MODEL SPECIFICATION AND UNIT ROOTS

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DECLARATION

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APPROVAL SHEET

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ABSTRACT

Despite a lot of research on unit roots, consensus on several important issues and implications has not emerged to date (Libanio, 2005). Conflicting opinions exist on the existence of unit root in economic series being investigated by multiple researchers. The development of literature on unit root and cointegration is mainly stimulated by two problems: (i) The results of unit root tests are often misleading; application of unit root test requires number of prior specification decisions and improper choice of the specification decisions results in misleading inference. (ii) Classical techniques for specification of economic model using time series data are often misleading; In the presence of unit root many of conventional inference procedures and tests used for model specification are invalid and hence result in misleading inference.

This thesis makes two contributions to obtain more reliable inference from unit root tests. First, the existing literature does not provide satisfactory solution for choice of deterministic part in a model, to be used for testing unit root. We propose a new method for specification of deterministic part. The performance of new method is compared with various alternatives via Monte Carlo simulations and results show that the method works better than the alternatives.

Second, for a given data series it is generally not possible to decide which of unit root tests would be the best. The performance of unit root tests depends on the type of data generating process (DGP), but for the real data we do not know the true DGP, hence, we cannot decide which of tests would perform best for a given time series. The bootstrap approach of Rudebusch (1993) offers an alternative to measure the performance of unit root test for any real time series with unknown DGP. Rudebusch (1993)'s approach is extended to measure and compare the performance of unit root tests for real GDP series of various countries. Our results show that unit root tests have very low probability to discriminate between best fitting trend stationary and difference stationary models for GDP series of most of countries. Phillips Perron test is superior to its rivals including Dickey-Fuller, DF-GLS and Ng-Perron tests. The results also support existence of unit root in real GDP series.

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If I travel back into history to the point when I was a student of grade 10, I would find it impossible to think of perusing PhD in Econometrics. I had never heard the word 'Econometrics' until my M.Sc. Then I managed to become enrolled in highest degree in the subject under guidance of world renowned scholar Dr. Asad Zaman.

In fact my efforts and talent played very little role in my enrollment in this degree program. I remember my class fellows in school and college life who were more brilliant than me in their academics, were eager to get higher education, but were unable to continue their studies. This is because it was not an easy job to be enrolled in higher studies for a student belonging to Neelum Valley. This area had been a war-field during 1990-2004. The proxy war between Indian and Pakistani troops across the line of control destroyed the social and educational infrastructure of the area. Even schools and hospitals were not safe from the Indian firing. So there were little chances for a student to continue the study.

I cannot ignore a series of incidents making it possible for me to get education. These incidents are:

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- Brave efforts of my sisters Amina & Safia to bear my educational expenditures when I was in college.
- Choice of Mathematics as subject in my M.Sc.
- Award of scholarship for doctoral studies from HEC

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CHAPTER 1

UNIT ROOT IN ECONOMICS & ECONOMETRICS

1.1: INTRODUCTION

Past three decades have witnessed very fast development in unit root literature. Perhaps, unit root and cointegration has been most debated issue in Econometrics. However, despite a lot of research, consensus on several important issues and implications has not emerged to date (Libanio, 2005). Conflicting opinions exist on the existence of unit root in economic series being investigated by various researchers. The growth of literature on unit root and cointegration is mainly stimulated by two problems:

i. The results of unit root tests are often misleading; application of unit root test requires a number of prior specification decisions, and improper choice of the specification decisions results in misleading inference.

ii. Classical techniques for specification of economic model using time series data are often misleading; In the presence of unit roots, many of conventional inference procedures and tests used for model specification are invalid, hence, result in misleading inference.

Our focus in this thesis is on first problem, which is reflected in size and power distortion of commonly used unit root tests. Performance of unit root tests depends on several specification decisions prior to their application e.g. whether or not to include a deterministic trend and how to choose the number of the included lags in a model. Practitioners routinely make several arbitrary specification decisions to specify the model used for testing unit root. For real data series, arbitrary specification decisions are often unjustifiable and sometimes incompatible with data [see Andreou & Spanos (2003) and Atiq-ur-Rehman & Zaman (2008)]. Specification of model before application of unit root tests is a major challenge in application of unit root tests.

This thesis makes two contributions to obtain more reliable inference from unit root tests.

First, we propose a method for specification of deterministic part in a model, to be used for testing unit root. Second, via extensive Monte Carlo experiments, we measure capability of various unit root tests to discriminate between trend and difference stationary models congruent to the best fitting models of either class for real time series. We use GDP data for a large pool of countries for this purpose and recommend a best test on the basis of these experiments.

The rest of this chapter is organized as follows; Section 2 of this chapter discusses importance of theme of unit roots in economics and econometrics. Implications of unit root regarding the economic theory, policy making and the econometric practices are discussed. Section 3 consists of brief history of major developments in unit root literature. Section 4 consists of discussion on inconclusiveness of unit root debate and the role of model specification. Section 5 describes contribution of the thesis in resolving problems posed by model specification and the outline of the thesis.

1.2: WHY ARE UNIT ROOTS IMPORTANT?

The presence or otherwise of unit root in a time series plays a very important role in determining statistical and economic properties of the series. Therefore, the literature on unit root developed very rapidly during past three decades. In the words of Haldrup and Jansson (2005), 'Since the mid-1980s there has been a *veritable explosion* of research on unit roots in the analysis of economic and other time series data'. To the question 'why do we care about unit roots?', Cribari-Neto (1996) provides the following interesting response:

> *...to a policymaker the answer could be: 'Because the policy implications are different.' To a macroeconomist, it could be answered that 'there are theoretical implications on several theories and models.' Finally, an econometrician would be satisfied with the answer: 'Because the asymptotics are different.'*

Cribari-Neto (1996) very beautifully summarized wide range of implications of unit roots. His view about importance of unit roots can be divided into three ideas: (i) Policy implications (ii) Macroeconomic theories (iii) Econometric implications. Presence or otherwise of unit root substantially affect all three areas discussed. For example, consider the econometric implication of unit roots, the distribution of conventional test statistics and estimators are simply invalid when there is unit root in the time series.

Although this thesis is not about the philosophy of unit root testing, it is important to mention some of the implications of unit roots to make it clear that the idea of unit root has deeper implications than apparent from a purely statistical perspective. In this section, we review some of the implications of unit root regarding econometric procedures and the economic theory in non-technical and intuitive way. Avoiding intricate algebraic detail, we focus on the main concepts related to the implications of unit root in empirical work. We elaborate the concepts using AR(1) model; however, similar logic applies to more complicated models.

1.2.1: IMPLICATIONS FOR ECONOMIC THEORY

Existence of unit root in a time series has serious implications regarding economic theory. Economic theory often predicts that a variable should be stationary or otherwise. Here we mention a few implications of unit roots in macroeconomic theories:

Let us start by a simple example; suppose we are interested in exploring whether or not there is long run relationship between two time series x_t and y_t. Suppose the relationship exists and it is given by: $y_t = a + bx_t + \varepsilon_t$. If ε_t is a unit root process, than it can deviate from its expectation for a long time unboundedly, so that y_t can be unboundedly away from its long run path for a long period of time. This contradicts with the existence of long run equilibrium relationship between x_t and y_t. Therefore, a long run relationship exists only, if ε_t is stationary; the answer to purely theoretical question of long run equilibrium relationship depends on the existence or otherwise of unit root. This line of argument led to the development of concept of the cointegration. Loosely speaking, cointegration occurs when a linear combination of some unit root series is stationary. Existence of Purchasing Power Parity (PPP) is another example of this type of question. If PPP exists, the real exchange rate should be stationary so that domestic currency always has same purchasing power in the foreign markets.

Here are some other economic theories whose validity depends on existence of unit root.

Efficient Markets Theory of Assets:

Efficient markets theory of asset pricing by Fama (1970) suggests that if future excess returns were predictable, they would provide chance of exploitations so that the price (or log price) would follow a random walk.

Real Business Cycle theory:

At first, evidence of unit roots in time series (Nelson and Plosser, 1982) was used to provide support for theories of fluctuations based on real factors. Nelson and Plosser (1982) argue that most of the fluctuations in output should be attributable to changes in the trend component, in a trend versus cyclical decomposition. The existence of unit roots implies that movements in output are persistent. Since the cyclical component is assumed to be stationary, it follows that output fluctuations are mostly associated with the stochastic component. The argument is supported by the idea that monetary shocks are necessarily temporary and so can only affect the cyclical component, and that the long run path of the economy is mainly guided by real factors such as tastes and technology.

New Keynesian Aggregate Fluctuation models:

First reaction to the conclusions of Nelson and Plosser can be seen as an attempt to promote New-Keynesian models of aggregate fluctuations, in which, GNP is expected to revert to a long run trend but the adjustment process can be very slow due to imperfections in goods and labor markets. A number of papers were published during the 1980s with different arguments in this direction. Campbell and Mankiw (1987), McCallum (1986) and others present various arguments in favor of/against the New Keynesian Model of aggregate fluctuations.

Models for Income and Consumptions:

If labor income has a unit root, then a simple version of the inter-temporal permanent income hypothesis (PIH) implies that consumption will also have a unit root, and that income minus consumption (savings) will not have a unit root, so that consumption and income are cointegrated (Stock, 1995).

1.2.2: ECONOMETRIC IMPLICATIONS

The econometric implications of unit roots cover almost many econometric procedures including estimation, testing and forecasting. A brief review is presented as under:

1.2.2.1: Point/Interval Estimation

Consider the unit root process

$$y_t = \rho y_{t-1} + e_t . \tag{1.1}$$

The least square estimator for ρ is given by

$$\hat{\rho} = \left(\sum_{t=2}^{T} y_{t-1}^2\right)^{-1} \sum_{t=2}^{T} y_t y_{t-1} \tag{1.2}$$

Mann and Wald (1943) proved that under the assumption of IID errors, when $|\rho| < 1$, $\sqrt{T}(\hat{\rho} - \rho)$ converges in distribution to $N(0, 1 - \rho^2)$. This suggests that expected value of the estimator $\hat{\rho}$ is equal to the true ρ, therefore, the estimator is unbiased. However, when $\rho = 1$, the least square estimator is biased towards negativity even asymptotically (Phillips, 1987). Hence if we are interested in the point estimates, use of OLS estimator can be problematic.

1.2.2.2: Hypothesis Testing

Dickey and Fuller (1979) observed that in regression model (1.1), the usual t-statistics for testing $\rho = 1$ does not have standard Student's t-distribution under the null. They tabulated the percentiles of t-statistics for the unit root processes. Phillips (1987) proved that the distribution of t-statistics is non-standard and converges to some function of Wiener process. This means that we cannot use conventional statistical tools for hypothesis testing, when there is unit root in the series and that the hypothesis $\rho = 1$ cannot be tested by conventional hypothesis testing techniques.

Consider the autoregressive model described in equation (1.1), the t-statistics for testing $\rho = \rho_0$. Under the null hypothesis, if $\rho = \rho_0$, and $|\rho_0| < 1$ then Mann & Wald theorem implies that the test statistics has asymptotic normal distribution $t_\rho \sim N\left(\rho_0, \frac{1}{T}(1-\rho_0^2)\right)$. If $|\rho_0| = 1$, it follows that $t_\rho \sim N(1,0)$, but this limit is obviously not valid. Therefore, the Mann & Wald theorem does not hold for the unit root processes. Phillips (1987) proves that the t-statistics converges in distribution to a function of Wiener process. Further analysis yield following interesting results:

i. $\hat{\rho}$ is super-consistent; that is, it converges to ρ more rapidly than conventional estimators.

ii. $\hat{\rho}$ is not asymptotically normally distributed and t_{ρ} is not asymptotically standard normal. Its limiting distribution is called the Dickey-Fuller (DF) distribution and does not have a closed form representation. Therefore, quintiles of the distribution must be computed by simulation or by numerical approximation.

The above discussion implies that conventional hypothesis tests are not valid for the unit root process and different hypothesis testing techniques are required when there is unit root in the time series.

1.2.2.3: Spurious Regression

The most important implication of unit root is, the presence or otherwise of spurious regression. A spurious regression occurs when a pair of independent series, with strong temporal properties, is found apparently to be related according to standard inference in an OLS regression (Granger et al. 1998). Yule (1926) showed that two unrelated economic time series might have strong correlation, but he was not sure of the reasons responsible for these results. He argues that spurious correlations appear when we drop some theoretically relevant series from the model, in particular the linear trend.

Consider two independent random walks

$$\begin{aligned} x_t &= x_{t-1} + e_t \qquad e_t \sim iid(0, \sigma_e^2) \\ y_t &= y_{t-1} + u_t \qquad u_t \sim iid(0, \sigma_u^2) \end{aligned}$$

Consider the regression $y_t = a + bx_t + e_t$, since the two series are independent, the true value of b is zero and the estimated coefficient should be insignificant in the regression output. Similarly, growth in one series is unrelated to growth in the other series so that R-square should be closer to zero. However, Granger & Newbold (1974) observed that in the above setup, the frequency of rejection of (true) hypothesis $b = 0$ is much greater than the nominal 5% significance level, and R-square is unusually high.

The results of Granger & Newbold (1974) were explained by Phillips (1987) who derived limiting distribution of regression coefficient $\hat{b}$ and proved that this coefficient does not have asymptotic normal distribution, as is the case for stationary variables.

In fact, there is always some possibility of spurious regression when there is some non-stationary variable involved in the regression. This fact is illustrated in the following table:

Series X/Series Y	Trend Stationary	Unit root
Trend Stationary	Valid Regression	Spurious Regression
Unit root	Spurious Regression	Spurious Regression unless the two series are cointegrated

The Table shows that the existence of unit root carries the risk of spurious regression. Therefore, to establish validity of regression output, it is very important to know whether or not the series is stationary.

1.2.3: POLICY IMPLICATIONS

We can divide the policy implications of unit roots into two types: indirect implications and direct implications. The two types of implications are discussed below:

1.2.3.1: Direct Policy Implications

The direct implications are due to characteristic of a single economic time series. Let us mention an intuitive example first. Suppose the GNP of a country contains unit root. This means the effect of a negative cut in income will persist over time, Therefore, the government will be more reluctant to induce a tax cut to facilitate some industry. Hamilton and Flavin (1986) argued that the economic notion of sustainability of budget deficit translates to the statistical notion of stationarity of series. That is, a stationary budgetary position is consistent with the idea that a government should run a sequence of discounted future non-interest budget surpluses capable of offsetting the current outstanding debt/deficit.

1.2.3.2: Indirect Policy Implications

Indirect implications are via the econometric models used in the policy making. Ultimate use of econometric models is the assessment of alternative economic policies [Ericsson et al (1998)]. Although classical econometric theory generally assumed stationary data, particularly constant means and variances across time periods, empirical evidence is strongly against the validity of this assumption. Stationary and non-stationary data need different sets of tools for developing a valid econometric model. As we have discussed in section (1.2.2) many classical econometric procedures are simply invalid in unit root regime. Hence data based economic policymaking rests on assumptions regarding existence of unit roots in the data. Therefore, the range of indirect implications of unit root in policy making is as wide as the implications of time series itself. For further details, reader is referred to Hendry and Juselius (1999) and Chinn (1991).

Chinn (1991) examines several cases where trend and difference stationary models lead to quite different policy implications. In general, it is assumed that depreciation in real exchange rate will enhance the exports and will lower the trade deficit, but in 1985-87, US$ depreciated by about 40% accompanied by a record trade deficit of $112 billion. Chinn argues that this happened because of belief of economists in so called J curve, which says that if exchange rate is decreased, the trade balance will deteriorate for short term and then it will rise. But Chinn (1991) argues that all econometric models supporting J curve are built on assumption of stationarity, whereas, the time series used has been proven to be non-stationary. So, he attributes failure of exchange rate policy to the model built on assumption of stationarity.

1.3: BRIEF HISTORY OF DEVELOPMENTS IN UNIT ROOTS MODELS AND TESTS

This section gives brief review of the history of application of unit root models in econometrics and the development of statistical theory/tests for unit root. The statistical models with autoregressive coefficient equal to or near unity are familiar to econometricians since 1940's but the research in this area got momentum after study of Nelson and Plosser (1982). The statistical theory of autoregressive process with root near unity has been active area of research during past three decades and a flood of articles emerged so far. Several good reviews and commentaries of literature in this area are already available. For example, Phillips (1988, 1992), Campbell and Perron (1992), Banerjee et al. (1993), Maddala and Kim (1998), Stock (1995) and Perron (2005) provide excellent commentary of development in this area. Therefore, a comprehensive survey of the development in unit root would be unnecessary and undesirable addition to the already prolific literature. We just go through major development in unit roots and the studies that are closely related to topics treated in this thesis.

Models with high persistence are familiar to econometricians since 1940's. Orcutt (1948) found high serial correlation in the econometric model of US economy developed by Tinbergen (1939). Orcutt examined a number of time series and concluded that they are better described by the model $\Delta y_t = 0.3\Delta y_{t-1} + e_t$, which is a unit root model. Mann and Wald (1943) developed theory of least square estimator of stationary autoregressive model which was extended to unit root and explosive models by White (1959), Anderson (1959) and Rao (1961). According to Stock (1995), it was customary in 1960's and 1970's to model economic relationship in differences which is more appropriate method for unit root process.

Formal test for unit root were developed by Fuller (1976), Dickey (1976) and Dickey & Fuller (1979). These tests test the null hypothesis $\rho = 1$ versus one sided alternative $\rho < 1$ in one of following autoregression:

$$y_t = \rho y_{t-1} + \varepsilon_t$$
$$y_t = \alpha + \rho y_{t-1} + \varepsilon_t$$
$$y_t = \alpha + \beta t + \rho y_{t-1} + \varepsilon_t$$

It was proved by Mann & Wald (1943) that if $\rho < 1$ and $\hat{\rho} = \left(\sum_{t=1}^{T} y_{t-1}^2\right)^{-1} \sum_{t=1}^{T} y_t y_{t-1}$, than $\sqrt{T}(\hat{\rho} - \rho) \Rightarrow N(0, 1 - \rho^2)$. Rubin (1950) proved that $\hat{\rho}$ is consistent estimator for all values of ρ. However, the distribution of $\hat{\rho}$ was unknown when $\rho = 1$. Dickey & Fuller (1979) derived limiting distribution of $\hat{\rho}$ and $t_{\hat{\rho}} = \hat{\rho} / SE(\hat{\rho})$ when $\rho = 1$. These distributions are nonstandard and converge to functions of Wiener process (Phillips, 1987). Dickey & Fuller (1979) provided various statistics for testing presence of autoregressive unit root. These statistics are:

K-statistics $T(\hat{\rho} - 1)$

t-statistics $t_{\hat{\rho}} = (\hat{\rho} - 1) / SE(\hat{\rho})$

The asymptotic distribution of $t_{\hat{\rho}} = (\hat{\rho} - 1) / SE(\hat{\rho})$ is nonstandard. The numerator is right skewed, but the ratio is left skewed.

Meanwhile, Meese & Singleton (1982) and Nelson & Plosser (1982) applied Dickey Fuller test to various economic time series and found that they were unable to reject presence of autoregressive unit root in most of the series. These findings enhanced the professional interest in unit root tests, since many econometric procedures and economic theories hinge on the existence of unit root. Findings of Nelson and Plosser have been supported by many authors in next few years. This led to faster development in theory of unit root tests.

The limiting distribution of $t_{\hat{\rho}}$ developed by Dickey & Fuller (1979) depends on assumption of independence of error structure. In case of serially correlated errors, the distribution tabulated by Dickey & Fuller (1979) is not valid. To get consistent estimator and test with serially correlated error structure, either the regression equation should be changed or the test statistics should be modified.

Modification to the regression equation is due to Dickey & Fuller (1981) and Said & Dickey (1984), whereas, modification to unit root test statistics is due to Phillips (1987) and Phillips & Perron (1988). Sargan & Bhargava (1983) generalize Durbin Watson

(DW) test and Brenblutt-Webb (WB) Test to use for unit root testing. Hall (1989, 1992) proposed Instrumental variable (IV) test for unit root in ARMA(p,q) models. Further modifications to this test are due to Li (1995), Lee and Schmidt (1994) and Choi (1992).

Elliott, Rothenberg & Stock (1996) used King (1988) approach to develop best point optimal test. They find out a test whose power function is tangent to the power envelop and never far below it. They then find a test which has power function closest to this test. This test is based on GLS detrending. Detrending is a procedure of subtracting deterministic part from a time series. There are several detrending techniques which differ in minute computational detail and exhibit a variety of characteristics.

Another approach to differentiate between trend and difference stationarity is to use stationarity as a null rather than the unit root. This type of tests are due to Tanaka (1991), Kwiatkowski et al. (1992) and Leybourne and McCabe (1994) etc. The most popular test of this kind is KPSS due to Kwaitkowsky, Phillips, Schmidt and Shin (1992). Consider the representation of a time series in terms of sum of unit root and stationary process:

$$y_t = \delta t + \xi_t + u_t$$

where u_t is stationary and ξ_t is random walk $\xi_t = \xi_{t-1} + e_t \;\; e_t \sim iidN(0, \sigma_u^2)$. If y_t is trend stationary than variance of random walk component would be zero, Therefore, test for stationarity is equivalent to testing $\sigma_u^2 = 0$ versus $\sigma_u^2 > 0$. Kwiatkowski et al. (1992)

used LM statistics developed by Nabeya and Tanaka (1988) for testing stationarity. The test statistics is:

$$LM = \left(\sigma_e^2\right)^{-1} \sum_{t=1}^{T} S_t^2$$

where e_t denotes the least square residuals from regression of y_t on constant and trend, σ_e^2 is variance of residuals and $S_t = \sum_{i=1}^{t} e_i$.

Perron (1989) opened a new avenue in the theory of unit root testing. Perron (1989) proves that unit root tests are biased against stationarity, if there is structural break in the deterministic part of the series. Perron suggests that the strong evidence for unit root observed by Nelson & Plosser (1982) and successors was due to failure to account for structural breaks in the series. Perron modified ADF test to incorporate structural breaks and using these tests, reversed the conclusion of Nelson & Plosser (1982) for many time series. An enormous literature emerged after the study of Perron (1989) analyzing impact of structural breaks, methods to find and test the breakpoints and to design powerful tests in presence of structural breaks. Zivot and Andrew (1992), Banejee et al. (1992) proposed tests for unit root with endogenized structural change. Further developments to unit root tests with known/unknown structural breaks date are due to Kunitomo and Sato (1995), Amsler and Lee (1995), Saikkonen and Lütkepohl (2001), & Lanne et al. (2002) etc. For a comprehensive survey of literature see Perron (2005).

1.4: UNIT ROOT DEBATE AND MODEL SPECIFICATION

Perhaps the issue discussed most in the history of econometric literature is the debate on trend versus difference stationarity, initiated by Nelson and Plosser (1982). We have described in detail the importance of information about existence of unit root with regard to econometric procedures and economic theory. The empirical relevance of unit root led to a huge amount of research in the past three decades, with no consensus on several basic questions. Even though vast numbers of unit root tests have been proposed and studied, conflicting opinions exist on the simplest of problems. For example, here is a list of the conclusions of authors who have studied the USA annual GNP series:

Difference stationary; Nelson and Plosser (1982),

Trend Stationary; Perron (1989),

Trend Stationary; Zivot and Andrews (1992),

Don't know; Rudebusch (1993),

Trend stationary; Diebold and Senhadji (1996),

Difference stationary; Murray and Nelson (2002), Kilian and Ohanian (2002),

Trend stationary; Papell and Prodan (2003)

Similarly, the so-called purchasing-power parity is another controversy in Econometrics that led to purchasing-power parity (PPP) puzzle. Purchasing-power parity puzzle takes one of two forms. In its first form, early tests of the PPP-hypothesis failed to reject unit roots in real exchange rates, thus rejecting the hypothesis of PPP holding in the long term. In the more recent literature, the literature on the PPP puzzle focused on stochastic real exchange rate models that allow long term PPP to hold. One can find and list a lot of controversial results on this issue with the conclusion that no definite answer could be found so far. The following quote by El-Gamal and Ryu (2003) reflects the ambiguity in the consequence of PPP debate:

> *... In particular, we show that it is possible to match desired "half-lives" for any of the most popular non-linear models recently proposed in the literature, at the expense of matching their more general dynamic structure. We conclude that depending on the models and criteria selected for investigating the PPP-puzzle, the puzzle may be in the eye of the beholder.*

Similarly, take any series that has been explored many times for stationarity, you will find number of conflicting conclusions.

A major reason responsible for ambiguity in the inference of unit root tests is the model specification prior to application of unit root tests. Performance of unit root tests depends on several specification decisions prior to application of unit root test e.g. whether or not to include a deterministic trend and how to choose the order of the included lags in the model. Practitioners routinely make several arbitrary specification decisions either implicitly or explicitly. Since Monte Carlo studies take these initial decisions as valid background information, such studies often overestimate the performance of tests on real data. In Monte Carlo, when the experiments condition on some implicit specification, the design of data generating process supports the implicit assumptions. But for the real data series, implicit assumptions/arbitrary specification decisions are often unjustifiable and sometimes incompatible with data (Rehman & Zaman, 2008).

For some of these specification decisions, there exist well documented and analyzed procedures and techniques e.g. selection of lag length and presence of structural break. The choice of lag length and presence of structural breaks have a lot of literature to their credit; see Ng and Perron (2001) and Perron (2005) for detailed surveys. However, existing literature does not provide satisfactory solution for choice of deterministic part in a model used for testing unit root (Elder and Kennedy, 2001). A part of this thesis is devoted to discuss the procedure for specification of deterministic part. A new procedure is proposed for specification of deterministic part and the performance of this procedure is illustrated via extensive Monte Carlo experiments.

Similarly, an absence of information about deterministic part in DGP of a series makes it difficult to choose the best test for a time series. This problem led to variety of opinion about stationarity of GDP series. Chapter 5 of this thesis is devoted to measure performance of unit root tests for GDP series of various countries and to compare various unit root tests on the basis of these measure of performances.

1.5: OUTLINE OF THE THESIS

The rest of this thesis is organized as follows:

Chapter 2 consists of discussion of technical background required to understand this thesis. It provides brief review of unit root tests utilized in the thesis, the asymptotic theory of unit root processes and other relevant background information.

Chapter 3 consists of brief review of literature most relevant to our study. The discussion mainly addresses the specification of deterministic part in a model used for testing unit root, and a discussion on the opinion of econometricians about stationarity of GNP series.

Chapter 4 is devoted to propose new procedure for the specification of deterministic part in autoregressive model. First, evidences are presented that the decision of deterministic part is very important to determine output of unit root test. Next, it is discussed that existing methods and techniques are incapable of specifying deterministic part even in stationary autoregressive series. Than we present new strategy for choice of deterministic part. The performance of this strategy is measured via extensive Monte Carlo experiments.

In chapter 5 we aim to propose best test when we do not know about the true form of data generating process. In particular, we compare various unit root tests for their ability to discriminate between best fitting trend stationary and best fitting difference stationary models of GDP series of various countries. The best test is recommended on the basis of performance in these Monte Carlo simulations.

Chapter 6 presents real application of the two contributions. The extent to which proposed techniques are helpful in solving specification issue is discussed. Conclusion, recommendations and directions for new research are presented in the last section of this chapter.

CHAPTER 2

ESSENTIALS OF UNIT ROOTS

This chapter consists of brief introduction to techniques, results and concepts utilized in the thesis. Most of these results can be found in any advanced Time Series Textbook, therefore, the proofs are not provided. However, reference to the source of results is mentioned. Outline of the Chapter is as follows:

Section 1 consists of the Functional Central Limit Theorem (FCLT) and its implications. Section 2 consists of the discussion on the unit root tests utilized in this thesis and their salient characteristic. Section 3 discusses pre-test model specification techniques utilized in the thesis. Section 4 discusses the approach of Rudebusch (1992) to analyze the performance of unit root tests.

2.1: ASYMPTOTIC THEORY FOR UNIT ROOT PROCESS

2.1.1: WIENER PROCESS

Let $W(.) : [0,1] \to \mathbb{R}$ be a continuous function such that

1. $W(0) = 0$
2. If $0 < r_1 < r_2 \ldots < r_s .. < 1$ than $W(r_i) - W(r_{i-1})$ is independent of $W(r_j) - W(r_{j-1}), i \neq j$
3. For any $t, s \in [0,1]$ $W(s) - W(t) \sim N(0, s-t), for \quad t < s$

Than $W(.)$ is called a wiener process

Wiener process is important because limiting distributions of most of functions of random walk processes converge to the functionals of wiener process. This detail could be found in number of econometric texts including Hamilton (1994).

2.1.2: CONTINUOUS MAPPING THEOREM

According to Continuous Mapping Theorem, Let $\{X_t\}_{t=1}^{\infty}$ be a sequence of random variables with $X_t \Rightarrow X$ and let $g:\mathbb{R}\rightarrow\mathbb{R}$ be a continuous functional, than $g(X_t)\Rightarrow g(X)$

2.1.3: FUNCTIONAL CENTRAL LIMIT THEOREM

According to the functional central limit theorem (FCLT),

Let $\varepsilon_t \sim N(0,\sigma^2)$, and $S_T=\sum_{i=1}^{T}\varepsilon_i$, than S_t is a random walk process. For any $r\in[0,1]$, Let $Tr*$ be the largest positive integer less than or equal to Tr. Define

$$X_T(r)=\frac{1}{T}\sum_{t=1}^{Tr^*}\varepsilon_t$$

Then $\frac{\sqrt{T}}{\sigma}X_T(r)\Rightarrow W(r)$

2.1.4: SOME IMPLICATION OF FCLT FOR UNIT ROOT PROCESSES

Let $y_t=y_{t-1}+\varepsilon_t$ where ε_t is i.i.d. with mean zero and variance σ^2 and $y_0=0$ then

a. $T^{-0.5}\sum_{t=1}^{T}\varepsilon_t \Rightarrow \sigma W(1)$

b. $T^{-1}\sum_{t=1}^{T}y_{t-1}\varepsilon_t \Rightarrow 0.5\sigma^2\left\{[W(1)]^2-1\right\}$

c. $T^{-1.5}\sum_{t=1}^{T}t\varepsilon_t \Rightarrow \sigma W(1)-\sigma\int_0^1 W(r)dr$

d. $T^{-1.5}\sum_{t=1}^{T}y_{t-1} \Rightarrow \sigma\int_0^1 W(r)dr$

e. $T^{-1.5}\sum_{t=1}^{T}y_{t-1}^2 \Rightarrow \sigma^2\int_0^1 [W(r)]^2dr$

The proof of these theorems and implications can be found in Maddala & Kim (1998) and Hamilton (1994), Levin and Lin (1992) etc.

2.2: TESTING FOR UNIT ROOT

In this thesis, we have utilized four univariate unit root tests: Augmented Dickey Fuller (ADF) test, Phillips Perron (PP) test, Dickey Fuller GLS test (DF-GLS) test and Ng-Perron (NP) test. The detail on computation of tests statistics and critical values is discussed in detail in following:

2.2.1: ADF TEST

ADF test is the modified version of test statistics proposed by Dickey and Fuller (1979). ADF test statistics is based on one of following regression equations.

M1	Without drift, trend	$\Delta y_t = \rho y_{t-1} + \sum_{i=1}^{k} \gamma_i \Delta y_{t-i} + e_t$	
M2	With drift, but no trend	$\Delta y_t = \alpha + \rho y_{t-1} + \sum_{i=1}^{k} \gamma_i \Delta y_{t-i} + e_t$	(2.1)
M3	With drift and trend	$\Delta y_t = \alpha + \beta\, t + \rho y_{t-1} + \sum_{i=1}^{k} \gamma_i \Delta y_{t-i} + e_t$	

Where $e_t \sim iid(0, \sigma^2)$

The test statistics is given by $t_{\hat{\rho}} = \frac{\hat{\rho}}{SE(\hat{\rho})}$, where $\hat{\rho}$ is OLS estimate of ρ.

Distribution of test Statistics:

Unlike the standard regression model, the t-statistics in regression equation 2.1 does not have Student's t-distribution and does not have asymptotic normal distribution under the null. Limiting distribution depends on the data generating process and the choice of regression equation used for testing unit root (Hamilton, 1994).

Summary of the limiting distributions is provided by Hamilton (1994) and is summarized below.

Case 1: True model and estimated equations are M1

The DF t-statistics has following limiting distribution

$$t_{\hat{\rho}} \Rightarrow \frac{\int_0^1 W(r)dW(r)}{\left[\int_0^1 [W(r)]^2 dr\right]^{1/2}}$$

Case 2: True model is M1, estimated equation is M2

$$t_{\hat{\rho}} \Rightarrow \frac{0.5\left\{[W(1)]^2 - 1\right\} - W(1)\int_0^1 W(r)dr}{\left\{\int_0^1 [W(r)]^2 dr - \left(\int_0^1 W(r)dr\right)^2\right\}^{0.5}}$$

Case 3: True model is M2, estimated equation is M2

In this case, the limiting distribution $t_{\hat{\rho}}$ is asymptotical Gaussian, can be approximated by standard normal critical values. The asymptotic distribution does not depend on variance and drift.

Case 4: True model is M2, estimated equation is M3

$$t_{\hat{\rho}} \Rightarrow \frac{AB^{-1}C(2,2)}{Q}$$

Where $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$B = \begin{pmatrix} 1 & \int_0^1 W(r)dr & 0.5 \\ \int_0^1 W(r)dr & \int_0^1 [W(r)]^2 dr & \int_0^1 rW(r)dr \\ 0.5 & \int_0^1 rW(r)dr & \frac{1}{3} \end{pmatrix}$$

$$C = \begin{pmatrix} W(1) \\ 0.5\left\{[W(1)]^2 - 1\right\} \\ W(1) - \int_0^1 W(r)dr \end{pmatrix}$$

And

$$Q = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} B^{-1} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Critical Values:

Asymptotic distribution of ADF test statistics is non-standard. Therefore, the critical values are to be computed by simulations or numerical approximations. The critical values of ADF test statistics are provided by McKinnon (1992) computed via Monte Carlo experiments.

2.2.2: PHILLIPS–PERRON TEST

Phillips-Perron test is a unit root test, based on the Dickey-Fuller regression equation. But unlike the Augmented Dickey–Fuller test, which extends the Dickey-Fuller test by including additional lags of variables as regressors in the model, the Phillips-Perron test makes a non-parametric correction to the t-test statistic to capture the effect of autocorrelation.

The Phillips Perron test statistics

The Phillips Perron test statistics is based on one of the three regression equation describe below:

M1 Without drift, trend $\Delta y_t = \rho y_{t-1} + e_t$

M2 With drift, but no trend $\Delta y_t = \alpha + \rho y_{t-1} + e_t$ (2.2)

M3 With drift and trend $\Delta y_t = \alpha + \beta\, t + \rho y_{t-1} + e_t$

Where e_t is stationary stochastic process

These three equations are similar to Dickey Fuller regression equations without any 'augmentation'.

The test statistics is given by:

$$\tilde{t}_{\rho} = t_{\hat{\rho}}\left(\frac{s^2}{\hat{f}(0)}\right) - T\left(\frac{\left(\hat{f}(0) - s^2\right)\left(SE(\hat{\rho})\right)}{2\left(\hat{f}(0)\sum_{t=2}^{T} e_t^2\right)^{0.5}}\right) \qquad (2.3)$$

where $t_{\hat{\rho}} = \frac{\hat{\rho}}{SE(\hat{\rho})}$, $s^2 = T^{-1}\sum_{t=2}^{T} e_t^2$ and e_t are the residuals of the regression. $\hat{f}(0)$ is estimate of spectral density at frequency zero whose estimation procedure is described as under:

Estimating Spectral Density at Frequency Zero

There are various ways of computing spectral density at frequency zero for a series. Following Ng & Perron (2001), we will use autoregressive estimate of spectral density, wherever needed in the thesis. This can be computed as follows:

Consider the ADF regression equation described in (2.1). Estimate number of lags included in ADF equation using some consistent criterion e.g. MAIC. Than the estimate of autoregressive spectral density at frequency zero is given by:

$$\hat{f}(0) = \frac{\hat{\sigma}^2}{\left(1 - \hat{\gamma}_1 - \hat{\gamma}_2 - \ldots \hat{\gamma}_k\right)} \qquad (2.4)$$

Where $\hat{\sigma}^2$ is estimate of error variance and $\hat{\gamma}_i, i = 1, \ldots k$ are the estimated coefficients from regression equation 2.1

Limiting Distribution of Phillips Perron Test Statistics and Critical Values

The limiting distributions of Phillips Perron test statistics are similar to corresponding distributions of Dickey Fuller test. Finite sample critical values are also same.

2.2.3: DF-GLS TEST

Elliott, Rothenberg & Stock (1996), use King (1988)'s approach to develop a best point optimal test. They find a test whose power function is tangent to the power envelope and never far below it. Then they find a test which has power function closest to this test. This test is based on GLS detrending whose procedure is as follows:

Let $y_1, y_2, \ldots y_t$ be the data series. The quasi differenced series is obtained as:

$$\nabla y_t = \begin{cases} y_t & \text{if} \quad t = 1 \\ y_t - a y_{t-1} & \text{if} \quad t > 1 \end{cases}. \tag{2.5}$$

Next considered following OLS regression:

$$\nabla y_t = \nabla x_t \beta + u_t \tag{2.6}$$

where x_t is the deterministic part; the GLS detrended series y_t^d is defined as:

$$y_t^d = y_t - x_t \hat{\beta} \tag{2.7}$$

$\hat{\beta}$ is the estimate of β from (2.6). The deterministic part x_t would be vector of ones, $\{1\} = (1, 1, \ldots 1)^T$ if series is assumed not to have linear trend and $\{1, t\} = \begin{pmatrix} 1, 1, \ldots.. 1 \\ 1, 2, \ldots.. T \end{pmatrix}'$ if series is assumed to have a linear trend. Value of a is chosen as under:

$a = \dfrac{-13.5}{T}$ if series is assumed to have linear trend

$a = \frac{-7}{T}$ if series does not have linear trend

This procedure is also called local to unity GLS detrending.

The DF-GLS statistics is then computed from following regression:

$$\Delta y_t^d = \rho y_{t-1}^d + \sum_{i=1}^{k} \gamma_j \Delta y_{t-i}^d + e_t \tag{2.8}$$

And the test statistics $\hat{t}_{GLS} = \frac{\hat{\rho}}{SE(\hat{\rho})}$

Limiting behavior and critical values of $\hat{t}_{GLS}$

Elliot et al. (1996) show that the power curve of $\hat{t}_{GLS}$ is tangent to asymptotic power envelop and is never far below it. The finite sample critical values can be found in Elliot at al. (1996).

2.2.4: NG-PERRON TEST

Elliott et al. (1996) and Dufour & King (1991) found that local GLS detrending of the data yields substantial power gains. Ng & Perron (2001) apply the idea of GLS detrending to some modified tests and show that substantial size and power gains can be made, when used in conjunction with an autoregressive spectral density estimator at frequency zero, provided the truncation lag is appropriately selected.

Elliott et al. (1996) showed that power function of their test is tangent to power envelop at 50% power. However, inappropriate choice of lag length can still lead to poor size/power properties. While the power gains of the DF from using GLS detrended data are impressive, simulations also show that the test exhibits strong size distortions when there is MA root with negative coefficient. Size distortions, however, are less of an issue with the M-tests in theory as shown by Perron and Ng (1996). In practice, it does require us to have a way to find the appropriate lag length. So, Ng & Perron kept these three things in mind and designed M test for GLS detrended data. They also designed a criterion for choice of appropriate lag length, which they show better than other existing criteria. Therefore, this test accumulates the intellectual wisdom of GLS detrending proposed by Elliot et al. (1996), and usage of M-estimators proposed by Stock (1990). M-type test use the estimate of spectral density of autoregressive process. Ng and Perron (2001) proposed a set of four tests all using M-estimator. Further detail on computation of these tests is as under:

Let $y_1, y_2, \ldots y_t$ be a time series to be tested for unit root. Compute GLS-detrended series $y_1^d, y_2^d, \ldots y_T^d$ as defined in equation 2.7

Consider the OLS regression equation 2.8 i.e. $\Delta y_t^d = \rho y_{t-1}^d + \sum_{j=1}^{k} \gamma_j y_{t-j}^d + e_{tk}$

Than spectral density estimate at frequency zero from equation 2.4 is:

$$\hat{f}(0) = \hat{\sigma}^2 \left(1 - \hat{\gamma}_1 - \hat{\gamma}_2 - \ldots \hat{\gamma}_k\right)$$

Define $\kappa = \sum_{t=2}^{T} (y_{t-1}^d)^2$

The set of tests proposed by Ng and Perron contain tests MZ_α, MZ_t, MSB and MP_T. These tests are defined as follows:

$$MZ_\alpha = \frac{\left(T^{-1}(y_T^d)^2 - \hat{f}(0)\right)}{2\kappa} \tag{2.9}$$

$$MSB = \left(\frac{\kappa}{\hat{f}(0)}\right)^{1/2} \tag{2.10}$$

$$MZ_t = MSB \times MZ_\alpha \tag{2.11}$$

$$MP_T = \begin{cases} \dfrac{1}{\hat{f}(0)}\left(a^2\kappa - aT^{-1}\left(y_T^d\right)^2\right) & \text{if} \quad x = \{1\} \\ \dfrac{1}{\hat{f}(0)}\left(a^2\kappa + (1-a)T^{-1}\left(y_T^d\right)^2\right) & \text{if} \quad x = \{1, t\} \end{cases} \tag{2.12}$$

where a is equal to -7 if $x = \{1\}$ and -13.5 if $x = \{1, t\}$.

Asymptotic Behavior and Critical Values of Ng-Perron Test

Ng & Perron claim that the four tests have optimal properties of DF-GLS test and M-estimator proposed by Stock (1990). They argue that asymptotic power curve of these tests is never far below the asymptotic power envelop. The asymptotic critical values of Ng-Perron test are provided by Ng & Perron (2001).

2.3: PRE-TEST MODEL SPECIFICATION

Before application of unit root test to a real data series, a researcher has to make number of specification decisions. There are various methods for making such decisions. Two important decisions are the choice of lag length and specification of deterministic regressors. Among many methods of these specification decisions, the methods utilized in the thesis are summarized below.

2.3.1: THE SEQUENTIAL TESTING STRATEGY

This strategy is to specify the deterministic regressors in a model to be used for testing unit root. The strategy is outlined in Enders (2004) and is summarized below:

1. Estimate the autoregression $\Delta y_t = \alpha + \beta t + \rho y_{t-1} + e_t$

1(a). Apply Dickey Fuller t-test to test $\rho = 0$ versus $\rho < 0$

i If $\rho = 0$ is rejected, conclude no unit root, model M1 or M2 or M3

ii If $\rho = 0$ is not rejected, go to 1(b)

1(b). Apply Dickey Fuller F –test to test $(\beta, \rho) = (0,0)$

i If $(\beta, \rho) = (0,0)$ is rejected, go to 1(c) ii If $(\beta, \rho) = (0,0)$ is not rejected, go to (2)

1(c). Test $\rho = 0$ versus $\rho < 0$ using conventional normal critical values

i If $\rho = 0$ is rejected, conclude no unit root, model M1 or M2

ii If $\rho = 0$ is not rejected, decide unit root, model M3

2. Estimate the autoregression $\Delta y_t = \alpha + \rho y_{t-1} + e_t$

2(a). Apply Dickey Fuller t-test to test $\rho = 0$ versus $\rho < 0$

i If $\rho = 0$ is rejected, conclude no unit root, model M1 or M2

ii If $\rho = 0$ is not rejected, go to 2(b)

2(b). Apply Dickey Fuller F –test to test $(\alpha, \rho) = (0,0)$

i If $(\alpha, \rho) = (0,0)$ is rejected, go to 2(c)

ii If $(\alpha, \rho) = (0,0)$ in not rejected, go to (3)

2(c). Test $\rho = 0$ versus $\rho < 0$ using conventional normal critical values

i If $\rho = 0$ is rejected, conclude no unit root, model M2

ii If $\rho = 0$ is not rejected, decide unit root, model M2

3. Estimate the autoregression $\Delta y_t = \rho y_{t-1} + e_t$

3(a). Apply Dickey Fuller t-test to test $\rho = 0$ versus $\rho < 0$

i If $\rho = 0$ is rejected, conclude no unit root, model M1

ii If $\rho = 0$ is not rejected, conclude unit root, model M1

2.3.2: CRITERION FOR CHOICE OF LAG LENGTH

Appropriate choice of truncation lag is important for the implementation of unit root test proposed by Dickey & Fuller (1981) and Said & Dickey (1984). It is also required to estimate the autoregressive spectral density at frequency zero. Several criteria exist for the choice of truncation lag. Ng & Perron (2001) compare performance of several criteria for the choice of lag length and show that Modified Akaike Information Criterion outperforms other criteria for the appropriate choice of lag length. Following Ng & Perron (2001), throughout this thesis we will use MAIC for the choice of lag length. This MAIC statistics is given as under:

For any of the autoregression defined in (1) the MAIC is computed as:

$$MAIC = \ln(\hat{\sigma}_k^2) + \frac{2\left(\tau_T(k) + k\right)}{T - k_{\max}} \qquad (2.11)$$

Here $\hat{\sigma}_k^2$ is the variance of residuals from regression equation 2.1 when k lags are included in the autoregression and $\tau_T(k) = (\hat{\sigma}_k^2)^{-1}\hat{\rho}^2 \sum_{t=k_{\max}+1}^{T} \tilde{y}_{t-1}^2$ .Also $\tilde{y}_t = y_t$ for ADF, PP and PP test whereas $\tilde{y}_t = y_t^d$ for DF-GLS and NP test.

2.4: RUDEBUSCH APPROACH TO EVALUATE PERFORMANCE OF UNIT ROOT TESTS

Rudebusch (1993) measures the ability of a unit root test to discriminate the best fitting trend stationary and best fitting difference stationary models estimated from given data series. This approach is outlined as under:

For a given real time series $\{y_t\}$, compute the best fitting trend stationary model by estimating following autoregression:

$$y_t = a + bt + \sum_{i=1}^{k} \phi_i y_{t-k} + \varepsilon_t \qquad (2.13)$$

For the same series, compute the best fitting difference stationary model by estimating following autoregression:

$$\Delta y_t = \alpha + \sum_{i=1}^{k} \gamma_i \Delta y_{t-k} + v_t \qquad (2.14)$$

Use the estimates of a, b, ϕ_i and σ_ε^2 to generate artificial data series analogues to DS model of the real data series. Compute the unit root test statistics for this series.

Use the estimates of α, γ_i and σ_v^2 to generate artificial data series analogues to TS model of the real data series. Compute the unit root test statistics for this series.

Repeating the above process for a large number of times one can estimate distribution of the test statistics for two types of models. If the two distributions are distant to each other than the unit root test would be able to discriminate between the two types of models whereas it would fail if the distributions are overlapping.

CHAPTER 3

LITERATURE REVIEW

The Literature on unit root related issues is so vast that it cannot be covered in several volumes of a book. There exist number of good summaries, reviews and commentaries on unit root literature. Interested reader is referred to these sources for detail; Libanio (2005), Chinn (1991), and Stock (1995) discuss the implications of unit root in economic theory, policy implications and econometric procedures. Stock (1995) and Patterson (2003) provide review of history of unit root models in economics and econometrics. Maddala and Kim (1998) provide excellent overview of contemporary approaches to unit root testing and cointegration. There are several surveys on specialized topics in unit roots e.g. Ng and Perron (2001) provide summary of available methods for choice of lag length. Perron (2005) provides a detailed review of literature on unit root in conjunction with structural breaks.

As we have discussed in Section 1.3, despite huge literature on theory related to autoregressive process near or equal to unity, there is no clarity on several important issues and implications regarding unit root. We have illustrated this by using example of widely studied GNP series of United States. Every author reaches a different conclusion after investigation of dynamics of the series.

A major reason responsible for ambiguity in the inference of unit root tests is the model specification prior to application of unit root tests. Before the application of unit root tests, a researcher has to make number of specification decisions (implicit or explicit) e.g.

a. decision about selection of lag length,

b. presence of structural break

c. deterministic part used in the model.

The choice of lag length and presence of structural breaks have a lot of literature to their credit and well documented methods exist for making these decision; see Ng and Perron (2001) and Perron (2005) for detail. However, existing literature does not provide satisfactory solutions for choice of deterministic part in a model used for testing unit root (Elder and Kennedy, 2001). Chapter 4 and 5 of this thesis are dedicated to present a systematic procedure for specification of deterministic trend in a model to be used for testing unit root. Therefore, necessary detail of literature related to this issue is provided in section 1.

Moreover, the question about existence of unit root in real GNP has been addressed by various authors, with each one reaching a different conclusion. The performance of unit root tests depends on the DGP but no one knows the DGP of real time series. So, it cannot be decided which of the tests is most reliable. A second contribution of this thesis is evaluates performance of unit root tests. An overview of the opinion of econometricians about existence of unit root in American real GNP series is presented in section 3.2.

3.1: SPECIFICATION OF DETERMINISTIC PART IN UNIT ROOT MODELS

The study of Dickey and Fuller (1979) is a premier in unit root testing. They developed various statistics for unit root testing and computed distributions of these statistics via Monte Carlo simulations. They used three different equations for computing unit root test statistics. These three equations are:

M1	Without drift, trend	$y_t = \delta\, y_{t-1} + \varepsilon_t$	
M2	With drift, but no trend	$y_t = \alpha + \delta\, y_{t-1} + \varepsilon_t$	(3.1)
M3	With drift and trend	$y_t = \alpha + \beta\, t + \delta\, y_{t-1} + \varepsilon_t$	

Where $\varepsilon_t \sim iid(0, \sigma^2)$

The parameters of interest in these equations is value of δ but the distribution of test statistics for testing $\delta = 1$ depends on the nuisance parameters α and β. That is, there are three different asymptotic distributions for various combinations of data generating process and regression model. This fact was realized by Dickey & Fuller (1979), therefore, they present three different sets of quintiles of distributions of statistic corresponding to the model used for testing unit root. Computation of critical value assumes that testing equation is congruent with data generating process. However, Dickey & Fuller do not provide any systematic procedure to choose between three test equations for real data sets.

The choice among three equations has very serious impact on the output of unit root tests. Campbell and Perron (1992) report following properties of unit root tests with regard to choice of deterministic part:

- When the estimated regression includes at least all deterministic elements in the actual data generating process, the distribution of test statistics is non-normal under the null. The distribution itself varies with the set of parameters included in the estimating equations.

- If the estimated regression includes deterministic regressors that are not in the actual data generating process, power of unit root test against a stationary alternative decreases as additional deterministic regressors are added.

- If the estimated regression omits an important deterministic trending variable present in the true data generating process, such as linear deterministic trend, the power of t-test goes to zero as the sample size increases. If the estimated regression omits a non-trending variable, (mean or a change in the mean), t-statistics is consistent but finite sample power is adversely affected and decreases as the magnitude of coefficient of omitted component increases (Campbell and Perron, 1992).

The remarks of Campbell and Perron (1992) can be summarized as follows:

If extraneous regressors are included in a model used for testing unit root, the power of unit root decreases and if some valid regressor is missing, the power of test goes to zero. This means that the unit root tests work better only, if we correctly specify the deterministic regressors. However, despite realizing importance of deterministic trend, in the early years of development of unit root test procedures, we don't find any systematic procedure for specification of deterministic part in a model used for testing unit root.

Nelson and Kang (1984) found that if conventional t-statistics for testing coefficient linear trend in Dickey fuller regression is heavily biased towards non-rejection. They conducted a Monte Carlo experiment in which simple random walks were generated by model $y_t = y_{t-1} + \varepsilon_t$. Then, they estimated following regression equation from the series thus generated: $y_t = \beta\, t + \varepsilon_t$. According to standard statistical theory, there is no predictable relationship between time path of simple random walk series and linear deterministic trend; therefore, its coefficient should be insignificant. But Nelson and Kang (1984) found that in 87% of the regressions, $t_{\hat{\beta}}$ appeared to be significant. This finding created skepticism about use of classical hypothesis testing procedure for deterministic component in the unit root models.

The first systematic procedure for specification deterministic part has been presented by Perron (1988), which is termed as sequential testing strategy. This strategy starts from most general model including drift and linear trend and number of deterministic regressors is reduced by successive testing.

Minor modifications to the sequential testing strategy were proposed by Dolado, Jenkinson, & Sosvilla-Rivero (1990) and Holden & Perman (1994). All these strategies start by the most general model [Elder and Kennedy, 2001], and then reduce the model in several steps. The latest version of sequential testing strategy is discussed in detail by Ender (2004) and is summarized in Section 2.2.

In fact, the literature on 'how to choose deterministic part' is much smaller than other decisions of similar nature; like choice of lag length. Elder & Kennedy (2001) noted the following weakness of the literature in the exposition of Dickey-Fuller test, which is most popular test of unit root, and it is being taught in almost all courses of time series econometrics.

> ....a crucial ingredient of this test, not well recognized in textbooks, is that a testing strategy is required, as opposed to mere calculation of a single test statistic. This strategy is necessary to determine if an intercept, an intercept plus a time trend, or neither an intercept nor a time trend, should be included in the regression run to conduct the unit-root test.

Elder & Kennedy (2001) oppose sequential testing strategies to be used in practice. Their arguments against use of these strategies are as follows:

- They do not exploit prior knowledge of the growth status of the variable under test, forcing their strategies to cover all possibilities. For example, unemployment clearly does not have a long-run growth trend, and so, for this variable, unit-root testing should begin by setting the trend coefficient equal to zero but these strategies never do so.

- They worry about outcomes that are not realistic, for example, simultaneous existence of a unit root and a trend. This is thought to be unrealistic as noted for example by Perron (1988).

- They double- and triple-test for unit root as they start by most general model and then testing is done at each step of reduction.

Elder & Kennedy (2001) proposed an alternate strategy, which differs from sequential testing strategy in following:

1. They recommend to start modeling by the graphical analysis of the data

2. They discard to consider some models which they consider economically implausible. In particular they discard the possibility of a model with simultaneous unit root and trend.

Given the crucial effect of these choices, it is surprising that there is a lack of studies on comparison of the performance of these strategies. Despite utilizing several search facilitators, we were unable to find any comparison of these strategies except an unpublished study of Hacker and Hatemi-J (2006). This study compares Ender's strategy with Elder and Kennedy's strategy with the conclusion that the latter strategy is superior. However, the study is restricted to Dickey-Fuller environment, since it utilizes Dickey-Fuller F-test for model reduction. A feasible strategy for choice of deterministic part evaluating more recent tests like Ng-Perron (2001)'s test remains to be explored.

The sequential testing strategy and the Elder & Kennedy (2001)'s strategy have many things common. In particular, both strategies utilize Dickey Fuller F-test in model reduction. Consider the Φ_3 test proposed by Dickey & Fuller (1981) which is the F test of joint null hypothesis $(\beta, \rho) = (0,1)$ in model M3 (Eq. 3.1); if the value ρ is closer to zero, this is against the null and value of test statistics would be greater than the critical value. However, it would be misleading to infer $\beta \neq 0$ from the output. Therefore the use of F test for specification of deterministic regressors must be justified.

3.1.1: CONTRIBUTION OF THE THESIS FOR SPECIFICATION OF DETERMINISTIC PART

This thesis proposes a solution to the problem of specification of deterministic part which is based on following principle:

The standard t-test of the hypothesis $\beta = 0$ in Dickey & Fuller (1979) type regressions is biased towards rejection, when a series has strong autocorrelation, but performs well when there is weak autocorrelation. The F-test of joint hypothesis $(\beta, \delta) = (0,1)$ works fine for testing $\beta = 0$ when δ is close to unity (strong autocorrelation) and biased towards rejection when correlation is very weak. Therefore, a suitable combination of two tests should be capable of specifying the deterministic part for any value of autocorrelation.

In this thesis, we propose a strategy which utilizes t-test and F-test simultaneously for specification of deterministic part. Performance of this strategy is measured for a variety of autoregressive processes via Monte Carlo experiments. The results show that the strategy we propose have reasonable probability of specifying correct form deterministic regressors thus can be used for specification of deterministic part for many unit root test.

3.2: STATIONARITY OF GDP AND PERFORMANCE OF UNIT ROOT TESTS

There was a consensus among the econometricians that the economic time series behave like stationary fluctuation around some deterministic trend. This consensus was challenged by Nelson & Plossor (1982). Nelson and Plossor applied Dickey Fuller unit root test to a number of American macroeconomic time series including real GNP and found that they are unable to reject unit root for most of these series. Due to important implications of their findings, the issue got interest of econometricians. The question about existence of unit root in real GNP has been addressed by various authors, with each one reaching a different conclusion. An overview of the opinion of econometricians about existence of unit root in American real GNP series is presented in section 1.4. Here we will discuss the arguments of econometricians for their different opinions.

The findings of Nelson and Plossor were supported by various authors including Stock & Watson (1986), Perron and Phillips (1987) and Evans (1989). The DF/ADF tests were unable to reject unit root for various transformations of the time series used by Nelson and Plossor.

The widespread acceptance of a unit root in GNP was challenged by Perron (1989). Perron (1989) suggested that Nelson and Plossor's strong evidence in support of the unit root hypothesis rested on a failure to account for structural change in the data, and demonstrated this through incorporating an exogenous structural break for the 1929 crash. In doing so he reversed the conclusion of Nelson-Plossor (1982) for 9 out of 13 series including real GNP series. This finding was supported by studies of Banerjee et al. (1992), Christiano (1992) and Zivot & Andrews (1992), using various methods for specification of the structural break.

Using bootstrap approach, Rudebusch (1993) investigated the ability of unit root tests to discriminate between trend stationary and difference stationary models. Rudebusch found that the distribution of unit root test statistics when the series is generated by trend stationary model is overlapping with the distribution of test statistics for series generated by difference stationary model. These results led him to the conclusion that little can be said about the relative likelihood of DS and TS models of (US) real GNP on the basis of conventional unit root tests. Diebold and Senhadji (1996), using long span data, argue that trend stationarity is supported by Rudebusch's procedure.

Murray and Nelson (2002) argue that the rejection of unit root in favor of trend stationarity is due to the bias in unit root tests with structural breaks for stationarity. They find that when a transitory component is added to underlying unit root process, the unit root hypothesis is (incorrectly) rejected too often. This finding was supported by Kilian and hanian (2002). However Papell and Prodan (2003) oppose the arguments of Murray and Nelson (2002) using the procedure of Murray and Nelson and tests developed by Lumsdaine and Papell (1997).

The variety of opinions on the existence of unit root in GNP exists because different people use different types of models for testing unit root, and for each model, there is a different optimal test. However, for the real time series, we have no access to the data generating process of the series. Therefore, the optimal test cannot be decided.

3.2.1: CONTRIBUTION OF THE THESIS FOR FINDING STATIONARITY OF GDP

For any given data series, different unit root tests give different results and it is generally not possible to decide which of unit root tests would be the most feasible for this series. The performance of unit root tests depends on the type of data generating process, but for the real data we do not know the true DGP. Rudebusch (1993) approach offers an alternative to measure the performance of unit root test for any real time series with unknown DGP. Rudebusch (1993) measures the ability of a unit root test to discriminate the best fitting trend stationary and difference stationary models estimated from given data series. An extension of Rudebusch (1993) approach is used to evaluate the performance of unit root tests for the GDP series of various countries. Rudebusch (1993) approach is extended in two directions:

i. Rudebusch (1993) procedure measures the performance of single unit root test; we use this approximation of the performance to compare various tests.

ii. Rudebusch (1993) estimates best fitting trend stationary and difference stationary model for single time series and then uses these estimates to evaluate size and power of unit root tests. We formulate two parametric spaces covering the estimated parameters of simplest of the best fitting difference stationary and trend stationary models of a large pool of countries. The performance of unit root tests is evaluated on these parametric spaces. Thus, the results can be generalized to any data series, whose estimated parameters fall into these parametric spaces.

Our results show that for most data series unit root tests are unable to discriminate between best fitting models of two types. However, for small number of series, it is possible to discriminate between two types of models and Phillips Perron test performs best for the purpose.

CHAPTER 4

SPECIFICATION OF DETERMINISTIC REGRESSORS IN UNIT ROOT TESTING

This chapter deals with the appropriate methodology to specify the deterministic regressors prior to testing for unit root. The discussion presented in section 3.1 reveals that no satisfactory solution exists for specification of deterministic regressors in unit root models. A new procedure is proposed for the specification of deterministic regressors. Performance of this new procedure is illustrated via Monte Carlo experiments.

The chapter is organized as follows:

Section 4.1 consists of introduction and brief description of the problem. Section 4.2 discusses impact of misspecified deterministic regressors on the output of unit root tests. Section 4.3 is about relationship between autoregression and deterministic regressors. Section 4.4 discusses properties of procedures for specification of deterministic regressors. Section 4.5 is describes new strategy proposed for specification of deterministic regressors. Section 4.6 presents results of Monte Carlo experiments for the evaluation of new strategy.

4.1: INTRODUCTION

Consider three models:

M1	Without drift, trend	$y_t = \delta y_{t-1} + \varepsilon_t$	(4.1a)
M2	With drift, but no trend	$y_t = \alpha + \delta y_{t-1} + \varepsilon_t$	(4.1b)
M3	With drift and trend	$y_t = \alpha + \beta\, t + \delta y_{t-1} + \varepsilon_t$	(4.1c)

Where $\varepsilon_t \sim iid(0, \sigma^2)$

The three models described in (4.1) are to be used as the data generating process and the tests equations. We will use Mi, i=1,2,3, when equations are used as data generating process, and $\bar{M}$i, i=1,2,3, when equation is used as a model for the data. Thus M2 means that equation (4.1b) is used as data generating process, whereas $\bar{M}$2 means equation (4.1b) is used as a model assumed to be the DGP by an econometrician.

The comments of Campbell & Perron (1991) summarized in section 3.1 reveal that unit root tests will give optimal performance only, if assumed data generating process is congruent with the actual data generating process. Unit root tests suffer power loss, if number of deterministic regressors is larger in the model than from the original data generating process. On the other hand, the power of tests converge to zero if the number of deterministic regressors in testing equation is smaller than the number of deterministic regressors in actual data generating process. This chapter quantifies the remarks of Campbell and Perron for ADF and PP test via Monte Carlo experiments. The effect of deterministic regressors on GLS-detrending based tests is also analyzed. It is shown that conventional methods and general to simple methods are inadequate to specify the deterministic regressors in autoregressive models, even with stationary roots. A new procedure for specification of deterministic regressors is presented and its performance is analyzed via Monte Carlos experiments.

4.2: IMPORTANCE OF DETERMINISTIC REGRESSORS IN UNIT ROOT TESTING

In this section, we demonstrate the practical importance of the proper specification of deterministic regressors in unit root testing. Dickey & Fuller (1979) designed unit root test for three specifications of deterministic trend and tabulated the percentiles of Dickey Fuller distribution. As discussed earlier, the computation of Dickey Fuller distribution assumes a match between the data generating process and the model used for testing unit root.

The classical tests of unit root including Phillips-Perron and Dickey-Fuller tests are based on one of three regression equations described in (4.1), whereas, another class of unit root tests which have been invented later, depends on detrending of the data prior to computation of tests statistics. This class includes DF-GLS test and Ng-Perron test etc. We discuss size and power properties of tests separately for these two types of tests, so as to illustrate the importance of proper specification of deterministic part in the model used for testing unit root.

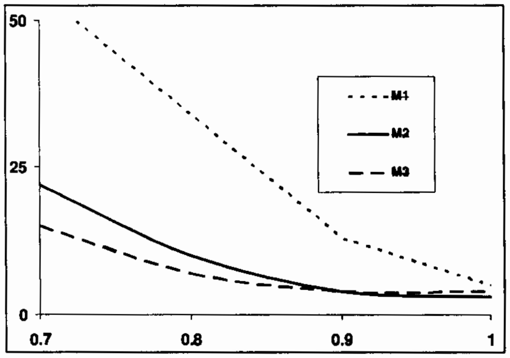
4.2.1: DETERMINISTIC REGRESSORS AND TESTS BASED ON DF EQUATION

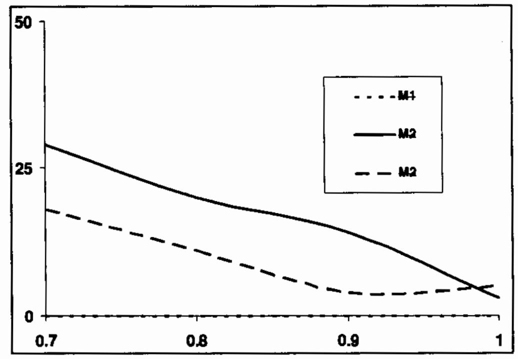
The tests based on Dickey-Fuller regression equations ~~M1~~, ~~M2~~ & ~~M3~~ include Phillips Perron and Dickey Fuller test. Recall the three model specifications described in (4.1).

The Figure 4.1 illustrates results of the Monte Carlo experiment for the sample of size 50. The Panel A summarizes percentage rejection by various tests when data is actually generated by model M1. The horizontal axis represents the value of autoregressive parameter δ. The tests applied to the data in all three scenarios i.e. ~~M1~~, ~~M2~~, and ~~M3~~. The power curve for various value of δ is plotted. The rejection percentage in panel A for model ~~M1~~ is about 55% when $\delta = 0.7$ and it is about 5% for $\delta = 1$. The power curves for ~~M2~~ and ~~M3~~ are far below the power curve of ~~M1~~.

Panel B and C give results for model M2 and M3 respectively. In Panel B and C, power curve of ~~M1~~ is almost identical with the X-axis. This means, if data was actually generated by M2, the DF test based on model ~~M1~~ is unable to reject the null of unit root for any value of δ in (0.7, 1). Therefore, this panel illustrates the intensity of the problem posed by the misspecification of model. Similarly, Panel C illustrates that the powers of DF regressions ~~M1~~ and ~~M2~~ are virtually zero. This means ~~M1~~ and ~~M2~~ have no power if the data was actually generated by M3. In all three panels, it is visible that power curve are highest when the DGP matches with test equations. Detailed Results are reported in Table 4.1.1 and 4.1.2. (in appendix)

Figure 4.1: Percentage Rejection of Null of Unit Root with different specification of DF Equation



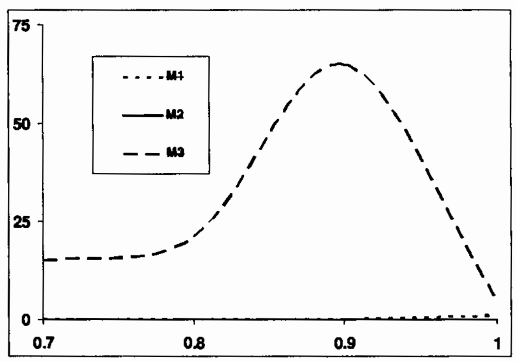


Panel A

DGP: M1

Panel B

DGP: M2 $\alpha = 0.5$



Panel C

DGP: M3, $\alpha = \beta = 0.5$

Power of Unit Root Tests is plotted against the value of autoregressive parameter. It can be seen that in all of three panels, highest power correspond to the test matching with the data generating process.

Similar results were obtained for Phillips-Perron test which are summarized in Table 4.1.3 and 4.1.4. Fig 4.1 and table 4.1 clearly show that unit root tests attain maximum power, when there is match between DGP and the test scenario.

4.2.2: DETERMINISTIC REGRESSORS AND TESTS BASED ON DETRENDING

Elliot et al. (1996) observed that detrending a series before application of Dickey-Fuller type test improves the performance of unit root tests. This finding originated a new class of unit root tests, which depends on detrending of the series before application of unit root tests. This class of tests includes Elliot, Rothenberg and Stock (1992)'s point optimal test, DF-GLS test and Ng-Perron test etc. The detrending is the procedure of subtracting the deterministic component from the time series. The detrending techniques differ in minute computational detail and exhibit a variety of characteristics.

Unfortunately, there is a misconceived perception that the dependence of detrending based tests on the proper specification is not as serious as for DF type tests. In fact, commonly used detrending methods are also of two types depending on the form of DGP with respect to the deterministic part. These two types are: with linear trend and without trend. Once the series has been detrended, the computation of test statistics is same for all specifications of deterministic part. However, the distribution of test statistics depends on both DGP and the detrending procedure. Detrending does not reduce the importance of information about appropriate specification of deterministic component in the model used for testing unit root. The distribution of test statistics still depends on the (i) deterministic component in DGP and (ii) the deterministic component used for detrending the data. This section is to show that, the tests show best performance when (i) and (ii) match with each other.

To investigate the impact of misspecified trend on the output of detrending based unit root tests, we have designed another Monte Carlo experiment on the lines similar to the previous experiment for DF and PP tests. The design of experiment is as under:

Consider the three models M1, M2 and M3 discussed in (1) above

1. Choose a particular model Mi, i=1,2,3
2. Generate the data series using Mi as a data generating process
3. Choose a unit root test
4. Apply the unit root test to series generated in step 2 under all possible variations of deterministic part and compute size/power of all variations for the series.

The experiments were performed for different sample sizes results of the experiments are summarized in Table 4.2. It can be seen from the table that the tests performance is best when there is appropriate match between DGP and the variant of test used.

4.3: RELATIONSHIP BETWEEN AUTOREGRESSION AND LINEAR TREND

Consider the data generated by the model

$$y_t = \alpha + \delta y_{t-1} + \varepsilon_t$$

The successive substitution yields:

$$y_t = \delta^t y_0 + \alpha \sum_{i=0}^{t} \delta^i + \sum_{i=0}^{t} \delta^i \varepsilon_{t-i}$$

So that

$$E(y_t) = \begin{cases} \dfrac{\alpha}{1-\delta} & if \quad \delta < 0 \\ \\ y_0 + \alpha\, t & if \quad \delta = 1 \end{cases} \qquad \text{when t is large}$$

If $\alpha = 0$, and $E(y_0) = 0$, than $E(y_t)$ does not depend on time trend, this means a data series generated by M1 is independent of t, regardless of value of δ and a regression of autoregressive series on time trend should be insignificant. Furthermore, if $\alpha \neq 0$ (series is generated by M2) and $\delta < 1$ than again there is no relationship between linear trend and the autoregressive series. But if $\alpha \neq 0$ and $\delta = 1$ i.e. the process is random walk with drift, then the expectation of autoregressive series depends on the time trend. Equation tells that if the data generating process is of the form of M2 and $\delta < 0$, there is no predictable relationship between the trend and the observations of the time series. However, above derivation is valid if the time series length is large enough. This is because we approximate $\sum_{i=0}^{t}\delta_i$ by $\frac{1}{1-\delta}$, whereas, $\sum_{i=0}^{t}\delta_i = \frac{1-\delta^t}{1-\delta}$ and δ^t should be close to zero to make the approximation work. Thus, for smaller t, $E(y_t)$ is not independent of time.

4.4: PROPERTIES OF PROCEDURES FOR SPECIFICATIONS OF DETERMINISTIC REGRESSORS

The models M1 and M2 are nested in model M3, therefore, the general to simple procedure is to start from M3 and reduce the models by testing significance of deterministic regressors. If the regressand is independent of regressor, then the regressor should not be significant. But as discussed in section 3.1, this is not true for random walk process. Nelson & Kang (1984) experiment reveals that, although there is no relation between random walk and linear trend, the regression on trend appear to be significant for about 87% of the times. We show that regression of stationary autoregressive series also produces spurious significance of trend even if autoregressive root is not close to unity. Therefore the General to Simple type methodology does not work for the specification of deterministic trend in series with autoregression.

Two types of equations are used to show that general to simple strategy does not work for autoregressive series.

First is Nelson & Kang (1984)'s equation [NK Equation hereafter] described by:

$$y_t = a + b\,t + \varepsilon_t \qquad (4.2)$$

where y_t is the autoregressive series. The discussion presented in section 4.3 reveals that if data was actually generated by M1, or M2 with $\delta < 1$, then there is no relation between trend and autoregressive series and trend and trend should be insignificant. But we show that this does not happen in the proceeding section.

Second is Dickey Fuller (1979)'s Equation [DF equation hereafter] described by:

$$y_t = \alpha + \beta t + \delta y_{t-1} + \varepsilon_t \qquad (4.3)$$

This regression equation should also produce insignificant coefficient of trend if data is generated by M1 or M2 with $\delta < 1$.

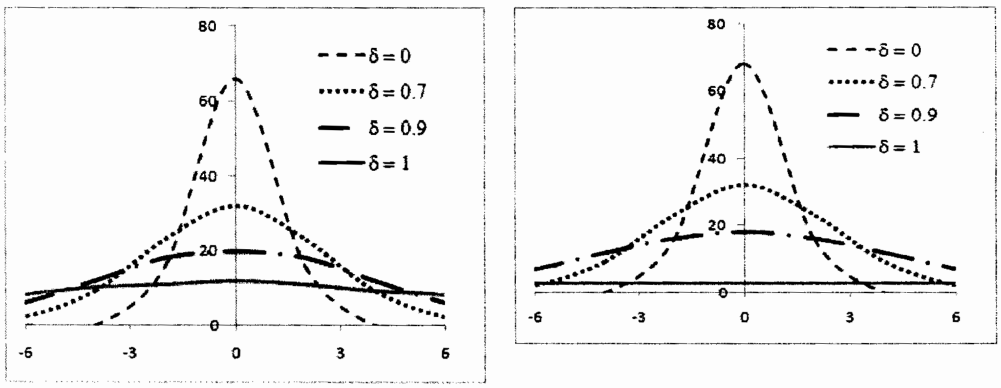
4.4.1: RESULTS OF MONTE CARLO USING NK EQUATION

To investigate the distribution of coefficient of linear trend in NK Equation (4.2), we performed Monte Carlo experiment with following design:

1. Generate a series according to the model Mi, i=1,2,3 described in eq. (4.1)
2. Estimate the least square regression equations $y_t = a + b\,t + \varepsilon_t$
3. Record the conventional t-statistics for coefficient b i.e. t_b..

The data generating process M1 becomes similar to the data generating process used by Nelson and Kang (1984) if $\delta = 1$. The study of Nelson & Kang (1984) is silent about the distribution of t-statistics for $\delta < 1$.

Fig 4.2: The distribution of t_b in NK Equation, DGP:M1

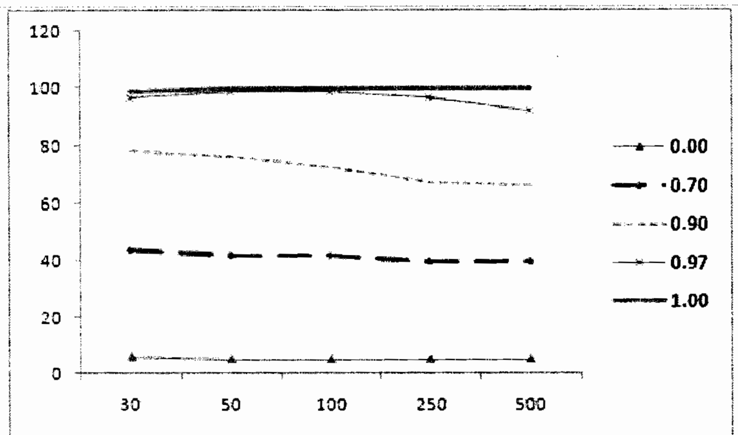


Sample Size 30 Sample size 500

The distribution of t-statistics for coefficient of linear trend is plotted for various values of autoregressive parameters in the NK Equation. It is visible that distribution of t-statistics is very different for different values of δ, the autoregressive coefficient.

The results on the distribution of t-statistics t_b are summarized in Table 4.3 for various sample sizes. The Fig 4.2 visually illustrates the distribution of t-statistics t_b for Sample of size 30and 500 and for various values of the autoregressive coefficients. The distribution of t-statistics is approximately identical with Standard Normal Distribution when $\delta = 0$ i.e. when series is serially independent (IID). But the distribution is very different from Standard Normal when $\delta > 0$. If we look at the distribution of t-statistics for $\delta = 0.7$ in the right panel (sample size 500, we see that the distribution ranges from -6 to +6. This clearly indicates that, the distribution of t-statistics is not standard normal, although the series is stationary with sufficiently large sample size. We have recorded percentage rejection of null of no relation with linear trend (See Table 4.4 in appendix) for various sample sizes ranging from 30 to 500. For $\delta = 0.7$, the percentage of t-statistics observed outside ± 2SE band was recorded to be 20% instead of 5% nominal probability of type-1 error.

Fig 4.3: Percentage Rejection of $b=0$ in NK Equation, GDP:M1



The probability of rejection of null of no relation between linear trend and autoregressive series is plotted for different sample sizes and different values of autoregressive parameters. It can be observed that the rejection percentage decreases very slowly with increase in sample size.

By looking at Fig 4.3, following facts can be observed about the rejection rate of no linear trend.

i. For δ closer to zero, percentage rejection of (true) null hypothesis $b=0$ decreases, however, the decrease is slow. If $\delta = 0.7$ the probability of rejection of $b=0$ is 42% when sample size is 30 & 40% for sample size 500.

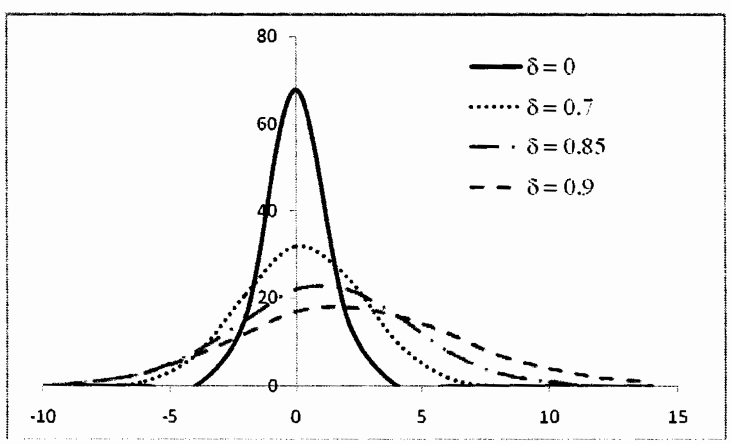
ii. For δ closer to 1 the probability of rejection of $b=0$ increases initially with the increase in sample size and eventually decreases.

This implies that the problem of spurious significance of linear trend exists regardless of the type of stationarity, if the series has autocorrelation. For the small and moderate sample sizes, the trend appears to be significant for a proportion much higher than the nominal significance level. Practically for all moderate sample sizes, the distribution of t-statistics is very different from the standard normal distribution.

The figure 4.4 demonstrates the distribution of t-statistic when data is generated by M2 and sample size is 100.

Figure 4.4 gives the pictorial summary of the distribution of t-statistics of for the autoregressive process. If drift parameter α is positive, the distribution of t-statistics is skewed towards right. The skewness increases with increase in value of autoregressive parameter. With increase of sample size, the magnitude of bias reduces.

Fig 4.4: The distribution of t_b in NK Equation, DGP:M2



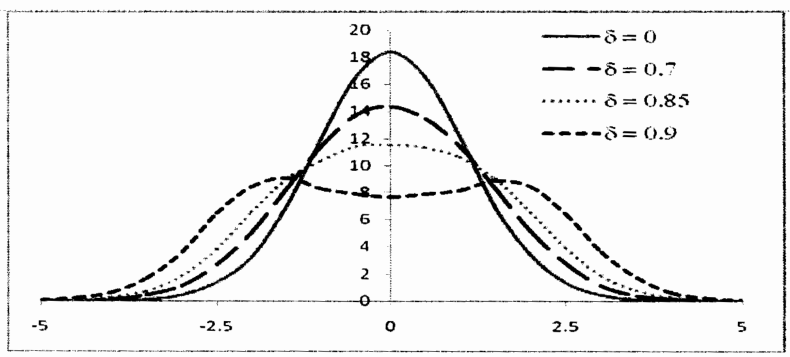
The distribution of t-statistics for coefficient of linear trend is plotted for various values of autoregressive parameters in the NK Equation. It is visible that distribution of t-statistics is skewed and is very different from standard normal distribution for nonzero autoregression.

However, for the moderate sample sizes the bias does not vanish. The distribution of t_b for the data generated by M2 is summarized in Table 4.5 and the percentage rejection of null $b = 0$ is tabulated in Table 4.6.

4.4.2: RESULTS OF MONTE CARLO USING DF EQUATION

The DF Equation is given by $y_t = \alpha + \beta\, t + \delta y_{t-1} + \varepsilon_t$. Fig 4.5 illustrates the distribution of t-statistics t_β in the DF Equation where the data is generated by M1;

Fig 4.5: The distribution of t_β in DF equation, DGP is M1

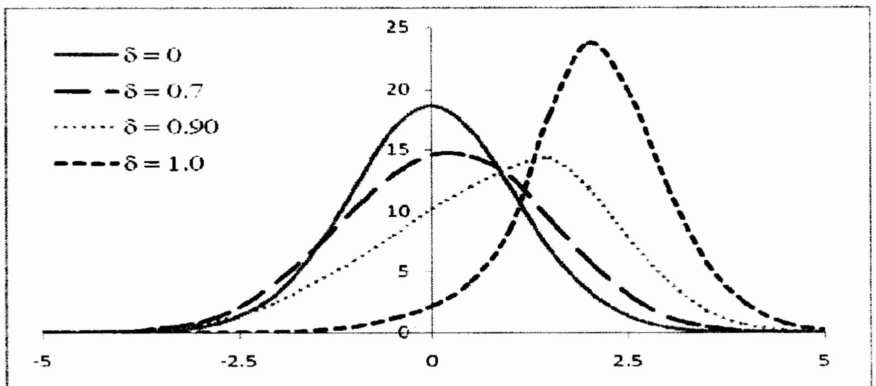


The distribution of t-statistics for coefficient of linear trend is plotted for various values of autoregressive parameters in the Dickey-Fuller Equation. It is visible that distribution of t-statistics is different from standard normal distribution. The distribution is bimodal for unit root case

For all non-zero values of autoregressive parameters, the distribution of t-statistics is very different from the conventional Student's distribution. The distribution is bimodal when the value of autoregressive parameter is 1. Table 4.8 summarizes the rejection probability of the null hypothesis of no relationship between trend and the time path of series.

But the distribution changes a lot when data is generated by the M2. Fig 4.6 plots the distribution of t-statistics when M2 is used as DGP:

Fig 4.6: The distribution of t_β in DF equation, DGP: M2

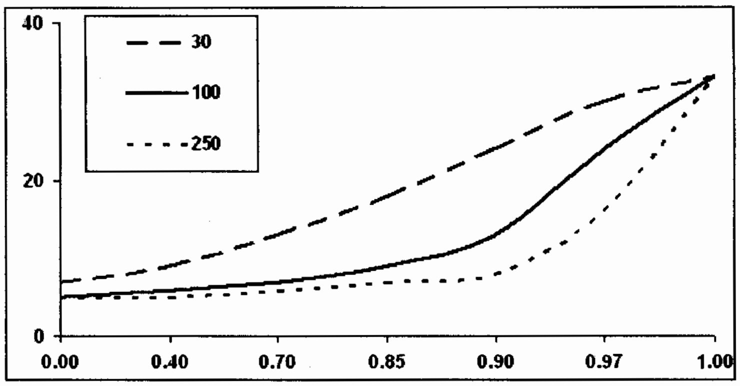


The distribution of t-statistics for coefficient of linear trend is plotted for various values of autoregressive parameters in the DF Equation when DGP is M2. It is visible that distribution of t-statistics is very different from standard normal distribution for nonzero autoregression. The skewness of distribution of t-statistics is clearly visible.

The distribution skewed towards infinity, if the positive drift coefficient is present in the data generating process. Figure 4.7 visualizes rejection probability for different sample sizes.

Fig 4.7 also shows that the rejection of $\beta = 0$ is close to nominal size if $\delta \in [0, 0.8]$. This implies that in DF Equation, t-test can be used to specify the deterministic regressor, if the autoregressive root δ is closer to zero. However, if autoregressive root is closer to 1 (but less than 1 i.e. stationary series), the false rejection of null $\beta = 0$ increases drastically.

Fig 4.7: Percentage Rejection of null $\beta = 0$ in DF equation, GDP is M1

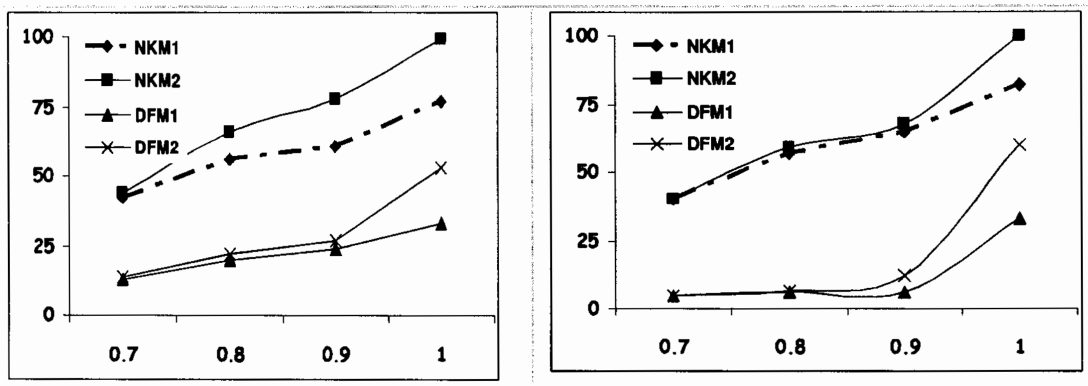


The probability of rejection of null of no relation between linear trend and autoregressive series is plotted for different sample sizes. It can be observed that the rejection percentage decreases with increase in sample size when value of autoregressive parameter is close to zero but doesnot decrease when it is close to unity

4.4.3: A COMPARISON OF NK EQUATION AND DF EQUATION FOR M1 AND M2

The Fig 4.8 presents a comparison of the rejection rates of coefficient of linear trend for model M1 and M2. It can be observed that rejection rates by NK Equation when M2 is used as DGP (NKM2) are always higher than the rejection rates for NKM1. This implies that in the drift model M2, there are more chances of spurious significance of the trend.

Fig 4.8: A comparison of Rejection Rate by Nelson & Kang an DF equations



Sample size 100 | Sample size 400

Percentage Rejection of the null hypothesis of no linear trend is given for two sample sizes. X-axis corresponds to different values of autoregressive parameter. Rejection Probability is always higher for NK Equation then from DF Equation.

Similarly, rejection rates for NKM1 are always higher than DFM1. This implies the NK Equation would produce more spurious results then DF Equation. The percentage rejection for $\delta = 0$ is approximately 5% for all the tests. This implies, if the series are serially independent then these methods perform equally well. When $\delta < 1$, the rejection probabilities are smaller in right panel than corresponding probabilities in the left panel. This means that the probability of spurious significance decreases as the sample size grows large. However, it can also be observed that the convergence of rejection probabilities to the nominal size is very slow. For example, if $\delta = 0.7$, DGP is M1 and sample size is 100, rejection probability of linear trend is 44% using NK Equation. The rejection probability reduces by 4% only, if the sample size is increased to 500.

4.5: PROPOSED STRATEGY FOR SPECIFICATION OF DETERMINISTIC PART

The aim of the new strategy is to test $\beta = 0$ in AR model ~~M3~~ and/or $\alpha = 0$ in ~~M2~~. The new strategy utilizes conventional t-test as well as the likelihood ratio tests Φ_1 and Φ_3 defined in 2.3 to test the two hypotheses. The idea starts from using likelihood ratio test of joint hypothesis $(\beta, \delta) = (0,1)$ for testing $\beta = 0$ in ~~M3~~ and hypothesis $(\alpha, \delta) = (0,1)$ for testing $\alpha = 0$ in model ~~M2~~. However, consider the joint hypothesis $(\beta, \delta) = (0,1)$, the likelihood ratio test works fine to test $\beta = 0$ if δ is close to unity. But if δ is away from unity, the joint hypothesis would be rejected even if $\beta = 0$. Therefore, the test cannot be utilized to specify the deterministic regressor.

Fortunately, the conventional t-test for hypothesis $\beta = 0$ in DF Equation works better when δ is close to zero. Therefore, a combination of two tests is capable for specifying the deterministic regressors. The strategy works in following sequence: the presence of deterministic regressors is tested by conventional t-test, if this test fails to reject null hypothesis of absence of deterministic regressor, the series is than referred to the likelihood ratio test. The deterministic regressor is finally included in the model if both tests reject the null of absence of regressor.

Therefore, the sequence of tests described in OSS and TSS have following properties:

No deterministic regressor; autoregressive root is close to zero:

In this case conventional t-test will be able to detect absence of deterministic regressor.

No deterministic regressor; autoregressive root is close to unity:

In this case conventional t-test will not be able to detect absence of deterministic regressor and will refer it to LR test. LR test performs better in finding deterministic regressor in neighborhood of unity.

Deterministic regressor is present; autoregressive root is close to zero:

In this case conventional t-test will be able to detect absence of deterministic regressor. Also LR test is biased towards non-rejection of deterministic regressor in the neighborhood of unity. Therefore, both tests would reject absence of deterministic regressor.

Deterministic regressor is present; autoregressive root is close to unity:

Conventional t-test will be biased towards non-rejection of deterministic regressor; therefore, will refer the case to LR test. LR test would be able to discriminate regressor in neighborhood of unity.

The four situations discussed above are summarized in following decision table:

Deterministic Regressors/AR root	Near Zero	Near 1
Present	t-test will accept deterministic regressor and will refer to LR test. LR have tendency to accept deterministic regressor when AR root is near zero, so deterministic regressor is included in the model	t-test will accept deterministic regressor and will refer to LR test. LR can make accurate decision about deterministic regressors in neighborhood of unity
Not Present	t-test will not accept deterministic regressor, so series would not be referred to LR test. So the decision is no deterministic regressors in model	t-test is biased for deterministic regressors, so it will accept deterministic regressor and will refer to LR test. LR can make accurate decision about deterministic regressors in neighborhood of unity

In any case, the strategy of using a combination of two tests is expected to perform better in finding out the performance of deterministic regressor. To support this claim, we perform several Monte Carlo experiments.

The new strategy utilizes following four simple tests:

TTT: the Trend t-test

Compute the following autoregression

$$y_t = \alpha + \beta\, t + \delta\, y_{t-1} + \varepsilon_t$$

Test $H_0 : \beta = 0$ versus $H_1 : \beta \neq 0$ using conventional t-test. Use the conventional Student's t-critical value for testing the hypothesis.

TLT: the Trend LR-test

Apply LR test to the following problem

$$
\begin{aligned}
H_0 &: \Delta y_t = a + u_t \\
H_1 &: y_t = \alpha + \beta\, t + \delta y_{t-1} + e_t
\end{aligned}
$$

Compare the computed test statistics with simulated critical values, and decide to reject H_0 if the test statistics is greater than one sided critical value.

DTT: the Drift t-test

Compute the following autoregression

$$y_t = \alpha + \delta\, y_{t-1} + \varepsilon_t$$

Test $\alpha = 0$ versus $\alpha \neq 0$using conventional t-test. Use the conventional Student's t-critical value for testing the hypothesis.

DLT: the Drift LR-test

Apply LR test to the following problem

$$
\begin{aligned}
H_0 &: \Delta y_t = u_t \\
H_1 &: y_t = \alpha + \delta y_{t-1} + e_t
\end{aligned}
$$

Compare the computed test statistics with simulated critical values, and decide to reject H_0 if the test statistics is greater than the critical value.

These four test arranged in a proper sequence formulate a strategy which is extremely useful in uncovering the original form of model. This sequencing is described as under:

4.5.1: ONE STEP STRATEGY (OSS)

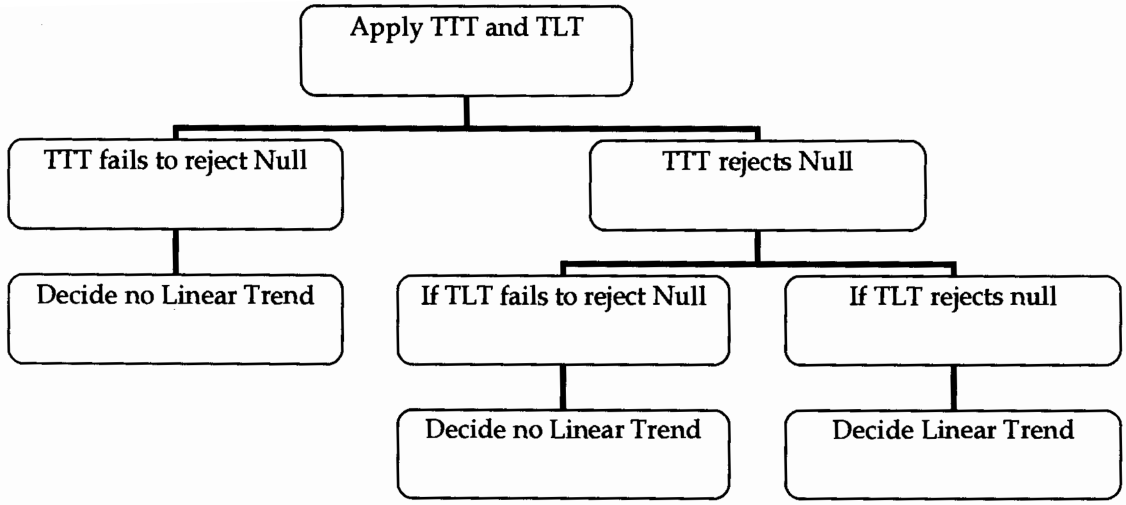
This strategy is aimed to specify model for tests based on detrending of the series. These tests usually have two variants with respect to the deterministic part i.e. with linear trend and without it. The strategy is outlined below

Apply TTT and TLT

1. If TTT does not rejects null: decide no linear trend
2. If TTT rejects null and
    a. TLT does not reject; decide no linear trend
    b. TLT reject; decide linear trend

This procedure is summarized in following flow chart:

The One Step Strategy for Specification of Linear Trend



4.5.2: TWO STEP STRATEGY (TSS)

This strategy is designed to specify model for tests based on Dickey Fuller Regression and it comprises two steps. Step 1 is to determine whether or not linear trend is present in the model. This strategy is to test the presence of drift in the model.

Step 1:

Apply TTT and TLT

3. If TTT does not rejects null: proceed to step2
4. If TTT rejects null and
    a. TLT reject; conclude M3
    b. TLT does not reject; conclude M2

Step 2:

Apply DTT and DLT

1. If DTT does not reject; conclude M1

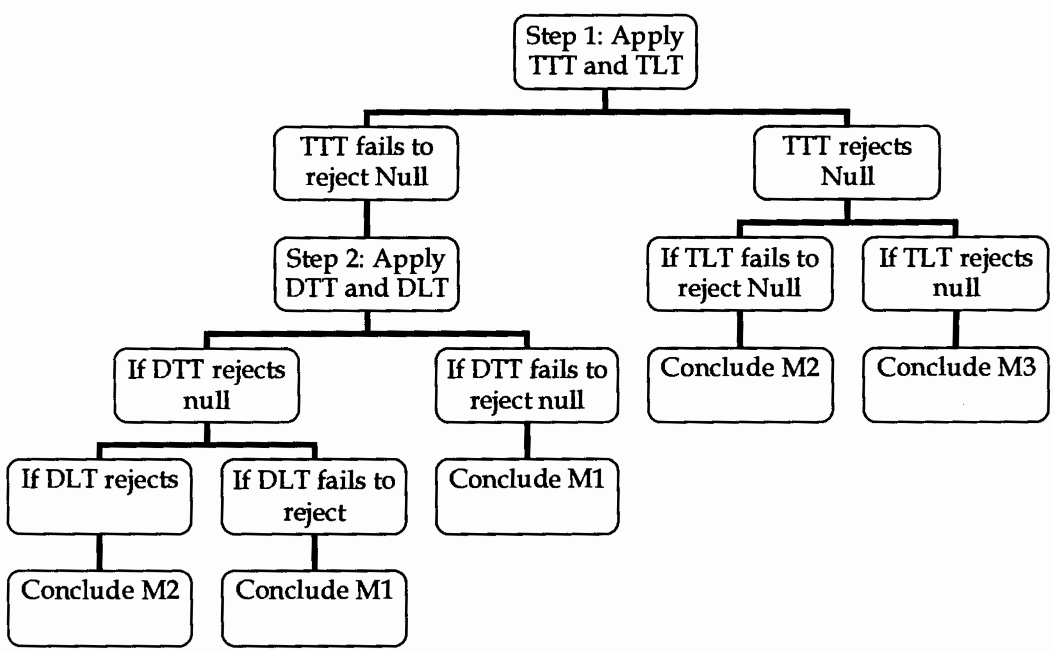
2. If DTT reject and

   a. DLT reject; M2

   b. DLT does not reject; M1

This procedure is summarized in following flow chart

The Two Step Strategy for Specification of Deterministic Part



4.5.3: ASYMPTOTIC DISTRIBUTION OF LR TESTS

We are utilizing two likelihood ratio tests in the proposed strategy. These tests are:

TLT: the Trend LR-test

LR test for the following problem

$$H_0 : \Delta y_t = a + u_t$$
$$H_1 : y_t = \alpha + \beta\, t + \delta y_{t-1} + e_t$$

DLT: the Drift LR-test

LR test for the following

$$H_0 : \Delta y_t = u_t$$
$$H_1 : y_t = \alpha + \delta y_{t-1} + e_t$$

We summarize here the asymptotic distributions of the two tests.

Theorem 1 (The limit distribution of DLT): Suppose null and alternative hypothesis are as defined in definition of DLT, the limiting distribution of test statistics is described as:

$$LR \Rightarrow \frac{\chi^2(T-1)}{\chi^2(T-1) - A'B^{-1}A}$$

where

$$A = \left(\sigma W(1) \quad 0.5\sigma^2 \left\{[W(1)]^2 - 1\right\}\right)$$

$$B = \begin{pmatrix} 1 & \sigma\int_0^1 W(r)dr \\ \sigma\int_0^1 W(r)dr & \sigma^2\int_0^1 [W(r)]^2 dr \end{pmatrix}$$

Proof:

The likelihood ratio test between two regression models can be described as:

$$LR = \left(\hat{\sigma}_1^2\right)^{-1} \hat{\sigma}_0^2$$

Where $\hat{\sigma}_0^2$ and $\hat{\sigma}_1^2$ are maximum likelihood estimates of variance under null and alternative respectively.

The regression model under the null is:

$$H_0 : \Delta y_t = u_t$$

The asymptotic distribution of the statistics is to be computed assuming null is true. When null is true, the right hand side of model is just errors, and $\hat{\sigma}_0^2$ is the estimate of variance of these errors having distribution $\chi^2(T-1)$.

However, the denominator of the regression model involves following regression: $y_t = a + \delta y_{t-1} + e_t$

This regression equation contains stochastic integrated regressor. Limiting distribution of the denominator can be computed as follows:

It is easy to show that the residual variance under the alternative is:

$$\hat{\sigma}_1^2 = \in' \in - \in' X(X'X)^{-1}X' \in$$

Where $X = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ y_0 & y_1 & \cdots & y_{T-1} \end{pmatrix}'$

Let D be the scaling matrix defined as: $D = \begin{pmatrix} \sqrt{T} & 0 \\ 0 & T \end{pmatrix}$, it is easy to see that

$$\in' X(X'X)^{-1}X' \in \quad = \quad \in' XD^{-1}(D^{-1}X'XD^{-1})^{-1}D^{-1}X' \in$$

Now simple computation yields:

$$\in' XD^{-1} = \left(T^{-0.5}\sum_{t=0}^{T} e_t \quad T^{-1}\sum_{t=0}^{T} y_{t-1}e_t\right)$$

$$\Rightarrow \left(\sigma W(1) \quad 0.5\sigma^2\left\{[W(1)]^2 - 1\right\}\right) = A' \tag{4.10}$$

Similarly it is straight forward to compute

$$D^{-1}X'XD^{-1} = \begin{pmatrix} 1 & T^{-1.5}\sum_{t=1}^{T} y_{t-1} \\ T^{-1.5}\sum_{t=1}^{T} y_{t-1} & T^{-2}\sum_{t=1}^{T} y_{t-1}^2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & \sigma\int_0^1 W(r)dr \\ \sigma\int_0^1 W(r)dr & \sigma^2\int_0^1 [W(r)]^2 dr \end{pmatrix} = B \tag{4.11}$$

Using 4.10 & 4.11

$$\in' XD^{-1}(D^{-1}X'XD^{-1})^{-1}D^{-1}X' \in \Rightarrow A'B^{-1}A$$

Therefore,

$$\begin{aligned} LR &= \frac{\hat{\sigma}_0^2}{\hat{\sigma}_1^2} = \frac{\in'\in}{\in'\in - \in' XD^{-1}(D^{-1}X'XD^{-1})^{-1}D^{-1}X' \in} \\ &\Rightarrow \frac{\chi^2(T-1)}{\chi^2(T-1) - A'B^{-1}A} \end{aligned}$$

Proposition 1 (Re-parameterization of the trend model):

Suppose $y_t = \alpha + \beta t + \delta y_{t-1} + e_t$

This model can be equivalently written as:

$$y_t = \alpha(1-\delta) + \delta(y_{t-1} - \alpha(t-1)) + (\beta + \delta\alpha)t + e_t$$

$$\xi_t = \alpha^* + \delta\xi_{t-1} + \beta^* t + e_t$$

Under the null hypothesis $(\beta, \delta) = (1, 0)$ it can be seen that

$$(\beta^*, \delta^*) = (1, 0)$$

And

$$\xi_t = y_t - t = y_0 + \sum_{t=1}^{T} u_t$$

Thus the process can be written as sum of deterministic polynomial plus a random walk without drift with same restriction on parameters under the null.

Theorem 2 (Limit Distribution of TLT): Suppose null and alternative hypothesis are as defined in definition of TLT, the limiting distribution of test statistics is described as:

$$LR \Rightarrow \frac{\chi^2(T-1)}{\chi^2(T-1) - C'D^{-1}C}$$

where

$$C = \left(\sigma W(1) \quad 0.5\sigma^2\left([W(1)]^2 - 1\right) \quad \sigma W(1) - \sigma \int_0^1 W(r)dr \right)$$

$$D = \begin{pmatrix} 1 & \sigma \int_0^1 W(r)dr & 1/2 \\ \sigma \int_0^1 W(r)dr & \sigma^2 \int_0^1 [W(r)]^2 dr & \sigma \int_0^1 rW(r)dr \\ 1/2 & \sigma \int_0^1 W(r)dr & 1/3 \end{pmatrix}$$

Proof:

As described above, the likelihood ratio test between two regression models can be described as:

$$LR = \left(\hat{\sigma}_1^2\right)^{-1} \hat{\sigma}_0^2$$

Where $\hat{\sigma}_0^2$ and $\hat{\sigma}_1^2$ are maximum likelihood estimates of variance under null and alternative respectively.

The regression model under the null is:

$$H_0 : \Delta y_t = a + u_t$$

The asymptotic distribution of the statistics is to be computed assuming null is true. When null is true, the right hand side of model contains only constant as a regressor, therefore the estimate of variance $\hat{\sigma}_0^2$ has distribution $\chi^2(T-1)$.

However, the denominator of the regression model involves following regression: $y_t = a + \beta t + \delta y_{t-1} + e_t$

This regression equation contains stochastic integrated regressors. Limiting distribution of the denominator can be computed as follows:

It is easy to show that the residual variance under the alternative is: $\hat{\sigma}_1^2 = \in' \in - \in' X(X'X)^{-1} X' \in$

The computation of limit distribution the first term in denominator is straightforward and it converges to $\chi^2(T-1)$. All what we have to compute is the limit distribution of second term in the denominator i.e. $\in' X(X'X)^{-1} X' \in$

Here matrix of Regressor X is given as:

$$X = \begin{pmatrix} 1 & 1 & \dots & 1 \\ \xi_0 & \xi_1 & \dots & \xi_{t-1} \\ 1 & 2 & \dots & T \end{pmatrix}'$$

We can write

$$\in' X(X'X)^{-1}X' \in \quad = \quad \in' XD^{-1}(D^{-1}X'XD^{-1})^{-1}D^{-1}X' \in$$

Where D is 3×3 scaling matrix given by:

$$D = \begin{pmatrix} T^{0.5} & 0 & 0 \\ 0 & T^{1.5} & 0 \\ 0 & 0 & T \end{pmatrix}$$

Now It can be seen that

$$\in' XD^{-1} = \left(T^{-1}\sum_{t=1}^{T} u_t \quad T^{-1.5}\sum_{t=1}^{T} \xi_{t-1}u_t \quad T^{-1}\sum_{t=1}^{T} tu_t \right)$$

$$\Rightarrow \left(\sigma W(1) \quad 0.5\sigma^2 \left([W(1)]^2 - 1 \right) \quad \sigma W(1) - \sigma \int_0^1 W(r)dr \right) = C \qquad (4.12)$$

Similarly it can be shown that:

$$D^{-1}X'XD^{-1} = \begin{pmatrix} 1 & T^{-1.5}\sum_{t=1}^{T}\xi_{t-1} & T^{-2}\sum_{t=1}^{T} t \\ T^{-1.5}\sum_{t=1}^{T}\xi_{t-1} & T^{-2}\sum_{t=1}^{T}\xi_{t-1}^2 & T^{-2.5}\sum_{t=1}^{T} t\xi_{t-1} \\ T^{-2}\sum_{t=1}^{T} t & T^{-2.5}\sum_{t=1}^{T} t\xi_{t-1} & T^{-3}\sum_{t=1}^{T} t^2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & \sigma\int_0^1 W(r)dr & 1/2 \\ \sigma\int_0^1 W(r)dr & \sigma^2\int_0^1 [W(r)]^2 dr & \sigma\int_0^1 rW(r)dr \\ 1/2 & \sigma\int_0^1 W(r)dr & 1/3 \end{pmatrix} = D \qquad (4.13)$$

Using 4.12 & 4.13, we get

$$\begin{aligned} \in' X(X'X)^{-1}X' \in &= \in' XD^{-1}(D^{-1}X'XD^{-1})^{-1}D^{-1}X' \in \\ &\Rightarrow C'D^{-1}C \end{aligned}$$

Therefore,

$$LR = \frac{\hat{\sigma}_0^2}{\hat{\sigma}_1^2} = \frac{\in'\in}{\in'\in - \in' XD^{-1}(D^{-1}X'XD^{-1})^{-1}D^{-1}X' \in}$$

$$\Rightarrow \frac{\chi^2(T-1)}{\chi^2(T-1) - C'D^{-1}C}$$

The two asymptotic distributions are non-standard and depend on functions of Wiener process. Therefore the critical values are to be computed via simulations.

4.6: PERFORMANCE OF NEW STRATEGY

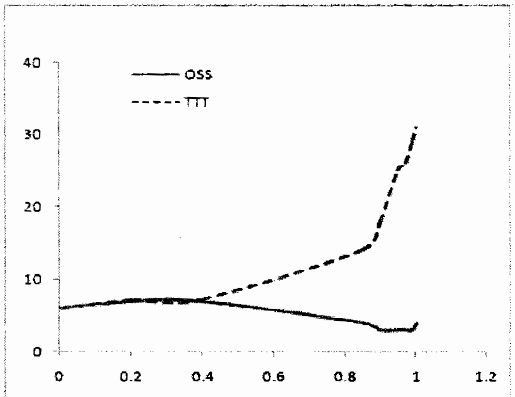
4.6.1: PERFORMANCE OF OSS

We measure the performance of one step methodology by the probability of uncovering the true form of data generating process. Consider the general model M3:

$$y_t = \alpha + \beta\, t + \delta\, y_{t-1} + \varepsilon_t$$

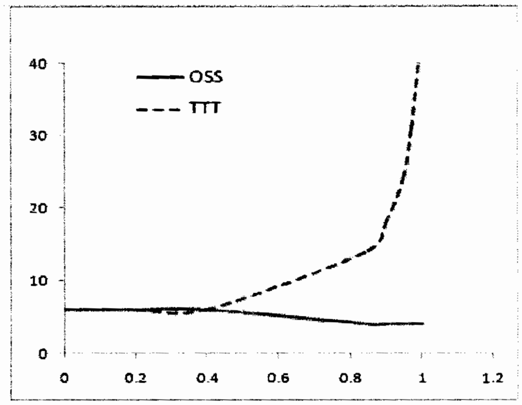
The purpose of the strategy is to test $H_0 : \beta = 0$ versus the alternative $\beta \neq 0$. We know that if in the DGP, $\beta = 0$ (model M1 and M2) and $\delta < 1$, then there is no predictable relationship between linear trend and the time series. However, as the discussion in section 4.2 reveals that a start from general equation of the form of NK Equation or the DF equation is biased towards non rejection of $\beta = 0$ for all sample sizes and for any $\delta > 0$. However, the t-test for trend in DF Equation, which we call TTT, always performs better then Nelson & Kang equation. Therefore, we compare TTT with one step strategy to find how much improvement is possible by using OSS.

Fig 4.9: Probability of Rejection of Null hypothesis of No trend for various DGP, sample size 50



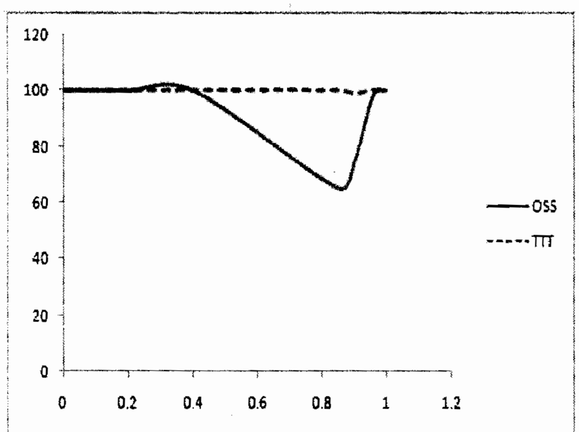
Panel A

DGP: M1 $\alpha = \beta = 0$



Panel B

DGP: M2 $\alpha = 0.5 \quad \beta = 0$



Panel C

DGP: M3 $\alpha = 0.5 = \beta$

Percentage Rejection of $\beta = 0$ is plotted for different DGPs and different values of autoregressive parameter. 'OSS' corresponds to rejection probability using one step strategy, whereas TTT corresponds to rejection probability using conventional t-test. It can be seen that in most of cases, probability of false rejection of $\beta = 0$ is larger when TTT is used

Figure 4.9 presents present rejection of $\beta = 0$ for the three types of DGPs. Panels A, B and C presents percentage rejection of $\beta = 0$ for data generated by M1, M2 and M3 respectively. First two panels correspond to size of the strategies, since they measure the rejection of null when it is true. It is visible from the first two panels that the size of TTT keeps on increasing as the value of autoregressive parameter δ approaches to 1. However, the OSS maintains its size (and even improves) when the value of α is close to 1. This experiment was repeated for various sample sizes which yield similar results (see Table 4.11).

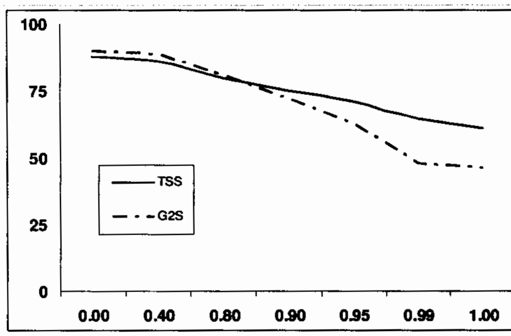
The B panel rejection probabilities of the (True) null hypothesis for different values of δ, where data is generated by M2. The probability of rejecting null is a bit higher when data is generated by the M2, however, the size characteristic remain almost same for M1 and M2. The CTT fails to perform in both the cases whereas the OSS performs exceptionally well for all values of autoregressive parameter.

Panel C gives probability of rejection of null hypothesis $b = 0$ when data is generated by Model C. This panel corresponds to the power of test since $\beta \neq 0$ in the actual data generating process. The TTT rejects null with approximately 100% probability. The power of OSS is reasonable and it is not smaller than power of TTT except in a small interval.

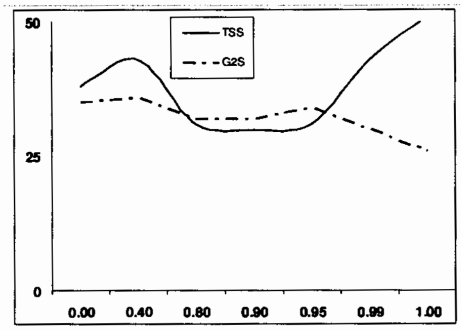
4.6.2: PERFORMANCE OF TSS

The purpose of two step strategy is to find out most appropriate model for the data series out of the three models ~~M1~~, ~~M2~~ and ~~M3~~. Given a time series, we apply 2-step procedure to find out which model is most appropriate for modeling the data series;

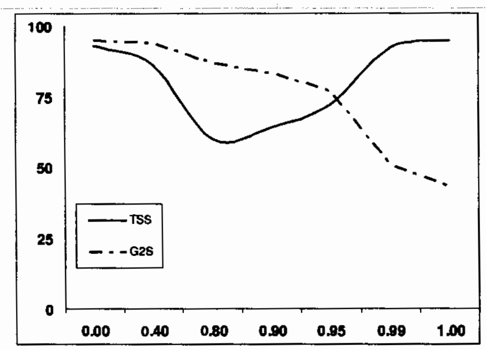
Fig 4.10: Probability of Choosing True model, Comparison of G2S and TSS



Panel A; DGP M1



Panel B; DGP M2



Panel C; DGP M3

Probability of finding true models using G2S and TSS methods is plotted. It can be seen that TSS performs better than G2S strategy, especially in the neighborhood of unity, where it is more important to have correct information of true DGP

Fig 4.10 summarizes performance of TSS for three types of DGP. The panel A gives probabilities of finding out correct model for different values of δ, where data is generated by M1. It can be seen that TSS works fine especially in the neighborhood of unity. This is desirable because unit root tests suffer power loss if the value of autoregressive parameter is close to unity, so it is more important to pick up true model to apply the test having maximum attainable power.

Similarly, Panel B and C summarize probability of picking up the correct model when data is actually generated by M2 and M3 respectively. Panel B reveals that OSS outperforms conventional tests in the neighborhood of unity, which is very useful in real life applications.

4.7: SUMMARY

Although the unit root tests are heavily dependent on the specification of deterministic part in the model, there is no systematic procedure for making decision about it, except the Sequential Testing Strategy, which is applicable to DF test only. We present the evidence that if General to Simple type methodology is adopted for the specification of deterministic part; it is unable to work even for the stationary processes. We present the evidences that if deterministic part is misspecified, the performance of unit root tests is badly affected. Then, we present two different strategies for specification of models for tests based on DF regression and the tests based on GLS detrending. We show that performance of this strategy is much better than the classical inference procedure in General to Simple type models especially in the neighborhood of unity.

4.8: APPENDIX

Table 4.1: Percentage rejection of unit root by DF and PP for various DGPs

Table 4.1.1: Percentage rejection of unit root by DF test for various DGPs Sample size 40

DGP	M1			M2			M3		
Test Scenario/ Value of δ	M1	M2	M3	M1	M2	M3	M1	M2	M3
0.7	**55**	22	15	0	**29**	18	0	0	**15**
0.8	**34**	10	7	0	**20**	11	0	0	**21**
0.9	**13**	4	4	0	**14**	4	0	0	**65**
1.0	**5**	3	4	0	**3**	5	1	0	**5**

Table 4.1.2: Percentage rejection of unit root by DF test for various DGPs, Sample size 80

DGP	M1			M2			M3		
Test Scenario/ Value of δ	M1	M2	M3	M1	M2	M3	M1	M2	M3
0.7	**89**	50	34	0	**60**	40	0	0	**52**
0.8	**86**	41	25	0	**54**	32	0	0	**55**
0.9	**19**	6	5	0	**13**	6	17	0	**35**
1.0	**4**	4	2	0	**5**	4	28	0	**4**

Table 4.1.3:Percentage rejection of unit root by PP test for various DGPs Sample size 40

DGP	M1			M2			M3		
Test Scenario/ Value of δ	M1	M2	M3	M1	M2	M3	M1	M2	M3
0.7	**87**	42	26	0	**60**	34	0	0	**27**
0.8	**57**	20	13	0	**42**	20	0	0	**38**
0.9	**23**	9	7	0	**28**	8	0	0	**98**
1.0	**5**	5	5	0	**3**	5	0	0	**5**

Table 4.1.4: Percentage rejection of unit root by PP test for various DGPs Sample size 80									
DGP	M1			M2			M3		
Test Scenario/ Value of δ	M1	M2	M3	M1	M2	M3	M1	M2	M3
0.7	**98**	77	56	0	**91**	69	0	0	**87**
0.8	**97**	66	42	0	**86**	58	0	0	**89**
0.9	**23**	11	8	0	**23**	11	0	0	**36**
1.0	**4**	5	5	0	**0**	5	0	0	**5**

Table 4.1 gives percentage rejection of unit root for different combination of data generating process (DGP) and the test Scenarios. Table 4.1.1 and 4.1.2 give rejection percentage of Dickey Fuller test for two different sample sizes 40 and 80 respectively whereas 4.1.3 and 4.1.4 give rejection percentage of Phillips Perron test for same sample sizes. The rejection Percentage is computed for $\delta = 0.7 : 0.1 : 1$. Therefore, first three rows of each table correspond to power of test and last row corresponds to power of test. It is clearly visible that the maximum power for each DGP is obtained when test equation is congruent with the data generating process. Fig 4.1 is based on this table.

Table 4.2: Percentage rejection of unit root by Ng-Perron test for various DGPs

Table 4.2:Percentage Rejection of Unit Root by Ng-Perron Test, Sample Size 100

DGP		M1			M2		
Test / δ		0.8	0.9	1.0	0.8	0.9	1.0
Without trend	MZA	100	96	7	25	15	6
	MZT	100	95	6	23	13	4
	MSB	100	93	6	32	16	5
	MPT	100	93	6	31	22	6
With trend	MZA	97	49	4	42	32	3
	MZT	97	49	4	38	45	5
	MSB	98	52	5	45	23	7
	MPT	98	53	5	33	28	6

Table 4.2 gives percentage rejection of unit root for different combination of data generating process (DGP) and the test scenario for the Ng-Perron test. Rejection percentage is calculated for $\delta = 0.8, 0.9, 1$. It can be observed that choice of test scenario plays very important in the power properties of test.

Table 4.4: Distribution of t_b in NK Equation (M1)

Table 4.3.1: Distribution of t_b in NK Equation

Value of δ / Bin	0	0.4	0.7	0.85	0.9	0.975	0.99	1.00
-10	0	0	0	1	2	6	8	11
-8	0	0	0	2	3	5	5	6
-6	0	0	2	5	6	8	8	8
-4	0	3	9	12	12	11	10	10
-2	16	23	23	19	18	14	12	11
0	66	47	32	23	20	15	13	12
2	16	23	23	20	18	14	12	11
4	0	3	9	11	12	11	10	9
6	0	0	2	5	6	8	8	8
8	0	0	0	2	3	5	5	5
10	0	0	0	1	1	6	8	11

Nelson & Kang Equation, M1, sample Size 30

Table 4.3.2: Distribution of t_b in NK Equation

Value of δ / Bin	0	0.4	0.7	0.85	0.9	0.975	0.99	1.00
-10	0	0	0	1	2	12	17	24
-8	0	0	0	2	4	6	6	5
-6	0	0	2	6	7	8	7	5
-4	0	3	9	12	12	9	8	6
-2	16	23	23	19	16	10	8	6
0	68	49	32	22	18	11	9	6
2	16	23	23	19	16	10	8	6
4	0	3	9	12	12	9	8	6
6	0	0	2	5	7	8	7	5
8	0	0	0	2	4	6	6	5
10	0	0	0	1	2	12	17	24

Nelson & Kang Equation for Data by M1, sample Size 80

Table 4.3.3: Distribution of t_b in NK Equation

Value of δ / Bin	0	0.4	0.7	0.85	0.9	0.975	0.99	1.00
-10	0	0	0	1	2	16	25	37
-8	0	0	0	2	4	6	5	3
-6	0	0	2	5	7	7	5	3
-4	0	3	9	12	12	8	6	3
-2	16	23	23	19	16	9	6	3
0	68	48	32	22	18	9	6	3
2	16	23	23	19	16	9	6	3
4	0	3	9	12	12	8	6	3
6	0	0	2	6	7	7	5	3
8	0	0	0	2	3	6	5	3
10	0	0	0	1	2	16	25	38

Nelson & Kang Equation for Data by M1, sample Size 500

Table 4.3 gives distribution of t-statistics for the coefficient of linear trend (t_b) in the NK Equation $y_t = a + bt + \varepsilon_t$ for three different sample sizes i.e. 30, 80 and 500. The DGP is M1 i.e. $y_t = \delta y_{t-1} + v_t$. Value of autoregressive parameter δ increases moving horizontally in any row of the Table. When δ is close to zero, the distribution of t-statistics is closer to standard normal and however, it deviates from normality with increase in value of δ. When $\delta = 1$ and sample size is 500, only 6% of the simulated values of t-statistics lie inside region 0 ± 2. Results of this table are visually illustrated in fig 4.2.

Table 4.4: Percentage rejection of null $b = 0$ in NK Equation (M1)

Table 4.4: Percentage Rejection of null $b = 0$ in NK Equation

Value of δ / Sample size	0.00	0.40	0.70	0.85	0.90	0.975	0.99	1.00
30	6	21	42	56	61	71	74	77
50	5	20	42	57	63	75	79	82
100	5	20	41	58	65	79	83	88
250	5	19	40	57	65	82	86	92
500	5	19	40	57	65	82	88	94

Percentage of Rejection of No Relation with Linear Trend, M1, different sample sizes NK Equation

Table 4.4 gives percentage rejection of null hypothesis $b = 0$ in the NK Equation $y_t = a + bt + \varepsilon_t$ for various sample sizes considering $t_b > 0$ as rejection. The DGP is M1 i.e. $y_t = \delta y_{t-1} + v_t$. Value of autoregressive parameter δ increases moving horizontally in any row. It is visible that rejection rate of (true) null hypothesis is much greater than 5% when autoregressive coefficient is nonzero. The probability of rejecting $b = 0$ decreases very slowly with the increase in sample size if δ is not very close to unity. Fig 4.3 is based on this table.

Table 4.5: Distribution of t_b in NK Equation (M2)

Table 4.5.1: Distribution of t_b in NK Equation								
Value of δ / Bin	0.00	0.40	0.70	0.85	0.90	0.975	0.99	1.00
-10	0	0	0	0	0	0	0	0
-8	0	0	0	0	0	0	0	0
-6	0	0	1	1	0	0	0	0
-4	0	2	5	3	1	0	0	0
-2	16	20	16	9	5	1	0	0
0	67	48	30	18	11	2	1	1
2	16	25	28	23	17	3	2	1
4	0	4	14	21	20	6	4	2
6	0	0	4	14	17	9	6	4
8	0	0	1	7	12	12	8	6
10	0	0	0	3	8	12	10	8
12	0	0	0	1	4	12	11	9
14	0	0	0	1	2	11	11	10
16	0	0	0	0	1	9	10	9
18	0	0	0	0	0	7	9	9
20	0	0	0	0	0	16	30	41
Nelson & Kang Equation for Data by M2, sample Size 30								

Table 4.5.2: Distribution of t_b in NK Equation								
Value of δ / Bin	0.00	0.40	0.70	0.85	0.90	0.975	0.99	1.00
-10	0	0	0	0	0	0	0	0
-8	0	0	0	1	1	0	0	0
-6	0	0	1	3	3	0	0	0
-4	0	2	7	8	7	0	0	0
-2	16	22	21	15	12	1	0	0
0	68	48	32	22	17	1	0	0
2	16	24	25	22	18	3	0	0
4	0	3	10	16	16	5	0	0
6	0	0	2	8	12	7	0	0
8	0	0	0	3	7	8	0	0
10	0	0	0	1	4	9	0	0
12	0	0	0	0	2	10	1	0
14	0	0	0	0	1	9	1	0
16	0	0	0	0	0	9	2	0
18	0	0	0	0	0	8	2	0
20	0	0	0	0	0	31	94	100

Nelson & Kang Equation for data generated by M2, sample Size 250

Table 4.5 gives distribution of t-statistics for the coefficient of linear trend (t_b) in the NK Equation $y_t = a + bt + \varepsilon_t$ for two different sample sizes i.e. 30 (4.5.1) and 250 (4.5.2). The DGP is M2 i.e. $y_t = \alpha + \delta y_{t-1} + v_t$ with $\alpha = 0.5$. The value of autoregressive parameter δ increases moving horizontally in any row. It is discussed in Section 4.2 that an autoregression has relationship with linear trend only if $\delta = 1$, however, the results of this table reveal that any nonzero value of autoregressive coefficient causes the distribution of t-statistics to deviate from normality. The distribution is not centered at zero when $\delta \neq 0$. These results are visually illustrated in fig 4.4

Table 4.6: Percentage rejection of null $b=0$ in NK Equation (M2)

Table 4.6: Percentage Rejection of null $b = 0$ in NK Equation								
Value of δ /	0.00	0.40	0.70	0.85	0.90	0.975	0.99	1.00
Sample size								
30	6	21	44	66	78	97	98	99
50	5	20	42	63	76	99	100	100
100	5	20	42	60	72	99	100	100
250	5	19	40	58	67	97	100	100
500	5	19	40	58	66	92	100	100

Percentage of Rejection of No Linear Trend for Data Generated by M2, Different Sample Sizes, NK Equation

Table 4.6 gives percentage rejection of null hypothesis $b = 0$ in the NK Equation $y_t = a + bt + \varepsilon_t$ for various sample sizes considering $|t_b| > 2$ as rejection. The DGP is M2 i.e. $y_t = \alpha + \delta y_{t-1} + v_t$. The value of autoregressive parameter δ increases moving horizontally in any row of the table. It is visible that rejection rate of (true) null hypothesis is much greater than 5% for nonzero autoregressive coefficient. The probability of rejecting $b = 0$ decreases very slowly with the increase in sample size if δ is not very close to unity.

Table 4.7: Distribution of t_b in DF Equation (M1)

Table 4.7.1: Distribution of t_b in DF Equation								
Value of δ / Bin	**0.00**	**0.40**	**0.70**	**0.85**	**0.90**	**0.975**	**0.99**	**1.00**
-4	0	0	0	1	1	1	1	1
-3	1	1	3	4	5	7	7	8
-2	7	8	10	13	14	15	16	17
-1	24	24	23	21	19	18	17	17
0	36	33	28	23	21	17	16	15
1	24	23	23	21	20	18	17	17
2	7	8	10	13	14	15	16	16
3	1	2	3	4	5	7	7	8
4	0	0	0	1	1	1	1	1
DF Equation for Data by M1, sample Size 30								

Table 4.7.2: Distribution of t_β in DF Equation								
Value of δ / Bin	**0.00**	**0.40**	**0.70**	**0.85**	**0.90**	**0.975**	**0.99**	**1.00**
-4	0	0	0	0	0	0	1	1
-3	1	1	1	1	1	3	5	7
-2	6	6	7	7	8	12	15	17
-1	24	24	24	24	23	22	19	17
0	38	38	37	35	34	25	20	15
1	24	24	24	24	24	22	19	17
2	6	6	7	8	8	12	15	17
3	1	1	1	1	1	3	5	7
4	0	0	0	0	0	0	1	1
DF Equation for Data by M1, sample Size 250								

Table 4.7 gives distribution of t-statistics for the coefficient of linear trend (t_β) in the Dickey-Fuller Equation $y_t = \alpha + \beta t + \delta y_{t-1} + \varepsilon_t$ for two different sample sizes i.e. 30 (4.7.1) and 250 (4.7.2). The DGP is M1 i.e. $y_t = \delta y_{t-1} + v_t$. When δ is close to zero, the distribution of t-statistics is closer to standard normal however, it deviates from standard normality with increase in value of δ. Fig 4.5 is based on this table.

Table 4.8: Percentage rejection of null $\beta = 0$ in DF Equation

Table 4.8: Percentage Rejection of null $\beta = 0$ in DF Equation								
Value of δ / Sample size	**0.00**	**0.40**	**0.70**	**0.85**	**0.90**	**0.975**	**0.99**	**1.00**
30	7	9	13	20	24	30	32	33
50	6	7	10	15	19	27	30	33
100	5	6	7	10	13	24	28	33
250	5	5	6	7	8	16	24	33
500	5	5	5	6	6	11	18	33

Percentage of Rejection of No Relation with Linear Trend, M1, different sample sizes DF equation

Table 4.8 gives percentage rejection of null hypothesis $\beta = 0$ in the Dickey-Fuller Equation $y_t = \alpha + \beta t + \delta y_{t-1} + \varepsilon_t$ for various sample sizes considering $|t_b| > 2$ as rejection. The DGP is M1 i.e. $y_t = \delta y_{t-1} + v_t$. Value of autoregressive parameter δ increases moving horizontally in any row of the Table. It is visible that rejection rate of (true) null hypothesis is much greater than 5% if there is nonzero autoregressive coefficient. However, the probability of rejecting $b = 0$ decreases with the increase in sample size.

Table 4.9: Distribution of t_β in DF Equation (M2)

Table 4.9.1: Distribution of t_β in DF Equation								
Value of δ/ Bin	**0.00**	**0.40**	**0.70**	**0.85**	**0.90**	**0.975**	**0.99**	**1.00**
-4	-4	1	1	2	2	1	0	0
-3	-3	7	8	9	7	5	1	0
-2	-2	24	23	20	15	12	4	2
-1	-1	36	33	29	23	20	10	7
0	0	24	24	25	26	27	24	21
1	1	7	9	12	19	23	38	42
2	2	1	2	3	6	9	19	22
3	3	0	0	0	1	1	4	4
4	4	0	0	0	0	0	0	1

DF Equation for Data by M2, sample Size 30

Table 4.9.2: Distribution of t_β in DF Equation								
Value of δ / Bin	**0.00**	**0.40**	**0.70**	**0.85**	**0.90**	**0.975**	**0.99**	**1.00**
-3	1	1	1	1	1	1	0	0
-2	6	6	7	7	7	4	1	0
-1	24	24	24	23	23	14	8	0
0	38	38	37	35	34	27	22	2
1	24	24	24	24	25	31	33	16
2	6	6	7	8	9	18	27	50
3	1	1	1	1	1	4	8	29
4	0	0	0	0	0	0	1	4
5	0	0	0	0	0	0	0	0

DF Equation for Data by M2, sample Size 250

Table 4.9 gives distribution of t-statistics for the coefficient of linear trend (t_β) in the Dickey-Fuller Equation $y_t = \alpha + \beta t + \delta y_{t-1} + \varepsilon_t$ for two different sample sizes i.e. 30 (4.9.1) and 250 (4.9.2). The DGP is M2 i.e. $y_t = \alpha + \delta y_{t-1} + v_t$ with $\alpha = 0.5$. Value of autoregressive parameter δ increases moving horizontally in any row. It is discussed in Section 4.2 that an autoregression has relationship with linear trend only if $\delta = 1$, however, the results of this Table reveal that the distribution of t-statistics for coefficient of linear trend deviates from normality when $\delta \neq 0$ and the distribution is not centered at zero. These results are illustrated via fig 4.6

Table 4.10: Percentage rejection of null $\beta = 0$ in DF Equation

Table 4.10: Percentage Rejection of null $\beta = 0$ in DF Equation								
Value of δ / Sample size	**0.00**	**0.40**	**0.70**	**0.85**	**0.90**	**0.975**	**0.99**	**1.00**
30	7	9	13	20	24	42	49	53
50	6	7	10	14	18	37	48	57
100	5	6	7	10	12	25	40	58
250	5	5	6	7	8	14	21	60
500	5	5	5	6	6	9	12	60

Percentage of Rejection of No Relation with Linear Trend, M1, different sample sizes DF equation

Table 4.10 gives percentage rejection of null hypothesis $\beta = 0$ in the Dickey-Fuller Equation $y_t = \alpha + \beta t + \delta y_{t-1} + \varepsilon_t$ for various sample sizes considering $|t_b| > 2$ as rejection. The DGP is M2 i.e. $y_t = \alpha + \delta y_{t-1} + v_t$ with $\alpha = 0.5$. The value of autoregressive parameter δ increases moving horizontally in any row of the table. It is visible that rejection rate of (true) null hypothesis is much greater than 5% if there is nonzero autoregressive coefficient. However, the probability of rejecting $\beta = 0$ decreases with the increase in sample size.

Table 4.11: Percentage rejection of null of No Linear Trend, CTS vs. OSS

Table 4.11.1: Percentage Rejection of Null of No Linear Trend, CTS versus OSS, DGP: M1 $\alpha = \beta = 0$

	Sample Size					
	50		100		200	
Value of δ	OSS	CTS	OSS	CTS	OSS	CTS
0.00	6	5	6	5	5	5
0.20	8	7	6	6	6	5
0.40	7	7	6	5	5	4
0.85	4	14	7	10	7	7
0.90	3	18	6	13	8	9
0.95	3	25	4	21	5	13
0.97	3	26	4	24	4	17
0.99	3	29	4	30	4	27
1.00	4	31	4	33	4	33

Table 4.11.2: Percentage Rejection of Null of No Linear Trend, CTS versus OSS, DGP: M2 $\alpha = 1.0 \quad \beta = 0$

	Sample Size					
	50		100		200	
Value of δ	OSS	CTS	OSS	CTS	OSS	CTS
0.00	6	5	6	6	4	4
0.20	7	7	6	6	5	4
0.40	7	7	6	5	6	6
0.85	5	12	8	9	7	7
0.90	6	17	7	9	8	8
0.95	6	22	8	12	8	9
0.97	5	30	7	16	9	10
0.99	4	45	6	32	11	14
1.00	5	60	5	60	5	58

Table 4.11.3: Percentage Rejection of Null of No Linear Trend, CTS versus OSS, DGP: M3 $\alpha = 0.5 \quad \beta = 0.25$

	Sample Size					
	50		100		200	
Value of δ	OSS	CTS	OSS	CTS	OSS	CTS
0.00	97	100	100	100	100	100
0.20	91	100	100	100	100	100
0.40	79	100	100	100	100	100
0.85	33	88	39	100	91	100
0.90	45	87	74	99	95	100
0.95	92	94	100	100	100	100
0.97	99	99	100	100	100	100
0.99	100	100	100	100	100	100
1.00	100	100	100	100	100	100

Table 4.11 compares the performance of One Step Strategy (OSS) with that of General to Simple Strategy using conventional t-test for specification of linear trend in a model. Table 4.11.1 & 4.11.2 correspond to data generating process M1 & M2 (no trend), whereas 4.11.3 corresponds to M3. Therefore, 4.11.1 & 4.11.2 give the empirical size of procedure for selection of linear trend whereas 4.11.3 gives power of the procedure. It can be seen that the size of OSS does not exceeds the nominal 5% size, but the size of TTT deviate a lot. The power of OSS is smaller for some values of autoregressive parameters but the not very small especially in the neighborhood of unity. Fig 4.9 is based on this table.

Table 4.12: Performance of Two Step Strategy

Table 4.12.1: Performance of Two Step Strategy
DGP: M1 $\alpha = \beta = 0$, Sample Size 50,

	Two Step Strategy				Classical Testing Strategy			
δ	M1	M2	M3	CD	M1	M2	M3	CD
0	88	6	6	88	90	6	5	90
0.4	86	7	7	86	89	5	6	89
0.8	80	16	5	80	81	7	12	81
0.9	75	21	3	75	72	9	19	72
0.95	71	26	3	71	63	13	24	63
0.99	65	32	3	65	48	21	31	48
1.00	61	35	3	61	46	23	32	46

CD: Percentage of correct decision

Table 4.12.2: Performance of Two Step Strategy
DGP: M1 $\alpha = \beta = 0$, Sample Size 150,

	Two Step Strategy				Classical Testing Strategy			
δ	M1	M2	M3	CD	M1	M2	M3	CD
0	90	5	5	90	91	5	4	91
0.4	87	8	6	87	90	5	5	90
0.8	70	23	7	70	88	5	7	88
0.9	69	25	6	69	84	6	10	84
0.95	76	20	4	76	76	9	15	76
0.99	70	26	4	70	56	19	25	56
1.00	64	31	5	64	46	23	32	46

CD: Percentage of correct decision

Table 4.12.3: Performance of Two Step Strategy

DGP: M2 $\alpha = 0.25$ $\beta = 0$, Sample Size 50,

	Two Step Strategy				Classical Testing Strategy			
δ	M1	M2	M3	CD	M1	M2	M3	CD
0	56	38	6	38	59	35	6	35
0.4	50	43	7	43	57	36	7	36
0.8	70	26	4	26	56	32	12	32
0.9	74	23	3	23	50	32	17	32
0.95	67	30	3	30	42	34	24	34
0.99	52	43	4	43	32	30	38	30
1.00	46	51	3	51	30	26	43	26

CD: Percentage of correct decision

Table 4.12.4: Performance of Two Step Strategy

DGP: M2 $\alpha = 0.25$ $\beta = 0$, Sample Size 150,

	Two Step Strategy				Classical Testing Strategy			
δ	M1	M2	M3	CD	M1	M2	M3	CD
0	12	82	6	82	13	81	6	81
0.4	11	84	6	84	16	78	6	78
0.8	9	84	7	84	20	73	7	73
0.9	44	49	7	49	24	66	10	66
0.95	68	28	5	28	30	56	14	56
0.99	53	42	5	42	20	46	34	46
1.00	20	76	5	76	18	28	55	28

CD: Percentage of correct decision

Table 4.12.5: Performance of Two Step Strategy DGP: M2 $\alpha = 0.25$ $\beta = 0$, Sample Size 50,								
	Two Step Strategy				Classical Testing Strategy			
δ	M1	M2	M3	CD	M1	M2	M3	CD
0	1	93	6	93	0	95	5	95
0.4	8	86	6	86	0	94	6	94
0.8	51	44	6	44	2	87	11	87
0.9	50	45	5	45	3	83	14	83
0.95	24	71	5	71	1	77	22	77
0.99	4	92	4	92	1	51	48	51
1.00	1	95	4	95	1	43	56	43
CD: Percentage of correct decision								

Table 4.12 (4.12.1 to 4.12.5) summarize the percentage of finding correct form of deterministic regressors for various DGPs using General to Simple Strategy (Classical Testing Strategy) and the Two Step Strategy developed in Section 4.5. If data generating process is M1 or M2, the TSS performs better than the General to Simple Strategy for almost all specifications of autoregressive parameter and for all sample sizes. Fig 4.10 is based on this table.

CHAPTER 5

DETECTING STATIONARITY OF GDP; A TEST OF UNIT ROOT TESTS

Although the stationarity of a time series has very important implications, the problem of testing stationarity of GDP series is still unresolved [see section 1.2 & 1.4 for detail]. There exist large numbers of studies each of which comes to a different conclusion regarding the stationarity of the annual GDP/GNP data. For any given data series, different unit root tests give different results and it is generally not possible to decide which of unit root tests would be the most feasible for this series. This is because the performance of unit root tests depends on the type of data generating process, but for the real data we do not know the true DGP [see section 3.1, 4.1 & 4.2]. Rudebusch (1993)'s bootstrap approach offers an alternative to measure the performance of unit root test for any real time series with unknown DGP. This approach is utilized to measure the performance of unit root tests for the GDP series of various countries, and to compare the tests with each other. Results show that for most data series, unit root tests are unable to discriminate between best fitting models of two types. However, for small number of series, it is possible to discriminate between two types of models and Phillips Perron test performs best for the purpose.

This chapter is organized as follows:

Section 5.1 consists of introduction; Section 5.2 gives the detail of methodology, data and results of the Monte Carlo experiment to compare unit root tests. Section 5.3 discusses implications of the finding of Monte Carlo experiments Section 5.4 concludes the discussion.

5.1: INTRODUCTION

There are a vast number of unit root tests, which often give conflicting results when applied to real data series. Power studies of unit root tests also do not settle the issue that which test is the best, since each test has its strengths and weaknesses. Our study is motivated by the idea that if the problem under study is restricted sufficiently, it may be possible to answer the question as to which test is the best, how much power it has under varying circumstances, and thereby come to a reliable conclusion regarding the original question of whether or not a given GNP series is Trend Stationary or Difference Stationary.

The performance of unit root test is heavily dependent on the type of DGP. Discussion presented in section 3.1 & 4.2 reveals that the tests have optimal performance only when number of deterministic regressors in unit root test equation is equal to the number of deterministic regressors in the true DGP. If there is mismatch, unit root tests suffer severe loss of power. Therefore, the optimal test for a real data series cannot be chosen since we don't know the number of deterministic regressors in true DGP of the data series. Moreover, the conventional methods for specification of deterministic regressors have different properties for trend and difference stationary; therefore, any approximation of number of deterministic regressors is invalid.

Rudebusch (1993) approach offers an alternative to measure the performance of a unit root test for a data series with unknown data generating process. Rudebusch (1993) approach which is summarized in section 2.3, starts by estimating best fitting trend stationary trend stationary and best fitting difference stationary model for the given time series. These models are then used as data generating process to compute the size and power of a unit root test.

An extension of Rudebusch (1993) approach is used to evaluate the performance of unit root tests for the GDP series of various countries. Rudebusch (1993) approach is extended in two directions

i. Rudebusch (1993) procedure measures the performance of single unit root test; we use this approximation of the performance to compare various unit root tests.

ii. Rudebusch (1993) estimates best fitting trend stationary and difference stationary model for single time series and then uses these estimates to evaluate size and power of unit root tests. We formulate two parametric spaces: Θ_{DS} covering the estimated parameters of simplest of the best fitting difference stationary models and Θ_{TS} covering the estimated parameters of simplest of the best fitting trend stationary models for GDP series. The performance of unit root tests is evaluated on these parametric spaces. Thus, the results can be generalized to any data series, whose estimated parameters fall into these parametric spaces.

Extensive Monte Carlo experiments were performed to compute the size and power of various unit root tests for models belonging to the two parametric spaces. Although, the scope of study is limited to the series whose parameters fall into these parametric spaces, our results give a fair measure of the ability of unit root tests to differentiate between trend stationary models. The simulation results lead to the following conclusions:

a. The Detrending based tests including DF-GLS and Ng-Perron tests are unable to discriminate between two types of models for the real GDP series.

b. The size of all unit root tests under consideration in this study does not exceed the nominal size, therefore, the probability of Type I error is not distorted.

c. Most of the tests have virtually zero power for the models under consideration. Phillips Perron test (with constant but no linear trend) maximizes the power for the trend stationary model. The Dickey Fuller test (with constant but no linear trend) comes second in the competition of tests.

d. For many of the series, Dicey Fuller tests and Phillips Perron tests are also unable to discriminate between two types of models with reliable level of confidence. However, for some series these tests have reasonable probability to differentiate between two types of models.

e. For a majority of the countries, nothing can be said about relative likelihood of TS and DS models for the DGP data. For the few countries, where sufficient power is available, the empirical evidence favors the DS model.

5.2: METHODOLOGY, DATA AND RESULTS

5.2.1: TESTS IN COMPETITION

The tests compared in this study are listed as under. The detail on computation of all of these tests is given in Section 2.2. Details about salient features of these tests and critical values are also given in Section 2.2.

- (Augmented) Dickey Fuller Test
  - Without drift and trend (DFN)
  - With drift but no trend (DFC)
  - With drift and Trend (DFT)
- Phillips Perron Test
  - Without drift and trend (PPN)
  - With drift but no trend (PPC)
  - With drift and Trend (PPT)
- Dickey Fuller GLS tests
  - Without Trend (DFGC)
  - With (DFGT)
- Ng Perron Test
  - MZA test without Trend (ZAC)
  - MZA test with Trend (ZAT)
  - MSB test without Trend (SBC)
  - MSB test with Trend (SBT)
  - MPT test without Trend (PTC)

- MPT test with Trend (PTT)
- MZT test without Trend (ZTC)
- MZT test with Trend (ZTT)

This list includes different versions of Dickey-Fuller and Phillips-Perron test from the classical tests and DF-GLS and Ng-Perron tests from the class of Detrending based tests.

5.2.2: DATA AND SAMPLE SIZE

Our main focus in this chapter is the annual GDP series, which share several common characteristics. One of the important characteristic is the small sample size. Most developing countries have small amount of macroeconomic data, which can be used for econometric analysis. The WDI database which is perhaps the largest data source for data on developing countries and is published by World Bank, consist of annual time series for various countries. This database has data starting from 1960; therefore, the length of data available today is smaller than 50 observations. However, for many countries, the available length of macroeconomic time series data does not exceed 20 observations.

The problem we have to study, is to decide whether a given GNP series is TS or DS, requires working with small samples. Samples larger than 50, are not available in our data set. This has important implication because the many tests which have good size/power in large/moderate sample sizes, fail to perform well in the small samples.

The data we use are GDP per capita (Constant US$) retrieved from WDI-2007 CDROM. We select the countries for which data is available from 1960 to 2006 and there is no evidence of structural break in this period. The structural break is inspected by applying Chow break point test to the following autoregression: $y_t = \alpha + \beta t + \sum_{i=1}^{3} y_{t-i} + \varepsilon_t$. Here y_t is the log transform of the GDP series. There were 96 countries for which we find full length data series. After discarding the data series with structural breaks we are left with following 55 countries:

Australia, Austria, Belgium, Benin, Botswana, Burkina Faso, Burundi, Cameroon, Central Africa, Chad, China, Cote d'Ivoire, Denmark, Dominican Rep, Ecuador, Egypt, Finland, France, Greece, Guyana, Honduras, Hong Kong, Iceland, Indonesia, Ireland, Italy, Japan, Kenya, Korea, Luxembourg, Malawi, Malta, Nepal, Netherlands, New Zealand, Nicaragua, Niger, Nigeria, Norway, Pakistan, Portugal, Seychelles, Sierra Leone, Spain, Sri Lanka, Sudan, Sweden, Switzerland, Syria, United Kingdom and Zimbabwe

5.2.3: ESTIMATING BEST FITTING MODELS AND EMPIRICAL PARAMETRIC SPACES

For the GDP of selected 55 countries, best fitting trend stationary and best fitting stationary model were estimated using Rudebusch (1993) approach described in section 2.3. The estimated models have various specifications, however, the simplest and most common trend stationary and difference stationary models were chosen to formulate the parametric spaces. Parametric space Θ_{DS} covers the estimated parameters of DS models and Θ_{TS} covers estimated parameters of TS models.

We report best fitting Difference Stationary models in Table 5.1. The simplest most common DS model was $\Delta y_t = a_0 + \varepsilon_t$, where $a_0 \in (0,.25)$ and $se(\varepsilon_t) \in (0,.027)$. Thus, the two dimensional parametric space for DS models is:

$$\Theta_{DS} = \{(a_0, \sigma_\varepsilon^2) : a_0 \in (0, 0.025), \sigma_\varepsilon \in (0,.027)\}. \quad (5.1)$$

This parametric space covers best fitting models for 22 out of 51 countries.

Best fitting Trend Stationary models are reported in Table 5.2. The simplest most common TS model was $y_t = a_1 + b_1 y_{t-1} + \zeta_t$, where $a_1 \in (0,.45), b_1 \in (0.85, 1)$ and $se(\zeta_t) \in (0, 0.3)$. Thus, the parametric space is:

$$\Theta_{TS} = \{(a_1, b_1, \sigma_\zeta^2) : a_0 \in (0,.45), b_1 \in (.85, 1), \sigma_\zeta \in (0,.027)\} \quad (5.2)$$

This parametric space covers best fitting models for 21 out of 51 countries. The intersection covers 9 countries.

5.2.4: MONTE CARLO DESIGN & RESULTS

5.2.4.1 Size of Tests

Parametric space for DS models i.e. Θ_{DS} was divided into multidimensional grid. Each point of this grid was used as parameter of data generating process. Size of unit root tests was computed at each point of this grid. Size of various unit root tests is reported in Table 5.3. For all tests, the size does not exceed the nominal size. Therefore, the probability of type I error is bounded above by the nominal size. No distortion of size was observed. Also it was observed that the size of tests is independent from the variance of error term σ^2.

5.2.4.2: Power of Tests

The parametric space for TS models Θ_{TS} was divided into another multidimensional grid and power of unit root tests was computed at each point of this grid. The powers of various unit root tests are reported in Table 5.4.

The powers of unit root tests show many unexpected results. Most surprising was the failure of tests based on GLS detrending including the Ng-Perron and the DF-GLS test. The DF-GLS is shown to have power closest to asymptotic power envelope (Elliot et al., 1996). Ng-Perron test is a test accumulating intellectual heritage of the DF-GLS test and M-estimator by King (1990). However, the optimality of these tests is based on asymptotic properties. The simulations show that these properties do not hold for small samples. For instance, the minimum sample size used for simulations by Ng & Perron (2001) is 100, whereas our sample size is 45. Anyway, these simulations show clear superiority of Dickey Fuller and Phillips Perron tests over the DF-GLS and the Ng-Perron tests in small samples. In fact an overview of Table 5.4 reveals that the power of detrending based tests i.e. the DF-GLS and Ng-Perron test rarely exceeds their size, so that these tests have no ability to discriminate between the trend and difference stationary processes for data under consideration. Furthermore, an overview of power of tests tabulated in table 5.4 reveals that ranking of tests according to average power for TS models is as follows: PPC, DFC, PPT and DFT.

5.2.5: THE RESPONSE SURFACE FOR POWER OF TESTS

The PPC test and DFC tests have maximum average power for the TS models, thus they have best overall performance in the context under consideration. The response surface function was estimated to decide better test among these two.

The response surface for DFC and PPC tests are given in Fig 5.1.

The response surfaces for the powers of two tests show similar behavior. The power of the tests is positively related to the difference of lag coefficient b_1 from unity i.e. its power increases if the value of b_1 goes to zero (distance from unity increases). The power is positively associated with the constant a_1 i.e. increases with the increase in value of a_1. However, it can be observed from table 5.4 that power of PPC test is higher than that of DFC test for entire parametric space.

We compute the approximate response surface functions for the powers of two tests by regressing the power of tests on various functions of $a_1 \quad and \quad b_1$. These response surface functions are:

$$\ln(P_{dfc} | a_1, b_1) = 374.394 - 17.392a_1 - 211.413b_1 - 8.03a_1^2 + 28.765a_1b_1 - 163.386b_1^{-1}$$

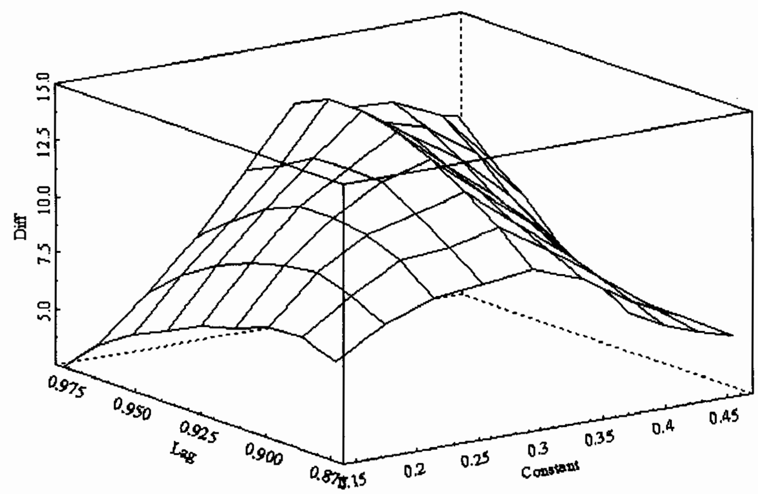
And

$$\ln(P_{ppc} | a_1, b_1) = 355.046 - 22.041a_1 - 201.320b_1 - 10.032a_1^2 + 34.762a_1b_1 - 154.032b_1^{-1}$$

where $P_{dfc} \quad and \quad P_{ppc}$ are the powers of DFC and PPC tests respectively. The two models are fairly similar to each other and both provide equal degree of fitness (R-square $\cong 92\%$ for the two models).

The numerical evaluation of the two functions reveals that value of difference $\ln(P_{ppc} | \theta) - \ln(P_{dfc} | \theta)$ is never smaller than zero for all $\theta \in \Theta_{TS}$. Fig 5.2 plots the difference between power of PPC test and DFC test i.e. $Diff = P_{ppc} - P_{dfc}$ estimated by using response surface function. Figure 5.2 confirms that power of PPC test is superior to that of DFC test, since the difference is always positive.

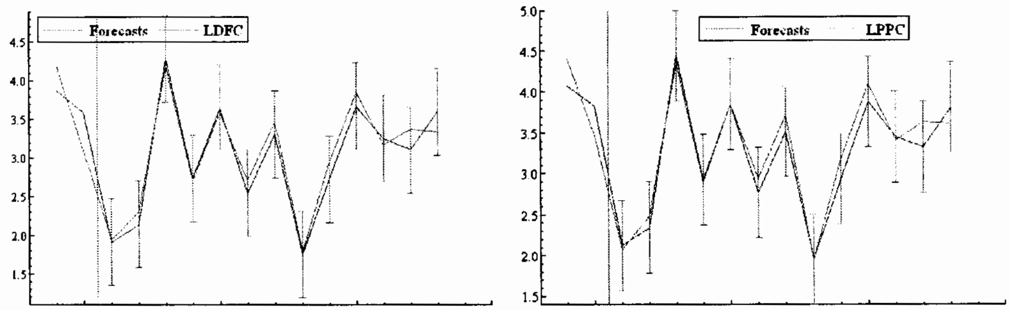
Fig 5.2: Difference Between powers of PPC and DFC



The figure plots the difference $Diff = P_{ppc} - P_{dfc}$. The difference is positive for all points in parametric space Θ_{TS} which shows that PPC test is superior to DFC test with regard to its power.

The estimated function was than used to predict power of tests for actual models for the real data.

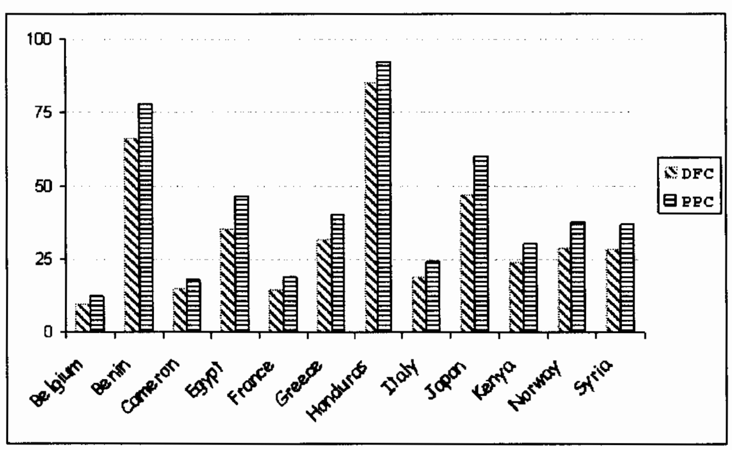
Fig 5.3: The predictions by estimated response surface functions of PPC and DFC



The estimation of Response Surface function was carried out using power of tests at regular grid and this function was than used to predict power of tests on some other points in the parametric space which corresponds to estimated models for real time series. The predictive performance of two tests seems reasonable

The Figure 5.3 gives the power of PPC and DFC tests for the estimated best fitting models for various countries. It is clear that the PPC test has better performance than DFC for all models. The powers of all other tests are much smaller than the powers of these two tests.

Fig 5.4: Power of PPC and DFC for best fitting TS models



Powers of DFC and PPC for TS models of various countries are plotted. The superiority of PPC to DFC is clearly visible

It can be seen that for all of the countries, the performance of PPC test is superior to that of DFC test. This leads to the conclusion that the PPC test is superior to other tests with regard to its power for testing stationarity of real GDP series. This superiority occurs without any distortion in the size of test, Therefore, the PPC test is superior to all other tests.

5.3: IMPLICATIONS

5.3.1: COMPARISON OF UNIT ROOT TESTS

Assume that for GDP of any country, the estimated best fitting trend stationary and difference stationary model are only two possible models. If the true data generating process was difference stationary, the tests should not reject unit root. Table 5.3 gives simulated probabilities of rejection of unit root for the DS models. It can be seen that the probability of rejection of unit root (Type I error) does not exceed 5% nominal size if the estimated parameters lie within the parametric space Θ_{DS}. Therefore, all unit root tests have capability of transmitting right message about stationarity of the series when true model is DS with parameters belonging to the parametric space.

Now if the true data generating process was trend stationary, than the unit root should be rejected. However, table 5.4 reveals that the GLS detrending based tests including DF-GLS and Ng-Perron test are unable to reject unit root for the trend stationary models with parameters belonging to Θ_{TS}. Detrending based tests have the tendency of not rejecting unit root, regardless of the type of data generating process. Therefore, these tests are unable to determine the type of stationarity for the data under consideration. Similarly DFN and PPN tests are also unable to reject unit root when true DGP is Trend Stationary. The PPT test and DFT tests also have low probability to reject unit root for trend stationary DGP.

However, PPC and DFC tests have maximum probabilities of rejecting unit root if the data was actually generated by TS model. Section 5.3 reveals that overall best performer test is PPC test.

The power of PPC test depends on the two parameters if the estimated model is generated from parametric space Θ_{TS}. Power depends on distance from the unity $1-b_1$ and on the lag coefficient a_1. Larger values of $1-b_1$ and a_1 lead to increased power (see Fig 5.2) and positively related to the value of drift coefficient.

5.3.2: RELIABILITY OF UNIT ROOT TESTS

The expected power of the best performing test i.e. the PPC tests for various countries based on response surface function is summarized in table 5.5. The simulation results in reported in section 5.3 and Fig 5.3 reveal that actual power of unit root tests does not deviate much from this approximation. Power of PPC test shows different characteristic for different models.

The TS models for various countries can be divided into three groups with respect to the power attained. For first group of countries, say Group I, PPC test has very low probability of rejecting unit root. This group contains the countries for which value of lag coefficient b_1 is close to unity and/or value of drift coefficient a_1 is close to zero. These countries include Malta, Nicaragua, Austria, Belgium, Guyana, Italy and Cameroon. For these countries the PPC test has less than 25% power. Since all other

5.3.3: STATIONARITY OF GDP SERIES

The analysis presented in 5.3.2 shows that the tests would be inconclusive for most of the countries. However, for Group III of countries containing Burundi, Chad, Malawi, Benin and Nigeria, we can determine the stationarity of data series with reasonable level of confidence using PPC test. Also for countries belonging to Group II, PPC test has power between 25%-75%.

When the unit root tests were applied to real data, all tests failed to reject unit root, for all of the countries including Group III. This implies the real data series have more resemblance with the DS model.

5.3.3: STATIONARITY OF GDP SERIES

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When the unit root tests were applied to real data, all tests failed to reject unit root, for all of the countries including Group III. This implies the real data series have more resemblance with the DS model.

5.4: CONCLUSION

A major problem in the comparison of various unit root tests is the absence of information about the data generating process of time series in hand. The properties of unit root tests crucially depend on the DGP, and for the real data, we have no information about the true DGP. The estimation of DGP via general to simple methodology is also not feasible since the performance of estimators depend on existence or otherwise of unit root.

Rudebusch (1993) approach offers an alternative to measure the performance of unit root test for any given series with unknown DGP. Rudebusch (1993) first estimates best fitting trend stationary and difference stationary models. The two models provide unbiased and consistent estimates of the parameters in general to simple specification procedure since they involve the stationary regressors [See Section 2.4].

Rudebusch (1993) approach is extended in various dimensions to use it to compare the unit root tests (see 5.1). Section 5.2 summarizes the results. This procedure gives fairly clear comparison of various unit root tests in terms of their size and power properties.

The findings of this study are summarized as under:

a. **Size of Unit Root Tests:** If we look at size of various unit root tests, it appears that actual size of all tests is smaller than the nominal size. This means that there is upper bound on probability of Type I error. No size distortion was observed for any of the tests.

b. **Power of Detrending based Unit Root Tests:** The simulated power of unit root tests gives some unexpected results. The most important is the failure of tests based on GLS detrending i.e. the DF-GLS and the Ng-Perron tests. DF-GLS test is assumed to have power closest to asymptotic power envelope and the Ng-Perron tests accumulates over the DF-GLS and some other tests. However, the optimality properties of these tests are based on asymptotic results and our study shows that these properties are not be valid for small samples.

c. **Power of ADF and PP Tests:** An overview of power of various unit root tests (Table 5.4) reveals that the clear winners in competition of unit root tests are PPC tests and DFC tests. The response surface analysis (Section 5.3) reveals that PPC test is superior to DFC test for all points in the parametric spaces Θ_{TS} .

d. **Reliability of Unit Root Tests:** The simulations show that most of tests have tendency to accept unit root even if series is generated by TS model. Only PPC and DFC test have reasonable power for TS models of few countries.

Therefore the tests have little ability to discriminate between TS and DS models.

e. **Stationarity of GDP:** The analysis in 5.4 shows that the tests would be inconclusive for most of the countries. However, for few countries we can determine the stationarity of data series with reasonable level of confidence using PPC test. We find that unit root cannot be rejected for any of these countries. Therefore, we conclude that the real GDP series is better described by a DS model. Unit root was also not rejected for the group of countries for which PPC test have power between 25% and 75%.

Limitations of Study: However, the limitations of this analysis are presented as under: This analysis is valid if the estimated parameters of best fitting DS and TS models of a series fall within the parametric spaces Θ_{DS} and Θ_{TS}. Also the length of time series was 46 throughout this analysis and results may not hold for longer time series.

5.5: APPENDIX:

Table 5.1: Best Fitting Difference Stationary Models for Various Countries

Table 5.1: Best Fitting Difference Stationary Models for Various Countries

Country	Const	Lag 1	Sigma	Country	Const	Lag 1	Sigma
Australia	0.009		0.009	Japan	0.005	0.660	0.011
Austria	0.012		0.008	Kenya	0.006		0.018
Belgium	0.011		0.008	Korea	0.025		0.015
Benin	0.002		0.013	Luxembourg	0.013		0.014
Botswana	0.008	0.734	0.013	Malawi	0.004		0.024
Burkina Faso	0.006	-0.332	0.013	Malta	0.007	0.673	0.013
Burundi	0.001		0.024	Nepal	-0.313	-0.313	0.012
Cameroon	0.004		0.026	Netherlands	0.006	0.363	0.007
Central Africa	-0.004	-0.076	0.018	New Zealand	0.006		0.013
Chad	0.001	0.098	0.037	Nicaragua	-0.002	0.301	0.029
China	0.020	0.340	0.020	Niger	-0.007	0.054	0.027
Cote d'Ivoire	-0.001	0.297	0.022	Nigeria	0.003	0.325	0.031
Denmark	0.009		0.009	Norway	0.008	0.394	0.006
Dominican Rep.	0.012		0.022	Pakistan	0.011		0.010
Ecuador	0.006		0.015	Portugal	0.007	0.495	0.014
Egypt	0.007	0.473	0.011	Seychelles	0.010		0.026
Finland	0.005	0.520	0.011	Sierra Leone	0.000		0.030
France	0.005	0.534	0.006	Spain	0.004	0.651	0.008
Greece	0.008	0.368	0.015	Sri Lanka	0.013		0.012
Guyana	0.003	0.309	0.023	Sudan	0.005		0.024
Honduras	0.004		0.013	Sweden	0.005	0.468	0.007
Hong Kong	0.021		0.018	Switzerland	0.004	0.328	0.009
Iceland	0.008	0.396	0.015	Syria	-0.265	0.009	0.032
Indonesia	0.011	0.324	0.017	United Kingdom	0.009		0.008
Ireland	0.009	0.465	0.010	Zimbabwe	-0.001	0.374	0.025
Italy	0.008	0.321	0.009				

Table 5.1 gives Best Fitting Difference Stationary models for GDP series of various countries. Detail about this estimation of models is given in Section 2.3.

Table 5.2: Best Fitting Trend Stationary Models for Various Countries

Table 5.2: Best Fitting Trend Stationary Models for Various Countries

Country	Const	Lag 1	Trend	Sigma	Country	Const	Lag 1	Trend	Sigma
Australia	0.700	0.83	0.001	0.008	Japan	0.257	0.95		0.010
Austria	0.119	0.97		0.007	Kenya	0.166	0.94		0.020
Belgium	0.135	0.97		0.007	Korea	0.502	0.84	0.004	0.015
Benin	0.311	0.87		0.013	Luxembourg	0.469	0.89	0.002	0.013
Botswana	0.070	0.99		0.019	Malawi	0.268	0.88		0.023
Burkina Faso	1.157	0.47	0.002	0.012	Malta	0.081	0.98		0.017
Burundi	0.222	0.89		0.023	Nepal	0.214	0.90	0.001	0.012
Cameroon	0.139	0.95		0.025	Netherlands	0.493	0.88	0.001	0.007
Central Africa	0.054	0.98		0.018	New Zealand	0.844	0.79	0.001	0.012
Chad	0.300	0.87		0.036	Nicaragua	0.077	0.97		0.031
China	0.445	0.74	0.008	0.026	Niger	0.670	0.74	-0.002	0.026
Cote d'Ivoire	0.297	0.90	-0.001	0.021	Nigeria	0.348	0.87		0.031
Denmark	1.001	0.76	0.002	0.008	Norway	0.200	0.96		0.007
Dominican Rep.	0.430	0.85	0.002	0.023	Pakistan	0.234	0.90	0.001	0.010
Ecuador	0.205	0.93		0.015	Portugal	0.271	0.93	0.001	0.014
Egypt	0.205	0.93	0.001	0.012	Seychelles	0.613	0.82	0.002	0.026
Finland	0.333	0.92	0.001	0.012	Sierra Leone	0.133	0.94		0.030
France	0.158	0.97	0.001	0.005	Spain	0.587	0.85	0.001	0.008
Greece	0.213	0.95		0.014	Sri Lanka	0.805	0.66	0.005	0.011
Guyana	0.115	0.96		0.023	Sudan	0.185	0.92	0.001	0.023
Honduras	0.438	0.85	0.001	0.012	Sweden	0.668	0.84	0.001	0.007
Hong Kong	0.114	0.98	0.001	0.017	Switzerland	0.784	0.82	0.001	0.008
Iceland	0.395	0.91	0.001	0.016	Syria	0.202	0.93		0.033
Indonesia	0.292	0.87	0.003	0.017	United Kingdom	0.989	0.76	0.002	0.007
Ireland	0.079	0.98	0.001	0.011	Zimbabwe	0.187	0.93		0.026
Italy	0.171	0.96		0.008					

Table 5.2 gives Best Fitting Trend Stationary models for GDP series of various countries. Detail about this estimation is given in Section 2.3.

Table 5.3: Probability of Rejection of Unit Root (size) for Various DS models

Table 5.3: Probability of Rejection of Unit Root (size) for Various DS models												
	Sigma=0.01						**Sigma=0.02**					
CNST	**DFN**	**DFC**	**DFT**	**PPN**	**PPC**	**PPT**	**DFN**	**DFC**	**DFT**	**PPN**	**PPC**	**PPT**
0	**4**	5	5	4	5	6	**4**	5	5	4	5	5
0.05	**0**	0	5	2	1	5	**1**	0	5	2	0	5
0.1	**0**	0	5	0	0	5	**0**	0	5	0	0	5
0.15	**0**	0	5	0	0	6	**0**	0	5	0	0	6
0.2	**0**	0	5	0	0	6	**0**	0	5	0	0	5
0.25	**0**	0	5	0	0	5	**0**	0	5	0	0	6

Table 5.3 gives probability of rejection (size) of unit root tests for Difference Stationary models belonging to parametric space Θ_{DS}. The variance does not seem to play any role in determining size of tests within the parametric space. The size of tests does not exceed nominal level of 5%.

Table 5.4: Probability of Rejection of Unit Root for Various TS models (power)

Table 5.4: Probability of Rejection of Unit Root (power) for Various TS models

CNST	LAG1	DFN	DFC	DFT	PPN	PPC	PPT	DFGC	DFGT	ZAC	ZTC	SBC	PTC	ZAT	ZTT	SBT	PTT
	0.86	7	32	13	12	39	16	13	12	11	11	11	5	4	4	4	4
	0.88	5	28	10	8	35	12	13	12	11	11	11	5	4	4	4	4
	0.9	3	24	8	6	29	10	10	9	8	8	8	5	3	3	4	3
	0.92	2	19	7	3	24	8	6	6	5	5	5	4	3	3	3	3
0.1	0.94	1	15	6	1	18	6	4	4	3	3	3	4	2	2	3	2
	0.96	1	10	5	1	12	5	1	1	1	1	1	4	3	3	3	2
	0.97	0	7	5	1	9	5	1	1	1	1	1	4	3	3	3	3
	0.98	0	5	5	0	6	6	0	0	0	0	0	5	3	3	3	3
	0.99	0	3	5	1	3	5	0	0	0	0	0	4	3	3	4	3
	0.86	0	48	18	1	58	23	7	7	6	6	6	3	2	2	2	2
	0.88	0	45	14	0	56	18	5	4	4	4	4	2	2	2	2	2
	0.9	0	44	11	0	54	13	3	3	2	2	2	2	1	1	1	1
	0.92	0	42	9	0	52	10	1	1	1	1	1	1	1	1	1	1
0.2	0.94	0	35	6	0	45	6	0	0	0	0	0	1	1	1	1	1
	0.96	0	25	4	0	33	5	0	0	0	0	0	1	1	1	1	1
	0.97	0	17	4	0	23	4	0	0	0	0	0	1	1	1	1	1
	0.98	0	11	4	0	14	4	0	0	0	0	0	2	2	1	2	2
	0.99	0	4	4	0	5	5	0	0	0	0	0	4	3	2	3	2
	0.86	0	67	26	0	78	36	2	2	2	2	2	1	1	1	1	1
	0.88	0	68	20	0	79	28	1	1	1	1	1	1	1	1	1	1
	0.9	0	69	15	0	80	21	0	0	0	0	0	0	0	0	0	0
	0.92	0	67	11	0	81	14	0	0	0	0	0	0	0	0	0	0
0.3	0.94	0	65	7	0	78	9	0	0	0	0	0	0	0	0	0	0
	0.96	0	53	4	0	68	5	0	0	0	0	0	0	0	0	0	0
	0.97	0	39	4	0	53	4	0	0	0	0	0	0	0	0	0	0
	0.98	0	24	4	0	33	4	0	0	0	0	0	1	1	0	1	0
	0.99	0	7	4	0	10	4	0	0	0	0	0	3	2	2	2	2

Table 5.4 (Continue)

	0.86	0	84	37	0	91	53	1	1	0	0	0	0	0	0	0	0
	0.88	0	85	29	0	93	43	0	0	0	0	0	0	0	0	0	0
	0.9	0	87	23	0	94	33	0	0	0	0	0	0	0	0	0	0
	0.92	0	87	15	0	96	22	0	0	0	0	0	0	0	0	0	0
0.4	0.94	0	87	8	0	96	12	0	0	0	0	0	0	0	0	0	0
	0.96	0	79	4	0	92	5	0	0	0	0	0	0	0	0	0	0
	0.97	0	68	3	0	83	3	0	0	0	0	0	0	0	0	0	0
	0.98	0	44	2	0	57	2	0	0	0	0	0	0	0	0	0	0
	0.99	0	14	3	0	19	3	0	0	0	0	0	1	1	1	1	1
	0.86	0	93	49	0	97	68	0	0	0	0	0	0	0	0	0	0
	0.88	0	95	43	0	98	61	0	0	0	0	0	0	0	0	0	0
	0.9	0	96	33	0	99	49	0	0	0	0	0	0	0	0	0	0
	0.92	0	97	22	0	99	34	0	0	0	0	0	0	0	0	0	0
0.5	0.94	0	97	11	0	100	17	0	0	0	0	0	0	0	0	0	0
	0.96	0	93	5	0	99	6	0	0	0	0	0	0	0	0	0	0
	0.97	0	86	3	0	96	3	0	0	0	0	0	0	0	0	0	0
	0.98	0	67	2	0	81	2	0	0	0	0	0	0	0	0	0	0
	0.99	0	22	3	0	32	3	0	0	0	0	0	1	0	0	1	0

Table 5.4 gives probability of rejection of unit root tests for trend stationary models (power) belonging to parametric space Θ_{TS}. Simulation was done for different values of variance but the variance did not effect power of tests..

Table 5.5: Power of PPC test for Various Countries

Table 5.5 Power of PPC test for Various Countries

Country	Estimated TS Model		Power	Country	Estimated TS Model		Power
	Const	Lag			Const	Lag	
Malta	0.081	0.98	5	Greece	0.213	0.95	36
Nicaragua	0.077	0.97	7	Zimbabwe	0.187	0.93	46
Austria	0.119	0.97	10	Japan	0.257	0.95	48
Belgium	0.135	0.97	11	Syria	0.202	0.93	51
Guyana	0.115	0.96	13	Ecuador	0.205	0.93	52
Italy	0.171	0.96	21	Burundi	0.222	0.89	78
Cameroon	0.139	0.95	21	Chad	0.3	0.87	89
Norway	0.2	0.96	26	Malawi	0.268	0.88	89
Sierra Leone	0.133	0.94	26	Benin	0.311	0.87	91
Kenya	0.166	0.94	33	Nigeria	0.348	0.87	98

Chapter 6

CONCLUSION, APPLICATIONS & DIRECTIONS FOR FURTHER RESEARCH

6.1: CONCLUDING REMARKS

As stated in the introduction, the development of unit root literature is stimulated by two problems:

1. The results of unit root tests are often misleading, which is obvious from the concerns about size and power of unit root tests. Unit root tests have optimal power/size properties when the model used for testing unit root is similar to the data generating process. Unit root tests suffer size/power distortion when there is mismatch between the two.

2. Classical techniques for specification of economic model using time series data are often misleading; this fact is reflected in the frequent encounter with the spurious regression. In the presence of unit roots, all conventional

various countries. (ii) Phillips Perron test with intercept (no linear trend) achieves maximum probability to discriminate two types of models (iii) the tests have limited ability to discriminate between Trend and Difference Stationary models. (iv) the difference stationary model is more appropriate for the GDP series (see Section 5.3 and 5.4).

6.2: APPLICATIONS

6.2.1: THE OSS AND TSS

We apply the Two Step Strategy to the unemployment rate of 12 high income OECD countries to specify drift and trend. The economic theory does not support existence of trend and drift in the unemployment rate [Elder & Kennedy, 2001]. The results show that the outcomes of TSS are in conformity with this for all countries under investigation, whereas the alternative methods often reject null of no deterministic regressor. The results are reported in Table 7.1 (in Appendix):

6.2.2: THE TEST OF UNIT ROOT TESTS

The discussion presented in chapter 5 reveals that in the time series with smaller sample sizes, the Ng-Perron test and the DF-GLS test have little probability to reject unit root and thus unable to discriminate between the trend and difference stationary model. At the same time Phillips Perron and ADF test do better job to discriminate trend and difference stationary model. Therefore, we predict that Ng-Perron and DF-GLS test will accept null hypothesis of unit root for time series of with small sample sizes. There are number of evidences to support this claim. We provide here some evidences from published results.

Shahbaz, Ahmad and Chaudhary (2007) analyze real GDP per capita, financial development, foreign direct investment, GDP, and annual inflation for Pakistan. Hye, Shahbaz and Butt (2008) analyze output, agricultural terms of trade and technology in agriculture, Hye and Riaz (2008) analyze energy consumption and economic growth for Pakistan using Ng-Perron test. Unit root null was not rejected for all of the series analyzed in three studies.

Soytas and Sari (2007) apply various unit root test to following Turkish economic timer series: Total employment in manufacturing, total electricity consumption in industry, value added-GNP manufacturing and total fixed investment in manufacturing. They apply DF, DFGLS, PP and Ng-Perron test to these series with two specifications of deterministic part i.e. including linear trend and without including linear trend. Their results are totally consistent with the results we computed and summarized in chapter 5. Phillips Perron test reject unit root for some of these series at 1% significance level but Ng Perron test and DF-GLS fail to reject unit root for the same series at 10% level of significance. For the remaining series, neither PP test nor remaining tests reject unit root.

6.3: DIRECTIONS FOR FURTHER RESEARCH

Here is the list of some problems which can be investigated further.

1. We have computed performance of OSS and TSS for AR(1) models. The performance of these strategies can be be analyzed for AR(p) model and ARMA(p,q) models

2. The methodology of OSS and TSS can be used in conjunctions with structural breaks

3. The methodology of 'Test of Unit Root Test' is applied to small samples where we saw that GLS-detrending based tests failed to perform. This analysis can be repeated for large sample data to explore the dominant test in large sample sizes.

6.4: APPENDIX

Table 6.1: Specification of Deterministic Regressors by G2S and TSS

Table 6.1: Specification of Deterministic Regressors via G2S and TSS

	Conventional Significance		Two Step Strategy	
	Drift	Trend	Drift	Trend
Australia	Yes	No	No	No
Canada	Yes	Yes	No	No
Finland	No	No	No	No
France	Yes	No	No	No
Italy	No	Yes	No	No
Japan	No	No	No	No
Korea	No	No	No	No
Norway	No	No	No	No
Portugal	No	No	No	No
Spain	Yes	Yes	No	No
Sweden	No	No	No	No
United States	No	No	No	No

Yes if coefficient is significant, No Otherwise

Table 6.1 gives result of specification of drift and trend for unemployment series of number of countries using sequential testing strategy and the two step strategy. The economic theory does not support existence of drift and trend in unemployment series. The results of two step strategy are consistent with the theory for all countries whereas results of sequential testing strategy often contradict.

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