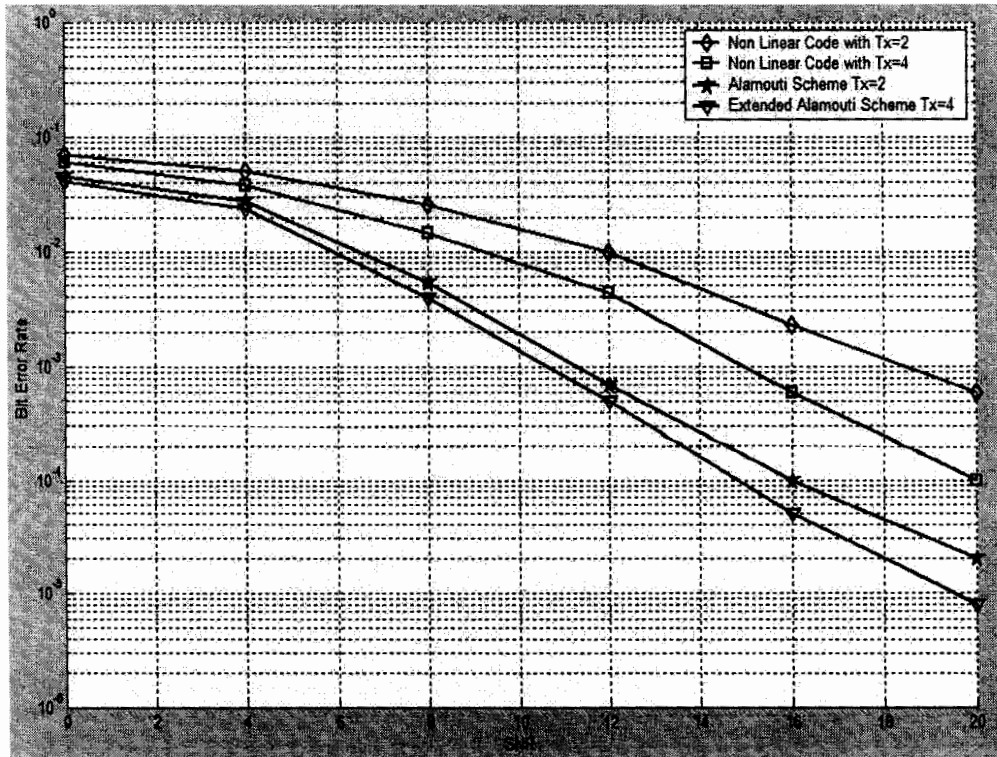


Figure 4.2 Bit error probabilities vs. SNR for two, four transmit and two receive antennas

In Figure 4.2 I have used four transmit antennas for Extended Alamouti, Non Linear STBC scheme and two transmit antenna for Alamouti, Non Linear STBC scheme and there are two antennas at receiver side for all cases. By increasing the number of receive antenna the BER decreases from low SNR and better results come after 8db SNR, which is much better, then the first scheme in which I have used only one receive antenna. By increasing the SNR the BER going significantly down and it can be seen from graph that after increasing 4db in SNR the improvement in data rate is going up ward by a factor. Although at 16db and at 20db the results of Non Linear STBC also have good results but Extended Alamouti scheme shows best results.



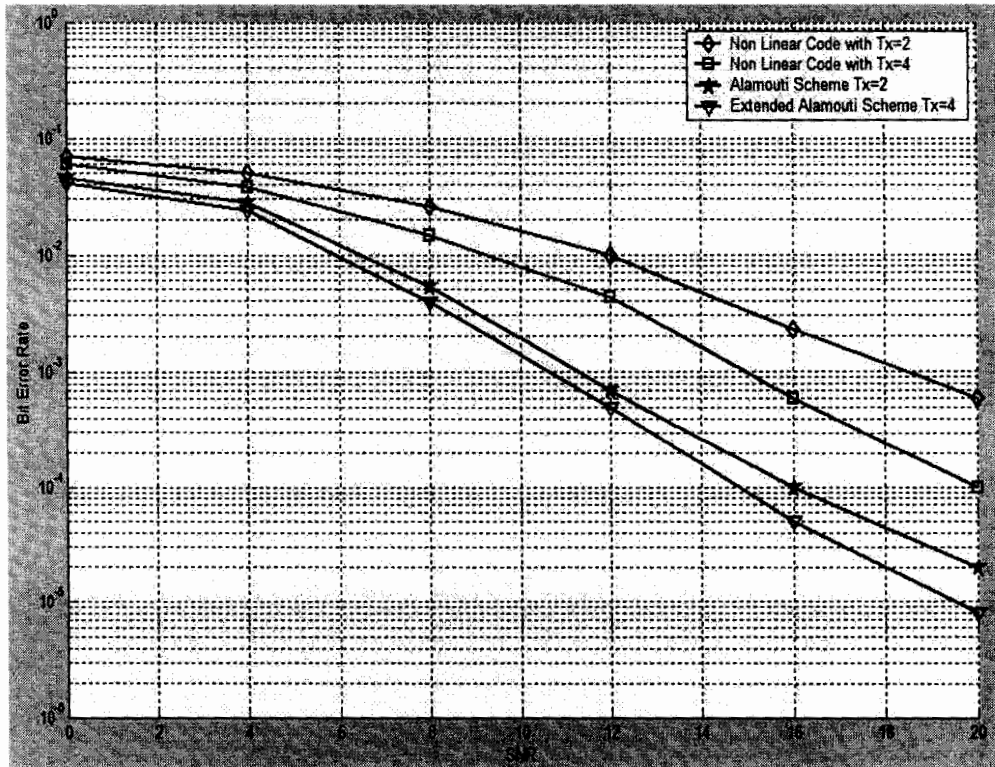


Figure 4.3 Bit error probabilities vs. SNR for two, four transmit and four receive antennas

In Figure 4.3 I have used four transmit antennas for Extended Alamouti, Non Linear STBC scheme and two transmit antenna for Alamouti, Non Linear STBC scheme and there are four antennas at receiver side for all cases. By increasing the number of receive antennas the BER decreases from very low SNR and better results come after 4db SNR, which is much better, then the two cases in which I have used one and two receive antennas. By increasing the SNR the BER going significantly down and it can be seen from graph that after increasing 4db in SNR the improvement in data rate is going upward by a factor. Although on 12db and 16db the results of Non Linear STBC also have good results but Extended Alamouti scheme provide significant improvement.

With four transmit and four receive antennas for both schemes there is much difference between BER at same SNR and if we see at 12db SNR the improvement is equal to one decay and at for eight antennas at same SNR the improvement is more than one decay. Similarly the BER at 16db for Non Linear scheme is 0.0006 for Non Linear STBC and 0.00005 for Extended Alamouti scheme. Similarly the BER at 20db for Non Linear scheme is 0.0001 and 0.000008 for Extended Alamouti scheme which is so much better.

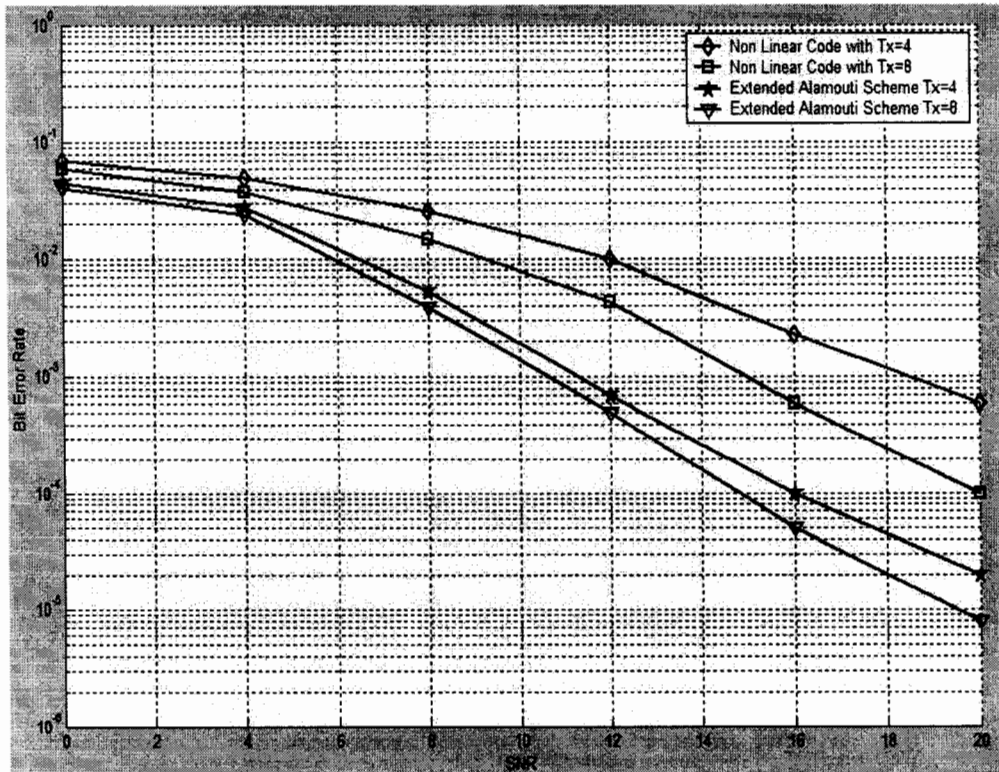


Figure 4.4 Bit error probabilities vs. SNR for four, eight transmit and only One receive antenna

In Figure 4.4 I have used four and eight transmit antennas for Extended Alamouti, Non Linear STBC scheme and only one receive antenna. In the low SNR region below 4db all gives almost same results but after this BER start decreasing. By increasing SNR there comes little improvement in data rate after 8db SNR the Extended Alamouti scheme shows much better results than Non Linear STBC scheme.

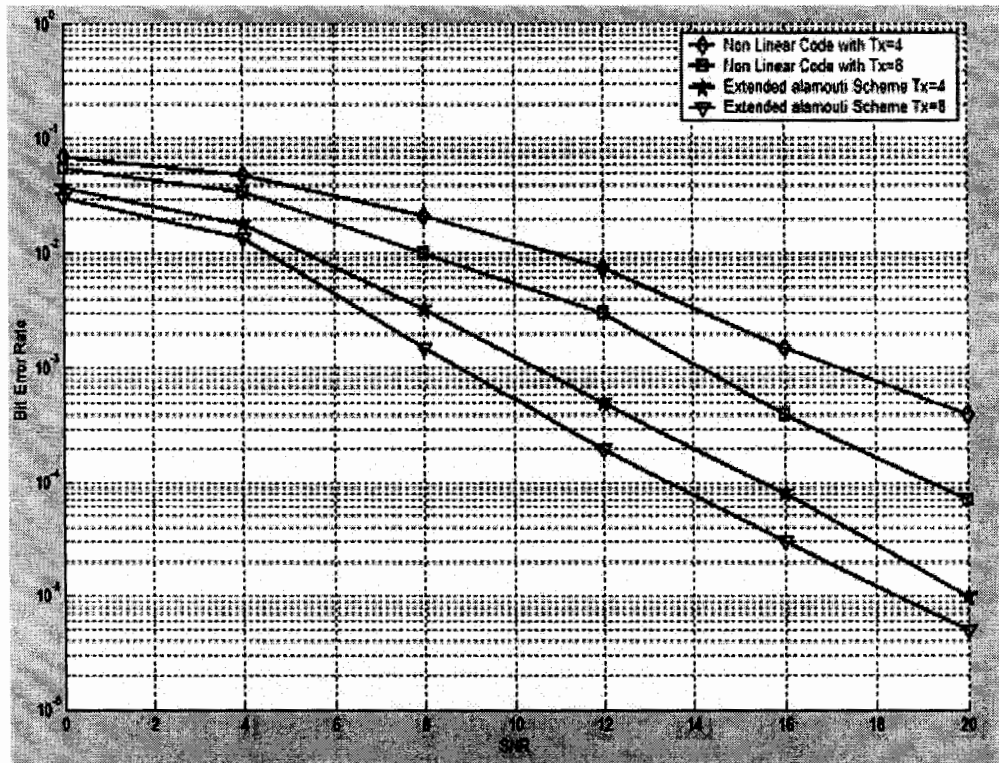


Figure 4.5 Bit error probabilities vs. SNR for four, eight transmit and two receive antenna

In Figure 4.5 I have used four and eight transmit antennas for Extended Alamouti, Non Linear STBC scheme and two receive antennas. By increasing the number of receive antenna the BER decreases from low SNR and better results come after 4db SNR, which is much better, then the first scheme in which I have used only one receive antenna. By increasing the SNR the BER going significantly down and it can be seen from graph that after increasing 4db in SNR the improvement in data rate is going up ward by a factor. Although at 16db and at 20db the results of Non Linear STBC also have good results but in Extended Alamouti scheme the BER decreases sharply.

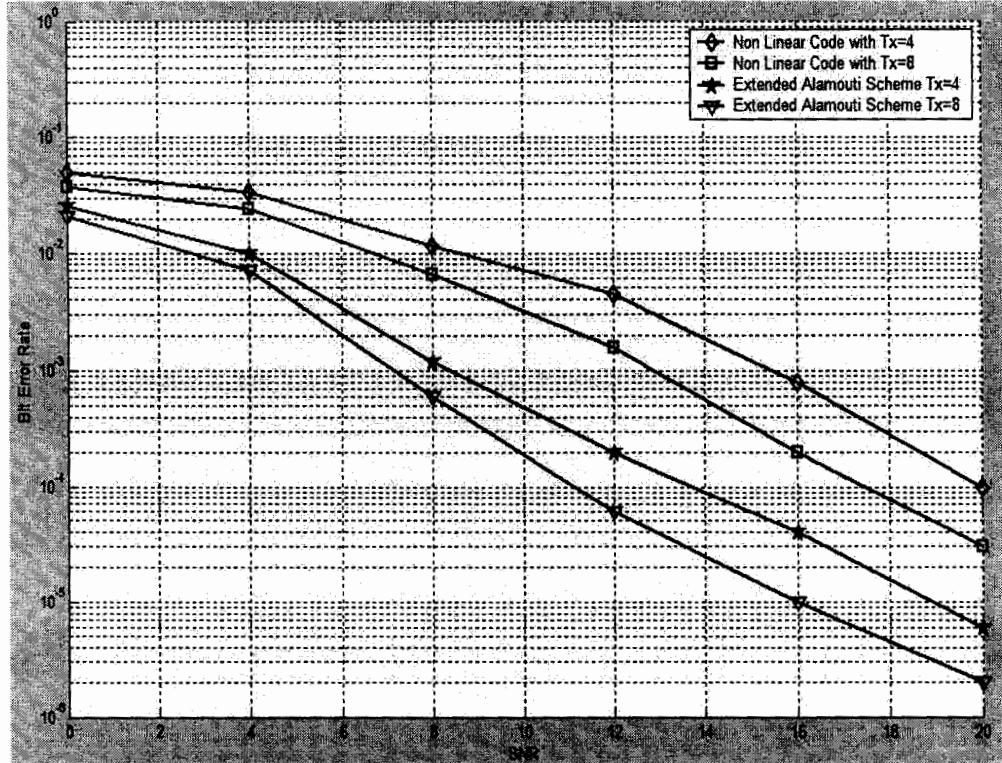


Figure 4.6 Bit error probabilities vs. SNR for four, eight transmit and four receive antennas

In Figure 4.6 I have used four and eight transmit antennas for Extended Alamouti, Non Linear STBC scheme and four receive antennas. By increasing the number of receive antennas the BER decreases from very low SNR and better results come after 4db SNR, which is much better, then the two cases in which I have used one and two receive antennas. By increasing the SNR the BER going significantly down and it can be seen from graph that after increasing 4db in SNR the improvement in data rate is going upward by a factor. Although on 8db the results of Non Linear STBC also have good results but Extended Alamouti scheme provide significant improvement.

With eight transmit and four receive antennas for both schemes there is much difference between BER at same SNR and if we see at 12db SNR the improvement is

equal to one decay and at for eight antennas at same SNR the improvement is more than one decay. Similarly the BER at 16db for Non Linear scheme is 0.0002 for Non Linear STBC and 0.00001 for Extended Alamouti scheme. Similarly the BER at 20db for Non Linear scheme is 0.00003 and 0.000002 for Extended Alamouti scheme which is so much better.

## CHAPTER 5

### Conclusions and Future Work

#### 5.1 Conclusions and Future Work

In this thesis i have used Extended Alamouti scheme in this scheme the transmissiom matrixs is divided into small groups and all columns with in each group are orthogonal to each other but these groups are not orthogonal to each other. I have plotted graphs between Extended Alamouti, Alamouti and Non Linear space time block codes for two, four and eight transmit antennas with one and two receive antennas.

From the simulation results given in last chapter, it is apparent that with increase in number of transmit antennas BER rate goes significantly down. Also by doubling the number of antennas, BER is reduced by a factor but at the cost of hardware used and power input. Also though 8-PSK is more valuable than QPSK, by using it with contrast to these scheme results of 8-PSK gives better results also bit rate go significantly high. This scheme can further be enhanced for different combination of number of transmit and receive antennas and according results can be investigated. Also this scheme can be very useful for Multi-carrier systems so can be utilized for them. Different detection schemes like MMSE, DD, SIC; PIC, MIC etc can be used for better results at the cost of affordable complexity. Also in this scheme I took flat-fading Rayleigh channel, whilst this can be extended to fast fading and disperse channel.

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## Glosary

SNR	Signal-to-noise Ratio
RF	Radio Frequency
BLAST	Bell labs Layered Space Time Architecture
STC	Space-Time Coding
LD	Linear-Dispersion
STC	Space-time Code
MIMO	Multiple-input Multiple-output
PEP	Pair-wise Error Probability
I.I.D	Identical Independent Distribution
OFDM	Orthogonal Frequency Division Multiplexing
AWGN	Additive White Gaussian Noise
DPSK	Differential Phase-shift Keying
ESTBC	Extended Alamouti Space-Time Block Code
TM	Time Diversity
FD	Frequency Diversity
AD	Antenna Diversity
SD	Space Diversity
BER	Bit Error Rate
MLSE	Maximum Likelihood Sequence Estimator
MMSE	Minimum Mean Squared Error
FEC	Forward Error Correction

BPSK	Binary Phase-shift Keying
PSK	Phase-shift Keying
PAM	Pulse Amplitude Modulation
QAM	Quadrature Amplitude Modulation

