Influence of Two Relaxation Times on P, SV and Thermal Waves at Interface with Magnetic Field and Temperature Dependent Elastic Moduli

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Magnetic fields - Mathematics.

Waves

Strain

Equations

Mathematical formulation

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Registration no: 267-FBAS/MSMA/F15

A THESIS SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF THE **MASTER OF SCIENCE** IN **MATHEMATICS**

We accept this thesis as conforming to the required standard.

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DEDICATED TO

MY PARENTS

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Declaration

I hereby declare that this thesis is my original work. I have not copied from any other student's work or from any other sources except where due reference or acknowledgement is made explicitly in the text, nor has any part been written for me by another person.

The work was done under the guidance of **Dr. Ambreen Afsar Khan** at International Islamic University, Islamabad.

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Preface

The generalized theory of thermoelasticity is one of the modified version of classical uncoupled and coupled theory of elasticity and have been developed in order to remove the paradox of physical impossible phenomena of infinite velocity of thermal signal in the classical coupled theory. Lord and Shulman [1] developed the generalized thermoelasticity theory involving one relaxation time. Green and Lindsay [2] formulated a temperature dependent thermoelasticity that includes two thermal relaxation times and does not violate the classical Fourier law of heat conduction.

The foundation of magnetoelasticity was proposed by Kaliski and Petykicwics [3]. Increasing attention on this topic is due to the interaction between magnetic field and strain field in a thermoelastic solid due to its many applications in the field of geophysics, plasma physics and related topics. A two dimensional problem for a half-space in magneto-thermoelasticity with thermal relaxation was discussed by Sherief and Kamal [4]. The phenomena of reflection and refraction of plane wave has been discussed by many authors e.g. Othman and Song [5] viewed the influence of temperature dependent elastic moduli on the reflection magneto thermoelastic waves with two relaxation times. Further [6-15] studied the reflection and refraction of magneto thermoelastic waves. Abd-Alla [16] considered the refraction and reflection of SV-waves at the solid liquid interface by considering primary stress and three thermoelastic theories. Kumar and Saini [17] illustrated the effect of refraction and reflection of waves at the interface between two different porous solids. Wei et al. [18] investigated the refraction and reflection of P-waves at thermoelastic and porous thermoelastic medium.

In the first chapter, we present some basic definitions. The second chapter deals with the reflection of magneto-thermoelastic waves with two relaxation times and temperature dependent elastic moduli. In the third chapter, we discuss the influence of two relaxation times on P, SV and Thermal waves at interface with magnetic field and temperature dependent elastic moduli.

Contents

Chapter 1

Preliminaries

In this chapter some basic definitions are discussed.

1.1 Definitions

1.1.1 Waves

A wave is an oscillation accompanied by transfer of energy through a medium.

1.1.2 Elastic Waves

A wave in which the propagated disturbance is an elastic deformation of the medium.

1.1.3 Elastic Modulus

Elastic Modulus is the ratio of the applied stress to the change in shape of an elastic body.

1.1.4 Reflection

Reflection is the change in direction of a wave front at an interface between two different media so that the wave front return into medium from which it is originated.

1.1.5 Refraction

Refraction is the change in direction of propagation of a wave due to change in its transmission media.

1.1.6 Interface

The point where two medium are separated.

1.2 Seismic Waves

Seismic waves produced by the energy released during an earthquake. There are two types of seismic waves.

Surface waves

Body waves

1.2.1 Surface Waves

A surface wave is a mechanical wave that propagates along the interface between differing media, usually as a gravity wave between two fluids with different densities. There are two types of surface waves.

Rayleigh waves

Love waves

1.2.2 Body Waves

These are the waves propagating inside the elastic material or travelling with in the medium. There are two types of body waves, P-waves and S-waves.

1.2.3 P-Waves

P-waves are known as primary, pressure or dilatational waves. These waves moves parallel to the direction of wave propagation. They are the first seismic wave to be felt during an earth quake. They can move through solid rock and fluids and they are the least destructive.

1.2.4 S-Waves

S-waves are known as secondary, rotational waves or shear waves. These waves move up and down or side to side as a sine wave perpendicular to the direction of wave propagation. They are the second seismic wave to be felt or recorded during an earthquake. These waves are the most destructive.

1.2.5 SV-Waves

If the displacement vector is along the vertical direction, we call such waves as SV-waves.

1.3 Strain

When an external force is applied on a body, there is some change occur in the dimension of the body. The ratio of this change of dimension in the body to its actual length is called strain. Strain is dimensionless quantity.

$$\text{Strain} = \frac{\text{Change in Dimension}}{\text{Original Dimension}}.$$

1.3.1 Types of Strain

Strain is of three types depending upon the change produced in a body when an elastic body is subjected to stress.

Longitudinal Strain

It is the ratio of the change in length of a body to the original length of the body. If L is the original length of a rod or a wire and the final length of the rod or the wire is $L + \Delta L$ under the action of a normal stress, the change in length is ΔL.

$$\text{Longitudinal Strain} = \frac{\Delta L}{L}.$$

Volume Strain

It is the ratio of the change in volume of a body to its original volume. If V is the original volume of a body and $V + \Delta V$ is the volume of the body under the action of a normal stress, the change in volume is ΔV.

$$\text{Volume Strain} = \frac{\Delta V}{V}.$$

1.3.2 Normal strain

Normal strain measures change in length along a specific direction. It is also called extensional strain as well as dimensional strain.

1.3.3 Shear strain

Shear strain measures change in angles with respect to two specific directions.

1.3.4 Normal Stress

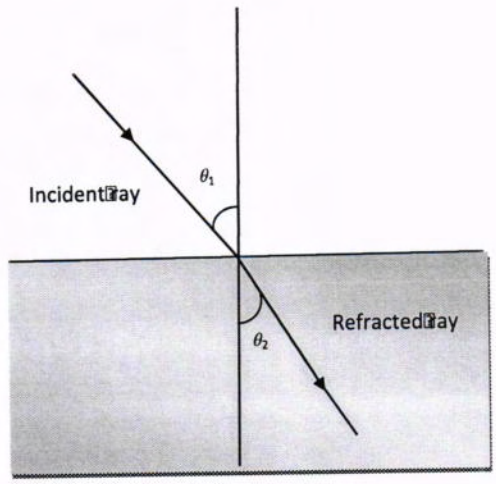
Stress that acts perpendicular to a surface. Can be either compressional or tensional.

1.4 Snell's Law

A law describes the refraction of seismic waves at the surface between two media, such that the product of the refractive index of the first medium and the sine of the angle of incidence equals the product of the refractive index of the second medium and the sine of the angle of refraction.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2.$$

where n_1 is index of refraction of 1^{st} medium, θ_1 is incidence angle, n_2 is index of refraction of 2^{nd} medium, θ_2 is refractive angle.



1.5 Equation of Motion

The equation of motion for an elastic medium is

$$\sigma_{ij,j} + f_i = \rho\left(\ddot{U}_i\right), \tag{1.1}$$

where $\sigma_{ij,j}$ is the stress force acting on the direction of x_i axis, f_i is the body force, ρ is the density of the medium and U_i is the displacement vector.

1.6 Energy Equation

The energy describe the distribution of heat in a given section over a time. The first law of thermodynamics i.e., the energy may neither be created nor destroyed. Therefore, the sum of all the energies in the system is a constant.

The law of conservation of energy is described as

$$K\nabla^2\dot{T} + K^*\nabla^2 T = \rho\tau_E\ddot{T} + \eta T_0 \operatorname{div} \ddot{\mathrm{u}}, \tag{1.2}$$

where η is the thermal modulus, ρ is the density of the medium, T is the temperature, T_0 is the uniform reference temperature, $\mathbf{u}$ is the displacement vector, τ_E is the specific heat of the medium at constant strain, K is the thermal conductivity and K^* is a material constant.

1.7 Maxwell Equations

The equations that relate elastic and magnetic field to their sources, charge and current density are known as Maxwell's equations. These equations are

$$\operatorname{curl}\mathbf{h} = \mathbf{J} + \epsilon_0\dot{\mathbf{E}}, \tag{1.3}$$

$$\operatorname{curl}\mathbf{E} = -\mu_0\frac{\partial\mathbf{h}}{\partial t}, \tag{1.4}$$

$$\mathbf{E} = -\mu_0\left(\dot{\mathbf{u}}\times\mathbf{H}_0\right), \tag{1.5}$$

$$\operatorname{div}\mathbf{h} = \mathbf{0}, \tag{1.6}$$

where μ_0 is magnetic permeability, $\mathbf{E}$ is induced electric field, $\mathbf{H}_0$ is initial uniform magnetic intensity vector, ϵ_0 is electric permeability and $\mathbf{J}$ is the current density vector.

Chapter 2

Reflection of Magneto-Thermoelastic Waves with Temperature Dependent Elastic Moduli and Two Relaxation Times

2.1 Introduction

This chapter is a review work of Ref [5]. This chapter studies the effect of magnetic field on the reflection of the plane thermoelastic wave from the boundary of elastic half space. The reflection coefficient ratios of different reflected waves with an incident angle have been obtained for Green and Lindsay theory and dynamical coupling theory. The effect of the permanent parameters on the distribution of reflection coefficient ratios are shown graphically.

2.2 Mathematical Formulation

Consider linear, an isotropic, homogeneous, perfectly conducting and thermally elastic medium whose mechanical properties depends on temperature having the half space

$G = \{(x,\ y,\ z)\} \mid -\infty < x, y < \infty, -\infty < z \geq 0\}.$

Let T_0 is the temperature of the half space and a constant magnetic field $\mathbf{H}_0 = (0,\ H,\ 0)$ is applied in the positive y-axis. The basic equations with respect to CD theory and GL theory in the absence of heat source are

(1) Equation of motion

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sigma_{ij,j} + f_i, \tag{2.1}$$

here f_i is the Lorentz force has a form

$$f_i = \mu_0 (\mathbf{J} \times \mathbf{H_0})_i, \tag{2.2}$$

where the derivative with respect to time represents a superposed dot and a comma after suffix shows material derivatives, $i,\ j = x,\ z$.

Using the magnetic field $\mathbf{h} = (0,\ h,\ 0)$. in Eqs. (1.3) – (1.6), we have

$$f_1 = -\mu_0 H \frac{\partial h}{\partial x} - \epsilon_0 \mu_0^2 H^2 u_{,tt}, \quad f_2 = 0, \quad f_3 = -\mu_0 H \frac{\partial h}{\partial z} - \epsilon_0 \mu_0^2 H^2 w_{,tt}. \tag{2.3}$$

(2) Under the GL theory, the constitutive law has the form

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij} \left[\lambda \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) - \eta \left(T - T_0 + v_0 \frac{\partial T}{\partial t} \right) \right]. \tag{2.4}$$

(3) Strain-displacement relation

$$e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}). \tag{2.5}$$

(4) Under GL theory, the heat conduction equation is

$$K \nabla^2 T = \rho \tau_E \left(1 + v_1 \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial t} + \eta T_0 \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right). \tag{2.6}$$

In the above equations, K is thermal conductivity, ρ is density, λ, μ are lame's constants, τ_E is specific heat at constant strain, u_i is components of displacement vector, T is absolute temperature, σ_{ij} is components of stress tensor, η is thermal parameter, t is time and v_0, v_1 are two relaxation times.

Consider the two dimensional unsteady problem in xz plane, the displacement component are given below.

$$u_x = u(x, z, t), \quad u_y = 0, \quad u_z = w(x, z, t). \tag{2.7}$$

The displacement potentials Φ and Ψ are given below

$$u = \frac{\partial \Phi}{\partial x} - \frac{\partial \Psi}{\partial z}, \quad w = \frac{\partial \Phi}{\partial z} + \frac{\partial \Psi}{\partial x}. \tag{2.8}$$

We assume that

$$E = E_0 f(T), \quad \lambda = \lambda_0 E_0 f(T), \quad \mu = \mu_0 E_0 f(T), \quad \eta = \eta_0 E_0 f(T). \tag{2.9}$$

Let the dimensionless temperature is $f(T)$. The modulus of elasticity is temperature independent when $f(T) = 1$ and $E = E_0$.

Using Eqs. (2.3) and (2.4) into Eq. (2.1) taking into consideration of Eqs. (2.5) and (2.9) we have the following form

$$\begin{aligned} \rho \frac{\partial^2 u}{\partial t^2} &= E_0 f(T) \left[\lambda_0 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z} \right) + 2\mu_0 \frac{\partial e_{xx}}{\partial x} - \eta_0 \frac{\partial}{\partial x} \left(T + v_0 \dot{T} \right) \right] \\ &\quad + 2E_0 f(T) \mu_0 \frac{\partial e_{xz}}{\partial z} - \mu_0 H \frac{\partial h}{\partial x} - \mu_0^2 H^2 \epsilon_0 \frac{\partial^2 u}{\partial t^2}, \end{aligned} \tag{2.10}$$

$$\begin{aligned} \rho \frac{\partial^2 w}{\partial t^2} &= E_0 f(T) \left[\lambda_0 \left(\frac{\partial^2 u}{\partial z \partial x} + \frac{\partial^2 w}{\partial z^2} \right) + 2\mu_0 \frac{\partial e_{zz}}{\partial z} - \eta_0 \frac{\partial}{\partial z} \left(T + v_0 \dot{T} \right) \right] \\ &\quad + 2E_0 f(T) \mu_0 \frac{\partial e_{zx}}{\partial x} - \mu_0 H \frac{\partial h}{\partial z} - \mu_0^2 H^2 \epsilon_0 \frac{\partial^2 w}{\partial t^2}. \end{aligned} \tag{2.11}$$

Putting Eq. (2.8) in Eqs. (1.3) – (1.6), we can obtain

$$h = -H \nabla^2 \Phi, \tag{2.12}$$

where ∇^2 is Laplace's operator.

We introduce the non-dimensional variables as follows

$$\begin{aligned} x_i^* &= \frac{x_i}{\omega_1 C_t}, \quad u_i^* = \frac{u_i}{\omega_1 C_t}, \quad t^* = \frac{t}{\omega_1}, \quad v_0^* = \frac{v_0}{\omega_1}, \quad v_1^* = \frac{v_1}{\omega_1}, \\ h^* &= \frac{h}{H}, \quad \sigma_{ij}^* = \frac{\sigma_{ij}}{\rho C_t^2}, \quad T^* = \frac{\eta_0 E_0 (T - T_0)}{\rho C_t^2}, \quad \beta = 1 + \frac{C_a^2}{c^2}, \\ \beta_1 &= \frac{1}{1 - \beta^* T_0} = \frac{1}{f(T_0)}. \end{aligned} \tag{2.13}$$

After non-dimensionalize, Eqs. (2.5), (2.6), (2.10), (2.11) and (2.12) are taking the following form

$$\beta\beta_1\frac{\partial^2\Phi}{\partial t^2} = (1+\beta_1 r_H)\nabla^2\Phi - \left(T + v_0\frac{\partial T}{\partial t}\right), \tag{2.14}$$

$$\beta\beta_1\frac{\partial^2\Psi}{\partial t^2} = (1-\alpha)\nabla^2\Psi, \tag{2.15}$$

$$\nabla^2 T = \left(\frac{\partial T}{\partial t} + v_1\frac{\partial^2 T}{\partial t^2}\right) + \varepsilon\nabla^2\dot{\Phi}, \tag{2.16}$$

$$h = -\nabla^2\Phi. \tag{2.17}$$

The constitutive equation reduce to

$$\beta_1\sigma_{ij} = (1-\alpha)\left(u_{i,j}+u_{j,i}\right) + \delta_{ij}\left[(2\alpha-1)\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right) - \left(T + v_0\frac{\partial T}{\partial t}\right)\right], \tag{2.18}$$

where

$$\begin{aligned} \alpha &= \frac{E_0\left(\lambda_0+\mu_0\right)}{\rho C_t^2}, \quad r_H = \frac{C_a^2}{C_t^2}, \quad \varepsilon = \frac{\eta_0 T_0}{\rho^2\tau_E C_t^2}, \quad C_a^2 = \frac{\mu_0 H^2}{\rho}, \\ C_t^2 &= \frac{E_0\left(\lambda_0+2\mu_0\right)}{\rho}, \quad c^2 = \frac{1}{\mu_0\epsilon_0}, \quad \omega_1 = \frac{K}{\rho C_t^2\tau_E}. \end{aligned}$$

Here r_H is the amount of magnetic pressure and E_0 is constant modulus of elasticity at $\beta^* = 0$. C_a is called Alfven speed, ε is the usual thermoelastic coupling parameter. Also we can see from Eqs. (2.14) – (2.16) the dilatational wave is affected due to the presence of the thermal effect and the magnetic field, while the coupled rotational waves remain unaffected.

We study the basic equations for the following two theories.

1) Classical and dynamical coupled theory

$$\tau_0 = 0, \quad \tau_1 = 0$$

2) Green and Lindsay's theory

$$\tau_1 \geq \tau_0 > 0$$

2.3 Solution of the Problem

For a harmonic wave propagated in the direction of xz-plane, and make an angle θ with the z-axis, we assume the solutions of the system of Eqs. (2.14) – (2.17) in the form:

$$\begin{aligned}\{\Phi,\, T,\, h\}(x,\, z,\, t) &= \{\Phi_1,\, T_1,\, h_1\} \exp\{i\xi\,(x\sin\theta + z\cos\theta) - \omega t\}\,,\\ \Psi(x,\, z,\, t) &= \Psi_1 \exp\{i\ell\,(x\sin\theta + z\cos\theta) - \omega t\}\,,\end{aligned} \tag{2.19}$$

where ξ and ℓ are the wave numbers and ω is the complex circular frequency.

Substituting Eq. (2.19) into Eqs. (2.14)–(2.17), we arrive at a system of three homogeneous equations:

$$\left(\xi^2\alpha_1 + \beta\beta_1\omega^2\right)\Phi_1 + pT_1 = 0, \tag{2.20}$$

$$\omega\varepsilon\xi^2\Phi_1 + \left(\xi^2 - \omega q\right)T_1 = 0, \tag{2.21}$$

$$-\xi^2\Phi_1 + h_1 = 0, \tag{2.22}$$

in which $\alpha_1 = 1 + \beta_1\tau_H$, $p = (1 - \omega v_0)$, $q = (1 - \omega v_1)$.

The system of Eqs. (2.20) – (2.22) has non-trivial solutions if and only if the determinant of the factor matrix vanishes. So

$$\begin{vmatrix} \left(\xi^2\alpha_1 + \beta\beta_1\omega^2\right) & p & 0 \\ \omega\varepsilon\xi^2 & \left(\xi^2 - \omega q\right) & 0 \\ -\xi^2 & 0 & 1 \end{vmatrix} = 0. \tag{2.23}$$

This yields

$$\mathrm{v}^4 - \frac{\beta\beta_1\omega - \alpha_1 q - p\varepsilon}{\beta\beta_1 q}\mathrm{v}^2 - \frac{\omega\alpha_1}{\beta\beta_1 q} = 0, \tag{2.24}$$

in which $\mathrm{v} = \frac{\omega}{\xi}$ is the velocity of P-waves.

From Eqs. (2.15) – (2.19), we can obtain

$$W^2 + \frac{(1-\alpha)}{\beta\beta_1} = 0, \quad W = \sqrt{\frac{(\alpha - 1)}{\beta\beta_1}}, \tag{2.25}$$

in which $W = \frac{\omega}{\ell}$ is the velocity of the SV-waves.

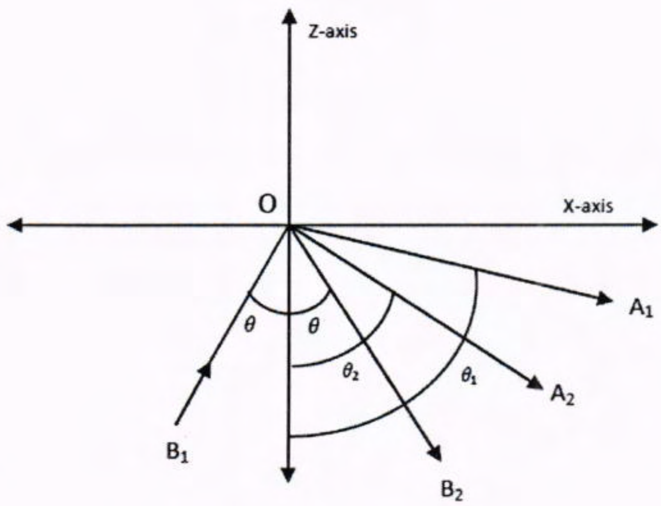


Fig. 2.1: Geometry of the Problem

2.3.1 For Incident SV-Wave

Eq.(2.24) is quadratic in v^2, there are two dilatational waves traveling with two different velocities. So assuming that when a SV-wave strikes at the boundary $z = 0$ within the elastic medium, it is reflected as SV-wave that makes an angle θ with the negative z-axis and two reflected dilatational waves (P-wave and thermal wave) that makes angles θ_1 and θ_2 with the same direction (see. Fig. 2.1). The displacement potentials Φ and Ψ can be written as

$$\Phi = A_1 \exp\{i\xi_1 (x \sin\theta_1 - z\cos\theta_1) - \omega t\} + A_2 \exp\{i\xi_2 (x\sin\theta_2 - z\cos\theta_2) - \omega t\}, \quad (2.26)$$

$$\Psi = B_1 \exp\{i\ell (x\sin\theta + z\cos\theta) - \omega t\} + B_2 \exp\{i\ell (x\sin\theta - z\cos\theta) - \omega t\}. \quad (2.27)$$

If the amplitude ratios of the reflected waves and incident wave $\frac{B_2}{B_1}$, $\frac{A_1}{B_1}$, $\frac{A_2}{B_1}$ give the corresponding reflection coefficients. The angles θ, θ_1, θ_2 and the corresponding wave numbers ℓ, ξ_1, ξ_2 are defined by the following relation:

$$\xi_1 \sin\theta_1 = \xi_2 \sin\theta_2 = \ell \sin\theta, \quad (2.28)$$

on the interface $z = 0$ of the medium, relation (2.24) may also be written as

$$\frac{\sin\theta_1}{\mathrm{v}_1} = \frac{\sin\theta_2}{\mathrm{v}_2} = \frac{\sin\theta'}{c'}, \tag{2.29}$$

in which

$$\mathrm{v}_1 = \frac{\omega}{\xi_1},\ \mathrm{v}_2 = \frac{\omega}{\xi_2},\ \acute{c} = \frac{\omega}{\ell} = W = \sqrt{\frac{(\alpha - 1)}{\beta\beta_1}}, \tag{2.30}$$

and v_1, v_2 are roots of Eq. (2.28).

2.3.2 For Incident P-Wave

Consider the incident P-wave falls on the boundary $z = 0$, it is reflected as SV-wave that makes an angle θ_2 with the negative z-axis and two reflected dilatational waves (P-wave and thermal wave) that makes angles θ and θ_1 with the same direction (see. Fig. 2.1). The displacement potentials Φ and Ψ can be written as

$$\begin{aligned}\Phi &= B_1 \exp\{i\xi_1 (x\sin\theta + z\cos\theta) - \omega t\} + B_2 \exp\{i\xi_1 (x\sin\theta - z\cos\theta) - \omega t\} \\ &\quad + A_1 \exp\{i\xi_2 (x\sin\theta_1 - z\cos\theta_1) - \omega t\},\end{aligned} \tag{2.31}$$

$$\Psi = A_2 \exp\{i\ell (x\sin\theta_2 - z\cos\theta_2) - \omega t\}, \tag{2.32}$$

also the angles θ, θ_1, θ_2 and the corresponding wave numbers ξ_1, ξ_2, ℓ are defined by the following relation:

$$\xi_1 \sin\theta = \xi_2 \sin\theta_1 = \ell \sin\theta_2, \tag{2.33}$$

on the interface $z = 0$ of the medium, relation (2.33) may also be written as:

$$\frac{\sin\theta}{\mathrm{v}_1} = \frac{\sin\theta_1}{\mathrm{v}_2} = \frac{\sin\theta_2}{c'}, \tag{2.34}$$

in which v_1, v_2, c' is the same as that obtained in Eq. (2.30).

2.4 Boundary Conditions

At the free surface, normal and tangential stress must vanishes.

$$\sigma_{zj} = 0 \quad (j = x,\, z) \quad \text{on } z = 0. \tag{2.35}$$

At the free surface, the boundary is thermally insulated.

$$\frac{\partial T}{\partial z} = 0 \quad \text{on } z = 0. \tag{2.36}$$

2.5 Expressions for the Reflection Coefficients

2.5.1 For Incident SV-Wave

Using the boundary conditions (2.35), (2.36) into Eqs. (2.26), (2.27), we have the following relations:

$$\frac{c'^2}{\mathrm{v}_1^2}\frac{A_1}{B_1}\sin 2\theta_1 + \frac{c'^2}{\mathrm{v}_2^2}\frac{A_2}{B_1}\sin 2\theta_2 - \frac{B_2}{B_1}\cos 2\theta = \cos 2\theta, \tag{2.37}$$

$$\frac{A_1 c'^2}{B_1}\left[\frac{2(\alpha-1)}{\mathrm{v}_1^2}\sin^2\theta_1 - \frac{\beta_1 r_H}{\mathrm{v}_1^2} - \beta\beta_1\right] + \frac{A_2 c'^2}{B_1}\left[\frac{2(\alpha-1)}{\mathrm{v}_2^2}\sin^2\theta_2 - \frac{\beta_1 r_H}{\mathrm{v}_2^2} - \beta\beta_1\right]$$

$$-\frac{B_2}{B_1}(\alpha-1)\sin 2\theta = (1-\alpha)\sin 2\theta, \tag{2.38}$$

$$\frac{A_1}{B_1}\left[\frac{\beta\beta_1}{\mathrm{v}_1} + \frac{\alpha_1}{\mathrm{v}_1^3}\right]\cos\theta_1 + \frac{A_2}{B_1}\left[\frac{\beta\beta_1}{\mathrm{v}_2} + \frac{\alpha_1}{\mathrm{v}_2^3}\right]\cos\theta_2 = 0. \tag{2.39}$$

The solution of this system for the reflection coefficient ratios of SV-wave $\frac{B_2}{B_1}$, reflection coefficient ratios of P and thermal waves $\frac{A_1}{B_1}$ and $\frac{A_2}{B_1}$ are

$$X_1 = \frac{A_1}{B_1} = \frac{P_1}{Q_1}, \quad X_2 = \frac{A_2}{B_1} = \frac{P_2}{Q_1}, \quad X_3 = \frac{B_2}{B_1} = \frac{P_3}{Q_1}, \tag{2.40}$$

in which

$$P_1 = (1-\alpha)\,\mathrm{v}_1^3\left[\beta\beta_1\mathrm{v}_2^2 + \alpha_1\right]\sin 4\theta\cos\theta_2, \tag{2.41}$$

$$P_2 = (\alpha-1)\,\mathrm{v}_2^3\left[\beta\beta_1\mathrm{v}_1^2 + \alpha_1\right]\sin 4\theta\cos\theta_1, \tag{2.42}$$

$$P_3 = -c'^2 v_2 \left[\beta\beta_1 v_1^2 + \alpha_1\right] \cos\theta_1 \begin{bmatrix} \cos 2\theta \left[1 - \alpha + \beta_1 r_H + \beta\beta_1 v_2^2\right] \\ -(1-\alpha)\cos 2(\theta + \theta_2) \end{bmatrix}$$
$$+c'^2 v_1 \left[\beta\beta_1 v_2^2 + \alpha_1\right] \cos\theta_2 \begin{bmatrix} \cos 2\theta \left[1 - \alpha + \beta_1 r_H + \beta\beta_1 v_1^2\right] \\ -(1-\alpha)\cos 2(\theta + \theta_1) \end{bmatrix}, \tag{2.43}$$

$$Q_1 = c'^2 v_2 \left[\beta\beta_1 v_1^2 + \alpha_1\right] \cos\theta_1 \begin{bmatrix} \cos 2\theta \left[1 - \alpha + \beta_1 r_H + \beta\beta_1 v_2^2\right] \\ -(1-\alpha)\cos 2(\theta - \theta_2) \end{bmatrix}$$
$$-c'^2 v_1 \left[\beta\beta_1 v_2^2 + \alpha_1\right] \cos\theta_2 \begin{bmatrix} \cos 2\theta \left[1 - \alpha + \beta_1 r_H + \beta\beta_1 v_1^2\right] \\ -(1-\alpha)\cos 2(\theta - \theta_1) \end{bmatrix}. \tag{2.44}$$

2.5.2 For Incident P-Wave

Using the boundary conditions (2.35), (2.36) into Eqs. (2.31), (2.32) we have the following relations:

$$\frac{B_2}{B_1}\sin 2\theta + \frac{A_1}{B_1}\frac{v_1^2}{v_2^2}\sin 2\theta_1 - \frac{A_1}{B_1}\frac{c'^2}{v_2^2}\cos 2\theta_2 = \sin 2\theta, \tag{2.45}$$

$$1 + \frac{B_2}{B_1} + \frac{A_1}{B_1}\frac{v_1^2}{v_2^2}\left[\frac{2(1-\alpha)\sin^2\theta_1 + \beta_1 r_H + \beta\beta_1 v_2^2}{2(1-\alpha)\sin^2\theta + \beta_1 r_H + \beta\beta_1 v_1^2}\right]$$
$$+\frac{A_2}{B_1}\frac{v_1^2}{c'^2}\left[\frac{(\alpha-1)\sin^2\theta_2}{2(1-\alpha)\sin^2\theta + \beta_1 r_H + \beta\beta_1 v_1^2}\right] = 0, \tag{2.46}$$

$$\frac{B_2}{B_1}\cos\theta + \frac{A_1}{B_1}\frac{v_1^3}{v_2^3}\left[\frac{\left(\beta\beta_1 v_2^2 + \alpha_1\right)\cos\theta_1}{\left(\beta\beta_1 v_1^2 + \alpha_1\right)}\right] = \cos\theta. \tag{2.47}$$

The solution of this system for the reflection coefficient ratios of P and thermal waves $\frac{B_2}{B_1}$ and $\frac{A_1}{B_1}$, the reflection coefficient ratio of SV-wave $\frac{A_2}{B_1}$, are

$$X_1 = \frac{B_2}{B_1} = \frac{R_1}{Q_2}, \quad X_2 = \frac{A_1}{B_1} = \frac{R_2}{Q_2}, \quad X_3 = \frac{A_2}{B_1} = \frac{R_3}{Q_2}, \tag{2.48}$$

in which

$$R_1 = -v_1^3\left(\beta\beta_1 v_2^2+\alpha_1\right)\cos\theta_1\left[\begin{array}{c}(\alpha-1)\cos(\theta+\theta_2)+\\ \left(1-\alpha+\beta_1 r_H+\beta\beta_1 v_1^2\right)\cos 2\theta_2\end{array}\right]$$
$$-v_1^2 v_2\left[\beta\beta_1 v_1^2+\alpha_1\right]\cos\theta\left[\begin{array}{c}(\alpha-1)\cos(\theta_1-\theta_2)+\\ \left(1-\alpha+\beta_1 r_H+\beta\beta_1 v_2^2\right)\cos 2\theta_2\end{array}\right], \tag{2.49}$$

$$R_2 = 2v_2^3\left[\beta\beta_1 v_2^2+\alpha_1\right]\left[2(1-\alpha)\sin^2\theta+\beta_1 r_H+\beta\beta_1 v_1^2\right]\cos\theta\cos 2\theta_2, \tag{2.50}$$

$$R_3 = 2c'^2\left\{v_1\left[\beta\beta_1 v_2^2+\alpha_1\right]\cos\theta_1\sin 2\theta - v_2\left[\beta\beta_1 v_1^2+\alpha_1\right]\cos\theta\sin 2\theta_1\right\}$$
$$.\left[2(1-\alpha)\sin^2\theta+\beta_1 r_H+\beta\beta_1 v_1^2\right], \tag{2.51}$$

$$Q_2 = v_1^3\left[\beta\beta_1 v_2^2+\alpha_1\right]\cos\theta_1\left[\begin{array}{c}(\alpha-1)\cos(\theta-\theta_2)+\\ \left(1-\alpha+\beta_1 r_H+\beta\beta_1 v_1^2\right)\cos 2\theta_2\end{array}\right]$$
$$-v_1^2 v_2\left[\beta\beta_1 v_1^2+\alpha_1\right]\cos\theta\left[\begin{array}{c}(\alpha-1)\cos(\theta_1-\theta_2)+\\ \left(1-\alpha+\beta_1 r_H+\beta\beta_1 v_2^2\right)\cos 2\theta_2\end{array}\right]. \tag{2.52}$$

2.6 Numerical Results

The copper material is chosen for numerical evaluations. In calculation process, the material constants necessary to be known can be found in ref. [4]. Let $\omega = \omega_0$, the other constants of the problem are taken as $T_0 = 300\ K$, $\alpha = 0.7472$, $v_0 = 0.05$, $v_1 = 0.02$, $\omega_0 = 5$.

Figs. 2.2 and 2.3 give the variation of the reflection coefficient ratios with the angle of incidence for the SV-wave and the P-wave under two theories. Here $r_H = 0.3$, $\beta = 1.275$, $\varepsilon = 0.03$, $\beta^* = 0.001$. We can observe that in the case of SV-wave, the reflection coefficient ratio $|X_1| = |X_2| = 0$ when $\theta = 0°$, $45°$, $90°$. $|X_3| = 1$ when $\theta = 0°$, $45°$. In the case of P-wave, the reflection coefficient ratio $|X_3| = 0$ when $\theta = 0°$, $90°$ and $|X_1|$ gets maximum value at $\theta = 0°$, $90°$.

Figs. 2.4 and 2.5 give the effect of the reference temperature dependent modulus on the reflection coefficient ratios for incident SV and P-waves. Here $r_H = 0.3$, $\beta = 1.275$, $\varepsilon = 0.03$,

$\beta^* = 0,\ 0.001,\ 0.002$, respectively. The reflection coefficient ratios $|X_1|$, $|X_2|$ for incident SV-wave and $|X_2|$, $|X_3|$ for incident P-wave decrease with the increase of β^*. While for incident SV-wave the reflection coefficient ratio $|X_3|$ decrease with the increase of β^* when $\theta < 45°$ and increases when $45° < \theta < 90°$. For P-wave, the reflection coefficient ratio $|X_1|$ increase with increase of β^*. It can be concluded that the reference of β^* plays an important role.

Figs. 2.6 and 2.7 give the effect of magnetic field on the reflection coefficient ratio. Here $\beta = 1.275$, $\varepsilon = 0.03$, $\beta^* = 0.001$, $r_H = 0.0, 0.3, 0.5$, respectively. We can see that the effect of the magnetic field has the same trend on reflection coefficient ratios as that reference temperature dependent modulus for both incident P-wave and SV-wave.

Figs. 2.8 and 2.9 give the variation of the angle of incidence with the reflection coefficient ratios under different values of coupling parameter for two type of incidence wave. $r_H = 0.3$, $\beta = 1.275$, $\beta^* = 0.001$ and $\varepsilon = 0,\ 0.03,\ 0.05$, respectively. We can see the reflection coefficient ratios $|X_2|$ increase with increase of coupling parameter, while the coupling parameter has small effect on reflection coefficient ratio $|X_1|$ for both cases. Also it can be observed that $|X_2|$ vanishes when $\varepsilon = 0.0$. While for incident SV-wave the reflection coefficient ratio $|X_3|$ increases with an increase of ε when $\theta < 45°$ and decreases with an increase of ε when $45° < \theta < 90°$.

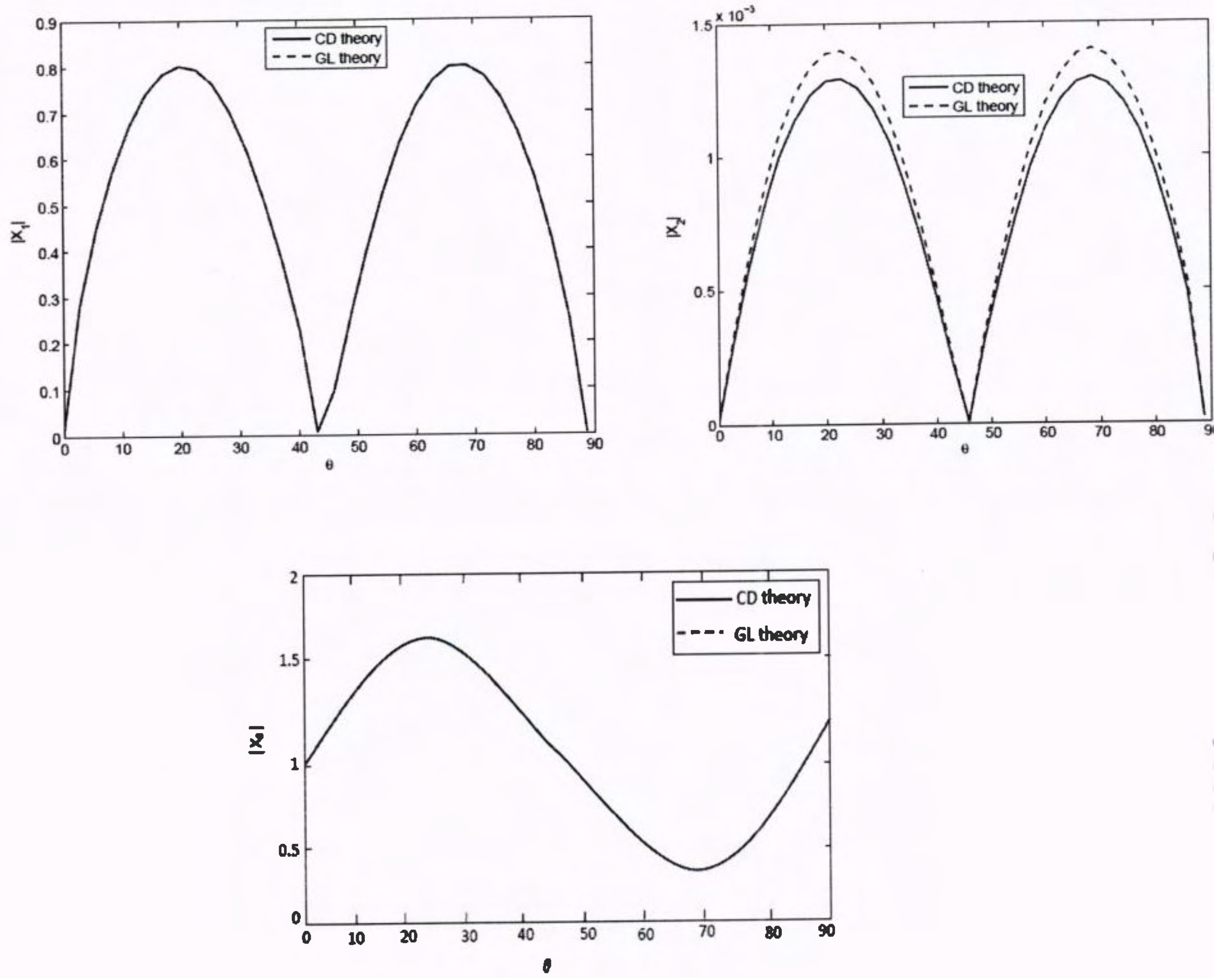


Fig. 2.2: Difference of the amplitudes of reflection coefficient ratios with incident angle of SV-wave under different theory.

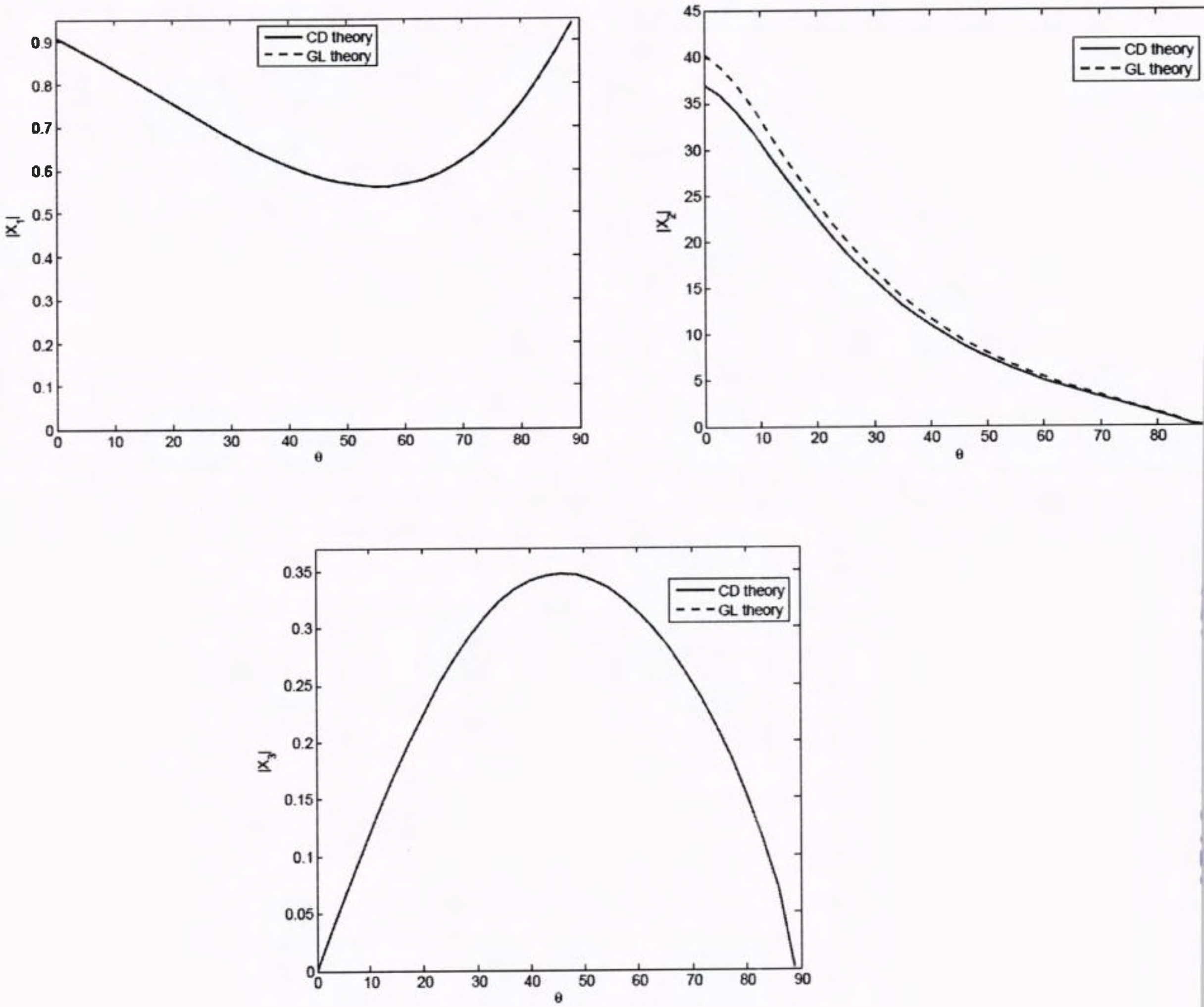


Fig. 2.3: Difference of the amplitudes of reflection coefficient ratios with incident angle of P-wave under different theory.

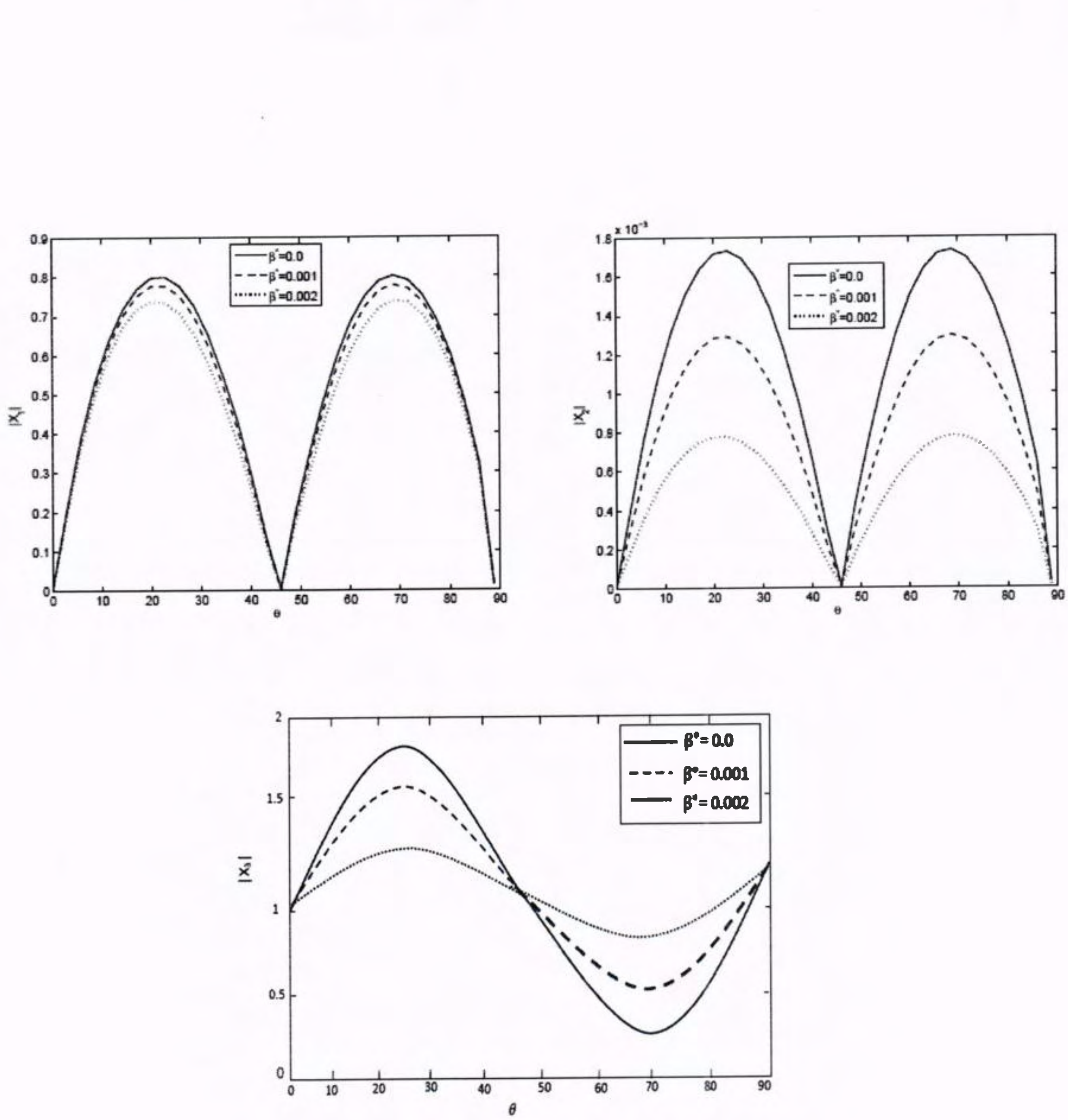


Fig 2.4: Difference of the amplitudes of reflection coefficient ratios with incident angle of SV-wave for reference of β^*.

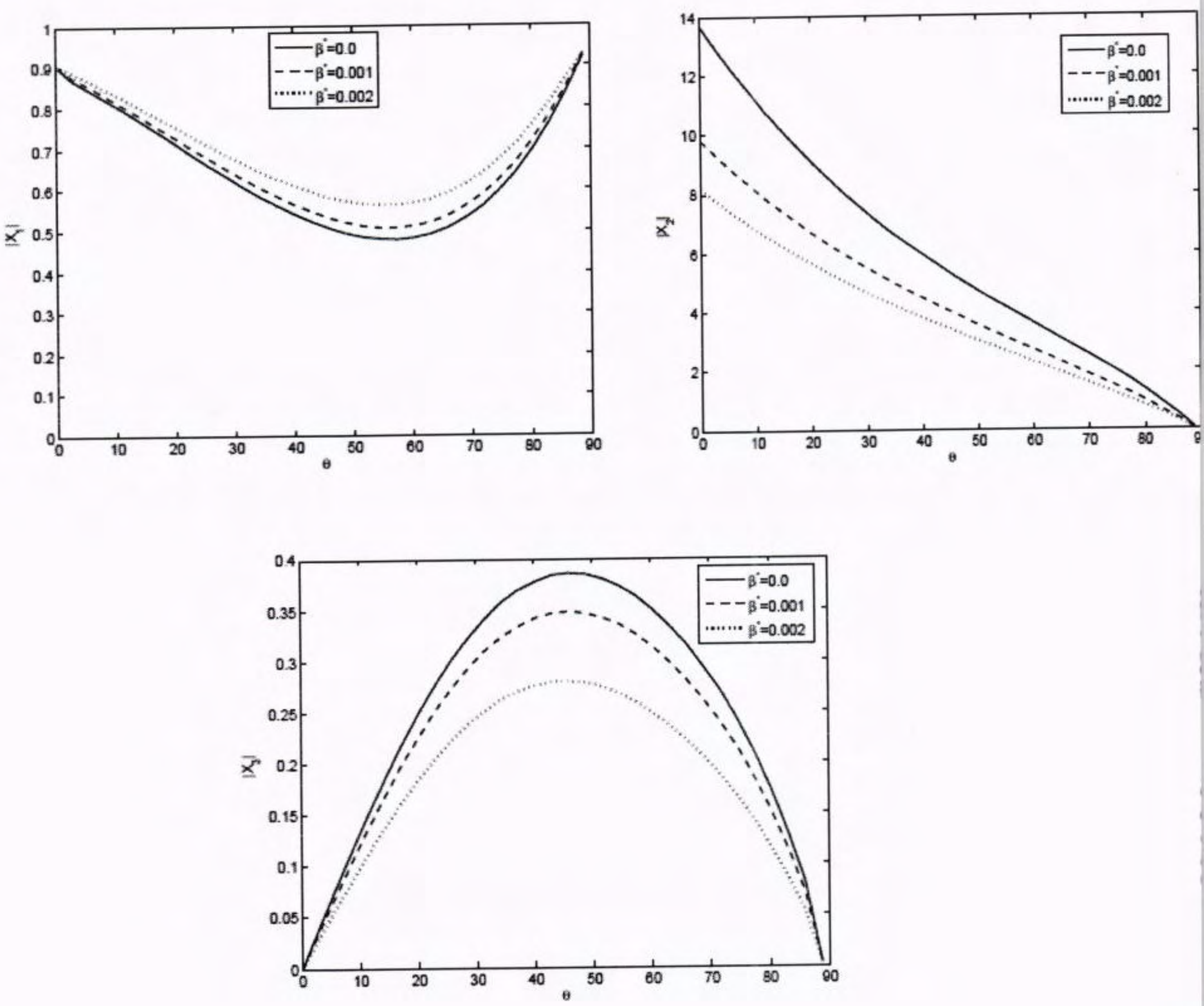


Fig 2.5: Difference of the amplitudes of reflection coefficient ratios with incident angle of P-wave for reference of β^*.

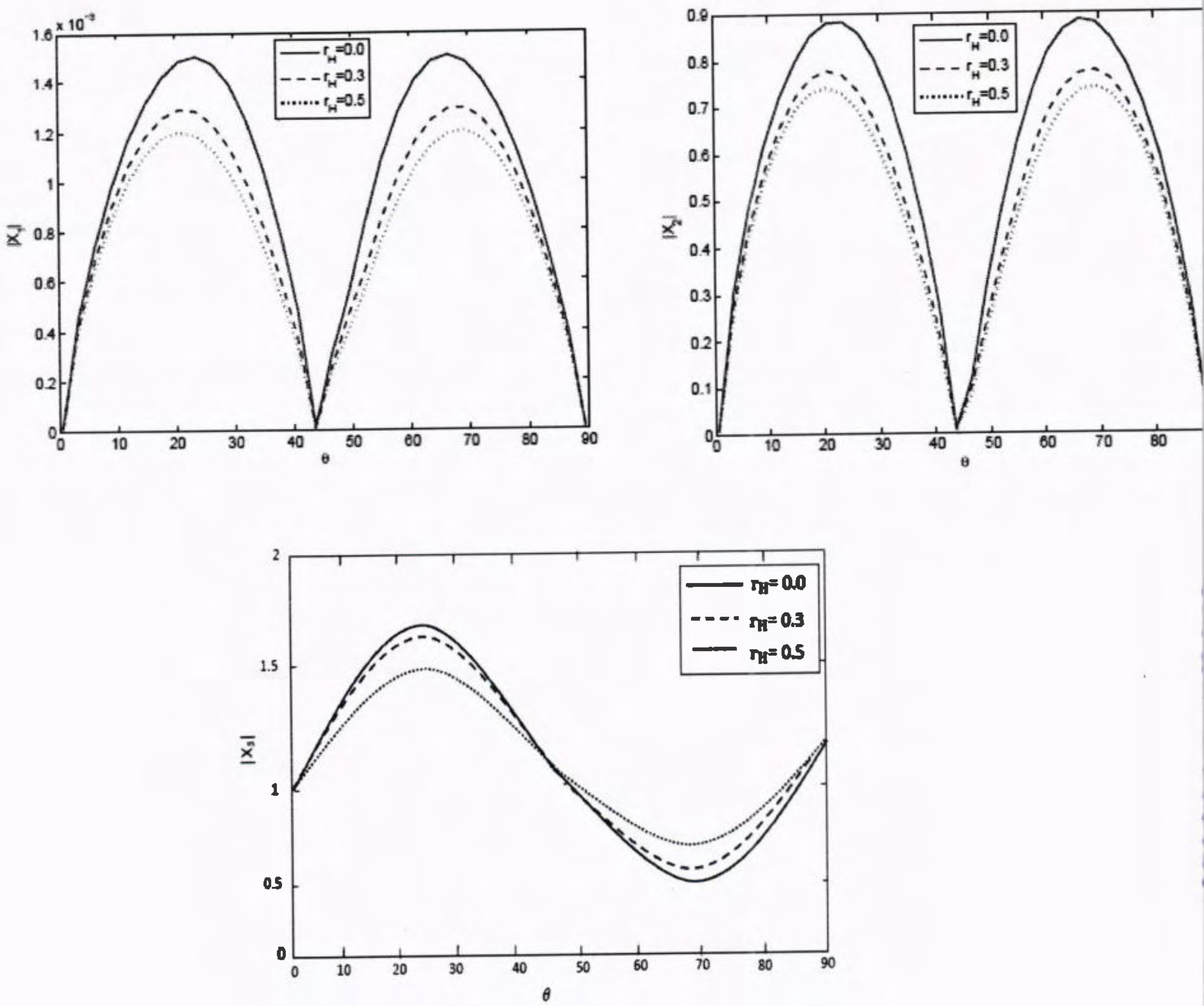


Fig 2.6: Difference of the amplitudes of reflection coefficient ratios with incident angle of SV-wave for effect of magnetic field.

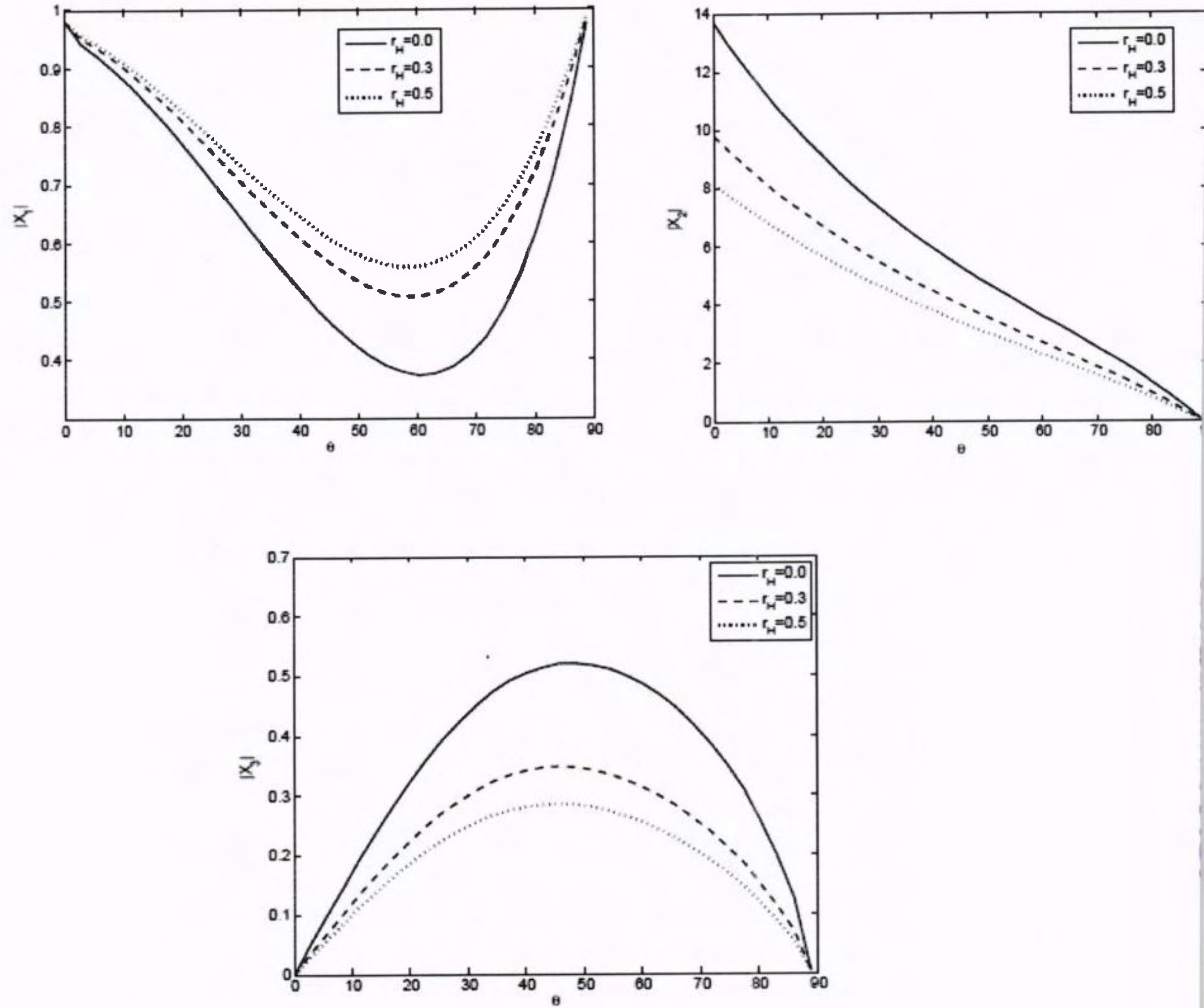


Fig 2.7: Difference of the amplitudes of reflection coefficient ratios with incident angle of P-wave for effect of magnetic field.

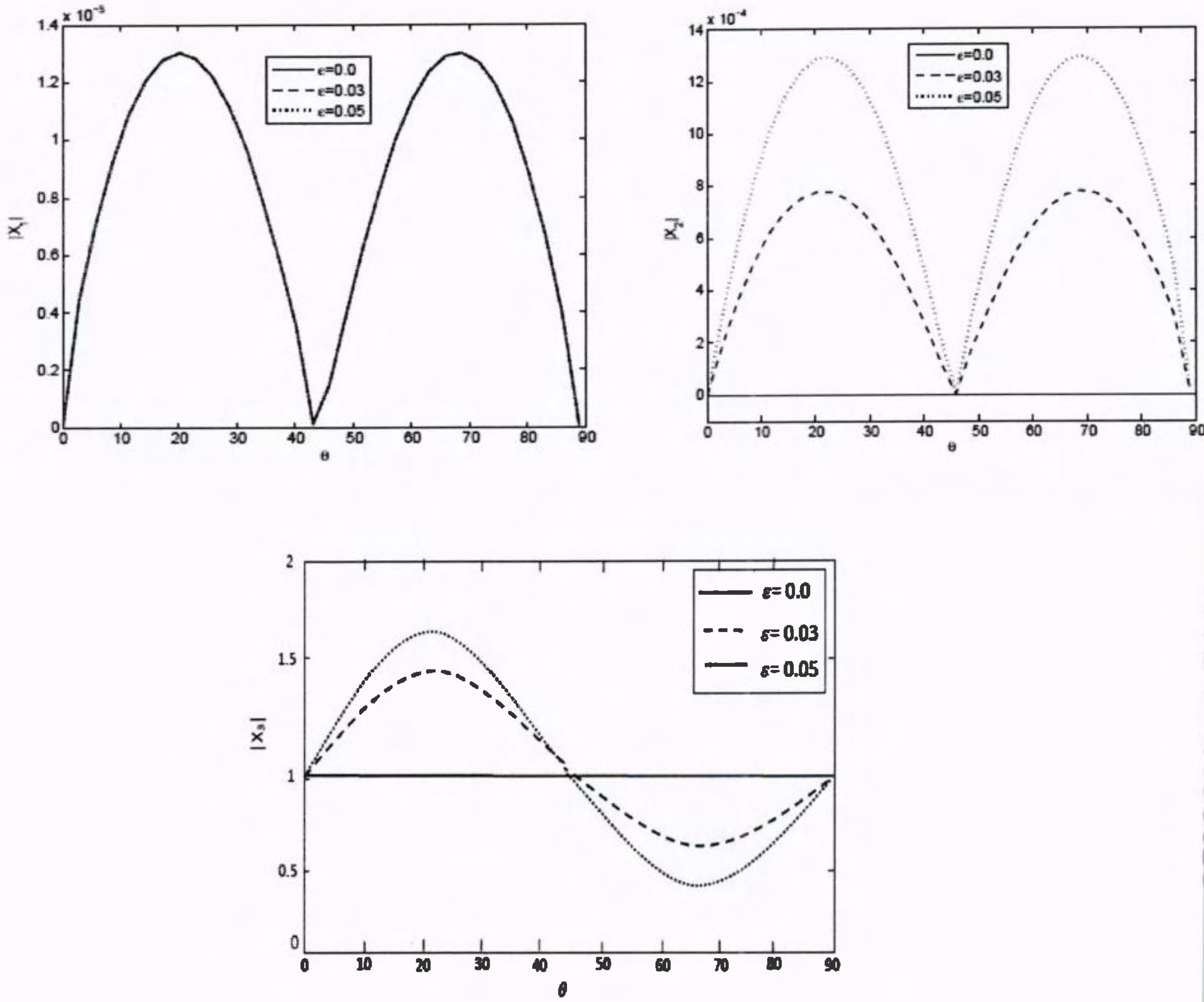


Fig 2.8: Difference of the amplitudes of reflection coefficient ratios with incident angle of SV-wave for coupling parameter.

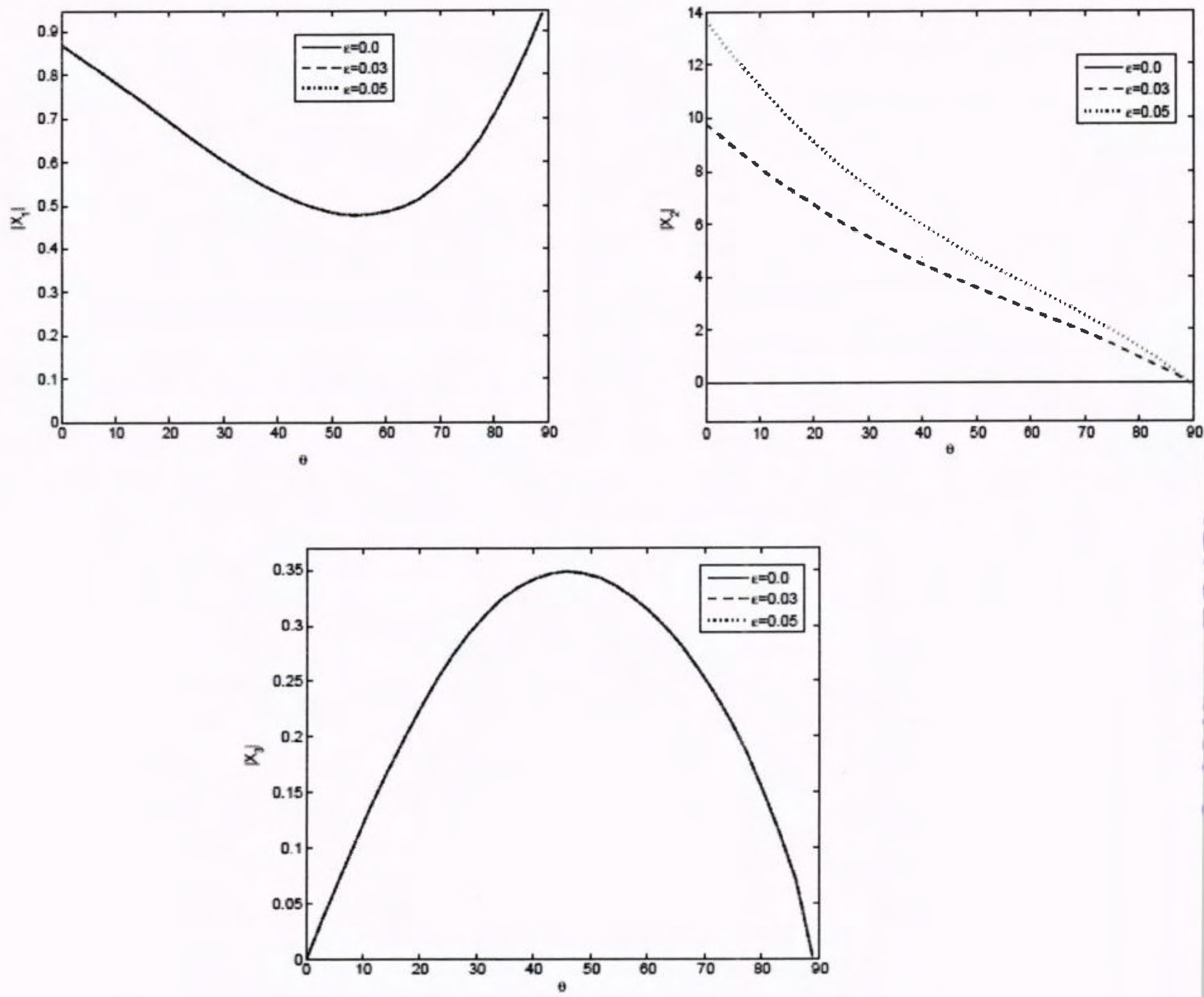


Fig 2.9: Difference of the amplitudes of reflection coefficient ratios with incident angle of P-wave for coupling parameter.

Chapter 3

Influence of Two Relaxation Times on P, SV and Thermal Waves at Interface with Magnetic Field and Temperature Dependent Elastic Moduli

3.1 Introduction

In this chapter, two models of the generalized thermoelasticity theory are used to see the influence on the refraction and reflection of the plane waves at the interface under a constant magnetic field. The elasticity modulus depends on the reference temperature. The elasticity modulus is considered as a linear function of reference temperature. The resulting problem is solved by using the boundary conditions at the interface. The matrix has been solved numerically.

3.2 Mathematical Formulation

We consider an isotropic, linear, homogeneous, perfectly conducting and thermally elastic medium whose mechanical properties depends upon temperature occupying the two half spaces.

We kept constant temperature T_0 throughout the body and is acted on all over by a uniform magnetic field $H_0 = (0,\ H,\ 0)$, which is applied in the positive y-axis.

Now for the medium M', we will use prime to describe all the quantities of the basic Eqs. (1.3) – (1.6) . Taking

$$u' = \frac{\partial \Phi'}{\partial x} - \frac{\partial \Psi'}{\partial z}, \quad w = \frac{\partial \Phi'}{\partial z} + \frac{\partial \Psi'}{\partial x}. \tag{3.1}$$

After non-dimensionalize, we obtain

$$\beta'\beta_1'\frac{\partial^2 \Phi'}{\partial t^2} = \left(1 + \beta_1' r_H'\right)\nabla^2\Phi' - \left(T' + v_0'\frac{\partial T'}{\partial t}\right), \tag{3.2}$$

$$\beta'\beta_1'\frac{\partial^2 \Psi'}{\partial t^2} = (1-\alpha')\nabla^2\Psi', \tag{3.3}$$

$$\nabla^2 T' = \left(\frac{\partial T'}{\partial t} + v_1'\frac{\partial^2 T'}{\partial t^2}\right) + \varepsilon'\nabla^2\dot{\Phi}', \tag{3.4}$$

$$h' = -\nabla^2\Phi'. \tag{3.5}$$

$$\beta_1'\sigma_{ij}' = (1-\alpha')\left(u'_{i,j} + u'_{j,i}\right) + \delta'_{ij}\left[\left(2\alpha' - 1\right)\left(\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z}\right) - \left(T' + v_0'\frac{\partial T'}{\partial t}\right)\right]. \tag{3.6}$$

3.3 Solution of the Problem

For a harmonic wave propagated in the direction of xz-plane, and make an angle θ' with the z-axis, we assume the solutions of the system of Eqs. (3.2) – (3.5) in the form:

$$\begin{aligned} \left\{\Phi', T', h'\right\}(x,\ z,\ t) &= \left\{\Phi_1', T_1', h_1'\right\}\exp\left\{i\xi'\left(x\sin\theta' + z\cos\theta'\right) - \omega t\right\}, \\ \Psi'(x,\ z,\ t) &= \Psi_1'\exp\left\{i\ell'\left(x\sin\theta' + z\cos\theta'\right) - \omega t\right\}, \end{aligned} \tag{3.7}$$

where ξ' and ℓ' are the wave numbers of refracted waves and ω is the complex circular frequency.

Substituting Eq. (3.7) into Eqs. (3.2) – (3.5), we arrive at a system of three homogeneous equations:

$$\left(\xi'^2\alpha_1' + \beta'\beta_1'\omega^2\right)\Phi_1' + mT_1' = 0, \tag{3.8}$$

$$\left(\xi'^2 - \omega n\right)T_1 + \omega\varepsilon'\xi'^2\Phi_1' = 0, \tag{3.9}$$

$$-\xi'^2\Phi_1' + h_1' = 0, \tag{3.10}$$

in which $\alpha_1' = 1 + \beta_1' r_H'$, $m = \left(1 - \omega v_0'\right)$, $n = (1 - \omega v_1')$.

The system of Eqs. (3.8) – (3.10) has non-trivial solutions if and only if the determinant of the factor matrix vanishes. So

$$\begin{vmatrix} \left(\xi'^2\alpha_1' + \beta'\beta_1'\omega^2\right) & m & 0 \\ \omega\varepsilon'\xi'^2 & \left(\xi'^2 - \omega n\right) & 0 \\ -\xi'^2 & 0 & 1 \end{vmatrix} = 0. \tag{3.11}$$

This yields

$$\mathbf{v}'^4 - \frac{\beta'\beta_1'\omega - \alpha_1' n - m\varepsilon'}{\beta'\beta_1' n}\mathbf{v}'^2 - \frac{\omega\alpha_1'}{\beta'\beta_1' n} = 0, \tag{3.12}$$

in which $\mathbf{v}' = \frac{\omega}{\xi'}$ is the velocity of refracted SV-waves.

From Eqs. (3.3) – (3.6), we can obtain

$$W'^2 + \frac{(1-\alpha')}{\beta'\beta_1'} = 0, \quad W' = \sqrt{\frac{(\alpha'-1)}{\beta'\beta_1'}}, \tag{3.13}$$

in which $W' = \frac{\omega}{\xi'}$ is the velocity of the refracted P-waves.

3.3.1 For Incident SV-Wave

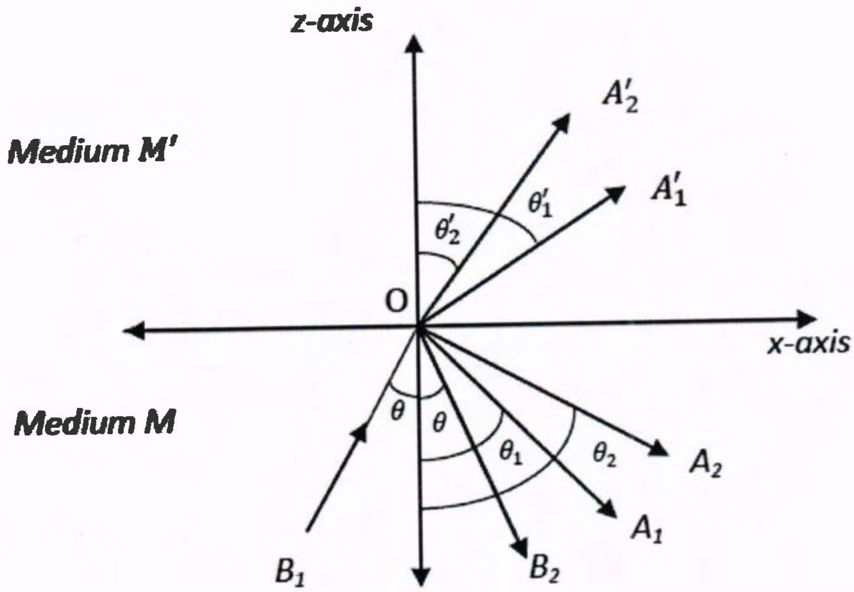


Fig 3.1: Relation between incident angle of SV-wave, reflect and the refract angles.

Consider a plane SV-wave propagating through a medium M and is incident at $z = 0$, three waves (SV, P and thermal) are reflected in the same medium M by making an angles θ, θ_1 and θ_2 with z-axis and P-wave, Thermal waves are transmitted into medium M' by making an angles θ_1' and θ_2'. The displacement potentials Φ, Ψ for medium M and Φ', Ψ' for medium M' will take the forms:

$$\Phi = A_1 \exp\{i\xi_1 (x \sin\theta_1 - z\cos\theta_1) - \omega t\} + A_2 \exp\{i\xi_2 (x\sin\theta_2 - z\cos\theta_2) - \omega t\}, \tag{3.14}$$

$$\Psi = B_1 \exp\{i\ell (x\sin\theta + z\cos\theta) - \omega t\} + B_2 \exp\{i\ell (x\sin\theta - z\cos\theta) - \omega t\}, \tag{3.15}$$

$$\Phi' = A_1' \exp\left\{i\xi_1' \left(x\sin\theta_1' + z\cos\theta_1'\right) - \omega t\right\} + A_2' \exp\left\{i\xi_2' \left(x\sin\theta_2' + z\cos\theta_2'\right) - \omega t\right\}, \tag{3.16}$$

$$\Psi' = 0, \tag{3.17}$$

where the angles θ, θ_1, θ_2, θ_1', θ_2' and the corresponding numbers ℓ, ξ_1, ξ_2, ξ_1', ξ_2' are joined with the following relation:

$$\xi_1 \sin\theta_1 = \xi_2 \sin\theta_2 = \ell \sin\theta = \xi_1' \sin\theta_1' = \xi_2' \sin\theta_2'. \tag{3.18}$$

On the interface $z = 0$, we have

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} = \frac{\sin\theta}{c'} = \frac{\sin\theta_1'}{v_1'} = \frac{\sin\theta_2'}{v_2'}, \tag{3.19}$$

in which

$$v_1 = \frac{\omega}{\xi_1}, \quad v_2 = \frac{\omega}{\xi_2}, \quad c' = \frac{\omega}{\ell}, \quad v_1' = \frac{\omega}{\xi_1'}, \quad v_2' = \frac{\omega}{\xi_2'}, \tag{3.20}$$

v_1, v_2 are the roots of Eq. (2.24) and v_1', v_2' are the roots of Eq. (3.12).

3.3.2 For Incident P-wave

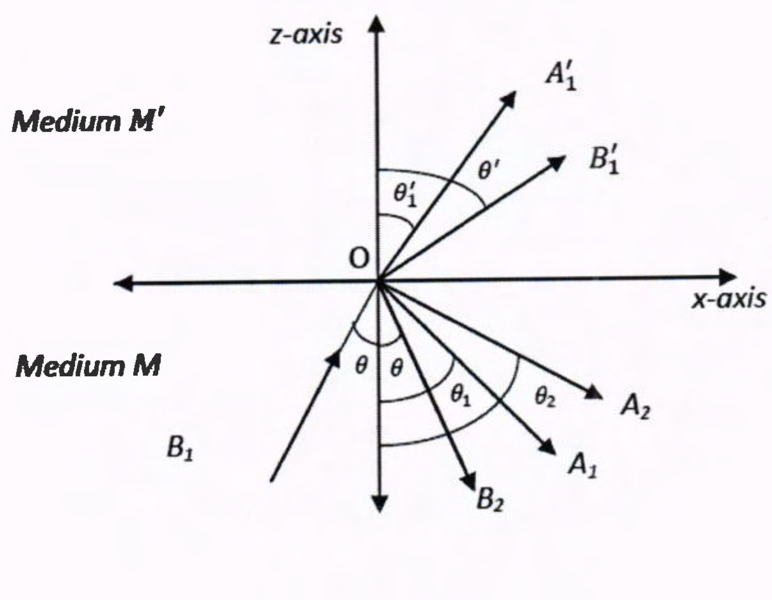


Fig 3.2: Relation between incident angle of P-wave, reflect and the refract angles.

A plane P-wave propagating through a medium M and is incident at $z = 0$, three waves (P, SV and thermal) are reflected in the same medium M by making an angles θ, θ_1 and θ_2 with z-axis and P-wave, Thermal waves are transmitted into medium M' by making an angles θ' and θ_1'

2) At the interface, the tangential displacement must disappear i.e., $u = 0$.

$$\frac{\partial \Phi}{\partial x} - \frac{\partial \Psi}{\partial z} = 0, \qquad \text{at } z = 0. \tag{3.29}$$

3) At the interface, normal force per unit primary area is continuous i.e., $\sigma_{33} = \sigma'_{33}$.

Substituting Eqs. (2.8), (2.14) into Eq. (2.18) and Eqs. (3.1), (3.2) into Eq. (3.6), we get at $z = 0$

$$\frac{1}{\beta_1}\left[(2\alpha - 2 - \beta_1 r_H)\nabla^2\Phi + 2(1-\alpha)\left(\frac{\partial^2\Phi}{\partial z^2} + \frac{\partial^2\Psi}{\partial x \partial z}\right) + \beta\beta_1\frac{\partial^2\Phi}{\partial t^2}\right]$$
$$= \frac{1}{\beta'_1}\left[\left(2\alpha' - 2 - \beta'_1 r'_H\right)\nabla^2\Phi' + 2\left(1-\alpha'\right)\left(\frac{\partial^2\Phi'}{\partial z^2} + \frac{\partial^2\Psi'}{\partial x \partial z}\right) + \beta'\beta'_1\frac{\partial^2\Phi'}{\partial t^2}\right]. \tag{3.30}$$

4) At the interface, tangential force per unit primary area must disappear i.e., $\sigma_{13} = 0$. Using Eq. (2.8) into Eq. (2.18), we get

$$2\frac{\partial^2\Phi}{\partial x \partial z} + \frac{\partial^2\Psi}{\partial x^2} - \frac{\partial^2\Psi}{\partial z^2} = 0, \qquad \text{at } z = 0. \tag{3.31}$$

5) At the interface, continuity of temperature i.e., $T = T'$. Substituting Eq. (2.14) into Eq. (2.19) and Eq. (3.2) into Eq. (3.7), we get

$$m\left[\alpha_1\nabla^2\Phi - \beta\beta_1\frac{\partial^2\Phi}{\partial t^2}\right] = p\left[\alpha'_1\nabla^2\Phi' - \beta'\beta'_1\frac{\partial^2\Phi'}{\partial t^2}\right], \qquad \text{at } z = 0. \tag{3.32}$$

3.5 Expressions for the Refraction and Reflection Coefficients

3.5.1 For Incident SV-Wave

Considering Eqs. (3.14) – (3.17) and then applying the boundary conditions (3.28) to (3.32). We obtain the following relations:

$$\frac{A_1}{B_1}\left(\frac{c'}{v_1}\cos\theta_1\right) + \frac{A_2}{B_1}\left(\frac{c'}{v_2}\cos\theta_2\right) - \frac{B_2}{B_1}(\sin\theta) + \frac{A'_1}{B_1}\left(\frac{c'}{v'_1}\cos\theta'_1\right)$$
$$+\frac{A'_2}{B_1}\left(\frac{c'}{v'_2}\cos\theta'_2\right) = \sin\theta, \tag{3.33}$$

$$\frac{A_1}{B_1}\left(\frac{c'}{\mathrm{v}_1}\sin\theta_1\right)+\frac{A_2}{B_1}\left(\frac{c'}{\mathrm{v}_2}\sin\theta_2\right)+\frac{B_2}{B_1}(\cos\theta)=\cos\theta, \tag{3.34}$$

$$\frac{A_1}{B_1}\left(\frac{c'^2}{\mathrm{v}_1^2}\frac{m_1}{\beta_1}\right)+\frac{A_2}{B_1}\left(\frac{c'^2}{\mathrm{v}_2^2}\frac{m_2}{\beta_1}\right)+\frac{B_2}{B_1}\left(\frac{(1-\alpha)}{\beta_1}\sin 2\theta\right)-\frac{A_1'}{B_1}\left(\frac{c'^2}{\mathrm{v}_1'^2}\frac{m_3}{\beta_1'}\right)$$

$$-\frac{A_2'}{B_1}\left(\frac{c'^2}{\mathrm{v}_2'^2}\frac{m_4}{\beta_1'}\right)=\frac{(1-\alpha)}{\beta_1}\sin 2\theta, \tag{3.35}$$

$$\frac{A_1}{B_1}\left(\frac{c'^2}{\mathrm{v}_1^2}\sin 2\theta_1\right)+\frac{A_2}{B_1}\left(\frac{c'^2}{\mathrm{v}_2^2}\sin 2\theta_2\right)+\frac{B_2}{B_1}(\cos 2\theta)=-\cos\theta, \tag{3.36}$$

$$\frac{A_1}{B_1}\left(\frac{n_1}{\mathrm{v}_1^2}\right)+\frac{A_2}{B_1}\left(\frac{n_2}{\mathrm{v}_2^2}\right)-\frac{A_1'}{B_1}\left(\frac{n_3}{\mathrm{v}_1'^2}\right)-\frac{A_2'}{B_1}\left(\frac{n_4}{\mathrm{v}_2'^2}\right)=0, \tag{3.37}$$

where

$$\begin{aligned}
m_1 &= \left[(2+\beta_1 r_H-2\alpha)+(2\alpha-2)\cos^2\theta_1+\frac{\beta\beta_1}{\xi_1^2}\omega^2\right],\\
m_2 &= \left[(2+\beta_1 r_H-2\alpha)+(2\alpha-2)\cos^2\theta_2+\frac{\beta\beta_1}{\xi_2^2}\omega^2\right],\\
m_3 &= \left[\left(2+\beta_1' r_H'-2\alpha'\right)+\left(2\alpha'-2\right)\cos^2\theta_1'+\frac{\beta'\beta_1'}{\xi_1'^2}\omega^2\right],\\
m_4 &= \left[\left(2+\beta_1' r_H'-2\alpha'\right)+\left(2\alpha'-2\right)\cos^2\theta_2'+\frac{\beta'\beta_1'}{\xi_2'^2}\omega^2\right].\\
n_1 &= m\omega^2\left(\alpha_1+\beta\beta_1\mathrm{v}_1^2\right), \qquad n_2=m\omega^2\left(\alpha_1+\beta\beta_1\mathrm{v}_2^2\right),\\
n_3 &= p\omega^2\left(\alpha_1'+\beta'\beta_1'\mathrm{v}_1'^2\right), \qquad n_4=p\omega^2\left(\alpha_1'+\beta'\beta_1'\mathrm{v}_2'^2\right).
\end{aligned}$$

Generalizing, we get a system of five nonhomogeneous equations for incident SV-wave.

$$\sum_{i=1}^{5} a_{ij}Y_j=p_j, \qquad (j=1,2,...,5)$$

where

$$\begin{aligned}
a_{11} &= \frac{c'}{\mathrm{v}_1}\cos\theta_1, \quad a_{12}=\frac{c'}{\mathrm{v}_2}\cos\theta_2, \quad a_{13}=-\sin\theta, \quad a_{14}=\frac{c'}{\mathrm{v}_1'}\cos\theta_1',\\
a_{15} &= \frac{c'}{\mathrm{v}_2'}\cos\theta_2', \quad a_{21}=\frac{c'}{\mathrm{v}_1}\sin\theta_1, \quad a_{22}=\frac{c'}{\mathrm{v}_2}\sin\theta_2, \quad a_{23}=\cos\theta,
\end{aligned}$$

$$
\begin{aligned}
a_{24} &= a_{25}=0, \quad a_{31}=\frac{c'^2}{\mathrm{v}_1^2}\frac{m_1}{\beta_1}, \quad a_{32}=\frac{c'^2}{\mathrm{v}_2^2}\frac{m_2}{\beta_1}, \quad a_{33}=\frac{(1-\alpha)}{\beta_1}\sin 2\theta,\\
a_{34} &= -\frac{c'^2}{\mathrm{v}_1'^2}\frac{m_3}{\beta_1'}, \quad a_{35}=-\frac{c'^2}{\mathrm{v}_2'^2}\frac{m_4}{\beta_1'}, \quad a_{41}=\frac{c'^2}{\mathrm{v}_1^2}\sin 2\theta_1, \quad a_{42}=\frac{c'^2}{\mathrm{v}_2^2}\sin 2\theta_2,\\
a_{43} &= \cos 2\theta, \quad a_{44}=a_{45}=0,\\
a_{51} &= \frac{n_1}{\mathrm{v}_1^2}, \quad a_{52}=\frac{n_2}{\mathrm{v}_2^2}, \quad a_{53}=0, \quad a_{54}=-\frac{n_3}{\mathrm{v}_1'^2}, \quad a_{55}=-\frac{n_4}{\mathrm{v}_2'^2},\\
Y_1 &= \frac{A_1}{B_1}, \quad Y_2=\frac{A_2}{B_1}, \quad Y_3=\frac{B_2}{B_1}, \quad Y_4=\frac{A_1'}{B_1}, \quad Y_5=\frac{A_2'}{B_1}.
\end{aligned}
$$

where$(j=1,2,...,5)$ represents the amplitude ratios of reflected P, T, SV-waves and refracted P, T-waves.

$$
p_1=\sin\theta, \quad p_2=\cos\theta, \quad p_3=\frac{(1-\alpha)}{\beta_1}\sin 2\theta, \quad p_4=-\cos\theta, \quad p_5=0
$$

3.5.2 For Incident P-Wave

Considering Eqs. (3.21) – (3.24) and then applying the boundary conditions (3.28) to (3.32). We obtain the following relations:

$$
\begin{aligned}
&\frac{B_2}{B_1}\left(\frac{\cos\theta}{\mathrm{v}_1}\right)+\frac{A_1}{B_1}\left(\frac{\cos\theta_1}{\mathrm{v}_2}\right)-\frac{A_2}{B_1}\left(\frac{\sin\theta_2}{c'}\right)+\frac{B_1'}{B_1}\left(\frac{\cos\theta'}{\mathrm{v}_1'}\right)+\frac{A_1'}{B_1}\left(\frac{\cos\theta_1'}{\mathrm{v}_2'}\right)\\
&= \frac{\cos\theta}{\mathrm{v}_1},
\end{aligned} \tag{3.38}
$$

$$
\frac{B_2}{B_1}\left(\frac{\sin\theta}{\mathrm{v}_1}\right)+\frac{A_1}{B_1}\left(\frac{\sin\theta_1}{\mathrm{v}_2}\right)+\frac{A_2}{B_1}\left(\frac{\cos\theta_2}{c'}\right)=-\frac{\sin\theta}{\mathrm{v}_1}, \tag{3.39}
$$

$$
\begin{aligned}
&\frac{B_2}{B_1}\left(\frac{m_2'}{\beta_1\mathrm{v}_1^2}\right)+\frac{A_1}{B_1}\left(\frac{m_3'}{\beta_1\mathrm{v}_2^2}\right)+\frac{A_2}{B_1}\left(\frac{(\alpha-1)}{\beta_1 c'^2}\sin 2\theta_2\right)-\frac{B_1'}{B_1}\left(\frac{m_4'}{\beta_1'\mathrm{v}_1'^2}\right)\\
&-\frac{A_1'}{B_1}\left(\frac{m_5'}{\beta_1'\mathrm{v}_2'^2}\right)=-\frac{m_1'}{\beta_1\mathrm{v}_1^2},
\end{aligned} \tag{3.40}
$$

$$
\frac{B_2}{B_1}\left(\frac{\sin 2\theta}{\mathrm{v}_1^2}\right)+\frac{A_1}{B_1}\left(\frac{\sin 2\theta_1}{\mathrm{v}_2^2}\right)+\frac{A_2}{B_1}\left(\frac{\cos 2\theta_2}{c'^2}\right)=\frac{\sin 2\theta}{\mathrm{v}_1^2}, \tag{3.41}
$$

$$
\frac{B_2}{B_1}\left(\frac{n_1}{\mathrm{v}_1^2}\right)+\frac{A_1}{B_1}\left(\frac{n_2}{\mathrm{v}_2^2}\right)-\frac{B_1'}{B_1}\left(\frac{n_3}{\mathrm{v}_1'^2}\right)-\frac{A_1'}{B_1}\left(\frac{n_4}{\mathrm{v}_2'^2}\right)=-\frac{m\omega^2}{\mathrm{v}_1^2}\left(\alpha_1+\beta\beta_1\mathrm{v}_1^2\right), \tag{3.42}
$$

where

$$\begin{aligned}
m_1' &= m_2' = \left[(2+\beta_1 r_H - 2\alpha) + (2\alpha - 2)\cos^2\theta + \frac{\beta\beta_1}{\xi_1^2}\omega^2\right], \\
m_3' &= \left[(2+\beta_1 r_H - 2\alpha) + (2\alpha - 2)\cos^2\theta_1 + \frac{\beta\beta_1}{\xi_2^2}\omega^2\right], \\
m_4' &= \left[\left(2+\beta_1' r_H' - 2\alpha'\right) + \left(2\alpha' - 2\right)\cos^2\theta' + \frac{\beta'\beta_1'}{\xi_1'^2}\omega^2\right], \\
m_5' &= \left[\left(2+\beta_1' r_H' - 2\alpha'\right) + \left(2\alpha' - 2\right)\cos^2\theta_1' + \frac{\beta'\beta_1'}{\xi_2'^2}\omega^2\right].
\end{aligned}$$

Generalizing, we get a system of five nonhomogeneous equations for incident P-wave.

$$\sum_{i=1}^{5} b_{ij} Z_j = s_j, \qquad (j = 1,\ 2, ..., 5)$$

where

$$\begin{aligned}
b_{11} &= \frac{\cos\theta}{v_1}, \quad b_{12} = \frac{\cos\theta_1}{v_2}, \quad b_{13} = -\frac{\sin\theta_2}{c'}, \quad b_{14} = \frac{\cos\theta'}{v_1'}, \quad b_{15} = \frac{\cos\theta_1'}{v_2'} \\
b_{21} &= \frac{\sin\theta}{v_1}, \quad b_{22} = \frac{\sin\theta_1}{v_2}, \quad b_{23} = \frac{\cos\theta_2}{c'}, \quad b_{24} = b_{25} = 0 \quad b_{31} = \frac{m_2'}{\beta_1 v_1^2}, \\
b_{32} &= \frac{m_3'}{\beta_1 v_2^2}, \quad b_{33} = \frac{(\alpha-1)}{\beta_1 c'^2}\sin 2\theta_2, \quad b_{34} = -\frac{m_4'}{\beta_1' v_1'^2}, \quad b_{35} = -\frac{m_5'}{\beta_1' v_2'^2}, \\
b_{41} &= \frac{\sin 2\theta}{v_1^2}, \quad b_{42} = \frac{\sin 2\theta_1}{v_2^2}, \quad b_{43} = \frac{\cos 2\theta_2}{c'^2}, \quad b_{44} = b_{45} = 0 \\
b_{51} &= \frac{n_1}{v_1^2}, \quad b_{52} = \frac{n_2}{v_2^2}, \quad b_{53} = 0, \quad b_{54} = -\frac{n_3}{v_1'^2}, \quad b_{55} = -\frac{n_4}{v_2'^2}, \\
Z_1 &= \frac{B_2}{B_1}, \quad Z_2 = \frac{A_1}{B_1}, \quad Z_3 = \frac{A_2}{B_1}, \quad Z_4 = \frac{B_1'}{B_1}, \quad Z_5 = \frac{A_1'}{B_1}.
\end{aligned}$$

where$(j = 1, 2, ..., 5)$ represents the amplitude ratios of reflected P, T, SV-waves and refracted P, T-waves.

$$s_1 = \frac{\cos\theta}{v_1}, \quad s_2 = -\frac{\sin\theta}{v_1}, \quad s_3 = -\frac{m_1'}{\beta_1 v_1^2}, \quad s_4 = \frac{\sin 2\theta}{v_1^2}, \quad s_5 = -\frac{m\omega^2}{v_1^2}\left(\alpha_1 + \beta\beta_1 v_1^2\right).$$

3.6 Numerical Results and Discussions

We considered the data for solid medium as crust and liquid medium as a water following Singh and Chakraborty [9], for the numerical analysis of previous section.

For solid medium (M crust "Granite"): which in geology is the uppermost solid shell of a rocky planet or natural satellite, and is chemically different from the underlying mantle.

$$\begin{aligned}\lambda &= \mu = 3\times10^{10}\ N.m^{-2}, \quad T_0 = 300\ K, \quad \omega = 7.5\times10^{13}\ S^{-1}, \quad \tau_E = 1100\ J.Kg^{-1}.K^{-1},\\ \rho &= 2900\ Kg.m^{-3}, \quad K = 3\ W.m^{-1}.K^{-1}, \quad E_0 = 2.6\times10^{5}.\end{aligned}$$

For fluid medium (M' "Water"):

$$\begin{aligned}\lambda' &= \mu' = 20.4\times10^{19}\ N.m^{-2}, \quad \tau'_E = 4187\ J.Kg^{-1}.K^{-1},\\ \rho' &= 1000\ Kg.m^{-3}, \quad K' = 0.6\ W.m^{-1}.K^{-1}, \quad E'_0 = 2.2\times10^{9}.\end{aligned}$$

Considering $v_0 = v'_0 = 0.8$, $v_1 = v'_1 = 0.9$ and $\epsilon_0 = \epsilon'_0 = 0.2$.

Fig. 3.3 and Fig. 3.4 give the effects of amplitude ratio with the incident angle for the SV and the P-waves under two theories. In the situation of SV-wave, $|Y_1|$, $|Y_2|$, $|Y_4|$ and $|Y_5|$ commenced from the maximum values and goes to zero at $\theta = 90°$ but for $|Y_3|$, it begins from unity and ends on as well a unity at $\theta = 90°$. It also indicates that GL theory in $|Y_1|$, $|Y_2|$ and $|Y_4|$ have the smaller values than CD theory, whereas GL theory in $|Y_3|$ and $|Y_5|$ have smaller values than CD theory after $\theta = 65°$.

In the case of P-wave, $|Z_1|$ begins from zero and reaches to unity at $\theta = 90°$. $|Z_2|$ and $|Z_5|$ begins from its extreme values and reaches to zero at $\theta = 90°$. Whereas for $|Z_2| = 0$ at $\theta = 90°$. $|Z_3| = 0$ when $\theta = 0°$ and $\theta = 90°$. $|Z_4|$ gets its highest value after $\theta = 50°$ and gradually it goes to zero at $\theta = 90°$.

Fig. 3.5 to Fig. 3.7 depict the effect of amplitudes with incident angle of SV-wave under the variation of two relaxation times to GL theory. Fig. 3.5 exhibits the variation of incident angle of SV-wave with the amplitude ratio under various values of ε . The amplitude ratio $|Y_1|$, $|Y_2|$, and $|Y_4|$ increases with increase of ε whereas amplitude ratio $|Y_3|$ and $|Y_5|$ initially decreases by increasing ε and after $\theta = 60°$, the amplitude ratio increases by increasing ε. Fig.

3.6 gives the difference of magnetic field on the amplitude ratio of SV-wave. It is seen that $|Y_1|$ and $|Y_2|$ increases with an increase of H, but $|Y_3|$, $|Y_4|$ and $|Y_5|$ decreases by increasing H. $|Y_4|$ has maximum value at $\theta = 45°$. Fig. 3.7 shows the influence of reference temperature modulus on amplitude ratio. We can see that the amplitude ratio $|Y_1|$ and $|Y_4|$ rises with rising β^* after $\theta = 20°$, $|Y_3|$ starts from unity and end on unity as well with increasing β^* and all the curve mix with each other after $\theta = 45°$. While $|Y_2|$ and $|Y_5|$ decreases with increasing β^* before $\theta = 45°$ and after $\theta = 45°$ it has opposite effect.

Fig. 3.8 to Fig. 3.10 gives the difference of amplitude with the incident angle of P-wave under the influence of two relaxation times to GL-theory. Fig. 3.8 shows the effect of ε on the amplitude ratio. $|Z_1|$, $|Z_2|$ and $|Z_5|$ decreases before $\theta = 30°$ and after $\theta = 45°$ increases by increasing ε, while $|Z_3|$ and $|Z_4|$ decreases before $\theta = 50°$ and after $\theta = 50°$ it start increasing and moves toward zero at $\theta = 90°$. We observed from Fig. 3.9 that $|Z_1|$ to $|Z_5|$ decreases as H increases. For $|Z_1|$ it moves towards unity at $\theta = 90°$ where as in $|Z_2|$, $|Z_3|$, $|Z_4|$ and $|Z_5|$ it moves toward zero at $\theta = 90°$. Fig. 3.10 exhibits the difference of reference temperature modulus on the amplitude ratio. We see that $|Z_1|$ to $|Z_5|$ have increasing and decreasing behavior for all values of β^*.

3.7 Conclusion

In this paper, we discussed the effect of temperature dependent elastic moduli, coupling parameter and magnetic field on the refraction and reflection of P and SV-waves at the interface. For SV and P-waves incident at the solid liquid interface, the effect of variation of temperature dependent modulus is more prominent than that of coupling parameter and magnetic field on the amplitude ratios of refracted and reflected P and thermal waves.

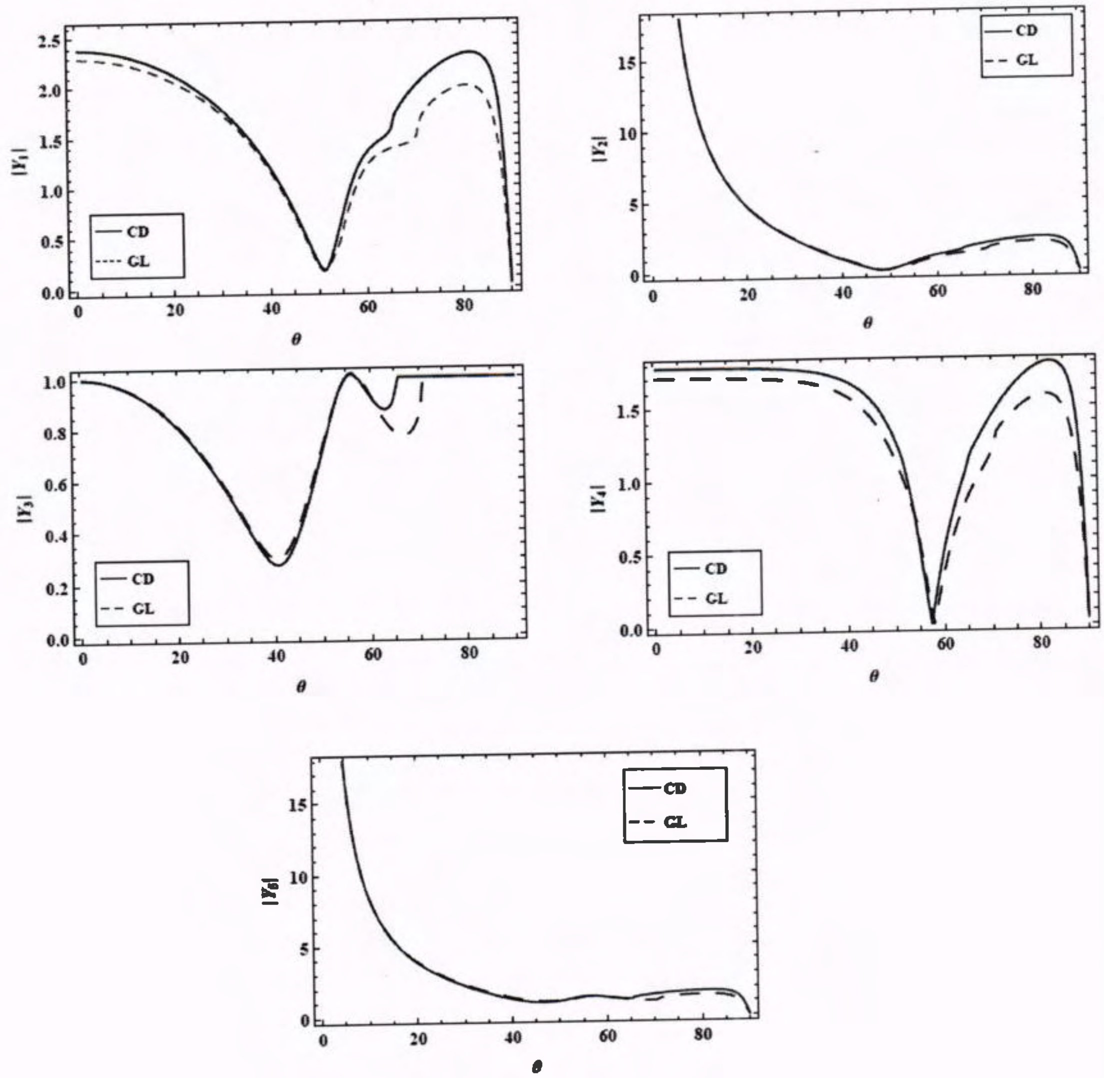


Fig. 3.3: Difference of the amplitudes $|Y_i|$, $(i = 1, 2, ..., 5)$ making an incident angle of SV-waves under different theory, $H = 0.4$, $\varepsilon = 0.08$, $\beta^* = 0.001$.

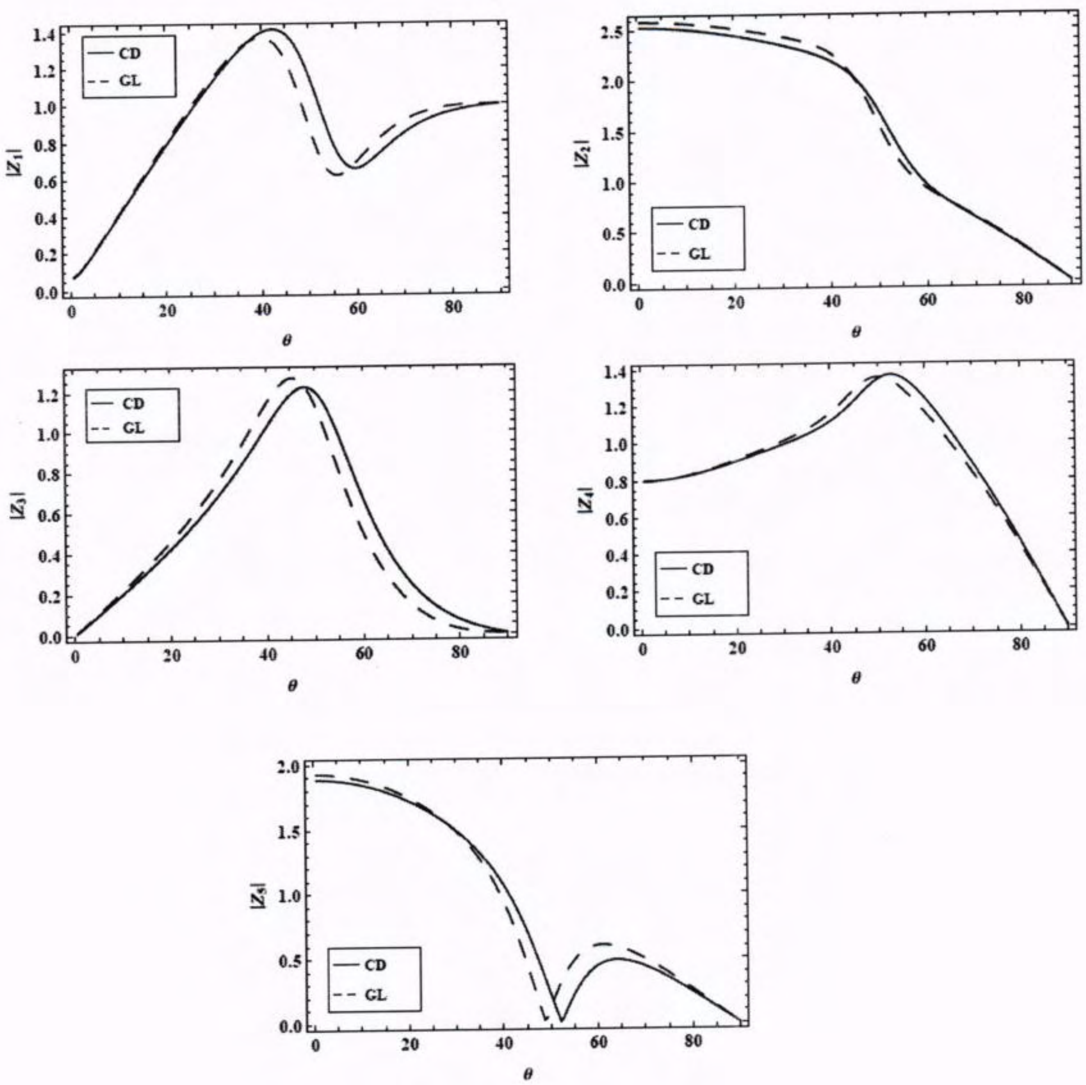


Fig. 3.4: Difference of the amplitudes $|Z_i|$, $(i = 1, 2, ..., 5)$ making an incident angle of P-waves under different theory, $H = 0.4$, $\varepsilon = 0.08$, $\beta^* = 0.001$.

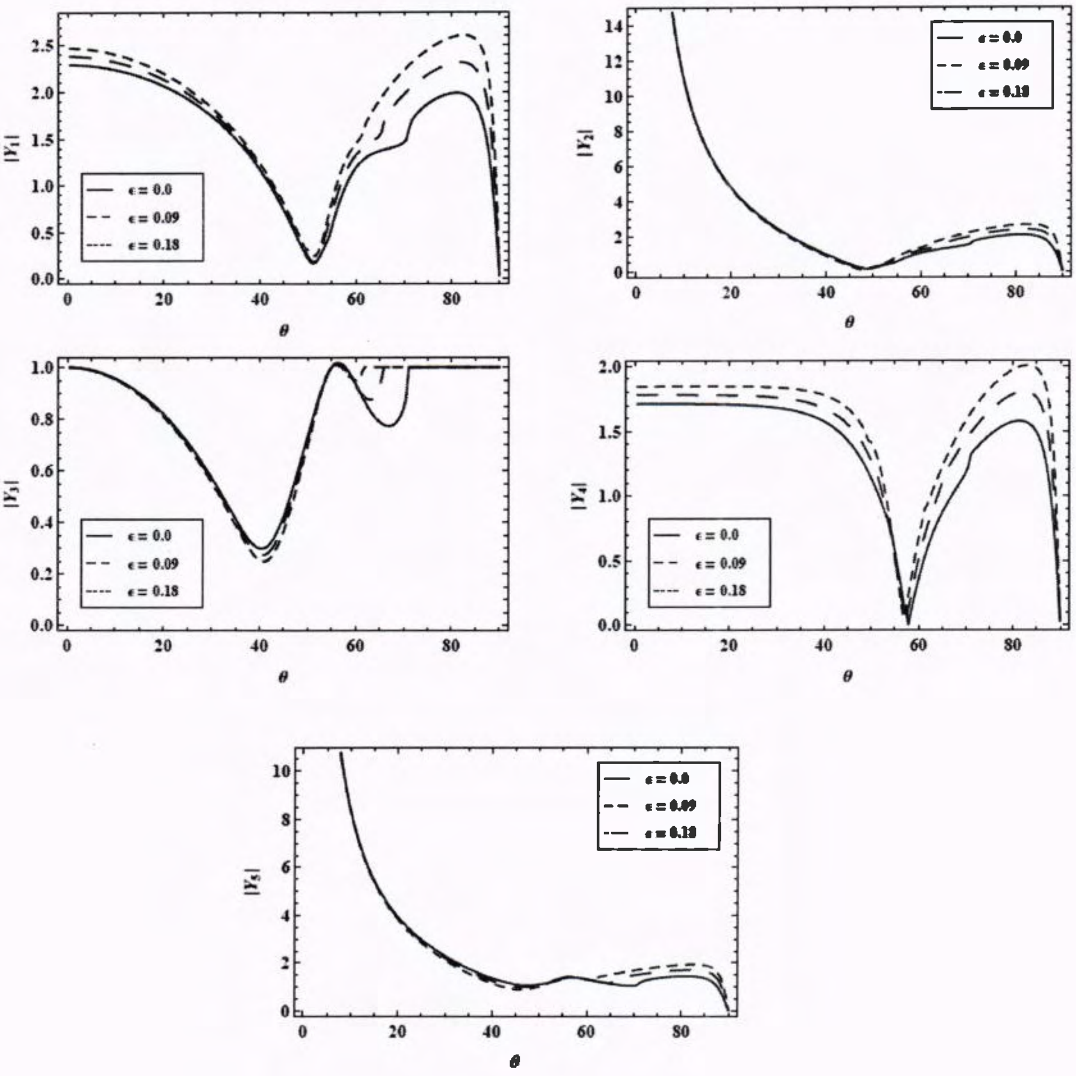


Fig. 3.5: Difference of the amplitudes $|Y_i|$, $(i = 1, 2, ..., 5)$ making an incident angle of SV-waves for effect of coupling parameter, $H = 0.5$, $\beta^* = 0.001$.

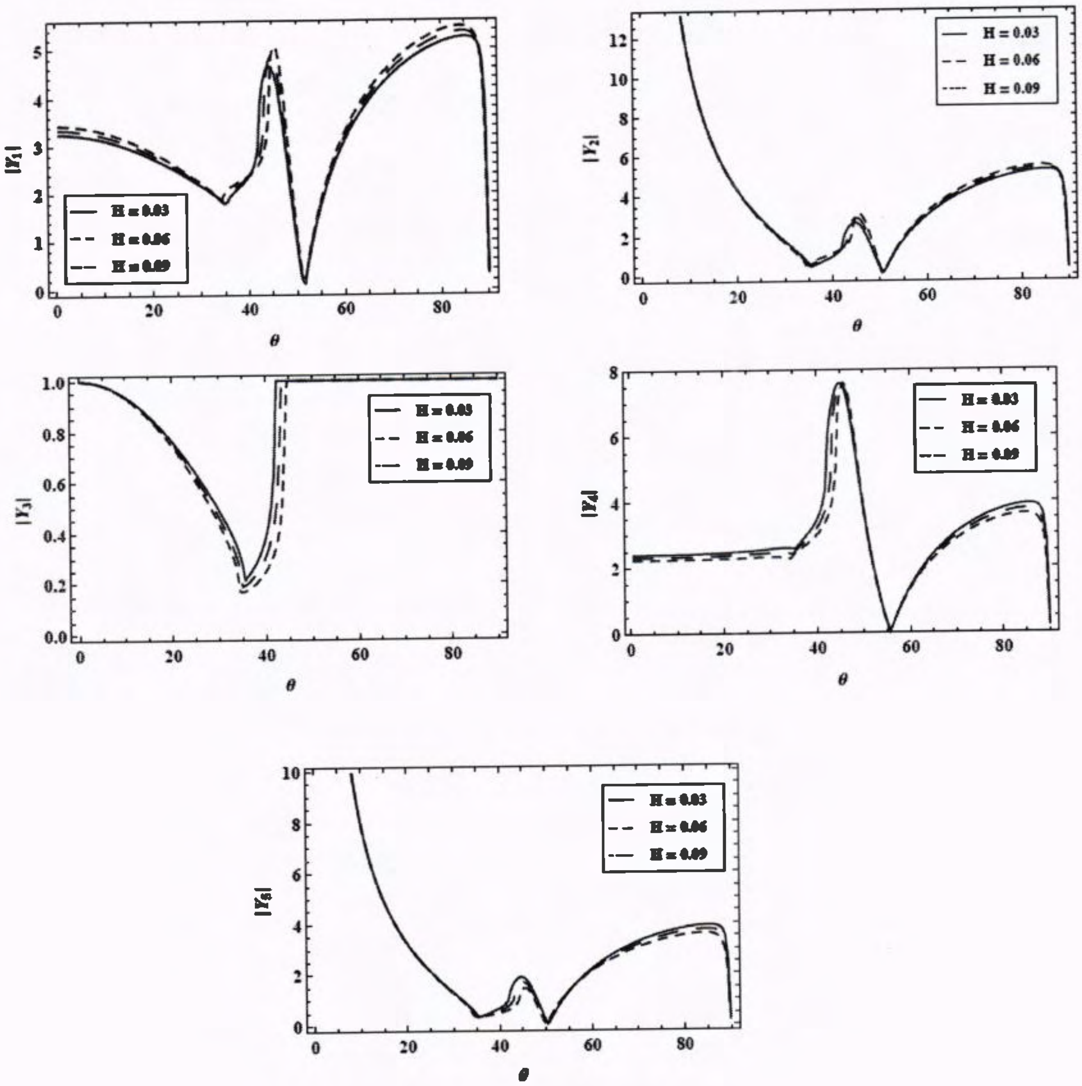


Fig. 3.6: Difference of the amplitudes $|Y_i|$, $(i = 1, 2, ..., 5)$ making an incident angle of SV-waves for effect of magnetic field, $\varepsilon = 0.7$, $\beta^* = 0.001$.

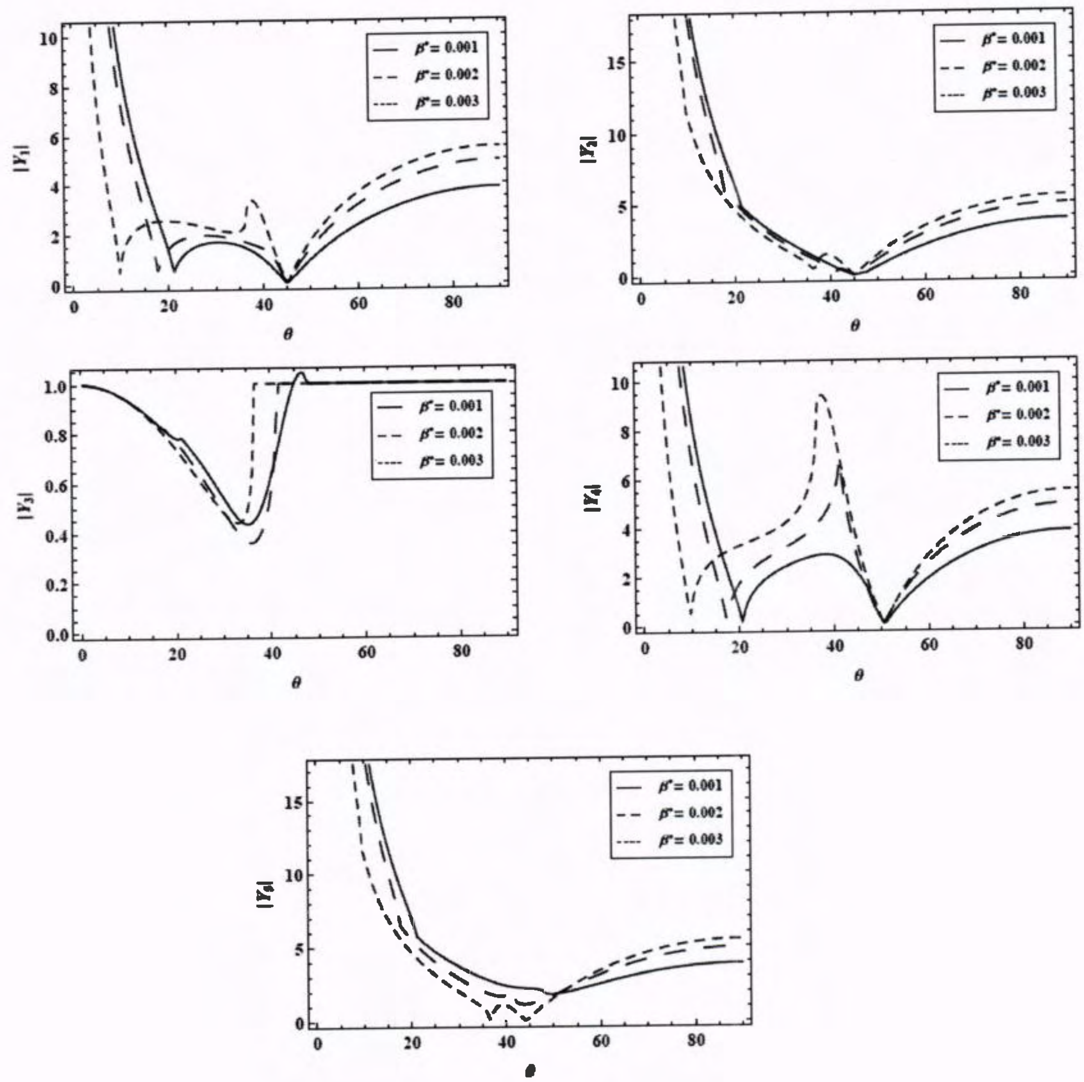


Fig. 3.7: Difference of the amplitudes $|Y_i|$, $(i = 1, 2, ..., 5)$ making an incident angle of SV-waves for temperature dependent modulus effect, $H = 0.0$, $\varepsilon = 0.7$.

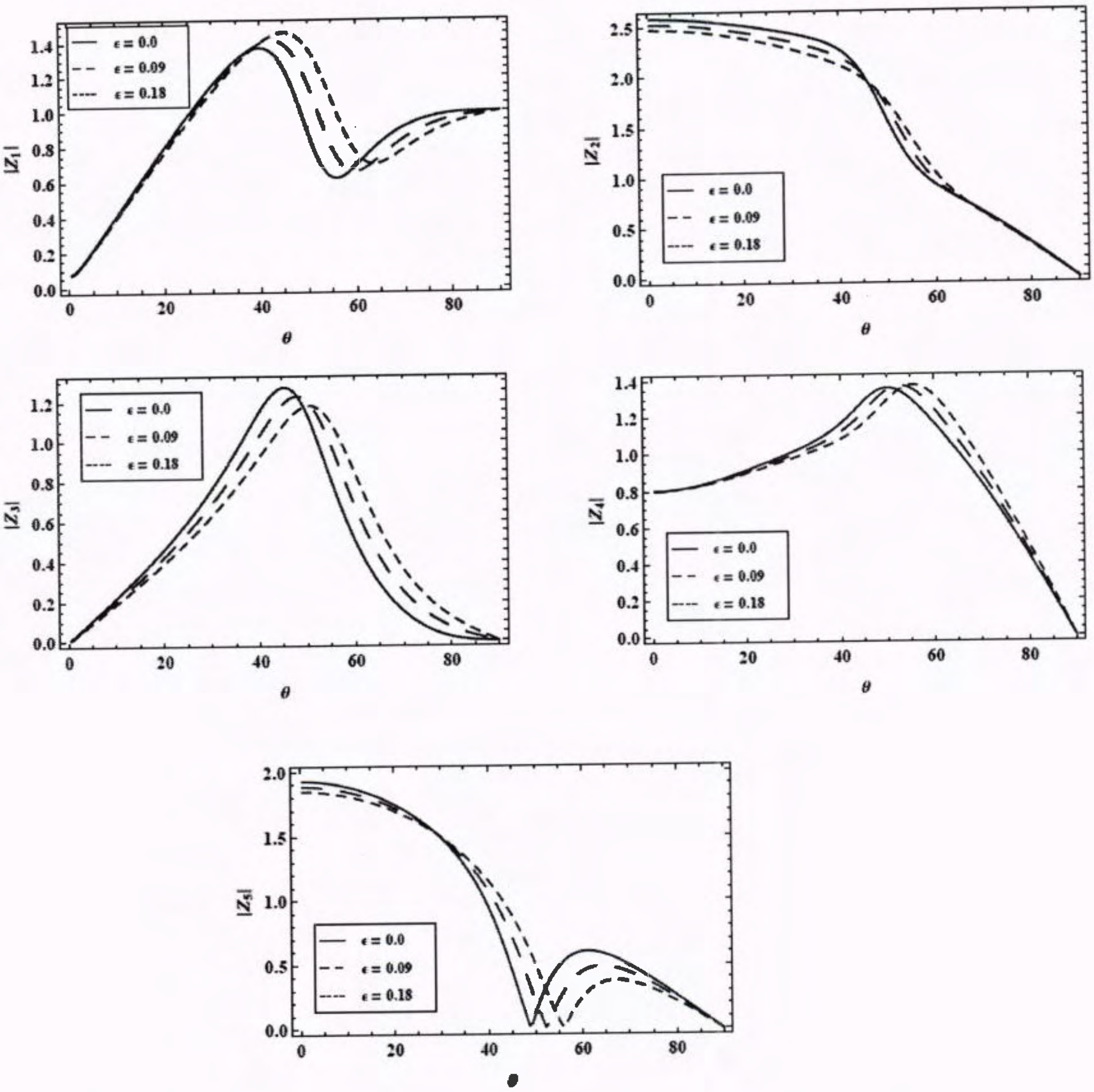


Fig. 3.8: Difference of the amplitudes $|Z_i|$, $(i = 1, 2, ..., 5)$ making an incident angle of P-waves for coupling parameter, $H = 0.5$, $\beta^* = 0.001$.

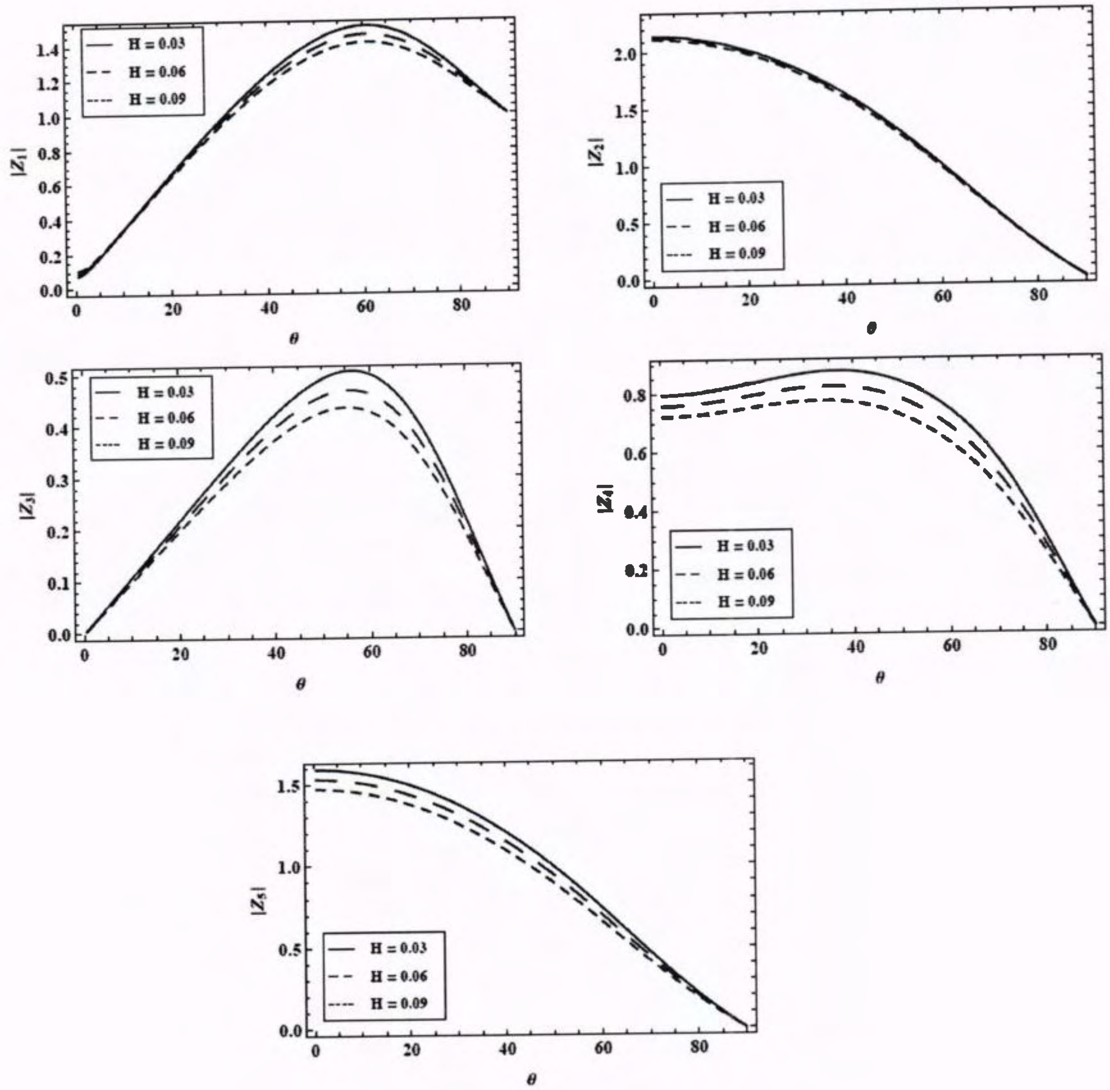


Fig. 3.9: Difference of the amplitudes $|Z_i|$, $(i = 1, 2, ..., 5)$ making an incident angle of P-waves for effect of magnetic field, $\varepsilon = 0.7$, $\beta^* = 0.001$.

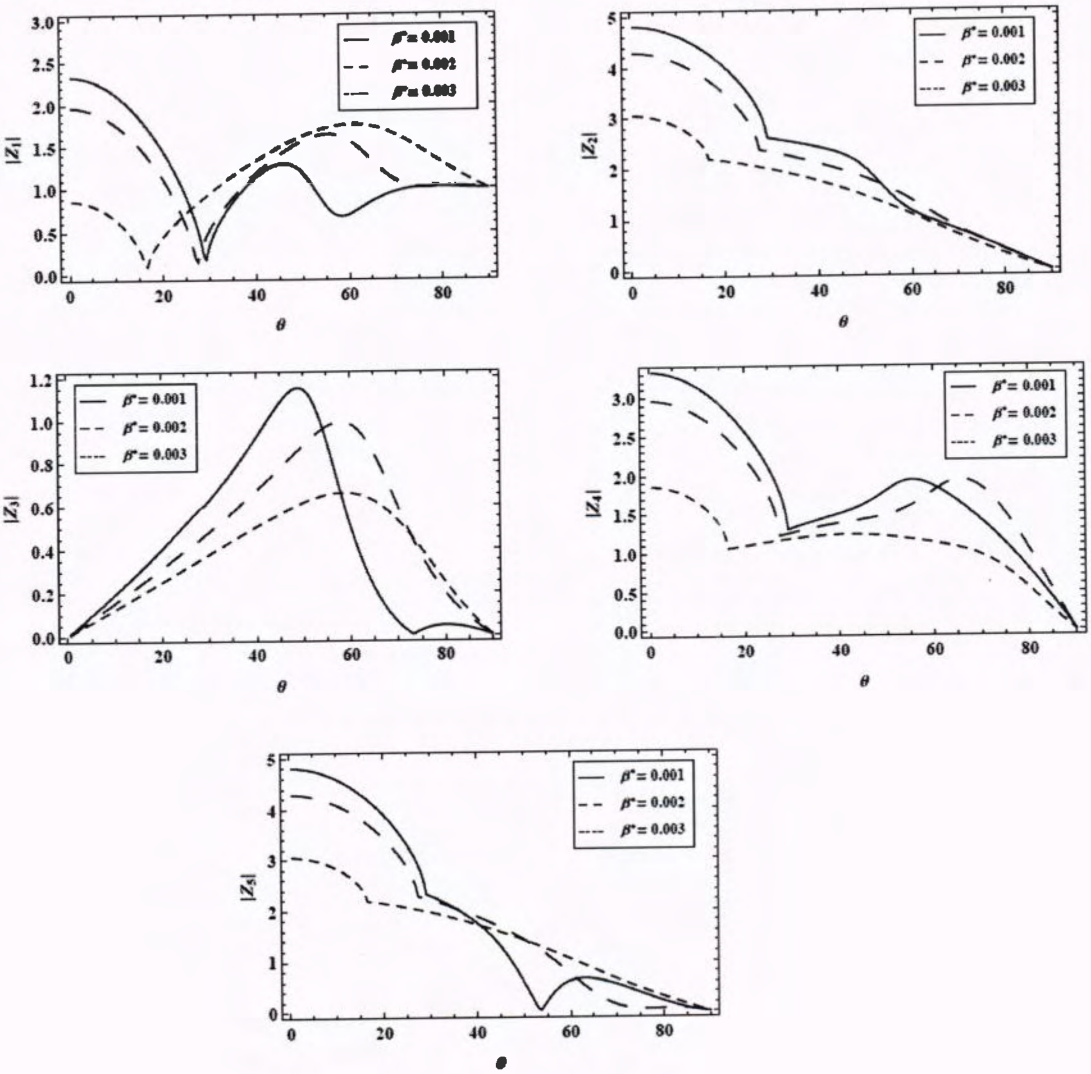


Fig. 3.10: Difference of the amplitudes $|Z_i|$, $(i = 1, 2, ..., 5)$ making an incident angle of P-wave for temperature dependent modulus effect, $H = 0.0$, $\beta^* = 0.001$.

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