Blind CMOE and LMMSE Detector for MC-CDMA

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Submitted by

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Submitted in partial fulfillment of the requirements for the MS in Electronic Engineering with specialization in Telecommunication at the faculty of Engineering, International Islamic University, Islamabad

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(2009)

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In the name of Allah (SWT) the most benificient and the most merciful.

Dedicated...

To a Person who is
"The Rehmat" for all the Universe,
and
To Our Parents,
and
To whom we love and respect.

Blind CMOE and LMMSE Detector for MC-CDMA

Dated: 12-12-09

It is certified that we have read the thesis submitted by **Mr. Muhammad Umair Reg. No. 92-FET/MSEE/F07**. It is our judgment that this thesis is of sufficient standard to warrant its acceptance by the International Islamic University, Islamabad for Masters Degree in Electronic Engineering (MSEE).

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Declaration

I hereby declare that this thesis, neither as a whole nor as a part thereof has been copied out from any source. It is further declared that I have developed this thesis entirely on the basis of my personal effort made under the sincere guidance of my supervisor (Dr.Aamer Saleem). No portion of the work presented in this thesis has been submitted in support of any application for any other degree or qualification of this or any other university or institute of learning.

Muhammad Umair

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Acknowledgement

All praise to Almighty Allah, who gave me the understanding, courage and patience to complete this thesis.

I express my gratitude to my kind supervisor **Dr. Aamer Saleem,** who provided me opportunity to learn and enhance my knowledge. As my research supervisor, he had been ready to help and guide me throughout the research.

I would also like to thank my teacher in the department **Dr. Aqdas Naveed Malik** for his moral support of our efforts.

And last but not the least; I would like to acknowledge the support of my family members. I would like to admit that I owe all my achievements to our truly, sincere and most loving parents, brothers and sisters, whose prayers are a source of determination for me.

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Abstract

Multi-carrier code division multiple access (MC-CDMA) is an efficient multicarrier transmission scheme for multiple access communication in which code division multiple access (CDMA) and orthogonal frequency division multiplexing (OFDM) techniques are combined. Due to sharing of frequency, multiple access interference (MAI) is a big challenge in MC-CDMA. MAI reduces the data rate and channel efficiency.

Many detectors have been proposed to extenuate MAI in MC-CDMA system. Few of them are blind constraint minimum output energy (CMOE) detector, subspace based least minimum mean square error (LMMSE) detector, robust constraint minimum output energy (CMOE) detector for MC-CDMA etc. Blind CMOE and subspace based LMMSE detector for MC-CDMA is investigated in this thesis. The performance of both detectors is evaluated in the presence of white Gaussian noise. The computational complexity of both of these detectors is also evaluated.

In the presence of the white Gaussian noise, the subspace based LMMSE detector performs better than the CMOE detector. The computational complexity of the subspace based LMMSE detector is very high than CMOE detector. The reason is that subspace based LMMSE detectors acquires the tracking of the signal subspace as well as noise subspace. But, blind CMOE detector only acquires the handling of signal subspace.

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Chapter 1
Introduction

Chapter 1

Introduction

1.1 Introduction

Multi-Carrier transmission schemes are very well supported by multi carrier code division multiple access (MC-CDMA) [1]. MC-CDMA supports the multiple access communication. In MC-CDMA, each user has its own spreading code that is used to separate the data from other users and many users share the same bandwidth simultaneously. Multicarrier modulation is applied for diminishing symbol rate. So it also reduces inter symbol interference (ISI) per sub channel. MC-CDMA is bounded to interference like the CDMA system. Due to concurrent sharing of the identical bandwidth by the multiple users, it gives rise to the multiple access interference (MAI). The linear minimum mean square error (LMMSE) detectors are usually productive to diminish MAI in MC-CDMA system.

A simpler LMMSE receiver called constrained minimum output energy (CMOE) detector is proposed to mitigate MAI in CDMA system [3]. The CMOE detector exaggerates signal to interference plus noise ratio (SINR) in the favorable condition. Using the CMOE based blind detector and channel estimator for the MC-CDMA is anticipated in [1]. A Blind CMOE detector to mitigate MAI in MC-CDMA system is anticipated in [1]. The recursive least squares (RLS) algorithm is used to refresh the correlation matrix. This helps to widely predict the channel by computing the least Eigen-vector of data sub-matrix.

In presence of white Gaussian noise, the execution of LMMSE detector (Subspace based) is better than the CMOE detector [2]. There are many developments in LMMSE detector like quasi synchronous MC-CDMA system without correct information of channel arrangement [9], subspace based LMMSE detector without cyclic prefix [4] etc. But, here the target is LMMSE detector with subspace method for MC-CDMA using efficient subspace tracking algorithm [2].

1.2 Objective

In this dissertation, Blind CMOE and subspace based LMMSE detectors are studied in presence of white Gaussian noise, implemented and then performance for both of them is compared. The ability of CMOE detector and LMMSE detector was examined by normalized channel estimation inaccuracy, number of data samples, SNR, average SINR, Average MSE. The ability is also analyzed for the Qausi-synchronous channel on the basis of normalized channel estimation error, number of data samples and SNR. These assessments gives information for using Blind CMOE and subspace based LMMSE detector according to requirements. Although, CMOE detector gives the best performance by updating of correlation matrix through RLS updating and the channel estimation is done by calculation of least Eigen-vector of a statistics sub matrix. In presence of white Gaussian noise, the CMOE detector performance diminishes considerably. The LMMSE detector using the subspace method performs better than the CMOE detector in presence of white Gaussian noise. MATLAB 7.3 has been used for the imitations and implementation of the algorithms and detection process has been implemented in functions.

1.3 Span of Thesis

Before studying the Blind CMOE detector, we must have the understanding of the MC-CDMA system. Therefore, to have the understanding of the Blind CMOE and subspace based LMMSE detector, the MC-CDMA system is also discussed. Firstly, the study of the MC-CDMA system is given that is followed by the Blind CMOE detector. After detailed study of the Blind CMOE detector, the LMMSE detector using subspace method is given. After that the mathematical model for Blind CMOE and LMMSE detector has been presented. Finally, both detectors have been reviewed and their mathematical equations are derived to evaluate their complexity.

1.4 Layout of thesis

The Chapter 1 is introductory which includes the objectives and extent of work. The Chapter 2 is basically the literature review, covers the relevant literature which I have studied in this thesis. In Chapter 3, the MC-CDMA system design is followed by problem statement and system design of both CMOE and subspace based LMMSE detector. The Chapter 4 presents algorithm and simulation results for both the detectors. In the end, a brief conclusion is given. Finally, in the end reference section is also given.

Chapter 2
Literature Review

Chapter 2

Literature Review

In this dissertation, Blind Constraint Minimum Output Energy (CMOE) and Linear Minimum Mean Square Error (LMMSE) detection in subspace mode has successfully applied to MC-CDMA detection process. As the base system is MC-CDMA, therefore background knowledge of the MC-CDMA system will be helpful. In this chapter CDMA system, OFDM system, MC-CDMA system and different multiuser detection techniques are presented. The Blind CMOE and Subspace based LMMSE detector is also discussed briefly.

2.1 MC-CDMA System

MC-CDMA is formed by uniting the CDMA (Code Division Multiple Access) and OFDM (Orthogonal Frequency Division Multiplexing) modulation. The CDMA allows simultaneous sharing of the same bandwidth among different users by allocating each user a unique signature that is different to the other used signatures. The OFDM is strong against multipath erasures with good spectral efficiency. To have the good understanding of MC-CDMA system, we are starting with CDMA and OFDM systems.

2.1.1 CDMA System

In CDMA, multiple users are allowed to share a channel or sub channel by DS-SSS (Direct-Sequence Spread Spectrum Signals) [10]. Each user is allocated an inimitable signature sequence often called as code sequence. This signature sequence allows each user to transmit the data signal along the allocated frequency band. Multiple user signals at the receiver are distributed by taking the received signal cross correlation with each possible user code sequences. These code sequences are formed such that there is small cross correlation between them. The demodulated multiple users received signals crosstalk problem is reduced by this fact.

The users access the channel randomly in the CDMA. This results in complete overlapping of signals transmission of multiple users in both domains frequency and time axis. The fact that each signal is spread in frequency by pseudorandom code sequence facilitates the demodulation and separation of the signals at the receiver.

CDMA Signal and Channel Model

Consider a simultaneous K user sharing CDMA system. A unique signature waveform $g_k(t)$ of duration T is assigned to each user, where T is the symbol duration. The code waveform is expressed as

$$g_k(t)=\sum_{n=o}^{L-1}a_k(n)p(t-nT_c), \quad 0\le t\le T \qquad (2.1)$$

The PN (Pseudo Noise) code sequence $\{a_k(n), 0\le n\le L-1\}$ is consisting of L chips that has values ± 1, *p(t)* is the pulse with interval T_C. The chip duration is T_C. Therefore, there are L chips per symbol and $T=LT_c$. The assumption is that the energy of K signature waveforms is unity, i.e.

$$\int_o^T g_k^2(t)dt=1 \qquad (2.2)$$

The cross correlation $\rho_{ij}(0)$ is only important for the synchronous transmission, that is

$$\rho_{ij}(0)=\int_0^T g_i(t)g_j(t)dt \qquad (2.3)$$

The binary antipodal signal is used to transmit information for simplicity. Suppose k^{th} user data sequence is presented by $\{b_k(m)\}$. Each bit may be ± 1. The length of the chunk of bits is *N*. After that, the data chunk from k^{th} user is

$$\mathbf{b}_k=[b_k(1).....b_k(N)]^t \qquad (2.4)$$

Hence the corresponding equivalent low pass waveform is

$$s_k(t)=\sqrt{\varepsilon_k}\sum_{i=1}^{N}b_k(i)g_k(t-iT) \qquad (2.5)$$

ε_k is the signal energy per bit. The composite signal of K users is represented as

$$s(t)=\sum_{k=1}^{K}\sqrt{\varepsilon_k}\sum_{i=1}^{N}b_k(i)g_k(t-iT-\tau_k) \qquad (2.6)$$

The transmission delays are τ_k with the condition $0\le\tau_k<T$. For synchronous transmission $\tau_k=0$ for $1\le k\le K$.

The transmitted signal is supposed to be corrupted by AWGN. The received signal after corruption by the AWGN is presented as

$$r(t)=s(t)+n(t) \qquad (2.8)$$

where $s(t)$ is given above and $n(t)$ is the noise with power spectral density $\frac{1}{2}N_0$.

CDMA Receiver with Synchronous Transmission

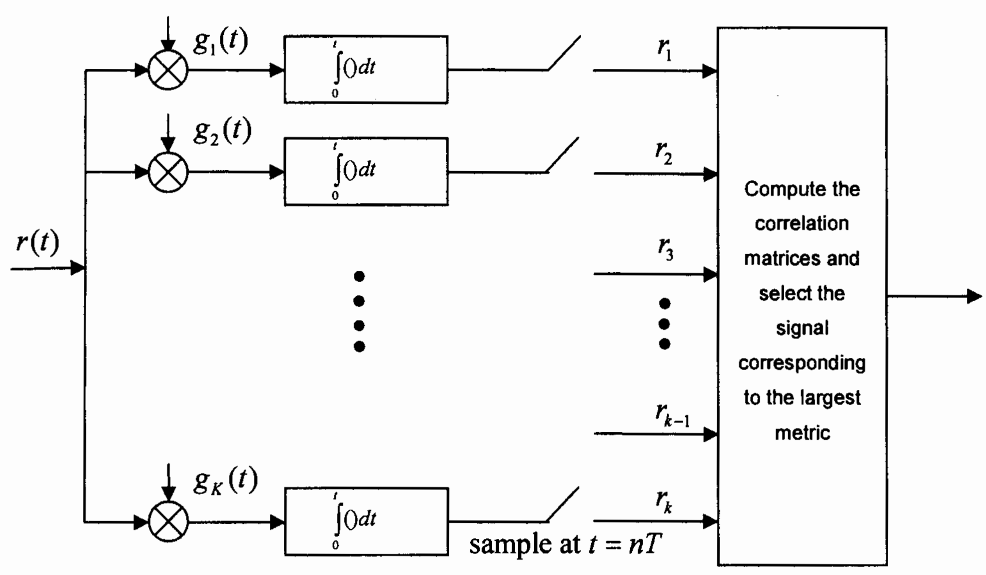

Figure 2.1: Receiver of CDMA system with Synchronous transmission

where $r(t)$ is received signal, $g_k(t)$ is code waveform of each user and the correlation matrices is

$$r(t)=\sum_{k=1}^{K}\sqrt{\varepsilon_k}b_k(t)+n(t), \qquad 0\le t\le T$$

The maximum-likelihood receiver combines the log likelihood detection function. This is presented in the form of the correlation matrices

$$c(\mathbf{r}_k;\mathbf{b}_k)=2\mathbf{b}_k^{'}\mathbf{r}_k-\mathbf{b}_k^{'}\mathbf{R}_s\mathbf{b}_k$$

where R_s is correlation metrics with elements $p_{jk}(0)$ and

$$\mathbf{r}_k=[r_1\,r_2\,....r_k]^{'},$$

$$b_k^{'}=[\sqrt{\varepsilon_1}b_1(1)....\sqrt{\varepsilon_k}b_k(1)]^{'}$$

The possible choices of the bits in the information sequence are 2^k. The correlation metrics is calculated for each sequence and the sequence that give up the largest correlation metric is selected. Hence, the complexity rises exponentially with increase of users.

2.1.2 OFDM System

In OFDM, each user is allocated a unique frequency slot. By using this frequency slot, each user is allowed to transmit data signal along the allocated frequency range. Each frequency slot is orthogonal to all other frequency slots.

In OFDM [11], the consecutive data rill of traffic channel is sent along a Serial to Parallel (S/P) converter. This splits the data into a number of parallel channels. Each channel data is adapted to a modulator, so there are N modulators for N channels with frequencies $f_0, f_1, ..., f_{n-1}$. Each channel is separated by a difference of Δf .So the overall modulated bandwidth W of the N channels is $N\Delta f$. The OFDM signal is formed by N modulated carriers.

OFDM Signal and Channel Model

A multi-carrier modulation communication system modulates N_c complex valued source symbols $S_n, n=0,....,N_c-1$ [11]. These source symbols are sent in parallel on N_c sub-carriers. Interleaving, symbol mapping, Source and channel coding may be applied to obtain the source symbols. The interval of the each source symbol is T_d. After S/P alteration the duration of the OFDM symbol is

$$T_s = N_c T_d \tag{2.8}$$

According to OFDM principle, the N_c subcarriers are modulated with the spacing of $F_s = \frac{1}{T_s}$ in order to achieve the orthogonality between the signals on the N_c subcarriers with a rectangular pulse shape. The N_c alongside transmitted source symbols S_n are denoted as the OFDM symbol. The OFDM symbol complex envelop with rectangular pulse shaping is

$$x(t) = \frac{1}{N_c} \sum_{n=0}^{N_c-1} S_n e^{j2\pi f_n t}, \quad 0 \le t \le T_s \tag{2.9}$$

The frequencies of the N_c sub-carriers are located at

$$f_n = \frac{n}{T_s}, \quad n = 0,1,\ldots\ldots, N_c - 1$$

In OFDM, the multicarrier modulation can be implemented by using the IFFT (Inverse Fast Fourier Transform) or IDFT (Inverse Discrete Fourier Transform). While sampling the complex envelop $x(t)$ of an OFDM symbol with rate $\frac{1}{T_d}$ the samples are

$$x_v = \frac{1}{N_c} \sum_{n=0}^{N_c-1} S_n e^{j2\pi nv/N_c}, \quad v = -L_g, \ldots\ldots N_c - 1 \tag{2.10}$$

The sampled sequence x_v is the IDFT of the source sequence symbol S_n.

By increasing the number of subcarriers, the OFDM symbol duration T_s becomes large compare to the duration of the impulse response $\tau_{\max}$ of the channel. This results in the reduction of the Inter Symbol Interference (ISI). The ISI is completely eliminated by allocation of guard interval of duration $T_g \ge \tau_{\max}$ between the adjacent OFDM symbols.

The cyclic addition of every OFDM symbol for the guard interval is accessed by increasing the interval of OFDM symbol to

$$T_s' = T_g + T_s \tag{2.11}$$

The OFDM sampled sequence with guard interval is

$$x_v = \frac{1}{N_c}\sum_{n=0}^{N_c-1} S_n e^{j2\pi nv/N_c}, \quad v = -L_g, ...N_c - 1 \tag{2.12}$$

The sequence is passed through the Digital to Analog (D/A) converter whose output is the waveform $x(t)$ with increased duration T_s'. The received signal waveform y(t) is given as

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau,t)d\tau + n(t) \tag{2.13}$$

In other words, the received symbol R_n obtained from frequency domain representation is

$$R_n = H_n S_n + N_n, \quad n = 0, ..., N_c - 1 \tag{2.14}$$

OFDM System for Simplified Multi-Carrier Transmission

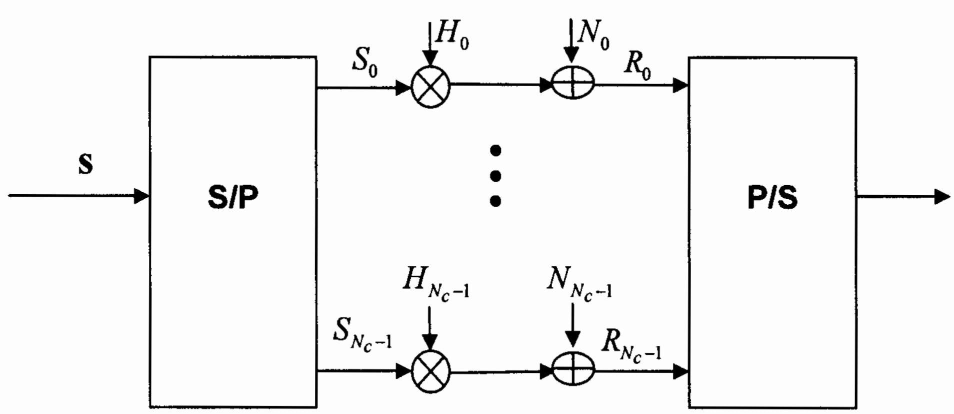

Figure 2.2: Multi-Carrier OFDM transmission system

where **s** is the source symbol, **H** is the $N_c \times N_c$ channel matrix and N is the additive noise. The received symbols **r** will be obtain after the inverse OFDM are given by the vector

$$\mathbf{r} = (R_0, R_1, ...R_{N_c-1})^T \quad (2.15)$$

and is obtained by

$$\mathbf{r} = \mathbf{Hs} + \mathbf{n} \quad (2.16)$$

where

$$\mathbf{H} = \begin{bmatrix} \mathrm{H}_{0,0} & 0 & . & . & . & 0 \\ 0 & H_{1,1} & & & & 0 \\ . & & & . & & . \\ . & & & & . & . \\ 0 & 0 & . & . & . & H_{N_c-1,N_c-1} \end{bmatrix},$$

$$\mathbf{s} = \left(\mathrm{S}_0, \mathrm{S}_1, ..., S_{N_c-1}\right)^T$$

and

$$\mathbf{n} = \left(N_0, \mathrm{N}_1, ..., N_{N_c-1}\right)^T$$

2.1.3 MC-CDMA System

Being a combination of the CDMA and OFDM, every chip of the DSS data symbol is chartered on distinct subcarrier in MC-CDMA. So, the spread data symbol chips are modulated in alongside on distinct subcarriers. There are K simultaneous active users in MC-CDMA adaptive transmission system.

The Figure 2.3 shows the Multi-carrier spread spectrum signal generation. The Figure 2.3 illustrates multi-carrier spectrum spreading of one complex valued data symbol $d^{(k)}$ is allocated to k user with serial data symbol rate $1/T_d$.

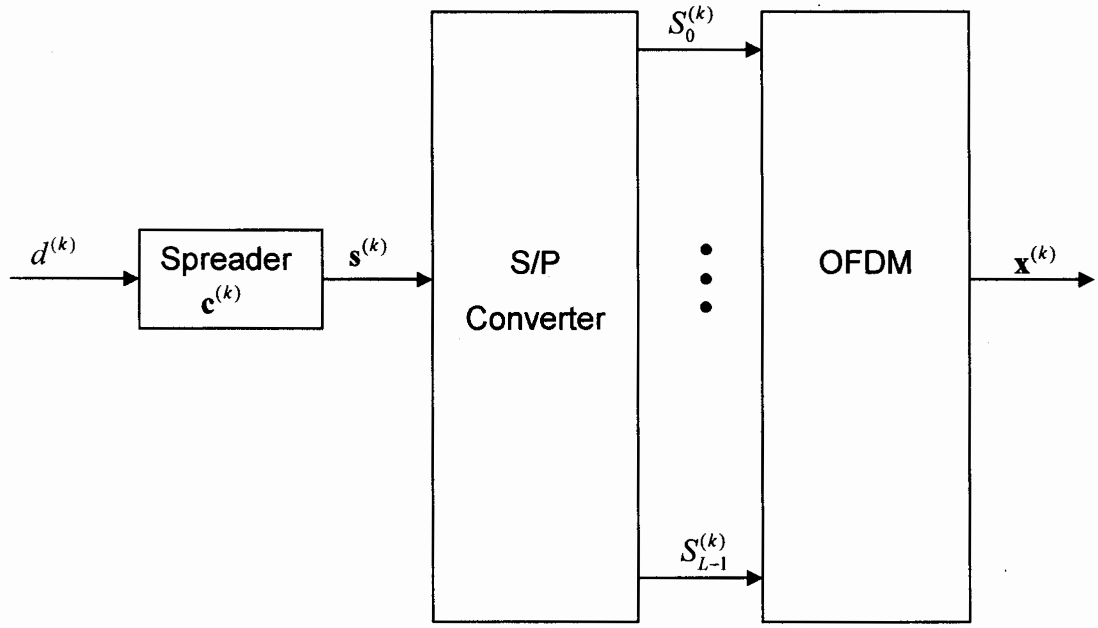

Figure 2.3: Multi-carrier spread spectrum signal generation

For shortness, the MC-CDMA signal generation is described for a one data symbol for each user. $d^{(k)}$ is the complex valued data symbol that is multiplied with the user explicit spreading code

$$\mathbf{c}^{(k)} = (c_0^k, c_1^k, \ldots, c_{L-1}^{(k)})^T \tag{2.17}$$

and the length of this spreading code is $L = P_G$. Before serial to parallel conversion, the consecutive spreading code $c^{(k)}$ has the chip rate

$$\frac{1}{T_c} = \frac{L}{T_d} \tag{2.18}$$

So this chip rate in (2.18) is more than the data symbol rate $1/T_d$. In vector form, the complex valued sequence after spreading is

$$\mathbf{s}^{(k)} = d^{(k)}\mathbf{c}^{(k)} = (S_0^{(k)}, S_1^{(k)}, \ldots, S_{L-1}^{(k)})^T \tag{2.19}$$

The modulation of components $s_l^{(k)}$ in parallel on to L subcarriers gives a multicarrier spread spectrum signal. By the multi-carrier spread spectrum, every data symbol is modulated on L subcarriers. The period of the OFDM symbol in multi-carrier spread spectrum with a guard interval after the equivalence in number of OFDM symbol subcarriers to the spreading code length L is

$$T_s^{'} = T_g + LT_c \tag{2.20}$$

where T_g is the guard interval, L is period of spreading code and T_c is spreading chip.

One data symbol for each user is sent in one OFDM symbol in previous discussion.

Downlink Signal

In downlink synchronous transmission, the spread signals for the total k users are added beyond to OFDM operation. The placement of the k sequences $\mathbf{s}^{(k)}$ results in the sequence

$$\begin{aligned}\mathbf{s} &= \sum_{k=0}^{K-1} \mathbf{s}^{(k)} \\ &= (S_0, S_1, \ldots, S_{L-1})^T\end{aligned} \tag{2.21}$$

The downlink is also represented for $\mathbf{s}$ is

$$\mathbf{s} = \mathbf{Cd} \tag{2.22}$$

where

$$\mathbf{d} = (d^{(0)}, d^{(1)}, \ldots, d^{(k-1)})^T$$

The $\mathbf{d}$ is the vector with the transmitted symbols of total k users and $\mathbf{C}$ is the spreading matrix given by

$$\mathbf{C} = (\mathrm{c}^{(0)}, \mathrm{c}^{(1)}, \ldots, \mathrm{c}^{(\mathrm{k}-1)})$$

The sequence $\mathbf{s}$ is processed in OFDM block according to (2.9) in order to obtain the MC-CDMA downlink signal. The Figure 2.4 illustrates downlink transmitter of MC-CDMA.

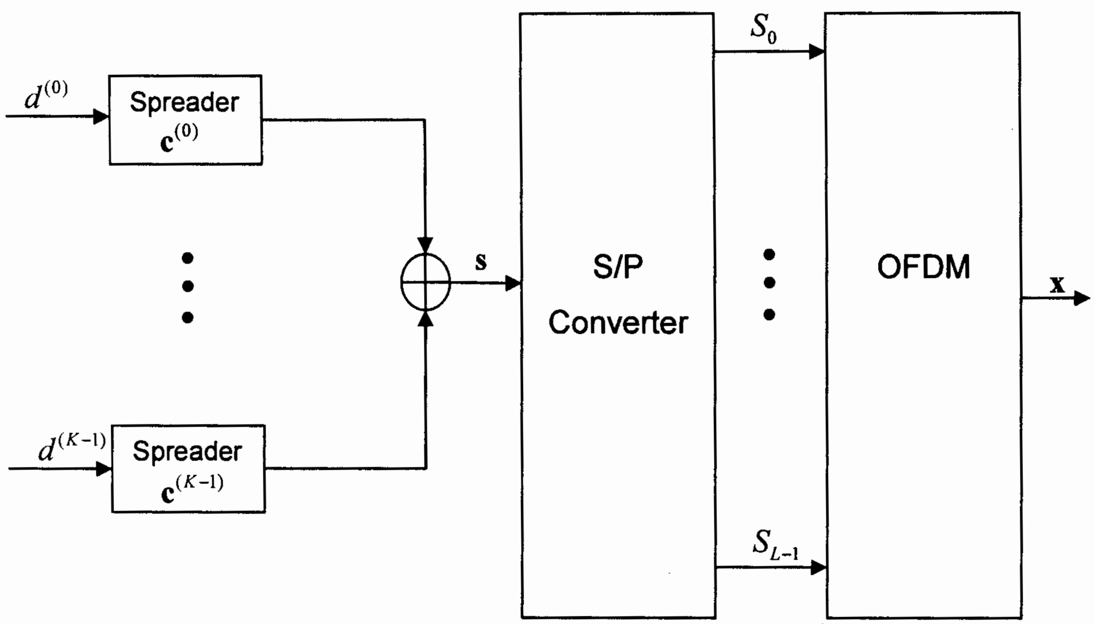

Figure 2.4: Downlink transmitter of MC-CDMA

The supposition is that guard intervals are long enough to eliminate the ISI. Therefore, the transmitted sequence **s** received vector after the inverse OFDM and de-interleaving is

$$\mathbf{r} = \mathbf{Hs} + \mathbf{n} = (R_0, R_1, \ldots, R_{L-1})^T \tag{2.23}$$

where **H** is $L \times L$ channel matrix and **n** is noise vector with length L. The **r** is catered to data detector for the purpose of getting the hard or soft estimate of sent data. The system matrix **A** is used instead of **H** for the multiuser detection. It can be written in the equivalent notation

$$\mathbf{r} = \mathbf{Ad} + \mathbf{n} = (R_0, R_1, \ldots, R_{L-1})^T \tag{2.24}$$

and the system matrix **A** is defined as

$$\mathbf{A} = \mathbf{HC}$$

Uplink Signal

The sequence $\mathbf{s}^{(k)}$ is processed in the OFDM block according to (2.9) for the uplink in order to get the MC-CDMA signal. The transmitted sequences $\mathbf{s}^{(k)}$ received vector after inverse OFDM and frequency de-interleaving is

$$\mathbf{r} = \sum_{k=0}^{K-1} \mathbf{H}^{(k)} \mathbf{s}^{(k)} + \mathbf{n} = (R_0, R_1, \ldots, R_{L-1})^T \tag{2.25}$$

where the sub-channels assigned to user k has coefficients contained in $\mathbf{H}^{(k)}$. The uplink is assumed to be synchronous for getting high spectral performance of the OFDM. The $\mathbf{r}$ is catered to data detector for getting hard or soft estimate of sent data. The system matrix is

$$\mathbf{A} = (\mathbf{a}^{(0)}, \mathbf{a}^{(1)}, \ldots, \mathbf{a}^{(k-1)}) \tag{2.26}$$

consists of k user specific vectors

$$\mathbf{a}^{(k)} = \mathbf{H}^{(k)} \mathbf{c}^{(k)} = (H_{0,0}^{(k)} c_0^{(k)}, H_{1,1}^{(k)} c_1^{(k)}, \ldots, H_{L-1,L-1}^{(k)} c_{L-1}^{(k)})^T$$

Spreading Codes

There are many spreading codes that can be used for the uplink and downlink signals. These codes vary for the synchronous and asynchronous transmission. Here, in MC-CDMA synchronous transmission, the Walsh codes are used.

2.2. Multiuser Detection Techniques

The multiuser detection had been divided in two categories. These categories are

1- Optimum detection

2- Sub-optimum detection

Here, few examples for both of them are discussed.

1- Optimum detection:

The optimum detection is the detection that selects the most likely sequence bits $\{b_k(n), 1 \le n \le N, 1 \le k \le K\}$ over the received signal $r(t)$ observed over the time interval $0 \le t \le NT + 2T$ [10]. In optimum multiuser detection technique, a posteriori (MAP) criterion or maximum likelihood (ML) criterion are the good tools.

One of the famous methods for the optimum multiuser detection is the Maximum Likelihood (ML) Detection. In here, two ML techniques are discussed. One is the Maximum Likelihood Sequence Estimate (MLSE) in which transmitted data sequence $\mathbf{d} = (d^{(0)}, d^{(1)}, \ldots, d^{(k-1)})^T$ is deduced [11]. The other is the Maximum Likelihood Symbol-by-Symbol Estimation (MLSSE) in which transmitted data symbol $d^{(k)}$ is deduced. All capable transmitted data symbol vectors are $\mathbf{d}_\mu$, $\mu = 0, \ldots, M^K - 1$, where the number of potential transmitted data symbol vectors is M^K. The M is potential realizations of $d^{(k)}$.

Maximum Likelihood Sequence Estimation (MLSE)

MLSE reduces error probability of the data sequence, i.e., the error probability of the data symbol vector. This probability is same as maximizing the dependent probability $\mathrm{P}(\mathbf{d}_\mu \mid \mathbf{r})$. $\mathrm{P}(\mathbf{d}_\mu \mid \mathbf{r})$ means that $\mathbf{d}_\mu$ was transmitted given the received vector $\mathbf{r}$. The $\mathbf{d}$ estimate resulted from MLSE is

$$\hat{\mathbf{d}} = \arg\max_{d_\mu} P(\mathbf{d}_\mu \mid \mathbf{r}) \qquad (2.27)$$

If consider a white Gaussian noise N_l, then (2.27) is same as calculating the data symbol vector $\mathbf{d}_\mu$ that results in the reduction of squared Euclidean distance

$$\Delta^2(\mathbf{d}_\mu \mathbf{.r}) = \| \mathbf{r} - \mathbf{A}\mathbf{d}_\mu \| \qquad (2.28)$$

where **A** is the system matrix discussed previously. The Euclidean distance is calculated among received and all attainable broadcasted sequences. The most probable broadcasted data vector is

$$\hat{\mathbf{d}} = \arg\min_{\mathbf{d}_\mu} \Delta^2(\mathbf{d}_\mu, \mathbf{r}) \tag{2.29}$$

There are total M^K squared Euclidean distances are calculated for data symbol vector $\hat{\mathbf{d}}$ prediction in MLSE.

Maximum likelihood Symbol by Symbol Estimation

The reduction in error probability of symbol is the basic theme of the MLSSE. This probability is same as maximizing the probability $P(\mathrm{d}_\mu^k \,|\, \mathbf{r})$ which means that d_μ^k was sent having received sequence **r**. The MLSSE estimated $d^{(k)}$ is

$$\hat{d}^{(k)} = \arg\max_{\mathbf{d}_\mu^{(k)}} P(\mathbf{d}_\mu^k \,|\, \mathbf{r}) \tag{2.30}$$

If N_l is the additive Gaussian noise then the most probable transmitted data symbol is

$$\hat{d}^{(k)} = \arg\max_{d_\mu^{(k)}} \sum_{\substack{\forall \mathbf{d}_\mu \text{ with same} \\ \text{realization of } \mathrm{d}_\mu^{(k)}}} \exp\left(-\frac{1}{\sigma^2}\Delta^2(\mathbf{d}_\mu . \mathbf{r})\right) \tag{2.31}$$

It has been seen from the above equation that the MLSSE is more complex as compare to MLSE.

2- Sub-optimum Detectors

In the above section, the observation is that intricacy of optimum detector increases exponentially with the increasing users [10]. Now, it will be seen that the intricacy of suboptimum detectors is less than the optimum detectors that grows linearly with number of users instead of the exponentially.

One of the famous sub-optimum detection techniques are the Block Linear equalization. The block linear equalization is the multiuser detection with low intricacy which requires the information of the system matrix **A** at receiver. There are further many abnormalities that are implemented to use this information at receiver for data detection. One is the Minimum Mean Square Error (MMSE) Block Linear Equalizer. In this thesis, the Linear Minimum Mean Square Error Block detectors are focused only. Therefore, the LMMSE [1] based technique are discussed here.

Linear MMSE Block Linear Equalizer

It is seen from section (2.1.3) that the transmitter of MC-CDMA transmits the infant data stream on distinct subcarriers by a known spreading code sometimes called user code in frequency domain [11][12]. There are distinct phase shifts and amplitude levels of all subcarriers in frequency selective fading channel. This reduces orthogonality among users and propagates the MAI. After Fast Fourier Transform (FFT) and the frequency de-interleaving, the equalization of received sequence by using the one tap equalizer per subcarrier to compensate for amplitude and phase distortion compensated by mobile radio channel. In order to compensate for the MAI, MMSE criterion applied independently to achieve better performance. LMMSE detection is bed on MMSE method pertained for each user which provides good performance.

The aim of the LMMSE detection is the reduction of mean square error among transmitted symbol b_k and the estimated symbol $\hat{b}_k$. Let the optimal weight vector is

$$\mathbf{W}_k^T = [W_k^0, W_k^1, ..., W_k^N] \tag{2.32}$$

where b_k is of size N . .There are total N weights in W_k^T .The k^{th} user estimated symbol is

$$\hat{b}_k = W_j^H \mathbf{R} = \mathbf{C}_k^H \mathrm{H}\mathbf{R} \tag{2.33}$$

where **C** is the spreading code for the k user, H is the impulse response and **R** is the received vector. The weight vector **W** is calculated differently in different MMSE algorithms. In the next chapter, the **W** for the CMOE detection and subspace based LMMSE detection will be discussed.

2.3 CMOE Detection

The blind detector and channel estimator based on CMOE for the MC-CDMA system will be formulated. As in the previous line, the word "Blind" had been used which means that there is no training sequence method in detection.

The Blind CMOE detector is based on RLS method to update the correlation matrix. After calculation of the correlation matrix, the channel can be estimated by solving the minimum eigen-vector of a data sub-matrix. Therefore, it is necessary to summarize RLS algorithm.

2.3.1 Recursive Least Squares Algorithm (RLS)

RLS algorithm is depicted like on the accession of the latest data the new estimate of the vector on iteration n is predicted by least squares estimate of the tap weight vector of the filter already calculated at iteration $n-1$ [13]. The RLS algorithm is given below.

Initialization

Initialization of the algorithm is done by setting

$$\hat{\mathbf{w}}(0) = \mathbf{0} \tag{2.34}$$

$$\mathbf{P}(0) = \delta^{-1}\mathbf{I} \tag{2.35}$$

with

$$\delta = \begin{pmatrix} \text{is the small positive constant for the high SNR} \\ \text{is the large positive constant for the low SNR} \\ \end{pmatrix}$$

where $\hat{\mathbf{w}}(0)$ is the desired tap weight vector at iteration zero and $\mathbf{P}(0)$ is the inverse correlation matrix for iteration zero. δ is the positive real number called the regularization parameter and $\mathbf{I}$ is the Identity matrix.

From now onward each instant is calculated for time, $n = 1, 2, 3, \ldots$

Gain Vector

$$\pi(n) = \mathbf{P}(\text{n-1})\mathbf{u}(\text{n}) \tag{2.36}$$

$$\mathbf{k}(\text{n}) = \frac{\pi(n)}{\lambda + \mathbf{u}^H(n)\pi(n)} \tag{2.37}$$

$\mathbf{k}(\text{n})$ is the gain M-by-1 vector. $\mathbf{k}(\text{n})$ help to refresh the gain vector value. λ is the positive constant but less than unity.

Filtering Operation

$$\xi(n) = d(n) - \hat{\mathbf{w}}^H(n-1)\mathbf{u}(n) \tag{2.38}$$

$\xi(n)$ is the priori estimation error. It helps in doing the filtering operation of the algorithm. $\hat{\mathbf{w}}^H(n-1)\mathbf{u}(n)$ represents an estimate of the desired response *d(n).*

Adaptive Operation

$$\hat{\mathbf{w}}(n) = \hat{\mathbf{w}}(n-1) + \mathbf{k}(n)\xi^*(n) \tag{2.39}$$

$\hat{\mathbf{w}}(n)$ is the desired tap weight vector which explains that operation of the algorithm which is adaptive, though which the refreshing of tap weight vector is done by increasing its previous value equal to complex conjugate product of the apriori estimation of channel error $\xi(n)$ and irregular time gain vector **k**(n).

Inverse correlation matrix

$$\mathbf{P}(\mathrm{n}) = \lambda^{-1}\mathbf{P}(\mathrm{n}\text{-}1) - \lambda^{-1}\mathbf{k}(n)\mathbf{u}^{H}(n)\mathbf{P}(\mathrm{n}\text{-}1) \qquad (2.40)$$

$\mathbf{P}(\mathrm{n})$ is termed as the inverse correlation matrix. It helps to enable the self updating of the gain vector.

2.3.2 CMOE Detection and Channel Estimation

In CMOE detection, "the overall variance of the receiver output is minimized while conserving the desired signals with the unit gain". For covariance, the sub fix will be declined from signal model in MC-CDMA system design. The assumption is that desired user is zero. The working of the CMOE detector is as follow:

Weight optimization

$$\mathbf{w}_0 = (\mathbf{d}_0^{\mathrm{H}}(n)\mathbf{R}_{\mathrm{y}}^{-1}(n)\mathbf{d}_0(n))^{-1}\mathbf{R}_{\mathrm{y}}^{-1}(n)\mathbf{d}_0(n) \qquad (2.41)$$

The above equation is the solution from the constraint optimization problem in [3]. Where $\mathbf{d}_0(n)$ is the effective signature waveform of the desired user is $\mathbf{d}_0(n) = \mathbf{C}_0\mathbf{h}_0(n)$ with $\mathbf{C}_0 = diag[C_{0,0}, C_{0,1}, ..., C_{0,M-1}]$ and $\mathbf{h}_0(n)$ is the impulse response of the desired user. $\mathbf{R}_{\mathrm{y}}(n)$ is the covariance matrix obtained by

$$\mathbf{R}_{\mathrm{y}}(n) = \lambda\mathbf{R}_{\mathrm{y}}(n-1) + \mathbf{y}(n)\mathbf{y}^{H}(n) \qquad (2.42)$$

The forgetting factor has values $0<\lambda<1$ and $\mathbf{y}(n)$ is the baseband MC-CDMA signal.

Minimum Output Variance

The minimum output variance of the detector can be given by [8]

$$P_{\min} = \mathbf{w}_0^{H}\mathbf{R}_{y}(n)\mathbf{w}_0 = (\mathbf{d}_0^{H}(n)\mathbf{R}_{y}^{-1}(n)\mathbf{d}_0(n))^{-1} \qquad (2.43)$$

$$P_{\min} = \mathbf{w}_0^H \mathbf{R}_y(n)\mathbf{w}_0 = (\mathbf{d}_0^H(n)\mathbf{R}_y^{-1}(n)\mathbf{d}_0(n))^{-1} \quad (2.43)$$

Effective Signature Waveform Estimation

The (2.43) will be maximized with respect to $\mathbf{d}_0(n)$ in order to predict the effective signature waveform $\mathbf{d}_0(n)$. Its effect is that the signal components at the output of receiver are exploited after suppression in interference. The effective signature waveform estimation equation is given by

$$\hat{\mathbf{d}}_0(n) = \arg \min_{\mathbf{d}_0(n)} \mathbf{d}_0^H(n)\mathbf{R}_y^{-1}(n)\mathbf{d}_0(n) \quad (2.44)$$

Channel Impulse Response

Suppose $g_0(n)$ represent the impulse response of channel for the first user with L paths. $g_0(n)$ has the impulse response $\mathbf{h}_0(n) = \mathbf{F}_m g_0(n)$ of the desired user. $\mathbf{F}_m$ is the $M \times L$ matrix. The $\mathbf{F}_m$ has the entries

$$[\mathbf{F}_m]_{m,l} = \exp(-j2\pi((P_M + p)l)/(M)), \qquad m = 0,\ldots,M-1, l = 0,\ldots,L-1$$

Hence the minimizing of the waveform estimation equation is

$$\hat{g}_0(n) = \arg \min_{\hat{g}_0(n) \in C^{L\times 1}} g_0^H(n)\boldsymbol{\Omega} g_0(n) \quad (2.45)$$

where

$$\boldsymbol{\Omega} = \mathbf{F}_m^H \mathbf{C}_0^H \mathbf{R}_y^{-1}(n)\mathbf{C}_0\mathbf{F}_m$$

and $\hat{g}_0(n)$ is the channel estimation associated with the smallest Eigen value of $\boldsymbol{\Omega} \in \mathbf{C}^{L\times L}$.

There are many methods to calculate the minimum Eigen vector of the $\mathbf{\Omega}$, such as Singular Value Decomposition (SVD), Eigen Value Decomposition (EVD) etc. But it has expensive calculations to keep the track of the desired Eigen-vector of $\mathbf{\Omega}$. However, the inverse iteration method is adopted for the calculation of minimum Eigen vector of matrix $\mathbf{\Omega}$.The inverse iteration method will discussed further in the next chapter. EVD is used in the subspace tracking that is also described briefly at the end of the chapter.

2.4 Subspace Based LMMSE Detection

In the presence of white Gaussian noise, the subspace based MMSE detectors has healthier performance than the typical CMOE detector. A particular concentration to the subspace based MMSE detector is paid. In the subspace tracking LMMSE detector, Lagrange multiplier is adopted for the calculation of weights. Therefore, it is necessary to discuss the Lagrange multiplier.

2.4.1 Lagrange Multiplier

Optimization determines the values of some variables that minimizes or maximizes an index of performance or the cost function [13]. These performances or cost functions combines the important properties of a system in to a single real valued number. The optimization has further two types. These are constrained and unconstrained which depends on whether the variables are also required to satisfy the side equations or not. The Lagrange multiplier is used for solving the complex version of the constrained optimization problem. Consider a case when the problem involves multiple side equations. Now, we are starting with the Lagrange multiplier.

Suppose a real valued function $f(\mathbf{w})$ that will be minimized. $f(\mathbf{w})$ is a quadratic function of a parameter vector $\mathbf{w}$, subject to the constraint

$$\mathbf{w}^H\mathbf{s}_k = \mathbf{g}_k,\ \ k = 1,2,3,...,K \tag{2.46}$$

with $\mathbf{s}$ a prescribed vector and $\mathbf{g}$ is a complex constant. The constraint by introducing a new function $c_k(\mathbf{w})$ is redefine here that is linear in $\mathbf{w}$. This is given by

$$\begin{aligned} c_k(\mathbf{w}) &= \mathbf{w}^H\mathbf{s}_k - \mathbf{g} \\ &= 0 + j0 \end{aligned} \tag{2.47}$$

Generally, the vectors $\mathbf{w}$, $\mathbf{s}$ and the function $c(\mathbf{w})$ are complex. It has been supposed that the problem is the minimization, the constrained optimization problem can be stated as following: "Minimize a real valued function $f(\mathbf{w})$ with respect to the constraint $c_k(\mathbf{w})$ =0+j0". The method of Lagrange multiplier changes the problem of the constrained minimization just defined in one of the unconstrained minimization by the introducing Lagrange multiplier.

Firstly, real function $f(\mathbf{w})$ is used and the complex constraints function $c_k(\mathbf{w})$ for defining a new real valued function. That is

$$h(\mathbf{w}) = f(\mathbf{w}) + \lambda_1 \operatorname{Re}[c_k(\mathbf{w})] + \lambda_2 \operatorname{Im}[c_k(\mathbf{w})] \tag{2.48}$$

where λ_1 and λ_2 are real Lagrange multipliers and

$$c_k(\mathbf{w}) = \operatorname{Re}[c_k(\mathbf{w})] + j\operatorname{Im}[c_k(\mathbf{w})] \tag{2.49}$$

Now defining a complex Lagrange multiplier

$$\lambda = \lambda_1 + j\lambda_2 \tag{2.50}$$

Rewriting the equation (2.48) as

$$h(\mathbf{w}) = f(\mathbf{w}) + \operatorname{Re}[\lambda^* c_k(\mathbf{w})] \tag{2.51}$$

The asterisk represents the complex conjugation. Now, the $h(\mathbf{w})$ can be minimized with respect to $\mathbf{w}$. This can be done by setting the conjugate derivative $\partial h / \partial \mathbf{w}^*$ equal to the null vector.

$$\frac{\partial f}{\partial \mathbf{w}^*} + \frac{\partial}{\partial \mathbf{w}^*}\left(\mathrm{Re}[\lambda^* c_k(\mathbf{w})]\right) = 0 \qquad (2.52)$$

The above equation will lead to the minimized value of the $h(\mathbf{w})$.

2.4.2 Subspace Based LMMSE Detector

The assumption is that each user has the independent data stream. The noise in this system is the white Gaussian noise. The working of the subspace based LMMSE Detector is as following.

Correlation Matrix

The correlation matrix $\mathbf{R}_y$ is given by

$$\mathbf{R}_y = E\{\mathrm{y(n)y^H(n)}\} = \sum_{k=0}^{K-1} P_\kappa \mathbf{d}_k \mathbf{d}_k^H + \sigma^2 \mathbf{I}_{M\times M} \qquad (2.53)$$

where, y(n) is the MC-CDMA baseband signal, P_κ is the chip energy of the k^{th} user, $\mathbf{d}_k$ is the effective signature of the k^{th} user.

The correlation matrix $\mathbf{R}_y$ can also written by using the Eigen-value decomposition. But, the need is to discuss the **Eigen-value decomposition** first.

By [14], Eigen-value problem for $m \times m$ matrix **A** is given by

$$\mathbf{Ax} = \lambda \mathbf{x} \qquad (2.54)$$

with Eigen-values λ and Eigen-vector $\mathbf{x}$ (nonzero). Eigen-value decomposition of A is defined by

$$\mathbf{A} = \mathbf{X\Lambda X}^{-1} \;\; or \;\; \mathbf{AX} = \mathbf{\Lambda X} \qquad (2.55)$$

with Eigen-vectors as columns of $\mathbf{X}$ and Eigen-values on diagonals of Λ.

Now coming back to the Correlation matrix $\mathbf{R}_y$ using the Eigen-value decomposition in (2.54) that is

$$\mathbf{R}_y = \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{U}_s^H + \mathbf{U}_n \mathbf{\Lambda}_n \mathbf{U}_n^H \qquad (2.57)$$

with $\Lambda_s = diag(\lambda_1, \lambda_2, ..., \lambda_K)$ that contains K largest Eigen-values, $\mathbf{U}_s \in \mathbf{C}^{M\times K}$ have the K signal Eigen-vectors which whirled the signal subspace and $\mathbf{U}_n \in \mathbf{C}^{M\times(M-K)}$ have $M-K$ noise Eigen-vectors which whirled the noise subspace.

Constrained Optimization Problem

The first user subspace based LMMSE detector is formulated as the following optimization problem

$$J(\mathbf{w}) = \min_{w} E\{|\sqrt{P_0}b_0(n) - \mathbf{w}^{\mathrm{H}}\mathbf{y}(n)|^2\} \text{ such that } \mathbf{w}^{\mathrm{H}}\mathbf{d}_0 = 1 \quad (2.58)$$

By Lagrange multiplier (2.4.1), the solution of the (2.58) is written as

$$\mathbf{w} = (\mathbf{U}_s\boldsymbol{\Lambda}_s^{-1}\mathbf{U}_s^H\mathbf{d}_0)/(\mathbf{d}_0^{\mathrm{H}}\mathbf{U}_s\boldsymbol{\Lambda}_s^{-1}\mathbf{U}_s^H\mathbf{d}_0) \quad (2.59)$$

Channel Vector Estimation

By the orthogonality between the noise and signal subspaces

$$\mathbf{U}_{\mathrm{n}}^{\mathrm{H}}\mathbf{d}_0 = \mathbf{U}_{\mathrm{n}}^{\mathrm{H}}\mathbf{C}_0\mathbf{F}_m\mathbf{g}_0 = 0 \quad (2.60)$$

The estimate of channel vector $\mathbf{g}_0$ is given by using the least square approach

$$\hat{\mathbf{g}}_0 = \arg\min_{\mathbf{g}_0 \in C^{L\times 1}} \mathbf{g}_0^H\boldsymbol{\Omega}_0\mathbf{g}_0 \quad (2.61)$$

where

$$\boldsymbol{\Omega}_0 = \mathbf{F}_m^H\mathbf{C}_0^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{C}_0\mathbf{F}_m \quad (2.62)$$

$\hat{\mathbf{g}}_0$ is the Eigen-vector associated with the minimum Eigen-value of the matrix $\boldsymbol{\Omega}_0$.

There are further many algorithms that are used for the signal and noise subspace tracking like PAST algorithm [15], Bi-SVD algorithm [16] and QR-inverse iteration method [17]. But, these are not the part of this thesis.

Chapter 3
System Design

Chapter 3

System Design

3.1 Introduction

Like the CDMA system, MC-CDMA system is a system bounded to the interference. It is sensible towards the multiple access interference (MAI) because it proceeds by the use of the similar frequency band between several users simultaneously. Therefore, the MAI mitigation is necessary for the good MC-CDMA system. The LMMSE multiuser detectors are good one to come over the MAI [18]-[20].

Here the aim is to design such LMMSE detectors for the MC-CDMA system in which the MAI is mitigated. The two types LMMSE detector for the MC-CDMA are focused. One is the CMOE Detector and the other one is the subspace based LMMSE detector for the MC-CDMA. As purposed techniques are for the MC-CDMA, therefore the sequence of the description for this chapter is as follow: "After brief problem statement the MC-CDMA system design is focused. The MC-CDMA system design is followed by the proposed CMOE detection method for the MC-CDMA. In the end the system design for the subspace based LMMSE detector for MC-CDMA system is given".

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3.2 Query Statement

Since in MC-CDMA system, multiple user shares the same frequency band at the same time. This gives arise to the MAI. MAI results in reduction of data rate and channel efficiency.

The target is to design a MC-CDMA system that over come the MAI in presence of white Gaussian noise. In presence of white Gaussian noise, the detector that is preferred for the MC-CDMA system is the subspace based LMMSE Detector as it has stronger performance than typical CMOE detector.

3.3 System Design of the MC-CDMA system

Suppose a synchronous MC-CDMA transmission system for K users. The uplink mode transmission is proceeding from the terminal to base station. The block diagram of transmitter for the MC-CDMA is given below Figure.3.1

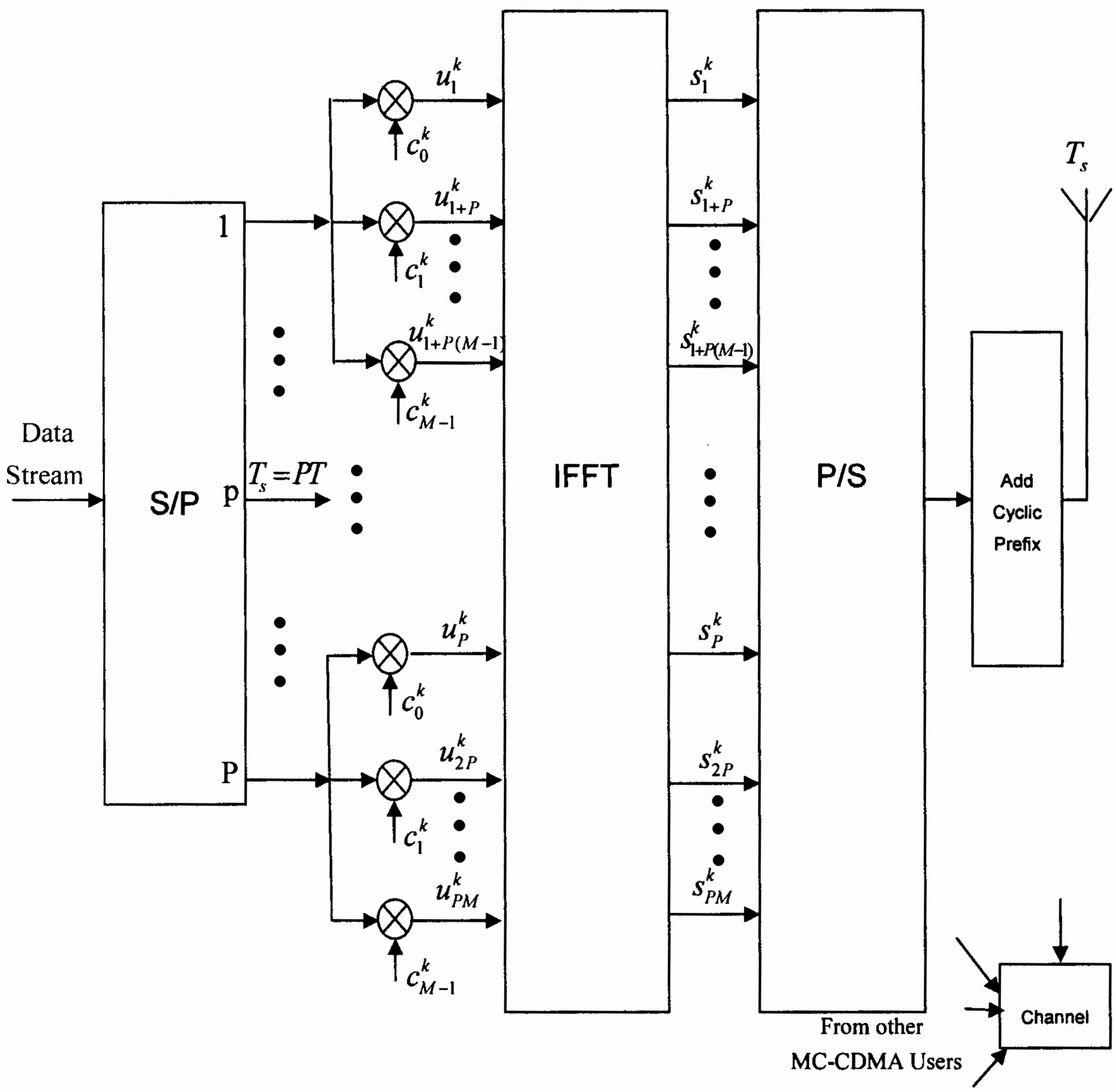


Figure 3.1 MC-CDMA Transmitter

The data stream of the user *1* at i^{th} time is

$$\mathbf{b}^1(i) = [b_0^1(i), b_1^1(i), b_2^1(i), \ldots, b_{p-1}^1(i)] \tag{3.1}$$

The data stream of the user *2* at i^{th} time is

$$\mathbf{b}^2(i) = [b_0^2(i), b_1^2(i), b_2^2(i), \ldots, b_{p-1}^2(i)] \quad (3.2)$$

Similarly, the data stream for the user *P* at i^{th} time is

$$\mathbf{b}^P(i) = [b_0^P(i), b_1^P(i), b_2^P(i), \ldots, b_{p-1}^P(i)] \quad (3.3)$$

The data stream of *k* users at the i^{th} time is

$$\mathbf{b}(i) = [\mathbf{b}^1(i), \mathbf{b}^2(i), \mathbf{b}^3(i), \ldots, \mathbf{b}^K(i)] \quad (3.4)$$

Firstly, the data stream of the k^{th} user is first converted into *P* alongside data sequences $\mathbf{b}(i)$ at time *i*. Suppose T and T_s are the period of the symbols before and after the S/P conversion with the $T_s = PT$. If the period of symbol is T_s at the subcarrier is larger as compare to the channel multipath delay spread than every subcarrier nearly suffers the flat fading.

After Serial to Parallel conversion, each output is spread using the spreading code $\mathbf{c}_k = [c_0^k, c_1^k, \ldots, c_{M-1}^k]^T, k = 0, \ldots, K-1$. After serial to parallel conversion, the data chips are converted into the M subcarriers.

The output of user *1* after spreading with the $\mathbf{c}_k$ is

$$\begin{aligned}
&b_0^1(i)[c_0^1, c_1^1, \ldots, c_{M-1}^1]^T = [b_0^1 c_0^1, b_0^1 c_1^1, \ldots, b_0^1 c_{M-1}^1]^T \\
&b_1^1(i)[c_0^1, c_1^1, \ldots, c_{M-1}^1]^T = [b_1^1 c_0^1, b_1^1 c_1^1, \ldots, b_1^1 c_{M-1}^1]^T \\
&\vdots \\
&b_{p-1}^1(i)[c_0^1, c_1^1, \ldots, c_{M-1}^1]^T = [b_{p-1}^k c_0^1, b_{p-1}^k c_1^1, \ldots, b_{p-1}^k c_{M-1}^1]^T
\end{aligned}$$

The output of user *k* after spreading with the $\mathbf{c}_k$ is

$$\begin{aligned}
&b_0^k(i)[c_0^k, c_1^k, \ldots, c_{M-1}^k]^T = [b_0^k c_0^k, b_0^k c_1^k, \ldots, b_0^k c_{M-1}^k]^T \\
&b_1^k(i)[c_0^k, c_1^k, \ldots, c_{M-1}^k]^T = [b_1^k c_0^k, b_1^k c_1^k, \ldots, b_1^k c_{M-1}^k]^T \\
&\cdot \\
&\cdot \\
&\cdot \\
&b_{p-1}^k(i)[c_0^k, c_1^k, \ldots, c_{M-1}^k]^T = [b_{p-1}^k c_0^k, b_{p-1}^k c_1^k, \ldots, b_{p-1}^k c_{M-1}^k]^T
\end{aligned}$$

$\mathbf{F}_1 \in \mathbf{C}^{N \times N}$ is Fourier matrix that performs Inverse Discrete Fourier Transform (IDFT). The $(u,v)^{th}$ element is $\exp(j2\pi uv/N)/N$. The k^{th} user 1^{st} element at the interval i^{th} is

$$\begin{aligned} s_0^K(i) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} u_0^k e^{jwi} di \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} b_0^k(i) c_0^k e^{jwi} di \\ &= \frac{c_0^k}{2\pi}\int_{-\infty}^{\infty} b_0^k(i) e^{jwi} di \\ &= \frac{c_0^k}{2\pi} \end{aligned} \qquad (3.8)$$

Similarly, $s_1^K,\ldots,s_{PM-1}^k$ can be formulated. The $\mathbf{s}^k(i)$ vector can be shown as

$$\mathbf{s}^k(i) = \begin{bmatrix} s_0^K(i), s_{0+P}^K(i), \ldots, s_{P(M-1)}^K(i), \\ s_1^K(i), s_{1+P}^K(i), \ldots, s_{1+p(M-1)}^K(i), \\ \ldots, s_{P-1}^K(i), s_{2P-1}^K(i), \ldots, s_{PM-1}^K(i) \end{bmatrix}_{N\times 1} \qquad (3.9)$$

The equation (3.7) can be derived from the above description.

There are L received paths for the user signal passing through a frequency selective channel. The channel is presented by the $(L\text{-}1)^{th}$ Finite Impulse Response (FIR) filter [5]. The channel impulse response is given by

$$h_k(t) = \sum_{l=0}^{L-1} g_{k,l}(t)\delta(t-\tau_{k,l}) \qquad (3.10)$$

where user index is k and $g_{k,l}(t)$ is the l^{th} path gain. The $g_{k,l}(t)$ is independent complex Gaussian random process with zero mean and variance $\sigma_{k,l}^2$. The l^{th} path propagation delay is $\tau_{k,l}$. The power delay profile is the supporter $\sigma_{k,l}^2$ $(l = 0,\ldots,L-1)$.

ISI also arises at this point. Therefore, it is necessary to overcome the ISI at this point. To eliminate the effect of the ISI, a cyclic prefix of N_g samples is further to each symbol, where $N_g \geq L-1$. After adding the cyclic prefix the MC-CDMA signal is transmitted to the channel.

The Figure 3.2 represents the configuration of the MC-CDMA receiver.

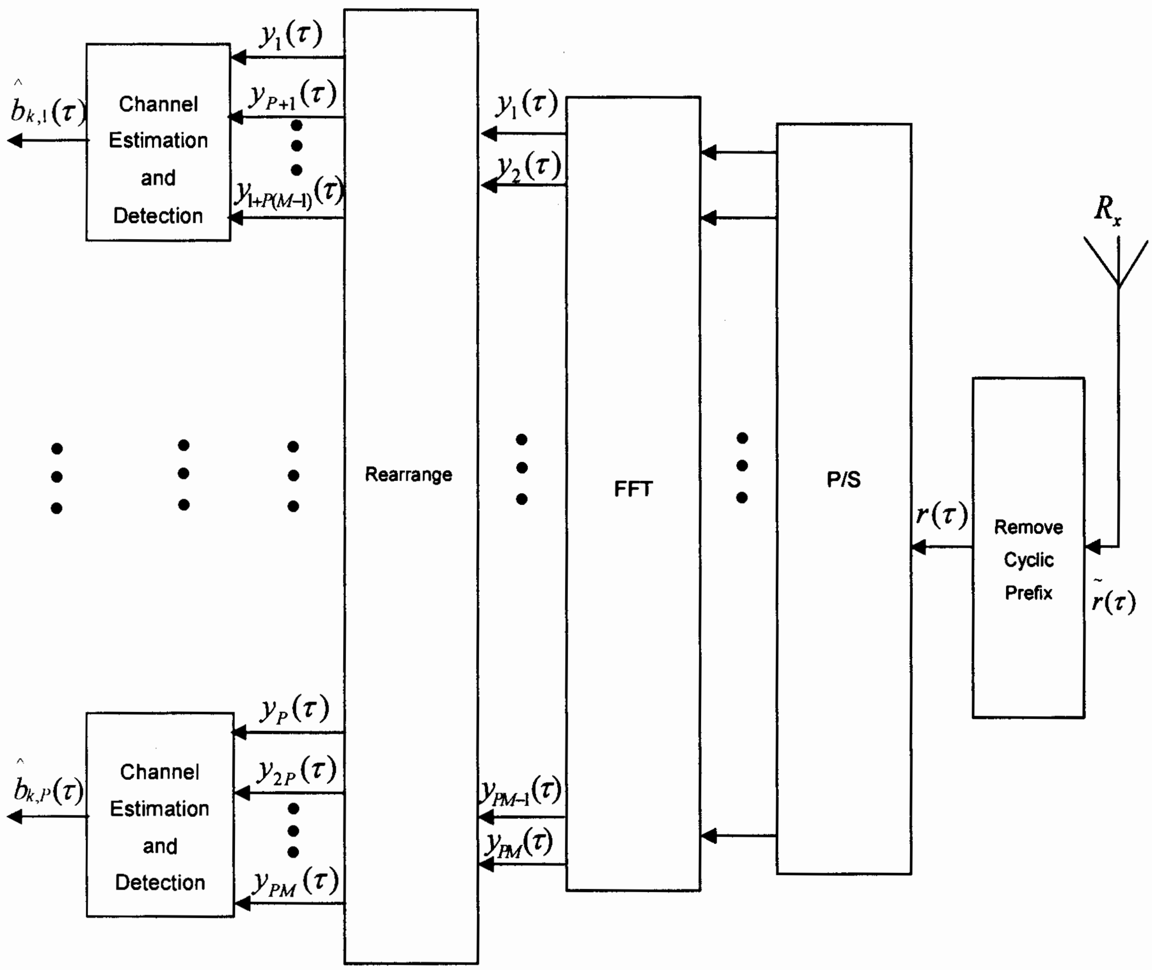


Figure 3.2 MC-CDMA Receiver

The supposition is that the communication channel remains fixed during the MC-CDMA symbol. The signal sampling rate is $(N+N_g)/T_s$ at the receiver. The k^{th} user received signal at the τth sample is

$$\begin{aligned}\hat{\mathbf{r}}(\tau) &= \int h_k(t)\hat{\mathbf{s}}_k(t-l)dt + \hat{\boldsymbol{\eta}}(\tau) \\ &= \int \sum_{l=0}^{L-1} g_{k,l}(t)\delta(t-\tau_{k,l})\hat{\mathbf{s}}_k(t-l)dt + \hat{\boldsymbol{\eta}}(\tau) \qquad (3.11)\end{aligned}$$

Therefore, the received baseband signal of the k^{th} user at the τth sample is

$$\hat{\mathbf{r}}_k(\tau) = \int \sum_{l=0}^{L-1} \sqrt{P_k}\, g_{k,l}(\tau)\hat{\mathbf{s}}_k(\tau-l)dt + \boldsymbol{\eta}_k \qquad (3.12)$$

where $\tau_{k,l} = t$ and $\int \delta(t - \tau_{k,l})dt = 1$. Now writing the received baseband signal at the τth sample is

$$\hat{\mathbf{r}}(\tau) = \sum_{k=0}^{K-1} \hat{\mathbf{r}}_k(\tau)$$
$$= \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} \sqrt{P_k}\, g_{k,l}(\tau)\, \hat{\mathbf{s}}_k(\tau - l)dt + \hat{\boldsymbol{\eta}}(\tau) \qquad (3.13)$$

where $\hat{\boldsymbol{\eta}}(\tau) = [\eta_1 \quad \eta_2 \quad \ldots \quad \eta_k]^T$ is the zero mean white Gaussian noise with variance σ_v^2, $g_{k,l}(\tau)$ is used for the complex envelop at the l^{th} path, $\hat{\mathbf{s}}_k(\tau) \in \mathbf{C}^{N_{tot} \times 1}$, $N_{tot} = N + N_g$ is the transmitted signal vector after with the cyclic prefix, the chip energy P_k is taken as unit in implementation.

Let the time and frequency synchronization has been attained at receiver and ISI has been eliminated due to the condition $N_g \geq L - 1$. The samples relevant to cyclic prefix are eliminated. After removing the cyclic prefix the synchronized τth MC-CDMA symbol is given by

$$\mathbf{r}(\tau) = \sum_{k=0}^{K-1} \sqrt{P_k}\, \mathbf{G}_k(\tau)\mathbf{s}_k(\tau) + \boldsymbol{\eta}(\tau) \qquad (3.14)$$

where

$$\mathbf{s}_k(\tau) = \left[\hat{\mathbf{s}}_k(\tau N_{tot} + N_g), \ldots, \hat{\mathbf{s}}_k(\tau N_{tot} + N_{tot} - 1) \right]^T,$$

$$\boldsymbol{\eta}(\tau) = \left[\hat{\boldsymbol{\eta}}(\tau N_{tot} + N_g), \ldots, \hat{\boldsymbol{\eta}}(\tau N_{tot} + N_{tot} - 1) \right]^T$$

and $\mathbf{G}_k(\tau)$ is the Toeplitz matrix of the form given below

$$\mathbf{G}_k = \begin{bmatrix} g_{k,0} & 0 & 0 & \dots & g_{k,l} \\ g_{k,l} & g_{k,0} & 0 & \dots & g_{k,2} \\ g_{k,2} & g_{k,l} & g_{k,0} & \dots & g_{k,3} \\ . & . & . & & . \\ . & . & . & . & . \\ . & . & . & & . \\ 0 & 0 & 0 & \dots & g_{k,0} \end{bmatrix}$$

∵ If all the elements on its main diagonal are equal and the elements on the any diagonal parallel to the main diagonal are also equal, than the matrix is called **Toeplitz Matrix**.

The fast Fourier transform of size N is executed at the detector. Suppose $\mathbf{F} \in \mathbf{C}^{N \times N}$ be the Fourier matrix. The $(u,v)th$ element of F is $\exp(-j2\pi uv/N)$. The baseband signal in the frequency domain can than be written in matrix notation as

$$\begin{aligned} \mathbf{y}(\tau) &= \mathbf{F}\,\mathbf{r}(\tau) \\ &= \mathrm{F}\left[\sum_{k=0}^{K-1} \sqrt{P_k}\,\mathbf{G}_k(\tau)\mathbf{s}_k(\tau) + \boldsymbol{\eta}(\tau)\right] \end{aligned} \qquad (3.15)$$

As $\mathbf{s}_k(\tau) \underset{F}{\overset{F^{-1}}{\rightleftarrows}} \mathbf{u}_k(\tau)$ *and* $\boldsymbol{\eta}(\tau) \underset{F^{-1}}{\overset{F}{\rightleftarrows}} \mathbf{n}(\tau)$, $\mathbf{G}_k(\tau)$ is Toeplitz matrix given above. The

$\mathbf{H}_k(\tau) = diag\left[h_{k,0}(\tau),\dots,h_{k,N-1}(\tau)\right]$,

where

$$h_{k,n}(\tau) = \sum_{l=0}^{L-1} g_{k,l}(\tau)\exp(-j2\pi(nl)/N),\ n = 0,\dots,N-1$$

or

$$h_n^k(\tau) = \sum_{l=0}^{L-1} g_l^k(\tau)\exp(-j2\pi(nl)/N),\ n = 0,\dots,N-1 \qquad (3.16)$$

The $h_0^k(\tau), h_1^k(\tau),\dots,h_{N-1}^k$ can be written from the (3.16) which is

$$h_0^k(\tau) = \sum_{l=0}^{L-1} g_l^k(\tau)\exp(-j2\pi((0)l)/N) = \sum_{l=0}^{L-1} g_l^k(\tau) \qquad (3.17)$$

$$h_1^k(\tau) = \sum_{l=0}^{L-1} g_l^k(\tau)\exp(-j2\pi l / N)$$

$$\vdots$$

$$h_{N-1}^k(\tau) = \sum_{l=0}^{L-1} g_l^k(\tau)\exp(-j2\pi(N-1)l / N)$$

As $\mathbf{G}_k(\tau) \xrightarrow{FFT} \mathbf{H}_k(\tau)$ .Then, the (3.15) becomes

$$\mathbf{y}(\tau) = \sum_{k=0}^{k-1} \sqrt{P_k}\mathbf{H}_k(\tau)\mathbf{u}_k(\tau) + \mathbf{n}(\tau) \tag{3.18}$$

$\mathbf{n}(\tau)$ is the zero mean white Gaussian noise with variance σ_n^2.

Let p^{th} data stream is transmitted by the every user. Therefore, the $M \times 1$ data vector $\mathbf{y}_p(\tau) = \left[y_p(\tau), y_{P+p}(\tau), \ldots, y_{P(M-1)+p}(\tau)\right]^T$ relevant to the symbol b_p^k is represented as

$$\begin{aligned}\mathbf{y}_p(\tau) &= \sum_{k=0}^{K-1} \sqrt{P_k}\mathbf{C}_k\mathbf{h}_p^k(\tau)b_p^k(\tau) + \mathbf{n}_p(\tau) \\ &= \sum_{k=0}^{K-1} \sqrt{P_k}\mathbf{d}_p^k(\tau)b_p^k(\tau) + \mathbf{n}_p(\tau)\end{aligned} \tag{3.19}$$

where $\mathbf{h}_p^k(\tau) = [h_p^k(\tau), h_{P+p}^k(\tau), \ldots, h_{P(M-1)+p}^k(\tau)]^T$ the channel coefficient at the subcarriers $(Pm + p)th$, $\mathbf{C}_k = diag\{c_0^k, c_1^k, \ldots, c_{M-1}^k\}$ is the code matrix, $\mathbf{d}_p^k(\tau) = \mathbf{C}_k\mathbf{h}_p^k(\tau)$ is the k^{th} user effective signature and $\mathbf{n}_p(\tau)$ is the zero mean white Gaussian noise with variance σ_n^2.

3.4 Blind CMOE Detector

The basic theme of the CMOE detector is that variance of the receiver output is minimized on the whole while saving the required signals with a gain equal to unity. The sub fix P in the signal model (3.19) is dropped for easy understanding. The assumption is that desired user is zero. The constrained optimization problem in [8] is used to formulate the CMOE detector as

$$\min_{\mathbf{w}} \sum_{i=1}^{n} \lambda^{n-i} \,|\, \mathbf{w}^{\mathrm{H}}(n)\mathbf{y}(i) \,|^2, \text{ such that } \mathbf{w}^{\mathrm{H}}(n)\mathbf{d}_0(n) = 1 \qquad (3.20)$$

where the forgetting factor has the value $0 < \lambda < 1$, $\mathbf{w}(\mathrm{n})$ is the weight vector at n^{th} time and the effective waveform is $\mathbf{d}_0(n) = \mathbf{C}_0\mathbf{h}_0(n)$ of preferred user.

The CMOE detector minimizes the cost function over b_0. The $|\, \mathbf{w}^{\mathrm{H}}(n)\mathbf{y}(i) \,|^2$ of (3.20) has been used which can also written as

$$MOE[\mathbf{b}_0] = E[< \mathbf{y}_0, \hat{\mathbf{d}}_0 + \mathbf{b}_0 >^2] \quad \therefore \mathbf{w}_0 = \hat{\mathbf{d}}_0 + \mathbf{b}_0$$

then

$$MOE[\mathbf{b}_0] = E[< \mathbf{y}_0, \mathbf{w}_0 >^2] \qquad \therefore < x, y >= \int_0^T x(t)y(t)dt \qquad (3.21)$$

where $\| \mathbf{b}_0 \|^2 = \chi \; and < \hat{\mathbf{d}}_0, \mathbf{b}_0 >= 0$ means that both of them are orthogonal to each other and $\int_0^T \mathbf{b}_0(t)\hat{\mathbf{d}}_0(t)dt = 0$. The optimization is now expressed in terms of $\mathbf{w}_0 = \hat{\mathbf{d}}_0 + \mathbf{b}_0$. The derivation has been taken w.r.t $\mathbf{w}_0$ of the associated Lagrange optimal condition that is

$$E[< \mathbf{y}_0, \mathbf{w}_0 > \mathbf{y}] + v_1\mathbf{w}_0 - v_2\hat{\mathbf{d}}_0 = 0 \qquad (3.22)$$

where v_1 & v_2 are the Lagrange multipliers chosen such that

$$\| \mathbf{w}_0 \|^2 = \| \hat{\mathbf{d}}_0 \|^2 + \| \mathbf{b}_0 \|^2 \qquad (3.23)$$

As given that $< \mathbf{w}_0^H, \hat{\mathbf{d}}_0 >= 1$, therefore

$$\int \mathbf{w}_0^{\mathrm{H}}\mathbf{d}_0(t)dt = 1$$

$$\int (\hat{\mathbf{d}}_0^H + \mathbf{b}_0^H)\mathbf{d}_0(t)$$

$$\int | \mathbf{d}_0(t) |^2 dt = 1$$

and $\|\mathbf{b}_0\|^2 = \chi$. So (3.23) can be written as

$$\|\mathbf{w}_0\|^2 = \chi + 1 \tag{3.24}$$

Let b_k bits are uncorrelated, the proceeding condition can be written as

$$\sum_{k=1}^{K} \mathbf{A}_k^2 < \mathbf{w}_0, \mathbf{d}_k > \mathbf{d}_k + (\nu_1 + \sigma^2)\mathbf{w}_0 - \nu_2 \hat{\mathbf{d}}_0 = 0 \tag{3.25}$$

where $\mathbf{A}$ is the outer product matrix such that $\mathbf{A} = \sum_{k=1}^{K} \mathbf{A}_k^2 \mathbf{d}_0 \mathbf{d}_0^T$. Now again writing the (3.25)

$$\mathbf{A} = \sum_{k=1}^{K} \mathbf{A}_k^2 \mathbf{d}_0 \mathbf{d}_0^T$$

$$\mathbf{A}\mathbf{w}_0 + (\nu_1 + \sigma^2)\mathbf{w}_0 - \nu_2 \hat{\mathbf{d}}_0 = 0$$

$$\text{let } \gamma = \nu_1 + \sigma^2, \text{then}$$

$$\mathbf{A}\mathbf{w}_0 + \gamma \mathbf{w}_0 - \nu_2 \hat{\mathbf{d}}_0 = 0$$

$$\mathbf{w}_0(\mathbf{A} + \gamma \mathbf{I}_N) = \nu_2 \hat{\mathbf{d}}_0$$

$$\mathbf{w}_0 = (\mathbf{A} + \gamma \mathbf{I}_N)^{-1} \nu_2 \hat{\mathbf{d}}_0 \tag{3.26}$$

$\mathbf{w}_0$ in (3.26) is the optimum weights that are orthogonal to all other than $\mathbf{d}_0$, $\mathbf{I}_N$ is the $N \times N$ identity matrix. The ν_2 is given by

$$\nu_2 = \frac{1}{\hat{\mathbf{d}}_0^H (\mathbf{A} + \gamma \mathbf{I}_N)^{-1} \hat{\mathbf{d}}_0} \tag{3.27}$$

The (3.26) can be written as

$$\mathbf{w}_0 = \frac{(\mathbf{A} + \gamma \mathbf{I}_N)^{-1} \nu_2 \hat{\mathbf{d}}_0}{\hat{\mathbf{d}}_0^H (\mathbf{A} + \gamma \mathbf{I}_N)^{-1} \hat{\mathbf{d}}_0} \tag{3.28}$$

ν_2 is resulted from the condition $< \mathbf{w}_0^H, \hat{\mathbf{d}}_0 > = 1$. The minimum value related to $\xi_{\min}$ output energy is get by taking inner product of (3.25) with $\mathbf{w}_0$

$$\xi_{\min} = \nu_2 - \nu_1(1+\chi) \tag{3.29}$$

As $<\mathbf{w}_0, \hat{\mathbf{d}}_0> = 1$. The linear product of (3.25) with w_0 gives

$$\sum_{k=1}^{K} \mathbf{A}_k^2 < \mathbf{w}_0, \mathbf{d}_k > \mathbf{d}_k + (\nu_1 + \sigma^2)\mathbf{w}_0 - \nu_2 \hat{\mathbf{d}}_0 = 0$$

$$\sum_{k=1}^{K} \mathbf{A}_k^2 < \mathbf{w}_0, \mathbf{d}_k >< \mathbf{w}_0, \mathbf{d}_k > + (\nu_1 + \sigma^2) \|\mathbf{w}_0\|^2 - \nu_2 < \mathbf{w}_0, \hat{\mathbf{d}}_0 >= 0$$

$$\sum_{k=1}^{K} \mathbf{A}_k^2 < \mathbf{w}_0, \mathbf{d}_k >^2 + (\nu_1 + \sigma^2)(1+\chi) - \nu_2 = 0$$

$$\sum_{k=1}^{K} \mathbf{A}_k^2 < \mathbf{w}_0, \mathbf{d}_k >^2 + \sigma^2(1+\chi) = \nu_2 - \nu_1(1+\chi)$$

$$\xi_{out} = \nu_2 - \nu_1(1+\chi) \tag{3.30}$$

where $\xi_{out} = \sum_{k=1}^{K} \mathbf{A}_k^2 < \mathbf{w}_0, \mathbf{d}_k >^2 + \sigma^2(1+\chi)$ and the Surplus energy is $\chi = \| \mathbf{w}_0 \|^2 - 1$, then it can be written again by using the (3.28)

$$\chi = \left\| \frac{(\mathbf{A} + \gamma \mathbf{I}_N)^{-1} \nu_2 \hat{\mathbf{d}}_0}{\hat{\mathbf{d}}_0^H (\mathbf{A} + \gamma \mathbf{I}_N)^{-1} \hat{\mathbf{d}}_0} \right\|^2 - 1$$

$$\chi = \frac{\hat{\mathbf{d}}_0^H (\mathbf{A} + \gamma \mathbf{I}_N)^{-1} \nu_2 \hat{\mathbf{d}}_0}{[\hat{\mathbf{d}}_0^H (\mathbf{A} + \gamma \mathbf{I}_N)^{-1} \hat{\mathbf{d}}_0]^2} - 1$$

$$\chi = \frac{\hat{\mathbf{d}}_0^H (\mathbf{A} + \gamma \mathbf{I}_N)^{-1} (\mathbf{A} + \gamma \mathbf{I}_N)^{-1} \hat{\mathbf{d}}_0}{[\hat{\mathbf{d}}_0^H (\mathbf{A} + \gamma \mathbf{I}_N)^{-1} \hat{\mathbf{d}}_0][\hat{\mathbf{d}}_0^H (\mathbf{A} + \gamma \mathbf{I}_N)^{-1} \hat{\mathbf{d}}_0]} - 1$$

$$\chi = \frac{(\mathbf{A} + \gamma \mathbf{I}_N)^{-1}}{\hat{\mathbf{d}}_0^H (\mathbf{A} + \gamma \mathbf{I}_N)^{-1} \hat{\mathbf{d}}_0} - 1 \tag{3.31}$$

The result can easily carry on relevant to the situation where there is no explicit constraint on the surplus energy by setting the $\nu_1 = 0$. Now, defining the covariance matrix $\mathbf{R}_y(n)$.

$$\mathbf{R}_y = E[\mathbf{y}\mathbf{y}^T]$$
$$= E\left[[\sum_{k=0}^{K-1} \sqrt{P_k}\mathbf{d}_k(i)b_k(i) + \mathbf{n}(i)][\sum_{j=0}^{K-1} \sqrt{P_j}\mathbf{d}_j(i)b_j(i) + \mathbf{n}(i)]^T \right]$$
$$= E\left[[\sum_{k=0}^{K-1} \sqrt{P_k}\mathbf{d}_k(i)b_k(i) + \mathbf{n}(i)][\sum_{j=0}^{K-1} \sqrt{P_j}b_j^H(i)\mathbf{d}_j^H(i) + \mathbf{n}^H(i)] \right]$$
$$= E[\sum_{k=0}^{K-1}\sum_{j=0}^{K-1} \sqrt{P_K}\sqrt{P_j}\mathbf{d}_k(i)b_k(i)b_j^H(i)\mathbf{d}_j^H(i)] + 0 + 0 + E[\mathbf{n}(i)\mathbf{n}^H(i)]$$
$$\therefore E[b_k(n)b_k^*(n)] = 1 \quad \therefore E[b_k(n)b_j^*] = 0, \text{ when } k \neq j$$
$$= \sum_{k=0}^{K-1} P_k\mathbf{d}_k(i)\mathbf{d}_k^H(i) + E[\mathbf{n}(i)\mathbf{n}^H(i)]$$
$$= \sum_{k=0}^{K-1} P_k\mathbf{d}_k(i)\mathbf{d}_k^H(i) + \sigma_n^2\mathbf{I}_N$$
$$\therefore \text{By condition of } \mathbf{A} \text{ in (3.26)}$$
$$\mathbf{R}_y = \mathbf{A} + \sigma_n^2\mathbf{I}_N \tag{3.32}$$

By using the (3.28) and (3.32) can be written as

$$\mathbf{w}_0 = \frac{\mathbf{R}_y^{-1}\hat{\mathbf{d}}_0}{\hat{\mathbf{d}}_0^H(\mathbf{A} + \sigma_n^2\mathbf{I}_N)\hat{\mathbf{d}}_0} \tag{3.33}$$

By (3.29), if $v_1 = 0$ the surplus energy is equal to zero

$$\xi_{\min} = v_2$$
$$= \hat{\mathbf{d}}_0^H(\mathbf{A} + \gamma\mathbf{I}_N)\hat{\mathbf{d}}_0 \tag{3.34}$$

then

$$\mathbf{w}_0 = \xi_{\min}\mathbf{R}_y^{-1}\hat{\mathbf{d}}_0 \tag{3.35}$$

Therefore, the constraint optimization problem solution can be written as

$$\mathbf{w}_0 = (\hat{\mathbf{d}}_0^H(\mathbf{A} + \gamma\mathbf{I}_N)^{-1}\hat{\mathbf{d}}_0)\mathbf{R}_y^{-1}\hat{\mathbf{d}}_0$$

or

$$\mathbf{w}_0 = (\hat{\mathbf{d}}_0^H\mathbf{R}_y^{-1}(n)\hat{\mathbf{d}}_0(n))^{-1}\mathbf{R}_y^{-1}(n)\mathbf{d}_0(n) \tag{3.36}$$

where the $\mathbf{R}_y^{-1}(n)$ is the covariance matrix obtained as

$$\mathbf{R}_y(n) = \lambda\mathbf{R}_y(n-1) + \mathbf{y}(n)\mathbf{y}^H(n) \tag{3.37}$$

By using the (3.36), the minimum output variance of the detector can be written as

$$
\begin{aligned}
\mathbf{P}_{\min} &= \mathbf{w}_0^H \mathbf{R}_y \mathbf{w}_0 \\
&= [(\mathbf{d}_0^H(n)\mathbf{R}_y^{-1}(n)\mathbf{d}_0(n))^{-1}\mathbf{R}_y^{-1}(n)\mathbf{d}_0(n)]^H \mathbf{R}_y(n)[(\mathbf{d}_0^H(n)\mathbf{R}_y^{-1}(n)\mathbf{d}_0(n))^{-1}\mathbf{R}_y^{-1}(n)\mathbf{d}_0(n)] \\
&= (\mathbf{d}_0^H(n)\mathbf{R}_y^{-1}(n)\mathbf{d}_0(n))^{-H}\mathbf{d}_0^H(n)\mathbf{R}_y^{-H}(n)\mathbf{R}_y(n)\mathbf{R}_y^{-1}(n)\mathbf{d}_0(n)(\mathbf{d}_0^H(n)\mathbf{R}_y^{-1}\mathbf{d}_0(n))^{-1} \\
&\therefore \mathbf{R}_y(n)\mathbf{R}_y^{-1}(n) = \mathbf{I} \\
&= (\mathbf{d}_0^H(n)\mathbf{R}_y^{-1}(n)\mathbf{d}_0(n))^{-2}(\mathbf{d}_0^H(n)\mathbf{R}_y^{-1}(n)\mathbf{d}_0(n)) \\
&= (\mathbf{d}_0^H(n)\mathbf{R}_y^{-1}(n)\mathbf{d}_0(n))^{-1}
\end{aligned} \tag{3.38}
$$

The estimate of effective signature waveform $\mathbf{d}_0(n)$ can be obtained by maximizing the (3.38) w.r.t $\mathbf{d}_0(n)$ such that the signal components are maximum after interference suppression at receiver output. In other words maximization of the (3.38) is same as minimum of its reciprocal. The estimate of effective signature waveform is given by

$$\hat{\mathbf{d}}_0(n) = \arg\min_{\mathbf{d}_0(n)} \mathbf{d}_0^H(n)\mathbf{R}_y^{-1}(n)\mathbf{d}_0(n) \tag{3.39}$$

where $\mathbf{d}_0(n) = \mathbf{C}_0\mathbf{h}_0(n)$ and $\mathbf{R}_y^{-1}(n)$ is the covariance matrix obtained in (3.37).

Let $g_0(n)$ is channel impulse response of 1st user with L paths. It has $\mathbf{h}_0(n) = \mathbf{F}_m\mathbf{g}_0(n)$, where $\mathbf{F}_m$ is $M \times L$ matrix. $\mathbf{F}_m$ have the entries

$$[\mathbf{F}_m]_{m,l} = \exp(-j2\pi(Pm+p)/(M)),\ m = 0,\dots,M-1, l = 0,\dots,L-1$$

By (3.39), it can be written

$$
\begin{aligned}
\mathbf{d}_0^H(n)\mathbf{R}_y^{-1}(n)\mathbf{d}_0(n) &= [\mathbf{C}_0\mathbf{h}_0(n)]^H \mathbf{R}_y^{-1}\mathbf{C}_0\mathbf{h}_0(n) \\
&= [\mathbf{C}_0\mathbf{F}_m\mathbf{g}_0(n)]^H \mathbf{R}_y^{-1}\mathbf{C}_0\mathbf{F}_m\mathbf{g}_0(n) \\
&= \mathbf{g}_0^H(n)\mathbf{F}_m^H\mathbf{C}_0^H\mathbf{R}_y^{-1}(n)\mathbf{C}_0\mathbf{F}_m\mathbf{g}_0(n) \\
&= \mathbf{g}_0^H(n)\mathbf{\Omega}\mathbf{g}_0(n) \quad \therefore \mathbf{\Omega} = \mathbf{F}_m^H\mathbf{C}_0^H\mathbf{R}_y^{-1}(n)\mathbf{C}_0\mathbf{F}_m
\end{aligned}
$$

Hence, the minimization of (3.39) with respect to $\mathbf{g}_0(n)$ gives

$$\hat{\mathbf{g}}_0(n) = \arg\min_{\hat{\mathbf{g}}_0(n)\in C^{L\times 1}} \mathbf{g}_0^H(n)\mathbf{\Omega}\mathbf{g}_0(n) \tag{3.40}$$

where

$$\mathbf{\Omega} = \mathbf{F}_m^H \mathbf{C}_0^H \mathbf{R}_y^{-1}(n) \mathbf{C}_0 \mathbf{F}_m$$

The $\hat{\mathbf{g}}_0(n)$ is the channel estimate which is Eigen-vector related to the smallest Eigen value of the $\mathbf{\Omega} \in \mathbf{C}^{L \times L}$. There are many methods to calculate the minimum Eigen-vector of the $\mathbf{\Omega}$, such as Singular Value Decomposition (SVD), Eigen Value Decomposition (EVD) etc. But it is computationally hard to follow the desired Eigen-vector of $\mathbf{\Omega}$. However, the inverse iteration method is used to calculate the minimum Eigen vector of the matrix $\mathbf{\Omega}$.

The inverse iteration method is adopted due to good tracking performance and less complexity [6]. The inverse iteration method is as following

Initialization: $\hat{\mathbf{g}}(0)$;

$\mathbf{\Omega}(n)\mathbf{a}(n) = \hat{\mathbf{g}}(n-1)$; LU decomposition to get $\mathbf{a}(n)$

$\hat{\mathbf{g}}(n) = \mathbf{a}(n) / \| \mathbf{a}^H(n)\mathbf{a}(n) \|^{1/2}$; Normalization

The above method had been used to calculate the minimum Eigen-vector of the $\mathbf{\Omega}$.

Further the data bit of the desired user is demodulated according to

$$\hat{b}_0 = \mathrm{sgn}(z) = \mathrm{sgn}\{\Re e(\mathbf{w}^H \mathbf{y})\} \tag{3.41}$$

3.5 Subspace based LMMSE detector

In subspace based LMMSE detector, suppose the data stream of each user is independent of the other such that

$$E\{b_i(t) b_j^*(t)\} = 0 \tag{3.42}$$

The noise in the system is the white Gaussian noise. The correlation matrix $\mathbf{R}_y$ is calculated as

The channel vector $\mathbf{g}_0$ can be estimated by the same approach formulating the

(3.40)

$$\hat{g}_0 = \arg \min_{\mathbf{g}_0 \in \mathbf{C}^{L\times 1}} \mathbf{g}_0^H \mathbf{\Omega}_0 \mathbf{g}_0 \tag{3.49}$$

where

$$\mathbf{\Omega}_0 = \mathbf{F}_m^H \mathbf{C}_0^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{C}_0 \mathbf{F}_m$$

$\hat{g}_0$ is the Eigen-vector relevant to the minimum Eigen-value of the matrix $\mathbf{\Omega}_0$.

The data bit of the desired user is demodulated according to

$$\hat{b}_0 = \mathrm{sgn}(z) = \mathrm{sgn}\{\Re e(\mathbf{w}^H \mathbf{y})\} \tag{3.50}$$

Chapter 4
Results & Discussion

Chapter 4

Results and Discussion

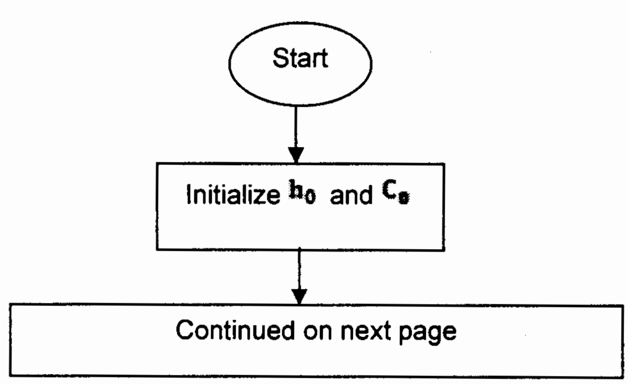
4.1 Introduction:

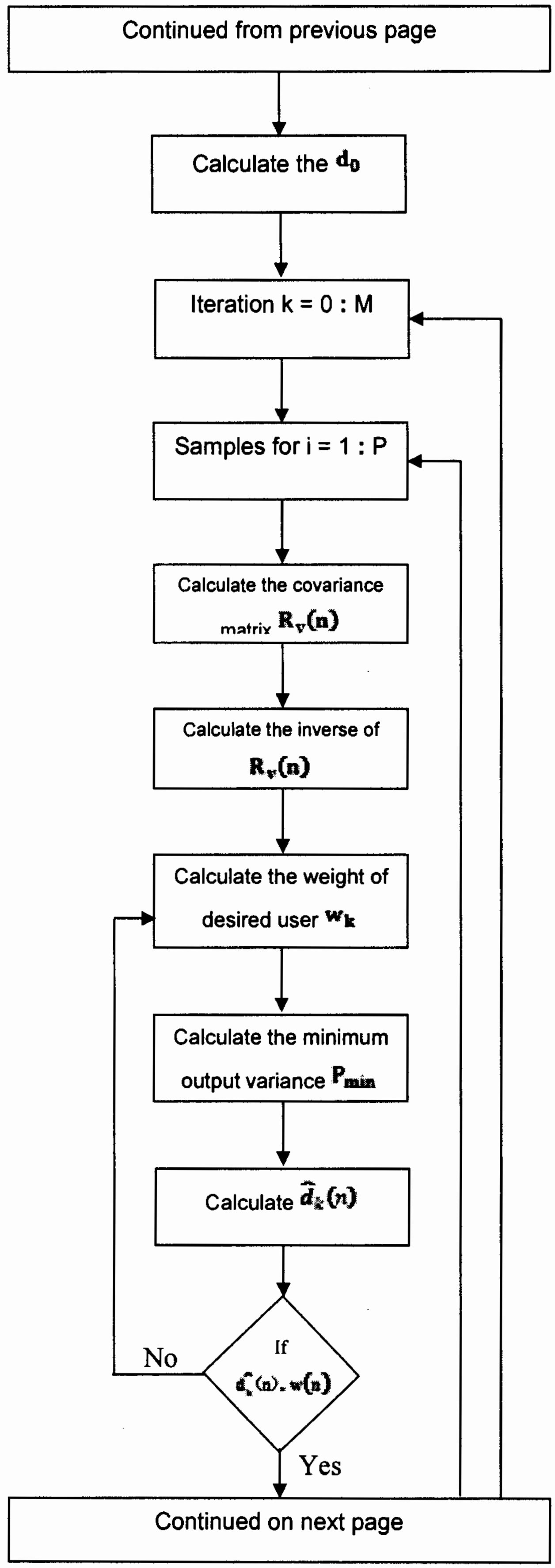
In this chapter, the simulations and results of work which were presented in the previous chapter are shown. The general algorithm of simulations is also presented there. All the simulations were done in Matlab 7.3 by a number of functions and programs written for CMOE and Subspace Based LMMSE Detector.

Both CMOE and Subspace Based LMMSE Detector were applied on the MC-CDMA detection process separately. The comparison of these results is based on different parameters such as SNR, number of iterations, SINR, etc.

4.2 The CMOE and subspace based LMMSE Detection Algorithm:

The algorithm in the form of flow chart for the working of CMOE Detection is shown Figure 4.1. The flow chart clearly explains the working of CMOE Detection for the MC-CDMA System.




Continued from previous page
Calculate the d_0
Iteration k = 0 : M
Samples for i = 1 : P
Calculate the covariance matrix $R_v(n)$
Calculate the inverse of $R_v(n)$
Calculate the weight of desired user w_k
Calculate the minimum output variance P_{min}
Calculate $\hat{d}_k(n)$
If
No
Yes
Continued on next page

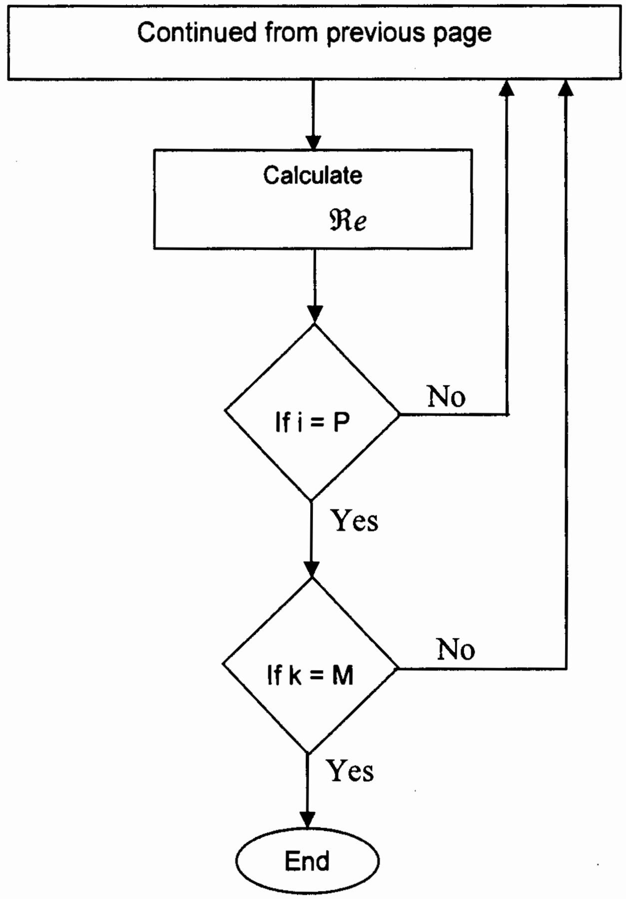
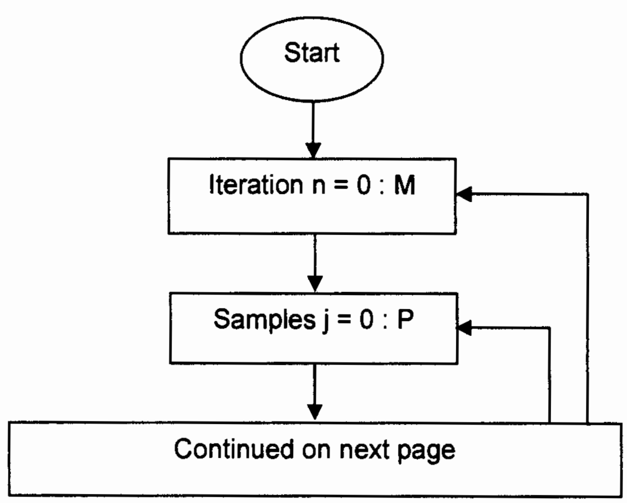


Figure 4.1: Flow chart of the CMOE Detection

The flow chart showing the algorithm for Subspace Based LMMSE Detection of the MC-CDMA system is shown in Figure 4.2.



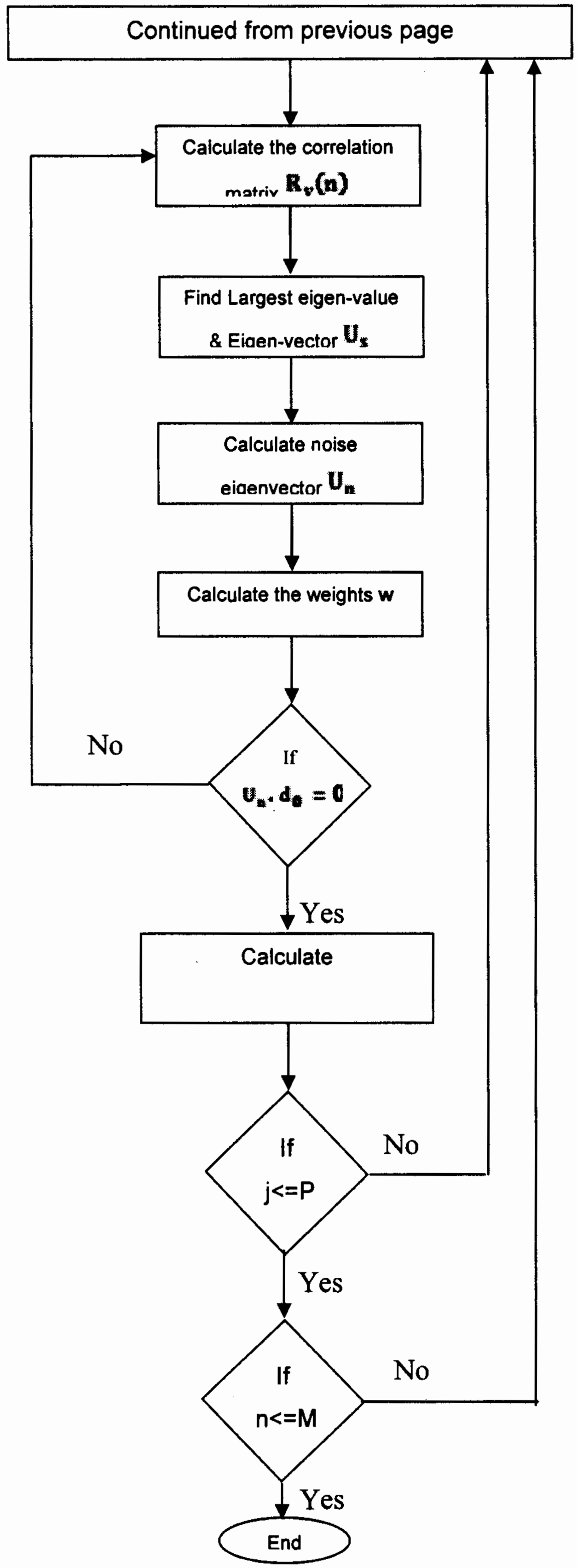


Figure 4.2: Flow chart of the Subspace LMMSE Detection

4.3 Simulations

In the simulation process the observations were done between the CMOE detection and subspace based LMMSE detection for the MC-CDMA system. The CMOE and LMMSE detection were applied for different values of SNR and many useful results were deducted from it.

With respect to the design of the MC-CDMA system, the values used in simulations are:

Parameter	Abbreviation	Value
Length of spreading code	M	16
Number of user bits	P	8
Number of users	K	8
Forgetting factor	λ	0.995
Delay factor	δ	0.1
Guard tones	N_g	8

Therefore, the unique data sequence is firstly changed to S/P in to 8 alongside data streams. Further additional guard tones of size $N_g = 8$ are added to avoid the ISI. The total simultaneous users in channel are 8. The preferred user is zero. The interference signal ratio of preferred user 10 dB lower than MAI, i.e.; $P_k / P_0 = 10$. While adopting the Rayleigh Fading assumption, the channel coefficients are randomly generated according to a complex Gaussian distribution. The SINR analysis is done on the basis of SINR analysis for the LMMSE detector in [1].

Based on the above data, Figure 4.3(a) and (b) shows the normalized channel estimation error of the CMOE and subspace based LMMSE detection versus data samples and SNR. The Figures show that at the high SNR, the CMOE based channel estimator and the subspace based LMMSE detector has the similar performance.

The Figure 4.4(a) and (b) shows the Average SINR and MSE for the CMOE and subspace based LMMSE detection versus the SNR.

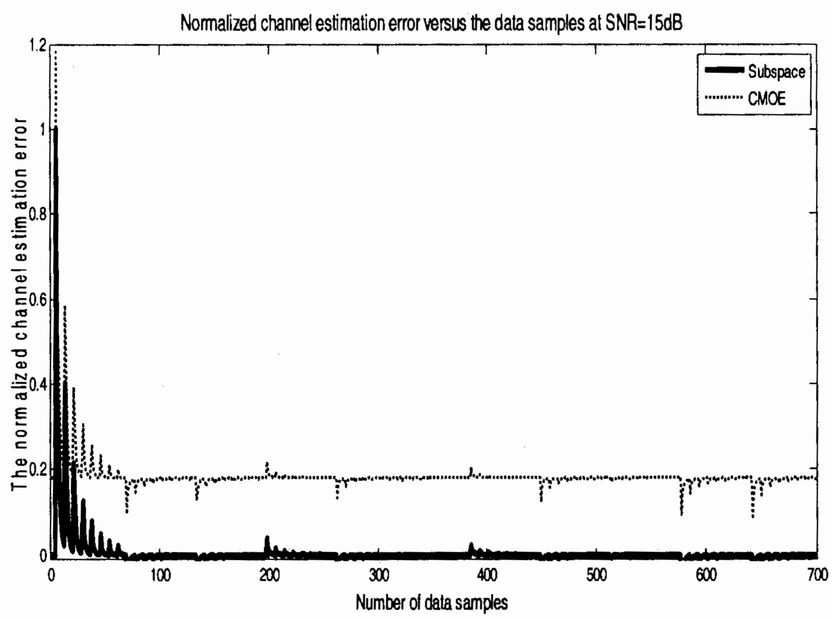


Figure 4.3(a): Normalized channel estimation w.r.t number of Data samples When SNR=15 dB.

The subspace based LMMSE detection performs better than the CMOE detection. It is seen that the MSE and output SNR of LMMSE detection is closer to the most favorable values.

The Figure 4.5(a) and (b) shows the normalized channel estimation error of the CMOE detection and subspace based LMMSE Detection versus number of data samples and SNR for the Qausi-synchronous MC-CDMA system.

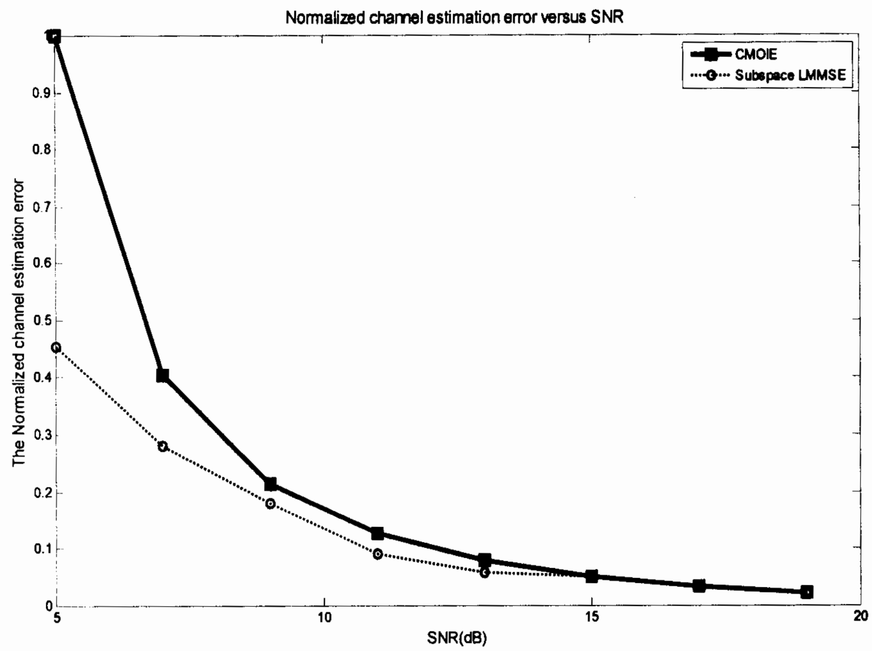


Figure 4.3(b): Normalized channel estimation error w.r.t SNR
M=16, P=8, K=8, ISR=10db, λ=0.995, δ=0.1.

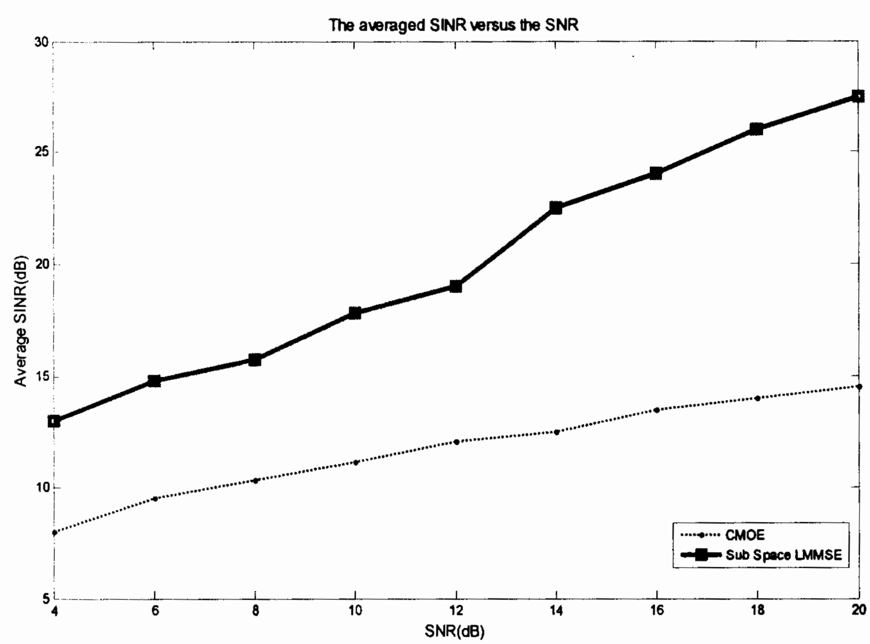


Figure.4.4(a): Average SINR w.r.t SNR. When
M=16, P=8, K=8, ISR=10db, λ=0.995, δ=0.1

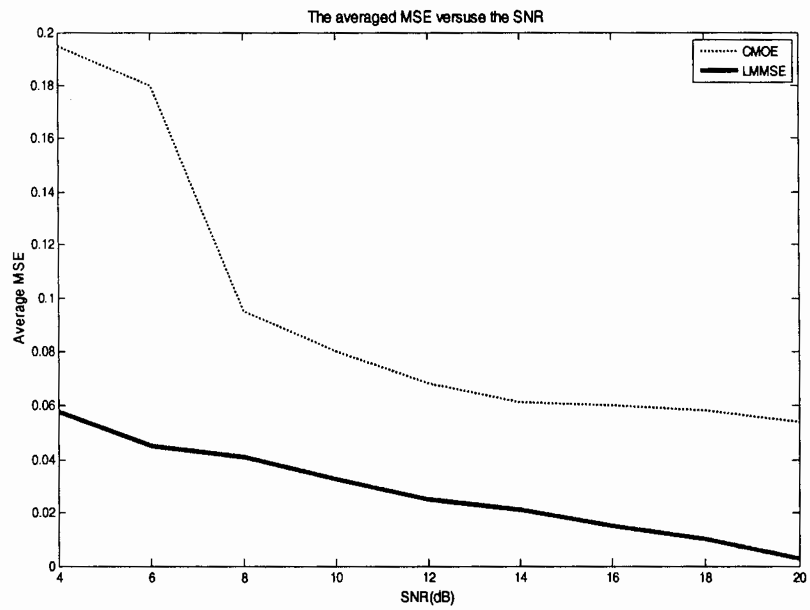


Figure 4.4 (b): Average MSE w.r.t SNR. When
M=16, P=8, K=8, ISR=10db, λ=0.995, δ=0.1.

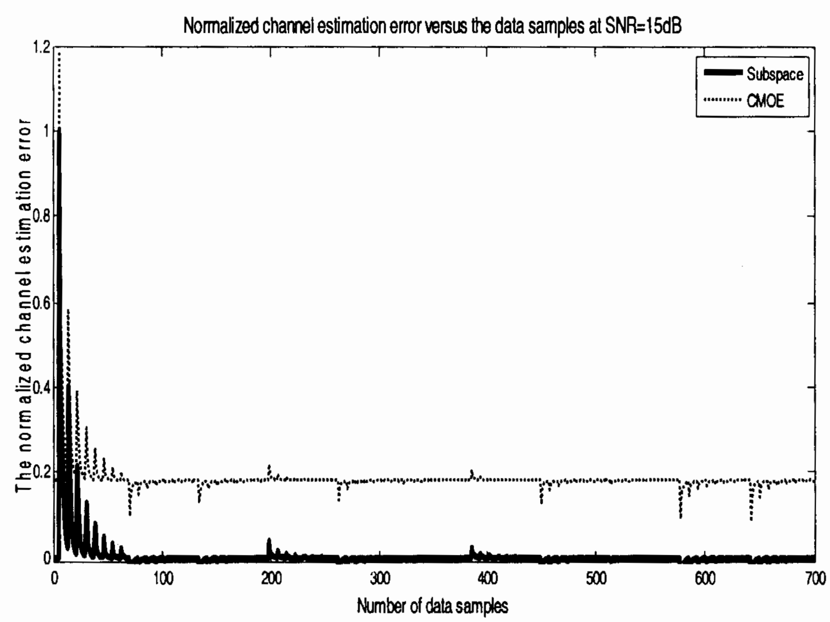


Figure 4.5 (a): The normalized channel estimation error w.r.t number of data samples for Qausi-synchoronous channel. When SNR=15 dB.

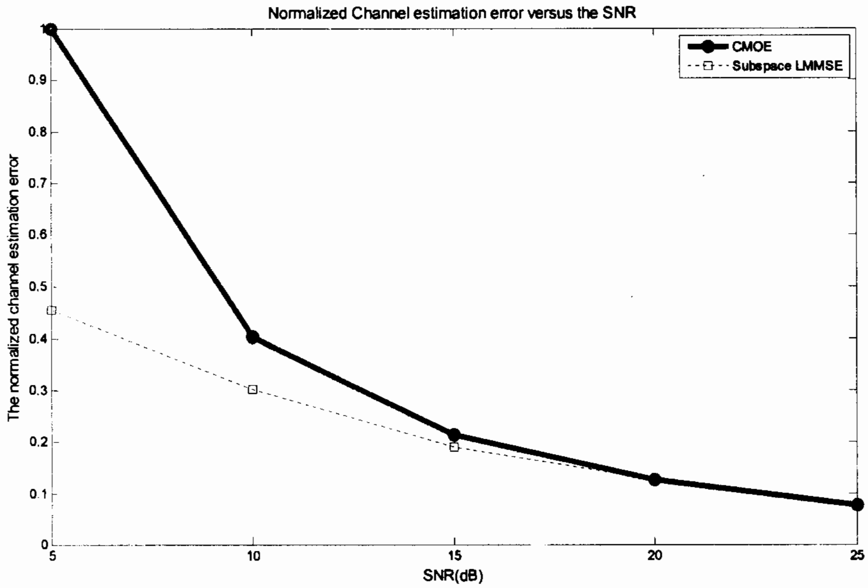


Figure 4.5 (b): The normalized channel estimation error w.r.t SNR for Qausi-synchoronous channel. When M=16, P=8, K=8, ISR=10db, λ=0.995, δ=0.1.

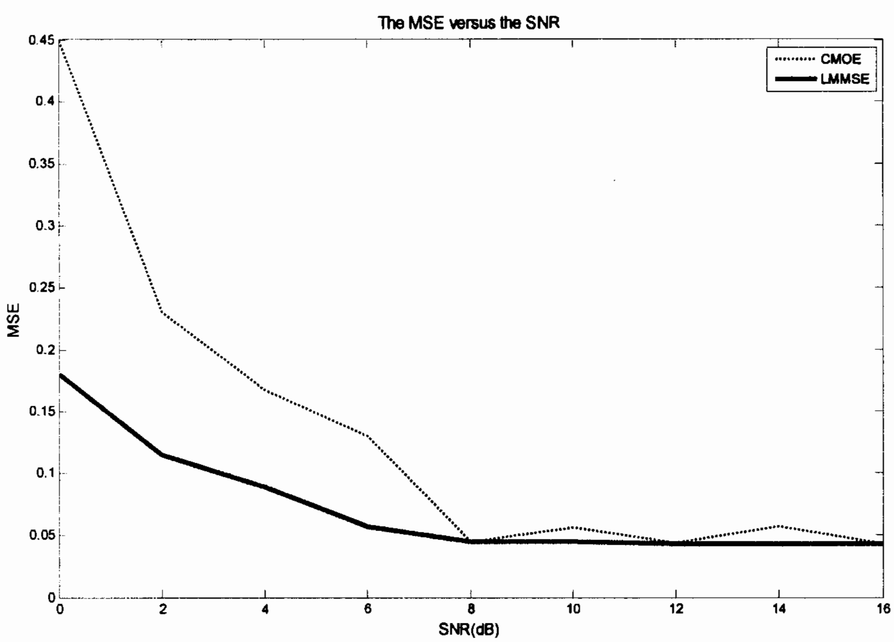


Figure 4.6: The MSE w.r.t SNR. When M=16, P=8, K=8, ISR=10db, λ=0.995, δ=0.1.

The Figures 4.5(b) illustrates that at low SNR the subspace LMMSE detection is better than the CMOE detection.

The Figure 4.6 shows the MSE performance versus the SNR of the CMOE detector and Subspace based LMMSE detection. The Figure 4.6 illustrate that the at the low SNR subspace LMMSE detection performs better than the CMOE detection.

4.4 Conclusion

In this thesis, the Blind CMOE and Subspace based LMMSE detector for MC-CDMA are studied. The subspace based LMMSE detector has better performance but, its computational complexity is higher than CMOE detector. The results show that the performance of the blind CMOE detector is same as subspace based LMMSE detector at medium SNR.

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