Advanced Adaptive Sliding Mode Controller Design Techniques For Coupled Tanks Liquid Level Control System

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RESEARCH COMPLETION CERTIFICATE

It is Certified that the research work contained in this thesis titled "**Advanced Adaptive Sliding Mode Controller Design Techniques For Coupled-Tank Liquid Level Control System**" has been carried out and completed by **Mr. Hur Abbas, Reg. No.179-FET/MSEE/F08** under my supervision.

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CERTIFICATE OF FINAL APPROVAL

It is certified that we have read the final thesis submitted by **Mr. Hur Abbas**, Registration Number: **179-FET/MSEE/F08** and it is our judgment that this thesis is of sufficient standard to warrant its acceptance by **INTERNATINAL ISLAMIC UNIVERSITY ISLAMABAD** in partial fulfillment of the requirement for the **Master of Philosophy Degree in Electronic Engineering (MSEE).**

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ABSTRACT

The process industries like the paper making, petro-chemical and water treatment industries have necessity that the liquid to be supplied in tanks, stored in tanks, and then transfer to other tanks as per requirement. The pumping of liquid in tanks, flow of liquid from one tank to another and supplying the liquid where it is required is a common and basic problem in the process industries. In this project the Sliding Mode Controller is designed for controlling the liquid level in the Coupled-Tank. The objective of the research is to design the different control techniques of Sliding Mode Control (SMC) for the level control in coupled tank plant. First of all, a dynamic model of plant is developed. The plant is simulated in MATLAB/SIMULINK and simulation studies are conducted based on the developed model of plant. A number of tests are conducted regarding tracking performance, disturbance rejection and plant parameter changes to analyze the performance of controller. A concise comparison of Sliding mode controller (SMC) tracking performance is made with PID tracking performance. It is found that SMC is a robust controller than PID controller when there is a variation in system parameters and in the presence of disturbance. In order to make the SMC also as an adaptive controller, the Model reference adaptive controller is merged with SMC. Hence a Robust and Adaptive controller is designed for coupled tank system. This controller is able to sense the change in the plant and is able to update its parameters according to the changes in the plant. The outcome of the project reveals that Model reference adaptive sliding mode controller is able to deal with non-linearity, disturbances and all sudden changes in the plant parameters. The framework of this project is so generic that the possible outcomes can be overviewed before implementing the SMC in real-time environment in the future.

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Nomenclature

M	Total mass
ρ	Density of fluid
V	Volume of fluid
U_i	Input flow rate
U_o	Output flow rate
A_i	Cross-sectional area of tanks
H_i	Liquid height in tanks
v	Velocity of liquid
g	Gravitational acceleration
β_i	Flow constants
h_i	Perturbation in tank levels
u_i	Perturbation in inflow rate
τ_1	Time constant of tank
K_1	Steady state gain
G_{DC}	DC gain
G(s)	Transfer function
U_{imax}	Maximum allowable volumetric flow rate
Y_m	Output of reference model
r	Reference input

e	Error
h_D	Desired height
S	Sliding surface
λ	Slope of sliding surface
U	Control input
U_c	Continuous control
U_D	Discontinuous control
U_{eq}	Equivalent control
η	Coefficient of viscosity
P	Pressure
L	Length of pipe
Y_p	Output of plant
K^*	Feedback gain
L^*	Feed forward gain
E_a	Activation energy

CHAPTER NO: 1

INTRODUCTION

1.1 COUPLED TANK SYSTEM

The process industries like petro-chemical, pharmaceutical, paper making and water treatment industries have necessity that the liquid to be supplied in tanks, may be stored in tanks, and afterwards transferred to other tanks according to the requirements. The supply of liquid in tanks, flow from one tank to another and supplying the liquid according to requirement is a most common problem in the process industries. Often it may be the required that the liquids be supplied in tanks with a chemical or mixing treatment but it is always of interest to maintain the liquid at a specific height or in a specific range. In order to achieve this goal, the liquid supplied to the tanks must be controlled properly. The applications of liquid level control in industries includes, food processing, boilers, dairy, beverage, filtration, spray coating, effluent treatment, pharmaceutical, nuclear power generation plants, industries, water purification systems, industrial chemical processing, automatic liquid dispensing and refilling devices [1]. The coupled-tank liquid level control system is regarded as the appropriate plant to imitate the process control in petrol and chemical industries. The processing plants in these industries mostly involve in controlling the liquid level and the flow rate from one tank to another in the presence of nonlinearity and disturbance [2].

1.2 OVERVIEW OF CONTROL SYSTEM

By the end of the 20th century, the field control system got a vital position in the field of engineering and became necessary element of modern society. Almost every system we are using is underpinned by control system. The examples of these systems range from simple household products (power regulator of electric fan, remote control of television, thermostats in hot water heaters, temperature regulation in air-conditioners etc.) to more complicated systems like the car (which has a number of control loops) to large scale systems (such as aircraft, industrial plants, atomic and chemical plants and manufacturing plants).

There is no denying the fact that industrial revolution had developed the engineering and resultantly control system achieved its progress. During that time, a lot of improvement and development came in machines and resultantly improved and efficient machines significantly improved the capacity to re-utilize the raw and waste materials into the useful and beneficiary products of people. The machines used for such purposes require a large amount of power and proper execution of operation. Then it was realized that this power should be controlled in an organized manner in order to maneuver the systems safely and efficiently [3].

Therefore, the control system engineering is concerned with designing of system, implementation and then operating these systems so as to function in an efficient manner. This is one of the most interesting and challenging area of modern engineering. Actually the control system engineering field is not independent. To implement a control on a system effectively, it is required to merge many other disciplines including mathematical modeling of the system (using the fundamental laws of physics and chemistry of the process), sensor technology (to detect and measure the current position of the system), actuators (to implement the correct action to the system), communications (to correspond the different parts of plant by transmitting data), computing (to execute the composite task of changing measured data into suitable actuator actions) and interfacing (to authorize the all of different parts of system to talk and respond to each other in a seem less way).

The control system is one of the most exciting multidisciplinary fields of engineering which has extremely large range of practical applications. The interest in the field of control and its application in new inventions have brought the mankind in a new era of modern science. Now it is to be expected that the control system engineering may become more essential because of the environmental concerns and increasing globalization of markets [3].

1.3 CONTROL SYSTEM CONFIGURATION

In the field of Control systems engineering, the systems can be categorized as self-correcting systems and non self-correcting systems.

The terminology self-correcting system pertains that the system has the ability to observe or measure an output or the variable of interest and correct it automatically without the involvement of a human operator. However, if the output or the variable of interest is above or below the specified limit, the systems automatically take right action. The systems that can execute such self-correcting actions are called feedback systems or closed-loop systems whereas non self-correcting systems are known as open loop systems [3].

1.3.1 OPEN LOOP LIQUID LEVEL CONTROL SYSTEM

As explained above the systems which do not monitor their output are known as open loop or non-self correcting systems. For the sake of maintaining the liquid in tanks at desired position, one would make use of a human operator who will adjust the manual valve so that the rate of liquid flow into the tank exactly balances the rate of liquid flow out of the tank. Resultantly, after a little bit of trial and error for the correct adjustment of valve, at last an intelligent human operator can adjust the proper valve setting. So if the operator continuously monitors the system for some time then he/she will observe that the liquid level remains constant at some specific height then he/she may conclude that the suitable adjustment of valve opening has been made for desired level [3].

But it is possible that some other disturbing factors can change or disturb the desired level again. For example, if any change occurs in temperature of fluid then the fluid viscosity would alter and hence the flow rate will be changed. Hence by the variation in outflow rate, the level of liquid in the tank will not remain at constant desired position. Also the leakage in the tank or an extra input may disturb the liquid level. In the presence of all these uncertainties, an operator must be there to adjust the valve again. So it would be considered as open loop level control system with a human operator.

1.3.2 CLOSED LOOP LIQUID LEVEL CONTROL SYSTEM

In the closed-loop systems, the output of the plant is monitored continuously and system has the ability to compare its output to the reference or desired value. The controller takes the action based on the error (between the actual and desired value) and controls the plant.

In case of level control in a tank, the error between actual height and desired height is continuously fed back to the controller. The controller operates on this error signal and adjusts the valve in order to minimize the error between actual height and desired height. So whenever there is a change in the reference signal or any other disturbance occurs, the controller itself adjusts the valve as per requirement. Hence the feedback control system diminishes the necessity of engaging a human operator.

1.4 PROBLEM STATEMENT

The delays present coupled-tank system might set obstacles for attaining high performance control. There are also several cases like coupled-tank in industry where the parameters of the plant parameters and disturbances acting on the system are continuously changing [4]. So it is the necessary to regulate the different variables simultaneously in order to achieve the desired high performance.

There are a number of control schemes that can be designed and implemented to fulfill the control task of the system. But there are number of issues such as tracking, stability, overshoot, reducing the effects of undesirable conditions, a certain rise-time, uncertainty and steady state tracking error which needs to be settled. The fixed gain controllers are not capable for achieving such high performance. Because fixed gain controllers do not senses any change in the plant and controls the plant conventionally. Also there may be some of unknown plant parameters (e.g. viscosity in coupled-tank) which cannot be modeled correctly in order to get a complete detailed model of a plant used for control intention.

Hence, it is necessary to have a Robust and Self adjusting (Adaptive) control scheme for a non-linear and complex system that can be utilized in all industrial plants.

1.5 OBJECTIVES

The objective of the research is to look forward the advanced techniques of Sliding Mode Control (SMC) for coupled-tank plant.

The objectives of the research work can be outlined as:

- **i.** To design the basic Sliding Mode Controller and Sliding Mode Controller with equivalent control technique for coupled tanks plant
- **ii.** To make the comparison of Sliding mode control with PID controller
- **iii.** To design a Model Reference Adaptive Controller (MRAC) for coupled tanks plant
- **iv.** To design a Model Reference Adaptive Sliding Mode Controller for coupled tanks plant
- **v.** To make the comparison of Model reference adaptive control and Model reference adaptive sliding mode control for coupled tanks plant

1.6 NOVELTY

The designing of Model Reference Adaptive Sliding Mode Controller (MRASMC) for coupled-tank system is the novelty of this research work. The designed controller is the combination of Model reference adaptive control and Sliding mode control. Hence the designed controller has the capabilities such as Robustness and self-correction.

1.7 ORGANIZATION OF THE WORK

The research work can be outlined likewise;

- The first Chapter briefly explains the Coupled-tank plant and its mode of operation.
- The second Chapter covers the research findings regarding coupled tanks system and Sliding mode controller.
- In the third Chapter, the mathematical model of the coupled-tank system is developed. The model is also linearized by using the linearization techniques.
- In the forth Chapter, the Sliding Mode Controller is designed for coupled-tank system. The Controller is applied on both linear and non-linear plant of coupled tanks system in MATLAB/SIMULINK. The results of Sliding mode Controller are also compared with PID controller.
- In the fifth Chapter, the Model Reference Adaptive Controller (MRAC) is designed for coupled-tank plant. This controller is self adjusting controller and based on a model which represents the desired performance specifications. This is an adaptive controller which updates its parameters according to the changes in plant.
- In the Sixth Chapter, the Model Reference Adaptive Sliding mode controller is designed for coupled tank plant. This controller is a hybrid controller which is the combination of Model reference adaptive controller (MRAC) and Sliding mode controller (SMC). This controller is the novelty of this research work.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

The research work relevant liquid level control in coupled-tank system is presented in this chapter. The research findings regarding Sliding mode control (SMC) and Model reference adaptive control (MRAC) are also presented. The researchers designed and implemented a number of control schemes for controlling the liquid level in the coupled-tank system. The control strategies demonstrated by various researchers are described in the literature.

2.2 RESEARCH WORK ON COUPLED TANK SYSTEM

1. J.A. Ramos and P Lopes dos Santos **[5]** presented a case study involving the level control in the coupled-tank system. They developed the mathematical model of the coupled-tank system and then estimated the plant parameters by system identification.
Also they introduced a physical system identification algorithm consisting of subspace identification and then similarity transformation computation to extort the parameters of physical plant. They also presented a method for parameter estimation for continuous time state space models with discrete data. At last they designed a micro-controller for the level control in the coupled-tank system by pole placement technique.

2. David Cartes and Lei Wu **[6]** designed the PID (Proportional-integral-differential) controller and Adaptive controllers of three different type for three tanks level control system and finally compared the performance of each controller with other controller. The adaptive controllers they designed are;

i. Direct model reference adaptive controller (DMRAC)

ii. Indirect Model reference adaptive controller (MRAC) with recursive least squares updating estimation

iii. Indirect Model reference adaptive controller (MRAC) with Lyapunov estimation

They took the sinusoidal wave as reference height. After implementing all these controllers they conclude that all these controllers exhibit somewhat varying performance in tracking the sinusoidal input. They compared the performance of all the designed controllers and made the conclusion that direct MRAC and indirect MRAC with Recursive Least squares RLS estimation gave the most excellent performance. They also showed that Recursive Least squares (RLS) estimation has a much faster convergence than the Lyaponov estimation. They also compared the results of these adaptive controllers with PID controller and made the conclusion that adaptive Control scheme for liquid level control is an improvement over conventional liquid level control scheme when accurate liquid level control in three inter-connected tanks is reqired.

3. Mohd Izzat B Dzolkafle [7] worked on coupled-tank system and wrote the research article ''Implementation of PID controller for controlling the liquid level of the coupled tank system".

In his research work, he developed a mathematical model of the coupled-tank system and then designed a PID controller to control the liquid level in coupled-tank system through simulation and implementation. Finally he compared the results of both simulation and implementation and concluded that simulation gave better results compared to implementation. In case of implementation the limitations of hardware such as the voltage at capacitive level sensor are not equal with the voltage that set at the coding of the controller and the DAQ card is needed to communicate well between software and coupled tank in order to get good implementation results.

4. K. Pirabakaran and V.M. Becerra [8] designed PID controllers using the Model Reference Adaptive Control (MRAC) approach. They utilized the application of artificial neural networks for automatic tuning PID. They tested the effectiveness of this control technique on coupled-tank liquid level control system and concluded that the use of neural networks allows non-linearity's in the controlled system to be considered for tuning purposes. Also, the use of the Model Reference Adaptive Control approach allows desired performance measures, such as natural frequency and damping ratio, to be specified.

5. Maruthai Suresh, Gunna Jeersamy Srinivasan, Ranganathan Rani Hemamalini **[9]** in their paper designed a fuzzy logic controller for three tanks system and the results were compared with those obtained using classical feedback control (PID) method. So they declared that the results of fuzzy logic controller are much improved than PID controller. They did all the work in MATLAB/SIMULINK.

6. Ahcene Boubakir, Fares Boudjema Salim Labiod **[10]** in their research work, designed a Neuro-fuzzy-sliding mode controller (NFSMC) for a coupled-tank system. They used the Non-linear sliding surface for sliding mode. The main focus of their research work was to diminish the chattering phenomenon present in Sliding mode control. They also tried to prevail over the setback of the computation of equivalent control. An appropriate 1^{st} order non-linear sliding surface was selected for sliding mode controller (SMC). They reduced the chattering phenomenon of sliding mode control by smoothing the switch signal. For the computation of equivalent controller, they used a feed-forward neural network (NN). This feed-forward neural network (NN) reduces the chattering which is an undesirable oscillation. They implemented the proposed control technique on coupled-tank system signify that the proposed control technique is excellent for control applications.

7. Muhammad Nasiruddin Mahyuddin, Mohd.Rizal Arshad and Zaharuddin Mohamed **[2]** applied direct model reference adaptive controller (DMRAC) on a nonlinear plant of coupled tank system through MATLAB/SIMULINK. They designed the model reference adaptive controller from the control topology ''command generator tracker'' or (CGT).

The main advantage of the direct adaptive model reference control scheme is that it does not need the precise identification of process parameters. The controller parameters are updated directly without the plant parameters estimation.

In model reference adaptive controller, a reference model with specified performance is introduced and the task is given to the controller to control the plant in such a manner to behave like reference model. So it is very important to select the appropriate reference model which closely resembles with real plant.. In case of a mechanical system, the reference model should be selected according to the situation where it should be possible for the plant to behave like reference model.

The authors in this paper selected three types of different reference models and controller

performance is tested for these three different models named as nominal response reference model, fast response reference model and slow response reference model.
Finally they designed the conventional PID controller by Zeigler Nicholas method and compared the performance of this fixed gain controller with model reference adaptive controller. The performance comparison was made on the basis of different set points tracking performance, disturbance rejection performance and the tracking when the plant parameters are varied.

In all these situations the model reference adaptive controller proved to be good and showed much better tracking performance than conventional PID controller.

8. Ivan Holic and Vojtech Vesely **[11]** designed a PID controller for the level control in coupled-tank system in the frequency domain. They used the two different techniques for designing the Robust PID controller. In the first technique, they applied the Edge Theorem and the Neimark's D -partition method and the second techniques was based on the modification of the Neimark's D -partition. The controller is tested for different set points tracking and proposed controller gave satisfactory results for these reference heights.

9. Marek Kubalcik and Vladimir Bobal **[12]** designed the Adaptive controllers for three interconnected tanks. They used the two different approaches for designing the adaptive controllers. In first approach, the controller is designed using polynomial method. While in the second approach, the model predictive control (MPC) method is applied to design the controller. Both control schemes are implemented on the same model of plant. So finally they made a statement that model predictive controller (MPC) scheme is much better regarding the asymptotic tracking of time varying reference signals.

10. Haizhou Pana, Hong Wonga, Vikram Kapilaa, **[1]** designed non-linear controllers using back-stepping for the liquid level control in coupled-tank system. First of all, they designed a model-based controller that showed very good tracking of reference signals. Then they designed an adaptive controller. The adaptive controller showed very good tracking in the presence of disturbance and uncertainties. They also compared the tracking performance of designed adaptive controller with PI controller. The designed adaptive controller gave much better tracking than PI controller in the presence of uncertainties.

11. Boonsrimuang, Numsomran and Kangwanrat **[13]** designed a PI model reference adaptive controller for coupled-tank system in order to control the level of liquid in tanks. The advantage of designing the PI controller by using Model Reference Adaptive Control technique is that the parameters of PI controller can be updated when there happen a change in the plant or any disturbance changes the behavior of plant. Hence they claimed that the proposed control scheme proved to be good for controlling the level in the coupled-tank plant.

12. Mohammad Khalid, Sarah K Spurgeon **[14]** designed a 2^{nd} order sliding mode controller (SMC) for level control in coupled-tank system. They observed the performance of designed controller in the presence of uncertainties such as variation in tank area, variation in admittance coefficients of various pipes, extra inflow or leakage in the tanks and uncertainty in the pump dynamics.

2.3 RESEARCH WORK ON SLIDING MODE CONTROL (SMC) AND MODEL REFERENCE ADAPTIVE CONTROL (MRAC)

13. Ruben Rojas, Oscar Camacho, Ramon Caceres, Alfredo Castellano **[15]** in their research article ''On Sliding Mode Control for Nonlinear Electrical Systems'' described the synthesis of a sliding mode controller (SMC) based on a second order linear model using an integral-differential surface of the tracking-error. They used a different strategy for tuning the parameters in order to keep a close relationship with the system dynamics in terms of conventional specifications of transient response. The proposed controller only used the output feedback of system and could be satisfactorily used in control of single input-single output nonlinear electric. They implemented the controller under reference and load step changes and observed that the controller gave zero steady state error without chattering.

14. Ahmed El-Bakly, A. Fouda, W. Sabry **[16]** in their research article '' A Proposed DC Motor Sliding Mode Position Controller Design using Fuzzy Logic and PID Techniques'' designed a sliding mode controller using fuzzy and PID techniques for the position control of a DC motor. According to their simulation results, the fuzzy PID sliding mode controller and fuzzy sliding mode controller gave the better results in the presence of external disturbance and uncertainties in the system and showed insensitivity and robustness to plant parameter variations. The tracking of fuzzy PID sliding mode controller and fuzzy sliding mode controllers against uncertainties and external disturbance proved to be the same. But the comparison between the position control of the DC motor by a fuzzy PID sliding mode controller and fuzzy sliding mode controller showed clearly that the Fuzzy PID sliding mode controller gave better performances than fuzzy sliding mode controller against external load. Hence their proposed controller proved to be a robust controller.

15. Oscar Camacho and Carlos A. Smith **[17]** in their paper '' Sliding Mode Control: An Approach to Regulate Nonlinear Chemical Processes'' designed a sliding mode controller (SMC) for two different non-linear chemical plants. There simulation results showed that the performance of sliding mode controller (SMC) is quite stable and satisfactory in presence of nonlinearities and disturbances over a wide range of operating conditions.

16. Juntao Fei **[18]** in his PhD thesis '' Adaptive sliding mode control with application to a MEMS vibratory gyroscope" worked on a gyroscope for the estimation of angular velocity and all the parameters of gyroscope. The control scheme which he used for this purpose is the Adaptive sliding mode control technique. The control techniques which he used are

i. Indirect adaptive sliding mode controller
ii. Direct adaptive sliding mode control with PI sliding surface
iii. Sliding mode observer based adaptive sliding mode control
iv. Adaptive sliding mode control for a tri-axial angular velocity sensor.

17. Amir Hossein, Samsul Bahari, Maryam Mohd, **[19]** in their research article ''Design of Integral Augmented Sliding Mode Control for Pitch Angle of a 3-DOF Bench-top Helicopter'' designed an integral augmented sliding mode controller (SMC) in order to improve the control performance of a plant with uncertainty. They used the example of bench-top helicopter and designed the sliding mode controller to solve the control problem of helicopter pitch angle control. The controller proved to be capable to control the plant effectively in spite of the plant uncertainty. Finally they compared the results of purposed control technique with the results obtained from conventional sliding mode control with and without a boundary layer. The whole work is carried out in MATLAB/SIMULINK environment.

18. Mariagrazia Dotoli, Biagio Turchiano **[20]** worked on fuzzy sliding mode controller and introduced a new fuzzy sliding mode control (SMC) technique for a class of 2^{nd} order dynamical systems.

They showed that the 2^{nd} order non-linear system is stable in canonical form with the proposed fuzzy sliding mode control technique. They also demonstrated the advantage of using this approach. They showed that this technique involve less control action with respect to sliding mode control and fuzzy sliding mode control techniques by selecting a linear sliding surface. Hence this control scheme showed its effectiveness in the presence of a saturated control input.

2.4 SUMMARY

The research work of researchers from all over the world regarding coupled-tank liquid level control system is presented in this chapter. By the deep study of literature, I have come to conclusion that the researchers applied different control design techniques for level control in coupled-tank system. Most of the researchers designed the PID controller and few of them used the Adaptive and Robust control techniques like Model reference adaptive control, Sliding mode control and Fuzzy control for coupled-tank system. In this research work, a hybrid controller is designed for coupled-tank system. The proposed controller is the combination of Model reference adaptive control (MRAC) and Sliding mode control (SMC). The designed controller is the novelty of this research work. The designed controller is Adaptive and Robust and showed excellent results in comparison to Sliding mode control and Model reference adaptive control. Also coupled-tank system is a benchmark problem and is a platform for testing and analyzing the control techniques. Many researchers tested the performance of their designed controllers with coupled-tank plant model. In next chapter, we will discuss the principle of operation of coupled-tank system and develop its mathematical model.

CHAPTER NO: 3

MATHEMATICAL MODEL OF COUPLED TANK SYSTEM

3.1 INTRODUCTION

This chapter presents the mathematical modeling of coupled tank system using the basic laws of Physics and Engineering. It is vital to develop a mathematical model which closely resembles the real plant. The chapter is organized as follows. First of all, the coupled tank system and its mode of operation are described briefly. Secondly, the non-linear mathematical model of the coupled tank system is developed and after that it is linearized by applying the linearization techniques. Thirdly, a first order model is developed by separating the two tanks and then second order model by joining both of them. Finally, the mathematical model is represented in transfer function and then in state space form. The structure of the coupled-tank plant is shown in figure 3.1.

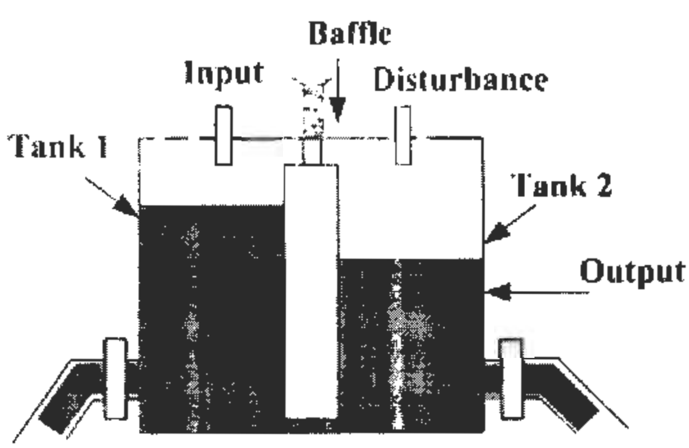

Figure 3.1: Schematic diagram of coupled-tank system

There are two tanks connected to each other. The liquid can easily flow from one tank to another. Both tanks have outlet drain pipes from which liquid can be continuously flow out of the tank. Also both the tanks have a source of liquid (i.e. a pump which can continuously feed liquid to the tanks). The coupled tank model we are using is the non-linear system model because the relationship of input to output is not linear. The outflow of liquid is the function of liquid height so when the height of liquid increases in the tank, the outflow of liquid increases but its relationship with height of the liquid is not linear.

The main focus of control in the coupled-tank system is to maintain the level of liquid at desired position in the 2^{nd} tank. The desired result can be achieved by controlling the input flow rate to the 1^{st} tank. The level will be maintained at a specific desired value as long the inflow rate and the outflow rate remains same.

In case of disturbances (increase in temperature of liquid or leakage in either tank) acting on the system, the inflow rate and outflow rate would change and level of liquid will not remain further more on desired position. The disturbance may be the leakage in the tanks or an extra input to the tanks or the increased outflow from tanks because of the increase in temperature, which is quite possible in physical plants. As the liquid flow heavily depends upon the temperature, so rise in temperature decreases the viscosity of the liquid and hence outflow increases. However, if the flow rate of liquid out of the tank is greater than the flow rate coming into the tank, then the liquid will settle at a lower level than reference position and this level will go to decrement continuously until the tank is empty. Similarly, when the flow rate into the tank is higher than the flow rate out of the tank, then the liquid will settle at a higher level than reference level and in this condition the liquid level goes on increasing until the tank overflows.

The intension of the control in coupled-tank system is that the input flow rate to the 1^{st} tank has to be adjusted in order to keep the liquid level at a desired position in 2^{nd} tank. If the liquid level in 2^{nd} tank is not at a desired point then a negative or a positive error will be generated that drives the controller to adjust the inflow rate to the 1^{st} tank. The inflow valve to 1^{st} tank can be fully opened or closed or partially opened or closed as per requirement. While the inflow rate to 2^{nd} tank is not a control variable and hence it will be considered as a disturbance. Similarly, the other type of disturbances like leakage in the tank 1 or tank 2 or the extra inflow to tank 1 or tank 2 may also be considered.

With the presence of such disturbances in the plant, it is required to have a suitable controller which recovers the desired plant output as quickly as possible in case of any uncertain happening and also controls the system as per requirement. Hence different types of controllers can be designed and implemented for the achievement of this goal.

By observing the efficiency and tracking performance of all the designed controllers in the presence of all above mentioned uncertainties, a suitable controller can be found. Hence coupled tank plant may be considered as a benchmark problem for which a number of controllers can be designed and their performances can be tested and compared with each other.

3.2 MATHEMATICAL MODELING OF COUPLED TANK SYSTEM

The mathematical model of the coupled tank system is developed in this section. The system behavior and its characteristics are expressed by a mathematical model which describes the steady state and dynamic attitude of the system. The mathematical model of any system is developed by using the basic laws of physics and engineering [2]. In order to design the controller and for the critical analysis of the system, the mathematical modeling of the system plays a vital role.

In the figure 1, the structure of coupled-tank control system apparatus is shown. H_1 and H_2 are taken to be the liquid levels in tank 1 and 2 respectively. While U_{i1} and U_{i2} are the inflows to tank 1 and tank 2 respectively. Similarly, U_{o1} and $U_{o2}$are the outflows from tank 1 and tank 2 respectively. While U_{o3} is the flow between the tanks. Applying the simple mass balance equation, the rate of change of liquid volume in each tank equals the input flow minus output flow of liquid.

{Rate of accumulation of total mass}= {Rate of mass entering the system}-{Rate of mass leaving the system}

$$\frac{dM}{dt} = \rho\frac{dV}{dt} = \rho(U_i - U_o)$$

$$\frac{dV}{dt} = (U_i - U_o)$$

So the general form of mass balance equation for the coupled tank plant will be

$$\frac{d(AH)}{dt} = (U_i - U_o)$$

Here A is the cross-sectional area of any tank and H is the height of fluid in the tank.

For tank 1;

$$A_1\frac{dH_1}{dt} = U_{i1} - U_{o1} - U_{o3} \quad (3.1)$$

For tank 2;

$$A_2\frac{dH_2}{dt} = U_{i2} - U_{o2} + U_{o3} \quad (3.2)$$

Where,

H_1, H_2 = Height of liquid in tank 1 and tank 2 respectively

A_1, A_2 = Cross-sectional area of tank 1 and tank 2 respectively

U_{i1}, U_{i2} = Liquid flow rate into tank 1 and tank 2 respectively

U_{o1}, U_{o2} = Liquid flow rate out of tank 1 and tank 2 respectively

U_{o3}= Flow rate of liquid between tanks.

According to Bernoulli's theorem the speed of fluid coming out from the out drain pipe of the tank is,

$$v = \sqrt{2gH}$$

Where g is the gravitational acceleration and H is the height of fluid in the tank.

Also from equation of continuity,

$$U_o = Av$$

Where A is the cross sectional area of the out drain pipe.

So

$$U_o = A\sqrt{2gH}$$

$$U_o = \beta\sqrt{H}$$

Where $\beta = A\sqrt{2g}$

Hence according to Bernoulli's theorem, the outflow of liquid from tank is directly proportional to the square root of the height of liquid in the tank for a non-viscous, steady and incompressible fluid. Similarly, the flow between the two tanks is directly proportional to the square root of the head differential.

$$U_{01} = \beta_1\sqrt{H_1} \qquad (3.3)$$

$$U_{02} = \beta_2\sqrt{H_2} \qquad (3.4)$$

$$U_{03} = \beta_3\sqrt{|H_1 - H_2|} \qquad \text{for } H_1 > H_2 \qquad (3.5)$$

$$U_{03} = -\beta_3\sqrt{|H_1 - H_2|} \qquad \text{for } H_2 > H_1 \qquad (3.5a)$$

Where β_1, β_2, β_3 are proportionality constants. These proportionality constants are dependent on the coefficients of discharge, gravitational constant and the cross-sectional area of each outlet.

Substituting the above equations (3.3), (3.4) and (3.5) into equations (3.1) and (3.2), in order to obtain the non-linear equation of the system. These equations completely describe the dynamics of system and its mode of operation. The equations are:

$$A_1 \frac{dH_1}{dt} = U_{i1} - \beta_1\sqrt{H_1} - \beta_3\sqrt{|H_1 - H_2|} \qquad \text{for} \quad H_1 > H_2 \tag{3.6}$$

$$A_2 \frac{dH_2}{dt} = U_{i2} - \beta_2\sqrt{H_2} + \beta_3\sqrt{|H_1 - H_2|} \qquad \text{for} \quad H_1 > H_2 \tag{3.7}$$

The equations (3.6) and (3.7) may also be expressed in the form

$$A_1 \frac{dH_1}{dt} = U_{i1} - \beta_1\sqrt{H_1} - \beta_3\sqrt{|H_1 - H_2|}\,[\text{sgn}(H_1 - H_2)] \tag{3.6a}$$

$$A_2 \frac{dH_2}{dt} = U_{i2} - \beta_2\sqrt{H_2} + \beta_3\sqrt{|H_1 - H_2|}\,[\text{sgn}(H_1 - H_2)] \tag{3.7a}$$

3.3 OPEN LOOP RESPONSE OF NON-LINEAR PLANT

The open loop response of non-linear coupled-tank plant is shown in figure 3.2. The input to the plant is 90 cm^3/s flow rate and output is the liquid level (8cm) in 2nd tank.

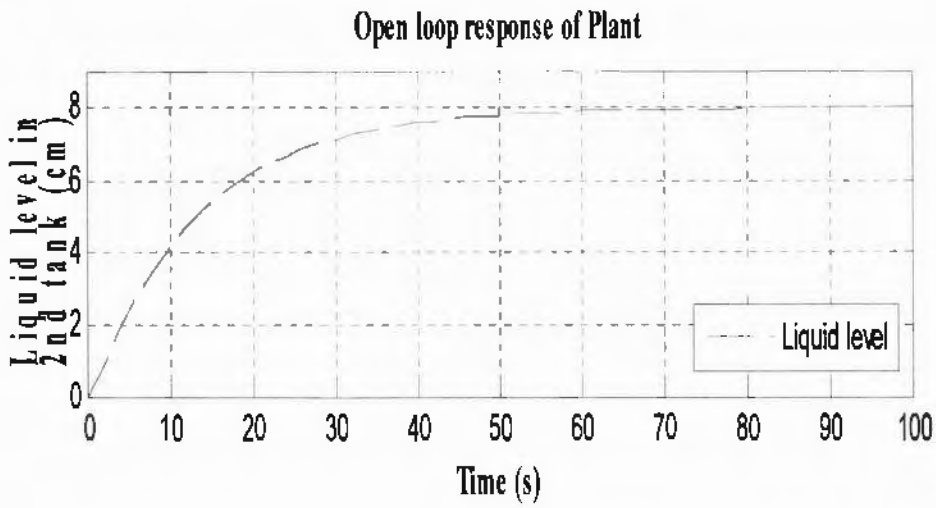

Figure 3.2: Coupled-tank open loop response

It is clear from the plot that the plant is stable but has large settling time. For maintaining the liquid at 8cm in 2nd tank, almost 80 seconds are required. Hence the control objective is to reduce the settling time. The open loop response is stable but it is unable to handle the uncertainties and disturbances acting on the system.

3.4 LINEARIZED PERTURBATION MODEL

Let us suppose that for a set of inflows U_{i1} and U_{i2} in tank 1 and tank 2, the liquid level in the tanks is at some steady state levels H_1 and H_2 and $H_1 > H_2$. Now we consider the small variations in each inflow U_{i1} and U_{i2}. Let u_1 be the small variation in inflow U_{i1} and u_2 be in U_{i2}. So because of these variations in inflows, the resulting perturbation in tank levels H_1 and H_2 be h_1 and h_2 respectively.

Hence the equations (3.6) and (3.7) will take the form:

For Tank 1,

$$A_1\frac{d(H_1+h_1)}{dt} = (U_{i1}+u_1) - \beta_1\sqrt{(H_1+h_1)} - \beta_3\sqrt{H_1-H_2+h_1-h_2} \quad (3.8)$$

For Tank 2,

$$A_2\frac{d(H_2+h_2)}{dt} = (U_{i2}+u_2) - \beta_2\sqrt{(H_2+h_2)} + \beta_3\sqrt{H_1-H_2+h_1-h_2} \quad (3.9)$$

Subtracting equations (3.6) and (3.7) from (3.8) and (3.9), the equation will take the form,

$$A_1\frac{dh_1}{dt} = u_1 - \beta_1\left[\sqrt{(H_1+h_1)} - \sqrt{H_1}\right] - \beta_3[\sqrt{H_1-H_2+h_1-h_2} - \sqrt{H_1-H_2}]$$

$$A_2\frac{dh_2}{dt} = u_2 - \beta_2\left[\sqrt{(H_2+h_2)} - \sqrt{H_2}\right] + \beta_3[\sqrt{H_1-H_2+h_1-h_2} - \sqrt{H_1-H_2}]$$

For small perturbations

$$\sqrt{(H_1+h_1)} = \sqrt{H_1}\left[1+\frac{h_1}{H_1}\right]^{0.5} \approx \sqrt{H_1}\,[1+\frac{h_1}{2H_1}]$$

Therefore consequently

$$\sqrt{(H_1+h_1)} - \sqrt{H_1} \approx \frac{h_1}{2\sqrt{H_1}}$$

Similarly,

$$\sqrt{(H_2+h_2)} - \sqrt{H_2} \approx \frac{h_2}{2\sqrt{H_2}}$$

and

$$[\sqrt{H_1-H_2+h_1-h_2} - \sqrt{H_1-H_2}] \approx \frac{h_1-h_2}{2\sqrt{H_1-H_2}}$$

Hence using these approximations in above equations, the equations (3.10) and (3.11) are established as

$$A_1\frac{dh_1}{dt} = u_1 - \frac{\beta_1}{2\sqrt{H_1}}h_1 - \frac{\beta_3}{2\sqrt{H_1-H_2}}(h_1 - h_2) \quad (3.10)$$

$$A_2\frac{dh_2}{dt} = u_2 - \frac{\beta_2}{2\sqrt{H_2}}h_2 + \frac{\beta_3}{2\sqrt{H_1-H_2}}(h_1 - h_2) \quad (3.11)$$

So the above equations can be expressed in state space form as

$$\begin{bmatrix} \frac{dh_1}{dt} \\ \frac{dh_2}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-\beta_1}{2A_1\sqrt{H_1}} - \frac{\beta_3}{2A_1\sqrt{H_1-H_2}} & \frac{\beta_3}{2A_1\sqrt{H_1-H_2}} \\ \frac{\beta_3}{2A_2\sqrt{H_1-H_2}} & \frac{-\beta_2}{2A_2\sqrt{H_2}} - \frac{\beta_3}{2A_2\sqrt{H_1-H_2}} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1} & 0 \\ 0 & \frac{1}{A_2} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$y = [0 \quad 1]\begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

It is important to note that, in the above derived linear coupled-tank plant; the coefficients of the perturbations in the level are the functions of the steady state operating points H_1 and H_2.

In the next sections, based on the developed linearised model, the two corresponding tank systems (single tank and coupled-tank) are described.

3.5 SINGLE TANK MODEL

In order to obtain a single-input single-output linear model of coupled-tank plant, the configuration can is achieved by completely depressing the baffle. The baffle in the coupled-tank system is used as a separating barrier between the two tanks. Hence under this condition, there will be no flow between the tanks. Hence both the tanks will become two separate single-input single-output systems. That is, the plant will have only one input and one output. So under this condition, two independent first order-systems are developed. In order to obtain a first order differential equation, we will simplify the equations (3.10) and (3.11), we get

$$A_1\frac{dh_1}{dt} = u_1 - \frac{\beta_1}{2\sqrt{H_1}}h_1 \quad (3.12)$$

$$A_2\frac{dh_2}{dt} = u_2 - \frac{\beta_2}{2\sqrt{H_2}}h_2 \quad (3.13)$$

The above first order differential equations are relating the perturbation in height of liquid in tanks to the perturbation in inflow of liquid in the tanks. Now there are two independent tanks having the same structure and principle of operation. So here we are taking the 1st tank for discussion purpose. Hence in the 1st tank, u_1 is a small variation in the steady state input flow rate U_{i1} to the 1st tank whereas the output variable h_1 represents a small variation in the steady state level H_1. Where H_1 is operating points around which we linearized the plant and is a constant. By taking the Laplace transform of equation (3.12) we have obtained an input-output relation which is also known as transfer function. The transfer function of 1st order system is developed below;

$$\frac{h_1(s)}{u_1(s)} = \frac{\frac{2\sqrt{H_1}}{\beta_1}}{1+\frac{2A_1\sqrt{H_1}}{\beta_1}s} = \frac{k_1}{1+\tau_1 s} \tag{3.14}$$

From equation (3.14), we can express the time constant of the tank dynamics as,

$$\tau_1 = \frac{2\,A_1\sqrt{H_1}}{\beta_1}$$

Similarly the expression for the steady state gain of the tank dynamics can be derived from equation (3.14).

$$k_1 = \frac{2\sqrt{H_1}}{\beta_1}$$

So it is clear from the above equations that the time constant and steady state gain are dependent on operating points and the constant factor β_1. The proportionality constant β_1 depends upon the cross-sectional area of outlet and gravitational constant. Where the time constant do not respond same for different steady-state operating points. The time constant depends upon the factor β. steady state operating points and cross-sectional area of the tank. By observing the time constant of a system, the transient performance of a system can be analyzed.

3.6 SINGLE-INPUT SINGLE-OUTPUT (SISO) SECOND ORDER PLANT

In order to make the same coupled tank plant as second order system, we will raise the baffle to a small height hence flow between the tanks will also be possible. With this second order arrangement, the height of the liquid h_2 in the 2nd tank will be the output variable while the inflow rate to the 1st tank u_1 will be the controlled variable. The flow into the 1st tank will be controlled accordingly in order to achieve the desired liquid height in the 2nd tank. The other input variable u_2 which is the input flow rate to the 2nd tank will be assumed zero because it is a single-input single-output system and u_2 will be considered as disturbance in this case. But here the model is derived under the conditions when no disturbance is acting on the system. In order to obtain the 2[nd] order transfer function of plant, we will take the Laplace Transforms of equations (3.10) and (3.11) and taking all the initial conditions at their steady state values.

$$A_1 s h_1(s) = u_1(s) - \left(\frac{\beta_1}{2\sqrt{H_1}} + \frac{\beta_3}{2\sqrt{H_1-H_2}}\right)h_1(s) + \frac{\beta_3}{2\sqrt{H_1-H_2}}h_2(s)$$

$$A_2 s h_2(s) = u_2(s) - \left(\frac{\beta_2}{2\sqrt{H_2}} + \frac{\beta_3}{2\sqrt{H_1-H_2}}\right)h_2(s) + \frac{\beta_3}{2\sqrt{H_1-H_2}}h_1(s)$$

By rearranging and rewriting in abbreviated manners

$$(\tau_1 s + 1)h_1(s) = k_1 u_1(s) + k_{12} h_2(s) \quad (3.15)$$

$$(\tau_2 s + 1)h_2(s) = k_2 u_2(s) + k_{21} h_1(s) \quad (3.16)$$

Where

$$\tau_1 = \frac{A_1}{\frac{\beta_1}{2\sqrt{H_1}} + \frac{\beta_3}{2\sqrt{H_1-H_2}}} \qquad \tau_2 = \frac{A_2}{\frac{\beta_2}{2\sqrt{H_2}} + \frac{\beta_3}{2\sqrt{H_1-H_2}}}$$

$$k_1 = \frac{1}{\frac{\beta_1}{2\sqrt{H_1}} + \frac{\beta_3}{2\sqrt{H_1-H_2}}} \qquad k_2 = \frac{1}{\frac{\beta_2}{2\sqrt{H_2}} + \frac{\beta_3}{2\sqrt{H_1-H_2}}}$$

$$k_{12} = \frac{\frac{\beta_3}{2\sqrt{H_1-H_2}}}{\frac{\beta_1}{2\sqrt{H_1}} + \frac{\beta_3}{2\sqrt{H_1-H_2}}} \qquad k_{21} = \frac{\frac{\beta_3}{2\sqrt{H_1-H_2}}}{\frac{\beta_2}{2\sqrt{H_2}} + \frac{\beta_3}{2\sqrt{H_1-H_2}}}$$

Simultaneously expressing the equations (3.15) and (3.16), into an input-output relation. Hence the final 2[nd] order transfer function equation can be obtained as,

$$\frac{h_2(s)}{u_1(s)} = \frac{k_1 k_{12}}{(\tau_1 s+1)(\tau_2 s+1) - k_{12}k_{21}} \quad (3.17)$$

$$\frac{h_2(s)}{u_1(s)} = \frac{k_1 k_{12}}{(\tau_1\tau_2 s^2 + (\tau_1+\tau_2)s + (1-k_{12}k_{21})}$$

$$\frac{h_2(s)}{u_1(s)} = \frac{\frac{k_1 k_{12}}{\tau_1\tau_2}}{s^2 + (\frac{\tau_1+\tau_2}{\tau_1\tau_2})s + \frac{(1-k_{12}k_{21})}{\tau_1\tau_2}}$$

$$G(s) = \frac{b}{s^2 + a_1 s + a_0} \quad (3.18)$$

Where

$$b = \frac{k_1 k_{12}}{\tau_1\tau_2} \quad , \quad a_1 = (\frac{\tau_1+\tau_2}{\tau_1\tau_2}) \quad , \quad a_0 = \frac{(1-k_{12}k_{21})}{\tau_1\tau_2}$$

3.7 PRESENCE OF LEAKAGE IN TANK 1 AND TANK 2

In this section, we will introduce the leakage in the tanks which will be considered like disturbance or an extra outflow from tanks. The increased outflow is often possible in physical plants because of the rise in temperature of the liquid. The rise in temperature decreases the viscosity of the liquid and hence the outflow increased from the tanks. The inclusion of leakage in the tanks will make the plant uncertain and the controller performance will be monitored under these uncertain conditions.

Suppose the leakage in 1st tank and 2nd tank is U_{j1} and U_{j2} respectively. Hence in the presence of leakage in the tank1 and tank2, equations (3.6) and (3.7) will take the form as,

$$A_1 \frac{dH_1}{dt} = U_{i1} - \beta_1\sqrt{H_1} - \beta_3\sqrt{|H_1 - H_2|} - U_{j1} \qquad \text{for} \quad H_1 > H_2 \qquad (3.19)$$

$$A_2 \frac{dH_2}{dt} = U_{i2} - \beta_2\sqrt{H_2} + \beta_3\sqrt{|H_1 - H_2|} - U_{j2} \qquad \text{for} \quad H_1 > H_2 \qquad (3.20)$$

3.8 DC GAIN

The DC Gain of any stable system with transfer function G(s) is given by

$$G_{DC} = \lim_{s\to 0} G(s)$$

$$G_{DC} = \lim_{s\to 0}\left(\frac{b}{s^2 + a_1 s + a_0}\right)$$

$$G_{DC} = \frac{b}{a_0}$$

$$G_{DC} = \frac{k_1 k_{12}}{(1 - k_{12}k_{21})} \qquad (3.21)$$

3.9 PARAMETERS OF COUPLED TANK PLANT

The coupled-tank plant parameters are given in Table 3.1.

Name	Symbol	Parametric Value		
Cross Sectional Area of each tank	A_1 & A_2	32 cm^2		
Proportionality constant which depends upon the cross sectional area of tank and gravitational constant	β_i Where subscript i is representing the respective tank	β_1	β_2	β_3
		14.30 $cm^{5/2}/sec$	14.30 $cm^{5/2}/sec$	20 $cm^{5/2}/sec$
Maximum allowable volumetric flow rate	U_{imax}	500cm^3/s		
Gravitational constant	g	981cm/sec^2		

Table: 3.1 Coupled-tank Parameters

3.10 STABILITY: OPEN LOOP POLES LOCATION

The open loop transfer function of coupled tank plant is given by

$$\frac{h_2(s)}{u_1(s)} = \frac{0.006989}{s^2+0.5539s+0.02785} \tag{3.22}$$

In order to find the location of open loop poles, the denominator term of transfer function is set equal to zero. i.e.

$$s^2 + 0.5539s + 0.02785 = 0$$

Hence the location of the poles is -0.05595 and -0.4979. As the poles are at LHP (left half plane), so the system is stable.

3.11 CONTROL OBJECTIVES

As our coupled tank plant is stable. So the objective of control design is to improve the performance of plant with designed controllers in order to track the time varying command signals. In our system, the transfer function is the ratio of inflow rate to the 1st tank and the desired liquid level in the 2nd tank. Hence in our single input single output system, the control objective is to control the inflow rate to the 1st tank (u_1) in order to maintain the desired liquid level in the 2nd tank (h_2).

3.12 STATE SPACE REPRESENTATION

Here in this section, the coupled tank system transfer function is transformed into the state space form. The state variables we have chosen are the liquid height in the second tank and its time derivative. So the state space representation of equation (3.18) is as follows:

Let $x_1 = h_2$ and $x_2 = \dot{h_2}$

So

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} u_1 \tag{3.23}$$

$$Y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

3.13 REFERENCE MODEL

Selection of the reference model dynamics is a very important part of the adaptive system design procedure that ultimately defines the desired system behavior. Different types of reference model can be formulated with each governs different response specifications. The transient response of reference models depends on the plant to be controlled which is in this case, a coupled-tank. Therefore, the selection of reference model is usually preceded by a careful study on the plant's boundary and physical limits of which it can operate.

In this study, the order of the formulated reference model is of second order.

Performance Specification	
Rise Time	7 sec
Settling Time	15 sec
Percentage overshoot	1%

Table 3.2. Plant Parameters

The corresponding reference model in state space matrix form is,

$$A_m = \begin{bmatrix} 0 & 1 \\ -0.1042 & -0.533 \end{bmatrix}, \quad B_m = \begin{bmatrix} 0 \\ 0.1042 \end{bmatrix},$$

$$C_m = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

3.14 SUMMARY

In this chapter, the non-linear mathematical model of the coupled-tank system is developed, which is then linearized by using the Taylor series linearization technique. After that, the linearized single tank and coupled tank models are expressed in the form of transfer function. Finally, the linear model of the system is expressed in state-space form.

CHAPTER NO: 4

SLIDING MODE CONTROL

4.1 INTRODUCTION

Sliding Mode Control (SMC) is a technique which is derived from Variable Structure Control (VSC). This control technique was originally studied by V. I. Utkin [21] in late 1960s. The most important characteristic of Sliding mode controller is to deal with non-linear and time varying systems. [22-23]. A controller which shows robustness to the uncertainties in the system is considered as a very useful controller. By designing a practical control system it is vital to have a robust controller. Hence Sliding mode control design can provide a robust solution.

The idea behind Sliding mode control is to choose a sliding surface along which the system can slide to its desired final value. Figure 4.1 is representing the principle of Sliding mode control.

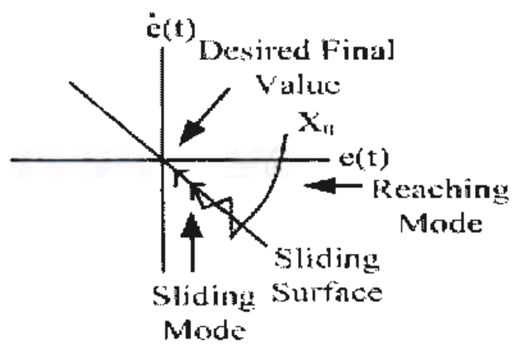

Figure 4.1 Principle of sliding mode control

Almost all practical processes exhibit non-linear behavior. Usually, their dynamic characteristics are not fully known. The commonly used PID controllers are simple to be implemented in different applications, but they may suffer from poor performance in the presence of uncertainties and nonlinearities in the system. In the near past, most of the researchers paid their attention for designing the robust controllers. The sliding mode controller is one of them [16].

Sliding Mode Control (SMC) is a robust control strategy and it has very simple procedure to design the controllers for linear and nonlinear plants. The main advantage of sliding mode control is that, it is robust to plant uncertainties and insensitive to disturbances acting on the system. Because of its excellent robustness to uncertainties, sliding mode control scheme has been recognized as a proficient technique for robust control of uncertain systems. Moreover, sliding mode controller is extensively used to achieve the good dynamic performance of controlled systems. Also. the sliding mode control (SMC) requires the mathematical model of the system with bounded uncertainties [16].

The structure of Sliding mode controller is deliberately changed when the system state trajectory crosses the sliding surface in accordance with a given control law. Hence first of all a sliding surface is selected for the designing of sliding mode controller. In a sliding mode control strategy, the system dynamics are forced by the controller to stay confined in a subset of pre-defined sliding surface $S(t)$. The system is directed toward and reaches the sliding surface at a finite time due to the control action. Finally, when the system dynamics reaches at the user defined sliding surface, it will behave according to that surface which is of a lower order than that of the system and independent of the model parametric uncertainties [21]. The system becomes insensitive to unknown external disturbances and certain parameters variations when it slides on the switching surface [17].

4.2 BASIC SLIDING MODE CONTROLLER DESIGN

In sliding mode control, the objective is to make the error and derivative of error equal to zero. In the coupled tank liquid level control system, the difference between actual height and desired height in 2nd tank is the system error. The designing of switching surface involves the construction of the switching function. If the sliding mode exists, we can find the transient response of the system from switching surface. As the system error is defined as the difference between actual height and desired height, mathematically

$$e(t) = h_D(t) - h_2(t) \tag{4.1}$$

Where $h_D(t)$ is the desired height in 2nd tank while $h_2(t)$ is the actual height in 2nd tank. The expression for the nth order sliding function (see reference [24, 25, 26]) is given by

$$S(t) = (d/dt + \lambda)^{n-1}e \tag{4.2}$$

A 2nd order sliding function (n=2) can be written as

$$S = \dot{e} + \lambda e \quad (4.3)$$

Where, $\lambda > 0$ is the slope of sliding surface. The basic control law of Sliding mode control is given by

$$U = Ksign(S) \quad (4.4)$$

Where the parameter K is known as the tuning parameter and is responsible for the reaching mode. This parameter should be tuned such that the control must have a sufficient energy to force the system onto the sliding surface and maintain its sliding motion. Sign is switching element and is non-linear part of control law. It is the discontinuous part of the controller and is discontinuous across the sliding surface. It allows for changes between the structures with a hypothetical infinitely fast speed, which results into a phenomenon known as chattering.

The main disadvantage of sliding mode control is the Chattering phenomena. Chattering is a high-frequency oscillation around the desired equilibrium point. In practical application, it is undesirable because it involves high control activity and can excite high-frequency dynamics ignored in the modeling of the system [22, 27, 28].

The gain K is responsible for the aggressiveness of the control system. Hence if K is greater, the controller is aggressive which can produce the chattering. In order to diminish the chattering, one way is to replace the relay-like function by saturation or sigma function [21].

The chattering problem could be solved satisfactorily if the control U(t) is designed according to [9]:

$$U(t) = K\frac{S}{|S| + \delta} \quad (4.5)$$

Where δ is to be adjusted to obtain the suppression of chattering.

Consider a Lyapunov function

$$V = \frac{1}{2}S^2 \quad (4.6)$$

From Lyapunov theorem, if $\dot{V}$ is negative semi definite then the system trajectory will be driven and attracted towards the sliding surface and remain sliding on it until the origin is reached.

A sufficient condition for stability of system is

$$V = \frac{1}{2}\frac{d}{dt}S^2 \leq -K\,|S| \quad (4.7)$$

4.3 SIMULATION RESULTS OF SMC WITH LINEAR COUPLED TANK PLANT MODEL

In this section, the simulation results of the simple basic sliding mode controller (SMC) with linear coupled tank plant model are discussed. The linear plant in the form of transfer function is given in equation (3.22). Where $\lambda = 2$ the slope of the sliding surface and K is the tuning parameter and is adjusted to 131. Also $\delta = 0.008$ is to be adjusted to obtain the suppression of chattering.

The controller tracking is observed by tracking the various set points. The input to the plant is the flow rate to the 1st tank and output of the plant is the liquid level in the 2nd tank. Hence the reference signal will be the desired liquid height in the 2nd tank. The reference signals taken are; step, random, square, saw tooth and sinusoidal. The controller tracking for step, random, square, saw tooth and sinusoidal wave like reference signals is shown in figure 4.2, 4.3, 4.4, 4.5 and 4.6.

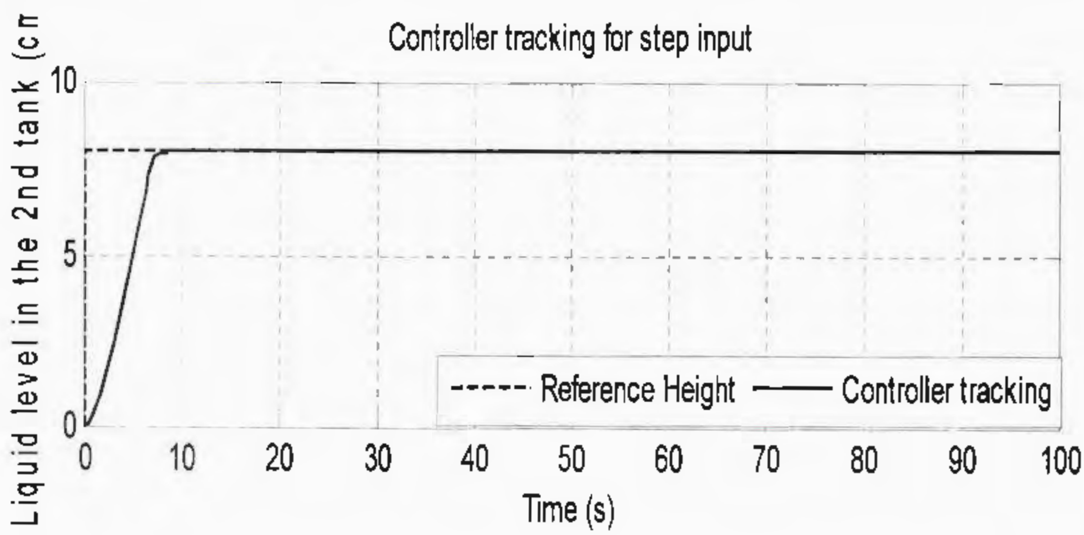

Figure 4.2: Controller tracking performance for step reference signal

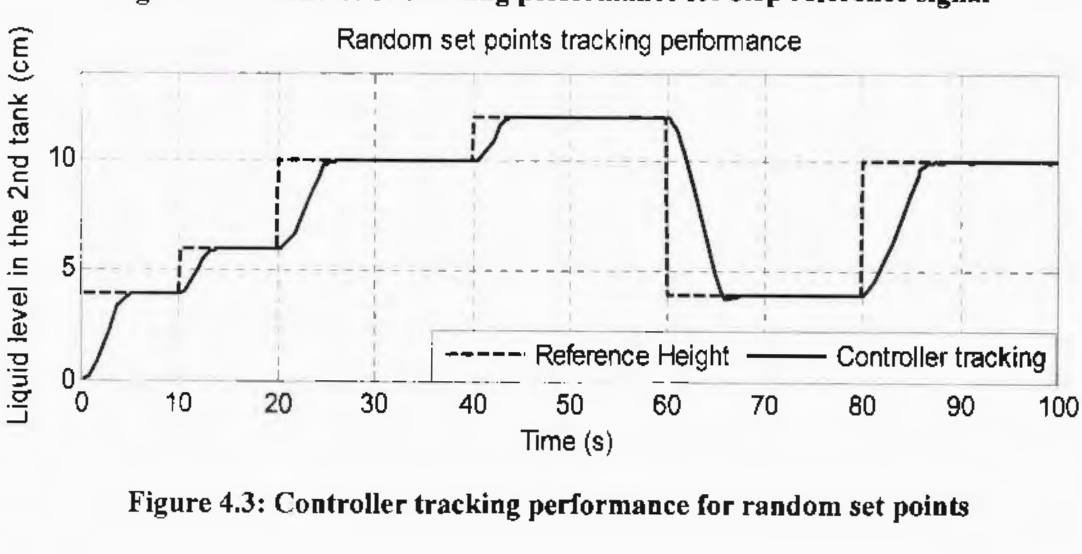

Figure 4.3: Controller tracking performance for random set points

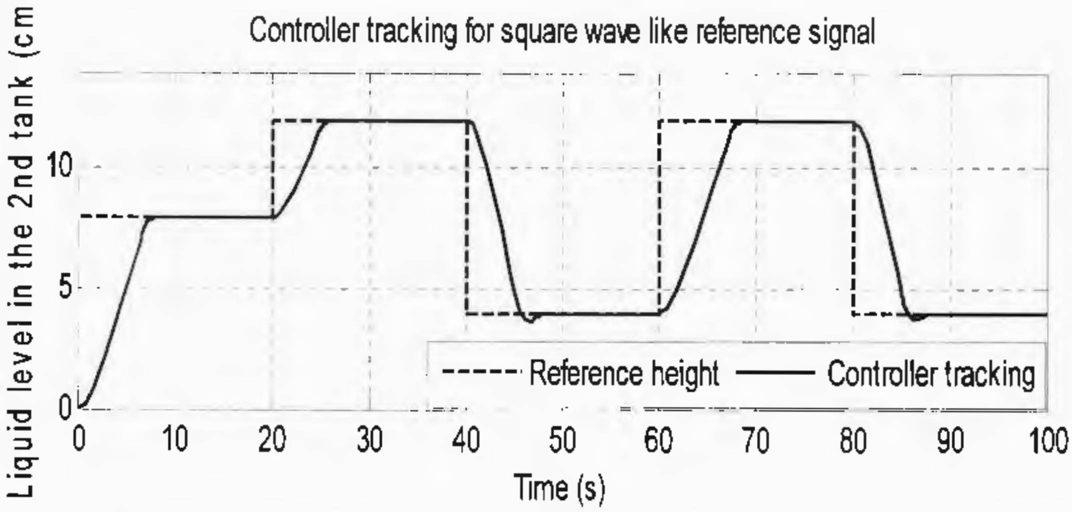

Figure 4.4: Controller tracking performance for square wave like reference signal

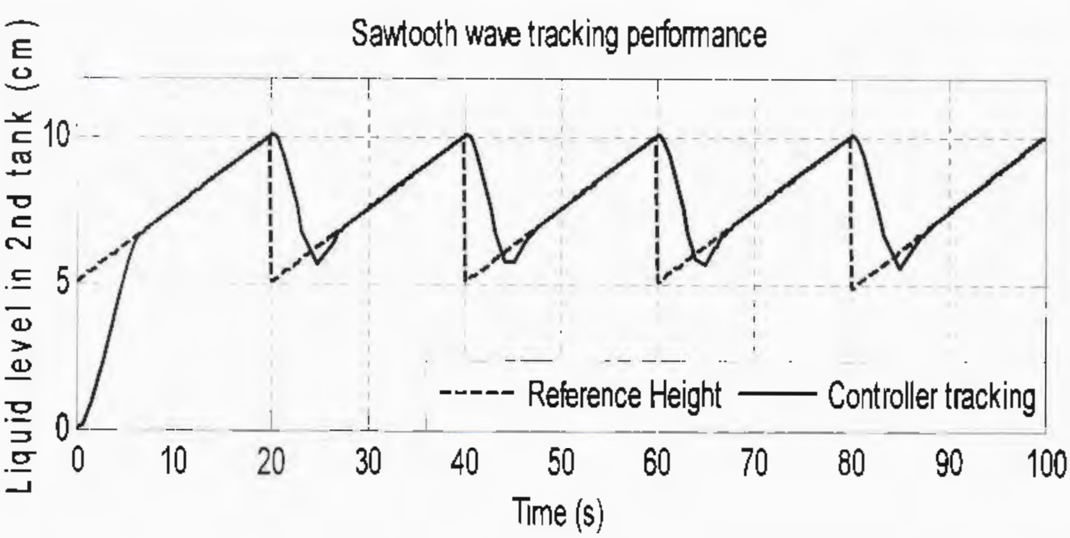

Figure 4.5: Controller tracking performance for saw tooth wave like reference signal

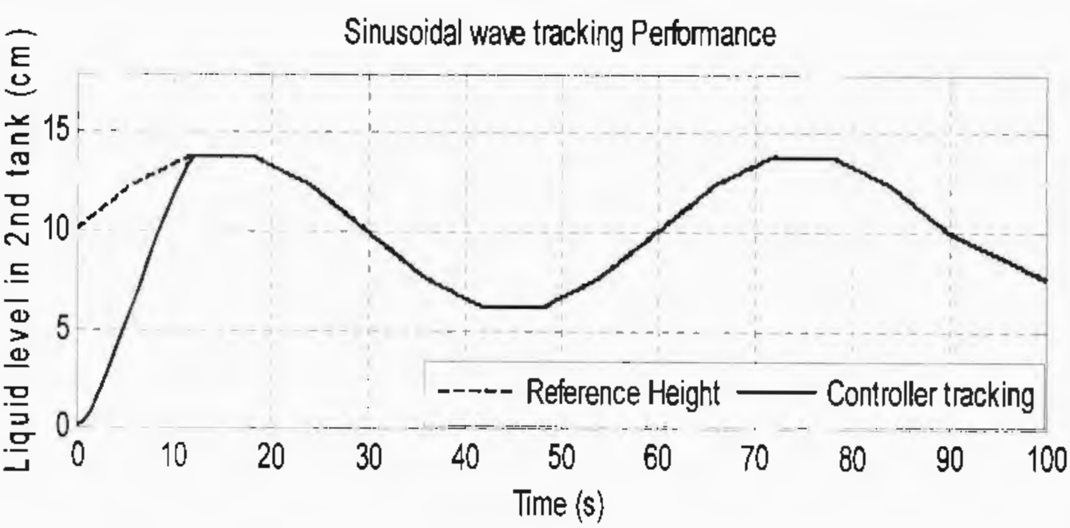

Figure 4.6: Controller tracking performance for sinusoidal wave like reference signal

4.4 SIMULATION RESULTS OF SMC WITH NON-LINEAR COUPLED TANK PLANT MODEL

In this section, the simulation results of the simple basic sliding mode controller are discussed with non-linear coupled tank plant model. Different set points tracking performance of the controller for different height levels in 2^{nd} tank is observed. We also checked the controller performance for time varying reference signals like square wave, sinusoidal wave and saw tooth wave in which the command signal is continuously changing.

4.4.1 STEP REFERENCE SIGNAL

The controller tracking performance for step reference signal is observed and is shown in figure 4.7. As the output of the plant is the height in the 2^{nd} tank so the reference signal is the height in the 2^{nd} tank. The reference step signal is taken as 8cm which is the reference height in the 2^{nd} tank. The controller showed a very good performance for tracking the command reference signal. For tracking the reference signal, the settling time of the controller is about 14 seconds. That is, the controller tracks the height of liquid 8 cm in the 2^{nd} tank in almost 14 seconds.

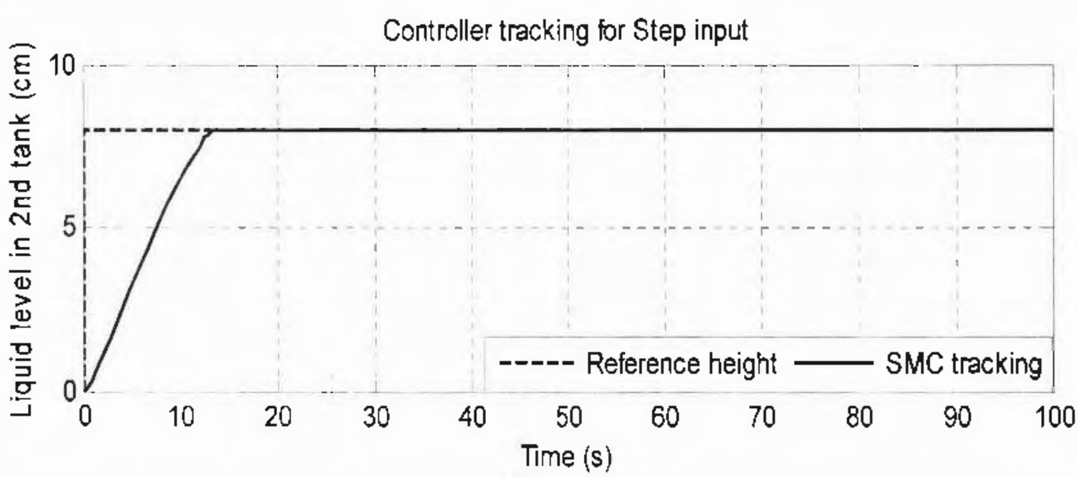

Figure 4.7: Controller tracking performance for step signal

4.4.2 INFLOW RATE TO THE 1^{ST} TANK

The inflow rate to the 1st tank is about $87cm^3/s$. The inflow rate to the 1st tank is shown in figure 4.8. The inflow rate is for the step reference value of height 8cm in the 2nd tank and its unit is cm^3/s. For maintaining the liquid at this level (8cm in the 2nd tank), the controller adjusted the input flow rate such that it is equals to the output flow rate so that the liquid level in the tank must be maintained at a height of 8cm.

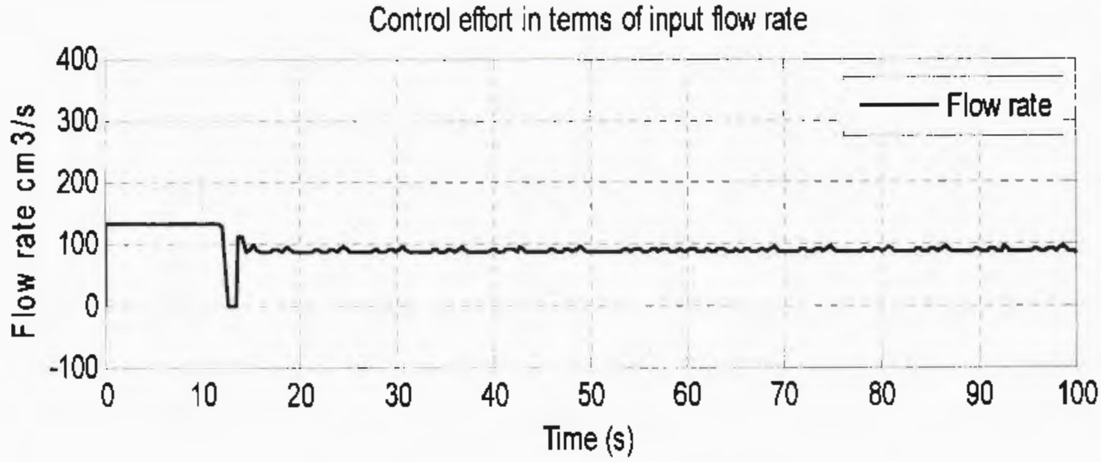

Figure 4.8: Control input for step signal

4.4.3 TRACKING ERROR FOR STEP SIGNAL

The tracking error of SMC for tracking the 8cm height in the 2nd tank is shown below in figure 4.9.

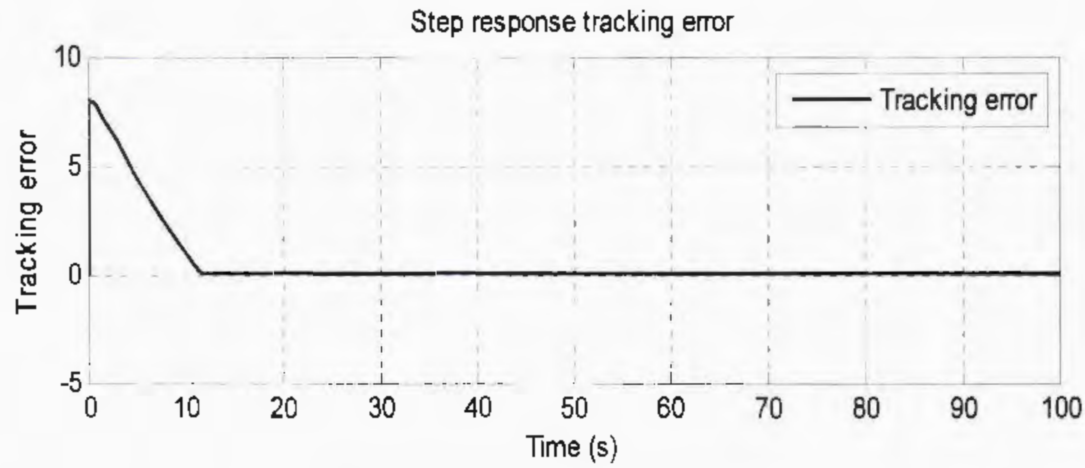

Figure 4.9: Controller tracking error

4.4.4 RANDOM SET POINTS TRACKING PERFORMANCE

Now the tracking performance of the designed controller is observed for the random reference set points in figure 4.10. The flow rate for tracking the random set points is shown in figure 4.11.

The reason for using the different reference set points is that, often it is required in industry to maintain the liquid at different level in tanks according to requirement. Hence it is necessary to check the performance of designed controller under these sudden changes in command signal.

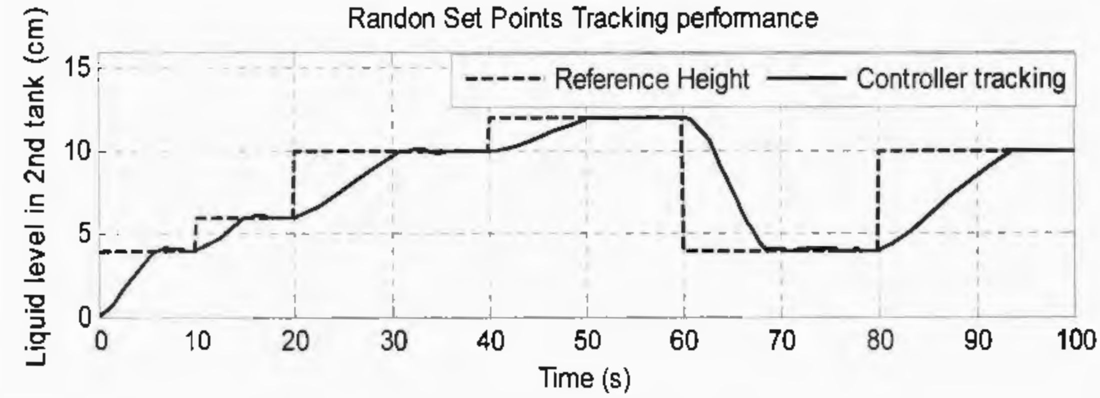

Figure 4.10: Controller tracking performance for random set points

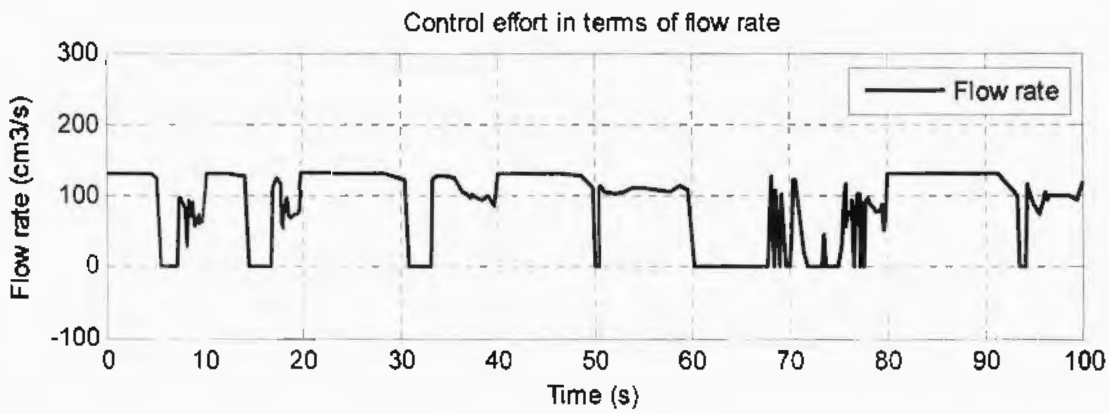

Figure 4.11: Flow rate for tracking random set points

4.4.5 TRACKING PERFORMANCE FOR SQUARE WAVE LIKE REFERENCE SIGNAL

The tracking performance of Sliding mode controller (SMC) for tracking the square wave like refernce signal is shown in figure 4.12 below. Also the control effort in terms of flow rate is shown in figure 4.13.

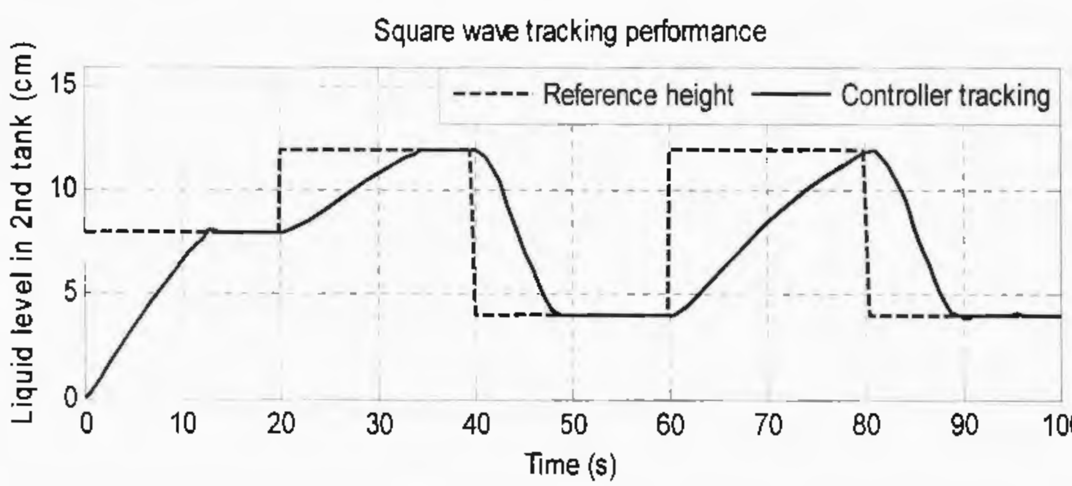

Figure 4.12: Controller tracking performance for square wave reference signal

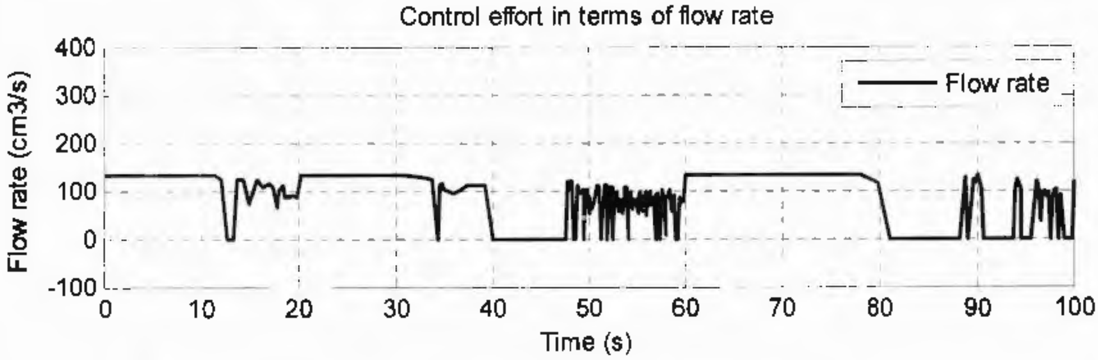

Figure 4.13: Flow rate for tracking square wave like reference signal

4.4.6 TRACKING PERFORMANCE FOR SAW TOOTH AND SINUSOIDAL WAVE LIKE REFERENCE SIGNALS

The SMC tracking performance is also observed for time varying refernce signals. The amplitude of the refernce signals is continously changing with time. These time varying signals are saw tooth and sinusoidal wave like refernce inputs. The changing amplitude of these waves is the desired height in the 2nd tank. The controller showed very good performance for tracking these refernce inputs. The tracking performance of controller for saw tooth and sinusoidal reference signals is shown in figure 4.14 and 4.16. The flow rate for these command signals is shown in figure 4.15 and 4.17.

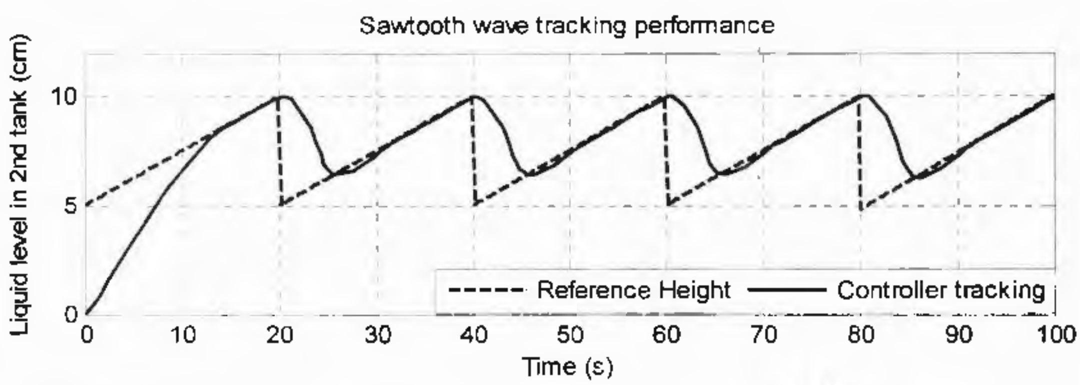

Figure 4.14: Controller tracking performance for sawtooth wave reference signal

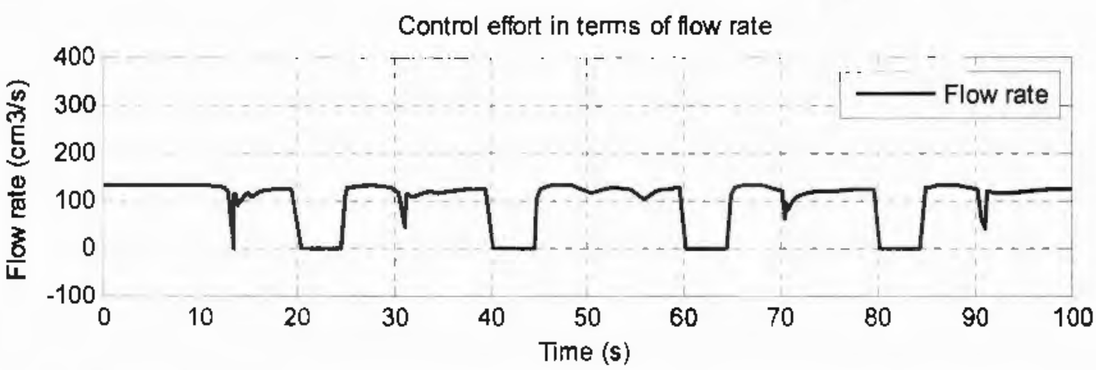

Figure 4.15: Flow rate for tracking the saw tooth wave like reference signal

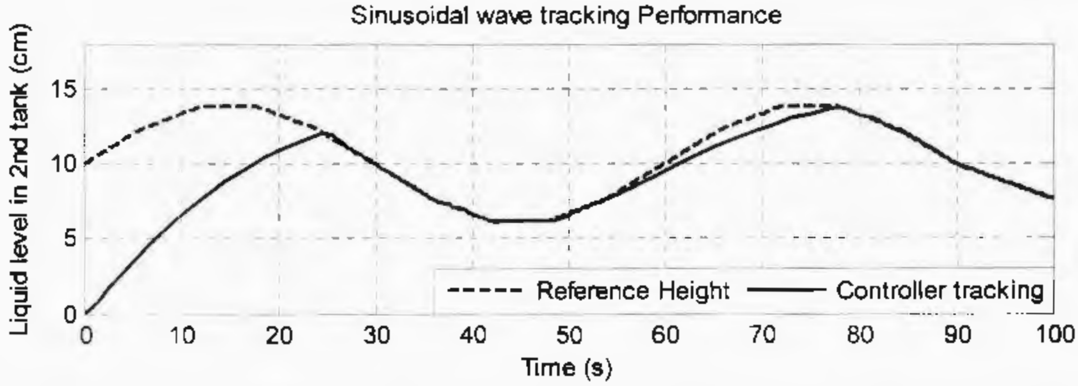

Figure 4.16: Controller tracking performance for sinusoidal wave reference signal

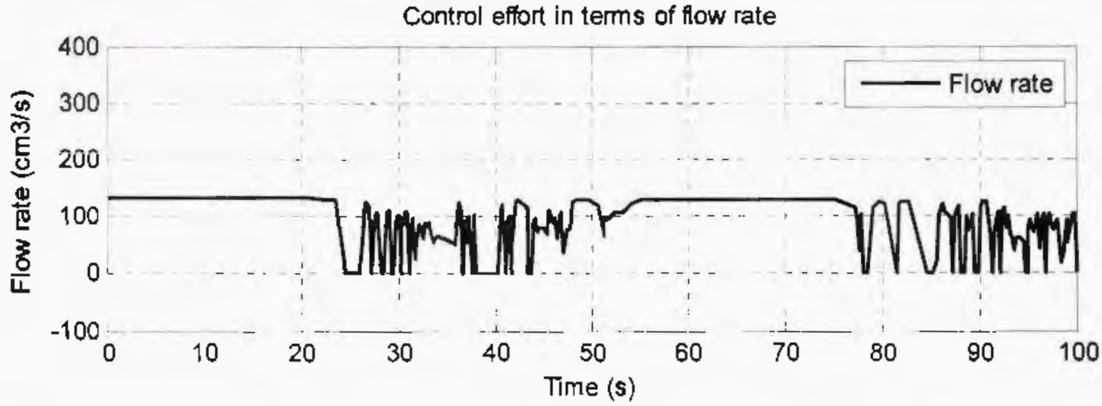

Figure 4.17: Flow rate for tracking the sinusoidal wave like reference signal

4.4.7 DISTURBANCE REJECTION PERFORMANCE OF SMC WHEN THERE IS AN INFLOW TO THE 2ND TANK

In this case, the performance of the controller is tested when the pump 2 is ON for some time. The pump 2 supplies liquid (50cm^3/sec) into the 2nd tank for 30 seconds (40 to 70 sec of total simulation time of 100 sec). This extra input will be considered as disturbance. In equation (3.7), the term U_{i2} represents the input to the 2nd tank. The disturbance rejection performance of the controller is shown in figure 4.18 and its control signal is shown in figure 4.19.

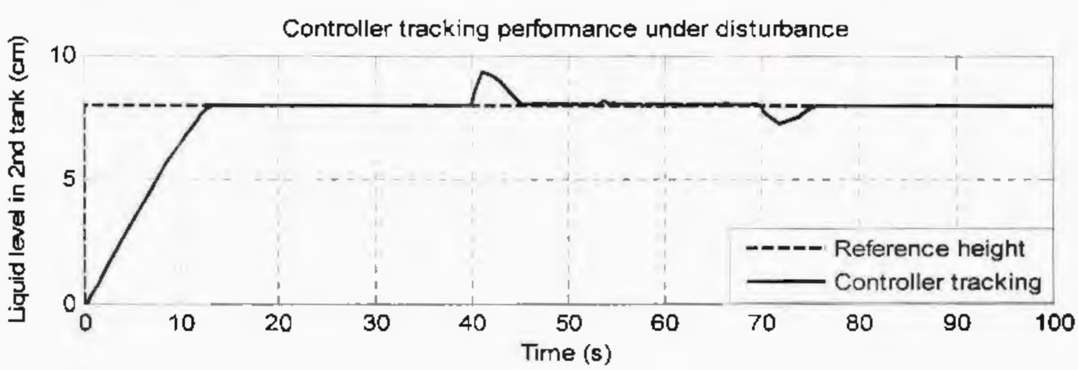

Figure 4.18: Disturbance rejection performance

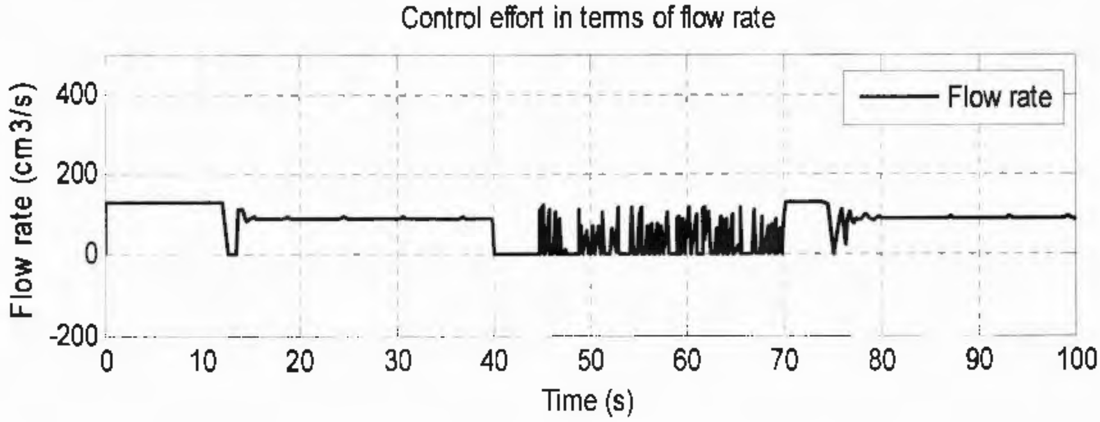

Figure 4.19: Flow rate when input to the 2nd tank

The SMC showed a satisfactory performance under disturbance. When the extra input is added at 40 seconds time, the level of the water rises but the controller recovered that error in almost 5 seconds by decreasing the flow rate into the 1st tank. Similarly, when the disturbance is removed at 70 seconds time, the level of the liquid goes on decreasing but again the controller recovered that error within 5 seconds by increasing the flow rate into the 1st tank.

4.4.8 SMC TRACKING PERFORMANCE WHEN THERE IS LEAKAGE IN THE 1ST TANK

Now the controller performance is tested when there is a leakage of $20cm^3/s$ in the 1st tank. The leakage of liquid is introduced in the 1st tank when the simulation time was 30 seconds. In equation (3.19). the term U_{j1} is representing the leakage in the 1st tank. The performance of controller and its control signal are shown in figure 4.20 and 4.21 respectively.

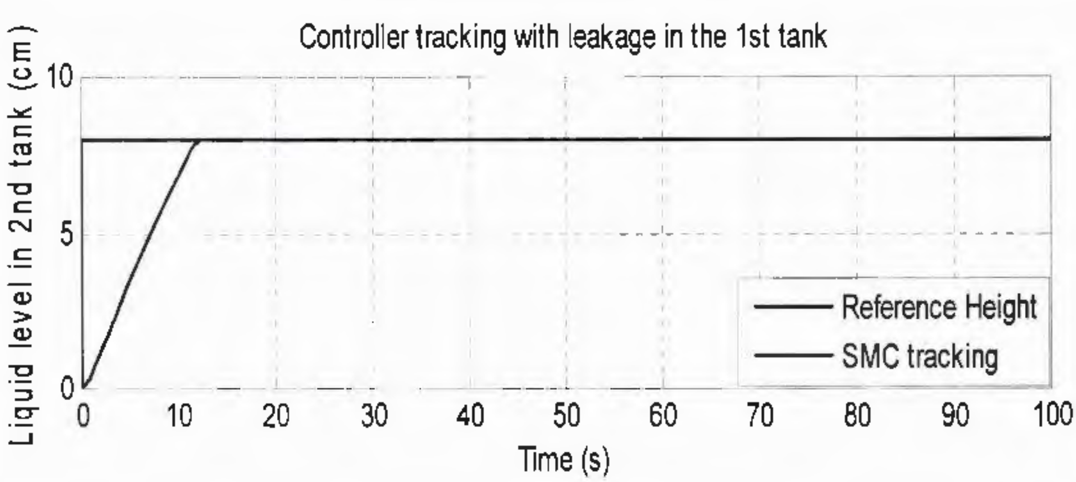

Figure 4.20: SMC performance with leakage in the 1st tank

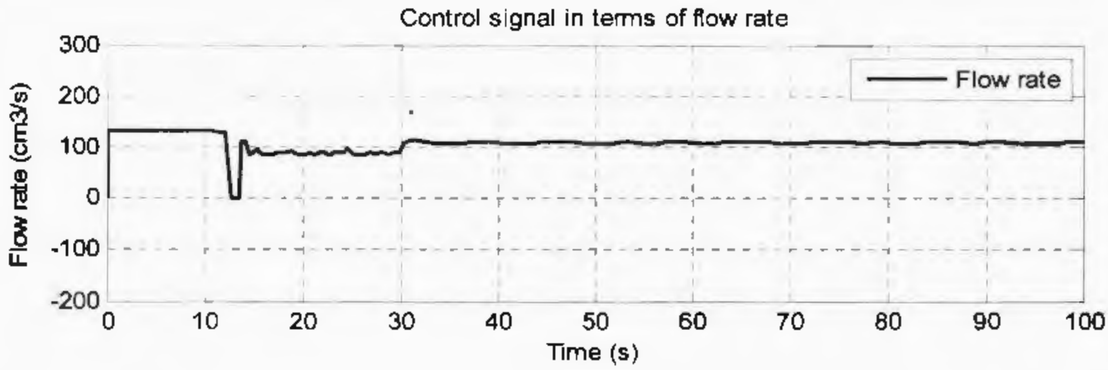

Figure 4.21: Flow rate when leakage in the 1st tank

4.4.9 SMC TRACKING PERFORMANCE WHEN THERE IS LEAKAGE IN THE 2ND TANK

Now the controller is tested for another uncertain happening, the leakage in the 2nd tank. When the simulation time was 30 seconds, the 20 cm^3/s extra liquid outflow or leakage from 2nd tank is introduced. The tracking performance of controller and its control signal are shown in figure 4.22 and 4.23 respectively.

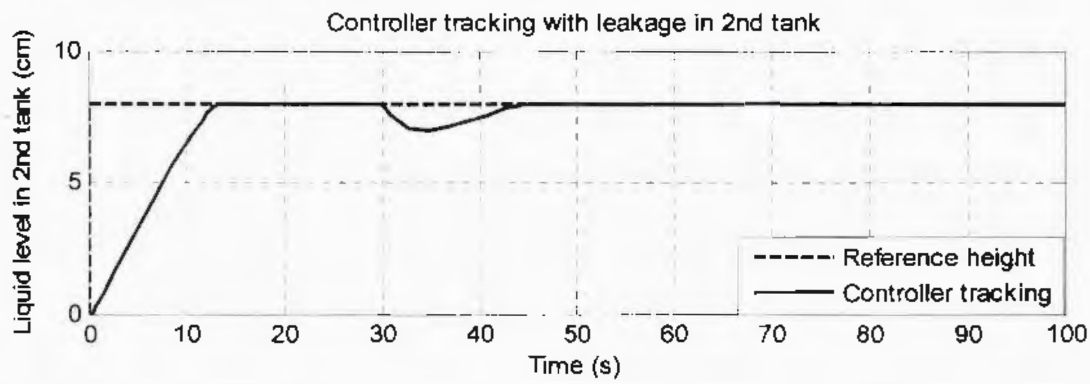

Figure 4.22: Controller performance with leakage in the 2nd tank

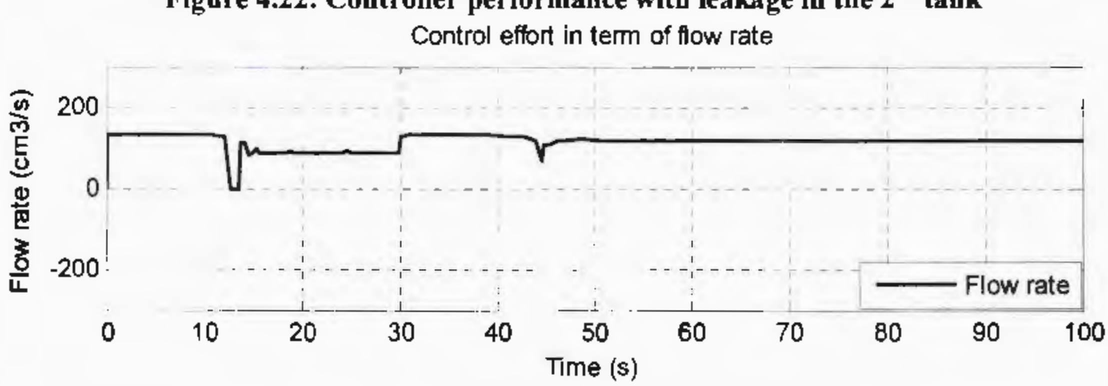

Figure 4.23: Flow rate when leakage in the 2nd tank

The controller showed good performance for the recovery of leakage or extra outflow from 2nd tank. The controller took almost 15 seconds for the recovery of 20cm^3/s liquid leakage in the 2nd tank.

4.5 SLIDING MODE CONTROLLER WITH EQUIVALENT CONTROL TECHNIQUE

In this section, the basic Sliding mode control is merged with equivalent control technique of sliding mode control.

4.5.1 CONTROLLER DESIGN

Now the basic sliding mode control is incorporated with equivalent control. The purpose of control is to make the controlled variables equal to reference set point, so that the error and its derivative remain zero all the time. When the actual position becomes equal to the reference value, that is error become zero and hence equation (4.2) indicates that the sliding surface S(t) reaches at a constant value. So when system remains on sliding surface that means *e (t)* is zero all times [17]. Hence it is desired to make

$$\frac{dS(t)}{dt} = 0 \tag{4.6}$$

First of all, a sliding surface has been selected, after that a suitable control law is designed so that the control variable is being driven to its reference value and satisfies equation (4.6). The structure of Sliding mode control law U(t) is based on two main parts; a continuous part $U_C(t)$ and a discontinuous part $U_D(t)$ **[27]**. That is

$$U(t) = U_c(t) + U_D(t) \tag{4.7}$$

$U_c(t) = U_{eq}(t)$, is the dominated equivalent control, represents the continuous part of the controller that maintains the output of the system restricted to the sliding surface. The continuous part of sliding mode control is given by,

$$U_c = f[R(t), Y(t)] \tag{4.8}$$

It is a function of the reference value R(t) and controlled variable Y(t).

The part U_D (discontinuous) of sliding mode control comprises a non-linear element that contains the switching element of the control law. This part (U_D) of the controller is discontinuous across the sliding surface. Due to the discontinuous part of the controller, the system states moves fast and reach at the sliding surface and this sliding motion proceed until a desired state is achieved. This discontinuous part was discussed briefly in previous section 4.2.

Now we will design the continuous part of Sliding mode controller for coupled-tank system. The coupled-tank system's model can be written as,

$$\frac{d^2h_2(t)}{dt^2} + a_1\frac{dh_2(t)}{dt} + a_0h_2(t) = bu_1(t) \tag{4.9}$$

Error is defined as the difference between actual height and desired height of liquid in 2nd tank. That is

$$e(t) = h_D(t) - h_2(t) \tag{4.10}$$

Its single and double derivative is

$$\dot{e}(t) = \frac{d}{dt}[h_D(t) - h_2(t)]$$

$$\dot{e}(t) = -\frac{dh_2(t)}{dt} \tag{4.11}$$

$$\ddot{e}(t) = -\frac{d^2h_2(t)}{dt^2} \tag{4.12}$$

The derivative of the sliding surface [equation (4.3)] is

$$\frac{d}{dt}S(t) = \ddot{e} + \lambda\dot{e} \tag{4.13}$$

In order to maintain S(t) at constant value. The time derivative of sliding surface S(t) should be equal to zero. That is, $\frac{dS(t)}{dt} = 0$, So

$$\ddot{e} + \lambda\dot{e} = 0 \tag{4.14}$$

Substituting the value of $\ddot{e}$ and $\dot{e}$, we get

$$\frac{d^2h_2(t)}{dt^2} + \lambda\frac{dh_2(t)}{dt} = 0 \tag{4.15}$$

Substituting the value of $\frac{d^2h_2(t)}{dt^2}$ from equation (4.9), we get

$$-a_1\frac{dh_2(t)}{dt} - a_0h_2(t) + bu_1 + \lambda\frac{dh_2(t)}{dt} = 0 \tag{4.16}$$

$$bu_1 = a_1\frac{dh_2(t)}{dt} + a_0h_2(t) - \lambda\frac{dh_2(t)}{dt}$$

Hence the continuous part of the control law will be

$$u_{1c} = \frac{1}{b}[(a_1 - \lambda)]\frac{dh_2(t)}{dt} + a_0h_2(t) \tag{4.17}$$

This procedure in which the continuous part of the controller derived is known in the Sliding mode control theory and the equivalent control procedure [21].

Then, the complete sliding mode control law can be represented as follows

$$U_1 = \frac{1}{b}[(a_1 - \lambda)\frac{dh_2(t)}{dt} + a_0h_2(t)] + K\frac{S}{|S|+\delta} \tag{4.18}$$

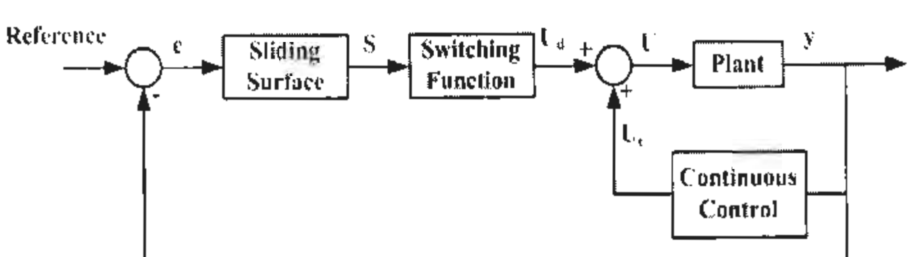

Figure 4.23: Schematic diagram of Sliding mode control

4.6 SIMULATION RESULTS OF SMC WITH LINEAR COUPLED TANK PLANT MODEL

In this section, the simulation results of Sliding Mode controller (SMC) with equivalent control technique are discussed. The linear model of the coupled tank plant in the form of transfer function is given by equation (3.22). Where $\lambda = 2$ the slope of the sliding surface and K is the tuning parameter and is adjusted to 450. Also $\delta = 0.32$ is to be adjusted to obtain the suppression of chattering.

The controller tracking is observed by tracking the various set points. The input to the plant is the flow rate to the 1st tank and output of the plant is the liquid level in the 2nd tank. Hence the reference signal will be the desired liquid height in the 2nd tank. The reference signals taken are; step, random, square, saw tooth and sinusoidal. The controller tracking performance for step, random, square, saw tooth and sinusoidal wave like reference signals is shown in figure 4.24, 4.25, 4.26, 4.27 and 4.28.

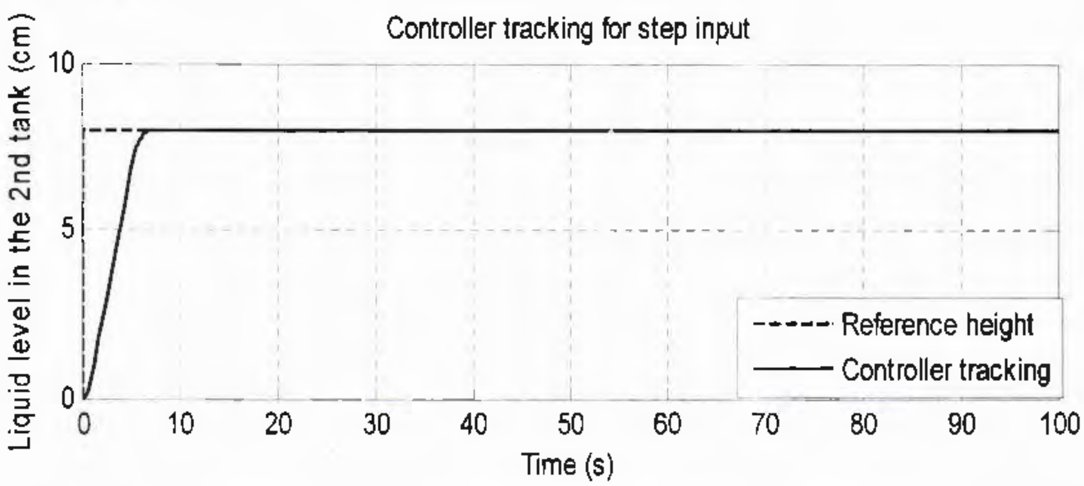

Figure 4.24: Controller tracking performance for step input

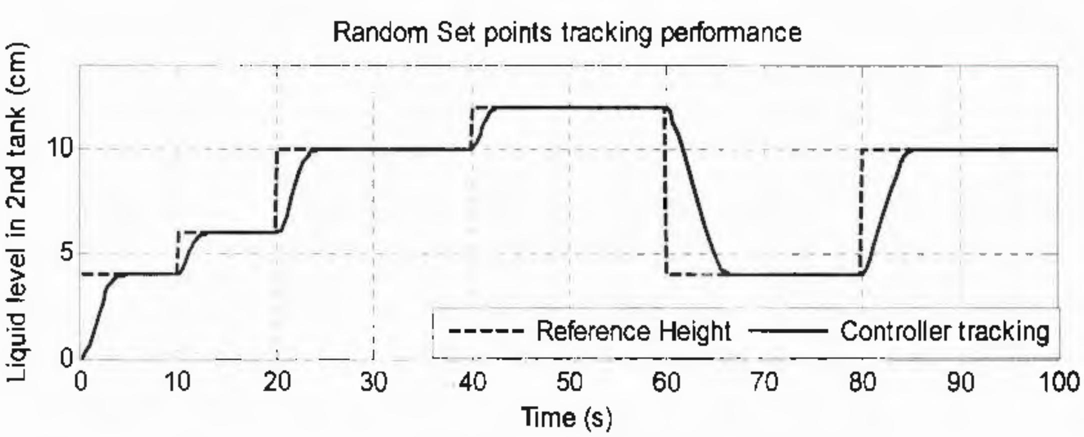

Figure 4.25: Controller tracking performance for random set points

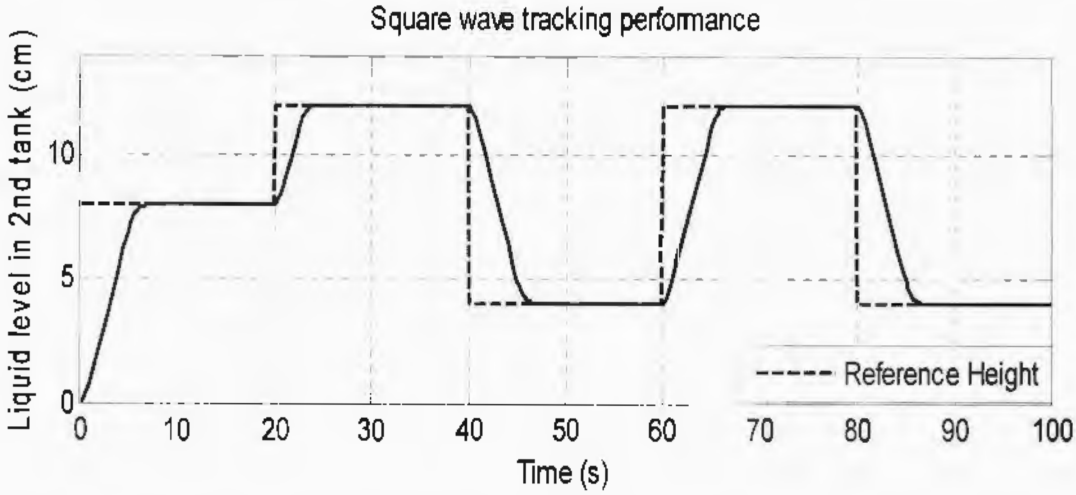

Figure 4.26: Controller tracking performance for square wave like reference signal

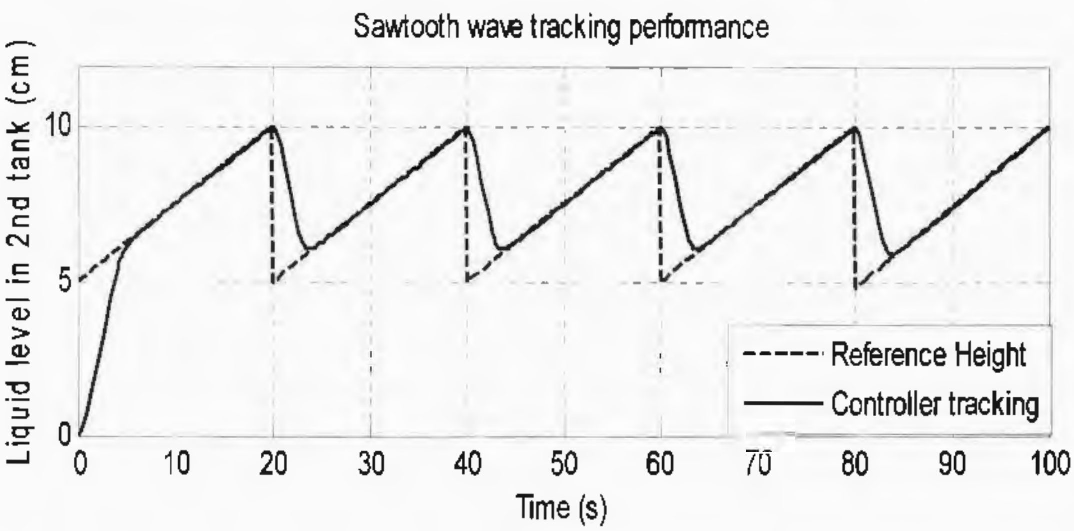

Figure 4.27: Controller tracking performance for saw tooth wave like reference signal

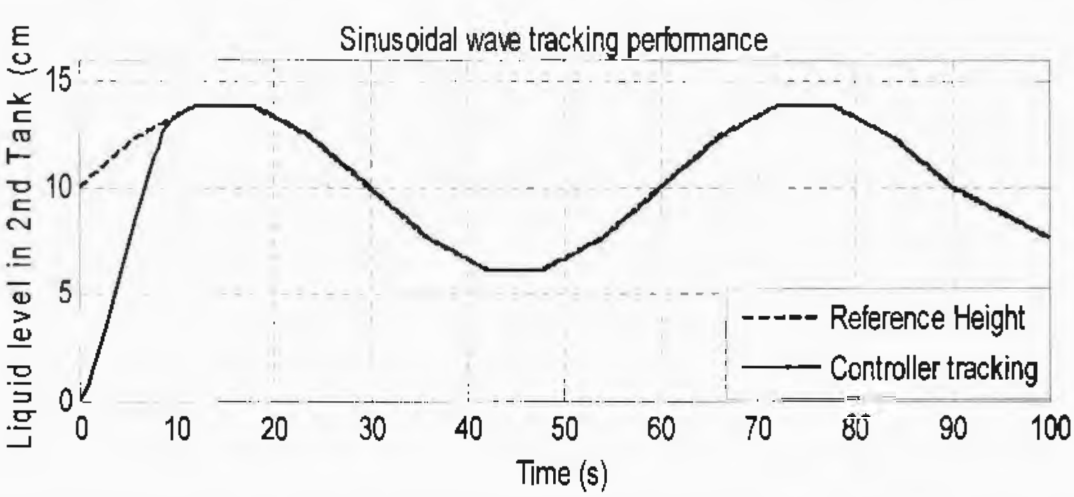

Figure 4.28: Controller tracking performance for sinusoidal wave like reference signal

4.7 SIMULATION RESULTS WITH NON-LINEAR PLANT MODEL

The simulation results of the sliding mode controller (SMC) for the liquid level control in coupled tank system are discussed in this section. The plant model used is the non-linear coupled tank plant model.

4.7.1 TRACKING PERFORMANCE FOR STEP INPUT

The controller tracking performance for step input is being observed in this section. The reference value is selected as 8 which is the required liquid level (in cm) in the 2^{nd} tank. Hence the controller is given a task to achieve and maintain the liquid level at 8cm height in the 2^{nd} tank. The response or the tracking performance of the controller for this height is shown in figure 4.29. The controller took almost 10 sec to achieve that reference height.

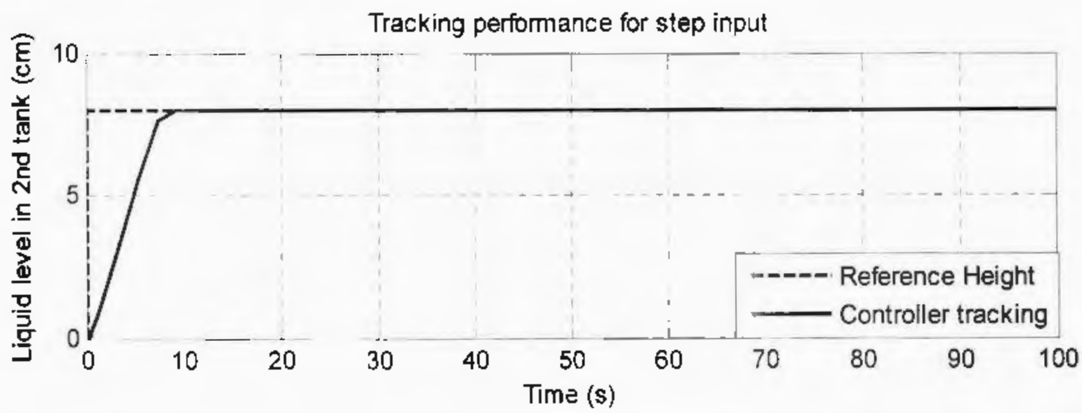

Figure 4.29: Tracking performance for step reference signal

4.7.2 INPUT FLOW RATE

The inflow rate U_{i1} into the 1^{st} tank is shown in figure 4.30. This flow rate is for tracking the 8cm height in the 2^{nd} tank. The flow rate is controlled by the controller and is adjusted in order to reduce the error between reference value and desired value.

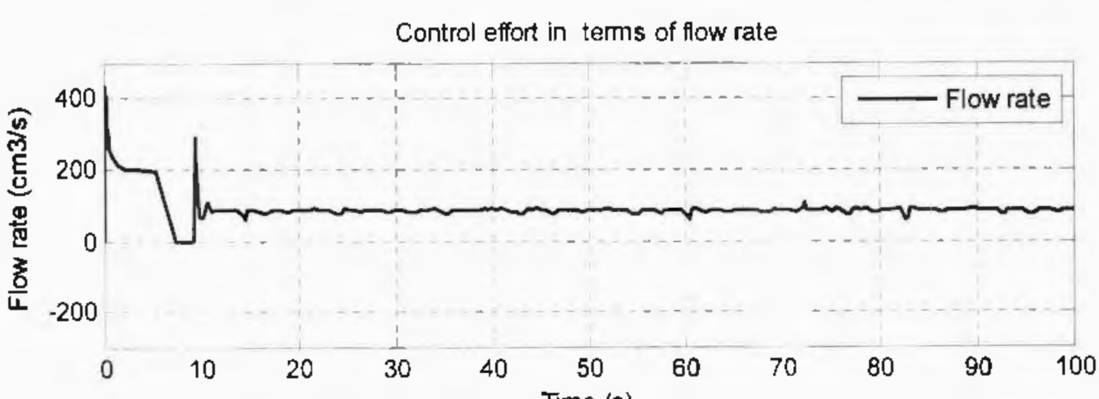

Figure 4.30: Control effort in terms of flow rate for step input

4.7.3 TRACKING ERROR

The tracking error of SMC for tracking the 8cm liquid level in the 2[nd] tank is shown in figure 4.31.

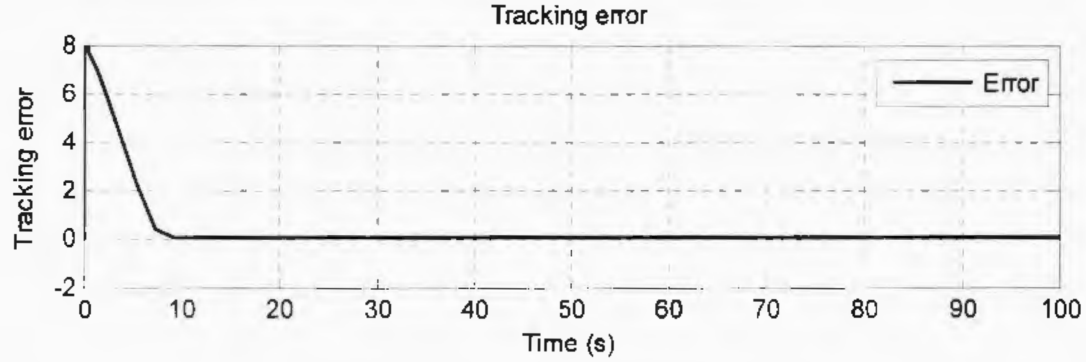

Figure 4.31: Tracking error for step input

4.7.4 RANDOM SET POINTS TRACKING PERFORMANCE

The random set points tracking performance of the sliding mode controller (SMC) is shown in figure 4.32. The Sliding mode controller (SMC) showed a good performance for tracking the random set points reference values of liquid height in 2[nd] tank. Also the flow rate for tracking the random set points is shown in figure 4.33.

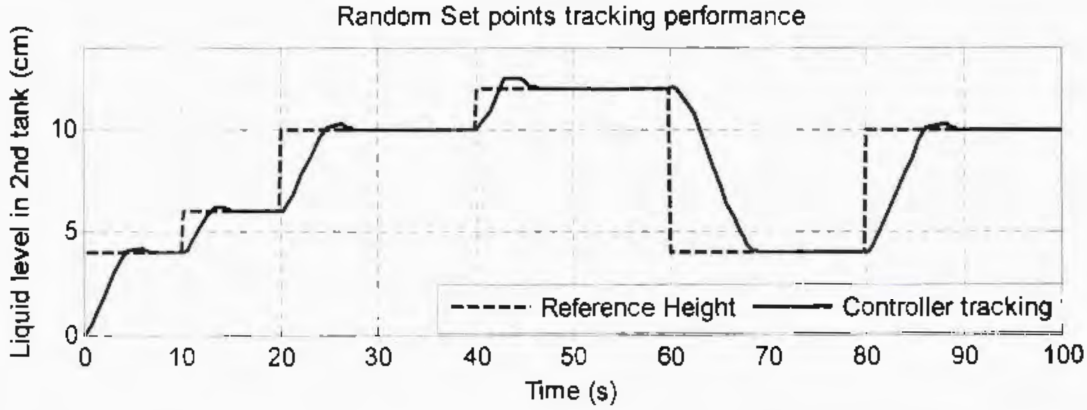

Figure 4.32: Tracking performance for random set points

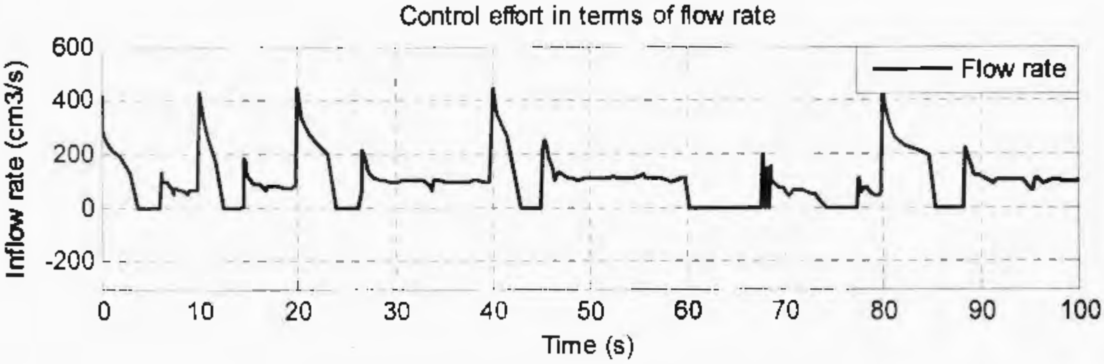

Figure 4.33: Flow rate for random set points

4.7.5 TRACKING PERFORMANCE FOR SQUARE, SINUSOIDAL AND SAWTOOTH WAVE LIKE REFERENCE SIGNALS

In the figures 4.34, 4.36 and 4.38, the efficiency of the Sliding mode controller in tracking the different time varying refernce signals is shown.These reference signals are square, sinusoidal and sawtooth wave like signals. The control input signal for tracking the square, sinusoidal and saw tooth wave like reference signals is shown in figures 4.35, 4.37 and 4.39. The control input is actually the inflow rate to the 1st tank.

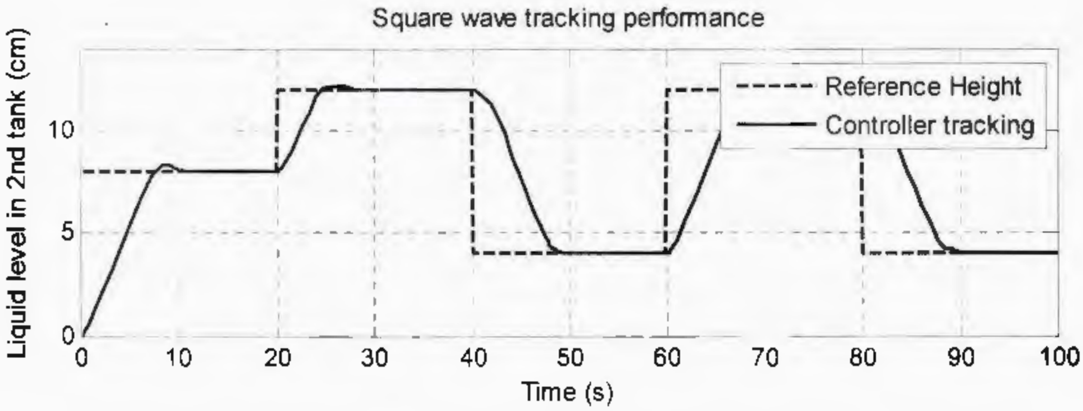

Figure 4.34: Tracking performance for square wave like reference signal

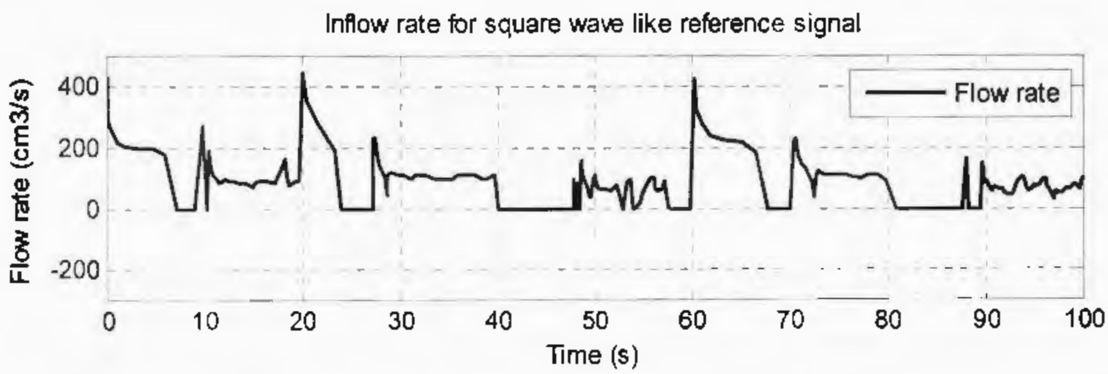

Figure 4.35: Control signal for tracking the square wave like reference signal

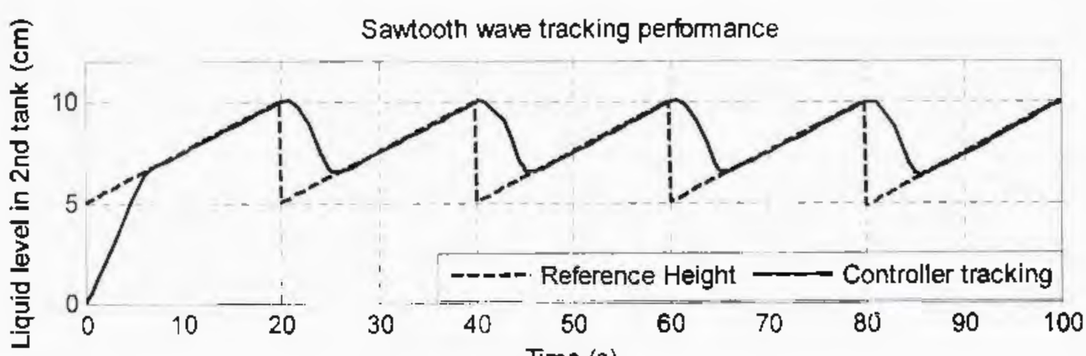

Figure 4.36: Tracking performance for sawtooth wave like reference signal

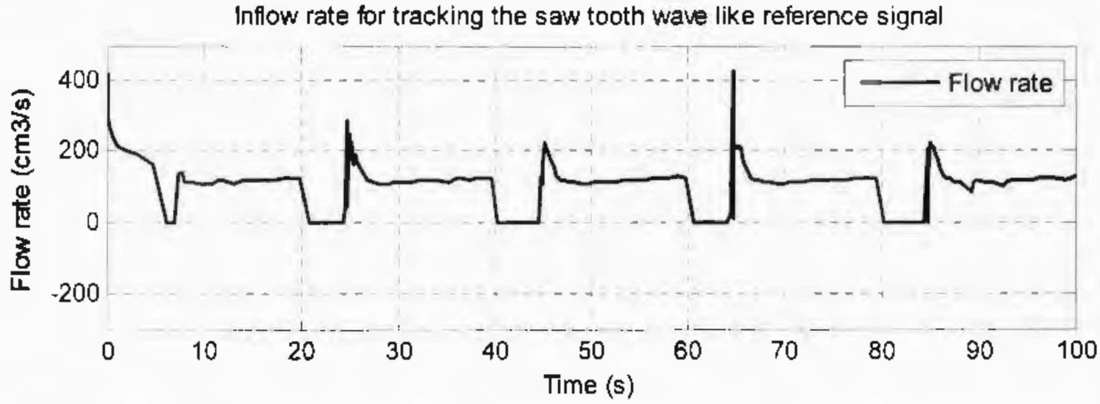

Figure 4.37: Control signal for tracking the sawtooth wave like reference signal

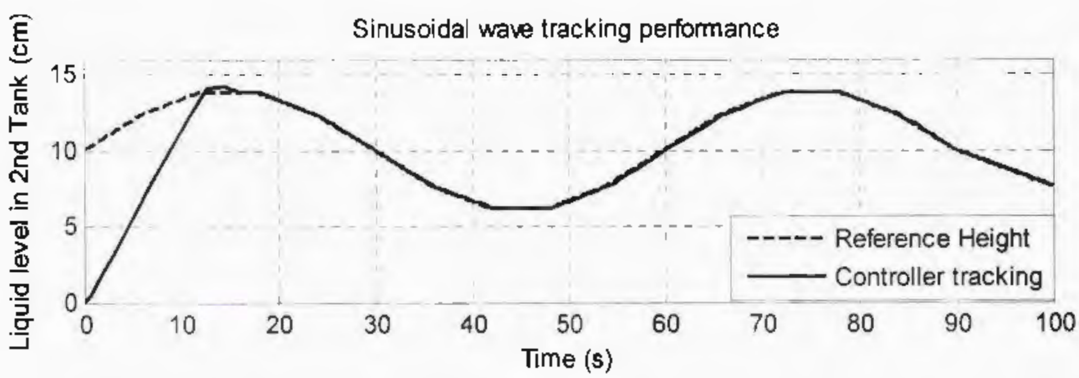

Figure 4.38: Tracking performance for sinusoidal wave like reference signal

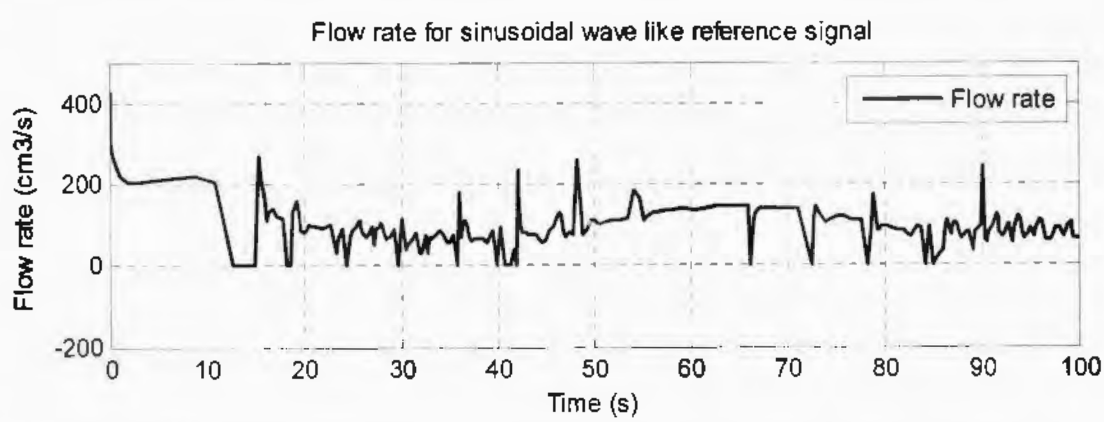

Figure 4.39: Control signal for tracking the sinusoidal wave like reference signal

4.7.6 DISTURBANCE REJECTION PERFORMANCE OF CONTROLLER WHEN PUMP2 IS ON FOR SOME TIME

In this section, the performance of the controller is observed when the pump 2 is ON for some time. The pump 2 supplies liquid at the rate of $50cm^3/s$ into the 2nd tank for 30 seconds (40 to 70 sec of total simulation time of 100 sec). This extra input U_{i2} term will be considered as disturbance in the model. In figure 4.40, the disturbance rejection performance of the controller is shown. In figure 4.41, the control input signal of the controller is shown.

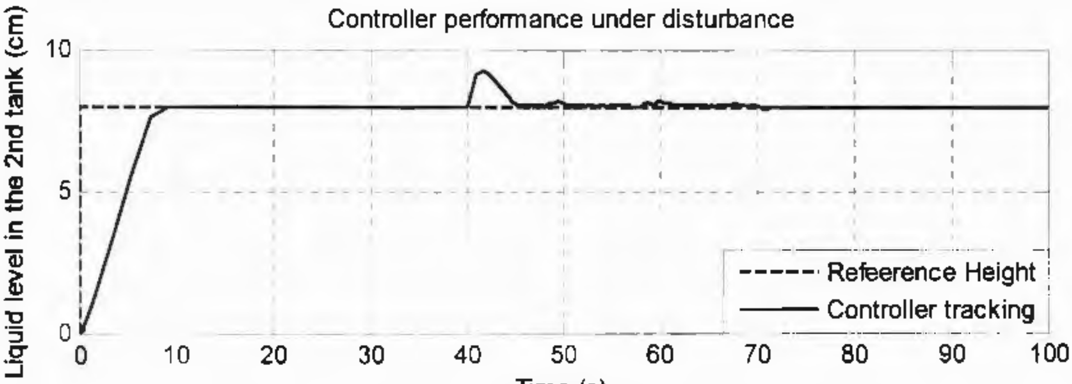

Figure 4.40: Disturbance rejection performance

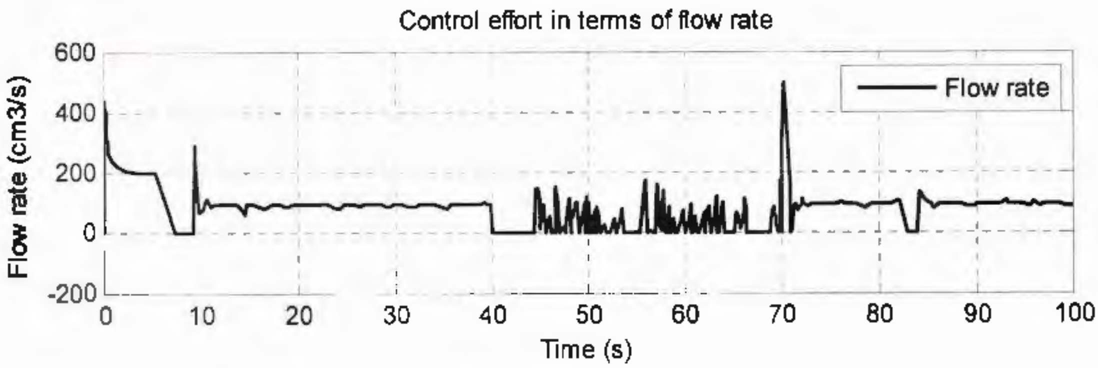

Figure 4.41: Control input for disturbance rejection

4.7.7 CONTROLLER PERFORMANCE WITH LEAKAGE IN THE 2ND TANK

Now the leakage in the 2nd tank is introduced and the performance of the controller is observed in this situation. When the simulation time was 30seconds, the 20cm^3/s extra liquid outflow or leakage from 2nd tank is introduced. The performance of the controller is shown in figure 4.42.

The controller showed very good tracking performance and recovered that error in the plant in a very short time of about 3 seconds. This is the much more improved performance than Basic Sliding Mode controller. Its control signal is shown in figure 4.43.

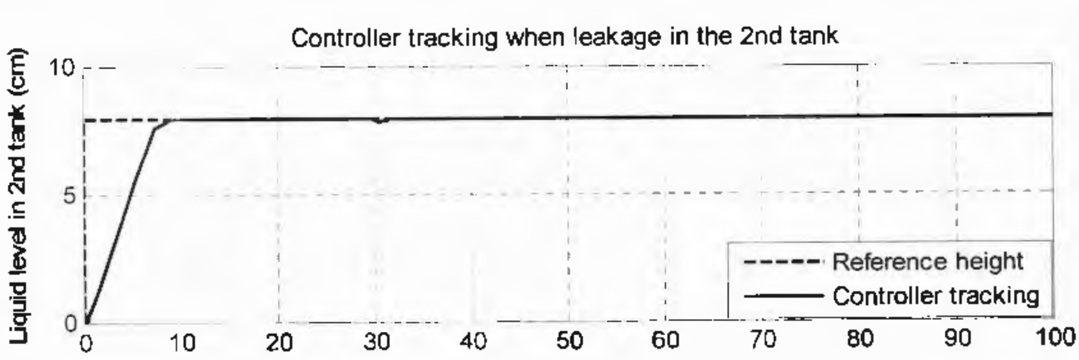

Figure 4.42: Tracking performance of controller with leakage in the 2nd tank

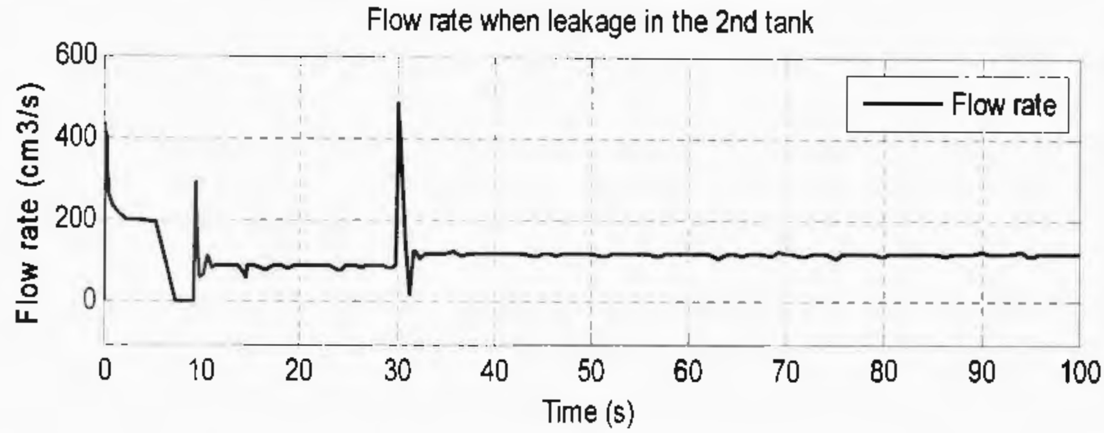

Figure 4.43: Flow rate when leakage in the 2nd tank

4.7.8 CONTROLLER PERFORMANCE WITH LEAKAGE IN THE 1ST TANK

In figure 4.44, performance of the controller is shown when leakage of liquid (20cm^3/s) is introduced in the 1st tank at the time of 30 sec. The controller showed very good results in tracking the reference input and rejected the disturbance. The control effort in terms of flow rate is shown in figure 4.45.

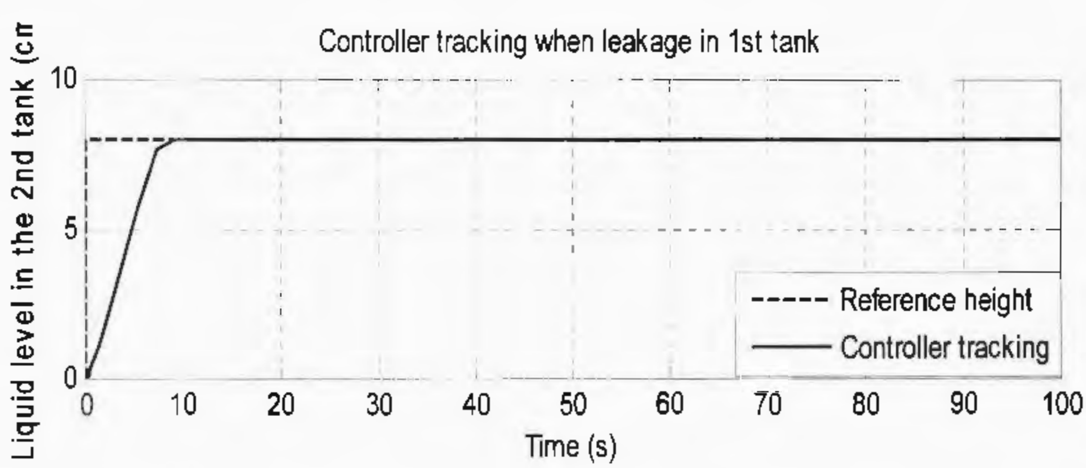

Figure 4.44: Tracking performance of controller with leakage in the 1st tank

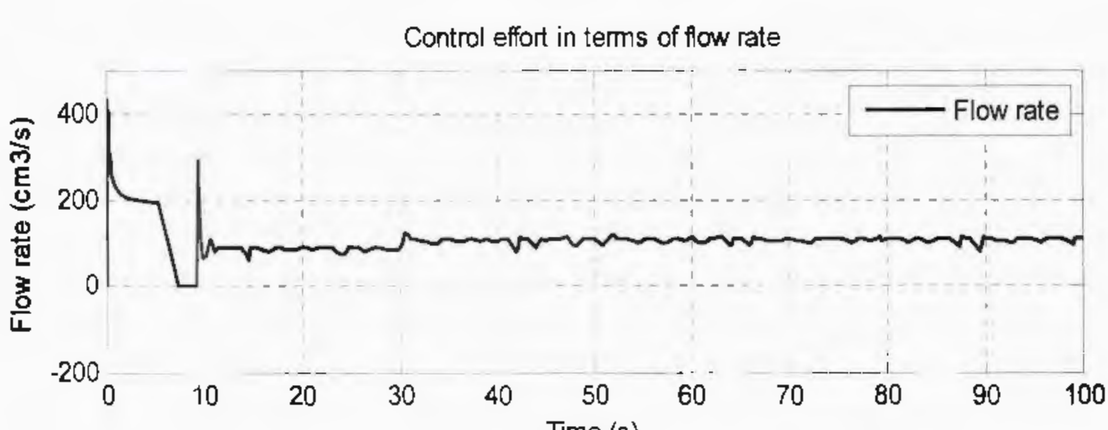

Figure 4.45: Flow rate when leakage in the 1st tank

4.8 PERFORMANCE COMPARISON OF BASIC SLIDING MODE CONTROLLER AND SLIDING MODE CONTROLLER WITH EQUIVALENT CONTROL TECHNIQUE

The brief comparison of both the controllers (Basic Sliding Mode controller and Sliding Mode controller with equivalent control technique) regarding tracking performance, transient response, settling time, and steady state error is made below. For simplicity, the Basic Sliding Mode controller and Sliding Mode controller with equivalent control technique are referred as SMC1 and SMC2 respectively. Overall the controller SMC2 showed a very good tracking in every aspect than SMC1. Its transient response, settling time and steady state error is much more improved than SMC1.

4.8.1 STEP REFERENCE SIGNAL

The tracking of both the controllers is compared for step reference signal. The settling time of controller SMC1 is almost 14 seconds while the settling time of controller SMC2 is 10 seconds. Also the controller SMC2 showed a very good transient response and no steady state error. The comparison is shown in figure 4.46.

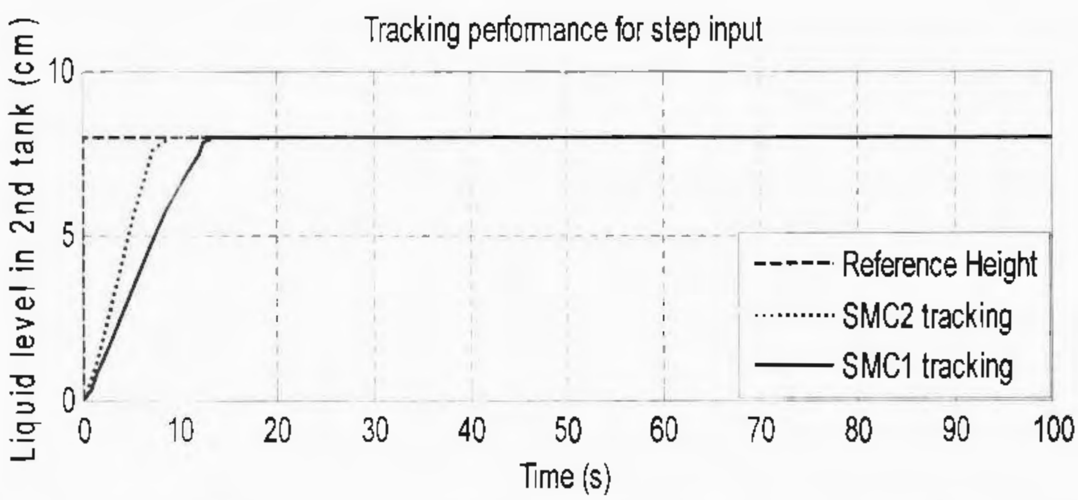

Figure 4.46: Comparison of SMC1 and SMC2 for step reference signal

4.8.2 RANDOM REFERENCE SET POINTS

The performance comparison of both the controllers SMC1 and SMC2 for tracking the different command set points (random, square, saw tooth, sinusoidal) is made below. Again, the controller SMC2 showed a very good tracking performance. The tracking of both the controllers for all different set points is shown in figures 4.47, 4.48, 4.49 and 4.50.

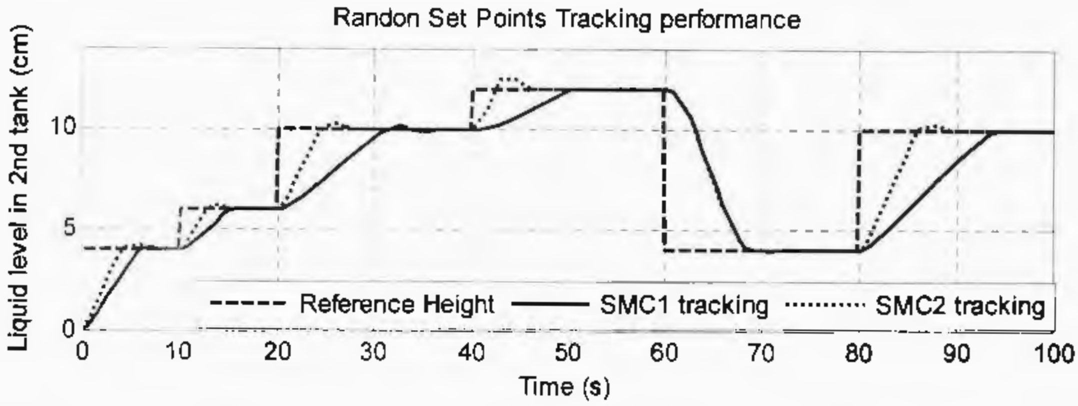

Figure 4.47: Comparison of SMC1 and SMC2 for random set point signal

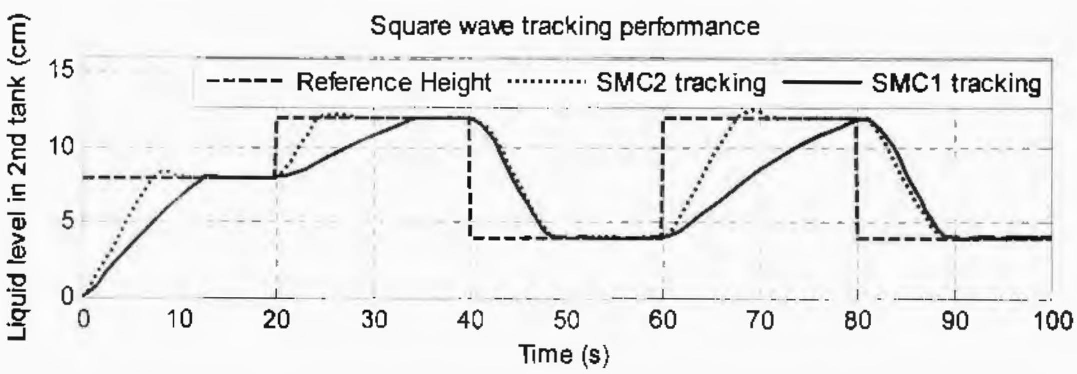

Figure 4.48: Comparison of SMC1 and SMC2 for Square wave like reference signal

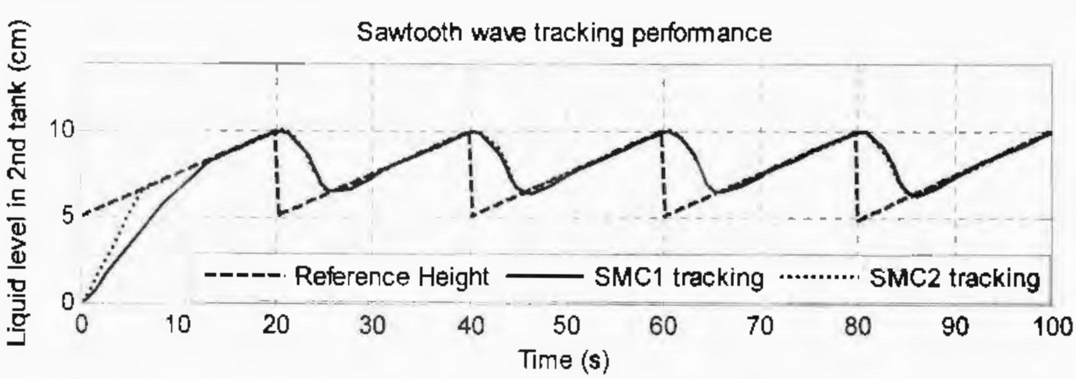

Figure 4.49: Comparison of SMC1 and SMC2 for Sawtooth wave like reference signal

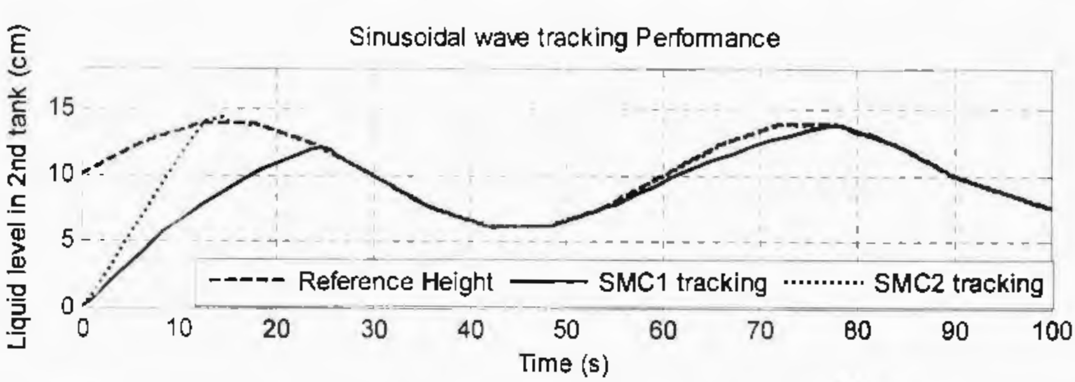

Figure 4.50: Comparison of SMC1 and SMC2 for Sinusoidal wave like reference signal

4.8.3 DISTURBANCE REJECTION PERFORMANCE COMPARISON OF SMC1 AND SMC2

The disturbance rejection performance of both SMC1 and SMC2 is compared below in this section.

4.8.3.1 INFLOW TO THE 2ND TANK

The disturbance introduced to the plant is the inflow of liquid (50cm^3/s) to the 2nd tank is made for some time (30 to 70 seconds of the total simulation time). It is clear from the figure 4.51 that controller SMC2 has much better performance than SMC1 in the presence of disturbance and recovered the error much quicker than SMC1.

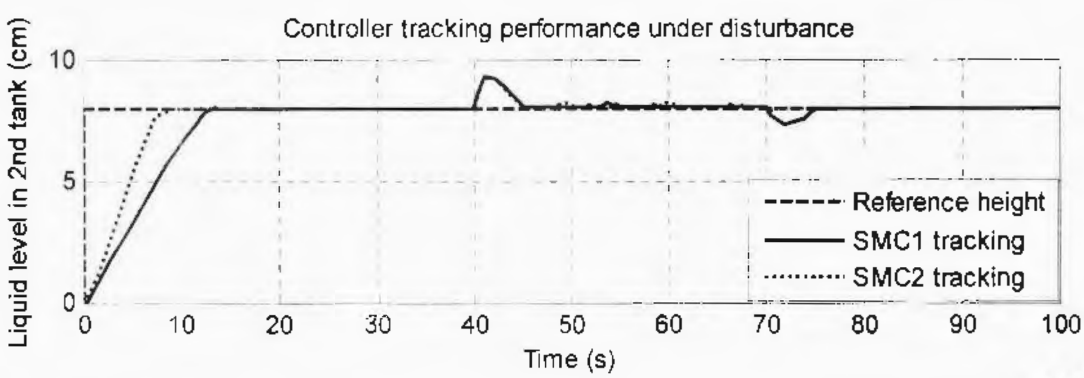

Figure 4.51: Disturbance (2nd input) rejection performance comparison of SMC1 and SMC2

4.8.3.2 LEAKAGE IN THE 2ND TANK

In this section, both the controllers are tested for another interesting situation i.e. the leakage in the 2nd tank or the increased outflow from 2nd tank which is quite possible in physical plants (with the rise in temperature). When the simulation time was 30 seconds, an extra outflow of 20cm^3/s is flowed out of the 2nd tank. Again the controller SMC2 showed a very good performance for this type of uncertainty in the plant. The comparison is shown in figure 4.52.

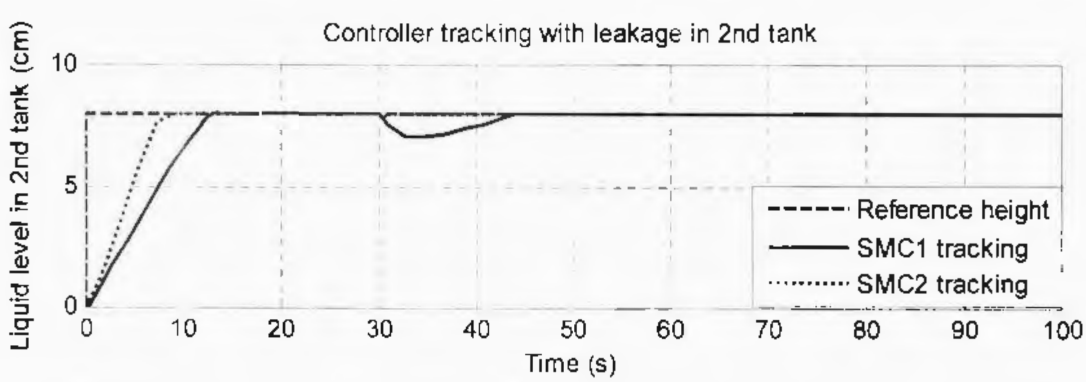

Figure 4.52: Disturbance (leakage in 2nd tank) rejection performance comparison of SMC1 and SMC2

4.9 TRACKING PERFORMANCE COMPARISON OF BOTH SLIDING MODE CONTROLLERS SMC1 AND SMC2 WITH PID CONTROLLER

MUHAMMAD NASIRUDDIN BIN MAHYUDDIN [2] used PID and Model reference adaptive controller for the same coupled tank plant model. So the results of his designed PID controller are compared with our designed Sliding Mode controllers SMC1 and SMC2. The comparison is made below in plots.

4.9.1 STEP REFERENCE SIGNAL

The tracking performance of the SMC1 and SMC2 is compared with PID controller for step reference signal and is shown in figure 4.53.

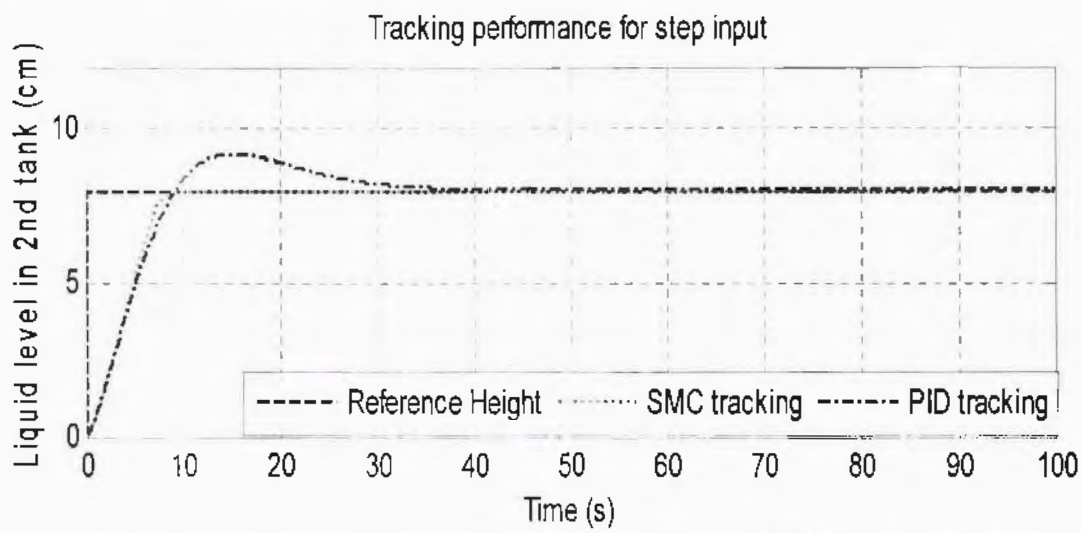

Figure 4.53: Performance comparison of SMC1 and SMC2 with PID controller for step signal

The settling time of SMC1 and SMC2 is 8 and 14 seconds respectively while for the PID controller, it about 40 seconds. So controller SMC1 and SMC2 have much better tracking performance than PID controller not only in terms of settling time but also transient response.

4.9.2 COMPARISON WITH DIFFERENT COMMAND SIGNALS

A brief comparison of Sliding Mode controllers with PID controller is made for tracking the random set points, square wave, saw tooth wave and sinusoidal wave like reference signals and is shown in figure 4.54, 4.55, 4.56 and 4.57 respectively. For all these different command inputs, the SMCs showed a very good tracking performance than PID controller. The PID controller was not able to adjust itself according to command or reference signal variations as the SMCs did.

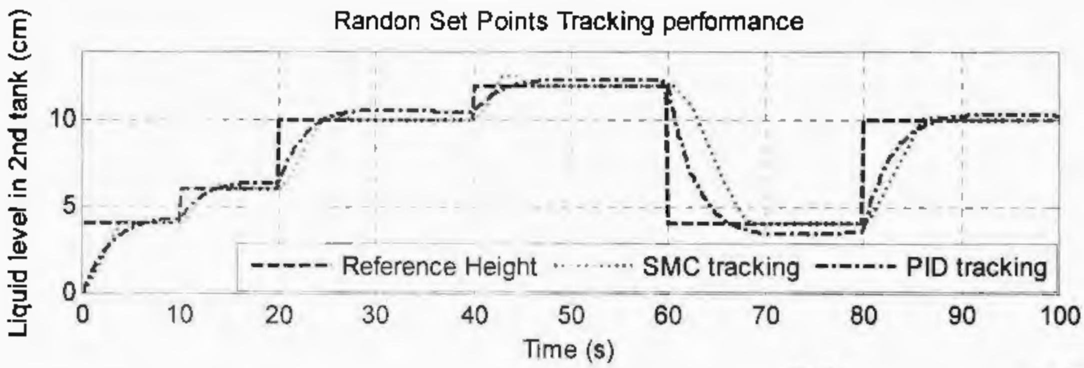

Figure 4.54: Performance comparison of SMC1 and SMC2 with PID controller for random command signal

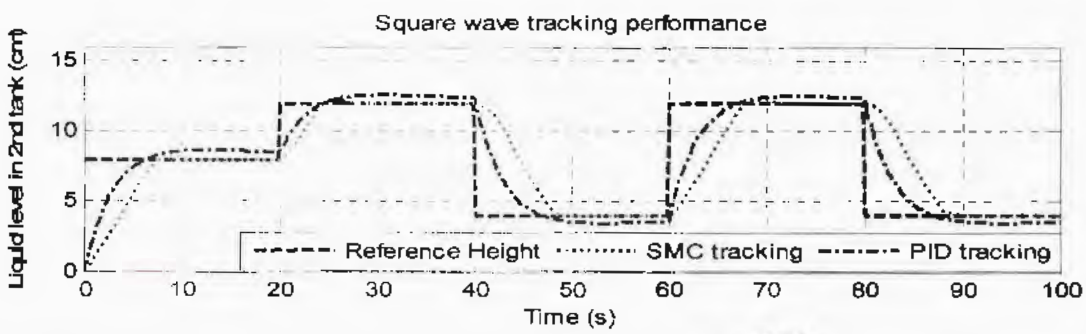

Figure 4.55: Performance comparison of SMC1 and SMC2 with PID controller for square wave like command signal

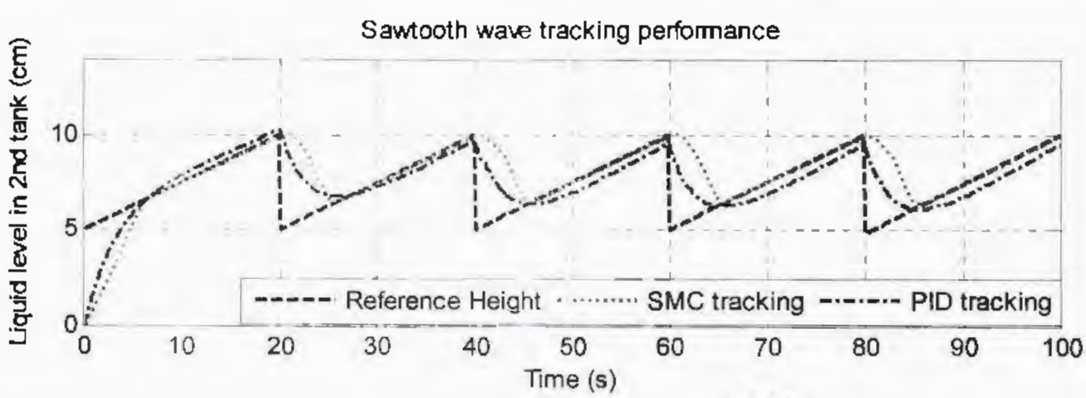

Figure 4.56: Performance comparison of SMC1 and SMC2 with PID controller for saw tooth wave like command signal

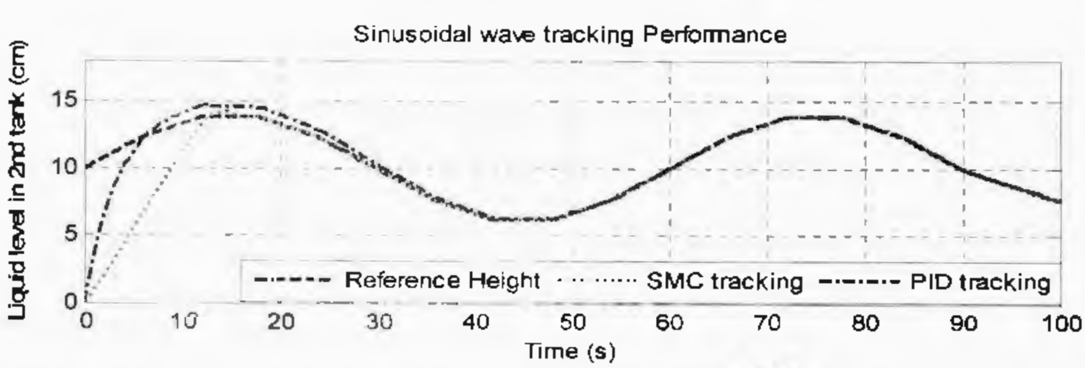

Figure 4.57: Performance comparison of SMC1 and SMC2 with PID controller for sinusidal wave like command signal

4.10 PERFORMANCE COMPARISON

The tracking performance of designed SMC for different time varying command signals is compared with conventional PID controller in terms of ITAE (Integral Time Absolute Error). The performance comparison is made in table 3.3.

PERFORMANCE TEST	SMC	PID
Step command signal	208	428
Random command signal	3986	4186
Square command signal	5163	6017
Saw tooth command signal	2268	3775
Sinusoidal command signal	558	607
Disturbance Rejection Performance ($50\ cm^3/sec$ inflow to 2nd tank)	210	442
Disturbance Rejection Performance ($20\ cm^3/sec$ leakage to 2nd tank)	277	416

Table 3.3: Performance index

The performance index comparison suggests that SMC has much better tracking and disturbance rejection performance than PID controller in controlling the liquid level in coupled-tank system.

4.11 SUMMARY

The designed controllers SMC1 and SMC2 are examined for various types of reference values. Most of these reference values were time varying command signals like sinusoidal, saw tooth wave, etc. In all the cases, the controller SMC2 showed a much better performance than SMC1 and the steady state error of SMC2 is much less than SMC1. Also the controllers are tested for uncertain happenings in the plant model. For example, extra inputs into the 2nd tank, leakage in the 2nd tank, the valve of the 2nd tank is opened more (practically in case of increment in temperature of liquid which increases the outflow). Both the controllers are tested under these uncertain conditions. Again the Controller SMC2 proved to be good to handle all these uncertainties. Finally the tracking performance of both the controllers is compared with simple PID controller and both the SMCs proved to be too good in comparison with fixed gain PID controller.

CHAPTER NO: 5

MODEL REFERENCE ADAPTIVE CONTROL

5.1 INTRODUCTION

In common sense, 'to adapt' means to change or modify a behavior to meet to new circumstances. However, in the sense of control system engineering, an adaptive controller is an "intelligent" controller that can adjust its behavior in response to the variations in the dynamics of the process and the nature of the disturbances [29].

Adaptive control systems have been in existence for over fifty years and a wide range of techniques have been developed so far. The most important feature of all the control techniques is that they have the capability to adept the controller to accommodate variations in the process. This authorizes the controller to be adaptive and maintain a prescribed level of performance regardless of any noise or instability in the process.

There are number of adaptive control methods which are currently in use but the objectives of the control are the same. That is to provide an accurate representation of the process at all the times. An adaptive system has a lot of applications. The adaptive controllers are quite suitable when the plant undergoes transitions or shows non-linear behavior. Also the adaptive controllers are very useful when the structure of the plant is not fully known. Another approach to adapt the controller's parameters is to use a model for the model identification error to tune the controller's parameters. Thus, tuning of the controller is indirect and essentially requires an accurate model of the process for sufficient performance [30].

In the past few decades, the adaptive control came as attention-grabbing field of engineering. There has been substantial interest in the development of adaptive control systems. The new techniques and control schemes are being developing that automatically adjust controller parameters more efficiently to remunerate for unexpected alterations in the plant dynamics. The capability of dealing with time-varying attribute, non-linearity and uncertainties permits adaptive control algorithms to have momentous prospective for the operation of such processes whose characteristics are poorly known or always subject to changes in random way [30].

There are many reasons of designing the adaptive controller for coupled tank plant liquid level control system. In coupled-tank system, the plant dynamics changes with time. Typical example is that, as the temperature increases, the viscosity decreases and hence the outflow increases. Similarly as the temperature decreases, the viscosity increases which results into the decreases in outflow. Therefore the characteristics of the plant changes with the variation in temperature of liquid. Also the coupled-tank system is a non-linear system. So this implies the need of an adaptive controller for the control of coupled-tank system.

In coupled tank system, there may be uncertainties in the system. The liquid used in the coupled tank may be the water, milk, oil or any other fluid. The viscosity of the liquid varies with different fluids. Hence the input and output flow rate has a lot of dependence on the viscosity of fluid. Similarly an extra input into the tanks or the leakage in the tanks makes the system uncertain. These uncertainties cannot be modeled correctly therefore the Model reference adaptive controller is designed to handle all these uncertainties.

The variation of viscosity with temperature in liquids is given by the following relation [35]

$$\eta = Ae^{\frac{E_a}{RT}}$$

Where η is the coefficient of viscosity. T is the temperature in Kelvin. A and R are constants and E_a is the activation energy. The temperature and viscosity coefficient are inversely proportional to each other, so the viscosity coefficient will decrease with rise in temperature and vice versa.

The relation between viscosity and flow of liquid from a pipe of radius r and length L is given by [36]

$$\frac{dV}{dt} = \frac{\pi P r^4}{8\eta L}$$

The above equation is known as Poiseullie,s equation. Where $\frac{dV}{dt}$ is the flow rate. It is clear from above equation that flow rate and coefficient of viscosity are inversely proportional to each other. Hence the flow rate depends upon the viscosity of liquids and the variation in temperature of liquid. Hence we cannot model the outflow correctly from tank when there is a variation in temperature. In such a condition when plant parameters are not fully known, adaptive controller is being used.

In an adaptive system, the parameters of the controller are tuned all the time. This implies that the controller parameters follow changes in plant behavior. It is not so easy to investigate the stability and convergence properties of such systems. To simplify the problem it can be assumed that the process has constant but unknown parameters. When the plant is known completely, the design procedure identifies a set of desired controller parameters. The adaptive controller should converge to these parametric values even when the plant is unknown. A controller having property is known as Self-Tuning controller. since it automatically tunes the controller to the desired performance [31].

A physical adaptive system is designed with an adaptive viewpoint. The Model Reference Adaptive Control System (MRACS) is an adaptive servo system in which the desired performance is expressed in terms of the reference model. [29]. The standard control scheme is the Model Reference Adaptive Control (MRAC) scheme in which it is required that a possibly "bad" plant should follow the behavior of a "good" Reference Model [32].

If the plant parameters were fully known, one could compute the corresponding controller gains that would force the plant to asymptotically follow the Model, or

$$x(t) \to x_m(t)$$

and correspondingly

$$y(t) \to y_m(t)$$

Because the entire plant state ultimately behaves exactly as the model state. When the plant parameters are not (fully or partially) known, one is naturally lead to use adaptive control gains. The basic idea is that the plant is fed a control signal that is a linear combination of the model state through some gains. If all gains are correct, the entire plant state vector would follow the model exactly. If, however, not all gains are correct, the measured plant output differs from the output of the Model Reference [32].

Model reference adaptive control is one of the main approach to adaptive control. The basic structure of a Model reference adaptive control scheme is shown in Figure 5.1. The reference model is chosen to generate the desired trajectory X_m that the plant state X_p has to follow. The tracking error $e \approx X_p - X_m$ is representing the divergence of the plant output from the desired set of performance. The closed loop plant is made up of an ordinary feedback control law that contains the plant and a controller and an adjustment mechanism that generates the on-line estimates controller parameters.

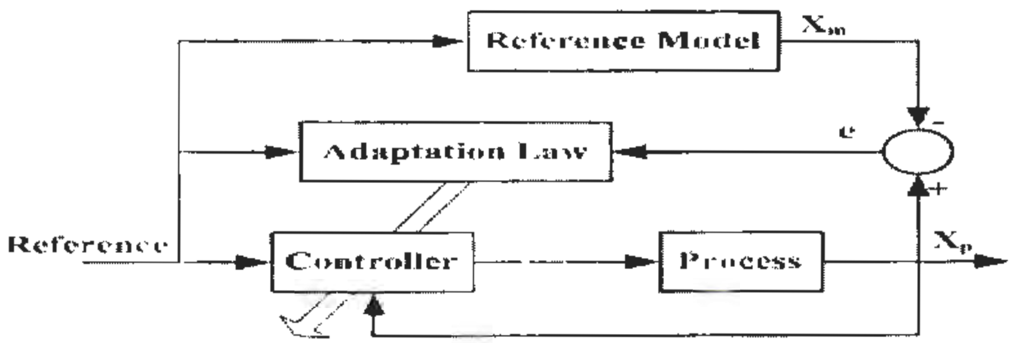

Figure 5.1: Principle of Model Reference Adaptive Control

The Model reference adaptive controller is used particularly when the parameters of the plant are not well known or partially known. This controller shows excellent performance for those systems which have uncertainties [34]. Suppose when any other disturbance is acting on the system or the reference signal is time varying, then Model reference adaptive controller updates its parameters and tracks the reference trajectory.

There are many reasons for adaptive control. The key factors are [33]:

i. Variations in the dynamics of plant. Plant Parameters may vary due to nonlinear actuators, changes in the operating conditions of the plant, and non-linear disturbances acting on the process.
ii. Variations in the nature of the disturbances.
iii. Engineering efficiency.

5.2 CONTROLLER DESIGN

Consider the single input single output plant in state space form as,

$$\dot{x}(t) = Ax(t) + Bu \qquad (5.1)$$

$$y = Cx(t)$$

Where $x(t) \in R^n$ the state vector and $u(t) \in R$ is the control signal. The matrices $A \in R^{n\times n}$ and $B \in R^n$ and $C \in R^{1\times n}$. Also state vector x(t) is available for measurement.

Similarly the reference model can be written in state space form as,

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t) \qquad (5.2)$$

$$y_m = C_m x_m(t)$$

Where $x_m(t) \in R^n$ the state vector and $r(t) \in R$ is the reference signal. Also $A_m \in R^{n\times n}$ and $B_m \in R^n$ and $C_m \in R^{1\times n}$.

The control objective is to determine the plant input *u*(*t*) so that all signals are bounded and the plant state $x(t)$ tracks the reference model state $x_m(t)$ as close as possible for any given reference input $r(t)$.

5.3 CONTROL LAW

If the matrices A and B were known, we can implement the control law

$$u = -K^*x + L^*r \tag{5.3}$$

The closed loop response could be obtained as

$$\dot{x} = (A - BK^*)x + BL^*r \tag{5.4}$$

Hence if $K^*$and L^* are chosen to satisfy the following equations

$$(A - BK^*) = A_m \tag{5.5}$$

$$BL^* = B_m \tag{5.6}$$

provided that $B \neq 0$, i.e., the plant (5.1) is controllable.

Then the transfer matrix of the closed-loop plant is the same as that of the reference model and plant state follows the reference model state. That is $x(t) \to x_m(t)$ exponentially fast for any bounded reference input signal $r(t)$. If the matrices A, B, A_m and B_m are known completely then K^* *and* L^* may exist and can be calculated to satisfy the matching conditions in equations (5.5) and (5.6). Hence the control law given in equation (5.3) may have enough structural flexibility to meet the control objective.

But if the plant parameters are not known, that is the matrices A and B are not known then control law given in equation (5.3) cannot be implemented. Then we propose the control law

$$u = -K(t)x + L(t)r \tag{5.7}$$

Where K, L are the estimates of K^*, L^* respectively that can be generated by an appropriate adaptive law.

5.4 ADAPTIVE LAW

By adding and subtracting the desired input term, namely, $-B(-k^*x + l^*r)$ in the plant equation (5.1), we obtain

$$\dot{x} = (A - BK^*)x + BL^*r + Bu - Bu \tag{5.8}$$

Using the matching conditions and from equation (5.3), we get

$$\dot{x} = A_m x + B_m r + Bu - B(-K^*x + L^*r)$$

$$\dot{x} = A_m x + B_m r + B(u + K^*x - L^*r) \tag{5.9}$$

Now we define the parameter estimation error as

$$\tilde{K} \approx K - K^* \qquad (5.10)$$

$$\tilde{L} \approx L - L^* \qquad (5.11)$$

Tracking error is

$$e(t) = x(t) - x_m(t) \qquad (5.12)$$

Its derivative will be

$$\dot{e} = \dot{x}(t) - \dot{x}_m(t)$$

From equation (5.2) and (5.9), we get

$$\dot{e} = A_m x + B_m r + B(u + K^* x - L^* r) - A_m x_m - B_m r \qquad (5.13)$$

$$\dot{e} = A_m(x - x_m) + B(u + K^* x - L^* r)$$

As $e = x - x_m$, So

$$\dot{e} = A_m e + Bu + BK^* x - BL^* r \qquad (5.14)$$

Putting the value of K^* and L^* from equations (5.10) and (5.11)

$$\dot{e} = A_m e + Bu + B(-\tilde{K} + K)x - B(-\tilde{L} + L)r \qquad (5.15)$$

$$\dot{e} = A_m e + Bu + B[-\tilde{K}x + \tilde{L}r] - B(-Kx + Lr)$$

From the purposed control law from equation (5.7)

$$\dot{e} = A_m e + Bu + B[-\tilde{K}x + \tilde{L}r] - Bu$$

Hence

$$\dot{e} = A_m e + B[-\tilde{K}x + \tilde{L}r] \qquad (5.16)$$

As above equation depends on the unknown matrix B, we manage to get away with the unknown B by assuming that its sign is known [34].

Hence

$$\dot{e} = A_m e + B_m L^{*-1}[-\tilde{K}x + \tilde{L}r] \qquad (5.18)$$

Now we propose the following Lyapunov function

$$V(e, \tilde{K}, \tilde{L}) = e^T P e + \frac{1}{|L^*|}[\tilde{K}\tilde{K}^T + \tilde{L}^2] \qquad (5.19)$$

Where $P = P^T > 0$

Satisfies the lyapunov equation

$$PA_m + A_m^T P = -Q \qquad (5.20)$$

For some $Q = Q^T > 0$

Now

$$\dot{V} = 2e^T P \dot{e} + \frac{2}{|L^*|}[\tilde{K}\dot{\tilde{K}}^T + \tilde{L}\dot{\tilde{L}}] \qquad (5.21)$$

$$\dot{V} = 2e^T P[A_m e + B_m L^{*-1}(-\tilde{K}x + \tilde{L}r)] + \frac{2}{|L^*|}\tilde{k}\,\widetilde{k^T} + \frac{2}{|L^*|}\tilde{L}\,\tilde{L}]$$

$$\dot{V} = 2e^T P A_m e + 2e^T P B_m L^{*-1}(-\tilde{K}x + \tilde{L}r)] + \frac{2}{|L^*|}\tilde{k}\,\widetilde{k^T} + \frac{2}{|L^*|}\tilde{L}\,\tilde{L}]$$

$$\dot{V} = -e^T Q e + 2e^T P B_m L^{*-1}(-\tilde{K}x + \tilde{L}r)] + \frac{2}{|L^*|}\tilde{k}\,\widetilde{k^T} + \frac{2}{|L^*|}\tilde{L}\,\tilde{L}]$$

To make $\dot{V} \leq 0$, We choose

$$-2e^T P B_m L^{*-1}\tilde{K}x + \frac{2}{|L^*|}\tilde{k}\,\widetilde{k^T} = 0 \tag{5.22}$$

and

$$2e^T P B_m L^{*-1}\tilde{L}r + \frac{2}{|L^*|}\tilde{L}\,\tilde{L}] = 0 \tag{5.23}$$

Solving equation (5.22), Adaptive law will be

$$2e^T P B_m L^{*-1}\tilde{K}x = \frac{2}{|L^*|}\tilde{k}\,\widetilde{k^T} \tag{5.24}$$

Hence

$$\tilde{K} = \dot{K} = x^T B_m^T P e \; sign(L^*) \tag{5.25}$$

Similarly solving equation (5.23), we get

$$\tilde{L} = \dot{L} = -e^T B_m P r \; sign(L^*) \tag{5.26}$$

Hence we have

$$\dot{V} = -e^T Q e \tag{5.27}$$

The matrix $B_m P$ acts as an adaptive gain matrix, where P is obtained by solving the Lyapunov equation

$$PA_m + A_m^T P = -Q$$

for some arbitrary $Q = Q^T >$

5.5 SIMULATION RESULTS OF MRAC WITH LINEAR COUPLED-TANK PLANT MODEL

In this section, the simulation results of the Model Reference Adaptive Controller (MRAC) for linear coupled-tank plant model are discussed. The plant is in the form of transfer function given in equation (3.22). The controller objective is to control the plant such that the plant model follows the output trajectory of ideal reference model.

The controller tracking is observed by tracking the various set points. The reference signals taken are; step, random, square, saw tooth and sinusoidal. The controller tracking for step, random, square, saw tooth and sinusoidal wave like reference signals is shown in figure 5.2.

5.5, 5.6, 5.7 and 5.8. The control signal for tracking the step input and adaptive gains are shown in figure 5.3 and 5.4 respectively.

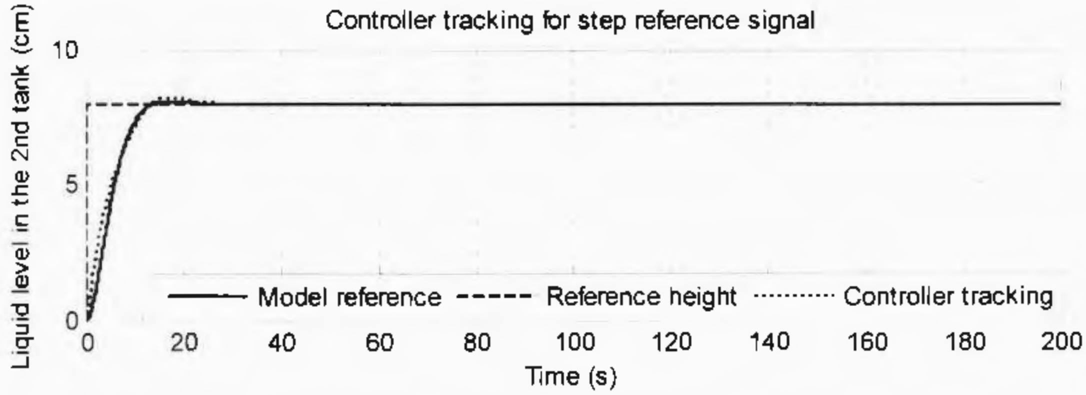

Figure 5.2: Controller tracking performance for step reference signal

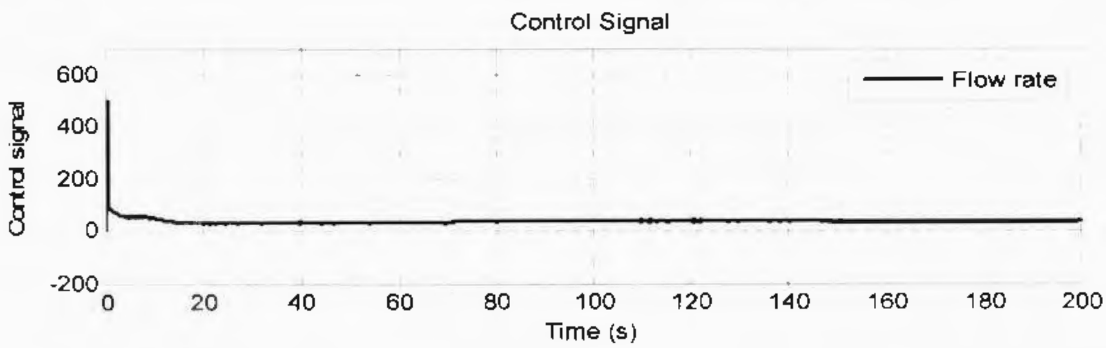

Figure 5.3: Control signal step reference signal

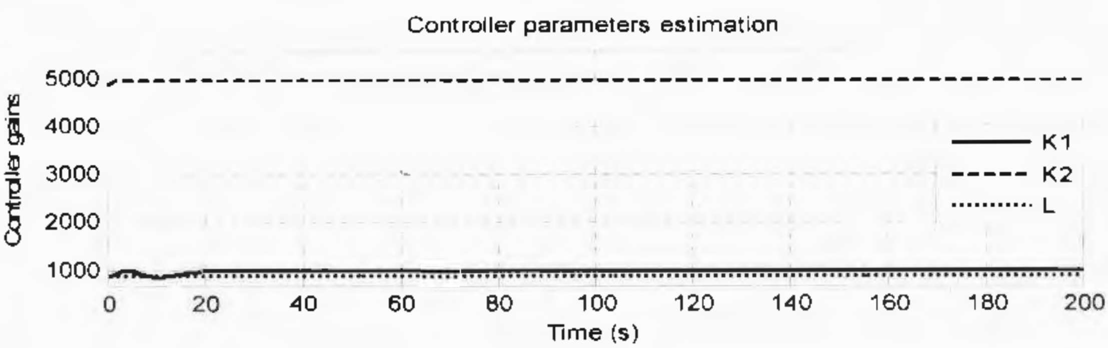

Figure 5.4: Adaptive Gains

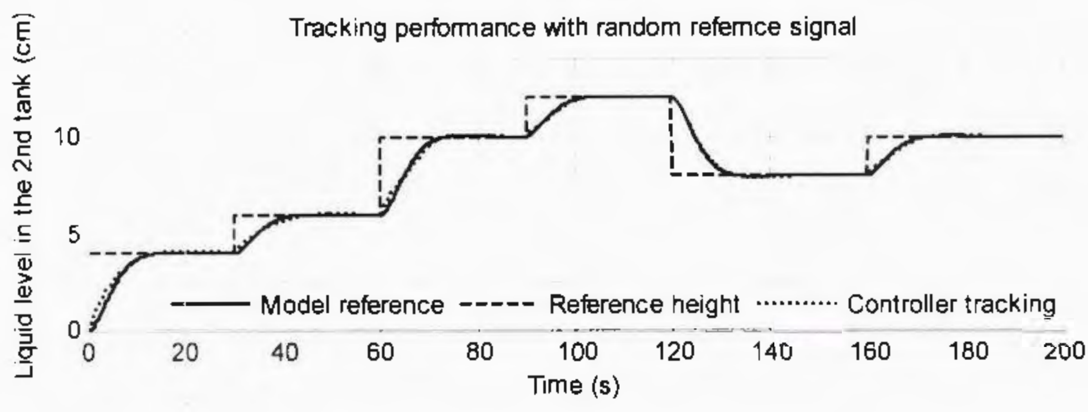

Figure 5.5: Controller tracking performance for random set points

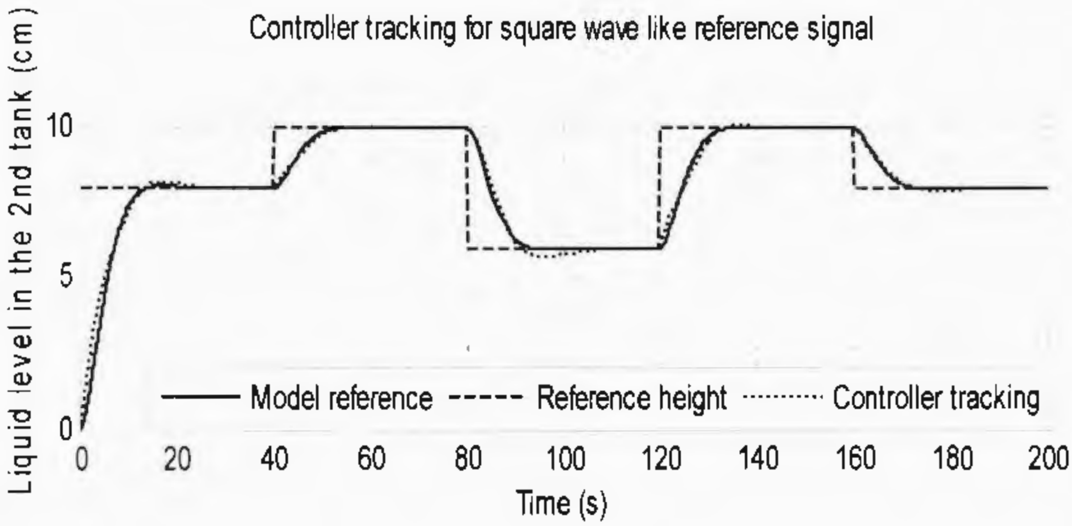

Figure 5.6: Controller tracking performance for square wave like reference signal

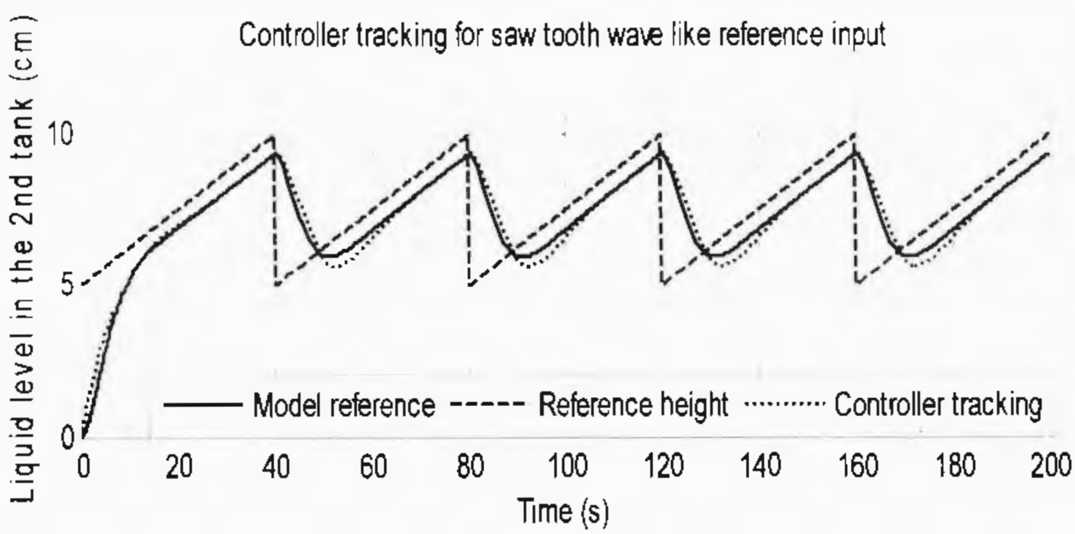

Figure 5.7: Controller tracking performance for saw tooth wave like reference signal

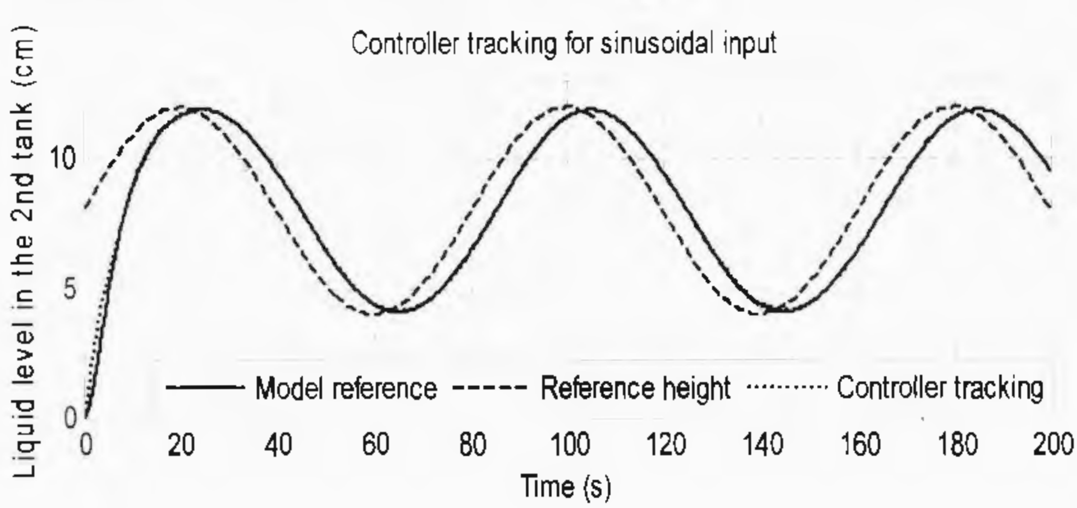

Figure 5.8: Controller tracking performance for sinusoidal wave like reference signal

5.6 SIMULATION RESULTS OF MRAC WITH NON-LINEAR COUPLED TANK PLANT MODEL

In this section, the simulation results of the Model Reference Adaptive Controller (MRAC) for non-linear coupled tank plant model are discussed.

5.6.1 STEP REFERENCE SIGNAL

The controller tracking performance for step reference signal is observed and is shown in figure 5.9. The output of the plant is the height of liquid in the 2nd tank. Hence the reference signal is the height in the 2nd tank. The step signal is taken as 8cm which is the reference height in the 2nd tank. The controller showed a very good performance for tracking the command reference signal.

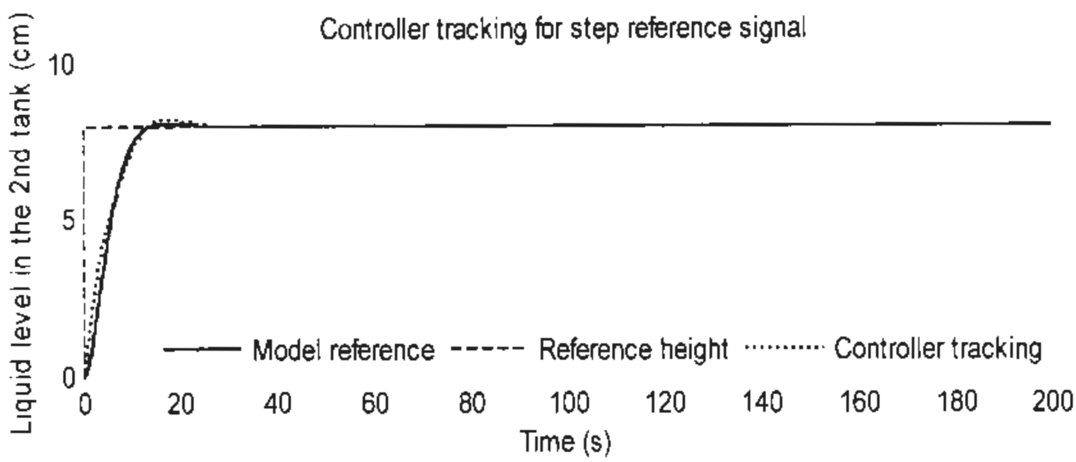

Figure 5.9: Controller tracking performance for step input

5.6.2 INFLOW RATE TO THE 1ST TANK

In order to maintain the liquid level in the 2nd tank at a height of 8cm, the inflow rate to the 1st tank is shown in figure 5.10.

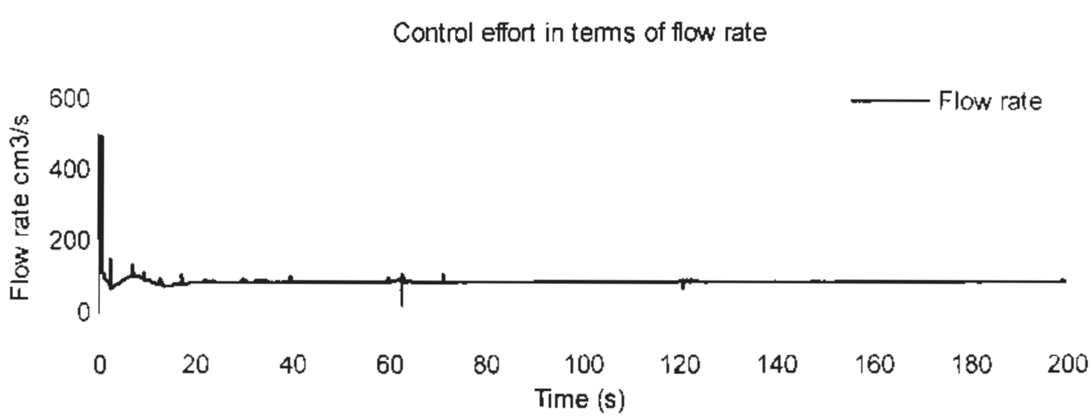

Figure 5.10: Control signal

5.6.3 TRACKING ERROR

The tracking error of controller for tracking the 8cm height in the 2[nd] tank is shown in figure 5.11.

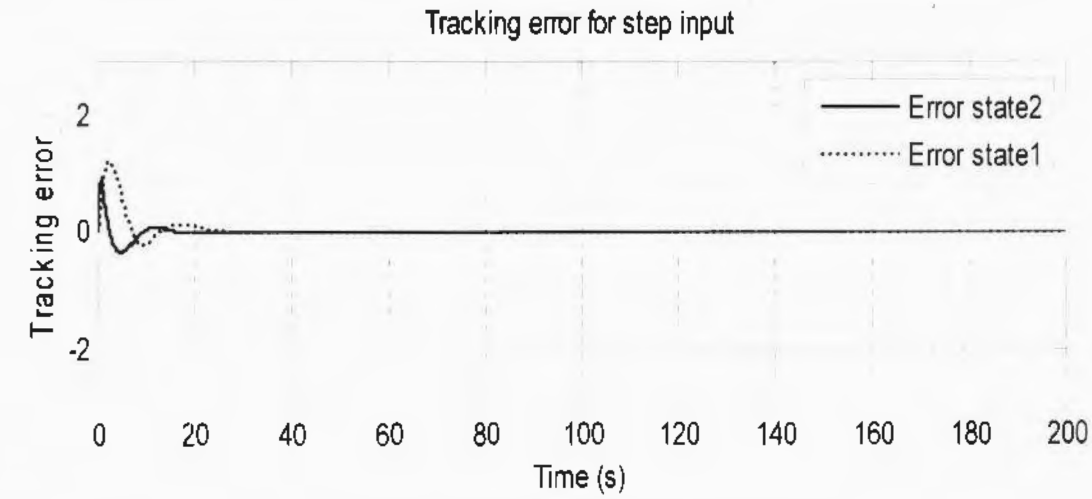

Figure 5.11: Controller tracking error

5.6.4 ADAPTIVE GAINS

In figure 5.12, the adaptive gains of the Model Refernce Adaptive controller are shown.

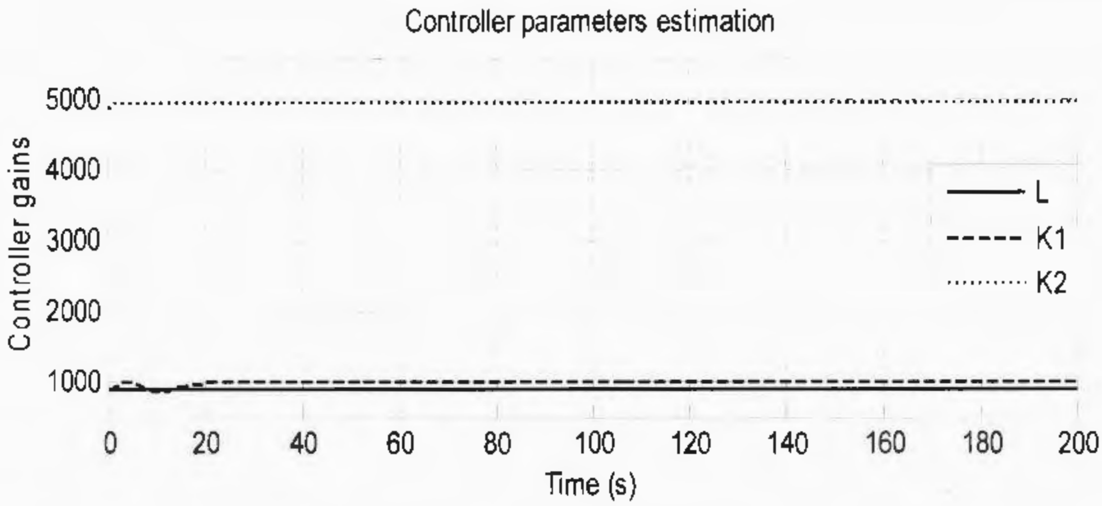

Figure 5.12: Adaptive gains

5.7 RANDOM SET POINTS TRACKING PERFORMANCE

Now the performance of the designed controller is tested for the random reference set points. The tracking performance of controller to track the random reference set points is shown in figure 5.13. The flow rate to the 1[st] tank and adaptive gains are shown in figure 5.14 and 5.15 respectivily.

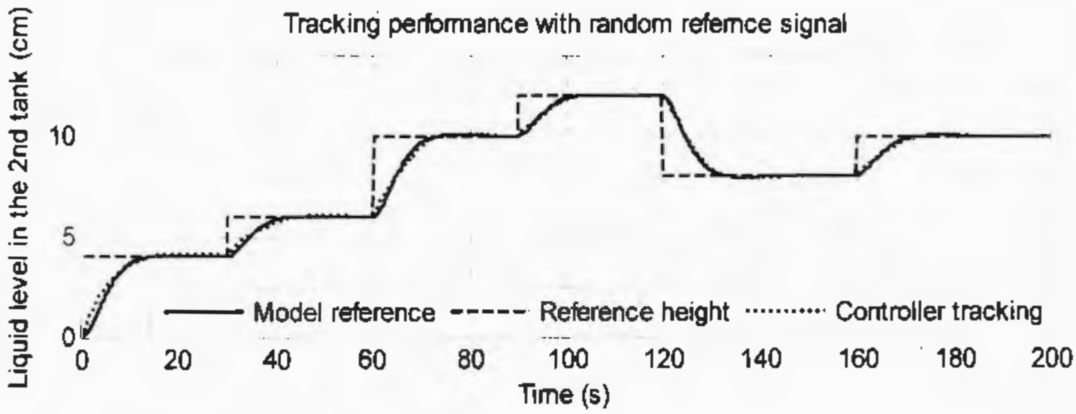

Figure 5.13: Controller tracking performance for random set points

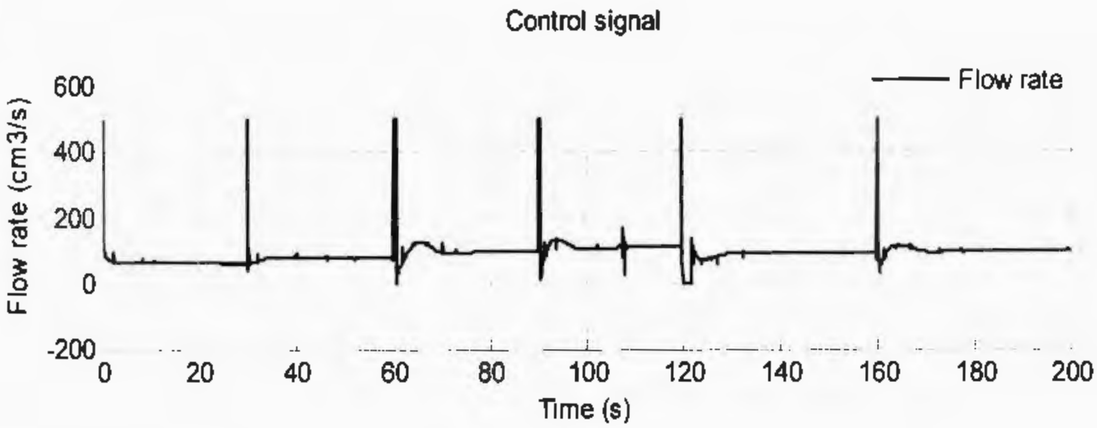

Figure 5.14: Control signal for random set points

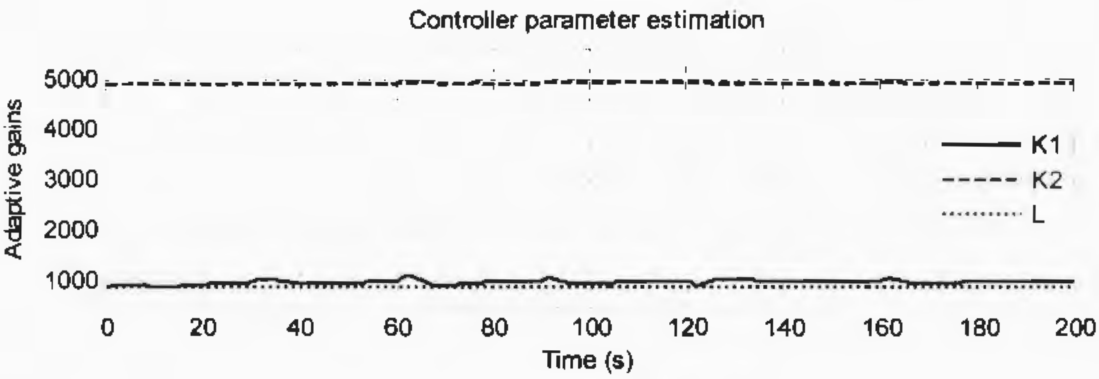

Figure 5.15: Adaptive gains for random set points

5.8 SQUARE WAVE TRACKING PERFORMANCE

In this section, the tracking performance of Model reference adaptive controller for tracking the square wave like reference signal is shown in figure 5.16. The flow rate to the 1st tank and its adaptive gains are shown in figure 5.17 and 5.18 respectively.

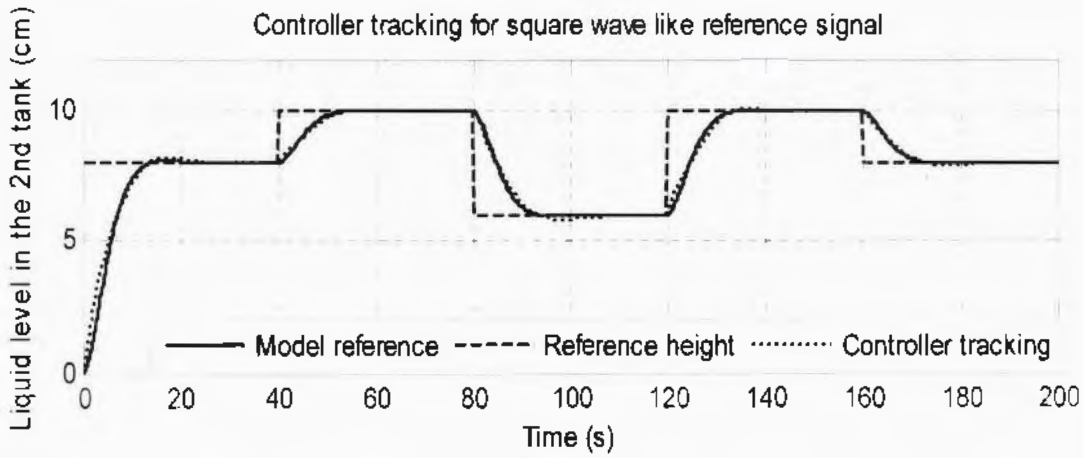

Figure 5.16: Controller tracking performance for square wave like reference signal

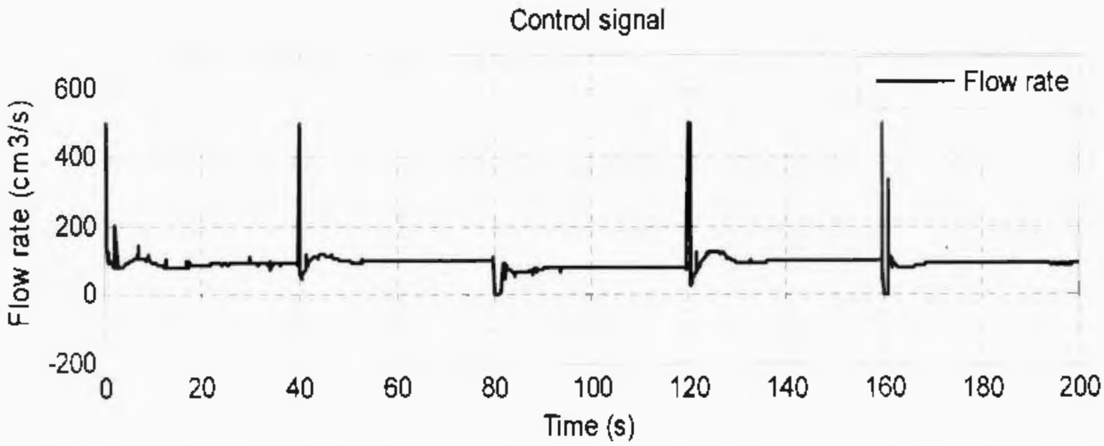

Figure 5.17: Control signal for square wave like reference signal

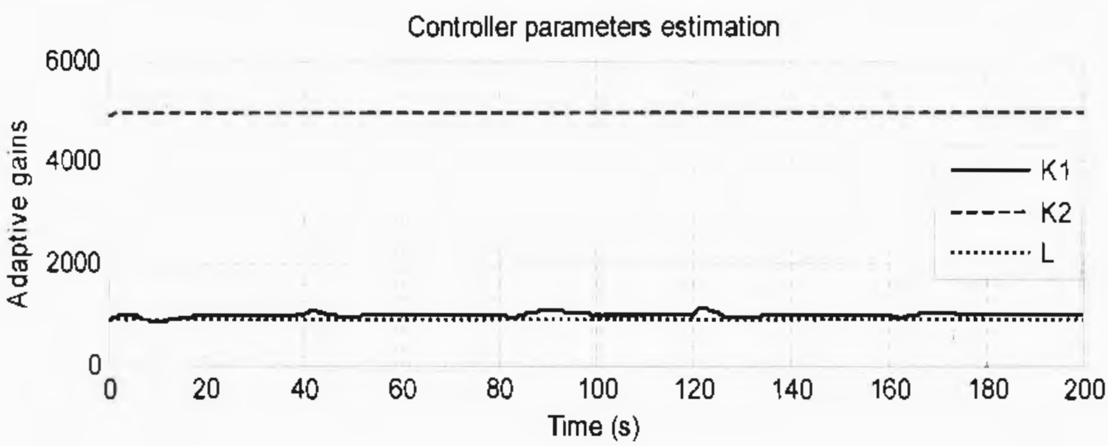

Figure 5.18: Adaptive gains for square wave like reference signal

5.9 SAW TOOTH WAVE TRACKING PERFORMANCE

In this section, the tracking performance of Model reference adaptive controller for tracking the saw tooth wave like reference signal is shown in figure 5.19. The flow rate to the 1st tank and adaptive gains are shown in figure 5.20 and 5.21 respectively.

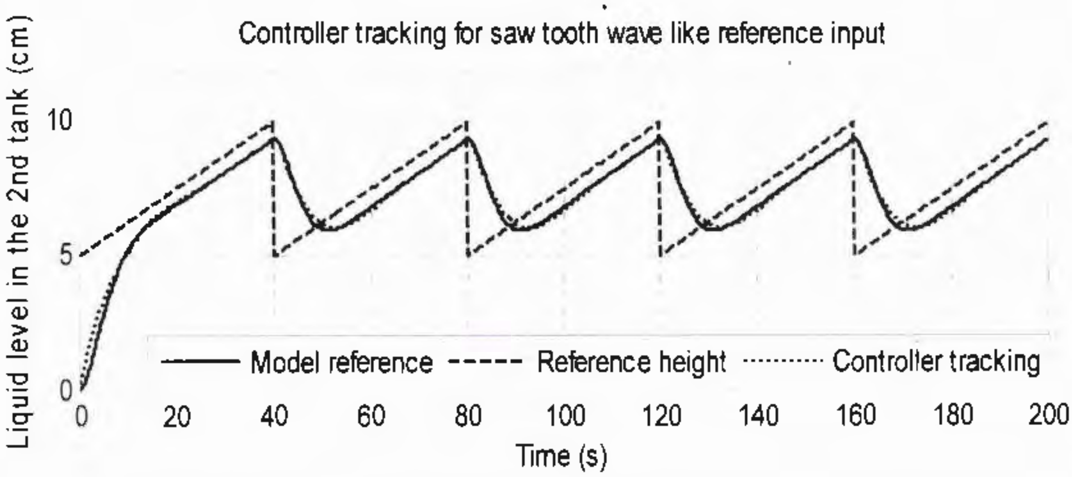

Figure 5.19: Controller tracking performance for sawtooth wave reference signal

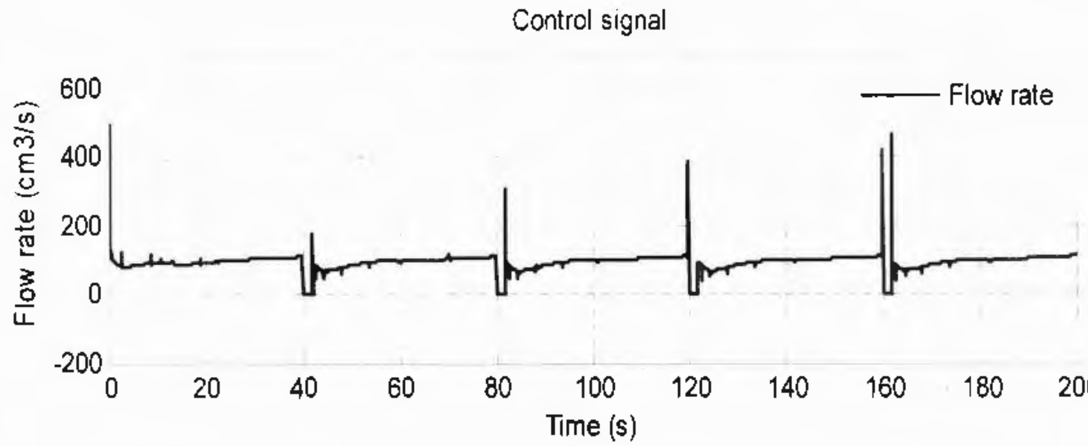

Figure 5.20: Control signal for saw tooth wave like reference signal

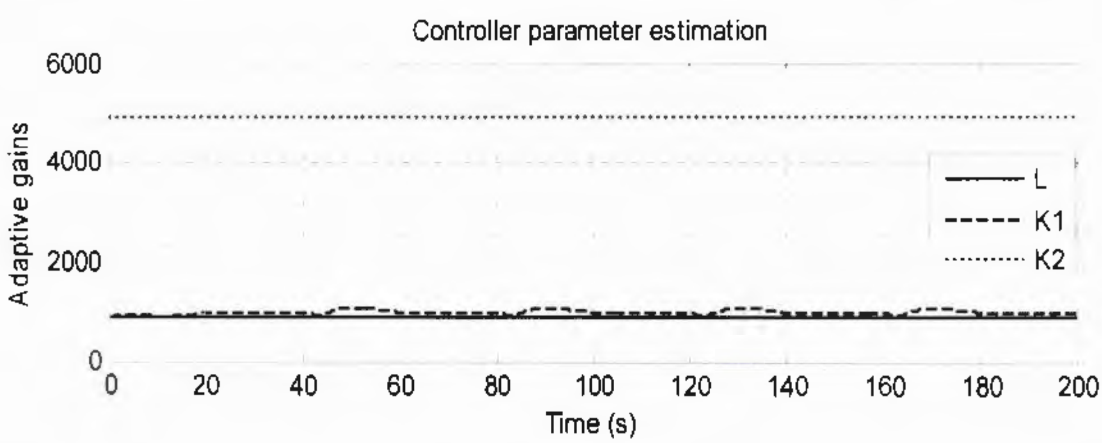

Figure 5.21: Adaptive gains for square wave like reference signal

5.10 SINUSOIDAL WAVE TRACKING PERFORMANCE

In this section, the tracking performance of Model reference adaptive controller for tracking the sinusoidal wave like reference signal is shown in figure 5.22. The flow rate to the 1st tank and adaptive gains are shown in figure 5.23 and 5.24 respectively.

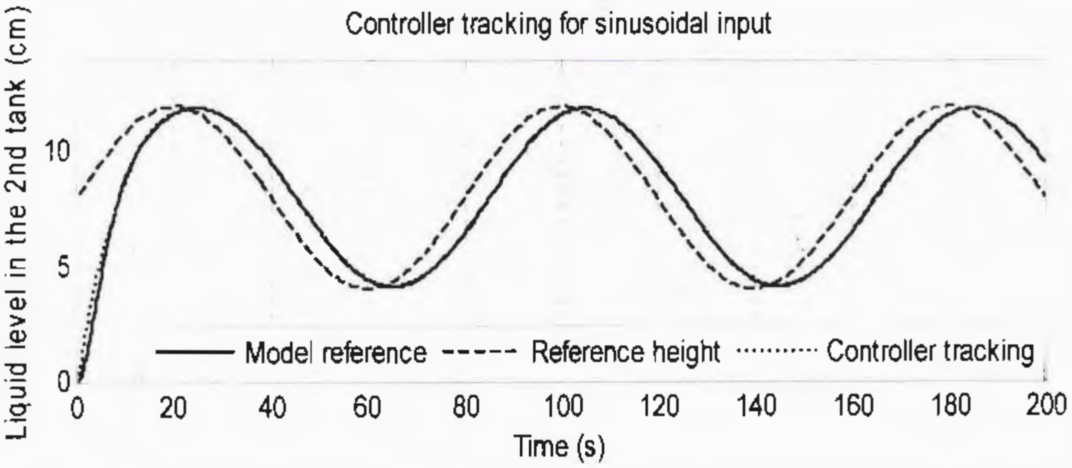

Figure 5.22: Controller tracking performance for sinusoidal wave reference signal

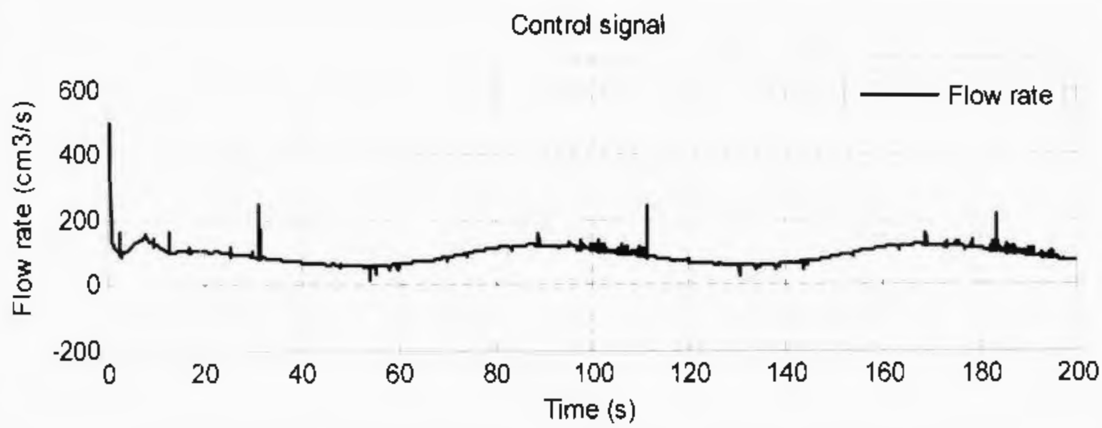

Figure 5.23: Control signal for sinusoidal wave like reference signal

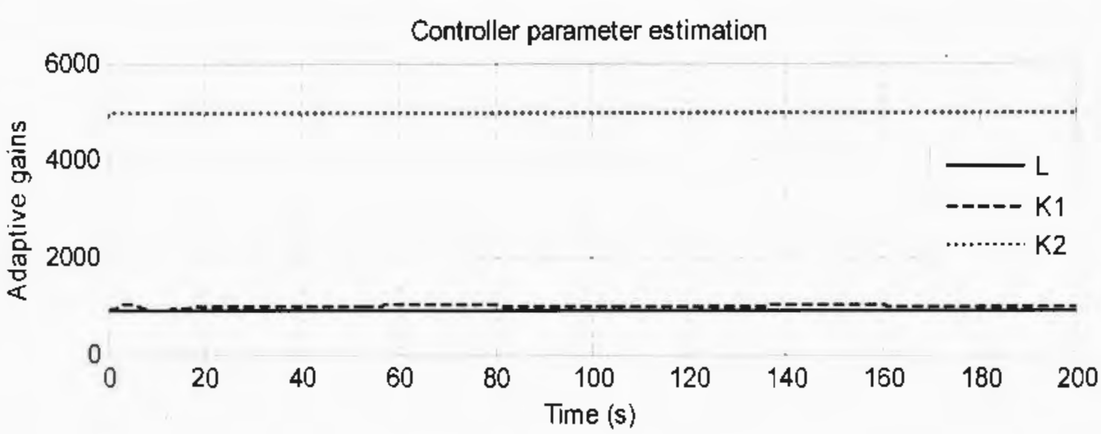

Figure 5.24: Adaptive gains

5.11 DISTURBANCE REJECTION PERFORMANCE OF CONTROLLER WHEN THERE IS AN INPUT TO THE 2ND TANK

In this section, the performance of the controller is observed when the pump 2 is ON for some time. The pump 2 supplies 20cm^3/s liquid into the 2nd tank for 40 seconds (40 to 80 sec of total simulation time of 200 sec). This extra input will be considered as disturbance. The disturbance rejection performance of the controller is shown below in figure 5.25. Also the flow rate is shown in figure 5.26.

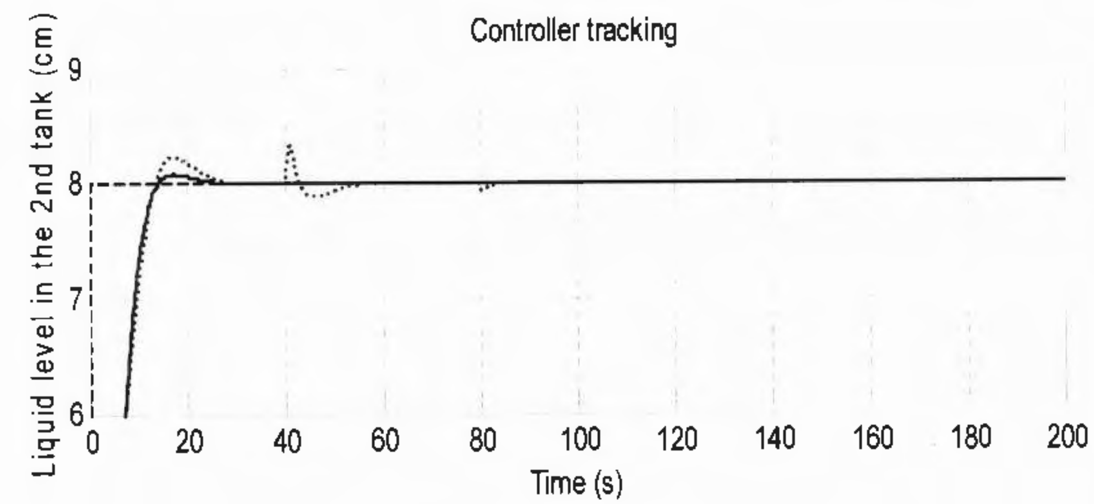

Figure 5.25: Disturbance rejection performance

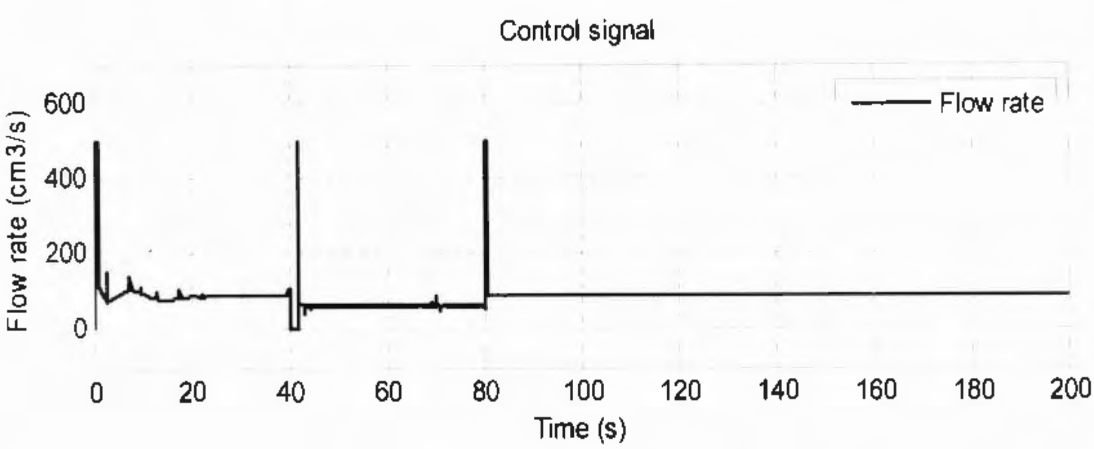

Figure 5.26: Control signal

The Model reference adaptive controller showed very performance under disturbance. When the extra input is added at 40 seconds time, the level of the water rises but the controller recovered that error in almost 15 seconds by decreasing the flow rate into the 1st tank. Similarly, when the flow rate is switched off at 80 seconds time, the level of the liquid goes on decreasing and the controller recovered that error within 7 seconds by increasing the flow rate into the 1st tank.

5.12 TRACKING PERFORMANCE WHEN THERE IS LEAKAGE IN THE 1ST TANK

Now the controller performance is tested when there is a leakage of $50cm^3/s$ in the 1st tank. The leakage of liquid is introduced in the 1st tank when the simulation time was 60 seconds.

The performance of controller is shown in figure 5.27. The controller showed satisfactory tracking results for maintaining the liquid level at desired height.

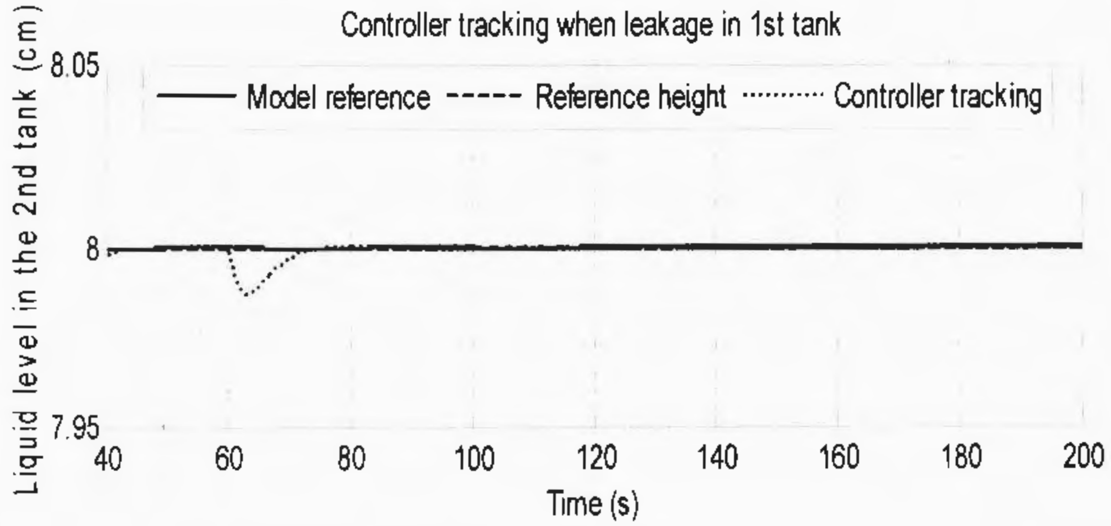

Figure 5.27: Tracking performance with leakage in the 1st tank

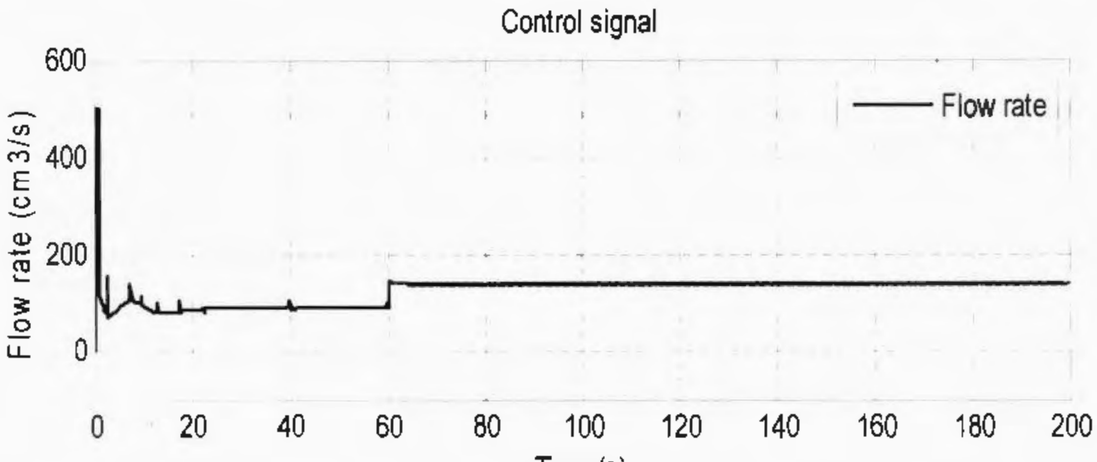

Figure 5.28: Control signal

5.13 TRACKING PERFORMANCE WITH LEAKAGE IN THE 2ND TANK

Now the controller is tested for another uncertain happening, the leakage in the 2nd tank. When the simulation time was 60 seconds, the 20 cm^3/s extra liquid outflow or leakage from 2nd tank is introduced. Because of the leakage or extra liquid outflow from 2nd tank, the level of the liquid in the 2nd tank decreases and hence the controller adjusted the inflow rate to the 1st tank in order to maintain the liquid level at desired position. In equation (3.20), the term U_{j2} is representing the leakage in the 2nd tank. Again the controller showed very good performance under the condition of leakage or extra outflow from 2nd tank. The performance of controller is shown in figure 5.29.

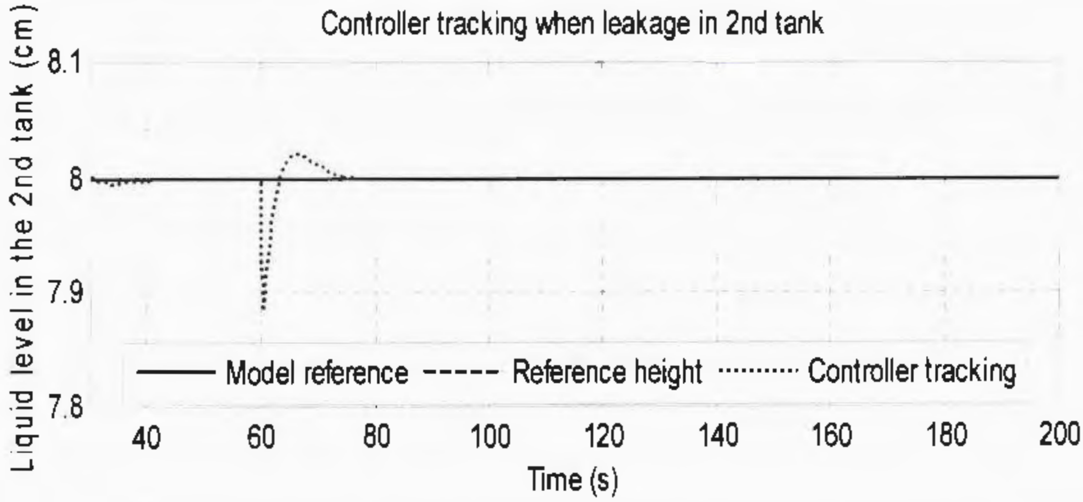

Figure 5.29: Controller performance with leakage in the 2nd tank

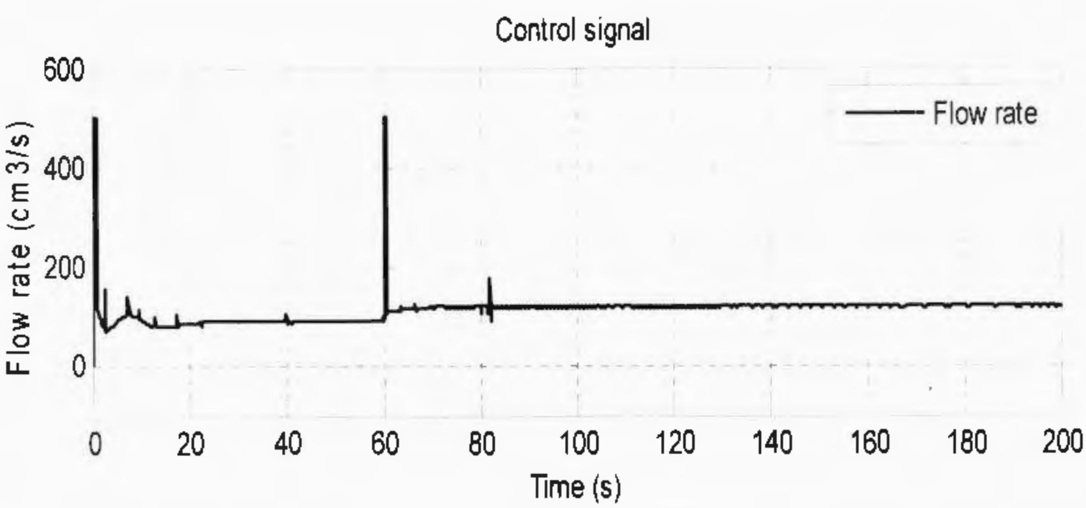

Figure 5.30: Control signal

5.14 SUMMARY

In this chapter, the Model reference adaptive controller is designed and implemented on linear and non-linear model of coupled-tank system. The controller showed very good tracking results for both the linear and non-linear plants. In the presence of disturbances and for time varying reference inputs, the controller exhibit adaptive behavior. The simulation results are briefly discussed and it is made the conclusion that the Model reference adaptive control is a very suitable control technique for coupled-tank system.

CHAPTER NO: 6

MODEL REFERENCE ADAPTIVE SLIDING MODE CONTROL

6.1 INTRODUCTION

In this chapter, the Model Reference Adaptive Sliding Mode Controller is designed for coupled-tank liquid level control system with a proportional and integral (PI) sliding surface. This controller is the combination of Model Reference Adaptive control and Sliding mode control. The designing of this controller for coupled-tank liquid level control system is the novelty of this thesis.

6.2 CONTROLLER DESIGN

Consider the single input single output (SISO) plant in state space form as,

$$\dot{x}(t) = Ax(t) + Bu \tag{6.1}$$

$$y = Cx(t)$$

Where $x(t) \in R^n$ is state vector and $u(t) \in R$ is the control signal. Matrices A, B and C have appropriate dimensions. That is $A \in R^{n\times n}$ and $B \in R^n$ and $C \in R^{1\times n}$.

Similarly the Reference model can be written in state space form as,

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t) \tag{6.2}$$

$$y_m = C_m x_m(t)$$

Where $x_m(t) \in R^n$ is state vector and $r(t) \in R$ is the reference signal. Also $A_m \in R^{n\times n}$ and $B_m \in R^n$ and $C_m \in R^{1\times n}$.

We make the following assumptions:

All the eigen values of A_m are in the open left-half plane (LHP), and $r(t)$ is bounded and continuous.

There exists a constant vector Q_1^* and a non-zero constant scalar Q_2^* such that the following conditions are satisfied.

$$A + BQ_1^{*T} = A_m \tag{6.3}$$

$$BQ_2^* = B_m \tag{6.4}$$

These conditions are known as matching conditions. The sign of the parameter Q_2^* is known.

The control objective is to determine the plant input U(t) so that the plant state *x*(*t*) follow the Reference model state $x_m(t)$ as close as possible for any given reference input *r*(*t*) .

The tracking error and its derivative are

$$e(t) = x(t) - x_m(t)$$

$$\dot{e} = \dot{x}(t) - \dot{x}_m(t) \tag{6.5}$$

$$\dot{e} = Ax + Bu - A_m x_m - B_m r$$

$$\dot{e} = Ax + Bu - A_m x_m - B_m r - A_m x + A_m x$$

$$\dot{e} = A_m(x - x_m) + (A - A_m)x + Bu - B_m r$$

$$\dot{e} = A_m e + (A - A_m)x + Bu - B_m r \tag{6.6}$$

The proportional integral sliding surface is defined as,

$$S(t) = \lambda e - \int_0^t \lambda A_m e \, d\tau \tag{6.7}$$

Where λ is a constant matrix having appropriate dimension such that,$|\lambda B| > \gamma$, where γ is known positive parameter.

The derivative of the sliding surface is

$$\dot{S} = \lambda \dot{e} - \lambda A_m e \tag{6.8}$$

$$\dot{S} = \lambda[A_m e + (A - A_m)x + Bu - B_m r] - \lambda A_m e$$

$$\dot{S} = \lambda A_m e + \lambda(A - A_m)x + \lambda Bu - \lambda B_m r - \lambda A_m e$$

$$\dot{S} = \lambda(A - A_m)x + \lambda Bu - \lambda B_m r \tag{6.9}$$

In order to find the equivalent control U_{eq}. Setting $\dot{S} = 0$ in equation (6.9).

$$0 = \lambda(A - A_m)x + \lambda Bu - \lambda B_m r$$

$$\lambda Bu = -\lambda(A - A_m)x + \lambda B_m r$$

$$U_{eq} = (\lambda B)^{-1}\lambda[B_m r - (A - A_m)x]$$

$$U_{eq} = (\lambda B)^{-1}\lambda B_m r - (\lambda B)^{-1}\lambda(A - A_m)x$$

$$U_{eq} = -(\lambda B)^{-1}\lambda(A - A_m)x + (\lambda B)^{-1}\lambda B_m r$$

$$U_{eq} = Q_1^{*T}x(t) + Q_2^* r(t) \tag{6.11}$$

The state tracking Adaptive sliding mode controller structure is proposed as

$$U(t) = Q_1^T x(t) + Q_2 r(t) - \rho sign(S) \tag{6.12}$$

Where $Q_1^T$and Q_2 are estimates of Q_1^* and Q_2^* respectively.

6.3 ADAPTIVE LAW

The goal of control design is to choose adaptive laws to update controller parameters so that the control objective of Model reference adaptive control is achievable. The main functions of adaptive update laws for Q_1 and Q_2 are to ensure the stability of the closed-loop system.

The parametric errors are

$$\widetilde{Q_1}(t) = Q_1(t) - Q_1^* \quad (6.13)$$

$$\widetilde{Q_2}(t) = Q_2(t) - Q_2^* \quad (6.14)$$

Substituting equations (6.12), (6.13) and (6.14) in equation (6.1)

$$\dot{x}(t) = Ax(t) + B[Q_1^T(t)x(t) + Q_2 r(t) - \rho sign(S)]$$

$$\dot{x}(t) = Ax(t) + B[Q_1^T(t)x(t) + Q_2 r(t)] - B\rho sign(S)$$

$$\dot{x}(t) = Ax(t) + B[\{\widetilde{Q_1^T}(t) + Q_1^{*T}\}x(t) + \{\widetilde{Q_2}(t) + Q_2^*\}r(t)] - B\rho sign(S)$$

$$\dot{x}(t) = Ax(t) + BQ_1^{*T}x(t) + BQ_2^* r(t) + B\left[\widetilde{Q_1^T(t)}x(t) + \widetilde{Q_2}(t)r(t)\right] - B\rho sign(S)$$

Using the assumptions in equation (6.3) and (6.4)

$$\dot{x}(t) = Ax(t) + (A_m - A)x(t) + B_m r(t) + \frac{B_m}{Q_2^*}\left[\widetilde{Q_1^T(t)}x(t) + \widetilde{Q_2}(t)r(t)\right] - B\rho sign(S)$$

$$\dot{x}(t) = A_m x(t) + B_m r(t) + \frac{B_m}{Q_2^*}\left[\widetilde{Q_1^T(t)}x(t) + \widetilde{Q_2}(t)r(t)\right] - B\rho sign(S)$$

Put above expression in equation (6.5)

$$\dot{e} = A_m x(t) + B_m r(t) + \frac{B_m}{Q_2^*}\left[\widetilde{Q_1^T}(t)x(t) + \widetilde{Q_2}(t)r(t)\right] - B\rho sign(S) - A_m x_m(t) - B_m r(t)$$

$$\dot{e} = A_m[x(t) - x_m(t)] + \frac{B_m}{Q_2^*}\left[\widetilde{Q_1^T}(t)x(t) + \widetilde{Q_2}(t)r(t)\right] - B\rho sign(S)$$

$$\dot{e} = A_m e + \frac{B_m}{Q_2^*}\left[\widetilde{Q_1^T}(t)x(t) + \widetilde{Q_2}(t)r(t)\right] - B\rho sign(S) \quad (6.15)$$

Put equation (6.15) in equation (6.8)

$$\dot{S}(t) = \lambda A_m e + \lambda\frac{B_m}{Q_2^*}\left[\widetilde{Q_1^T}(t)x(t) + \widetilde{Q_2}(t)r(t)\right] - \lambda B\rho sign(S) - \lambda A_m e$$

$$\dot{S}(t) = \lambda\frac{B_m}{Q_2^*}\left[\widetilde{Q_1^T}(t)x(t) + \widetilde{Q_2}(t)r(t)\right] - \lambda B\rho sign(S) \quad (6.16)$$

Define a Lyapunov function

$$V = \frac{1}{2}S^2 + \frac{1}{2|Q_2^*|}\widetilde{Q_1^T}\tau^{-1}\widetilde{Q_1} + \frac{1}{2|Q_2^*|}\widetilde{Q_2^2}\gamma^{-1} \quad (6.17)$$

Differentiating with respect to time gives

$$\dot{V} = S\dot{S} + \frac{1}{|Q_2^*|}\widetilde{Q_1^T}\tau^{-1}\widetilde{\dot{Q_1}} + \frac{1}{|Q_2^*|}\widetilde{Q_2}\gamma^{-1}\widetilde{\dot{Q_2}}$$

$$\dot{V} = S\left[\lambda\frac{B_m}{Q_2^*}\{\widetilde{Q_1^T}(t)x(t) + \widetilde{Q_2}(t)r(t)\} - S\lambda B\rho sign(s)\right] + \frac{1}{|Q_2^*|}\widetilde{Q_1^T}\tau^{-1}\widetilde{\dot{Q_1}} + \frac{1}{|Q_2^*|}\widetilde{Q_2}\gamma^{-1}\widetilde{\dot{Q_2}}$$

$$\dot{V} = -\lambda B\rho|S| + \left[S\,\lambda\frac{B_m}{Q_2^*}\widetilde{Q_1^T}(t)x(t) + \frac{1}{|Q_2^*|}\widetilde{Q_1^T}\tau^{-1}\widetilde{\dot{Q_1}}\right] + [S\,\lambda\frac{B_m}{Q_2^*}\widetilde{Q_2}(t)r(t) + \frac{1}{|Q_2^*|}\widetilde{Q_2}\gamma^{-1}\widetilde{\dot{Q_2}}] \quad (6.18)$$

To make $\dot{V} \leq 0$. We choose

$$\left[S\,\lambda\frac{B_m}{Q_2^*}\widetilde{Q_1^T}(t)x(t) + \frac{1}{|Q_2^*|}\widetilde{Q_1^T}\tau^{-1}\widetilde{\dot{Q_1}}\right] = 0 \quad (6.19)$$

And

$$\left[S\,\lambda\frac{B_m}{Q_2^*}\widetilde{Q_2}(t)r(t) + \frac{1}{|Q_2^*|}\widetilde{Q_2}\gamma^{-1}\widetilde{\dot{Q_2}}\right] = 0 \quad (6.20)$$

Solving (6.19), so the adaptive law would be

$$\frac{1}{|Q_2^*|}\widetilde{Q_1^T}\tau^{-1}\widetilde{\dot{Q_1}} = -S\,\lambda\frac{B_m}{Q_2^*}\widetilde{Q_1^T}(t)x(t)$$

$$\widetilde{Q_1^T}\tau^{-1}\widetilde{\dot{Q_1}} = -\frac{|Q_2^*|}{Q_2^*}\,S\,\lambda B_m\widetilde{Q_1^T}(t)x(t)$$

$$\widetilde{\dot{Q_1}} = -sign(Q_2^*)\tau S\lambda B_m x(t)$$

So

$$\dot{Q}_1 = -sign(Q_2^*)\tau S\lambda B_m x(t) \quad (6.21)$$

Similarly solving equation (6.20), we get the adaptive law as

$$\dot{Q}_2 = -sign(Q_2^*)\gamma S\lambda B_m r(t) \quad (6.22)$$

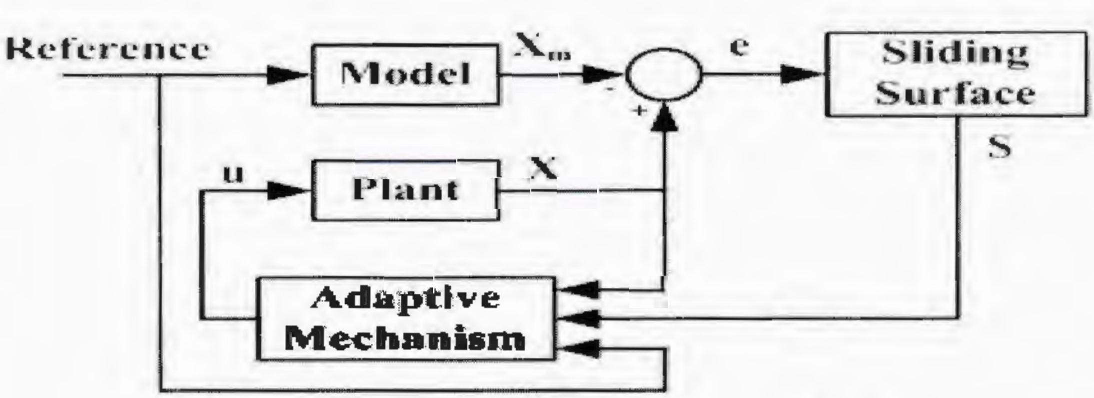

Figure 6.1a: Structure of Model reference Adaptive Sliding mode controller

6.4 SIMULATION RESULTS OF MODEL REFERENCE ADAPTIVE SLIDING MODE CONTROLLER WITH LINEAR COUPLED TANK PLANT MODEL

In this section, the simulation results of the Model Reference Adaptive Sliding Mode Controller for linear coupled tank plant model are discussed.

The controller performance is monitored by tracking the various reference set points. The reference signals are; step, random, square, saw tooth and sinusoidal. The controller tracking for step, random, square, saw tooth and sinusoidal wave like reference signals is shown in figure 6.1, 6.4, 6.5, 6.6 and 6.7 respectively. The control signal for tracking the step input and adaptive gains are shown in figure 6.2 and 6.3 respectively.

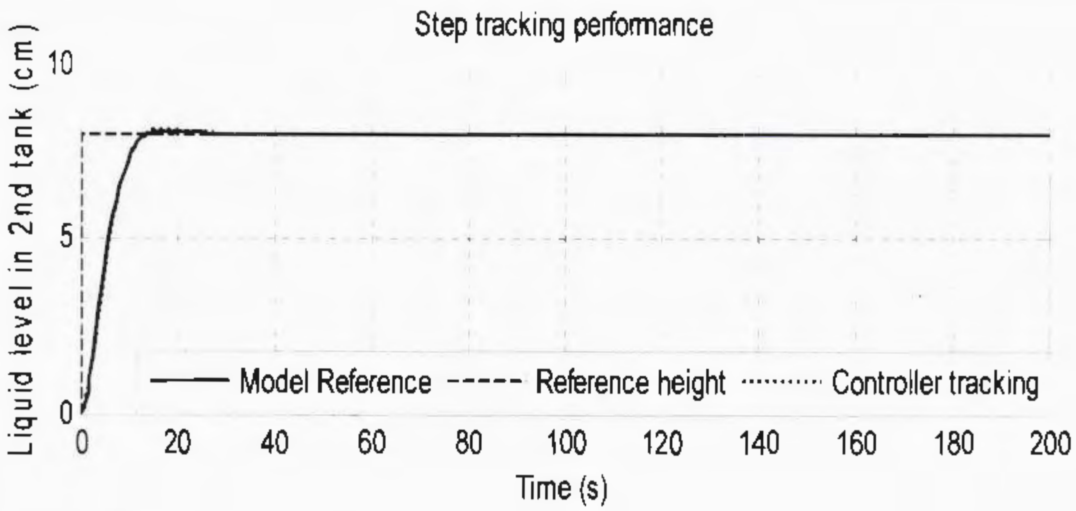

Figure 6.1: Controller tracking performance for step reference signal

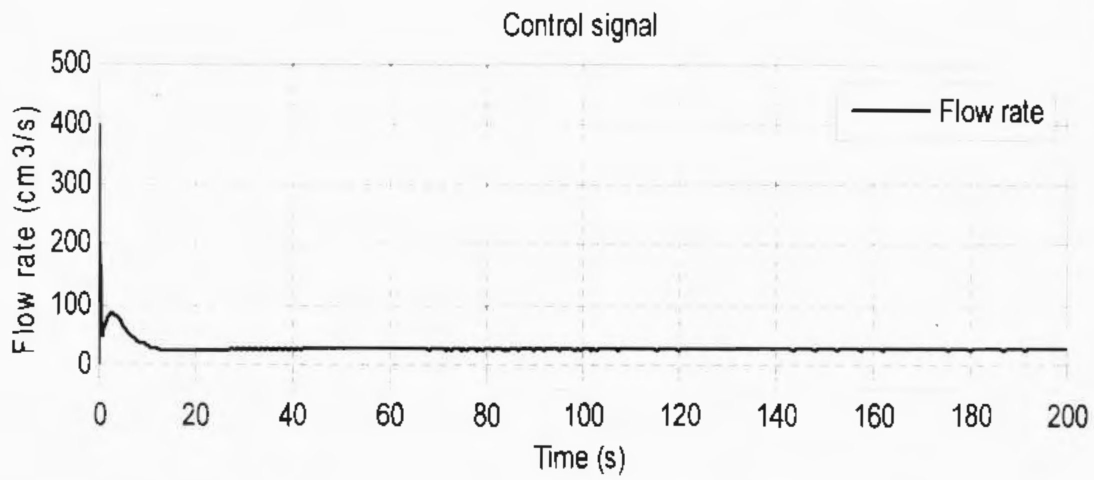

Figure 6.2: Control signal step reference signal

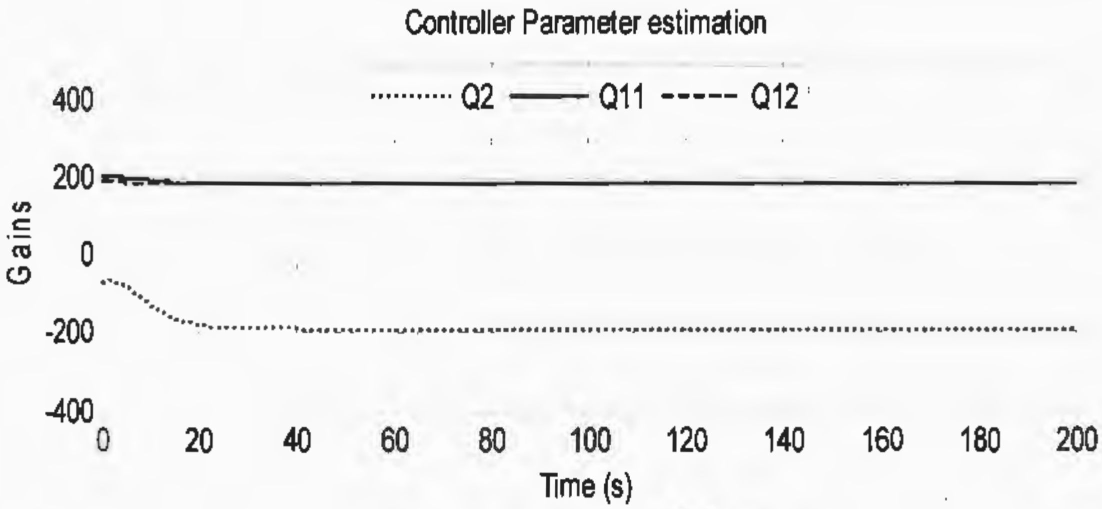

Figure 6.3: Adaptive Gains

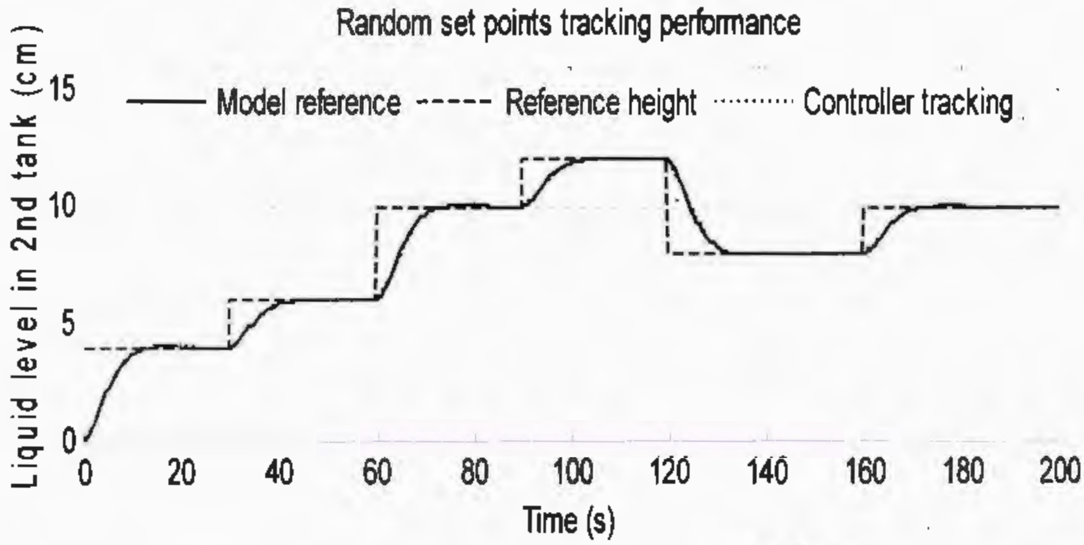

Figure 6.4: Controller tracking performance for random set points

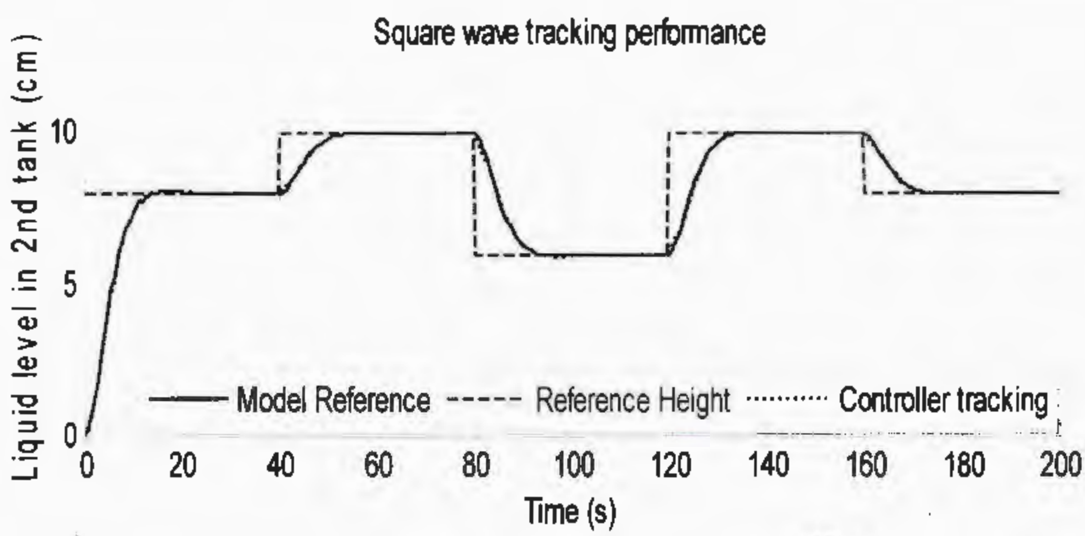

Figure 6.5: Controller tracking performance for square wave like reference signal

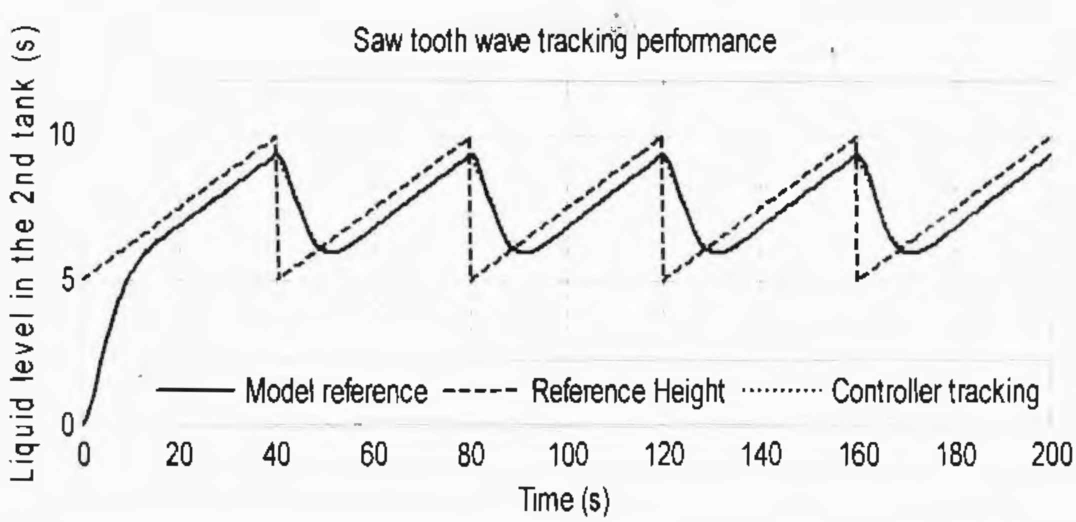

Figure 6.6: Controller tracking performance for saw tooth wave like reference signal

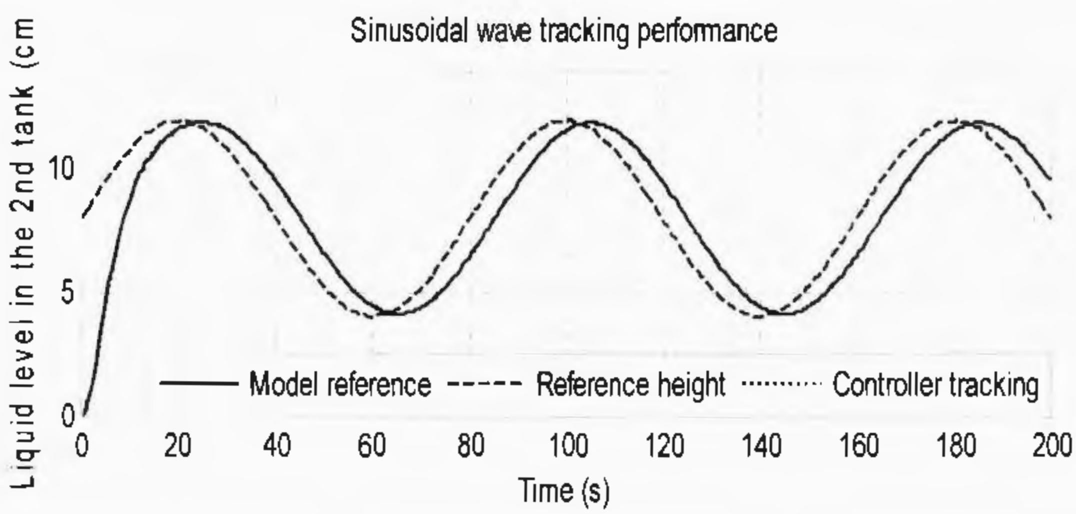

Figure 6.7: Controller tracking performance for sinusoidal wave like reference signal

6.5 SIMULATION RESULTS OF MRAC WITH NON-LINEAR COUPLED TANK PLANT MODEL

The simulation results of the Model reference adaptive sliding mode controller for the non-linear coupled tank plant model are discussed briefly in this section. The control objective is that the non-linear plant model follows the state trajectory of ideal reference model.

6.5.1 STEP REFERENCE SIGNAL

The controller tracking performance for step reference signal is observed and is shown in figure 6.8. As the output of the plant is the height in the 2nd tank so the reference signal is the height in the 2nd tank. The step signal is taken as 8cm which is the reference height in the 2nd tank. The controller showed very good performance for tracking the command reference signal.

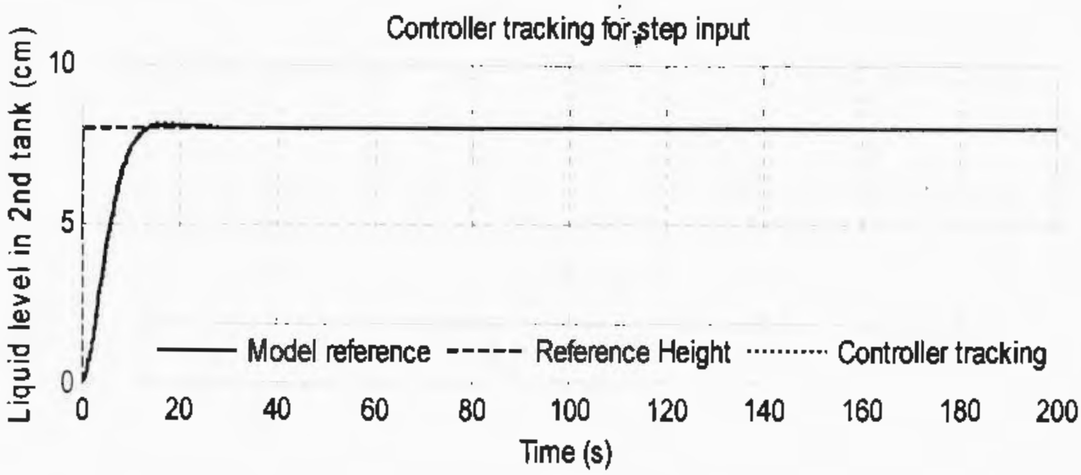

Figure 6.8: Controller tracking performance for step input

6.5.1.1 INFLOW RATE TO THE 1ST TANK

In order to maintain the liquid level in the 2nd tank at a height of 8cm, the inflow rate to the 1st tank is shown in figure 6.9.

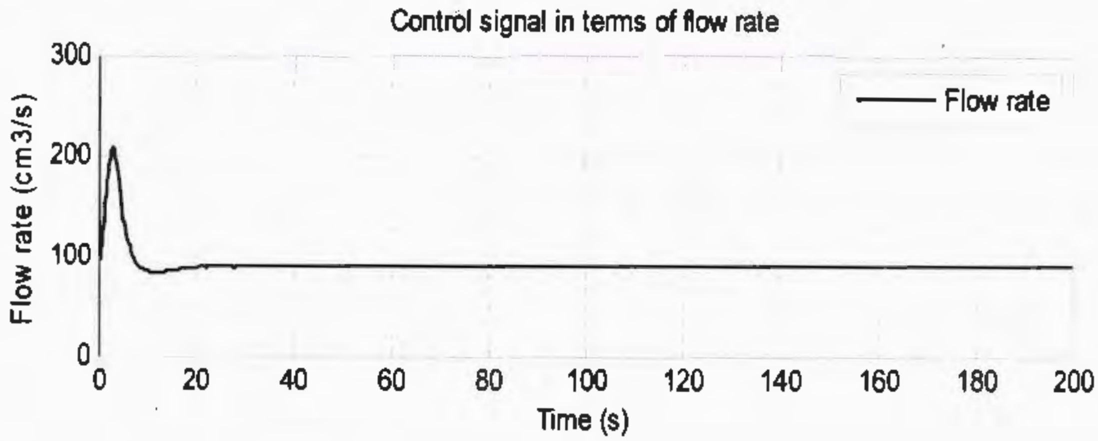

Figure 6.9: Control signal

6.5.1.2 TRACKING ERROR

The tracking error of controller for tracking the 8cm height in the 2nd tank is shown in figure 6.10.

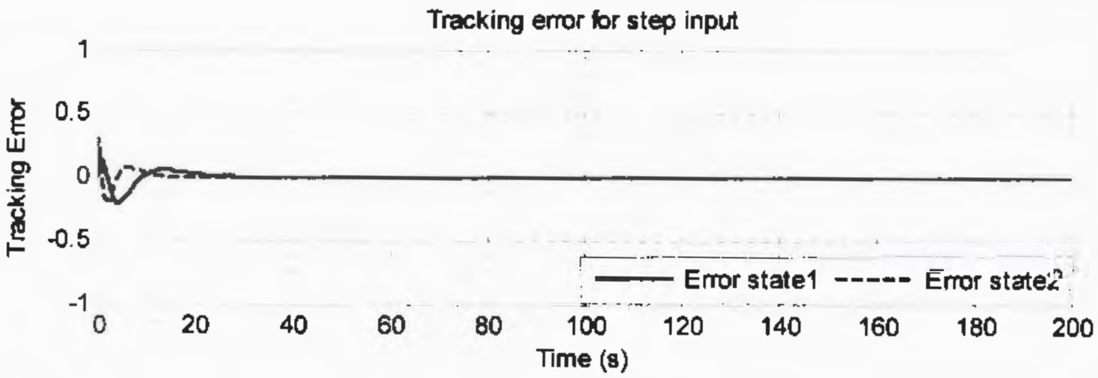

Figure 6.10: Controller tracking error

6.5.1.3 ADAPTIVE GAINS

In figure 6.11, the adaptive gains of the Model Refernce Adaptive Sliding Mode Controller for tracking the step input are shown below.

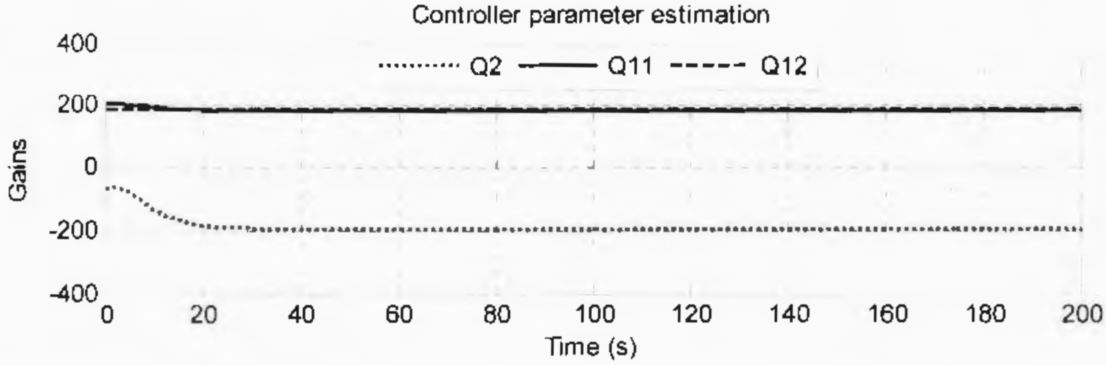

Figure 6.11: Adaptive gains

6.6 RANDOM SET POINTS TRACKING PERFORMANCE

Now the performance of the designed controller is tested for the random reference set points. The tracking performance of controller to track the random reference set points is shown in figure 6.12. The flow rate to the 1st tank is shown in figure 6.13. The tracking error and the adaptive gains are shown in figure 6.14 and 6.15 respectively.

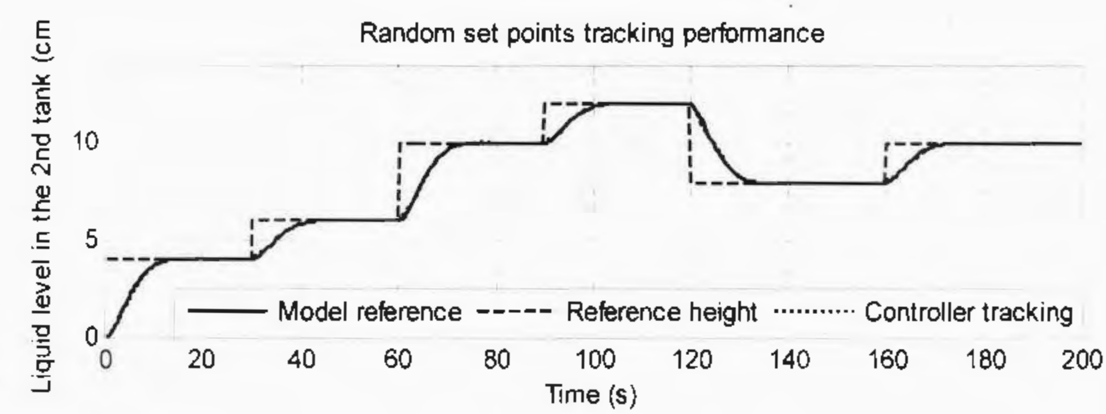

Figure 6.12: Controller tracking performance for random set points

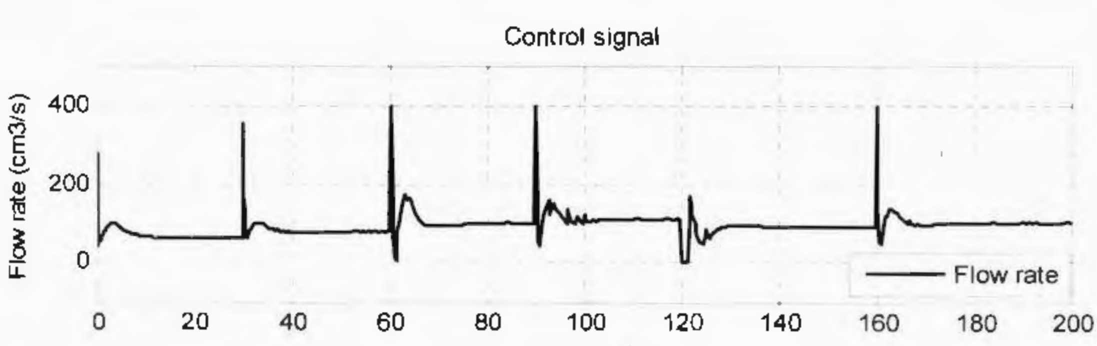

Figure 6.13: Control signal for random set points

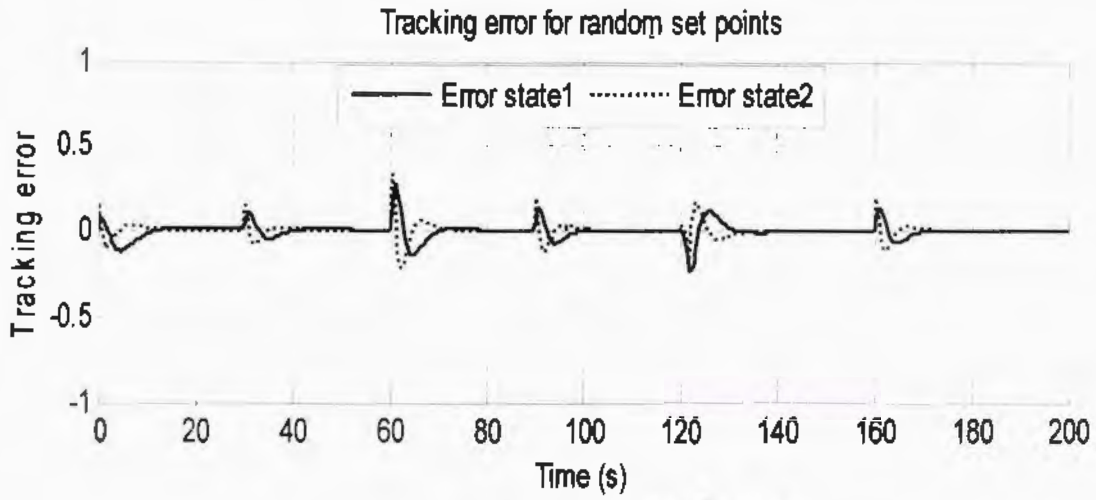

Figure 6.14: Tracking error for random set points

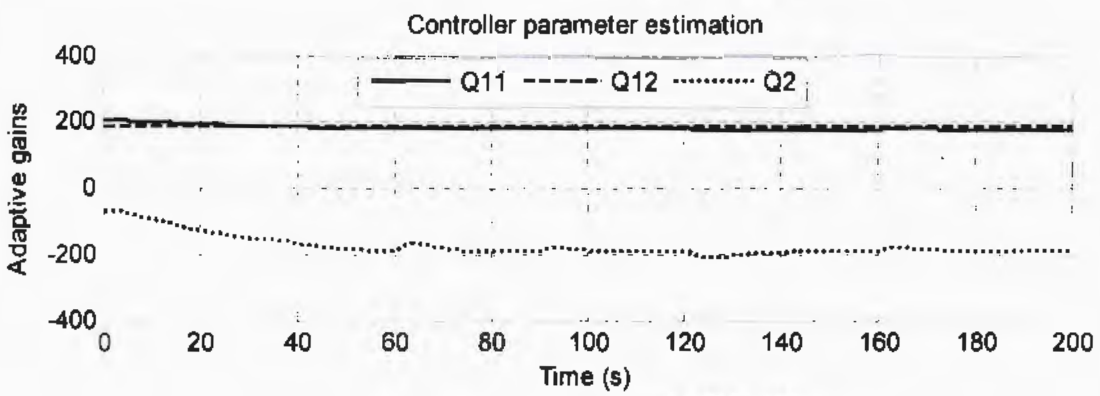

Figure 6.15: Adaptive gains for random set points

6.7 SQUARE WAVE TRACKING PERFORMANCE

In this section, the tracking performance of Model Reference Adaptive Sliding Mode controller for tracking the square wave like reference signal is shown in figure 6.16. The flow rate to the 1st tank and adaptive gains are shown in figure 6.17 and 6.18 respectively.

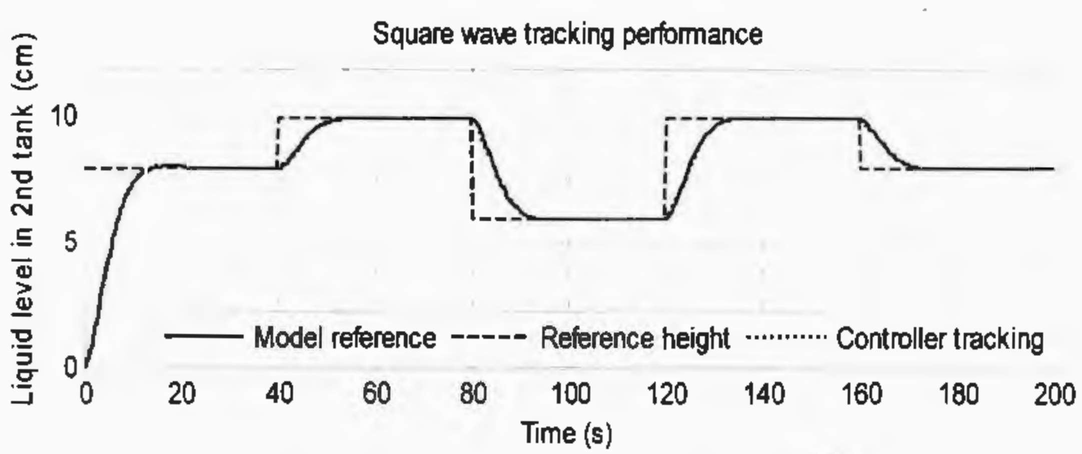

Figure 6.16: Controller tracking performance for square wave like reference signal

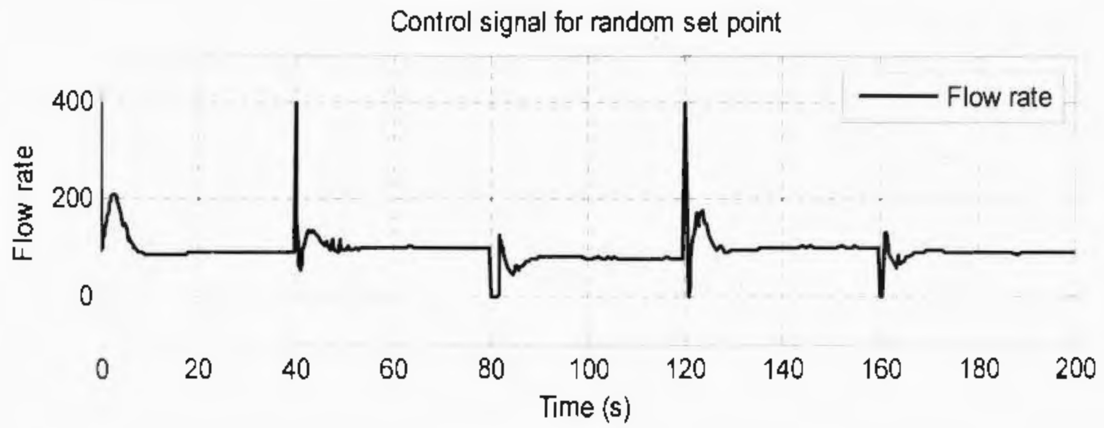

Figure 6.17: Control signal for square wave like reference signal

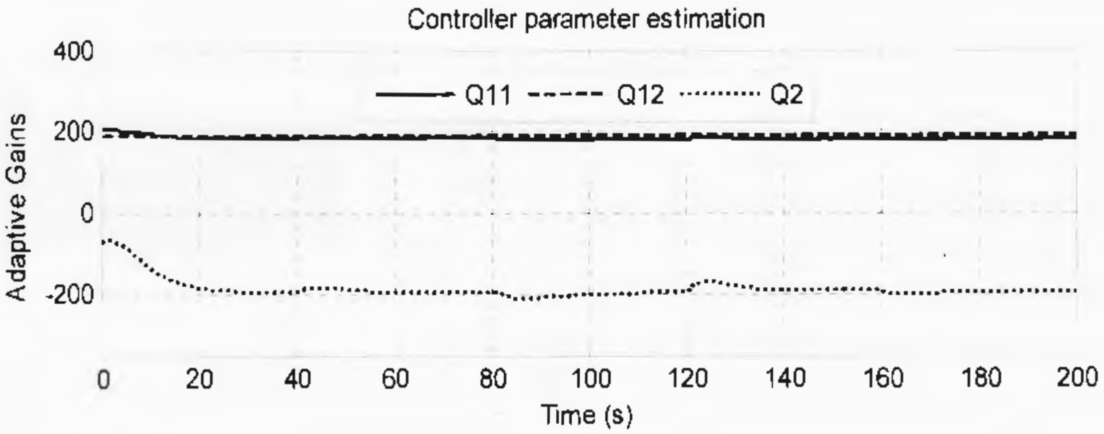

Figure 6.18: Adaptive gains for square wave like reference signal

6.8 SAW TOOTH WAVE TRACKING PERFORMANCE

In this section, the tracking performance of Model Reference Adaptive Sliding Mode controller for tracking the saw toth wave like reference signal is shown in figure 6.19. The flow rate to the 1[st] tank and adaptive gains are shown in figure 6.20 and 6.21 respectively

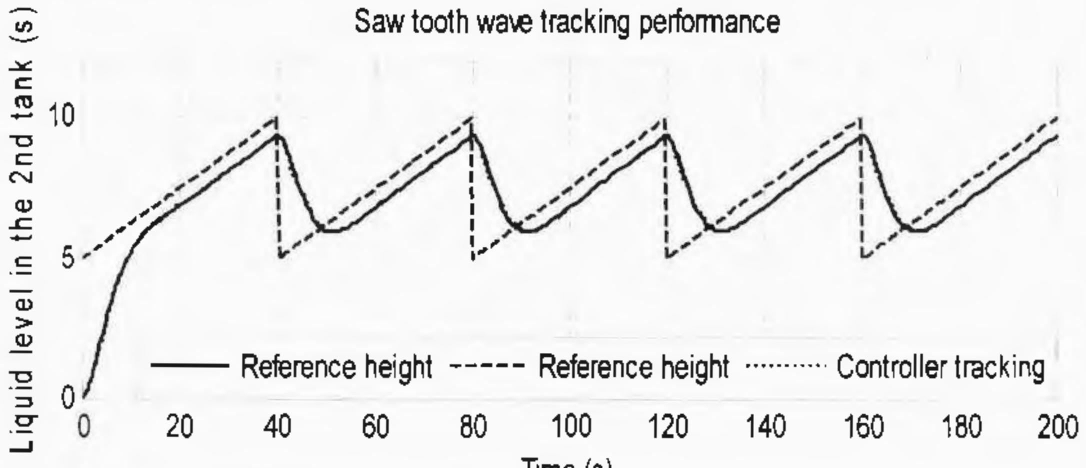

Figure 6.19: Controller tracking performance for sawtooth wave reference signal

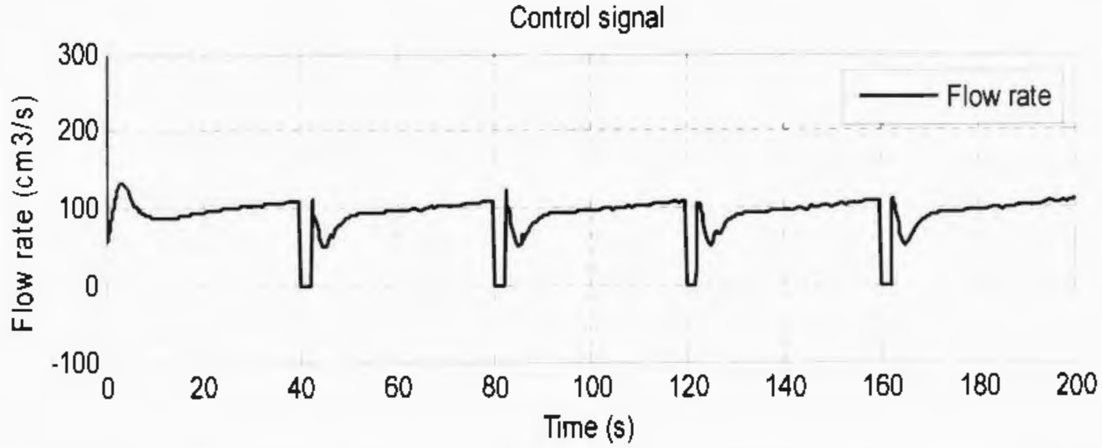

Figure 6.20: Control signal for saw tooth wave like reference signal

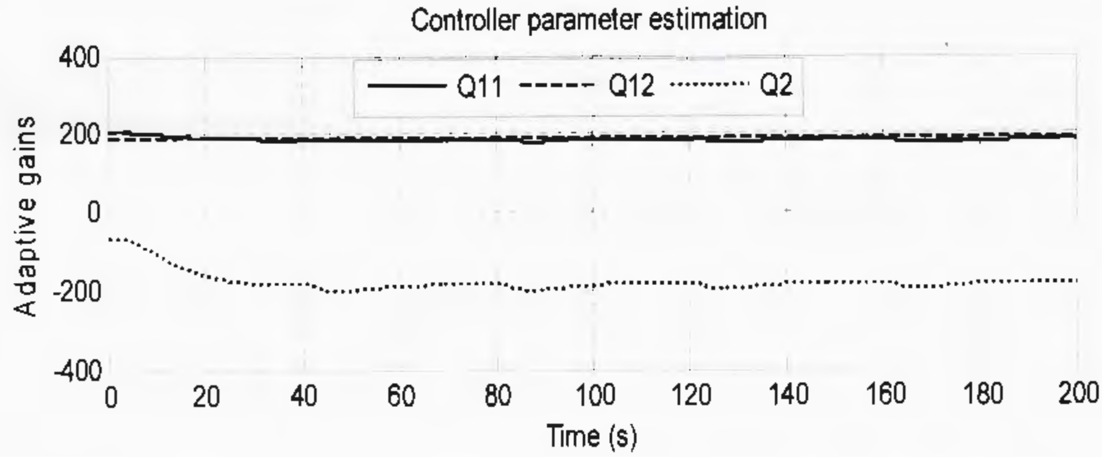

Figure 6.21: Adaptive gains for saw tooth wave like reference signal

6.9 SINUSOIDAL WAVE TRACKING PERFORMANCE

In this section, the tracking performance of Model Reference Adaptive Sliding Mode controller for tracking the sinusoidal wave like reference signal is shown in figure 6.22. The control signal and Adaptive gains are shown in figure 6.23 and 6.24 respectively.

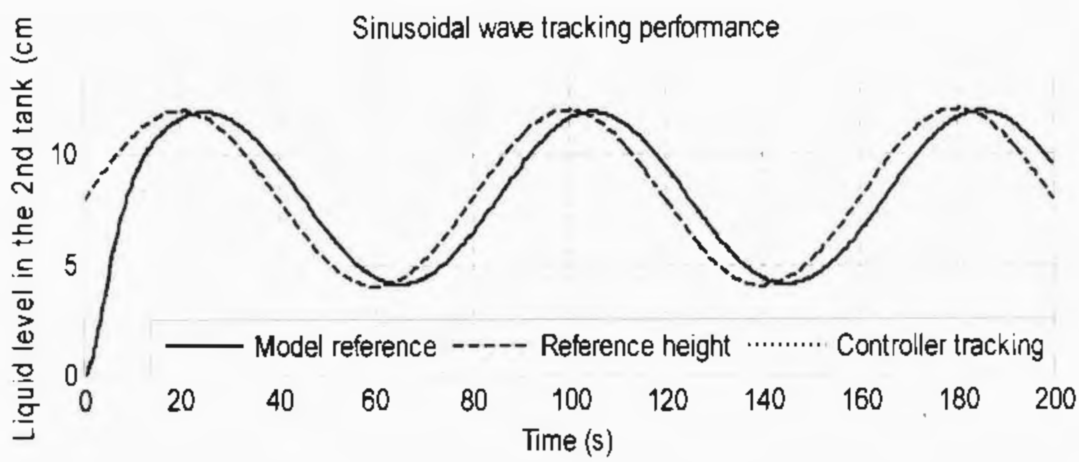

Figure 6.22: Controller tracking performance for sinusoidal wave reference signal

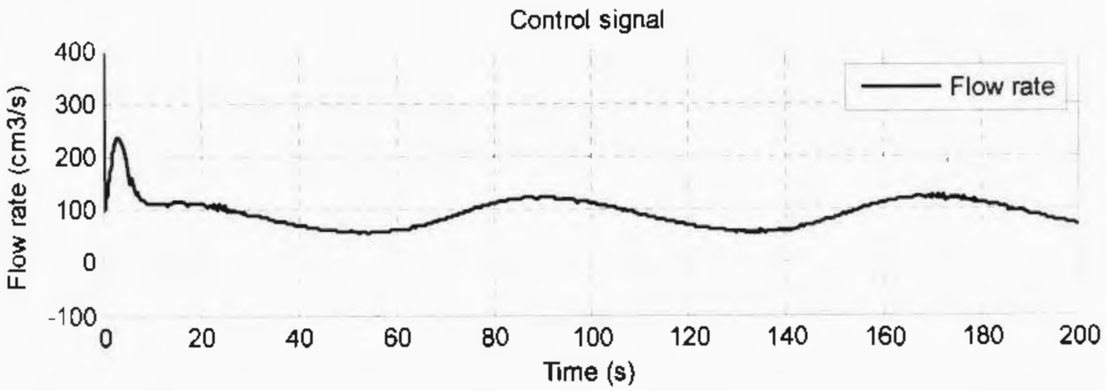

Figure 6.23: Control signal for sinusoidal wave reference signal

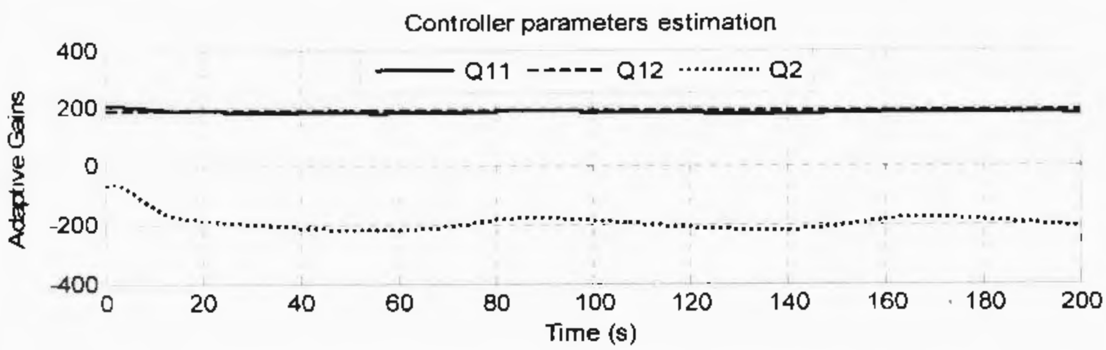

Figure 6.24: Adaptive gains for sinusoidal wave reference signal

6.10 DISTURBANCE REJECTION PERFORMANCE OF CONTROLLER WHEN THERE IS AN INPUT TO THE 2ND TANK

In this section, the performance of the controller is analyzed when the pump 2 (input to 2nd tank) is ON for some time. The pump 2 supplies 20cm^3/s liquid into the 2nd tank for 40 seconds (40 to 80 sec of total simulation time of 200 sec). This extra input will be considered as disturbance. The disturbance rejection performance of the Model reference adaptive controller is shown below in figure 6.24 and its control signal in terms of flow rate is shown in figure 6.25.

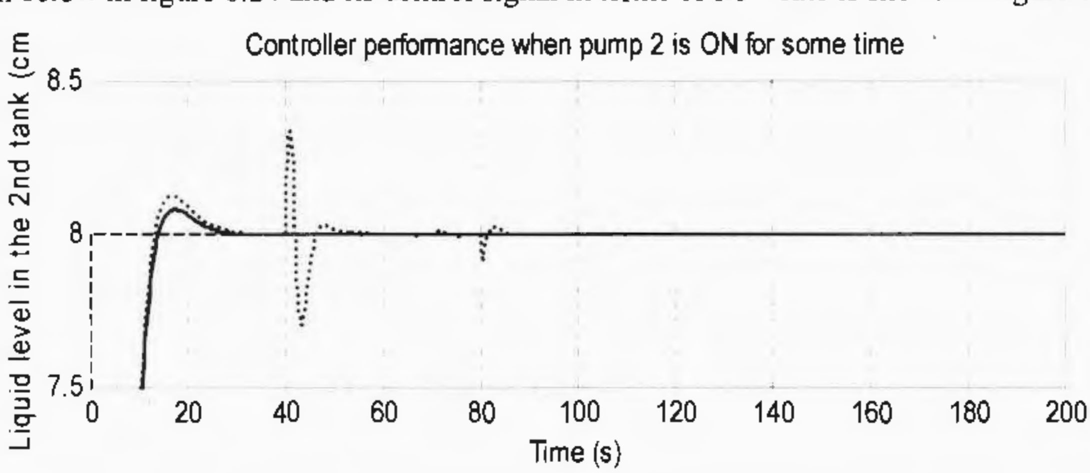

Figure 6.25: Disturbance rejection performance

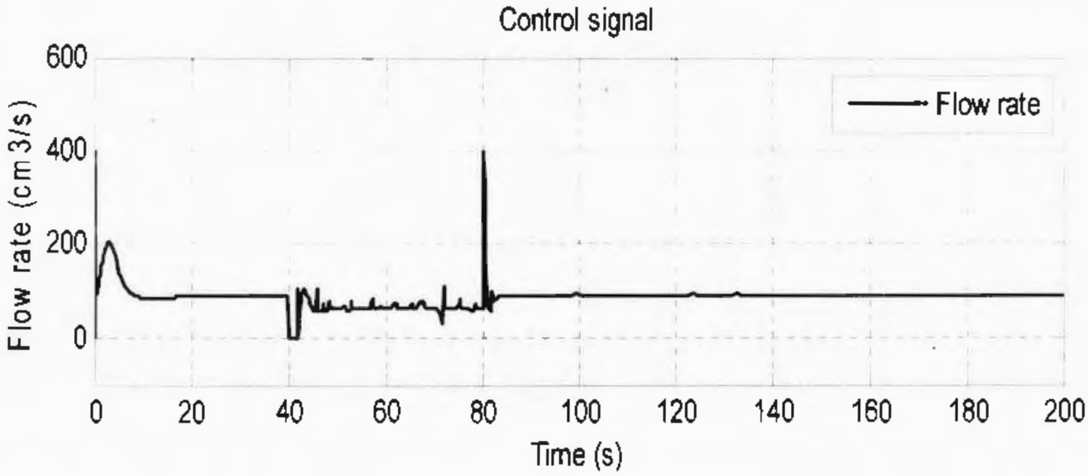

Figure 6.26: Control signal

The model reference adaptive sliding mode controller showed very performance under disturbance. When the extra input is added at 40 seconds time, the level of the water rises but the controller recovered that error in almost 8 seconds. Similarly, when the flow is switched off at 80 seconds time, the level of the liquid goes on decreasing but again the controller recovered that error in almost 5 seconds by increasing the flow rate into the 1st tank.

6.11 TRACKING PERFORMANCE WHEN THERE IS LEAKAGE IN THE 1ST TANK

Now the controller performance is tested when there is a leakage of $50cm^3/s$ in the 1st tank. The leakage of liquid is introduced in the 1st tank when the simulation time was 60 seconds. The performance of controller is shown in figure 6.27. The controller showed very good tracking performance and recovered that error in almost 15 seconds and again maintained the liquid at desired level. The control signal is shown in figure 6.28.

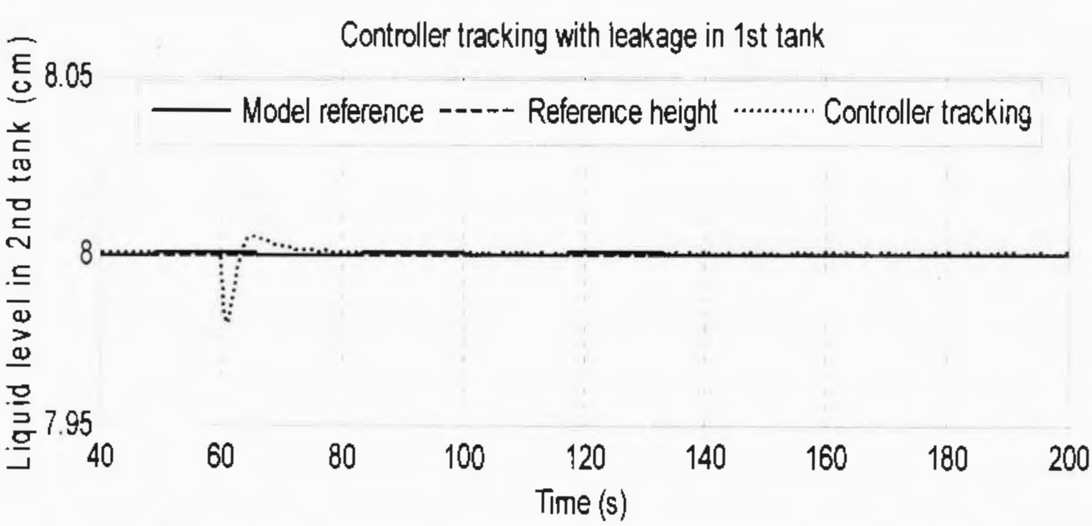

Figure 6.27: Tracking performance with leakage in the 1st tank

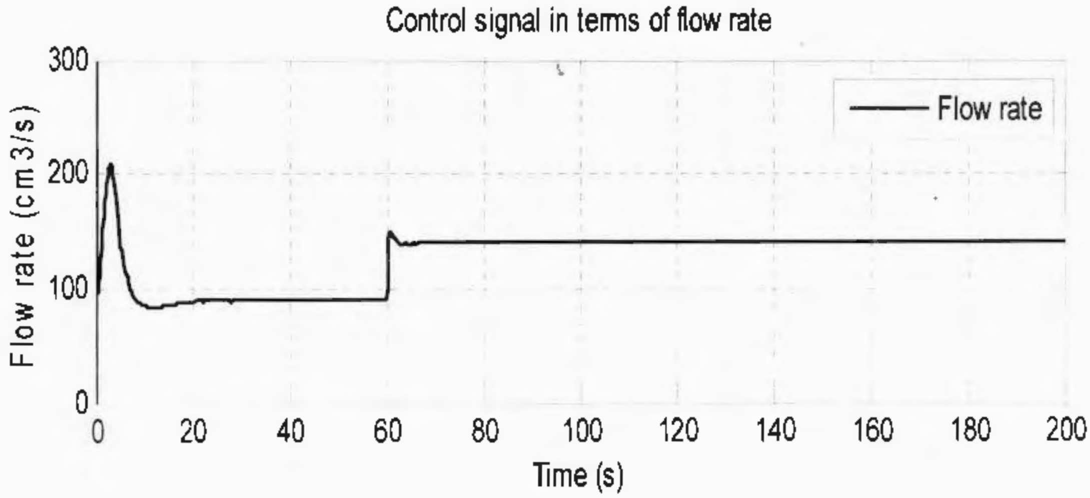

Figure 6.28: Control signal

6.12 TRACKING PERFORMANCE WITH LEAKAGE IN THE 2ND TANK

Now the controller is tested for another uncertain happening, the leakage in the 2nd tank. When the simulation time was 60 seconds, the 20 cm^3/s extra liquid outflow or leakage from 2nd tank is introduced. Because of the leakage or extra liquid outflow from 2nd tank, the level of the liquid in the 2nd tank decreases and hence the controller adjusted the inflow rate to the 1st tank in order to maintain the liquid level at desired position. In equation (3.20), the term U_{j2} is representing the leakage in the 2nd tank. Again the controller showed very good performance under the condition of leakage or extra outflow from 2nd tank and recovered that error in almost 6 seconds by adjusting the flow rate to the 1st tank. The performance of controller is shown in figure 6.29 and its control signal is shown in figure 6.30.

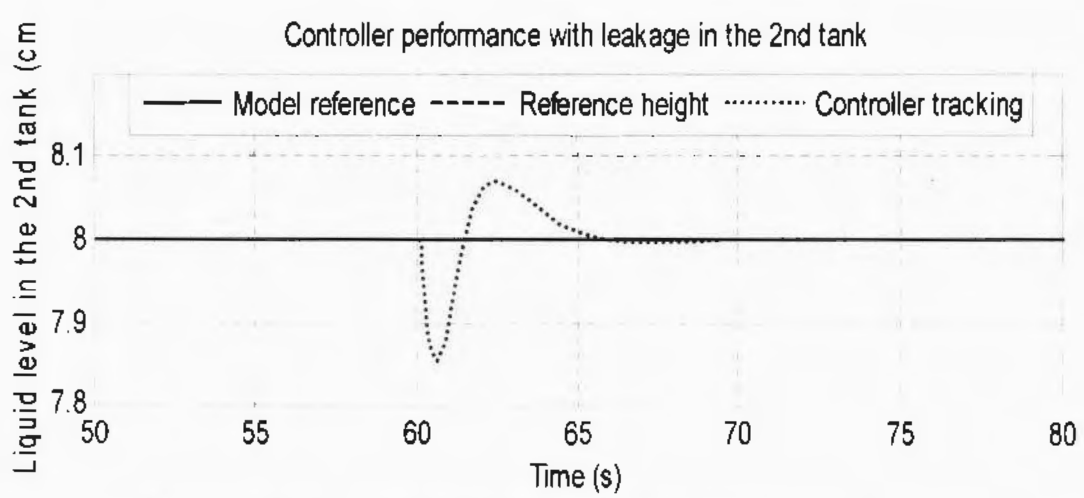

Figure 6.29: Controller performance with leakage in the 2nd tank

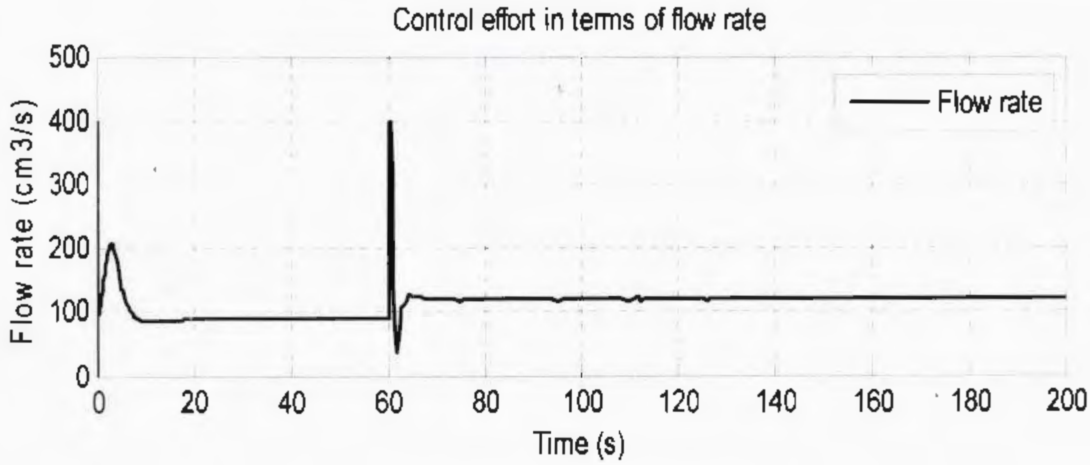

Figure 6.30: Control signal

6.13 MODEL REFERENCE ADAPTIVE CONTROL Vs MODEL REFERENCE ADAPTIVE SLIDING MODE CONTROL

Model reference Adaptive controller is designed in Chapter 5 and Model reference adaptive sliding mode controller is designed in this Chapter. Both controllers are simulated on linear and non-linear model of coupled tank system for the liquid level control in the system. The simulation results are briefly discussed and it is found that the Model reference adaptive sliding mode controller gave extremely good results in comparison to all the controllers designed for the system. Its tracking performance, disturbance rejection performance, robustness to parameters variation and control effort is excellent in comparison to the other designed controllers.

The tracking performance comparison of MRAC and MRA-SMC in terms of ITAE (Integral Time Absolute Error) is made in table 3.4.

PERFORMANCE TEST	MRAC	MRA-SMC
Step command signal	49	16
Random command signal	917	349
Square command signal	961	353
Saw tooth command signal	1470	482
Sinusoidal command signal	485	140
Disturbance Rejection Performance ($20\ cm^3/sec$ inflow to 2^{nd} tank)	128	89
Disturbance Rejection Performance ($20\ cm^3/sec$ leakage to 2^{nd} tank)	69	36

Table 3.4: Performance index

The performance index comparison suggests that MRA-SMC has much better tracking and disturbance rejection performance than MRA controller in controlling the liquid level in coupled-tank system.

6.14 SUMMARY

In this Chapter, Model reference Adaptive Sliding mode controller is designed for coupled-tank plant. The controller is simulated on both linear and non-linear model of coupled-tank plant in order to control the flow rate and hence achieving the desired liquid level. The controller gave excellent tracking performance for linear and non-linear plant model in the presence of disturbance. This novel control scheme proved to be an excellent control strategy for liquid level control in coupled-tank system.

CHAPTER NO: 7

CONCLUSION AND FUTURE WORK

Sliding Mode Controller exhibits excellent performance in controlling the coupled-tank plant in comparison to fixed gain controllers like PID. The controller proved to be Robust for the variation and the sudden changes in Plant parameters. The control objective in coupled-tank plant was to control the flow rate in order to keep the liquid level at desired position. The inclusion of Model Reference Adaptive Controller to Sliding mode controller made it Adaptive also in tracking the time varying reference signals. The plant (Coupled-tank) is operated with different realistic situations. For example, leakage in the tanks, extra input to tanks, increased outflow, etc. A number of different reference set points are taken for analyzing the tracking performance of controllers. Sliding mode controller as well Model Reference Adaptive Sliding Mode Controller are implemented on both the linear and non-linear plant models of coupled-tank system. The controllers exhibit excellent tracking performance for both linear and non-linear models. All the work is done in MATLAB/SIMULINK but all the real encounters are taken into consideration so that simulated plant almost resembles with the real plant environment. The Model reference Adaptive sliding mode controller technique is proved to be good control technique for coupled tank liquid level control system. The tracking performance and disturbance rejection performance of this control technique is much better than other control techniques.

FUTURE WORK

- Higher order Sliding mode controllers can be designed for Coupled-tank Plant.
- Different techniques can be implemented for chattering reduction in Sliding mode control.
- Sliding mode control can be designed by selecting different Sliding surfaces.
- A suitable Sliding mode observer can also be designed for coupled-tank system.
- In coupled-tank plant, temperature effects can be added in flow rate which resembles the plant with boiler.
- The designed controllers can be implemented on coupled-tank apparatus in real time environment

Reference

[1] Haizhou Pana, Hong Wonga, Vikram Kapilaa, Marcio S.de Queirozb "Experimental validation of a nonlinear backstepping liquid level controller for a state coupled two tank system" Control Engineering Practice 13 (2005) 27–40

[2] Muhammad Nasiruddin Mahyuddin, Mohd. Rizal Arshad and Zaharuddin Mohamed "Simulation of Direct Model Reference Adaptive Control on a Coupled-Tank System using Nonlinear Plant Model" International Conference on Control, Instrumentation and Mechatronics Engineering (CIM'07), Johor Bahru, Johor, Malaysia, May 28-29,2007

[3] A tutorial ''Control Systems and the Task of a Control Engineer''

[4] Mohd Shahizan Bin Shapii ''Implementation of LQR Controller on Coupled Tank Liquid Level System'' Faculty of Electrical & Electronics Engineering University Malaysia Pahang November, 2008

[5] J. A. Ramos and P. Lopes dos Santos "Mathematical Modeling, System Identification, and Controller Design of a Two Tank System" Proceedings of the 46th IEEE Conference on Decision and Control New Orleans, LA, USA, Dec. 12-14, 2007

[6] David Cartes, Lei Wu "Experimental Evaluation of Adaptive three-tank level control" ISA Transactions 44 ~2005! 283–293

[7] Mohd Izzat B Dzolkafle "Implementation of PID Controller for Controlling the Liquid Level of the Coupled Tank System" University Malaysia Pahang

[8] K. Pirabakaran and V.M. Becerra "PID Auto-tuning Using Neural Networks And Model Reference Adaptive Control" 15th Triennial World Congress, Barcelona, Spain. 2002

[9] Maruthai Suresh, Gunna Jeersamy Srinivasan, Ranganathan Rani Hemamalini "Integrated Fuzzy Logic Based Intelligent Control of Three Tank System" Serbian Journal Of Electrical Engineering Vol. 6. No. 1, May 2009, pp. 1-14

[10] Ahcene Boubakir; Fares Boudjema Salim Labiod "A Neuro-fuzzy-sliding Mode Controller Using Nonlinear Sliding Surface Applied to the Coupled Tanks System" International Journal of Automation and Computing 06(1), February 2009, pp. 72-80

[11] Ivan Holic and Vojtech Vesely "Robust PID Controller Design For Coupled-Tank Process" 18th International Conference on Process Control June 14–17, 2011, Slovakia

[12] Marek Kubalcik. Vladimir Bobal "Adaptive Control of Three – Tank – System: Comparison of Two Methods" 16th Mediterranean Conference on Control and Automation Congress Centre, Ajaccio, France June 25-27, 2008

[13] Boonsrimuang P, Numsomran A.and Kangwanrat S. "Design of PI Controller Using MRAC Techniques For Couple-Tanks Process" World Academy of Science, Engineering and Technology 2009

[14] Mohammad Khalid Khan, Sarah K Spurgeon "Second Order Sliding Mode Control of Coupled Tanks" Control & Instrumentation Research Group Department of Engineering University of Leicester, U.K. 2005

[15] Ruben Rojas, Oscar Camacho, Ramon Caceres, Alfredo Castellano "On Sliding Mode Control For Nonlinear Electrical Systems" Universidad de Los Andes. Mérida VENEZUELA

[16] Ahmed El-Bakly, A. Fouda, W. Sabry "A Proposed DC Motor Sliding Mode Position Controller Design using Fuzzy Logic and PID Techniques" 13th International Conference on Aerospace Sciences & Aviation Technology, ASCVIIAT- 13, May 26 – 28, 2009

[17] Oscar Camacho, Carlos A. Smith "Sliding Mode Control: An Approach To Regulate Nonlinear Chemical Processes" University of South Florida Merida 5101. Venezuela

[18] Juntao Fei and Hongfei Ding " System Dynamics and Adaptive Control for MEMS Gyroscope Sensor" International Journal of Advanced Robotic Systems, Vol. 7, No. 4 (2010) ISSN 1729-8806, pp. 81-86

[19] Amir Hossein Zaeri, Samsul Bahari Mohd Noor, Maryam Mohd Isa, Farah Saleena Taip "Design of Integral Augmented Sliding Mode Control for Pitch Angle of a 3-DOF Bench-top Helicopter" Majlesi Journal of Electrical Engineering Vol. 4, No. 3, September 2010

[20] Mariagrazia Dotoli, Biagio Turchiano "Piecewise Linear Fuzzy Sliding Mode Control" Dipartimento di Elettrotecnica ed Elettronica, Politecnico di Bari, Via Re David, Bari, Italy

[21] Utkin. V. I., "Variable structure systems with sliding modes". Transactions of IEEE on Automatic Control. 1997, AC-(22), pp. 212-222.

[22] Sira-Ramirez, Llanes-Santiago "Dynamical Discontinuous Feedback Strategies In The Regulation of Nonlinear Chemical Processes", IEEE Transactions on Control Systems Technology, pp. 11 – 21. (1994)

[23] Colantino, Maria C, Alfredo C. Desages, Jose A. Romagnoli, and Ahmet Palazoglu, "Nonlinear Control of a CSTR: Disturbance rejection using sliding mode control", Industrial & Engineering Chemistry Research, 34, pp. 2383-2392. (1995).

[24] Amir Hossein Zaeri, Samsul Bahari Mohd Noor, Maryam Mohd Isa, Farah Saleena Taip "Design of Integral Augmented Sliding Mode Control for Pitch Angle of a 3-DOF Bench-top Helicopter" Majlesi Journal of Electrical Engineering Vol. 4, p.No. 3, September 2010

[25] Mariagrazia Dotoli, Biagio Turchiano "Piecewise Linear Fuzzy Sliding Mode Control" Dipartimento di Elettrotecnica ed Elettronica, Politecnico di Bari, Via Re David, Bari, Italy

[26] Eduardo Sebastian, Miguel A. Sotelo "Adaptive Fuzzy Sliding Mode Controller For The Snorkel Underwater Vehicle" Intelligent Control Systems And Optimization- ICINCO 2005

[27] Zinober ''Variable Structure And Liapunov Control'', Spring – Verlag, London (1994).

[28] Slotine, J. J and Li, W.''Applied Nonlinear Control'' Prentice-Hall, New Jersey, 1991

[29] R. Prakash1 and R. Anita "Model Reference Adaptive PI Control" International Journal of Electronic Engineering Research ISSN 0975 - 6450 Volume 2 Number 2 (2010) pp. 189–199

[30] Abubeker Yimam "Adaptive Control Design for a MIMO Chemical Reactor" Addis Ababa University School of Graduate Studies Faculty of Technology Department of Chemical Engineering, july 2004

[31] Nguyen Duc Hoang and Bui Thanh Huyen ''Application Of Self-Tuning Controller Using Pole Assignment Method In Controlling Electric Oven'' International Symposium on Electrical & Electronics Engineering 2007 - Oct 24, 25 2007 - HCM City, Vietnam

[32] Itzhak Barkana "Simple Adaptive Control - A Stable Direct Model Reference Adaptive Control Methodology - Brief Survey" 2007 IFAC

[33] Dr. Mohammed Y. Hassan "Notes of Lecture Course in Adaptive Control for the 4th Class of Control Engineering in the Control and Systems''

[34] Petros A. Ioannou Jing Sun ''Robust Adaptive Control''

[35] http://www2.stetson.edu.

[36] http://www. wikipedia.org/wiki/Hagen–**Poiseuille** _equation