EFFECT OF POLYA'S PROBLEM SOLVING METHOD OF TEACHING ON ACHIEVEMENT OF REVISED BLOOM'S TAXONOMY IN MATHEMATICS AT ELEMENTARY LEVEL

By

SHEIKH TARIQ MEHMOOD

(82-FSS/PHDEDU/F11)

DEPARTMENT OF EDUCATION

FACULTY OF SOCIAL SCIENCES

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By

Researcher:

Sheikh Tariq Mehmood

Reg. No. 82-FSS/PHDEDU/F11

Supervisor:

Prof. Dr. N. B. Jumani

Co-Supervisor:

Dr. Irshad Ahmed Arshad

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SHEIKH TARIQ MEHMOOD

82-FSS/PHDEDU/F11

A thesis submitted in partial fulfillment of the requirements for the Degree of

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FORWARDING SHEET

This thesis entitled, " Effect of Polya's Problem Solving Method of Teaching on Achievement of Revised Bloom's Taxonomy in Mathematics at Elementary Level" submitted by Sheikh Tariq Mehmood in partial fulfillment of Doctor of Philosophy in Education has been completed under my guidance and supervision. I am satisfied with the quality of student's research work and allow him to submit this thesis for further process as per rules and regulations.

Date: 27/5/15

Signature: ----------
Supervisor Name: Prof. Dr. N. B. Jumani

Signature: ----------
Co-Supervisor: Dr. Irshad Ahmed Arshad

APPROVAL SHEET

Thesis titled " **EFFECT OF POLYA'S PROBLEM SOLVING METHOD OF TEACHING ON ACHIEVEMENT OF REVISED BLOOM'S TAXONOMY IN MATHEMATICS AT ELEMENTARY LEVEL**" submitted by Sheikh Tariq Mehmood, Registration No. 82-FSS/PHDEDU/F11 in partial fulfillment for the requirement of PhD degree in Education with specialization in Teacher Education is accepted by the Department of Education, Faculty of Social Sciences, International Islamic University Islamabad for the award of Ph.D degree in Education.

Viva Voce Committee:

Supervisor: _____
(Prof. Dr. N.B. Jumani)

Co-Supervisor: _____
(Dr. Irshad Ahmed Arshad)

Internal Examiner : _____
(Dr. SaminaYasmeen Malik)

External Examiner I: _____
(Dr. Wasim Qazi)

External Examiner II: _____
(Dr. Ayesha Akbar)

Dated: 29/04/2015

Chairman _____
Department of Education,
International Islamic University,
Islamabad

Dean _____
Faculty of Social Sciences,
International Islamic University,
Islamabad

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Sheikh Tariq Mehmood

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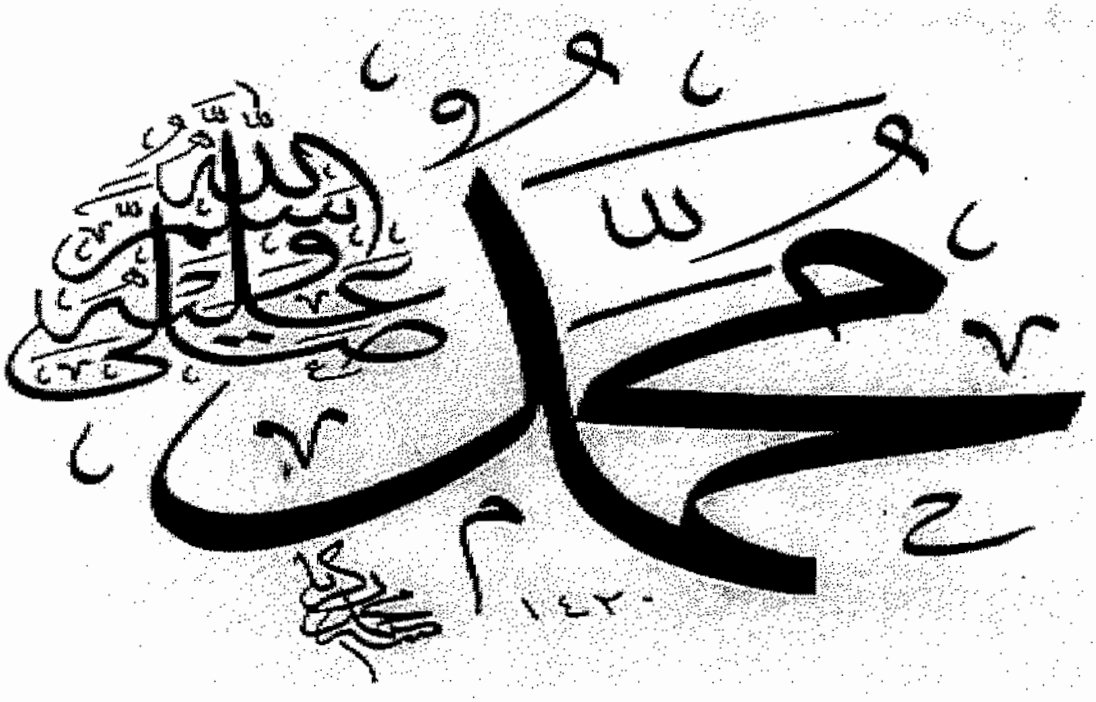


ABSTRACT

In Pakistan, mathematics is taught as one of the compulsory subject in all public and private schools from primary to secondary level. But unfortunately, this subject is not of much interest for the Pakistani students studying in public and private schools; it is rather a nightmare for them. There may be many reasons for this effect but among one of them is the teaching method. Problem solving method is a source of developing problem solving ability, through which students could be able to solve daily life problems. Most of the mathematicians were inspired by the classical work of Polya (1981) and Dewey (1933). Problem Solving Method is helpful in developing Higher Order Thinking Skills (HOTS). According to Anderson (2001), to measure these skills, the Revised Bloom's Taxonomy is the best evaluation source. The intent of this true experimental study was to find out the effect of George Polya's problem solving method of teaching on revised Bloom's Taxonomy of Educational Objectives, in the subject of mathematics at elementary level. The objectives of the study were i) To find out the effect of problem solving method on conceptual knowledge sub-level remembering. ii) To evaluate the effect of problem solving method on conceptual knowledge sub-level understanding. iii) To check the effect of problem solving method on conceptual knowledge sub-level applying. iv) To determine the effect of problem solving method on conceptual knowledge sub-level analyzing. v) To find out the effect of problem solving method on conceptual knowledge sub-level evaluating.vi) To check the effect of problem solving method on conceptual knowledge sub-level creating. vii)To find out the effect of problem solving method on Higher Order Thinking skills of Revised Bloom's Taxonomy. To achieve the objectives, eight research questions were developed. To verify the objectives, eight null hypotheses were developed.

True experimental pretest-posttest (double control group) design was used. An experiment was performed in a school situated in urban area of Islamabad. Multistage sampling technique used for the selection of school. 132 8^{th} grade students were used in the experiment. Pre-test which consisted of 60 items of six cognitive processes was developed. Validity of the test was checked by the experts and reliability was checked through Alpha reliability analysis which was $\alpha = .89$. On the basis of pre-test scores, three groups were formed through proportionate random sampling. The Experimental Group was taught by the researcher through Problem Solving Method, and other two groups were taught through Conventional Method by the same school teachers. All possible efforts were made to control the internal and external threats. Forty lessons were taught during the 8-week experiment to all three groups. Post-test was developed by changing the order of the questions of pre-test. Post-test was administered and marked according to the rubric design for marking. Pre-test and post-test scores were analyzed on SPSS. One Way ANOVA and t-test were applied on the data. In the light of the analyses of data it was found that by teaching through Problem Solving Method, students performed better on Revised Bloom Taxonomy as compared to Conventional method. It was concluded on the bases of findings that Problem Solving Method showed significant improvement at all levels of knowledge dimension and hence all null hypotheses were not accepted. It was also concluded that Problem Solving Method works better than Conventional Method for teaching of Mathematics. In the light of these conclusions, Problem Solving Method was recommended for teaching of Mathematics at Elementary level and also suggested to add it in teachers training programmes as well.

TABLE OF CONTENTS

LIST OF TABLES

CHAPTER 4 PRESENTATION ANDANALYSIS OF DATA

LIST OF FIGURES

LIST OF ABBREVIATIONS

AAMT	Australian Association of Mathematics Teachers
ANOVA	Analyses of Variance
df	Degree of Freedom
FDE	Federal Directorate of Education
FG	Federal Government
HOTS	Higher Order Thinking Skills
LOTS	Lower Order Thinking Skills
MoE	Ministry of Education
OECD	Organization for Economic Co-operation and Development
PIAASC	Programme for International Assessment Audit Competencies
PISA	Programme for International Students Assessment
PSM	Problem Solving Method
RBT	Revised Bloom's Taxonomy
SPSS	Statistical Package for Social Sciences
TALIS	Teaching and Learning International Survey

CHAPTER 1

INTRODUCTION

1.1 Introduction

Science and technology do essentially contribute to national development and progress. The base for science and technology development is Mathematics. Stepelman (2004) as cited by Andrews (2006) argued that among all the sciences cultivated by mankind, none had been more useful than Mathematics (Amirali & Halai, 2002; VanHattum, 2011; cited by Yaun, 2013).

Christian Wolff's mind theory as cited by Andrews (2006), states that mind has the mental powers or faculties, such as reason, memory, judgment, will, attention, observation, and the like, each of which functions as a separate entity that can be improved through exercise or use. This fact indicates that mental abilities can easily be developed through Mathematics especially in solving problems (Aravena and Caamaño, 2007; Betne, 2010; Ellis, 2011).

The aims of teaching of Mathematics are also framed according to the faculties of mind and societal needs. Mostly the aims are utilitarian aim, disciplinary aim, cultural aim, adjustment aim, social aim, international aim, vocational aim, educational preparation aim and development of mental power aim. In the light of

these aims, objectives are drawn such as knowledge and understanding objectives, skill objectives, application objectives, attitude objectives and appreciation and interest objectives (Startalk, 2009; Balki, 2010).

The objectives stated by the National Curriculum for Mathematics (2006) in Pakistan focus on creating the abilities among the students such as to communicate Mathematically, reason and analyze, think and act in positive ways, comprehend the key concepts, evaluate the effectiveness of using different strategies to address the same problem, use a variety of strategies to problem solving and to make Mathematical connections, discriminate between relevant and irrelevant attributes of a concept in selecting examples, integrate and to make sense of Mathematical concept and procedures and examine real life situations by reasoning Mathematically. This has the reflection of international objectives in the curriculum (Government of Pakistan, 2006, 2009).

In Pakistan, Mathematics is taught as one of the compulsory subjects in all public and private schools at elementary and secondary levels. The curriculum covers a wide range of topics and concepts related to subject as well as to daily life. But unfortunately, this subject is not of much interest for students studying in public and private schools but it is a nightmare in Pakistan. The curriculum of Mathematics covers a wide range of knowledge which are desired specific attitude, abilities, analytical and logical thinking and practices form the learner's part (e.g. Rojan, 2008; Government of Pakistan, 2009, Ellis,2011 cited by Amirali, 2011).

It was observed that after completing ten years of education in Mathematics students fail to make connection with the subject in their life. There may be many

reasons for this effect, but one of them is the way of teaching cited by Amirali (2011) in his analysis stated about exploring the learning problems of the students in Mathematics that most of the teachers transfer the knowledge to the students according to the textbook through 'rote memorization' and traditional ways, to asses' students' learning through lower level of cognition. In Pakistan, Mathematics learning consists mainly of rules for solution, memorization, and solution of textbook problems. Surface information/knowledge is provided to the students without in-depth understanding of the subject (Halai, 2008; VanHattum, 2011; cited by Yaun, 2013).

According to the reports of OECD, (2014) and PIAS, (2012) that in the present world of knowledge, almost every country is trying to improve the quality of society through the quality of education introducing appropriate measures in curriculum, school systems, assessment and teachers' training. Likewise, Pakistan is also the part of these activities, the Ministry of Education and its supporting sister institution are working for the improvement of quality education. The new Mathematics curriculum 2006 was designed in accordance with international standards which promote conceptual understanding and logic among the students and make them able to use all that in practical life.(Government of Pakistan, 2009; Ellis, 2011 cited by Amirali, 2011).

According to the analysis presented by Amirali (2011) about the status of teaching of Mathematics, despite the improvement in curriculum of Mathematics, the in-depth learning is not usually the focal point of all the classroom activities and interactions in Pakistan. Overview of the research studies eludes a gap between

curricular goals and what actually is happening in reality in the classrooms in Pakistan (Aravena and Caamaño, 2007; Betne, 2010; Ellis, 2011).

For the empirical evidence a number of studies conducted on national, provincial and district level in Pakistan endorsed the fact that achievement of students in the subject of Mathematics is very poor. These studies have been consistent in justifying the fact of low level achievement in the subject. The studies indicated many reasons for this low achievement and one of them is the teaching method of the teacher (e.g. Academy of Planning and Management, 1999; Benoliel & Miske, 1999; Government of Pakistan, 1999, Government of Baluchistan, 1999; Government of Sindh, 2000; Abdeen & Jone, 2000; Samo,2009 cited by Amirali, 2011).

For the effective achievement of all these objectives mentioned in the National Curriculum (2006), supportive pedagogy is required. These could not be achieved by the traditional method of teaching. This fact is evident from Sadia (2010), investigation of Mathematics achievement of middle grade students in Pakistan. It was found that students could only solve or attempt simple routine item requiring simple Mathematical skills and showed poor performance in items that required reasoning.

According to National Research Council's cited by Zaman (2011), much of the failure in the subject of Mathematics at school level is due to the traditional ways of teaching of Mathematics that is inappropriate to the way most students learn. Several scientific studies have shown the ineffectiveness of the traditional method of teaching. In order to make students knowledgeable, teachers have to equip themselves

with new teaching methods. Learning is not limited within the four walls of the classroom. Being resourceful is a big help to update what has previously been learnt.

In the words of Yaun (2013), "The most basic concept of all is that math is merely a problem solving technique. If students can learn to see math problems as just a formal, codified version of any other kind of problem, perhaps their phobias will disappear. But how can a math instructor get students to make this connection? To answer that question I turned to a Hungarian Mathematician himself obsessed with teaching problem solving".

According to Schoenfeld (2001) cited by Tutkun & Okay (2012), Problem solving is a method through which students encounter a problem for which they are not ready to solve it immediately. Then they do efforts and examine carefully, work step by step and then find a solution by themselves. In this process the students read the problem carefully, analyze it and collect all the possible information regarding the problem, try to find out the relationship between known and unknown, and then devise a plan keeping in view his/her Mathematical knowledge. After these steps starts implementation and finally he/she becomes able to solve similar examples (Tutkun & Okay, 2012).

Problem solving method is a source of developing problem solving ability, through which students could be able to solve daily life problems. It is also a source of developing in-depth knowledge about the subject. According to Lester & Kehle (2003), cited by Yaun (2013), most of the Mathematicians were inspired by the classical work of Polya (1981) and Dewey (1933). Teaching of Mathematics through problems solving method (PSM) is a source of making the students independent

discoverers or learners. PSM is helpful in developing Higher Order Thinking (HOT). Many research studies (Paris, 1991, Schoenfeld, 1992, Marzano, 2000, Weber, 2008, Anthony, 2010, Caballero, 2011) cited by Walshaw (2012), argued that problem solving method was an effective method of teaching for Mathematics.

Different studies have opted different strategies of inquiry like qualitative and quantitative e.g. Blnco, 2000, Riasat, 2011, Zaman, 2011, Mustafa, 2011 carried out the experimental studies, Herreid, 2003, Roh, 2003, Tick, 2007 undertook the survey studies, while Schoenfeld, 1992, Yager, 2003, Walker, 2007 worked on the qualitative studies to find out the effect of problem solving method of teaching Mathematics on Mathematics (Walshaw, 2012).

George Polya was known as a great Mathematician due to his work on problem solving. He was Hungarian and taught at the Swiss Federal Institute of technology Zurich. He also served the Stanford University. He was respected and considered an authority in Mathematics' pedagogy. He was born in Budapest, Hungary on 13th December, 1887.

George Polya is known as the father of the modern focus on problem solving in Mathematics education. He was a renowned Mathematician and had written many books on the subject of Mathematics but "How to solve" was his famous book. In that book Polya suggested the four step method of solving a problem. According to Schoenfeld, 1992; Marzano, 2000; Weber, 2008, that this method of teaching gained a lot of currency. This method is the most used method to develop the skill of problem solving. This great Mathematician died on 7th September, 1985 in California, USA (Yaun, 2013).

According to Polya (1976), "Problem solving is a method to develop the ability of Mathematical problem solving. It enables the students to become independent discoverer and by this method they can solve daily life Mathematical problems. It helps in developing higher level abilities like analysis, synthesis, evaluation and creativity."

It is a four step method which works systematically to reach the solution of a Mathematical problem.

Step-1 Understanding the problem

In this step, the given problem is understood with respect to given data. For understanding several questions, figures and diagrams may be asked and constructed. All these quires depend upon the nature of the problem (Polya, 1976).

Step-2 Devising a Plan

At this step, students are motivated to find out links between data given and the unknown. This stage provides deeper understanding about the problem. For this, students are asked about a similar nature problem. If students know about a similar problem, they are asked to recall and solve that one. The way from understanding the problem to conceiving a plan may be long and difficult. The plan for a problem can be prepared through a 'bright idea' or an 'auxiliary problem' (Polya, 1976).

Step-3 Carrying out the Plan

After a careful planning at the step-2 what has been decided is now implemented for reaching a solution. To devise a plan and to conceive the idea for the solution is not an easy task. It takes a long exercise. Contrary to this, carrying out is

easy. The plan provides the general outlines for implementation and we have to put data into it in order to find out either it is working or not (Polya, 1976).

If the students develop the plan to solve the problem, it is good for the teacher because much of work has been done now. The major danger at this stage is that students may forget the plan, but it happens when students borrow it. If they develop themselves, it gives them satisfaction (Polya, 1976).

Step-4 Looking back

This is the step where students have to confirm their solution by applying it in a new situation. In this step, students seek new arguments and try to recheck their findings by comparing the known with unknown. By looking back means reconsidering, re-examining the results and the process of solution for the consolidation of their knowledge and to develop the ability to solve such examples independently in their daily life (Polya, 1976)

Teacher has to explain that this solution of the problem is not the destination, but it is merely a milestone which is travelled (Polya 1976).

There is a paradigm shift from teaching problem solving to teaching through problem solving (Taplin, 2005; Lester, Msainglla, Mau Landin, 1994 cited by Kolawole & Popoola, 2011). By problem solving method students can construct a deep understanding of conjecturing, exploring, testing, analyzing, synthesizing, evaluating and creating (Schoenfeld, 1994). The problem solving method is an important teaching tool in Mathematics because it is the source through which functional, logical and aesthetic objectives of teaching of Mathematics can be achieved (Schoenfeld, 1994). According to Polya (1980), "If education fails to

contribute to development of the intelligence, it is obviously incomplete. Yet intelligence is essentially the ability to solve problems: everyday problems, and personal problems."

Many research studies have proved the effectiveness of problem solving method on different faculties of mind (Polya, 1980; Thompson, 1985; Carpenter, 1985; Groves, 1985; Schoenfeld, 1995; Stigler, 2003; Betne, 2010 cited by D'Agostino, 2011). Following are the few faculties of mind on which this method works very effectively:

- Cognition
- Meta-cognition
- Knowledge
- Critical thinking
- Creative thinking
- Synthesis
- Analysis

Although Polya's problem solving method helps in developing many mental abilities, but these abilities may be presented in learning outcomes. The taxonomy of educational objectives is a frame work through which we can get the results of instruction from the students. Hence to confirm the effectiveness of the Polya's problem solving method, it is necessary that it may be checked against some standard taxonomy (Aravena and Caamaño, 2007; Betne, 2010; Ellis, 2011).

1.2 Bloom's Taxonomy of Educational Objectives

Benjamin S. Bloom, Associate Director of the Board of Examination of the University of Chicago and his team were assigned the job to create item banks for the different universities to measure the same educational objectives. For this assignment, many eminent measurement specialists were included across the United States. The whole team under the leadership of Bloom met twice a year in 1949. The work was completed and published in 1956, the final draft was titled "Taxonomy of Educational Objective: The Classification of Educational Goals. Handbook 1: "Cognitive Domain" (Bloom, Engelhart, Furst, Hill, & Krathwohl, 1956 cited by Tutkun & Okay, 2012).

According to Kolawole & Popoola (2011), this taxonomy served as a measurement tool and a common language for describing educational objectives for the subjects and learning outcomes. Bloom's taxonomy was developed for the following purposes:

- It can be used as a source of communication among the subject matter, teachers and evaluators.
- It can be used for determining the aims of the curriculum according to national, state, and local standards.
- It can be used for measurement and evaluation of the course and program.
- On the basis of it, range of educational possibilities can be determined.

The Bloom's taxonomy consisted of six levels of cognitive domain. Every domain was clearly divided into its sub levels. The order of proceeding was based on simple to complex and from concrete to abstract. It is presented in a complete

hierarchy; each category was prerequisite for the next category (Kolawole & Popoola, 2011).

1.2.1 Rational for Revised Bloom's Taxonomy

In the 21st century, Psychological and educational researchers have developed many learning theories and approaches. According to Zimmernan cited by Amer (2006), among these new theories of learning are the constructivism, Meta-cognition and Self structured learning. In the light of these theories, learning is considered now as "a proactive activity" which required self-initiated motivation and behavioral processes along with Meta-cognition.

In this present era of knowledge, educators have more knowledge about the processes of learning and its evaluation; teachers have new methods of teaching (Startalk, 2009, cited by Tutkun & Okay, 2012). As stated by Tutkus (2012), "The Bloom's taxonomy was designed to classify objectives of curriculum in terms of implicit and explicit cognitive skills and abilities. Taxonomy is accepted as one of the important studies that affect the curriculum in 21st century. For instance, a search engine shows more than 455,000 results for Bloom's taxonomy."

This taxonomy was so comprehensive that it survived for a very long time. Despite the various taxonomies, only Bloom's taxonomy is accepted (Forehand, 2005). But due to the advancement in the field of Psychology and Education, it was felt that this taxonomy required some revision to meet the diversified needs of the 21st century (Tutkun & Okay, 2012).The advancement in the field of assessment and evaluation also describes some limitations of the Bloom's taxonomy; few of them are as follows:

1.2.2 Emerging demands of new learning theories

According to Marzona (2008), who is the author of his own taxonomy "Marzona Taxonomy of Educational Objectives", it is not fit for the emerging demands of new learning theories. The hierarchy structure of the Bloom's taxonomy, moving from simplest level to difficult level of evaluation has no empirical evidences. This fact is not satisfied by the cognitive processes in Bloom's Taxonomy. The new theories of Constructivism, Meta-cognition and Self Structure Learning (SSL) required new learning domains which were not fulfilled by the old Bloom's Taxonomy. The theory of Constructivism focused on how students create knowledge while they are engaged in learning. Creation of new knowledge requires both comparison of new learning with old learning by using various cognitive processes for it. This framework changes the entire learning process and students are exposed to a new learning environment. This environment provides an opportunity to select knowledge according to their requirement. In the present scenario, it is expected that every learner may make progress on periodic bases. Due to this reason, combination of educational objectives and teaching evaluation is more vital than ever merged. Therefore, it was the need of time that it might be revised (Pickard, 2007).

1.3 Revised Bloom's Taxonomy

Anderson who was the Bloom's student, with a team of eminent cognitive psychologists revisited the Bloom's Taxonomy and provided its reyised version in 2001. Keeping in view the new developments in the field of education, students' learning ways, new assessment and evaluation methods and teachers' lesson planning,

the old taxonomy was revisited and the Revised Bloom's Taxonomy (RBT) was introduced (Anderson & Krathwohl, 2001).

Tutkun & Okay (2012) argued that in Revised Bloom's Taxonomy; significant changes were made to address the limitation of old Bloom's taxonomy. The revised Bloom's Taxonomy has three main areas of changes with respect to the old taxonomy. Through these changes, it became more comprehensive and relevant to modern learning theories (Aravena and Caamaño, 2007; Betne, 2010; Farzad, 2010; Ellis, 2011).

Following are the main areas of changes:

1-Changes in Terminologies

The revised version of Bloom's Taxonomy has changed noun to verb. The process of synthesis is replaced with creativity and it is kept at the highest level in the Revised Bloom's Taxonomy, because creativity is the result of combination of many parts of knowledge. According to Farzad (2010), Revised Bloom's Taxonomy (RBT) provided a very comprehensive view of all terms in verbs (Anderson & Krathwohl, 2012).

2-Structural Changes

According to the change with respect to structure between the old and the revised taxonomy, there is the difference of dimensions. The original Bloom's Taxonomy was one dimensional which was moving in a hierarchy from simple to complex, while the RBT is two dimensional. One dimension represents the 'knowledge dimension' while the other is 'the cognitive process dimension'. Hence, the RBT divided the noun and verb into two separate dimensions. The knowledge

dimension is divided into four parts which is 'noun aspect' and the learning outcome on the student's part is divided into six parts which is 'verb aspect' (Forehand, 2005; Amer, 2006 & Answer, 2012 cited by Tutkun & Okay, 2012).

3-Changes in Emphasis

There were three categories in old Bloom's Taxonomy which were (Factual, Conceptual and Procedural) presented with a cutline in RBT with a new addition of category 'Meta-cognitive knowledge'. It is the way how knowledge can be acquired by someone. Students can adjust their ways of learning and can come to know about their strengths and weaknesses. The RBT provided an authentic and reliable tool to the curriculum developer, educational planner, teachers for instructional delivery and evaluators for assessment and evaluation (Forehand, 2005 cited by Tutkun & Okay, 2012).

1.3.1 Cognitive Process Dimension of Revised Bloom Taxonomy

This dimension of the taxonomy is divided into six categories which are described in 'verb'. It represents the process of learning and students are expected to learn in the result of teaching (Anderson, 2001).

1-Remembering

In this dimension it is expected that students may recognize and recall the relevant information and knowledge from the long term memory. This dimension consists of two main sub classes i-e recognizing and recalling or it is an ability to remember the previous knowledge and recall it at the time when it is required (Anderson, 2001).

2-Understanding

This dimension of the cognitive process deals with the ability among the students that they can grasp meaning, explain and restate ideas, can provide their own meaning to the knowledge. It has its sub-levels which include interrelating, exemplifying, comparing and summarizing (Anderson, 2001).

3-Applying:

This dimension of cognitive process deals with the ability of using acquired knowledge in a similar or new situation. It shows its learning outcome in the form of executing and implementing in either new or old situation (Anderson & Krathwohl, 2012).

4-Analysing:

This is the cognitive process which deals with the division of knowledge into its parts and studies the parts to understand the whole. The learning outcomes of this ability are differentiating, organizing and attributing. It is also included in higher order thinking skill (Anderson & Krathwohl, 2012).

5- Evaluating:

This ability also belongs to higher order thinking skill. The learning outcome of this ability includes checking and critiquing (Anderson & Krathwohl, 2012).

6- Creating:

This dimension is new in Revised Bloom's Taxonomy; it was not included in the old taxonomy. This is the highest ability and replaced by the synthesis of the previous taxonomy. It involves putting knowledge together for the creation of new knowledge and developing something new. It includes generating, planning and

producing. The learning outcomes of this ability are based upon combining the pieces of knowledge for the creation of new ideas and things. For assessment of this ability students have to accomplish tasks like creating plan and produce new things (Anderson, 2001).

1.3.2 The Knowledge Dimension of Revised Bloom Taxonomy

In the two ways RBT the second dimension is of knowledge. It has four dimensions and every dimension represent its unique features. Mathematics is the very important subject for individual's development as well as for the society development. But the performance of students in this subject in Pakistan is not good. Among reasons one of them is the traditional way of teaching which develops Low Order thinking Skills (LOTS), while curriculum demands Higher Order Thinking Skills (HOTS). Polya's Problem Solving Method is a well-known method for the development of HOTS and it can be accessed through Revised Bloom's Taxonomy. So an effort was made through this study to find out the effect of this method on Revised Bloom's Taxonomy in Pakistan context (Anderson & Krathwohl, 2012).

1.4 Statement of the problem

Importance of Mathematics cannot be denied in the present era of science and technology OECD (2014). It is the subject, which has not only a relationship with other subjects but also a strong relationship with daily life as well. To achieve its real objects successful Mathematics education is subject to the interlinking of the elements like content, pedagogy, teacher and student interactions in a shared learning environment that encourages Mathematical sense making PISA (2012). The role of the Mathematics teacher and the method of teaching are very significant in learning

of this subject, but facts provide empirical evidences that Mathematics teachers often focus on drilling Mathematical formulas, developing computational algorithms, and symbol manipulation without stressing understanding as to why these things work. According to Amirali (2012), there is only surface learning which causes rote learning among students. The students cannot apply the subject knowledge in their real life and hence do not continue with the subject PISA (2012). A good teaching method is that method which addresses all the domains of learning. The subject of Mathematics provides it fruits, if all domains of learning are addressed like cognitive, affective and psychomotor. Traditional method of teaching which can be **operationally defined** as the, 'talk and chalk method'; in which teacher is active most of the time while students are passive or just keep themselves busy in noting. Conventional method of teaching works on Lower Order Thinking (LOTs).According to Farzad (2010), George Polya's Problem solving method is the method by which students can be made independent discoverers and through this method of teaching interest and attitude of learning can be developed. This method works on Higher Order Thinking (HOTs).Problem solving Method has an effect on cognition but few studies discussed its effect on revised Bloom's taxonomy of educational objectives. The intent of this true experimental study was to find out the effect of George Polya's problem solving method of teaching on revised Bloom's Taxonomy of Educational Objectives, in the subject of Mathematics at elementary level.

1.5 Objectives of the study

The objectives of the study were:

1. To find out the effect of problem solving method on conceptual knowledge sub-level remembering.
2. To evaluate the effect of problem solving method on conceptual knowledge dimension sub-level understanding.
3. To check the effect of problem solving method on conceptual knowledge dimension sub-level applying.
4. To determine the effect of problem solving method on conceptual knowledge dimension sub-level analyzing.
5. To find out the effect of problem solving method on conceptual knowledge dimension sub-level evaluating.
6. To check the effect of problem solving method on conceptual knowledge dimension sub-level creating.
7. To find out the effect of problem solving method on Higher Order Thinking skills of Revised Bloom's Taxonomy.
8. To check the difference of achievement of experimental Group with Control Groups through comparisons of Groups.

1.6 Research Questions

Following research questions were answered through this study:

1. Does problem solving method of teaching effect the conceptual knowledge dimension sub-level remembering?
2. Does problem solving method of teaching does develops the conceptual knowledge dimension sub-level understanding?

3. Does problem solving method of teaching develops the ability of applying sub-level of conceptual knowledge dimension?
4. Does problem solving method of teaching work on ability of Analyzing sub-level of conceptual knowledge dimension?
5. To what extent the problem solving method develops the ability of evaluating, sub-level of conceptual knowledge?
6. To what extent the problem solving method develops the ability creating, sub-level of conceptual knowledge?
7. Does problem solving method develop the Higher Order thinking skills?

1.7 Hypotheses of the study

Following null hypotheses for quantitative analyses were checked through the study:

Ho1: There is no significant effect of problem solving teaching method on the mean achievement scores at conceptual knowledge dimension sub-level remembering in Mathematics.

Ho2: There is no significant effect of problem solving teaching method on the mean achievement scores at conceptual knowledge dimension sub-level understanding in Mathematics.

Ho3: There is no significant effect of problem solving teaching method on the mean achievement scores at conceptual knowledge dimension sub-level applying in the subject of Mathematics.

Ho 4: There is no significant effect of problem solving teaching method on the mean achievement scores at conceptual knowledge dimension sub-level analyzing in Mathematics.

Ho5: There is no significant effect of problem solving teaching method on the achievement scores at conceptual knowledge dimension sub-level evaluating in Mathematics.

Ho6: There is no significant effect of problem solving teaching method on the mean achievement scores at conceptual knowledge dimension sub-level creating in Mathematics.

Ho7: There is no significant effect of problem solving teaching method on the mean achievement scores at Higher Order Thinking Skills.

Ho8: There is no significant difference on the mean achievement scores between the Experimental Group with Control Groups

1.8 Significance of the study

1. The study could be helpful for the Mathematics teachers at Elementary level to use of problem solving method for teaching.
2. It would help the teacher to develop lesson plan according to problem solving method.
3. It would be useful for Mathematics master trainers for training.
4. It would be useful for the curriculum developers to develop the courses which could help to select the content on problem solving method.
5. It would help to introduce new heuristics in the subject of Mathematics.

6. It may help the teacher training institutions to use this method to teach according to the Revised Blooms' Taxonomy to the prospective teachers.
7. It may be helpful to develop new heuristics in Mathematics at elementary level.

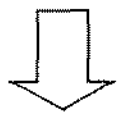
1.9 Delimitations of the study

Due to time and space constraints the study was delimited as follow:

1. The study was delimited to conceptual knowledge sub-levels remembering, understanding, applying, analyzing, evaluating and creating of Revised Bloom's Taxonomy.
2. The study was delimited to urban Islamabad Model schools for Boys in Islamabad having more than 120 students at elementary level.
3. The study was delimited to the content of word problems, practical geometry and theoretical geometry of the Mathematics of book of class 8^{th}.

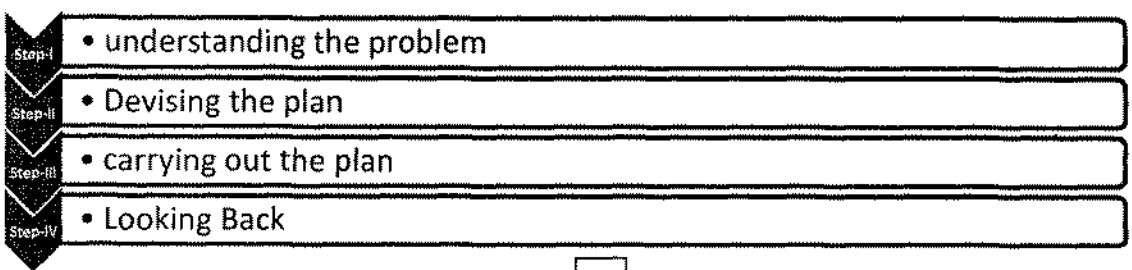
1.10 Theoretical framework of the study

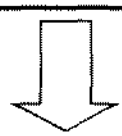
Mathematical Problem



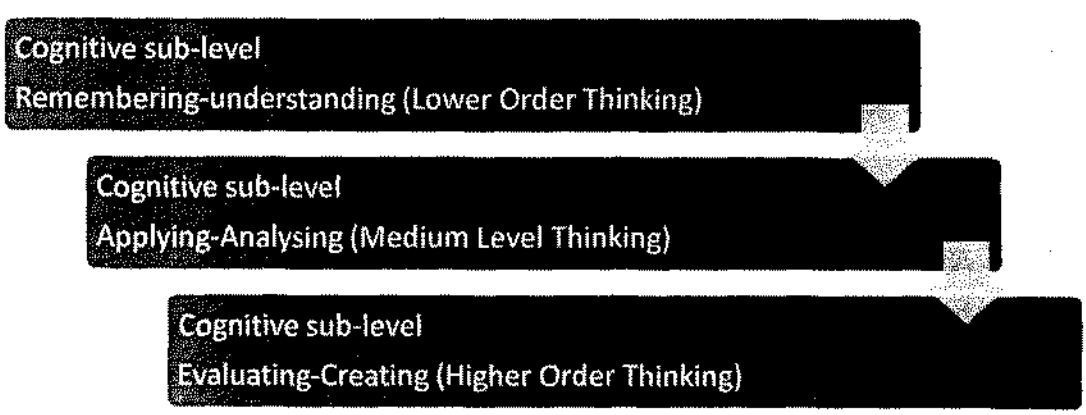
INPUT

Application of Polya's problem solving method



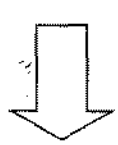


PROCESS



Development of conceptual knowledge

Solution



OUTPUT

1.11 Research Design and Methodology

The present study was an applied study with respect to application and quantitative by paradigm (Dewey, 1938). The study was based on deductive theory with positivist's philosophy. According to Cohen, Manion, & Morrison, (2000) the control group design helps the researcher to quantify the impact that could be attributed to extraneous variables, it does not separate out the other effects that may be due to the research instruments (such as the reactive effects), or, respondents(such as the maturation or regression effects). When the researcher needs to identify and separate out these effects, a double-control design is required. In a double-control study, the researcher has two control groups instead of one. To quantify, say, the reactive effect of an instrument (Kumar, 2009).

1.11.1 Population of the Study

The population of the study consisted of all 6932, 8th grade boys students studying in thirty seven Federally administrative institutions in urban areas of Islamabad. The population of the study was taken from Islamabad Model Schools, because institutions were easily accessible.

1.11.2 Sample

The present study was based on true experimental (Double control group) pretest -posttest design. A probability sample of 132 male students from randomly selected Islamabad Boys Schools I-8, Islamabad was taken. After pretest they were divided into groups according to scores and then through multistage sampling, they were divided into three groups.

1.11.3 Strategy of inquiry

For collecting the quantitative data regarding the effect of Polya's problem solving method on conceptual knowledge dimension sub-levels, the true experimental design: the pretest-posttest (double control group) design was used (Aravena and Caamaño, 2007).

One problem that has been identified in single control group design was the interaction effect of testing (Kumar, 2009). This threat can be controlled by adding a third group. A sample of 120 students was divided through randomization in to three equal groups one group was given treatment and two groups were kept controlled. To overcome the problem of interaction effect of testing the following design was used.

Experimental	RO_1	X	O_2
Control	RO_3	C	O_4
Control	RO_5	C	O_6

1.12 variables of the Study

Following were the variables in the study:

1.12.1 Independent Variables

Poly's Problem solving method of teaching was used as independent variable in the study.

1.12.2 Dependent Variables

Achievement scores of students on Conceptual knowledge dimension sub-levels (remembering, understanding, applying, analyzing, evaluating and creating) were the dependent variables in the study.

1.12.3 Intervening Variables

Four steps of Poly's problem solving method (understanding the problem, devising a plan, carrying out the plan and looking back) was the intervening variables in the study.

1.12.4 Extraneous Variables

These might be the extraneous variables (Age, I.Q level, Experience of teachers, qualifications of teachers, home tuitions) of the study.

1.12.5 Threats to Internal Validity

Threats like history, maturation, statistical regression, testing, instrumentation, selection; mortality were controlled through randomization and experimental design.

1.12.6 Threats to External Validity

Threats like failure to describe independent variables explicitly, lack of available and target, Hawthorne effect, inadequate operationalzing of dependent variables, sensitization to experimental conditions and interaction effect were controlled trough different measures like introducing third group.

1.13 Instruments of Data Collection

Pre-test and post-test were used as instruments to collect data. Test consisted of the items on word problems, theoretical geometry and practical geometry. The same pre-test by changing the order of the items was used as post-test.

1.13.1 Validity of Instruments

Instruments were validated through expert's comments and pilot testing.

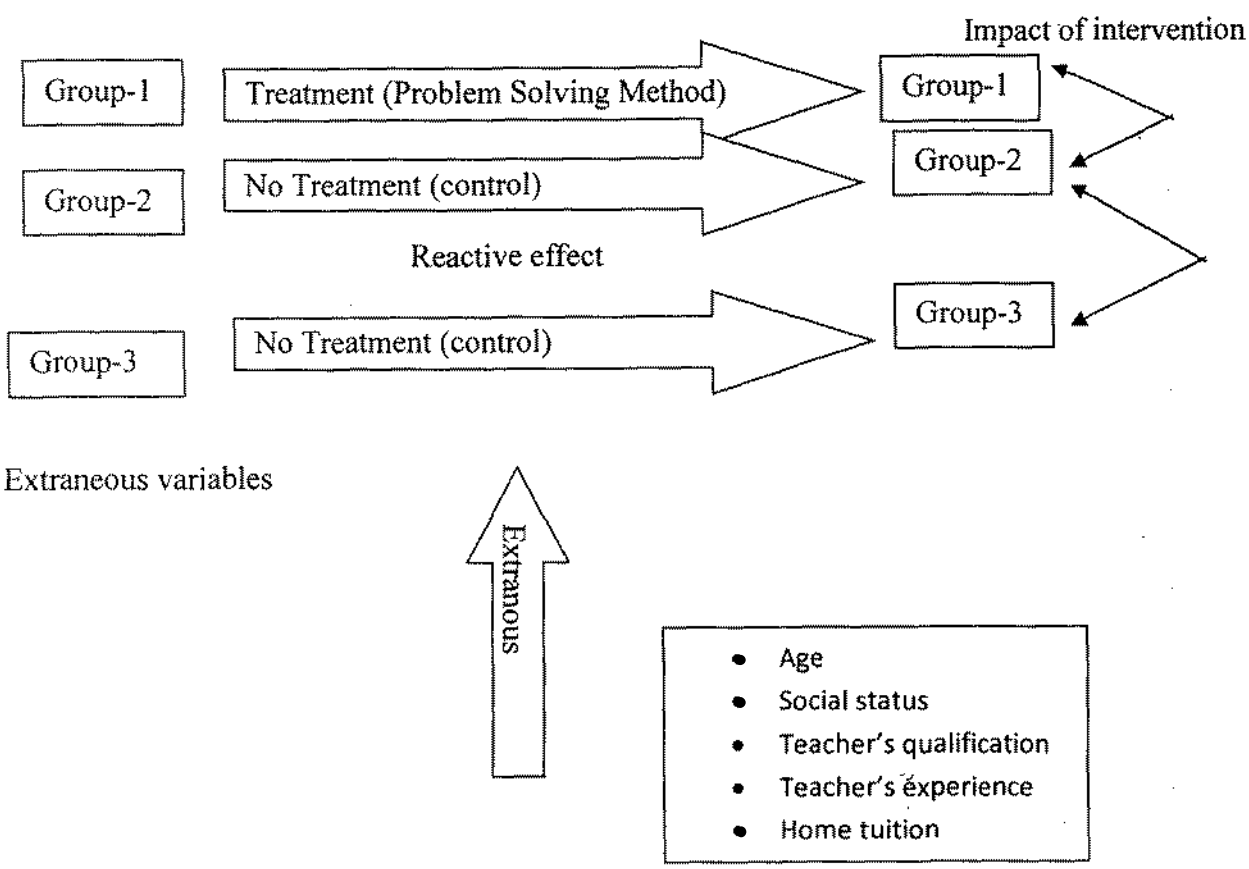
1.13.2 Reliability of the Instruments

The reliability of the instrument was determined through Alpha Reliability Analysis method.

1.14Analysis of data

Quantitative data were analyzed by applying One Way ANOVA and t-test through SPSS.

1.14.1 Operational Framework



CHAPTER 2
REVIEW OF RELATED LITERATURE

2.1 Etymology of Mathematics

Development of mental abilities and Mathematics has a close relationship Andrews, (2006). The word Mathematics is derived from the ancient Greek language (mathema) standing for "that which is learnt" or "what one gets to know," and in Modern Greek just "lesson." The term mathema is driven from the word (manthano). The word Mathematics has more technical meaning "Mathematical study" in Greece. Its adjective is (mathematikos) which stands for related to learning or studious (Singh & Rohatgi, 2005).

Singh & Rohatigi (2002, p.10) quoted that the world Mathematics in Latin means "the Mathematical art". The word Mathematics was used for the study of "astrology" instead of studying Mathematics in Latin and English until around 1700 century. The term Mathematics remained for some time under the process of definition and redefinition. Sometimes it was mistranslated as well, particularly

notorious one is Saint Augustine's "Christians should beware of 'matematici' means astrologers which stands for as a condemnation of Mathematics" (Alamolhodaei, 2009).

2.2 Uses of Mathematical Sciences

Advancement in the Mathematical sciences are helping several disciplines like, to predict the path and strength of a tsunami following an earthquake or other oceanic event, to map the topography of the ocean floor and infer large-scale wave behavior from independent ocean tide gauges that are irregularly spaced and can be hundreds of miles apart, to find the brain images, to find the chemical formulae of proteins, to trace the secret signals, to develop precise images, for developing simulations, and unlimited new ways. For instance, the Mathematical sciences are used in planning logistics, deployments, and scenario evaluations for complex operations, Mathematical simulations allow predictions of the spread of smoke and chemical and biological agents in urban terrain, from it, the importance of Mathematics is clear in daily life(National Academy of Sciences, 2012).

According to the report of National Academy of Sciences, (2012) the Mathematics is used to design advanced armor, underpin tools for control and communications in tactical operations. Signal analysis and control theory are essential for drones. Large-scale computational codes are used to design aircraft, simulate flight paths, and train personnel, Mobile translation systems employ voice, recognition software to reduce language barriers when human linguists were not available More generally, math based simulations are used in mission and specialty training, Satellite-guided weapons utilize GPS for highly-precise targeting, while

Mathematical methods improve ballistic, modeling and simulation facilitates trade-off analysis during vehicle design, while statistics underpins test and evaluation. It can be easily inferred that the subject of Mathematics has it utility not only in daily life but also in complex phenomenon, by this subject all other disciplines are taking benefits. It is a supporting subject with all other subjects. Its scope is unlimited and in the future it will prove its significance and many other utilizations of Mathematics will unfold, this fact reveals the importance of Mathematics for the national development (National Academy of Sciences, 2012; PISA, 2012; OECD, 2014).

2.3 Importance of Teaching of Mathematics

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As per the statement of Balki, (2010), that nature talks in the language of Mathematics, means all the nature is consisted in shapes, number and diagrams, so for the understanding of nature, Mathematics is required. So its teaching is also very important. To understand the necessity of teaching of Mathematics as a compulsory subject at the schools stage one has to be familiar with the utility or importance of this subject in schools curriculum or the value of this subject (Singh & Rohatgi, 2005; Consortium on Reaching Excellence in Education, 2013).

Keeping in view the importance of teaching of Mathematics following are the few points:

2.3.1 Practical Value

The practical value of Mathematics in our daily life has no limits. It is simply impossible to lead our life without the use of fundamental processes of Mathematics. The knowledge of this subject is essential for the house wives to prepare their budget according to their respective income and expenditure. All of us poor or rich, skilled or

unskilled worker, labor, shop keeper etc. have to make some calculations and so all of us need the knowledge of Mathematics to a certain extent. We may be able to lead a life without learning how to read and write but can never get on without knowing how to calculate. So as to avoid confusion and chaos in the present day complex society, it is essential to develop some system for fixing timings, ratios, prices, wages, fares, percentages, interests, exchange, commissions, discounts, and length etc. Knowledge of Mathematics helps in this entire world's entire commercial system (Singh & Rohatgi, 2005).

Singh & Rohatgi (2005) cited Bacon that, he was right when he said the "Mathematics is the gate and key of all sciences". Whatever comforts science has given us that are all due to the knowledge of Mathematics. Singh quoted the Young statement that, "Mathematics has been the pioneer, where its backbone removed our material civilization would inevitably collapse".

Any person ignorant to Mathematics will be at the mercy of others and will be easily cheated. According to Bacon quoted by Singh & Rohatgi (2005), the neglect of Mathematics works does injury to all knowledge since, he who is ignorant of it, cannot know other sciences or the things of the world.

Natural phenomena follow Mathematical principles. For instance, the rising and setting of the sun and appearing of stars, the change of seasons, speed of rotation of planes, etc. According to Naploeon cited by Singh & Rohatgi (2005) "The progress and the improvement of Mathematics are linked to the prosperity of the state". The validity of the statement processes were invented in accordance with the requirement of mankind. In order to spend life through a systematic way people has

to fixed many issues regarding daily life dealings which are based on Mathematics. In this complex world passing through scientific and technological age and rapidly moving towards computer age, the practical value of Mathematics is going to be increasingly felt and recognized.

2.3.2 Disciplinary Value

Weber (2008) quoted Startalk, (2009), who was a great Mathematician, said that "If Mathematics, however, had no disciplinary value; its teaching in schools could hardly be justified solely on ground of its bread and butter value". Mathematics is a way to settle in mind a habit of reasoning. So if taught properly it develops reasoning and thinking power and discourages memorization. The various characteristics that are needed for training the minds of students are simplicity, accuracy, certainty, originality, verification and application of knowledge. This subject teaches that facts when expressed in simple language can easily be understood. The answer is either right or wrong in Mathematics so there is no possibility of disagreement of the teacher with the student, hence students develops a habit of confidence among themselves. In students original thinking can be developed through this subject. As the results in this subject can be easily verified, so it develops the habit of self-evaluation, achievement and confidence (Startalk, 2009; Consortium on Reaching Excellence in Education, 2013).

Thus from above, it is clear that habits of simplicity and clearness, accuracy, certainty in expression, originality in thinking and verification of results are formed and strengthened by the study of Mathematics. Besides these habits of concentration, hardworking, punctuality, neatness, cleanliness and orderliness can also be

developed. These are the days of rapid progress and quick changes. In every field of life new ideas and new methods are being introduced at a very high speed. So the most important thing in this ever advancing society is not only to learn facts but to know how to learn facts. The main is not the acquisition of knowledge but the power of acquiring knowledge. This is the discipline of mind that each one try to learn Mathematics. The knowledge becomes real and applicable only when the mind is able to apply it in a new situation. The subject of Mathematics provides a great opportunity for this. By its teaching latent power of mind can be developed (Weber, 2008).

2.3.3 Intellectual Value

Many intellectual traits such as power of thinking, power of reasoning, induction, analysis, synthesis, originality, generalization, decision making, hypnotizing, etc., can be developed through the teaching of Mathematics. Hence it is an effective tool for the development of such attributes in an individual. Problem solving is an intellectual ability and help an individual throughout his/her life can be developed by the Mathematics (Consortium on Reaching Excellence in Education, 2013).

According to the Professor Schultz cited by Balki (2010), "Mathematics is primarily taught on account of the mental training it affords and secondarily on account of the facts it imparts". This fact of intellectual value of Mathematics is also supported by the Balki (2010), "Mathematics sharpens the minds of the people in the same way as some stone sharpens the tools". In other words Mathematics helps in the

development of power of imagination, observation, creativity and systematic thinking (Association of Mathematics Teachers, 2009).

2.3.4 Cultural Value

According to the statement given by Farayola (2011) 'Culture has many definitions but in the context of Mathematics it means mode of living of people. Society is the result of interrelations of individuals'. Mathematics plays an important role in determining the culture and civilization of a country. It affects our ways of thinking and also the way of living.

The fact is also defended by the words of Hogben cited by Farayola (2011), "The development of Mathematics at any time in any country is the true mirror of its civilization and progress at that time." Society draws huge benefits, when Mathematics makes its contribution to the advancement of science and technology. It is due to the progress of various avocations like engineering, medicine, industry, railways, rods, computers, buildings, agriculture etc. that we are living in such an advance culture. Mathematics also helps in preserving our social and cultural heritage in the form of Mathematical knowledge that could be better understood by students of Mathematics (Berggren, 2007; Faroyloa, 2011; PISA, 2012).

2.3.5 Psychological Value

The teaching and learning process of mathematics is based on fundamental principles of practical learning, learning by observation, from concrete to abstract, in which needs, interests, ability and motives of the child are taken into consideration. Mathematics satisfies our common instincts like creativeness, self-assertion and authority. It utilizes the best interests, innate tendencies and aptitudes of the students.

This subject provides them inner satisfaction, self-esteem and confidence (Farayola, 2011).

2.3.6 Moral Value

In the light of the discussion by the Kolawole & Popoola & Paplla (2011), on the moral value of mathematics means, how it is linked with right and wrong conduct, person's moral habits and encouragement. It also stands for an individual's confidence and manners. It is not a fact that the aim of education is to produce persons who are physically fit, mentally alert and morally sound. It is the moral influence which is laid on our youth by the study of mathematics or by teaching mathematics to them, at the early stages of their schooling. Mathematics unlike History and allied subjects, does not preach character formation, but it has its own way of building and molding the character of youngster (Berggren, 2007; Betne, 2010; Burnett, 2010).

The main emphasis of teaching of mathematics along with other affects are: openness of mind; using neat and clean methods of solving problems; maintain objectivity; being brief; and exercising full control on the self-bias. In mathematics, there is no room for doubts, half-truths, exaggerations, under valuations, double answers, etc. Mathematics learning takes upon itself to improve the attention of young boys and girls towards the subject. It was justified by the Hamilton cited by Singh that "The study of mathematics cures the vice of mental distraction and cultivates the habit of continuous attention" (Kolawole & Popoola, 2011).

According to Locke the great thinker quoted by Kolawole (2011), "Mathematics is a way to settle in the mind a habit of reasoning." It is unfortunate

that most of the students resort to rote learning, but to get them back on the track is best possibility to encourage them to learn the art of reasoning. It is the subject which develops learning is a systematic way and the learners acquire the habit of step by step learning. Mathematics is such a subject that demands hard work, develops proper scientific attitude towards mathematical riddles, problems, postulates etc., and it subsequently prepares youngsters to face the knotty problems of life. The inculcation of hard work among the rising generation of mathematics learners results into the burning of the cognitive flame. The habit of hard work gradually helps students to tap their creative faculties so that they equip them with the art of discovery and invention which is extremely prized in the world (Kolawole & Popoola, 2011).

According to John quoted by VanHattum (2011), a well-known mathematician, elaborates this in these words, "I have taught mathematics to almost every kind of students. In my experience there is hardly any man who may not become a discoverer." This subject provides a moral tone of young seekers of knowledge. The study of mathematics always inculcates in a human mind the habit of economy reflected in the economy of thought, actions and time. All these faculties on mind developed by this subject, lead towards moral development of the learners (Berggren, 2007; Betne, 2010; Burnett, 2010).

2.3.7 Artistic and Aesthetic Value

Artistically creative subject like painting, sculpture, drawing, dance, music etc. have certainly a mathematical feedback of concepts and theories ingrained in them to give artistic satisfaction to the respective followers while they are mastering the same. All these arts are related to directly or indirectly to the subject of

mathematics. Most of the people failed to appreciate the artistic influence of mathematics on its learners. But once the hurdle in the teaching of mathematics is removed, learner begins to appreciate its harmony, beauty, symmetry, art and even music. It is an enjoyable experience for the mathematics teachers when students solve the question by themselves and become happy (Burnett, 2010; VanHattum, 2011).

The Archimedes rushed nakedly in the streets to reach the king to tell him that he had proved, what became known as Archimedes Principle. This was justified by the words of Leibnitz cited by VanHattum (2011), "Music is a hidden exercise in arithmetic of a mind, unconscious of dealing with numbers." While planning public or private gardens, one has to depend on mathematical concepts to add the beauty and charm to them. A gold-smith also use mathematic to provide artistic and aesthetic value to its ornaments. Mathematical games played on video, serve as a great recreation and joy to its users. Regarding the artistic and aesthetic, mathematics had three fold roles namely creating, saving and nourishing the precious pieces of art in our art galleries, exhibitions, and elsewhere. Therefore, it is said that "Mathematics is an art of all arts." It is rightly said that "Beauty is the result of ratio and proportion" (VanHattum, 2011).

2.3.8 Vocational value

The knowledge of mathematics is helpful in a number of professions. It forms the basis of so many studies which are purely vocational in nature, for instance, a student of mathematics can choose medical, engineering, technical profession, teaching, agriculture, banking, computer engineering, or some statistical profession in which he/she is interested in and fit for. So it can be inferred that this subject provides

basis for many vocational professions along with the matters of daily life. It also provides effectiveness and efficiency in vocational professions. Almost every profession involves investment, loan, interest, profit and loss, percentage, commission and a lot more, all depends on mathematics. In short, it can be concluded that a sound vocational life demands a sound mathematical background (Rani, 2011).

2.3.9 International Value

In the book "Teaching of Mathematics" by Rani (2011), discussed that all mathematicians, irrespective of their castes, colors, creeds or places have contributed towards the progress of mathematics. Any new idea brought in the field of this subject does not take much time to become an international property. All the latest developments are shared by community all around the world. Its language is common in the world. The books of Mathematics and research journals are exchanged or circulated among almost all the countries of the world. Through exchange programme, experts provide services in the world. Many software engineers developed their links through internet to promote the cause of mathematics.

Form the points cited above, it can be sum up that mathematics teaching confers on its students valuable information and knowledge, varied skills and logical thinking, disciplines the mind, ensure cultural advancement, grants practical dignity and intellectual growth, helps character building, encourages aesthetic and artistic supremacy, grants social in-sight, international understanding etc. It may also be concluded that it is the subject which makes the life easy and affective. From the above discussion, importance on mathematics cannot be denied. So we have to pay special attention while setting its aim and objectives (Rani, 2011).

2.4 Aims and Objectives of Teaching of Mathematics

As narrated by Suneetha & Rao (2010), in their book "Methods of Teaching Mathematics" that the 'Aims are like ideas and their attainments require a long term planning. The setting of aims is not an easy task. And so far their realization, they are divided into some definite, functional units, called objectives'.

Education is a conscious effort to impart knowledge in order to achieve certain ends and goals. Various subjects of school curriculum are different means to achieve these goals. By aims of teaching of mathematics we mean the broader purpose that may be fulfilled by the teaching of mathematics. The aims and values are inter-related and independent. Aims help in the realization of values possessed by a subject. Aims enable us to act with meaning and give direction to the activity. Mathematics may be studied in such a way that nature can be converted into an understandable language (Suneetha & Rao, 2010).

Objective in any area of curriculum may be regarded as direction of growth, and not an ultimate ends to be reached completely. Objectives also provide the direction in the selection of an appropriate teaching method. The aims and objectives of mathematics teaching are under the process of change in the last few decades because of the developments in the field of science and technology in the 21st century. The basis for setting aims and objectives of mathematics may be on philosophical, sociological, science and technology. In contrast, while setting aims and objectives for teaching of mathematics, we may consider the utility, appropriateness, practicability and timeliness. We have to select those objectives which help in the growth and development of an individual (Suneetha & Rao, 2010).

Following are the major sources or bases for the formulation of objectives of mathematics:

- Learner's capabilities and need
- Demand of the society
- The nature of the subject matter
- The working environment and available resources
- The international situation (Suneetha & Rao, 2010)

2.4.1 Aims of Teaching of Mathematics in School

According to Barton, (2005), the main aims of teaching of mathematics in schools are:

- To help the learners to acquire knowledge and understanding of concepts, fundamentals, operations in number and quantity needed in daily life.
- To develop students power of thinking and reasoning, and to inculcate habits of self-confidence, enquiry, concentration and observation.
- To develop in pupils an awareness of the mathematical principles, and operations which will enable them to understand and participate in general, social and economic life of the society.
- To prepare the pupil for school and post-school education in allied disciplines of physical, biological and social sciences.
- To enhance the habit of hard work.
- To enrich the students power of expression and decision.
- To inculcate the skills for solving problems.

- To bridge up the gap between nature and mathematics.
- To provide the aesthetic ability through mathematics.
- To expose the students to their culture and values.

From all these points, it may be inferred that the real aim of teaching of mathematics is to develop the power, not knowledge. It is therefore, not learning, but learning the art of learning. The knowledge acquired through reasoning power, which leads in discovering new facts. According to the importance of the mathematics, it has different objectives at different levels because every level is based on next level (Suneetha & Rao, 2010; PIAAC, 2013; OECD, 2013, 2014).

2.5 Aims of Mathematics at Elementary School Level

- To introduce the subject to the students in a very happy and receptive manner, keeping in mind that the first impression is the last impression.
- To teach all basic concepts and processes with absolute clarity and demonstration.
- To give glimpse of the vast advantages of the subject in daily life.
- To develop the interest among the students about the subject.
- To inculcate discipline among the students
- To familiarize the pupils with mathematical language and symbols.
- To develop the habits of patience, self-confidence, drill and regularity etc.
- To inform the students, the relationship of the subjects with other subjects.

- To elaborate the scope of the subject.
- To create the ability of problem solving.
- To lay a foundation for undertaking any profession or vocation after schooling or ever before.
- To help the students in making generalizations after being exposed to varied and numerous problems.
- To develop a taste for mathematics (Rani, 2011; Christopher, 2010).

The broader aims are divided in some definite, functional, measurable and workable units named as objectives. The achievement of objects leads towards the achievement of aims. Thus, the objectives of mathematics teaching are those short-term immediate goals or purposes that may be achieved within the specified class room resources by a teacher. These objectives help in bringing appropriate behavioral changes in the learner for the ultimate realization of various aims of mathematics teaching (Suneetha & Rao, 2010).

The various objectives of teaching mathematics at various levels of school education are as follows:

2.51 Knowledge Objectives

- To enable the students to have a good start in learning mathematics.
- To develop the basic concepts required for other concepts.
- To provide knowledge about misconceptions about basic concepts.
- To expose them with the terms and symbols of mathematics.
- To help students to know about the relationship of various topics in mathematics.

2.5.2 Skill Objectives

- To enhance the skills of reading, writing and counting of numbers.
- To inculcate the four fundamental operation of numbers.
- To develop speed and accuracy in performing different operations.
- To disseminate the skill of estimations.
- To enable students to recognize and construct various geometrical figures neatly

2.5.3 Application objectives

- To enable the students to solve the problems of daily life independently.
- To apply the knowledge in different fields.
- To prepare themselves for higher classes.
- To use the knowledge for reasoning.
- To enable them to express the problems in different ways.

2.5.4 Attitude Objectives

- To promote the power of observation.
- To enable the students to verify the results.
- To develop the qualities of tolerance, punctuality and honesty.
- To inculcate the objective approach to problems.
- To develop a habit of systematic thinking to solve the problems.

2.5.5 Ability Objectives

- To develop the ability to sense a problem.
- To increase the ability to analyze and generalize.

- To develop the ability to estimate.
- To develop the ability of recognize the different shapes.
- To develop the ability to construct and reconstruct the figures.

2.5.6 Appreciation Object

- To enable the pupils to appreciate the contribution of mathematics in the progress of civilization and culture.
- To enable the students to appreciate the recreational and amusement value of mathematics to utilize leisure time.
- To acknowledge the role of mathematics in the development of science and technology.
- To appreciate the role of mathematics in art and music.
- To enable the students to develop aesthetic ability to admire the natural beauty (Alamolhodaei, 2009; Association of Mathematics Teachers, 2009; Betne, 2010; Rani, 2011).

It is quite obvious from the aims and objectives of mathematics that the main objective of teaching mathematics is our ultimate goal, but to what extent these are kept in mind while teaching the subject. Many of the objectives are ignored which are against the spirit of the teaching of the subject. Mathematics is an indispensable agent of change which leads us from joy to joy, and as such a place of honor is earned by the subject to reduce the ills of society in its race for prosperity, plenty, peace and progress (Suneetha & Rao, 2010).

2.6Mathematics Curriculum in Pakistan

Mathematics in the context of Pakistan has a clear vision about its teaching and objectives. Pakistan started its efforts for developing and revising curriculum from 1975. First national curriculum was developed in 1975-76 in the guidance of the Education policy 1971. This curriculum was revised and modified in 1984. To continue the process of improvement in curriculum the next revision was made in curriculum on 1994. After this, the revision was done in 2000 and 2002 respectively. This showed continuity in the revision process of curriculum in Pakistan since 1975 to 2002 (Government of Pakistan, 2006, 2009; Suneetha & Rao, 2010).

Now days, we are applying the curriculum which was revised in 2006. The curriculum of 2006 is an effort to make the curriculum more vibrant and related to the need of socio-economic, technical, professional, and the needs of labor market of the country. It is developed keeping in view the modern utilization of mathematics in comparative perspective. Its reflection can be seen in the document of curriculum 2006.

"Mathematical structures, operations and processes provide students with a framework and tools of reasoning, justifying conclusions and expressing ideas clearly. As students identify relationships between mathematical concepts and everyday situations and make connections between mathematics and other subjects, they develop the ability to use mathematics to extend and apply their knowledge in other fields" (Government of Pakistan, 2006, 2009).

The above statement of the curriculum covers the all domains of cognition which is described by the revised Bloom's Taxonomy. The objectives set in the

curriculum of 2006 are also a reflection of those objectives which are international, and most of the developed nations are following. The salient feature of this curriculum is its focus on the content by which different objectives can be achieved. The following topics are suggested in curriculum 2006 for the students:

- Such concepts may be added to curriculum by which solid conceptual foundation is developed that will enable the students to apply acquired knowledge to acquire further knowledge of different subjects.
- To develop the abilities of thinking logically, reasoning, conjecture astutely, and geometrical concepts.
- To enhance the skills to visualize and interpret mathematical expressions.
- To provide the knowledge about emerging and current technologies, such concepts are recommended in examples and questions through which students can come to know about them.
- To create the ability to solve the daily life problems and problem solving ability.

Content on daily life problems may be added in to the curriculum (Government of Pakistan, 2006; Suneetha & Rao, 2010).

To maintain the quality of the National curriculum for mathematics 2006, it was organized about the five basic standards which are not only broad but also flexible. Following are the standards:

1. Numbers and Operations
2. Algebra
3. Measurement and Geometry

4. Information handling
5. Reasoning and logical thinking

The importance of mathematics has it$s empirical evidences through researches. It is an excellent vehicle for development of mental abilities, logical reasoning, spatial visualization, analysis and problem solving. An emphasis on mathematics education is an indicator that nation has intention to develop a workforce to meet the challenges of the 21st century. Mathematics is also the subject that provides students an opportunity to become independent discoverers. It has its utility not only in the classroom but also beyond the classroom. The development of a highly skilled scientifically and technologically based manpower requires a strong grounding in mathematics. Keeping in view the importance of the subject, the National curriculum for mathematics 2006 is focused to develop the following abilities among the students:

- Students can understand and communicate mathematically
- Students can comprehend and elaborate the basic concepts
- Students can use these concepts effectively in solving mathematical problems
- Students can use these concepts in their daily life problems
- Students can differentiate between different problems like problem to solve and problem to prove
- Students can use different methods through integration
- Students can be able to examine the real life logically and mathematically (Alamolhodaei, 2009; PISA, 2012; OECD, 2014).

The above mentioned points show the intentions and efforts for the achievement of mathematics to meet the challenges of 21st century. The review of the National curriculum for mathematics 2006 provides a clear picture that focuses on quality. But in spite of all these efforts, results reflect a different story. In the same way, the classroom teaching of mathematics portrays some different picture of teaching. Innovative content and dissemination of knowledge demand some innovative methods of teaching. Quality of curriculum depends on quality of teaching. According to Education Policy 2009, "The foregoing analysis reveals that Pakistan has made progress on a number of education indicators in recent years. Notwithstanding the progress, education in Pakistan suffers from two key deficiencies: at all levels of education, accesses to educational opportunities remain low and the quality of education is weak, not only in relation to Pakistan's goals themselves but also in international comparisons with the reference countries."

The Policy identifies the two major causes of these poor performances:

(i) Lack of commitment to education – the commitment gap

(ii) The implementation gap (Government of Pakistan,2009)

The second gap indicates the ways and means in which the things are implemented which is not good. The expected outcomes in education especially in mathematics are not due to lack of commitment to education and the poor way of implementation. This fact is also endorsed by National Plan of Action to Accelerated Education-Related to MDGs 2301-16 by the statement that, in Pakistan, there are two keys challenges in Education. These are as under:

(i) Lack of access to education/high drop-out rate

(ii) Poor quality of education

One of the responsible factors for poor quality of education is the use of outdated teaching methods. This problem can be addressed through the use of new teaching methods. From the above mentioned facts, it is quite obvious that teaching methods play a vital role in the dissemination of knowledge especially in teaching of mathematics (Government of Pakistan, 2009; 2013).

2.7Concept of Methods for Teaching Mathematics

Methods stand for "How" effectively the content can be transferred to students. The learning depends upon the method adopted by the teacher. Methods provide the way how to impart mathematical knowledge and how to enable the students to learn mathematics. Methods of teaching have an intimate relationship with teaching and instructional objectives. So the main aim of teaching is about socially desirable behavioral changes in the students. Teaching is considered to be an art than methods which is the way to understand and practice it. This becomes more important that teacher has a variety of methods with him (Barkatsas& Malone, 2005).

The etymology of word 'method' has its roots in Latin language. The word has been derived from Latin word means "Mode" or "Way". It means that the methods are the means of transferring or delivering knowledge. Those are the ways of transmitting mathematical skills by a teacher to his students and their comprehension and application by the students in the process of learning of mathematics. In short, methods mean "what to teach?" and "how to teach mathematics?" or "how to approach it?" Hence, the process of interpreting the world of knowledge to students'

mind is called the methods of teaching (Barkatsas& Malone, 2005; Alamolhodaei, 2009).

It can also be described through the diagram as:

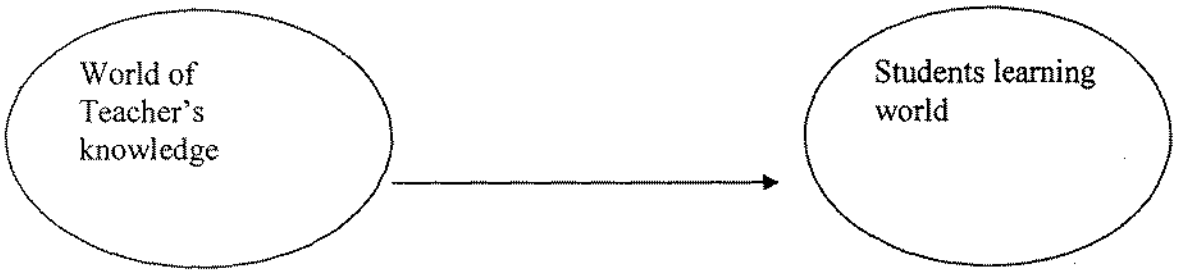


Figure 1 Concept of Teaching Method

The world of knowledge means; the content knowledge, interest, attitudes, skills and what we want to transfer to the students or the information about cognitive domain of knowledge (Remember, Understand, Apply, Analyze, Evaluate and Create). Methods can be interpreted through another approach for the sake of understanding (Barkatsas & Malone, 2005).

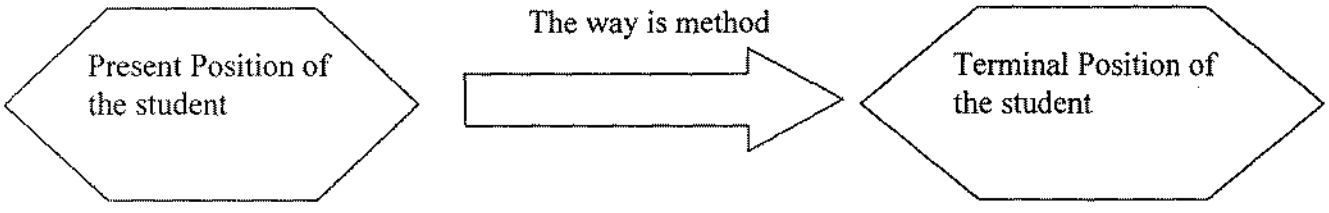


Figure 2 Concept of Method

In the words of Brandy (1963), quoted by Brewer & Hunter (2006), the method refers to the formal structure of the sequence of acts commonly denoted by instruction. The terms, strategies and techniques fall under the umbrella of method. The selection of a method of teaching of mathematics depends upon the abilities and interest of the teacher. While selecting the method, the teacher kept in mind the aims of teaching of mathematics and the objectives of the topic. These objectives may

include the mental, social, moral, intellectual, aesthetic and artistic values. A teacher who does not use suitable method to teach the subject matter according to the requirement of the children, is not supposed to be a good teacher. Among the many reasons that students do not like the subject of mathematics, one of them is the teaching method used by the teacher (Graybed, 2010; Christopher, 2010; Ellis, 2011).

It is important to note that a method should not become an end in itself but should be used as a means to achieve the determined aims and objectives of teaching mathematics. However, the teacher is free to use a variety of the teaching methods according to his/her abilities, training, interest, support system and experiences (Brewer & Hunter, 2006).

2.7.1 Selection Criteria for an Appropriate Method

Following consideration may be used in the selection of a method:

- What to teach (content)
- Why to teach (objectives)
- Whom to teach (students)
- How to teach (various methods)
- What are the problems while using the method (physical support system)
- How can these problems be overcome (alternative options)
- Which is the best method (select one suitable method) (Brewer & Hunter, 2006)

2.7.2 Fundamental of a Teaching Method for Mathematics

According to Herbart Spencer, quoted by Brewer &Hunter (2006), methods of teaching may be based on the following principles:

i) From simple to complex

ii) From concrete to abstract

iii) From known to unknown

iv) From direct to indirect

v) Form definite to indefinite

vi) From empirical to rational

vii) Focused on self-learning

viii) Develop interest among the students

ix) Create motivation among the students

x) Make the students independent discoverer

The selected method for teaching of mathematics may fulfill the basic principles of teaching as well, because the terminal behavior is the result of the required learning (Graybed, 2010; Christopher, 2010; Ellis, 2011).

Following are the some basic principles:

- Principle of learning by doing
- Principle of individual differences
- Principle of motivation
- Principle of correlation
- Principle of linking with previous knowledge of the student
- Principle of distribution

- Principle of repetition
- Principle of certain objectives
- Principle of evaluation (Brewer & Hunter, 2006)

2.8 Relationship of Philosophy with Teaching Method

There is a very close relationship between education and philosophy. Philosophy determines the destination and education leads to destination. Philosophy plays an important role in the selection of method. There are different philosophies and have their own ways/methods to disseminate knowledge. Philosophy is the experiment and schools are the labs in which these experiments are done. Education is the source of propagation of any philosophy. Every teaching method has its origin in a philosophical thought. All the actions of methods reflect the thinking which is working as a base of this method. It is a fact that every method is inspired by a philosophy (Booker, 2007; Graybed, 2010; Christopher, 2010; Ellis, 2011).

2.8.1 Idealist's Philosophy and Teaching Method

Although Idealist have not adopted any specific method of teaching, but they recommended some methods like Lecture method, Discussion method, Conversation, Dialogue, Questions answer, Argumentation etc (Booker, 2007).

2.8.2Realist's Philosophy and Teaching Method

Realists believe in reality and they accept reality on the scientific bases. So, they suggested the teaching methods like Heuristic, Experimental, Self-experience, Research, and correlation method of teaching (Booker, 2007).

2.8.3 Naturalist's Philosophy and Teaching Method

They follow the natural course of actions and emphasize learning by doing, learning by experience and learning by playing. They recommend the method through which the desired objectives can be achieved like Montessori, Dalton plan, and Kindergarten method etc (VanHattum, 2011).

2.8.4 Pragmatist's Philosophy and Teaching Method

They are the followers of Naturalists. So the recommended methods by them are: Project method; Problem solving method; cooperative method etc. According to VanHattum (2011), there are several techniques of teaching of mathematics along with methods. But there are differences between method and technique. For instance, Homework, Oral work, Group work, Self-study, Supervised studies, Assignments and Brain Storming are the techniques of teaching of mathematics

2.9 Difference between Methods and Techniques

Both teaching methods and teaching techniques are affective in teaching of mathematic. All content of Mathematics required different methods and techniques. According to Grek (2009), there are following differences between two of them. Those are as follows:

Table 2.1

Difference between Methods and Techniques

Teaching Methods	Teaching Techniques
They have direct relation with aims of teaching	They have indirect relation with aims of teaching
They are based on classical theory of human	They are based on modern theory of human

organization	organization
They are focused on 'How"	They are focused on 'With what'
They required mastery on content for evaluation	They required acquisition of objectives for evaluation
The assumption for teaching methods is that teaching is an art	The assumption for teaching technique is that teaching is a result of behavior
The objective of method is the impressive presentation of the content	The objective of technique is to create complete learning situation
The methods focused on task and it presentations	The technique focused on behavior and its relationship
The teaching objectives are not considered to be important in methods	The teaching objectives are considered to be important in techniques

2.10Teaching Methods for Teaching Mathematics

The success of teaching is depending on the teaching methods. There are many approved methods for teaching of Mathematics; every method has its potential as well as its limitations. Every method has its classification with respect to Teacher Centered and student centered (Rani, 2007).

Following are the few methods commonly used for teaching of Mathematics:

2.10.1Demonstration Method

In the demonstration method both teacher and students are active. The teacher performs the experiments while teaching in the class and the students acquire knowledge with careful observation of the experiment. It can also be done through team teaching. In demonstration most of the time control remains in the hand of

teacher. For the effective implementation of this method teacher requires training (Musttfa, 2011).

Merits

Following are its merits:

- Teaching becomes effective with small number of students
- Ability of observation and reasoning can be developed
- It is an economical method It is useful when apparatus is costly
- It uses less time and teacher control the demonstration

Limitations

Following are few limitations of this method:

- Students can only observe but not performs
- Students cannot learn by doing
- Students cannot repeat the same experience
- Students cannot observe minutely in large classes
- Not helpful in developing different mental abilities

2.10.2 Analytic Method

The original meaning of the world Analysis is un-loose or to separate things that are together. With the help of this method, the difficult parts of any problem can be analyzed to find out the solution of the given problem. This method moves from unknown to known or from conclusion to hypothesis. It is not used in every situation. It has its specific conditions which are as follow:

- To prove any theorem
- For construction work in geometry

- Find out the solution of some new arithmetical problems

Merits

Following are its merits:

- It is based on psychological principles
- It is based on heuristic approach
- It develops the scientific attitude
- It is the process of thinking
- It is based on inductive reasoning

Limitations

- Following are its limitations:
- It is time consuming method
- It cannot be administered without training
- This method is not applicable for all content
- It requires prior knowledge of known and unknown
- Students cannot solve daily life problems (Musttfa, 2011)

2.10.3 Synthesis Method

It is the reverse of analytic method. Synthesis means to place together things that are apart to join separate parts. This method moves from known to unknown or from hypothesis to conclusion. It begins from the data and connects them with the conclusion. It is the method used for formulation, recording and presenting concisely the discovered solution omitting the trials and errors (Hattie, 2009; Graybed, 2010; Jenkins, 2010).

Merits

Following are few merits of this method:

- It is short and speedy
- It develops the memory of the students
- It develops the logical thinking
- It omits the trails and errors
- It takes less time

Limitation

Following are the few limitations of this method:

- Students cannot learn the art of discovery
- It develop rote memorization among the students
- It leads towards doubts in mind
- It leads towards dissatisfaction
- Different abilities like reasoning and creativity cannot be developed (Mustufa, 2011).

2.10.4 Heuristic Method

According to Balki (2010), in his book titled "Application of Heuristic Methods to study of Mathematics at Schools" stated that this is a modern method for teaching of Mathematics. It is taken from the Greek word "Heurisco" which means "I find out". This method was given by Professor. Henery Edward Armstrong cited by Yuan (2013), this method focuses on the way how discoveries can be done by. It is focused on independent work and discoverer. It is useful to develop scientific and Mathematical attitudes among the students. This method is intended to change the passive recipient of knowledge into an active independent enquirer and discoverer of

knowledge. The main purpose of this method is to train the students to think. The Polya's Problem Solving method is also some time known as Heuristic Method (Yuan, 2013).

Merits:

Following are it plus points:

- It is based on psychological grounds
- It helps in developing confidence among the students
- It develop scientific attitude
- It helps in the process of discovery
- It helps in learning by doing

Limitations

Following are its few limitations:

- It is not fit for all levels of students
- It is time consuming method
- It is not economical
- Material is not available for this method
- This is a formational method rather than informational (Yuan, 2013).

2.10.5 Project Method

This method was given by Kilpatrick, an American educationist. It is based on the philosophy of Pragmatism. It is based of activities about a topic. It works in group and in collaboration as well as individual. According to its author "A project is a whole-hearted purposeful activity proceeding in a social environment". There are four types of project:

1. Creative or constructive projects
2. Artistic projects
3. Problematic projects
4. Drill projects

It has its specific steps for its implementation which depends upon the nature and type of the projects. Most common are the following steps:

1. Proposing and choosing the project
2. Planning about the project (identification of activities)
3. Execution of the project
4. Evaluation of the project
5. Recording of the process of project for feedback

Merits

Following are its salient features:

- Methods is based on psychological principles
- It helps in developing confidence among the students
- It is scientific and democratic
- It develops discovery attitude in the students
- It develops socialization

Limitations

- It is time consuming method
- Acquired knowledge is not sequential
- Text books and helping material is not available
- It is not economical

- All students cannot have equal opportunity for learning (Sadia, 2010)

2.10.6 Conventional Method of Teaching

According to Rehman (2011), this is the most practiced method in our schools system. Lecturing is one of its most popular types. It is used to teach different disciplines at all levels. It is a teacher-centered method. Teacher plays a dominant role in teaching. Teacher becomes more active than students. It is an authoritative method. Some of its salient features are as under:

- The focus of this method is on listing of students.
- Its instructions focused on memorization.
- It provides surface knowledge to the students.
- It mostly works on Lower Order Thinking Skills.
- The motivational force in this method is threat of punishment or grades; therefore, students' works in an environment of command and obey.
- It works one one-way communication system.
- There is lake of interaction between students and teacher.
- This method promotes abstract perceiving and reflective processing.
- The students are passive learners.
- During lecturing students remained busy in noting most of the time.
- It is an oral presentation of content.
- It develops rote learning among the students.
- Students feel boredom through this method (Adebule, 2004; Amazigo, 2000; Rehman, 2011).

2.11 Conclusion

All the above mentioned methods are equally good and effective in achieving the preset objectives for teaching of Mathematics. But the formal schools system are facing many problems among one of them is the poor performance of the students in Mathematics (e.g.Nwangu, 2005; Adebule, 2004; Amazigo, 2000; Alonge, 1998 cited by Kolawole et al, 2013).

In this 21st century every nation wants to shapes its future through science and technology and therefore, Mathematics is the integral part of schools curriculum. Mathematics is an instrument for development and through this human endeavor becomes true (Kolawole & Popoola, 2011).

Farayola (2011), validating the work of Ajayi (2010) suggested that Mathematics is an aid to representing and attempting to resolve problem situation in all disciplines. Mathematics is an inter-disciplinary logical tool and the language of research. The knowledge of Mathematics is central to the economic growth and technological development of every nation. The economic, social, moral and intellectual development depends of Mathematics. According to National Research Council (2001) cited by Mustfa (2011), the achievement of objective in the subject of Mathematics required skillful teaching. Through these teaching methods required skills for the smooth learning of Mathematics can be achieved. Through the competent methods of teaching the skill of problem solving can be developed.

According to description given by McLaren (2010), regarding the performance in the subject of Mathematics, "contrary to needed strands, unfortunately teaching of Mathematics has been criticized for its formal and artificial

appearance where much attention has been paid to the drilling of certain calculation methods, algebra and the mechanical use of formulas." Using the reference of Piaget (1896-1980) theory of learning McLaren (2010), interpret that it is necessary for a teacher to understand the psychology of the students so that they can learn the concepts through doing by themselves. So it can be inferred from this that the teacher may use such teaching method which provide an opportunity to learn not verbally but conceptually. For this one has to use appropriate method. In the era of knowledge explosion, there are plenty of methods for teaching of Mathematics.

The statement of Voltaire and Spencer quoted by Mustafa (2011), that, "every method has some good in it and no method is all good". Students may be told in such a way that they conceive the concept by doing and use the knowledge in their daily life. As the topics in the Text book are not same and hence different methods are required to transmit the content knowledge. This is also endorsed by (Herrer, Koss, Kanold, Rayan&Speer,2007) cited by Mustfa (2011), there is no accepted and authentic method which can be equally fit for all the content of Mathematics and by which several objectives can be achieved.

According to the (Kolawole et al, 2103) "Conventional or traditional method is the widely-used traditional teachers'-centered instructional method where the teacher is seen as the absolute reservoir of knowledge, who always delivers a pre-planned body of knowledge to learners." Through this method students are getting used to rote memorization and cannot polish their mental abilities and not solve the daily life problems.

The National Curriculum for Mathematics (2006) set five standards they are:

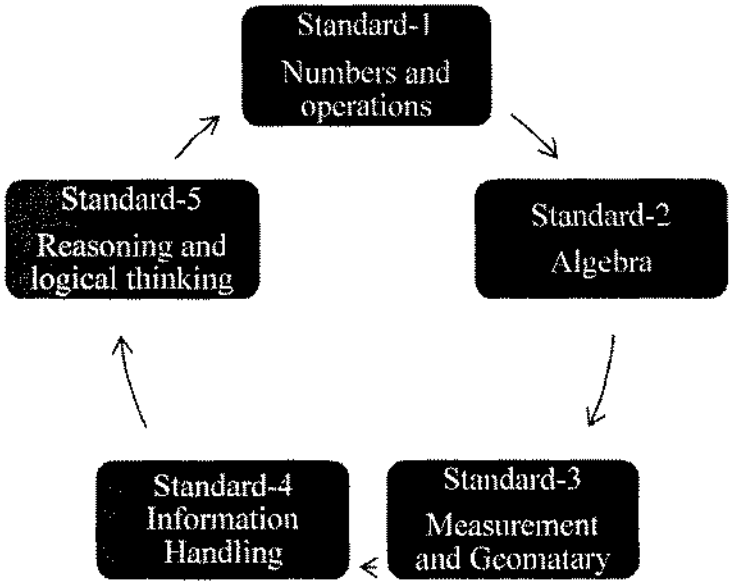


Figure 3 Standards for curriculum, 2006 (*National Curriculum for Mathematics 2006– Standards and Benchmarks*)

Every standard has some benchmarks and indicators to show the particular developmental level of attainment. Standard number five which is related to Reasoning and Logical thinking is the highest level among the standards and the indicators for the attainment of this level make the students able to:

- Analyze, synthesize, use patterns, and known facts, properties and relationships about Mathematics
- Examine real life situations by identifying, mathematically valid arguments and use it for solving daily life problems (Government of Pakistan, 2006).

Problem solving is ability and this can be developed through the method of problem solving method and students can become able to solve daily life problems and become independent discoverers. The problem solving method given by Polya

gain the popularity in last few decades and proved that it is the best method for creating the ability of problem solving (Yuan, 2013).

As cited by Yuan (2013), that, Turkish researchers acknowledge the importance of problem solving in Mathematics. Many empirical researches have proved that through problem solving method problem solving ability can be developed (e.g. Sani, 1997; Balcıoglu, 1998; Budak, 1999; Saygılı, 2000; Yurttas, 2001; Ciftci. 2001; Eroglu, 2001; Sonmaz, 2002; Cetinkale, 2006; Kayan, 2007 cited by Yaun, 2013). This fact was further proved by the number of researches, that problem solving makes the students to solve daily life problems (Polya, 1954; Schoenfeld, 1992; Bailey, 2009; Betne, 2010; VanHattum, 2011 cited by Yaun, 2013).

According to Orhan & Ruhan quoted by Amirali (2010), in the learning of Mathematics telling is not teaching and listening is not learning, it requires some practical work.

According to the work of (e.g. Walker & Lofton, 2003; Chin & Chia, 2004 cited by Amirali, 2010), "Human beings face a multiple dimensional problems in their lives and they try to solve these problems in a particular way in the light of their previously gained knowledge and experiences. In this regard it is essential for the students to be prepared for future or near future challenges by facing real life, or real like, problems in their learning environment, and finding appropriate solution of these problems. Each society expects from its education system that it enables the individuals to become an effective problem solver in their real life."

According to Betne, (2010), the roots of problem solving learning are found in Dewey's philosophy in which learning by experimentation or doing is more lasting (Dewy, 1938). Actually the problem solving is to know how to learn independently. It is the most suitable approach to achieve the aims of teaching learning process in any educational system.

According to experiences shared by Yagerand quoted by Amirali (2010), "We live in a dynamic society in which social political and technological conditions are changing constantly, so educationalists should analyze and evaluate the developments in order to decide an appropriate curricula and method of instruction which will make students ready for real life situation. Today, it is recognized that every person must be empowered to suggest possible explanations, to propose ways to test personal or class, to collect and interpret data obtained, to communicate the process and results to others."

In the light of the studies by Seng, (2010) cited by VanHattum, (2011), "This era of unprecedented breakthroughs in technology and constant change in many aspects of life, educators are challenged more than ever before with the need to develop students who will be adaptable in fast-changing environments. This calls for equipping students with better thinking skills and learning abilities. Concomitant with the quest for the development of skills pertaining to creativity and enterprise is the call for a paradigm shift in education" (VanHattum, 2011).

From the above discussion it becomes quite obvious that problem solving method is a reliable method for the development of problem solving ability and higher order skills. The Polya's Problem Solving method is the method through

which problem solving ability can be produced among the students in order to make them independent learner and show good achievements (Betne, 2010).

Before proceeding further it is essential to elaborate the following terms and then problem solving method:

2.12 Concept of Mathematical Problem

According to Answer (2012), Mathematical problem is defined to be a situation when there are some known and unknown variables in it and it requires some calculations to reach a solution. It challenges the curiosity of the student and let him/her to start from some known method move some new discoveries.

In his famous book How to solve it: "A New Aspect of Mathematical Method" Polya (1973), described two types of problems.

1. Problem to find
2. Problem to prove

2.121 Problem to find

The main focus of the 'problem to find' is to find the unknown certain value, object or the unknown of the problem. The unknown is also called "quaesutum" or the thing sought, or the variable required. The problem relevant to find something may be theoretical, practical, concrete, abstract, serious problem or some puzzles etc. While solving the problems which are related to find out some unknown, one has to find all sorts of unknowns, to acquire, to produce, to construct, to note, to infer all kinds of variables. Following are the few examples of the problem to find:

1. In the story of a murder case the unknown is the murderer.
2. In the chess game the unknown is the move of the opponent.

3. In the game of riddle the unknown is the missing word.
4. In a word problem about cost price and sale price the unknown is profit percentage.
5. In the algebra the unknown may be a number.
6. To find the LCM of two numbers the unknown is the LCM.
7. One angle is 30^0in supplementary angle to fine the 2^{nd} angle.
8. In construction problem the unknown is the figure.
9. In the problem of in-com tax the unknown is the tax.
10. In the problem of circle of given radios the unknown is area (Polya, 1973).

The problem related to find something has three basic components they are as:

- Unknown
- Data
- The condition

As Aslan (2012), quoted (Polya, 1954) says:

"A problem is composed of three components, "*data, objectives and processes*". Data corresponds to information provided as a part of the problem. Objectives define the finalization required for solving the problem. Finally, processes are potential activities regarding accomplished objectives of the solution."

2.12.2 Problem to Prove

The main focus of this type is to show categorically that a given claim is true or not true. The 'problem to prove' has to answer the stated question either true or false. For showing the worth of the claim either true or false it has to provide

evidence which is a proof. The problem to prove has the following basic components:

- Hypothesis
- Conclusion (which is to be proved)

Following are the few examples of the problem to prove:

1. Prove that the sum of the angles of triangle is 180^0.
2. Diameter of circle is twice than the radius of a circle.
3. Prove the theorem of Pythagoras.
4. Prove the De-Morgan's laws.
5. $(A \cup B)' = A' \cap B'$
6. If all the sides of a quadrilateral are equal, then the two diagonals are perpendicular.
7. If two parallel lines are cut by a line then the alternative angles are congruent.
8. Prove all the angles of a square are right angle.
9. Prove that the exterior angle of a triangle is greater in measure than its non-adjacent interior angles.
10. (AUB)UC=AU (BUC) (Polya, 1973).

The problem to find is more valuable at elementary level and problem to prove are more important at higher level. For solving a problem relevant to 'find' requires that one recalls the previous knowledge and facts by which it can be solved that this act is known as mobilization of knowledge. After mobilizing the knowledge,

the next step is organizing the knowledge. These are the two basic interlinked processes which are although complex in nature but exist simultaneously (Betne, 2010).

A good Mathematical problem is that which enhance the student's intellectual abilities and problem solving skills. Problem also captures the student's interest and curiosity. This fact is endorsed by, (Hiegert & Walle, 2003; Marcus & Fey, 2003; NCTM, 1991cited by Betne, 2010). This fact was also recommended through many researches that students should be exposed to such problems which are Mathematical in nature and develop problem solving skills (Walker & Lofton, 2003; Chin & Chia, 2004 cited by Answer, 2012).

Basic criteria for a Mathematical problem through which problem solving skill can be developed was described in the light of the finding of the studies by (Bailey, 2009;Betne, 2010; D'Agostino, 2011; cited by Yuan, 2013) is as under:

1. The problem has some calculations, and Mathematical procedures.
2. The problem must contain high level thinking and problem solving skills.
3. The problem helps the students in clarity of concepts.
4. The problem guides the teachers in assessment of students' progress and their learning difficulties.
5. The problem must has many solutions and develop the decision power of students in selecting the right one.
6. The problem must develop motivation among the students.
7. The problem develops higher abilities of cognition like application, analysis, evaluation and creativity.

8. The problem becomes a source of learning of different Mathematical concepts.
9. The problem creates the use of skill in their daily life.
10. The problem enhances the opportunity of using and practicing the skill in Mathematics.

Hence problem can be defined as an obstacle which obstructs and hinders an individual in achieving his/her goals. To turn disadvantage into an opportunity, individuals need to activate certain cognitive capabilities and psychological supports such as determination, courage and analytical thinking. So for solving problems students may need ability and the ability through which Mathematical problems can be addressed is known as problem solving ability (Adebule, 2004; Amazigo, 2000; Aslan, 2012).

2.13 Concept of problem solving

Problem solving is ability like swimming, riding, mountain climbing etc. Problem solving has several definitions but actually it is the brain's internal inborn function which can be developed through several problem solving methods (Polya, 1954; Wallas, 1926, cited Aslan, 2001; Wang, Wang, Patel, & Patel, 2006; Wilson & Clark, 1988; Kayan, 2007cited Yuan, 2013).

According to the facts found by Aslan (2012), regarding the problem solving, it means such a thinking which divides a problem or issue into sub-topics and then examining each of them one by one and reaching towards a solution. According to Ozden (2005), analytical thinking has a strong relationship with problem solving. Problem solving ability helps in improving self-confidence. Problem solving is a cognitive as well

as a behavioral process in which individual tries to manage him/her, and find out the ways to identify and cope with problem situations faced in daily life (Bailey, 2009).

In a research article Rizvi (2007), investigated the fact regarding the development of problem solving skill. He says "Problem solving skills can be acquired by means of teaching curriculum content by means of different teaching methods. A problem is an obstacle thrown before an individual. The individual is expected to remove the obstacle. In other words, the problem refers to the current status, and problem solving to required situation. Thanks to the problem solving method, students are able to cope with real life problems outside schools".

Problem solving ability is also helpful in creating a number of other skills among the students, they are as follow:

- Problem solving develops scientific thinking skills
- It helps in creating collaborative working skills
- By problem solving ability students can communication Mathematically
- Problem solving helps in building confidence and prediction skills
- It also helps in time management skills
- It promote skills of comparing and contrasting the real world
- It generates skills of visualizing information and reporting Mathematically
- It also enables students expressing and assessing themselves before the community (Rizvi, 2007).

So the problem solving ability development in classroom environment required a specific teaching method. The investigation of Canturk and Baser (2009),

"*Effects of Problem Based Learning on Students' Critical Thinking Skills* "cited by Anghileri, (2005), proves that,

"Problem solving skills can be taught by specialists in the field and by means of special methods. As a thinking style underscored in teaching Mathematics, critical thinking today is necessary for an individuals' success in any area."

2.14 Problem Solving Method (Introduction)

The ability of problem solving can be developed through problem solving method and the content for developing this ability is provided by the content of Mathematics (Yuan, 2013). According to Polya (1973), in his famous book "*How to solve it*" described problem solving is a practical skill like swimming. For acquiring any practical skill imitation and practice are required. For acquiring the ability to swim one needs to learn it though imitation of his trainer by keeping his head above the water and use of hands and feet. In the same way problem solving can be learnt through imitation what other people do when solving problems. The problem solving method introduced by Polya is the mostly used method. Before proceeding to the problem solving method; there is a brief look on the life history of this great Mathematician known as

2.14.1 George Polya

He was born in Budapest, Hungary on 13 December, 1887. His father Jakab Polya was a lawyer. Jakab Pólya died in 1897. At that time he left a wife, Anna, and five children. Polya completed his elementary education for the schools in Budapest after that he joined the Daniel Berzsenyi Gymnasium institute form that he learnt different languages like, Greek, Latin, German and Hungarian. He was not good in Mathematics at that time and according to Polya the reasons was his Mathematics

teachers and he called them as "despicable teachers". In 1905 Polya took admission in the University of Budapest. At that time he was interested in Philosophy but on the advice of his teacher Bernat Alexander he opted the subjects of Physics and Mathematics and it was the turning point in his life. According to statement given by Polya (1976), that,

> "I thought I am not good enough for Physics and I am too good for Philosophy, Mathematics is in between".

He was very much impressed by his Mathematics teacher Mr.Fejer, and praised his efforts and collaboration in his studies. He studied at the University of Vienna in the years 1910-11. He got his doctorate degree in Mathematics from the University of Budapest on the topic of 'A problem in the Theory of Geometric Probability". He got married in 1918 with a Swiss girl, Stella Vera Weber. In his academic career he wrote many research works on Mathematics. His immense contribution in field of Mathematics is difficult to mention. Geometric symmetry, enumeration of symmetry classes of objects was area of his interest. After spending a bright career at Stanford University, 1953 he retired. After his retirement he remained attached with the university and did a lot of work in Mathematics (Adebule, 2004; Amazigo, 2010).

He has many honors in his life. He was given the honor of as elected an honorary member of the Hungarian Academy Association, the London Mathematical Society, the Mathematical Association of Great Britain, and the Swiss Mathematical Society. He was also an elected member of the National Academy of Sciences of the United States. He was also given the opportunity to join the American Academy of

Arts and Sciences, the Academia International de Philosophie des Sciences de Bruxelles, and the California Mathematics Council. Heworked as the corresponding member of the Académie des Sciences in Paris (Yan, 2013).

As cited by Yaun (2013), George Polya is known as the father of the modern focus on problem solving in Mathematics education. He was a renowned Mathematician he has written many books on the subject of Mathematics but "How to Solve" is his famous book. In that book Polya suggested the four step method of solving a problem. This method is the most used method to develop the skill of problem solving. This great Mathematician died on 7th September, 1985 in California, USA.

2.15 Problem Solving Method

The human beings have curiosity, which compel them for asking questions and seeking its answers. The curiosity remained unsatisfied until it gets its answer. As life is full of problems and successful man in life is he, who is fully equipped with adequate knowledge and reasoning power to tackle these problems. The solution of these difficulties/problems enables him/her to have a mastery over his environment. There are several definitions of problem solving for instance:

According to Gagne (2008) cited by Sadia (2010) "Problem solving is a set of events in which human being try to achieve some goals"

According to Risk (2005) quoted by Bailey (2009); "Problem solving may be defined as a process of raising a problem in the minds of students in such a way as to stimulate purposeful reflective thinking in arriving at a rational solution"

According to Ausubel referenced by Betne (2010), "Problem solving involves concept formation and discovery learning" According to Polya (1976), "Problem solving is a method to develop the ability of Mathematical problem solving. It enables the students to become independent discoverer and by this method they can solve daily life Mathematical problems. It helps in developing higher level abilities like analysis, synthesis, evaluation and creativity."

2.15.1 Steps of Problem Solving Method

It is a four steps method which works systematically to reach the solution of a Mathematical problem.

2.15.2 Step-1 Understanding the Problem

In this step, the given problem is understood with respect to the given data. For the understanding several questions, figures and diagrams may be asked and construct. All these depend upon the nature of the problem.

Example:

For the illustration of step-1 regarding the understanding of the problem, take an example given by the author in his book "How to solve it", that find the diagonal of a rectangular parallelepiped of which the length, the width, and the height are unknown.

Before starting the first step it is essential that students may have some prior knowledge about Pythagoras theorem and its uses in plan Geometry. So it is the point for the teachers who want to use this method before solving a problems students must have some prior knowledge about the variables in the problem, it may not be new altogether for them. For providing the clear concept about the problem teacher

may use concrete example so students may conceive that what is the actual problem and what is actually given (Polya, 1976).

The problem can be made interesting and concrete by giving the example of classroom, which is in a rectangle parallelepiped shape and whose measurements can be calculated. So keeping in view the classroom teacher may ask the following question for the understanding of the problem:

- What is given, the length, the width, and the height of parallelepiped?
- What is unknown, the length of diagonals of the parallelepiped.
- Introduce the notations for known and unknowns
- Let's length=a, width=b, height=c, and length of diagonal=x
- What is the condition, if we know length, width and height than diagonals can be find
- Do you think it is enough data to precede further students?
- If yes than we have to move towards the next stage (Polya, 1976)

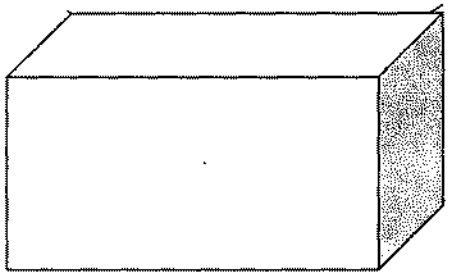


Figure 4 Find the area of parallelepiped

In the first step while understanding the problem there may be the conditions. These conditions may be of three types;

1. Insufficient condition

2. Redundant condition

3. Contradictor condition

In short, at this stage, three type of basic information are required to solve the problem they are:

1. What is the given data?

2. What is to find?

3. What is the condition (Polya, 1976 p.40).

2.15.3Step-2 (Devising a Plan)

At this step students are motivated to find out links between data given and the unknown. This stage provides deeper understandings about the problem. For this student are asked about a similar nature problem. If students came to know about a similar problem they are asked to recall and solve that one. The way from understanding the problem to conceiving a plan may be long and difficult. The plan for a problem can be prepared through a 'bright idea' or an 'auxiliary problem'.

The bright idea

The bright idea may be the result of sudden idea or generated through series of trials. For developing a bright idea teacher has to help the students unobtrusively. It can be provided through discussion with students. How the students can be helped in creating bright idea teacher has to consider him/herself as a student. Ideas required to solve the problem need experience and practice. Therefore, it is recommended that teacher starts on this by asking the student that "Do you know an example related to this problem?" The difficulty is that there are numerous relevant problems to the given problem. It is difficult to select the right one. Sometimes we have to modify

the idea according the problem. For reaching to a right idea, teacher may ask the following questions:

- By knowing the unknown, do you know or have some such example
- Can you solve such a problem?
- Look carefully before you remember problem like that (Polya, 1976).

The Auxiliary problem

An auxiliary problem is which has to be considered for devising plan for the solution of the given problem. The solution of the original problem is the end and the auxiliary problem is the way or means by which we reach to the solution (Polya, 1976 p.51).

An auxiliary example can be illustrated by Polya (1976), that an insect tries to come into a room through a windowpane which is close and the insect tries again and again and does not try the next window which is open and through which it comes into the room. Contrary to insect, human beings are more intelligent. Human has intellect and by using this he/she tries to overcome an obstacle directly as well as indirectly by devising plan. So for solving a problem directly sometimes we need an auxiliary example to solve it. For instance if we have to find the value of 'x' for the equation:

$$X^4-12X^2+20=0$$

For solving this problem we can change $X^4=(X^2)^2$ and develop an auxiliary example equation which is known to students, that is

$$\mathbf{Y=X^2}$$

Then new equation will be $Y^2-12Y+20=0$

So students are used to solving the auxiliary equation and by the help of this they can find the solution of the given problem.

For example in a word problem in which profit or loss percentage was unknown, auxiliary example for this question may be the question of finding percentages of different numbers.

Following are the advantages of auxiliary examples for solving the original problem:

- Guidance to solve the original problem
- Time can be save through it
- Provide insight into the original problem
- Easy to follow the procedures
- Calculations become easy

There are also some risks with these auxiliary problems which are as follows:

- Time and efforts may be wasted
- Choice is based on judgments
- No criteria for selecting the auxiliary problem
- Equivalent problem selection is a risk
- Difficult to differentiate between two problems for instance two theorems are sometimes not identical.

Hence in the step-2 of problem solving of 'devising plan' is the most important step. It is the step which leads from original problem to the auxiliary problem and this process is known as convertible reduction or bilateral reduction or

equivalent reduction. Another way to reaching towards a bright idea or an auxiliary problem is the use of heuristics.

Heuristics:

According to Polya (1973), cited by Tutkun & Okay (2012), "heuristics are rules of thumb for making progress on difficult problems". According to Schoenfeld (1985), the heuristics are the general suggestions on strategy which leads towards the solution of a problem. These are the ways and methods which are used previously to solve the problems and by the use of them new relevant problems can be solved (Bruner, 1960).

To solve the current problem in Mathematics with the help of the way which has been used somewhere heuristic is the basic part of problem solving. It has become synonymous with problem solving (polya, 1973; Schoenfeld, 1985; Rubinstein, 1986; Mayer, 2003; cited by Tutkun & Okay (2012). Heuristics is the branch of study in which rules of discoveries are taught it is not a discovery itself (Polya 1976, p.112). In the process of devising plan for solving the problem in Mathematics heuristics play a vital role; it helps in finding a bright idea to solve the original problem (Schoenfeld, 1985).

Not all heuristics can be used to solve a problem; in fact sometime we use heuristics according to our own experiences, or learn by other people by observing them. According to Schoenfeld (1985), explained the process as "Occasionally the person solves a problem using a technique that was successful earlier, and something clicks...If that method succeeds twice, the individual may use it when faced with another similar problem. Over a period of years each individual problem solver comes to rely".

Keeping in view the importance of heuristics in problem solving process, quoted by Tiong (2005), the Singapore Mathematics syllabuses developed by Curriculum Planning and Developing Division (CPDD), Ministry of Education Singapore (MOE), have identified thirteen heuristics, they are as follows:

1. Act it out
2. Construct a figure/model/diagram
3. Apply guess and check
4. Try to develop a check list to solve problem
5. Identify the similar patterns
6. Work in reverse order
7. Apply before-after approach
8. Make assumptions
9. Develop some auxiliary problem
10. Solve the problem
11. Solve different parts of the problem
12. Work on relevant problems
13. Use different equations for solution

So the step-2 is the most important part of the process of problem solving.

2.15.4 Step-3 (Carrying out the Plan)

After a careful planning at the step-2, what has been decided is now implemented to reach at a solution. To devise a plan, to conceive the idea for the solution is not an easy task; it takes a long exercise. Contrary to carry out is easy. The plan provides the general outlines for implementation and we have to put data into it

and find either it is working or not .If the students develop the plan to solve the problem it is good for the teacher because much of work has been done now. The major danger at this stage is that students may forget the plan, but it happens when student borrow it if they develop themselves it provides them satisfaction (Polya 1976, p.13).

Students reach to the correct plan either through intuitively or formally, but every step of the plan may be clear for implementation. For instance in the example

$$X^4-12X^2+20=0$$

It was conceived that the solution plan would be as:

Let $X^4=(X^2)^2$ and $\mathbf{Y=X^2}$

Then new equation will be $Y^2-12Y+20=0$

Now check it so $Y^2-2Y-10Y+20=0$

$$Y(Y-2)-10(Y-2)=0$$

$$(Y-2)(Y-10)=0$$

Then $(Y-2)=0$ and $(Y-10)=0$

$Y=2$ and $Y=10$

By this it can be understood that the plan is working and problem can be solved by this plan. Teacher has to play a facilitator role while students are implementing the plan (Polya, 1976).

2.15.5 Step-4 (Looking Back)

This is the step where students have to confirm their solution by applying it in a new situation. In this step students seek new arguments and try to recheck their

findings by changing the known with unknown. By looking back means reconsidering, reexamining the results and the process of solution for the consolidation of their knowledge and develop the ability to solve such examples independently in their daily life (Polya 1976, p.15).

Teacher has to explain that this solution of the problem is not the destination, but it is merely a milestone which is travelled. For further improvement, teacher has to ask the following questions from the students:

- Do you check the results once again step by step?
- Do you have the understanding of all the variables regarding known and unknown?
- Can you derive the results differently?
- Do you examine all in a glance?
- Are you able to use this method to solve some other examples?
- Are you able to change the condition among the variables?
- Can you develop some example from daily life? (Polya 1976, p.16).

These are the four steps of problem solving method. By these the ability to solve problem can be developed among the students. The success of this innovative method depends upon the training of the teacher. For this purpose, teacher has to ask the special question which helps the students in learning of this method (Goldrick, 2007).

There is a paradigm shift from teaching problem solving to teaching through problem solving (Cited by Taplin, 2005; Lester, Msainglla, Mau Landin, 1994). By problem solving method students can construct a deep understanding of conjecturing,

exploring, testing, analysis, synthesis, evaluation and creating (Schoenfeld, 1994). The problem solving method is an important teaching tool in Mathematics because it is the source through which functional, logical and aesthetic objectives of teaching of Mathematics can be achieved (Schoenfeld, 1994). According to Polya (1980) "If education fails to contribute to development of the intelligence, it is obviously incomplete. Yet intelligence is essentially the ability to solve problems: everyday problems, personal problems."

Many researchers have proved the effectiveness of problems solving method on different faculties of mind Polya, 1980; Thompson, 1985; Carpenter, 1985; Groves, 1985; Schoenfeld, 1995; Stigler, 2003; Betne, 2010 cited by D'Agostino, 2011). Following are the few faculties of mind on which this method works very effectively:

- Cognition
- Metacognition
- Knowledge
- Critical thinking
- Creative thinking
- Synthesis
- Analysis
- Practical mindedness
- Ability to solve daily life problems
- Ability to correlate
- Ability of observation

- Interest in the subject

Although Polya problem solving method helps in developing many mental abilities, but these abilities may be presented in learning outcomes. The taxonomy of educational objectives is a frame work through which we can get the results of instruction from the students. Hence to confirm the effectiveness of the Polya problem solving method it is necessary that it may be checked against some standard taxonomy.

2.16 Bloom's Taxonomy of Educational Objectives

Benjamin S. Bloom, Associate Director of the Board of Examination of the University of Chicago and his team were assigned the job to create item banks for the different universities for measuring the same educational objectives. For this assignment many eminent measurement specialist were included across the United States. The whole team under the leadership of Bloom met twice in a year in1949. The work was completed and published in 1956, and the final draft was titled "Taxonomy of Educational Objective: The Classification of Educational Goals Handbook 1:Cognitive Domain" (Bloom, Engelhart, Furst, Hill, &Krathwohl, 1956 cited by Tutkun & Okay, 2012).

According to Krathwohl (2002), this taxonomy served as a measurement tool and a common language for describing educational objectives for the subjects and learning outcomes. Bloom's taxonomy was considered to be used for the following purposes:

- It can be used as a source of communication among the subject matter, teachers and evaluators.

- It can be used for determining the aims of the curriculum according to national, state, and local standards.
- It can be used for measurement and evaluation of the course and program.
- On these bases, range of educational possibilities can be determined.

The Bloom's taxonomy consisted of six levels of cognitive domain; every domain was clearly divided into its sub levels. The order of proceeding was based on simple to complex and from concrete to abstract. It is presented in a complete hierarchy; each category was prerequisite for the next category (Krathwol, 2002).

As stated by Tutkus (2012), in the beginning of the Bloom's taxonomy term 'Taxonomy' was a new term in education. The educationists did not know the meaning of this; therefore, it was given less attention. But with the passage of time this term was gain the currency and widely known and cited. Bloom's taxonomy was translated into 25 languages. This taxonomy consisted of six major classes and each class has its sub-levels.

2.16.1 Structure of the Taxonomy

Cognitive Domain

Sub-level

1-Knowledge

- Knowledge of specifics
- Knowledge of terminology
- Knowledge of specific facts
- Knowledge of ways and means of dealing with specifics
- Knowledge of conventions

- Knowledge of trends and sequences
- Knowledge of classifications and categories
- Knowledge of criteria
- Knowledge of methodology
- Knowledge of universals and abstractions in a field
- Knowledge of principles and generalizations
- Knowledge of theories and structures

2-Comprehension:

- Translation
- Interpretation
- Extrapolation

3-Application

- Use of knowledge, facts, rules in different ways
- Put theory into practice
- Solve the problem
- Use of facts and knowledge in a new situation
- Use knowledge to infer and predict new things
- Provide convergent thinking

4-Analysis

- Analysis of elements
- Analysis of relationships
- Analysis of organizational principles

5-Synthesis

- Production of a unique communication
- Production of a plan, or proposed set of operations
- Derivation of a set of abstract relations

6-Evaluation

- Evaluation in terms of internal evidence
- Judgments in terms of external criteria (Krathwol, 2002).

Table 2.2

The action verb used against each sub-level

Skill	Definition	Key Words
Knowledge	Recall information	Identify, describe, name, label, recognize, reproduce, follow
Comprehension	Understand the meaning, paraphrase a concept	Summarize, convert, defend, paraphrase, interpret, give examples
Application	Use the information or concept in a new situation	Build, make, construct, model, predict, prepare
Analysis	Break information or concepts into parts to understand it more fully	Compare/contrast, break down, distinguish, select, separate
Synthesis	Put ideas together to form something new	Categorize, generalize, reconstruct
Evaluation	Make judgments about value	Appraise, critique, judge, justify, argue, support

Source: Cossta (2000): *Developing minds: A resource book for teaching thinking.* Alexandria

2.17 Rational for Revised Bloom's Taxonomy

In 21st century Psychological and educational researchers have developed many learning theories and approaches. According to Alagic (2009), among these new theories of learning are the constructivism, Metacognition and Self structured learning. In the light of these theories learning is considered now as "a proactive activity" which required self-initiated motivation and behavioral processes along with meta-cognition.

In the present era of knowledge educators have more knowledge about the processes of learning and its evaluation; teachers have new methods of teaching (Startalk, 2009, cited by Tutkus, 2012).

As stated by Tutkus (2012), "The Bloom's taxonomy was designed to classify objectives of curriculum in terms of implicit and explicit cognitive skills and abilities. Taxonomy is accepted as one of the important studies that affect the curriculum in 21st century. For instance, a search engine shows 455,000 results for Bloom's taxonomy."

This taxonomy was so comprehensive that it survived for a very long time. Despite the various taxonomies, only Bloom's taxonomy is accepted (Forehand, 2005). But due to the advancement in the field of Psychology and Education it was felt that this taxonomy required some revision to meet the diversified needs of the 21st century (Tutkus, 2012).

The advancement in the field of assessment and evaluation also describes some limitations of the Bloom's taxonomy. Few of them are as follows:

2.17.1 Emerging Demands of New Learning Theories

According to Marzona (2000), the author of his own taxonomy "Marzona Taxonomy of Educational Objectives" pointed out that it is not fit for the emerging demands of new learning theories. The hierarchy structure of the Bloom's taxonomy, moving from simplest level to difficult level of evaluation has no empirical evidences. This fact is not satisfied by the cognitive processes in Bloom's Taxonomy.

The new theories of Constructivism, Met cognition and Self Structure Learning (SSL) required new learning domains which were not fulfilled by the old Bloom's Taxonomy. The theory of Constructivism focused how students create knowledge while they are engage in learning. Creation of new knowledge requires both comparison of new learning with old learning and using various cognitive processes for this learning. This framework changes the entire learning process and students are exposed to a new learning environment. This environment provides an opportunity to select knowledge according to his/her requirement. In the present scenario, it is expected that every learner may make progress on periodic bases. Due to this reason combination of educational objectives and teaching evaluation is more vital than ever merged. Therefore, it was the need of time that it may be revised (Pickard, 2007).

2.17.2 Uni-dimensionality of Bloom's Taxonomy

The Bloom's Taxonomy was consisted of both noun and verbs forms. The target dimension are described as noun form situated in the bottom steps of knowledge while verb forms used to describe cognitional process is defined as students' remembrance of knowledge (Krathwohl, 2002).

The old Taxonomy was Unidimenstional and unable to explain the two way cognitive process of learning. This fact also became a cause of revision of old Bloom's Taxonomy (Pickard, 2007).

2.17.3 Cumulative Structure

Among many limitations one of them was its cumulative structure. It moves according to the rule simple to complex. Every proceeding stage required the completion of next stage. The Bloom's Taxonomy has rigidity in its cognitive process. It is also difficult to draw demarcation among the levels of cognitions because they are overlapping, for instance terms analysis and evaluation are overlapping (Krathwohl, 2002).

According to Farzad & Hassan (2010), Mathematics is a very important subject in the society and every member who is contributing towards society must know Mathematics. However Mathematical achievement is based on the learning of the students (Suurkamm, & Vezian, 2010).

As cited by Farzad & Hassan (2010), the statement of (Kilpatrick, Swafford &Findell, 2001), that "All students must learn to think mathematically, and they must think mathematically to learn". It was become a big challenge for the Mathematicians to design and implement standard based Mathematics curriculum, reliable assessment process.

According to the criticism on Bloom's taxonomy given by Tutkun & Okay (2012), numerous changes have been taken place due to the advancement in knowledge and culture which influenced the thinking patron and assessment in education, the Bloom's taxonomy is not fulfilling the new requirements of

assessment. This taxonomy was developed with the aim to assist the teachers, administrators and researchers to tackle the problems of education with greater precision (Bloom, 1994).

By citing (Smith, Coupland,& Stephen,1996; Zimmerman, 1998; Anderson et al, 2001) Tutkun & Okay (2012), described that innovation in learning theories such as Information Processing Theory (IPT) , Constructivism and Metacognition has changed the learning patrons and students have their own ways of thinking and learning. Learning has become as proactive and needs new ways and techniques of learning and assessment. The old Bloom's taxonomy was not compatible with these new modifications in learning and assessing students' achievements in Mathematics. In the light of these changes it was suggested that this Taxonomy may be revised and made compatible to the new changes in learning and assessment field (Anderson et al, 2001).

2.18 Revised Bloom's Taxonomy

Anderson who was the Bloom's student, with a team of eminent cognitive psychologists revisited the Bloom's Taxonomy and provided its revised version in 2001. Keeping in view the new developments in the field of education, students learning ways, new assessment and evaluation methods, and teacher's lesson planning the old taxonomy was revisited and Revised Bloom's Taxonomy (RBT) was introduced (Anderson, 2001).

According to the words by Tutkun & Okay (2012), in Revised Bloom's Taxonomy significant changes were made to address the limitation of old Bloom's taxonomy. The revised Bloom's Taxonomy has three main areas of changes with

respect to old taxonomy (Forehand, 2005). Through these changes it became more comprehensive and relevant to modern learning theories (Anderson, 2003). Following are the main areas of changes:

2.18.1 Changes in Terminologies

The revised version of Bloom's Taxonomy has changed noun to verb. The process of synthesis is replaced with creativity and it is kept at the highest level in the Revised Bloom's Taxonomy because creativity is the result of combination of many parts of knowledge. According to Farzad (2010), Revised Bloom's Taxonomy (RBT) provided a very comprehensive view of all terms term in verb which is as under;

Table 2.3

Comparisons of old and Revised Bloom's Taxonomy

Old Bloom's Taxonomy	Revised Bloom's Taxonomy
Knowledge	Remembering
Comprehension	Understanding
Application	Applying
Analysis	Analyzing
Synthesis	Evaluating
Evaluation	Creating

Source: A Study on Mathematical Performance Based on RBT (Farzad, 2010)

2.18.2 Structural Changes

According to the change with respect to structure between the old and revised taxonomy is the difference of dimensions. The original Bloom's Taxonomy was one dimensional which was moving in a hierarchy from simple to complex, while the

RBT is of two dimensional. One dimension represent the 'knowledge dimension' while the other is 'the cognitive process dimension'. Hence the RBT divided the noun and verb into two separate dimensions. The knowledge dimension is divided into four parts which is 'noun aspect' and the learning outcome on the student's part is divided into six parts which is 'verb aspect' (Forehand, 2005; Amer. 2006 & Answer, 2012 cited by Tutkun & Okay, 2012).

Revised Bloom's taxonomy table is given below:

Table 2.4

Structural changes in Revised Bloom Taxonomy

Knowledge dimension ⇩	⇨ THE COGNITIVE PROCESS DIMENSION					
	1	2	3	4	5	6
	Remembering	Understanding	Applying	Analyzing	Evaluating	creating
FACTUAL KNOWLEDGE						
CONCEPTUAL KNOWLEDGE						
PROCEDURAL KNOWLEDGE						
METACOGNITIVE KNOWLEDGE						

Source: A Study on Mathematical Performance Based on RBT (Krathwohl, 2002; cited in Tutkun& Okay, 2011).

2.18.3 Changes in Emphasis

There were three categories in old Bloom's Taxonomy which were (Factual, Conceptual and Procedural) presented with a cutline in RBT with a new addition of

category 'Met cognitive knowledge'. It is the way how knowledge can be acquired by someone. Students can adjust their ways of learning and can come to know about their strength and weaknesses. The RBT provided an authentic and reliable tool to the curriculum developer, educational planner, teachers for instructional delivery and evaluators for assessment and evaluation. (Forehand, 2005 cited by Tutkun & Okay, 2012).

The RBT has provided a large range of teaching activities for teaching and assessment and redefine the teaching learning process. This fact is justified by the words of Pockard (2007), that "RBT enables educators to identify which knowledge they expect students to use and to determine which cognitive process dimension is used".

2.18.4 Cognitive process dimension of RBT

This dimension of the taxonomy is divided into six categories which are described in 'verb'. It represents the process of learning and students are expected to learn in the result of teaching (Anderson, 2000).

1-Remembering:

In this dimension it is expected that students may recognize and recall the relevant information and knowledge from the long term memory. This dimension is consisted of two main sub classes i-e recognizing and recalling or it is an ability to remember the previous knowledge and recall it at the time when it is required (Anderson, 2000).

Table 2.5

Remembering Cognitive Process

Cognitive Processes	Examples
Remembering—Produce the right information from memory	
Recognizing	• Identify the shapes of geometry. • Find rectangular shapes in your neighborhood. • Answer any true-false or multiple-choice questions or fill in the blanks.
Recalling	• write types of angles. • Write the multiplication facts. • Reproduce the formula for finding loss and profit.

2-Understanding

This dimension of the cognitive process deals with the ability among the students that they can grasp meaning, explain and restate ideas, can provide their own meaning to the knowledge. It has its sub-levels include interrelating, exemplifying, comparing and summarizing (Anderson, 2000).

Table 2.6

Understanding Cognitive Process

Cognitive Processes	Examples
Understanding—Make meaning from educational materials or experiences	
Interpreting	• Translate a story problem into an algebraic equation. • Draw a diagram with the given measurements.
Exemplifying	• Construct a parallelogram. • What should be added to 89 becomes a complete square.
Classifying	• Label the sides of right angle triangle. • List the types of angles.
Summarizing	• Make two similar triangles • List the key components required in solving problems related to circle.
Inferring	• In a right angle triangle ABC $c^2=a^2+b^2$ is example of the theorem known as. • Look at a series of numbers and

	predict what the next number will be.
Comparing	• Separate the quadratic equations from the given data. • Use a Venn diagram to demonstrate how two books by Charles Dickens are similar and different.
Explaining	• Draw a diagram explaining the equilateral triangle. • Describe how interest rates affect the rate of return.

3-Applying:

This dimension of cognitive process deals with the ability of using acquired knowledge in a similar or new situation. It shows its learning outcome in the form of executing and implementing in a situation either new or old.

Table 2.7

Applying Cognitive Process

Cognitive Processes	**Examples**
Applying—Use a procedure	
Executing	• Add a column of two-digit numbers.

	•write a biggest number in four digits.
Implementing	• Design a singular matrix. • solve the equation with the use of quadratic formula.

4-Analyzing

Analyzing is the cognitive process, of dividing knowledge into its parts and studies the parts to understand the whole. The learning outcomes of this ability are differentiating, organizing and attributing. It is also included in Higher Order Thinking Skill (HOTS).

Table 2.8

Analyzing Cognitive Process

Cognitive Processes	**Examples**
Break a concept down into its parts and describe how the parts relate to the whole	
Differentiating	• List the important information in a Mathematical word problem. • Construct a diagram showing the diagonals of a square are equal.
Organizing	• Make a list of fractions in ascending order.

	• Arrange the numbers according to the condition given.
Attributing	• Enlist the characteristics of even numbers. • Discuss the properties of rational numbers

5- Evaluating:

This ability also belongs to higher order thinking skill. The learning outcome of this ability includes checking and critiquing.

Table 2.9

Evaluating Cognitive Process

Evaluating—Make judgments based on criteria and syllabus guidelines	
Checking	• solve the quadratic equation and check it by formula. •which number does not belong to the series.
Critiquing	•Evaluate how well a project meets the criteria of a rubric. •Choose the best method for solving a complex Mathematical problem.

6- Creating:

This dimension is new in RBT; it was not included in the old taxonomy. This is the highest ability and replaced by the synthesis of the previous taxonomy. It involves putting knowledge together for the creation of new knowledge and developing something new. It includes generating, planning and producing. The learning outcomes of this ability are based upon combining the pieces of knowledge for the creation of new ideas and things. For assessment of this ability students have to accomplish tasks like creating plan and produce new things (Anderson, 2000).

Table 2.10

Creating Cognitive Process

Creating-Put pieces together to form something new or recognize components of a new structure.	
Generating	• Draw a square whose area is equal to the perimeter of the rectangle. • Generate a formula for finding the value of unknown.
Planning	•Outline the steps of construction of a RAM, ROM • Design a diagram having circle in a triangle.
Producing	

	• Write the importance of Mathematics in daily life. • Draw inclined plan with different angles.

2.18.5 The Knowledge Dimension of RBT

In the two ways RBT the second dimension is of knowledge. It has four dimensions and every dimension represent its unique features. It is represented in the following table:

Table 2.11

Knowledge dimensions in RBT

The Knowledge Dimension Factual Knowledge—Basic information	
Knowledge of terminology	Vocabulary terms, Mathematical symbols, musical notation, alphabet
Knowledge of specific details and elements	Components of the Food Pyramid, names of congressional representatives, major battles of WWII
Conceptual Knowledge—The relationships among pieces of a larger structure that make them function together	
Knowledge of classifications and categories	Species of animals, different kinds of arguments, geological eras
Knowledge of principles and generalizations	Types of conflict in literature, Newton's Laws of Motion, principles of democracy

Knowledge of theories, models, and structures	Theory of evolution, economic theories, DNA models
Procedural Knowledge—How to do something	
Knowledge of subject-specific skills and algorithms	Procedure for solving quadratic equations, mixing colors for oil painting, serving a volleyball
Knowledge of subject-specific techniques and methods	Literary criticism, analysis of historical documents, Mathematical problem-solving methods
Knowledge of criteria for determining when to use appropriate procedures	Methods appropriate for different kinds of experiments, statistical analysis procedures used for different situations, syllabus guidelines for different genres of writing
Metacognitive Knowledge—Knowledge of thinking in general and your thinking in particular	
Strategic knowledge	Ways of memorizing facts, reading comprehension strategies, methods of planning a Website
Knowledge about cognitive tasks, including appropriate contextual and conditional knowledge	Different reading demands of textbooks and novels; thinking ahead when using an electronic database; differences between writing emails and writing business letters
Self-knowledge	Need for a diagram or chart to understand

	complex processes, better comprehension in quiet environments, need to discuss ideas with someone before writing an essay

Source: Bloom's Taxonomy: A New Look at an Old Standby (2012)

2.19 Conclusion

The original taxonomy is still in action and used by curriculum developers, researchers, teachers and evaluation experts. The old/original version of taxonomy has some criticisms on it that were:

- Internal inconsistency in the hierarchy
- Overlapping of the cognitive levels
- Learning does not follow a sequential progression patron
- Taxonomy does support the modern learning theories (Suurkamm & Vezian, 2010)

Keeping in view the limitations of the old taxonomy, the revision published by Lorin Anderson and his fellows in 2001, by the name "*A Taxonomy for Learning, Teaching, and Assessing: A Revision of Bloom's Taxonomy of Educational Objectives*". It also gained the widest acceptance as the old one. The revised Bloom's Taxonomy is the extension of the original old framework and not a new taxonomy (Munzenmaier & Rubin, 2013).

Among many reasons for revision following are the major reasons:

1. To enhance the new understanding of learning and new methods of teaching.

2. To aligning curriculums and assessment and evaluation.
3. Addition of new category that is 'creating'.
4. Conversion of nouns into verbs.
5. One dimension to two dimensions.
6. Revised taxonomy represents students thinking process not the behaviors.
7. Cognitive process and knowledge dimension are separate.

The intent of the present study was to find out the effect of Polya's Problem Solving method on the revised Bloom's Taxonomy. Polya's Problem Solving method claims that different abilities among the students can be produced among the students with respect to Mathematics. These abilities are well explained in the revised Bloom's Taxonomy. The learning outcomes are very important in teaching learning process. Before applying any method there may be some learning outcomes. Evaluation is based on these pre-determined objectives (Krathwol, 2002).

So it is important to check the effectiveness of Polya problem solving method against some set learning out comes. The intent of the present study was to check the effect of Polya Problem solving method on the revised Bloom Taxonomy. The method has been checked on Bloom's Taxonomy, but the present study was innovative in the way that it was checking the effect of this method on revised Bloom Taxonomy (Krathwol, 2002).

2.20 Related studies

According to the result of the studies of Kloawole, Olasosu & Ajetunmobi (2013), showed that Problem Solving Method of instructions was significantly better than the conventional method for the achievement of Higher Order Thinking Skills.

For this they concluded that method of instruction used by the teacher has effect on student's achievement in Mathematics. The results were endorsed by the findings of Nikandrov (1990) cited by Kloawole (2013), which showed that ability to solve problem can be developed through a very good strategy of teaching.

The 'chalk and talk' or conventional method is no longer effective for the development of higher level ability in Mathematics. Traditional methods of teaching cannot satisfy the complexity of the modern technological society. So it may be replaced with new and approved methods of teaching of Mathematics (Ubuz, 1994; Dongpong, 2000; Wang, 2001Carson, 2007; cited by Kloawole, 2013).

The findings of the study of Peng (2002), showed that the problem solving method was found not influenced by sex difference. From this it can be concluded that this method could be used to improve the performances of male and female students in Mathematics. This fact was supported by the findings of Kolawole and Popoola (2011), Adebule (2004) and Ibitola (2003) cited by Kolavole (2013), that sex of students has no significant influence on the effectiveness of a very good instructional method.

While discussing the results of his research study Yuan (2013), states that through problem solving method, students can be made to learn the abilities of reasoning and solve daily life problems. According to the conclusion drawn by Amirali (2011), in their research study that the reforms in the curriculum of Pakistan and other parts of the world strongly recommend the problem solving approaches for teaching of Mathematics at schools level. Conventional methods cannot stay longer to address the new advancement.

In the study of (Amirali & et al, 2010) the objectives tested were:

(i) To determine the role of problem solving method in the academic achievement of students in Mathematics at elementary level.

(ii) To compare the achievement of students taught by problem solving method and students taught by traditional method.

The results were in the favor of problem solving method and on the basis of the findings in that study, the following conclusions were made:

- Students taught through problem solving method achieved better than those taught by traditional method.
- There exists a significant difference in the achievement of Mathematics students taught through problem solving method and traditional method.
- Difference between the achievements level is due to problem based strategy, otherwise both groups have equal basic knowledge of Mathematics (Amirali, 2011).

The recommendation given in the light of findings and conclusions showed that problem solving method of teaching Mathematics is more effective than the traditional method and teachers of Mathematics may use problem solving method for the better results and understanding in Mathematics.

According to Hutt (2009), the student's achievement is the main concern of educational psychologists over the last decades. The achievement is based on the knowledge provided to the student's before the assessment. This is a fact that achievement may be assessed on different cognitive levels. As Anderson (1995), cited by Hutt (2009), factual knowledge means knowledge of facts and procedural

knowledge means knowledge about process now they may be checked through different methods. Therefore, taking into account the different nature of knowledge different assessment levels are required. Similarly, in the revised Bloom's taxonomy (Anderson & Krathwohl, 2001) quoted by Hutt (2009), the division is made between different types of knowledge and cognitive processes which are four knowledge levels and six levels of cognitive process. The taxonomy presents two ways to the desired product of learning, that is, the kind of knowledge to be learned, and the cognitive process along which the knowledge can differ. The taxonomy provides a viewpoint on how different types and levels of knowledge could be assessed at different levels and sub levels. From that it can be inferred that the revised Bloom's Taxonomy is a suitable tool for the assessment of achievement at different levels (Hailikari et al, 2007).

Effect of any instructional method can be assessed through achievements scores by applying reliable tool for assessment, this fact is endorsed by many researches like (Portier, 1995; Dochy, 1996; Dochy& McDowell, 1997; Anderson & Krathwohl, 2001 cited by Hutt, 2009).

The Revised Bloom's Taxonomy is a reliable tool for the assessment of achievement of different levels.

According to Takbir (2010), in his research paper regarding the learning difficulties of students in the subject of Mathematics described the worth of in-depth learning for students, that through it a number of personal needs can be fulfilled like satisfaction and it accelerates the process of further learning for the mastering of new material. The traditional method of teaching is a source of surface learning through it

in-depth knowledge could not be developed for this some specialized method may be used for instruction, like problem solving. Polya's problem solving method claims that through this higher level cognitive abilities can be developed in students (Polya, 1976).

This fact has been proven through the empirical evidences by the researchers (e.g. Johson, 1985; Prawat, 1989; Kilbourn,1992; Halfords, 1993; Sierpinska, 1994; Newton, 2002, cited by Takbir, 2010), points out that students who acquired the subject knowledge through understanding show an enhanced ability to think flexibly dealing with new problems. It makes the learner independent discoverer who can interact with the world and think for themselves and solve daily life problems.

In-depth learning in the subject of Mathematics of students is an essential goal but according to Takbir (2010), "it is and has not been a central concern in every classroom in the world in general and in Pakistan in particular". According to a survey reported by many researchers that "The current literature reveals that understanding avoidance is not unique to the context of Pakistan and other developing countries where the quality of education is considered to be poor; it is and has been a matter of concern in the context of developed countries as well, where there is a tendency to emphasize memorization and reproduction of information" (e.g. Wildy & Wallace, 1992; McLaughlin& Talbert, 1993; Das & Barunah, 2010 cited by Takbir, 2010).

To inculcate the in-depth learning is a difficult task because deep cognitive process is required for this. Students do not engage themselves in this process except that they are motivated towards this process. Students have many potential, abilities,

skills, perception, expectations and preferences which may be suitable or not suitable for in-depth understanding (Takbir, 2010). It is due to two factors, first external factor and second is the internal factor. The external factors includes parents, career aspirations, different influences and teachers, while the internal factors are intrinsic motivation and interests (Takbir, 2010). Researchers (Stoll, 1999; Lockheed & Verspoor, 2000; cited by Takbir) argued that "The teacher's interaction with learners as the axis on which education quality of learning turns". In the same way the National Curriculum 2006 and the Education Policy 2009 states that a marked change is required in teacher's role from transmitter of information to creator of learning environment in the classroom in which students develop rational thinking and understanding of the Mathematical concepts. The teacher can do it by using innovative methods of teaching, like problem solving. Teacher's pedagogical skills play a vital role in the developing in-depth learning among the students (Joseph & Yoe, 2010).

According to Amirali (2008), Pakistan makes an effort to improve the quality of education through the initiatives, like change in curriculum, and through teachers training programmes. The new Mathematics curriculum emphasized on conceptual understanding, logical reasoning and problems solving skills. It can only be possible if teachers change their current Mathematics teaching. The National Curriculum for Mathematics, 2006, demands that teacher role has to change from dispensing information to planning investigative tasks, conducive learning environment and in-depth development of higher skills in Mathematics (Iqbal, 2009).

In the research article on topic "Teachers' knowledge about the nature of Mathematics" Amirali (2011), stated that "the Mathematics curriculum reforms in all over the world including Pakistan strongly recommend problem solving approaches to schools Mathematics". This fact is endorsed by the empirical evidences of the researches by (Leramn, 1990; Pajares, 1992; Pehkonen, 2002; Azam & Rana, 2009; Goldin, Rosken & Torner, 2009 cited by Amirali, 2011).

In exploring the answer of the question that 'how can problems solving be used as a tool for cognitive development?' Jenkins (2010), found that problem solving method is a powerful tool for the development of cognition and higher level abilities. In its conclusion it is also found that if problem solving method is used in elementary level it is more effective because students early Mathematical years are very important with respect to attitudes and motivation. This fact is endorsed by Jenkins (2010).

According to the research article written by Munzenmaier Rubin (2013), regarding the Bloom's Taxonomy of Educational Objectives and problem solving method for teaching the Higher Order Thinking Skills (HOTS) can be produced by the problem solving method. HOTS refer to the skill to collecting, analyzing and creating.

Rote learning is considered as deceptive and could not be retained for a longer time period, and it also does not let students to develop rational thinking and to solve the daily life problems. In-depth learning in Mathematics through problem solving allows the students to develop HOTS (Rizivi, 2007).

In the conclusion of their research Munzenmaier & Rubin (2013) stated that problem solving method and Bloom's Taxonomy can be used as a tool in the process of teaching and learning.

According to the research study conducted by Rudman & Peter (2007) on the topic "the method of problem solving based of the Japanese and Polya's Model: A classroom experience in Chilean Schools" the main objective of the study was to validate a methodological proposal based on Japanese model of lesson study with the support of Polya problem solving method, the result showed an improvement in the achievement results by teaching through Polya problem solving method. By this method students showed improvement in the comprehension and problem solving with oral and written forms. Students also showed improvements in solving the daily life problems.

In the research done by Yeo & Yeap (2012), by the titled "Characterizing the cognitive process in Mathematical investigation" showed that teaching of Mathematics involved development of cognition. In the same way the problem solving method of teaching Mathematics improve cognitive abilities of the students. According to Canturk& Baser (2009), in his research article about 'challenges in Mathematics teacher education' contributes that poor performance in Mathematics is the result of poor teaching. For the development of higher abilities among the students problem solving approaches are required.

According to the recommendation given by the Aslan (2012), in his research paper about comparison of thinking styles in teaching of social sciences and Mathematics; recommended, that studies are required to find out the effect of

problem solving method on the development of higher level abilities in teaching of Mathematics.

In the conclusion presented by Tutkun & Okay (2012), revision in the old taxonomy has become inevitable because of change in the learning concepts in 21st century. All credit goes to Anderson and Krathwohl for their revised taxonomy. Now it is the duty of educators to explore its utilization in teaching and learning.

Form the above literature review it could be inferred that Mathematics is the subject through which cognition can be developed. By applying the appropriate method in-depth conceptual knowledge can be transferred. Polya's Problem solving method is an approved method for developing higher level abilities among the students and makes them able to solve daily life problems. Through this method problem solving ability can be developed. On the other hand Revised Bloom's Taxonomy is more refined tool for assessment.

Keeping in view the situation of Mathematics in Pakistan the intent of the present study was to check the effect of Polya's Problem Solving Method on Revised Bloom Taxonomy. A lot of work has been done on effect of Polya's Problem Solving Method and Bloom' Taxonomy.

Very few studies were done to find out the effect of Polya's Problem Solving Method on Revised Bloom's Taxonomy.

Through this study, an effort was made to check the effect of this method of teaching on Mathematics at elementary level with Revised Bloom's Taxonomy through double control group experimental method.

CHAPTER 3

RESEARCH DESIGN

This chapter discusses the research design, research paradigm, methods of inquiry, research instruments and the validation and reliability ways of instruments. The study was true experimental study which was focused on to find the Polya's Problem Solving Method which has four steps of teaching on Revised Bloom Taxonomy. The main focus of the revised taxonomy is 'what to learn' and 'how to learn'. Revised Bloom Taxonomy is a new innovation in learning outcomes. It has four knowledge levels i.e. Factual knowledge, Conceptual knowledge, Procedural knowledge and Meta-cognitive knowledge. Every knowledge domain has six levels of cognition i.e. Remembering, Understanding, Applying, Analyzing, Evaluating and Creating. The cognitive levels are also divided into two categories; they are LOTS (Lower Order Thinking Skills), and HOTS (Higher Order Thinking Skills). Poor results in the subject of Mathematics in all over the world and especially in Pakistan are due to traditional and conventional ways of teaching. The Polya's Problem Solving Method is a method through which higher order mental abilities can be developed. So this experimental study was design to find out the answer of the following questions:

1. How does problem solving method of teaching effect the conceptual knowledge dimension sub-level remembering?
2. How does problem solving method of teaching develop the conceptual knowledge dimension sub-level understanding?
3. How does problem solving method of teaching develop the ability of applying sub-level of conceptual knowledge dimension?
4. How does problem solving method of teaching works on ability of Analyzing sub-level of conceptual knowledge dimension?
5. To what extent the problem solving method develop the ability of evaluating, sub-level of conceptual knowledge dimension?
6. To what extent the problem solving method develop the ability of creating, sub-level of conceptual knowledge dimension?
7. Does problem solving method develop the Higher Order thinking skills?

To answers these questions following null hypotheses were formed:

Ho1: There is no significant effect of problem solving teaching method on the mean achievement scores at conceptual knowledge dimension sub-level remembering in Mathematics.

Ho2: There is no significant effect of problem solving teaching method on the mean achievement scores at conceptual knowledge dimension sub-level understanding in Mathematics.

Ho3: There is no significant effect of problem solving teaching method on the mean achievement scores at conceptual knowledge dimension sub-level applying in the subject of Mathematics.

Ho 4: There is no significant effect of problem solving teaching method on the mean achievement scores at conceptual knowledge dimension sub-level analyzing in Mathematics.

Ho5: There is no significant effect of problem solving teaching method on the achievement scores at conceptual knowledge sub-level dimension evaluating in Mathematics.

Ho6: There is no significant effect of problem solving teaching method on the mean achievement scores at conceptual knowledge dimension sub-level creating in Mathematics.

Ho7: There is no significant effect of problem solving teaching method on the mean achievement scores at Higher Order Thinking Skills.

Ho8: There is no significant difference on the mean achievement scores between the Experimental Group with Control Groups

To find out the answers of the above mentioned questions and to check the hypotheses a true experimental study with double control group was conducted. Pre-test post-test was used to conduct the experiment.

According to Cohen, Manion & Morrison, (2003), there are two types of experimental designs. They are;

1. Single variable design (True experimental design)

   In this design one independent variable which is manipulated and the effect of this independent variable is checked on single depended variable.

2. Multiple variable design (Factorial design)

In this design two or more than two independent variable are used and at least one variable is manipulated.

3.1 Research Paradigm

According to Kerlinger (1986), quoted by Creswell (2009), a research paradigm is a lens through which problem is seen and solved. It is a comprehensive beliefs system, world view or framework. It provides guidance in the respective field. The present study paradigm of research was found in the post positivist paradigm. Post positivist believe that researcher requires believing on present existence of realities endorsed by Phillips & Burbules, (2000) cited by Zaman (2012), by saying that according to Post Positivist, researcher has to prove that these existence of multiple realities, challenging the conventional notion of absolute truth of knowledge'. This paradigm requires providing empirical evidences to prove something by data. It means that any conclusions drawn are based upon hard evidence gathered from information collected from real life experiences. Keeping in view the usefulness of Polya's Problem Solving Method, it was required to check it in real life experience i.e. in the classroom situation. So the researcher selected the Post Positivist paradigm of research.

According to Willis (2000) cited by Zaman (2012), the emphasis of Post Positivist approach is on objective and on reliable facts supported by data which is valid and reliable, therefore, instruments used for data collection required tests of validity and reliability. So the pre-test and post-tests were checked with respect to validity and reliability.

As in this paradigm human knowledge is not considered unchallengeable, rather flexible or conjectural. So the current study developed the null hypotheses for

testing statistically. The Post Positivists paradigm either accepts or not accepts the conjectures on the basis of data. Therefore, experiment is conducted in the present study to collect the data in real situation for the acceptance or not acceptance of the hypotheses. To make the results more reliable and authentic double control group was used so that the treatment could show its real effect. As the Post Positivists paradigm required evidence and standard facts for confirmation, therefore to check the effect of Polya's teaching method Revised Bloom's Taxonomy was used which is a well-known source of evaluation (Zaman, 2012).

3.2 Research Design

According Creswell (2009), the research design is a plan and strategy of investigation. The researcher conceived it as answers to research questions. The design is a complete scheme of the research. It is also a blueprint or detailed plan for how a research study is to be completed. A plan contains how the hypotheses are established and how they will be measured. What will be the respondents of the study and how representative sample will be selected for study? Moreover, what type of instruments was used and how data was analyzed and interpreted.

According to Jahoda quoted by Zaman (2012), "A research design is the arrangement of conditions for the collection and analysis of data in a manner that aims to combine relevance to the research purpose with economy in procedure".

The main functions of a research design may be as follow:

- Conceptualize an operational plan to undertake the various procedures, and task needed to complete the study.
- Confirm that the procedures are valid and reliable (Kerlinger cited by Zaman, 2012).

The design of the present study was developed in such a way that the effect of the independent variable (cause variable) could become visible. In an experimental study the dependent variable can be affected by the 'extraneous' as well as 'chance variables'. It can be expressed as the total change in dependent variable is due to:

- Change because of cause variable
- Change because of extraneous variable
- Change because of chance variable

As the total change measures the combined effect of all three components it is difficult to isolate the impact of each of them. The present study is designed in such a way that the change can be attributed to the independent variable (Polya's Problem Solving Method) and also to minimize the effect of extraneous and chance variables. This is what known as 'maxmincon' principle of variance (Kerlingr cited by Zaman, 2012).

Table 3.12

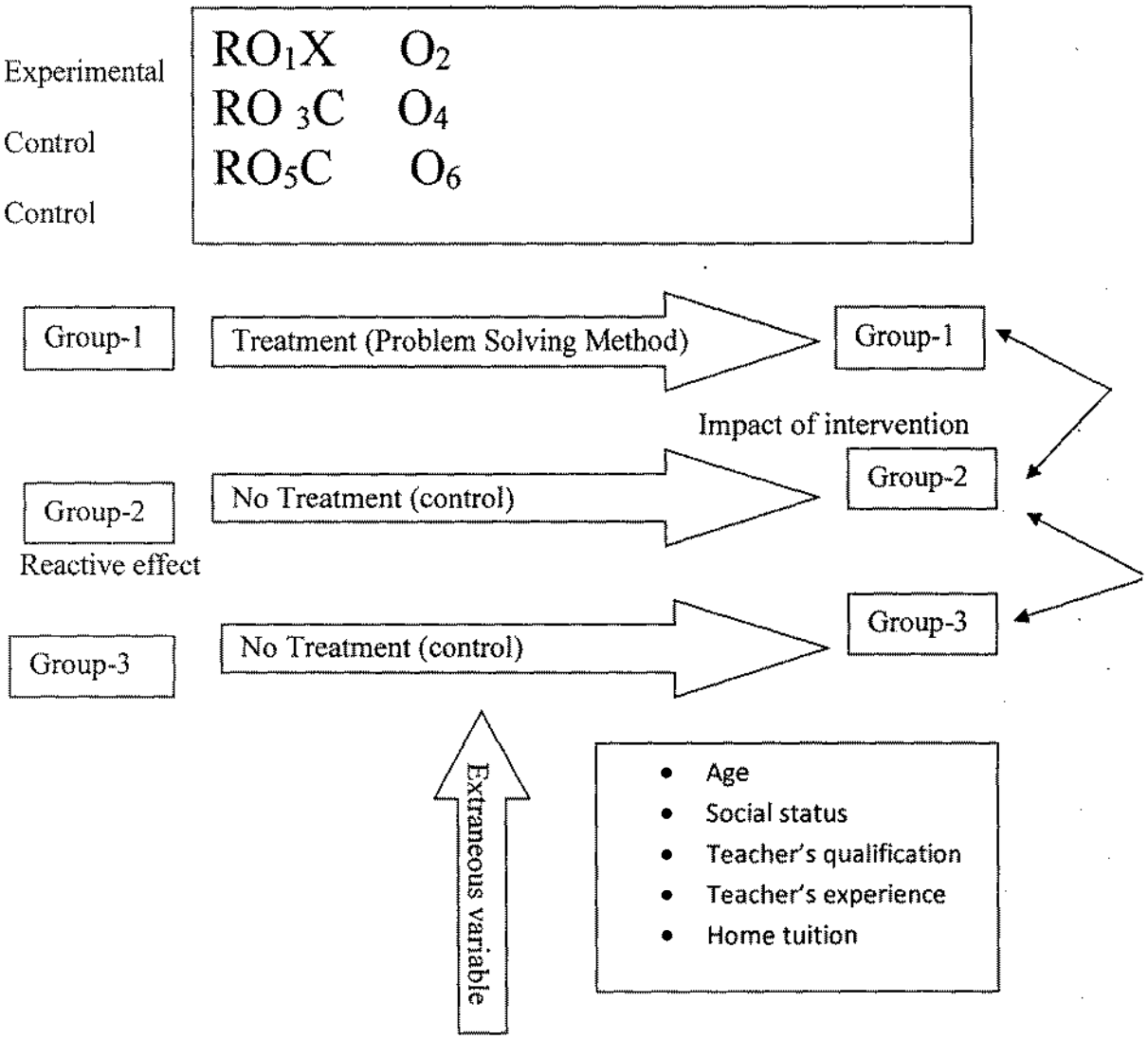
The proportion attributable to the three components (Maxmincon, principle of variance)

Change attributed to extraneous variable	Change attributed to independent variable	Change attributed to chance variable

The main concern of the researcher in the present study was how to minimize the effect of extraneous variable, chance variable and maximize the effect of

independent variable. For this purpose the effects of extraneous variable and chance variables were quantified. Chance variation is due to two reasons; first is due to respondents and second is due to instrument. In population some respondents effect positively, others effects negatively. So to make the effect minimum respondents were divided into groups according to achievements and randomly divided into three groups. Hence they tend to cancel each other out so the net effect is assumed to be zero. In this way the first reason of chance variable is minimized. For controlling or minimizing the second reason the instrument was made reliable and valid.

To address the extraneous variable effect on experimental and control groups, the present study forms the group through 'Randomization' method. To minimize the effect of extraneous variable double control group was introduced. Although the control group design is suitable to quantify the impact that can be attributed to extraneous variables but it does not separate out other effects that may be due to the research instrument like or instance the reactive effect or respondents such as maturation, regression effect.



The true experimental single control group studies were done by many researchers. For example (Blanco 2000, Riasat 2011; Zaman, 2011; Mustafa, 2011) carried out the experimental studies to find out the effect of problem solving method of teaching Mathematics on Mathematics. The double control group experimental design was used by Bhadwal and Panda (1991), in their investigation cited by Cohen et al (2003).

3.3 Variables of the Study

As the study was an experimental study and the basic variables of the study were as:

3.3.1 Independent Variable

In the present study the independent variable or the treatment or the cause variable was the Polya's Problem Solving Method. This method has four steps.

3.3.2 Dependent Variable

In the present study the depended variable was the achievement of students on Revised Bloom's Taxonomy knowledge level (conceptual Knowledge) with six cognition level (Remembering, Understanding, Applying, Analyzing, Evaluating and creating).

3.3.3 Chance Variable

The population and instrument in the present study were the chance variables.

3.3.4 Extraneous Variables

In the study the possible extraneous variables were, age, IQ levels, social status, teachers qualifications, teachers experience, home tuitions and interest of students.

3.4 Population of the Study

Islamabad is the capital of Pakistan is working under the jurisdiction of Federal Government of Pakistan. It is divided into two main streams i.e. urban and ruler areas. The study was delimited to the urban area with boys' high schools having 8^{th} class. The total high schools in the urban area of Islamabad are thirty seven according to the Pakistan Education Statistics 2008-2009, (Government of Pakistan,

2009) and the total number of boys students were 6,932. So the population of the study was the total students (6932) studying in the class 8th in F.G Boys High Schools (VIII-X) in urban areas.

3.4.1 Sample of the Study

The present study was based on true experimental (Double control group) pretest -posttest design. A probability sample of 132 male students from randomly selected Islamabad Boys Schools I-8/1, Islamabad was taken and after pretest they were divided into three groups through multistage sampling.

3.4.2 Sampling of the Schools

The Federal territory is divided administratively into five sectors as:

1) Islamabad City
2) Bhara Kahu Sector
3) Tarnol Sector
4) Nilore Sector
5) Sihala Sector

The Islamabad City is known as the urban area, while the other four areas are known as the rural areas. For the present study the researcher selected the F.G. Boy Model Schools I.8/1 Islamabad. The selection was made due to certain reasons, including the better infrastructure, sufficient number of students for study (132), equipped classrooms and sufficient number of Mathematics teachers. These schools were selected among the 37 schools through purposive sampling form Islamabad City sector. Further reasons for the selection of these particular schools were as follow:

- The availability of more than 120 students.

- Availability of three sections for class 8th
- Willingness and facilitation on the behalf of schools principal, teachers and students to conduct the experiment smoothly.
- Willingness of the principal for making three groups on the basis of pre-test
- Permission for making the same timetable and content for the experiment
- Location of the schools was at a manageable distance
- Conducive environment for conducting experiment in a peaceful manner

3.4.3 Groups Formation after Pre-Test

Pre-test of 100 marks was administrated with 132, 8th grade students. On the basis of marks obtained they were divided into four sampling frames for randomization as under:

Table 3.13

Distribution of Marks into Groups of Pre-Test

Groups	Less than 40	41-49	50-59	More than 60
Number of students	32	60	30	10
Proportionately randomly selected respondents from each groups	10+10+12	20+20+20	10+10+10	3+3+4

Table3.14

Groups Formation for Post-Test

Group-1 (control)	Group-2 (control)	Group-3 (Experimental)
10+20+10+3=43	10+20+10+3=43	12+20+10+4=46

On the basis of pre-test scores four sampling frames were formed. From each sampling frame proportionate numbers of students were randomly selected and hence three groups were formed. The selected three groups were named three sections of routine classes (Section-A, B and C). Section-A was treated as Experimental group, while Section-B and Section-C were treated as control groups. Each control group has 43 students, while experimental group has 46 students, while two students were dropped and size became 44. According to the researches done (e.g. Zaman, 2011; Arif, 2011; Yan, 2012; Tuckman, 2012) used the experimental design and used experimental and control group of 40 to 45 students, and the duration of the experiment was 60 days.

3.5 Selection of Content for Experiment

The content for the experiment was selected form the Textbook of Mathematics for class-8th printed by National Book Foundation, Islamabad 13th print 2013. The researcher selected the following units for the entire groups:

1) **Unit No-6: Percentage**

Sub topics and exercises

- Profit and loss
- Profit and loss involving successive transactions and comparison of two transactions

- Commission
- Exercise 6.1 (Thirteen word problems)
- Concept of insurance
- Life insurance
- Exercise 6.2 (Four words problems)
- Vehicle and property insurance
- Exercise 6.3 (Eight word problems)
- Gross income, net income, taxable income
- Exercise 6.4 (Three word problems)

2) **Unit No-8 Geometry (Theoretical)**

Sub Topics and Exercises

- Application of Pythagoras Theorem
- Exercise 8.1 (Seven questions)
- Area of triangular region (Hero's formula)
- Exercise 8.2 (Three question with sub-questions)
- Area of Quadrilateral by using Hero's formula
- Exercise 8.3 (two questions with sub-questions)
- Sphere, surface area of sphere
- Exercise 8.4 (Eight questions)
- Volume of Sphere
- Exercise 8.5 (Six questions)
- Cone, surface area of a right circular cone
- Exercise 8.6 (Six questions

- Volume of right circular cone
- Exercise 8.7 (Seven questions)

3) Unit No-8 Geometry (Practical)

Sub Topics and Exercises

- Dividing a line segment into a given number of equal parts
- Exercise 8.8 (Two question with sub-questions)
- Constructing a triangle when perimeter and ratio among the lengths of sides are given
- Exercise 8.9 (Five questions)
- Construction of Polygons (Regular Pentagon, Octagon
- Exercise 8.10 (Four questions)
- Construction of a square when measure of the diagonal is given
- Exercise 8.11(Seven questions)

3.6 Development of Lesson Plans for Experimental Group

For the present study forty lesson Plans were constructed (Appendix-A) for the experimental group according to four steps of Polya's Problem Solving Method in accordance with Revised Bloom's Taxonomy. The study was delimited to the conceptual level of Revised Bloom's Taxonomy and according to Anderson (2001), the conceptual knowledge deals with the relationships among pieces of a larger structure that make them function together, Knowledge of classifications and categories, Knowledge of principles and generalizations, Knowledge of theories, models, and structures. So keeping in view the concept of conceptual knowledge all the lesson plans were developed through this approach. The learning outcomes were

in accordance with the conceptual level. All selected topics were the part of lesson plans. The other two controlled groups were also taught the same topic according to the decided schedule with conventional method of teaching by the regular teachers of the same school. All the lesson plans for controlled groups were discussed with the teachers.

3.6.1 Implementation Strategy of Lesson Plans

The three groups of class 8th were taught the same, topic on the same day and in the same period within the specified classrooms. Lessons plans for the control groups were designed in the light of conventional teaching method by the respective teachers, whereas the lesson plans for the experimental group was prepared in the perspective of Revised Bloom's Taxonomy, at conceptual level and taught with Polya's Problem Solving Method.

3.7 Controls on Internal Threats

There are many conditions or threats which can jeopardize the validity of the experiments (Cambell & Stanley, 1963; Glass, 1968; Lewis-Beck, 1993; cited Cohen et al, 2003). The internal validity is concerned with the questions, do the experimental treatments, in fact, make a difference in the specific experiments under scrutiny? These threats include History, Maturation, Statistical regression, Testing, Instrumentation, Selection, Experimental mortality, Instrument reactivity and Selection-maturation interaction. According to Creswell (2009), the best way to address these internal threats is the design of the experiment. The present study was a true experimental research with randomization, which was the best way to control the internal threats. Following internal threats were controlled as:

1) **History and Maturation:** During the experiment no such incident occurred which might put influence on the treatment or on the results. Hence the history did not affect the internal validity of the experiment. In the same way the maturation factor was the same for three groups because it was occurring at the same rate in the all groups.

2) **Testing:** There was a two months gaps between pre-test and post-test. The test was administered as a general test. The item of the test was content based not text based. The students were kept unconscious about the post-test to control the testing threat to internal validity.

3) **Instrumentation:** To address the threat of instrumentation, the researcher used the same pre-test as post-test by changing the order of the questions. The instrument was also checked through statistically and was found reliable and valid. The study was also designed in such a way as to control the threat of instrumentation.

4) **Statistical Regressions:** These effects become dominant with the time interval between pre-test and post-test. This factor of internal threat was controlled through normal distribution of the students. There were three groups one experimental and other two control groups to measure the statistical regression.

5) **Selection:** This internal threat was controlled by the researcher through including all the students in pre-test and the selection of all groups was made randomly. The selection was made through the systematic way to address the threat.

6) **Mortality:** This threat was controlled by ensuring the attendance of all students by the help of teachers during the duration of the experiment. Luckily no students fell ill or remained absent during the study.

3.8 Controls on External Threats

According to Bracht & Glass (1993), cited by Cohen et al, (2003), the external validity concerned with the questions, regarding generalization of the results. The external threats include failure to describe independent variables explicitly, lack of representativeness of available and target populations, Hawthoren effect, Interaction effect, Ecological validity and Reactive effect. To control the most of the threats the present study used randomization as well as the double control group for the experiment. Following measures were done to control the external threats:

1) **Pretest-Treatment Interaction:** This external threat was controlled by the researcher by making the pre-test which was not text-book based. The test was unknown to all the respondents. The items were based on conceptual knowledge and students were solving it first time. The same pre-test was used as post-test as well by changing the order of the questions.
2) **Multiple-Treatment Interference:** This threat was controlled by confirming that no respondent of the experiment was exposed to any other experiment. For further measure to control this experimental and control groups were of the same schools and not informed about any treatment.
3) **Selection-Treatment Interaction:** This external threat was addressed by the randomization sampling technique. The groups were formed after pre-test on the basis of their achievement scores. The researcher did not distribute the

intact classes into control or experimental groups for the study. Groups were formed on normal distribution principle.

4) **Specificity of the Variables:** The study was started with the opening of new section on 10th April to 11th June about two months in Islamabad. The intent of the study was to check the effect of Polya's Problem Solving Method of Mathematics on Revised Bloom's Taxonomy. For this purpose 40 lesson plans were made with the consultation of the schools teachers. The experiment was started with the start of new classes in 2014 and students were divided into three groups which were assigned three sections as per practice in the schools. In these three groups two groups were controlled and taught by the school's regular teachers and the third group which was experimental taught by the researcher himself. For students the researcher was also like a school teacher, because it was the begging of the new session. All of such specific conditions which could influence the experiment were controlled by adaptation of the validated procedures of sampling, pre-test, post-test, rubrics, duration of the study. Students were taught in normal routine and post-test was taken from all students as exam requirement. All the students were taught the same lesson according to the timetable and course requirement.

5) **Experimenter Effects:** To minimize the effect of experimenter the researcher did all the precautionary measures because the experimental group was taught by the researcher himself. For this same lesson plans were decided for all three groups rubrics were made for objective scoring of pre-test and

post-test, duration of the experiment, time and venue of the treatment, topics and exercise of the textbook, sex and socioeconomic status, homogeneity among the groups during their formation, and the time of the period of Mathematics. Hence in this way an effort was made to control the experimenter effect.

6) **Reactive Arrangements:** To control this external threat, the double control group was taken. For further precautionary measures the two control groups were studying as usual by their own teachers by the conventional teaching method. Moreover, all the students of control and experimental group were kept unaware of any sort of comparison. So all the students were taught the same lesson as per routine and they were told to cover this much syllabus before the summer vocations. So in this way they were unconscious about any sort of treatment. The same time of the treatment which was 8-weeks, provided control over the Hawthorne effect. To address the threat, the principal of the schools has assured of the strict implementation of the specified lesson plans by the control group teachers. This measure also played important role in controlling the ambitious performance of students because during experiment time no test was conducted in normal routine work. There was no Placebo effect in the study because two control groups were taught by conventional methods and experimental group with Poly's Problems Solving Method indiscriminately. The duration of the study also helped the researcher to mitigate the effect of treatment.

3.9 Construction of pre-test for the experiment

A pre-test was constructed on the topics of word problems, practical geometry and theoretical geometry from the Mathematics book of class 8th. The construction of the pre-test was based on Revised Bloom's Taxonomy, conceptual level. As the Revised Bloom's Taxonomy was based on two ways as shown:

Table 3.15

Revised Bloom's Taxonomy

Knowledge dimension ⇩ / THE COGNITIVE PROCESS DIMENSION ⇨	1	2	3	4	5	6
	Remembering	Understanding	Applying	Analyzing	Evaluating	creating
FACTUAL KNOWLEDGE	1	1	1	1	1	1
CONCEPTUAL KNOWLEDGE	1	1	1	1	1	1
PROCEDURAL KNOWLEDGE	1	1	1	1	1	1
METACOGNITIVE KNOWLEDGE	1	1	1	1	1	1

It has four levels of knowledge dimension and six levels of cognitive process dimension if all levels are addressed than twenty four (6*4=24) items are required and every row and column has only one item. If researcher wants to explore all dimensions then at least every row needs five items and hence they become 30*4=120. It was not feasible to construct a test on all dimensions, represented in the following table:

Table 3.16

Pre-Test Items Construction

Knowledge dimension ⇩	⇨ THE COGNITIVE PROCESS DIMENSION					
	1	2	3	4	5	6
	Remembering	Understanding	Applying	Analyzing	Evaluating	creating
FACTUAL KNOWLEDGE	5	5	5	5	5	5
CONCEPTUAL KNOWLEDGE	5	5	5	5	5	5
PROCEDURAL KNOWLEDGE	5	5	5	5	5	5
METACOGNITIVE KNOWLEDGE	5	5	5	5	5	5

Keeping in view the treatment to the experimental group the Revised Bloom's Taxonomy one knowledge dimension was taken i-e conceptual knowledge because of the following reasons:

- For teaching a concept 24 objectives could not be set as learning outcomes; therefore, one dimension is taken and six objectives were set for each topic
- Polya's Problem Solving Method of teaching can be used to achieve a specific dimension.
- Time for normal class period (45 minutes) did not allow achieving multiple dimensions.
- To make the study more purposeful it was required to be specific to one dimension.

- It was convenient to develop Pre-test and post-test on one dimension

  So a pre-test based on conceptual knowledge dimension with six cognition process was developed of 100 marks as under:

Table 3.17

Numbers of Items in Pre-Test and Post-Test

Knowledge dimension ⇩	THE COGNITIVE PROCESS DIMENSION ⇨					
	1	2	3	4	5	6
	Remembering	Understanding	Applying	Analyzing	Evaluating	Creating
CONCEPTUAL KNOWLEDGE	10 items	10 items	10 items	10 items	10 items	10 items

3.9.1 Rubric for Marking:

- Remembering=10*1=10
- Understanding=10*1=10
- Applying = 10*2=20
- Analyzing =10*2=20
- Evaluating = 10*2=20
- Creating =10*2=20

100

3.10 Validity of the Pre-Test

Conceptual knowledge dimension in Revised Bloom's Taxonomy covers the concept knowledge about the topics. This dimension deals with the relationships among pieces of a larger structure that make them function together. It explores the

knowledge about the concept. It covers the knowledge about classifications, categories, principles, generalizations, theories, formulae, models and structures. Keeping in view the conceptual knowledge dimension and the cognitive process levels which has six levels (i.e. Remembering, Understanding, Applying, Analyzing, Evaluating and Creating), test items were constructed on the topics of word problems, practical geometry and theoretical geometry from the Mathematics book of class 8^{th}.

Validity is the ability of an instrument to measure what is designed to measure. It can also be described as 'the degree to which the researcher has measured what the researcher has set out to measure.' According to Kerlinger (1973) cited by Mustfa (2011), 'validity is epitomized by question: Are we measuring what we think we are measuring?'

In most of the cases validity is concerned with two fundamental questions:

1. Who decides that an instrument is measuring what it is supposed to measure?
2. How can it be established that an instrument is measuring what it is supposed to measure? (Babbie, 1990).

The answer of the first question is given by the researcher because he has to prepare the instrument according to the objective of the study. The second question which is related to validity is the most important question and it can be answered by some experts of the field. So the validity of the test was confirmed through the opinions of the experts in the field of Mathematics. (Appendix-D)

3.10.1 Content Validity

This is an important attribute of an instrument with respect to validity. It shows relevancy of instrument to the objective of the study. It can be stated as

'content validity can be measured by showing how well the content of the test is developed by which the results can be measured according to the objective of the study'. According to Lennon (1956) cited by Zaman (2011) that content validity represents by a description of the universe of items from which selection was made with a clear selection criterion for the study objectives. For the validation of test for the students formed on the basis of conceptual knowledge dimension were sent to 10 experts for comments (Appendix-D).

In the light of the comments of the experts the test items were revised and teat was refined. The major concern shown by experts regarding the test was the use of action verb for different levels which were improved.

3.10.2 Subscale Total Correlation for Validity

Table 3.18

Subscale Total Correlation for Validity of Scale and its Subscales

Variables	1	2	3	4	5	6	7
1. Remembering	--	.901**	.907**	.970**	.973**	.970**	.943**
2. Understanding		--	.951**	.849**	.849**	.839**	.795**
3. Applying			--	.892**	.847**	.840**	.778**
4. Analyzing				--	.941**	.916**	.889**
5. Evaluating					--	.935**	.899**
6. Creating						--	.918**
7. Overall abilities							--

**$p < .01$

Table 6 shows correlation analysis for assessing validity of the scale. Result shows that all the subscales have significant positive correlation (i.e. $p < .01$) with the total scale (i.e., Overall abilities) indicates high validity of the scale and subscales.

3.10.3 Pilot Testing

For the further improvement of the test with respect to students a pilot study was conducted on 30 students of the same level at F. G. Boys School I-9/4, Islamabad, to know the understanding levels of the students about the test items. Pilot study has the advantage that it provides information about the shortcoming in data collection instruments and other research protocols (Zaman, 2011).

The responses gained by the students were noted and test was tailored accordingly. The major concerns shown by the students were as follows:

- Some words problems were not understandable, they were reworded.
- Some figures were not clear, they were changed.
- Some questions were not related to their course, they were replaced.
- Some data was not giving answer in wholes that was changed.
- Units of some question were not same, they were managed.

In the light of the feedback of students, required changes were done in the test.

3.11 Reliability Analysis

Table 3.19

Alpha reliability analysis for the Scale and its Subscales

Scale/Subscale	No. of items	Alpha Reliability
Remembering	10	.89
Understanding	10	.91
Applying	10	.91
Analyzing	10	.95
Evaluating	10	.93
Creating	10	.94
Overall abilities	60	.95

Table 20 shows Alpha reliability analysis for the scale and its subscales. Results shows that reliability among all the scales and subscales ranges from α = .89 (i.e. Remembering) to α = .95 (i.e. Analysis, Overall abilities). This indicates that all scale and subscales have Hughs alpha reliability (i.e., α>.89) and therefore they are reliable for use.

3.12 Selection of Teachers for Control and Experimental Groups

The two control groups were taught by the regular teacher of the schools; both teachers were having the same qualification and were taking classes for more than ten years. Section-A was taught by Mr. Hamid (B.Sc/B.Ed). Section-B was taught by Mr. Yaseen (B.Sc/B.Ed). Section-C was taught by the researcher himself because the Polya's Problem Solving Method is a quite new concept in our perspective. To design the lesson plan in accordance to RBT is also a new concept. The working teachers were not well aware about the new method and new taxonomy. To train and engage

the teachers for teaching for experimental group may cause gaps between theory and practice. The researcher was in touch with this concept and with the RBT through review of literature, watching videos clips. Researcher had been practicing this method with his students in the courses of 'Teaching of Mathematics' at B.S and M.A level classes. There were some other reasons which were as follows:

- The Polya's Problem Solving Method needs training and clarity for teaching so to save time of provide training researcher prefer to teach by him.
- To keep the treatment according to the objectives of research it was considered better to teach the group by the researcher.
- To make the group intact from other groups and to tackle the other prevailing situation on the spot it was necessary that researcher had to be there.
- The previous researches done by (e.g. Zaman, 2011; Arif, 2011; Yan, 2013; Tuckman, 2012) have shown that the experimental groups were taught by the researcher himself.

The content for the teaching was decided before the start of the experiment among the teachers. The word problems, practical geometry and theoretical geometry of the book of class 8th were agreed upon for teaching.

3.13 Duration of the Study

The experiment for the present study was executed from the start of new secession of 2014-2015. The new session started after the announcement of annual results of 2013. The new classes started on 10th April, 2014. The experiment started on 10th April, 2014 to 11th June, 2014. The pre-test was conducted on 10th April and post-test was conducted on 11th June before the summer vacations.

3.14 Teaching Method for Control Groups

The two control groups were taught through conventional method which can be **operationally defined** as the, 'talk and chalk method'; in which teacher is active most of the time while students are passive or just keep themselves busy in noting. The other main features of the conventional method of teaching were as follow:

- promoting rote learning among the students
- lecturing for most of the time
- compulsion for taking notes from board
- less interaction between teacher and students
- less interaction among the students
- Homework and completion of exercises
- Promoting individual work
- Forcing students to complete notebooks
- Punishment as a source of encouragement
- Teacher is more active
- Students are supposed to be passive
- Explaining the concept and solving the exercises by the teachers
- Teacher centered environment

3.15 Teaching Method for Experimental Group

The experimental group was taught by the Polya's Problem Solving Method of teaching; the main focus of the method is to make the learner an independent learner. It works on to create the ability of problem solving. The learners become used to this ability and can solve the problems of daily routine as well. This method is

considered more effective in the development of Higher Order Thinking abilities like applying, analyzing, evaluating and synthesis. Teacher's role is the facilitator and provides maximum opportunity to the students to learn independently. Problem solving is ability like swimming which needs practice in the same way problem solving is a method through which this ability can be produced. Following main features were dominant during the experiment:

- Students were taught through Polya's four step method
- Students were more active
- Students were provided maximum opportunity to learn independently
- Positive reinforcement was provided for motivation
- Habit of problem solving was developed
- Auxiliary examples were solved instead of text exercises
- Maximum work was done in the classroom
- Students were allowed for maximum participation
- Students were allowed to help each other
- Conceptual clarity was the focus of the lesson
- Students were agreed upon the importance of Mathematics
- Open environment was provided for learning
- Students were not conscious about the treatment
- Students were kept out of fear of any test or punishments
- Self-accountability was developed among the students
- Non routine examples were used for the clarification of the concepts
- Jolly classroom environment was kept during the sections

- Heuristics were used in teaching
- Students were provided alternative chances to solve the problems on boards (Polya, 1976)

3.16 Analyses of Data

The data consisted of the followings:

3.16.1 Pre-Test

A pre-test consisted of 60 items developed on Revised Bloom's Taxonomy, conceptual dimension, was taken form 132 students. The test was developed on conceptual knowledge dimension with six cognitive dimensions i.e. Remembering, Understanding, Applying, Analyzing, Evaluating and Creating from the 8th class book from the topic of word problems, Practical Geometry and theoretical Geometry. Rubric was made for scoring the pre-test. On the basis of the scores obtained by the students they were divided into four sampling frames for instance less than forty, between 41-50, between 51-60 and more than 60.

On the basis of these sampling frames through proportionate random sampling three groups were formed. One group was treated as Experimental Group, while two groups were kept controlled.

3.16.2 Post-Test

After changing the order of the question of pre-test, post-test was prepared with the same scoring rubric. After 8-weeks the post-test was conducted. Scores of achievement of post-test of all three groups were obtained.

3.16.3 Achievement Scores

The difference of each student's scores from post-test to pre-test was calculated and named as achievement scores i.e. Achievement scores= Post-test score-pre-test score

Following tests were applied for testing the null hypotheses:

- Normal distribution analysis on each group to check the normality
- P-P Plots for Normal Distribution Analysis (Appendix-)
- Reliability and Validity Analysis
- One way ANOVA
- Post Hock Analysis (Tuckey HSD method)
- Means for groups in Homogeneous subsets
- Descriptive Statistics among Study variables
- T-test applied on the Results for Problem Solving Method on Outcome variables
- Descriptive Statistics among Study variables for Control Group-I and II

For the analysis of data SSPS 12 was used. Test of Homogeneity of Variances (Levene Test)of Experimental Group, Control Group-I and Control Group-II were conducted and Results showed that among all groups Levene test results were non-significant (p>.05 = Equal Variance) which is fulfilling the assumption of homogeneity of variance. Alpha reliability analysis for the Scale and its Subscales were applied and found the scale and subscale reliable.

Subscale Total Correlation for Validity of Scale and its Subscales were calculated and result shows that all the subscales have significant positive correlation

(i.e. $p < .01$) with the total scale (i.e. Overall abilities) indicates high validity of the scale and subscales.

One Way ANOVA Analysis for effect of Problem Solving Method on Outcome variables for Experimental Group, Control Group 1 and Control Group 2 was also calculated. Post Hock Analysis (Tuckey HSD method) for effect of Problem Solving Method on Outcome variables for Experimental Group, Control Group 1 and Control Group 2 was calculated through SSPS. Means for groups in Homogeneous subsetsfor effect of Problem Solving Method on Outcome variables for Experimental Group, Control Group 1 and Control Group 2was selected for analysis. The t-test Results for Pretest and Posttest Differences on Outcome variables for Experimental Group were calculated and represented through graphs. Descriptive Statistics among Study variables for Control Group-I, Control Group-II and Experimental Group was done to analyze mean and variations.

On the basis of these obtained results findings were made and on the basis of finding conclusions and recommendations were drawn.

CHAPTER 4

PRESENTATION ANDANALYSIS OF DATA

This chapter deals with presentation of data and analyses of data. The intent of present study was to find out the effect of Polya's Problem Solving Method of teaching Mathematics at elementary level on the Revised Bloom's Taxonomy. The PSM was consisted of four steps first understanding the problem, second devising the plan, third carry out the plan and fourth was feedback. The effect of this method was checked on Revised Bloom's Taxonomy. Revised Bloom's Taxonomy has four knowledge dimensions i.e. Factual, Conceptual, Procedural and Meta-Cognition with six sub-levels of cognitive processes. The study was delimited to the conceptual knowledge dimension with six sub-levels of cognitive process. The objectives of the study were:

1. To find out the effect of problem solving method on conceptual knowledge sub-level remembering.
2. To evaluate the effect of problem solving method on conceptual knowledge sub-level understanding.
3. To check the effect of problem solving method on conceptual knowledge sub-level applying.

4. To determine the effect of problem solving method on conceptual knowledge sub-level analyzing.
5. To find out the effect of problem solving method on conceptual knowledge sub-level evaluating.
6. To check the effect of problem solving method on conceptual knowledge sub-level creating.
7. To find the effect of Problem Solving Method on the development of Higher Order Thinking Skills.
8. To check the difference of achievement of experimental Group with Control Groups through comparisons of Groups.

To address the above objectives researcher made seven null hypotheses which were as follow:

Ho1: There was no significant effect of problem solving teaching method on the mean achievement scores at conceptual knowledge sub-level remembering in Mathematics.

Ho2: There was no significant effect of problem solving teaching method on the mean achievement scores at conceptual knowledge sub-level understanding in Mathematics.

Ho3: There was no significant effect of problem solving teaching method on the mean achievement scores at conceptual knowledge sub-level applying in the subject of Mathematics.

Ho 4: There was no significant effect of problem solving teaching method on the mean achievement scores at conceptual knowledge sub-level analyzing in Mathematics.

Ho5: There was no significant effect of problem solving teaching method on the achievement scores at conceptual knowledge sub-level evaluating in Mathematics.

Ho6: There was at significant effect of problem solving teaching method on the mean achievement scores of conceptual knowledge sub-level creating in Mathematics.

Ho7: There is no significant effect of problem solving teaching method on the mean achievement scores at Higher Order Thinking Skills.

H_o8: There is no significant difference on the mean achievement scores between the Experimental Group with Control Groups.

To accept or not accept the null hypotheses an experimental study was conducted by the researcher on 132 students of 8thgrade in the urban area of Islamabad territory. The design of the experiment was double control group design. This design was used to control the external threats of the experiment. An 8-week experiment was conducted in I/8-4 F.G Boys Schools Islamabad form 10th April to 11th June. On the bases of pre-test results through randomization three groups were formed one experimental and two control group. Experimental Group was taught by the PSM and two Control Groups were taught by conventional method. Selected content form the 8th class book was taught to three groups. The Control Groups were taught by the local schools teachers while the Experimental Group was taught by the researcher himself.

The post-test was conducted after 8-weeks. The achievement scores were treated through SPSS. The validity and reliability of the test were checked through statistically, by the pilot testing, and through expert opinion. The pre-test and post-test were consisted of items based on RBT conceptual dimension having six cognitive sub-levels i.e. Remembering, Understanding, Applying, Analyzing, Evaluating and Creating. Forty lesson plans were prepared by the researcher to teach according to PSM to the Experimental Group and same lesson were delivered through conventional method to the two Control Groups.

Normal Distribution Analysis for Study Variables among Control Group-I, Control Group-II and Experimental Group was performed and entire study variable and overall abilities for each group was found within the assumption of normal distribution. Every study variables i.e. Remembering, Understanding, Applying, Analyzing, Evaluating, Creating and overall abilities were checked through p-p plot of probability and all points were lying near the straight line and showing the characteristics of normal distribution. Which showed the finding was based on mean and SD was reliable and valid. (Appendix-E)

Normal Distribution Analysis for Study Variables among Control Group-I, Control Group-II and Experimental Group was performed and every study variable and overall abilities for each group was found within the assumption of normal distribution. Every study variables i.e. Remembering, Understanding, Applying, Analyzing, Evaluating, Creating and overall abilities were checked through p-p plot of probability and all points were lying near the straight line and showing the characteristics of normal distribution which showed the finding based on mean and

SD were reliable and valid. Test of Homogeneity of Variances (Levene Test) Experimental Group, Control Group-I and Control Group-II was conducted and all the three groups fulfilled the condition.

To make the analyses more valid and reliable following statistical tests were applied:

- Normal distribution analysis on each group to check the normality
- P-P Plots for Normal Distribution Analysis
- Reliability and Validity Analysis
- One way ANOVA
- Post Hock Analysis (Tuckey HSD method)
- Means for groups in Homogeneous subsets
- Descriptive Statistics among Study variables
- For checking the effect of Problem Solving Method on Outcome variables, t-test was applied
- Descriptive Statistics among Study variables for Control Group-I and II

The results of these tests were as follows:

4.1Normal Distribution Analysis

Table 4.20

Normal Distribution Analysis of Study Variables among Control Group-I

	Kolmogorov-Smirnov			Shapiro-Wilk		
Variables	Statistic	*Df*	*P*	Statistic	*Df*	*P*
Remembering	.20	43	.67	.49	43	1.22
Understanding	.24	43	.25	.55	43	3.23

Applying	.19	43	1.00	.49	43	.45
Analyzing	.28	43	3.02	.31	43	.56
Evaluating	.16	43	.98	.13	43	1.53
Creating	.25	43	.09	.01	43	.29
Overall abilities	.12	43	.67	.20	43	.11

Table 4.20 shows Normal Distribution Analysis for Study Variables on Conceptual dimension of Revised Bloom's Taxonomy among Control Group-I. Results show that on both tests including Kolmogorov-Smirnov and Shapiro-Wilk, results were non-significant (p>.05 = Normal Distribution). In the same way the overall abilities were also non-significant (p>.05 = Normal Distribution) which fulfilled the assumptions of normal distribution of the data.

Table 4.21

Normal Distribution Analysis for Study Variables among Control Group-II

	Kolmogorov-Smirnov			Shapiro-Wilk		
Variables	Statistic	*Df*	*P*	Statistic	*Df*	*P*
Remembering	.24	43	.34	.37	43	3.45
Understanding	.01	43	.12	.08	43	.33
Applying	.15	43	.85	.45	43	.40
Analyzing	.18	43	1.01	.27	43	1.09
Evaluating	.24	43	2.23	.38	43	5.04
Creating	.23	43	1.45	.07	43	2.12
Overall abilities	.16	43	.21	.39	43	.23

Table 4.21 shows Normal Distribution Analysis for Study Variables of conceptual dimension of Revised Bloom's Taxonomy among Control Group-II. Results show that on both tests including Kolmogorov-Smirnov and Shapiro-Wilk, results were non-significant (p>.05 = Normal Distribution). The overall abilities variable results was also non-significant which fulfilled the assumptions of normal distribution of the data.

Table 4.22

Normal Distribution Analysis for Study Variables among Experimental Group

	Kolmogorov-Smirnov			Shapiro-Wilk		
Variables	Statistic	*df*	*P*	Statistic	*Df*	*P*
Remembering	.16	44	.21	.13	44	.57
Understanding	.03	44	.33	.16	44	.98
Applying	.07	44	1.21	.19	44	.87
Analyzing	.04	44	.40	.18	44	5.24
Evaluating	.07	44	.67	.12	44	3.27
Creating	.03	44	.32	.14	44	.80
Overall abilities	.18	44	.56	.13	44	2.13

Table 4.22 shows Normal Distribution Analysis for Study Variables among Experimental Group. Results show that on both tests including Kolmogorov-Smirnov and Shapiro-Wilk, results of study variables and overall abilities were non-significant (p >.05 = Normal Distribution) which fulfilled the assumptions of normal distribution of the data.

4.2 Homogeneity of Variance

Table 4.23

Test of Homogeneity of Variances (Levene Test) Experimental Group, Control Group-I and Control Group-II

Groups	Levene Statistic	*df1*	*df2*	*P*
Experimental Group	1.29	2	127	.37
Control Group-I	1.45	2	127	.23
Control Group-II	1.87	2	127	.45

Table 4.23 shows Test of Homogeneity of Variances (Levene Test) Experimental Group (P=.37), Control Group-I (P=.23) and Control Group-II (P=.45). Results show that among all groups Levene test results were non-significant (p>.05 = Equal Variance) which fulfilled the assumption of homogeneity of variance.

4.3 Hypotheses Testing

Table 4.24

One Way ANOVA Analysis for effect of Problem Solving Method & Conventional Method on Outcome variables for Experimental Group, Control Group 1 and Control Group 2

	Experimental Group		Control Group-I		Control Group-II				
Variable	*M*	*SD*	*M*	*SD*	*M*	*SD*	*F*	*P*	*H*
Outcome	65.34	18.24	50.16	10.34	47.07	10.77	22.37	.00	.34

Table 4.24 shows One Way ANOVA Analysis for effect of Problem Solving Method on Outcome variables for Experimental Group, Control Group 1 and Control Group 2. Results show that experimental group had significant greater mean (M = 65.34, SD =18.24; F= 22.7, p<.01, $\dot{\eta}$ = .34), than control group 1 (M = 50.16, SD =10.34; t = 6.65,) and control group 2 (M = 47.07, SD =10.77). However, Post Hoc analysis was applied for seeing many differences among Experimental Group, Control Group 1 and Control Group 2. The results show that the Experimental Group with respect to mean and standard deviation performed better in achievements scores as compared to Control Group-I and Control Group-II. So the Problem Solving Method of teaching worked better in teaching of Mathematics than the conventional method of teaching. The control group 1 (M = 50.16, SD =10.34; t = 6.65,) and control group 2 (M = 47.07, SD =10.77), the mean value and Standard Deviation values are close to one another shows the similar change. Which indicates the that the performances of Group-I and Group-II were the same to some extent and the change was the result of teachers teaching style, because role of teacher could not be ignored.

Table 4.25

Post Hock Analysis (Tuckey HSD method) for effect of Problem Solving Method on Outcome variables for Experimental Group, Control Group 1 and Control Group 2

Dependent Variable	(I) Groups	(J) Groups	Mean Difference (I-J)	*p*	95% CI	
					LL	*UL*
Outcome	Control Group 1	Control Group 2	3.09	.54	-3.89	10.08
		Experimental Group	-15.17*	.00	-22.12	-8.23
	Control Group 2	Control Group 1	-3.09	.54	-10.08	3.89
		Experimental Group	-18.27*	.00	-25.22	-11.33
	Experimental Group	Control Group 1	15.17*	.00	8.23	22.12
		Control Group 2	18.27*	.00	11.33	25.22

*$p < .01$

Table 4.25 shows Post Hock Analysis (Tuckey HSD method) for effect of Problem Solving Method on Outcome variables for Experimental Group, Control Group 1 and Control Group 2. Results show that Problem Solving Method of teaching provide good results on Outcome variables i.e.at Remembering, Understanding, Applying, Analyzing, Evaluating and Creating as compared to Control Group 1 and Control Group 2 which were taught with conventional method of teaching. So the data in the above table supports that the Problem Solving Method had its effect on the Revised Bloom's Taxonomy.

Table 4.26

Means for groups in Homogeneous subsets for effect of Problem Solving Method on Outcome variables for Experimental Group, Control Group 1 and Control Group 2

		Subset for alpha = 0.05	
Groups	*N*	1	2
Control 2	43	47.07	
Control 1	43	50.16	
Experimental Group	44		65.34
Sig.		.54	1.00

In table 4.26, means for groups in homogeneous subsets are displayed. Uses Harmonic Mean Sample Size was 43.32.The group sizes were unequal. The harmonic mean of the group sizes was used. Type I error levels were not guaranteed.

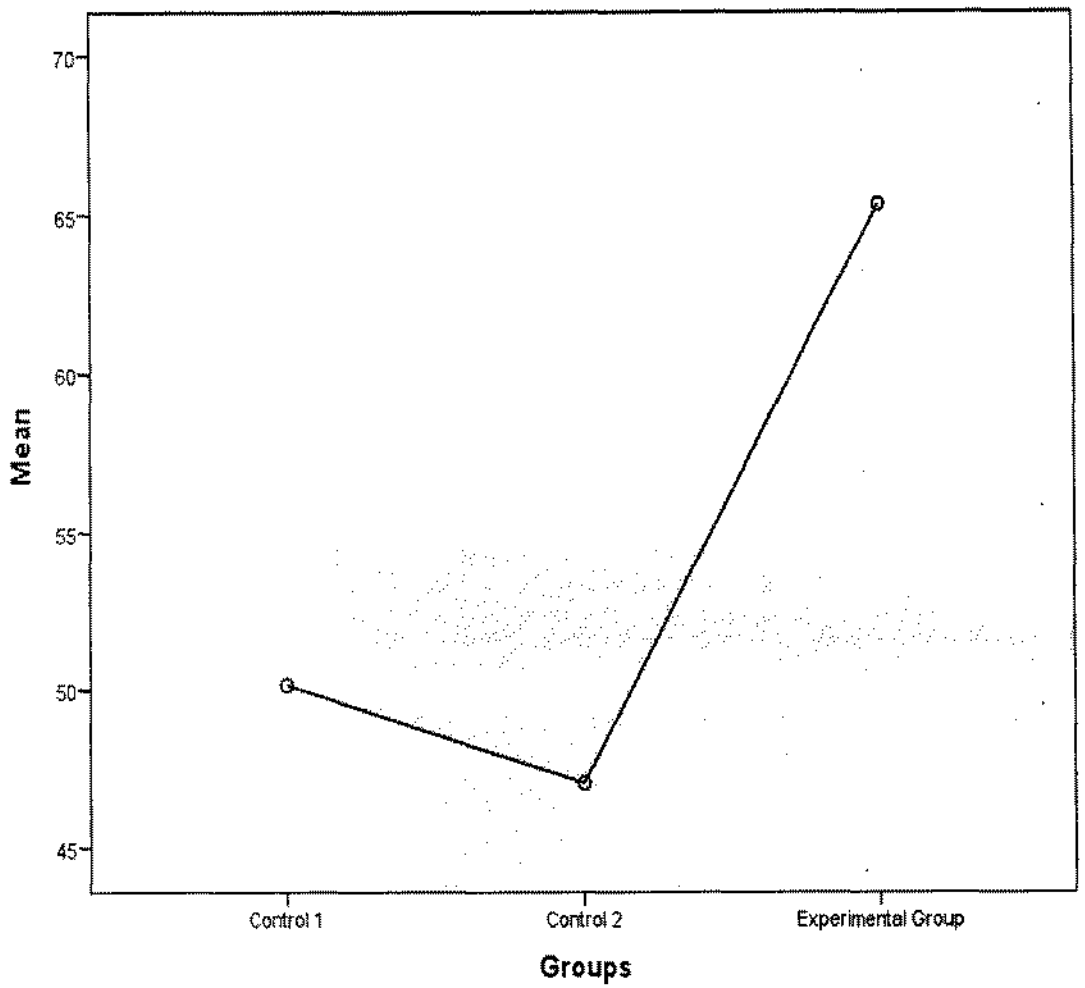


Figure 25 The effect of Problem Solving Method on Outcome variables for Experimental Group, Control Group 1 and Control Group 2

The figure shows that there was improvement in the Experimental Group which was taught by the Polya's Problem Solving Method of teaching on the outcome variables i.e. Revised Bloom's Taxonomy. On the contrary, the Control Group-I and Control Group-II which were taught by the Conventional Method showed little improvement. The Control Group-I and Control Group-II which were taught by the Conventional Method also showed the consistency in achievement scores. Hence Problem Solving Method showed better performance on achievements scores as

compared to Conventional Method on Revised Bloom's Taxonomy. Hence all the null hypotheses were not accepted.

Table 4.27

Descriptive Statistics among Study Variables for Experimental Group

Variables	*N*	Minimum	Maximum	*M*	*SD*	Skewness	Kurtosis
Remembering	130	4	10	8.42	1.52	-.79	.18
Understanding	130	4	10	8.48	1.49	-.77	.01
Applying	130	4	18	11.54	3.20	.12	-.62
Analysis	130	4	20	10.83	3.30	.51	-.10
Evaluation	130	2	20	9.83	3.48	.77	.11
Creating	130	0	16	5.17	4.29	.69	-.50
Overall abilities	130	20	94	54.28	15.79	.46	-.48

Table 2.27 shows Descriptive Statistics among Study variables for Experimental Group. Results showed that the data was normally distributed (i.e., Skewness < 2, Kurtosis< 2) and therefore, there was no issue of symmetry and poutiness in Experimental Group which provide a base for further analysis and validity to calculated results.

Table 4.28

Descriptive Statistics and t-test Results for Problem Solving Method on Outcome variables for Experimental Group

	Pretest (n = 43)		Posttest (n = 43)				95% CI		
Outcome	*M*	*SD*	*M*	*SD*	*R*	*t*(42)	*LL*	*UL*	*Cohen's d*
Remembering	5.77	2.34	8.80	1.72	.82**	14.98**	-3.43	-2.61	1.49
Understanding	5.80	2.49	8.68	1.70	.86**	14.36**	-3.29	-2.48	1.36
Applying	7.73	4.18	12.91	3.79	.93**	22.47**	-5.64	-4.71	1.31
Analyzing	7.02	4.31	12.77	4.25	.94**	25.76**	-6.20	-5.30	1.35
Evaluating	5.48	3.11	12.50	4.22	.91**	24.86**	-7.59	-6.45	1.91
Creating	2.36	2.48	9.68	3.42	.77**	22.53**	-7.97	-6.66	2.47
Overall abilities	34.16	17.98	65.34	18.24	.96**	42.12**	-32.67	-29.68	1.74

**p< .01

Table 4.28 showed descriptive statistics and t-test results for pretest and posttest on outcome variables for Experimental Group. Results shows that there were significant differences found in posttest on remembering (M = 8.80, SD =1.72; t = 14.98, p<.01, *Cohen'sd* = 1.49), understanding (M = 8.68, SD = 1.70; t = 14.36,

$p<.01$, *Cohen's d* = 1.36), applying(M = 12.91, SD = 3.79; t = 22.47, $p<.01$, *Cohen's d* = 1.31), analysis (M = 12.77, SD = 4.25; t = 25.76, $p<.01$, *Cohen's* d = 1.35), evaluation (M = 12.50, SD = 4.22; t = 24.86, $p<.01$, *Cohen's* d = 1.91), creating (M = 9.68, SD = 3.42; t = 22.53, $p<.01$, *Cohen's* d = 2.47), and on overall abilities (M = 65.34, SD = 18.24; t = 42.12, $p<.01$, *Cohen's d* = 1.74). It could be concluded that results showed significant differences and high effect sizes were found among all study variables.

Form the table 4.28 following can also be inferred:

1. The achievement scores at sub-level Remembering were (Mean difference= 8.80-5.77=3.03), through Problem solving method of teaching reflects, the improvement. So the first null hypothesis (There is no significant effect of problem solving teaching method on the mean achievement scores at conceptual knowledge sub-level remembering in Mathematics) is not accepted.
2. There was an improvement in the achievement scores at sub-level Understanding (Mean difference=8.68-5.80=2.88), through Problem Solving Method of teaching. So the second null hypothesis (There is no significant effect of problem solving teaching method on the mean achievement scores at conceptual knowledge sub-level understanding in Mathematics) is not accepted.
3. There was an improvement in achievement scores at sub-level Applying (Mean difference=12.91-7.73=5.18) through Problem Solving Method. So the third hypothesis (There is no significant effect of problem solving teaching

method on the mean achievement scores at conceptual knowledge sub-level applying in the subject of Mathematics) is not accepted.

4. The improvement in achievement scores at sub-level Analyzing were (Mean difference=12.77-7.02=5.75), through Problem Solving Method of teaching. Hence the fourth hypothesis (There is no significant effect of problem solving teaching method on the mean achievement scores at conceptual knowledge sub-level analyzing in Mathematics) is not accepted.
5. There was an improvement in the achievement scores at sub-level Evaluating (Mean difference=12.50-5.48=7.02), through Problem Solving Method. Hence the fifth hypothesis (There is no significant effect of problem solving teaching method on the achievement scores at conceptual knowledge sub-level evaluating in Mathematics)is not accepted.
6. The improvement in achievement scores at sub-level Creating were (Mean difference=9.68-2.36=7.32), through Problem Solving Method of teaching. Hence the sixth hypothesis (There is no significant effect of problem solving teaching method on the mean achievement scores at conceptual knowledge sub-level creating in Mathematics)is not accepted.
7. The improvement in the achievement scores at overall abilities on knowledge dimension, Conceptual knowledge with six sub-levels i.e. Remembering, Understanding, Applying, Analyzing, Evaluating and Creating of Revised Bloom's Taxonomy were (Mean difference=65.34-34.16=31.18), through Problem Solving Method of teaching.. It shows the effectiveness of Poly's Problems Solving Method of teaching on Revised Bloom's Taxonomy.

The data in table 4.28 shows that the difference in achievements scores of Higher Order Thinking Skills (HOTS) is large than the Lower Order Thinking Skills (LOTS). The mean difference of Remembering, Understanding and Applying was (3.03, 2.88 & 5.18). The average mean change was 3.7. While on the other side the mean difference of Analyzing, Evaluating and Creating was (5.75, 7.02 & 7.32) was 6.7. hence the seventh null hypothesis (There is no significant effect of problem solving teaching method on the mean achievement scores at Higher Order Thinking Skills.) was not accepted, which also provide an answer to the research question (Does problem solving method develop the Higher Order thinking skills?). So it can be inferred that Poly's Problem Solving Method teaching works on Lower Order Thinking skills as well as on Higher Order Thinking skills.

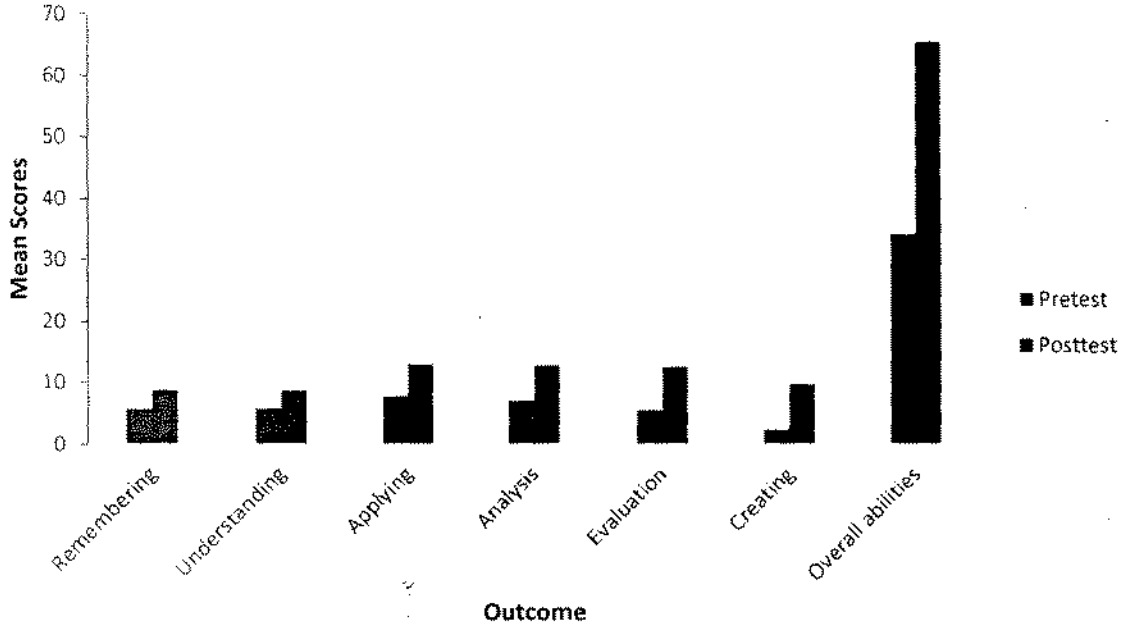


Figure 26 The t-test Results for Pretest and Posttest Differences on Outcome variables for Experimental Group

The graph in the above figure showed that there was improvement in overall abilities through the teaching of Problem Solving Method. The method showed effectiveness not only on Lower Order Thinking Skills (Remembering, Understanding & Applying) but more effective on Higher Order Thinking Skills (Analyzing, Evaluation & Creating). So the claim made by the method that it worked on Higher Order Thinking was evident from the difference of pre-test scores and post-test scores.

Table 4.29

Descriptive Statistics among Study Variables for Control Group-I

Variables	*N*	Minimum	Maximum	*M*	*SD*	Skewness	Kurtosis
Remembering	43	5	10	8.58	1.51	-.70	-.69
Understanding	43	5	10	8.63	1.54	-.63	-.89
Applying	43	4	16	10.23	2.80	-.05	-.69
Analysis	43	4	14	9.26	2.26	.18	.05
Evaluation	43	6	14	8.00	1.95	.96	.89
Creating	43	0	10	2.37	2.27	1.41	2.72
Overall abilities	43	26	72	47.07	10.77	.30	-.44

Table 4.29 showed Descriptive Statistics among Study variables for Control Group-I. Results showed that the data was normally distributed (i.e., Skewness< 2, Kurtosis< 2) and therefore, there was no issue of symmetry in Control Group-I and data is normally distributed.

Table 4.30

Descriptive Statistics and t-test Results for Pretest and Posttest Differences on Outcome Variables in Control Group-1

	Pretest (n = 43)		Posttest (n = 43)				95% CI		
Outcome	*M*	*SD*	*M*	*SD*	*r*	*t*(42)	*LL*	*UL*	*Cohen's d*
Remembering	6.58	1.78	7.88	1.17	.69**	6.65**	1.69	.90	.87
Understanding	7.35	1.46	8.14	1.14	.79**	5.83**	1.06	.51	.61
Applying	10.42	1.93	11.44	2.28	.81**	5.02**	1.43	.61	.48
Analysis	10.09	1.79	10.42	1.77	.91**	2.85**	.55	.09	.18
Evaluation	8.14	1.47	8.93	1.86	.54**	3.15**	1.29	.28	.47
Creating	3.26	2.50	3.35	2.60	.95**	.81	.32	.13	.03
Overall abilities	45.84	9.57	50.16	10.34	.94**	8.43**	5.36	3.29	.43

*p< .05, **p< .01

Table 4.30 shows descriptive statistics and t-test results for pretest and posttest on outcome variables for Control Group-I. Results showed that there were significant differences found in posttest on remembering (M = 7.88, SD =1.17; t = 6.65, p<.01, *Cohen'sd* = .87), understanding (M = 8.14, SD = 1.14; t = 5.83, p<.01, *Cohen's d* =

.61), applying(M = 11.44, SD = 2.28; t = 5.02, p<.01, *Cohen's d* = .48), analysis(M = 10.42, SD = 1.77; t = 2.85, p<.05, *Cohen's* d = .18),evaluating(M = 8.93, SD = 1.86; t = 3.15, p<.01, *Cohen's* d = .47), and on overall abilities (M = 50.16, SD = 10.34; t = 8.43, p<.01, *Cohen's d* = .43). The results were non-significant on creating (M = 3.35, SD = 2.60; t = .81, p>.05, *Cohen's d* = .48). It could be concluded that results showed significant differences but differences were too small and also their effect sizes were low as compared to experimental group.

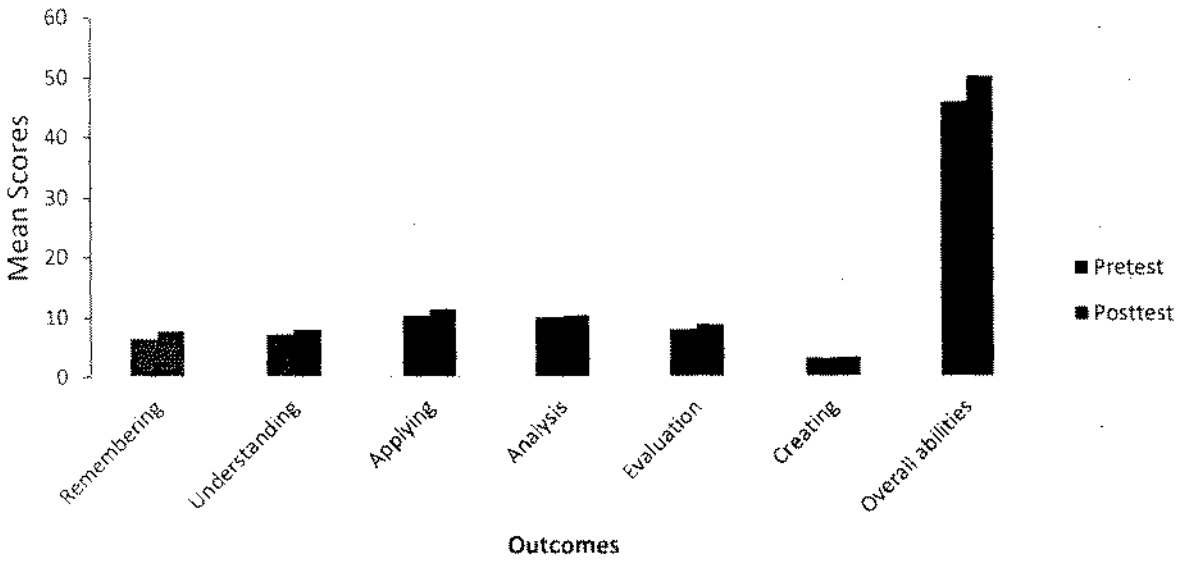


Figure 27 ***The t-test Results for Pretest and Posttest Differences on Outcome variables in Control Group-I***

The above figure showed the minor change in the Lower Order Thinking Skills i.e. in Remembering, Understanding and Applying, while the Higher Order Thinking Skills did not showed any significant improvement in the Control Group-I which was taught by the Conventional Method. Hence it could be inferred that Conventional Teaching Method was not effective for Higher Order Thinking Skills.

Table 4.31

Comparison of Mean difference of Experimental Group with Control Group-I

Outcome Variables	Experimental Group		Difference	Control Group-I		Difference
	Pre-test	Post-test		Pre-test	Post-test	
	Mean	Mean		Mean	Mean	
Remembering	5.77	8.80	3.03	6.58	7.88	1.3
Understanding	5.80	8.68	2.88	7.35	8.14	0.79
Applying	7.73	12.91	5.18	10.42	11.44	1.02
Analyzing	7.02	12.77	5.75	10.09	10.42	0.33
Evaluating	5.48	12.50	7.02	8.14	8.93	0.79
Creating	2.36	9.68	7.32	3.26	3.35	0.09
Average			5.2			0.72

From the table 4.31 the net effect on the outcome variables i.e. conceptual knowledge dimension at six sub-levels by the teaching through Problem Solving Method was 5.2 and the net effect by teaching through Conventional Method was 0.72 which is low. The difference of averages between Experimental Group and Control Group-I (5.2, 0.72)=4.48 showed that Experimental Group performed better with was thought through Problem Solving Method than the Control Group- I which was taught by Conventional Method. Hence it could be inferred that Problem Solving Method of teaching was better than Conventional Method for teaching of Mathematics.

Table 4.32

Descriptive Statistics among Study Variables for Control Group-II

Variables	*N*	Minimum	Maximum	*M*	*SD*	Skewness	Kurtosis
Remembering	44	4	10	8.80	1.72	-1.42	1.28
Understanding	44	4	10	8.68	1.70	-1.26	.99
Applying	44	4	18	12.91	3.79	-.44	-1.04
Analysis	44	4	20	12.77	4.25	-.44	-.88
Evaluation	44	2	20	12.50	4.22	-.48	-.44
Creating	44	2	16	9.68	3.42	-.20	-.34
Overall abilities	44	20	94	65.34	18.24	-.60	-.56

Table 4.32 showed Descriptive Statistics among Study variables for Control Group-II. Results shows that the data was normally distributed (i.e., Skewness< 2, Kurtosis< 2) and therefore there was no issue of symmetry in Control Group-II.

Table 4.33

Descriptive Statistics and t-test Results for Pretest and Posttest Differences on Outcome Variables in Control Group-II

	Pretest (n = 43)		Posttest (n = 43)				95% CI		
Outcome	*M*	*SD*	*M*	*SD*	*r*	*t*(42)	*LL*	*UL*	*Cohen's d*
Rememberin g	6.63	1.87	8.58	1.51	.92**	16.97**	-2.18	-1.72	1.16
Understandin g	6.30	2.05	8.63	1.54	.89**	15.73**	-2.62	-2.02	1.30
Applying	9.67	2.68	10.23	2.80	.90**	3.09**	-.92	-.19	.20
Analysis	8.88	2.06	9.26	2.26	.93**	3.09**	-.61	-.13	.17
Evaluation	7.67	1.44	8.00	1.95	.91**	2.46*	-.59	-.05	.19
Creating	2.51	2.00	2.37	2.27	.93**	1.13	-.10	.38	.06
Overall abilities	41.67	10.91	47.07	10.77	.98**	21.44**	-5.90	-4.88	.50

*p< .05, **p< .01

Table 4.33 shows descriptive statistics and t-test results for pretest and posttest on outcome variables for Control Group-II. Results showed that there were significant differences found in posttest on remembering (M = 8.58, SD =1.51; t = 16.97, p<.01, *Cohen'sd* = 1.16), understanding (M = 8.63, SD = 1.54; t = 15.73, p<.01, *Cohen's d* = 1.30), applying(M = 10.23, SD = 2.80; t = 3.09, p<.01, *Cohen's d* = .20), analysis (M = 9.26, SD = 2.26; t = 3.09, p<.01, *Cohen's* d = .17), evaluation (M = 8.00, SD = 1.95; t = 3.09, p<.01, *Cohen's* d = .17), and on overall abilities (M = 47.07, SD = 10.77; t = 21.44, p<.01, *Cohen's d* = .50). The results were non-significant on creating (M = 2.37, SD = 2.27; t = 1.13, p>.05, *Cohen's d* = .06). It could be concluded that results shows significant differences but differences were too small and some variables have low effect sizes as compared to experimental group

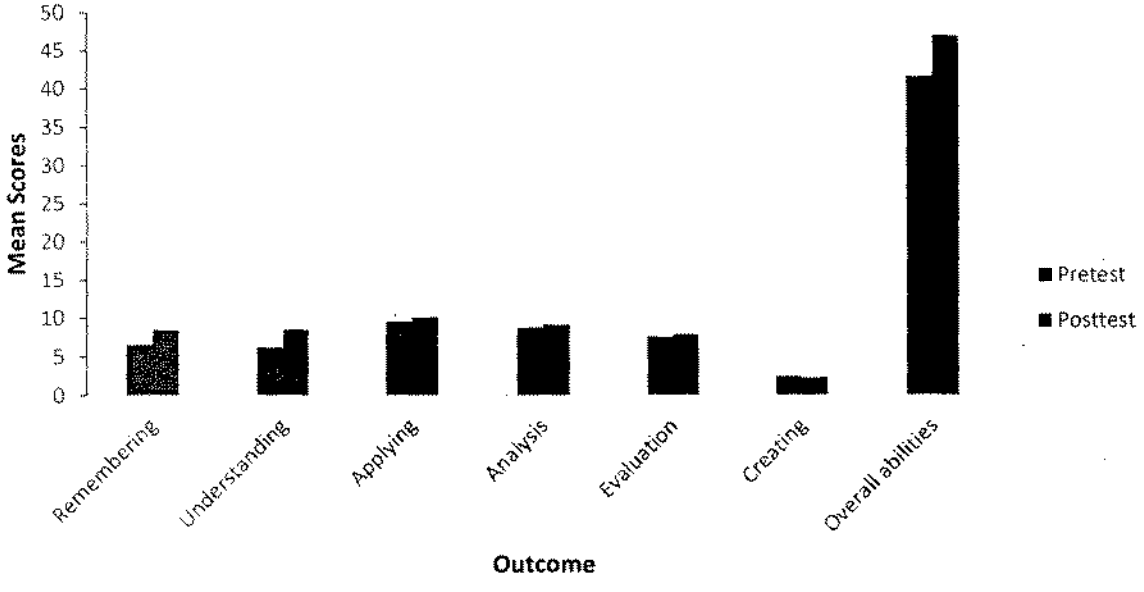


Figure 28 ***The t-test Results for Pretest and Posttest Differences on Outcome variables in Control Group-II***

The above figure showed the minor change in the Lower Order Thinking Skills i.e. in Remembering, Understanding and Applying, while the Higher Order Thinking Skills did not show any significant improvement in Control Group-II taught by the Conventional Method. Hence it could be inferred that Conventional Teaching Method was not effective for developing Higher Order Thinking Skills.

Table 4.34

Comparison of Mean Difference of Experimental Group with Control Group-II

Outcome Variables	Experimental Group		Difference	Control Group-II		Difference
	Pre-test Mean	Post-test Mean		Pre-test Mean	Post-test Mean	
Remembering	5.77	8.80	3.03	6.63	8.58	1.95
Understanding	5.80	8.68	2.88	6.30	8.63	2.33
Applying	7.73	12.91	5.18	9.67	10.23	0.56
Analyzing	7.02	12.77	5.75	8.88	9.26	0.38
Evaluating	5.48	12.50	7.02	7.67	8.00	0.33
Creating	2.36	9.68	7.32	2.51	2.37	-0.14
Average			5.2			0.9

From the table-4.34 the net effect on the outcome variables i.e. conceptual knowledge dimension at six sub-levels by the teaching through Problem Solving Method was 5.2 and the net effect by teaching through Conventional Method was 0.9 which is very low. The difference of averages between Experimental Group and Control Group-II was(5.2, 0.9)=43 shows that Experimental Group performed better taught through Problem Solving method than the Control Group- II taught with Conventional Method.

Table 4.35

Comparison of Mean Difference of Group-I with Control Group-II

Outcome Variables	Control Group-I		Difference	Control Group-II		Difference
	Pre-test Mean	Post-test Mean		Pre-test Mean	Post-test Mean	
Remembering	6.58	7.88	1.3	6.63	8.58	1.95
Understanding	7.35	8.14	0.79	6.30	8.63	2.33
Applying	10.42	11.44	1.02	9.67	10.23	0.56
Analyzing	10.09	10.42	0.33	8.88	9.26	0.38
Evaluating	8.14	8.93	0.79	7.67	8.00	0.33
Creating	3.26	3.35	0.09	2.51	2.37	-0.14
Average			0.7			0.9

The comparison of Control Group-I and Control Group-II which were taught by the Conventional Method showed the same results to some extent. It means that there was no effect of extraneous variable. It also showed that there was no Placebo effect, Hawthorne effect, Interaction effect, Ecological validity and Reactive effect.. So the result produced by the Experimental Group was the effect of treatment which was the Polya's Problem Solving Method of teaching. The slight difference in achievement result maybe the effect of teachers teaching style.

4.4 Discussion

In present world of knowledge, almost every country was trying to improve the quality of society through the quality of education by the introduction of appropriate measures in curriculum, schools system, assessment and teacher's training OECD, (2014). Likewise Pakistan is also doing the same activities for quality improvements. The new Mathematics curriculum, 2006 is designed in accordance to international standards which promotes conceptual understanding and logic among the students and makes them able to use it in daily life (Amirali, 2012). According to the analysis presented by Amirali (2012), about the status of teaching of Mathematics, tells a different story that despite of the improvement in curriculum of Mathematics the in-depth learning was not usually the focal point of all the classroom activities and interactions in Pakistan. Overview of the research studies alludes a gap between curricular goals and what actually happening in reality in the classrooms in Pakistan. For the empirical evidence a number of studies conducted on national, provincial and district level in Pakistan, which endorsed the fact that achievement of students in the subject of Mathematics, is very poor. These studies have been consistent in justifying the fact of low level achievement in subject of Mathematics. The studied indicated many reasons for this low achievement in the subject of Mathematics and among one of them is the teaching method of the teacher (e.g. Academy of Planning and Management, 1999; Benoliel &Miske, 1999; Government of Pakistan, 1999, Government of Baluchistan, 1999; Government of Sindh, 2000; Abdeen&Jone,2000; Samo,2009 cited Amirali, 2011).

For the effective achievement of all these objectives mentioned in the National Curriculum, 2006 supportive pedagogy was required. They could not be achieved by the traditional method of teaching (UNESCO,2001; Williams, 2005; PISA, 2012).

This fact was evident from Sadia investigation of Mathematics achievement of middle grade students in Pakistan that students could only solve or attempt simple routine item requiring simple Mathematical skills and showed poor performance in items that require reasoning (2010). As cited by Zaman (2011), that according to National Research Council, much of the failure in the subject of Mathematics at schools level was due to a traditional way of teaching of Mathematics that was inappropriate to the way most students learn.

According Schoenfeld cited by Tutkun & Okay (2012), Problem solving was a method of solving problems, in which students encounter a problem for which students are not ready to solve it immediately. Then they do efforts and examine carefully, work step by step and then find a solution by themselves. In this process the students read the problem carefully, analyze it and collect all the possible information regarding the problem, try to find out the relationship between known and unknown, then devise a plan keeping in view his/her Mathematical knowledge. After this step implementation starts and at final step he/she become able to solve similar examples (Tutkun & Okay, 2012).

Problem solving method was a source of developing problem solving ability, through which students could be able to solve daily life problems. It was also a source of developing in-depth knowledge about the subject. According to Lester &Charles (2003) cited Yaun (2013), the most of the Mathematicians were inspired by the

classical work of Polya (1981) and Dewey (1933). Teaching of Mathematics through problems solving Method (PSM) was a source of making the students independent discover. PSM is helpful in developing Higher Order Thinking Skills (HOTS).

A one Way ANOVA analysis for effect of PSM on outcome variables for three groups was applied. Results showed that experimental group has significant greater mean (M = 65.34, SD =18.24; F= 22.7, p<.01, $\dot{\eta}$ = .34), than control group 1 (M = 50.16, SD =1034; t = 6.65,) and control group 2 (M = 47.07, SD =10.77). This showed that PSM has performed better than the conventional method. These results were supported by the results of the studies done by Kloawole, Olasosu & Ajetunmobi (2013) which showed that Problem Solving Method of instructions was significantly better than the conventional method. For this they concluded that method of instruction used by the teacher has effect on student's achievement in Mathematics. The same results were endorsed by the findings of Nikandrov (1990) cited by Kloawole (2013), which showed that ability to solve problem could be developed through a very good strategy of teaching. This result also indicates that the conventional method was not effective for teaching of Mathematics This finding was supported by Kloawole (2013), quoted that, The 'chalk and talk' or conventional method was no longer effective for the development of higher level ability in Mathematics. Traditional methods of teaching cannot satisfy the complexity of the modern technological society (Ubuz, 1994; Dongpong, 2000; Wang, 2001Carson,2007; cited by Kloawole, 2013).

Post Hock Analysis (Tuckey HSD method) for effect of Problem Solving Method on Outcome variables for Experimental Group, Control Group 1 and Control

Group 2 was applied on the three groups. Results show that Problem Solving Method on Outcome variables which were Remembering, Understanding, Applying, Analyzing, Evaluating and Creating works better in Experimental Group than Control Group 1 and Control Group 2 which were taught with conventional method. So the data was in supports of the hypothesis that the Problem Solving Method has effect on the Revised Bloom's Taxonomy. This fact was supported by the findings of Kolawole and Popoola (2011), Adebule (2004) and Ibitola (2003) cited by Kolavole (2013), problem solving method was better than conventional method.

Descriptive Statistics and t-test Results for Problem Solving Method on Outcome variables for Experimental Group was performed. Results showed that there were significant differences found in posttest on remembering (M = 8.80, SD =1.72; t = 14.98, p<.01, *Cohen'sd* = 1.49), understanding (M = 8.68, SD = 1.70; t = 14.36, p<.01, *Cohen's d* = 1.36), applying (M = 12.91, SD = 3.79; t = 22.47, p<.01, *Cohen's d* = 1.31), analysis (M = 12.77, SD = 4.25; t = 25.76, p<.01, *Cohen's* d = 1.35), evaluation (M = 12.50, SD = 4.22; t = 24.86, p<.01, *Cohen's* d = 1.91), creating (M = 9.68, SD = 3.42; t = 22.53, p<.01, *Cohen's* d = 2.47), and on overall abilities (M = 65.34, SD = 18.24; t = 42.12, p<.01, *Cohen's d* = 1.74). It could be concluded that results shows significant differences and high effect sizes were found among all study variables. These results were supported by the research study of Yuan (2013), through problem solving method students could be made able to learn the abilities of reasoning and solve daily life problems. According to the conclusion drawn by Amirali (2011), in his research study that the reforms in the curriculum of Pakistan and other parts of the world strongly recommend the problem solving approaches for

teaching of Mathematics at schools level. Conventional methods cannot stay longer to address the new advancement. The same results were found by the study of (Amirali& at el, 2010) the objectives tested were:

(i) To determine the role of problem solving method in the academic achievement of students in Mathematics at elementary level.

(ii) To compare the achievement of students taught by problem solving method and students taught by traditional method. The results were in the favor of problem solving method and on the basis of the findings in that study, the following conclusions were made:

- Students taught through problem solving method achieved better than those taught by traditional method.
- There exists a significant difference in the achievement of Mathematics students taught through problem solving method and traditional method.
- Difference between the achievements level was due to problem based strategy, otherwise both group had equal basic knowledge of Mathematics (Amirali, 2011; PISA, 2012; OECD, 2014).

The recommendation given in light of findings and conclusions were that problem solving method of teaching Mathematics was more effective than the traditional method and teachers of Mathematics may use problem solving method for the better results and understanding in Mathematics. In the light of the t-test the null hypotheses were not accepted, because the treatment showed the significant difference in mean scores of pre-test and post-test (Ubuz, 1994; Dongpong, 2000; Wang, 2001Carson, 2007; cited by Kloawole, 2013).

The improvement in overall abilities was (Mean difference=65.34-34.16=31.18) through Problem Solving Method. It showed the effectiveness of Poly's Problems Solving Method on Revised Bloom's Taxonomy. It was also noted that the difference in Higher Order Thinking Skills (HOTS) was larger than the Lower Order Thinking Skills (LOTS). The mean difference of Remembering, Understanding and Applying was (3.03, 2.88 &5.18). The average change was 3.7. While on the other side the mean difference of Analyzing, Evaluating and Creating was (5.75, 7.02 & 7.32). The average change was 6.7.which provides answer to the research question (Does problem solving method develop the Higher Order thinking skills?). So it could be inferred that Polya's Problem Solving Method works on HOTS. According to the research article written by Narayanan & Munirathanam (2012), regarding the Bloom's Taxonomy of Educational Objectives and problem solving method for teaching stated that the Higher Order Thinking Skills (HOTS) could be produced by the problem solving method. HOTS refer to the skill of collecting, analyzing and creating. Hence the finding of the present study is in support of the study conducted by Narayanan & Munirathanam (2012). This was also according to the Vyjayanthi (2011), that Rote learning was considered as deceptive and could not be retained for a longer time period, and also did not let students to develop rational thinking and not solved the daily life problems. While in-depth learning in Mathematics through problem solving allows the students to develop HOTS. In the conclusion of their research Narayanan &Munirathanam (2012), stated that problem solving method and Bloom's Taxonomy could be used as a tool in the process of teaching and learning.

Descriptive Statistics and t-test Results for Pretest and Posttest Differences on Outcome variables in Control Group-I and Control Group-II showed change in the results from which that it could be concluded that results showed significant differences but differences were too small and also their effect sizes were low as compared to experimental group. So the conventional method effect as compared to PSM was not much significant on RBT. It was also inferred that through Conventional method HOTS could not be developed.

Revised Bloom's Taxonomy was used to assess the both methods and pre-test and post-test provided the difference of both methods on RBT. This fact was supported by Haklikari et al (2007), the student's achievement was the main concern of educational psychologists over the last decades. The achievement was based on the knowledge provided to the student's before the assessment. This was a fact that achievement may be assessed on different cognitive levels. As Anderson (1995) cited by Hailikari et al (2007), states that factual knowledge means knowing what and procedural knowledge means knowing how may be assessed through different methods. Therefore, taking into account the different nature of knowledge different assessment levels were required. Similarly, in the revised Bloom's taxonomy Anderson & Krathwohl (2001) quoted by Hailikari et al (2007), the division was made between different types of knowledge and cognitive processes which were four knowledge level and six level of cognitive process. The taxonomy presents two ways to the desired product of learning that was, the kind of knowledge to be learned, and the cognitive process along which the knowledge could differ. The taxonomy provides a viewpoint on how different types and levels of knowledge could be

assessed at different levels and sub levels. It could be infer form that the revised Bloom's Taxonomy was a suitable tool for the assessment of achievement at different levels (Hailikari et al, 2007). Effect of any instructional method could be assessed through achievements scores by applying reliable tool for assessment, this fact was endorsed by many researches like (Portier, 1995; Dochy, 1996; Dochy& McDowell, 1997; Anderson & Krathwohl, 2001 cited by Hailikari et al, 2007).

The Revised Bloom's Taxonomy was a reliable tool for the assessment of achievement of different levels.The finding of the study was also supporting the statement given by Takbir (2010), the learning difficulties of students in the subject of Mathematics described the worth of in-depth learning for students, and that through it a number of personal needs could be fulfilled like satisfaction and it accelerates the process of further learning for the mastering of new material. The traditional method of teaching was a source of surface learning through which in-depth knowledge could not be developed for this some specialized method may be used for instruction, like problem solving. Polya's problem solving method claims that through this higher level cognitive abilities could be developed in students (Polya, 1976). The finding of the study also matched with the researchers (e.g. Johson, 1985; Prawat, 1989; Kilbourn,1992; Halfords, 1993; Sierpinska, 1994; Newton,2002 cited by Takbir, 2010) that students who acquired the subject knowledge through understanding show an enhanced ability to think flexibly while dealing with new problems. It makes the learner independent discoverer who could interact with the world and think for themselves and solve daily life problems. The study also provided the evidence that conventional method was not working properly in the subject of Mathematics as it

was showing low performance of the students. The in-depth skills were not addressed by this method. The same was reported by Takbir (2010), " it was and has not been a central concern in every classroom in the world in general and in Pakistan in particular"(PISA, 2012;OECD, 2014).

According to a survey reported by Takbir (2010), "The current literature reveals that understanding avoidance was not unique to the context of Pakistan and other developing countries where the quality of education was considered to be poor; it was and had been a matter of concern in the context of developed countries as well, where there was a tendency to emphasize memorization and reproduction of information" (e.g. Wildy & Wallace, 1992;McLaughlin& Talbert, 1993;Das & Barunah, 2010 cited by Takbir, 2010).

To inculcate the in-depth learning was a difficult task because deep cognitive process was required for this. Students did not engage themselves in this process except that they were motivated towards this process. Students had many potential, abilities, skills, perception, expectations and preferences which may be suitable or not suitable for in-depth understanding (Takbir, 2010). It was due to two factors first external factor and second was the internal factor. The external factors includes parents, career aspirations, different influences and teachers, while the internal factors were intrinsic motivation and interests (Takbir, 2010). Researchers (Stoll, 1999; Lockheed & Verspoor, 2000; cited by Takbir), argued that "the teacher's interaction with learners is the axis on which education quality of learning turns". In the same way the National Curriculum 2006 and the Education Policy 2009 states that a marked change was required in teacher's role from transmitter of information to

creator of learning environment in the classroom in which students develop rational thinking and understanding of the Mathematical concepts. The teacher could do it by using innovative methods of teaching, like problem solving. Teacher's pedagogical skills play a vital role in the developing in-depth learning among the students (Joseph &Yoe, 2010). Study also reveals the fact that PSM is essential to develop in-depth skills. The same was supported by Amirali (2008), Pakistan made an effort of improving the quality of education through the initiatives like change in curriculum, through teachers training programmes. The new Mathematics curriculum emphasized on conceptual understanding, logical reasoning and problems solving skills.

It could only be possible if teachers change their current Mathematics teaching. The National Curriculum for Mathematics, 2006, demands that teacher role has to change from dispensing information to planning investigative tasks, conducive learning environment and in-depth development of higher skills in Mathematics (Iqbal, 2009). The finding of the study was also endorsed by exploring the answer of the question that 'how problem solving could be used as a tool for cognitive development?' Tripathi (2010), found that problem solving method was a powerful tool for the development of cognition and higher level abilities. In its conclusion it was also found that if problem solving method was used in elementary level it was more effective because students early Mathematical years were very important with respect to attitudes and motivation. This fact was endorsed by Selden and Selden (1995). According to Chapman (2012), in his research article about 'challenges in Mathematics teacher education' contributes that poor performance in Mathematics was the result of poor teaching. For the development of higher abilities among the

students problem solving approaches are required. It was also proved by the present study. The recommendation given by the Arslan (2012), in his research paper about comparison of thinking styles in teaching of social sciences and Mathematics said that studies are required to find out the effect of problem solving method on the development of higher level abilities in teaching of Mathematics (Ubuz, 1994; Dongpong, 2000; Wang, 2001;Carson, 2007; cited by Kloawole, 2013).

Hence form the above discussion it may be concluded that the Polya's Problem Solving Method of teaching Mathematics is better than the Conventional Method of teaching .The findings of the present study provided the empirical evidences in the favor of Problem Solving Method that it is an effective method of developing Lower Order Thinking Skills and as well as Higher Order Thinking Skills in the subject of Mathematics. This study also proved that through PSM method objectives of Revised Bloom's Taxonomy can be achieved. This method is also helpful in developing the Problem Solving ability among the students. The study does not rule out the role of the innovative teacher, because the improvement in the achievements scores of Control Groups reflects the contribution of teachers. The study also confirms the effect of Conventional Method of teaching on Mathematics but it only effective to develop Lower Order Thinking Skills. So for development of Higher Order Thinking Skills required for further study of Mathematics this is a suitable method. It also helps the students to learn the problem solving ability by which he/she can solve the daily life problems. It is also a useful method of developing in-depth learning of Mathematics.

CHAPTER 5

SUMMARY, FINDINGS, CONCLUSIONS AND RECOMMENDATIONS

5.1 SUMMARY

Mathematics remained an important subject in the entire curriculum at all level. It plays a vital role in the development of mental faculties, like reasoning, observing and analyzing. It also helps in the understanding of other subject. OECD (2014), Development of every county depends upon the development of science and technology and development of science and technology depends upon the subject of Mathematics. In this world of knowledge, almost every country is trying to improve the quality of society through the quality of education by the introduction of appropriate measures in curriculum, schools system, assessment and teacher's training. Likewise Pakistan is also the part of these activities, the Ministry of Education and its supporting sister institution are working for the improvement of quality education.

The new Mathematics curriculum, 2006 is designed accordance with the international standards which promotes conceptual understanding and logic among the students and makes them able to use it in daily life. Most of the teachers transfer

the knowledge to the students according to the textbook through 'rote memorization' and traditional ways, asses' students' learning through lower level of cognition. In Pakistan Mathematics learning consists mainly of rules for solution, memorization, and solution of textbook problems. Surface information/knowledge is provided to the students without in-depth understanding of the subject. (Government of Pakistan, 2006, 2009)

According to the reports of PISA (2012) and OECD (2014), the performance of the student in this subject was not good all over the world. In Pakistan it is considered as a nightmare for the students. Mathematics is considered as a dull and boring subject. Many talented students were asked about the Mathematics they replied that it was a difficult subject and they faced difficulties in the subject. There are many reasons of disinterest of students in Mathematics among one of them is the teaching method (Tayyaba, 2010).

Polya's Problem solving is a method of solving problems, in which students encounter a problem for which students are not ready to solve it immediately. Then they do efforts and examine carefully, work step by step and then find a solution by themselves. In this process the students read the problem carefully; analyze it and collect all the possible information regarding the problem; try to find out the relationship between known and unknown, then devise a plan keeping in view his/her Mathematical knowledge. In the third step implementation starts and at final step he/she becomes able to solve similar examples of that problem. Problem solving method is a source of developing problem solving ability, through which students could be able to solve daily life problems. It is also a source of developing in-depth

knowledge about the subject. According to Lester &Charles (2003) cited Yaun (2013), the most of the Mathematicians were inspired by the classical work of Polya (1981) and Dewey (1933). Teaching of Mathematics through problems solving Method (PMS) is a source of making the student's independent discoverer. PSM is helpful in developing Higher Order Thinking (HOT). The Revised Bloom's Taxonomy is the reliable source for evaluation. The RBT is two way taxonomy of educational objective, it has for dimensions of knowledge and six sub-levels of cognition process The focus of RBT is not 'what to learn' but 'how to learn'.

The present study was designed to find out the Polya's Problem Solving Method of Teaching Mathematics at elementary level on Revised Bloom's Taxonomy. The study was consisted of eight objectives. For the present study eight null hypotheses were developed. The study was quantitative in nature. The study was true experimental. The design of the study was double control experimental design. This design helps in controlling many external threats.

To perform the experiment FG boys Schools I-8/4 Islamabad was selected from the urban area. A pre-test which was consisted of sixty items based on conceptual dimension of knowledge was prepared. The items of the test were taken from the topics of word problems, practical Geometry and theoretical Geometry. The test was of hundred marks. After making the test valid and reliable it was administrated to 132 students of 8^{th} grade. On the bases of rubric made for scoring the students were divided into three groups through randomization. One group was named as an Experimental Group that was taught by the researcher himself through PSM, while the other two Control Groups were taught by the local schools teachers

through Conventional Method. The content was selected with the consultations of local teachers and forty lesson plans were decided for the 8-week experiment.

Researcher developed forty lesson plans based on Revised Bloom's Taxonomy. The post-test was developed by changing the order of the questions of post-test. After collecting the achievement scores of post-test the difference was collected and the acquired data was treated through SPSS. The chapter four represented the analyses of data. The data was analyzed through One Way ANOVA and t-test. The present chapter is about the finding, conclusions and recommendation on the bases of chapter four.

5.2 Findings of the Study

Following findings were obtained from the analyses of data:

1. Normal distribution analysis for study variables (Remembering, Understanding, Applying, Analyzing, Evaluating, Creating and overall abilities) among Control Group-I was conducted, Kolmogorov-Smirnov and Shapiro-Wilk, results were non-significant (p>.05 = Normal Distribution). In the same way the overall abilities were also non-significant (p>.05 = Normal Distribution) which is fulfilling the assumptions of normal distribution of the data (Table-4.20)
2. Normal distribution analysis for study variables (Remembering, Understanding, Applying, Analyzing, Evaluating, Creating and overall abilities) among Control Group-II was conducted Kolmogorov-Smirnov and Shapiro-Wilk, results were non-significant (p>.05 = Normal Distribution). In the same way the overall abilities were also non-significant (p>.05 = Normal

Distribution) which is fulfilling the assumptions of normal distribution of the data (Table-4.21)

3. Normal distribution analysis for study variables (Remembering, Understanding, Applying, Analyzing, Evaluating, Creating and overall abilities) among Experimental Group was conducted Kolmogorov-Smirnov and Shapiro-Wilk, results were non-significant (p>.05 = Normal Distribution). In the same way the overall abilities were also non-significant (p>.05 = Normal Distribution) which is fulfilling the assumptions of normal distribution of the data (Table-4.22)
4. P-P plots for normal distribution analysis of experimental group was done for outcome variables (Remembering, Understanding, Applying, Analyzing, Evaluating, Creating and overall abilities) all the outcome variables fulfill the assumption of normal distribution (Appendix-E).
5. P-P plots for normal distribution analysis of Control Group-I was done for outcome variables (Remembering, Understanding, Applying, Analyzing, Evaluating, Creating and overall abilities) all the outcome variables fulfill the assumption of normal distribution (Appendix-E).
6. P-P plots for normal distribution analysis of Control Group-II was done for outcome variables (Remembering, Understanding, Applying, Analyzing, Evaluating, Creating and overall abilities) all the outcome variables fulfill the assumption of normal distribution (Appendix-E).
7. Test of Homogeneity of Variances (Levene Test)for all groups was done. Experimental Group with (P=.37), Control Group-I (P=.23) and Control

Group-II (P=.45). Results shows that among all groups Levene test results were non-significant (p>.05 = Equal Variance) which is fulfilling the assumption of homogeneity of variance (Table-4.23)

8. One Way ANOVA analysis for effect of Problem Solving Method on outcome variables for Experimental Group was done. Results show that experimental group has significant mean (M = 65.34, SD =18.24; F = 22.7, p<.01, $\acute{\eta}$ = .34) (Table-4.24).
9. One Way ANOVA analysis for effect of Conventional Method on outcome variables for Control Group-I was done. Results showed that experimental group has significant greater mean (M = 65.34, SD =18.24; F = 22.7, p<.01, $\acute{\eta}$ = .34), than the Control Group-I(M = 50.16, SD =10.34; t = 6.65,) (Table-4.24).
10. One Way ANOVA analysis for effect of Conventional Method on outcome variables for Control Group-II was done. Results showed that experimental group has significant greater mean (M = 65.34, SD =18.24; F = 22.7, p<.01, $\acute{\eta}$ = .34), than the Control Group-II(M = 47.07, SD =10.77) (Table-4.24).
11. Post Hock analysis (Tuckey HSD method) for effect of PSM on outcome variables for Experimental Group, Control Group-I and Control Group-II was done. Results showed the better effect of Problem Solving Method of teaching on Outcome variables i.e. Remembering, Understanding, Applying, Analyzing, Evaluating and Creating in Experimental Group than Control Group 1 and Control Group 2 which were taught by conventional method. So

the data in the table-5 supported that the Problem Solving Method has an effect on the Revised Bloom's Taxonomy (Table-4.25)

12. Means for groups in Homogeneous subsets for effect of Problem Solving Method on Outcome variables for Experimental Group, Control Group 1 and Control Group 2 was done. Uses Harmonic Mean Sample Size is 43.32.The group sizes are unequal. The harmonic mean of the group sizes used. Type I error levels were not guaranteed (Table-4.26)
13. The effect of Problem Solving Method on Outcome variables for Experimental Group, Control Group 1 and Control Group II. There was improvement in the Experimental Group which was taught by the Polya's Problem Solving Method on the outcome variables i.e. Revised Bloom's Taxonomy. In contrast the Control Group-I and Control Group-II which were taught by the Conventional Method showed little improvement. Hence PSM showed better performance as compared to Conventional Method on RBT. Hence all the null hypotheses were not accepted (Figure-25)
14. Descriptive Statistics among Study variables for Experimental Group was analyzed. Results showed that the data was normally distributed (i.e., Skewness< 2, Kurtosis< 2) and therefore there was no issue of symmetry in Experimental Group (Tabele-4.27).
15. Descriptive Statistics and t-test Results for Problem Solving Method on Outcome variables for Experimental Group was analyzed. Results showed that there were significant differences in posttest on remembering (M = 8.80, SD =1.72; t = 14.98, p<.01, *Cohen'sd* = 1.49), understanding (M = 8.68, SD =

1.70; t = 14.36, p<.01, *Cohen's d* = 1.36), applying(M = 12.91, SD = 3.79; t = 22.47, p<.01, *Cohen's d* = 1.31), analysis (M = 12.77, SD = 4.25; t = 25.76, p<.01, *Cohen's* d = 1.35), evaluation (M = 12.50, SD = 4.22; t = 24.86, p<.01, *Cohen's* d = 1.91), creating (M = 9.68, SD = 3.42; t = 22.53, p<.01, *Cohen's* d = 2.47), and on overall abilities (M = 65.34, SD = 18.24; t = 42.12, p<.01, *Cohen's d* = 1.74). It was concluded that results showed significant differences and high effect sizes among all study variables (Table-4.28).

16. The improvement in Remembering was (Mean difference= 8.80-5.77=3.03) through Problem solving method. So the first null hypothesis (There is no significant effect of problem solving teaching method on the mean achievement scores at conceptual knowledge sub-level remembering in Mathematics) was not accepted (Table-4.28).
17. The improvement in Understanding was (Mean difference=8.68-5.80=2.88) through Problem Solving Method. So the second null hypothesis (There is no significant effect at problem solving teaching method on the mean achievement scores of conceptual knowledge sub-level understanding in Mathematics) was not accepted.(Table-4.28).
18. The improvement in Applying was (Mean difference=12.91-7.73=5.18) through Problem Solving Method. So the third hypothesis (There is no significant effect of problem solving teaching method on the mean achievement scores at conceptual knowledge sub-level applying in the subject of Mathematics) was not accepted.(Table-4.28).

19. The improvement in Analyzing was (Mean difference=12.77-7.02=5.75) through Problem Solving Method. Hence the fourth hypothesis (There is no significant effect of problem solving teaching method on the mean achievement scores at conceptual knowledge sub-level analyzing in Mathematics) was not accepted.(Table-4.28).
20. The improvement in Evaluating was (Mean difference=12.50-5.48=7.02) through Problem Solving Method. Hence the fifth hypothesis (There is no significant effect of problem solving teaching method on the achievement scores at conceptual knowledge sub-level evaluating in Mathematics) was not accepted (Table-4.28).
21. The improvement in Creating was (Mean difference=9.68-2.36=7.32) through Problem Solving Method. Hence the sixth hypothesis (There is no significant effect of problem solving teaching method on the mean achievement scores at conceptual knowledge sub-level creating in Mathematics) was not accepted (Table-4.28).
22. The difference in Higher Order Thinking Skills (HOTS) is large than the Lower Order Thinking Skills (LOTS). The mean difference of Remembering, Understanding and Applying was (3.03, 2.88 & 5.18). The average mean change was 3.7. While on the other side the mean difference of Analyzing, Evaluating and Creating was (5.75, 7.02& 7.32) = 6.7. Hence the hypothesis (There is no significant effect of problem solving teaching method on the mean achievement scores at Higher Order Thinking Skills) was not accepted. Provided answer to the research question (Does problem solving method

develop the Higher Order thinking skills?). So it could be inferred that Poly's Problem Solving Method worked on HOTS. Hence null hypothesis seventh was not accepted (Table-4.28).

23. The improvement in overall abilities was (Mean difference=65.34-34.16=31.18) through Problem Solving Method. It showed the effectiveness of Poly's Problems Solving Method effectiveness on Revised Bloom's Taxonomy (Table-4.28).
24. There was improvement in overall abilities through the teaching of Problem Solving Method. The method showed its effectiveness not only on Lower Order Thinking Skills (Remembering, Understanding & Applying) but more effective on Higher Order Thinking Skills (Analyzing, Evaluation & Creating). So the claim made by the method that it works on Higher Order Thinking was evident from the difference of pre-test scores and post-test scores (Figure-26)
25. Descriptive Statistics among Study variables for Control Group-I. Results showed that the data was normally distributed (i.e., Skewness < 2, Kurtosis< 2) and therefore there was no issue of symmetry in Control Group-I (Table-4.29).
26. Descriptive statistics and t-test results for pretest and posttest differences on outcome variables in control group-I was analyzed. Results showed that there were significant differences found in posttest on remembering (M = 7.88, SD =1.17; t = 6.65, p<.01, *Cohen'sd* = .87), understanding (M = 8.14, SD = 1.14; t = 5.83, p<.01, *Cohen's d* = .61), applying(M = 11.44, SD = 2.28; t = 5.02,

$p<.01$, *Cohen's d* = .48), analysis ($M = 10.42$, $SD = 1.77$; $t = 2.85$, $p<.05$, *Cohen's* d = .18), evaluation ($M = 8.93$, SD = 1.86; $t = 3.15$, $p<.01$, *Cohen's* d = .47), and on overall abilities ($M = 50.16$, $SD = 10.34$; $t = 8.43$, $p<.01$, *Cohen's d* = .43). The results were non-significant on creating ($M = 3.35$, $SD = 2.60$; $t = .81$, $p>.05$, *Cohen's d* = .48). It was concluded that results showed significant differences but differences were too small and also their effect sizes were low as compared to experimental group (Table-4.30).

27. The graph showed the minor change in the Lower Order Thinking Skills i.e. in Remembering, Understanding and Applying, while the Higher Order Thinking Skills did not show any significant improvement in the Control Group-I taught by the Conventional Method. Hence it was inferred that Conventional Teaching Method was not effective for HOTS (Figure-27).

28. Comparison of Mean difference of Experimental Group with Control Group-I was analyzed. The net effect on the outcome variables i.e. conceptual knowledge dimension six sub-levels by the teaching through Problem Solving Method was 5.2 and the net effect by teaching through Conventional Method was 0.72 which was very low. The difference of averages between Experimental Group and Control Group-I (5.2, 0.72), showed that Experimental Group performed better with PSM than the Control Group- I that was taught by Conventional Method. Hence it was inferred that PSM was better than Conventional Method (Table-4.31).

29. Descriptive Statistics among Study variables for Control Group-II was analyzed. Results showed that the data was normally distributed (i.e.,

Skewness < 2, Kurtosis < 2) and therefore, there is no issue of symmetry Control Group-II (Table-13)

30. Descriptive statistics and t-test results for pretest and posttest differences on outcome variables in control group-II were analyzed. Results showed that there were significant differences found in posttest on remembering (M = 8.58, SD =1.51; t = 16.97, p<.01, *Cohen'sd* = 1.16), understanding (M = 8.63, SD = 1.54; t = 15.73, p<.01, *Cohen's d* = 1.30), applying(M = 10.23, SD = 2.80; t = 3.09, p<.01, *Cohen's d* = .20), analysis (M = 9.26, SD = 2.26; t = 3.09, p<.01, *Cohen's* d = .17), evaluation (M = 8.00, SD = 1.95; t = 3.09, p<.01, *Cohen's* d = .17), and on overall abilities (M = 47.07, SD = 10.77; t = 21.44, p<.01, *Cohen's d* = .50). The results were non-significant on creating (M = 2.37, SD = 2.27; t = 1.13, p>.05, *Cohen's d* = .06). It was concluded that results showed significant differences but differences were too small and some variables have low effect sizes as compared to experimental group (Table-4.33)

31. The graphical representation showed the minor change in the Lower Order Thinking Skills i.e. in Remembering, Understanding and Applying, while the Higher Order Thinking Skills did not show any significant improvement in Control Group-II taught by the Conventional Method. Hence it was inferred that Conventional Teaching Method was not effective for HOTS (Figure-27).

32. Comparison of Mean difference of Experimental Group with Control Group-II was analyzed. The net effect on the outcome variables i.e. conceptual knowledge dimension six sub-levels by the teaching through Problem Solving

Method was 5.2 and the net effect by teaching through Conventional Method was 0.9 which is very low. The difference of averages between Experimental Group and Control Group-I (5.2, 0.9), showed that Experimental Group performed better with PMS than the Control Group- II with Conventional Method. Hence it was inferred that PSM is better than Conventional Method (Table-4.34).

33. Comparison of Mean difference of Group-I with Control Group-II was analyzed. The comparison of Control Group-I and Control Group-II which were taught by the Conventional Method showed the same results. It means there was no effect of extraneous variable. It also showed that there was no Placebo effect, Hawthorne effect, Interaction effect, Ecological validity and Reactive effect. So the result produced by the Experimental Group is the effect of treatment which was the Polya's Problem Solving Method. (Table-4.35)

5.3 Conclusions

Following conclusions were drawn on the basis of findings of the study:

1. From the finding 8 represented by Table-4.24, it was concluded on the basis of One Way ANOVA that Experimental Group taught by PSM showed better results on pre-test. Hence PSM was an effective method for teaching Mathematics at Elementary level.
2. From the finding 9 represented by table-4.24, it was concluded that the achievement scores of Control Group-I that was taught by Conventional Method, showed lower results than the Experimental Group taught by PSM on the basis of One Way ANOVA.

3. From the finding 10 represented by table-4.24, it was concluded that the Control Group-II taught by Conventional Method showed lower results than the Experimental Group taught by PSM on the basis of One Way ANOVA.
4. With accordance to the finding 11 represented by table-4.25, it was concluded that on Post Hock analyses of Experimental Group, Control Group-I and Control Group-II on outcome variables (Remembering, Understanding, Applying, Analyzing, Evaluating and Creating). PSM showed better results as compared to Conventional Method. Hence PSM was better than Conventional Method on Revised Bloom's Taxonomy.
5. According to finding 13 showed by figure-4.24, it was concluded that PSM performed better on outcome variables as compared to Conventional Method. Hence it supported that all null hypotheses may not be accepted.
6. According to finding 15 represented in table-4.28, it was concluded on the bases of t-test that PSM showed significant difference of performance in achievement scores as compared to Conventional Method and had high effect size on all outcome variables.
7. On the bases of finding 16 represented in table-4.28, it was concluded that first null hypothesis was not accepted and hence PSM has significant effect on conceptual dimension sub-level Remembering at RBT.
8. On the bases of finding 17 represented in table-4.28, it was concluded that second null hypothesis was not accepted and hence PSM has significant effect on conceptual dimension sub-level Understanding at RBT.

9. On the bases of finding 18 represented in table-4.28, it was concluded that third null hypothesis was not accepted and hence PSM had significant effect on conceptual dimension sub-level Applying at RBT.
10. On the bases of finding 19 represented in table-4.28, it was concluded that forth null hypothesis was not accepted and hence PSM had significant effect on conceptual dimension sub-level analyzing at RBT.
11. On the bases of finding 20 represented in table-4.28, it was concluded that fifth null hypothesis was not accepted and hence PSM had significant effect on conceptual dimension sub-level Evaluating at RBT.
12. On the bases of finding 21 represented in table-4.28, it was concluded that sixth null hypothesis was not accepted and hence PSM had significant effect on conceptual dimension sub-level creating at RBT.
13. On the bases of finding 22 represented in table-4.28, it was concluded that seventh null hypothesis was not accepted and hence PSM has significant effect on Higher Order Thinking Skills.
14. According to finding 23 represented by table-4.28, it was concluded that PSM had effect on overall abilities.
15. In the light of finding 24 shown in figure -26, it was concluded that PSM is equally effective on LOTS as well as on HOTS at Revised Bloom's Taxonomy.
16. According to finding 26 represented by table-4.30, it was concluded that Control Group-I taught by Conventional Method showed improvement in

post-test on outcome variables but difference and effect size was low as compared to PSM in Experimental Group.

17. The finding 27 showed in figure-27, provided the conclusion that Conventional Method worked on LOTS, while it did not show any significant improvement in HOTS.
18. The finding 28 represented by table-4.31 provided the conclusion that PSM net effect on outcome variables were 5.2 and the net effect of Conventional Method in Control Group-I was 0.72 (less than one). Hence it was inferred that PMS was better than Conventional Method.
19. In the light of finding 30 represented by table-4.33, it was concluded that Control Group-II taught by Conventional Method showed improvement in post-test on outcome variables but difference and effect size was low as compared to PSM in Experimental Group.
20. The finding 31 shown in figure-28 provided the conclusion that Conventional Method worked on LOTS, while it did not show any significant improvement in HOTS in Control Group-II.
21. The finding 32 represented by table-4.34 provided the conclusion that PSM net effect on outcome variables was 5.2 and the net effect of Conventional Method in Control Group-I was 0.9 (less than one). Hence it was inferred that PMS was better than Conventional Method.
22. According to finding 33 shown in table-4.35, it was concluded that the performance of Control Group-I and Control Group-II which were taught by the Conventional Method showed the same results. It means that there was no

effect of extraneous variable. It was also concluded that there was no Placebo effect, Hawthorne effect, Interaction effect, Ecological validity and Reactive effect. So the result produced by the Experimental Group was the effect of treatment which was the Polya's Problem Solving Method.

5.4 Recommendations

Keeping in view the finding and conclusion of the study the following recommendation are suggested:

1. As PSM has shown its strength on the Conventional Method so it may be suggested that teacher use this method in the classroom for teaching of Mathematics. For this it may be added in pre-service and in-service training programes.
2. The Problem Solving Method has proved its strength on Higher Order Thinking Skills so it is suggested that it may be used specifically for developing HOTS. For this In-Service training may be provided to Mathematics teachers to apply this method in their teaching.
3. As PSM helps in developing the ability of problem solving, therefore, it is suggested it may be used to develop the ability among the students. For this In-Service training may be provided to Mathematics teachers to apply this method in their teaching.
4. PSM is effective for teaching of Mathematics so it may be used for making the base of students at elementary level. For this it may be recommended that this method of teaching may be included in all teachers training programmes at Elementary level.

5. It has an effect on Revised Bloom's Taxonomy, so it is suggested that lesson plans may be prepared according to RBT.
6. As PSM is an effective method of teaching Mathematics so it is suggested that it may be included in teacher's training programmes.
7. As Revised Bloom's Taxonomy is used widely therefore, it is suggested that teachers may be provided training in this field. For this it is suggested this may be included in the curriculum of teachers training programmes.
8. As PSM is an effective method of teaching, so it is suggested that it may be included in teachers guide.
9. To provide training of PSM to the Mathematics teachers at elementary level, it is suggested that Master Trainer be prepared for imparting training. For this it may be included Mater Trainer's programmes.
10. Keeping in view the importance of PSM it may be suggested that this method may be included in pre-service, in-service and induction training programmes of teachers.

5.5 Replications of the study

Keeping in view the limitations of the study, the following studies are recommended for future studies:

1) The researcher conducted the study on boys at elementary level; therefore, the study may be repeated on girls.
2) The present study was conducted at elementary level so therefore, the study may be conducted on secondary and Higher-Secondary levels.

3) The researcher used the true experimental design with two control groups; therefore, research design may be changed to find out the effectiveness of PSM.

4) The researcher used the word problems, Practical Geometry, Theoretical Geometry as content of experiment; therefore, the study may be conducted on different areas of Mathematics.

5) The researcher used the urban area of Islamabad for the study; therefore, the study may be repeated with other geographical areas.

6) The researcher used the subject of Mathematics therefore; the study can be repeated with other subjects.

7) The evaluation parameter was used RBT in the present. The study was on the conceptual dimension of RBT; therefore, it the study can be repeated with other dimensions of RBT.

8) The researcher used RBT for the PSM; the study can be repeated with other available Taxonomies.

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Appendix-A

Lesson Plan No-1

Class: 8th
Time: 45M
Date:11/4/14
Subject: Mathematics
Topic: profit& Profit Percentage

Behavioral Objectives/Learning outcomes:
After studying this topic students will be able to:

Cognitive process	*Conceptual Knowledge*
Remembering	Describe the concept of profit
Understanding	Demonstrate the concept of profit with examples
Applying	Find the profit from the given questions
Analyzing	Compare the different profits
Evaluating	Conclude the profit percentage
Creating	Develop the formula of profit and profit percentage

A.V Aids: White Board, Marker, Pointer, Duster, Book

Teaching Method: Polya's Problem Solving Method

Presentation:
Step-1: Understand the problem
Read the problem carefully and find what is given and what to find?
Step-2: Devising a Plan
How the unknown can be find e.g by adding or by subtracting.
Step-3: Carry out the Plan.
Let's do it and find a common formula for solving the problems.
Step-4: Looking back
Can we do a similar question? Change the data and do it.

Assessment: Questions will be asked according to the objectives.

Lesson Plan No-2

Class: 8th
Time: 45M
Date:12/4/14
Subject: Mathematics
Topic: Loss & Loss Percentage

Behavioral Objectives/Learning outcomes:
After studying this topic students will be able to:

Cognitive process	*Conceptual Knowledge*
Remembering	Describe the concept of Loss
Understanding	Demonstrate the concept of Loss with examples
Applying	Find the loss from the given questions
Analyzing	Compare the different losses
Evaluating	Conclude the loss percentage
Creating	Develop the formula of loss and loss percentage

A.V Aids: White Board, Marker, Pointer, Duster, Book

Teaching Method: Polya's Problem Solving Method

Presentation:
Step-1: Understand the problem
Read the problem carefully and find what is given and what to find?
Step-2: Devising a Plan
How the unknown can be find e.g. by adding or by subtracting.
Step-3: Carry out the Plan.
Let's do it and find a common formula for solving the problems.
Step-4: Looking back
Can we do a similar question? Change the data and do it.

Assessment: Questions will be asked according to the objectives.

Lesson Plan No-3

Class: 8th
Time: 45M
Date: 14/4/14
Subject: Mathematics
Topic: Commission
Behavioral Objectives/Learning outcomes:
After studying this topic students will be able to:

Cognitive process	*Conceptual Knowledge*
Remembering	Define the concept of commission
Understanding	Explain the circumstances of some conditions of commission
Applying	Calculate the commission according to given rates
Analyzing	Classify the different commissions on different articles
Evaluating	Compare the commissions of different companies
creating	Generate the formula for finding commission

A.V Aids: White Board, Marker, Pointer, Duster, Book
Teaching Method: Polya's Problem Solving Method
Presentation:
Step-1: Understand the problem
Read the problem carefully and find what is given and what to find?
Step-2: Devising a Plan
How the unknown can be find e.g by adding or by subtracting.
Step-3: Carry out the Plan.
Let's do it and find a common formula for solving the problems.
Step-4: Looking back
Can we do a similar question? Change the data and do it.
Assessment: Questions will be asked according to the objective

Lesson Plan No-4

Class: 8th
Time: 45M
Date: 15/4/14
Subject: Mathematics
Topic: Insurance
Behavioral Objectives/Learning outcomes:
After studying this topic students will be able to:

Cognitive process	*Conceptual Knowledge*
Remembering	Outline the concept
Understanding	Demonstrate the concept with examples
Applying	Solve the problems
Analyzing	Classify the types
Evaluating	Determine the high and low rates
creating	Choose the right plan

A.V Aids: White Board, Marker, Pointer, Duster, Book
Teaching Method: Polya's Problem Solving Method

Presentation:
Step-1: Understand the problem
Read the problem carefully and find what is given and what to find?
Step-2: Devising a Plan
How the unknown can be find?
Step-3: Carry out the Plan.
Let's do it and find a common formula for solving the problems.
Step-4: Looking back
Can we do a similar question? Change the data and do it.

Assessment: Questions will be asked according to the objectives.

Lesson Plan No-5

Class: 8th
Time: 45M
Date:16/4/14
Subject: Mathematics
Topic: Tax

Behavioral Objectives/Learning outcomes:
After studying this topic students will be able to:

Cognitive process	*Conceptual Knowledge*
Remembering	Outline the concept
Understanding	Demonstrate the concept with examples
Applying	Solve the problems
Analyzing	Classify the types
Evaluating	Compare the different taxes
creating	Compose a chart for tax

A.V Aids: White Board, Marker, Pointer, Duster, Book
Teaching Method: Polya's Problem Solving Method
Presentation:
Step-1: Understand the problem
Read the problem carefully and find what is given and what to find?
Step-2: Devising a Plan
How the unknown can be find e.g by adding, subtracting, multiplying or dividing.
Step-3: Carry out the Plan.
Let's do it and find a common formula for solving the problems.
Step-4: Looking back
Can we do a similar question? Change the data and do it.

Assessment: Questions will be asked according to the objectives.

Lesson Plan No-6

Class: 8th
Time: 45M
Date:17/4/14
Subject: Mathematics
Topic: Concept of Pythagoras Theorem
Behavioral Objectives/Learning outcomes:
The accepted outcomes will be:

Cognitive process	*Conceptual Knowledge*
Remembering	Outline the history of the concept
Understanding	interpret the concept with diagrams
Applying	Identify the sides of a right angled triangle
Analyzing	Point out the Pythagoras theorem
Evaluating	Judge the difference
creating	Compose formula with different right angled triangles

A.V Aids: White Board, Marker, Pointer, Duster, Book
Teaching Method: Polya's Problem Solving Method
Presentation:
Step-1: Understand the problem
Historic concept will be given in a paragraph, with diagrams and formula.
Step-2: Devising a Plan
Prepare an outline of the concept
Step-3: Carry out the Plan.
Write the formula of different right angled triangles.
Step-4: Looking back
Rewrite the formula with different format.
Assessment: Questions will be asked according to the objectives.

Lesson Plan No-7

Class: 8th
Time: 45M
Date:18/4/14
Subject: Mathematics
Topic: Application of Pythagoras Theorem

Behavioral Objectives/Learning outcomes:
After studying this topic students will be able to:

Cognitive process	*Conceptual Knowledge*
Remembering	Outline the concept
Understanding	Demonstrate the concept with examples
Applying	Solve the problems
Analyzing	Arrange the data
Evaluating	Test the answer
creating	Solve the daily life problems

A.V Aids: White Board, Marker, Pointer, Duster, Book
Teaching Method: Polya's Problem Solving Method

Presentation:
Step-1: Understand the problem
Read the problem carefully and find what is given and what to find?
Step-2: Devising a Plan
How the unknown can be found?
Step-3: Carry out the Plan.
Let's do it and find a formula for solving the problems.
Step-4: Looking back
Can we do a similar question? Change the data and do it.

Assessment: Questions will be asked according to the objectives.

Lesson Plan No-8

Class: 8th
Time: 45M
Date: 19/4/14
Subject: Mathematics
Topic: Area of Triangular Region

Behavioral Objectives/Learning outcomes:
After studying this topic students will be able to:

Cognitive process	*Conceptual Knowledge*
Remembering	Outline the concept
Understanding	Demonstrate the concept with examples
Applying	Solve the problems
Analyzing	Arrange the data
Evaluating	Test the answer
creating	Solve the daily life problems

A.V Aids: White Board, Marker, Pointer, Duster, Book
Teaching Method: Polya's Problem Solving Method
Presentation:
Step-1: Understand the problem
Read the problem carefully and find what is given and what to find?
Step-2: Devising a Plan
How the unknown can be found?
Step-3: Carry out the Plan.
Let's do it and find a formula for solving the problems.
Step-4: Looking back
Can we do a similar question? Change the data and do it.

Assessment: Questions will be asked according to the objectives.

Lesson Plan No-9

Class: 8th
Time: 45M
Date:18/4/14
Subject: Mathematics
Topic: Area of Quadrilateral by using Hero's formula
Behavioral Objectives/Learning outcomes:
After studying this topic students will be able to:

Cognitive process	*Conceptual Knowledge*
Remembering	Outline the concept
Understanding	Demonstrate the concept with examples
Applying	Solve the problems from the exercise
Analyzing	Arrange the data
Evaluating	Test the answer given in the book
creating	Solve the daily life problems

A.V Aids: White Board, Marker, Pointer, Duster, Book
Teaching Method: Polya's Problem Solving Method
Presentation:
Step-1: Understand the problem
Read the problem carefully and find what is given and what to find?
Step-2: Devising a Plan
How the unknown can be found?
Step-3: Carry out the Plan.
Let's do it and find a formula for solving the problems.
Step-4: Looking back
Can we do a similar question? Change the data and do it.

Assessment: Questions will be asked according to the objectives.

Lesson Plan No-10

Class: 8th
Time: 45M
Date:/4/14
Subject: Mathematics
Topic: Surface area of Sphere
Behavioral Objectives/Learning outcomes:
After studying this topic students will be able to:

Cognitive process	*Conceptual Knowledge*
Remembering	Describe the concept
Understanding	Demonstrate the concept with examples
Applying	Solve the problems
Analyzing	Arrange the data
Evaluating	Test the answer
creating	Solve the daily life problems

A.V Aids: White Board, Marker, Pointer, Duster, Book
Teaching Method: Polya's Problem Solving Method
Presentation:
Step-1: Understand the problem
Read the problem carefully and find what is given and what to find?
Step-2: Devising a Plan
How the unknown can be found?
Step-3: Carry out the Plan.
Let's do it and find a formula for solving the problems.
Step-4: Looking back
Can we do a similar question? Change the data and do it.
Assessment: Questions will be asked according to the objectives.

Lesson Plan No-11

Class: 8th
Time: 45M
Date:/4/14
Subject: Mathematics
Topic: Volume of Sphere
Behavioral Objectives/Learning outcomes:
After studying this topic students will be able to:

Cognitive process	*Conceptual Knowledge*
Remembering	Outline the concept
Understanding	Demonstrate the concept with examples
Applying	Solve the problems
Analyzing	Arrange the data
Evaluating	Test the answer
creating	Solve the daily life problems

A.V Aids: White Board, Marker, Pointer, Duster, Book
Teaching Method: Polya's Problem Solving Method
Presentation:
Step-1: Understand the problem
Read the problem carefully and find what is given and what to find?
Step-2: Devising a Plan
How the unknown can be found?
Step-3: Carry out the Plan.
Let's do it and find a formula for solving the problems.
Step-4: Looking back
Can we do a similar question? Change the data and do it.
Assessment: Questions will be asked according to the objectives.

Lesson Plan No-12

Class: 8th
Time: 45M
Date:/4/14
Subject: Mathematics
Topic: Surface Area of Right Circular Cone
Behavioral Objectives/Learning outcomes:
After studying this topic students will be able to:

Cognitive process	*Conceptual Knowledge*
Remembering	Outline the concept
Understanding	Demonstrate the concept with examples
Applying	Solve the problems
Analyzing	Arrange the data
Evaluating	Test the answer
creating	Solve the daily life problems

A.V Aids: White Board, Marker, Pointer, Duster, Book
Teaching Method: Polya's Problem Solving Method
Presentation:
Step-1: Understand the problem
Read the problem carefully and find what is given and what to find?
Step-2: Devising a Plan
How the unknown can be found?
Step-3: Carry out the Plan.
Let's do it and find a formula for solving the problems.
Step-4: Looking back
Can we do a similar question? Change the data and do it.
Assessment: Questions will be asked according to the objectives.

Lesson Plan No-13

Class: 8th
Time: 45M
Date:/4/14
Subject: Mathematics
Topic: volume of Right Circular Cone
Behavioral Objectives/Learning outcomes:
After studying this topic students will be able to:

Cognitive process	*Conceptual Knowledge*
Remembering	Outline the concept
Understanding	Demonstrate the concept with examples
Applying	Solve the problems
Analyzing	Arrange the data
Evaluating	Test the answer
creating	Solve the daily life problems

A.V Aids: White Board, Marker, Pointer, Duster, Book
Teaching Method: Polya's Problem Solving Method
Presentation:
Step-1: Understand the problem
Read the problem carefully and find what is given and what to find?
Step-2: Devising a Plan
How the unknown can be found?
Step-3: Carry out the Plan.
Let's do it and find a formula for solving the problems.
Step-4: Looking back
Can we do a similar question? Change the data and do it.
Assessment: Questions will be asked according to the objectives.

Lesson Plan No-14

Class: 8th
Time: 45M
Date:/4/14
Subject: Mathematics
Topic: Dividing a line Segment (First Method)
Behavioral Objectives/Learning outcomes:
After studying this topic students will be able to:

Cognitive process	*Conceptual Knowledge*
Remembering	Outline the concept
Understanding	Demonstrate the concept with examples
Applying	Practically do the process
Analyzing	Arrange the data
Evaluating	Test the answer
creating	Solve the daily life problems

A.V Aids: White Board, Marker, Pointer, Duster, Book
Teaching Method: Polya's Problem Solving Method
Presentation:
Step-1: Understand the problem
Read the problem carefully and find what is given and what to find?
Step-2: Devising a Plan
How the unknown can be found?
Step-3: Carry out the Plan.
Let's do it and find a formula for solving the problems.
Step-4: Looking back
Can we do a similar question? Change the data and do it.
Assessment: Questions will be asked according to the objectives.

Lesson Plan No-15

Class: 8th
Time: 45M
Date:/4/14
Subject: Mathematics
Topic: Dividing a line Segment (Second Method)
Behavioral Objectives/Learning outcomes:
After studying this topic students will be able to:

Cognitive process	*Conceptual Knowledge*
Remembering	Outline the concept
Understanding	Demonstrate the concept with examples
Applying	Solve the problems
Analyzing	Arrange the data
Evaluating	Test the answer
creating	Solve the daily life problems

A.V Aids: White Board, Marker, Pointer, Duster, Book
Teaching Method: Polya's Problem Solving Method
Presentation:
Step-1: Understand the problem
Read the problem carefully and find what is given and what to find?
Step-2: Devising a Plan
How the unknown can be found?
Step-3: Carry out the Plan.
Let's do it and find a formula for solving the problems.
Step-4: Looking back
Can we do a similar question? Change the data and do it.
Assessment: Questions will be asked according to the objectives.

Lesson Plan No-16

Class: 8th
Time: 45M
Date: /4/14
Subject: Mathematics
Topic: Construction of Triangle (when perimeter and ratio among the sides are given)
Behavioral Objectives/Learning outcomes:
After studying this topic students will be able to:

Cognitive process	*Conceptual Knowledge*
Remembering	Outline the concept
Understanding	Demonstrate the concept with examples
Applying	Solve the problems
Analyzing	Arrange the data
Evaluating	Test the answer
creating	Solve the daily life problems

A.V Aids: White Board, Marker, Pointer, Duster, Book
Teaching Method: Polya's Problem Solving Method
Presentation:
Step-1: Understand the problem
Read the problem carefully and find what is given and what to find?
Step-2: Devising a Plan
How the unknown can be found?
Step-3: Carry out the Plan.
Let's do it and find a formula for solving the problems.
Step-4: Looking back
Can we do a similar question? Change the data and do it.
Assessment: Questions will be asked according to the objectives.

Lesson Plan No-17

Class: 8th
Time: 45M
Date: /4/14
Subject: Mathematics
Topic: Concept of Polygons
Behavioral Objectives/Learning outcomes:
After studying this topic students will be able to:

Cognitive process	*Conceptual Knowledge*
Remembering	Outline the concept
Understanding	Demonstrate the concept with examples
Applying	Solve the problems
Analyzing	Arrange the data
Evaluating	Test the answer
creating	Solve the daily life problems

A.V Aids: White Board, Marker, Pointer, Duster, Book
Teaching Method: Polya's Problem Solving Method
Presentation:
Step-1: Understand the problem
Read the problem carefully and find what is given and what to find?
Step-2: Devising a Plan
How the unknown can be found?
Step-3: Carry out the Plan.
Let's do it and find a formula for solving the problems.
Step-4: Looking back
Can we do a similar question? Change the data and do it.
Assessment: Questions will be asked according to the objectives.

Lesson Plan No-18

Class: 8th
Time: 45M
Date: /4/14
Subject: Mathematics
Topic: Construction of a regular Pentagon
Behavioral Objectives/Learning outcomes:
After studying this topic students will be able to:

Cognitive process	***Conceptual Knowledge***
Remembering	Outline the concept
Understanding	Demonstrate the concept with examples
Applying	Solve the problems
Analyzing	Arrange the data
Evaluating	Test the answer
creating	Solve the daily life problems

A.V Aids: White Board, Marker, Pointer, Duster, Book
Teaching Method: Polya's Problem Solving Method
Presentation:
Step-1: Understand the problem
Read the problem carefully and find what is given and what to find?
Step-2: Devising a Plan
How the unknown can be found?
Step-3: Carry out the Plan.
Let's do it and find a formula for solving the problems.
Step-4: Looking back
Can we do a similar question? Change the data and do it.
Assessment: Questions will be asked according to the objectives.

Lesson Plan No-19

Class: 8th
Time: 45M
Date: /4/14
Subject: Mathematics
Topic: Construction of regular Hexagon
Behavioral Objectives/Learning outcomes:
After studying this topic students will be able to:

Cognitive process	*Conceptual Knowledge*
Remembering	Outline the concept
Understanding	Demonstrate the concept with examples
Applying	Solve the problems
Analyzing	Arrange the data
Evaluating	Test the answer
creating	Solve the daily life problems

A.V Aids: White Board, Marker, Pointer, Duster, Book
Teaching Method: Polya's Problem Solving Method
Presentation:
Step-1: Understand the problem
Read the problem carefully and find what is given and what to find?
Step-2: Devising a Plan
How the unknown can be found?
Step-3: Carry out the Plan.
Let's do it and find a formula for solving the problems.
Step-4: Looking back
Can we do a similar question? Change the data and do it.
Assessment: Questions will be asked according to the objectives.

Lesson Plan No-20

Class: 8th
Time: 45M
Date: /4/14
Subject: Mathematics
Topic: Construction of regular Octagon
Behavioral Objectives/Learning outcomes:
After studying this topic students will be able to:

Cognitive process	*Conceptual Knowledge*
Remembering	Outline the concept
Understanding	Demonstrate the concept with examples
Applying	Solve the problems
Analyzing	Arrange the data
Evaluating	Test the answer
creating	Solve the daily life problems

A.V Aids: White Board, Marker, Pointer, Duster, Book
Teaching Method: Polya's Problem Solving Method
Presentation:
Step-1: Understand the problem
Read the problem carefully and find what is given and what to find?
Step-2: Devising a Plan
How the unknown can be found?
Step-3: Carry out the Plan.
Let's do it and find a formula for solving the problems.
Step-4: Looking back
Can we do a similar question? Change the data and do it.
Assessment: Questions will be asked according to the objectives.

Lesson Plan No-21

Class: 8th
Time: 45M
Date: /4/14
Subject: Mathematics
Topic: Concept of diagonals of a Square and Rhombus
Behavioral Objectives/Learning outcomes:
After studying this topic students will be able to:

Cognitive process	*Conceptual Knowledge*
Remembering	Outline the concept
Understanding	Demonstrate the concept with examples
Applying	Solve the problems
Analyzing	Arrange the data
Evaluating	Test the answer
creating	Solve the daily life problems

A.V Aids: White Board, Marker, Pointer, Duster, Book
Teaching Method: Polya's Problem Solving Method
Presentation:
Step-1: Understand the problem
Read the problem carefully and find what is given and what to find?
Step-2: Devising a Plan
How the unknown can be found?
Step-3: Carry out the Plan.
Let's do it and find a formula for solving the problems.
Step-4: Looking back
Can we do a similar question? Change the data and do it.
Assessment: Questions will be asked according to the objectives.

Lesson Plan No-22

Class: 8th
Time: 45M
Date: /4/14
Subject: Mathematics
Topic: Construction of Square
Behavioral Objectives/Learning outcomes:
After studying this topic students will be able to:

Cognitive process	*Conceptual Knowledge*
Remembering	Outline the concept
Understanding	Demonstrate the concept with examples
Applying	Solve the problems
Analyzing	Arrange the data
Evaluating	Test the answer
creating	Solve the daily life problems

A.V Aids: White Board, Marker, Pointer, Duster, Book
Teaching Method: Polya's Problem Solving Method
Presentation:
Step-1: Understand the problem
Read the problem carefully and find what is given and what to find?
Step-2: Devising a Plan
How the unknown can be found?
Step-3: Carry out the Plan.
Let's do it and find a formula for solving the problems.
Step-4: Looking back
Can we do a similar question? Change the data and do it.
Assessment: Questions will be asked according to the objectives.

Lesson Plan No-23

Class: 8th
Time: 45M
Date: /4/14
Subject: Mathematics
Topic: Construction of Rhombus
Behavioral Objectives/Learning outcomes:
After studying this topic students will be able to:

Cognitive process	*Conceptual Knowledge*
Remembering	Outline the concept
Understanding	Demonstrate the concept with examples
Applying	Solve the problems
Analyzing	Arrange the data
Evaluating	Test the answer
creating	Solve the daily life problems

A.V Aids: White Board, Marker, Pointer, Duster, Book
Teaching Method: Polya's Problem Solving Method
Presentation:
Step-1: Understand the problem
Read the problem carefully and find what is given and what to find?
Step-2: Devising a Plan
How the unknown can be found?
Step-3: Carry out the Plan.
Let's do it and find a formula for solving the problems.
Step-4: Looking back
Can we do a similar question? Change the data and do it.
Assessment: Questions will be asked according to the objectives.

Appendix-B

Pre-Test

Level-Elementary

Time: 120 Minutes Marks: 100

1. Net profit means is equal to:
   a- Cost price- Sale price
   b- Sale price- Cost price
   c- Sale price=cost price
   d- Sale price + cost price

2. Net loss can be find as:
a- price- Sale price
b- Sale price- Cost price
c- Sale price=cost price
d- Sale price + cost price

3. Profit % can be calculate by the formula:
a- Net profit/Cost price*100
b- Cost price/ Net profit*100
c- Net profit/Cost price
d- Net profit/Cost price*1000

4. Loss% is equal to:
a- Net profit/Cost price*1000
b- Net profit/Cost price*100
c- Cost price/ Net profit*100
d- Net profit/Cost price

5. The price reduced to a fix percentage for the buyer is known as:
a- Tax
b- Rebate
c- Commission
d- Discount

6. Hero was the famous --------- mathematician.
a- German

b- Indian
c- Greek
d- French

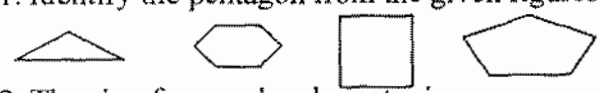
7. The triangle with two sides equal in length isknown as:
a- Equilateral triangle
b- Isosceles triangle
c- Scalene triangle
d- Right angle triangle

8. The opposite side to angle 90 in right angle triangle is known as:
a- Base
b- Altitude
c- Hypotenuse
d- Diagonal

9. A quadrilateral having opposite sides equal in length and all right angles known as:
a- Square
b- Rhombus
c- Rectangle
d- Equilateral

10. A regular Polygon having six sides known as:
a- Pentagon
b- Hexagon
c- Octagon
d- Polygon

11. Identify the pentagon from the given figures.



12. The sign for equal and greater is.
a- $\leq$
b- $\geq$
c- $\neq$
d- $\pm$

13. The diameter of the circle is
a- Equal to radius

b- Half of the radius
c- Twice of the radius
d- Thrice of the radius

14. The diagonals in the square are
a- Equal in length
b- Unequal in length
c- Congruent
d- Non of them

15. The obtuse angle means the angle which is
a- Greater than 90
b- Equal to 90
c- Less than 90
d- Greater and equal to 90

16. The shortest distance between two points is known as
a- Displacement
b- Ray
c- Line
d- Hypotenuse

17. The Pythagoras Theorem for the given triangle is as.
a- $(\text{Hypotenuse})^2=(\text{Base})^2+(\text{Perpendicular})^2$
b- $(\text{Base})^2=(\text{Hypotenuse})^2+()^2$
c- $(\text{Perpendicular})^2=(\text{Base})^2+(\text{Hypotenuse})^2$
d- $(\text{Hypotenuse})^2=(\text{Altitude})^2+(\text{Perpendicular})^2$

18. The conical solids consist of two parts.
a- Circular base, curved surface
b- Circular base, square surface
c- Curved surface, vertex
d- Vertex, circular base

19. Match Box is an example of
a- Rectangle
b- Square
c- Cube
d- Cone

20. The sum of measurement of all interior angles of a 'n' sided polygon is.

a- (n-2)*180
b- (n-3)*180
c- (n-4)*90
d- (n-2)*360

21. The area of square room is 252meter. Find the length of its side.

22. The price of three dozen eggs is Rs.150. find the price of five dozen eggs.

23. Tariq bought six pens for Rs.60 and sold tree pens for RS.50 and remaining pens for Rs.40. How much profit or loss he earned.

24. The written price of a pair of shoe is RS.250 and it has 10% off of the written price. How one has to pay to buy it.

25. In a factory ten workers can prepare 100 bats in a day. If 200 bats are required in a day how many workers has to be added.

26. Newspaper agents sold newspapers for Rs.10800 in a month and got a total commission of Rs.1620. calculate the percentage of the commission.

27. The lengths of the sides of a triangle are 12m, 31m and 25m. Calculate its area.
28. The diameter of a circle is 44m. Find its area.

29. How many liters of water a spherical tank can contain whose radius is 21m.

30. A tent in the form of a right circular come is 5m high and its base is of radius 12m. Find the area of the tent.

31. Two cars started their journey for Lahore at the same time; the speed of the first car is 100Km/h and the speed of second the car is 60Km/h. The distance is 300Km. Analyze their time difference.

32. A rectangular field has an area of 28900sq.meter. Its length is twice as long as its width. What is the length of its sides?

33. The cost of ploughing in a square field is Rs.2450 at the rate of Rs.2 per100 sq.meter. Find the length of the side of the square.

34. Group 1225 students in rows in such a way that the number of rows is equal to the number of students in a row.

35. A shopkeeper sold five TVs for Rs.10, 000 in a day. One TV was returned due to damaged screen. What is the actual sale of the day?

36. Find that last number which when subtracted from 58780, the answer is a complete square.

37. Anwar charged 12% from a landlord for selling rice. What amount did he get if he sold rice worth Rs.10, 05,600?

38. Justify the reason that why a right angle triangle cannot be constructed with these measurements. mAB=6cm, mBC=5cm, m<B=95.

39. The area of a square field is $14400m^2$. Calculate the length of its diagonal.

40. The area of circle is $154m^2$. Find the length of it diameter.

41. A salesman bought one dozen eggs for Rs.36 and sold each egg at the rate of Rs. 3. What would be the rate of each egg if he wants to earn the double his profit.

42. Drive a formula for finding the circumference of a circle.

43. Create the formula for loss percentage.

44. Draw a triangle having one angle of 90° and others two are in 1:2 ratio.

45. Develop a quadrilateral having all sides equal.

46. Four sisters have one brother each, how many brother and sisters are altogether.

47. What will be length of a ladder, standing against a wall of 12m high and the distance between ladder and wall is 5m.
48. Write an equation for finding the surface area of a Sphere.

49. Draw the diagram of a square whose area is equal to the area of a rectangle. The area of the rectangle is $144m^2$.
50. If your age is 12year, 09months and 24days. Calculate your age in days.
51. What would be the length of the square if its area is 2500 square meters?
52. Use your imagination about two such positive digits whose product is 230496 and one of the number is 6 times the other.
53. Which is the better transition if C.P=RS.2 profit=RS.1 or C.P=R.S.25 profit=RS.10
54. What would be area of a square if its perimeter is 16 square meter.
55. Identify two such numbers whose sum is 35 and their difference is 13.
56. How many times do the volume of a sphere becomes if radius is made double.
57. If the length of a rectangular region is 12meter and width is 9meter what would be the length of its diagonal.
58. Determine the height of an isosceles triangle if length of base is 6cm and length of each side is 5cm.
59. An agent sold goods for Rs.50000and got a commission of Rs.3000. calculate the percentage of commission and if commission percentage is reduced to half what would be the commission.
60. If the sale price of an article is Rs.1000 and profit percentage is 25% what would be the sale price if profit percentage made doubled?

Appendix-C

Post-Test

Level-Elementary

Time: 120 Minutes Marks:
100

1. Net profit means is equal to:
   e- Cost price- Sale price
   f- Sale price- Cost price
   g- Sale price=cost price
   h- Sale price + cost price

2. Net loss can be find as:
e- price- Sale price
f- Sale price- Cost price
g- Sale price=cost price
h- Sale price + cost price

3. Profit % can be calculate by the formula:
e- Net profit/Cost price*100
f- Cost price/ Net profit*100
g- Net profit/Cost price
h- Net profit/Cost price*1000

4. Loss% is equal to:
e- Net profit/Cost price*1000
f- Net profit/Cost price*100
g- Cost price/ Net profit*100
h- Net profit/Cost price

5. The price reduced to a fix percentage for the byer is known as:
e- Tax
f- Rebate
g- Commission
h- Discount

6. Hero was the famous ---------- mathematician.
e- German
f- Indian
g- Greek

h- French

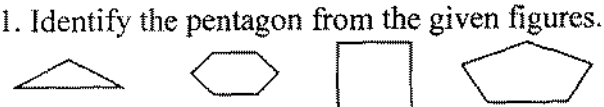
7. The triangle with two sides equal in length is known as:
e- Equilateral triangle
f- Isosceles triangle
g- Scalene triangle
h- Right angle triangle

8. The opposite side to angle 90 in right angle triangle is known as:
e- Base
f- Altitude
g- Hypotenuse
h- Diagonal

9. A quadrilateral having opposite sides equal in length and all right angles known as:
e- Square
f- Rhombus
g- Rectangle
h- Equilateral

10. A regular Polygon having six sides known as:
e- Pentagon
f- Hexagon
g- Octagon
h- Polygon

11. Identify the pentagon from the given figures.



12. The sign for equal and greater is.
e- $\leq$
f- $\geq$
g- $\neq$
h- $\pm$

13. The diameter of the circle is
e- Equal to radius
f- Half of the radius
g- Twice of the radius
h- Thrice of the radius

14. The diagonals in the square are
e- Equal in length
f- Unequal in length
g- Congruent
h- Non of them

15. The obtuse angle means the angle which is
e- Greater than 90
f- Equal to 90
g- Less than 90
h- Greater and equal to 90

16. The shortest distance between two points is known as
e- Displacement
f- Ray
g- Line
h- Hypotenuse

17. The Pythagoras Theorem for the given triangle is as.
e- $(\text{Hypotenuse})^2=(\text{Base})^2+(\text{Perpendicular})^2$
f- $(\text{Base})^2=(\text{Hypotenuse})^2+()^2$
g- $(\text{Perpendicular})^2=(\text{Base})^2+(\text{Hypotenuse})^2$
h- $(\text{Hypotenuse})^2=(\text{Altitude})^2+(\text{Perpendicular})^2$

18. The conical solids consist of two parts.
e- Circular base, curved surface
f- Circular base, square surface
g- Curved surface, vertex
h- Vertex, circular base

19. Match Box is an example of
e- Rectangle
f- Square
g- Cube
h- Cone

20. The sum of measurement of all interior angles of a 'n' sided polygon is.
e- (n-2)*180
f- (n-3)*180
g- (n-4)*90
h- (n-2)*360

21. The area of square room is 252meter. Find the length of its side.

22. The price of three dozen eggs is Rs.150. find the price of five dozen eggs.

23. Tariq bought six pens for Rs.60 and sold tree pens for RS.50 and remaining pens for Rs.40. How much profit or loss he earned.

24. The written price of a pair of shoe is RS.250 and it has 10% off of the written price. How one has to pay to buy it.

25. In a factory ten workers can prepare 100 bats in a day. If 200 bats are required in a day how many workers has to be added.

26. Newspaper agents sold newspapers for Rs.10800 in a month and got a total commission of Rs.1620. calculate the percentage of the commission.

27. The lengths of the sides of a triangle are 12m, 31m and 25m. Calculate its area.
28. The diameter of a circle is 44m. Find its area.

29. How many liters of water a spherical tank can contain whose radius is 21m.

30. A tent in the form of a right circular come is 5m high and its base is of radius 12m. Find the area of the tent.

31. Two cars started their journey for Lahore at the same time; the speed of the first car is 100Km/h and the speed of second the car is 60Km/h. The distance is 300Km. Analyze their time difference.

32. A rectangular field has an area of 28900sq.meter. Its length is twice as long as its width. What is the length of its sides?

33. The cost of ploughing in a square field is Rs.2450 at the rate of Rs.2 per100 sq.meter. Find the length of the side of the square.

34. Group 1225 students in rows in such a way that the number of rows is equal to the number of students in a row.

35. A shopkeeper sold five TVs for Rs.10, 000 in a day. One TV was returned due to damaged screen. What is the actual sale of the day?

36. Find that last number which when subtracted from 58780, the answer is a complete square.

37. Anwar charged 12% from a landlord for selling rice. What amount did he get if he sold rice worth Rs.10, 05,600?

38. Justify the reason that why a right angle triangle cannot be constructed with these measurements. mAB=6cm, mBC=5cm, m<B=95.

39. The area of a square field is $14400m^2$. Calculate the length of its diagonal.

40. The area of circle is $154m^2$. Find the length of it diameter.

41. A salesman bought one dozen eggs for Rs.36 and sold each egg at the rate of Rs. 3. What would be the rate of each egg if he wants to earn the double his profit.

42. Drive a formula for finding the circumference of a circle.

43. Create the formula for loss percentage.

44. Draw a triangle having one angle of 90^o and others two are in 1:2 ratio.

45. Develop a quadrilateral having all sides equal.

46. Four sisters have one brother each, how many brother and sisters are altogether.

47. What will be length of a ladder, standing against a wall of 12m high and the distance between ladder and wall is 5m.
48. Write an equation for finding the surface area of a Sphere.

49. Draw the diagram of a square whose area is equal to the area of a rectangle. The area of the rectangle is $144m^2$.
50. If your age is 12year, 09months and 24days. Calculate your age in days.
51. What would be the length of the square if its area is 2500 square meters?
52. Use your imagination about two such positive digits whose product is 230496 and one of the number is 6 times the other.
53. Which is the better transition if C.P=RS.2 profit=RS.1 or C.P=R.S.25 profit=RS.10
54. What would be area of a square if its perimeter is 16 square meter.
55. Identify two such numbers whose sum is 35 and their difference is 13.
56. How many times do the volume of a sphere becomes if radius is made double.
57. If the length of a rectangular region is 12meter and width is 9meter what would be the length of its diagonal.
58. Determine the height of an isosceles triangle if length of base is 6cm and length of each side is 5cm.
59. An agent sold goods for Rs.50000and got a commission of Rs.3000. calculate the percentage of commission and if commission percentage is reduced to half what would be the commission.
60. If the sale price of an article is Rs.1000 and profit percentage is 25% what would be the sale price if profit percentage made doubled?

Appendix-D

LIST OF EXPERTS

Professor Dr. N.B Jumani, Dean, Faculty of Social Sciences, International Islamic University, Islamabad

Dr. Irshad Ahmad Arshad, Chairman Department of Mathematics and stat, International Islamic University, Islamabad

Dr. Zarina Akhtar, Assisstant Professor, Department of Education, International Islamic University, Islamabad

Dr. Thair Javed, Assisstant Professor, Department of Mathematics and stat, International Islamic University, Islamabad

Dr. Nsir Ali, Assisstant Professor, Department of Mathematics and stat, International Islamic University, Islamabad

Dr. Zhaid Iqbal, Assisstant Professor, Department of Mathematics and stat, International Islamic University, Islamabad

Dr. Mushtaq Ahmad, M.sc (Mathematics) Principal FGBMS F-8/3, Islamabad

Dr. Javed Awan, Principal Govt. Muslim Higher Secondary School, Rawalpindi, Head examiner, sub examiner (Mathematics) in the Board of Intermediate &Secondary Education, Sargodha and Rawalpindi.

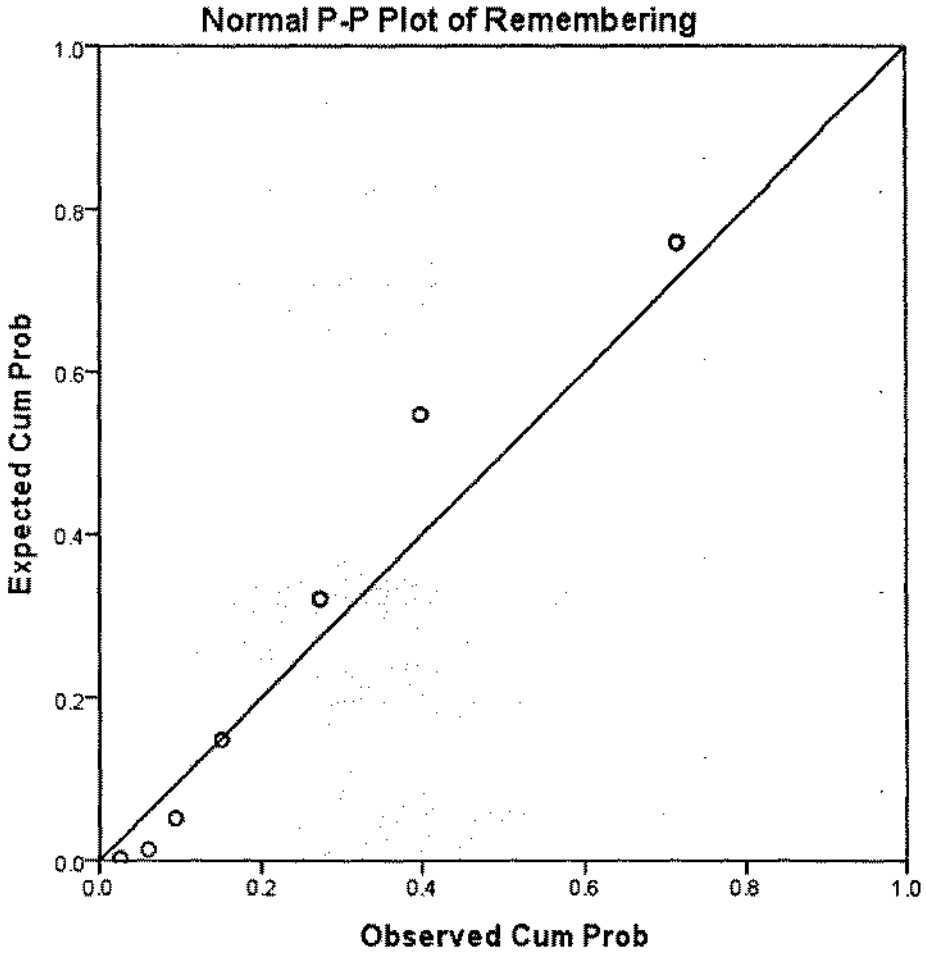
Dr.Aziz-ur-Rehman, Subject Specialist (Mathematic) Govt. Muslim Higher Secondary School, Rawalpindi, Head examiner, sub examiner (Mathematics) in the Board of Intermediate &Secondary Education, Sargodaha and Rawalpindi.

Dr. Afzal Mehmood, M.sc (Mathematics) Principal FGBMS G-8/3, Islamabad

Mr. Mustaq Ahmad Sial, Senior Subject Specialist, Govt. College of Elementary Teachers, Rawalpindi.Paper Setter in the Board of Intermediate &Secondary Education, Sargodha and Rawalpindi.

Appendix-E

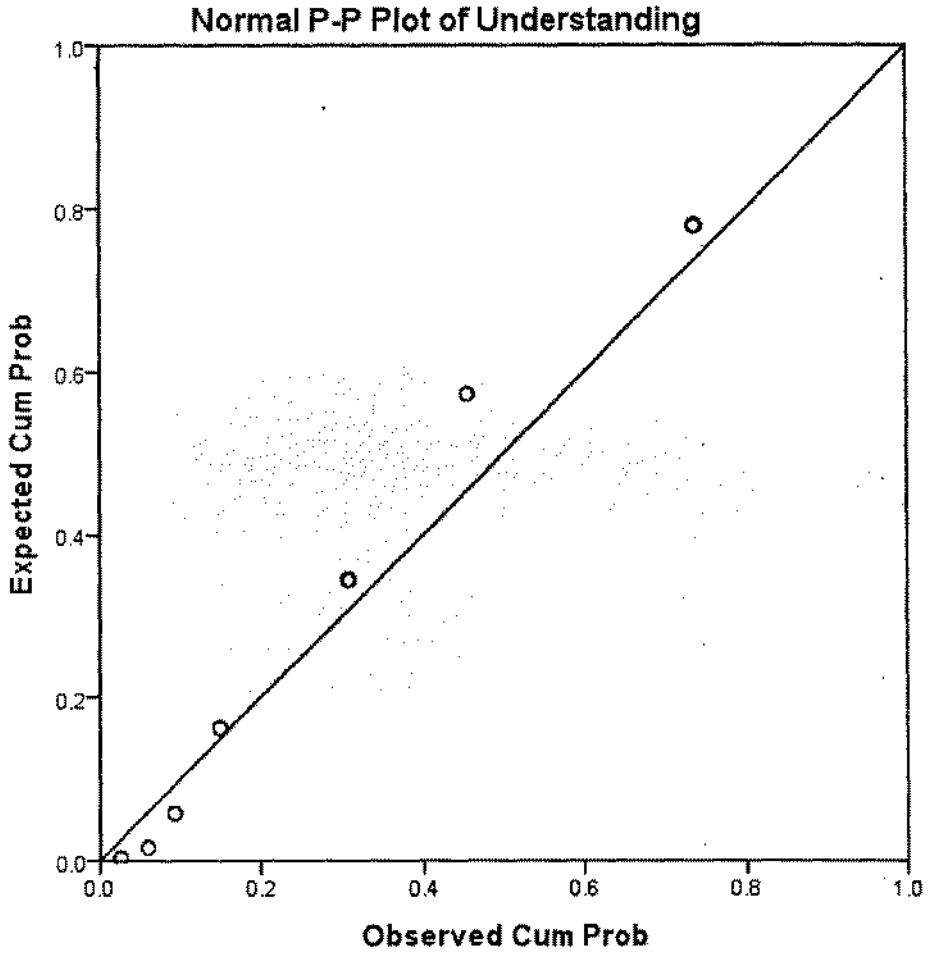
P-P Plots for Normal Distribution Analysis (Experimental Group)



Remembering consisted of recognizing and recalling relevant information from long-term memory. Ten items were made on remembering for experimental group which showed near to the straight line and had the characteristics of normal

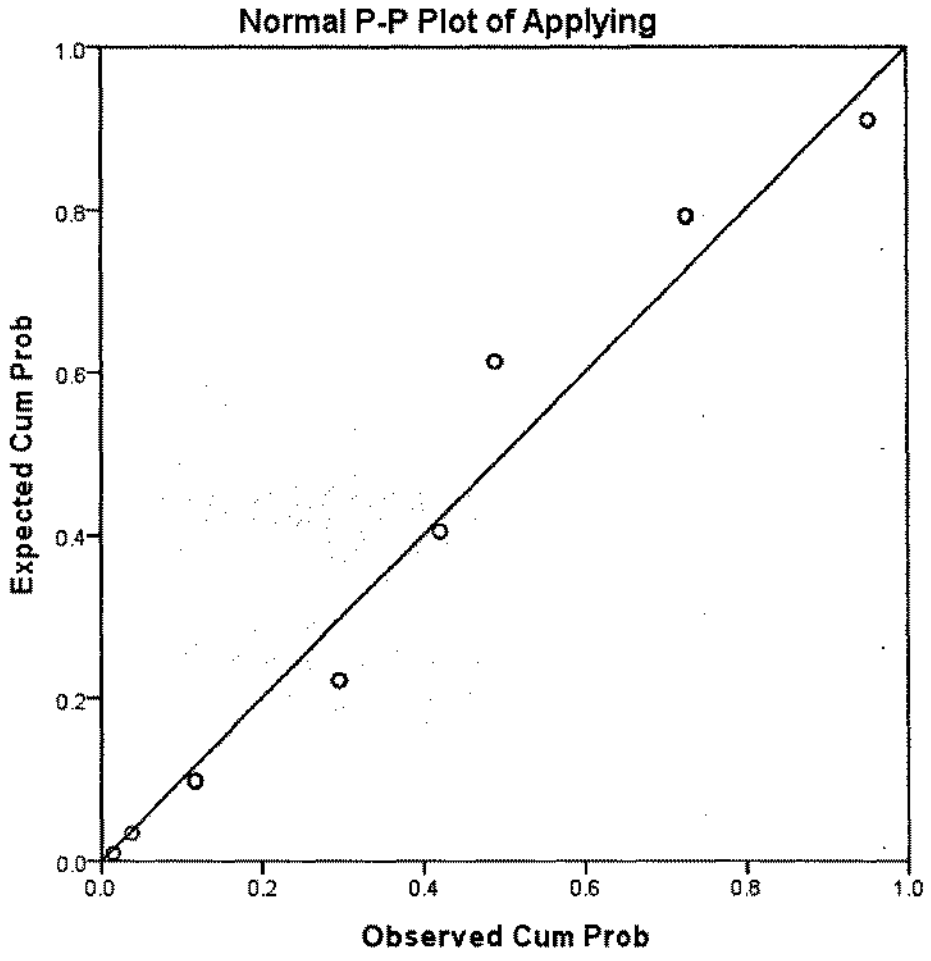
distribution. It also addressed the external threat of Selection and Treatment Interaction.

P-P Plot for Normal Distribution on Understanding **(Experimental Group)**



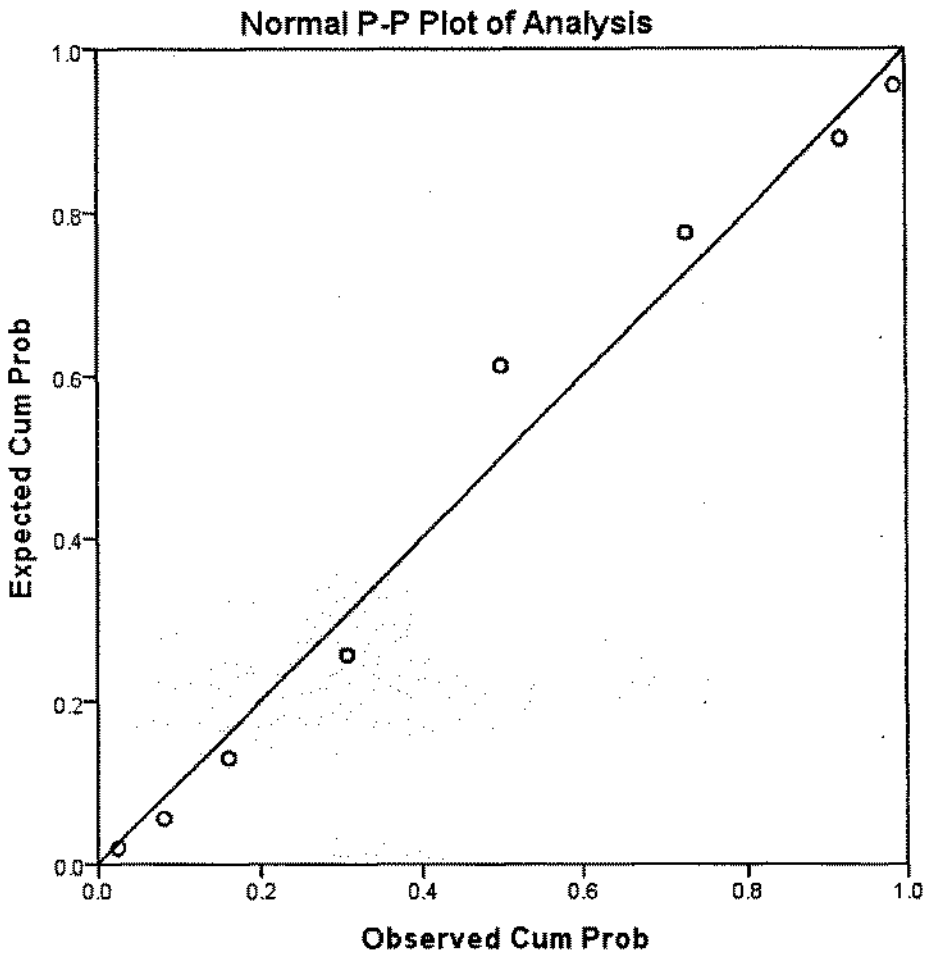
Understanding meant the ability to make your own meaning from educational material such as reading and teacher explanations. The sub-skills for this process include interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining. The figure shows that items were having the characteristics of normal distribution because points were not deviating from the straight line.

P-P Plot for Normal Distribution on Applying (**Experimental Group**)



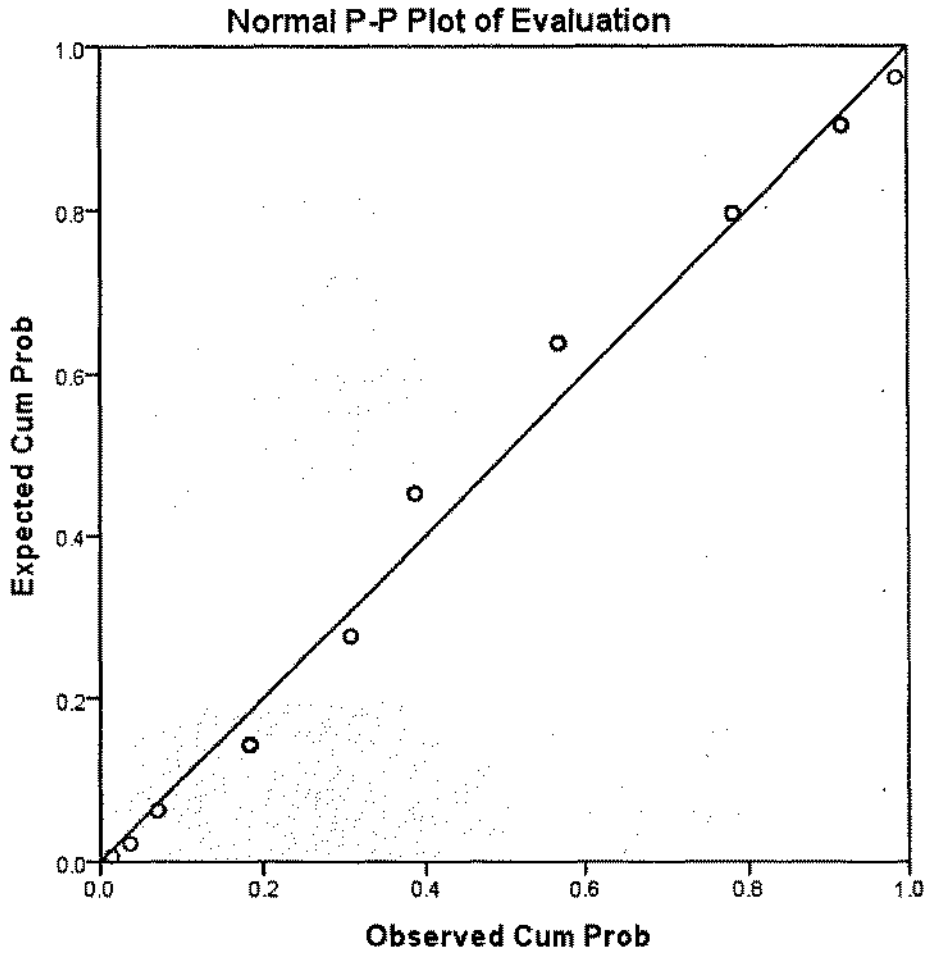
Applying referred to using a learned procedure either in a familiar or new situation. The items developed on Applying were near the expected straight line which showed that the data had the characteristics of the normal distribution.

P-P Plot for Normal Distribution on Analyzing **(Experimental Group)**



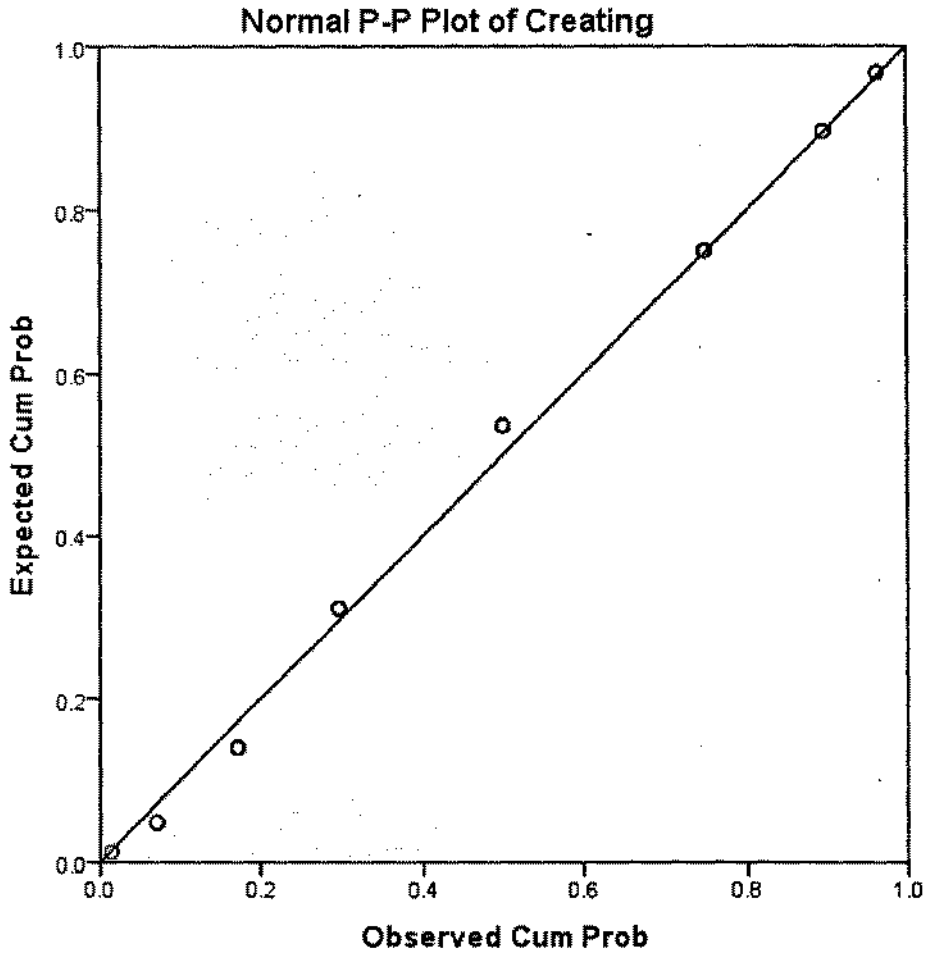
Analysis consisted of breaking knowledge down into its parts and thinking about how the parts were related to its overall structure. It could be exhibited by differentiating, organizing and attributing. The item formed on analyzing, according to figure-4 showed the characteristics of a normal distribution.

P-P Plot for Normal Distribution on Evaluating **(Experimental Group)**



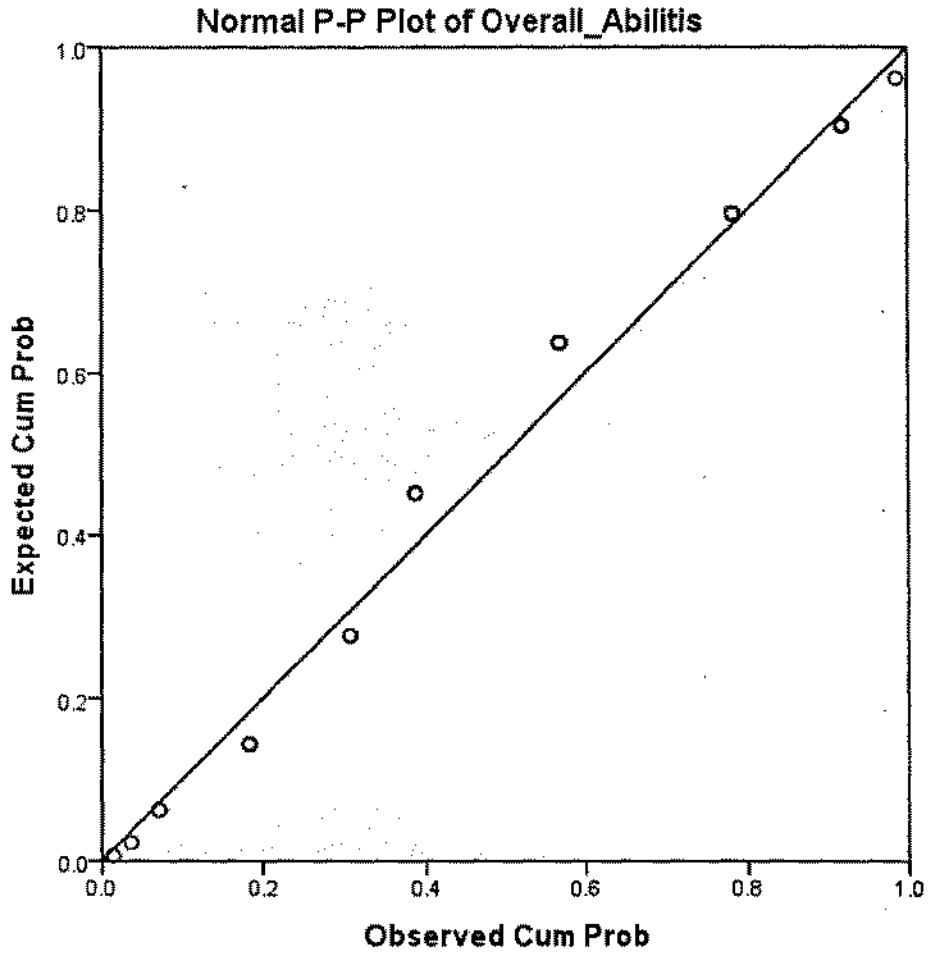
Evaluation included in Higher Order Thinking Skills like checking and critiquing. It also included to infer and reached to some decision. All the items for checking the evaluation were near the straight line, hence the items developed on evaluation showed the normal distribution.

***P-P Plot for Normal Distribution on Creating* (Experimental Group)**



This skill of Creating involved putting things together to make something new. To accomplish creative tasks, the learners generated, planned and produced. This was the highest level in RBT. The items developed on this showed the characteristics of normal distribution.

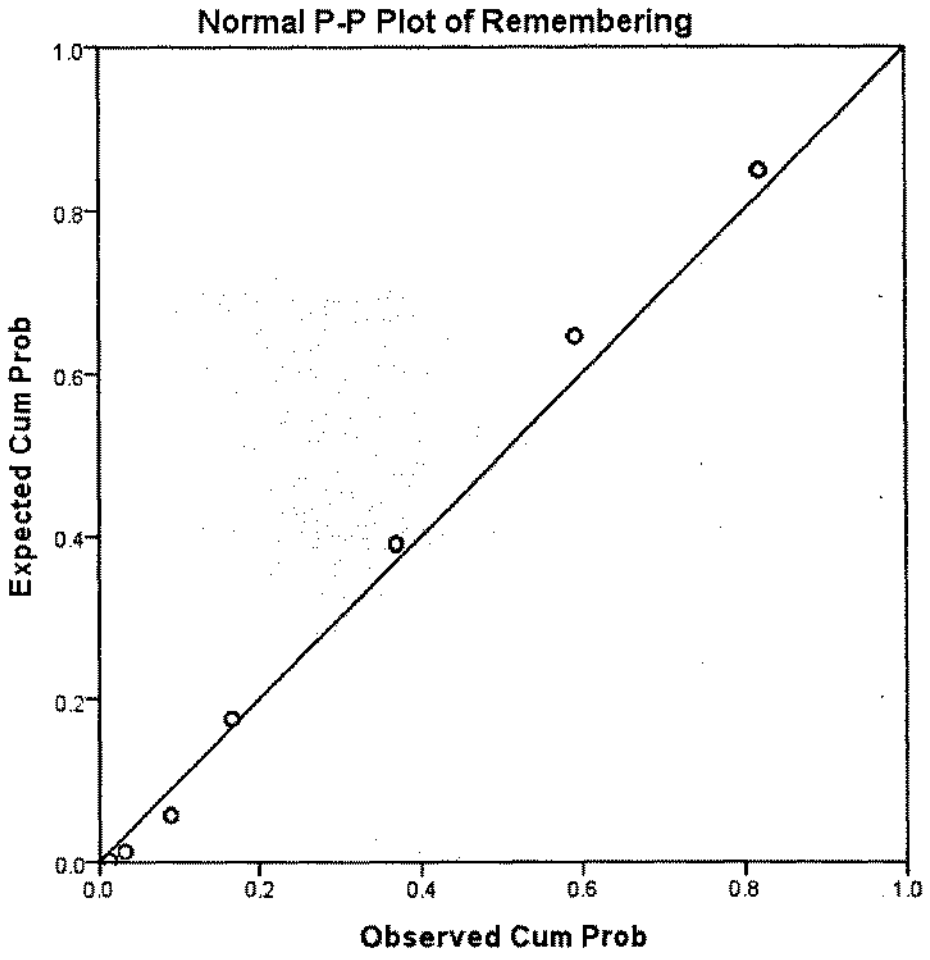
P-P Plot for Normal Distribution on Overall Abilities **(Experimental Group)**



The figure shows the overall points developed on conceptual dimension of cognitive process i.e. Remembering, Understanding, Applying, analyzing, Evaluating and creating also had the characteristics of normal distribution.

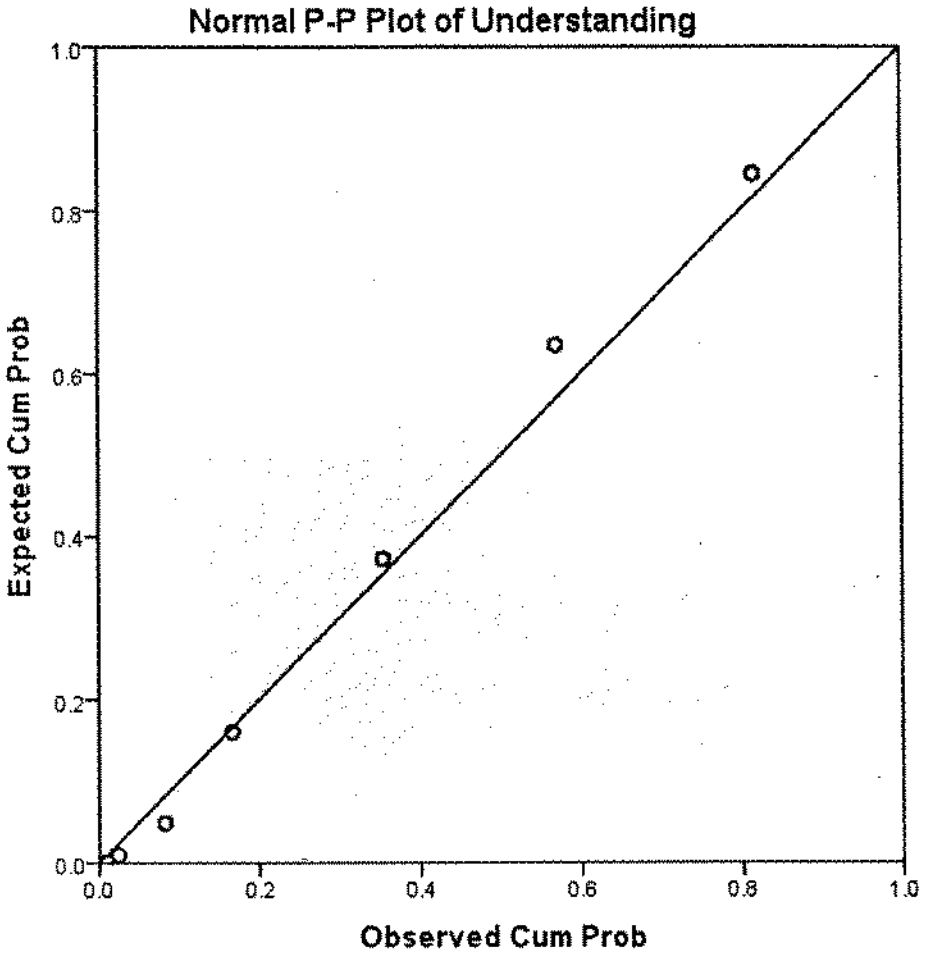
Plots forNormal Distribution Analysis (Control Group-I)

P-P Plot for Normal Distribution on Remembering



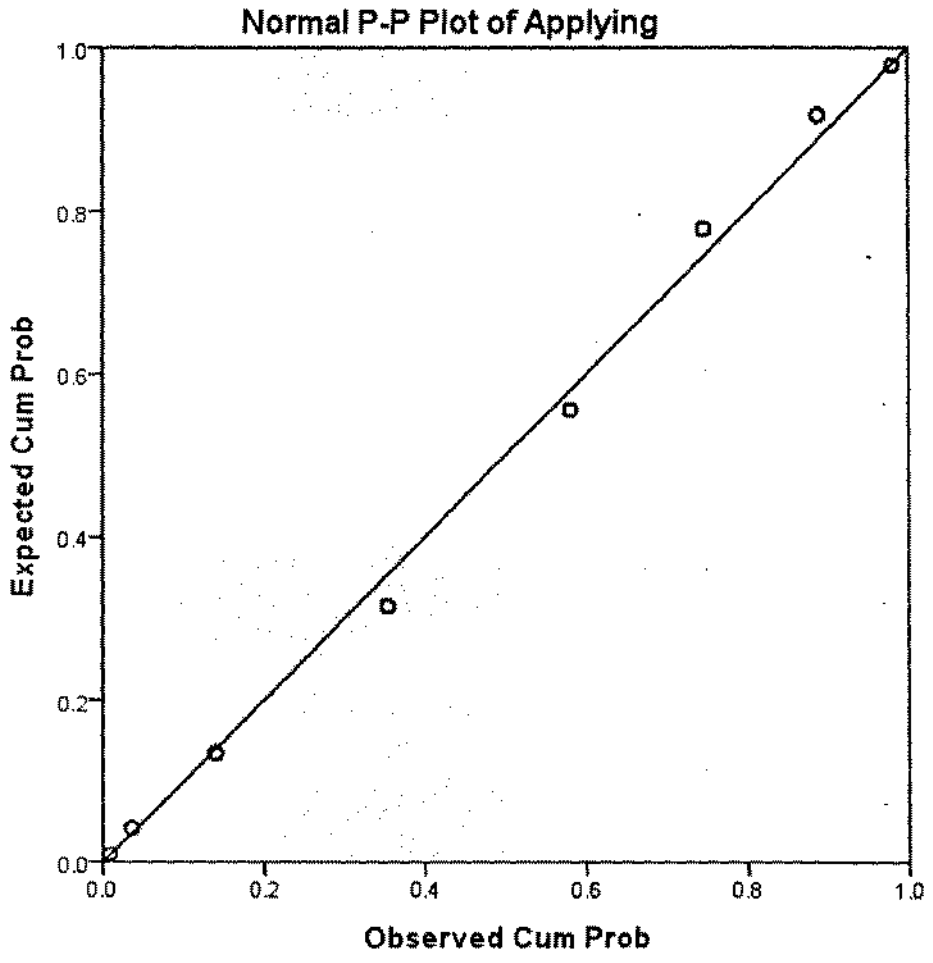
Remembering consisted of recognizing and recalling relevant information from long-term memory. Ten items were made on remembering for control group-1 which lied near to the straight line and had the characteristics of normal distribution.

P-P Plot for Normal Distribution on Understanding (Control Group-I)



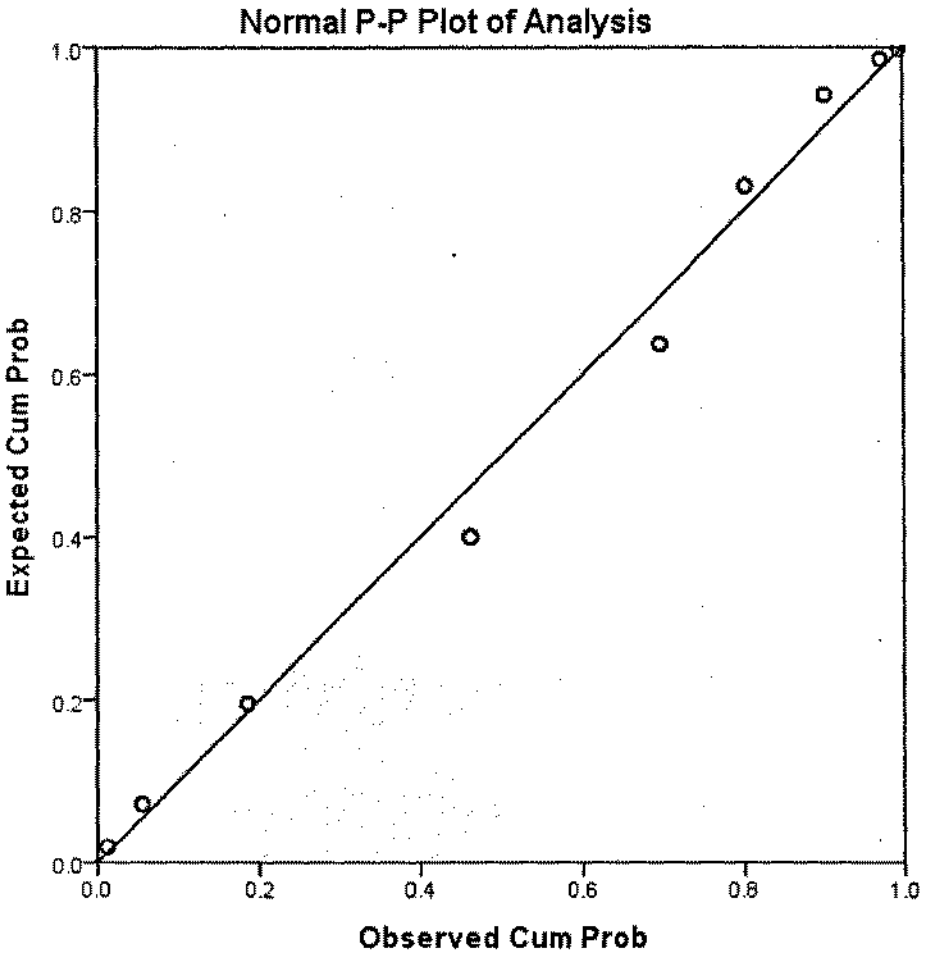
Understanding meant the ability to make your own meaning from educational material such as reading and teacher explanations. The sub-skills for this process include interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining. The figure shows that items were having the characteristics of normal distribution because points were not deviating from the straight line.

P-P Plot for Normal Distribution on Applying **(Control Group-I)**



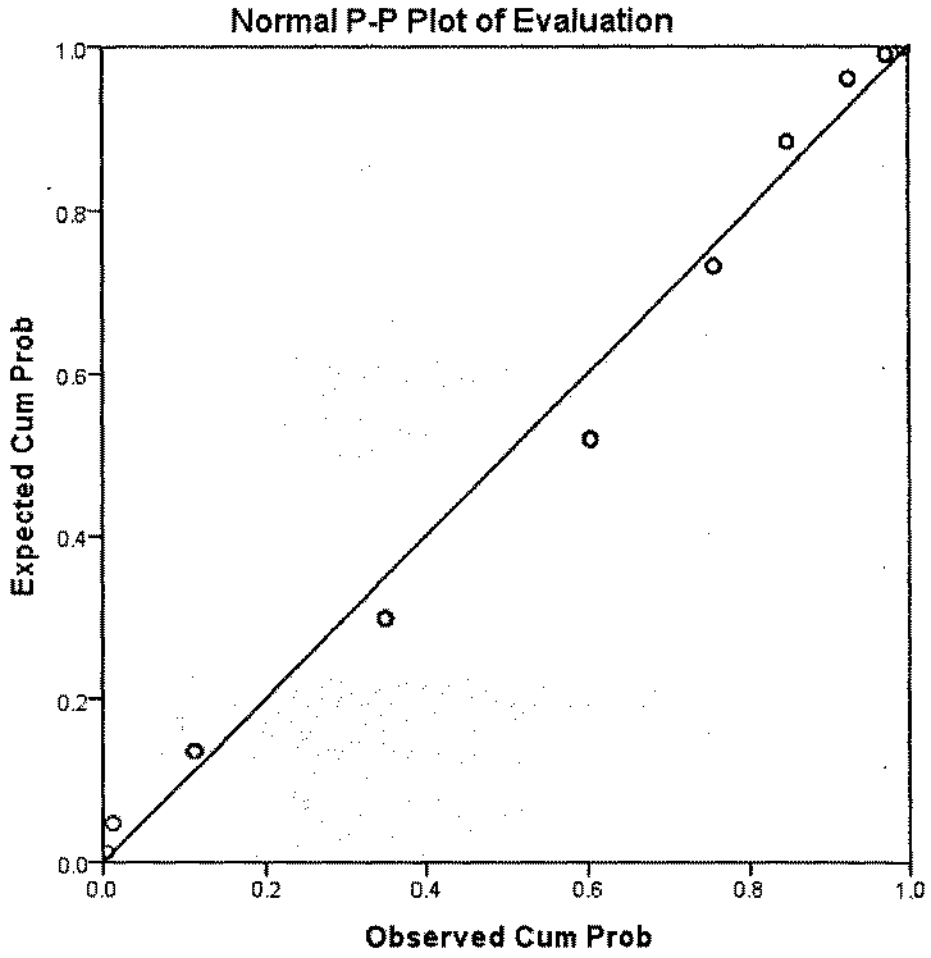
Applying referred to using a learned procedure either in a familiar or new situation. The items developed on Applying were near the expected straight line which showed that the data had the characteristics of the normal distribution; hence the control group-1 shows normal distribution of applying items.

P-P Plot for Normal Distribution on Analyzing (Control Group-I)



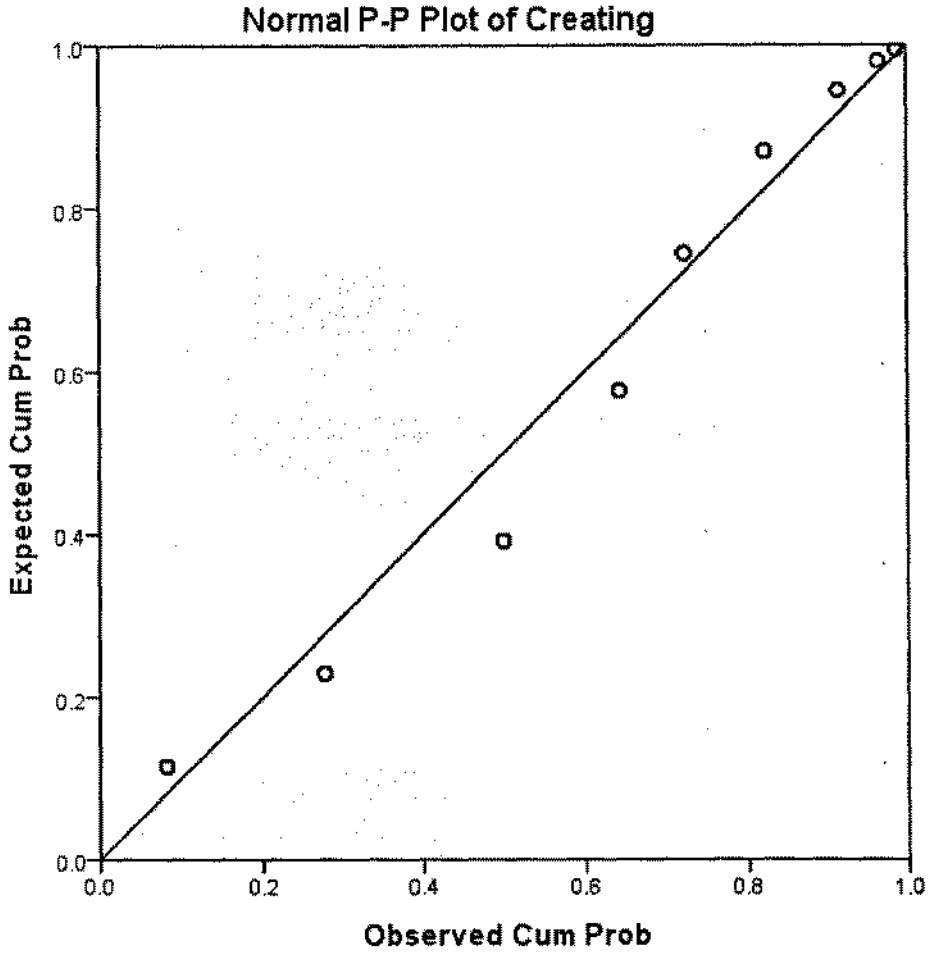
The analysis of items on Analyzing shows the attributes of normal distribution, because locations of the points were close to the expected straight line, hence the control group-1 shows normal distribution of analyzing items of control group-1.

P-P Plot for Normal Distribution on Evaluating (Control Group-I)



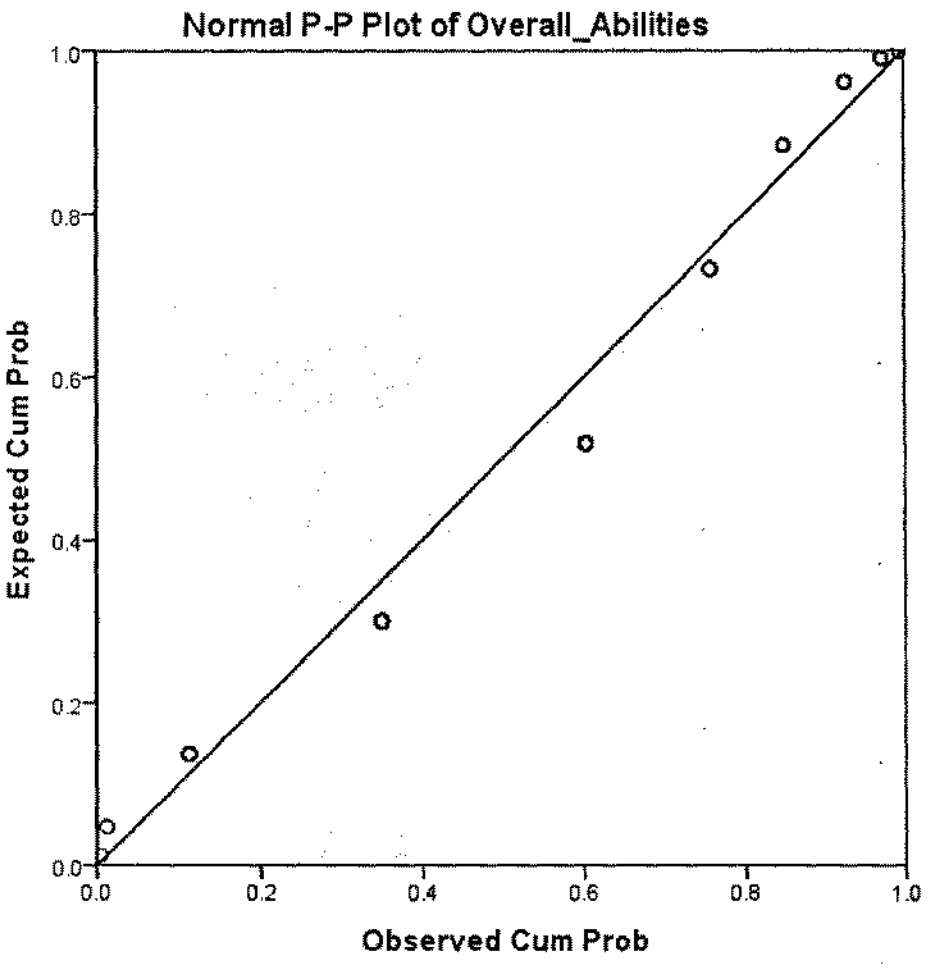
Evaluation included in Higher Order Thinking Skills like checking and critiquing. It also included to infer and reached to some decision. The items developed on evaluation showed the normal distribution

P-P Plot for Normal Distribution on Creating **(Control Group-I)**



The location of the points near the straight line in the above figure means that the data had the characteristics of the normal distribution.

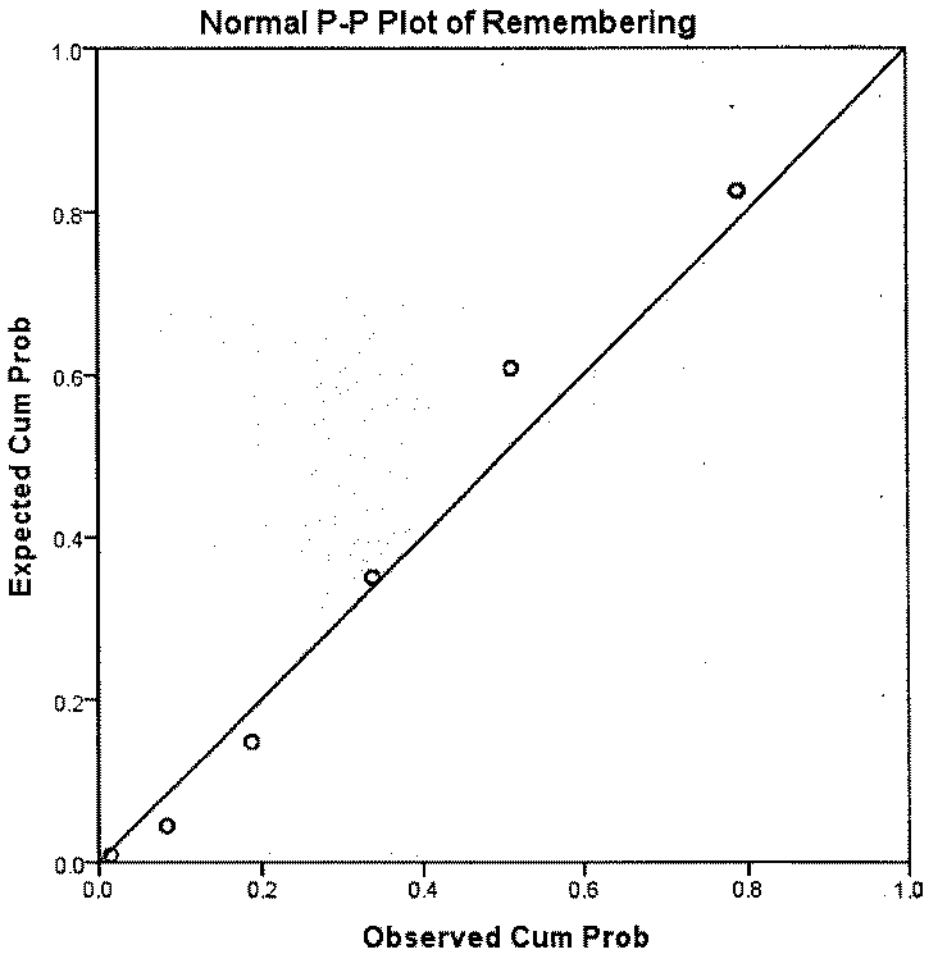
P-P Plot for Normal Distribution on Overall Abilities **(Control Group-I)**



The figure shows the overall points developed on conceptual dimension of cognitive process i.e. Remembering, Understanding, Applying, analyzing, Evaluating and creating also had the characteristics of normal distribution. The analysis of overall abilities in the control group-1 also had the characteristics of normal distribution.

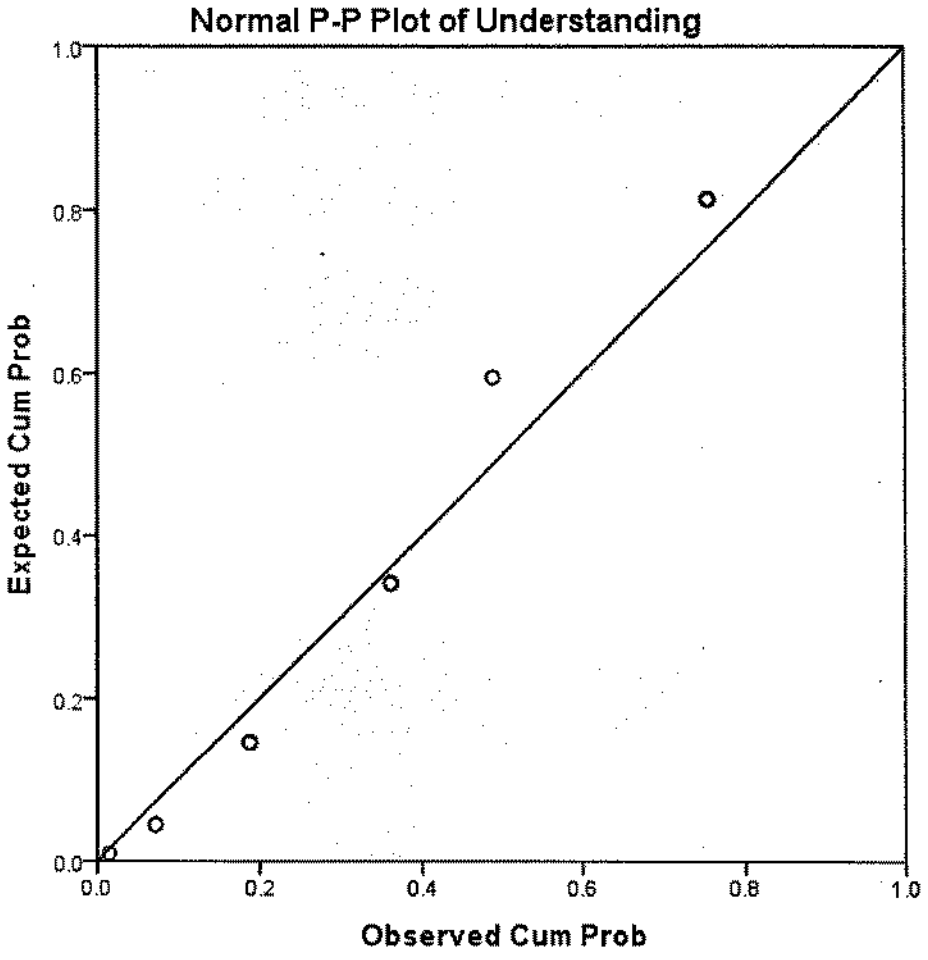
P-P Plots for Normal Distribution Analysis (Control Group-II)

P-P Plot for Normal Distribution on Remembering



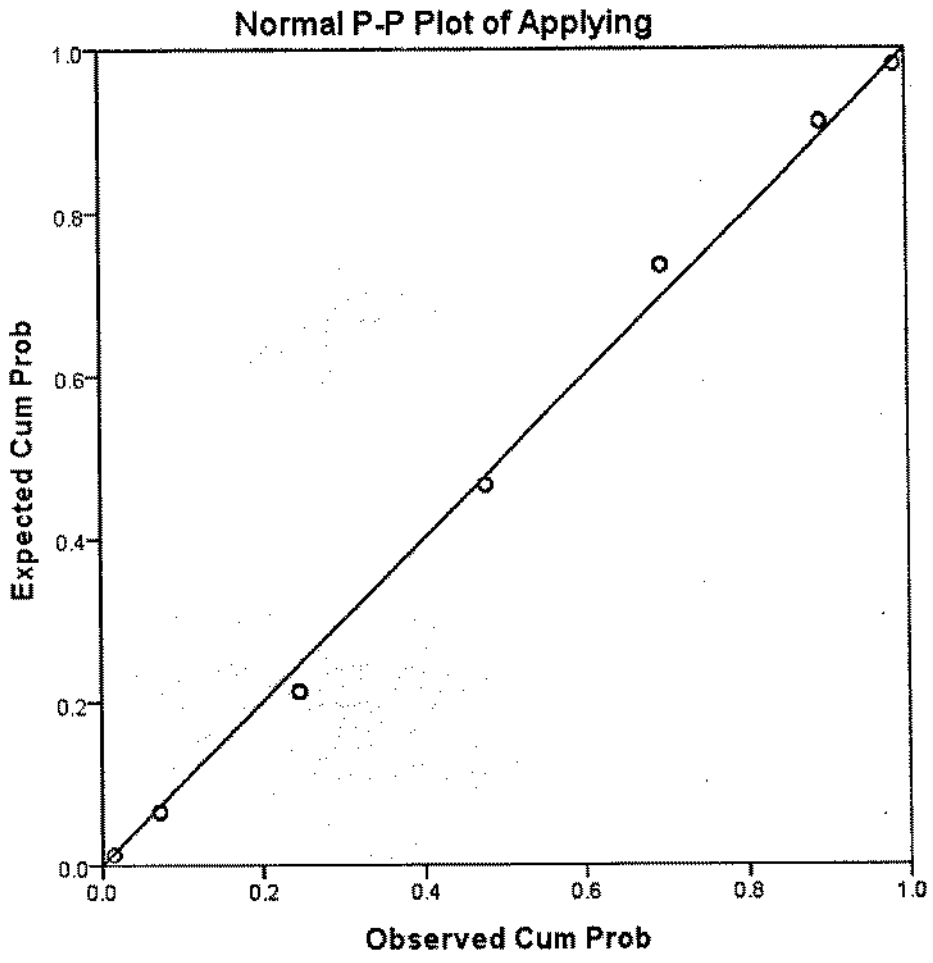
Remembering consisted of recognizing and recalling relevant information from long-term memory. Ten items were made on remembering for experimental group which showed near to the straight line and had the characteristics of normal distribution. The points in the figure represented the characteristics of normal distribution for the items on remembering for control group-II

P-P Plot for Normal Distribution on Understanding (Control Group-II)



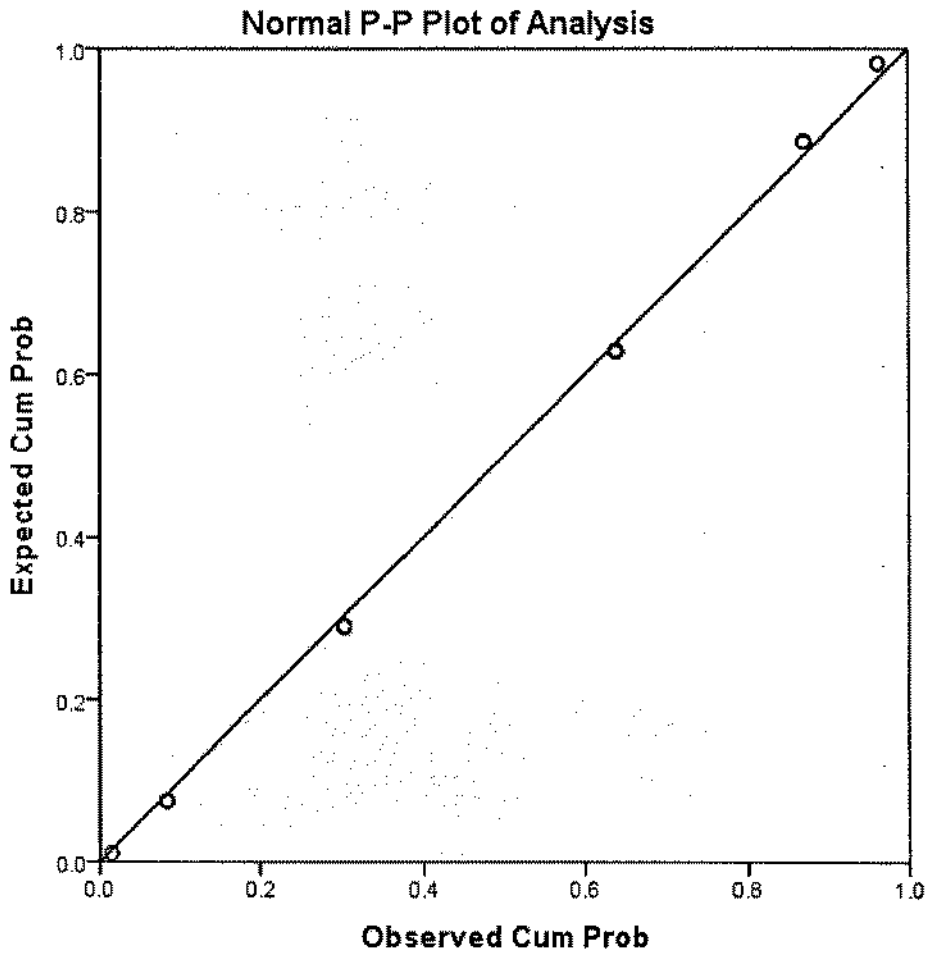
Understanding meant the ability to make your own meaning from educational material such as reading and teacher explanations. The sub-skills for this process include interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining. The analysis of the items regarding Understanding in the control group-II in above figure showed the attributes of normal distribution.

P-P Plot for Normal Distribution on Applying (Control Group-II)



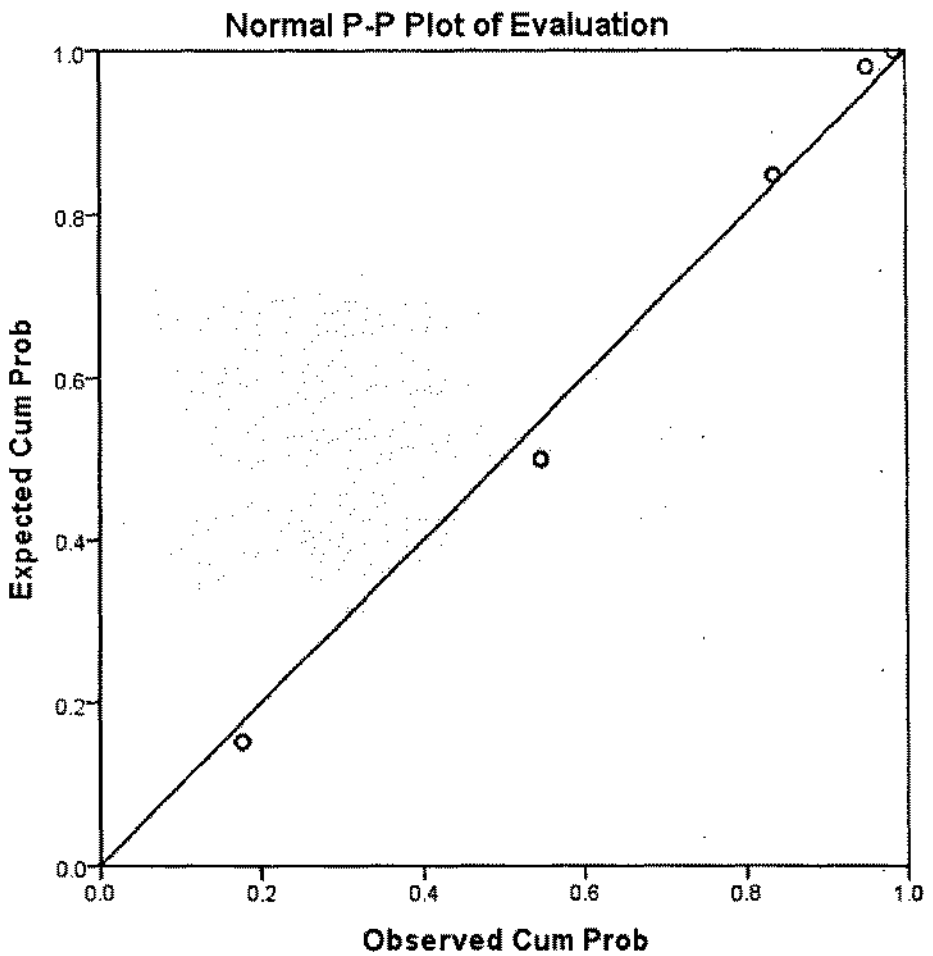
Applying referred to using a learned procedure either in a familiar or new situation. The items developed on Applying were near the expected straight line which showed that the data had the characteristics of the normal distribution. The points showed the attributes of normal distribution for control group-II for applying items.

P-P Plot for Normal Distribution on Analyzing **(Control Group-II)**



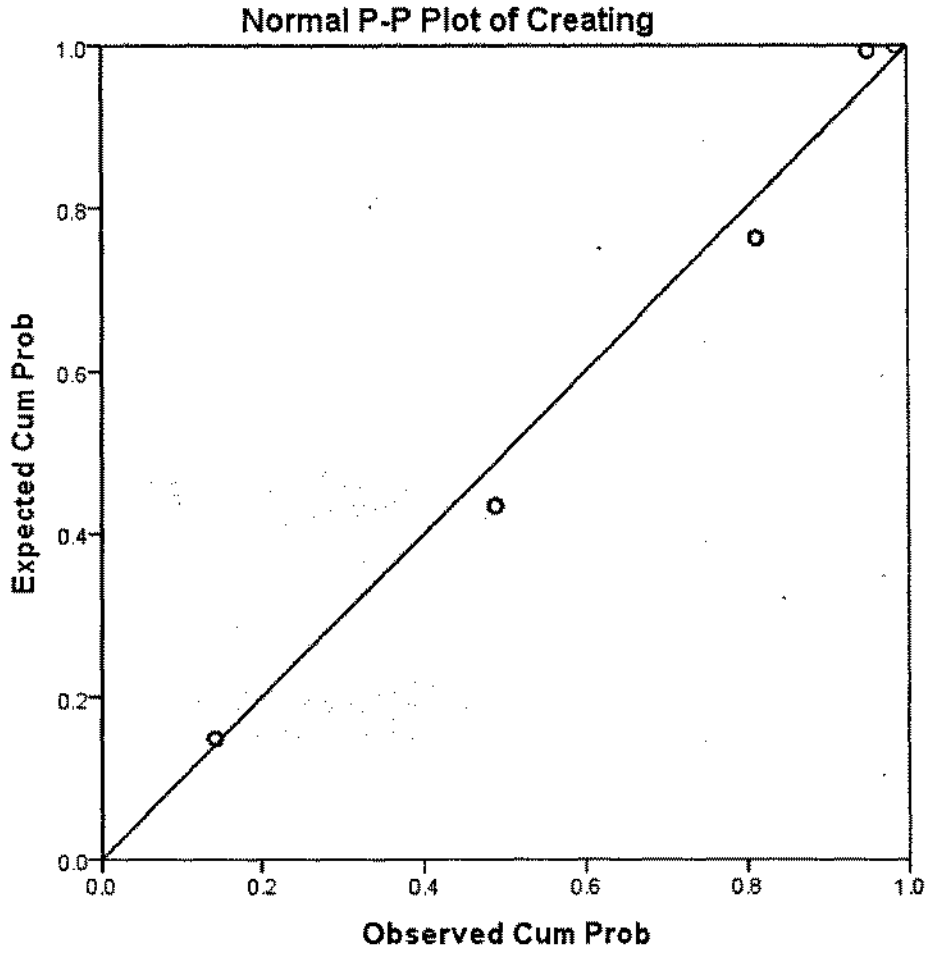
Analyzing consisted of breaking knowledge down into its parts and thinking about how the parts were related to its overall structure. It could be exhibited by differentiating; organizing and attributing. The points showed the attributes of normal distribution for control group-II for analyzing items.

P-P Plot for Normal Distribution on Evaluating **(Control Group-II)**



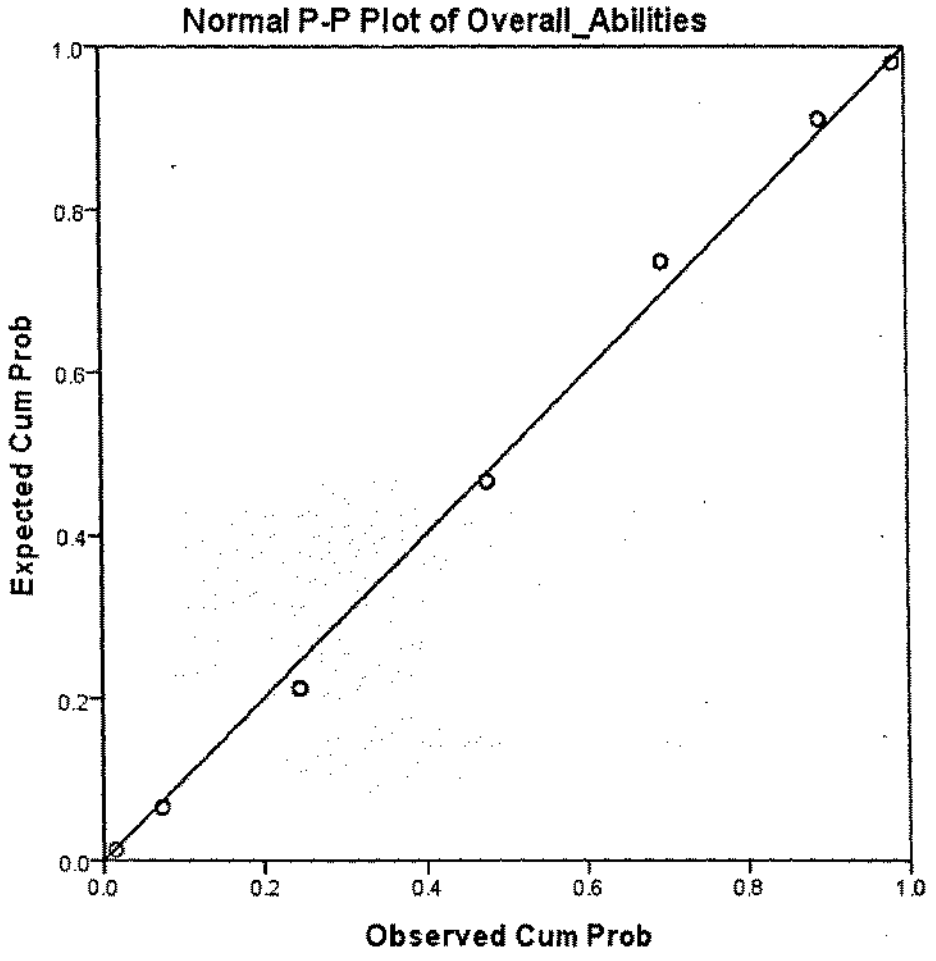
Evaluation included in Higher Order Thinking Skills like checking and critiquing. It also included to infer and reached to some decision. The items developed on evaluation showed the normal distribution. The analyses of evaluating items for the control group-II showed the characteristics of normal distribution.

P-P Plot for Normal Distribution on Creating **(Control Group-II)**



This skill of Creating involved putting things together to make something new. To accomplish creative tasks, the learners generated, planned and produced. This was the highest level in RBT. The items developed on this showed the characteristics of normal distribution. The points in the above figure meant the attributes of normal distribution for the item of creating for the control group-II.

P-P Plot for Normal Distribution on Overall Abilities (Control Group-II)



The analysis for the overall abilities regarding Revised Bloom's Taxonomy showed the characteristics of normal distribution for control group-II. Hence the results of all three groups Experimental, Control group-I and Control group-II showed the normal distribution.