MHD PERISTALTIC FLOW OF FOURTH GRADE FLUID IN A CURVED CHANNEL

By

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Pakistan

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A Dissertation

Submitted in the Partial Fulfillment of the

Requirements for the Degree of

MASTER OF SCIENCE

IN

MATHEMATICS

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Certificate

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A DISSERTATION SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN MATHEMATICS

We accept this dissertation as confirming to the required standard.

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2015

Dedication

"I dedicate this dissertation to my parents"

<u>Forwarding Sheet by Research Supervisor</u>

The Thesis entitled ***"MHD peristaltic flow of fourth grade fluid in a curved channel"*** submitted by ***Fouzia Masood Reg. No 150-FBAS/MSMA F13*** in partial fulfillment of MS Degree in Mathematics has been completed under my guidance and supervision. I am satisfied with the quality of her research work and allow her to submit this thesis for further process to graduate with Master of Science degree from the Department of Mathematics & Statistics, as per IIUI rules and regulations.

Date 27-8-15

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I would not able to enhance my performance without strenuous efforts, continue support, guidance and prayers of my parents, teachers and friends. Especially, my parents always encourage not only for higher education but also supported financially for which it was possible to complete my job. I am really grateful to all my well-wishers for their sincere guidance.

Declaration

I hereby declare that this thesis is my original work. I have not copied from any other student's work or from any other sources except where due reference or acknowledgement is made explicitly in the text, nor has any part been written for me by another person.

The work was done under the guidance of Dr. Ambreen Afsar Khan at International Islamic University, Islamabad.

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Preface

Recently considerable attention has been focused on the peristaltic flow through tube/channels. Such flows are significant in both theoretical and industrial contexts. The peristaltic action also occurs in many biological system for example, food swallowing through the esophagus, intra-urine fluid motion, circulation of blood in small blood vessels and the flows of many other glandular ducts. The mechanism of peristaltic transport has been exploited for industrial application like sanitary fluid transport, blood pumps in heart lung machine. Extensive literature on the topic is now available and we can only mention a few recent investigations [1-10]. The subject of bio magnetic fluid dynamics has become more and more evident during the past few decades. The example include the development of magnetic devices for cell separation, magnetic wound or cancer tumor treatment causing magnetic hyperthermia, reduction of bleeding during surgeries etc. In view of importance of MHD, some recent attempts [11-17] in this direction.

In chapter one, the basic definitions of fluids and equations are discussed.

Chapter is devoted to the study of the peristaltic flow of a fourth grade fluid in an inclined asymmetric channel. Analytic solution for the problem using perturbation is studied.

The objective of chapter three is to discuss the peristaltic flow of an electrically conducting fourth grade fluid in a curved channel. The solutions for the stream function and pressure gradient are calculated by using perturbation method. Results of pressure rise is analyzed through the use of numerical integration.

Contents

Chapter 1

Preliminaries

Some basic definitions and concept of various types of fluid are described in this chapter.

1.1 Basic definitions

1.1.1 Fluid

A fluid is a substance that deforms continuously under the influence of shear stress.

1.1.2 Newtonian fluid

A Newtonian fluid is a fluid in which shear stress is directly proportional to rate of deformation in a linear manner is called Newtonian fluid

$$\tau_{yx} = \mu \frac{du}{dy}, \tag{1.1}$$

where μ is the dynamic viscosity and du/dy is the shear rate.

1.1.3 Non-Newtonian fluid

A non Newtonian fluid is a fluid in which shear stress is directly proportional to rate of deformation in a non linear manner is called non Newtonian fluid,

$$\tau_{yx} = \mu \left(\frac{du}{dy}\right)^n \qquad n \neq 1, \tag{1.2}$$

where n is the flow behavior index and μ is the dynamic viscosity, du/dy is the shear rate.

1.1.4 Density

Density of a fluid is defined as its mass per unit volume. Mathematically it can be written as

$$\rho = \frac{m}{v}, \tag{1.3}$$

where ρ is the density, m is the mass and v is the volume.

1.1.5 Viscosity

Viscosity is the property of a fluid that resists the force tending to cause the fluid to flow.

1.1.6 Kinematic Viscosity

Kinematic viscosity is the ratio of dynamic viscosity to density. Kinematic viscosity can be obtained by dividing the absolute viscosity of a fluid with the fluid mass density

$$\nu = \frac{\mu}{\rho}, \tag{1.4}$$

where

ν = kinematic viscosity

μ = absolute or dynamic viscosity

ρ = density.

1.2 Types of Flows

1.2.1 Uniform Flow

The flow in which the velocity field and other hydrodynamic parameters do not change from point to point at any level.

1.2.2 Non-Uniform Flow

The flow in which the velocity and other hydrodynamic parameters changes from one point to another the flow is defined as non-uniform flow.

1.2.3 Steady Flow

If the flow pattern is not changed with respect to time then it is called steady flow.

1.2.4 Unsteady Flow

If the flow pattern is changed with respect to time then it is called unsteady flow.

1.2.5 Incompressible and Compressible Flow

If density ρ of a fluid particle does not change as it moves in the flow field, then flow is incompressible otherwise it is compressible.

1.3 Dimensionless number

1.3.1 Reynolds number

The Reynolds number (Re) is a measure of the ratio of inertial forces to viscous forces. Mathematically, the Reynolds number is defined below

$$Re = \frac{\text{inertial forces}}{\text{viscous forces}}. \tag{1.5}$$

1.3.2 Wave number

Its define as

$$\delta = \frac{2\pi d_1}{\lambda}, \tag{1.6}$$

where d_1 is the half width of the channel and λ is the wave length of the peristaltic wave.

1.4 Magnetohydrodynamics

MHD is the study of the dynamics of electrically conducting fluids.

1.5 Maxwell equation

Maxwell's Equations are a set of four equations that describe the world of electromagnetic. These equations describe how electric and magnetic fields propagate, interact, and how they are influenced by objects, these equations are known as Gauss's law, Gauss's magnetism law, Faraday's law and Ampere's law. These equation are describes as

$$\begin{aligned}
\boldsymbol{\nabla}.\mathbf{E} &= \frac{\rho}{\epsilon_0}, \\
\boldsymbol{\nabla}.\mathbf{B} &= 0, \\
\boldsymbol{\nabla}\times\mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, \\
\boldsymbol{\nabla}\times\mathbf{B} &= \mu_0\mathbf{J} + \mu_0\epsilon_0\frac{\partial \mathbf{E}}{\partial t}.
\end{aligned} \tag{1.7}$$

In the above equation ϵ_0 is the permittivity of the free space also called electric constant, μ_0 is the permeability of free space which is also called magnetic constant, ρ is the electric charge density and $\mathbf{J}$ is the electric current density.

1.6 Equation of motion

When some bodies constituting an isolated system act upon one another, the total momentum of the system remains same. In an inertial frame of reference, the general form of equations of fluid motion is

$$\rho\frac{d\mathbf{V}}{dt} = \operatorname{div}\mathbf{T} + \rho\mathbf{b}, \tag{1.8}$$

where $\frac{d}{dt}$ is the material time derivative, $\mathbf{V}$ is the velocity, $\mathbf{T}$ is the Cauchy stress tensor, ρ is a density and $\mathbf{b}$ is a body force. In component form Eq. (1.8) yields

$$\rho\left[\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right]u = \frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{xy}}{\partial y} + b_x, \tag{1.9}$$

$$\rho\left[\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right]v = \frac{\partial T_{yx}}{\partial x} + \frac{\partial T_{yy}}{\partial y} + b_y. \tag{1.10}$$

1.7 Perturbation Method

This is one of the most powerful technique to solve nonlinear ordinary and partial differential equations of fluid mechanics in which one of the physical parameter can be assumed very small.

1.8 Regular perturbation

Differential equations with small parameter yields. A series of terms of decreasing magnitude that approximate the solution of the original differential equation. When an equation is changed by only a small amount, the solution will often only change by a small amount. Expand the dependent variable in a power series depending on the small parameter in the problem. Substitute this series into the original equations, the boundary conditions and the initial conditions. Expand these equations in a Taylor series, equate the terms corresponding to different powers of small parameter and solve equations sequentially.

Chapter 2

Peristaltic flow of a fourth grade fluid in an inclined asymmetric channel

2.1 Introduction

This chapter is a review work of [9]. This chapter studies the peristaltic flow of a fourth grade fluid in an inclined asymmetric channel. Analysis is carried out under long wave length approximation. Expression of the pressure gradient per unit wave length, ΔP and the stream function ψ are obtained. Several graphs of physical interest are displayed and discussed.

2.2 Physical Model

Extra stress tensor $\overline{\mathbf{S}}$ for fourth grade fluid model is given by

$$\overline{\mathbf{T}} = -p\overline{\mathbf{I}} + \overline{\mathbf{S}}, \tag{2.1}$$

where

$$\begin{aligned}
\overline{\mathbf{S}} &= \mu\overline{\mathbf{A}}_1+\alpha_1\overline{\mathbf{A}}_2+\alpha_2\overline{\mathbf{A}}_1^2+\beta_1\overline{\mathbf{A}}_3+\beta_2\left(\overline{\mathbf{A}}_2\overline{\mathbf{A}}_1+\overline{\mathbf{A}}_1\overline{\mathbf{A}}_2\right)+\beta_3\left(tr\overline{\mathbf{A}}_1^2\right)\overline{\mathbf{A}}_1 \\
&+\gamma_1\overline{\mathbf{A}}_4+\gamma_2\left(\overline{\mathbf{A}}_3\overline{\mathbf{A}}_1+\overline{\mathbf{A}}_1\overline{\mathbf{A}}_3\right)+\gamma_3\overline{\mathbf{A}}_2^2+\gamma_4\left(\overline{\mathbf{A}}_2\overline{\mathbf{A}}_1^2+\overline{\mathbf{A}}_1^2\overline{\mathbf{A}}_2\right)+\gamma_5\left(tr\overline{\mathbf{A}}_2\right)\overline{\mathbf{A}}_2 \\
&+\gamma_6\left(tr\overline{\mathbf{A}}_2\right)\overline{\mathbf{A}}_1^2+\left[\gamma_7\left(tr\overline{\mathbf{A}}_3\right)+\gamma_8\left(tr\overline{\mathbf{A}}_2\overline{\mathbf{A}}_1\right)\right]\overline{\mathbf{A}}_1, \qquad (2.2)\\
\overline{\mathbf{A}}_1 &= \left(\operatorname{grad}\overline{V}\right)+\left(\operatorname{grad}\overline{V}\right)^T, \\
\overline{\mathbf{A}}_i &= \frac{d\overline{\mathbf{A}}_{i-1}}{dt}+\overline{\mathbf{A}}_{i-1}\left(\operatorname{grad}\overline{V}\right)+\left(\operatorname{grad}\overline{V}\right)^T\overline{\mathbf{A}}_{i-1},\ i>1, \qquad (2.3)
\end{aligned}$$

in which $\mathbf{I}$, p, μ, $\mathbf{S}$, $\mathbf{A}_i\,(i=1-4)$ respectively stands for identity tensor, the pressure, the viscosity, the extra stress tensor and the Rivlin Ericksin tensor in which $\alpha_i\,(i=1-2)$, $\beta_i\,(i=1-3)$ and $\gamma_i\,(i=1-8)$ are the material parameters.

2.3 Mathematical formation

Consider fourth grade fluid in an asymmetric channel of width d_1+d_2 and inclined at an angle α to the horizontal. Let c be the speed by which sinusoidal wave trains propagate along the channel walls. The $\overline{X}$ and $\overline{Y}$ are taken parallel and transverse to the direction of wave propagation, respectively. The wall surface satisfy

$$\begin{aligned}
h_1\left(\overline{X},\overline{t}\right) &= d_1+a_1\sin\left[\frac{2\pi}{\lambda}\left(\overline{X}-c\overline{t}\right)\right], \\
h_2\left(\overline{X},\overline{t}\right) &= -d_2-a_2\sin\left[\frac{2\pi}{\lambda}\left(\overline{X}-c\overline{t}\right)+\phi\right], \qquad (2.4)
\end{aligned}$$

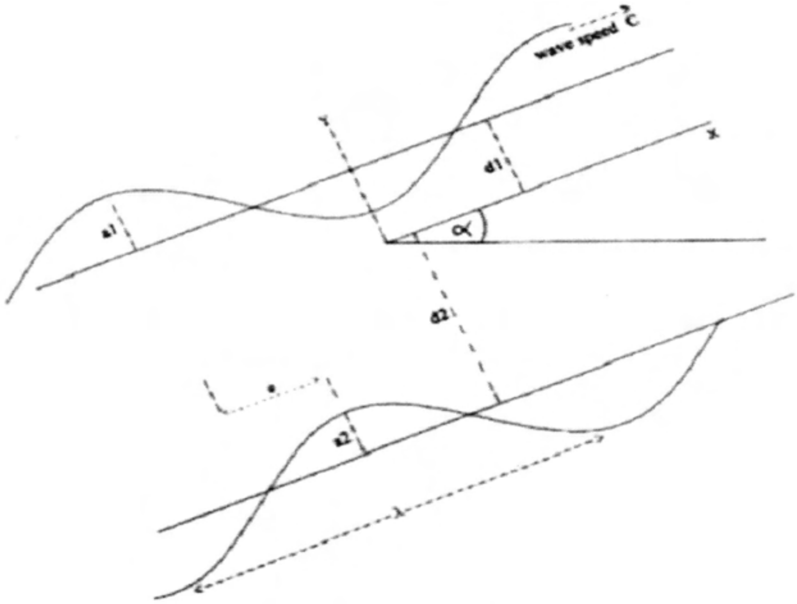

Fig. (2.1) : Geometry of the problem

where a_1 and a_2 are the amplitudes of the waves, λ is the wave length, c is the wave speed and ϕ is the phase difference which varies in the range $0 \leq \phi \leq \pi$. Denoting the velocity component $\overline{U}$ and $\overline{V}$ along the $\overline{X}$ and $\overline{Y}$ direction in the fixed frame, one can write $\mathbf{V}$ as

$$\mathbf{V} = \left[\overline{U}\left(\overline{X}, \overline{Y}, t\right),\ \overline{V}\left(\overline{X}, \overline{Y}, t\right),\ 0\right]. \tag{2.5}$$

The fundamental equations governing the flow of an incompressible fluids are

$$\frac{\partial \overline{U}}{\partial \overline{X}} + \frac{\partial \overline{V}}{\partial \overline{Y}} = 0, \tag{2.6}$$

$$\rho\left(\frac{\partial}{\partial \overline{t}} + \overline{U}\frac{\partial}{\partial \overline{X}} + \overline{V}\frac{\partial}{\partial \overline{Y}}\right)\overline{U} = \frac{\partial \overline{p}}{\partial \overline{X}} + \frac{\partial \overline{S}_{\overline{XX}}}{\partial \overline{X}} + \frac{\partial \overline{S}_{\overline{XY}}}{\partial \overline{Y}} + \rho g \sin\alpha, \tag{2.7}$$

$$\rho\left(\frac{\partial}{\partial \overline{t}} + \overline{U}\frac{\partial}{\partial \overline{X}} + \overline{V}\frac{\partial}{\partial \overline{Y}}\right)\overline{V} = \frac{\partial \overline{p}}{\partial \overline{Y}} + \frac{\partial \overline{S}_{\overline{XY}}}{\partial \overline{X}} + \frac{\partial \overline{S}_{\overline{YY}}}{\partial \overline{Y}} - \rho g \cos\alpha, \tag{2.8}$$

in which ρ denotes the density and g the acceleration due to gravity.

To facilitate the analysis, we introduce the following transformation between fixed and wave frames

$$\begin{aligned}
\overline{x} &= \overline{X} - c\overline{t}, \;\; \overline{y} = \overline{Y}, \;\; \overline{u} = \overline{U} - c, \\
\overline{v} &= \overline{V}, \;\; \overline{p}\left(\overline{x}, \overline{y}\right) = \overline{P}\left(\overline{X}, \overline{Y}, \overline{t}\right),
\end{aligned} \tag{2.9}$$

in which $(\overline{u}, \overline{v})$ and $\left(\overline{U}, \overline{V}\right)$ are the velocity components in wave frame and fixed frame of references respectively. We now introduce the following dimensionless variables

$$\begin{aligned}
x &= \frac{2\pi\overline{x}}{\lambda}, \;\; y = \frac{y}{d_1}, \;\; u = \frac{\overline{u}}{c}, \;\; v = \frac{\overline{v}}{c}, \;\; t = \frac{2\pi c\overline{t}}{\lambda}, \\
p &= \frac{2\pi d_1}{\lambda\mu c}\overline{p}, \;\; h_1 = \frac{\overline{h}_1}{d_1}, \;\; h_2 = \frac{\overline{h}_2}{d_1}, \;\; S = \frac{d_1}{\mu c}\overline{S}.
\end{aligned} \tag{2.10}$$

Now using the dimensionless variables (2.10) and introducing the stream function through

$$u = \frac{\partial\psi}{\partial y}, v = -\delta\frac{\partial\psi}{\partial x}. \tag{2.11}$$

The continuity equation is identically satisfied and momentum equations are

$$\delta R\left[\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\left(\frac{\partial\psi}{\partial y}\right)\right] + \frac{\partial p}{\partial x} = \delta\frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} + \frac{R\sin\alpha}{Fr}, \tag{2.12}$$

$$-\delta^3 R\left[\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\left(\frac{\partial\psi}{\partial x}\right)\right] + \frac{\partial p}{\partial y} = \delta^2\frac{\partial S_{xy}}{\partial x} + \delta\frac{\partial S_{yy}}{\partial y} - \frac{\delta R\cos\alpha}{Fr}, \tag{2.13}$$

in which δ the dimensionless wave number, R the Reynolds number, Fr the Froud number, the material coefficients $\lambda_i\,(i = 1 - 2)\,,\, \xi_i\,(i = 1 - 3)$ and $\eta_i\,(i = 1 - 8)$ are defined respectively as

$$\begin{aligned}
\delta &= \frac{2\pi d_1}{\lambda}, \;\; R{=}\frac{\rho c d_1}{\mu}, \;\; Fr = \frac{c^2}{g d_1}, \;\; \lambda_1 = \frac{\alpha_1 c}{\mu d_1}, \;\; \lambda_2 = \frac{\alpha_2 c}{\mu d_1}, \\
\xi_1 &= \frac{\beta_1 c^2}{\mu d_1^2}, \;\; \xi_2 = \frac{\beta_2 c^2}{\mu d_1^2}, \;\; \xi_3 = \frac{\beta_3 c^2}{\mu d_1^2}, \;\; \eta_1 = \frac{\gamma_1 c^3}{\mu d_1^3}, \;\; \eta_2 = \frac{\gamma_2 c^3}{\mu d_1^3}, \\
\eta_3 &= \frac{\gamma_3 c^3}{\mu d_1^3}, \;\; \eta_4 = \frac{\gamma_4 c^3}{\mu d_1^3}, \;\; \eta_5 = \frac{\gamma_5 c^3}{\mu d_1^3}, \;\; \eta_6 = \frac{\gamma_6 c^3}{\mu d_1^3}, \;\; \eta_7 = \frac{\gamma_7 c^3}{\mu d_1^3}, \\
\eta_8 &= \frac{\gamma_8 c^3}{\mu d_1^3}.
\end{aligned} \tag{2.14}$$

Eliminating the pressure gradient from Eqs. (2.12) and (2.13) is

$$\delta R\left[\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\nabla^2\psi\right]=\left[\left(\frac{\partial^2}{\partial y^2}-\delta^2\frac{\partial^2}{\partial x^2}\right)S_{xy}\right]+\delta\left[\frac{\partial^2}{\partial x\partial y}\left(S_{xx}-S_{yy}\right)\right], \quad (2.15)$$

where

$$\nabla^2=\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right),$$

and then applying long wavelength approximation in Eqs. (2.12), (2.13) and (2.15), we have

$$\frac{\partial p}{\partial x}=\frac{\partial S_{xy}}{\partial y}+\frac{R\sin\alpha}{Fr}, \quad (2.16)$$

$$\frac{\partial p}{\partial y}=0, \quad (2.17)$$

$$\frac{\partial^2 S_{xy}}{\partial y^2}=0, \quad (2.18)$$

where

$$\begin{aligned} S_{xy} &= \frac{\partial^2\psi}{\partial y^2}+2\Gamma\left(\frac{\partial^2\psi}{\partial y^2}\right)^3, \\ \Gamma &= \xi_2+\xi_3. \end{aligned} \quad (2.19)$$

The boundary conditions in dimensionless form becomes

$$\begin{aligned} \psi &= \frac{F}{2}, \quad \frac{\partial\psi}{\partial y}=-1 \quad \text{for } y=h_1(x), \\ \psi &= -\frac{F}{2}, \quad \frac{\partial\psi}{\partial y}=-1 \quad \text{for } y=h_2(x), \end{aligned} \quad (2.20)$$

in which the dimensionless time mean flows θ and F respectively in fixed and wave frame by the following expressions

$$\theta=F+1+d, \quad (2.21)$$

where

$$F=\int_{h_2}^{h_1}\frac{\partial\psi}{\partial y}dy. \quad (2.22)$$

2.4 Perturbation solution

We look for a regular perturbation in terms of small parameter Γ as follows

$$\begin{aligned}
\psi &= \psi_0 + \Gamma\psi_1 + \dots \\
F &= F_0 + \Gamma F_1 + \dots \\
p &= p_0 + \Gamma p_1 + \dots
\end{aligned} \tag{2.23}$$

Substituting Eq. (2.23) into Eqs. (2.16) to (2.20), we find the following system.

2.4.1 Zeroth order system

$$\frac{\partial^4 \psi_0}{\partial y^4} = 0, \tag{2.24}$$

$$\frac{\partial p_0}{\partial x} = \frac{\partial^3 \psi_0}{\partial y^3} + \frac{R\sin\alpha}{Fr}, \tag{2.25}$$

with the boundary conditions

$$\begin{aligned}
\psi_0 &= \frac{F_0}{2}, \quad \frac{\partial \psi_0}{\partial y} = -1 \quad \text{for } y = h_1(x), \\
\psi_0 &= -\frac{F_0}{2}, \quad \frac{\partial \psi_0}{\partial y} = -1 \quad \text{for } y = h_2(x).
\end{aligned} \tag{2.26}$$

2.4.2 First order system

$$\frac{\partial^4 \psi_1}{\partial y^4} = -2\frac{\partial^2}{\partial y^2}\left[\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)^3\right], \tag{2.27}$$

$$\frac{\partial p_1}{\partial x} = \frac{\partial^3 \psi_1}{\partial y^3} + 2\frac{\partial}{\partial y}\left[\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)^3\right], \tag{2.28}$$

with the boundary conditions

$$\begin{aligned}
\psi_1 &= \frac{F_1}{2}, \quad \frac{\partial \psi_1}{\partial y} = 0 \quad \text{for } y = h_1(x), \\
\psi_1 &= -\frac{F_1}{2}, \quad \frac{\partial \psi_1}{\partial y} = 0 \quad \text{for } y = h_2(x).
\end{aligned} \tag{2.29}$$

2.4.3 Second order system

$$\frac{\partial^4 \psi_2}{\partial y^4} = -6\frac{\partial^2}{\partial y^2}\left[\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)^2\left(\frac{\partial^2 \psi_1}{\partial y^2}\right)\right], \tag{2.30}$$

$$\frac{\partial p_2}{\partial x} = \frac{\partial^3 \psi_2}{\partial y^3} + 6\frac{\partial}{\partial y}\left[\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)^2\left(\frac{\partial^2 \psi_1}{\partial y^2}\right)\right], \tag{2.31}$$

with the boundary conditions

$$\begin{aligned} \psi_2 &= \frac{F_2}{2}, \quad \frac{\partial \psi_2}{\partial y} = 0 \quad \text{for } y = h_1(x), \\ \psi_2 &= -\frac{F_2}{2}, \quad \frac{\partial \psi_2}{\partial y} = 0 \quad \text{for } y = h_2(x). \end{aligned} \tag{2.32}$$

2.4.4 Zeroth order solution

Solving Eq. (2.24) and using boundary conditions (2.26), we get

$$\begin{aligned} \psi_0 &= \frac{1}{(h_2 - h_1)^3}\left[(h_1 - h_2 + F_0)\left(2y^3 - 3(h_1 + h_2)y^2 + 6h_1h_2y\right)\right] \\ &\quad -y - \frac{F_0}{2} + h_2 + \frac{1}{(h_2 - h_1)^3}\left[h_2^2(h_2 - 3h_1)(h_1 - h_2 + F_0)\right]. \end{aligned} \tag{2.33}$$

The pressure gradient of this order is

$$\frac{\partial p_0}{\partial x} = -12\left[\frac{1}{(h_1 - h_2)^2} + \frac{F_0}{(h_1 - h_2)^3}\right] + 12G. \tag{2.34}$$

2.4.5 First order solution

Substituting the zeroth order solution (2.33) into (2.27) and using the boundary condition (2.29), we obtain

$$\begin{aligned} \psi_1 &= -\frac{1}{10}\left(\frac{\partial p_0}{\partial x} - 12G\right)^3 y^5 + \frac{(h_1 + h_2)}{4}\left(\frac{\partial p_0}{\partial x} - 12G\right)^3 y^4 + \frac{B_1}{6}y^3 \\ &\quad -\frac{(h_1 + h_2)^2}{4}\left(\frac{\partial p_0}{\partial x} - 12G\right)^3 y^3 - \frac{(h_1 + h_2)}{8}\left[2B_1 - (h_1 + h_2)^2\left(\frac{\partial p_0}{\partial x} - 12G\right)^3\right] y^2 \\ &\quad + \left[\frac{h_1h_2B_1}{2} - S_3\right] y + \frac{1}{2}\left[\frac{h_1^2(h_1 - 3h_2)B_1}{6} + F_1 - 2S_1 + 2h_1S_3\right], \end{aligned} \tag{2.35}$$

$$
\begin{aligned}
\frac{\partial p_1}{\partial x} &= -\frac{12F_1}{(h_1-h_2)^3} - \frac{2592\,(h_1-h_2)^2\,G^3}{5} + \frac{648\,(h_1-h_2)^2\,G^2}{5}\left(\frac{\partial p_0}{\partial x}\right) \\
&\quad -\frac{54\,(h_1-h_2)^2\,G}{5}\left(\frac{\partial p_0}{\partial x}\right)^2 + \frac{3\,(h_1-h_2)^2}{10}\left(\frac{\partial p_0}{\partial x}\right)^3. \qquad (2.36)
\end{aligned}
$$

2.4.6 Second order solution

Substituting the zeroth order and first order into Eq. (2.30) and using the boundary condition (2.32), we have

$$
\begin{aligned}
\psi_2 &= K_1y^7 + K_2y^6 + K_3y^5 + K_4y^4 + (K_5+K_6)\,y^3 + (K_7+K_8)\,y^2 \\
&\quad +K_9y + K_{10}, \qquad (2.37)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial p_2}{\partial x} &= -\frac{6}{(h_1-h_2)^3}\left[\frac{3\,(h_1-h_2)^7}{350}\left(\frac{\partial p_0}{\partial x} - 12G\right)^5\right. \\
&\quad \left.+\frac{9\,(h_1-h_2)^2\,F_1}{5}\left(\frac{\partial p_0}{\partial x} - 12G\right)^2\right] - \frac{12F_2}{(h_1-h_2)^3}. \qquad (2.38)
\end{aligned}
$$

The non dimensional pressure rise per wave length is defined as

$$
\Delta p = \int_0^{2\pi} \frac{\partial p}{\partial x}dx, \qquad (2.39)
$$

where

$$
\begin{aligned}
B_1 &= -\frac{12F_1}{(h_1-h_2)^3} + \frac{12\,(S_1-S_2)}{(h_1-h_2)^3} - \frac{12S_3}{(h_1-h_2)^2}, \\
S_1 &= \frac{h_1^2\left(h_1^3 + 5h_2\left(h_1^2 + h_1h_2 + h_2^2\right)\right)}{40}\left(\frac{\partial p_0}{\partial x} - 12G\right)^3, \\
S_2 &= \frac{h_2^2\left(h_2^3 + 5h_1\left(h_2^2 + h_1h_2 + h_1^2\right)\right)}{40}\left(\frac{\partial p_0}{\partial x} - 12G\right)^3, \\
S_3 &= \frac{h_1h_2\left(h_2^2 + h_1^2\right)}{4}\left(\frac{\partial p_0}{\partial x} - 12G\right)^3,
\end{aligned}
$$

$$
\begin{aligned}
K_1 &= \frac{2}{7}\left(\frac{\partial p_0}{\partial x} - 12G\right)^5, \quad K_2 = -(h_1+h_2)\left(\frac{\partial p_0}{\partial x} - 12G\right)^5, \\
K_3 &= \frac{3}{10}\left(\frac{\partial p_0}{\partial x} - 12G\right)^2 \left[\frac{12F_1}{(h_1-h_2)^3}\right. \\
&\quad \left. + \frac{(47h_1^2 + 106h_1h_2 + 47h_2^2)}{10}\left(\frac{\partial p_0}{\partial x} - 12G\right)^3\right], \\
K_4 &= -\frac{(h_1+h_2)}{2}\left(\frac{\partial p_0}{\partial x} - 12G\right)^2 \left[\frac{18F_1}{(h_1-h_2)^3}\right. \\
&\quad \left. + \frac{(41h_1^2 + 118h_1h_2 + 41h_2^2)}{20}\left(\frac{\partial p_0}{\partial x} - 12G\right)^3\right], \\
K_5 &= (h_1+h_2)^2\left(\frac{\partial p_0}{\partial x} - 12G\right)^2 \left[\frac{9F_1}{(h_1-h_2)^3}\right. \\
&\quad \left. + \frac{(4h_1^2 + 17h_1h_2 + 4h_2^2)}{10}\left(\frac{\partial p_0}{\partial x} - 12G\right)^3\right], \\
K_6 &= \frac{1}{(h_1-h_2)^3}\left[-2(K_{11}-K_{12}) + (h_1-h_2)(K_{13}+K_{14})\right], \\
K_7 &= \frac{1}{(h_1-h_2)^3}\left[3(h_1+h_2)(K_{11}-K_{12})\right. \\
&\quad \left. - (h_1-h_2)((h_1+2h_2)K_{13} + (2h_1+h_2)K_{14})\right], \\
K_8 &= -\frac{3(h_1+h_2)^3}{4}\left(\frac{\partial p_0}{\partial x} - 12G\right)^2 \left[\frac{6F_1}{(h_1-h_2)^3}\right. \\
&\quad \left. + \frac{(h_1^2 + 8h_1h_2 + h_2^2)}{10}\left(\frac{\partial p_0}{\partial x} - 12G\right)^3\right], \\
K_9 &= K_{14} - 3h_2^2K_6 - 2h_2K_7, \\
K_{10} &= K_{11} - h_1K_{14} - h_1\left(h_1^2 - 3h_2^2\right)K_6 - h_1(h_1-2h_2)K_7, \\
K_{11} &= -h_1^2K_8 + \frac{F_2}{2} - h_1^7K_1 - h_1^6K_2 - h_1^5K_3 - h_1^4K_4 - h_1^3K_5, \\
K_{12} &= -h_2^2K_8 - \frac{F_2}{2} - h_2^7K_1 - h_2^6K_2 - h_2^5K_3 - h_2^4K_4 - h_2^3K_5, \\
K_{13} &= -2h_1K_8 - 7h_1^6K_1 - 6h_1^5K_2 - 5h_1^4K_3 - 4h_1^3K_4 - 3h_1^2K_5, \\
K_{14} &= -2h_2K_8 - 7h_2^6K_1 - 6h_2^5K_2 - 5h_2^4K_3 - 4h_2^3K_4 - 3h_2^2K_5.
\end{aligned}
$$

2.5 Results and discussion

In this section, we briefly describe the effects of various emerging behavior of included parameters on the pressure rise per wave length and trapping. The variation of pressure rise ΔP with flow rate θ for various values of Γ, $\sin\alpha$, Fr, ϕ and d is studied in figure 2.2 (a) to 2.2 (e). Fig 2.2 (a) shows that the effect of Γ on ΔP and it is observed that for a Newtonian fluid $\Gamma = 0$, there is a linear relation between the pressure rise and the flow rate. For $\Gamma \neq 0$ it is non linear. Fig 2.2 (b) describes the variation of $\sin\alpha$ on ΔP. It is observed that the pressure rise increase with an increase of $\sin\alpha$. Fig 2.2 (c) depicts the variation of Fr on ΔP. It is notice that flow rate and pressure rise decreases with an increase of Fr. Fig 2.2 (d) and Fig 2.1 (e) shows the effect of ϕ and d on ΔP. It is found that pressure rise decreases by increasing ϕ and d. Fig 2.2 (f) studies the effect of Γ on variation of ΔP with d. It shows that pressure rise increases with increasing Γ. Fig 2.2 (g) is a graph of ΔP with the upper wave amplitude a for different values of $\sin\alpha$. It indicate that the pressure rise increases as $\sin\alpha$ increases. Fig 2.2 (h) describes the effect of Fr on variation of ΔP with lower wave amplitude b. It is notice that pressure rise decreases as Fr increases. Fig 2.2 (i) presented the effect of phase difference ϕ on variation of ΔP with R. It is observed that pressure rise decreases as ϕ increases and increases with an increase of R.

The trapping for different values of Γ, ϕ, $\sin\alpha$, Fr are shown in Figures 2.3 to 2.6. The effect of Γ on trapping is illustrated in Fig 2.3. It is notice that the size of the trapped bolus decreases with increasing Γ. Fig 2.4 describes for different values of ϕ on trapping. It is observed that the bolus appearing in the center region for $\phi = 0$ and it moves towards left and decreases in size as ϕ increases. Fig 2.5 depicts the variation of $\sin\alpha$ on trapping. For $\sin\alpha = 0$ the trapped bolus does not exist, when $\sin\alpha \neq 0$ the trapping bolus exist and the bolus volume increases with an increase in $\sin\alpha$. Fig 2.6 present the effect of Fr on trapping. It is notice that the size of the trapped bolus decreases with increases Fr.

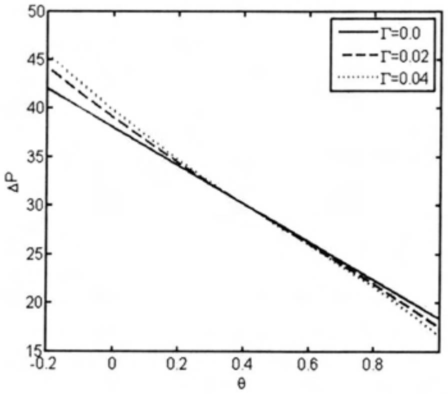

Fig. 2.2:(a) Effect of Γ on ΔP when $a = 0.2$, $b = 0.5$, $d = 0.8$, $R = 10$, $Fr = 1$, $\sin\alpha = 0.5$, $\phi = \frac{\pi}{4}$.

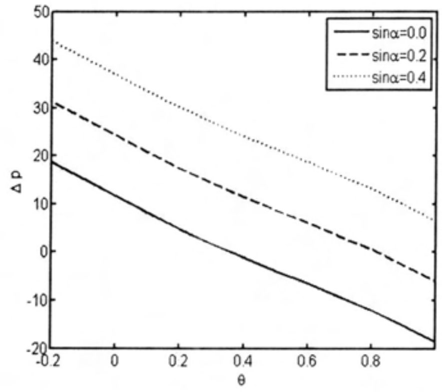

Fig. 2.2:(b) Effect of $\sin\alpha$ on ΔP when $a = 0.2$, $b = 0.5$, $d = 0.7$, $R = 10$, $Fr = 1$, $\Gamma = 0.04$, $\phi = \frac{\pi}{4}$.

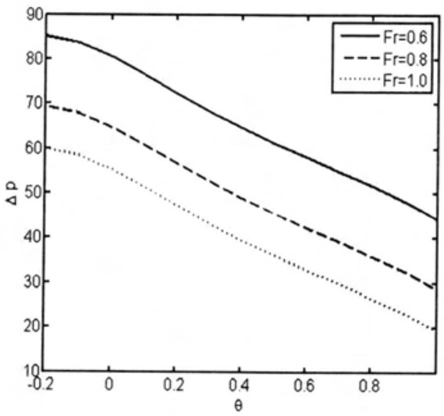

Fig. 2.2:(c) Effect of Fr on ΔP when $a = 0.3$, $\Gamma = 0.04$, $b = 0.5$, $d = 0.7$, $R = 10$, $\phi = \frac{\pi}{4}$, $\sin\alpha = 0.6$.

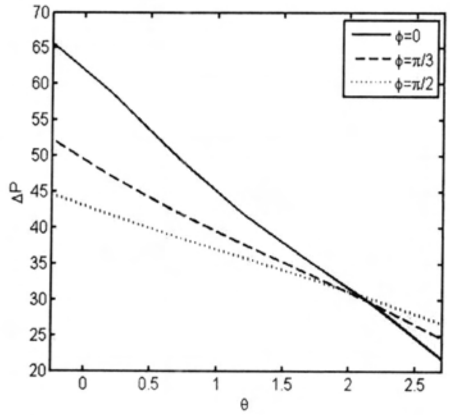

Fig. 2.2:(d) Effect of ϕ on ΔP when $a = 0.7$, $\Gamma = 0.01$, $b = 1.2$, $d = 2$, $Fr = 1$, $R = 10$, $\sin\alpha = 0.6$.

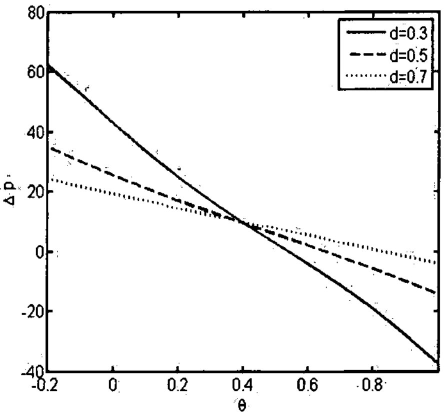

Fig. 2.2:(e) Effect of d on ΔP when $a = 0.2$, $b = 0.4$, $\phi = \frac{\pi}{6}$, $R = 10$, $Fr = 1$, $\sin\alpha = 0.2$, $\Gamma = 0.01$.

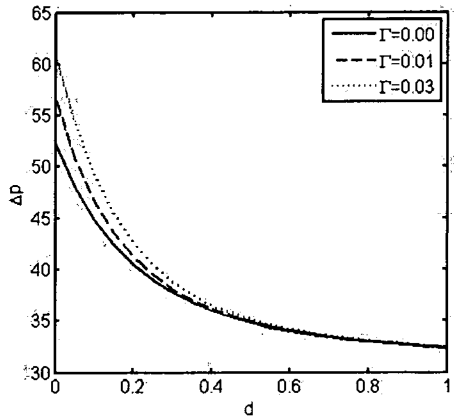

Fig. 2.2:(f) Effect of Γ on ΔP when $a = 0.3$, $b = 0.2$, $\phi = \frac{\pi}{2}$, $R = 10$, $Fr = 1$, $\sin\alpha = 0.5$.

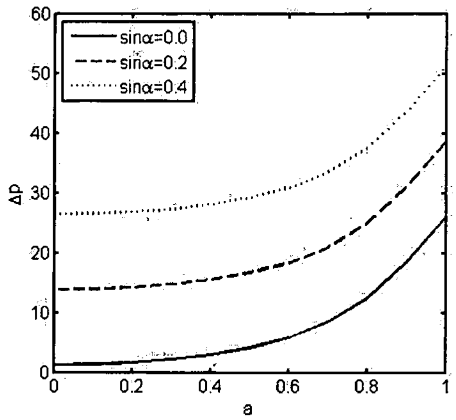

Fig. 2.2:(g) Effect of Fr on ΔP when $b = 0.4$, $d = 1, \phi = \frac{\pi}{2}$, $R = 10$, $\Gamma = 0.02, Fr = 1$.

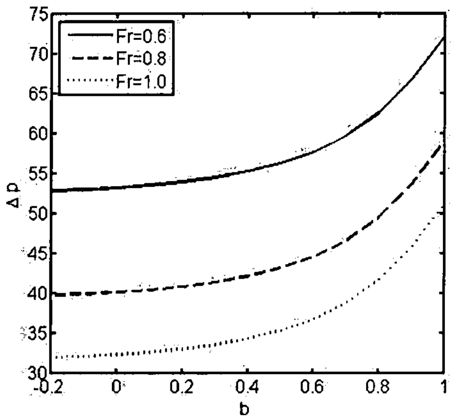

Fig. 2.2:(h) Effect of α on ΔP when $a = 0.5$, $b = 0.5$, $d = 1.5$, $\phi = \frac{\pi}{4}$, $R = 10$, $\Gamma = 0.02$, $\sin\alpha = 0.5$.

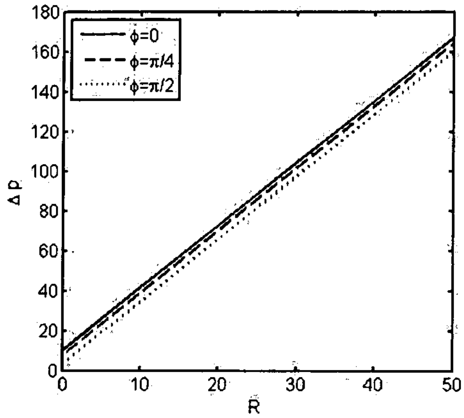

Fig. 2.2:(i) Effect of ϕ on ΔP when $a = 0.5$, $\Gamma = 0.02$, $b = 0.7$, $d = 1.5$, $Fr = 1$, $\sin\alpha = 0.5$.

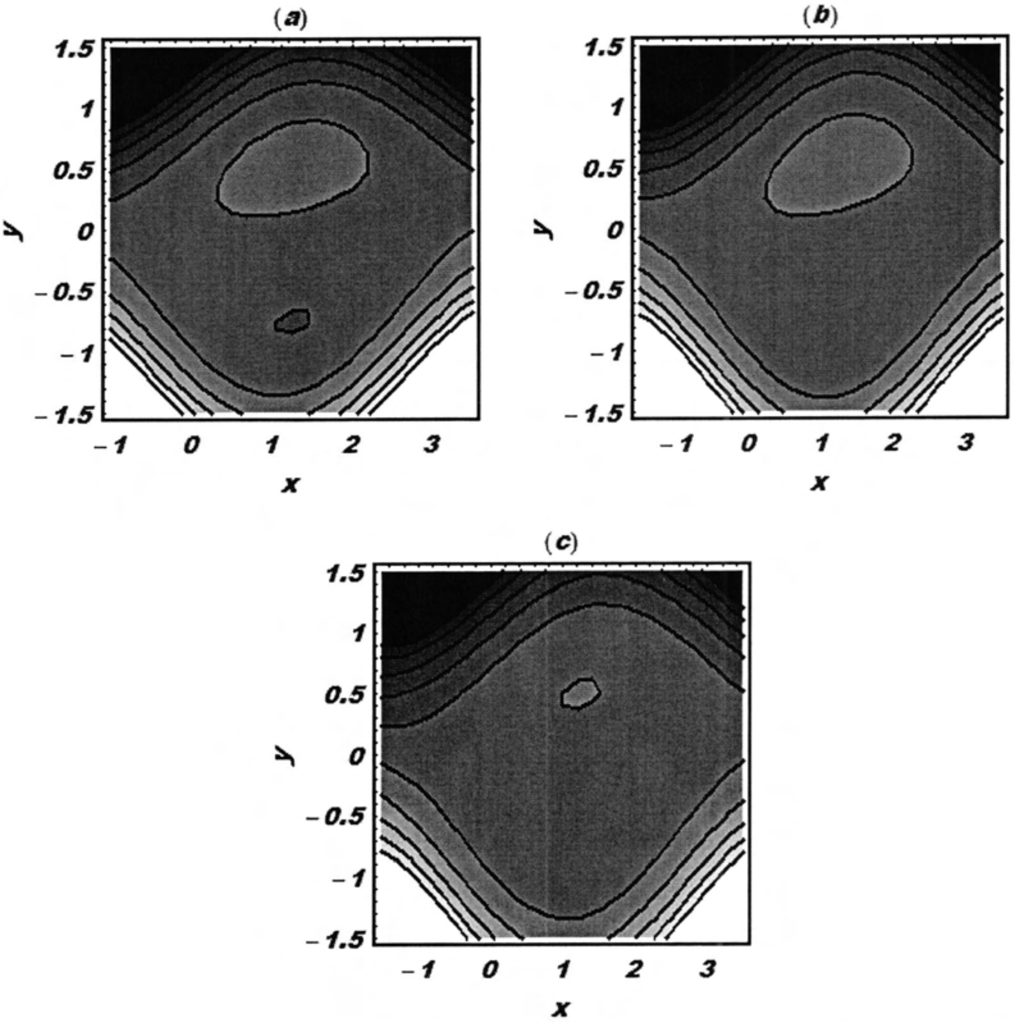

Fig. 2.3 : Streamlines for $a = 0.5$, $b = 0.7$, $R = 10$, $Fr = 1$, $\phi = \frac{\pi}{6}$, $\sin\alpha = 0.2$, $\frac{\partial p}{\partial x} = 1$, $d = 1$, with different Γ : (a) $\Gamma = 0.00$, (b) $\Gamma = 0.01$, (c) $\Gamma = 0.03$.

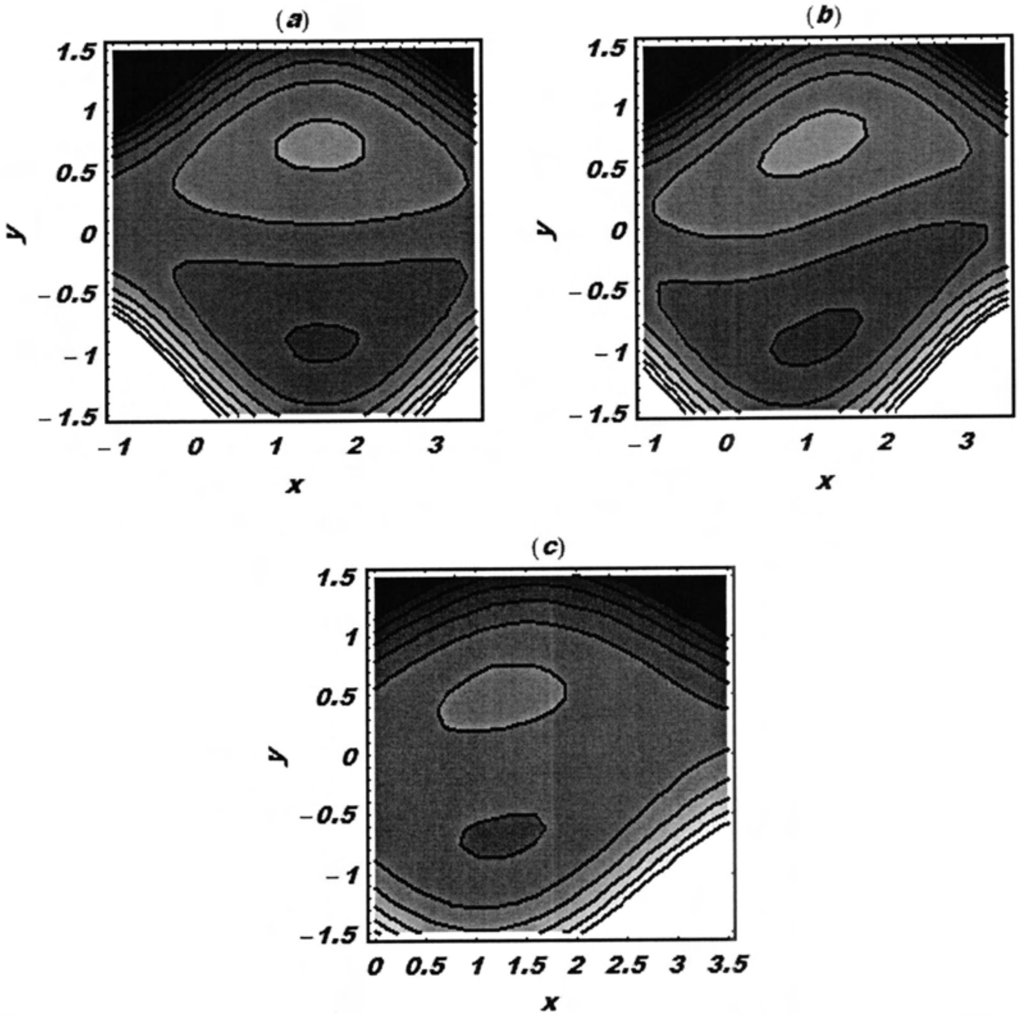

Fig. 2.4 : Streamlines for $a = 0.5$, $b = 0.7$, $R = 10$, $Fr = 0.6$, $\Gamma = 0.01$, $\sin\alpha = 0.2$, $\frac{\partial p}{\partial x} = 2$, $d = 1$, with different ϕ : (a) $\phi = 0$, (b) $\phi = \frac{\pi}{4}$, (c) $\phi = \frac{\pi}{2}$.

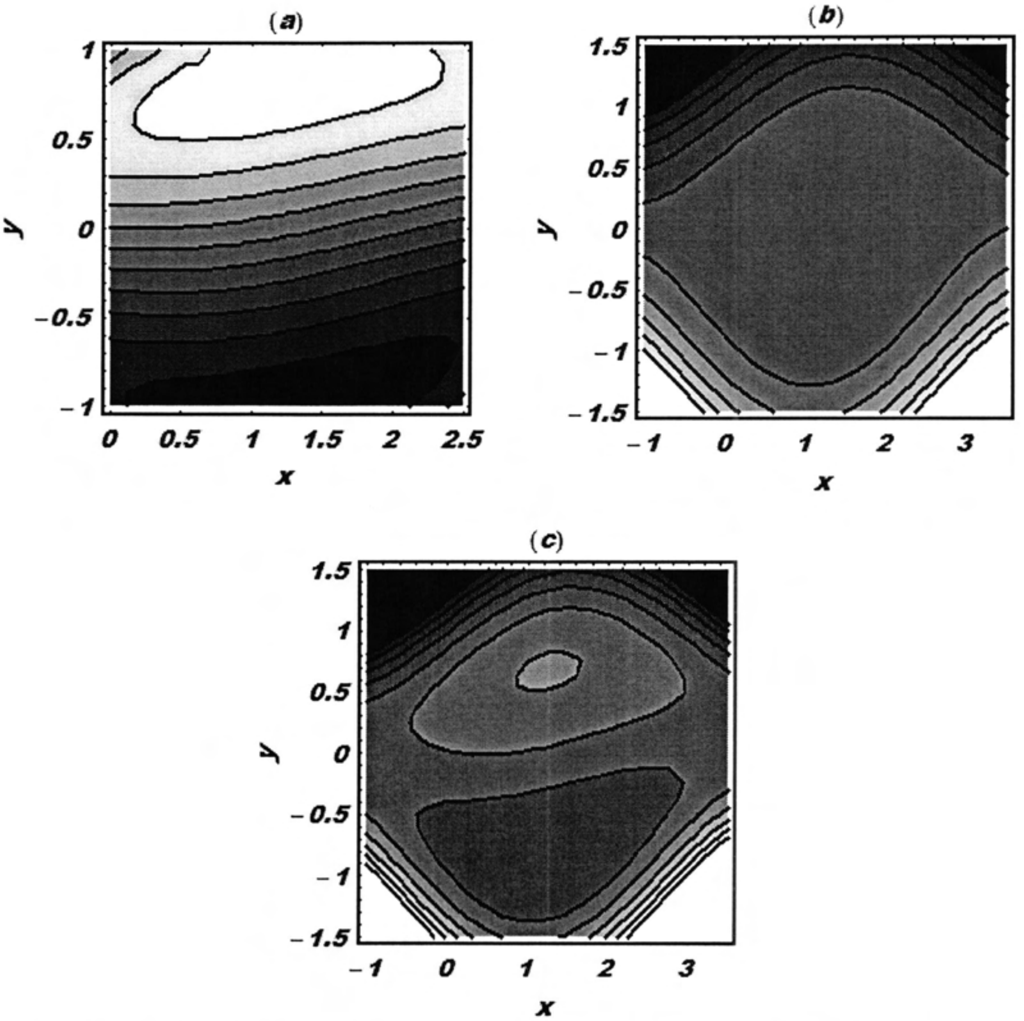

Fig. 2.5 : Streamlines for $a = 0.5$, $b = 0.7$, $R = 10$, $Fr = 1$, $\Gamma = 0.01$, $d = 1$, $\frac{\partial p}{\partial x} = 1.2$, $\phi = \frac{\pi}{6}$, with different $\sin\alpha$: (a) $\sin\alpha = 0$, (b) $\sin\alpha = 0.2$, (c) $\sin\alpha = 0.25$.

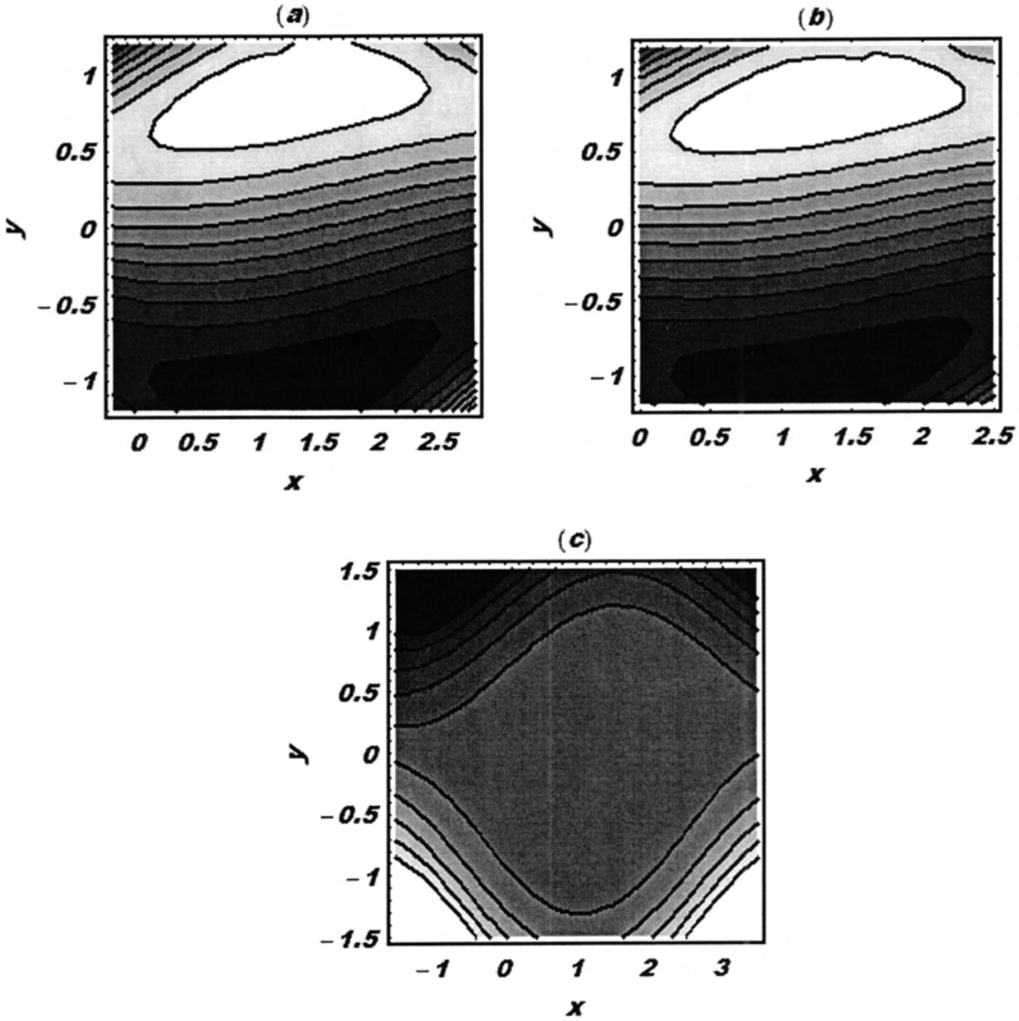

Fig. 2.6 : Streamlines for $a = 0.5$, $b = 0.7$, $R = 10$, $\sin\alpha = 0.3$, $\Gamma = 0.01$, $d = 1$, $\frac{\partial p}{\partial x} = 1.2$, $\phi = \frac{\pi}{6}$, with different Fr : (a) $Fr = 0.5$, (b) $Fr = 1$, (c) $Fr = 1.5$.

Chapter 3

MHD peristaltic flow of fourth grade fluid in a curved channel

3.1 Introduction

This chapter concerned with the influence of magnetic field on the peristaltic flow of a fourth grade fluid in a curved channel. The governing equations of the fourth grade fluid are modelled. Series solution is carried out under long wave length assumption. The expression for stream function and pressure gradient are obtained. Numerical results are graphically discussed for various values of physical parameters of interest.

3.2 Mathematical formulation

Consider a two dimensional magnetohydrodynamic (MHD) flow of an incompressible fourth grade fluid in a curved channel of radius R^* and uniform thickness $2d_1$ coiled in a circle with centre O. The flow is electrically conducting in the presence of constant magnetic field B_0 applied in the radial direction. Due to small magnetic Reynolds number, the induced magnetic field is neglected. The flow is in the azimuthal direction X and R is in radial direction, U and V are the components of velocity in the azimuthal and radial directions respectively. The walls of curved channel are

$$r = \pm h(x,t) = \pm\left[d_{1+}a\sin\frac{2\pi}{\lambda}(x-ct)\right], \tag{3.1}$$

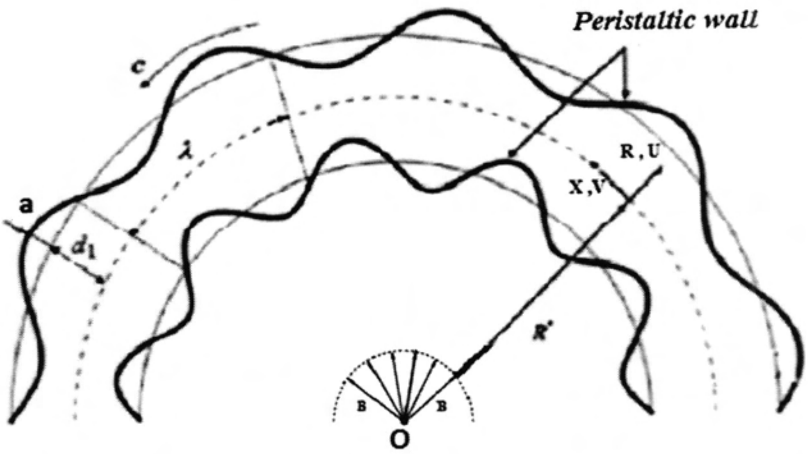

Fig. (3.1) : Geometry of the problem

where c is the wave speed, a and λ are the wave amplitude and wavelength respectively. Eqs. (2.6)-(2.8) in curvilinear coordinates becomes

$$\frac{\partial\overline{U}}{\partial\overline{R}}+\frac{R^*}{\overline{R}+R^*}\frac{\partial\overline{V}}{\partial\overline{X}}+\frac{\overline{U}}{\overline{R}+R^*} = 0, \tag{3.2}$$

$$\rho\left[\frac{\partial\overline{U}}{\partial\overline{t}}+\overline{U}\frac{\partial\overline{U}}{\partial\overline{R}}+\frac{R^*}{\overline{R}+R^*}\overline{V}\frac{\partial\overline{U}}{\partial\overline{X}}-\frac{\overline{V}^2}{\overline{R}+R^*}\right] = -\frac{\partial\overline{p}}{\partial\overline{R}}+\frac{1}{\overline{R}+R^*}\frac{\partial}{\partial\overline{R}}\left(\left(\overline{R}+R^*\right)\overline{S}_{\overline{RR}}\right) +\frac{R^*}{\overline{R}+R^*}\frac{\partial\overline{S}_{\overline{RX}}}{\partial\overline{X}}-\frac{\overline{S}_{\overline{XX}}}{\overline{R}+R^*}, \tag{3.3}$$

$$\rho\left[\frac{\partial\overline{V}}{\partial\overline{t}}+\overline{U}\frac{\partial\overline{V}}{\partial\overline{R}}+\frac{R^*}{\overline{R}+R^*}\overline{V}\frac{\partial\overline{V}}{\partial\overline{X}}+\frac{\overline{U}\,\overline{V}}{\overline{R}+R^*}\right] = -\frac{R^*}{\overline{R}+R^*}\frac{\partial\overline{p}}{\partial\overline{X}}+\frac{1}{\left(\overline{R}+R^*\right)^2}\frac{\partial}{\partial\overline{R}}\left(\overline{R}+R^*\right)^2\overline{S}_{\overline{RX}} +\frac{R^*}{\overline{R}+R^*}\frac{\partial\overline{S}_{\overline{XX}}}{\partial\overline{X}}-\frac{\sigma B_0^2}{\left(\overline{R}+R^*\right)^2}\overline{V}. \tag{3.4}$$

where p the pressure, ρ density of the fluid, R^* the curvature parameter and $\mathbf{S}$ is the extra stress tensor. If $(\overline{u},\ \overline{v})$ and $(\overline{U},\ \overline{V})$ are the velocity components in the wave and laboratory frames then the relations between these frames are

$$\overline{x}=\overline{X}-c\overline{t}, \overline{r}=\overline{R}, \overline{u}=\overline{U}, \overline{v}=\overline{V}-c. \tag{3.5}$$

We introduce the non dimensional variables

$$
\begin{aligned}
x &= \frac{\overline{x}}{\lambda},\, r=\frac{\overline{r}}{a},\, u=\frac{\overline{u}}{c\delta},\, v=\frac{\overline{v}}{c},\, p=\frac{a^2\overline{p}}{c\lambda\mu},\, \delta=\frac{a}{\lambda},\, \varepsilon=\frac{a}{d_1}, \\
k &= \frac{R^*}{a},\, t=\frac{c\overline{t}}{\lambda},\, S_{ij}=\frac{a}{\mu c}\overline{S_{ij}},\, Re=\frac{\rho c a}{\mu},\ m=\sqrt{\frac{\sigma}{\mu}}B_0 a.
\end{aligned} \tag{3.6}
$$

and introduce the dimensionless stream function through

$$
u=\delta\frac{k}{r+k}\frac{\partial\psi}{\partial r},\ v=-\frac{\partial\psi}{\partial r}. \tag{3.7}
$$

Making use of Eqs. (3.5) to (3.7). Eq. (3.2) is identically satisfy and after using long wavelength and low Reynolds number approximations. Eqs. (3.3) and (3.4) in terms of stream function are

$$
\frac{\partial p}{\partial r}=0, \tag{3.8}
$$

$$
\frac{\partial p}{\partial x}=\frac{1}{k\left(r+k\right)}\frac{\partial}{\partial r}\left(\left(r+k\right)^2 S_{rx}\right)-\frac{m^2\left(-\psi_r+1\right)}{k\left(r+k\right)}, \tag{3.9}
$$

where

$$
\begin{aligned}
S_{rx} &= -\psi_{rr}-\frac{1-\psi_r}{r+k}-2\Gamma\left(\psi_{rr}+\frac{1-\psi_r}{r+k}\right)^3, \\
\Gamma &= \xi_2+\xi_3.
\end{aligned} \tag{3.10}
$$

We note from Eq. (3.8) that $p\neq p\left(r\right)$ and above equations after rearrangement yield

$$
\frac{\partial}{\partial r}\left[\frac{1}{k\left(r+k\right)}\frac{\partial}{\partial r}\left(\left(r+k\right)^2 S_{rx}\right)-\frac{m^2\left(-\psi_r+1\right)}{k\left(r+k\right)}\right]=0, \tag{3.11}
$$

and with the boundary conditions

$$
\begin{aligned}
\psi &= \frac{F}{2},\, \frac{\partial\psi}{\partial r}=1 \text{ at } r=-h=-\left(1+\epsilon\sin 2\pi\left(x-t\right)\right), \\
\psi &= -\frac{F}{2},\, \frac{\partial\psi}{\partial r}=1 \text{ at } r=h=\left(1+\epsilon\sin 2\pi\left(x-t\right)\right),
\end{aligned} \tag{3.12}
$$

where

$$F = -\int_{-h}^{h} \frac{\partial \psi}{\partial r} dr, \tag{3.13}$$

is the dimensionless mean flow rate in the wave frame which is related to the dimensionless mean flow rate θ in the laboratory frame as in the following expression

$$\theta = F + 2. \tag{3.14}$$

3.3 Solution procedure

Employing Eq. (2.23) into Eqs. (3.9) – (3.12), we get following systems:

3.3.1 Zeroth order system

$$\frac{\partial}{\partial r}\left[\frac{1}{k(r+k)}\frac{\partial}{\partial r}\left((r+k)^2 S_{0rx}\right) - \frac{m^2(-\psi_{0r}+1)}{k(r+k)}\right] = 0, \tag{3.15}$$

$$\frac{\partial p_0}{\partial x} = \frac{1}{k(r+k)}\frac{\partial}{\partial r}\left((r+k)^2 S_{0rx}\right) - \frac{m^2(-\psi_{0r}+1)}{k(r+k)}, \tag{3.16}$$

where

$$S_{0rx} = -\psi_{0rr} - \frac{1-\psi_{0r}}{r+k}, \tag{3.17}$$

$$\begin{aligned} \psi_0 &= \frac{F_0}{2}, \frac{\partial \psi_0}{\partial r} = 1 \text{ at } r = -h, \\ \psi_0 &= -\frac{F_0}{2}, \frac{\partial \psi_0}{\partial r} = 1 \text{ at } r = h. \end{aligned} \tag{3.18}$$

3.3.2 First order system

$$\frac{\partial}{\partial r}\left[\frac{1}{k(r+k)}\frac{\partial}{\partial r}\left((r+k)^2 S_{1rx}\right) - \frac{m^2(-\psi_{1r})}{k(r+k)}\right] = 0, \tag{3.19}$$

$$\frac{\partial p_1}{\partial x} = \frac{1}{k(r+k)}\frac{\partial}{\partial r}\left((r+k)^2 S_{1rx}\right) - \frac{m^2(-\psi_{1r})}{k(r+k)}, \tag{3.20}$$

where

$$S_{1rx} = -\psi_{1rr} + \frac{\psi_{1r}}{r+k} - 2\left(\psi_{0rr} + \frac{1-\psi_{0r}}{r+k}\right)^3, \tag{3.21}$$

$$
\begin{aligned}
\psi_1 &= -\frac{F_1}{2}, \frac{\partial \psi_1}{\partial r} = 0 \text{ at } r = h, \\
\psi_1 &= \frac{F_1}{2}, \frac{\partial \psi_1}{\partial r} = 0 \text{ at } r = -h.
\end{aligned} \tag{3.22}
$$

3.3.3 Second order system

$$
\frac{\partial}{\partial r}\left[\frac{1}{k(r+k)}\frac{\partial}{\partial r}\left((r+k)^2 S_{2rx}\right) - \frac{m^2(-\psi_{2r})}{k(r+k)}\right] = 0, \tag{3.23}
$$

$$
\frac{\partial p_2}{\partial x} = \frac{1}{k(r+k)}\frac{\partial}{\partial r}\left((r+k)^2 S_{2rx}\right) - \frac{m^2(-\psi_{2r})}{k(r+k)}, \tag{3.24}
$$

where

$$
\begin{aligned}
S_{2rx} = {} & -\psi_{2rr} + \frac{\psi_{2r}}{r+k} - 6\psi_{0rr}^2\psi_{1rr} - \frac{6}{(r+k)^3}\left(2\psi_{0r}\psi_{1r} - \psi_{1r} - \psi_{0r}^2\psi_{1r}\right) \\
& -\frac{12}{(r+k)}\left(\psi_{0rr}\psi_{1rr} - \psi_{0r}\psi_{0rr}\psi_{1rr} - \psi_{0rr}^2\psi_{1r}\right) - \frac{6}{(r+k)^2} \\
& \left(\psi_{1rr} - 2\psi_{0r}\psi_{0rr}\psi_{1r} + \psi_{0r}^2\psi_{1rr} - 2\psi_{0r}\psi_{1rr} - 2\psi_{0rr}\psi_{1r}\right),
\end{aligned} \tag{3.25}
$$

$$
\begin{aligned}
\psi_2 &= -\frac{F_2}{2}, \frac{\partial \psi_2}{\partial r} = 0 \text{ at } r = h, \\
\psi_2 &= \frac{F_2}{2}, \frac{\partial \psi_2}{\partial r} = 0 \text{ at } r = -h.
\end{aligned} \tag{3.26}
$$

3.3.4 Zeroth order solution

$$
\psi_0 = c_1 + c_2(r+k)^2 + c_3(r+k)^{1-\sqrt{1+m^2}} + c_4(r+k)^{1+\sqrt{1+m^2}} + (r+k). \tag{3.27}
$$

$$
\frac{dp_0}{dx} = \frac{1}{k}\left(2m^2 c_2\right). \tag{3.28}
$$

3.3.5 First order solution

$$
\begin{aligned}
\psi_1 = {} & c_5 + c_6(k+r)^2 + c_7(k+r)^{1-\sqrt{1+m^2}} + c_8(k+r)^{1+\sqrt{1+m^2}} \\
& -c_9(k+r)^{-1-3\sqrt{1+m^2}} - c_{10}(k+r)^{-1-\sqrt{1+m^2}} - c_{11}(k+r)^{-1+\sqrt{1+m^2}} \\
& -c_{12}(k+r)^{-1+3\sqrt{1+m^2}},
\end{aligned} \tag{3.29}
$$

$$
\frac{dp_1}{dx} = \frac{1}{k}\left(2m^2 c_6\right). \tag{3.30}
$$

3.3.6 Second order solution

$$\begin{aligned}
\psi_2 &= c_{13} + (k+r)^2 c_{14} + c_{15}(k+r)^{1-\sqrt{1+m^2}} + c_{16}(k+r)^{1+\sqrt{1+m^2}} \\
&\quad + c_{17}(k+r)^{-1-3\sqrt{1+m^2}} + c_{18}(k+r)^{-1-\sqrt{1+m^2}} + c_{19}(k+r)^{-1+\sqrt{1+m^2}} \\
&\quad + c_{20}(k+r)^{-1+3\sqrt{1+m^2}} + c_{21}(k+r)^{-3-5\sqrt{1+m^2}} + c_{22}(k+r)^{-3-3\sqrt{1+m^2}} \\
&\quad + c_{23}(k+r)^{-3-\sqrt{1+m^2}} + c_{24}(k+r)^{-3+\sqrt{1+m^2}} + c_{25}(k+r)^{-3+3\sqrt{1+m^2}} \\
&\quad + c_{26}(k+r)^{-3+5\sqrt{1+m^2}} + c_{27}(k+r)^{-2\sqrt{1+m^2}} + c_{28}(k+r)^{2\sqrt{1+m^2}} \\
&\quad + c_{29}(k+r)^{-2-4\sqrt{1+m^2}} + c_{30}(k+r)^{-2} + c_{31}(k+r)^{-2+2\sqrt{1+m^2}} \\
&\quad + c_{32}(k+r)^{-2-2\sqrt{1+m^2}} + c_{33}(k+r)^{-2+4\sqrt{1+m^2}} + c_{34}(k+r)^{\sqrt{1+m^2}} \\
&\quad + c_{35}(k+r)^{-2-3\sqrt{1+m^2}} + c_{36}(k+r)^{-2-\sqrt{1+m^2}} + c_{37}(k+r)^{-2+\sqrt{1+m^2}} \\
&\quad + c_{38}(k+r)^{-2+3\sqrt{1+m^2}} + c_{39}(k+r) + c_{40}(k+r)^{-\sqrt{1+m^2}} \\
&\quad + c_{41}(k+r)^{-3-4\sqrt{1+m^2}} + c_{42}(k+r)^{-3-2\sqrt{1+m^2}} + c_{43}(k+r)^{-3} \\
&\quad + c_{44}(k+r)^{-3+2\sqrt{1+m^2}} + c_{45}(k+r)^{-1-2\sqrt{1+m^2}} + c_{46}(k+r)^{-1} \\
&\quad + c_{47}(k+r)^{-1+2\sqrt{1+m^2}} + c_{48}(k+r)^{-3+4\sqrt{1+m^2}} + c_{49}\ln(k+r) \\
&\quad (k+r)^{1-\sqrt{1+m^2}} + c_{50}\ln(k+r)(k+r)^{1+\sqrt{1+m^2}}, \qquad (3.31) \\
\frac{dp_2}{dx} &= \frac{1}{k}\left(2m^2 c_{14}\right). \qquad (3.32)
\end{aligned}$$

The dimensionless pressure rise per wave length is defined as

$$\Delta p = \int_0^1 \frac{dp}{dx}dx, \qquad (3.33)$$

where

$$\begin{aligned}
c_1 &= \frac{1}{2c_{51}}(F_0+2h)\left[c_{51} - \left(m^2+2\right)(k+h)^2\left((k+h)^{2\sqrt{1+m^2}} - (k-h)^{2\sqrt{1+m^2}}\right)\right] \\
&\quad -\frac{1}{2}\left(F_0 + 2(h+k)\right), \\
c_2 &= \frac{1}{2c_{51}}\left[(F_0+2h)\left(-m^2\right)\left((k-h)^{2\sqrt{1+m^2}} - (k+h)^{2\sqrt{1+m^2}}\right)\right], \\
c_3 &= \frac{1}{c_{51}}\left[(F_0+2h)\left(1+\sqrt{1+m^2}\right)\left((k+h)^{2\sqrt{1+m^2}}(k-h)^{1+2\sqrt{1+m^2}}\right.\right.
\end{aligned}$$

$$
\begin{aligned}
& - (k+h)^{1+2\sqrt{1+m^2}} (k-h)^{2\sqrt{1+m^2}} \Big) \Big] , \\
c_4 &= \frac{1}{c_{51}} \Big[(F_0 + 2h) \Big(1 - \sqrt{1+m^2}\Big) \Big((k+h)^{1+\sqrt{1+m^2}} - (k-h)^{1+\sqrt{1+m^2}} \Big) \Big] , \\
c_5 &= -c_6 (k+h)^2 - c_7 (k+h)^{1-\sqrt{1+m^2}} - c_8 (k+h)^{1+\sqrt{1+m^2}} + c_9 (k+h)^{-1-3\sqrt{1+m^2}} \\
& \quad + c_{10} (k+h)^{-1-\sqrt{1+m^2}} + c_{11} (k+h)^{-1+\sqrt{1+m^2}} + c_{12} (k+h)^{-1+3\sqrt{1+m^2}} - \frac{F_1}{2} , \\
c_6 &= F_1 \Big(-\frac{1}{2hk} + \frac{m^2}{2c_{51}} \Big(-(k+h)^{2\sqrt{1+m^2}} + (k-h)^{2\sqrt{1+m^2}} \Big) \Big) - \frac{1}{4hkc_{51}c_{56}} \Big[-c_9 \\
& \quad \Big(c_{51} \Big(\Big(1 - \sqrt{1+m^2}\Big) \Big((k+h)^{-2\sqrt{1+m^2}} - (k+h)^{-1-3\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}} \\
& \quad - (k-h)^{-1-3\sqrt{1+m^2}} (k+h)^{1+\sqrt{1+m^2}} + (k-h)^{-2\sqrt{1+m^2}} \Big) + \Big(-1 - 3\sqrt{1+m^2}\Big) \\
& \quad \Big((k+h)^{-1-3\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}} - (k-h)^2 (k+h)^{-2-2\sqrt{1+m^2}} - (k+h)^2 \\
& \quad (k-h)^{-2-2\sqrt{1+m^2}} + (k-h)^{-1-3\sqrt{1+m^2}} (k+h)^{1+\sqrt{1+m^2}} \Big) \Big) - c_{51}c_{52} + 2hkm^2 \\
& \quad \Big(-(k+h)^{2\sqrt{1+m^2}} + (k-h)^{2\sqrt{1+m^2}} \Big) c_{52} \Big) - c_{10} \Big(c_{51} \Big(\Big(1 - \sqrt{1+m^2}\Big) \\
& \quad \Big(2 - (k+h)^{1+\sqrt{1+m^2}} (k-h)^{-1-\sqrt{1+m^2}} - (k-h)^{1+\sqrt{1+m^2}} (k+h)^{-1-\sqrt{1+m^2}} \Big) \\
& \quad + \Big(-1 - \sqrt{1+m^2}\Big) \Big((k-h)^{1+\sqrt{1+m^2}} (k+h)^{-1-\sqrt{1+m^2}} - (k+h)^2 \\
& \quad (k-h)^{-2} - (k-h)^2 (k+h)^{-2} + (k+h)^{1+\sqrt{1+m^2}} (k-h)^{-1-\sqrt{1+m^2}} \Big) \Big) - c_{51}c_{53} \\
& \quad + 2hkm^2 \Big(-(k+h)^{2\sqrt{1+m^2}} + (k-h)^{2\sqrt{1+m^2}} \Big) c_{53} \Big) - c_{11} \Big(c_{51} \Big(\Big(1 - \sqrt{1+m^2}\Big) \\
& \quad \Big((k+h)^{2\sqrt{1+m^2}} - (k+h)^{-1+\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}} - (k-h)^{-1+\sqrt{1+m^2}} \\
& \quad (k+h)^{1+\sqrt{1+m^2}} + (k-h)^{2\sqrt{1+m^2}} \Big) + \Big(-1 + \sqrt{1+m^2}\Big) \Big((k+h)^{-1+\sqrt{1+m^2}} \\
& \quad (k-h)^{1+\sqrt{1+m^2}} - (k-h)^2 (k+h)^{-2+2\sqrt{1+m^2}} - (k+h)^2 (k-h)^{-2+2\sqrt{1+m^2}} \\
& \quad + (k-h)^{-1+\sqrt{1+m^2}} (k+h)^{1+\sqrt{1+m^2}} \Big) \Big) + c_{51}c_{54} - 2hkm^2 \Big(-(k+h)^{2\sqrt{1+m^2}} \\
& \quad + (k-h)^{2\sqrt{1+m^2}} \Big) c_{54} \Big) - c_{12} \Big(c_{51} \Big(\Big(1 - \sqrt{1+m^2}\Big) \Big((k+h)^{4\sqrt{1+m^2}} \\
& \quad - (k+h)^{-1+3\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}} - (k-h)^{-1+3\sqrt{1+m^2}} (k+h)^{1+\sqrt{1+m^2}} \\
& \quad + (k-h)^{4\sqrt{1+m^2}} \Big) + \Big(-1 + 3\sqrt{1+m^2}\Big) \Big((k+h)^{-1+3\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}} \\
& \quad - (k+h)^2 (k-h)^{-2+4\sqrt{1+m^2}} - (k-h)^2 (k+h)^{-2+4\sqrt{1+m^2}} + (k-h)^{-1+3\sqrt{1+m^2}} \\
& \quad (k+h)^{1+\sqrt{1+m^2}} \Big) \Big) + c_{51}c_{55} - 2hkm^2 \Big(-(k+h)^{2\sqrt{1+m^2}} + (k-h)^{2\sqrt{1+m^2}} \Big) \\
& \quad c_{55} \Big) \Big] , \\
c_7 &= -\frac{1}{c_{51}c_{56}} \Big(F_1 c_{56} \Big(1 + \sqrt{1+m^2}\Big) \Big((k+h)^{2\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}}
\end{aligned}
$$

$$
\begin{aligned}
& -(k+h)^{1+\sqrt{1+m^2}}(k-h)^{2\sqrt{1+m^2}}\Big) + c_9 c_{51}\Big(\Big(1+3\sqrt{1+m^2}\Big) \\
& \Big((k+h)^{-2-2\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}}(k-h)^{-2-2\sqrt{1+m^2}}\Big) \\
& -\Big(1+\sqrt{1+m^2}\Big)\Big(\Big((k+h)^{2\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}}\Big) \\
& (k-h)^{2\sqrt{1+m^2}}\Big) c_{52}\Big) + c_{10}\Big(c_{51}\Big(1+\sqrt{1+m^2}\Big)\Big((k+h)^{-2} \\
& \Big((k-h)^{1+\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}}(k-h)^{-2}\Big) - \Big(1+\sqrt{1+m^2}\Big) \\
& \Big((k+h)^{2\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}}(k-h)^{2\sqrt{1+m^2}}\Big) c_{53}\Big) \\
& -c_{11}\Big(c_{51}\Big(-1+\sqrt{1+m^2}\Big)\Big((k+h)^{-2+2\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}} \\
& -(k+h)^{1+\sqrt{1+m^2}}(k-h)^{-2+2\sqrt{1+m^2}}\Big) + \Big(1+\sqrt{1+m^2}\Big)\Big((k+h)^{2\sqrt{1+m^2}} \\
& (k-h)^{1+\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}}(k-h)^{2\sqrt{1+m^2}}\Big) c_{54}\Big) - c_{12}\,(c_{51} \\
& \Big(-1+3\sqrt{1+m^2}\Big)\Big((k+h)^{-2+4\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}} + (k+h)^{1+\sqrt{1+m^2}} \\
& (k-h)^{-2+4\sqrt{1+m^2}}\Big) + \Big(1+\sqrt{1+m^2}\Big)\Big((k+h)^{2\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}} \\
& (k+h)^{1+\sqrt{1+m^2}}(k-h)^{2\sqrt{1+m^2}}\Big) - c_{55}\Big)\Big),
\end{aligned}
$$

$$
\begin{aligned}
c_8 &= \frac{1}{c_{51}}\Big[F_1\Big(1-\sqrt{1+m^2}\Big)\Big((k+h)^{1+\sqrt{1+m^2}} - (k-h)^{1+\sqrt{1+m^2}}\Big) - c_9 c_{52} \\
&\quad -c_{10}c_{53} + c_{11}c_{54} + c_{12}c_{55}\Big], \\
c_9 &= \frac{(m^6 c_3^3)}{\Big(6+6\sqrt{1+m^2}+4m^2\Big)}, \quad c_{10} = \frac{3m^6 c_3^2 c_4}{2+2\sqrt{1+m^2}}, \\
c_{11} &= \frac{3m^6 c_3 c_4^2}{2-2\sqrt{1+m^2}}, \quad c_{12} = \frac{(m^6 c_4^3)}{\Big(6-6\sqrt{1+m^2}+4m^2\Big)}, \\
c_{13} &= -\frac{F_2}{2} - (k+r)^2 c_{14} - c_{15}(k+r)^{1-\sqrt{1+m^2}} - c_{16}(k+r)^{1+\sqrt{1+m^2}} \\
&\quad -c_{17}(k+h)^{-1-3\sqrt{1+m^2}} - c_{18}(k+h)^{-1-\sqrt{1+m^2}} - c_{19}(k+h)^{-1+\sqrt{1+m^2}} \\
&\quad -c_{20}(k+h)^{-1+3\sqrt{1+m^2}} - c_{21}(k+h)^{-3-5\sqrt{1+m^2}} - c_{22}(k+h)^{-3-3\sqrt{1+m^2}} \\
&\quad -c_{23}(k+h)^{-3-\sqrt{1+m^2}} - c_{24}(k+h)^{-3+\sqrt{1+m^2}} - c_{25}(k+h)^{-3+3\sqrt{1+m^2}} \\
&\quad -c_{26}(k+h)^{-3+5\sqrt{1+m^2}} - c_{27}(k+h)^{-2\sqrt{1+m^2}} - c_{28}(k+h)^{2\sqrt{1+m^2}} \\
&\quad -c_{29}(k+h)^{-2-4\sqrt{1+m^2}} - c_{30}(k+h)^{-2} - c_{31}(k+h)^{-2+2\sqrt{1+m^2}} \\
&\quad -c_{32}(k+h)^{-2-2\sqrt{1+m^2}} - c_{33}(k+h)^{-2+4\sqrt{1+m^2}} - c_{34}(k+h)^{\sqrt{1+m^2}} \\
&\quad -c_{35}(k+h)^{-2-3\sqrt{1+m^2}} - c_{36}(k+h)^{-2-\sqrt{1+m^2}} - c_{37}(k+h)^{-2+\sqrt{1+m^2}}
\end{aligned}
$$

$$
\begin{aligned}
&-c_{38}(k+h)^{-2+3\sqrt{1+m^2}}-c_{39}(k+h)-c_{40}(k+h)^{-\sqrt{1+m^2}}\\
&-c_{41}(k+h)^{-3-4\sqrt{1+m^2}}-c_{42}(k+h)^{-3-2\sqrt{1+m^2}}-c_{43}(k+h)^{-3}\\
&-c_{44}(k+h)^{-3+2\sqrt{1+m^2}}-c_{45}(k+h)^{-1-2\sqrt{1+m^2}}-c_{46}(k+h)^{-1}\\
&-c_{47}(k+h)^{-1+2\sqrt{1+m^2}}-c_{48}(k+h)^{-3+4\sqrt{1+m^2}}-c_{49}\ln(k+h)\\
&(k+h)^{1-\sqrt{1+m^2}}-c_{50}\ln(k+h)(k+h)^{1+\sqrt{1+m^2}},\\
c_{14}\;=\;&F_2\left(-\frac{1}{2hk}+\frac{m^2}{2c_{51}c_{56}}\left(-(k+h)^{2\sqrt{1+m^2}}+(k-h)^{2\sqrt{1+m^2}}\right)\right)-\frac{1}{4hkc_{51}c_{56}}\\
&\Big[c_{17}\Big(c_{51}\Big(\Big(1-\sqrt{1+m^2}\Big)\Big((k+h)^{-2\sqrt{1+m^2}}-(k+h)^{-1-3\sqrt{1+m^2}}\\
&(k-h)^{1+\sqrt{1+m^2}}-(k-h)^{-1-3\sqrt{1+m^2}}(k+h)^{1+\sqrt{1+m^2}}+(k-h)^{-2\sqrt{1+m^2}}\Big)\\
&+\Big(-1-3\sqrt{1+m^2}\Big)\Big((k+h)^{-1-3\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}}-(k-h)^2\\
&(k+h)^{-2-2\sqrt{1+m^2}}-(k+h)^2(k-h)^{-2-2\sqrt{1+m^2}}+(k-h)^{-1-3\sqrt{1+m^2}}\\
&(k+h)^{1+\sqrt{1+m^2}}\Big)\Big)+c_{51}c_{57}-2hkm^2\Big(-(k+h)^{2\sqrt{1+m^2}}+(k-h)^{2\sqrt{1+m^2}}\Big)\\
&c_{57}\Big)+c_{18}\Big(c_{51}\Big(\Big(1-\sqrt{1+m^2}\Big)\Big(2-(k+h)^{1+\sqrt{1+m^2}}(k-h)^{-1-\sqrt{1+m^2}}\\
&-(k-h)^{1+\sqrt{1+m^2}}(k+h)^{-1-\sqrt{1+m^2}}\Big)+\Big(-1-\sqrt{1+m^2}\Big)\Big((k-h)^{1+\sqrt{1+m^2}}\\
&(k+h)^{-1-\sqrt{1+m^2}}-(k+h)^2(k-h)^{-2}-(k-h)^2(k+h)^{-2}+(k+h)^{1+\sqrt{1+m^2}}\\
&(k-h)^{-1-\sqrt{1+m^2}}\Big)\Big)+c_{51}c_{58}-2hkm^2\Big(-(k+h)^{2\sqrt{1+m^2}}+(k-h)^{2\sqrt{1+m^2}}\Big)c_{58}\Big)\\
&+c_{19}\Big(c_{51}\Big(\Big(1-\sqrt{1+m^2}\Big)\Big((k+h)^{2\sqrt{1+m^2}}-(k+h)^{-1+\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}}\\
&-(k-h)^{-1+\sqrt{1+m^2}}(k+h)^{1+\sqrt{1+m^2}}+(k-h)^{2\sqrt{1+m^2}}\Big)+\Big(-1+\sqrt{1+m^2}\Big)\\
&\Big((k+h)^{-1+\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}}-(k-h)^2(k+h)^{-2+2\sqrt{1+m^2}}-(k+h)^2\\
&(k-h)^{-2+2\sqrt{1+m^2}}+(k-h)^{-1+\sqrt{1+m^2}}(k+h)^{1+\sqrt{1+m^2}}\Big)\Big)+c_{51}c_{59}-2hkm^2\\
&\Big(-(k+h)^{2\sqrt{1+m^2}}+(k-h)^{2\sqrt{1+m^2}}\Big)c_{59}\Big)+c_{20}\Big(c_{51}\Big(\Big(1-\sqrt{1+m^2}\Big)\\
&\Big((k+h)^{4\sqrt{1+m^2}}-(k+h)^{-1+3\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}}-(k-h)^{-1+3\sqrt{1+m^2}}\\
&(k+h)^{1+\sqrt{1+m^2}}+(k-h)^{4\sqrt{1+m^2}}\Big)+\Big(-1+3\sqrt{1+m^2}\Big)\Big((k+h)^{-1+3\sqrt{1+m^2}}\\
&(k-h)^{1+\sqrt{1+m^2}}-(k+h)^2(k-h)^{-2+4\sqrt{1+m^2}}-(k-h)^2(k+h)^{-2+4\sqrt{1+m^2}}\\
&+(k-h)^{-1+3\sqrt{1+m^2}}(k+h)^{1+\sqrt{1+m^2}}\Big)\Big)+c_{51}c_{60}-2hkm^2\\
&\Big(-(k+h)^{2\sqrt{1+m^2}}+(k-h)^{2\sqrt{1+m^2}}\Big)c_{60}\Big)+c_{21}\Big(c_{51}\Big(\Big(1-\sqrt{1+m^2}\Big)
\end{aligned}
$$

$$\left((k+h)^{-2-4\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}}(k-h)^{-3-5\sqrt{1+m^2}} - (k-h)^{1+\sqrt{1+m^2}}\right.$$
$$\left.(k+h)^{-3-5\sqrt{1+m^2}} + (k-h)^{-2-4\sqrt{1+m^2}}\right) + \left(-3-5\sqrt{1+m^2}\right)$$
$$\left((k+h)^{-3-5\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}} - (k+h)^2(k-h)^{-4-4\sqrt{1+m^2}} - (k-h)^2\right.$$
$$\left.\left.(k+h)^{-4-4\sqrt{1+m^2}} + (k-h)^{-3-5\sqrt{1+m^2}}(k+h)^{1+\sqrt{1+m^2}}\right)\right) + c_{51}c_{61} - 2hkm^2$$
$$\left(-(k+h)^{2\sqrt{1+m^2}} + (k-h)^{2\sqrt{1+m^2}}\right)c_{61}\Big) + c_{22}\left(c_{51}\left(\left(1-\sqrt{1+m^2}\right)\right.\right.$$
$$\left((k+h)^{-2-2\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}}(k-h)^{-3-3\sqrt{1+m^2}} - (k-h)^{1+\sqrt{1+m^2}}\right.$$
$$\left.(k+h)^{-3-3\sqrt{1+m^2}} + (k-h)^{-2-2\sqrt{1+m^2}}\right) + \left(-3-3\sqrt{1+m^2}\right)\left((k+h)^{-3-3\sqrt{1+m^2}}\right.$$
$$(k-h)^{1+\sqrt{1+m^2}} - (k+h)^2(k-h)^{-4-2\sqrt{1+m^2}} - (k-h)^2(k+h)^{-4-2\sqrt{1+m^2}}$$
$$\left.\left.+(k-h)^{-3-3\sqrt{1+m^2}}(k+h)^{1+\sqrt{1+m^2}}\right)\right) + c_{51}c_{62} - 2hkm^2\left((k-h)^{2\sqrt{1+m^2}}\right.$$
$$\left.-(k+h)^{2\sqrt{1+m^2}}\right)c_{62}\Big) + c_{23}\left(c_{51}\left(\left(1-\sqrt{1+m^2}\right)\left((k+h)^{-2} - (k+h)^{1+\sqrt{1+m^2}}\right.\right.\right.$$
$$\left.(k-h)^{-3-\sqrt{1+m^2}} - (k-h)^{1+\sqrt{1+m^2}}(k+h)^{-3-\sqrt{1+m^2}} + (k-h)^{-2}\right) + \left(-3-\sqrt{1+m^2}\right)$$
$$\left((k+h)^{-3-\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}} - (k+h)^2(k-h)^{-4} - (k-h)^2(k+h)^{-4}\right.$$
$$\left.\left.+(k-h)^{-3-\sqrt{1+m^2}}(k+h)^{1+\sqrt{1+m^2}}\right)\right) + c_{51}c_{63} - 2hkm^2\left((k-h)^{2\sqrt{1+m^2}}\right.$$
$$\left.-(k+h)^{2\sqrt{1+m^2}}\right)c_{63}\Big) + c_{24}\left(c_{51}\left(\left(1-\sqrt{1+m^2}\right)\left((k+h)^{-2+2\sqrt{1+m^2}}\right.\right.\right.$$
$$\left(-(k+h)^{1+\sqrt{1+m^2}}(k-h)^{-3+\sqrt{1+m^2}} - (k-h)^{1+\sqrt{1+m^2}}(k+h)^{-3+\sqrt{1+m^2}}\right.$$
$$\left.+(k-h)^{-2+2\sqrt{1+m^2}}\right) + \left(-3+\sqrt{1+m^2}\right)\left((k+h)^{-3+\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}}\right.$$
$$-(k+h)^2(k-h)^{-4+2\sqrt{1+m^2}} - (k-h)^2(k+h)^{-4+2\sqrt{1+m^2}} + (k-h)^{-3+\sqrt{1+m^2}}$$
$$\left.\left.(k+h)^{1+\sqrt{1+m^2}}\right)\right) + c_{51}c_{64} - 2hkm^2\left(-(k+h)^{2\sqrt{1+m^2}} + (k-h)^{2\sqrt{1+m^2}}\right)$$
$$c_{64}) + c_{25}\left(c_{51}\left(\left(1-\sqrt{1+m^2}\right)\left((k+h)^{-2+4\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}}\right.\right.\right.$$
$$\left.(k-h)^{-3+3\sqrt{1+m^2}} - (k-h)^{1+\sqrt{1+m^2}}(k+h)^{-3+3\sqrt{1+m^2}} + (k-h)^{-2+4\sqrt{1+m^2}}\right)$$
$$+\left(-3+3\sqrt{1+m^2}\right)\left((k+h)^{-3+3\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}} - (k-h)^{-4+4\sqrt{1+m^2}}\right.$$
$$\left.\left.(k+h)^2 - (k-h)^2(k+h)^{-4+4\sqrt{1+m^2}} + (k-h)^{-3+3\sqrt{1+m^2}}(k+h)^{1+\sqrt{1+m^2}}\right)\right)$$
$$+c_{51}c_{65} - 2hkm^2\left(-(k+h)^{2\sqrt{1+m^2}} + (k-h)^{2\sqrt{1+m^2}}\right)c_{65}\Big) + c_{26}\,(c_{51}$$
$$\left(\left(1-\sqrt{1+m^2}\right)\left((k+h)^{-2+6\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}}(k-h)^{-3+5\sqrt{1+m^2}}\right.\right.$$
$$\left.-(k-h)^{1+\sqrt{1+m^2}}(k+h)^{-3+5\sqrt{1+m^2}} + (k-h)^{-2+6\sqrt{1+m^2}}\right) + \left(-3+5\sqrt{1+m^2}\right)$$
$$\left((k+h)^{-3+5\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}} - (k+h)^2(k-h)^{-4+6\sqrt{1+m^2}} - (k-h)^2\right.$$

$$(k+h)^{-4+6\sqrt{1+m^2}} + (k-h)^{-3+5\sqrt{1+m^2}} (k+h)^{1+\sqrt{1+m^2}} \Big)\Big) + c_{51}c_{66} - 2hkm^2$$

$$\Big(-(k+h)^{2\sqrt{1+m^2}} + (k-h)^{2\sqrt{1+m^2}}\Big) c_{66}\Big) + c_{27}\Big(c_{51}\Big(\Big(1-\sqrt{1+m^2}\Big)$$

$$\Big((k+h)^{1-\sqrt{1+m^2}} - (k+h)^{-2\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}} - (k-h)^{-2\sqrt{1+m^2}}$$

$$(k+h)^{1+\sqrt{1+m^2}} + (k-h)^{1-\sqrt{1+m^2}}\Big) + \Big(-2\sqrt{1+m^2}\Big)\Big((k+h)^{-2\sqrt{1+m^2}}$$

$$(k-h)^{1+\sqrt{1+m^2}} - (k+h)^2 (k-h)^{-1-\sqrt{1+m^2}} - (k-h)^2(k+h)^{-1-\sqrt{1+m^2}}$$

$$+ (k-h)^{-2\sqrt{1+m^2}}(k+h)^{1+\sqrt{1+m^2}}\Big)\Big) + c_{51}c_{67} - 2hkm^2\Big((k-h)^{2\sqrt{1+m^2}}$$

$$-(k+h)^{2\sqrt{1+m^2}}\Big)c_{67}\Big) + c_{28}\Big(c_{51}\Big(\Big(1-\sqrt{1+m^2}\Big)\Big((k+h)^{1+3\sqrt{1+m^2}}$$

$$-(k+h)^{1+\sqrt{1+m^2}}(k-h)^{2\sqrt{1+m^2}} - (k+h)^{2\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}}$$

$$+(k-h)^{1+3\sqrt{1+m^2}}\Big) + 2\sqrt{1+m^2}\Big((k+h)^{2\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}}$$

$$-(k+h)^2(k-h)^{-1+3\sqrt{1+m^2}} - (k-h)^2(k+h)^{-1+3\sqrt{1+m^2}} + (k-h)^{2\sqrt{1+m^2}}$$

$$(k+h)^{1+\sqrt{1+m^2}}\Big)\Big) + c_{51}c_{68} - 2hkm^2\Big(-(k+h)^{2\sqrt{1+m^2}} + (k-h)^{2\sqrt{1+m^2}}\Big)$$

$$c_{68}\big) + c_{29}\Big(c_{51}\Big(\Big(1-\sqrt{1+m^2}\Big)\Big((k+h)^{-1-3\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}}$$

$$(k-h)^{-2-4\sqrt{1+m^2}} - (k+h)^{-2-4\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}} + (k-h)^{-1-3\sqrt{1+m^2}}\Big)$$

$$+\Big(-2-4\sqrt{1+m^2}\Big)\Big((k+h)^{-2-4\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}} - (k+h)^{-3-3\sqrt{1+m^2}}$$

$$(k-h)^2 - (k+h)^2(k-h)^{-3-3\sqrt{1+m^2}} + (k-h)^{-2-4\sqrt{1+m^2}}(k+h)^{1+\sqrt{1+m^2}}\Big)\Big)$$

$$+c_{51}c_{69} - 2hkm^2\Big(-(k+h)^{2\sqrt{1+m^2}} + (k-h)^{2\sqrt{1+m^2}}\Big)c_{69}\Big) + c_{30}\,(c_{51}$$

$$\Big(\Big(1-\sqrt{1+m^2}\Big)\Big((k+h)^{-1+\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}}(k-h)^{-2} - (k+h)^{-2}$$

$$(k-h)^{1+\sqrt{1+m^2}} + (k-h)^{-1+\sqrt{1+m^2}}\Big) - 2\Big((k+h)^{-2}(k-h)^{1+\sqrt{1+m^2}} - (k-h)^2$$

$$(k+h)^{-3+\sqrt{1+m^2}} - (k+h)^2(k-h)^{-3+\sqrt{1+m^2}} + (k-h)^{-2}(k+h)^{1+\sqrt{1+m^2}}\Big)\Big)$$

$$+c_{51}c_{70} - 2hkm^2\Big(-(k+h)^{2\sqrt{1+m^2}} + (k-h)^{2\sqrt{1+m^2}}\Big)c_{70}\Big) + c_{31}\,(c_{51}$$

$$\Big(\Big(1-\sqrt{1+m^2}\Big)\Big((k+h)^{-1+3\sqrt{1+m^2}} - (k+h)^{-2+2\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}}$$

$$-(k-h)^{-2+2\sqrt{1+m^2}}(k+h)^{1+\sqrt{1+m^2}} + (k-h)^{-1+3\sqrt{1+m^2}}\Big) + \Big(-2+2\sqrt{1+m^2}\Big)$$

$$\Big((k+h)^{-2+2\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}} - (k-h)^2(k+h)^{-3+3\sqrt{1+m^2}} - (k+h)^2$$

$$(k-h)^{-3+3\sqrt{1+m^2}} + (k-h)^{-2+2\sqrt{1+m^2}}(k+h)^{1+\sqrt{1+m^2}}\Big)\Big) + c_{51}c_{71} - 2hkm^2$$

$$\Big(-(k+h)^{2\sqrt{1+m^2}} + (k-h)^{2\sqrt{1+m^2}}\Big)c_{71}\Big) + c_{32}\Big(c_{51}\Big(\Big(1-\sqrt{1+m^2}\Big)$$

$$\Big((k+h)^{-1-1\sqrt{1+m^2}} - (k+h)^{-2-2\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}} - (k-h)^{-2-2\sqrt{1+m^2}}$$

$$(k+h)^{1+\sqrt{1+m^2}} + (k-h)^{-1-\sqrt{1+m^2}}\Big) + \Big(-2-2\sqrt{1+m^2}\Big) \Big((k+h)^{-2-2\sqrt{1+m^2}}$$

$$\Big((k-h)^{1+\sqrt{1+m^2}} - (k-h)^2 (k+h)^{-3-\sqrt{1+m^2}} - (k+h)^2 (k-h)^{-3-\sqrt{1+m^2}}$$

$$+ (k-h)^{-2-2\sqrt{1+m^2}} (k+h)^{1+\sqrt{1+m^2}}\Big)\Big) + c_{51}c_{72} - 2hkm^2 \Big((k-h)^{2\sqrt{1+m^2}}$$

$$- (k+h)^{2\sqrt{1+m^2}}\Big) c_{72}\Big) + c_{33} \Big(c_{51} \Big(\Big(1-\sqrt{1+m^2}\Big) \Big((k+h)^{-1+5\sqrt{1+m^2}}$$

$$- (k+h)^{-2+4\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}} - (k-h)^{-2+4\sqrt{1+m^2}} (k+h)^{1+\sqrt{1+m^2}}$$

$$+ (k-h)^{-1+5\sqrt{1+m^2}}\Big) + \Big(-2+4\sqrt{1+m^2}\Big) \Big((k+h)^{-2+4\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}}$$

$$- (k-h)^2 (k+h)^{-3+5\sqrt{1+m^2}} - (k+h)^2 (k-h)^{-3+5\sqrt{1+m^2}} + (k-h)^{-2+4\sqrt{1+m^2}}$$

$$(k+h)^{1+\sqrt{1+m^2}}\Big)\Big) + c_{51}c_{73} - 2hkm^2 \Big(-(k+h)^{2\sqrt{1+m^2}} + (k-h)^{2\sqrt{1+m^2}}\Big)$$

$$c_{73}) + c_{34} \Big(c_{51} \Big(\Big(1-\sqrt{1+m^2}\Big) \Big((k+h)^{1+2\sqrt{1+m^2}} - (k+h)^{\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}}$$

$$- (k-h)^{\sqrt{1+m^2}} (k+h)^{1+\sqrt{1+m^2}} + (k-h)^{1+2\sqrt{1+m^2}}\Big) + \sqrt{1+m^2} \Big((k+h)^{\sqrt{1+m^2}}$$

$$(k-h)^{1+\sqrt{1+m^2}} - (k-h)^2 (k+h)^{-1+2\sqrt{1+m^2}} - (k+h)^2 (k-h)^{-1+2\sqrt{1+m^2}}$$

$$+ (k-h)^{\sqrt{1+m^2}} (k+h)^{1+\sqrt{1+m^2}}\Big)\Big) + c_{51}c_{74} - 2hkm^2 \Big((k-h)^{2\sqrt{1+m^2}}$$

$$- (k+h)^{2\sqrt{1+m^2}}\Big) c_{74}\Big) + c_{35} \Big(c_{51} \Big(\Big(1-\sqrt{1+m^2}\Big) \Big((k+h)^{-1-2\sqrt{1+m^2}}$$

$$- (k+h)^{-2-3\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}} - (k-h)^{-2-3\sqrt{1+m^2}} (k+h)^{1+\sqrt{1+m^2}}$$

$$+ (k-h)^{-1-2\sqrt{1+m^2}}\Big) + \Big(-2-3\sqrt{1+m^2}\Big) \Big((k+h)^{-2-3\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}}$$

$$- (k-h)^2 (k+h)^{-3-2\sqrt{1+m^2}} - (k+h)^2 (k-h)^{-3-2\sqrt{1+m^2}} + (k-h)^{-2-3\sqrt{1+m^2}}$$

$$(k+h)^{1+\sqrt{1+m^2}}\Big)\Big) + c_{51}c_{75} - 2hkm^2 \Big(-(k+h)^{2\sqrt{1+m^2}} + (k-h)^{2\sqrt{1+m^2}}\Big)$$

$$c_{75}) + c_{36} \Big(c_{51} \Big(\Big(1-\sqrt{1+m^2}\Big) \Big((k+h)^{-1} - (k+h)^{-2-\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}}$$

$$- (k-h)^{-2-\sqrt{1+m^2}} (k+h)^{1+\sqrt{1+m^2}} + (k-h)^{-1}\Big) + \Big(-2-\sqrt{1+m^2}\Big) \Big(-(k-h)^2$$

$$(k+h)^{-3} + (k+h)^{-2-\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}} (k-h)^{-2-\sqrt{1+m^2}} (k+h)^{1+\sqrt{1+m^2}}$$

$$- (k+h)^2 (k-h)^{-3} +\Big)\Big) + c_{51}c_{76} - 2hkm^2 \Big(-(k+h)^{2\sqrt{1+m^2}} + (k-h)^{2\sqrt{1+m^2}}\Big)$$

$$c_{76}) + c_{37} \Big(c_{51} \Big(\Big(1-\sqrt{1+m^2}\Big) \Big((k+h)^{-1+2\sqrt{1+m^2}} - (k+h)^{-2+\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}}$$

$$- (k-h)^{-2+\sqrt{1+m^2}} (k+h)^{1+\sqrt{1+m^2}} + (k-h)^{-1+2\sqrt{1+m^2}}\Big) + \Big(-2+\sqrt{1+m^2}\Big)$$

$$\left(\left(k+h\right)^{-2+\sqrt{1+m^2}}\left(k-h\right)^{1+\sqrt{1+m^2}}-\left(k-h\right)^2\left(k+h\right)^{-3+2\sqrt{1+m^2}}-\left(k+h\right)^2\right.$$
$$\left.\left(k-h\right)^{-3+2\sqrt{1+m^2}}+\left(k-h\right)^{-2+\sqrt{1+m^2}}\left(k+h\right)^{1+\sqrt{1+m^2}}\right)\Big)+c_{51}c_{77}-2hkm^2$$
$$\left(-\left(k+h\right)^{2\sqrt{1+m^2}}+\left(k-h\right)^{2\sqrt{1+m^2}}\right)c_{77}\Big)+c_{38}\Big(c_{51}\Big(\left(1-\sqrt{1+m^2}\right)\Big(\left(k+h\right)^{-1+4\sqrt{1+m^2}}$$
$$-\left(k+h\right)^{-2+3\sqrt{1+m^2}}\left(k-h\right)^{1+\sqrt{1+m^2}}-\left(k-h\right)^{-2+3\sqrt{1+m^2}}\left(k+h\right)^{1+\sqrt{1+m^2}}$$
$$+\left(k-h\right)^{-1+4\sqrt{1+m^2}}\Big)+\left(-2+3\sqrt{1+m^2}\right)\Big(\left(k+h\right)^{-2+3\sqrt{1+m^2}}\left(k-h\right)^{1+\sqrt{1+m^2}}$$
$$-\left(k-h\right)^2\left(k+h\right)^{-3+4\sqrt{1+m^2}}-\left(k+h\right)^2\left(k-h\right)^{-3+4\sqrt{1+m^2}}+\left(k-h\right)^{-2+3\sqrt{1+m^2}}$$
$$\left(k+h\right)^{1+\sqrt{1+m^2}}\Big)\Big)+c_{51}c_{78}-2hkm^2\left(-\left(k+h\right)^{2\sqrt{1+m^2}}+\left(k-h\right)^{2\sqrt{1+m^2}}\right)c_{78}\Big)$$
$$+c_{39}\Big(c_{51}\Big(\left(1-\sqrt{1+m^2}\right)\left(2h\left(k+h\right)^{1+\sqrt{1+m^2}}-2h\left(k-h\right)^{1+\sqrt{1+m^2}}\right)$$
$$-2h\left(\left(k+h\right)\left(k-h\right)^{\sqrt{1+m^2}}-\left(k-h\right)\left(k+h\right)^{\sqrt{1+m^2}}\right)\Big)+c_{51}c_{79}-2hkm^2$$
$$\left(-\left(k+h\right)^{2\sqrt{1+m^2}}+\left(k-h\right)^{2\sqrt{1+m^2}}\right)c_{79}\Big)+c_{40}\Big(c_{51}\Big(\left(1-\sqrt{1+m^2}\right)$$
$$\Big(\left(k+h\right)-\left(k+h\right)^{-\sqrt{1+m^2}}\left(k-h\right)^{1+\sqrt{1+m^2}}-\left(k-h\right)^{-\sqrt{1+m^2}}\left(k+h\right)^{1+\sqrt{1+m^2}}$$
$$+\left(k-h\right)\Big)-\sqrt{1+m^2}\Big(\left(k+h\right)^{-\sqrt{1+m^2}}\left(k-h\right)^{1+\sqrt{1+m^2}}-\left(k-h\right)^2\left(k+h\right)^{-1}$$
$$-\left(k+h\right)^2\left(k-h\right)^{-1}+\left(k-h\right)^{-\sqrt{1+m^2}}\left(k+h\right)^{1+\sqrt{1+m^2}}\Big)\Big)+c_{51}c_{80}-2hkm^2$$
$$\left(-\left(k+h\right)^{2\sqrt{1+m^2}}+\left(k-h\right)^{2\sqrt{1+m^2}}\right)c_{80}\Big)+c_{41}\Big(c_{51}\Big(\left(1-\sqrt{1+m^2}\right)$$
$$\Big(\left(k+h\right)^{-2-3\sqrt{1+m^2}}-\left(k+h\right)^{-3-\sqrt{1+m^2}}\left(k-h\right)^{1+\sqrt{1+m^2}}-\left(k-h\right)^{-3-4\sqrt{1+m^2}}$$
$$\left(k+h\right)^{1+\sqrt{1+m^2}}+\left(k-h\right)^{-2-3\sqrt{1+m^2}}\Big)+\left(-3-4\sqrt{1+m^2}\right)\Big(\left(k+h\right)^{-3-4\sqrt{1+m^2}}$$
$$\left(k-h\right)^{1+\sqrt{1+m^2}}-\left(k-h\right)^2\left(k+h\right)^{-4-3\sqrt{1+m^2}}-\left(k+h\right)^2\left(k-h\right)^{-4-3\sqrt{1+m^2}}$$
$$+\left(k-h\right)^{-3-4\sqrt{1+m^2}}\left(k+h\right)^{1+\sqrt{1+m^2}}\Big)\Big)+c_{51}c_{81}-2hkm^2\Big(-\left(k+h\right)^{2\sqrt{1+m^2}}$$
$$+\left(k-h\right)^{2\sqrt{1+m^2}}\Big)c_{81}\Big)+c_{42}\Big(c_{51}\Big(\left(1-\sqrt{1+m^2}\right)\Big(\left(k+h\right)^{-2-\sqrt{1+m^2}}$$
$$-\left(k+h\right)^{-3-2\sqrt{1+m^2}}\left(k-h\right)^{1+\sqrt{1+m^2}}-\left(k-h\right)^{-3-2\sqrt{1+m^2}}\left(k+h\right)^{1+\sqrt{1+m^2}}$$
$$+\left(k-h\right)^{-2-\sqrt{1+m^2}}\Big)+\left(-3-2\sqrt{1+m^2}\right)\Big(\left(k+h\right)^{-3-2\sqrt{1+m^2}}\left(k-h\right)^{1+\sqrt{1+m^2}}$$
$$-\left(k-h\right)^2\left(k+h\right)^{-4-\sqrt{1+m^2}}-\left(k+h\right)^2\left(k-h\right)^{-4-\sqrt{1+m^2}}+\left(k-h\right)^{-3-2\sqrt{1+m^2}}$$
$$\left(k+h\right)^{1+\sqrt{1+m^2}}\Big)\Big)+c_{51}c_{82}-2hkm^2\left(-\left(k+h\right)^{2\sqrt{1+m^2}}+\left(k-h\right)^{2\sqrt{1+m^2}}\right)$$
$$c_{82}\Big)+c_{43}\Big(c_{51}\Big(\left(1-\sqrt{1+m^2}\right)\Big(\left(k+h\right)^{-2+\sqrt{1+m^2}}-\left(k+h\right)^{-3}\left(k-h\right)^{1+\sqrt{1+m^2}}$$

$$-(k-h)^{-3}(k+h)^{1+\sqrt{1+m^2}}+(k-h)^{-2+\sqrt{1+m^2}}\Big)-3\Big((k+h)^{-3}(k-h)^{1+\sqrt{1+m^2}}$$
$$-(k-h)^{2}(k+h)^{-4+\sqrt{1+m^2}}-(k+h)^{2}(k-h)^{-4+\sqrt{1+m^2}}+(k-h)^{-3}$$
$$(k+h)^{1+\sqrt{1+m^2}}\Big)\Big)+c_{51}c_{83}-2hkm^2\left(-(k+h)^{2\sqrt{1+m^2}}+(k-h)^{2\sqrt{1+m^2}}\right)c_{83}\Big)$$
$$+c_{44}\Big(c_{51}\Big(\Big(1-\sqrt{1+m^2}\Big)\Big((k+h)^{-2+3\sqrt{1+m^2}}-(k+h)^{-3+2\sqrt{1+m^2}}$$
$$(k-h)^{1+\sqrt{1+m^2}}-(k-h)^{-3+2\sqrt{1+m^2}}(k+h)^{1+\sqrt{1+m^2}}+(k-h)^{-2+3\sqrt{1+m^2}}\Big)$$
$$+\Big(-3+2\sqrt{1+m^2}\Big)\Big((k+h)^{-3+2\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}}-(k+h)^{-4+3\sqrt{1+m^2}}$$
$$(k-h)^{2}-(k+h)^{2}(k-h)^{-4+3\sqrt{1+m^2}}+(k-h)^{-3+2\sqrt{1+m^2}}(k+h)^{1+\sqrt{1+m^2}}\Big)\Big)$$
$$+c_{51}c_{84}-2hkm^2\left(-(k+h)^{2\sqrt{1+m^2}}+(k-h)^{2\sqrt{1+m^2}}\right)c_{84}\Big)+c_{45}\,(c_{51}$$
$$\Big(\Big(1-\sqrt{1+m^2}\Big)\Big((k+h)^{-\sqrt{1+m^2}}-(k+h)^{-1-2\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}}$$
$$-(k-h)^{-1-2\sqrt{1+m^2}}(k+h)^{1+\sqrt{1+m^2}}+(k-h)^{-\sqrt{1+m^2}}\Big)+\Big(-1-2\sqrt{1+m^2}\Big)$$
$$\Big((k+h)^{-1-2\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}}-(k+h)^{2-\sqrt{1+m^2}}(k-h)^{-2-\sqrt{1+m^2}}$$
$$-(k+h)^{-2-\sqrt{1+m^2}}(k-h)^{2-\sqrt{1+m^2}}+(k-h)^{-1-2\sqrt{1+m^2}}(k+h)^{1+\sqrt{1+m^2}}\Big)\Big)$$
$$+c_{51}c_{85}-2hkm^2\left(-(k+h)^{2\sqrt{1+m^2}}+(k-h)^{2\sqrt{1+m^2}}\right)c_{85}\Big)+c_{46}\,(c_{51}$$
$$\Big(\Big(1-\sqrt{1+m^2}\Big)\Big((k+h)^{\sqrt{1+m^2}}-(k+h)^{-1}(k-h)^{1+\sqrt{1+m^2}}-(k-h)^{-1}$$
$$(k+h)^{1+\sqrt{1+m^2}}+(k-h)^{\sqrt{1+m^2}}\Big)-\Big((k+h)^{-1}(k-h)^{1+\sqrt{1+m^2}}-(k-h)^{2}$$
$$(k+h)^{-2+\sqrt{1+m^2}}-(k+h)^{2}(k-h)^{-2+\sqrt{1+m^2}}+(k-h)^{-1}(k+h)^{1+\sqrt{1+m^2}}\Big)\Big)$$
$$+c_{51}c_{86}-2hkm^2\left(-(k+h)^{2\sqrt{1+m^2}}+(k-h)^{2\sqrt{1+m^2}}\right)c_{86}\Big)+c_{47}\,(c_{51}$$
$$\Big(\Big(1-\sqrt{1+m^2}\Big)\Big((k+h)^{3\sqrt{1+m^2}}-(k+h)^{-1+2\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}}$$
$$-(k-h)^{-1+2\sqrt{1+m^2}}(k+h)^{1+\sqrt{1+m^2}}+(k-h)^{3\sqrt{1+m^2}}\Big)+\Big(-1+2\sqrt{1+m^2}\Big)$$
$$\Big((k+h)^{-1+2\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}}-(k+h)^{2}(k-h)^{-2+3\sqrt{1+m^2}}-(k-h)^{2}$$
$$(k+h)^{-2+3\sqrt{1+m^2}}+(k-h)^{-1+2\sqrt{1+m^2}}(k+h)^{1+\sqrt{1+m^2}}\Big)\Big)+c_{51}c_{87}-2hkm^2$$
$$\left(-(k+h)^{2\sqrt{1+m^2}}+(k-h)^{2\sqrt{1+m^2}}\right)c_{87}\Big)+c_{48}\Big(c_{51}\Big(\Big(1-\sqrt{1+m^2}\Big)$$
$$\Big((k+h)^{5\sqrt{1+m^2}-2}-(k+h)^{-3+4\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}}-(k-h)^{-3+4\sqrt{1+m^2}}$$

$$
\begin{aligned}
& (k+h)^{1+\sqrt{1+m^2}} + (k-h)^{5\sqrt{1+m^2}-2}\Big) + \left(-3+4\sqrt{1+m^2}\right)\Big((k+h)^{-3+4\sqrt{1+m^2}} \\
& \Big((k-h)^{1+\sqrt{1+m^2}} - (k+h)^2 (k-h)^{-4+5\sqrt{1+m^2}} - (k+h)^{-4+5\sqrt{1+m^2}} (k-h)^2 \\
& + (k-h)^{-3+4\sqrt{1+m^2}} (k+h)^{1+\sqrt{1+m^2}}\Big)\Big) + c_{51}c_{88} - 2hkm^2 \Big(-(k+h)^{2\sqrt{1+m^2}} \\
& + (k-h)^{2\sqrt{1+m^2}}\Big) c_{88}\Big) + c_{49}\Big(c_{51}\Big(\left(1-\sqrt{1+m^2}\right)\Big(\ln(k+h)(k+h)^2 \\
& - \ln(k+h)(k+h)^{1-\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}} - \ln(k-h)(k-h)^{1-\sqrt{1+m^2}} \\
& (k+h)^{1+\sqrt{1+m^2}} + \ln(k-h)(k-h)^2\Big) + \Big(\Big((k+h)^{1-\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}} \\
& - (k+h)^2 - (k-h)^2 + (k-h)^{1-\sqrt{1+m^2}} (k+h)^{1+\sqrt{1+m^2}}\Big) + \left(1-\sqrt{1+m^2}\right) \\
& \Big(\ln(k+h)(k+h)^{1-\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}} - \ln(k-h)(k+h)^2 - \ln(k+h) \\
& (k-h)^2 + \ln(k-h)(k-h)^{1-\sqrt{1+m^2}} (k+h)^{1+\sqrt{1+m^2}}\Big)\Big)\Big) + c_{51}c_{89} - 2hkm^2 \\
& \Big(-(k+h)^{2\sqrt{1+m^2}} + (k-h)^{2\sqrt{1+m^2}}\Big) c_{89}\Big) + c_{50}\Big(c_{51}\Big(\left(1-\sqrt{1+m^2}\right) \\
& \Big(\ln(k+h)(k+h)^{2+2\sqrt{1+m^2}} - \ln(k+h)(k+h)^{1+\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}} \\
& - \ln(k-h)(k-h)^{1+\sqrt{1+m^2}} (k+h)^{1+\sqrt{1+m^2}} + \ln(k-h)(k-h)^{2+2\sqrt{1+m^2}}\Big) \\
& + \Big(\Big((k-h)^{1+\sqrt{1+m^2}} (k+h)^{1+\sqrt{1+m^2}} - (k+h)^2 (k-h)^{2\sqrt{1+m^2}} - (k-h)^2 \\
& (k+h)^{2\sqrt{1+m^2}} + (k-h)^{1+\sqrt{1+m^2}} (k+h)^{1+\sqrt{1+m^2}}\Big) + \left(1+\sqrt{1+m^2}\right) \\
& \Big((k+h)^{1+\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}} \ln(k+h) - \ln(k-h)(k+h)^2 (k-h)^{2\sqrt{1+m^2}} \\
& - \ln(k+h)(k-h)^2 (k+h)^{2\sqrt{1+m^2}} + \ln(k-h)(k-h)^{1+\sqrt{1+m^2}} (k+h)^{1+\sqrt{1+m^2}}\Big)\Big)\Big) \\
& + c_{51}c_{90} - 2hkm^2\Big(-(k+h)^{2\sqrt{1+m^2}} + (k-h)^{2\sqrt{1+m^2}}\Big) c_{90}\Big)\Big],
\end{aligned}
$$

$$
\begin{aligned}
c_{15} \;=\; & \frac{1}{c_{51}c_{56}}\Big[F_2 c_{56}\left(1+\sqrt{1+m^2}\right)\Big((k+h)^{2\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}} - (k-h)^{2\sqrt{1+m^2}} \\
& (k+h)^{1+\sqrt{1+m^2}}\Big) + c_{17}\Big(c_{51}\left(-1-3\sqrt{1+m^2}\right)\Big((k-h)^{1+\sqrt{1+m^2}} (k+h)^{-2-2\sqrt{1+m^2}} \\
& \Big(-(k+h)^{1+\sqrt{1+m^2}} (k-h)^{-2-2\sqrt{1+m^2}}\Big) - c_{91}c_{57}\Big) + c_{18}\Big(c_{51}\left(-1-\sqrt{1+m^2}\right) \\
& \Big((k-h)^{1+\sqrt{1+m^2}} (k+h)^{-2} - (k+h)^{1+\sqrt{1+m^2}} (k-h)^{-2}\Big) - c_{91}c_{58}\Big) + c_{19}\big(c_{51} \\
& \left(-1+\sqrt{1+m^2}\right)\Big((k-h)^{1+\sqrt{1+m^2}} (k+h)^{-2+2\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}} \\
& (k-h)^{-2+2\sqrt{1+m^2}}\Big) - c_{91}c_{59}\Big) + c_{20}\Big(c_{51}\left(-1+3\sqrt{1+m^2}\right)\Big((k-h)^{1+\sqrt{1+m^2}}
\end{aligned}
$$

$$(k+h)^{-2+4\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}}(k-h)^{-2+4\sqrt{1+m^2}}\Big) - c_{91}c_{60}\Big) + c_{21}\,(c_{51}$$

$$\Big(-3-5\sqrt{1+m^2}\Big)\Big((k-h)^{1+\sqrt{1+m^2}}(k+h)^{-4-4\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}}$$

$$(k-h)^{-4-4\sqrt{1+m^2}}\Big) - c_{91}c_{61}\Big) + c_{22}\Big(c_{51}\Big(-3-3\sqrt{1+m^2}\Big)\Big((k-h)^{1+\sqrt{1+m^2}}$$

$$(k+h)^{-4-2\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}}(k-h)^{-4-2\sqrt{1+m^2}}\Big) - c_{91}c_{62}\Big) + c_{23}\,(c_{51}$$

$$\Big(-3-\sqrt{1+m^2}\Big)\Big((k-h)^{1+\sqrt{1+m^2}}(k+h)^{-4} - (k+h)^{1+\sqrt{1+m^2}}(k-h)^{-4}\Big)$$

$$-c_{91}c_{63}) + c_{24}\Big(c_{51}\Big(-3+\sqrt{1+m^2}\Big)\Big((k-h)^{1+\sqrt{1+m^2}}(k+h)^{-4+2\sqrt{1+m^2}}$$

$$-(k+h)^{1+\sqrt{1+m^2}}(k-h)^{-4+2\sqrt{1+m^2}}\Big) - c_{91}c_{64}\Big) + c_{25}\Big(c_{51}\Big(-3+3\sqrt{1+m^2}\Big)$$

$$\Big((k-h)^{1+\sqrt{1+m^2}}(k+h)^{-4+4\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}}(k-h)^{-4+4\sqrt{1+m^2}}\Big)$$

$$-c_{91}c_{65}) + c_{26}\Big(c_{51}\Big(-3+5\sqrt{1+m^2}\Big)\Big((k-h)^{1+\sqrt{1+m^2}}(k+h)^{-4+6\sqrt{1+m^2}}$$

$$-(k+h)^{1+\sqrt{1+m^2}}(k-h)^{-4+6\sqrt{1+m^2}}\Big) - c_{91}c_{66}\Big) + c_{27}\Big(c_{51}\Big(-2\sqrt{1+m^2}\Big)$$

$$\Big((k-h)^{1+\sqrt{1+m^2}}(k+h)^{-1-\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}}(k-h)^{-1-\sqrt{1+m^2}}\Big)$$

$$-c_{91}c_{67}) + c_{28}\Big(c_{51}\Big(2\sqrt{1+m^2}\Big)\Big((k-h)^{1+\sqrt{1+m^2}}(k+h)^{-1+3\sqrt{1+m^2}}$$

$$-(k+h)^{1+\sqrt{1+m^2}}(k-h)^{-1+3\sqrt{1+m^2}}\Big) - c_{91}c_{68}\Big) + c_{29}\Big(c_{51}\Big(-2-4\sqrt{1+m^2}\Big)$$

$$\Big((k-h)^{1+\sqrt{1+m^2}}(k+h)^{-3-3\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}}(k-h)^{-3-3\sqrt{1+m^2}}\Big)$$

$$-c_{91}c_{69}) + c_{30}\Big(c_{51}\,(-2)\Big((k-h)^{1+\sqrt{1+m^2}}(k+h)^{-3+\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}}$$

$$(k-h)^{-3+\sqrt{1+m^2}}\Big) - c_{91}c_{70}\Big) + c_{31}\Big(c_{51}\Big(-2+2\sqrt{1+m^2}\Big)\Big((k-h)^{1+\sqrt{1+m^2}}$$

$$(k+h)^{-3+3\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}}(k-h)^{-3+3\sqrt{1+m^2}}\Big) - c_{91}c_{71}\Big) + c_{32}\,(c_{51}$$

$$\Big(-2-2\sqrt{1+m^2}\Big)\Big((k-h)^{1+\sqrt{1+m^2}}(k+h)^{-3-\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}}$$

$$(k-h)^{-3-\sqrt{1+m^2}}\Big) - c_{91}c_{72}\Big) + c_{33}\Big(c_{51}\Big(-2+4\sqrt{1+m^2}\Big)\Big((k-h)^{1+\sqrt{1+m^2}}$$

$$(k+h)^{-3+5\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}}(k-h)^{-3+5\sqrt{1+m^2}}\Big) - c_{91}c_{73}\Big) + c_{34}$$

$$\Big(c_{51}\Big(\sqrt{1+m^2}\Big)\Big((k-h)^{1+\sqrt{1+m^2}}(k+h)^{-1+2\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}}$$

$$(k-h)^{-1+2\sqrt{1+m^2}}\Big) - c_{91}c_{74}\Big) + c_{35}\Big(c_{51}\Big(-2-3\sqrt{1+m^2}\Big)\Big((k-h)^{1+\sqrt{1+m^2}}$$

$$(k+h)^{-3-2\sqrt{1+m^2}} - (k+h)^{1+\sqrt{1+m^2}}(k-h)^{-3-2\sqrt{1+m^2}}\Big) - c_{91}c_{75}\Big) + c_{36}$$

$$\Big(c_{51}\Big(-2-\sqrt{1+m^2}\Big)\Big((k-h)^{1+\sqrt{1+m^2}}(k+h)^{-3} - (k+h)^{1+\sqrt{1+m^2}}(k-h)^{-3}\Big)$$

$$-c_{91}c_{76}) + c_{37}\Big(c_{51}\Big(-2+\sqrt{1+m^2}\Big)\Big((k-h)^{1+\sqrt{1+m^2}}(k+h)^{-3+2\sqrt{1+m^2}}$$

$$-(k+h)^{1+\sqrt{1+m^2}}(k-h)^{-3+2\sqrt{1+m^2}}\Big) - c_{91}c_{77}\Big) + c_{38}\Big(c_{51}\Big(3\sqrt{1+m^2}-2\Big)$$

$$
\begin{aligned}
&\left(\left(k-h\right)^{1+\sqrt{1+m^2}}\left(k+h\right)^{-3+4\sqrt{1+m^2}}-\left(k+h\right)^{1+\sqrt{1+m^2}}\left(k-h\right)^{-3+4\sqrt{1+m^2}}\right)-c_{91}c_{78}\Big)\\
&+c_{39}\left(c_{51}\left(-2h\left(k+h\right)^{\sqrt{1+m^2}}\left(k-h\right)^{\sqrt{1+m^2}}\right)-c_{91}c_{79}\right)+c_{40}\left(c_{51}\left(-\sqrt{1+m^2}\right)\right.\\
&\left.\left(\left(k-h\right)^{1+\sqrt{1+m^2}}\left(k+h\right)^{-1}-\left(k+h\right)^{1+\sqrt{1+m^2}}\left(k-h\right)^{-1}\right)-c_{91}c_{80}\right)+c_{41}\left(c_{51}\right.\\
&\left(-3-4\sqrt{1+m^2}\right)\left(\left(k-h\right)^{1+\sqrt{1+m^2}}\left(k+h\right)^{-4-3\sqrt{1+m^2}}-\left(k-h\right)^{-4-3\sqrt{1+m^2}}\right.\\
&\left.\left.\left(k+h\right)^{1+\sqrt{1+m^2}}\right)-c_{91}c_{81}\right)+c_{42}\left(c_{51}\left(-3-2\sqrt{1+m^2}\right)\left(\left(k-h\right)^{1+\sqrt{1+m^2}}\right.\right.\\
&\left.\left.\left(k+h\right)^{-4-\sqrt{1+m^2}}-\left(k+h\right)^{1+\sqrt{1+m^2}}\left(k-h\right)^{-4-\sqrt{1+m^2}}\right)-c_{91}c_{82}\right)+c_{43}\left(c_{51}\left(-3\right)\right.\\
&\left.\left(\left(k-h\right)^{1+\sqrt{1+m^2}}\left(k+h\right)^{-4+\sqrt{1+m^2}}-\left(k+h\right)^{1+\sqrt{1+m^2}}\left(k-h\right)^{-4+\sqrt{1+m^2}}\right)-c_{91}c_{83}\right)\\
&+c_{44}\left(c_{51}\left(-3+2\sqrt{1+m^2}\right)\left(\left(k-h\right)^{1+\sqrt{1+m^2}}\left(k+h\right)^{-4+3\sqrt{1+m^2}}-\left(k+h\right)^{1+\sqrt{1+m^2}}\right.\right.\\
&\left.\left.\left(k-h\right)^{-4+3\sqrt{1+m^2}}\right)-c_{91}c_{84}\right)+c_{45}\left(c_{51}\left(-1-2\sqrt{1+m^2}\right)\left(\left(k-h\right)^{1+\sqrt{1+m^2}}\right.\right.\\
&\left.\left.\left(k+h\right)^{-2-\sqrt{1+m^2}}-\left(k+h\right)^{1+\sqrt{1+m^2}}\left(k-h\right)^{-2-\sqrt{1+m^2}}\right)-c_{91}c_{85}\right)+c_{46}\left(c_{51}\left(-1\right)\right.\\
&\left.\left(\left(k-h\right)^{1+\sqrt{1+m^2}}\left(k+h\right)^{-2+\sqrt{1+m^2}}-\left(k+h\right)^{1+\sqrt{1+m^2}}\left(k-h\right)^{-2+\sqrt{1+m^2}}\right)-c_{91}c_{86}\right)\\
&+c_{47}\left(c_{51}\left(-1+2\sqrt{1+m^2}\right)\left(\left(k-h\right)^{1+\sqrt{1+m^2}}\left(k+h\right)^{-2+3\sqrt{1+m^2}}-\left(k+h\right)^{1+\sqrt{1+m^2}}\right.\right.\\
&\left.\left.\left(k-h\right)^{-2+3\sqrt{1+m^2}}\right)-c_{91}c_{87}\right)+c_{48}\left(c_{51}\left(-3+4\sqrt{1+m^2}\right)\left(\left(k-h\right)^{1+\sqrt{1+m^2}}\right.\right.\\
&\left.\left.\left(k+h\right)^{-4+5\sqrt{1+m^2}}-\left(k+h\right)^{1+\sqrt{1+m^2}}\left(k-h\right)^{-4+5\sqrt{1+m^2}}\right)-c_{91}c_{88}\right)+c_{49}\left(c_{51}\right.\\
&\left(\left(\left(k-h\right)^{1+\sqrt{1+m^2}}-\left(k+h\right)^{1+\sqrt{1+m^2}}\right)+\left(1-\sqrt{1+m^2}\right)\left(\ln\left(k+h\right)\left(k-h\right)^{1+\sqrt{1+m^2}}\right.\right.\\
&\left.\left.\left.-\ln\left(k-h\right)\left(k+h\right)^{1+\sqrt{1+m^2}}\right)\right)-c_{91}c_{89}\right)+c_{50}\left(c_{51}\left(\left(\left(k-h\right)^{1+\sqrt{1+m^2}}\left(k+h\right)^{2\sqrt{1+m^2}}\right.\right.\right.\\
&\left.-\left(k+h\right)^{1+\sqrt{1+m^2}}\left(k-h\right)^{2\sqrt{1+m^2}}\right)+\left(1+\sqrt{1+m^2}\right)\left(\ln\left(k+h\right)\left(k-h\right)^{1+\sqrt{1+m^2}}\right.\\
&\left.\left.\left.\left(k+h\right)^{2\sqrt{1+m^2}}-\ln\left(k-h\right)\left(k+h\right)^{1+\sqrt{1+m^2}}\left(k-h\right)^{2\sqrt{1+m^2}}\right)\right)-c_{91}c_{90}\right)\Big],
\end{aligned}
$$

$$
\begin{aligned}
c_{16} \;=\;& \frac{1}{c_{51}}\left[F_2c_{56}-c_{17}c_{57}-c_{18}c_{58}-c_{19}c_{59}-c_{20}c_{60}-c_{21}c_{61}-c_{22}c_{62}-c_{23}c_{63}-c_{24}c_{64}\right.\\
&-c_{25}c_{65}-c_{26}c_{66}-c_{27}c_{67}-c_{28}c_{68}-c_{29}c_{69}-c_{30}c_{70}-c_{31}c_{71}-c_{32}c_{72}-c_{33}c_{73}\\
&-c_{34}c_{74}-c_{35}c_{75}-c_{36}c_{76}-c_{37}c_{77}-c_{38}c_{78}-c_{39}c_{79}-c_{40}c_{80}-c_{41}c_{81}-c_{42}c_{82}\\
&\left.-c_{43}c_{83}-c_{44}c_{84}-c_{45}c_{85}-c_{46}c_{86}-c_{47}c_{87}-c_{48}c_{88}-c_{49}c_{89}-c_{50}c_{90}\right],\\
c_{17} \;=\;& \frac{1}{12+12\sqrt{1+m^2}+8m^2}\left[-6\left(1+m^2\right)\sqrt{1+m^2}\left(1-\sqrt{1+m^2}\right)^3c_3^2c_7-72c_9\right.\\
&\left.\left(1+3\sqrt{1+m^2}\right)\left(1+2\sqrt{1+m^2}\right)c_2^2+6\left(1-2\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)^3c_3^2c_7\right],
\end{aligned}
$$

$$
\begin{aligned}
c_{18} &= \frac{1}{4+4\sqrt{1+m^2}}\left[6\left(1+m^2\right)\left(-4+\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)^2\left(1+\sqrt{1+m^2}\right)c_3^2c_8\right.\\
&+12m^2\left(1+m^2\right)\left(1-\sqrt{1+m^2}\right)\left(2+\sqrt{1+m^2}\right)c_3c_4c_7+12\left(1-\sqrt{1+m^2}\right)^3\\
&\left(1+\sqrt{1+m^2}\right)c_3c_4c_7+120c_{10}\left(1+\sqrt{1+m^2}\right)c_2^2+6\left(1-3\sqrt{1+m^2}\right)\left(1+\sqrt{1+m^2}\right)\\
&\left.\left(1-\sqrt{1+m^2}\right)^2c_3^2c_8-12\sqrt{1+m^2}\left(1-\sqrt{1+m^2}\right)c_7\right],\\
c_{19} &= \frac{1}{4-4\sqrt{1+m^2}}\left[6\left(1+m^2\right)\left(2+\sqrt{1+m^2}\right)\left(1+\sqrt{1+m^2}\right)^2\left(1-\sqrt{1+m^2}\right)c_4^2c_7\right.\\
&+6\left(1-\sqrt{1+m^2}\right)\left(1+\sqrt{1+m^2}\right)^3c_4^2c_7+24\left(1+\sqrt{1+m^2}\right)c_2^2c_8-12\\
&\left(1-\sqrt{1+m^2}\right)^2\left(1+\sqrt{1+m^2}\right)^2c_3c_4+12m^2\left(1+m^2\right)\left(1+\sqrt{1+m^2}\right)c_3c_4c_8\\
&\left.-72c_{11}\left(-1+\sqrt{1+m^2}\right)c_2^2-12\sqrt{1+m^2}\left(1-\sqrt{1+m^2}\right)\left(1+\sqrt{1+m^2}\right)^2c_3c_4c_8\right],\\
c_{20} &= \frac{1}{12-12\sqrt{1+m^2}+8m^2}\left[6\left(1+m^2\right)\left(2-\sqrt{1+m^2}\right)\left(1+\sqrt{1+m^2}\right)^3c_4^2c_8-120c_{12}\right.\\
&\left.\left(-1+3\sqrt{1+m^2}\right)c_2^2+6\left(1+\sqrt{1+m^2}\right)^4c_4^2c_8\right],\\
c_{21} &= \frac{1}{40+40\sqrt{1+m^2}+24m^2}\left[-6c_9\left(1+m^2\right)\left(1-\sqrt{1+m^2}\right)^2\left(1+3\sqrt{1+m^2}\right)\right.\\
&\left(4+3\sqrt{1+m^2}\right)c_3^2+6c_9\left(1+2\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)^2\left(1+3\sqrt{1+m^2}\right)\\
&\left.\left(2+3\sqrt{1+m^2}\right)c_3^2+6c_9\left(-1+2\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)^2\left(-1-3\sqrt{1+m^2}\right)c_3^2\right],\\
c_{22} &= \frac{1}{24+24\sqrt{1+m^2}+8m^2}\left[-6c_{10}\left(1+m^2\right)\left(1-\sqrt{1+m^2}\right)^2\left(1+\sqrt{1+m^2}\right)\right.\\
&\left(4+\sqrt{1+m^2}\right)c_3^2+12c_9m^2\left(1+m^2\right)\left(1+3\sqrt{1+m^2}\right)\left(4+3\sqrt{1+m^2}\right)c_3c_4+6c_9\\
&\left(-1-3\sqrt{1+m^2}\right)+12c_{10}\sqrt{1+m^2}\left(1-\sqrt{1+m^2}\right)^2\left(1+\sqrt{1+m^2}\right)^2c_3^2+6c_{10}\\
&\left(1-\sqrt{1+m^2}\right)^2\left(1+\sqrt{1+m^2}\right)\left(3+\sqrt{1+m^2}\right)c_3^2+36c_9\left(1+3\sqrt{1+m^2}\right)\\
&\left.\left(1-\sqrt{1+m^2}\right)\left(1+\sqrt{1+m^2}\right)^2c_3c_4\right],\\
c_{23} &= \frac{1}{16+8\sqrt{1+m^2}}\left[-6c_{11}\left(1+m^2\right)\left(1-\sqrt{1+m^2}\right)^2\left(-1+\sqrt{1+m^2}\right)\left(-4+\sqrt{1+m^2}\right)\right.\\
&c_3^2+6c_9\left(1+m^2\right)\left(1+\sqrt{1+m^2}\right)^2\left(1+3\sqrt{1+m^2}\right)c_4^2+12c_{10}\left(1+\sqrt{1+m^2}\right)\\
&\left(1-\sqrt{1+m^2}\right)c_3c_4+6c_9\left(1-2\sqrt{1+m^2}\right)\left(1+\sqrt{1+m^2}\right)^2\left(1+3\sqrt{1+m^2}\right)\\
&\left(2+3\sqrt{1+m^2}\right)c_4^2+12c_{11}\left(1-\sqrt{1+m^2}\right)^2\left(-1+\sqrt{1+m^2}\right)\left(-2+\sqrt{1+m^2}\right)c_3^2
\end{aligned}
$$

$$-12c_9\left(1+m^2\right)\left(1+\sqrt{1+m^2}\right)^2\left(-1-3\sqrt{1+m^2}\right)^2 c_4^2+12c_{11}\sqrt{1+m^2}$$
$$\left(-1+\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)^2 c_3^2-6c_9\left(1+2\sqrt{1+m^2}\right)\left(1+\sqrt{1+m^2}\right)^2$$
$$\left(-1-3\sqrt{1+m^2}\right)c_4^2-6c_{11}\left(1-\sqrt{1+m^2}\right)^3\left(1+\sqrt{1+m^2}\right)c_3^2+12c_{10}\sqrt{1+m^2}$$
$$\left(1+\sqrt{1+m^2}\right)^2\left(1-\sqrt{1+m^2}\right)\left(2+\sqrt{1+m^2}\right)c_3c_4+12c_{10}m^2\left(1+m^2\right)$$
$$\left(1+\sqrt{1+m^2}\right)\left(4+\sqrt{1+m^2}\right)c_3c_4\Big],$$

$$c_{24} = \frac{1}{16-8\sqrt{1+m^2}}\Big[-6c_{12}\left(1+m^2\right)\left(1-\sqrt{1+m^2}\right)^2\left(-1+3\sqrt{1+m^2}\right)\left(-4+3\sqrt{1+m^2}\right)$$
$$c_3^2+6c_{10}\left(1+m^2\right)\left(1+\sqrt{1+m^2}\right)^3\left(4+\sqrt{1+m^2}\right)c_4^2+12c_{11}m^2\left(1+m^2\right)$$
$$\left(-1+\sqrt{1+m^2}\right)\left(4+\sqrt{1+m^2}\right)c_3c_4+6c_{10}\left(1-2\sqrt{1+m^2}\right)\left(1+\sqrt{1+m^2}\right)^3$$
$$\left(2+\sqrt{1+m^2}\right)c_4^2+12c_{12}\sqrt{1+m^2}\left(1-\sqrt{1+m^2}\right)^2\left(-1+3\sqrt{1+m^2}\right)^2 c_3^2+12c_{11}$$
$$\left(-1+\sqrt{1+m^2}\right)\left(-2+\sqrt{1+m^2}\right)+6c_{10}\left(1+2\sqrt{1+m^2}\right)\left(1+\sqrt{1+m^2}\right)^3 c_4^2+6c_{12}$$
$$\left(1-\sqrt{1+m^2}\right)^2\left(-1+3\sqrt{1+m^2}\right)\left(-3+3\sqrt{1+m^2}\right)c_3^2-12c_{11}\left(1-\sqrt{1+m^2}\right)^2$$
$$\left(-3+\sqrt{1+m^2}\right)\left(1+\sqrt{1+m^2}\right)c_3c_4\Big],$$

$$c_{25} = \frac{1}{24-24\sqrt{1+m^2}+8m^2}\Big[6c_{11}\left(1+m^2\right)\left(1+\sqrt{1+m^2}\right)^2\left(-1+\sqrt{1+m^2}\right)$$
$$\left(-4+\sqrt{1+m^2}\right)c_4^2-2c_{12}m^2\left(1+m^2\right)\left(-1+3\sqrt{1+m^2}\right)\left(-11+18\sqrt{1+m^2}\right)c_3c_4$$
$$-12c_{11}\sqrt{1+m^2}\left(-1+\sqrt{1+m^2}\right)\left(1+\sqrt{1+m^2}\right)^3 c_4^2+12c_{12}\left(1-\sqrt{1+m^2}\right)$$
$$\left(1+\sqrt{1+m^2}\right)\left(-1+3\sqrt{1+m^2}\right)\left(-3+3\sqrt{1+m^2}\right)c_3c_4-6c_{11}\left(1+\sqrt{1+m^2}\right)^2$$
$$\left(1-\sqrt{1+m^2}\right)\left(-3+\sqrt{1+m^2}\right)c_4^2\Big],$$

$$c_{26} = \frac{1}{40-40\sqrt{1+m^2}+24m^2}\Big[6c_{12}\left(1+m^2\right)\left(1+\sqrt{1+m^2}\right)^2\left(-1+3\sqrt{1+m^2}\right)$$
$$\left(-4+3\sqrt{1+m^2}\right)c_4^2+12c_{12}\sqrt{1+m^2}\left(1+\sqrt{1+m^2}\right)^2\left(-1+3\sqrt{1+m^2}\right)$$
$$\left(-3+3\sqrt{1+m^2}\right)c_4^2+6c_{12}\left(1+\sqrt{1+m^2}\right)^2\left(-1+3\sqrt{1+m^2}\right)\left(-3+3\sqrt{1+m^2}\right)c_4^2\Big],$$

$$c_{27} = \frac{1}{4+4\sqrt{1+m^2}+3m^2}\Big[-60c_{12}\left(1+m^2\right)\left(1-\sqrt{1+m^2}\right)^2 c_3^2c_6-24\left(1+m^2\right)$$
$$\left(1-\sqrt{1+m^2}\right)^2 c_2c_3c_7+24\left(2-\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)^3 c_2c_3c_7$$
$$-24\sqrt{1+m^2}\left(1-\sqrt{1+m^2}\right)^2 c_3^2c_6\Big],$$

$$
\begin{aligned}
c_{28} &= \frac{1}{4-4\sqrt{1+m^2}+3m^2}\left[12\left(1+m^2\right)\left(1+\sqrt{1+m^2}\right)^2 c_4^2c_6+48\sqrt{1+m^2}\left(1+\sqrt{1+m^2}\right)^2\right.\\
&\left. c_4^2c_6+24\left(1+\sqrt{1+m^2}\right)^4 c_2c_4c_8+24\left(1-\sqrt{1+m^2}\right)^3 c_2c_4c_8\right],\\
c_{29} &= \frac{1}{24+24\sqrt{1+m^2}+15m^2}\left[-72\sqrt{1+m^2}\left(1-\sqrt{1+m^2}\right)\left(1+3\sqrt{1+m^2}\right)\right.\\
&\left(1+\sqrt{1+m^2}\right)c_2c_3-24c_9\left(1-\sqrt{1+m^2}\right)^2\left(1+3\sqrt{1+m^2}\right)^2 c_2c_3+24c_9\\
&\left(1-\sqrt{1+m^2}\right)\left(1+3\sqrt{1+m^2}\right)\left(3+3\sqrt{1+m^2}\right)c_2c_3+12c_9\sqrt{1+m^2}\\
&\left.\left(1-\sqrt{1+m^2}\right)\left(-1-3\sqrt{1+m^2}\right)\right],\\
c_{30} &= \frac{1}{8-m^2}\left[-24c_{11}\sqrt{1+m^2}\left(1-\sqrt{1+m^2}\right)\left(-1+\sqrt{1+m^2}\right)^2\left(-4+\sqrt{1+m^2}\right)\right.\\
&c_2c_3+24c_{10}\left(-1+\sqrt{1+m^2}\right)\left(1+\sqrt{1+m^2}\right)^2\left(2+\sqrt{1+m^2}\right)c_2c_4+6c_{11}\\
&\left(-5+\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)^2\left(-2+\sqrt{1+m^2}\right)c_2c_3+24c_{10}\\
&\left.\left(1+\sqrt{1+m^2}\right)^3 c_2c_4+24c_{10}\left(1+\sqrt{1+m^2}\right)^2\left(3+3\sqrt{1+m^2}\right)c_2c_4\right],\\
c_{31} &= \frac{1}{12-12\sqrt{1+m^2}+3m^2}\left[24c_{12}\left(1-\sqrt{1+m^2}\right)^2\left(-1+3\sqrt{1+m^2}\right)\left(-2+3\sqrt{1+m^2}\right)\right.\\
&c_2c_3+24c_{11}\left(-1+\sqrt{1+m^2}\right)^2\left(1+\sqrt{1+m^2}\right)\left(-2+\sqrt{1+m^2}\right)c_2c_4-24c_{12}\\
&\left(1-\sqrt{1+m^2}\right)^2\left(-1+3\sqrt{1+m^2}\right)^2 c_2c_3-24\left(1-2\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)\\
&\left(-1+3\sqrt{1+m^2}\right)c_2c_3-24c_{11}\left(2+3\sqrt{1+m^2}\right)\left(-1+\sqrt{1+m^2}\right)\left(1+\sqrt{1+m^2}\right)\\
&\left.c_2c_4+24\left(-1+\sqrt{1+m^2}\right)\left(1+\sqrt{1+m^2}\right)\left(-2+\sqrt{1+m^2}\right)c_2c_4\right],\\
c_{32} &= \frac{1}{12+12\sqrt{1+m^2}+3m^2}\left[-24c_{10}\left(1+\sqrt{1+m^2}\right)^2\left(1-\sqrt{1+m^2}\right)^2 c_2c_3+24\right.\\
&\left(1-2\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)\left(-1-\sqrt{1+m^2}\right)c_2c_3-48c_9\left(2-\sqrt{1+m^2}\right)\\
&\left(1+\sqrt{1+m^2}\right)\left(-1-3\sqrt{1+m^2}\right)c_2c_4+24c_{10}\left(1-\sqrt{1+m^2}\right)\left(1+\sqrt{1+m^2}\right)\\
&\left.\left(3+\sqrt{1+m^2}\right)c_2c_3\right],\\
c_{33} &= \frac{1}{24-24\sqrt{1+m^2}+15m^2}\left[-24c_{12}\left(1+2\sqrt{1+m^2}\right)\left(1+\sqrt{1+m^2}\right)\left(-1+3\sqrt{1+m^2}\right)\right.\\
&\left.c_2c_4-24c_{12}\sqrt{1+m^2}\left(1+\sqrt{1+m^2}\right)\left(-1+3\sqrt{1+m^2}\right)\left(-2+3\sqrt{1+m^2}\right)c_2c_4\right],\\
c_{34} &= \frac{1}{1-2\sqrt{1+m^2}}\left[48m^2\left(1+\sqrt{1+m^2}\right)^2 c_2c_4c_8+48\sqrt{1+m^2}\left(1+\sqrt{1+m^2}\right)c_2^2\right.
\end{aligned}
$$

$$+48\sqrt{1+m^2}\left(1+\sqrt{1+m^2}\right)c_4c_6+24\left(1+\sqrt{1+m^2}\right)^2c_2c_8\Big],$$

$$c_{35} = \frac{1}{17+18\sqrt{1+m^2}+8m^2}\left[-48c_9\left(-1-3\sqrt{1+m^2}\right)c_2\right],$$

$$c_{36} = \frac{1}{9+6\sqrt{1+m^2}}\Big[48c_{10}\left(1+\sqrt{1+m^2}\right)c_2-12c_{10}\sqrt{1+m^2}\left(1+\sqrt{1+m^2}\right)$$
$$\left(1-\sqrt{1+m^2}\right)\left(2+\sqrt{1+m^2}\right)c_3-24c_{10}\left(1+\sqrt{1+m^2}\right)\left(2+\sqrt{1+m^2}\right)c_2\Big],$$

$$c_{37} = \frac{1}{9-6\sqrt{1+m^2}}\left[-24c_{11}\left(-1+\sqrt{1+m^2}\right)c_2\right],$$

$$c_{38} = \frac{1}{17-18\sqrt{1+m^2}+8m^2}\Big[-72c_{12}\sqrt{1+m^2}\left(-1+3\sqrt{1+m^2}\right)c_2+12\left(1+m^2\right)$$
$$\left(1+\sqrt{1+m^2}\right)^3c_4^2c_8\Big],$$

$$c_{39} = \frac{1}{1+m^2}\left[-48c_2c_6\right],$$

$$c_{40} = \frac{1}{1+2\sqrt{1+m^2}}\Big[48\left(1-\sqrt{1+m^2}\right)^2c_2c_3c_6+48\left(1-\sqrt{1+m^2}\right)^2c_2^2c_7-24$$
$$\sqrt{1+m^2}\left(1-\sqrt{1+m^2}\right)c_3c_6+24\left(1-\sqrt{1+m^2}\right)\left(2-\sqrt{1+m^2}\right)c_2c_7\Big],$$

$$c_{41} = \frac{1}{31+32\sqrt{1+m^2}+15m^2}\Big[-12c_9\left(1-\sqrt{1+m^2}\right)\left(-1-3\sqrt{1+m^2}\right)+12c_9$$
$$\left(1+\sqrt{1+m^2}\right)\left(-1-3\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)c_3\Big],$$

$$c_{42} = \frac{1}{19+16\sqrt{1+m^2}+3m^2}\Big[12c_{10}\sqrt{1+m^2}\left(1+\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)$$
$$\left(2+\sqrt{1+m^2}\right)c_3-12c_9\sqrt{1+m^2}\left(1+\sqrt{1+m^2}\right)\left(-1-3\sqrt{1+m^2}\right)c_4$$
$$-24c_9\left(1+3\sqrt{1+m^2}\right)\left(2+3\sqrt{1+m^2}\right)c_2-12c_9\sqrt{1+m^2}\left(1+\sqrt{1+m^2}\right)$$
$$\left(-1-3\sqrt{1+m^2}\right)\Big],$$

$$c_{43} = \frac{1}{15-m^2}\Big[8c_{10}\left(3+\sqrt{1+m^2}\right)\left(1+\sqrt{1+m^2}\right)^2\left(2+\sqrt{1+m^2}\right)c_4+24c_{11}$$
$$\left(1-\sqrt{1+m^2}\right)^2\left(-1+\sqrt{1+m^2}\right)c_3\Big],$$

$$c_{44} = \frac{1}{19-16\sqrt{1+m^2}+3m^2}\Big[12c_{12}\left(-1+3\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)(-3$$
$$+2\sqrt{1+m^2})\left(-5+2\sqrt{1+m^2}\right)c_3+24c_{11}\left(1-\sqrt{1+m^2}\right)\left(1+\sqrt{1+m^2}\right)c_4$$
$$+12c_{12}\left(1-\sqrt{1+m^2}\right)^2\left(-1+3\sqrt{1+m^2}\right)c_3-24c_{11}\sqrt{1+m^2}\left(1+\sqrt{1+m^2}\right)$$
$$\left(-1+\sqrt{1+m^2}\right)c_4-12c_{12}\left(-1+3\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)\Big],$$

$$
\begin{aligned}
c_{45} &= \frac{1}{7+8\sqrt{1+m^2}+3m^2}\left[-24\sqrt{1+m^2}\left(1-\sqrt{1+m^2}\right)^2 c_3c_7\right], \\
c_{46} &= \frac{1}{3-m^2}\Big[-12\left(1-\sqrt{1+m^2}\right)\left(1+\sqrt{1+m^2}\right)^2 c_3c_8 - 12\left(1-2\sqrt{1+m^2}\right) \\
&\quad \left(1-\sqrt{1+m^2}\right)\left(1+\sqrt{1+m^2}\right)c_4c_7 + 24\left(1+m^2\right)\left(1-\sqrt{1+m^2}\right)\left(1+\sqrt{1+m^2}\right) \\
&\quad c_4c_7 - 12\left(1+m^2\right)\left(1+\sqrt{1+m^2}\right)^2 c_3c_8 - 12\sqrt{1+m^2}\left(1-\sqrt{1+m^2}\right) \\
&\quad \left(1+\sqrt{1+m^2}\right)c_3 - 24\sqrt{1+m^2}\left(1-\sqrt{1+m^2}\right)c_3c_6\Big], \\
c_{47} &= \frac{1}{7-8\sqrt{1+m^2}+3m^2}\left[24\sqrt{1+m^2}\left(1+\sqrt{1+m^2}\right)^2 c_4c_8\right], \\
c_{48} &= \frac{1}{31-32\sqrt{1+m^2}+15m^2}\left[-24c_{12}\sqrt{1+m^2}\left(1+\sqrt{1+m^2}\right)\left(-1+3\sqrt{1+m^2}\right)c_4\right], \\
c_{49} &= \Big[48\sqrt{1+m^2}\left(1-\sqrt{1+m^2}\right)c_2^2c_7 + 24\left(1-\sqrt{1+m^2}\right)c_2^2 + 48\left(1-2\sqrt{1+m^2}\right) \\
&\quad \left(1-\sqrt{1+m^2}\right)c_2c_3c_6 - 48c_{10}\left(-1-\sqrt{1+m^2}\right)^2 c_2^2c_7\Big], \\
c_{50} &= \Big[-48\sqrt{1+m^2}\left(1+\sqrt{1+m^2}\right)c_2^2c_8 + 40\left(1+\sqrt{1+m^2}\right)c_2c_4c_6 + 96\left(1+\sqrt{1+m^2}\right) \\
&\quad c_2^2c_8 + 48\sqrt{1+m^2}\left(1+\sqrt{1+m^2}\right)c_2c_4c_6 + 48\left(1+\sqrt{1+m^2}\right)^2 c_2c_4c_6 + 24\sqrt{1+m^2} \\
&\quad c_2c_8\Big], \\
c_{51} &= 2\sqrt{1+m^2}\left[k^2\left((k-h)^{\sqrt{1+m^2}}-(k+h)^{\sqrt{1+m^2}}\right)^2 + h^2\left((k-h)^{\sqrt{1+m^2}}+(k+h)^{\sqrt{1+m^2}}\right)^2\right] \\
&\quad +2kh\left(m^2+2\right)\left((k-h)^{2\sqrt{1+m^2}}-(k+h)^{2\sqrt{1+m^2}}\right), \\
c_{52} &= 2hk\left(1+3\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)\left((k+h)^{-2-2\sqrt{1+m^2}}-(k-h)^{-2-2\sqrt{1+m^2}}\right) \\
&\quad +\left(1-\sqrt{1+m^2}\right)\left((k+h)^{-2\sqrt{1+m^2}}-(k+h)^{-1-3\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}}\right. \\
&\quad \left.-(k+h)^{1+\sqrt{1+m^2}}(k-h)^{-1-3\sqrt{1+m^2}}+(k-h)^{-2\sqrt{1+m^2}}\right)-\left(1+3\sqrt{1+m^2}\right) \\
&\quad \left((k+h)^{-1-3\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}}-(k-h)^2(k+h)^{-2-2\sqrt{1+m^2}}-(k+h)^2\right. \\
&\quad \left.(k-h)^{-2-2\sqrt{1+m^2}}+(k+h)^{1+\sqrt{1+m^2}}(k-h)^{-1-3\sqrt{1+m^2}}\right), \\
c_{53} &= -2hkm^2\left((k+h)^{-2}-(k-h)^{-2}\right)+\left(1-\sqrt{1+m^2}\right)\left(2-(k+h)^{-1-\sqrt{1+m^2}}\right. \\
&\quad \left.(k-h)^{1+\sqrt{1+m^2}}-(k+h)^{1+\sqrt{1+m^2}}(k-h)^{-1-\sqrt{1+m^2}}\right)-\left(1+\sqrt{1+m^2}\right) \\
&\quad \left((k-h)^{1+\sqrt{1+m^2}}(k+h)^{-1-\sqrt{1+m^2}}-(k-h)^{-2}(k+h)^2-(k+h)^{-2}(k-h)^2\right. \\
&\quad \left.+(k+h)^{1+\sqrt{1+m^2}}(k-h)^{-1-\sqrt{1+m^2}}\right), \\
c_{54} &= -2hk\left(1-\sqrt{1+m^2}\right)^2\left((k+h)^{-2+2\sqrt{1+m^2}}-(k-h)^{-2+2\sqrt{1+m^2}}\right) \\
&\quad +\left(1-\sqrt{1+m^2}\right)\left((k+h)^{2\sqrt{1+m^2}}-(k+h)^{-1+\sqrt{1+m^2}}(k-h)^{1+\sqrt{1+m^2}}\right.
\end{aligned}
$$

$$
\begin{aligned}
& - (k+h)^{1+\sqrt{1+m^2}} (k-h)^{-1+\sqrt{1+m^2}} + (k-h)^{2\sqrt{1+m^2}}\Big) + \Big(-1+\sqrt{1+m^2}\Big) \\
& \Big((k+h)^{-1+\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}} - (k-h)^2 (k+h)^{-2+2\sqrt{1+m^2}} - (k+h)^2 \\
& (k-h)^{-2+2\sqrt{1+m^2}} + (k+h)^{1+\sqrt{1+m^2}} (k-h)^{-1+\sqrt{1+m^2}}\Big), \\
c_{55} &= 2hk\Big(-1+3\sqrt{1+m^2}\Big)\Big(1-\sqrt{1+m^2}\Big)\Big((k+h)^{-2+4\sqrt{1+m^2}} - (k-h)^{-2+4\sqrt{1+m^2}}\Big) \\
& + \Big(1-\sqrt{1+m^2}\Big)\Big((k+h)^{4\sqrt{1+m^2}} - (k+h)^{-1+3\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}} \\
& - (k+h)^{1+\sqrt{1+m^2}} (k-h)^{-1+3\sqrt{1+m^2}} + (k-h)^{4\sqrt{1+m^2}}\Big) + \Big(-1+3\sqrt{1+m^2}\Big) \\
& \Big((k+h)^{-1+3\sqrt{1+m^2}} (k-h)^{1+\sqrt{1+m^2}} - (k+h)^2 (k-h)^{-2+4\sqrt{1+m^2}} - (k-h)^2 \\
& (k+h)^{-2+4\sqrt{1+m^2}} + (k+h)^{1+\sqrt{1+m^2}} (k-h)^{-1+3\sqrt{1+m^2}}\Big), \\
c_{56} &= \Big(1-\sqrt{1+m^2}\Big)\Big((k+h)^{1+\sqrt{1+m^2}} - (k-h)^{1+\sqrt{1+m^2}}\Big), \\
c_{57} &= \frac{1}{2(h+k)}\Big[4hk\Big(-1-3\sqrt{1+m^2}\Big)\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{-1-2\sqrt{1+m^2}} - (h+k) \\
& (-h+k)^{-2-2\sqrt{1+m^2}}\Big) - 2\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{1-2\sqrt{1+m^2}} - (h+k)^{2+\sqrt{1+m^2}} \\
& (-h+k)^{-1-3\sqrt{1+m^2}} - (h+k)^{-3\sqrt{1+m^2}} (-h+k)^{1+\sqrt{1+m^2}} + (h+k)(-h+k)^{-2\sqrt{1+m^2}}\Big) \\
& -2\Big(-1-3\sqrt{1+m^2}\Big)\Big((h+k)^{-3\sqrt{1+m^2}} (-h+k)^{1+\sqrt{1+m^2}} - (-h+k)^{-2-2\sqrt{1+m^2}} \\
& (h+k)^3 - (-h+k)^2 (h+k)^{-1-2\sqrt{1+m^2}} + (h+k)^{2+\sqrt{1+m^2}} (-h+k)^{-1-3\sqrt{1+m^2}}\Big)\Big], \\
c_{58} &= \frac{1}{2(h+k)}\Big[4hk\Big(-1-\sqrt{1+m^2}\Big)\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{-1} - (h+k)(h+k)^{-2}\Big) \\
& -2\Big(1-\sqrt{1+m^2}\Big)\Big(2(h+k) - (h+k)^{2+\sqrt{1+m^2}} (-h+k)^{-1-\sqrt{1+m^2}} - (h+k)^{-\sqrt{1+m^2}} \\
& (-h+k)^{1+\sqrt{1+m^2}}\Big) - 2\Big(-1-\sqrt{1+m^2}\Big)\Big((h+k)^{-\sqrt{1+m^2}} (-h+k)^{1+\sqrt{1+m^2}} - (h+k)^3 \\
& (-h+k)^{-2} - (-h+k)^2 (h+k)^{-1} + (h+k)^{2+\sqrt{1+m^2}} (-h+k)^{1-\sqrt{1+m^2}}\Big)\Big], \\
c_{59} &= \frac{1}{2(h+k)}\Big[4hk\Big(-1+\sqrt{1+m^2}\Big)\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{-1+2\sqrt{1+m^2}} - (h+k) \\
& (-h+k)^{-2+2\sqrt{1+m^2}}\Big) - 2\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{1+2\sqrt{1+m^2}} - (h+k)^{2+\sqrt{1+m^2}} \\
& (-h+k)^{1+\sqrt{1+m^2}} - (h+k)^{\sqrt{1+m^2}} (-h+k)^{1+\sqrt{1+m^2}} + (h+k)(-h+k)^{2\sqrt{1+m^2}}\Big) \\
& -2\Big(-1+\sqrt{1+m^2}\Big)\Big((h+k)^{\sqrt{1+m^2}} (-h+k)^{1+\sqrt{1+m^2}} - (h+k)^3 (-h+k)^{-2+2\sqrt{1+m^2}} \\
& - (-h+k)^2 (h+k)^{-1+2\sqrt{1+m^2}} + (h+k)^{2+\sqrt{1+m^2}} (-h+k)^{-1+\sqrt{1+m^2}}\Big)\Big], \\
c_{60} &= \frac{1}{2(h+k)}\Big[4hk\Big(-1+3\sqrt{1+m^2}\Big)\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{-1+4\sqrt{1+m^2}} - (h+k) \\
& (-h+k)^{-2+4\sqrt{1+m^2}}\Big) - 2\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{1+4\sqrt{1+m^2}} - (h+k)^{2+\sqrt{1+m^2}}
\end{aligned}
$$

$$
\begin{aligned}
& (-h+k)^{-1+3\sqrt{1+m^2}} - (h+k)^{3\sqrt{1+m^2}} (-h+k)^{1+\sqrt{1+m^2}} + (h+k)(-h+k)^{4\sqrt{1+m^2}} \Big) \\
& -2\Big(-1+3\sqrt{1+m^2}\Big)\Big((h+k)^{3\sqrt{1+m^2}} (-h+k)^{1+\sqrt{1+m^2}} - (-h+k)^{-2+4\sqrt{1+m^2}} \\
& (h+k)^3 - (-h+k)^2 (h+k)^{-1+4\sqrt{1+m^2}} + (h+k)^{2+\sqrt{1+m^2}} (-h+k)^{-1+3\sqrt{1+m^2}}\Big)\Big], \\
c_{61} = & \frac{1}{2(h+k)}\Big[4hk\Big(-3-5\sqrt{1+m^2}\Big)\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{-3-4\sqrt{1+m^2}} - (h+k) \\
& (-h+k)^{-4-4\sqrt{1+m^2}}\Big) - 2\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{-1-4\sqrt{1+m^2}} - (h+k)^{2+\sqrt{1+m^2}} \\
& (-h+k)^{-3-5\sqrt{1+m^2}} - (h+k)^{-2-5\sqrt{1+m^2}} (-h+k)^{1+\sqrt{1+m^2}} + (-h+k)^{-1-4\sqrt{1+m^2}} \\
& (h+k)) - 2\Big(-3-5\sqrt{1+m^2}\Big)\Big((h+k)^{-2-5\sqrt{1+m^2}} (-h+k)^{1+\sqrt{1+m^2}} - (h+k)^3 \\
& (-h+k)^{-4-4\sqrt{1+m^2}} - (-h+k)^2 (h+k)^{-3-4\sqrt{1+m^2}} + (h+k)^{2+\sqrt{1+m^2}} \\
& (-h+k)^{-3-5\sqrt{1+m^2}}\Big)\Big], \\
c_{62} = & \frac{1}{2(h+k)}\Big[4hk\Big(-3-3\sqrt{1+m^2}\Big)\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{-3-2\sqrt{1+m^2}} - (h+k) \\
& (-h+k)^{-4-2\sqrt{1+m^2}}\Big) - 2\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{-1-2\sqrt{1+m^2}} - (h+k)^{2+\sqrt{1+m^2}} \\
& (-h+k)^{-3-3\sqrt{1+m^2}} - (h+k)^{-2-3\sqrt{1+m^2}} (-h+k)^{1+\sqrt{1+m^2}} + (-h+k)^{-2-2\sqrt{1+m^2}} \\
& (h+k)) - 2\Big(-3-3\sqrt{1+m^2}\Big)\Big((h+k)^{-2-3\sqrt{1+m^2}} (-h+k)^{1+\sqrt{1+m^2}} - (h+k)^3 \\
& (-h+k)^{-4-2\sqrt{1+m^2}} - (-h+k)^2 (h+k)^{-3-2\sqrt{1+m^2}} + (h+k)^{2+\sqrt{1+m^2}} \\
& (-h+k)^{-3-3\sqrt{1+m^2}}\Big)\Big], \\
c_{63} = & \frac{1}{2(h+k)}\Big[4hk\Big(-3-\sqrt{1+m^2}\Big)\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{-3} - (h+k)(-h+k)^{-4}\Big) \\
& -2\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{-1} - (h+k)^{2+\sqrt{1+m^2}} (-h+k)^{-3-\sqrt{1+m^2}} - (h+k)^{-2-\sqrt{1+m^2}} \\
& (-h+k)^{1+\sqrt{1+m^2}} + (h+k)(-h+k)^{-2}\Big) - 2\Big(-3-\sqrt{1+m^2}\Big)\Big((h+k)^{-2-\sqrt{1+m^2}} \\
& (-h+k)^{1+\sqrt{1+m^2}} - (h+k)^3(-h+k)^{-4} - (-h+k)^2(h+k)^{-3} + (h+k)^{2+\sqrt{1+m^2}} \\
& (-h+k)^{-3-\sqrt{1+m^2}}\Big)\Big], \\
c_{64} = & \frac{1}{2(h+k)}\Big[4hk\Big(-3+\sqrt{1+m^2}\Big)\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{-3+2\sqrt{1+m^2}} - (h+k) \\
& (-h+k)^{-4+2\sqrt{1+m^2}}\Big) - 2\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{-1+2\sqrt{1+m^2}} - (h+k)^{2+\sqrt{1+m^2}} \\
& (-h+k)^{-3+\sqrt{1+m^2}} - (h+k)^{-2+\sqrt{1+m^2}} (-h+k)^{1+\sqrt{1+m^2}} + (-h+k)^{-2+2\sqrt{1+m^2}} \\
& (h+k)) - 2\Big(-3+\sqrt{1+m^2}\Big)\Big((h+k)^{-2+\sqrt{1+m^2}} (-h+k)^{1+\sqrt{1+m^2}} - (h+k)^3 \\
& (-h+k)^{-3+2\sqrt{1+m^2}} - (-h+k)^2 (h+k)^{-2+2\sqrt{1+m^2}} + (h+k)^{2+\sqrt{1+m^2}}
\end{aligned}
$$

$$\left.\left.(-h+k)^{-3+\sqrt{1+m^2}}\right)\right],$$

$$\begin{aligned}
c_{65} &= \frac{1}{2(h+k)}\left[4hk\left(-3+3\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)\left((h+k)^{-3+4\sqrt{1+m^2}}-(h+k)\right.\right.\\
&\left.(-h+k)^{-4+4\sqrt{1+m^2}}\right)-2\left(1-\sqrt{1+m^2}\right)\left((h+k)^{-1+4\sqrt{1+m^2}}-(h+k)^{2+\sqrt{1+m^2}}\right.\\
&(-h+k)^{-3+3\sqrt{1+m^2}}-(h+k)^{-2+3\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}}+(-h+k)^{-2+4\sqrt{1+m^2}}\\
&(h+k))-2\left(-3+3\sqrt{1+m^2}\right)\left((h+k)^{-2+3\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}}-(h+k)^3\right.\\
&(-h+k)^{-4+4\sqrt{1+m^2}}-(-h+k)^2(h+k)^{-3+4\sqrt{1+m^2}}+(h+k)^{2+\sqrt{1+m^2}}\\
&\left.\left.(-h+k)^{-3+3\sqrt{1+m^2}}\right)\right],\\
c_{66} &= \frac{1}{2(h+k)}\left[4hk\left(-3+5\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)\left((h+k)^{-3+6\sqrt{1+m^2}}-(h+k)\right.\right.\\
&\left.(-h+k)^{-4+6\sqrt{1+m^2}}\right)-2\left(1-\sqrt{1+m^2}\right)\left((h+k)^{-1+6\sqrt{1+m^2}}-(h+k)^{2+\sqrt{1+m^2}}\right.\\
&(-h+k)^{-3+5\sqrt{1+m^2}}-(h+k)^{-2+5\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}}+(-h+k)^{-2+6\sqrt{1+m^2}}\\
&(h+k))-2\left(-3+5\sqrt{1+m^2}\right)\left((h+k)^{-2+5\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}}-(h+k)^3\right.\\
&(-h+k)^{-4+6\sqrt{1+m^2}}-(-h+k)^2(h+k)^{-3+6\sqrt{1+m^2}}+(h+k)^{2+\sqrt{1+m^2}}\\
&\left.\left.(-h+k)^{-3+5\sqrt{1+m^2}}\right)\right],\\
c_{67} &= \frac{1}{2(h+k)}\left[4hk\left(-2\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)\left((h+k)^{-\sqrt{1+m^2}}-(h+k)\right.\right.\\
&\left.(-h+k)^{-1-\sqrt{1+m^2}}\right)-2\left(1-\sqrt{1+m^2}\right)\left((h+k)^{2-\sqrt{1+m^2}}-(-h+k)^{-2\sqrt{1+m^2}}\right.\\
&\left.(h+k)^{2+\sqrt{1+m^2}}-(h+k)^{1-2\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}}+(h+k)(-h+k)^{1-\sqrt{1+m^2}}\right)\\
&+4\sqrt{1+m^2}\left((h+k)^{1-2\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}}-(h+k)^3(-h+k)^{-1-\sqrt{1+m^2}}\right.\\
&\left.\left.-(-h+k)^2(h+k)^{-\sqrt{1+m^2}}+(h+k)^{2+\sqrt{1+m^2}}(-h+k)^{-2\sqrt{1+m^2}}\right)\right],\\
c_{68} &= \frac{1}{2(h+k)}\left[4hk\left(2\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)\left((h+k)^{3\sqrt{1+m^2}}-(h+k)\right.\right.\\
&\left.(-h+k)^{-1+3\sqrt{1+m^2}}\right)-2\left(1-\sqrt{1+m^2}\right)\left((h+k)^{2+3\sqrt{1+m^2}}-(-h+k)^{2\sqrt{1+m^2}}\right.\\
&\left.(h+k)^{2+\sqrt{1+m^2}}-(h+k)^{1+2\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}}+(h+k)(-h+k)^{1+3\sqrt{1+m^2}}\right)\\
&-4\sqrt{1+m^2}\left((h+k)^{1+2\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}}-(h+k)^3(-h+k)^{-1+3\sqrt{1+m^2}}\right.\\
&\left.\left.-(-h+k)^2(h+k)^{3\sqrt{1+m^2}}+(h+k)^{2+\sqrt{1+m^2}}(-h+k)^{2\sqrt{1+m^2}}\right)\right],\\
c_{69} &= \frac{1}{2(h+k)}\left[4hk\left(-2-4\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)\left((h+k)^{-2-3\sqrt{1+m^2}}-(h+k)\right.\right.\\
&\left.(-h+k)^{-3-3\sqrt{1+m^2}}\right)-2\left(1-\sqrt{1+m^2}\right)\left((h+k)^{-3\sqrt{1+m^2}}-(h+k)^{2+\sqrt{1+m^2}}\right.
\end{aligned}$$

$$
\begin{aligned}
&(-h+k)^{-2-4\sqrt{1+m^2}} - (h+k)^{-2-4\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}} + (-h+k)^{-1-3\sqrt{1+m^2}} \\
&(h+k)) - 2\left(-2-4\sqrt{1+m^2}\right)\left((h+k)^{-1-4\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}} - (h+k)^3\right. \\
&(-h+k)^{-3-3\sqrt{1+m^2}} - (-h+k)^2(h+k)^{-2-3\sqrt{1+m^2}} + (h+k)^{2+\sqrt{1+m^2}} \\
&\left.\left.(-h+k)^{-2-4\sqrt{1+m^2}}\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
c_{70} \;=\;& \frac{1}{2(h+k)}\left[4hk(-2)\left(1-\sqrt{1+m^2}\right)\left((h+k)^{-2+\sqrt{1+m^2}} - (-h+k)^{-3+\sqrt{1+m^2}}\right.\right. \\
&(h+k)) - 2\left(1-\sqrt{1+m^2}\right)\left((h+k)^{\sqrt{1+m^2}} - (h+k)^{2+\sqrt{1+m^2}}(-h+k)^{-2}\right. \\
&\left.-(h+k)^{-1}(-h+k)^{1+\sqrt{1+m^2}} + (h+k)(-h+k)^{-1+\sqrt{1+m^2}}\right) + 4\left((h+k)^{-1}\right. \\
&(-h+k)^{1+\sqrt{1+m^2}} - (h+k)^3(-h+k)^{-3+\sqrt{1+m^2}} - (-h+k)^2(h+k)^{-2+\sqrt{1+m^2}} \\
&\left.\left.+(h+k)^{2+\sqrt{1+m^2}}(-h+k)^{-2}\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
c_{71} \;=\;& \frac{1}{2(h+k)}\left[4hk\left(-2+2\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)\left((h+k)^{-2+3\sqrt{1+m^2}} - (h+k)\right.\right. \\
&\left.(-h+k)^{-3+3\sqrt{1+m^2}}\right) - 2\left(1-\sqrt{1+m^2}\right)\left((h+k)^{3\sqrt{1+m^2}} - (h+k)^{-1+2\sqrt{1+m^2}}\right. \\
&(-h+k)^{1+\sqrt{1+m^2}} - (h+k)^{2+\sqrt{1+m^2}}(-h+k)^{-2+2\sqrt{1+m^2}} + (-h+k)^{-1+3\sqrt{1+m^2}} \\
&(h+k)) - 2\left(-2+2\sqrt{1+m^2}\right)\left((h+k)^{-1+2\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}}\right. \\
&-(-h+k)^{-3+3\sqrt{1+m^2}}(h+k)^3 - (-h+k)^2(h+k)^{-2+3\sqrt{1+m^2}} + (h+k)^{2+\sqrt{1+m^2}} \\
&\left.\left.(-h+k)^{-2-2\sqrt{1+m^2}}\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
c_{72} \;=\;& \frac{1}{2(h+k)}\left[4hk\left(-2-2\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)\left((h+k)^{-2-\sqrt{1+m^2}} - (h+k)\right.\right. \\
&(-h+k)^{-3-\sqrt{1+m^2}} - 2\left(1-\sqrt{1+m^2}\right)\left((h+k)^{-\sqrt{1+m^2}} - (h+k)^{2+\sqrt{1+m^2}}\right. \\
&(-h+k)^{-2-2\sqrt{1+m^2}} - (h+k)^{-1-2\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}} + (-h+k)^{-1-\sqrt{1+m^2}} \\
&(h+k)) - 2\left(-2-2\sqrt{1+m^2}\right)\left((h+k)^{-1-2\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}} - (h+k)^3\right. \\
&(-h+k)^{-3-\sqrt{1+m^2}} - (-h+k)^2(h+k)^{-2-\sqrt{1+m^2}} + (h+k)^{2+\sqrt{1+m^2}} \\
&\left.\left.(-h+k)^{-2-2\sqrt{1+m^2}}\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
c_{73} \;=\;& \frac{1}{2(h+k)}\left[4hk\left(-2+4\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)\left((h+k)^{-2+5\sqrt{1+m^2}} - (h+k)\right.\right. \\
&\left.(-h+k)^{-3+5\sqrt{1+m^2}}\right) - 2\left(1-\sqrt{1+m^2}\right)\left((h+k)^{5\sqrt{1+m^2}} - (h+k)^{2+\sqrt{1+m^2}}\right. \\
&(-h+k)^{-2+4\sqrt{1+m^2}} - (h+k)^{-1+4\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}} + (-h+k)^{5\sqrt{1+m^2}} \\
&(h+k)) - 2\left(-2+4\sqrt{1+m^2}\right)\left((h+k)^{-1+4\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}} - (h+k)^3\right.
\end{aligned}
$$

$$
\begin{aligned}
& (-h+k)^{-3+5\sqrt{1+m^2}} - (-h+k)^2 (h+k)^{-1+4\sqrt{1+m^2}} + (h+k)^{2+\sqrt{1+m^2}} \\
& (-h+k)^{-2+4\sqrt{1+m^2}}\Big)\Big], \\
c_{74} &= \frac{1}{2(h+k)}\Big[4hk\sqrt{1+m^2}\left(1-\sqrt{1+m^2}\right)\Big((h+k)^{2\sqrt{1+m^2}} - (-h+k)^{-1+2\sqrt{1+m^2}} \\
& (h+k)\Big) - 2\left(1-\sqrt{1+m^2}\right)\Big((h+k)^{2+2\sqrt{1+m^2}} - (h+k)^{2+\sqrt{1+m^2}}(-h+k)^{\sqrt{1+m^2}} \\
& - (h+k)^{1+\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}} + (-h+k)^{1+2\sqrt{1+m^2}}(h+k)\Big) - 2\sqrt{1+m^2} \\
& \Big((h+k)^{1+\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}} - (h+k)^3(-h+k)^{-1+2\sqrt{1+m^2}} - (-h+k)^2 \\
& (h+k)^{2\sqrt{1+m^2}} + (h+k)^{2+\sqrt{1+m^2}}(-h+k)^{\sqrt{1+m^2}}\Big)\Big], \\
c_{75} &= \frac{1}{2(h+k)}\Big[4hk\left(-2-3\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)\Big((h+k)^{-2-2\sqrt{1+m^2}} - (h+k) \\
& (-h+k)^{-3-2\sqrt{1+m^2}}\Big) - 2\left(1-\sqrt{1+m^2}\right)\Big((h+k)^{-2\sqrt{1+m^2}} - (h+k)^{2+\sqrt{1+m^2}} \\
& (-h+k)^{-2-3\sqrt{1+m^2}} - (h+k)^{-1-3\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}} + (-h+k)^{-1-2\sqrt{1+m^2}} \\
& (h+k)\Big) - 2\left(-2-3\sqrt{1+m^2}\right)\Big((h+k)^{-1-3\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}} - (h+k)^3 \\
& (-h+k)^{-3-2\sqrt{1+m^2}} - (-h+k)^2(h+k)^{-2-2\sqrt{1+m^2}} + (h+k)^{2+\sqrt{1+m^2}} \\
& (-h+k)^{-2-3\sqrt{1+m^2}}\Big)\Big], \\
c_{76} &= \frac{1}{2(h+k)}\Big[4hk\left(-2-\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)\left((h+k)^{-2} - (h+k)(h+k)^{-3}\right) \\
& -2\left(1-\sqrt{1+m^2}\right)\Big(1 - (h+k)^{-1-\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}} - (h+k)^{2+\sqrt{1+m^2}} \\
& (-h+k)^{-2-\sqrt{1+m^2}} + (h+k)(-h+k)^{-1}\Big) - 2\left(-2-\sqrt{1+m^2}\right)\Big((h+k)^{-1-\sqrt{1+m^2}} \\
& (-h+k)^{1+\sqrt{1+m^2}} - (h+k)^3(-h+k)^{-3} - (-h+k)^2(h+k)^{-2} + (h+k)^{2+\sqrt{1+m^2}} \\
& (-h+k)^{-1-\sqrt{1+m^2}}\Big)\Big], \\
c_{77} &= \frac{1}{2(h+k)}\Big[4hk\left(-2+\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)\Big((h+k)^{-2+2\sqrt{1+m^2}} - (h+k) \\
& (-h+k)^{-3+2\sqrt{1+m^2}}\Big) - 2\left(1-\sqrt{1+m^2}\right)\Big((h+k)^{2\sqrt{1+m^2}} - (h+k)^{2+\sqrt{1+m^2}} \\
& (-h+k)^{-2+\sqrt{1+m^2}} - (h+k)^{-1+\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}} + (-h+k)^{-1+2\sqrt{1+m^2}} \\
& (h+k)\Big) - 2\left(-2+\sqrt{1+m^2}\right)\Big((h+k)^{-1+\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}} - (h+k)^3 \\
& (-h+k)^{-3+2\sqrt{1+m^2}} - (-h+k)^2(h+k)^{-2+2\sqrt{1+m^2}} + (h+k)^{2+\sqrt{1+m^2}} \\
& (-h+k)^{-2+\sqrt{1+m^2}}\Big)\Big], \\
c_{78} &= \frac{1}{2(h+k)}\Big[4hk\left(-2+3\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)\Big((h+k)^{-2+4\sqrt{1+m^2}} - (h+k)
\end{aligned}
$$

$$
\begin{aligned}
& (-h+k)^{-3+4\sqrt{1+m^2}}\Big) - 2\left(1-\sqrt{1+m^2}\right)\Big((h+k)^{4\sqrt{1+m^2}} - (h+k)^{-1+3\sqrt{1+m^2}} \\
& (-h+k)^{1+\sqrt{1+m^2}} - (h+k)^{2+\sqrt{1+m^2}}(-h+k)^{-2+3\sqrt{1+m^2}} + (-h+k)^{-1+4\sqrt{1+m^2}} \\
& (h+k)\Big) - 2\left(-2+3\sqrt{1+m^2}\right)\Big((h+k)^{-2+3\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}} - (h+k)^3 \\
& (-h+k)^{-3+4\sqrt{1+m^2}} - (-h+k)^2(h+k)^{-2+4\sqrt{1+m^2}} + (h+k)^{2+\sqrt{1+m^2}} \\
& (-h+k)^{-2+3\sqrt{1+m^2}}\Big)\Big],
\end{aligned}
$$

$$
\begin{aligned}
c_{79} \;=\; & \frac{1}{2(h+k)}\Big[4hk\left(1-\sqrt{1+m^2}\right)\Big((h+k)^{1+\sqrt{1+m^2}} - (-h+k)^{1+\sqrt{1+m^2}} \\
& -2h(-h+k)^{\sqrt{1+m^2}}\Big) - 2\left(1-\sqrt{1+m^2}\right)\Big(2h(h+k)^{2+\sqrt{1+m^2}} - 2h(h+k) \\
& (-h+k)^{1+\sqrt{1+m^2}}\Big) + 4h\Big((h+k)^2(-h+k)^{\sqrt{1+m^2}} - (h+k)^{1+\sqrt{1+m^2}} \\
& (-h+k)\Big)\Big],
\end{aligned}
$$

$$
\begin{aligned}
c_{80} \;=\; & \frac{1}{2(h+k)}\Big[4hk\left(-\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)\left(1-(h+k)(-h+k)^{-1}\right) \\
& -2\left(1-\sqrt{1+m^2}\right)\Big((h+k)^2 - (h+k)^{1-\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}} \\
& -(h+k)^{2+\sqrt{1+m^2}}(-h+k)^{-\sqrt{1+m^2}} + (h+k)(-h+k)\Big) + 2\sqrt{1+m^2} \\
& \Big((h+k)^{1-\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}} - (h+k)^3(-h+k)^{-1} - (-h+k)^2 \\
& +(h+k)^{2+\sqrt{1+m^2}}(-h+k)^{-\sqrt{1+m^2}}\Big)\Big],
\end{aligned}
$$

$$
\begin{aligned}
c_{81} \;=\; & \frac{1}{2(h+k)}\Big[4hk\left(-3-4\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)\Big((h+k)^{-3-3\sqrt{1+m^2}} \\
& -(-h+k)^{-4-3\sqrt{1+m^2}}(h+k)\Big) - 2\left(1-\sqrt{1+m^2}\right)\Big((h+k)^{-1-3\sqrt{1+m^2}} \\
& -(h+k)^{-2-4\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}} - (h+k)^{2+\sqrt{1+m^2}}(-h+k)^{-3-4\sqrt{1+m^2}} \\
& +(h+k)(-h+k)^{-2-3\sqrt{1+m^2}}\Big) - 2\left(-3-4\sqrt{1+m^2}\right)\Big((h+k)^{-2-4\sqrt{1+m^2}} \\
& (-h+k)^{1+\sqrt{1+m^2}} - (h+k)^3(-h+k)^{-4-3\sqrt{1+m^2}} - (h+k)^{-3-3\sqrt{1+m^2}} \\
& (-h+k)^2 + (h+k)^{2+\sqrt{1+m^2}}(-h+k)^{-3-4\sqrt{1+m^2}}\Big)\Big],
\end{aligned}
$$

$$
\begin{aligned}
c_{82} \;=\; & \frac{1}{2(h+k)}\Big[4hk\left(-3-2\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)\Big((h+k)^{-3-\sqrt{1+m^2}} \\
& -(-h+k)^{-4-\sqrt{1+m^2}}(h+k)\Big) - 2\left(1-\sqrt{1+m^2}\right)\Big((h+k)^{-1-\sqrt{1+m^2}} \\
& -(h+k)^{-2-2\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}} - (h+k)^{2+\sqrt{1+m^2}}(-h+k)^{-3-2\sqrt{1+m^2}} \\
& +(h+k)(-h+k)^{-1-\sqrt{1+m^2}}\Big) - 2\left(-3-2\sqrt{1+m^2}\right)\Big((h+k)^{-2-2\sqrt{1+m^2}} \\
& (-h+k)^{1+\sqrt{1+m^2}} - (h+k)^3(-h+k)^{-4-\sqrt{1+m^2}} - (h+k)^{-3-\sqrt{1+m^2}}
\end{aligned}
$$

$$
\begin{aligned}
& (-h+k)^2+(h+k)^{2+\sqrt{1+m^2}}(-h+k)^{-3-2\sqrt{1+m^2}}\Big)\Big], \\
c_{83} &= \frac{1}{2(h+k)}\Big[4hk(-3)\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{-3+\sqrt{1+m^2}}-(h+k) \\
& (-h+k)^{-4+\sqrt{1+m^2}}\Big)-2\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{-1+\sqrt{1+m^2}}-(h+k)^{-2} \\
& (-h+k)^{1+\sqrt{1+m^2}}-(h+k)^{2+\sqrt{1+m^2}}(-h+k)^{-3}+(-h+k)^{-2+\sqrt{1+m^2}}(h+k)\Big) \\
& +6\Big((h+k)^{-2}(-h+k)^{1+\sqrt{1+m^2}}-(-h+k)^{-4+\sqrt{1+m^2}}(h+k)^3-(-h+k)^2 \\
& (h+k)^{-3+\sqrt{1+m^2}}+(h+k)^{2+\sqrt{1+m^2}}(-h+k)^{-3}\Big)\Big], \\
c_{84} &= \frac{1}{2(h+k)}\Big[4hk\Big(-3+2\sqrt{1+m^2}\Big)\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{-3+3\sqrt{1+m^2}} \\
& -(-h+k)^{-4+3\sqrt{1+m^2}}(h+k)\Big)-2\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{-1+3\sqrt{1+m^2}} \\
& -(h+k)^{-2+2\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}}-(h+k)^{2+\sqrt{1+m^2}}(-h+k)^{-3+2\sqrt{1+m^2}} \\
& +(h+k)(-h+k)^{-2+3\sqrt{1+m^2}}\Big)-2\Big(-3+2\sqrt{1+m^2}\Big)\Big((h+k)^{-2+2\sqrt{1+m^2}} \\
& (-h+k)^{1+\sqrt{1+m^2}}-(h+k)^3(-h+k)^{-4+3\sqrt{1+m^2}}-(h+k)^{-3+3\sqrt{1+m^2}} \\
& (-h+k)^2+(h+k)^{2+\sqrt{1+m^2}}(-h+k)^{-3+2\sqrt{1+m^2}}\Big)\Big], \\
c_{85} &= \frac{1}{2(h+k)}\Big[4hk\Big(-1-2\sqrt{1+m^2}\Big)\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{-1-\sqrt{1+m^2}} \\
& -(-h+k)^{-2-\sqrt{1+m^2}}(h+k)\Big)-2\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{1-\sqrt{1+m^2}} \\
& -(h+k)^{-2\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}}-(h+k)^{2+\sqrt{1+m^2}}(-h+k)^{-1-2\sqrt{1+m^2}} \\
& +(h+k)(-h+k)^{-\sqrt{1+m^2}}\Big)-2\Big(-1-2\sqrt{1+m^2}\Big)\Big((h+k)^{-2\sqrt{1+m^2}} \\
& (-h+k)^{1+\sqrt{1+m^2}}-(h+k)^{3-\sqrt{1+m^2}}(-h+k)^{-2-\sqrt{1+m^2}}-(-h+k)^2 \\
& (h+k)^{-1-\sqrt{1+m^2}}+(h+k)^2(-h+k)^{-1-2\sqrt{1+m^2}}\Big)\Big], \\
c_{86} &= \frac{1}{2(h+k)}\Big[-4hk\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{-1+\sqrt{1+m^2}}-(-h+k)^{-2+\sqrt{1+m^2}} \\
& (h+k))-2\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{1+\sqrt{1+m^2}}-(-h+k)^{1+\sqrt{1+m^2}} \\
& -(h+k)^{2+\sqrt{1+m^2}}(-h+k)^{-1}+(h+k)(-h+k)^{\sqrt{1+m^2}}\Big)+2\Big((-h+k)^{1+\sqrt{1+m^2}} \\
& -(h+k)^3(-h+k)^{-2+\sqrt{1+m^2}}-(-h+k)^2(h+k)^{-1+\sqrt{1+m^2}}+(-h+k)^{-1} \\
& (h+k)^{2+\sqrt{1+m^2}}\Big)\Big], \\
c_{87} &= \frac{1}{2(h+k)}\Big[4hk\Big(-1+2\sqrt{1+m^2}\Big)\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{-1+3\sqrt{1+m^2}} \\
& -(-h+k)^{-2+3\sqrt{1+m^2}}(h+k)\Big)-2\Big(1-\sqrt{1+m^2}\Big)\Big((h+k)^{1+3\sqrt{1+m^2}}
\end{aligned}
$$

$$
\begin{aligned}
& - (h+k)^{2\sqrt{1+m^2}} (-h+k)^{1+\sqrt{1+m^2}} - (h+k)^{2+\sqrt{1+m^2}} (-h+k)^{-1+2\sqrt{1+m^2}} \\
& + (h+k)(-h+k)^{3\sqrt{1+m^2}}\Big) - 2\left(-1+2\sqrt{1+m^2}\right)\Big((h+k)^{2\sqrt{1+m^2}} \\
& (-h+k)^{1+\sqrt{1+m^2}} - (h+k)^3(-h+k)^{-2+3\sqrt{1+m^2}} - (-h+k)^2 \\
& (h+k)^{-1+3\sqrt{1+m^2}} + (h+k)^{2+\sqrt{1+m^2}}(-h+k)^{-1+2\sqrt{1+m^2}}\Big)\Big],
\end{aligned}
$$

$$
\begin{aligned}
c_{88} &= \frac{1}{2(h+k)}\Big[4hk\left(-3+4\sqrt{1+m^2}\right)\left(1-\sqrt{1+m^2}\right)\Big((h+k)^{-3+5\sqrt{1+m^2}} \\
& - (-h+k)^{-4+5\sqrt{1+m^2}}(h+k)\Big) - 2\left(1-\sqrt{1+m^2}\right)\Big((h+k)^{-1+5\sqrt{1+m^2}} \\
& - (h+k)^{-2+4\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}} - (h+k)^{2+\sqrt{1+m^2}}(-h+k)^{-3+4\sqrt{1+m^2}} \\
& + (h+k)(-h+k)^{-2+5\sqrt{1+m^2}}\Big) - 2\left(-3+4\sqrt{1+m^2}\right)\Big((h+k)^{-2+4\sqrt{1+m^2}} \\
& (-h+k)^{1+\sqrt{1+m^2}} - (h+k)^3(-h+k)^{-3+5\sqrt{1+m^2}} - (-h+k)^2 \\
& (h+k)^{-3+5\sqrt{1+m^2}} + (h+k)^{2+\sqrt{1+m^2}}(-h+k)^{-3+4\sqrt{1+m^2}}\Big)\Big],
\end{aligned}
$$

$$
\begin{aligned}
c_{89} &= \frac{1}{2(h+k)}\Big[4hk\left(1-\sqrt{1+m^2}\right)^2(\ln(h+k)(h+k) - \ln(-h+k)(h+k)) \\
& -2\left(1-\sqrt{1+m^2}\right)\Big(\ln(h+k)(h+k)^3 - \ln(h+k)(h+k)^{-2-\sqrt{1+m^2}} \\
& (-h+k)^{1+\sqrt{1+m^2}} - \ln(-h+k)(h+k)^{2+\sqrt{1+m^2}}(-h+k)^{1-\sqrt{1+m^2}} \\
& + \ln(-h+k)(h+k)(-h+k)^2\Big) - 2\Big(\Big((-h+k)^{1+\sqrt{1+m^2}}(h+k)^{2-\sqrt{1+m^2}} \\
& - (h+k)^3 - (-h+k)^2(h+k) + (h+k)^{2+\sqrt{1+m^2}}(-h+k)^{1-\sqrt{1+m^2}}\Big) \\
& + \left(1-\sqrt{1+m^2}\right)\Big(\ln(h+k)(h+k)^{2-\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}} \\
& - \ln(-h+k)(h+k)^3 - \ln(h+k)(-h+k)^2(h+k) + \ln(-h+k) \\
& (h+k)^{2+\sqrt{1+m^2}}(-h+k)^{1-\sqrt{1+m^2}}\Big)\Big)\Big],
\end{aligned}
$$

$$
\begin{aligned}
c_{90} &= \frac{1}{2(h+k)}\Big[4hk\left(1-\sqrt{1+m^2}\right)\Big(\Big((h+k)^{1+2\sqrt{1+m^2}} - (-h+k)^{2\sqrt{1+m^2}} \\
& (h+k)\Big) + \left(1-\sqrt{1+m^2}\right)\Big(\ln(h+k)(h+k)^{1+2\sqrt{1+m^2}} - \ln(-h+k) \\
& (h+k)(-h+k)^{2\sqrt{1+m^2}}\Big)\Big) - 2\left(1-\sqrt{1+m^2}\right)\Big((h+k)^{3+2\sqrt{1+m^2}} \\
& \ln(h+k) - \ln(h+k)(h+k)^{2+\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}} - (h+k)^{2+\sqrt{1+m^2}} \\
& \ln(-h+k)(-h+k)^{1+\sqrt{1+m^2}} + \ln(-h+k)(h+k)(-h+k)^{-2+2\sqrt{1+m^2}}\Big) \\
& -2\Big(\Big((-h+k)^{1+\sqrt{1+m^2}}(h+k)^{2+\sqrt{1+m^2}} - (h+k)^3(-h+k)^{2\sqrt{1+m^2}} \\
& - (h+k)^{1+2\sqrt{1+m^2}} + (h+k)^{2+\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}} - (-h+k)^2\Big)
\end{aligned}
$$

$$
\begin{aligned}
& -2\left(1+\sqrt{1+m^2}\right)\left(\ln(h+k)(h+k)^{2+\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}}\right. \\
& -\ln(-h+k)(h+k)^3(-h+k)^{2\sqrt{1+m^2}}-\ln(h+k)(h+k)^{1+2\sqrt{1+m^2}} \\
& \left.\left.(-h+k)^2+\ln(-h+k)(h+k)^{2+\sqrt{1+m^2}}(-h+k)^{1+\sqrt{1+m^2}}\right)\right)\Big], \\
c_{91} \;=\; & \left(1+\sqrt{1+m^2}\right)\left((-h+k)^{1+\sqrt{1+m^2}}(h+k)^{2\sqrt{1+m^2}}-(h+k)^{1+\sqrt{1+m^2}}\right. \\
& \left.(-h+k)^{2\sqrt{1+m^2}}\right).
\end{aligned}
$$

3.4 Results and discussion

Our main interest in this section is to examine the pressure rise per wave length, pressure gradient and stream function for the influence of various patient parameters i.e. Γ, ε, k and m.

Fig. 3.2(a) represents the variation of ΔP for various values of Γ. It is noted that ΔP is increases with increasing Γ in the pumping region, while in copumping region the pressure decreases by increases Γ. Fig. 3.2(b) depicts that ΔP is decreases with increases ε in the pumping region. Fig. 3.2(c) shows the effect of k on ΔP. It is found that the pressure rise decreases by increasing k in the pumping region where as the effect is quite opposite in the copumping region. The effect of m on ΔP can be seen from Fig. 3.2(d), here it is revealed from this figure that an increase in the m leads to an increase in pressure rise in the pumping region and a decrease in pressure rise in the copumping region.

Figs. 3.3(a) to 3.3(d) provide the variation of the pressure gradient with respect to the x-axis which it has oscillatory in the whole range of the x-axis for different values of ε, Γ, k and m. It is clear from these figures that the pressure gradient increases with an increase ε, Γ and k while it decreases with increasing of m.

The formulation of an internally circulating of fluid by closed stream lines is called trapping and this is pushed along with the peristaltic wave. Fig. 3.4(a) illustrates the effect of the Γ on the stream line. The trapped bolus does not exist for $\Gamma = 0$, it appears when $\Gamma \neq 0$ and its size increases with increasing Γ. The stream lines for different ε are shown in Fig. 3.4(b). The similar effect shown as we seen in Fig. 3.4(a). Fig. 3.4(c) depicts the effects of k on the trapping. In this case it is seen that the size of trapped bolus increases with increasing k. To see the effects of m on the stream lines, we have sketched Fig. 3.4(d). It reveals that an increasing in m results in the increase in the size of the trapped bolus.

3.5 Conclusion

In this chapter, we have studied a mathematical model to investigate the peristaltic transport of a magnetohydrodynamic (MHD) fourth grade fluid in a curved channel. The main finding have been listed as below:

- Increase in Γ leads to increase in the magnitude of $\frac{dp}{dx}$ and ΔP.
- The increase in ε reduces the pressure gradient.
- Free pumping flux almost remains invariant for change in Γ, ε, k and m.
- The size of the trapped bolus increases with an increase in Γ, ε, k and m.
- The results for the case of planar channel can be recovered by choosing $k \rightarrow \infty$.

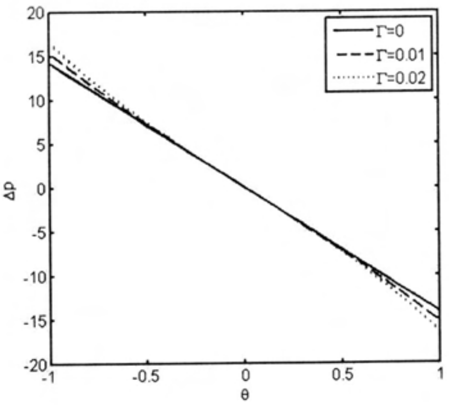

Fig. 3.2:(a) Effect of Γ on ΔP when $\varepsilon = 0.01$, $k = 1.1$, $m = 0.8$.

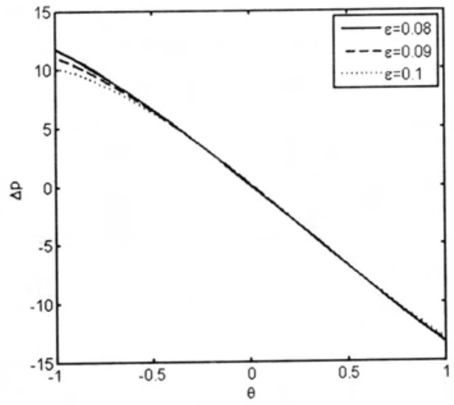

Fig. 3.2:(b) Effect of ε on ΔP when $\Gamma = 0.01$, $k = 1.1$, $m = 0.7$.

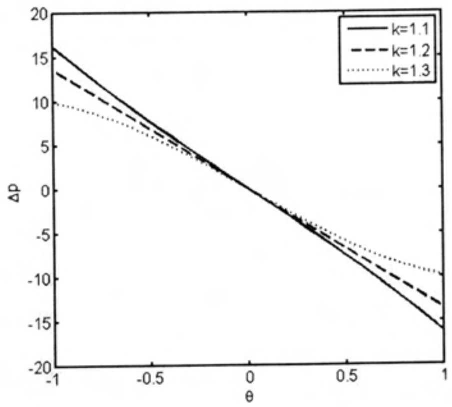

Fig. 3.2:(c) Effect of k on ΔP when $\varepsilon = 0.01$, $\Gamma = 0.01$, $m = 0.9$.

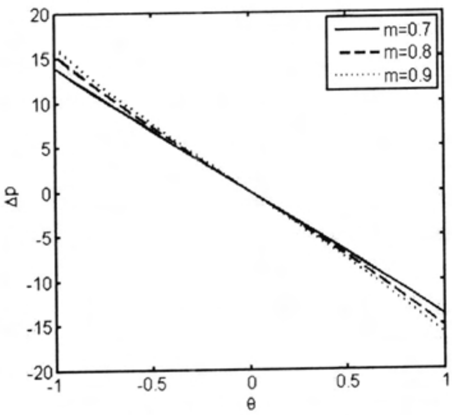

Fig. 3.2:(d) Effect of m on ΔP when $\varepsilon = 0.01$, $k = 1.1$, $\Gamma = 0.01$.

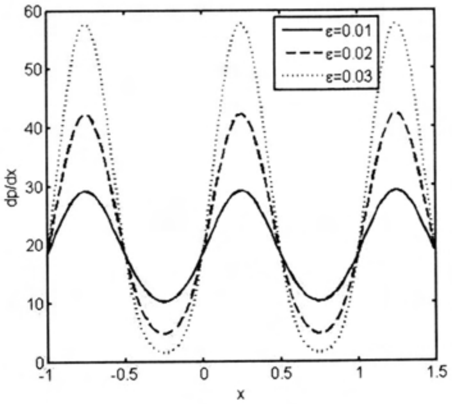

Fig. 3.3:(a) Effect of ε on $\frac{dp}{dx}$ when $k=5$, $\theta=0.1$, $m=0.7$, $\Gamma=0.01$.

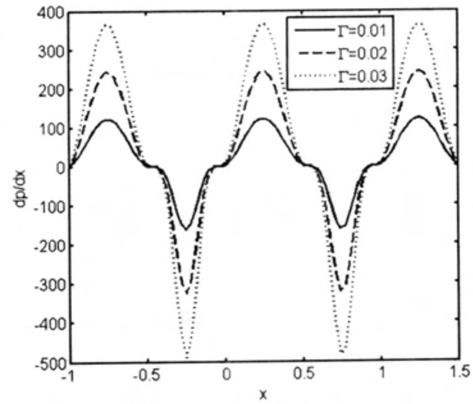

Fig. 3.3:(b) Effect of Γ on $\frac{dp}{dx}$ when $k=5$, $m=0.7$, $\theta=0.05$, $\varepsilon=0.1$.

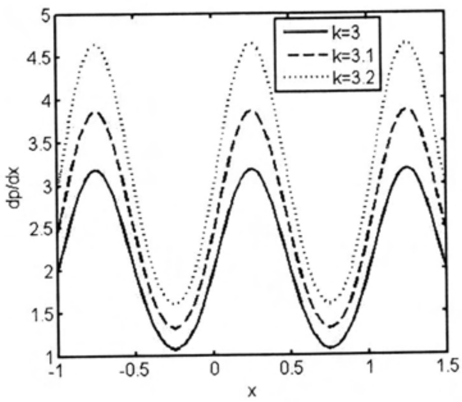

Fig. 3.3:(c) Effect of k on $\frac{dp}{dx}$ when $m=0.6$, $\Gamma=0.01$, $\varepsilon=0.01$, $\theta=0.1$.

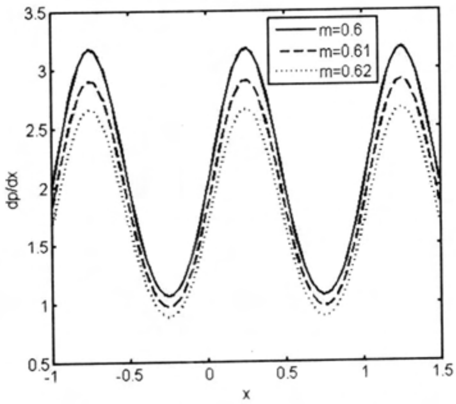

Fig. 3.3:(d) Effect of m on $\frac{dp}{dx}$ when $\theta=0.1$, $k=3$, $\Gamma=0.01$, $\varepsilon=0.01$.

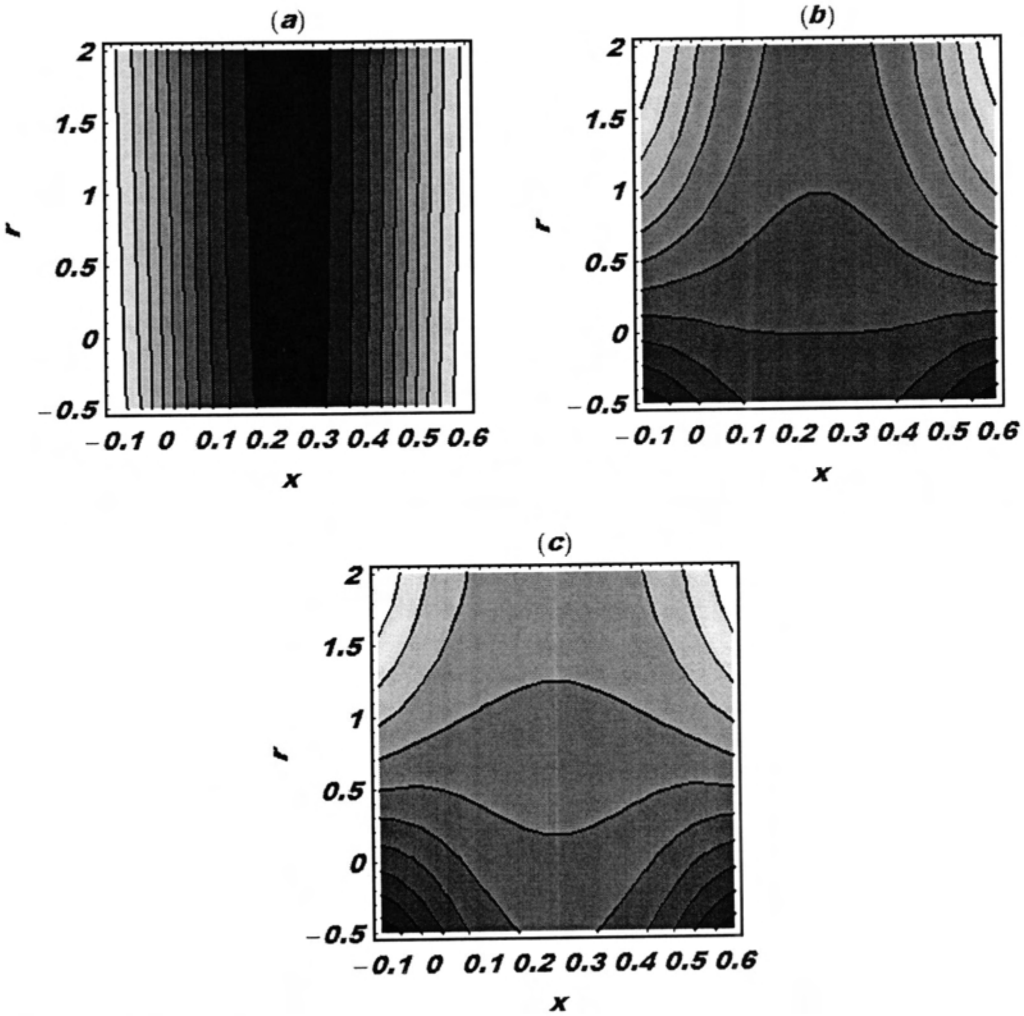

Fig. 3.4 : (a) Streamlines for $\varepsilon = 0.3$, $k = 2.5$, $m = 0.3$, $\theta = 1$, with different Γ : (a) $\Gamma = 0.00$, (b) $\Gamma = 0.001$, (c) $\Gamma = 0.005$.

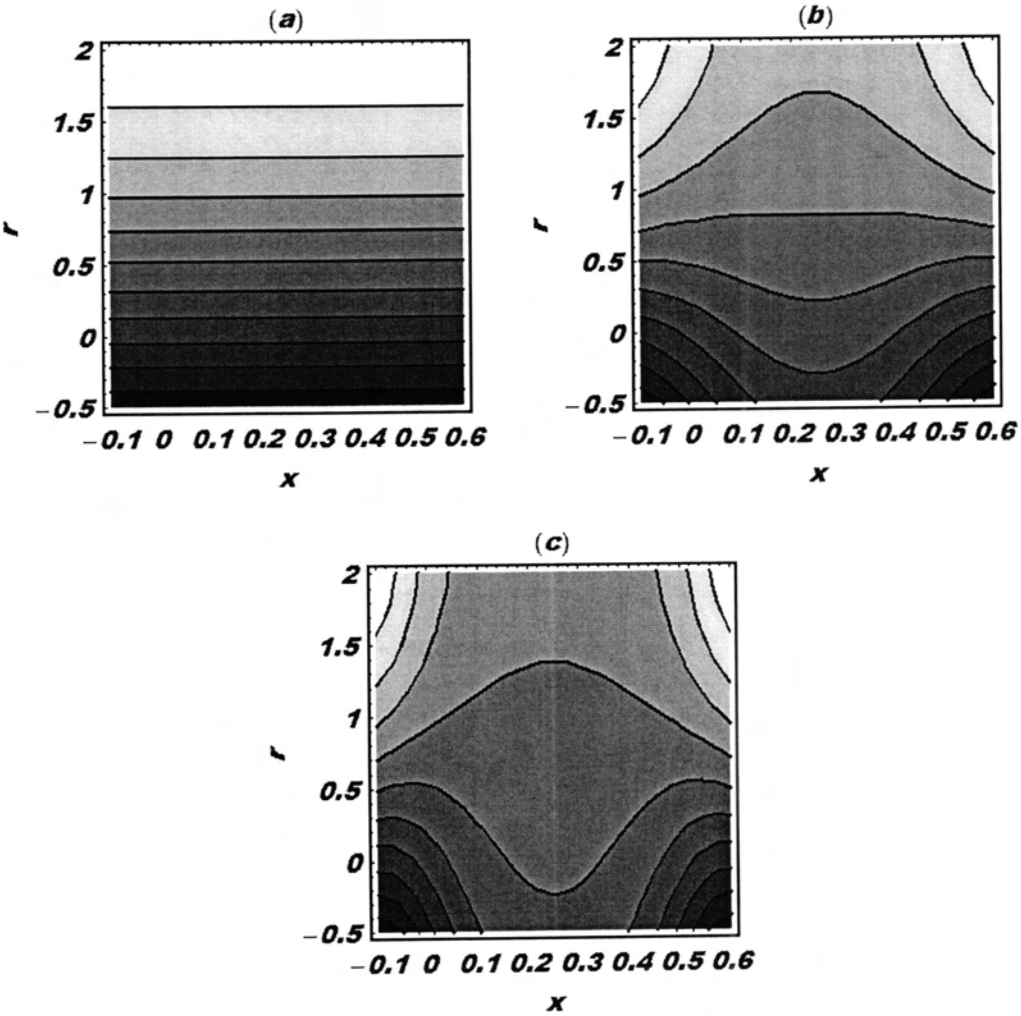

Fig. 3.4 : (b) Streamlines for $k = 2.5$, $m = 0.3$, $\theta = 1$, $\Gamma = 0.01$, with different ε : (a) $\varepsilon = 0$, (b) $\varepsilon = 0.2$, (c) $\varepsilon = 0.4$.

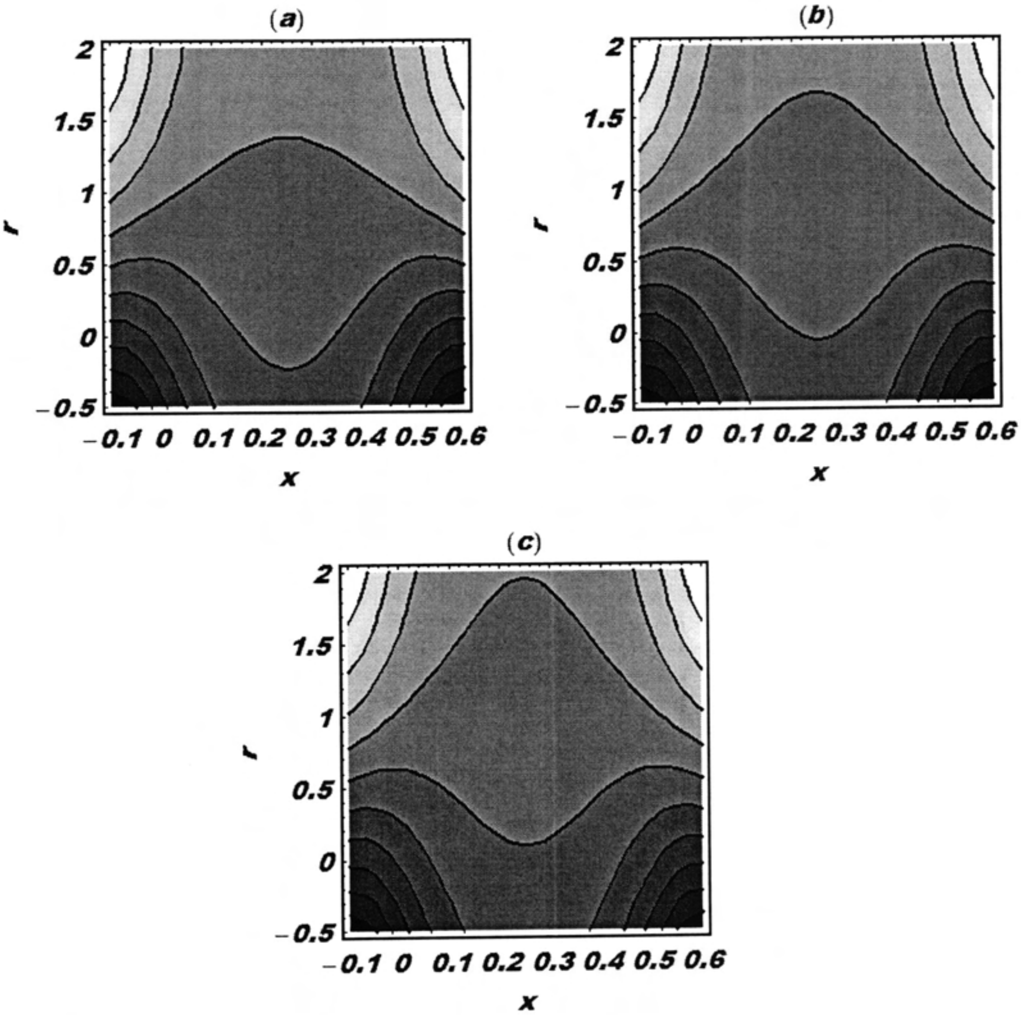

Fig. 3.4 : (c) Streamlines for $\varepsilon = 0.4$, $\Gamma = 0.01$, $m = 0.3$, $\theta = 1$, with different k : (a) $k = 2.5$, (b) $k = 2.7$, (c) $k = 2.9$.

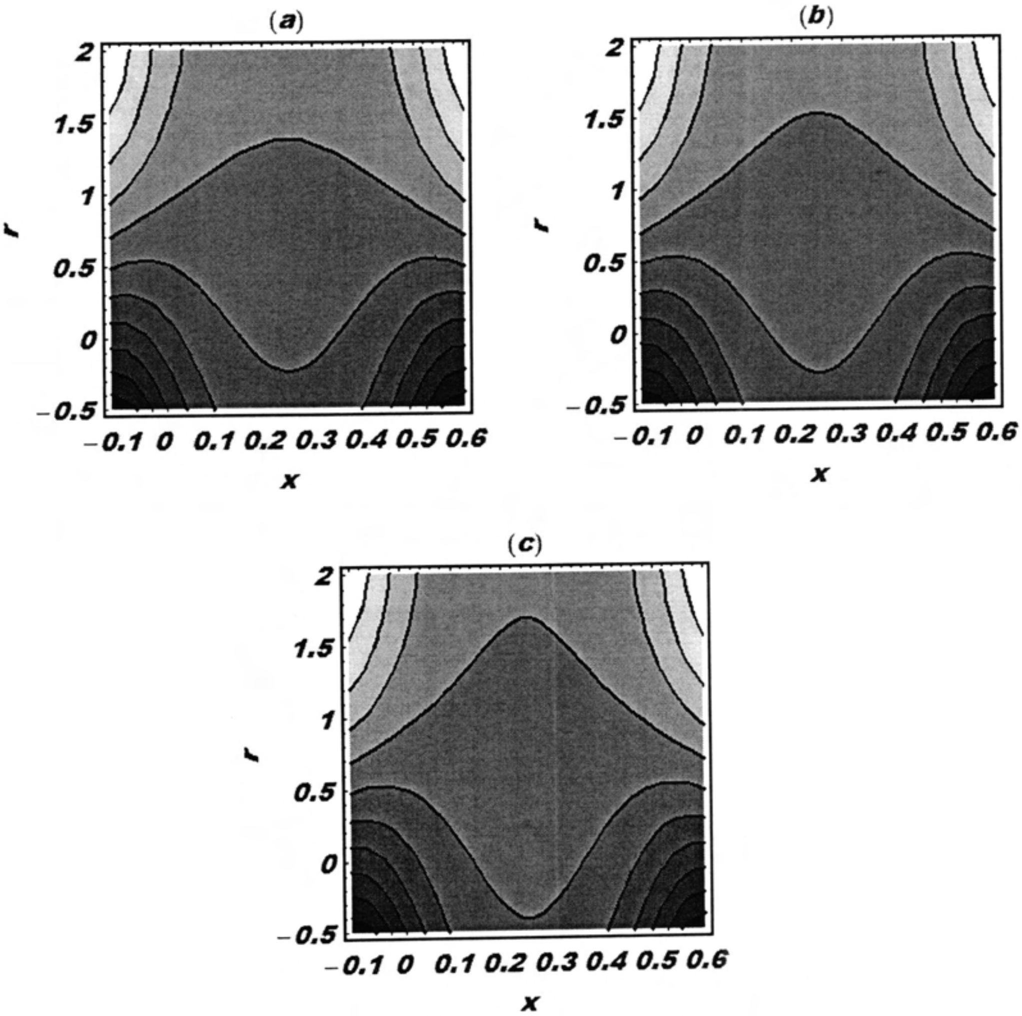

Fig. 3.4 : (d) Streamlines for $\varepsilon = 0.4$, $\Gamma = 0.01$, $k = 2.5$, $\theta = 1$, with different m : (a) $m = 0.3$, (b) $m = 0.35$, (c) $m = 0.38$.

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