Channel Estimation for Massive MIMO Communication Systems

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DEDICATED TO

Parents,

My Teachers,

Wife, Children,

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ABSTRACT

Massive multiple input multiple output (MIMO) is believed to be a very important technology to get 1000x data rates in wireless communication systems. Massive MIMO occupies a very large number of antennas placed at the base station (BS) to communicate and serve multiple users simultaneously. It has appeared as a promising technique to realize high-throughput green wireless communications. Massive MIMO exploits the higher degree of spatial freedom, to extensively improve the capacity and energy efficiency of the system. Thus, massive MIMO systems have been broadly accepted as an important enabling technology for 5th Generation (5G) systems. In massive MIMO communication systems, a precise acquirement of the channel state information (CSI) is needed for beamforming, signal detection, resource allocation, etc. Yet, having large antennas at the BS, users have to estimate channels linked with hundreds of transmit antennas. Consequently, pilot overhead gets prohibitively high. Hence, realizing the correct channel estimation with the reasonable pilot overhead has become a challenging issue, particularly for frequency division duplex (FDD) in massive MIMO systems. In this thesis, by taking advantage of spatial and temporal common sparsity of massive MIMO channels in delay domain, nonorthogonal pilot design and channel estimation schemes are proposed under the frame work of structured compressive sensing (SCS) theory that considerably reduces the pilot overheads for massive MIMO FDD systems. The proposed pilot design is fundamentally different from conventional orthogonal pilot designs based on Nyquist sampling theorem. Finally, simulations have been performed to verify the performance of the proposed methods and schemes. Compared to its

conventional counterparts with fewer pilots overhead, the proposed schemes get better performance of the system.

LIST OF ALL PUBLICATIONS

[1]. **Athar Waseem**, Aqdas Naveed, Sardar Ali, Muhammad Arshad, Haris Anis, Ijaz Mansoor Qureshi "Compressive Sensing based Channel Estimation for Massive MIMO Communication Systems," *Wireless Communications and Mobile Computing*, Article ID 6374764, Volume 2019, DOI: 10.1155/2019/6374764 **(Impact Factor 1.6)**

[2]. **Athar Waseem**, Aqdas Naveed, "Performance Analysis of Compressive Sensing based Channel Estimation Approaches for 5G Communication Systems" (to be submitted)

(The above two papers present the PhD research work. This PhD thesis is based on these two papers)

[3]. S. M. Ali, **Athar Waseem,** Zahid Ullah, "Energy Management Models: A Game-Theoretic Optimization Techniques for Energy Management in Smart Grid," in *Proceedings of the 5th International Conference on Power Generation System and Renewable Energy Technology* (PGSRET '19), August 2019

[4]. Muhammad Umar Khan, Muhammad Atif Imtiaz, Sumair Aziz, Zeeshan Kareem, **Athar Waseem**, Muhammad Ammar Akram, "System Design for Early Fault Diagnosis of Machines using Vibration Features," in *Proceedings of the 5th International Conference on Power Generation System and Renewable Energy Technology* (PGSRET '19), August 2019

[5]. Abdul Basit, Shaddrack Yaw Nusenu, Wasim Khan , Shafqatullah Khan , Samad Wali , Muhammad Arshad , **Athar Waseem**, "Adaptive Main Lobe/Sidelobes Controls Selection in FDA based Joint Radar-Communication Design," in *Proceedings of the Ist International Conference on Electrical, Communication, and Computer Engineering* (ICECCE '19), July 2019

[6]. N. Shaukat , Zahid Ullah , Bilal Khan , S. M. Ali , **Athar Waseem**, "An Information-Based Waste Management Approach for Pakistan," in *Proceedings*

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[12]. Muhammad Umar Khan, Sumair Aziz, Javeria Mumtaz Ch., Anber Shahjehan, Atif Imtiaz, **Athar Waseem**, "Detection of Acute Coronary Syndrome using Electrocardiogram Signal Analysis" in *Proceedings of the 2nd International Conference on Electrical, Communication, and Computer Engineering* (ICECCE '20), June 2020

[13]. Kaleem Ullah, Zahid Ullah, Irfanullah Khan, Fazale Wahab, Waqar Uddin, **Athar Waseem**, Aun Haider, Ghulam Hafeez, Sahibzada Muhammad Ali, Khadim Ullah Jan, "Load Forecasting Schemes and Demand Response Programs within

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[14]. Zahid Ullah, Rafiq Asghar, Irfanullah Khan, Kaleem Ullah, **Athar Waseem**, Fazale Wahab, Aun Haider, Sahibzada Muhammad Ali, Khadim Ullah Jan, "Renewable Energy Resources Penetration within Smart Grid: An Overview," in *Proceedings of the 2nd International Conference on Electrical, Communication, and Computer Engineering* (ICECCE '20), June 2020

[15]. Sumair Aziz, Muhammad Ahmed, Syed Zohaib Hassan Naqvi, Muhammad Umar Khan, Atif Imtiaz, **Athar Waseem**, "Machine Bearing Fault Diagnosis System using Tri-Axial Accelerometer," in *Proceedings of the 2nd International Conference on Electrical, Communication, and Computer Engineering* (ICECCE '20), June 2020

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[18]. Muhammad Umar Khan, Sumair Aziz, Areeba Zainab, Hamza Tanveer, Khushbakht Iqtidar, **Athar Waseem**, "Biometric System using PCG Signal Analysis: A New Method of Person Identification," in *Proceedings of the 2nd International Conference on Electrical, Communication, and Computer Engineering* (ICECCE '20), June 2020

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(Athar Waseem)

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LIST OF ABBREVIATIONS

5G	Fifth generation
MIMO	Multiple input multiple output
CS	Compressive sensing
4G	Fourth generation
CIR	Channel impulse response
MU-MIMO	Multiuser MIMO
(MAC)	Medium access control
CSI	channel state information
FDD	frequency division duplex
TDD	time division duplex
BS	base station
RF	radio frequency
SUCoSaMP	Sparsity update CoSaMP (SUCoSaMP)
TFDT	Time and frequency-domain training
OFDM	Orthogonal frequency-division multiplexing
SCS	structured compressive sensing
BDCS	Block distributed compressive sensing
CBP	Continuous basis pursuit
PWSP	probability-weighted subspace pursuit
RIP	Restricted isometric property

RMT	Random matrix theory
SRIP	Structured restricted isometric property
OMP	Orthogonal matching pursuit
BSAMP	Block sparsity adaptive matching pursuit
SSP	Structured Subspace Pursuit
ASSP	Adaptive Structured Subspace Pursuit
QAM	Quadrature amplitude modulation
SNR	Signal to noise ratio
MSE	Mean squared error

LIST OF NOTATIONS

Upper-case	Matrices
lower-case	Vectors
$(.)^T$	Matrix transpose
$(.)^{-1}$	Matrix inversion
$(.)^*$	Conjugate transpose
$(.)^\dagger$	Moore-Penrose inversion of matrix
$\|\|.\|\|_2$	l2-norm operation
$\|\|.\|\|_F$	Frobenius-norm operation
$\|.\|_C$	Cardinality of a set
$\psi^{(j)}$	j-th column vector of the matrix ψ
T^C	Complementary set of the set T
$\lfloor . \rfloor$	Integer floor operator

LIST OF SYMBOLS

A list of commonly used symbols/variables in this dissertation is given below.

$\boldsymbol{d}_{z,m}$	CIR between m-th transmit antenna and one single-antenna user
L	Channel length equivalent to the maximum delay spread
$P_{z,m}$	Support of $\boldsymbol{d}_{z,m}$
R	Number of adjacent OFDM symbols
M	Number of antennas placed at the BS in a uniform linear antenna array
N_g	Number of subgroups
M_{ng}	Number of antennas in one group
D_{max}	Maximum resolvable distance
d_m	Spacing between two antennas
d_T	The maximum distance between two antennas
$d_{M_{ng}}$	The maximum distance between two antennas in a single group
f_c	Carrier Frequency
ξ_n	Represents the index set of pilot subcarriers for nth antenna group
$\mathbf{s}_{m,n}$	Pilot sequence of m-th transmit antenna in n-th antenna group
$\mathbf{y}_{z,n}$	The received vector of the pilot sequence of z-th OFDM Symbol of n-th antenna group at user for N_g antenna groups
$\mathbf{R}$	The Discrete Fourier Transform matrix (DFT)
$\mathbf{d}_{m,z,n}$	The CIR vector of m^{th} transmit antenna of nth antenna group for z^{th} OFDM symbol

$\mathbf{R}\vert_{\xi_n,\mathrm{n}}$	Submatrix comprised of N_p rows selected according to ξ_n
$\mathbf{w}_{\mathrm{z,n}}$	The Additive White Gaussian Noise of nth antenna group for zth OFDM symbol
$\boldsymbol{\psi}_{\mathrm{n}}$	Aggregate CIR vector of nth antenna group
$\tilde{\mathbf{h}}_{\mathrm{z,n}}$	Structured sparse equivalent CIR vector
β_p	Pilot overhead ratio
ϵ	Noise variance
s	Initial sparsity level

Chapter 1. Introduction

This chapter gives background, motivation and an overview to understand the current important research issues of the fifth generation (5G) wireless systems. Massive multiple input multiple output (MIMO) communication is one of major technology and issues of 5G. Furthermore, the problem statement and research objectives are presented. The focus of our research is on channel estimation for massive MIMO communication systems. A compressive sensing (CS) based channel estimation approach is proposed in this thesis. Finally, research philosophy, hypothesis, research methodology and thesis outlines are presented.

1.1 Background and Motivation

The fourth generation (4G) wireless communication system has been set up or is soon to be set up in many countries. Yet, with a sudden raise in number of wireless devices and services, there are still several challenges and issues that are not being resolved and accommodated by 4G, such as the energy consumption issue and spectrum crisis. Due to new wireless applications the demand of more data rate and mobility has been significantly increased for wireless system researchers and designers. Therefore, designers have started investigation on fifth generation (5G) system. The 5G is expected to be set up in 2020. Right now, it is very difficult to say or describe that what will be 5G. However, there is an overall agreement across the world that the 5G network will achieve 10 times the data rate (the peak rate upto 1-10 Gb/s depending upon the mobility), energy efficiency, spectral efficiency, 1000 times the capacity of system and approximately the average cell throughput will be 25 times as compared to 4G systems. The target is to achieve the ubiquitous and seamless communication by connecting the whole world in terms of machine to people, people to machine, people to people and machine to machine anywhere, whenever and wherever, by any type of services, devices and networks. This means that 5G systems will provide communications in some special scenarios that are not carried out by 4G systems [1].

The various feature requirements for 5G, promising key technologies and their future challenges for 5G networks are summarized in the following categories:

1.1.1 Engineering Requirements of 5G

In order to identify the 5G engineering issues, and to prepare to understand and resolve them, it is essential to initially recognize the important requirements for the 5G system. It is significant to keep in mind that not all of these are necessary to be fulfilled at the same time. The key requirements are mentioned below:

i. Data Rate: Data rate may be characterized into various types, and there is a goal target for each item in 5G:

a) Collective data rate is the entire measure of data a network can provide, described in bits/s/area. The broader agreement is that this number should be increased by approximately 1000 times from 4G to 5G [2].

b) Edge rate, which is known as 5% rate shows the worst amount of data rate that a network provides to a user. Targeted edge rate for 5G is about 100 Mbps (sufficient to carry out high definition (HD) streaming) upto 1 Gbps. Achieving 100 Mbps for 90 to 95 percent of the users is extremely challenging itself, even through major advancement in technology. This is about 100 times the current 4G edge rate that is about 1 Mbps, though the exact number depends on the load, radius of cell, and some other factors [2].

c) Peak rate, the maximum rate that a user expect to experience, it should be in the around tens of Gbps [2].

ii. Latency: The expected total latency in 5G is about 1 ms that is much more faster than 4G (i.e. on the order of about 15 ms) [2].

iii. Energy and Cost: Since the data rates per-link will increase 100 times therefore costs and energy consumption should significantly decrease, the cost per bit and Joules per bit must be reduced by minimum 100 times) [2].

1.1.2 Heterogeneous Networks

A simple but enormously efficient method to raise the network capacity and to get demand of high data rates is to deploy smaller cells. By shrinking the size of a cell, spectral efficiency per area can be substantially enhanced with the help of high frequency reuse factor, whereas transmit power can also be minimized (i.e. the loss of power due to transmission will be lesser). Moreover, by deploying smaller cells indoors, coverage can be better where reception is not possible to be high [3].

Furthermore, modifying the architecture of operational access network will allow data as well as control signals to channel through the Internet (i.e. to deploy small cells anywhere through Internet connectivity). These small cells usually have several types such as femtocells classically deployed in enterprise and residential areas, and the higher picocells that are generally powered to achieve broader outdoor coverage or covering the holes within a coverage of a macro cell. Such networks are called heterogeneous networks shown in Figure 1.1 which provide a synchronized operation of various categories of base stations which are femto, pico, micro and macro base stations [3].

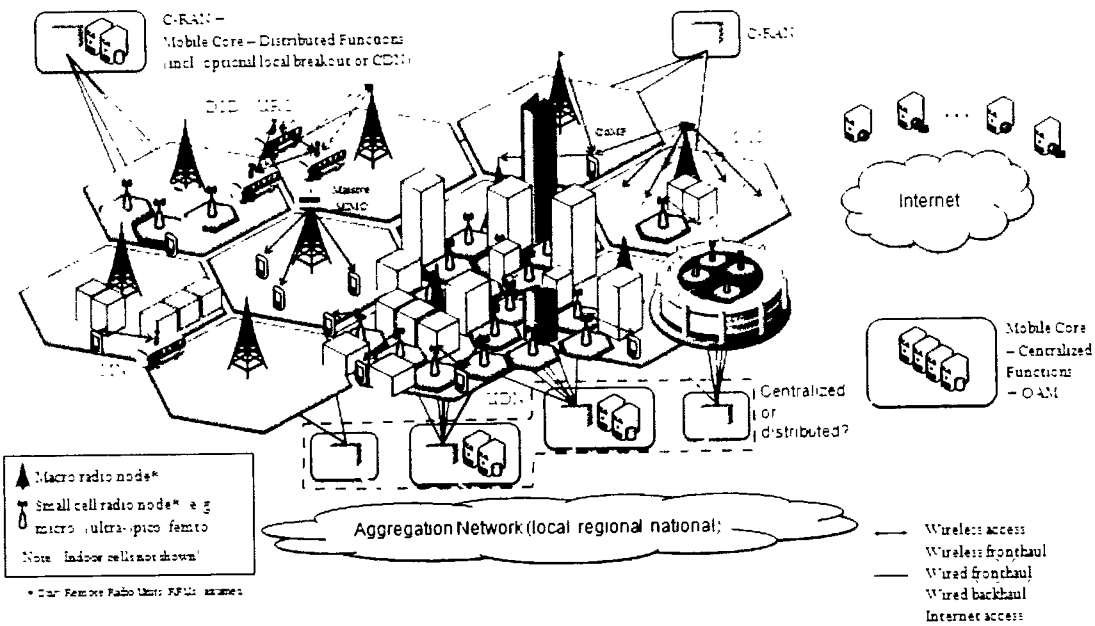


Figure 1.1 Graphical representation of a Heterogeneous Network [3]

1.1.3 Millimeter Wave

A clear way of achieving higher throughput is by bandwidth extension. Still, the bandwidth below 6 GHz is restricted. Designers are trying to stare beyond 6 GHz. Specially millimetre wave frequencies are under study to assess their practicability to employ in 5G systems [3].

The characteristics and advantages of higher frequencies are yet to be studied. The measurement operations along with channel modeling for required environments and scenarios is essential well before transmission advancements are made and deployed for them. There is a strong consensus that millimetre wave communication will be a promising technology in getting targets of 5G, and efforts are already started to make this a possibility. Frequency range is form 25 GHz to 300 GHz and Wavelength is between 10mm to 1mm shown in Figure 1.2 and Figure 1.3 shows the Millimeter wave enabled network with macro cell [3].

28 GHz and 38 GHz millimetre wave frequencies are under study in [4] to realize their transmission behavior in different propagation scenarios, paving the way to use them in 5G wireless networks [3].

The major advantages Millimeter Wave are listed below:

- The radio spectrum is still rather undeveloped
- More bandwidth is available
- High data rates can be achieved
- Security and privacy is better

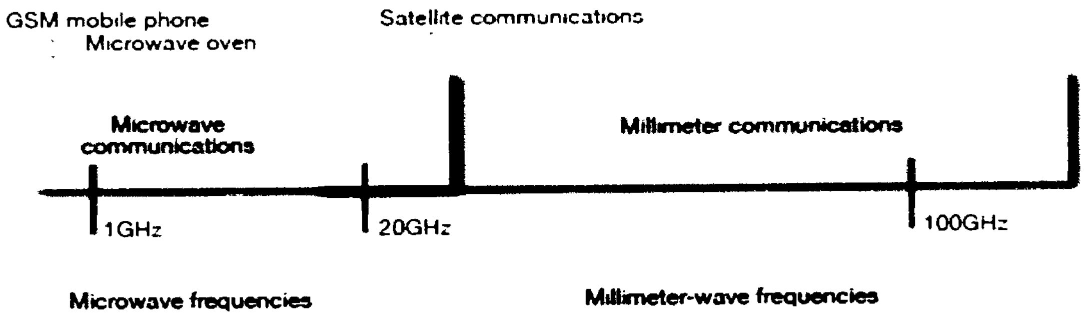


Figure 1.2 Frequency ranges for Microwave, Satellite and Millimeter wave communications [4]

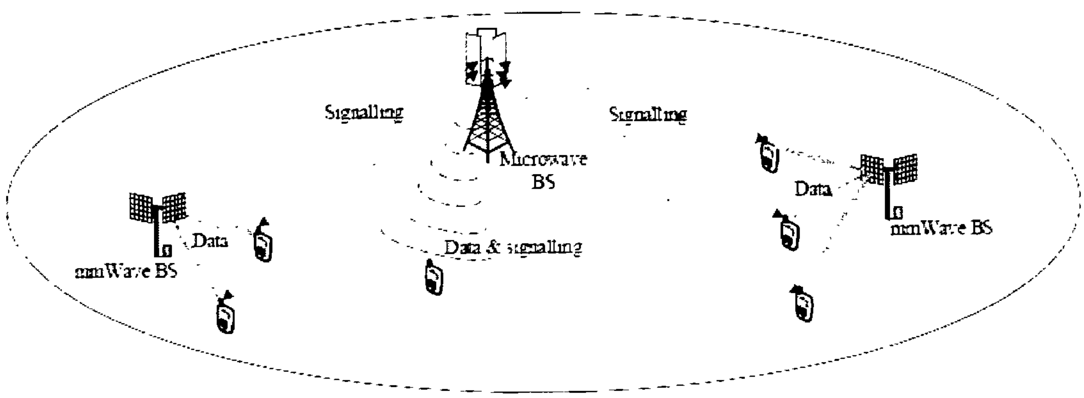
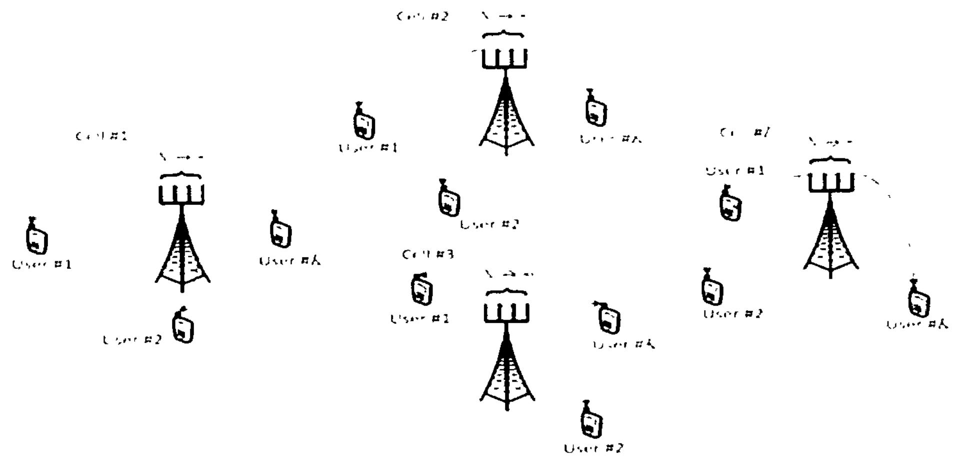


Figure 1.3 Millimeter wave enabled network with macro cell [2]

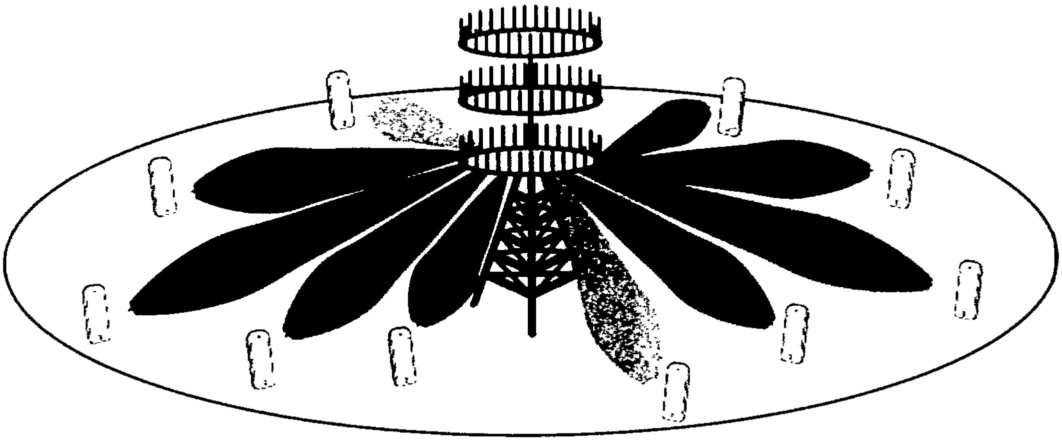
1.1.4 Massive MIMO

Multiple input multiple output (MIMO) systems have several antennas at both the transmitter and the receiver sides. By addition of multiple antennas, higher degree of freedom in wireless communication channels (in terms of time and frequency dimensions) can be obtained in order to achieve target of high data rates. For this reason, major performance progress can be attained in terms of system reliability, energy and spectral efficiency. And also, these higher degrees of freedom may be further subjugated using beamforming given that channel state information is available. There are high number of antenna elements (around tens or hundreds) deployed at both sides, the transmitter and receiver. It is very important to consider that the transmit antennas can be collocated or distributed for various functions. Moreover, the huge number of receive antennas can be obtained by single device or distributed into several devices [1],[5]. Additionally, massive MIMO systems help in minimizing the results of fast fading and noise, and also interference in intra-cell scenario can be reduced using straightforward detection and linear precoding methods. By appropriately implementing multiuser MIMO (MU-MIMO) in massive MIMO communication, the layer design of medium access control (MAC) may be more simplified by getting rid of complex scheduling algorithms [5]. With MU-MIMO, signals to individual users can be easily separated utilizing the identical time-frequency resource, at base station. Therefore, these main advantages make it feasible to introduce the massive MIMO system as a potential candidate for 5G wireless systems.

Figure 1.4 a) demonstrates Massive MIMO deployments in multi cell scenario and Figure 1.4 b) is showing a beamforming scenario in single cell. In Figure 1.5 a 5G heterogeneous network along with large MIMO is shown.



a)



b)

Figure 1.4 Massive MIMO deployments: a) multi cell scenario b) beamforming in single cell

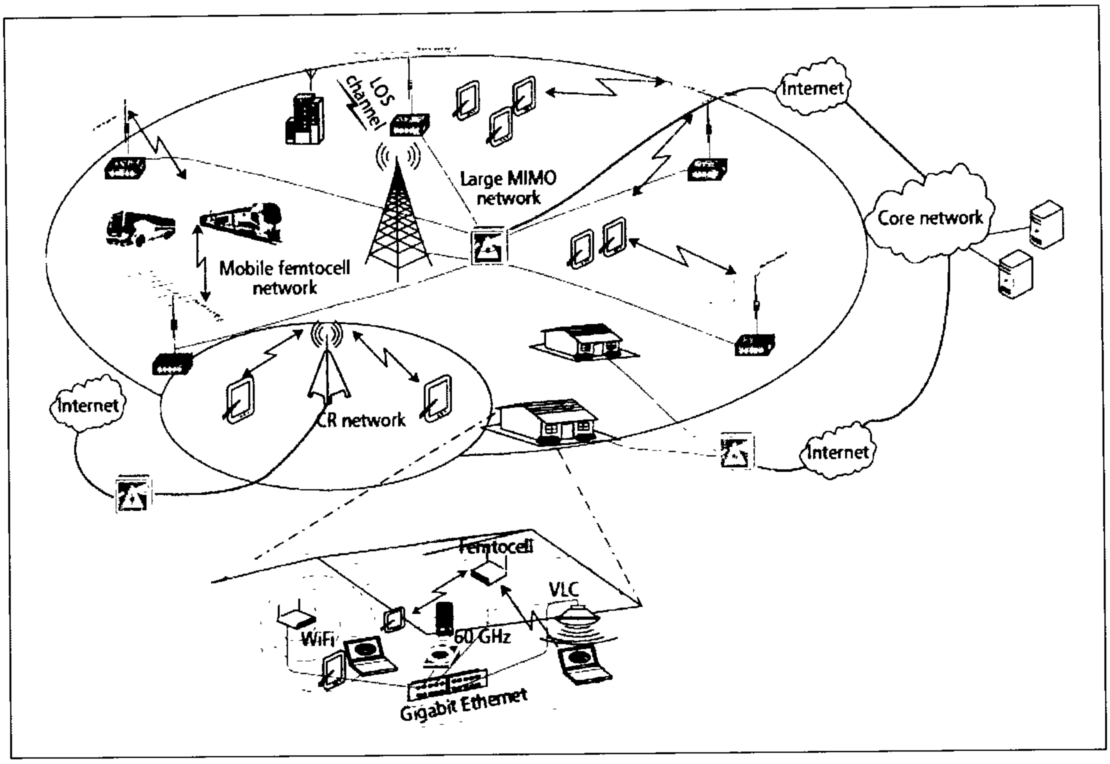


Figure 1.5 A 5G heterogeneous network along with large MIMO [1]

1.2 Research Problem

One of the major issues in massive MIMO systems is the exact attainment of the channel state information (CSI) for beamforming, resource allocation, signal detection, etc. Due to large antennas placed at the BS, the estimation of channels linked with hundreds of transmit antennas is required at users which results in large pilot overhead. Hence, the precise channel estimation with the low pilot overhead is a tough task [5],[6].

There are numerous research issues and challenges which are required to be resolved prior to massive MIMO can be fully included into upcoming wireless networks [2],[3]:

(i) MIMO can be unfeasible for frequency division duplex (FDD) systems, but can be employed in time division duplex (TDD) systems due to having channel reciprocity [2],[3].

(ii) Conventional channel estimation approaches require a large pilot and feedback overhead, which typically scales proportionally with number of base station (BS) transmit antennas, which results in unfeasible condition for large-scale FDD MIMO systems [2],[3].

(iii) There is a need for channel models of massive MIMO systems, without which it will be difficult for researchers to perfectly validate the algorithms and techniques [2],[3].

(iv) For channel estimation, TDD scenarios are only taken for massive MIMO due to the high cost of feedback and channel estimation. Even for TDD to work, massive MIMO channel calibration can prove to be a big achievement. New methods and schemes will be needed for the task of channel estimation in massive MIMO systems [2],[3].

(v) Extremely fast processing algorithms will be required for processing the massive amount of data from the radio frequency (RF) chains [2],[3].

1.3 Research Objectives

The goal and approach of this thesis is to propose and examine several methodologies, algorithms and configurations to fight against the problem of channel estimation for massive MIMO systems. Channel estimation is usually done by two approaches that are supervised (training based) and unsupervised (blind modes). We have analyzed the application of compressive sensing which require comparatively short pilot overhead and follow the supervised (training mode) for the appropriate channel estimation.

Furthermore, the objective is to provide an efficient solution of the following important issues and research challenges for deployment of massive MIMO systems in 5G technology:

Research Objective 1: To employee massive MIMO system for (FDD) systems, without having channel reciprocity property. To get rid of conventional channel estimation approaches since they require a large pilot and feedback overhead specially for handling a large number of BS transmit antennas which are unfeasible for large-scale FDD MIMO systems.

Research Objective 2: To avoid pilot contamination, arise from neighboring cells specially when transmit power is high. To provide a channel model for massive MIMO systems based on realistic assumption for researchers to perfectly validate the algorithms and techniques.

Research Objective 3: To propose new methods and schemes needed for the task of channel estimation in massive MIMO systems which can work for both FDD and TDD scenarios with reduced cost. Even for TDD to work, massive MIMO channel calibration can prove to be a big achievement. To propose extremely fast processing algorithms by incorporating the theory of compressive sensing for processing the massive amount of data from the radio frequency (RF) chains.

1.4 Hypothesis

Many experimental studies have shown that massive MIMO channels for TDD protocol demonstrate spatial and temporal sparsity in its delay domain. These studies have also shown that the channel state information (CSI) in the downlink can be directly tacked by exploiting the sparse nature of massive MIMO channels. Furthermore, recent researches have also demonstrated that the pilot overhead to estimate Rician MIMO channels can be compacted by making use of the spatial correlation of MIMO channels.

Moreover, compressive sensing theory has shown significant results in recent studies to achieve the targets of channel estimation for different types of wireless channels having sparse nature. In few more studies the spatial correlation in sparsity of delay domain MIMO channels is utilized to estimate channels achieving the reduced pilot overhead, but the assumption level of the known channel sparsity to the user is impractical.

Therefore, keeping in view the sparse nature of massive MIMO channels there is a huge room for further research to achieve good channel estimation results by considering the several other parameters and assumptions closer to practical systems. Moreover, one can apply compressive sensing theory to perform channel estimation in massive MIMO systems since there exist a large number of transmitting and receiving antennas for FDD protocol. Also pilot overhead reduction is achievable by integrating the framework of compressing sensing theory.

1.5 Research Methodology

In order to accomplish the highest success rate in research, a correct and good methodology is required. In this thesis following important points have been considered:

(i) Develop an analytical overall system model, simulation models for massive MIMO communication channels and corresponding compressive sensing-based recovery algorithms.

(ii) Use the models and algorithms to construct specific massive MIMO channel estimator for simulations.

(iii) Execute simulations in Matlab to verify and validate the results.

Specifically speaking in this thesis, we developed an overall massive MIMO communication systems model based on CS theory then we proposed a CS based efficient non-orthogonal pilot scheme for the developed model by exploring the temporal and spatial sparsity of massive MIMO channels. The proposed pilot scheme is significantly different from the conventional schemes and substantially reduces the pilot overhead.

Additionally we proposed a channel estimation scheme, i.e., sparsity update CoSaMP (SUCoSaMP), Compared with the conventional CS algorithms subspace pursuit (SP) and orthogonal matching pursuit (OMP) and with other available CS based algorithms, the proposed CS based pilot and channel estimation scheme is verified through simulations over the developed system model with different parameters. It was validated that the proposed schemes provided improved channel estimation performance.

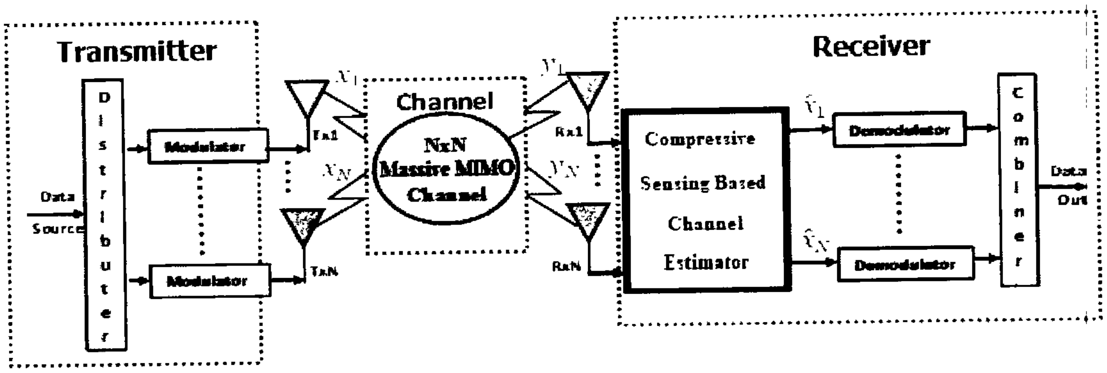


Figure 1.6 Massive MIMO Communication System with CS based Channel Estimator

Figure 1.6 illustrates the general graphical representation of the complete communication system. The system consists of *N*x*N* massive MIMO communication channel along with CS based channel estimator and compensator at the receiver end. The similar type of CS based estimator is incorporated for each individual user. Each CS based estimator works independently on the combined received signals for recovery and detection of transmitted signals from their respective transmitters (i.e. Tx1 to TxN).

1.6 Thesis Outline

The organization of the thesis is as follows. Chapter 2 provides the literature review including the general background, related research works and contribution of this thesis. Chapter 3 demonstrates the materials, methods and experimentation details. It provides the details about delay domain spatial and temporal sparsity of massive MIMO communication channels, the proposed nonorthogonal pilot scheme based on CS theory, the CS based massive MIMO channel estimation scheme, the simulation setup and the summary. Chapter 4 presents the simulation results and discussion.

Finally, Chapter 5 winds up the thesis by presenting the conclusion and future suggestions.

Chapter 2. Literature Review

This chapter presents the literature review. A general overview of compressive sensing (CS) theory, related work based on CS theory and the brief details about methods proposed in this thesis are included.

2.1 Background of Compressed Sensing (CS)

The basic idea presented by compressed sensing is to recover and detect a signal which is sparse in some domain from extremely a small amount of non adaptive, linear measurements by applying convex optimization. Having a different opinion, it relates the precise recovery of a sparse vector of high dimension by reducing its dimension. From another point of view, the problem can be considered as calculation of a signal's sparse coefficient with respect to an over complete system. The basics of compressed sensing are related to different other areas such as, frame theory, applied harmonic analysis, numerical linear algebra, geometric functional analysis, random matrix theory and optimization theory also explore some new methodologies. The concept of compressed sensing was primarily applied for random sensing matrices, as which allow for a reduced amount of non-adaptive, linear measurements. These days, the idea of compressed sensing has been generally replaced by sparse recovery.

2.2 Related work

The challenges for massive MIMO communication systems mentioned in chapter 1 have been addressed up to some extent in the last few years. To date, many researches on massive MIMO avoided the challenge of considering FDD systems by simply assuming the TDD protocol. At BS the uplink CSI is easy to obtain due to few one antenna users at the BS and the strong capability of processing. Afterwards by leveraging an important channel reciprocity property, the CSI in the downlink can be directly tacked [7],[8]. However, due to the fact that radio frequency chains suffer from calibration error and restricted coherence time, the CSI obtained in the uplink is usually not correct for the downlink [9],[10]. Additionally FDD systems have low latency as compared to TDD; therefore the communication is more efficient [11]. Hence, it is significant to discover the challenges and major issues in channel estimation for FDD systems, which can assist massive MIMO to be compatible with existing FDD dominated cellular systems. There has been wide analysis on channel estimation for traditional small-scale FDD scenarios in MIMO systems [12],[13]. It was established that the equally spaced and equally powered orthogonal pilots can be ideally suitable to approximate the noncorrelated Rayleigh distributed MIMO channels for single OFDM symbol, where it increases the pilot overhead as the number of transmitters increases [12]. The pilot overhead to estimate Rician MIMO channels can be compacted by making use of the spatial correlation of MIMO channels [14],[15]. Furthermore, by taking advantage of the temporal channel correlation, more reduced pilot overhead can be attained to estimate MIMO channels linked to multiple OFDM symbols [16],[17],[18]. Presently, orthogonal pilots have been extensively introduced in the current MIMO systems; due to small number of transmit antennas (i.e., up to eight antennas in LTE-Advanced system), they have reasonable pilot overhead [13],[19]. Though, this is a critical issue in massive MIMO systems due to massive figure of antennas at the BS (i.e., up to 128 antennas or even more at the BS [20]). An approach of exploiting the temporal correlation and the delay domain sparsity of channels for achieving the reduced pilot overhead has been presented for FDD massive MIMO systems in [21],[22], but with large number of transmit antennas, the interference cancellation of pilot sequences of different transmit antennas will be difficult. In [23],[24],[25],[26] the spatial correlation in sparsity of delay domain MIMO channels is utilized to estimate channels achieving the reduced pilot overhead, but the assumption level of the known channel sparsity to the user is impractical. In [27],[28],[29], the compressive sensing (CS) based channel estimation schemes are presented by exploiting the spatial channel correlation; however due to having nonideal antenna array, the leveraged spatial correlation can be impaired. In [30] a new structured compressive sensing (SCS) channel estimation scheme for doubly selective MIMO-OFDM systems is proposed with time and frequency-domain training (TFDT). In [32] a channel estimator based on block distributed compressive sensing (BDCS) is proposed for the large-scale MIMO systems. BDCS exploits structured sparsity to reduce the pilot overhead. In [33] a new way to estimate sparse channels by construction of beamforming dictionary matrices is presented. Continuous basis pursuit (CBP) algorithm that exploits the sparse nature of channels to adaptively estimate the multipath mmWave channels is proposed. In [34] a channel estimation scheme based on an open-loop and a closed-loop system for massive MIMO is presented, but the channel statistics cannot be perfectly known to the user in the long term, whereas in conventional wideband wireless systems, delay domain channels basically exhibit the sparse nature due to the large channel delay spread and having limited number of major scatterers in the transmission environments [22],[35].

In the meantime, MIMO communication systems have similar scatterers in the transmission environment; therefore BS channels associated with one user and several transmit antennas experience similar type of path delays, which shows that these delay domain channels share common sparsity specially in the case of not having very large aperture of antenna array [5],[36]. Furthermore, throughout the coherence time, such sparsity is nearly unchanged which is due to the fact that path delays fluctuate at very slow rate as compared to path gains because of temporal correlation of channels [37]. In [38] CS based block sparsity adaptive matching pursuit (BSAMP) technique is presented, which targets the estimation of channels of MIMO system with unknown number of channels paths. The BSAMP exploits the joint sparse nature of massive MIMO channels. In [39] CS based probability-weighted subspace pursuit (PWSP) algorithm is proposed, which exploits the probability information attained from previously estimated CIRs to recover the uplink channels in massive MIMO scenario. In [40] such properties of channels in MIMO are considered as the spatiotemporal common sparsity, which is generally ignored in current work. A general idea of the CS method is presented in [41], in which fundamental setup, recovery techniques, and guarantee of performance are discussed. Furthermore, different sub problems of CS, i.e., sparse approximation, identification of support, and sparse identification, are discussed, with respect to some applications of wireless systems. Design issues of wireless systems, limitations and potentials of CS algorithms, useful tips, and prior information are also discussed to achieve maximum performance. In [41] a valuable guidance is provided for researchers working in the area of wireless communication systems. In [42] authors proposes a CS based detection and channel estimation scheme by making use of the sparsity of massive MIMO in virtual angle domain. Authors in [43] proposed the block sparsity adaptive matching pursuit (B-SAMP) algorithm for the purpose of Doubly Selective Channel

Estimation for Massive MIMO Systems. Finally a structured turbo compressed sensing (Turbo-CS) method is proposed in [44] for design and investigation of channel estimation schemes based on structured sparsity.

Before getting into the detail of spatiotemporal common sparsity, we present an overview of compressive sensing.

2.3 Thesis Contribution

In this thesis we propose a CS based efficient nonorthogonal pilot scheme for massive MIMO communication systems by exploring the temporal and spatial sparsity of massive MIMO channels. And then we propose a channel estimation scheme, i.e., sparsity update CoSaMP (SUCoSaMP), which exploits the temporal and spatial sparsity of massive MIMO channels. The proposed pilot scheme is significantly different from the conventional schemes and substantially reduces the pilot overhead. The proposed pilot scheme employs fully identical subcarriers for pilots of several transmit antennas in a specific antenna group. The antennas placed at base station (BS) are subdivided into groups based on the observation that the coherence time of path gains and system carrier frequency are inversely proportional to each other and the variation in path delays and signal bandwidth are inversely proportional to each other. Therefore, the decision of making antenna groups and determining the number of antennas to be included in one antenna group is taken according to the given system parameters, i.e., systems frequency, system bandwidth, and antenna spacing at BS. Furthermore, considering the antenna array geometry of BS, the proposed non-orthogonal pilot scheme is a space-time adaptive pilot scheme that adaptively changes its design according to the given system parameters.

The proposed CS algorithm-based channel estimation scheme SUCoSaMP considers the initial sparsity level as 1 and then regularly updates the sparsity level until the stopping criteria are met or a correct sparsity level is achieved in a scenario where sparsity level is unknown. Compared with the conventional CS algorithms subspace pursuit (SP) and orthogonal matching pursuit (OMP) and with other available CS based algorithms, the proposed CS based pilot and channel estimation scheme is tested through simulations on systems with different parameters. It was verified that the proposed schemes provided improved channel estimation performance.

2.4 Summary

This chapter provided the brief background of CS theory which is a base of methods proposed in this thesis. Furthermore, most relevant related works are briefly discussed and finally an overview of the contribution of this thesis is given.

Chapter 3. Methodological Details of Compressive Sensing Based Channel Estimation for Massive MIMO Communication Systems

3.1 Introduction

This chapter provides complete details of methodology which includes the overview of a general compressive sensing problem, proposed non-orthogonal pilot scheme which include a method for creating the antenna groups and a pilot design based on delay domain spatial and temporal sparsity. Furthermore, this chapter includes a massive MIMO system model for experimentation purpose and a proposed massive MIMO channel estimation approach based on compressive sensing. Finally, experimentation details and summary of chapter are presented.

3.2 Materials and Methods

3.2.1 The Compressed Sensing Problem

To specify the problem in terms of mathematics, let consider a signal of interest as $x = (x_i)_{i=1}^{n} \in \mathbb{R}^n$. Where x has a small number of nonzero coefficients i.e. x is assumed to be a sparse signal as $\|x\|_0 := \#\{i: x_i \neq 0\}$.

An orthonormal basis Φ exist such that $x = \Phi c$ with c as sparse. Where Φ is a matrix containing elements as frame or the orthonormal basis as column vectors. A frame is usually more flexible as compared to orthonormal basis due to having redundancy and this directs to better sparsifying properties, therefore customarily frames are preferred than orthonormal bases. At times, the idea of sparsity is weaken, which refer to as *nearly sparse*. In addition, there exist a matrix known as *sensing matrix,* let A be sensing matrix of size $m \times n$. where $m < n$ and there does not exist any zero colums in A, yet if not clearly mentioned.

Then the problem can be stated as follows: Recover x from knowledge of

$$y = Ax \tag{3.1}$$

or recover c from knowledge of

$$y = A\Phi c \tag{3.2}$$

Above mentioned, cases are underdetermined systems having prior information of sparsity about the vector to be recovered. The support is yet not known, given that the trivial solution could be obtained.

CS techniques allow reliable recovery of the sparse signal x. In case of nearly sparse and the noise is also there, yet CS theory provides a guarantee stable solution and strong result of reconstructed sparse signal if restricted isometric property (RIP) is satisfied [45] [46] and the problem can be expressed as

$$\min \|x\|_1 \ s.t. \|y - A\Phi c\|_2^2 < \varepsilon \tag{3.3}$$

Where ε is related to variance of noise.

3.2.2 Delay Domain Spatial Sparsity

Considerable experimental researches have explored that in delay domain massive MIMO channels demonstrate spatial sparsity. This is due to the fact that number of significant scaterrers is limited in wireless communication in fading environments. While communication between Base Station (BS) and users, the transmission distance is very large as compared to the distance between several antennas in an antenna array placed at BS. That is to say that CIRs associated with several transmitting antennas and single user exhibit exactly same path delays, therefore they also share identical common support of CIRs [40].

Let's consider a massive MIMO-OFDM system where M transmit antennas are placed at BS. The CIR between m-th transmit antena and one single-antenna user for z-th OFDM symbol is expressed by

$$\boldsymbol{d}_{z,m} = [d_{z,m}(1), d_{z,m}(2), \dots, d_{m,r}(L)]^T \qquad (3.4)$$

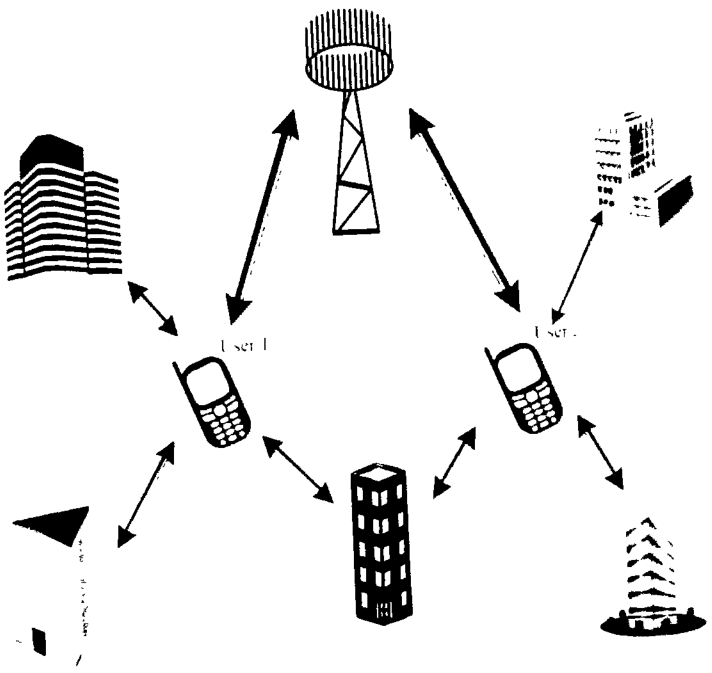
Where $1 \leq m \leq M$, L is the channel length equivalent to the maximum delay spread. Let $S_{z,m}$ be the sparsity level of CIR between one transmit-receive antenna pair i.e. the number of non zero elements in $\boldsymbol{d}_{z,m}$, the support of $\boldsymbol{d}_{z,m}$ can be expressed as

$$P_{z,m} = supp\{\boldsymbol{d}_{z,m}\} = \{l: |d_{z,m}[l]| > 0\}\, with\, 1 \leq l \leq L \qquad (3.5)$$

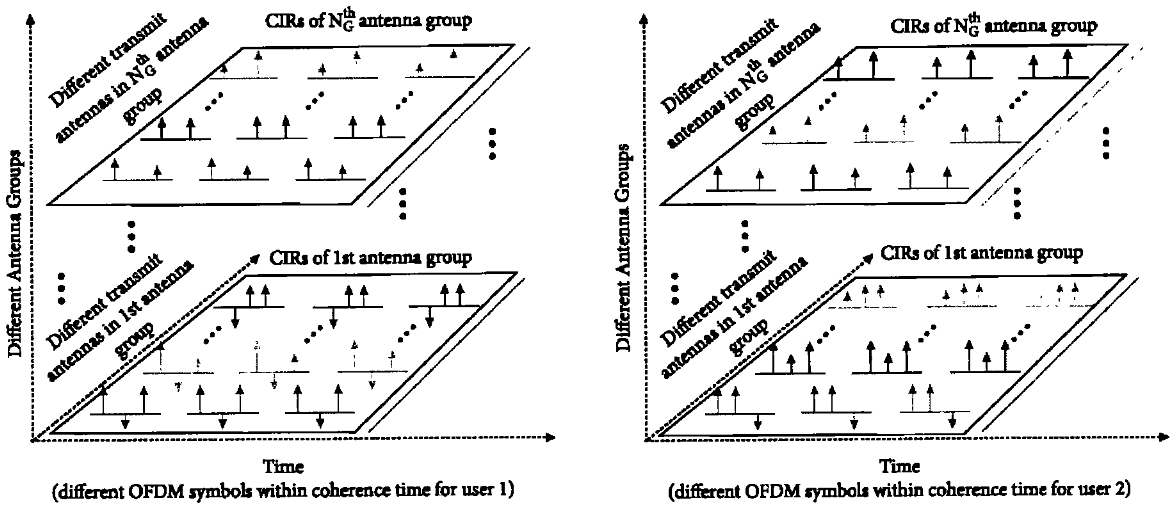
where $S_{z,m} = |P_{z,m}|_c$ fulfilling $S_{z,m} \ll L$. And due to spatial sparsity, we have

$$P_{z,1} = P_{z,2} = \cdots = P_{z,M} \qquad (3.6)$$

The delay domain spatial sparsity with specific system parameters will be detailed in section 3.2.8.



(a)



(b)

Figure 3.1 Delay domain temporal and spatial sparsity of massive MIMO channels:

(a) Limited number of scatterers and common scatterers in wireless communication scenario;
(b) Temporal and Spatial sparsity of massive MIMO channels in delay domain between two users and co-located antenna array

Figure 3.1 shows the common sparse pattern of CIRs for different transmit-receive antenna pairs. The details about temporal and spatial sparsity are given in the following two subsections.

3.2.3 Delay Domain Temporal Sparsity

In [40], the authors have explored that fast time varying channels to illustrate temporal correlation. That is to say that the variations in path gains are significant whereas the path delays are almost invariant for various consecutive OFDM symbols. The reason is that path delays variation duration over time-varying channels and the signal bandwidth are inversely proportional to each other while the coherence time of path gains and carrier frequency of the system are inversely proportional [29].

Consider a system with signal bandwidth B= 10 MHz and carrier frequency of system f_c= 2 GHz, the path delays fluctuate slower than the path gains [16]. Therefore, due to the nearly invariant path delays the CIRs share the common sparsity for R adjacent OFDM symbols over the coherence time of path delays. The supports of CIRs associated with R successive OFDM symbols signify that:

$$P_{1,m} = P_{z,m} = \cdots = P_{R,m} \ , 1 \leq m \leq M \qquad (3.7)$$

Figure 3.1. demonstrates the temporal sparsity of massive MIMO channels.

3.2.4 Proposed Non-orthogonal Pilot Scheme

In this section we shall propose a nonorthogonal pilot scheme based on the above-mentioned two observations. First the basic idea to divide the antennas placed at BS into subgroups is proposed. The proposed scheme of dividing the antennas into subgroups is based on system parameters and the temporal and spatial sparsity of massive MIMO channels. Then after creating the antenna groups, a specific nonorthogonal pilot scheme is proposed.

3.2.5 Proposed Scheme for Creating the Antenna Groups

There will be the uniform distribution of antennas to be included in sub-groups. Consider a system with signal bandwidth B, system carrier frequency f_c, total number of antennas placed at the BS in a uniform linear antenna array *M*, number of subgroups N_g, number of antennas in one group M_{ng}. The maximum resolvable distance D_{max} must be smaller than c/10B [39], that is to say, to have two channel taps resolvable, the time interval of arrival must be less than 1/10B, where c is the speed of light [39]. The two successive antennas are spaced by the distance $d_m = \lambda/2$, therefore the maximum distance between two antennas can be $d_T = d_m(M-1)$ and the maximum distance between two antennas in a single group can be $d_{M_{ng}} = d_m(M_{ng}-1)$. In order to guarantee the spatial sparsity and based on the condition that $D_{max}/c < 1/10B$, the $d_{M_{ng}}$ must satisfy $d_{M_{ng}} < D_{max}$.

And the formula for number of antennas M_{ng} in each sub-group N_g can be derived as follow.

$$d_m\left(M_{ng}-1\right) \le \frac{c}{10B} \qquad (3.8)$$

$$\lambda/2\left(M_{ng}-1\right) \le \frac{c}{10B} \qquad (3.9)$$

$$M_{ng} \le \frac{c}{5B(\frac{c}{fc})} + 1 \qquad (3.10)$$

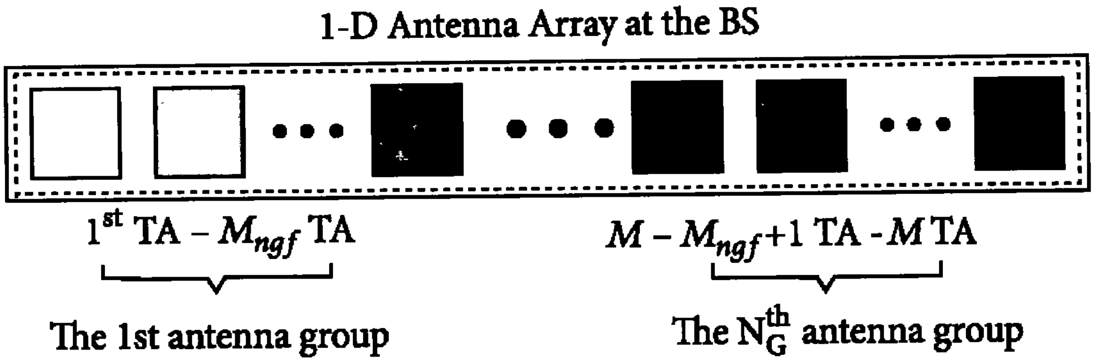


Figure 3.2 1-D antenna array at the BS

The N_g antenna groups are shown in Figure 3.2, where TA denotes the transmit antenna.

In the section 3.2.6, first we will derive a massive MIMO system model. And then specific pilot design will be proposed in section 3.2.7.

3.2.6 Massive MIMO System Model

Consider a massive MIMO OFDM system with M transmit antennas placed at BS communicating with one user, ξ_n represents the index set of pilot subcarriers for nth antenna group, the choice of ξ_n will be detailed in the section ii C). There are total N subcarriers in one OFDM symbols out of which N_p corresponds to pilot subcarriers and the pilot sequence of m-th transmit antenna in n-th antenna group is denoted by $\mathbf{s}_{m,n} \in \mathbb{C}^{N_p x 1}$. $\mathbf{y}_{z,n} \in \mathbb{C}^{N_p x 1}$ is the received vector of the pilot sequence of z-th OFDM Symbol of n-th antenna group at user for N_g antenna groups, $\mathbf{y}_{z,n}$ can be expressed as.

$$\mathbf{y}_{z,n} = \sum_{m=1}^{M_{ngf}} \mathrm{diag}\{\mathbf{s}_{m,n}\}\mathbf{R}|_{\xi_{n},n} \begin{bmatrix} \mathbf{d}_{m,z,n} \\ \mathbf{0}_{(N-L)\times 1} \end{bmatrix} + \mathbf{w}_{z,n} \quad (3.13)$$

Which clearly shows that M_{ng} is a function of B and f_c. Therefore the N_g can be given by

$$N_g = \left\lfloor \frac{M}{M_{ng}} \right\rfloor + \alpha \qquad (3.11)$$

The constant α represents an integer to make sure the uniform distribution of antennas in each sub-group i.e. M is completely divisible by N_g. Similarly, actual number of antennas M_{ngf} in each group can be calculated by

$$M_{ngf} = M/N_g \qquad (3.12)$$

The formula for N_g will ensure the spatial sparsity for any massive MIMO system with large 1-D antenna array placed at BS. Moreover, since the M_{ng} is a function of system parameters B and f_c, therefore if the B and f_c are changed but the N_g still ensures the spatial sparsity for antennas. And also N_g is the minimum number of sub-groups a system must have to ensure spatial sparsity, moreover by increasing N_g, spatial sparsity of system will remain preserved. For example, consider a system with following specifications:

M=128 1-D antenna array, f_c= 2 GHz, B= 20 MHz, M_{ng} given in eq. (3.10) can be calculated as 21. N_g will be equal to 8 with $\alpha = 2$ and M_{ngf} will be equal to 16. Based on these calculations the initial condition is satisfied that $d_{M_{ng}} < D_{max}$, hence the spatial sparsity is preserved for the system.

With $1<n<N_g$

$$\mathbf{y}_{z,n} = \sum_{m=1}^{M_{ngf}} \mathbf{S}_{m,n}\mathbf{R}_L|_{\xi_n,n}\, \mathbf{d}_{m,z,n} + \mathbf{w}_{z,n} = \sum_{m=1}^{M_{ngf}} \boldsymbol{\psi}_{m,n}\mathbf{d}_{m,z,n} + \mathbf{w}_{z,n} \qquad (3.14)$$

Where $\mathbf{R} \in \mathbb{C}^{NxN}$ is the Discrete Fourier Transform matrix (DFT), $\mathbf{y}_{z,n}$ is expressed after exclusion of guard interval, $\mathbf{d}_{m,z,n} \in \mathbb{C}^{Lx1}$ is the CIR vector of m_{th} transmit antenna of nth antenna group for z_{th} OFDM symbol, $\mathbf{R}|_{\xi_n,n} \in \mathbb{C}^{N_pxN}$ is a submatrix comprised of N_p rows selected according to ξ_n, $\mathbf{R}_L|_{\xi_n,n} \in \mathbb{C}^{N_pxL}$ contains the first L columns of $\mathbf{R}|_{\xi_n,n} \in \mathbb{C}^{N_pxN}$, $\mathbf{S}_{m,n} = \text{diag}\{\mathbf{s}_{m,n}\} \in \mathbb{C}^{N_pxN_p}$, $\boldsymbol{\psi}_{m,n} = \mathbf{S}_{m,n}\mathbf{R}_L|_{\xi_n,n} \in \mathbb{C}^{N_pxL}$ and $\mathbf{w}_{z,n}$ represents the Additive White Gaussian Noise of nth antenna group for zth OFDM symbol.

The equation can be rearranged as

$$\mathbf{y}_{z,n} = \boldsymbol{\psi}_n\tilde{\mathbf{d}}_{z,n} + \mathbf{w}_{z,n} \qquad (3.15)$$

Where $\boldsymbol{\psi}_n = [\boldsymbol{\psi}_{1,n}, \boldsymbol{\psi}_{2,n}, \dots, \boldsymbol{\psi}_{M_{ngf},n}] \in \mathbb{C}^{Np\times M_{ngf}L}$ and the aggregate CIR vector of nth antenna group is given by $\tilde{\mathbf{d}}_{z,n} = \left[\mathbf{d}_{1,z,n}, \mathbf{d}_{2,z,n}, \dots, \mathbf{d}_{M_{ngf},z,n}\right]^T \in \mathbb{C}^{M_{ngf}L\times 1}$. As explained earlier that the CIR vector $\mathbf{d}_{m,z,n}$ exhibit spatial and temporal sparsity therefore $\tilde{\mathbf{d}}_{z,n}$ is also a sparse signal. The system in equation is an under determined system due to the fact that $N_p < M_{ngf}L$ and cannot be reliably solved using the traditional channel estimation methods [40].

Moreover the equation can be rearranged into structured sparse form according to [40] as

$$\mathbf{y}_{z,n} = \mathbf{A}_n\tilde{\mathbf{h}}_{z,n} + \mathbf{w}_{z,n} \qquad (3.16)$$

Where $\tilde{\mathbf{h}}_{\mathrm{z,n}} = \left[\mathbf{h}_{1,\mathrm{z,n}}^{\mathrm{T}}, \mathbf{h}_{2,\mathrm{z,n}}^{\mathrm{T}}, \dots, \mathbf{d}_{\mathrm{L,z,n}}^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{M_{ngf}L\times 1}$ is an structured sparse equivalent CIR vector, $\mathbf{h}_{\mathrm{l,z,n}} = \left[d_{1,\mathrm{z,n}}[l], d_{2,\mathrm{z,n}}[l], \dots, d_{M_{ng},\mathrm{z,n}}[l]\right]^{\mathrm{T}}$ for $1 \leq l \leq L$ and after reformulation $\boldsymbol{\psi}_{\mathrm{n}}$ can be converted into $\mathbf{A}_{\mathrm{n}}$, where $\mathbf{A}_{\mathrm{n}}$ can be written as

$$\mathbf{A}_{\mathrm{n}} = \left[\mathbf{A}_{1,\mathrm{n}}, \mathbf{A}_{2,\mathrm{n}}, \dots \mathbf{A}_{\mathrm{L,n}}\right] \in \mathbb{C}^{N_p \times M_{ngf}L} \qquad (3.17)$$

Where $\mathbf{A}_{l,n} = \left[\boldsymbol{\psi}_{1,n}^{(l)}, \boldsymbol{\psi}_{2,n}^{(l)}, \dots, \boldsymbol{\psi}_{M_{ngf},n}^{(l)}\right] = \left[A_{1,n,l}, A_{2,n,l}, \dots A_{M_{ngf},n,l}\right] \in \mathbb{C}^{Np\times M_{ngf}}$

After converting to structured form, we will have N_g such equations to be solved simultaneously corresponding to each sub-group. The received pilot vectors of z_{th} OFDM symbol for N_g sub-groups can be expressed as

$$\left.\begin{aligned} \mathbf{y}_{\mathrm{z,1}} &= \mathbf{A}_1\tilde{\mathbf{h}}_{\mathrm{z,1}} + \mathbf{w}_{\mathrm{z,1}} \\ \mathbf{y}_{\mathrm{z,2}} &= \mathbf{A}_2\tilde{\mathbf{h}}_{\mathrm{z,2}} + \mathbf{w}_{\mathrm{z,2}} \\ &\vdots \\ \mathbf{y}_{\mathrm{z,n}} &= \mathbf{A}_{\mathrm{n}}\tilde{\mathbf{h}}_{\mathrm{z,n}} + \mathbf{w}_{\mathrm{z,n}} \\ &\vdots \\ \mathbf{y}_{\mathrm{z},N_g} &= \mathbf{A}_{N_g}\tilde{\mathbf{h}}_{\mathrm{z},N_g} + \mathbf{w}_{\mathrm{z},N_g} \end{aligned}\right\} \qquad (3.18)$$

The system equation of each sub-group exhibits structured sparsity, which provides the motivation to apply CS theory to recover the high dimension structured sparse signal $\tilde{\mathbf{h}}_{\mathrm{z},N_g}$ from the low dimension received pilot vector $\mathbf{y}_{\mathrm{z},N_g}$. The CS based sparse signal recovery/channel estimation will be explained in section 3.2.8.

3.2.7 Proposed Pilot Design

Mostly wireless communication systems detect the data with the aid of pilot signals. Specifically the purpose of pilot signals is to assist the receiver to estimate the wireless channels and then the receiver coherently detects the data on the basis of estimated channel [47] .

In massive MIMO communication systems, due to huge number of transmit antennas at BS, the number of channels turn into prohibitively high and thus result in high pilot overhead for estimation of these channels. Therefore, there is need of methods to reduce the high pilot overhead in massive MIMO communication systems to achieve the target of high data rate.

In this thesis CS theory is implemented to reduce the high pilot overhead based on the fact that the wireless channels undergo spatial and temporal sparsity. CS based pilot design significantly reduces the pilot overhead as compare to conventional pilot design. Figure 3.3 demonstrated the proposed, uniformly distributed and identical pilot subcarrier for multiple antennas, while Figure 3.4 shows the convention pilot design (i.e orthogonal pilots for different antennas). The Proposed pilot design allows the system to provide more resources to data.

The CS based pilot design permits the antennas of each sub-group to occupy identical subcarriers for pilots within a group while pilot subcarriers for each sub-group are completely different from each other as shown in Figure 3.3. The design is based on the choice of ξ_n. The guard interval can be calculated by $N_G = \frac{N}{B\text{x(maximum channel delay spread)}}$. Consider a set $O = \{1,2,\dots,N-N_G\}$, The ξ_n is a subset of O, represents the index of pilot subcarriers for n-th sub-group and it is identical for all the antennas within that group, the formula for creating the ξ_n can be given by

$$\xi_n = \{n + spacing * q: \left\lfloor \frac{M}{M_{ng}} \right\rfloor - \alpha \leq N_g \leq spacing, \quad 0 < q < N_p - 1 \text{ and } sapcing = \left\lfloor \frac{N-N_G}{N_p} \right\rfloor \geq \left\lfloor \frac{M}{M_{ng}} \right\rfloor - \alpha\} \qquad (3.19)$$

Where $N_p > L$ and by solving the inequality $\left\lfloor \frac{N-N_G}{N_p} \right\rfloor \geq \left\lfloor \frac{M}{M_{ng}} \right\rfloor - \alpha$ we have

$$M_{ngf} \leq N_p \leq \frac{(N-N_G)}{(M-\alpha M_{ng})} M_{ng} \qquad (3.20)$$

There are N_G unique sets associated with ξ_n i.e. $\xi_1 \ldots\ldots \xi_n \ldots. \xi_{Ng}$, There will be total $N_g N_p$ subcarriers occupied for pilot vectors for M transmit antennas at BS. Figure 3.3 demonstrates the proposed pilot design.

$N_p = |\xi_n|_C$ is number of subcarriers for pilot vector in OFDM symbol of n_{th} sub-group.

Pilot subcarriers of n_{th} group are the null pilots for all the other $N_g - 1$ sub-groups where as $N - N_G - N_g N_p$ subcarriers are available for data.

Pilot overhead ratio is defined by $\beta_p = \frac{N_g N_p}{N}$.

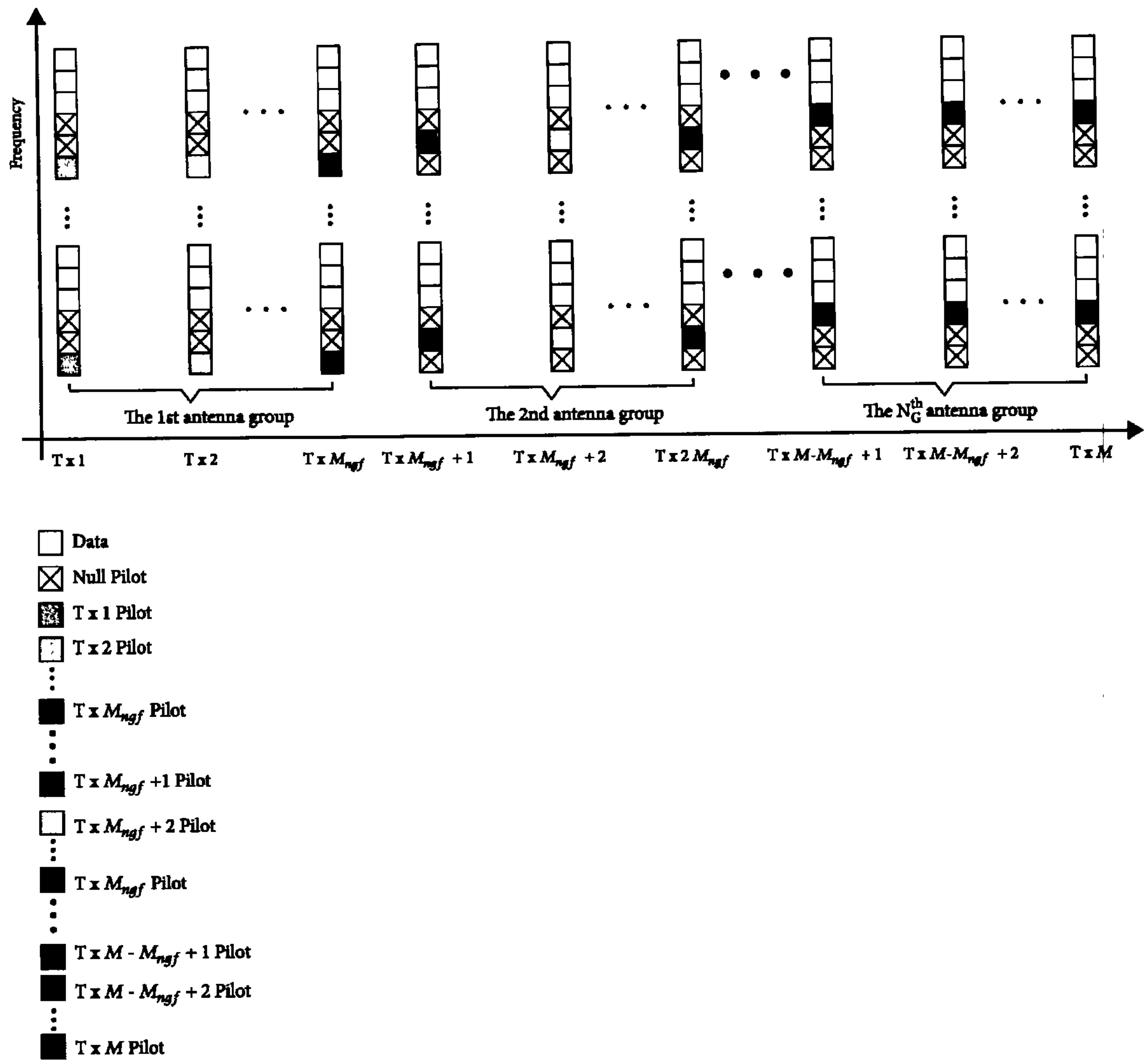


Figure 3.3 Proposed pilot design.

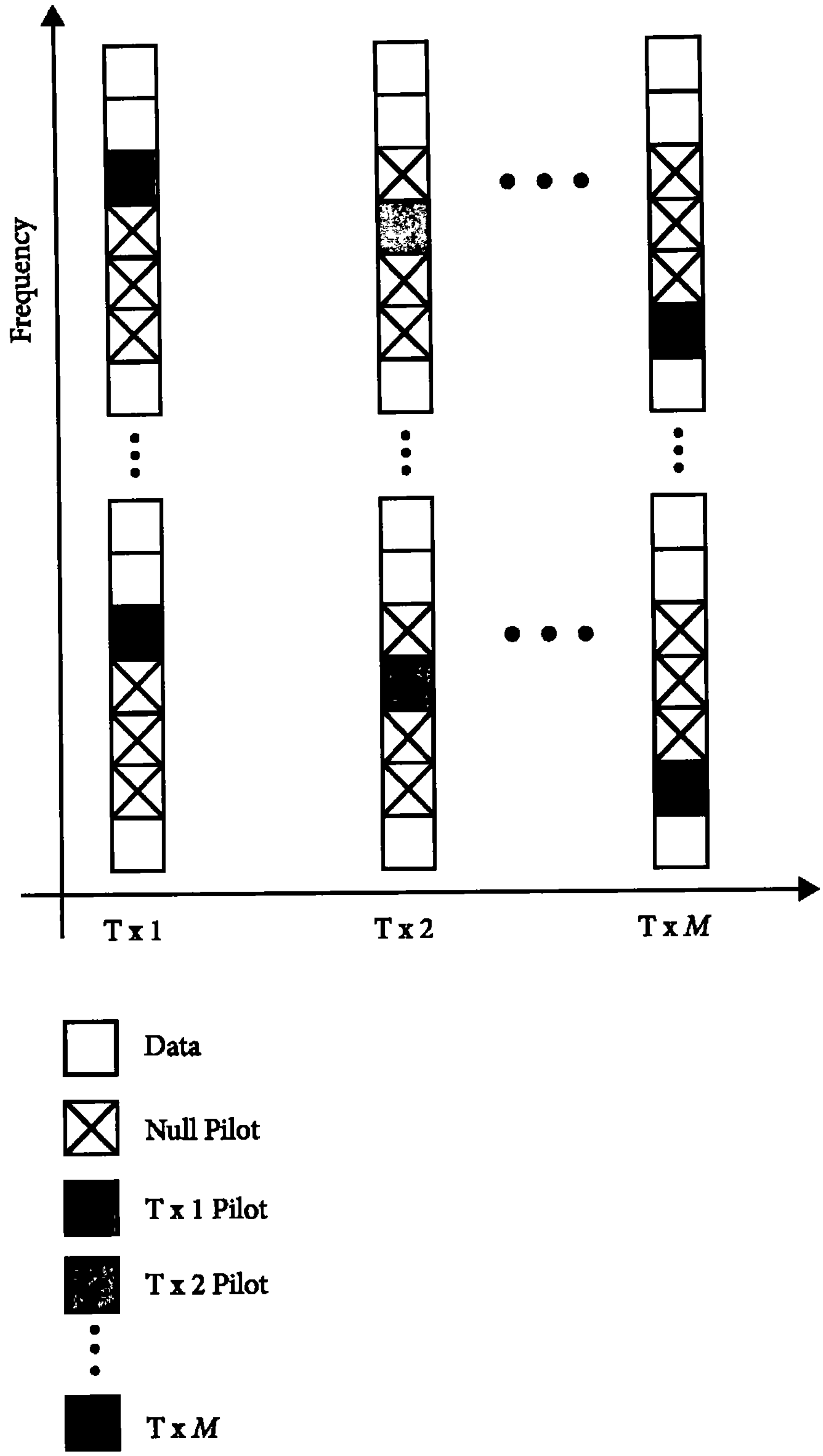


Figure 3.4 Conventional orthogonal pilot design.

Figure 3.4 shows the conventional pilots in which different pilots are allocated to different antennas resulting in very high pilot overhead.

3.2.8 Massive MIMO Channel Estimation Based on Compressive Sensing

The basic theory of CS is already presented in section 3.2.1. The idea is to consider the task of channel estimation as sparse recovery problem.

In eq. (3.16) it can be seen $\tilde{\mathbf{h}}_{\mathrm{z,n}}$ demonstrate structured sparsity in delay domain, where $\tilde{\mathbf{h}}_{\mathrm{z,n}}$ is $S_{z,m,n}$-sparse vector due to

$$P_{z,m,n} = supp\{\tilde{\mathbf{h}}_{\mathrm{z,n}}\} = \{l: |\tilde{h}_{z,n}[l]| > 0\}\, with\, 1 \leq l \leq L \qquad (3.21)$$

where $S_{z,m,n} = |P_{z,m,n}|_C$ fulfilling $S_{z,m,n} \ll L$. And due to spatial sparsity we have

$$P_{z,1,n} = P_{z,2,n} = \cdots = P_{z,M,n} \qquad (3.22)$$

The desired small correlation of $\mathbf{A}_{\mathrm{n}}$ according to CS theory is achieved which is described in [40], therefore reliable sparse recovery is guaranteed. It is further shown in [40] that the any two columns of $\mathbf{A}_{\mathrm{n}}$ attain excellent cross correlation between them according to RMT since pilot design proposed in [40] is the special case of proposed pilot design. Therefore the proposed pilot design is simple to employ and also supportive in terms of compatibility with current wireless networks [40].

For K adjacent OFDM symbols having identical pattern of pilots, we have

$$\mathbf{Y}_{\mathrm{n}} = \mathbf{A}_{\mathrm{n}}\mathbf{H}_{\mathrm{n}} + \mathbf{W}_{\mathrm{n}} \qquad (3.23)$$

Where $\mathbf{Y}_{\mathrm{n}} = [\mathbf{y}_{\mathrm{z,n}}, \mathbf{y}_{\mathrm{z+1,n}}, \dots, \mathbf{y}_{\mathrm{z+}K\mathrm{-1,n}}]\mathbb{C}^{Np\times K}$

The $\mathbf{A}_{\mathrm{n}}$ derived in massive MIMO model satisfies the Structure Restricted Isometric Property (SRIP) condition. Specifically, SRIP can be given by

$$\sqrt{1-\delta}\left\|\mathbf{A}_{N_g}\right\|_F \leq \left\|\mathbf{A}_{\mathrm{n}}\tilde{\mathbf{h}}_{\mathrm{z,n}}\right\|_F \sqrt{1-\delta}\left\|\mathbf{A}_{N_g}\right\|_F \qquad (3.24)$$

The definition and justification for SRIP is discussed in detail in [40]. Where $\delta \in [0,1)$.

Our aim is to recover $\tilde{\mathbf{h}}_{\mathrm{z,n}}$, given $\mathbf{y}_{\mathrm{z,n}}$. Under the framework of CS theory, the massive MIMO channels $\tilde{\mathbf{h}}_{\mathrm{z,n}}$ are estimated by means of following problem

$$\hat{\mathbf{h}} = \arg\min\left\|\tilde{\mathbf{h}}_{\mathrm{z,n}}\right\|_1 \text{ s.t. } \left\|\mathbf{y}_{\mathrm{z,n}} - \mathbf{A}_{\mathrm{n}}\tilde{\mathbf{h}}_{\mathrm{z,n}}\right\|_2 < \epsilon \qquad (3.25)$$

Where ϵ is the noise variance. There are many algorithms that can solve the problem. For example, projected gradient methods or interior point methods can be used for applying convex optimization. A famous greedy algorithm is orthogonal matching pursuit (OMP).

For channel estimation purpose we propose the SUCoSaMP algorithm derived from basic CoSaMP as described in Algorithm 1. There will be N_g similar parallel processing required for estimating the massive MIMO channels with N_g sub-antenna groups i.e. the same algorithm will be working simultaneously at user to estimate channels of N_g sub-groups.

Algorithm 1. Proposed SUCoSaMP Recovery Algorithm

Input: Sensing matrix $\mathbf{A}_n$ and noisy measurement vector $\mathbf{y}_{\mathrm{z,n}}$
Output: An sparse estimation $\hat{\mathbf{h}}$ of channels $\{\mathbf{d}_{\mathrm{m,z,n}}\}_{m=1,\ \ n=1}^{m=M_{ngf},\ n=N_g}$

Step 1 (Initialization)

1. $\check{\mathbf{h}}^0 \leftarrow 0$

Trivial initial approximation

2. $v \leftarrow \mathbf{y}_{\mathrm{z,n}}$

Current samples = input samples

3. $k \leftarrow 0$

Iterative index

4. $s \leftarrow 1$

Initial sparsity level

Step 2 Solve the structure sparse vector $\tilde{\mathbf{h}}_{\mathrm{z,n}}$ to eq. (2.18)

Repeat

1. $k \leftarrow k + 1$
2. $y \leftarrow \mathbf{A}_{\mathrm{n}}^{*}\mathbf{v}$

Make the signal proxy

3. $\Omega \leftarrow \mathrm{supp}(\mathbf{y}_{2s})$

Identify large components

4. $\mathrm{T} \leftarrow \Omega \cup \mathrm{supp}(\check{\mathbf{h}}^{k-1})$

Merge supports

5. $\check{\boldsymbol{h}}|_T \leftarrow \boldsymbol{A}_{nT}^{\dagger}\mathbf{y}_{\mathrm{z,n}}$

Signal estimation by least squares

6. $\check{\boldsymbol{h}}|_{T^C} \leftarrow 0$
7. $\check{\mathbf{h}}^k \leftarrow \check{\boldsymbol{h}}_s$

Prune to get next approximation

8. $v \leftarrow \mathbf{y}_{\mathrm{z,n}} - \mathbf{A}_n\check{\mathbf{h}}^k$

Update the current samples

if $\|\boldsymbol{v}^k\|_2 < \|\boldsymbol{v}^{k+1}\|_2$

9. Iteration with fixed sparsity level

else

10. Update sparsity level $\check{\mathbf{h}}_s \leftarrow \check{\mathbf{h}}^k$; $\boldsymbol{v}_s \leftarrow \boldsymbol{v}^k$; $s \leftarrow s+1$

end if

Until stopping criterion true

Step 3 Obtain channels $\widehat{\boldsymbol{h}} = \check{\mathbf{h}}_{s-1}$ and obtain estimation of channels $\{\mathbf{d}_{\mathrm{m,z,n}}\}_{m=1,\ \ n=1}^{m=M_{ngf},\ n=N_g}$ according to (3.14)-(3.18)

There are many natural approaches of stopping the algorithm. We follow the stopping criterion as if $\|\boldsymbol{v}^{k+1}\|_2 > \|\boldsymbol{v}_{s-1}\|_2$ the iteration is stopped [40]. The information of correct sparsity level $S_{z,m,n}$ is usually not available and also it is practically not possible to have prior knowledge of correct sparsity level. Whereas information about sparsity level plays the significant role in compressive sensing problem of solving underdetermined system and it is also required as prior information by most of the CS based algorithms. The proposed SUCoSaMP algorithm does not require prior information of sparsity level because it adaptively acquires the sparsity level and avoids the unrealistic assumption of having prior information of correct sparsity level.

In steps 2.1 to 2.9 the target of SUCoSaMP algorithm is to obtain the solution $\tilde{\mathbf{h}}_{\mathrm{z,n}}$ to eq. (3.18) with fixed sparsity level s similar to conventional CoSaMP. The condition $\|\boldsymbol{v}^k\|_2 \leq \|\boldsymbol{v}^{k+1}\|_2$ shows that the solution $\tilde{\mathbf{h}}_{\mathrm{z,n}}$ to eq. (3.18) has been acquired and then the new iteration is started with updated sparsity level s+1. This process is repeated until the stopping criteria is true and then the iteration is stopped. We get the solution to eq. (3.18) with updated sparsity level and obtain the channels i.e. $\widehat{\boldsymbol{h}} = \check{\mathbf{h}}_{s-1}$.

Note that the proposed SUCoSaMP algorithm works in the same way as the CoSaMP algorithm works. The CoSaMP algorithm is basically based on basic OMP. The SUCoSaMP algorithm has incorporated some other ideas from the literature to present strong guarantees that OMP and CoSaMP cannot satisfy and to speed up the algorithm as compared to OMP.

The major advantage of SUCoSaMP over OMP, CoSaMP and basic subspace pursuit (SP) algorithms is that it does not require prior acquaintance of sparsity level, which is an unrealistic assumption, specifically in wireless communication scenario.

There are two differences between SUCoSaMP and CoSaMP. Firstly, SUCoSaMP estimates the channels i.e. it recovers the high dimensional sparse vector by utilizing structured sparsity of massive MIMO channels from one vector of low dimension. Secondly, SUCoSaMP adaptively acquires the sparsity level. In contrast, the CoSaMP recovers the sparse vector without exploiting the structured sparsity and it requires prior information of correct sparsity level.

The proposed SUCoSaMP algorithm is exactly the same as CoSaMP algorithm up to step 2.9. The proposed algorithm stops the iteration with fixed sparsity level (i.e. the current value of s) if $\|\boldsymbol{v}^k\|_2 > \|\boldsymbol{v}^{k+1}\|_2$ and then it performs an additional step that is step # 2.10 to update the sparsity level by adding 1 to current value of sparsity level (i.e. $s \leftarrow s + 1$).

There are two different types of iterations in SUCoSaMP, one on k and one on s and finally the iterations on s are stopped if $\|\boldsymbol{v}^{k+1}\|_2 > \|\boldsymbol{v}_{s-1}\|_2$. Table 1 elaborates some further details of major steps involved in CoSaMP and SUCoSaMP algorithms. And Table 2 demonstrates the hypothesis of CoSaMP and SUCoSaMP algorithms.

Table 1. Major steps involved in CoSaMP and SUCoSaMP		
	CoSaMP	**SUCoSaMP**
1. **Classification.**	The algorithm creates a replacement of the residual from the existing samples and places the biggest components of the replacement.	Follows the same step of **CoSaMP**.
2. **Support union.**	The set of recently recognized components is joined with the set of com- ponents that emerge in the present approximation.	Follows the same step of **CoSaMP**.
3. **Estimation.**	The algorithm finds the solution of a least-squares problem to estimate the objective signal on the combined set of components.	Follows the same step of **CoSaMP**.
4. **Pruning.**	The algorithm generates a fresh estimation by keeping only the biggest entries in this least-squares approximation of signal.	Follows the same step of **CoSaMP**.
5. **Sample Update.**	Finally, the algorithm updates the samples such that they reflect the residual, the un-approximated elements of the signal.	Follows the same step of **CoSaMP**.
6. **Stopping Criterion**	Until stopping criterion true.	Until first stopping criterion true.
7. **Sparsity level Update**	None	The algorithm updates the sparsity level
8. **Stopping Criterion**	None	Until second stopping criterion true.

Table 2. CoSaMP and SUCoSaMP Hypothesis		
	CoSaMP	**SUCoSaMP**
1.	The sparsity level s is fixed. (i.e. initialization with fixed value of sparsity level).	The sparsity level s is not fixed (i.e. initialization with sparsity level equals to 1). It adaptively acquires the correct sparsity level.
2.	The sensing matrix $\mathbf{A}_n$ has restricted isometry constant $\delta_{4s} \leq 0.1$.	$\mathbf{A}_n$ satisfies the Structure Restricted Isometric Property (SRIP) condition according to [40].
3.	The signal $\tilde{\mathbf{h}}_{\mathrm{z,n}} \in \mathbb{C}^{M_{ngf}L\times 1}$ is random, except where prominent.	The signal $\tilde{\mathbf{h}}_{\mathrm{z,n}} \in \mathbb{C}^{M_{ngf}L\times 1}$ is a structured sparse equivalent CIR vector.
4.	$\mathbf{w}_{\mathrm{z,n}}$ represents arbitrary noise vector	$\mathbf{w}_{\mathrm{z,n}}$ represents the Additive White Gaussian Noise of nth antenna group for zth OFDM symbol

3.3 Experimentation Details

This section provides complete details of simulations setup. Simulations have been performed in MATLAB in order to verify the effectiveness of the proposed methods. Mean square error performance of proposed scheme is compared with the conventional OMP, CoSaMP, Structured Subspace Pursuit (SSP) and Adaptive Structured Subspace Pursuit (ASSP) algorithms. Simulation parameters are mentioned in the Table 3 for the proposed system. Two types of antenna arrays have been deployed, in first case the BS has 1-D 1x128 antenna array (M= 128) and in second case the BS has 1-D 1x256 antenna array (M= 256). The system bandwidth and carrier frequency are set to B = 20 MHz and f_c = 2 GHz respectively. There are N_g =8 sub-antenna groups when M= 128, while there are N_g = 16 sub-antenna groups when M= 256 with 16 transmit antennas in each group to make sure the spatial channel sparsity with-in group. The OFDM subcarriers are set as N = 2048, guard interval is N_G = 16 which could fight the delay spread up to 6.4 μs and 16QAM and 64QAM modulation is used. The numbers of pilot subcarrier N_p in OFDM symbol transmitted by each antenna in each antenna group and channel length L are varied over a reasonable range to verify the performance of the proposed system. The pilot positions are uniformly distributed according to eq. (16) and are identical for the entire antennas with-in one group. The number of multipath is randomly chosen and the channel multipath amplitudes and positions follow Rayleigh and random distribution, respectively.

Table 3. System Parameters

Parameter	**Type/Value**
Total number of transmit antenna	128, 256
Number of transmit antennas in one sub-group	16
Number of antennas groups (N_g)	8,16
Modulation	16 QAM, 64 QAM
Guard Interval	16
Number of pilot subcarriers (N_p)	16, 32, 64
System bandwidth	20 MHz
DFT size	2048

3.4 Summary

This chapter proposes a non-orthogonal pilot scheme and a CS based algorithm SUCoSaMP to estimate the massive MIMO wireless channels. The reduction of high pilot overhead in massive MIMO systems and the recovery capability when the sparsity level of massive MIMO channels is unknown are the focus of this research. The methods have been proposed by exploiting the spatial and temporal common sparsity of massive MIMO channels in the delay domain. Finally, experimentation details have been included in order to implement and verify the proposed methods.

Chapter 4. Results and Discussion

4.1 Simulation Results

In this section simulation results are presented and discussed. Simulations have been performed in MATLAB to validate the proposed methods. Mean square error performance of proposed scheme is compared with the conventional OMP, CoSaMP, Structured Subspace Pursuit (SSP) and Adaptive Structured Subspace Pursuit (ASSP) algorithms. Details about simulation setup and parameters for the proposed system are given in the Table 3.

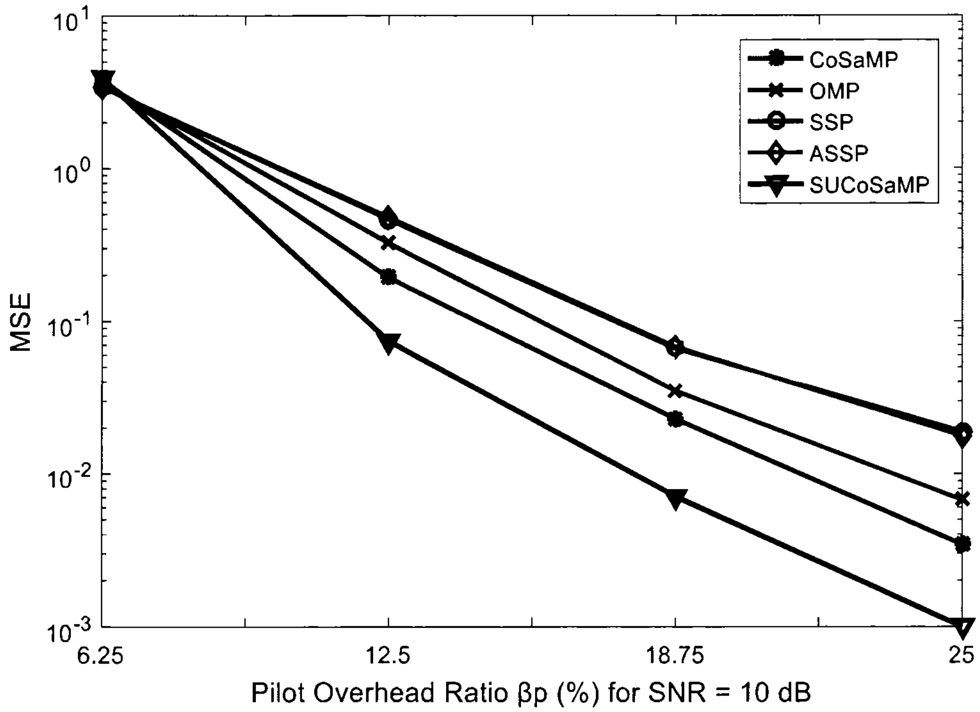


Figure 4.1 MSE Comparison of each algorithm under different Pilot Overhead Ratios: For SNR=10 dB, M=128, N_g = 8 and 16QAM

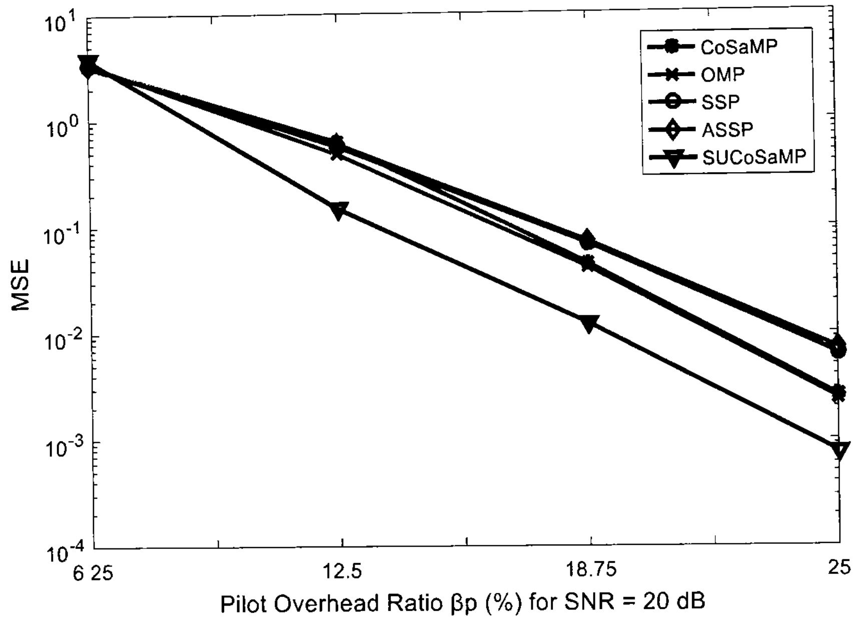


Figure 4.2 MSE Comparison of each algorithm under different Pilot Overhead Ratios: For SNR=20 dB, M=128, N_g = 8 and 16QAM

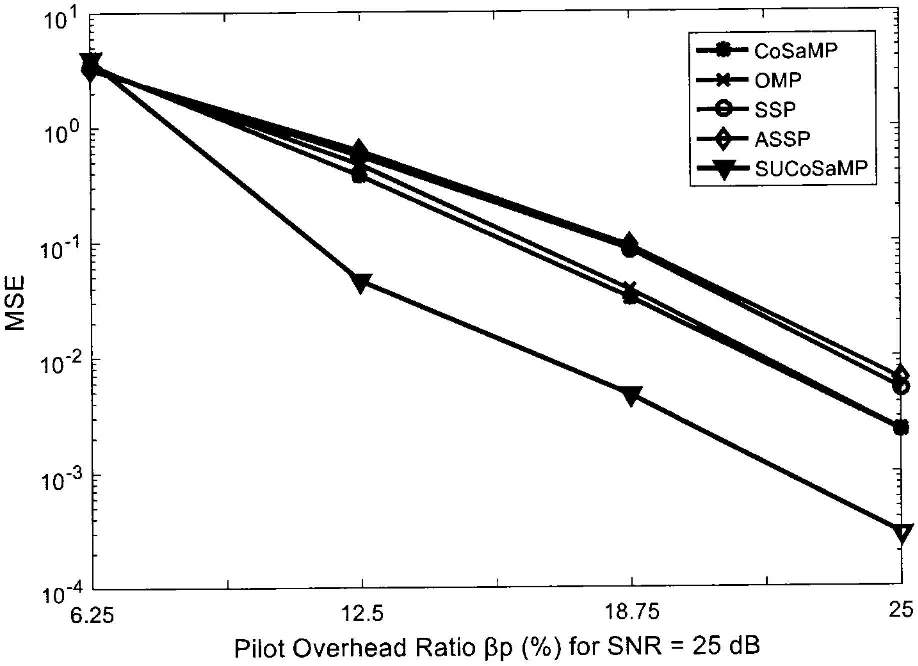


Figure 4.3 MSE Comparison of each algorithm under different Pilot Overhead Ratios: For SNR=25 dB, M=128, N_g = 8 and 16QAM

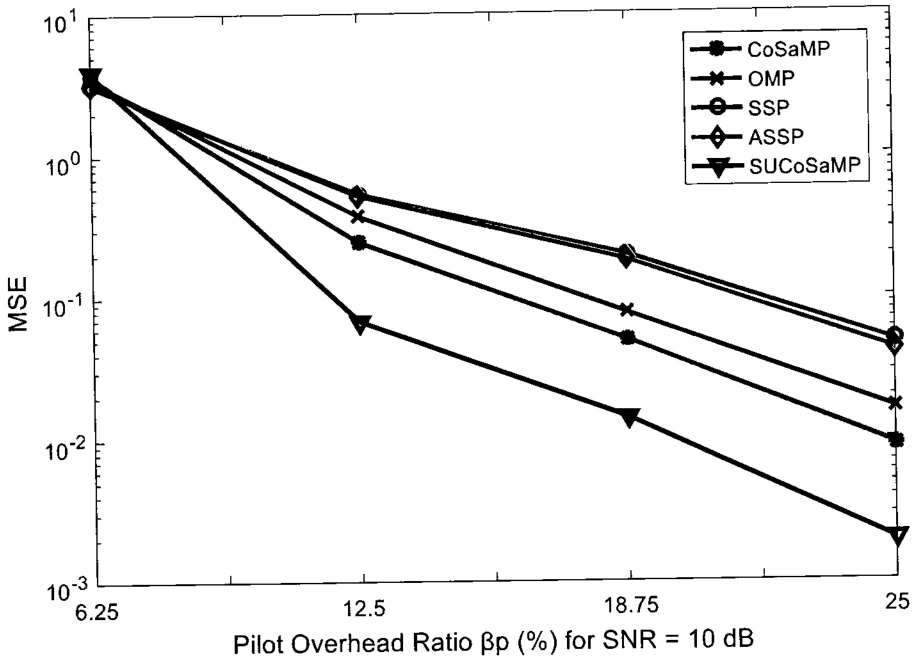


Figure 4.4 MSE Comparison of each algorithm under different Pilot Overhead Ratios: For SNR=10 dB, M=128, N_g = 8 and 64QAM

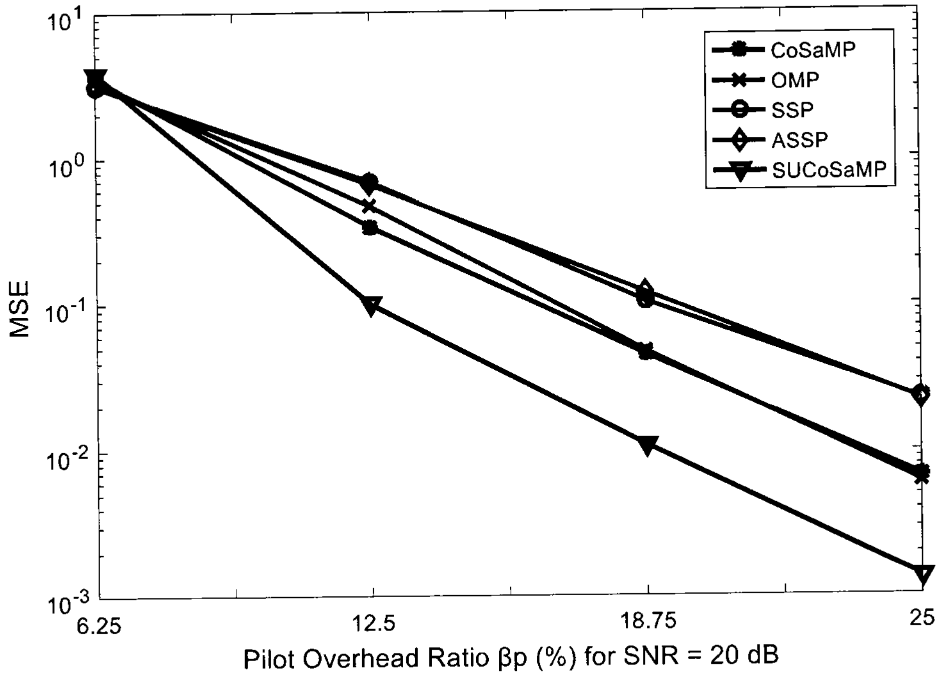


Figure 4.5 MSE Comparison of each algorithm under different Pilot Overhead Ratios: For SNR=20 dB, M=128, N_g = 8 and 64QAM

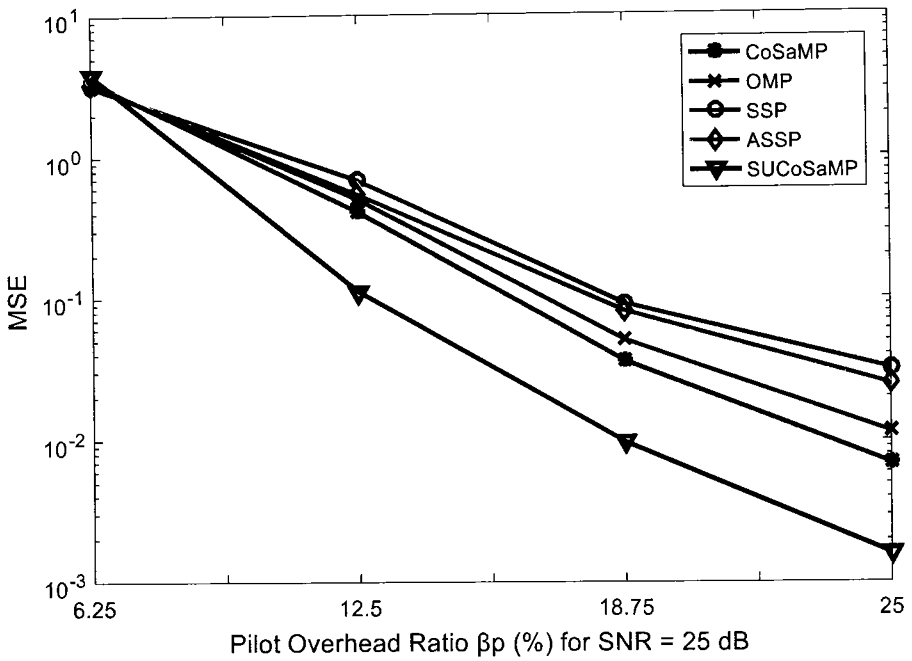


Figure 4.6 MSE Comparison of each algorithm under different Pilot Overhead Ratios: For SNR=25 dB, M=128, N_g = 8 and 64QAM

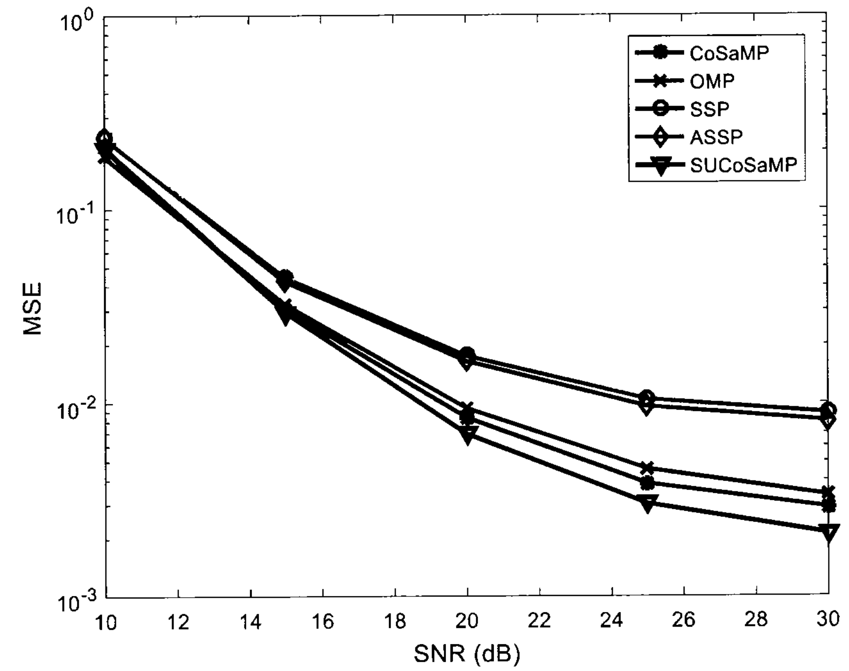


Figure 4.7 MSE Comparison of each algorithm under different SNRs: N_p = 32 Pilot M=128, N_g = 8 and 16QAM

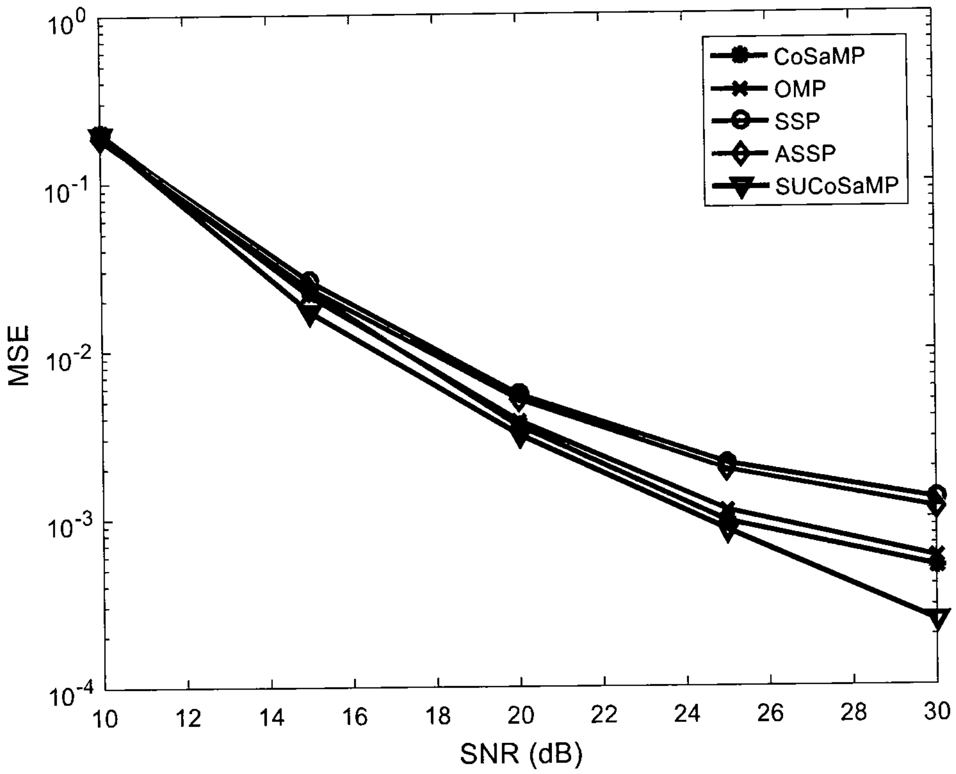


Figure 4.8 MSE Comparison of each algorithm under different SNRs: With $N_p = 64$ M=128, $N_g = 8$ and 16QAM

Figure 4.1 – 4.3 show the MSE Comparison of each algorithm under different Pilot Overhead Ratios with $M = 128$, $N_g = 8$ and 16QAM for SNR = 10, 20 & 25 dB respectively.

Figure 4.4 – 4.6 demonstrate the MSE Comparison of each algorithm under different Pilot Overhead Ratios with $M = 128$, $N_g = 8$ and 64QAM for SNR = 10, 20 & 25 dB respectively.

Figure 4.7 – 4.8 show the MSE performance of proposed SUCoSaMP algorithm with different values of N_p i.e. 32 and 64 respectively versus signal to noise ratio (SNR). In

Figures 4.1 to 4.8, the performance of SUCoSaMP is compared with the conventional algorithms OMP and CoSaMP and with SSP and ASSP. The sparsity level for OMP, CoSaMP and SSP is known in simulations whereas the ASSP and proposed SUCoSaMP adaptively acquire the correct sparsity level. It is observed that the channel estimation performance of all the algorithms is improved by increasing pilot subcarriers N_p. And overall, the proposed SUCoSaMP outperforms all the other algorithms. This is because the SUCoSaMP takes full advantage of structured sparsity of massive MIMO channels. Since the prior information of sparsity level of massive MIMO wireless channel is not a realistic assumption, therefore the proposed SUCoSaMP algorithm has a clear advantage over the conventional algorithms. Furthermore, the MSE gap between the proposed SUCoSaMP and the conventional algorithm remains almost constant as the SNR increases which makes SUCoSaMP superior in both low and high SNR wireless communication scenarios.

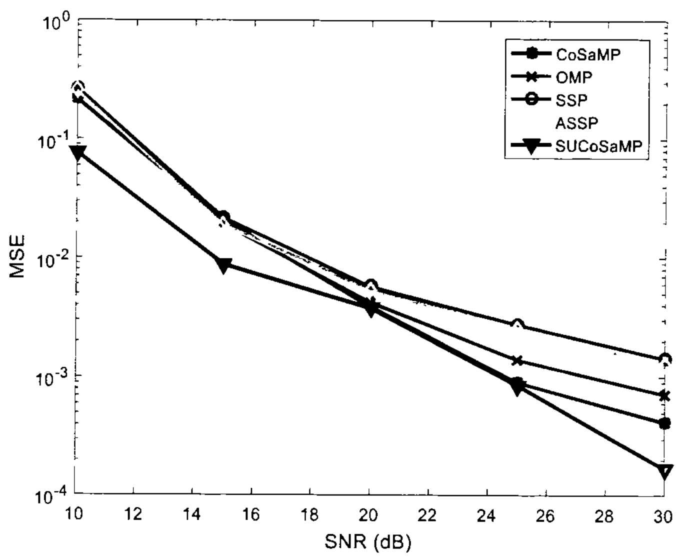


Figure 4.9 MSE performance comparison with different SNRs of 1st antenna group (N_1) with $N_p = 64$, M=128, $N_g = 8$ and 16QAM

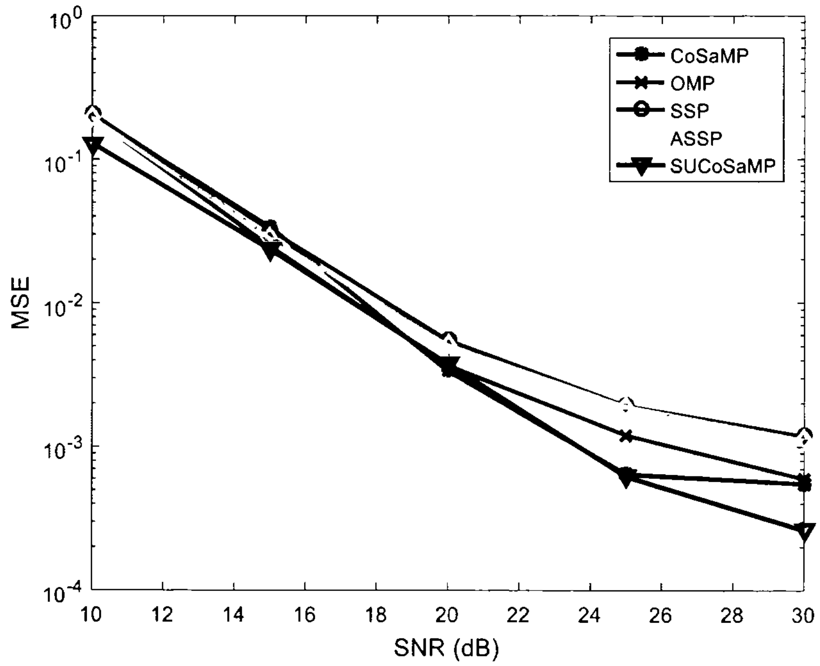


Figure 4.10 MSE performance comparison with different SNRs of 2nd antenna group (N_2) with $N_p = 64$, M=128, $N_g = 8$ and 16QAM

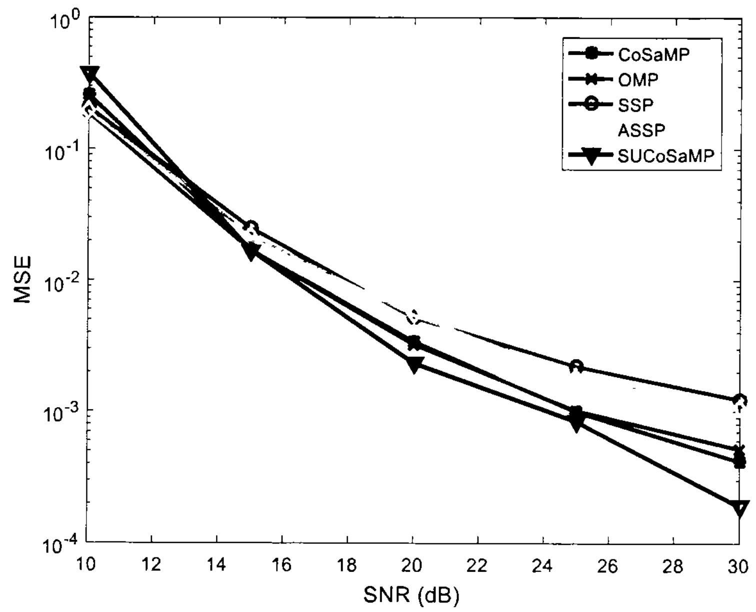


Figure 4.11 MSE performance comparison with different SNRs of 3rd antenna group (N_3) with $N_p = 64$, M=128, $N_g = 8$ and 16QAM

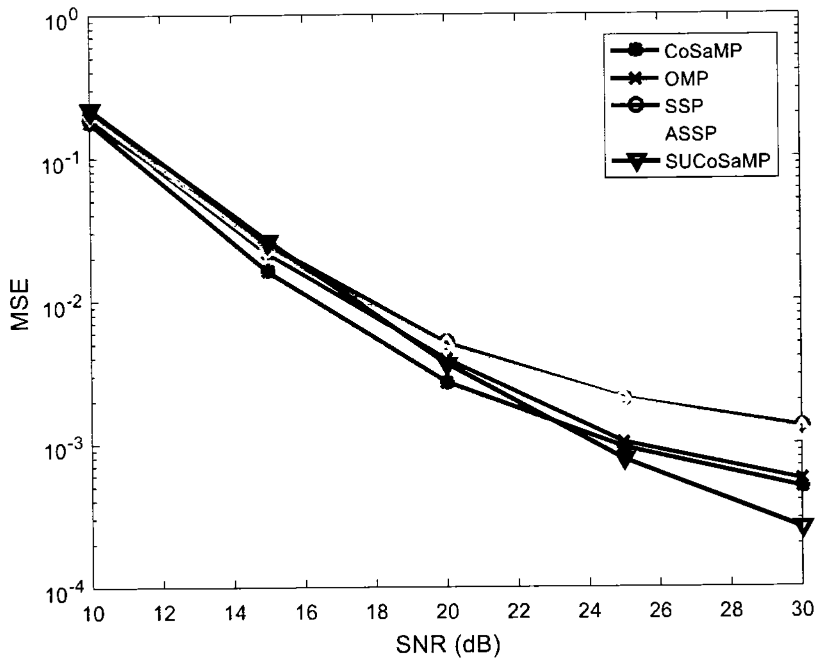


Figure 4.12 MSE performance comparison with different SNRs of 4th antenna group (N_4) with N_p = 64, M=128, N_g = 8 and 16QAM

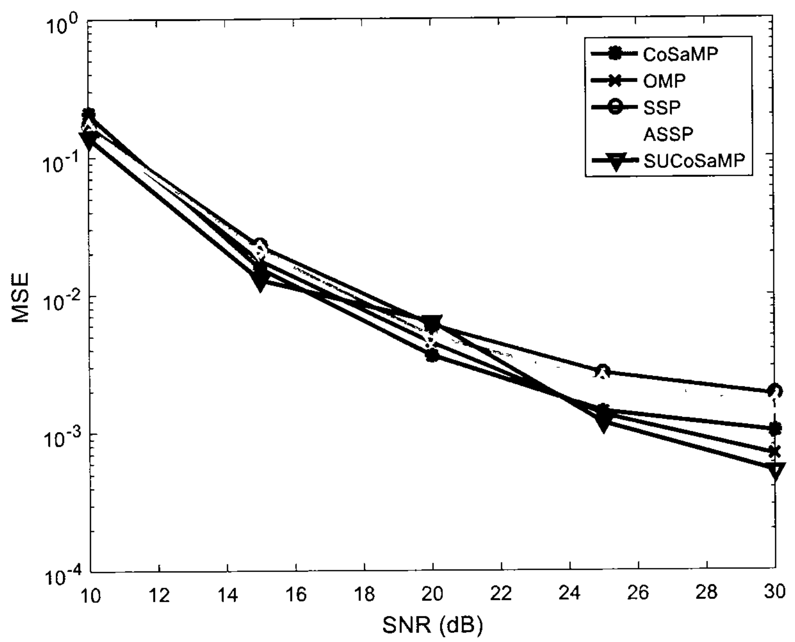


Figure 4.13 MSE performance comparison with different SNRs of 5th antenna group (N_5) with N_p = 64, M=128, N_g = 8 and 16QAM

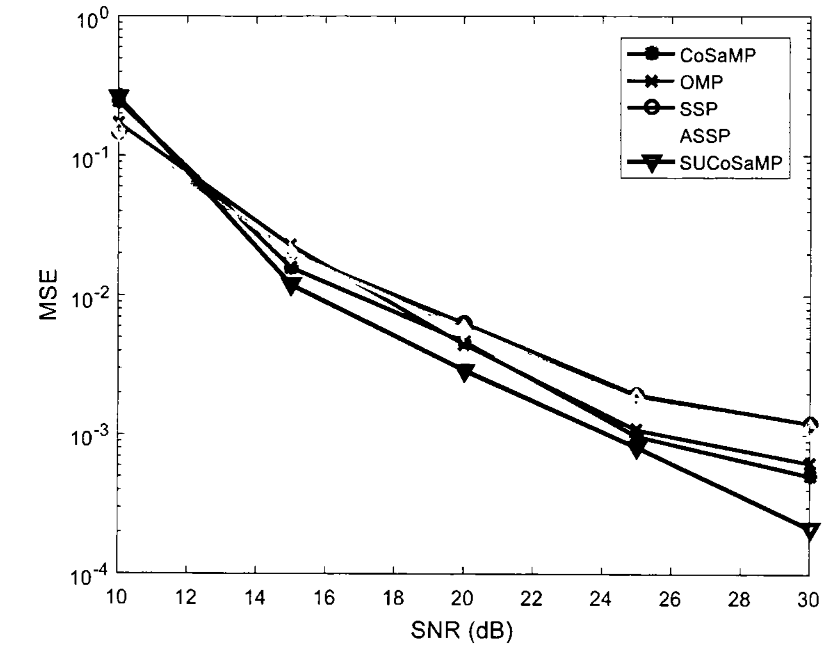


Figure 4.14 MSE performance comparison with different SNRs of 6th antenna group (N_6) with $N_p = 64$, M=128, $N_g = 8$ and 16QAM

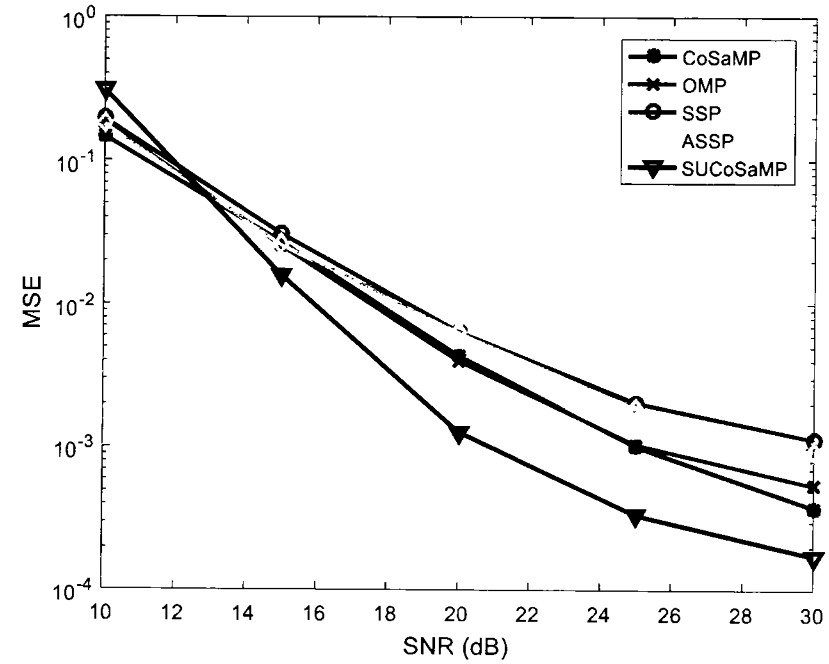


Figure 4.15 MSE performance comparison with different SNRs of 7th antenna group (N_7) with $N_p = 64$, M=128, $N_g = 8$ and 16QAM

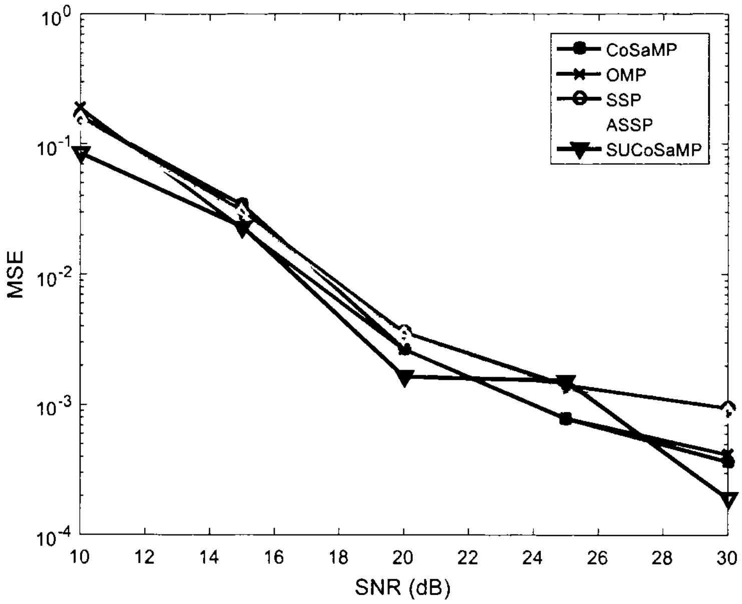


Figure 4.16 MSE performance comparison with different SNRs of 8th antenna group (N_8) with $N_p = 64$, M=128, $N_g = 8$ and 16QAM

In Figure 4.9 – 4.16, individual MSE performance versus SNR of different sub antenna groups (i.e. from antenna group N_1 to N_8) is presented with $N_p = 64$ for $M = 128$, $N_g = 8$ and 16QAM and is compared with conventional algorithms OMP and CoSaMP and with SSP and ASSP. It can be seen in the figures that SUCoSaMP is evidently superior to conventional algorithms. This is because the provided sparsity level to conventional algorithms cannot be the actual sparsity of massive MIMO wireless channels. However, the adaptive process in ASSP and SUCoSaMP is realized by fixed increment in sparsity level, therefore the sparsity level estimation in these algorithms is slightly over-estimated or under-estimated. It is again observed that with high number of pilot subcarriers N_p the performance of SUCoSaMP is improved along with the other algorithms. Furthermore, it is observed that the MSE gap between the proposed SUCoSaMP and the conventional algorithm slightly increases or remains almost constant with the increase in SNR resulting in SUCoSaMP better in different wireless communication environments with respect to SNR.

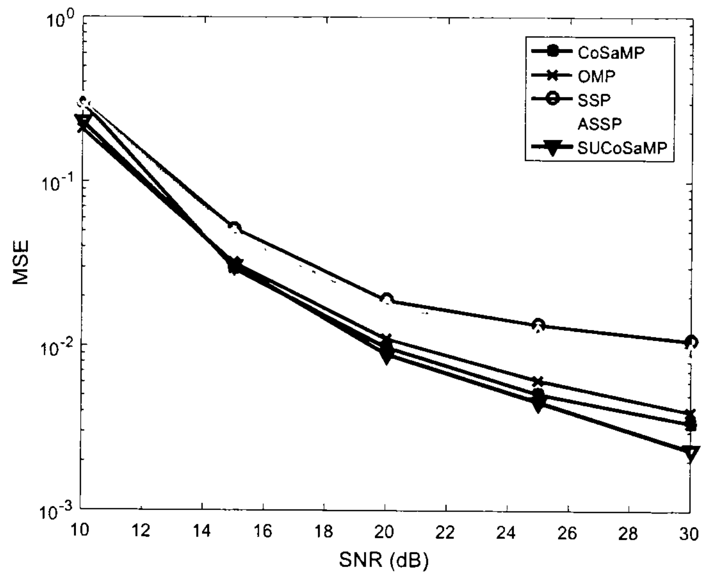


Figure 4.17 MSE performance comparison with different SNRs of 1st antenna group (N_1) with $N_p = 32$, M=128, $N_g = 8$ and 16QAM

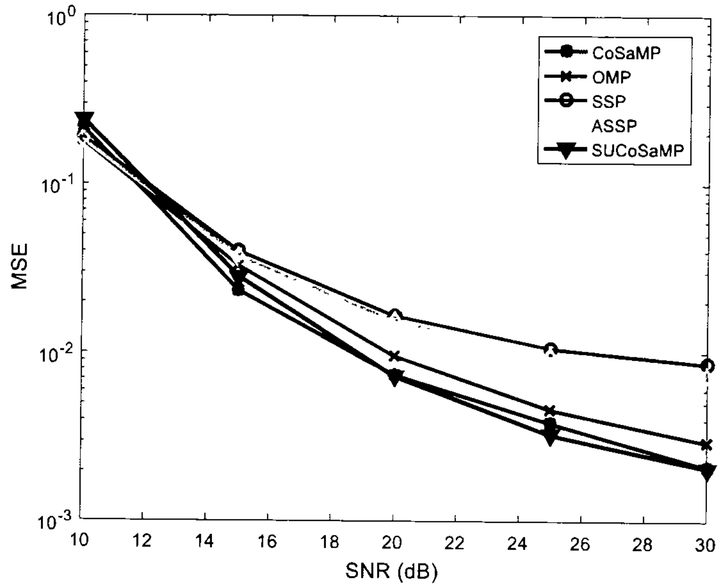


Figure 4.18 MSE performance comparison with different SNRs of 2nd antenna group (N_2) with $N_p = 32$, M=128, $N_g = 8$ and 16QAM

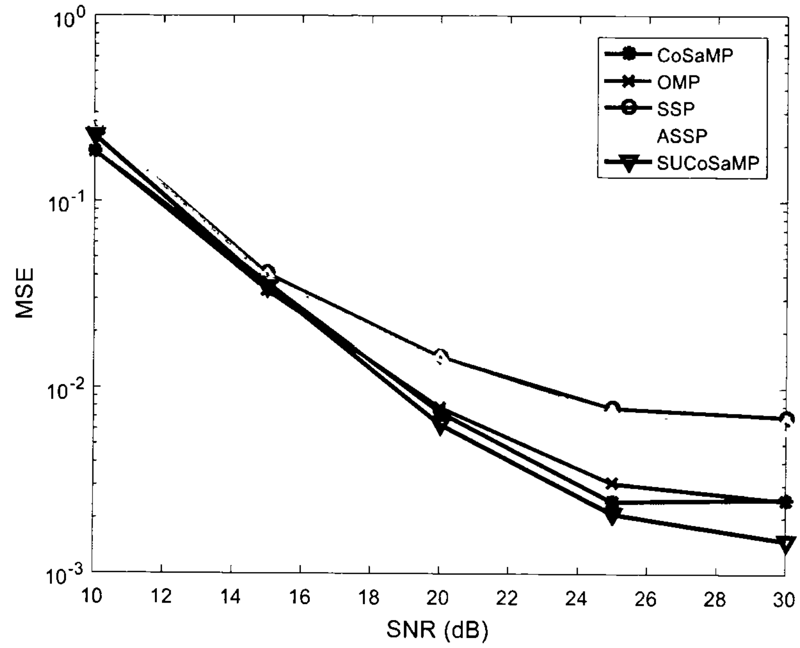


Figure 4.19 MSE performance comparison with different SNRs of 3rd antenna group (N_3) with $N_p = 32$, M=128, $N_g = 8$ and 16QAM

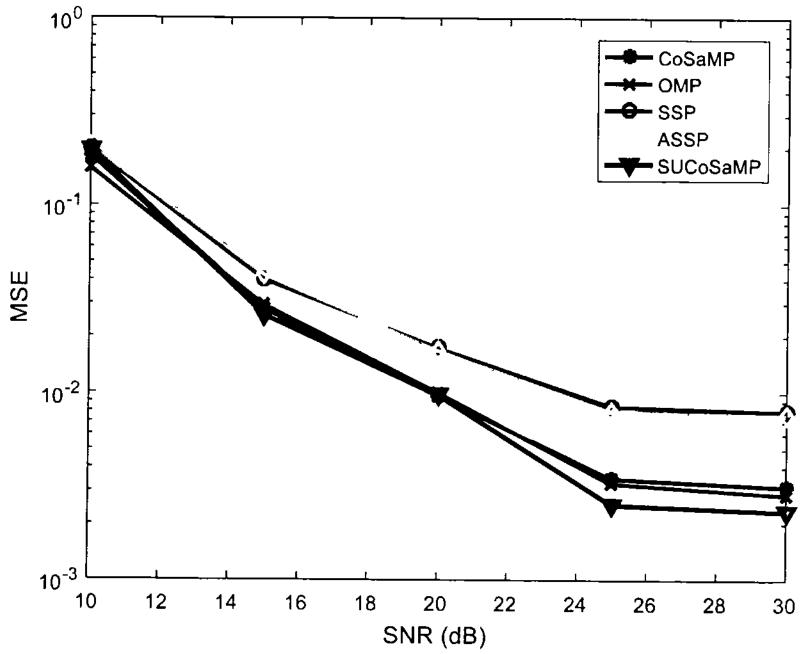


Figure 4.20 MSE performance comparison with different SNRs of 4th antenna group (N_4) with $N_p = 32$, M=128, $N_g = 8$ and 16QAM

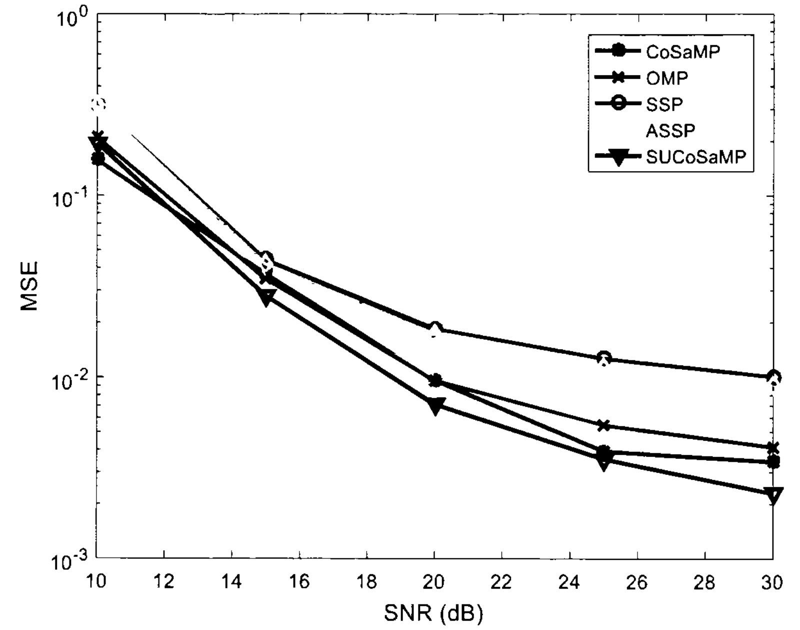


Figure 4.21 MSE performance comparison with different SNRs of 5th antenna group (N_5) with $N_p = 32$, M=128, $N_g = 8$ and 16QAM

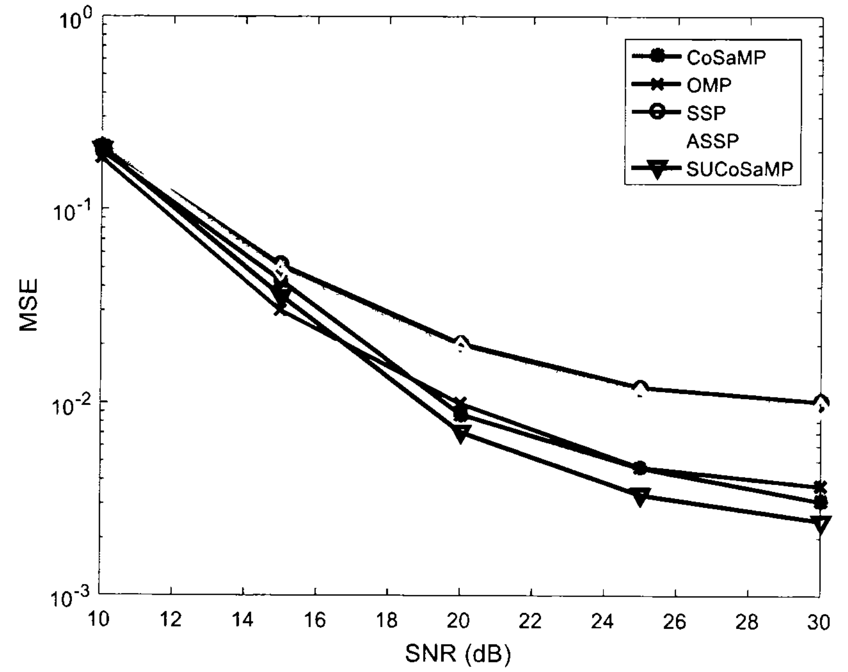


Figure 4.22 MSE performance comparison with different SNRs of 6th antenna group (N_6) with $N_p = 32$, M=128, $N_g = 8$ and 16QAM

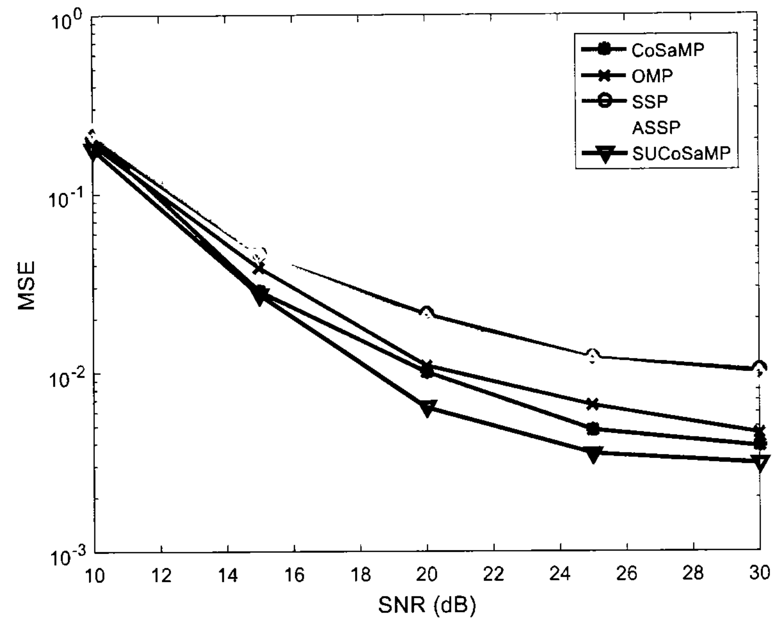


Figure 4.23 MSE performance comparison with different SNRs of 7[th] antenna group (N_7) with $N_p = 32$, M=128, $N_g = 8$ and 16QAM

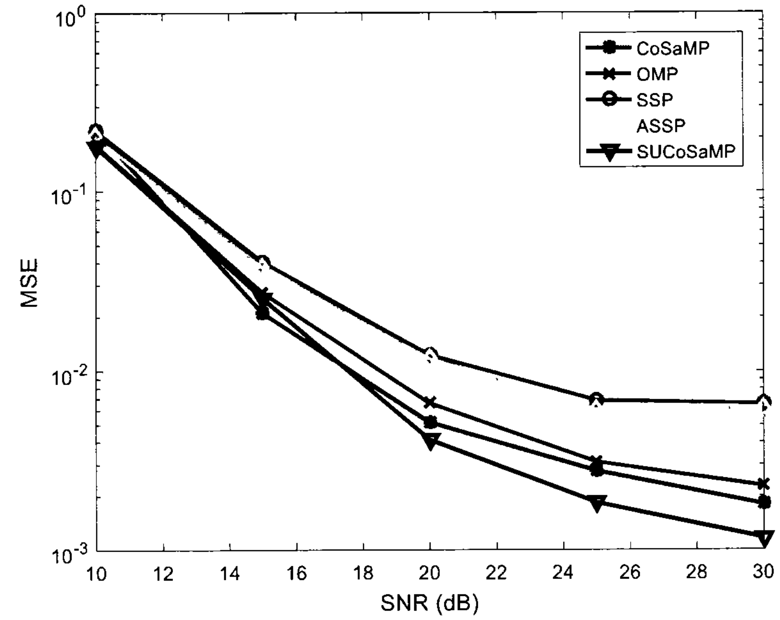


Figure 4.24 MSE performance comparison with different SNRs of 8[th] antenna group (N_8) with $N_p = 32$, M=128, $N_g = 8$ and 16QAM

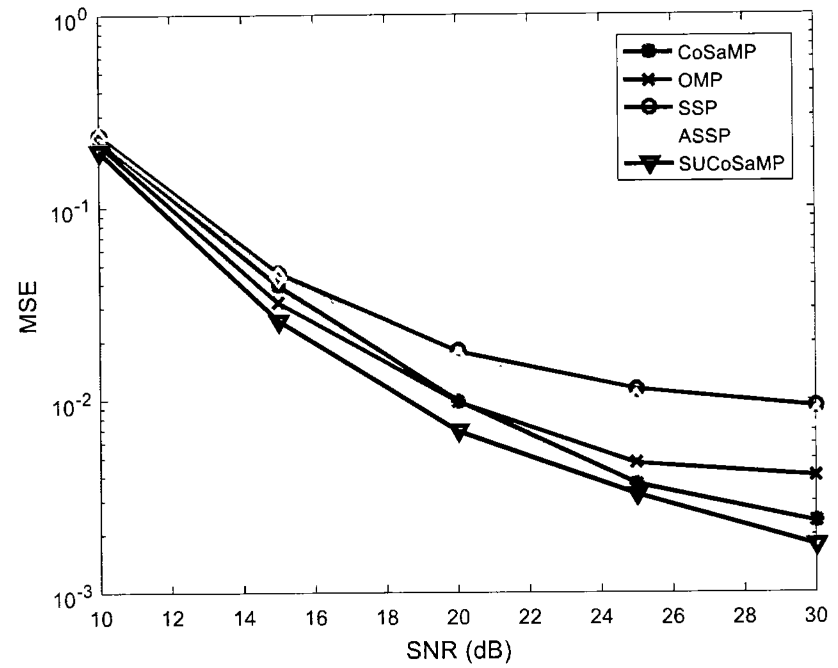


Figure 4.25 MSE performance comparison with different SNRs of 1st antenna group (N_1) with $N_p = 32$, M=128, $N_g = 8$ and 64QAM

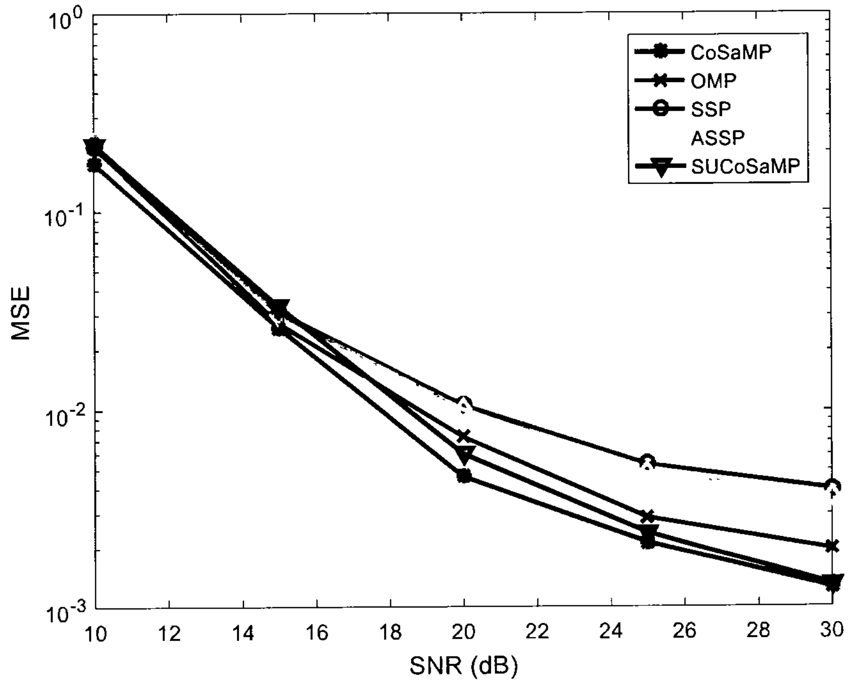


Figure 4.26 MSE performance comparison with different SNRs of 2nd antenna group (N_2) with $N_p = 32$, M=128, $N_g = 8$ and 64QAM

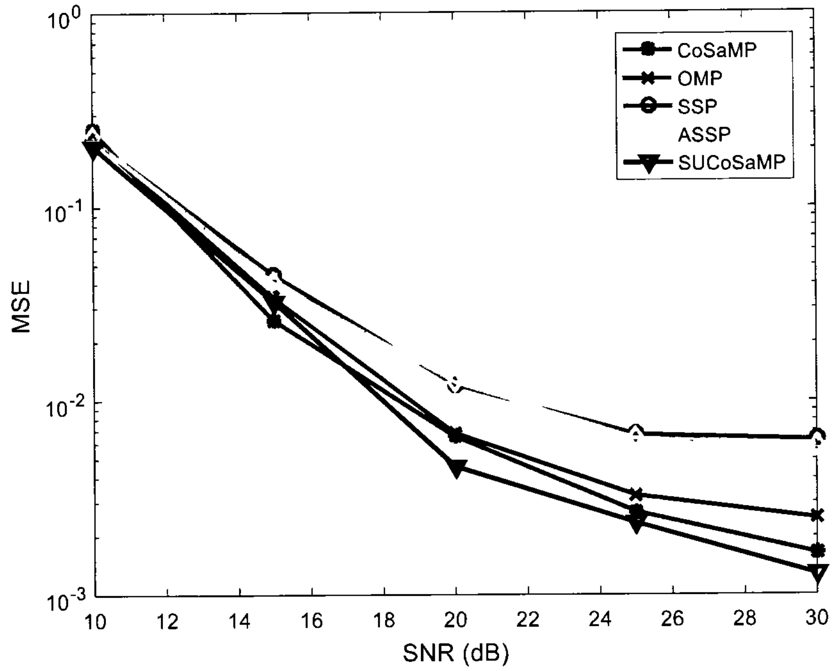


Figure 4.27 MSE performance comparison with different SNRs of 3rd antenna group (N_3) with $N_p = 32$, M=128, $N_g = 8$ and 64QAM

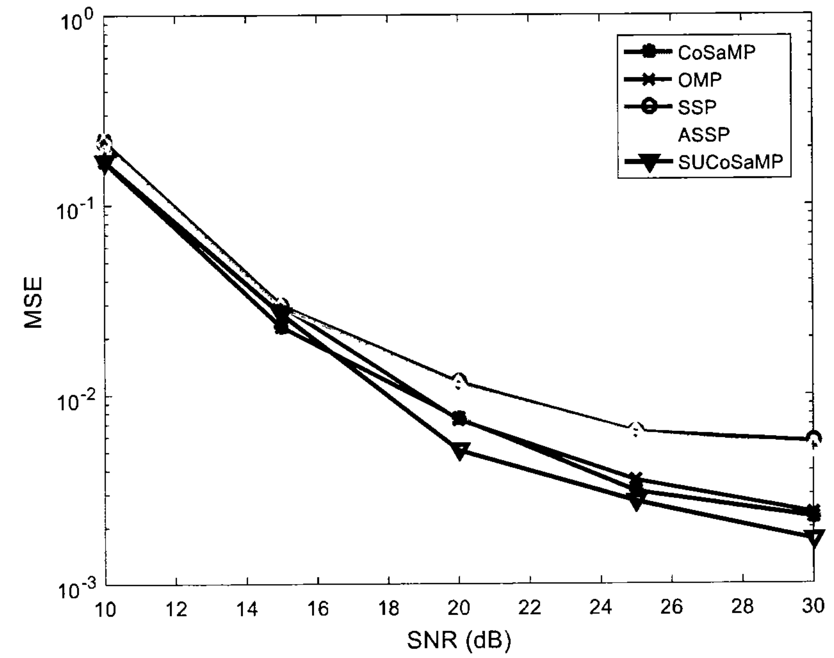


Figure 4.28 MSE performance comparison with different SNRs of 4th antenna group (N_4) with $N_p = 32$, M=128, $N_g = 8$ and 64QAM

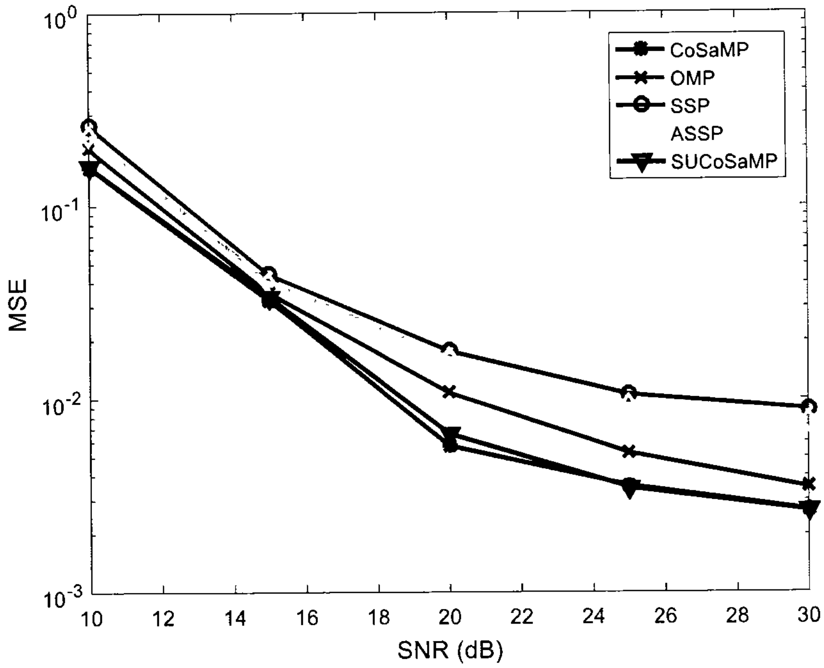


Figure 4.29 MSE performance comparison with different SNRs of 5[th] antenna group (N_5) with $N_p = 32$, M=128, $N_g = 8$ and 64QAM

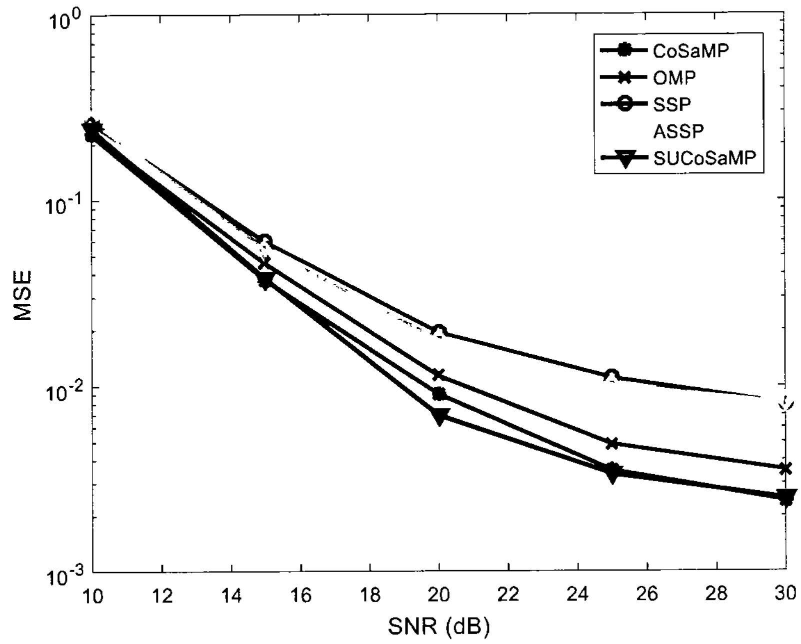


Figure 4.30 MSE performance comparison with different SNRs of 6[th] antenna group (N_6) with $N_p = 32$, M=128, $N_g = 8$ and 64QAM

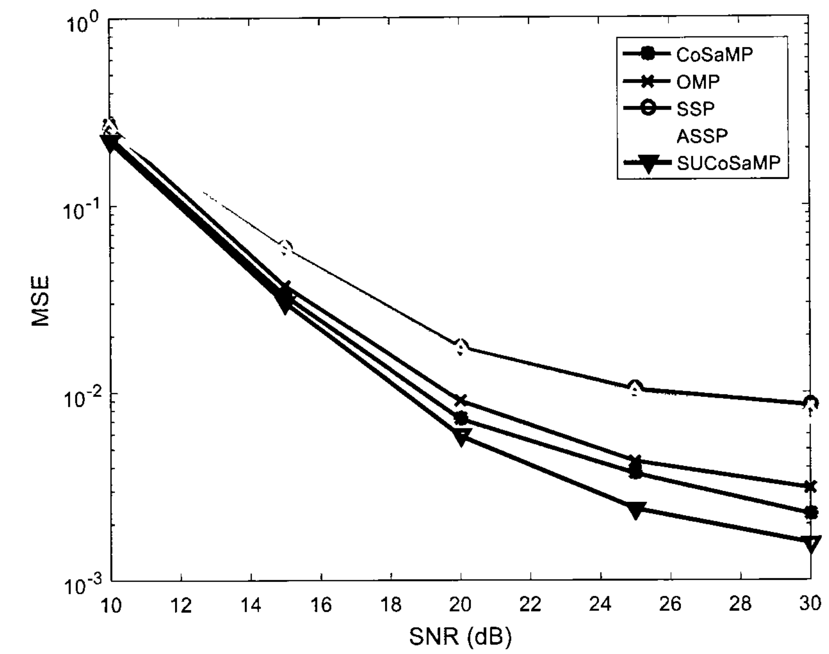


Figure 4.31 MSE performance comparison with different SNRs of 7th antenna group (N_7) with N_p = 32, M=128, N_g = 8 and 64QAM

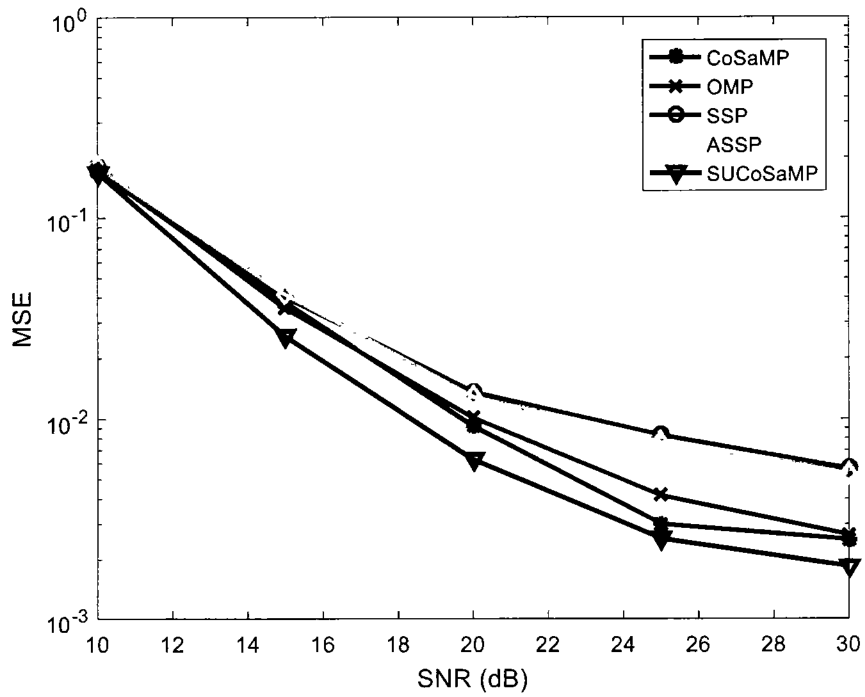


Figure 4.32 MSE performance comparison with different SNRs of 8th antenna group (N_8) with N_p = 32, M=128, N_g = 8 and 64QAM

In Figures 4.17 – 4.32, MSE performance versus SNR of individual antenna groups (i.e. from antenna group N_1 to N_8) is presented with N_p= 32, M = 128, N_g = 8 for 16 QAM and 64 QAM respectively.

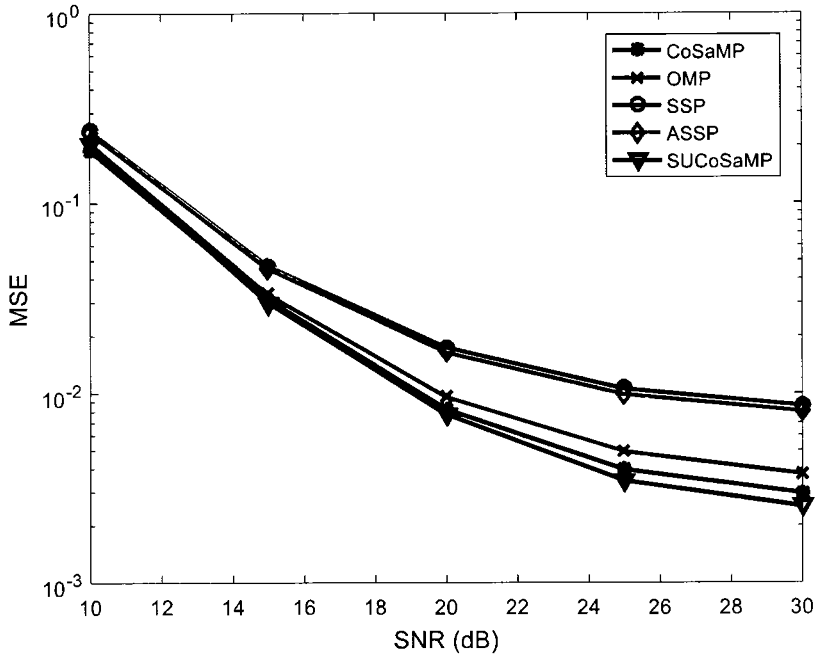


Figure 4.33 MSE Comparison of each algorithm under different SNRs: With $N_p = 32$ M=256, $N_g = 16$ and 16QAM

In Figure 4.33, combined MSE performance of 256 antennas versus SNR is presented with N_p= 32, M = 256, N_g = 16 for 16 QAM.

In these figures, comparison of SUCoSaMP is provided with conventional algorithms OMP and CoSaMP and with SSP and ASSP. It can be seen from the figures that the SUCoSaMP algorithm for each antenna group can achieve huge MSE gains over conventional algorithms. However, it is observed that the MSE performance of all algorithms slightly degraded with decrease in number of pilot subcarriers N_p. Results shows that SUCoSaMP efficiently consider the effect of SNR and performs better in both high and low SNR scenarios.

4.2 Resource Requirements and Execution

This thesis illustrates a modified iterative recuperation algorithm called SUCoSaMP that provides the similar assurances as the top optimization centered approaches. Additionally, SUCoSaMP presents thorough constraints on storage and computational complexity cost. SUCoSaMP is particularly suitable and efficient for practical scenarios and challenges since it simply needs matrix/vector multiplication with the sensing matrix. The execution time for compressible signals is just $O(M_{ngf}Llog^2M_{ngf}L)$, where $M_{ngf}L$ represents the size of the signal [48].

As described in Chap 3, SUCoSaMP is derived from basic CoSaMP [48]. The resource requirement analyses presented in [48] for CoSaMP has been revisited in this section in context of SUCoSaMP. CoSaMP was developed for practical solutions to recover signals. A methodical execution of the algorithm needs several concepts from linear algebra, along with a few basic practices from the theory and techniques of algorithms. This section reviews the important factors and concerns and builds up an evaluation of the execution time for the two most common scenarios [48].

The focus of CoSaMP/SUCoSaMP is on solving the least-squares problem during estimation step as this step is the key hurdle to a rapid functioning of algorithms. The CoSaMP/SUCoSaMP make it sure that the matrix $\boldsymbol{A}_{n\ T}$ does not have higher than 3s columns, thus the hypothesis of CoSaMP/SUCoSaMP i.e. $\delta_{4s} \leq 0.1$ entails that the matrix $\boldsymbol{A}_{n\ T}$ is particularly in perfect state. Therefore, the pseudoinverse $\boldsymbol{A}_{nT}^{\dagger} =$

$(\boldsymbol{A}_{nT}^{*}\boldsymbol{A}_{n\,T})^{-1}\boldsymbol{A}_{nT}^{*}$ can be employed exceedingly fast by applying any iterative method like conjugate gradient or Richardson's iteration. The further benefit of the techniques is that the interaction with the matrix $\boldsymbol{A}_{n\,T}$ is simply through action on vectors. Thus, it develops that the fast matrix-vector multiplication for sensing matrix help algorithms to perform better [48].

The solving cost of least-squares problem is $O(\mathcal{L})$ where, $\mathcal{L}$ restraints the cost of matrix–vector multiplication with $\boldsymbol{A}_{n\,T}$ or $\boldsymbol{A}_{nT}^{*}$. Whereas the least-squares approach is initialized with the existing approximation $\check{\mathbf{h}}^{k-1}$ [48].

In these arrangements the direct methods for least-square problems are highly ineffective due to the following three major issues:

i) The first issue is that every single least-square problem can include considerably unique sets of columns from $\boldsymbol{A}_{n}$. Consequently, the cost of SVD or QR factorization required to be executed throughout every single iteration becomes $O(s^2N_p)$ [48].

ii) The second issue is that computing SVD or QR factorization normally requires to access the columns of the matrix directly. Whereas it becomes challenging while accessing the matrix through actions on vectors [48].

iii) The third issue is that the storage cost for direct methods becomes $O(sN_p)$, which unfeasible for large scale scenarios [48].

The algorithm includes typical approaches for the other steps, the process counts are as follows as discussed in [48]:

Signal proxy: The cost of making signal proxy is controlled by matrix-vector multiplication $\mathbf{A}_{\mathrm{n}}^{*}\mathbf{v}$ [48].

Classification: The largest 2s entries in a vector can be found in time $O(M_{ngf}L)$. Practically, sorting of entries in a signal can be quicker at cost $O(M_{ngf}L \log M_{ngf}L)$ in declining order of scale and then choose first 2s of them. The concluding process can be completed by heapsort or quick sort. Whereas, for the algorithm to be implemented as per requirement, the sorting should be stable enough [48].

Support union: The two sets of size $O(s)$ can be united using randomized hashing methods in estimated time $O(s)$. The other option is that to use elementary merge procedure after sorting both sets first for a cost $O(s\log s)$ [48].

Signal estimation by least-square: Conjugate gradient or Richardson's iteration can be used to compute $\boldsymbol{A}_{n_T}^{\dagger}\mathbf{y}_{\mathrm{z,n}}$. Initialization of least-squares approach needs a matrix-vector multiplication with $\boldsymbol{A}_{n_T}^{*}$. A matrix-vector multiplication is essential for least-squares approach during iteration each with $\boldsymbol{A}_{n_T}^{*}$ and $\boldsymbol{A}_{n_T}$ [48].

Pruning: It can be executed in time $O(s)$, since it is exactly the similar step to classification, however the better approach can be by sorting the vector components by scale and then first s can be chosen at cost $O(s\log s)$ [48].

Sample update: The cost of this step is controlled by multiplication $\boldsymbol{A}_n\check{\mathbf{h}}^{k-1}$.

Two cases are presented in Table 4. The first case is for standard way of applying sensing matrix $\boldsymbol{A}_n$ to vectors. The second case is that how both sensing matrix $\boldsymbol{A}_n$ and its adjoint $\boldsymbol{A}_n^{*}$ execute fast multiplication with cost $\mathcal{L}$ by considering the assumption $\mathcal{L} \geq M_{ngf}L$. Whereas the value $O(M_{ngf}L \log M_{ngf}L)$ is considered as a conventional value. Partial Fourier matrices particularly comply with this constraint [48].

It has been further noted that the storage needs are also satisfactory for the algorithm. Apart from storage requirement for the sensing matrix, the signal proxy is formed by taking only one vector of length $M_{ngf}L$. The vectors $\boldsymbol{v}$ and $\mathbf{y}_{\mathrm{z,n}}$ require $\mathrm{O}(N_p)$ storage as they are of length N_p. The approximations of the signal may be stored as structured data, requiring maximum storage $\mathrm{O}(s \log M_{ngf}L)$. In the same way the indexes set needs simply $\mathrm{O}(s \log M_{ngf}L)$ storage. Finally, the total worst storage is $\mathrm{O}(M_{ngf}L)$ [48].

Table 4. Execution-operation count for SUCoSaMP: $N_p \times M_{ngf}L$ are the dimensions of sensing matrix $\boldsymbol{A}_n$, s is the sparsity level, $\mathcal{L}$ restraints the cost of matrix–vector multiplication with $\boldsymbol{A}_n$ or $\boldsymbol{A}_n^*$, For the purpose of reading clarity, Big-O notation is excluded.

Step	Multiplication (Standard)	Multiplication (Fast)
Make proxy	$N_p M_{ngf}L$	$\mathcal{L}$
Classification	$M_{ngf}L$	$M_{ngf}L$
Support union	s	s
Estimation (least-squares)	sN_p	$\mathcal{L}$
Pruning	s	s
Sample update	sN_p	$\mathcal{L}$
Total per iteration	$\mathrm{O}(N_p M_{ngf}L)$	$\mathrm{O}(\mathcal{L})$

4.3 Comparative Performance Analysis

Table 5 presents the comparative performance of different algorithms based on the following points discussed in [48]:

Table 5. Assessment of various recovery algorithms: N_p is the length of measurement vector (i.e. the received vector of the pilot sequence) $M_{ngf}L$ is the length of signal (i.e. the aggregate CIR vector of n[th] antenna group) and the sparsity level is denoted by s. [48]

	SUCoSaMP	OMP	ROMP	Fourier Sampling	HHS Pursuit	Convex Optimization	CoSaMP
Broad Samples	SRIP	Subgauss.	RIP	no	no	RIP	RIP
Stability	yes	?	yes	yes	yes	yes	yes
Ideal Number of Samples	yes	yes	no	no	no	yes	yes
Consistency	yes	no	yes	no	yes	yes	yes
Execution Time	$O(N_p M_{ngf} L)$	$O(sN_p M_{ngf} L)$	$O(sN_p M_{ngf} L)$	s polylog$(M_{ngf}L)$	poly$(s \log M_{ngf}L)$	$LP(M_{ngf}L, N_p)$	$O(N_p M_{ngf} L)$

Broad Samples: This describes that how an algorithm behaves for a variety of sampling methods i.e. whether it needs structured or unstructured samples? SRIP means that the algorithm holds structured isometric property [40], whereas RIP means that the algorithm satisfies the restricted isometry constant bound. Finally, "Subgauss." represents that the algorithm can be successfully incorporated particularly for sub-gaussian sampling matrices [48].

Stability: This describes two important situations in which an algorithm assures success or not; (a) A signal is not sparse but compressible (b) the samples suffers from noise contamination [48].

Ideal Number of Samples: This is about possible recovery of s-sparse signal from $O(s \log M_{ngf}L)$ measurements i.e. Does the algorithm require higher sampling by logarithmic factor? [48]

Consistency: Can all the signals be recovered by the algorithm given a fixed sensing matrix? Or does it need to derive a sensing matrix at random for each signal? [48]

Execution Time: This is about the worst-scenario cost to recuperate a signal with sparsity level "s" by the algorithm up to some comparative accuracy, without having any specific structure of a sensing matrix. The term $LP(M_{ngf}L, N_p)$ represents the cost (i.e $O({N_p}^2 M_{ngf} L^{1.5})$ for an interior point approach) of linear program solution with N_p constraints and $M_{ngf}L$ variables. Furthermore, it can be easily noted that mostly algorithms take good advantage of rapid matrix-vector multiplication to achieve improved execution times. [48]

Finally, it can be concluded that the SUCoSaMP/CoSaMP performs better amongst the super-linear and linear approaches for the given metrics. While the sub-linear approaches are faster than SUCoSaMP/CoSaMP, however the SUCoSaMP/CoSaMP overcomes the limitation by accepting the broad variety of sensing matrices with rarer samples [48].

4.4 Summary

By taking advantage of spatial and temporal common sparsity of massive MIMO channels in delay domain, the proposed nonorthogonal pilot design and channel estimation scheme under the frame work of CS theory significantly reduce the pilot overheads for massive MIMO systems and also outperform the conventional algorithms in performance. This research may be extended by incorporating the proposed schemes in the multicell scenarios.

Chapter 5. Conclusion and Future Suggestions

This chapter presents the conclusion of each method proposed in chapter 3. The methods include proposed nonorthogonal pilot scheme and proposed SUCoSaMP recovery algorithm. This chapter also describes future research suggestions and potential directions.

5.1 Conclusion

The targets of 5G can be met by using ultra-wide bandwidth, highly dense deployment of BSs and hundreds of antennas at BS, however using these techniques will result in unexpected power consumption, prohibitively high complexity and very large pilot overheads.

Therefore, CS theory has motivated whole wireless communication community to address these issues, and in this thesis, we also attempted to investigate the effectiveness of CS by employing CS theory for the purpose of channel estimation for massive MIMO wireless systems.

5.1.1 Proposed Nonorthogonal Pilot Scheme

- A nonorthogonal pilot scheme is proposed based on the experimentally observed spatio-temporal common sparsity of massive MIMO communication channels.
- Initially the scheme divides the antennas placed at BS into subgroups. The division of the antennas into subgroups is based on the system parameters and the temporal and spatial sparsity of massive MIMO channels.
- Then after creating the antenna groups, a specific equally spaced nonorthogonal pilot scheme is proposed.

- The proposed scheme dynamically reduces the pilot overhead and save the resources.
- Simulation results demonstrate that the proposed method has significant low computational complexity. Moreover, the channel estimation using the proposed scheme attains good estimation results.

5.1.2 Proposed SUCoSaMP Recovery Algorithm

- For the task of channel estimation, the basic idea of CS theory is implemented that is to recover a signal which is sparse in some domain from extremely small amount of nonadaptive linear measurements by applying convex optimization.
- We proposed the SUCoSaMP algorithm derived from basic CoSaMP as described in Algorithm 1.
- The information of correct sparsity level is usually not available and also it is practically not possible to have prior knowledge of correct sparsity level, whereas information about sparsity level plays a significant role in compressive sensing problem of solving underdetermined system and it is also required as prior information by most of the CS based algorithms.
- The proposed SUCoSaMP algorithm does not require prior information of sparsity level because it adaptively acquires the sparsity level and avoids the unrealistic assumption of having prior information of correct sparsity level.
- Note that the proposed SUCoSaMP algorithm works in the same way as the CoSaMP algorithm. The CoSaMP algorithm is basically based on basic OMP. The SUCoSaMP algorithm has incorporated some other ideas from the literature

to present strong guarantees that OMP and CoSaMP cannot provide and to speed up the algorithm as compared to OMP.

- The major advantage of SUCoSaMP over OMP, CoSaMP, and basic subspace pursuit (SP) algorithms is that it does not require prior knowledge of sparsity level, which is an unrealistic assumption, specifically in wireless communication scenario.
- There are two differences between SUCoSaMP and CoSaMP. Firstly, SUCoSaMP estimates the channels, i.e. it recovers the high dimensional sparse vector by utilizing structured sparsity of massive MIMO channels from one vector of low dimension. Secondly, SUCoSaMP adaptively acquires the sparsity level.
- In contrast, the CoSaMP recovers the sparse vector without exploiting the structured sparsity and it requires prior information of correct sparsity level.
- Simulation results shows that the proposed nonorthogonal pilot design and channel estimation scheme under the frame-work of CS theory significantly reduce the pilot overheads for massive MIMO systems and also outperform the conventional algorithms in performance.
- Moreover, the proposed schemes are easy to implement in practical systems with high-quality adaptive modification capacity and secure estimation results.

5.2 Future Recommendations

Following are some future suggestions and directions for compressive sensing-based channel estimation for massive MIMO communication systems which could help to further improve the system.

- CS based interference cancellations schemes can be implemented to avoid the interference from neighboring cells and to overcome the pilot contamination issue. Also, the feedback systems can be deployed to reduce the high pilot overhead. The idea is to formulate the CS based recovery algorithm to be implemented at the BS side by receiving the channel feedback information using the uplink control channel.
- This research may be extended by incorporating the proposed schemes in the multicell and multiuser scenarios.
- The virtual angular domain sparsity property of massive MIMO channels can be exploited to further reduce the access latency. Moreover, the structured sparsity in the angle-frequency domain can be exploited to design CS based channel estimator.

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