Electromagnetic Waves in Spin Magnetized Plasmas with Temperature Degeneracy Effects

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Plasmas.

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This Thesis submitted to Department of Physics International Islamic University, Islamabad for the award of degree of MS Physics.

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Dedicated
to
My beloved
Parents
Brothers
And
My
Respected teachers

Declaration

I, Mr. **Umar khitab**(Registration # 400-FBAS/MSPHY/S16), student of MS Physics (session 2016-2018), hereby declare that the work presented in the thesis entitled **"Electromagnetic Waves in Spin Magnetized Plasmas with Temperature Degeneracy Effects"** in partial fulfillment of MS degree in Physics from International Islamic University Islamabad, is my own work and has not been published or submitted as research work or thesis in any form in any other university or institute in Pakistan or aboard.

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The thesis entitled **"Electromagnetic Waves in Spin Magnetized Plasmas with Temperature Degeneracy Effects"**(Registration # 400-FBAS/MSPHY/S16) in partial fulfillment of MS degree in Physics has been completed under my guidance and supervision. I am satisfied with the quality of his research work and allow him to submit this thesis for further process to graduate with Master of Science degree from Department of Physics, as per IIU Islamabad rules and regulations.

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Acknowledgment

All glory be to ALLAH, the unfailing fountain and containing source of knowledge, who conferred upon us both the ability to contribute to the eternally, flowing oceanof knowledge, a few drops which in its turn helps man in his, universe and ultimatelyALLAH recognition; indeed, our drops can easily be picked out an account of themunique color, flavor and worth. The role model life of the holy prophet (**PBUH**) hasremained a beacon house in spurring the successful accomplish meant of this glorioustask.

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Abstract

Spin magnetization, Bohm potential and arbitrary temperature degeneracy effects on the electromagnetic waves propagating in arbitrary direction in spin magnetized quantum plasmas are considered. We study the high frequency modes in the presence of the above mentioned quantum corrections, and that's why because of large inertia of ions these are assumed to be static and act as the neutralizing background. While the electrons are considered dynamically and quantum mechanical.

Using the Fermi-Dirac distribution the generalized temperature degeneracy formula for pressure is derived in terms of polylogarithm function. The expansion of these poly log for $\xi << 1$ and $\xi >> 1$, ($\xi = e^{\beta\mu}$ with $\beta = \frac{1}{k_B T_e}$ and μ is the chemical potential), corresponds to nearly nondegenerate and nearly degenerate quantum plasmas respectively. The parallel and perpendicular modes dispersion relations is then derived for both nearly nondegenerate and nearly degenerate limits of the plasma. The effects of arbitrary degeneracy in the presence of Bohm potential and spin magnetization, on the dispersion curve of parallel and perpendicular modes are discussed for both degenerate limits.

The resonant and cut-off frequencies for perpendicular (x-mode) in the presence of all quantum corrections are derived analytically and discussed numerically. The effects of number density, degeneracy factor and that of magnetic field are discussed with their physical significance.

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Chapter 1

Introduction

The word plasma originates from Greek $\pi\lambda\alpha\sigma\mu\alpha$, which represents something fabricated or jellylike. In the mid nineteenth century the Czech physiologist Jan Evangelista Purkinje (1787-1869) has used this word for the first time for the remaining clear fluid after separation of all the corpuscular things in blood. In 1929 Tonks and Langmuir have used the word to inscribe the internal portion of a glowing ionized gas produced in electric discharge tube. In 1930 few researchers, each motivated by a specific problem, developed the field of plasma physics. Their work was mainly focused on understanding of radio wave absorption and distortion through ionospheric plasma (partially ionized gas lies at a distance of about 60 km in the upper atmosphere) and gaseous electron (vacuum) tubes used for rectification, amplification, and regulation of voltage in the early days of electronics [1]. Astrophysicists come to know that most part of the known universe is in plasma state , and thus a better understanding of astrophysical phenomena requires an advancement in plasma physics. Hannes Alfven was one of the main contributer, who presented a theory of hydromagnetic waves (Alfvén waves) around in 1940. And suggested that this theory would be significant in astrophysical plasmas and this theory has been successfully used to investigate sunspots, solar flares, the solar wind, star formation, and many other areas in astrophysics [2]. In 1950 fusion energy research started simultaneously in the USA, Britain, and Russia. Since this work was a part of thermonuclear weapon research, it was initially classified but, because of less progress in each country's effort and the realization that controlled fusion research was unlikely to be of military value. Nowadays many other countries cotribuited in fusion research as well [3].

In 1952 the creation of hydrogen bomb by US (one of the major applications of plasma physics) increased the interest in controlled thermonuclear fusion reaction as a possible energy source for future. Fusion physics is mostly concerned with understanding the trapping of plasma (mostly this trapping is done with the help of magnetic field) and searching many instabilities in plasma which may cause escape of particles. In 1958 James A. Van Allen's systematically explored the Earth's magnetosphere by discovering the Van Allen radiation belts surrounding the Earth [4], in addition to explore area contained in space plasma physics. In 1960's LASER plasma interaction was developed, when solid object is strike with a high power laser beam the object condensed to a shorter volume and than fused with creation of energy. Another application of laser plasma interaction is the generation of extremely strong polarized electric fields when an intense laser pulse passes through a plasma to accelerate the charge species (which separate the charge particles at some distances) [5].

1.1 What is Plasma Physics?

Plasma is a quasi-neutral mixture of charged and neutral particles which exhibits collective behavior. Quasi-neutrality means that contain approximately equal number of positive and negative charge particles ($n_i \approx n_e \approx n$) on scale length long as compared to Debye length (small deviation from quasi-neutrality can be developed on scale shorter than Debye length). Where n_i is number density of positive ions , n_e is number density of electrons and n is a common density of species called plasma density .Collective behavior is because of long range coulombic force involved in plasma. The variation in coulombic force (that describe the electrostatic interaction between the charged particles) occur very slowly with distance as r^{-2} which makes it a long range force. This means that each particle interact with a large number of particles. Therefore the plasma particles shows a significant response to an external electric field, such a behavior is called collective behavior. On the other hand in neutral gases, particles interact via short range Vander Waal's force i.e. during binary collisions. This force decay very rapidly with distance as r^{-6} [6].

It is believed that about 99% of the known matter of our universe lies in the plasma state. Plasma may be fully ionized or partially ionized. Fully ionized plasma occurs in the solar wind and in fusion reaction experiments while partially ionized plasmas occur in different types of gas discharges (fluorescent lamps, gas lasers, arc discharges, plasma for materials processing) and the earth's ionosphere. Naturally, occurring plasmas includes the stellar objects (sun and stars), aurora borealis, radiation belts, comet tails, Earth's ionosphere, solar corona and interstellar media, etc. Moreover, man-made plasmas are, plasma lamps, gas discharges, gas laser, the welding sparks, controlled fusions, the arcs and plasma screens [7].

1.2 Importance of plasma physics

The earliest research work in plasmas was inspired by the need to develop gas discharge tubes (feld with ionized gases) that could carry large currents. These vacuum tubes are nowadays, used in mercury rectifiers, ignitrons, spark gaps, welding arcs and lightning discharges. The observation of the earth's environment in space such as that of earth magnetosphere (which prevent us from solar winds) is another significant application of plasma [7]. The field of quantum plasma physics has vast applications in modern technology, metallic and semiconductor nanostructures such as metallic nanoparticles, thin-metal films, quantum wells and quantum dots, Nano-plasmonic devices, quantum X-ray free-electron lasers etc. [8]. Last but never least one of the most important application of plasma physics is controlled thermonuclear fusion reaction which is one of the possible solution of all the energy crises [9].

1.3 Classification of Plasmas

Plasma is generally classified into, classical plasma and quantum plasma on the basis of certain specific parameters such as plasma frequency, velocity scales and characteristic length. By using basic dimensional analysis the above mentioned parameters can be find out. Certainly, better scientific studies will be important to know about the interceding of above mentioned parameters in real phenomena.

1.3.1 Classical Plasmas

Classical plasma is well-defined by high temperature and low number density regimes. Classical plasma obey the laws of Newtonian mechanics, in which no overlapping of quantum surfaces occur in plasma particles [10], as show in Fig 1.1.

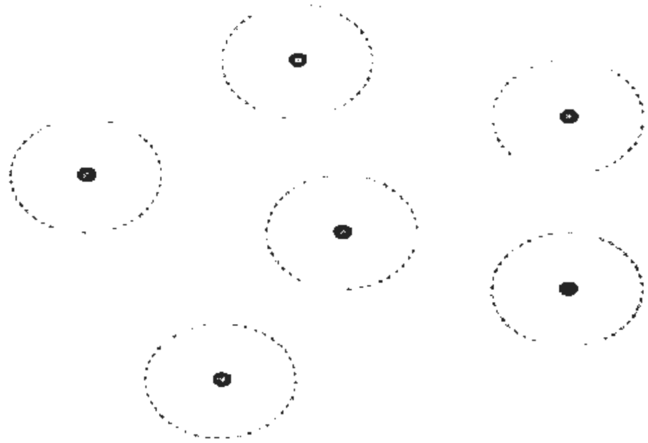


Figure 1.1: Particles behavior in low density, high temperature plasma, that's classical Maxwellian plasma.

The coupling parameter, for classical plasma which contain particles concentrations n, electric charge q and temperature T can be written as [11].

$$\Gamma c = \frac{q^2 n^{\frac{1}{3}}}{\varepsilon_o k_B T} \tag{1.1}$$

Where ε_o is the permittivity of free space and k_B is the Boltzmann's constant. For collisionless plasma it is necessary that $\Gamma c \ll 1$.On the basis of plasma constituents such as neutral particles, electron density, and ion density we can define other quantities, like plasma ionization degree which is the ratio between numbers of charged particles in a certain unit volume to the total number of particles in the same unit volume. The number density variation changes the degree of ionization. Ionization degree is in the range of $10^{-3} - 10^{-6}$ for ordinary classical plasma at low pressure in the absence of external magnetic field, while by applying external magnetic field (used for confinement) the ionization degree could be increased up to 10^{-2} [12]. Because of free charge carrier plasma behave like an excellent conductor. By introducing an external test charge to the plasma, the charge carriers inside the plasma shield off its field and preserve quasineutrality. Therefore, the field of the test charge particle does not fallow the inverse square law, but follow exponential law. This phenomenon is called Debye shielding, and the typical length scale of the exponentially decreasing field from the test charge is defined as the Debye length [13]

$$\lambda_{Dx} = \sqrt{\frac{\epsilon_0 k_B T_x}{e^2 n_{0x}}}. \quad (1.2)$$

Where λ_{Dx} is the Debye length, n_{0x} is the density and T_x is the temperature for any particle x with a Boltzmann's constant k_B. Classical plasma shields out the potential of the charge particle in a vacuum and this shielding can be observed on scale length that is Debye length. Usually in Debye sphere of radius λ_{Dx} the value of thermal energy exceed the value of potential energy. The number of particles inside a certain volume with linear size of a Debye length gives the definition of plasma parameter as

$$N_D = n\lambda_{Dx}^3 \gg 1 \quad (1.3)$$

If there is a finite temperature of plasma then, we can construct the average thermal velocity V_x of a particle with mass m_x

$$\mathbf{V}_x = \sqrt{\frac{k_B T_x}{m_x}} \quad (1.4)$$

By taking the ratio λ_{Dx}/v_x gives the definition of time scale $\sqrt{\frac{\epsilon_0 m_x}{e^2 n_{0x}}}$. Its inverse defines the plasma frequency which is the characteristic frequency for fluctuations of concentration in plasma

$$\omega_{px} = \sqrt{\frac{e^2 n_{0x}}{\epsilon_0 m_x}} \quad (1.5)$$

Alternative important time scale is specified by cyclotron frequency

$$\omega_{cx} = \frac{e\mathbf{B}_0}{m_x} \tag{1.6}$$

which is an angular frequency for cyclotron motion of a charged particle in the magnetic field $\mathbf{B}_0$ [7]. Applications of classical plasmas are those mentioned in introduction.

1.3.2 Quantum Plasmas

In the above section we have discuised the plasmas of low densities and high temperatures, that classical explanation is enough to understand the dynamics of particles. But there are certain situation where quantum effects come into play and the model needs to be extended to comprise quantum physics [14]. The quantum plasma has low temperature and high number density. This type of plasma show microscopic behavior and their constituent particles obey the laws of quantum mechanics in which quantum surfaces overlapping is occurring as shown in Fig. 1.2.

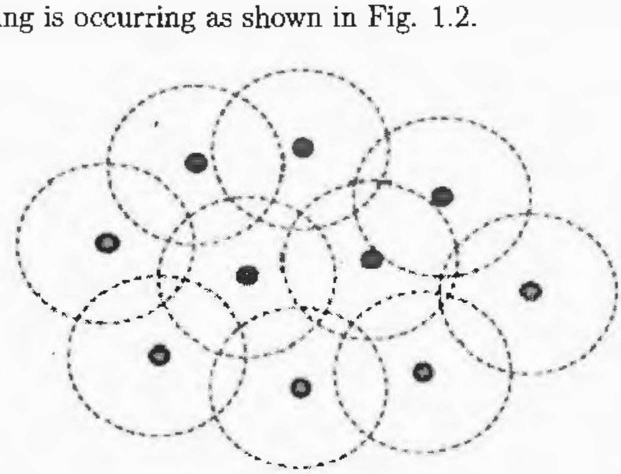


Figure 1.2: Particles behavior in low temperature, high density plasma, that's quantum fermionic plasma

To study quantum effects an additional length scale is needed, specfecally the de-Broglie wavelength of charged species [15].

$$\lambda_{Bx} = \frac{\hbar}{m_x \mathbf{V}_x}, \tag{1.7}$$

where $\hbar$ is reduced Plank constant, m_x is the mass and $\mathbf{V}_x$ is the velocity of the specie x. This wavelength shows the spatial extension of particles wave function and also clarifies that electrons show quantum mechanical behavior much more easily as compared to ions due to larger mass difference. However if we treat the ion quantum mechanically their dynamics show very less contribution. That's why in practical situation, we consider that ions dynamic are classical, and only the electrons are treated quantum mechanically. When interparticle distance of plasma is on scale of de-Broglie wavelength, which is a quantum wave like effect, then microscopic effects are important to be considered. In case of weakly coupled systems, where the charge carriers lies far away from one another, then the de-Broglie wavelength is the characteristic quantum length, while in strongly coupled system the de-Broglie wavelength is replaced by the particles wave function. As the temperature of classical plasma decreases or density increases, it can enter in a regime where quantum nature of its constituent start to affect its macroscopic properties and dynamics quite naturally, such plasma is then termed as quantum plasma [16]. Furthermore, on the basis of particles velocity, quantum plasma can be classified into non-degenerate and degenerate quantum plasmas. When the velocity for quantum plasma is interpret as thermal velocity then quantum plasma is non-degenerate one, while for degenerate quantum plasma the velocity is surely be the Fermi velocity [17]. In quantum plasma the mean inter-particle distance in comparison to the de-Broglie wavelength of lightest particles and the effect of degeneracy (it is quantum degeneracy due to Pauli's exclusion principle) is taken in accounts. In solid state physics we know that a temperature which qualifies the degeneracy in quantum plasma is Fermi temperature and can be written as

$$T_F = \frac{\hbar\,(3\pi^2 n_e)^{\frac{2}{3}}}{2m_e k_B}, \tag{1.8}$$

When $T_e \to T_F$, then statistical distribution changes from Maxwell-Boltzmann to Fermi-Dirac. The thermal transition between these two regimes is related with the scaled parameter $n\lambda_B^3$.

$$\chi = \frac{T_F}{T_e} = \frac{1}{2}(3\pi^2)^{\frac{2}{3}}(n\lambda_B^3)^{\frac{2}{3}}, \tag{1.9}$$

In order to establish a relevant set of typical scales (time, space and velocity) for quantum plasma as well as the relevant parameters. We must considered that simple expressions can only be found in the limiting cases i.e. $T_e >> T_F$ correspond to classical plasma and $T_e << T_F$ leads to quantum plasma regime. It is important to keep in mind that there is transition between these two regimes but this can not be treated using straight forward dimensional arguments. The time scale for collective behavior in quantum plasma can be write in terms of plasma frequency i.e. $t_{Qe} = \frac{1}{\omega_{pe}}$. For deep degenerate quantum limits $\chi >> 1$, then Eq. (1.9) becomes much significant.

In very low temperature limit thermal velocity of electron should be replaced by a typical velocity known as Fermi-velocity, and in terms of $E_F = k_B T_F = (\frac{\hbar^2}{2m})(3\pi^2 n)^{\frac{2}{3}}$, it is defined by

$$V_{Fe} = (\frac{2E_F}{m_e})^{\frac{1}{2}} = \frac{\hbar\,(3\pi^2 n_e)^{\frac{1}{3}}}{m_e}, \tag{1.10}$$

Using plasma frequency and Fermi-velocity we can define a length scale, called the Fermi length

$$\lambda_{Fe} = \frac{\mathbf{V}_{Fe}}{\omega_{pe}}, \tag{1.11}$$

which is quantum picture of Debye length. Since it shows the length above which a positive charge is completely screened and thus a plasma approximation is valid. The quantum coupling parameter is defined in term of Fermi temperature T_F as,

$$\Gamma_Q = \left(\frac{1}{n\lambda_{Fe}^3}\right)^{\frac{2}{3}}, \tag{1.12}$$

while the classical coupling parameter is defined by

$$\Gamma_C = \left(\frac{1}{n\lambda_D^3}\right)^{\frac{2}{3}}. \tag{1.13}$$

Eq. (1.12) and Eq. (1.13) are equivalent, but the first has no classical part and inscribes the quantum coupling parameter in the form of ratio between the energy of an elementary excitation and Fermi energy. By means of quantum plasma models we can study the motions of electrons in metals as well as metallic nanostructures like nanoparticles, thin films and clusters etc. [18].

In these days, there has been a great deal of attention in examining quantum plasma which is described by high number density of particle and low temperature in comparison to the classical plasma which has high temperatures and low plasma particle concentration [19]. Quantum plasmas are common in dissimilar atmospheres, e.g. in intense laser-solid density plasma experiments [20, 21] and in ultra-small electronic devices [22], quantum dots and nanowires [23], carbon nanotubes [24], quantum diodes [25], in super dense astrophysical bodies [26] (i.e. the inner portion of Jupiter and massive white dwarfs magnetors, and neutron stars), micro plasmas [27], ultra-cold plasmas [28], biophotonics [29] and Laser produced plasma [30]. Different region of plasma is shown in the Fig 1.3.

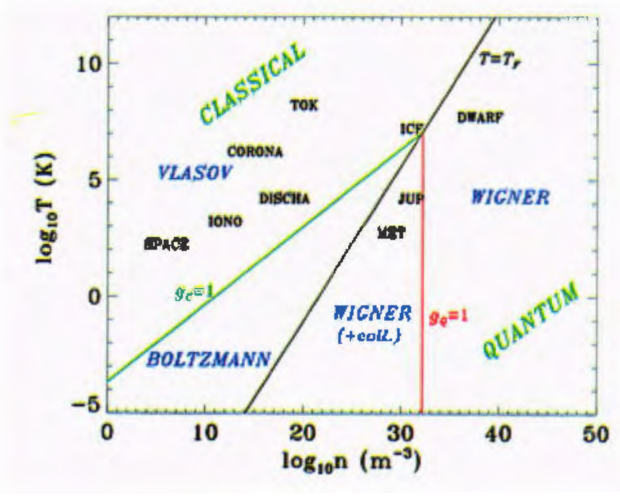


Figure 1.3: Plasma diagram in the log T - log n plane. IONO: ionospheric plasma; SPACE: interstellar space; CORONA: solar corona; DISCHA: typical electric discharge; TOK: tokamak experiment (magnetic confinement fusion); ICF: inertial confinement fusion; MET: metals and metal clusters; JUP: Jupiter's core; DWARF: white dwarf star

1.4 Degeneracy Effect in Quantum Plasma

Degenerate plasma have vast applications in strong laser produced plasmas, dense and compact astrophysical plasmas likewise white dwarfs, neutron stars and high density electronic devices [31]. In degenerate plasma the equilibrium distribution of electrons/positrons changes from Maxwellian (classical plasmas) to Fermi-Dirac distribution (quantum plasmas). For electrons, Fermi-Dirac distribution function can be written as

$$f(v,r,t) = \frac{A}{1+e^{\beta(\varepsilon-\mu_e)}} \tag{1.14}$$

Where $\beta = \frac{1}{k_B T_e}$, $\varepsilon = \frac{1}{2}m_e v_e^2$, k_B is Boltzmann's constant, T_e is the electron temperature, v_e is velocity of electron, A is normalization constant and μ_e is a chemical potential regarded as a function of space and time. On the basis of above equation the number of electrons (n_e) can be written as

$$n_e = \int_0^\infty f(v,r,t)dvdrdt \tag{1.15}$$

inserting the value of $f(v,r,t)$ from (1.14) in (1.15), yields

$$n_e = 2\int_0^\infty f(v,r,t)dv \times 2\int_0^\infty f(v,r,t)dr \times 2\int_0^\infty f(v,r,t)dt = \{2\int_0^\infty f(v,r,t)dv\}^3$$

Where 2 on the right side is because of electron spin

$$n_e = \{2\int_0^\infty \frac{A}{1+e^{\beta(\frac{1}{2}m_e v_e^2)\times e^{-\beta\mu}}}\}^3$$

Let $s = \beta(\frac{1}{2}m_e v_e^2) \Rightarrow ds = \beta(m_e v_e)dv \Rightarrow dv = \frac{ds}{\beta m_e v_e}$ and $\frac{1}{v_e} = \sqrt{\frac{m_e\beta}{2s}}$
So $dv = \frac{ds}{\beta\mu_e} \times \sqrt{\frac{m_e\beta}{2}} \times s^{-1/2}$, then

$$n_e = \{2A\int_0^\infty \frac{\sqrt{\frac{m_e\beta}{2}}}{m_e\beta} \times \frac{s^{-1/2}ds}{1+e^s \times e^{-\beta\mu_e}}\}^3$$

By simplification, we get

$$n_e = \{-A\sqrt{\frac{2\pi}{m_e\beta}} \times \frac{1}{\Gamma 1/2}\int_0^\infty \frac{s^{1/2-1}}{\frac{e^s}{-e^{\beta\mu_e}}-1}ds\}^3$$

Where $\frac{\sqrt{\pi}}{\Gamma 1/2}\int_0^\infty \frac{s^{-1/2-1}}{\frac{e^s}{-e^{\beta\mu_e}}-1}ds = Li_{\frac{3}{2}}(e^{-\beta\mu_e})$,then

$$A = -\frac{n_e}{Li_{\frac{3}{2}}(-\xi)}(\frac{\beta m_e}{2\pi})^{\frac{3}{2}} = 2(\frac{m_e}{2\pi\hbar})^3 \quad (1.16)$$

$Li_{\frac{3}{2}}$ represents polylogarithmic function of argument ($-\xi = -e^{\beta\mu}$) and order $\frac{3}{2}$. Polylogarithm is a special function $Li_s(Z)$ of order s and argument Z. Only for special value of "s" (like $s = 1$) the polylogarithm reduces to an elementary function such as natural logarithm or rational logarithm function. In quantum statics the poly logarithmic function appear as the closed form of integral of Fermi-Dirac distribution and the Bose-Einstein distribution and it is also known as the Fermi-Dirac integral or Bose-Einstein integral. It can also be written in term of power series (for order s, argument Z and $k = 1, 2, 3....$).

$$Li_s(Z) = \sum_{k=1}^{\infty} \frac{Z^k}{k^s} = Z + \frac{Z^2}{2^s} + \frac{Z^3}{3^s} + ... \quad (1.17)$$

Pauli exclusion Principle for fermions obeying Fermi-Dirac distribution function and Heisenberg uncertainty principle due to the wave like nature of particles give rise to the pressure in quantum plasma. The distribution function for completely degenerate limit is a constant below the Fermi speed, v_F and zero for speeds $v > v_F$, or alternatively for energies $E = \frac{1}{2}mv^2$ below and above the Fermi energy or temperature, $T_F = \frac{1}{2}m_e v_F^2$ respectively. The dense astrophysical quantum objects contain degenerate particles like electrons and holes as well as non-degenerate particles like ions. The degenerate particles exist at high number densities, and low temperature, i.e. the characteristics of all the particles are identical, They do not lose their individuality at a distance from each other [32]. In quantum plasmas, the electron and positrons/holes are considered to be degenerate particles while the ions are considered as non-degenerate particles due to their much smaller de-Broglie wavelength as compared to the electrons. Because of electron degeneracy pressure many electrons are caused to closed in a little volume. The parameter $\xi = e^{\beta\mu_e}$, which describes the degeneracy level, is defined in term of chemical potential μ_e. In complete non degenerate case $\xi \to 0$ which mean that $\beta\mu_e$, is greater than 1 and negative. While for complete degenerate case $\xi \to \infty$ which mean that $\beta\mu_e$ is greater than 1 and positive. With $\mu_e \to T_F = \frac{1}{2}mv_F^2$, The completely degenerate limit corresponds to, $\ln \xi = \frac{T_F}{T_e}$ [33]. At $\xi = 1$, which separates the nearly non degenerate, $\frac{T_F}{T_e} \ll 1$ and nearly degenerate, $\frac{T_F}{T_e} \gg 1$ limits. The polylogarithm is well explain for arbitrary degeneracy, with its asymptotic from $\xi \to \infty$ corresponding to the completely degenerate limit, $\mu_e \to T_F$, $T_e = 0$. In the other way round in the limit $T_e \to 0$ of the Fermi Dirac distribution $n(\varepsilon) = \frac{1}{e^{\frac{\varepsilon}{T_e}}/\xi+1}$. This gives $n(\varepsilon) = 0$ for $\xi \to 0$. For a complete degenerate Fermi gas $\mu_e = \varepsilon$ shows that Fermi energy become equal to the chemical potential. For nearly non degenerate we have $\xi << 1$ when an expansion in powers converges rapidly. The expansion i.e. $G_n(\xi) = \frac{-Li_{\frac{3}{2}+n}(-\xi)}{-Li_{\frac{3}{2}}(-\xi)}$ can be derived directly without the intermediate step of expanding and is exact in the sense that it applies for arbitrary degeneracy. Here $Li_{\frac{3}{2}}$ is the polylogarithm function of order 3/2 [34].

1.5 Quantum plasma Application

Along with the applications discussed in the previous section, quantum plasma have wide applications in dense Astrophysical objects. These objects are composed of degenerate particles like electrons and holes as well as non-degenerate particles like ious (due to their much smaller de-Broglie wavelength as compared to electrons). These degenerate particles exists at low temperature and high number density, i.e. the de-Broglie wavelength actually exceeds the average inter-particle distance($n_0^{\frac{-1}{3}}$). Some of the important application of quantum plasma in dense Astrophysical objects are discussed below in detail.

1.5.1 White Dwarf

A star that is in hydrostatic equilibrium because of electron degeneracy pressure rather than due to thermal pressure . The white dwarf shows a spectacular case for the occurrence of complete relativistic degenerate plasmas. They are thought to be the last phase of around 97% of all stars including our Sun. The white dwarf are developed from very massive stars specially from those stars that have mass in the range $6 \sim 8M_\odot$ [34], where $M_\odot$ is the mass of sun.

These stars have offetnally fascinating composition where the quantum properties combine the degeneracy of thermal electrons on micron scales to the stability of the white dwarf stars on large scales through the equilibrium between gravitational force and the pressure of degeneracy of thermal electrons. The white dwarf star have mass smaller than one solar mass and the radius smaller than 10^{-6} solar radius due to high surface emissivity and low luminosity, which has average volume densities in the range of $\sim 10^{9} - 10^{12}$ $kg\ m^{-3}$.

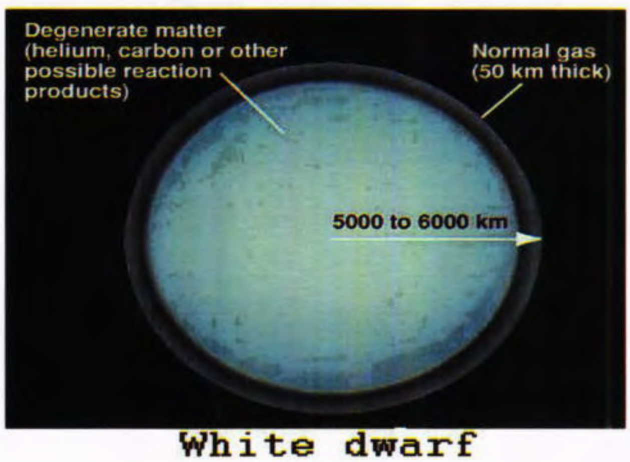


Figure 1.4: sketch of white dwarf.

Several interesting properties exist for degenerate matter. For example, if the mass of the white dwarf star is increased (which enhance the gravitional force), then the electrons are forced to "squeeze" together even more as a result the radius of star actually decrease. Due to electromagnetic radiation, physical and dynamically characteristics of white dwarf stars are studied [35]. Nowadays, there are numerous comments of oscillating white dwarf stars. Non-radial gravity oscillation mode (g-mode) is assigned to the pulsation rate that lies in the range 2-25 minutes. The asteroseismology is investigating the rotation time period, mass in addition to the equation of State due to theoretical and observational clues of white dwarf, where ions are inertial and the degenerate thermal electrons provide the restoring force [36, 37]. The duration of oscillation of universally propagating acoustic mode (p-mode) is recognized through the time for the wave to travel over the stars as well as comes within the range of a couple of seconds, two orders of magnitude smaller than the non-radial gravity oscillation modes (g-modes). Experimental study has not been carried out so far on these modes [38]. But the lack of information did not suggest the non appearance of acoustic modes, however may be concerned with the dynamics below the detection boundary [39]. The chance of the formation of the finite amplitude sound waves is also proposed in the case of events such as supernova explosions. [40].

1.5.2 Neutron Star

A neutron star is an astrophysical object that contain complete relativistic degenerate quantum plasmas. They are over dense degenerate matter which exist in the galaxies. The mass of neutron star is about 1.4 times of our sun and having diameter approximately $20 \times 10^3\ m$. Because of its less size and large density we can guess that neutron star is highly dense that one tea spoon full of neutron star would weight a billion tons. The gravitational field of neutron star is about 2×10^{11} times stronger than that of earth and also the magnetic field of neutron star is million times stronger as compared to the magnetic fields produced on earth. Actually these types of stars contain massive amount of neutrons in its core that's why they are termed as neutron stars. Along with huge amount of neutrons in its core, hopefully they will contain slight amount of protons as well as electrons to neutralize the matter. Most of the neutron stars contain iron as their main component, but the outer layers which comes in contact with outer atmosphere might collect lighter elements above the core of iron layer [41]. An image of neutron star is shown in the Fig 1.5.

The neutron star radius is estimated in the range of $R \sim 10-14\ km$ and a basic mass M around $1-2M_{\odot}$, the density is calculated up to $\rho \sim 10^{17} - 5 \times 10^{18} kg.m^{-3}$ [35], which is round about 3 times the typical density of atomic nucleus, $\rho_0 = 2 \cdot 8 \times 10^{17} kg.m^{-3}$. These stars can change its density, which in some cases are higher than the atomic density ρ_0. Because of their small size these stars are collapsed more by inward gravitational force as compared to white dwarf stars. In cases where the neutron star's core shrinks, a neutron degeneracy pressure is created due to the flattening of neutrons. Due to this flattening effect the neutron star resists the degeneracy and that creates the more stable neutron star [41].

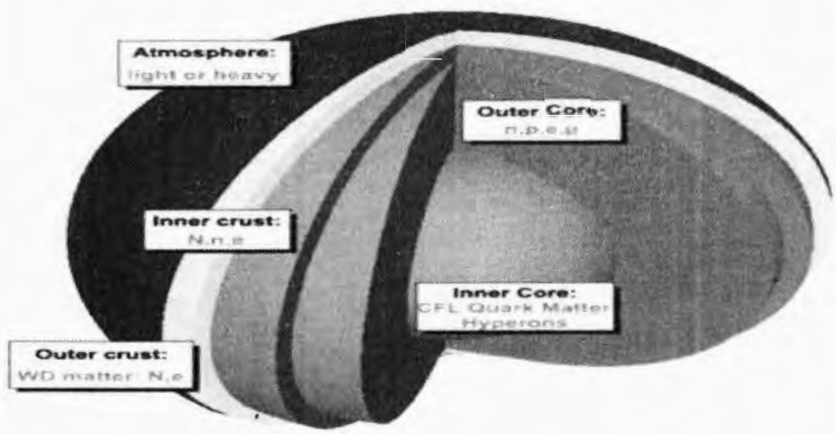


Figure 1.5: Cross-section through the interior of a neutron star. The neutron star is surrounded by a thin atmosphere and an outer crust consisting of heavy nuclei and electrons. The inner crnst consists of nuclei, neutrons and electrons, which at nuclear density make a transition to a neutron fluid. The composition of the central core is still unclear, but certainly consists in the outer part only of neutrons, protons, electrons and muons.

1.5.3 Supernova

Supernova is the decay of massive stars, one of the most energetic events (explosions) that take place in our Universe [42]. The energy which a supernova can emit in few days is more than that of the sun radiates in its whole lifetime [43]. They are also considered as the primary cause of heavy elements in the universe. According to NASA, the largest explosion that takes place in our universe is supernova [44]. On average, in every 50 years a supernova will happen once in a galaxy. The energy emitted in a core-collapse supernova explosion is around 10^{46} joule [45]. Ahout 99% of this energy is radiated in the form of neutrinos, which leaves a mechanical energy around 10^{44}joule [46]. This energy is still enough to eject matter at velocities up to 104000 m/s. Apart from being spectacular events in the Universe, many types of information can be obtained from supernova explosions and their after effects. For example, estimates of late stellar evolution can be made, help us to understand shock physics and triggered star formation [47]. An image of supernova is shown in Fig 1.6.

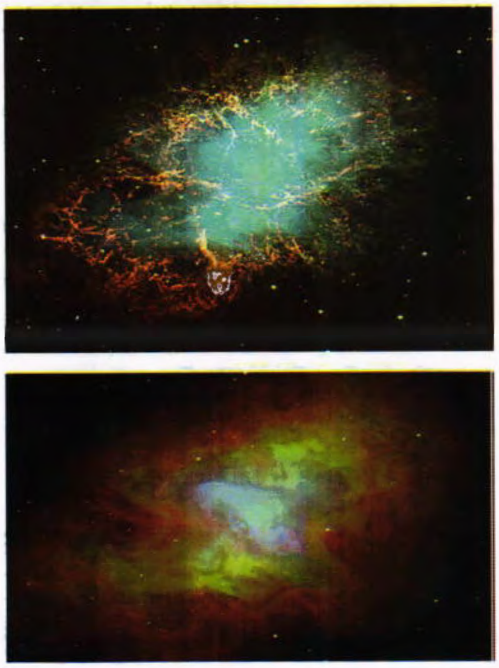


Figure 1.6: The Crab Nebula, filled with gaseous filaments, is the result of a star that was seen to explode in 1054 AD. Red indicates the electrons are recombining with protons to form neutral hydrogen, while blue indicates synchrotron emission from the inner nebula. The lower image shows synchrotron emission of plasma accelerated by tremendous electric voltages created by the central pulsar (red: radio emission, green: visible emission, blue: X-ray emission). The inner ring, with prominent knots, of this X-ray nebula is about one lightyear in diameter. The Crab pulsar is the hot spot in the center of the torus-like structure.

1.5.4 Black Hole

Mathematically a black hole is the portion of space-time showing such a high gravitational force that no particle or EM radiation can escape from it. For the first time black holes were predicted by Albert Einstein, on the basis of his general theory of relativity. The theory of general relativity shows that a huge and dense object can deform space-time to form a black hole. However, for proper understanding of black holes we need to include quantum effects [48].

In 1987 two scientists Roger Penrose and Stephen Hawking suggested that on the basis of general theory of relativity any object that collapses to make a black hole will keeps on to collapse to a singularity inside the black hole. This means that on small distance inside a black hole there exist strong gravitational effects. To understand the collapsing matter inside black hole on smaller distance, we surely need a quantum theory. The existence of a singularity in the classical theory similarly means that once we go appropriately far inside the black hole, we will no longer be able to guess that what will happen. Hopefully this letdown of the classical physics can be preserved by quantizing gravity as well [49].

Black holes are strange and most attractive matters found in outer space. There are high number of much smaller black holes lies around the universe. Collapsing of stars is not their only reason while black holes may be formed by collapsing of highly compressed matter in the hot, dense medium that is believed to exist shortly after the "big bang" in which the universe came into existence. Such "primitive" black holes are of greatest interest on quantum point of view. Although black hole are very large in number in universe among them the well-known black holes are three. Thesc are primordial (miniature) black holes, stellar black holes and supermassive black holes. An image of black hole as shown in Fig 1.7.

A black hole having weight of billion tons (near the mass of a mountain) would have a radius of about 10^{-13} cm which is approximately equal to the size of a neutron or a proton. It could be in orbit whichever around the sun or about the center of the galaxy [49].

1.6 Quantum Hydrodynamic Model

The formulation of quantum hydrodynamic, which shows behavior in quantum mechanical subsystems is under study from the time of Schrodinger wave equation. It was Madelung in the begiuning era of quantum mechanics, who suggested that the Schrodinger wave equation for spinless uni-electron problems can be converted into the form of quantum hydrodynamics (QHD) equations. By considering the wave function of the form $\Psi = \alpha e^{i\beta}$ with real valued α and β, which depends on time. From Schrodinger wave equation Madelung obtained continuity equation and Euler equation . After a long break, Bohm and others played an important role in the development of quantum hydrodynamic equations. The so-called Madelung hydrodynamics is commonly considered as an initiative of the Bohmian mechanics, a quantum theory deals with reasonable explanation in terms of hidden parameters [50] in which the reinterpre-

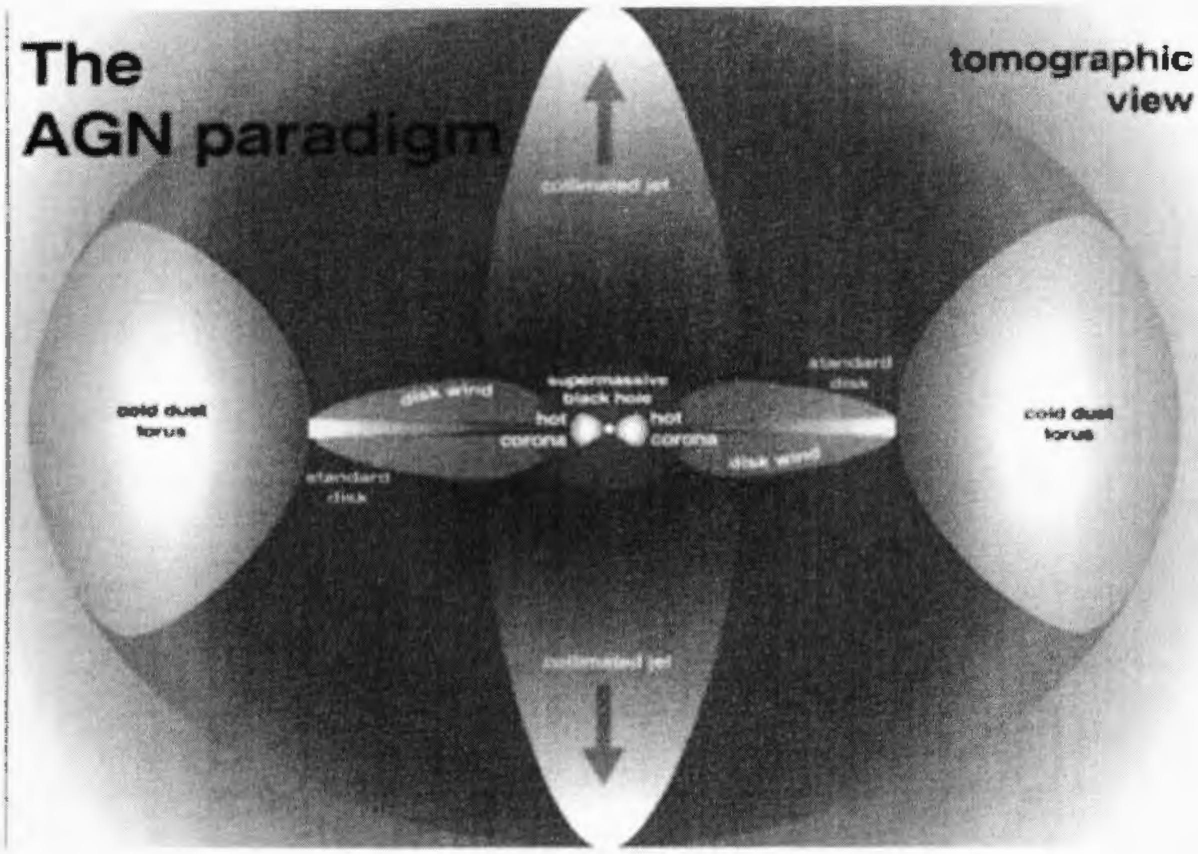


Figure 1.7: An artist's conception of a supermassive black hole sitting in the center of a galaxy. Credits: Andreas Muller (ZAH, LSW Heidelberg)

tation of the solution of the Schrodinger equation and concerned events on the lines of classical mechanics was suggested. This explanation is also known as de Broglie-Bohm theory on the basis of the idea of the pilot-wave by Louis de Broglie and Bohm put forward this explanation to its logical conclusion. Quantum hydrodynamic (QHD) are applicable to only quantum degenerate electron. So few conditions are applied to the electron component. The plasma is ideally weakly coupled which means that all types of interactions are much smaller than the quantum kinetic energy $r_s \ll 1$, where r_s is weak coupling parameter. The interconnection of the particles are dealing in the mean field approximation and inscribed by the induced electrostatic potential, no EM and quantization effects are analyzed. The phase velocity of wave alone with the velocities of particles are non-relativistic i.e. $\frac{\omega}{k} < c$, and no energy transport equation is take into accounts. This could be possible by considering the second order moment of the Wigner function. However, the inclusion of a magnetic field in QHD is straight forward by starting from a quantum kinetic equation with an EM field inclusion. QHD model has been broadly used in resounding burrowing diode, super liquid, and electrical conduction [51]. New interest in QHD models are found in gaseous quantum plasmas [52], and other important applications to the dense plasmas found in some astrophysical objects like white dwarfs, giant planets and in experiments on inertial confinement fusion (ICF) [53]. Following are the mathematical formulation of quantum hydrodynamic (QHD).

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{v}_e) = 0 \tag{1.18}$$

$$n_e m_e \frac{\partial \mathbf{v}_e}{\partial t} = -e n_e (\mathbf{E} + \mathbf{v}_e \times \mathbf{B}) - \nabla p_e + \frac{\hbar^2}{2m_e} \nabla \left(\frac{\nabla^2 n_e^{\frac{1}{2}}}{n_e^{\frac{1}{2}}} \right) + \frac{2\mu}{\hbar} \nabla (\mathbf{B}_0 . \mathbf{S}_e) \tag{1.19}$$

And maxwell's equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}. \tag{1.20}$$

$$\nabla \times \mathbf{B} = \mu_o (\mathbf{j}_e + \mathbf{j}_m + \epsilon_o \frac{\partial \mathbf{E}}{\partial t}) \tag{1.21}$$

where n_e is number density, $\mathbf{v}_e$ is the velocity , m_e is the mass of electron, p_e is the pressure of electrons respectively, μ_0 is magnetic permeability, ϵ_o is permittivity of free space, $\mathbf{j}_e$ is the free electron current density and $\mathbf{j}_m$ is electron magnetization current density.

1.7 Waves in plasmas

Oscillation of fields and particles in regular interval generate waves in plasma. Basically plasma contain thermal electrons and positive ions, but it may contains different ion species like positrons, negative ions, dust and neutral particles. Due to complex set of particles and fields Plasma can supports a larger variety of waves. The detail study of these waves in plasma are very important for plasma diagnostics, because the wave modes of a plasma depend on the plasma characteristics. On the basis of oscillating magnetic field waves in plasmas can be classified as electrostatic or EM waves. By using a plane wave solution Faraday's law of induction can be interpreted as, $\mathbf{k} \times \mathbf{E} = \omega \mathbf{B}$, which verify that an electrostatic wave should be longitudinal and on other hand an EM wave must have a transverse component, but may also be partially longitudinal [54]. On the basis of oscillating species electrostatic and electromagnetic waves can further be classified as under.

1.7.1 Electrostatic waves

Those waves having only electric field component with zero magnetic field component are termed as electrostatic waves. Vlasov equation has been verified the existence of electrostatic waves. In small amplitude limit this wave is time and space dependent, which can be written as

$$\exp(ik \cdot x - i\omega t) + c.c(\text{complex} \ \rightarrow \text{conjugate}) \tag{1.22}$$

Where $\mathbf{k}$ is propagation vector (number) and ω is angular frequency. $\mathbf{k}$ represents the propagation direction of wave which is always parallel to the electric field $\mathbf{E}$. i.e. $\mathbf{k}||\mathbf{E}$ for electrostatic wave [4]. By adopting the above statement ($\mathbf{k}||\mathbf{E}$) the electrostatic waves can also be proved from one of the Maxwell's equation.

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{1.23}$$

using plane wave solution Eq. (1.23) becomes

$$\mathbf{k} \times \mathbf{E} = \omega \mathbf{B}. \tag{1.24}$$

Thus it is clear from the Maxwell's Equation, that for wave to show an electrostatic nature $\mathbf{k}$ must be parallel to $\mathbf{E}$.

To understand quantum mechanically an electrostatics wave (high frequency longitudinal waves), considering a wave having propagation vector $\mathbf{k}$ and angular frequency

ω propagating in a plasma, given by the quantum Vlasov equation, as

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial x} = \int dvk_{\phi}[\phi\,|\mathbf{v}' - \mathbf{v}, \mathbf{x}, t]\;\; f(\mathbf{x}, \mathbf{v}, t) \tag{1.25}$$

Where ϕ is the electrostatics potential and $k_{\phi}[\phi\,|\mathbf{v}' - \mathbf{v}, \mathbf{x}, t] = \frac{iem}{\hbar}\int \frac{ds}{2\pi\hbar}\exp(\frac{iem(\mathbf{v}'-\mathbf{v})s}{\hbar}) \times (\phi(\mathbf{x}+\frac{s}{2}, t) - \phi(\mathbf{x} - \frac{s}{2}, t))$

for first-order disturbances it is supposed an equilibrium quantum Vlasov function $f = f_0(\mathbf{v})$ such that $\int dv f_0(\mathbf{v}) = n_0$ and a zero equilibrium electrostatic potential. Linearization and simplification of Eq. (1.25), give the result as

$$\omega^2 = \omega_p^2 + 3\mathbf{k}^2\mathbf{v}^2 + \frac{\hbar^2\mathbf{k}^4}{4m^2} \tag{1.26}$$

Eq. (1.26) is the Bohm–Pines dispersion relation, the quantum counterpart of the Bohm–Gross dispersion relation of classical high frequency longitudinal plasma waves [12]. Some other examples of electrostatic waves in plasma include ions acoustic waves (IAW), lower hybrid waves (LHW), uper hybrid waves (UHW), ion cyclotron waves (ICW) and electrostatic drift waves etc.

1.7.2 Electromagnetic Waves

A Scottish mathematician named James clerk Maxwell in 1873 give a new idea about electromagnetic waves. According to his idea electromagnetic wave consist of oscilating electric and magnetic field , both of which are perpendicular to the propagation vector **k**. This idea combined the subjects of electricity and magnetism and form the foundation of modern electromagnetism as shown in Fig 1.8.

Maxwell had the advantage of his ancestor's brilliant experiments and that must not be neglected. He kept the experimental laws of Faraday, Ampere and Gauss on affirming the mathematical foundation and made an important contribution to ampere's law that allowed the prediction of electromagnetic waves. Electromagnetic waves carry energy or momentum from one point in space to another point by means of electric and magnetic field. An accelerated charge particle is necessary for the generation of EM waves [55].

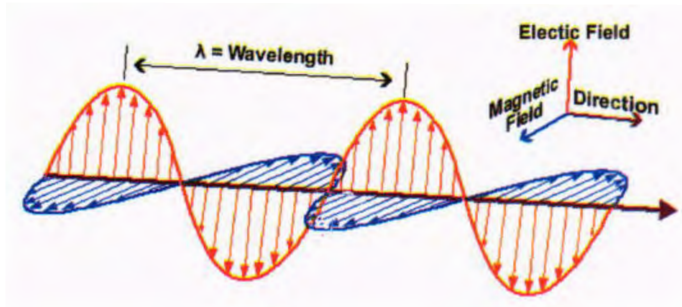


Figure 1.8: Geometry of electromagnetic waves with electric field $\mathbf{E}$, and magnetic field $\mathbf{B}$.

Thus any electromagnetic waves consist of perturb electric and magnetic field both of which are always perpendicular to the direction of propagation. By using Maxwell's Equation Eq. (1.23) we can differentiate between electrostatic and electromagnetic waves, as discussed and proved previously that a wave with zero perturbation in magnetic field i.e. ($\mathbf{B}_1 = 0$) is called an electrostatic wave. while for an electromagnetic wave the perturb magnetic field should not equal to zero ($\mathbf{B}_1 \neq 0$), because in Maxwell's equation $\mathbf{k}$ must be perpendicular to $\mathbf{E}$.

To study EM wave quantum mechanically, assuming Eq. (1.18), (1.19) and Poisson equation along with fermi pressure ($p_F = \frac{m\mathbf{v}_F^2}{3n_0^2}n^3$, where $\mathbf{v}_F = \frac{\pi\hbar n_0}{2m}$) for a wave having frequency ω and wave number $\mathbf{k}$. Linearizing around the homogeneous equilibrium

$n = n_0, \qquad \mathbf{v} = 0, \qquad e\phi = m\mathbf{v}_F^2/2,$

give the following dispersion relation

$$\omega^2 = \omega_p^2 + \mathbf{k}^2\mathbf{v}_F^2 + \frac{\hbar^2\mathbf{k}^4}{4m^2} \tag{1.27}$$

This is a further indication that the effective Schrodinger–Poisson system is a good approximation to the complete Wigner–Poisson system for long wavelengths. In the same way for $\Gamma_Q \to 0$, the dispersion relation become

$$\omega^2 = \omega_p^2 + \mathbf{k}^2\mathbf{v}_F^2$$

This is exactly the dispersion relation derived from the classical Vlasov–Poisson equation with a zero-temperature Fermi–Dirac equilibrium. Alternatively, when the quantum coupling parameter is vanishingly small, a classical dynamic equation can be used, as the only quantum effects come from the Fermi–Dirac statistics [12].

Ordinary and Extraordinary Waves

Assuming a magnetized plasma with external magnetic field $\mathbf{B}_0$ and wave electric field $\mathbf{E}_1$. Electromagnetic (EMW) waves can further be classified into ordinary waves (O-waves) and extraordinary waves (X-waves), on basis of relative propagation of electric and magnetic fields. If both fields are parallel to each other then such waves are called Ordinary waves or (O-waves). Generally ordinary wave is not affected by the magnetic field. When electromagnetic waves propagate along the magnetic field $\mathbf{B}_0$ then circularly polarized waves are formed likewise, right hand circular polarized (R-wave) and left hand, L-wave. The electric field vector for R-waves rotate clockwise in time in the direction of $\mathbf{B}_0$ and L-waves rotate anticlokwise in the direction of $\mathbf{B}_0$. In this case the direction of rotation vector $\mathbf{E}_1$ and wave number $\mathbf{k}$ are independent of each other as shown in Fig 1.9 [7].

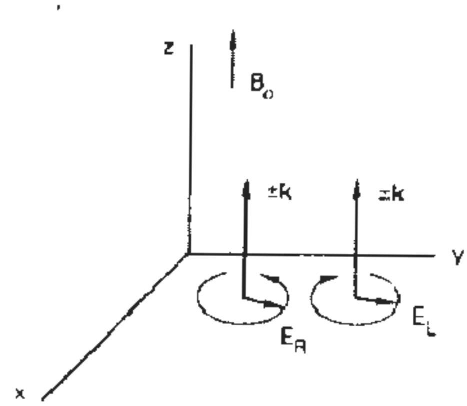


Figure 1.9: Geometry of circular right and left hand waves

Furthermore when electric and magnetic field are perpendicular to one another then electron dynamics will he effected by magnetic field $\mathbf{B}_0$ which will effect the dispersion relation . The X-waves with electric $\mathbf{E}_1 \perp \mathbf{B}_0$ tend to be elliptically polarized and did not show plane polarization. Such a waves propagate into a plasma, develop an $\mathbf{E}_{1x}$ component along wave number $\mathbf{k}$, thus the waves become partly longitudinal and partly transverse. To treat this X-mode properly E_1 should have both x and y component. The electric field vector of extraordinary waves has elliptically polarized component E_{1x} and E_{1y} which oscillate out of phase so that the total electric field vector has a tip that moves in an ellipse, once in each wave time period as shown in Fig 1.10 [8].

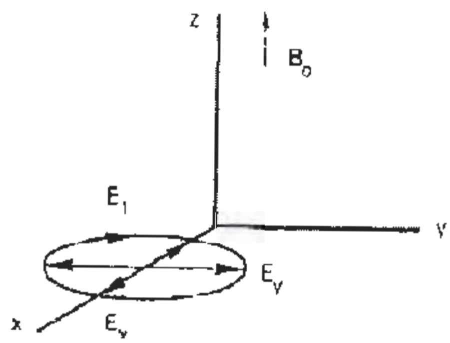


Figure 1.10: Geometry of electric field components for elliptically polarized waves

1.8 Layout of Thesis

Outline of thesis has been described in the following way: The first chapter of dissertation deals basically to the introduction of the subject, its application and importance. We give the detail of the definition, historical background, and occurrence of plasma. The difference between classical and quantum plasmas is discussed. Arbitrary temperature degeneracy effects in quantum plasma are discussed, Application of quantum plasma in dense astrophysical objects are discussed .The suitable approach i.e. quantum hydrodynamic model detail are given. Different wave modes i.e. electrostatic and electromagnetic waves are studied.

The chapter 2 is divided into two sections. In first section, we derive the dispersion relation for EM wave modes for parallel and perpendicular propagation, for both nearly non degenerate and nearly degenerate quantum plasma by using quantum hydrodynamic model. And discuss the effects of Bohm potential and arbitrary temperature degeneracy on the wave dispersion. In 2nd section, we derive the resonant and cut-off frequencies only for perpendicular propagation, along with discussion of the effects of Bohm potential and arbitrary temperature degeneracy.

In chapter 3 the analytical results are discussed numerically. The effect of temperature, number density and magnetic field are observed on different modes, for both nearly non degenerate and nearly degenerate quantum plasma

In chapter 4 we give the summary and conclusion of the work.

Chapter 2

Electromagnetic waves in spin magnetized plasmas with temperature degeneracy effects

2.1 Introduction

The wave function of particles start overlapping in case of plasma containing large number of charge carrier (high density) and having low temperature. In such situation we can say that the wavelength associated with particles (de Broglie wavelength) is large as compared to the interparticle distances, then quantum effects start playing a role. For plasma to behave quantum mechanically it should obey certain condition as discussed by Manfredi [66]. It must be mentioned that the equilibrium distribution of the degenerate electrons obey the Fermi-Dirac statistics in dense quantum plasmas and there are new aspects of collective behavior due to the forces involved electron tunneling as a result of quantum Bohm potential and electron spin effects. Quantum plasma can be described by three known models the Wigner-Poisson (WP) model (in the presence of magnetic fields the so called Wigner-Maxwell model), Hartree model and quantum hydrodynamic (QHD) model. The WP model explain the statistical aspects of quantum plasmas whereas the Hartree model explain the hydrodynamic aspects. The QHD model, illustrating the transport of some microscopic variables such as charge momentum and energy in plasmas, has been introduced to deal with some issues in semiconductor physics [67]. The concept of spin magneto hydrodynamics introduced by Brodin and Marklund has attracted much interest [68]. Similarly quantum effects including the Bohm potential and electron spin effects have been studied by many researchers [69]. The temperature degeneracy can be investegated via chemical potential μ in Fermi distribution (FD), discussed with detail in chapter 1.

2.2 Literature Review

M. Marklund and G. Brodin for the first time presented fully nonlinear governing equations for spin1/2 quantum electron plasmas. Started from the Pauli equation describing the nonrelativistic electron behavior, they have shown that the electron-ion plasma equations are subjected to spin-related terms, and under certain circumstances the collective spin effects can dominate the plasma dynamics [68]. Later on Marklund and Brodin have shown that these particles may constitute electrons, positrons (albeit non-relativistic) and holes [70]. Shukla et al. [71] have derived a set of nonlinear equations for finite amplitude EM waves in highly dense self-gravitating dense magnetoplasmas. Furthermore, they shown that the electron 1/2 spin force should play a role in plasma assisted nanoscale structures in plasmonic devices . Mushtaq et al. have investigated the dispersive characteristic of low frequency magnetosonic waves in magnetized degenerate quantum EPI (electron, positron and ion) plasmas with effects of quantum diffraction, temperature degeneracy, positron concentration and spin-1/2 magnetization. They have observed that quantum effects, and in particular the electron and positron spin, may produce new and interesting aspects in plasma theories and experiments [72]. Ren, Wu, and Chu have studied the dispersion properties of linear waves in cold quantum magnetized plasmas. Their results have shown that the quantum corrections have an important influence on the dispersion properties of the Langmuir wave. Due to quantum influence, the electrostatic oscillation becomes a Langmuir wave, which has weak propagation and group velocity enhancing with the frequency [73]. Mushtaq and Qamar have studied linear and nonlinear dispersion of obliquely dispersive magnetosonic waves in quantum plasma, in the presence of magnetic field. The linear dispersion relations for slow and fast quantum magnetosonic waves have been inscribed in detail especially with reference to the effects of quantum corrections and obliqueness [74]. Haas and Mahmood have studied the linear and nonlinear ion-acoustic waves in a non-relativistic quantum plasma with arbitrary degeneracy of electrons. Along with degeneracy, the quantum diffraction effect of electrons was also considered in terms of the Bohm potential. They concluded, that derivation covers both the basic quantum effects in plasmas (arising from quantum statistics and wave-like behavior of the charge carriers), in both the dilute and dense regimes [75]. Maroof et al. have investegated the propagating aspects of magnetohydrodynamic waves in spin magnetized dusty quantum plasma by incorporating the effects of exchange and correlation on electrons. Started from one-dimensional quantum hydrodynamic equations, incorporating the terms of spin magnetization and exchange correlation, they derived a generalized dispersion relations [76]. Andreev has analyzed the non-linear spin-electron acoustic waves, by developing a generalization of the separate spin evolution quantum hydrodynamics. This generalization included the Coulomb exchange interaction, which appears from the interaction of the spin-down electrons being in states occupied by one electron only [77]. Maroof et al. have studied the properties of high and low frequency (MHD) waves in a magnetized degenerate (EPI) plasmas by including the relativistic degeneracy pressure effects of electrons and positrons. They have obtained a generalized dispersion relation with oblique geometry by considering the inertia of all three species. The electrons and positrons due to their low inertia were treated quantum mechanically, whereas the ions were taken as classical and non-degenerate [78]. Li et al. have studied the quantum corrections to the elliptically polarized x-waves while ignoring quantum electrodynamics (QED) and relativistic effects. They have shown the modification of the group velocity of the x-wave due to the quantum forces and magnetization effects within a certain range of wave numbers. It means that the quantum spin1/2 effects can minimize the transformation of energy in such quantum plasma [79]. Hu et al. have investigated the spin effects on the electromagnetic (EM) wave propagating in arbitrary direction in plasma, in the presence of external magnetic field. They have derived the dispersion relations for EM wave propagating parallel and perpendicular to the applied magnetic field. They have shown that the spin effects influence the lower frequency modes in comparison to high frequency modes. The significance of work in understanding the dense and compact astrophysical objects and the condensed matter physics were pointed out [63].

To the best of our knowledge no one has analyzed this problem [63] with inclusion of Bohm potential and degenerate finite temperature effects. The motivation of our work is to study electromagnetic waves in spin magnetized plasma with arbitrary temperature degeneracy effects. The cutoff and resonant frequencies are also investigated in spin quantum plasma (in case of $\theta = \frac{\pi}{2}$), for both nearly non degenerate and nearly degenerate plasma. The arbitrary degeneracy in temperature has been taken from the expansion of chemical potential μ involved in Fermi-Dirac distribution function. The detail of these calculation were discussed by Melrose and Mushtaq [65], and by Hass and Shahzad [75], and all these were discussed in chapter 1.

2.3 Basic Formulation and Dispersion Relation

Consider a quantum electron-ion plasma with quantum corrections. The ions are supposed to be stationary, since their inertia is too high for response to high-frequency electromagnetic wave. On the basis of this reason, ions are treated classically, while one component electrons plasma is considered to be placed in an external background magnetic field $\mathbf{B}_0$ in $\hat{z}-$direction, wave vector $\mathbf{k}$ lies is in xz plane i.e. $\mathrm{k} = k_\perp \hat{x} + k_\parallel \hat{z}$. And electric field $\mathbf{E}$ has components in all three directions as shown in Fig 2.1 [63].

In order to analyze the electromagnetic wave in spin magnetized quantum plasmas with arbitrary temperature degeneracy effects, we start with the linearized form of Maxwell's equations.

The Faraday law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{2.1}$$

The Ampere law

$$\nabla \times \mathbf{B} = \mu_o(\mathbf{j}_e + \mathbf{j}_m + \epsilon_o \frac{\partial \mathbf{E}}{\partial t}) \tag{2.2}$$

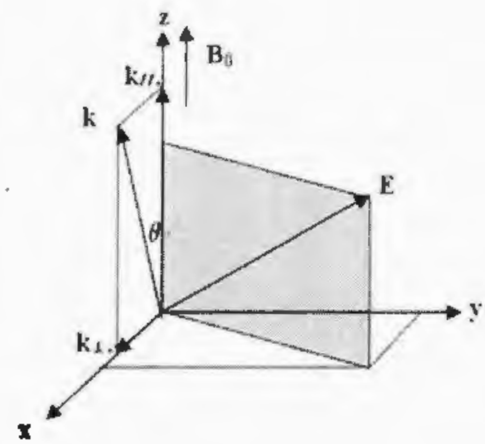


Figure 2.1: Cartesian coordinate system, chosen such that B_0 is along $\hat{z}$, and $\hat{y}$ is perpendiculer to the plane formed by **k** and B_0.

Where μ_0 is the permeability of free space, ϵ_0 is the permittivity of free space, $\mathbf{j}_e$ is the conduction current density and $\mathbf{j}_m$ is magnetization current density, due to spin.

We separate the external magnetic field and the wave magnetic field by letting $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}$ and using the plane wave solution i.e. putting $\nabla = i\mathbf{k}$ and $\frac{\partial}{\partial t} = -i\omega$ Eq. (2.1) can be written as

$$\mathbf{k} \times \mathbf{E} = \omega \mathbf{B} \tag{2.3}$$

where ω is the frequency

And Eq. (2.2) can be expressed as

$$i\mathbf{k} \times \mathbf{B} = \mu_o(\mathbf{j}_e + \mathbf{j}_m - i\omega\epsilon_o\mathbf{E}) \tag{2.4}$$

By putting value of **B** from Eq. (2.3) in (2.4), gives us

$$i\mathbf{k} \times \frac{\mathbf{k} \times \mathbf{E}}{\omega} + \mu_0\epsilon_o\omega^2\mathbf{E} = -i\omega\mu_o(\mathbf{j}_e + \mathbf{j}_m)$$

$$\mathbf{k} \times \mathbf{k} \times \mathbf{E} + \mu_0\epsilon_o\omega^2\mathbf{E} = -i\omega\mu_o(\mathbf{j}_e + \mathbf{j}_m) \tag{2.5}$$

If we use the angle (θ) between $\mathbf{B}_0$ and **k** by choosing Cartesian coordinate system. Then from Fig. 2.1

Since $\mathbf{B}_0 = |B_0|\,\hat{z}$

$$\mathbf{k} = k\sin\theta\hat{\mathbf{x}} + k\cos\theta\hat{\mathbf{z}} \tag{2.6}$$

Then first term on left side of Eq. (2.5) give the result in the following form

$$\mathbf{k}\times\mathbf{k}\times\mathbf{E} = k^2\cos\theta(E_z\sin\theta - E_x\cos\theta)\hat{x} - k^2E_y\hat{y} + k^2\sin\theta(-E_z\sin\theta + E_x\cos\theta)\hat{z} \tag{2.7}$$

Using Eq. (2.7) in (2.5) , and making equation for x, y and z components, we get the below relation by following exactly the procedure of [63], as

$$\begin{pmatrix} 1-\eta^2\cos\theta^2 & 0 & \eta^2\sin\theta\cos\theta \\ 0 & 1-\eta^2 & 0 \\ \eta^2\sin\theta\cos\theta & 0 & 1-\eta^2\sin\theta^2 \end{pmatrix} \begin{pmatrix} \mathrm{E}_x \\ \mathrm{E}_y \\ \mathrm{E}_z \end{pmatrix} = \left(\frac{-i}{\varepsilon_0\omega}\right) \begin{pmatrix} \mathbf{J}_{mx} \\ \mathbf{J}_{my} \\ \mathbf{J}_{mz} \end{pmatrix} - \left(\frac{i}{\varepsilon_0\omega}\right) \begin{pmatrix} \mathbf{J}_{ex} \\ \mathbf{J}_{ey} \\ \mathbf{J}_{ez} \end{pmatrix} \tag{2.8}$$

Where $\eta^2 = \frac{c^2k^2}{\omega^2}$, $\mathbf{j}_{ex}$, $\mathbf{j}_{ey}$ and $\mathbf{j}_{ez}$ represents the current density in their respective direction.

The first term on right side of Eq. (2.8) is magnetization current density, which can be calculated by taking start with spin evolution equation for spin quantum plasma, as

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_e.\nabla\right)\mathbf{S}_e = \frac{-2\mu}{\hbar}\mathbf{B}\times\mathbf{S}_e \tag{2.9}$$

Writing $\mathbf{S}_e = \mathbf{S}_{e0} + \mathbf{S}_e$, where $\mathbf{S}_{\uparrow 0} = -\mathbf{S}_{\downarrow 0} = \frac{\hbar}{2}\hat{z}$, and $\mathbf{S}_e$ is the perturbation of spin, the lowest order of approximation of Eq. (2.9) gives

$$-i\omega\mathbf{S}_e = \frac{-2\mu}{\hbar}(\mathbf{B}_0\times\mathbf{S}_e + \mathbf{B}\times\mathbf{S}_{e0}) \tag{2.10}$$

Making x, y and z components of Eq. (2.10), we get the below relation by following exactly the procedure of [63].

$$\begin{pmatrix} \mathbf{S}_{ex} \\ \mathbf{S}_{ey} \\ \mathbf{S}_{ez} \end{pmatrix} = \frac{g\mu_B \mathbf{S}_{e0}}{\hbar\omega(1-Y'^2)} \begin{pmatrix} -Y' & i & 0 \\ -i & -Y' & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{B}_x \\ \mathbf{B}_y \\ \mathbf{B}_z \end{pmatrix} \tag{2.11}$$

Where $Y' = \frac{\omega_g}{\omega}$, with $\omega_g = \frac{g\mu_B \mathbf{B}_0}{\hbar} = (g/2)\Omega$. By letting (g) the electron g factor equal to 2, we get $Y' = Y = \frac{\Omega}{\omega}$.

Additionally from Eq. (2.3), we can calculate the x, y and z components of $\mathbf{B}$, as

$$\begin{pmatrix} \mathbf{B}_x \\ \mathbf{B}_y \\ \mathbf{B}_z \end{pmatrix} = \frac{\mathbf{k}}{\omega} \begin{pmatrix} 0 & -\cos\theta & 0 \\ \cos\theta & 0 & -\sin\theta \\ 0 & \sin\theta & 0 \end{pmatrix} \begin{pmatrix} \mathbf{E}_x \\ \mathbf{E}_y \\ \mathbf{E}_z \end{pmatrix} \tag{2.12}$$

Substituting Eq. (2.12) into (2.11), gives

$$\begin{pmatrix} \mathbf{S}_{ex} \\ \mathbf{S}_{ey} \\ \mathbf{S}_{ez} \end{pmatrix} = \frac{g\mu_B \mathbf{S}_{e0}\mathbf{k}}{\hbar\omega^2(1-Y^2)} \begin{pmatrix} i\cos\theta & Y\cos\theta & -i\sin\theta \\ -Y\cos\theta & i\cos\theta & Y\sin\theta \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{E}_x \\ \mathbf{E}_y \\ \mathbf{E}_z \end{pmatrix} \tag{2.13}$$

Considering $\mathbf{S}_{\uparrow 0} = -\mathbf{S}_{\downarrow 0}$, Eq. (2.13) implies that $\mathbf{S}_\uparrow = -\mathbf{S}_\downarrow$. So, the magnetization current density can be written as

$$\mathbf{j}_m = \nabla \times (\mathbf{M}\uparrow + \mathbf{M}\downarrow) = \frac{2\mu}{\hbar}\nabla \times (n_\uparrow \mathbf{S}_\uparrow + n_\downarrow \mathbf{S}_\downarrow) = \frac{2\mu}{\hbar}(n_\uparrow - n_\downarrow)\nabla \times \mathbf{S}_\uparrow \tag{2.14}$$

In the nondegenerate plasmas ($T >> T_F$), $n_\uparrow - n_\downarrow = n\tanh(\frac{\mu_B \mathbf{B}_0}{k_B T_e})$, and in the degenerate plasmas ($T << T_F$), $n_\uparrow - n_\downarrow = \frac{3n\mu_B \mathbf{B}_0}{2k_B T_F}$, where T_F is the Fermi temperature of the electrons.

Making x, y and z components of Eq. (2.14), we get the below relation by following exactly the procedure of [63], as

$$\begin{pmatrix} \mathbf{J}_{mx} \\ \mathbf{J}_{my} \\ \mathbf{J}_{mz} \end{pmatrix} = \frac{-ig\mu_B(n_\uparrow - n_\downarrow)k}{\hbar} \begin{pmatrix} 0 & -\cos\theta & 0 \\ \cos\theta & 0 & -\sin\theta \\ 0 & \sin\theta & 0 \end{pmatrix} \begin{pmatrix} \mathbf{S}_{ex} \\ \mathbf{S}_{ey} \\ \mathbf{S}_{ez} \end{pmatrix} \tag{2.15}$$

Using Eq. (2.13), Eq. (2.15) can be written as

$$\begin{pmatrix} \mathbf{J}_{mx} \\ \mathbf{J}_{my} \\ \mathbf{J}_{mz} \end{pmatrix} = -\frac{ig^2\mu_B^2\mathbf{S}_0(n_e\uparrow - n_e\downarrow)\mathbf{k}^2}{\hbar^2\omega^2(1-Y^2)} \begin{pmatrix} Y\cos\theta^2 & -i\cos\theta^2 & -Y\sin\theta\cos\theta \\ i\cos\theta^2 & Y\cos\theta^2 & -i\sin\theta\cos\theta \\ -Y\sin\theta\cos\theta & i\sin\theta\cos\theta & Y\sin\theta^2 \end{pmatrix} \begin{pmatrix} \mathbf{E}_x \\ \mathbf{E}_y \\ \mathbf{E}_z \end{pmatrix} \tag{2.16}$$

In order to find the second term on right side of Eq. (2.8) , let's take equation of force with an electrons spin and arbitrary temperature degeneracy effects as

$$n_{0e}m_e\frac{\partial \mathbf{v}}{\partial t} = -en_{e0}(\mathbf{E} + \mathbf{v}_e \times \mathbf{B}_0) - \nabla p_e + n_eF_Q \tag{2.17}$$

In equation (2.17) m_e, n_e, $\mathbf{v}_e$ is the mass, number density and velocity of electron respectively, $\mathbf{B}_0$ is applied magnetic field, $\mathbf{E}$ is electric field, Pe represents the arbitrary pressure of electrons which can be written in term of arbitrary degeneracy G (detail is given in chapter 1) as

$$p_e = Gn_e \tag{2.18}$$

Where $G = \frac{1}{\beta}\frac{Li_{3/2}(-\xi)}{Li_{1/2}(-\xi)}$ is the arbitrary degeneracy factor in term of polylogarithm function $Li_s(-\xi)$ with $\xi = e^{\frac{\mu}{k_BT_e}}$ for nearly non degenerate $\xi << 1$ and for nearly degenerate $\xi >> 1$. and F_Q is quantum force given by

$$F_Q = \frac{\hbar^2}{2m_e}\nabla(\frac{\nabla^2 n_e^{\frac{1}{2}}}{n_e^{\frac{1}{2}}}) + \frac{2\mu}{\hbar}\nabla(\mathbf{B}_0.\mathbf{S}_e) = \frac{\hbar^2}{4m_e}\nabla\nabla^2 n_e + \frac{2\mu}{\hbar}\nabla(\mathbf{B}_0.\mathbf{S}_e) \tag{2.19}$$

Where the first term on the right side of equation (2.19) is quantum Bohm potential with $\hbar$ being the reduced plank constant and the second term is spin term with $\mathbf{S}_e$ being the spin of electron and $\mu = \frac{-g\mu_B}{2}$ with $\mu_B = \frac{e\hbar}{2m}$. By taking gradient of Eq. (2.18) and put $\beta = \frac{1}{k_B T_e}$, we get

$$\nabla p_e = G\nabla n_e \tag{2.20}$$

Where G is arbitrary degeneracy term and is given by [75].

$$G = \frac{1}{\beta}\frac{Li_{3/2}(-\xi)}{Li_{1/2}(-\xi)} \tag{2.21}$$

where $\xi = e^{\frac{\mu}{k_B T_e}}$ is the degeneracy factor depends on chemical potential μ and electrons temperature T_e for nearly non degenerate case $\xi << 1$ and for nearly degenerate case $\xi >> 1$. The poly logarithmic functionin in arbitrary degeneracy can be expanded under the assumption $\xi << 1$ as [65].

$$Li_{3/2}(-\xi) = -\xi + \frac{(-\xi)^2}{2^{\frac{3}{2}}}$$

$$Li_{1/2}(-\xi) = -\xi + \frac{(-\xi)^2}{2^{\frac{1}{2}}}$$

By putting the expanded value of the $Li_{3/2}(-\xi)$ and $Li_{1/2}(-\xi)$ in Eq. (2.21), gives arbitrary degenerate term G in the following form

$$G = \frac{1}{\beta}(\frac{-\xi + \frac{(-\xi)^2}{2^{\frac{3}{2}}}}{-\xi + \frac{(-\xi)^2}{2^{\frac{1}{2}}}})$$

$$G = \frac{1}{\beta}(1 - \frac{\xi}{2^{\frac{3}{2}}})(1 - \frac{\xi}{2^{\frac{1}{2}}})^{-1}$$

Expanding arbitrary Degeneracy term by using the binomial expansion (keeping term up to first order) we have $(1-\frac{\xi}{2^{\frac{1}{2}}})^{-1}=1+\frac{\xi}{2^{\frac{1}{2}}}$, then arbitrary degeneracy term G for nearly non degenerate case becomes

$$G=\frac{1}{\beta}(1-\frac{\xi}{2^{\frac{3}{2}}}+\frac{\xi}{2^{\frac{1}{2}}})$$

By putting $\frac{1}{\beta}=k_BT_e$, the above relation gives

$$G=(1+\frac{\xi}{2^{\frac{3}{2}}})\frac{k_BT_e}{m_e}m_e=(1+\frac{\xi}{2^{\frac{3}{2}}})v_{th}^2m_e \tag{2.22}$$

Relation (2.22) shows the arbitrary degeneracy factor for nearly non-degenerate limit ($\xi \ll 1$). The general formula of polylogarithmic expansion in the nearly degenerate case ($\xi >> 1$) can be expanded as [64]

$$-Li_s(-\xi)=\frac{(\ln\xi)^s}{\Gamma(s+1)}$$

Or

$$Li_s(-\xi)=\frac{-(\ln\xi)^s}{\Gamma(s+1)}$$

For $s=3/2$ and $s=1/2$ the $Li_s(-\xi)$ become

$$Li_{\frac{3}{2}}(-\xi)=\frac{-(\ln\xi)^{\frac{3}{2}}}{\Gamma\frac{5}{2}}$$

$$Li_{\frac{1}{2}}(-\xi)=\frac{-(\ln\xi)^{\frac{1}{2}}}{\Gamma\frac{3}{2}}$$

The ratio of the two relations gives

$$\frac{Li_{\frac{3}{2}}(-\xi)}{Li_{\frac{1}{2}}(-\xi)}=(\frac{-(\ln\xi)^{\frac{3}{2}}}{\Gamma\frac{5}{2}})(\frac{\Gamma\frac{3}{2}}{-(\ln\xi)^{\frac{1}{2}}})$$

$$\frac{Li_{\frac{3}{2}}(-\xi)}{Li_{\frac{1}{2}}(-\xi)}=\frac{(\ln\xi)^{\frac{3}{2}}((\ln\xi)^{-\frac{1}{2}})\Gamma\frac{3}{2}}{3/2\Gamma\frac{3}{2}}$$

$$\frac{Li_{\frac{3}{2}}(-\xi)}{Li_{\frac{1}{2}}(-\xi)} = \frac{\ln\xi}{3/2} \tag{2.23}$$

Using the expanded value of the $Li_{3/2}(-\xi)$ and $Li_{1/2}(-\xi)$ under the assumption ($\xi >> 1$) from Eq. (2.23) in Eq. (2.21) we can write

$$G = \frac{\ln\xi}{\frac{3}{2}\beta} \tag{2.24}$$

Where $\xi = e^{\beta\mu} \Rightarrow \ln\xi = \mu\beta$ then μ will be $\ln\xi/\beta$. Using the Sommerfeld's lemma as given in ([65]), the value of $\ln\xi$ for nearly degenerate quantum plasma can be written as

$$\ln\xi = \frac{T_{Fe}}{T_e}\{1 - \frac{\pi^2}{12}(\frac{T_e}{T_{Fe}})^2\} \tag{2.25}$$

Inserting the value of Eq. (2.25) in (2.24), gives

$$G = \frac{\frac{T_{Fe}}{T_e}}{\frac{3}{2}\beta}\{1 - \frac{\pi^2}{12}(\frac{T_e}{T_{Fe}})^2\}$$

Putting $\beta = \frac{1}{k_B T_e}$, for nearly degenerate case the simplified form of arbitrary degenerate factor G becomes

$$G = \frac{2}{3}(1 - \frac{\pi^2}{12}\delta^2)m_e v_F^2 \tag{2.26}$$

Where $v_F^2 = \frac{k_B T_F}{m_e}$, $\delta = \frac{T_e}{T_{Fe}}$.

Dividing Eq. (2.17) by m_e, and n_{0e} and neglecting the spin term. Then put Eqs. (2.19), (2.21) in Eq. (2.17), we get

$$\frac{\partial \mathbf{v}_e}{\partial t} = -\frac{e}{m_e}(\mathbf{E} + \mathbf{v}_e \times \mathbf{B}_0) - \frac{G\nabla n_e}{n_{0e}m_e\beta} + \frac{\hbar^2}{4n_{0e}m_e^2}\nabla\nabla^2 n_{e1} \tag{2.27}$$

Using the plane wave solution Eq. (2.27) can be written as

$$-i\omega\mathbf{v}_e = \frac{-e}{m_e}(\mathbf{E} + \mathbf{v}_e \times \mathbf{B}_0) - i(G + \frac{\hbar^2\mathbf{k}^2}{4m_e})\frac{\mathbf{k}}{m_e}\left(\frac{n_{e1}}{n_{e0}}\right) \tag{2.28}$$

To find $\frac{n_{e1}}{n_{0e}}$, we use the continuity equation for electrons as

$$\frac{\partial n_e}{\partial t} + \nabla.(n_e \mathbf{v}_e) = 0 \tag{2.29}$$

Incorporating the plane wave solution, in Eq. (2.29), we obtained

$$\frac{n_{e1}}{n_{0e}} = \frac{1}{\omega}(\mathbf{k}_x \mathbf{v}_{ex} + \mathbf{k}_z \mathbf{v}_{ez}) \tag{2.30}$$

In the same way if $\mathbf{v}_e$ have x, y and z component and $\mathbf{B}_0$ have only z component then $(\mathbf{v}_e \times \mathbf{B}_0)$ can be written as

$$\mathbf{v}_e \times \mathbf{B}_0 = \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_{ex} & v_{ey} & v_{ez} \\ 0 & 0 & B_0 \end{pmatrix} \tag{2.31}$$

The determinant of Eq. (2.31), gives

$$\mathbf{v}_e \times \mathbf{B}_0 = v_{ey} B_0 \hat{x} - v_{ex} B_0 \hat{y} \tag{2.32}$$

Putting the values of Eq. (2.30) and (2.32) in Eq. (2.28), we get

$$-i\omega \mathbf{v}_e = \frac{-e}{m_e}\mathbf{E} - \frac{e}{m_e}(v_{ey} B_0 \hat{x} - v_{ex} B_0 \hat{y}) - i(G + \frac{\hbar^2 \mathbf{k}^2}{4m_e})\frac{\mathbf{k}}{m_e \omega}(\mathbf{k}_x v_{ex} + \mathbf{k}_z v_{ez}) \tag{2.33}$$

The x-component of Eq. (2.33) will be

$$-i\omega \mathbf{v}_{ex} = \frac{-e}{m_e}\mathbf{E}_x - \frac{e}{m_e} v_{ey} \mathbf{B}_0 - i(G + \frac{\hbar^2 \mathbf{k}^2}{4m_e})\frac{\mathbf{k}_x}{m_e \omega}(\mathbf{k}_x v_{ex} + \mathbf{k}_z v_{ez})$$

Re-arranging the terms, we get

$$\{1 - (G + \frac{\hbar^2 \mathbf{k}^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e \omega^2}\}\mathbf{v}_{ex} = \frac{-ie}{m_e \omega}\mathbf{E}_x + \frac{ie}{m_e \omega}\mathbf{v}_{ey}\mathbf{B}_0 + (G + \frac{\hbar^2 \mathbf{k}^2}{4m_e})\frac{\mathbf{k}_x \mathbf{k}_z}{m_e \omega^2}\mathbf{v}_{ez}$$

Further simplification gives

$$\{1-(G+\frac{\hbar^2\mathbf{k}^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e\omega^2}\}\mathbf{v}_{ex}+\frac{ie}{m_e\omega}\mathbf{v}_{ey}\mathbf{B}_0-(G+\frac{\hbar^2\mathbf{k}^2}{4m_e})\frac{\mathbf{k}_x\mathbf{k}_z}{m_e\omega^2}\mathbf{v}_{ez}=\frac{-ie}{m_e\omega}\mathbf{E}_x \quad (2.34)$$

Similarly y-component of Eq. (2.33) is,

$$\mathbf{v}_{ey}=\frac{-ie}{m_e\omega}\mathbf{E}_y+\frac{ie}{m_e\omega}\mathbf{v}_{ex}\mathbf{B}_0$$

Further simplification gives

$$\mathbf{v}_{ey}-\frac{ie}{m_e\omega}\mathbf{v}_{ex}\mathbf{B}_0+0v_{ez}=\frac{-ie}{m_e\omega}\mathbf{E}_y \quad (2.35)$$

For z component Eq. (2.33) can be simplified as

$$\{1-(G+\frac{\hbar^2\mathbf{k}^2}{4m_e})\frac{\mathbf{k}_z^2}{m_e\omega^2}\}\mathbf{v}_{ez}+0v_{ey}-(G+\frac{\hbar^2\mathbf{k}^2}{4m_e})\frac{\mathbf{k}_x\mathbf{k}_z}{m_e\omega^2}\mathbf{v}_{ex}=\frac{-ie}{m_e\omega}\mathbf{E}_z \quad (2.36)$$

Let $\Omega=\frac{e\mathbf{B}_0}{m_e}$ represents the electron cyclotron frequency and $Y=\frac{\Omega}{\omega}$ is the ratio of cyclotron to wave frequency then Eq. (2.34) to (2.36) can be simplified in matrix notation as

$$\begin{pmatrix}\mathbf{v}_{ex}\\ \mathbf{v}_{ey}\\ \mathbf{v}_{ez}\end{pmatrix}\begin{pmatrix}1-(G+\frac{\hbar^2\mathbf{k}^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e\omega^2} & iY & -(G+\frac{\hbar^2\mathbf{k}^2}{4m_e})\frac{\mathbf{k}_x\mathbf{k}_z}{m_e\omega^2}\\ -iY & 1 & 0\\ -(G+\frac{\hbar^2\mathbf{k}^2}{4m_e})\frac{\mathbf{k}_x\mathbf{k}_z}{m_e\omega^2} & 0 & 1-(G+\frac{\hbar^2\mathbf{k}^2}{4m_e})\frac{\mathbf{k}_z^2}{m_e\omega^2}\end{pmatrix}=\frac{-ie}{m_e\omega}\begin{pmatrix}\mathbf{E}_x\\ \mathbf{E}_y\\ \mathbf{E}_z\end{pmatrix} \quad (2.37)$$

Suppose N=$1-(G+\frac{\hbar^2\mathbf{k}^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e\omega^2}$, M=$(G+\frac{\hbar^2\mathbf{k}^2}{4m_e})\frac{\mathbf{k}_x\mathbf{k}_z}{m_e\omega^2}$ and P=$1-(G+\frac{\hbar^2\mathbf{k}^2}{4m_e})\frac{\mathbf{k}_z^2}{m_e\omega^2}$, then above Eq. (2.37) become

$$\begin{pmatrix}\mathbf{v}_{ex}\\ \mathbf{v}_{ey}\\ \mathbf{v}_{ez}\end{pmatrix}\begin{pmatrix}N & iY & -M\\ -iY & 1 & 0\\ -M & 0 & P\end{pmatrix}=\frac{-ie}{m_e\omega}\begin{pmatrix}\mathbf{E}_x\\ \mathbf{E}_y\\ \mathbf{E}_z\end{pmatrix} \quad (2.38)$$

Suppose

$$L = \begin{pmatrix} N & iY & -M \\ -iY & 1 & 0 \\ -M & 0 & P \end{pmatrix}$$

To find the inverse of matrix L of Eq. (2.38) we need matrix of minor which is

$$L_{\min} = \begin{pmatrix} P & iPY & M \\ iPY & NP - M^2 & iMY \\ M & -iMY & N - Y^2 \end{pmatrix}$$

Matrix of cofactor

$$L_{cof} = \begin{pmatrix} P & iPY & M \\ -iPY & NP - M^2 & -iMY \\ M & iMY & N - Y^2 \end{pmatrix}$$

Adjoint of matrix

$$L_{adj} = \begin{pmatrix} P & -iPY & M \\ iPY & NP - M^2 & iMY \\ M & -iMY & N - Y^2 \end{pmatrix}$$

determinant of cofactor matrix, which is

$$L_{\mathrm{det}} = NP - PY^2 - M^2 = \{1 - (G + \tfrac{\hbar^2\mathbf{k}^2}{4m_e})\tfrac{\mathbf{k}_x^2}{m_e\omega^2}\}\{1 - (G + \tfrac{\hbar^2\mathbf{k}^2}{4m_e})\tfrac{\mathbf{k}_z^2}{m_e\omega^2}\} - \{1 - (G + \tfrac{\hbar^2\mathbf{k}^2}{4m_e})\tfrac{\mathbf{k}_z^2}{m_e\omega^2}\}Y^2 - \{(G + \tfrac{\hbar^2\mathbf{k}^2}{4m_e})\tfrac{\mathbf{k}_x\mathbf{k}_z}{m_e\omega^2}\}^2$$

In simplified form determinant can be written as

$$L_{\mathrm{det}} = P(1 - Y^2) + (N - 1)$$

Let $P(1 - Y^2) + (N - 1) = Q$, so the inverse which is $\frac{\text{Adjoint of matrix}}{\text{determinant of matrix}}$ can be found as

$$L_{inv} = \frac{1}{Q}\begin{pmatrix} P & -iPY & M \\ iPY & NP - M^2 & iMY \\ M & -iMY & N - Y^2 \end{pmatrix} \tag{2.39}$$

Now multiply both side of Eq. **2.38** by inverse of matrix L, we get

$$\begin{pmatrix} \mathbf{v}_{ex} \\ \mathbf{v}_{ey} \\ \mathbf{v}_{ez} \end{pmatrix} = \frac{1}{Q}(\frac{-ie}{m_e\omega})\begin{pmatrix} P & -iPY & M \\ iPY & NP - M^2 & iMY \\ M & -iMY & N - Y^2 \end{pmatrix}\begin{pmatrix} \mathbf{E}_x \\ \mathbf{E}_y \\ \mathbf{E}_z \end{pmatrix} \tag{2.40}$$

Using the definition of current **density** $\mathbf{j}_e = -en_e v_e$, we can modified **Eq. (2.40) as**

$$\begin{pmatrix} \mathbf{J}_{ex} \\ \mathbf{J}_{ey} \\ \mathbf{J}_{ez} \end{pmatrix} = \frac{n}{Q}(\frac{ie^2}{m_e\omega})\begin{pmatrix} P & -iPY & M \\ iPY & NP - M^2 & iMY \\ M & -iMY & N - Y^2 \end{pmatrix}\begin{pmatrix} \mathbf{E}_x \\ \mathbf{E}_y \\ \mathbf{E}_z \end{pmatrix} \tag{2.41}$$

Putting values of Eq. (2.41) and (2.16) in Eq. (2.8) , we get

$$\begin{pmatrix} 1 - \eta^2\cos\theta^2 & 0 & \eta^2\sin\theta\cos\theta \\ 0 & 1 - \eta^2 & 0 \\ \eta^2\sin\theta\cos\theta & 0 & 1 - \eta^2\sin\theta^2 \end{pmatrix}$$
$$+\frac{\omega_s\eta^2}{\omega(1-Y^2)}\begin{pmatrix} Y\cos\theta^2 & -i\cos\theta^2 & -Y\sin\theta\cos\theta \\ i\cos\theta^2 & Y\cos\theta^2 & -i\sin\theta\cos\theta \\ -Y\sin\theta\cos\theta & i\sin\theta\cos\theta & Y\sin\theta^2 \end{pmatrix} \tag{2.42}$$
$$-\frac{\omega_p^2}{Q\omega^2}\begin{pmatrix} P & -iPY & M \\ iPY & NP - M^2 & iMY \\ M & -iMY & N - Y^2 \end{pmatrix}\begin{pmatrix} \mathbf{E}_x \\ \mathbf{E}_y \\ \mathbf{E}_z \end{pmatrix} = 0$$

Where $\omega_s = \frac{g^2\hbar\omega_p^2}{8m_ec^2}\frac{(n_e\uparrow - n_e\downarrow)}{n_e}$ is spin magnetized frequency for both spin up and spin down electrons and $\eta = \frac{ck}{\omega}$ is the refractive index.

2.3.1 Parallel propagation mode

For parallel propagation ($\theta = 0$) leads $\mathbf{k}_x = 0$ and $\mathbf{k}_z \neq 0$ which consequently makes $M = 0, \mathbf{k} = \mathbf{k}_z$ and $N = 1$, so Eq. (2.42) becomes

$$\begin{pmatrix} 1-\eta^2+\frac{\omega_s Y\eta^2}{\omega(1-Y^2)}-\frac{X}{(1-Y^2)} & \frac{-i\omega_s\eta^2}{\omega(1-Y^2)}+\frac{iXY}{(1-Y^2)} & 0 \\ \frac{i\omega_s\eta^2}{\omega(1-Y^2)}-\frac{iXY}{(1-Y^2)} & 1-\eta^2+\frac{\omega_s Y\eta^2}{\omega(1-Y^2)}-\frac{X}{(1-Y^2)} & 0 \\ 0 & 0 & 1-\frac{X}{P} \end{pmatrix} \begin{pmatrix} \mathbf{E}_x \\ \mathbf{E}_y \\ \mathbf{E}_z \end{pmatrix} = 0 \tag{2.43}$$

Where $X = \frac{\omega_p^2}{\omega^2}$ shows the ratio of plasma frequency to wave frequency. In limiting case, if we put Bohm term and G equal to zero, then we obtain the result given in [63]. In order to have a nontrivial solution, the determinant of Eq. (2.43) must vanish which gives the dispersion relation of EM wave mode propagating parallel in spin magnetized plasmas, with the effect of arbitrary temperature degeneracy effect as

$$\{1-\frac{X}{P}\}\{(1-\eta^2+\frac{\omega_s Y\eta^2}{\omega(1-Y^2)}-\frac{X}{(1-Y^2)})^2+(\frac{-i\omega_s\eta^2}{\omega(1-Y^2)}+\frac{iXY}{(1-Y^2)})^2\} = 0 \tag{2.44}$$

Either

$$1-\frac{X}{P} = 0 \tag{2.45}$$

Or

$$(1-\eta^2+\frac{\omega_s Y\eta^2}{\omega(1-Y^2)}-\frac{X}{(1-Y^2)})^2+(\frac{-i\omega_s\eta^2}{\omega(1-Y^2)}+\frac{iXY}{(1-Y^2)})^2 = 0 \tag{2.46}$$

Eq. (2.45) can be simplified as

$$P - X = 0$$

By putting the values of P and X we get

$$1-(G+\frac{\hbar^2\mathbf{k}_z^2}{4m_e})\frac{\mathbf{k}_z^2}{m_e\omega^2}-\frac{\omega_p^2}{\omega^2} = 0$$

$$\frac{m_e\omega^2 - (G + \frac{\hbar^2 \mathbf{k}_z^2}{4m_e})\mathbf{k}_z^2}{m_e\omega^2} - \frac{\omega_p^2}{\omega^2} = 0$$

Which can be simplified as

$$\omega = \omega_p[1 + (G + \frac{\hbar^2 \mathbf{k}_z^2}{4m_e})\frac{\mathbf{k}_z^2}{m_e\omega_p^2}]^{1/2} \tag{2.47}$$

This is Langmuir mode clearly modified by the Bohm potential and arbitrary degeneracy factor G. In the limiting case i.e. by putting Bohm potential and G terms equal to zero we get the longitudinal electron plasma oscillation [63].

We can expand this result for nearly non degenerate (N.n.D) and nearly degenerate (N.D) limits by using the value of G from Eq. (2.22) and (2.26) respectively. For (N.n.D) case with $G = (1 + \frac{\xi}{2^{\frac{3}{2}}})v_{th}^2 m_e$ Eq. (2.47), become

$$\omega_{(N.n.D)} = \sqrt{\omega_p^2 + (1 + \frac{\xi}{2^{\frac{3}{2}}})\mathbf{k}_z^2\mathbf{v}_{th}^2 + (\frac{\hbar^2\mathbf{k}_z^4}{4m_e^2})} \tag{2.48}$$

For complete non degenerate plasma we have $\xi \to 0$ and we get

$$\omega_{(c.n.D)} = \sqrt{\omega_p^2 + \mathbf{k}_z^2\mathbf{v}_{th}^2 + (\frac{\hbar^2\mathbf{k}_z^4}{4m_e^2})}$$

For (N.D) case with G$=\frac{2}{3}\{1 - \frac{\pi^2}{12}(\frac{T_e}{T_{Fe}})^2\}m_e v_{Fe}^2$ Eq. (2.47), can be written as

$$\omega_{(N.D)} = \sqrt{\omega_p^2 + \frac{2}{3}(1 - \frac{\pi^2}{12}\delta^2)\mathbf{k}_z^2\mathbf{v}_{Fe}^2 + (\frac{\hbar^2\mathbf{k}_z^4}{4m_e^2})} \tag{2.49}$$

For complete degenerate pressure we put $\delta \to 0$, which leads

$$\omega_{(c.D)} = \sqrt{\omega_p^2 + \frac{2}{3}\mathbf{k}_z^2\mathbf{v}_{Fe}^2 + (\frac{\hbar^2\mathbf{k}_z^4}{4m_e^2})+}$$

Which is Langmuir mode in quantum plasma derived earlier by Shukla et al. [57]. For second root Eq. (2.46), gives

$$1 - \eta^2 + \frac{\omega_s Y\eta^2}{\omega(1-Y^2)} - \frac{X}{(1-Y^2)} = \pm(\frac{XY}{(1-Y^2)} - \frac{\omega_s\eta^2}{\omega(1-Y^2)}) \tag{2.50}$$

For positive sign Eq. (2.50) gives the dispersion relation as

$$\eta^2 = \frac{(1 - \frac{\omega_p^2}{\omega^2} - \frac{\Omega}{\omega})}{(1 - \frac{\Omega}{\omega} - \frac{\omega_s}{\omega})} \tag{2.51}$$

Which is right-hand secularly polarized mode (RCP). If we set $\omega_s = 0$, Eq. (2.51) reduced to the dispersion relation of RCP mode in classical magnetized plasmas [4].

For negative sign Eq. (2.50) gives the dispersion relation as

$$\eta^2 = \frac{(1 - \frac{\omega_p^2}{\omega^2} + \frac{\Omega}{\omega})}{(1 + \frac{\Omega}{\omega} - \frac{\omega_s}{\omega})} \tag{2.52}$$

Which is left -hand circularly polarized mode (LCP).

2.3.2 Perpendicular Propagation mode

For perpendicular mode ($\theta = \frac{\pi}{2}$), we have $\mathbf{k}_z = 0$ and $\mathbf{k}_x \neq 0$, and that makes again $M = 0$, $P = 1$ and $\mathbf{k} = \mathbf{k}_x$. These then make Eq. (2.42) as

$$\begin{pmatrix} 1 - \frac{X}{(1-Y^2)+(N-1)} & \frac{iXY}{(1-Y^2)+(N-1)} & 0 \\ -\frac{iXY}{(1-Y^2)+(N-1)} & 1 - \eta^2 - \frac{XN}{(1-Y^2)+(N-1)} & 0 \\ 0 & 0 & 1 - \eta^2 + \frac{\omega_s Y \eta^2}{\omega(1-Y^2)} + X \end{pmatrix} \begin{pmatrix} \mathrm{E}_x \\ \mathrm{E}_y \\ \mathrm{E}_z \end{pmatrix} = 0 \tag{2.53}$$

In limiting case i.e. by putting Bohm term and G equal to zero we obtain exactly equation (48) of [63]. In order to have a nontrivial solution, the determinant of Eq. (2.53) must vanish which gives the dispersion relation of EM wave propagating perpendicular in spin magnetized plasmas with the effect of temperature degeneracy, as

$$\{1 - \eta^2 + \tfrac{\omega_s Y \eta^2}{\omega(1-Y^2)} + X\}\{(1 - \tfrac{X}{(1-Y^2)+(N-1)})(1 - \eta^2 - \tfrac{XN}{(1-Y^2)+(N-1)}) + (\tfrac{iXY}{(N-1)})^2\} = 0 \tag{2.54}$$

Either

$$1 - \eta^2 + \frac{\omega_s Y \eta^2}{\omega(1 - Y^2)} + X = 0 \tag{2.55}$$

Or

$$(1-\frac{X}{(1-Y^2)+(N-1)})(1-\eta^2-\frac{XN}{(1-Y^2)+(N-1)})+(\frac{iXY}{(1-Y^2)+(N-1)})^2=0 \tag{2.56}$$

Eq. (2.55) gives the dispersion relation as

$$\eta^2=\frac{(\omega^2-\omega_p^2)(\omega^2-\omega_g^2)}{\omega^2(\omega^2-\omega_g^2-\omega_s\omega_g)} \tag{2.57}$$

This relation is exactly the same as equation (52) of [63]. In case of $\omega_s \to 0$ Eq. (2.57), reduced to the dispersion relation of ordinary mode in classical magnetized plasmas [4]. Eq. (2.56), can be simplified as

$$(1-\frac{X}{(N-Y^2)})(1-\eta^2-\frac{XN}{(N-Y^2)})-(\frac{XY}{(N-Y^2)})^2=0$$

Which gives

$$(1-\frac{X}{(N-Y^2)})(1-\eta^2-\frac{XN}{(N-Y^2)})=(\frac{XY}{(N-Y^2)})^2 \tag{2.58}$$

From this equation the value of η is

$$\eta^2=\frac{-(\frac{XY}{(N-Y^2)})^2+(1-\frac{X}{(N-Y^2)})(1-\frac{XN}{(N-Y^2)})}{(1-\frac{X}{N-Y^2})} \tag{2.59}$$

This shows the refractive index of extra ordinary mode (x-mode) clearly effected by Bohm potential and arbitrary temperature degeneracy factor G. In limiting case i.e. by putting ($N=0$), means Bohm term and G equal to zero we obtain the dispersion relation as

$$\eta^2=\frac{(\omega^2-\omega_p^2-\omega\Omega)(\omega^2-\omega_p^2+\omega\Omega)}{\omega^2(\omega^2-\omega_p^2-\Omega^2)} \tag{2.60}$$

Which is exactly same as equation (50) of [63], and Eq. (2.60) further shows that it is not effected by spin effects.

Eq. (2.59) can be simplified as

$$\eta^2(1-\frac{X}{(N-Y^2)}) = (1-\frac{X}{(N-Y^2)})(1-\frac{XN}{(N-Y^2)}) - (\frac{XY}{(N-Y^2)})^2$$

$$\eta^2(N-Y^2-X)(N-Y^2) = (N-Y^2-X)(N-Y^2-XN) - (XY)^2$$

Simplification, yields

$$\eta^2(N^2-2NY^2-XN+Y^4+XY^2) = -(XY)^2+(N^2-2NY^2-XN+Y^4+XY^2-XN^2-XNY^2+X^2N)$$

Putting the values of N, X, Y and η, and keeping ω power up to 4, gives us

$$\begin{gathered}\frac{y^2}{\omega^2}[\{1-(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e\omega^2}\}^2 - 2\{1-(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e\omega^2}\}\frac{\Omega^2}{\omega^2} - \frac{\omega_p^2}{\omega^2}\{1-(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e\omega^2}\} \\ +\frac{\Omega^4}{\omega^4} + \frac{\Omega^2}{\omega^2}\frac{\omega_p^2}{\omega^2}] = -\frac{\Omega^2}{\omega^2}\frac{\omega_p^4}{\omega^4} + \{1-(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e\omega^2}\}^2 - 2\{1-(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e\omega^2}\}\frac{\Omega^2}{\omega^2} \\ -\frac{\omega_p^2}{\omega^2}\{1-(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e\omega^2}\} + \frac{\Omega^4}{\omega^4} + \frac{\Omega^2}{\omega^2}\frac{\omega_p^2}{\omega^2} - \frac{\omega_p^2}{\omega^2}\{1-(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e\omega^2}\}^2 \\ -\frac{\omega_p^2}{\omega^2}\{1-(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e\omega^2}\}\frac{\Omega^2}{\omega^2} + \frac{\omega_p^4}{\omega^4}\{1-(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e\omega^2}\}\end{gathered}$$

Where $y = ck$, re-arranging the terms on the basis of ω power, gives us

$$\begin{gathered}\omega^4[y^2+2(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}+2\omega_p^2] - \omega^2[2(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}y^2 + 2\Omega^2y^2 + \omega_p^2y^2 \\ +\{(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}\}^2 + 2(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}\Omega^2 + \omega_p^2(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e} + \Omega^4 + \Omega^2\omega_p^2 \\ +2\omega_p^2(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e} - \Omega^2\omega_p^2 + \omega_p^4] + \{(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}\}^2y^2 + \Omega^4y^2 \\ +2(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}\Omega^2y^2 + (G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}\omega_p^2y^2 + \Omega^2\omega_p^2y^2 + \Omega^2\omega_p^4 \\ +\omega_p^2\{(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}\}^2 - \omega_p^2\{(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}\}\Omega^2 + (G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}\omega_p^4 = 0\end{gathered} \tag{2.61}$$

Eq. (2.61) is biquadratic in ω, and solution gives

$$\omega^2 = \frac{b \pm \sqrt{b^2-4ac}}{2a} \tag{2.62}$$

Where a, b and c is given by Eq. (2.63) to (2.65), respectively

$$a = y^2 + 2(G + \frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e} + 2\omega_p^2 \tag{2.63}$$

$$\begin{gathered} b = 2(G + \tfrac{\hbar^2\mathbf{k}_x^2}{4m_e})\tfrac{\mathbf{k}_x^2}{m_e}y^2 + 2\Omega^2y^2 + \omega_p^2y^2 + \{(G + \tfrac{\hbar^2\mathbf{k}_x^2}{4m_e})\tfrac{\mathbf{k}_x^2}{m_e}\}^2 + 2(G + \tfrac{\hbar^2\mathbf{k}_x^2}{4m_e})\tfrac{\mathbf{k}_x^2}{m_e}\Omega^2 + \\ \omega_p^2(G + \tfrac{\hbar^2\mathbf{k}_x^2}{4m_e})\tfrac{\mathbf{k}_x^2}{m_e} + \Omega^4 + \Omega^2\omega_p^2 + 2\omega_p^2(G + \tfrac{\hbar^2\mathbf{k}_x^2}{4m_e})\tfrac{\mathbf{k}_x^2}{m_e} - \Omega^2\omega_p^2 + \omega_p^4 \end{gathered} \tag{2.64}$$

$$\begin{gathered} c = \{(G + \tfrac{\hbar^2\mathbf{k}_x^2}{4m_e})\tfrac{\mathbf{k}_x^2}{m_e}\}^2y^2 + \Omega^4y^2 + 2(G + \tfrac{\hbar^2\mathbf{k}_x^2}{4m_e})\tfrac{\mathbf{k}_x^2}{m_e}\Omega^2y^2 + (G + \tfrac{\hbar^2\mathbf{k}_x^2}{4m_e})\tfrac{\mathbf{k}_x^2}{m_e}\omega_p^2y^2 + \Omega^2\omega_p^2y^2 \\ +\Omega^2\omega_p^4 + \omega_p^2\{(G + \tfrac{\hbar^2\mathbf{k}_x^2}{4m_e})\tfrac{\mathbf{k}_x^2}{m_e}\}^2 - \omega_p^2\{(G + \tfrac{\hbar^2\mathbf{k}_x^2}{4m_e})\tfrac{\mathbf{k}_x^2}{m_e}\}\Omega^2 + (G + \tfrac{\hbar^2\mathbf{k}_x^2}{4m_e})\tfrac{\mathbf{k}_x^2}{m_e}\omega_p^4 \end{gathered} \tag{2.65}$$

We can expand this result for nearly non degenerate (N.n.D) limit and nearly degenerate (N.D) limit by using the value of G from Eq. (2.22) and (2.26), respectively. Thus for (N.n.D) case Eq. (2.62), become

$$\omega^2_{(N.n.D)} = \frac{b' \pm \sqrt{b'^2 - 4a'c'}}{2a'} \tag{2.66}$$

Where a', b' and c' is given by Eq. (2.67) to (2.69), respectively

$$a' = y^2 + 2(1 + \frac{\xi}{2^{\frac{3}{2}}})\mathbf{k}_x^2\mathbf{v}_{th}^2 + \frac{\hbar^2\mathbf{k}_x^4}{4m_e^2} + 2\omega_p^2 \tag{2.67}$$

$$\begin{gathered} b' = 2(1 + \tfrac{\xi}{2^{\frac{3}{2}}})\mathbf{k}_x^2\mathbf{v}_{th}^2y^2 + \tfrac{\hbar^2\mathbf{k}_x^4}{4m_e^2}y^2 + 2\Omega^2y^2 + \omega_p^2y^2 + \{(1 + \tfrac{\xi}{2^{\frac{3}{2}}})\mathbf{k}_x^2\mathbf{v}_{th}^2 + \tfrac{\hbar^2\mathbf{k}_x^4}{4m_e^2}\}^2 + \\ 2\{(1 + \tfrac{\xi}{2^{\frac{3}{2}}})\mathbf{k}_x^2\mathbf{v}_{th}^2 + \tfrac{\hbar^2\mathbf{k}_x^4}{4m_e^2}\}\Omega^2 + \omega_p^2\{(1 + \tfrac{\xi}{2^{\frac{3}{2}}})\mathbf{k}_x^2\mathbf{v}_{th}^2 + \tfrac{\hbar^2\mathbf{k}_x^4}{4m_e^2}\} + \Omega^4 + \Omega^2\omega_p^2 + \\ 2\omega_p^2\{(1 + \tfrac{\xi}{2^{\frac{3}{2}}})\mathbf{k}_x^2\mathbf{v}_{th}^2 + \tfrac{\hbar^2\mathbf{k}_x^4}{4m_e^2}\} - \Omega^2\omega_p^2 + \omega_p^4 \end{gathered} \tag{2.68}$$

$$c' = \{(1+\tfrac{\xi}{2^{5/2}})\mathbf{k}_z^2\mathbf{v}_{th}^2 + \tfrac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}^2 y^2 + \Omega^4 y^2 + 2\{(1+\tfrac{\xi}{2^{5/2}})\mathbf{k}_z^2\mathbf{v}_{th}^2 + \tfrac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}\Omega^2 y^2 +$$
$$\{(1+\tfrac{\xi}{2^{5/2}})\mathbf{k}_z^2\mathbf{v}_{th}^2 + \tfrac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}\omega_p^2 y^2 + \Omega^2\omega_p^2 y^2 + \Omega^2\omega_p^4 + \omega_p^2\{(1+\tfrac{\xi}{2^{5/2}})\mathbf{k}_z^2\mathbf{v}_{th}^2 + \tfrac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}^2 -$$
$$\omega_p^2\{(1+\tfrac{\xi}{2^{5/2}})\mathbf{k}_z^2\mathbf{v}_{th}^2 + \tfrac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}\Omega^2 + \{(1+\tfrac{\xi}{2^{5/2}})\mathbf{k}_z^2\mathbf{v}_{th}^2 + \tfrac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}\omega_p^4 \tag{2.69}$$

For complete non degenerate case $\xi \to 0$, which will modify Eq. (2.67) to (2.69), accordingly.

And for nearly degenerate (N.D) case Eq. (2.62), can be written as

$$\omega_{(N.D)}^2 = \frac{b'' \pm \sqrt{b''^2 - 4a''c''}}{2a''} \tag{2.70}$$

Where a'', b'' and c'' is given by Eq. (2.71) to (2.73), respectively

$$a'' = y^2 + \frac{4}{3}(1 - \frac{\pi^2}{12}\delta^2)\mathbf{k}_z^2 v_{Fe}^2 + \frac{\hbar^2\mathbf{k}_z^4}{4m_e^2} + 2\omega_p^2 \tag{2.71}$$

$$b'' = \tfrac{4}{3}(1 - \tfrac{\pi^2}{12}\delta^2)\mathbf{k}_z^2\mathbf{v}_{Fe}^2 y^2 + 2\tfrac{\hbar^2\mathbf{k}_z^4}{4m_e^2}y^2 + 2\Omega^2 y^2 + \omega_p^2 y^2 + \{\tfrac{2}{3}(1 - \tfrac{\pi^2}{12}\delta^2)\mathbf{k}_z^2\mathbf{v}_{Fe}^2 + \tfrac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}^2 +$$
$$2\{\tfrac{2}{3}(1 - \tfrac{\pi^2}{12}\delta^2)\mathbf{k}_z^2\mathbf{v}_{Fe}^2 + \tfrac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}\Omega^2 + \omega_p^2\{\tfrac{2}{3}(1 - \tfrac{\pi^2}{12}\delta^2)\mathbf{k}_z^2\mathbf{v}_{Fe}^2 + \tfrac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}$$
$$+\Omega^4 + \Omega^2\omega_p^2 + 2\omega_p^2\{\tfrac{2}{3}(1 - \tfrac{\pi^2}{12}\delta^2)\mathbf{k}_z^2\mathbf{v}_{Fe}^2 + \tfrac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\} - \Omega^2\omega_p^2 + \omega_p^4 \tag{2.72}$$

$$c'' = \{\tfrac{2}{3}(1 - \tfrac{\pi^2}{12}\delta^2)\mathbf{k}_z^2\mathbf{v}_{Fe}^2 + \tfrac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}^2 y^2 + \Omega^4 y^2 + 2\{\tfrac{2}{3}(1 - \tfrac{\pi^2}{12}\delta^2)\mathbf{k}_z^2\mathbf{v}_{Fe}^2 + \tfrac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}\Omega^2 y^2 +$$
$$\{\tfrac{2}{3}(1 - \tfrac{\pi^2}{12}\delta^2)\mathbf{k}_z^2\mathbf{v}_{Fe}^2 + \tfrac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}\omega_p^2 y^2 + \Omega^2\omega_p^2 y^2 + \Omega^2\omega_p^4 + \omega_p^2\{\tfrac{2}{3}(1 - \tfrac{\pi^2}{12}\delta^2)\mathbf{k}_z^2\mathbf{v}_{Fe}^2 +$$
$$\tfrac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}^2 - \omega_p^2\{\tfrac{2}{3}(1 - \tfrac{\pi^2}{12}\delta^2)\mathbf{k}_z^2\mathbf{v}_{Fe}^2 + \tfrac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}\Omega^2 + \{\tfrac{2}{3}(1 - \tfrac{\pi^2}{12}\delta^2)\mathbf{k}_z^2\mathbf{v}_{Fe}^2 + \tfrac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}\omega_p^4 \tag{2.73}$$

For complete degenerate case $\delta \to 0$, which will modify Eq. (2.71) to (2.73), accordingly.

Resonant Frequency

Resonance occurs when refractive index $\eta = (\frac{ck}{\omega}) \to \infty$, a wave is generally absorbed at resonance. Thus for x-mode in quantum spin plasma with effect of Bohm potential and arbitrary temperature degeneracy the resonant condition occurred in the following form

$$(1 - \frac{X}{N - Y^2}) = 0$$

$$N - Y^2 - X = 0$$

$$N = Y^2 + X$$

Putting the values of N, Y and X gives

$$1 - (G + \frac{\hbar^2 \mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e \omega^2} = \frac{\Omega^2}{\omega^2} + \frac{\omega_p^2}{\omega^2}$$

$$\frac{m_e \omega^2 - (G + \frac{\hbar^2 \mathbf{k}_x^2}{4m_e})\mathbf{k}_x^2}{m_e \omega^2} = \frac{\Omega^2}{\omega^2} + \frac{\omega_p^2}{\omega^2}$$

Solving for ω, we get the following resonant frequency

$$\omega_R = \sqrt{\Omega^2 + \omega_p^2 + (G + \frac{\hbar^2 \mathbf{k}_x^4}{4m_e^2})} \tag{2.74}$$

This resonance mode for perpendicular case ($\mathbf{k} \perp \mathbf{B}_0$) is clearly effected by Bohm potential and arbitrary temperature degeneracy factor G. We can expand this result for nearly non degenerate (N.n.D) case and nearly degenerate (N.D) case by using the values of G from Eq. (2.22) and (2.26) respectively. So for (N.n.D) case Eq. (2.74), becomes

$$\omega_{R(N.n.D)} = \sqrt{\Omega^2 + \omega_p^2 + (1 + \frac{\xi}{2^{\frac{3}{2}}})\mathbf{k}_x^2 \mathbf{v}_{th}^2 + \frac{\hbar^2 \mathbf{k}_x^4}{4m_e^2}} \tag{2.75}$$

For complete non degeneracy, $\xi \to 0$, yields

$$\omega_{R(c.n.D)} = \sqrt{\Omega^2 + \omega_p^2 + \mathbf{k}_x^2 \mathbf{v}_{th}^2 + \frac{\hbar^2 \mathbf{k}_x^4}{4m_e^2}} \tag{2.76}$$

And for (N.D) case Eq. (2.74), can be written as

$$\omega_{R(N.D)} = \sqrt{\Omega^2 + \omega_p^2 + \frac{2}{3}(1 - \frac{\pi^2}{12}\delta^2)\mathbf{k}_x^2 \mathbf{v}_{Fe}^2 + \frac{\hbar^2 \mathbf{k}_x^4}{4m_e^2}} \tag{2.77}$$

For complete degeneracy, $\delta \to 0$, yields

$$\omega_{R(c..D)} = \sqrt{\Omega^2 + \omega_p^2 + \frac{2}{3}\mathbf{k}_x^2 \mathbf{v}_{Fe}^2 + \frac{\hbar^2 \mathbf{k}_x^4}{4m_e^2}} \tag{2.78}$$

Cut-off Frequency

Cut-off occur when refractive index $\eta = (\frac{ck}{\omega}) \to 0$, a wave is generally reflected at cut-off. Thus for x-mode in quantum spin plasma with effect of Bohm potential and arbitrary temperature degeneracy the cut-off condition occurred in the following form

$$-(\frac{XY}{N - Y^2})^2 + (1 - \frac{X}{N - Y^2})(1 - \frac{XN}{N - Y^2}) = 0$$

By taking L.C.M, and simplify

$$-(XY)^2 + N^2 + Y^4 - 2NY^2 - X(N - Y^2) - XN(N - Y^2) + X^2 N = 0$$

Putting the values of X and Y gives

$$-\omega_p^4\Omega^2 + N^2\omega^6 + \Omega^4\omega^2 - 2N\Omega^2\omega^4 - \omega_p^2(N\omega^2 - \Omega^2)\omega^2 - N\omega_p^2(N\omega^2 - \Omega^2)\omega^2 + N\omega_p^4\omega^2 = 0$$

Putting the value of N, and maintaining ω power up to to 4, we get

$$\omega_p^4\Omega^2 - \{(G + \tfrac{\hbar^2\mathbf{k}_x^2}{4m_e})\tfrac{\mathbf{k}_x^2}{m_e}\}^2\omega^2 + 2(G + \tfrac{\hbar^2\mathbf{k}_x^2}{4m_e})\tfrac{\mathbf{k}_x^2}{m_e}\omega^4 - \Omega^4\omega^2 + 2\Omega^2\omega^4 - 2(G + \tfrac{\hbar^2\mathbf{k}_x^2}{4m_e})\tfrac{\mathbf{k}_x^2}{m_e}\Omega^2\omega^2$$
$$+\omega_p^2\omega^4 - (G + \tfrac{\hbar^2\mathbf{k}_x^2}{4m_e})\tfrac{\mathbf{k}_x^2}{m_e}\omega_p^2\omega^2 - \Omega^2\omega_p^2\omega^2 + \omega_p^2\omega^4 + \{(G + \tfrac{\hbar^2\mathbf{k}_x^2}{4m_e})\tfrac{\mathbf{k}_x^2}{m_e}\}^2\omega_p^2$$
$$-2(G + \tfrac{\hbar^2\mathbf{k}_x^2}{4m_e})\tfrac{\mathbf{k}_x^2}{m_e}\omega_p^2\omega^2 - \Omega^2\omega_p^2\omega^2 + (G + \tfrac{\hbar^2\mathbf{k}_x^2}{4m_e})\tfrac{\mathbf{k}_x^2}{m_e}\Omega^2\omega_p^2 - \omega_p^4\omega^2 + (G + \tfrac{\hbar^2\mathbf{k}_x^2}{4m_e})\tfrac{\mathbf{k}_x^2}{m_e}\omega_p^4 = 0$$

Rearranging the terms on the basis of ω power

$$\omega^4\{2(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}+2\Omega^2+2\omega_p^2\}-\omega^2[\{(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}\}^2+\Omega^4+2(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}\Omega^2$$
$$+(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}\omega_p^2+2\Omega^2\omega_p^2+2(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}\omega_p^2+\omega_p^4]+\omega_p^4\Omega^2+\{(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}\}^2\omega_p^2$$
$$+(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}\Omega^2\omega_p^2+(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}\omega_p^4=0 \tag{2.79}$$

Eq. (2.79), is biquadratic in ω which can be written as

$$\omega^2=\frac{B\pm\sqrt{B^2-4AC}}{2A} \tag{2.80}$$

Where A, B and C is given by Eq. (2.81) to (2.44), respectively

$$A=2(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}+2\Omega^2+2\omega_p^2 \tag{2.81}$$

$$B=\{(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}\}^2+\Omega^4+2(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}\Omega^2+(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}\omega_p^2$$
$$+2\Omega^2\omega_p^2+2(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}\omega_p^2+\omega_p^4 \tag{2.82}$$

$$C=\omega_p^4\Omega^2+\{(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}\}^2\omega_p^2+(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}\Omega^2\omega_p^2+(G+\frac{\hbar^2\mathbf{k}_x^2}{4m_e})\frac{\mathbf{k}_x^2}{m_e}\omega_p^4 \tag{2.83}$$

We can expand this result for nearly non degenerate (N.n.D) and nearly degenerate (N.D) limits by using the value of G from Eq. (2.22) and(2.26) respectively. So for (N.n.D) case Eq. (2.80), become

$$\omega^2_{(N.n.D)}=\frac{B'\pm\sqrt{B'^2-4A'C'}}{2A'} \tag{2.84}$$

Where A', B' and C' is given by Eq. (2.85) to (2.87), respectively

$$A'=2(1+\frac{\xi}{2^{\frac{3}{2}}})\mathbf{k}_x^2\mathbf{v}_{th}^2+\frac{\hbar^2\mathbf{k}_x^4}{4m_e^2}+2\Omega^2+2\omega_p^2 \tag{2.85}$$

$$B' = \{(1+\frac{\xi}{2^{\frac{5}{2}}})\mathbf{k}_z^2\mathbf{v}_{th}^2 + \frac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}^2 + \Omega^4 + 2\{(1+\frac{\xi}{2^{\frac{5}{2}}})\mathbf{k}_z^2\mathbf{v}_{th}^2 + \frac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}\Omega^2 + \{(1+\frac{\xi}{2^{\frac{5}{2}}})\mathbf{k}_z^2\mathbf{v}_{th}^2 + \frac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}\omega_p^2 + 2\Omega^2\omega_p^2 + 2\{(1+\frac{\xi}{2^{\frac{5}{2}}})\mathbf{k}_z^2\mathbf{v}_{th}^2 + \frac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}\omega_p^2 + \omega_p^4 \tag{2.86}$$

$$C' = \omega_p^4\Omega^2 + \{(1+\frac{\xi}{2^{\frac{5}{2}}})\mathbf{k}_z^2\mathbf{v}_{th}^2 + \frac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}^2\omega_p^2 + \{(1+\frac{\xi}{2^{\frac{5}{2}}})\mathbf{k}_z^2\mathbf{v}_{th}^2 + \frac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}\Omega^2\omega_p^2 + \{(1+\frac{\xi}{2^{\frac{5}{2}}})\mathbf{k}_z^2 v_{th}^2 + \frac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}\omega_p^4 \tag{2.87}$$

For complete non degeneracy, $\xi \to 0$, which modify Eq. (2.85) to (2.87), accordingly

And for nearly degenerate (N.D) case Eq. (2.80), become

$$\omega_{(N.D)}^2 = \frac{B'' \pm \sqrt{B''^2 - 4A''C''}}{2A''} \tag{2.88}$$

Where A'', B'' and C'' is given by Eq.

$$A'' = 2\{\frac{2}{3}(1-\frac{\pi^2}{12}\delta^2)\mathbf{k}_z^2\mathbf{v}_{Fe}^2 + \frac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\} + 2\Omega^2 + 2\omega_p^2 \tag{2.89}$$

$$B'' = \{\frac{2}{3}(1-\frac{\pi^2}{12}\delta^2)\mathbf{k}_z^2\mathbf{v}_{Fe}^2 + \frac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}^2 + \Omega^4 + 2\{\frac{2}{3}(1-\frac{\pi^2}{12}\delta^2)\mathbf{k}_z^2\mathbf{v}_{Fe}^2 + \frac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}\Omega^2 + \{\frac{2}{3}(1-\frac{\pi^2}{12}\delta^2)\mathbf{k}_z^2\mathbf{v}_{Fe}^2 + \frac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}\omega_p^2 + 2\Omega^2\omega_p^2 + 2\{\frac{2}{3}(1-\frac{\pi^2}{12}\delta^2)\mathbf{k}_z^2\mathbf{v}_{Fe}^2 + \frac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}\omega_p^2 + \omega_p^4 \tag{2.90}$$

$$C'' = \omega_p^4\Omega^2 + \{\frac{2}{3}(1-\frac{\pi^2}{12}\delta^2)\mathbf{k}_z^2\mathbf{v}_{Fe}^2 + \frac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}^2\omega_p^2 + \{\frac{2}{3}(1-\frac{\pi^2}{12}\delta^2)\mathbf{k}_z^2\mathbf{v}_{Fe}^2 + \frac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}\Omega^2\omega_p^2 + \{\frac{2}{3}(1-\frac{\pi^2}{12}\delta^2)\mathbf{k}_z^2\mathbf{v}_{Fe}^2 + \frac{\hbar^2\mathbf{k}_z^4}{4m_e^2}\}\omega_p^4 \tag{2.91}$$

For complete degeneracy, $\delta \to 0$, which will modify Eq. (2.89) to (2.91), accordingly

Chapter 3

Results and Discussion

We have calculated the analytical result of electromagnetic waves in magnetized spin quantum plasmas with effect of arbitrary temperature degeneracy. In this regard we have calculated the generalized dispersion relation (for parallel and perpendicular propagation to external magnetic field B_0) for both nearly non-degenerate (N.n.D) and nearly degenerate (N.D) quantum plasma. The $\xi = e^{\frac{\mu_e}{k_B T_e}}$ has been expanded for nearly non-degenerate and nearly degenerate limits. The resonant and cut-off frequencies are derived (in case of $\theta = \frac{\pi}{2}$) for both {N.n.D} and {N.D} limits, and their limiting cases for complete non-degenerate and complete degenerate are discussed. In order to analyze these results parametrically and graphically we are going to discuss some results (equation (2.48, 2.49, 2.66, 2.70, 2.75, 2.77, 2.84 and 2.88) numerically. These numerical results are plotted for various values of degeneracy factor, electron densities and magnetic field B_0. To visualize the complete picture of quantum effects including spin of electron, the temperature arbitrary degeneracy term ξ (incorporated in the from of pressure term), some typical parameter for degenerate quantum plasma in the interiors of neutron stars, white dwarfs and black holes are presented in S.I system [80], $m_e = 9.11 \times 10^{-31} kg$, $e = 1.6 \times 10^{-19} c$, $n_e = 10^{16}/m^3 \rightarrow 10^{28}/m^3$, $B_0 = 10^2 T \rightarrow 10^4 T$, $T_e = 10^4 k \rightarrow 10^8 k$, $\varepsilon_0 = 8.85 \times 10^{-12} c^2/Nm^2$, $\hbar = 1.054 \times 10^{-34} js$, $k_B = 1.3807 \times 10^{-23} j/k$. If we put these values into the defined parameters, we obtained some interesting and significant effects in the system. These all are explained graphically in the following paragraphs.

Fig 3.1 shows the dispersion plot for EM wave modes propagating along the external magnetic field for both (a) nearly non degenerate and (b) nearly degenerate quantum plasma. The number density for (a) is $n = 10^{12}/m^3$, and for (b) is $n = 10^{26}/m^3$. Fig 3.1 (a) shows the parallel mode variation with respect to degeneracy factor i.e. $\xi = e^{\beta\mu}$. The solid line ($\xi = 0$) represents complete non degenerate plasmas, while the dashed ($\xi = 0.4$) and dotted ($\xi = 0.9$) lines give some degeneracy. From plot (a) of Fig 3.1, it is clear that increasing the degeneracy factor ξ causes the frequency to enhance. Plot (b) of Fig 3.1, shows the variation of deep degenerate factor i.e. $\delta = \frac{T_e}{T_{Fe}}$ in nearly degenerate plasma, and it shows that mode frequency decreases with increasing the value of δ. Fig 3.1 overall indicates that in nondegenerate plasma if we enhance the probability of some degeneracy of fermionic factor then phase velocity of parallel mode of EM waves increases, while viceversa phenomena is observed if we enhance the probability of non degeneracy of Maxwellian factor in degenerate plasma.

Fig 3.2 gives the dispersion plot for EM wave modes propagating along the external magnetic field for both (a) nearly non degenerate and (b) nearly degenerate quantum plasma for different values of number density. It shows that for plot (a) the frequency increases by increasing the value of number density. Frequency trend for plot (b) is same as for (a). Thus for increasing number density, the plasma frequency is increasing, which consequently enhance the mode frequency both in nearly nondegenerate and nearly degenerate quantum plasmas.

Fig 3.3 reveals the dispersion plot for EM wave modes propagating perpendicular the external magnetic field for both (a) nearly non degenerate and (b) nearly degenerate quantum plasma for different values of ξ and δ. The number density and magnetic field for plot (a) are $n = 10^{12}/m^3$, $B_0 = 1 \times 10^{-7}T$ and for (b) are $n = 10^{31}/m^3$, $B_0 = 2$ T. Plot (a) of Fig 3.3, shows the variation of degeneracy factor i.e. $\xi = e^{\beta\mu}$ that's solid line ($\xi = 0$) show complete non degenerate plasmas, dashed line ($\xi = 0.4$) and dotted line ($\xi = 0.9$) is for nearly non degenerate quantum plasmas. Plot (a) shows that frequency increases by increasing the value of ξ. Plot (b) shows the variation of deep degeneracy factor i.e. $\delta = \frac{T_e}{T_{Fe}}$. It is clear from Fig 3.3 that for plot (b) the frequency decreases by decreasing the value of δ.

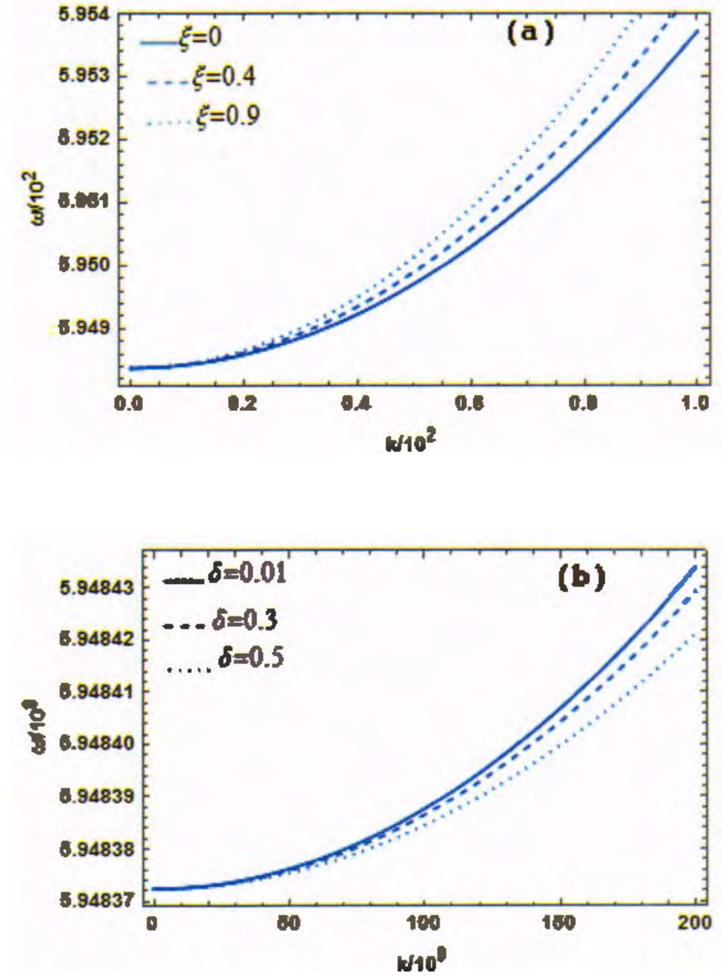


Figure 3.1: Dispersion plots for EM wave modes propagating parallel to the magnetic feild ($\mathbf{k}\|\mathbf{B}_0$) for both (a) nearly non degenerate and (b) nearly degenerate quantum plasma for various values of (a) $\xi = 0$ (solid line), 0.4 (dashed line) and 0.9 (dotted line) and of (b) $\delta = 0.01$ (solid line), 0.3 (dashed line) and 0.5 (dotted line). The number density for plot (a) is $n = 10^{12}/m^3$, and for (b) is $n = 10^{26}/m^3$, For case (a) we have $T_e = 100\ T_{Fe}$ and for case (b) $T_e \ll T_{Fe}$.

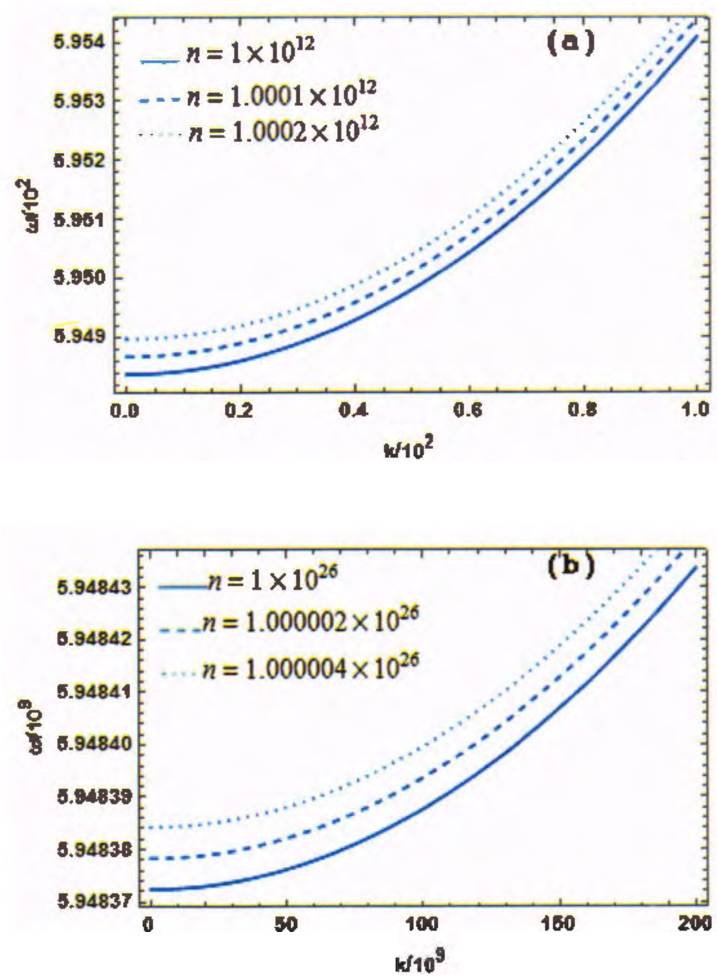


Figure 3.2: Dispersion plots for EM wave modes propagating along the magnetic feild ($\mathbf{k}\|\mathbf{B}_0$) for both (a) nearly non degenerate and (b) nearly degenerate quantum plasma for various values of (a) $n = 1\times10^{12}/m^3$ (solid line), $1.0001\times10^{12}/m^3$ (dashed line) and $1.0002\times10^{12}/m^3$ (dotted line) and of (b) $1\times10^{26}/m^3$ (solid line),$1.000002\times10^{26}/m^3$ (dashed line) and $1.000004\times10^{26}/m^3$ (dotted line). The value of ξ for plot (a) is $\xi = 0.2$ and δ for (b) is $\delta = 0.02$. For case (a) we have $T_e = 100\ T_{Fe}$ and for case (b) $T_e \ll T_{Fe}$.

Fig 3.4 shows the dispersion plot for EM wave modes propagating perpendicular to external magnetic field for both (a) nearly non degenerate and (b) nearly degenerate quantum plasma for different values of number density. The value of magnetic field and ξ for plot (a) are $\mathbf{B}_0 = 0.0000001T$, $\xi = 0.1$ while for (b) are $\mathbf{B}_0 = 10^2\ T$, $\delta = 0.1$. Plot (a) of Fig 3.4 shows that the perpendicular x-mode frequency with increasing trend with respect to $\mathbf{k}$ is further increasing for greater values of number density, for nearly non degenerate quantum plasma. Similar trend and behavior but with different magnitude have shown in Fig 3.4 (b) for nearly degenerate quantum plasma. These plots manifests, that since x-mode depend both on plasma and cyclotron frequency. So for fixed cyclotron frequency, by increasing number density since increase the plasma frequency and that consequently enhance the mode frequency.

Fig 3.5 depicts the dispersion plot for EM wave modes propagating perpendicular the external magnetic field for both (a) nearly non degenerate and (b) nearly degenerate quantum plasma for different values of external magnetic field. The number density and ξ for plot (a) of Fig 3.5 are $n = 10^{12}/m^3$, $\xi = 0.3$ and number density and δ for (b) are $n = 10^{30}/m^3$, $\delta = 0.2$. Plot (a) shows the magnetic field variation. It is clear from Fig 3.5 (a) that by increasing the value of magnetic field increases the frequency in nearly non degenerate limits for x-mode. While Fig 3.5 (b) shows that x-mode frequency decreases with increasing magnetic field for nearly degenerate limits. this interesting behavior is because of high number density compared to magnetic field. Also magnetic field variation increment is very small.

Fig 3.6 shows the behavior of resonant frequency as a function of $\mathbf{k}$ for x-modes perpendicular the external magnetic field for various values of ξ and δ respectively, for both (a) nearly non degenerate and (b) nearly degenerate quantum plasma. The variation in Fig 3.6 (a) with respect to ξ is such that solid line ($\xi = 0$) show complete non degenerate plasmas, dashed line ($\xi = 0.4$) and dotted line ($\xi = 0.8$) represents nearly non degenerate quantum plasmas. It is observed that resonant frequency increases for increased values of ξ. Physically increased ξ means that degeneracy increases, which consequently enhance the probability of wave particle interaction and hence the resonant frequency. The opposite phenomena occurs in Fig 3.6 (b), where resonant frequency decreases with increased value of $\delta = \frac{T_e}{T_{Fe}}$, for nearly degenerate limits. For fixed T_e, the decreasing values of T_{Fe}, shows increasing δ. Thus for decreasing T_{Fe} the degeneracy in the system decrease, this then decrease the probability of wave particle interaction and hence the resonant frequency.

Fig 3.7 shows dispersion plot for Resonant frequency of EM wave modes propagating perpendicular to external magnetic field for various values of number density for both (a) nearly non degenerate and (b) nearly degenerate quantum plasma. The value of magnetic field and ξ for plot (a) are $\mathbf{B}_0 = 0.5T$, $\xi = 0.1$ and value of magnetic and δ for (b) are $\mathbf{B}_0 = 10^3T$, $\delta = 0.001$. Both Fig 3.7 (a) and (b) shows that resonant frequency increase with increased number density for nearly non degenerate and nearly degenerate quantum plasma. Physically it is obvious that with increasing number density enhance the probability of wave particle interaction and hence the resonant frequency.

Fig 3.8 shows the dispersion plot for Resonant frequency of EM wave modes propagating perpendicular to external magnetic field for various values of magnetic field for both (a) nearly non degenerate and (b) nearly degenerate quantum plasma. The number density and ξ for plot (a) are $n = 10^{22}/m^3$, $\xi = 0.2$ and number density and δ for (b) are $n = 10^{36}/m^3$, $\delta = 0.001$. Both Fig 3.8 (a) and (b) shows that resonant frequency increases with increased magnetic field for nearly non degenerate and nearly degenerate quantum plasma. Physically it is obvious that with increasing magnetic field enhance the probability of wave particle interaction and hence the resonant frequency.

Fig 3.9 depicts the behavior of cut-off frequency as a function of $\mathbf{k}$ for x-modes perpendicular the external magnetic field for different values of ξ and δ respectively, for both (a) nearly non degenerate and (b) nearly degenerate quantum plasma. The variation in Fig 3.9 (a) with respect to ξ is such that solid line ($\xi = 0$) show complete non degenerate plasmas, dashed line ($\xi = 0.3$) and dotted line ($\xi = 0.7$) represents nearly non degenerate quantum plasmas. It is observed that cut-off frequency increases for increased values of ξ. Physically increased ξ means that degeneracy increases, which consequently enhance the probability of wave particle interaction and hence the cut-off frequency. The opposite phenomena occurs in Fig 3.9 (b), where cut-off frequency decreases with increased value of $\delta = \frac{T_e}{T_{F_e}}$, for nearly degenerate limits. For fixed T_e, the decreasing values of T_{Fe}, shows increasing δ. Thus for decreasing T_{Fe} the degeneracy in the system decrease, this then decrease the probability of wave particle interaction and hence the cut-off frequency.

Fig 3.10 reveals the dispersion plot for cut-off frequency of EM wave modes propagating perpendicular to external magnetic field for various values of number density for both (a) nearly non degenerate and (b) nearly degenerate quantum plasma. The values magnetic field and ξ for plot (a) are $\mathbf{B}_0 = 0.008T$, $\xi = 0.1$ and values of magnetic and δ for (b) are $\mathbf{B}_0 = 10^3 T$, $\delta = 0.0003$. Both Fig 3.10 (a) and (b) shows that cut-off frequency increase with increased number density for nearly non degenerate and nearly degenerate quantum plasma. Physically it is obvious that with increasing number density enhance the probability of wave particle interaction and hence the cut-off frequency.

Fig 3.11 shows the dispersion plot for cut-off frequency of EM wave modes propagating perpendicular to external magnetic field for different values of magnetic field for both (a) nearly non degenerate and (b) nearly degenerate quantum plasma. The number density and ξ for plot (a) are $n = 10^{21}/m^3$, $\xi = 0.2$ and number deusity and δ for (b) are $n = 10^{34}/m^3$, $\delta = 0.03$. Both Fig 3.11 (a) and (b) shows that cut-off frequency increases with increased magnetic field for nearly non degenerate and nearly degenerate quantum plasma. Physically it is obvious that with increasing magnetic field enhance the probability of wave particle interaction and hence the cut-off frequency.

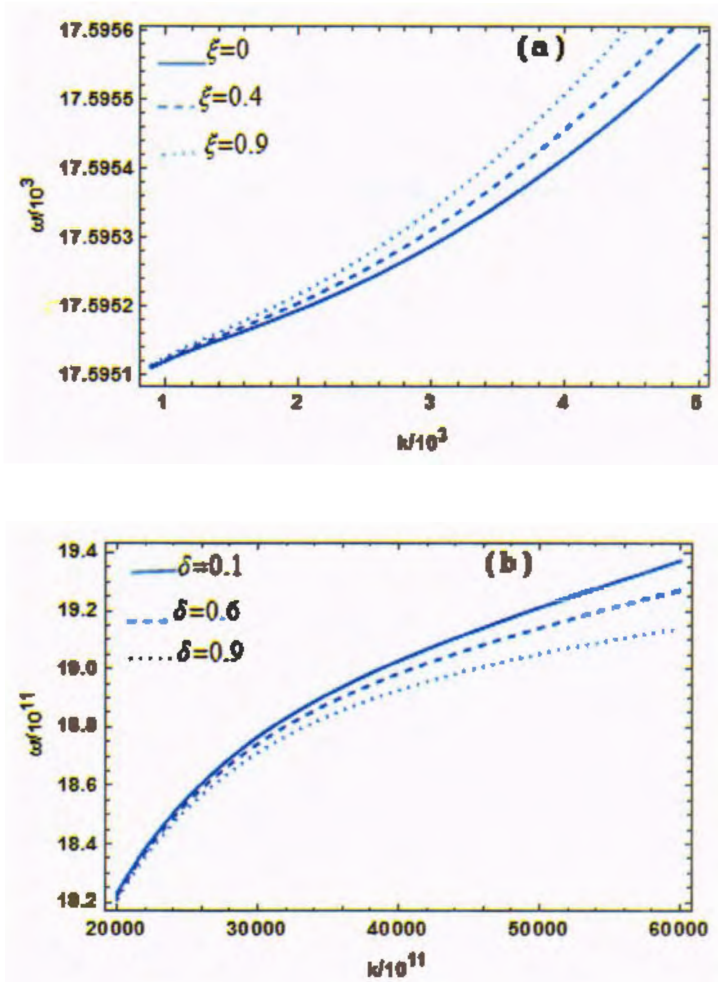
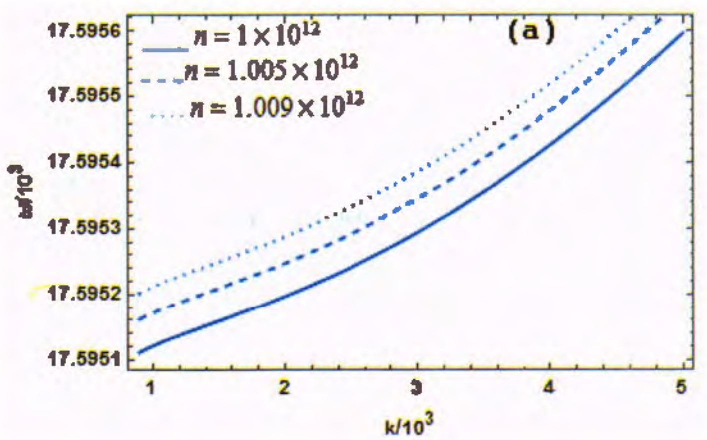


Figure 3.3: Dispersion plots for EM wave modes propagating perpendiculer to the magnetic feild ($\mathbf{k}\perp\mathbf{B}_0$) for both (a) nearly non degenerate and (b) nearly degenerate quantum plasma for various values of (a) $\xi = 0$ (solid line), 0.4 (dashed line) and 0.9 (dotted line) and of (b) $\delta = 0.1$ (solid line), 0.6 (dashed line) and 0.9 (dotted line). The number density and magnetic feild for plot (a) are $n = 10^{12}/m^3$, $\mathbf{B}_0 = 0.0000001T$ and number density and magnetic field for (b) are $n = 10^{31}/m^3$, $\mathbf{B}_0 = 2T$ For case (a) we have $\mathrm{T}_e = 100\ T_{Fe}$ and for case (b) $T_e \ll T_{Fe}$.



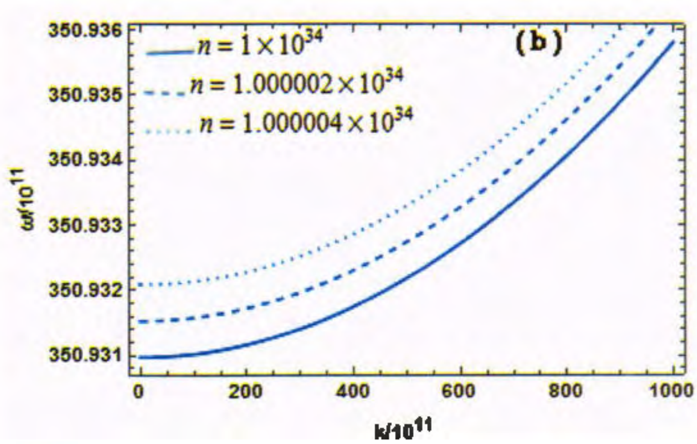


Figure 3.4: Dispersion plots for EM wave modes propagating perpendiculer to the magnetic feild ($\mathbf{k}\perp\mathbf{B}_0$) for both (a) nearly non degenerate and (b) nearly degenerate quantum plasma for various values of (a) $n = 1\times10^{12}/m^3$ (solid line), $1.005\times10^{12}/m^3$ (dashed line) and $1.009\times10^{12}/m^3$ (dotted line) and of (b) $1\times10^{34}/m^3$ (solid line), $1.000002\times10^{34}/m^3$ (dashed line) and $1.000004\times10^{34}/m^3$ (dotted line). The vale magnetic feild and ξ for plot (a) are $\mathbf{B}_0 = 0.0000001T$, $\xi = 0.1$ and value of magnetic and δ for (b) are $\mathbf{B}_0 = 10^2T$, $\delta = 0.1$. For case (a) we have $T_e = 100\ T_{Fe}$ and for case (b) $T_e \ll T_{Fe}$.

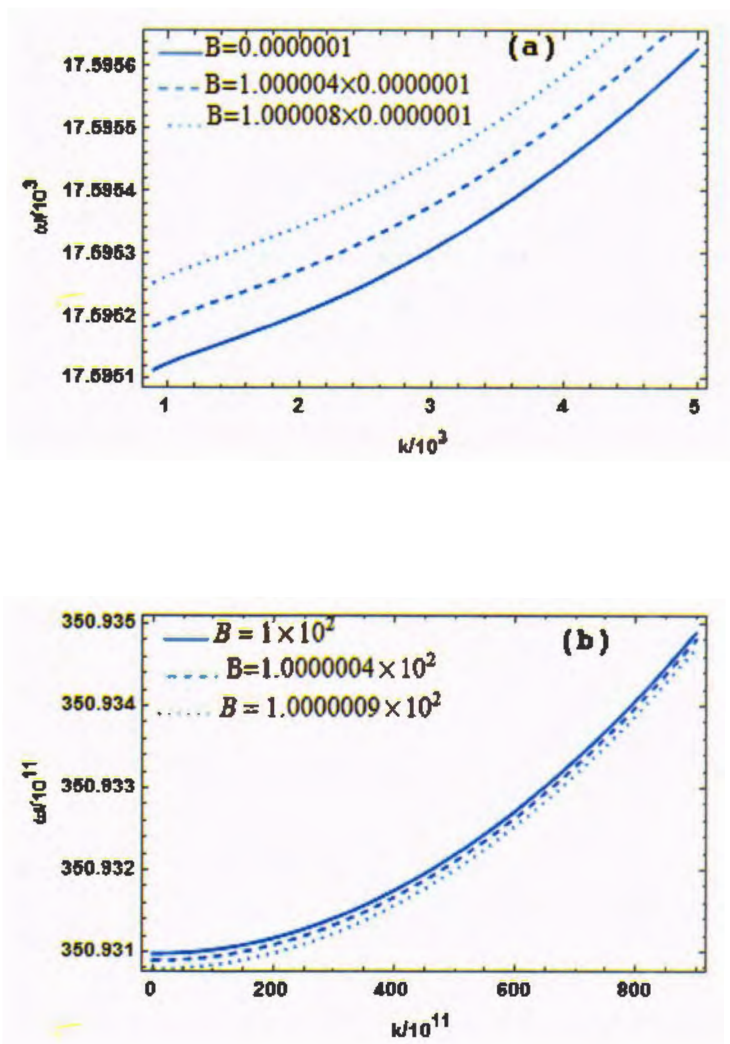


Figure 3.5: Dispersion plots for EM wave modes propagating perpendiculer to the magnetic feild ($\mathbf{k} \perp \mathbf{B}_0$) for both (a) nearly non degenerate and (b) nearly degenerate quantum plasma for various values of (a) $\mathbf{B}_0 = 0.0000001T$ (solid line), $1.000004 \times 0.0000001$ (dashed line) and $1.000008 \times 0.0000001T$ (dotted line) and of (b) $\mathbf{B}_0 = 1 \times 10^2 T$ (solid line), $1.0000004 \times 10^2 T$ (dashed line) and $1.0000009 \times 10^2 T$ (dotted line). The number density and ξ for plot (a) are $n = 10^{12}/m^3$, $\xi = 0.3$ and number density and δ for (b) are $n = 10^{30}/m^3$, $\delta = 0.2$. For case (a) we have $T_e = 100 T_{Fe}$ and for case (b) $T_e \ll T_{Fe}$.

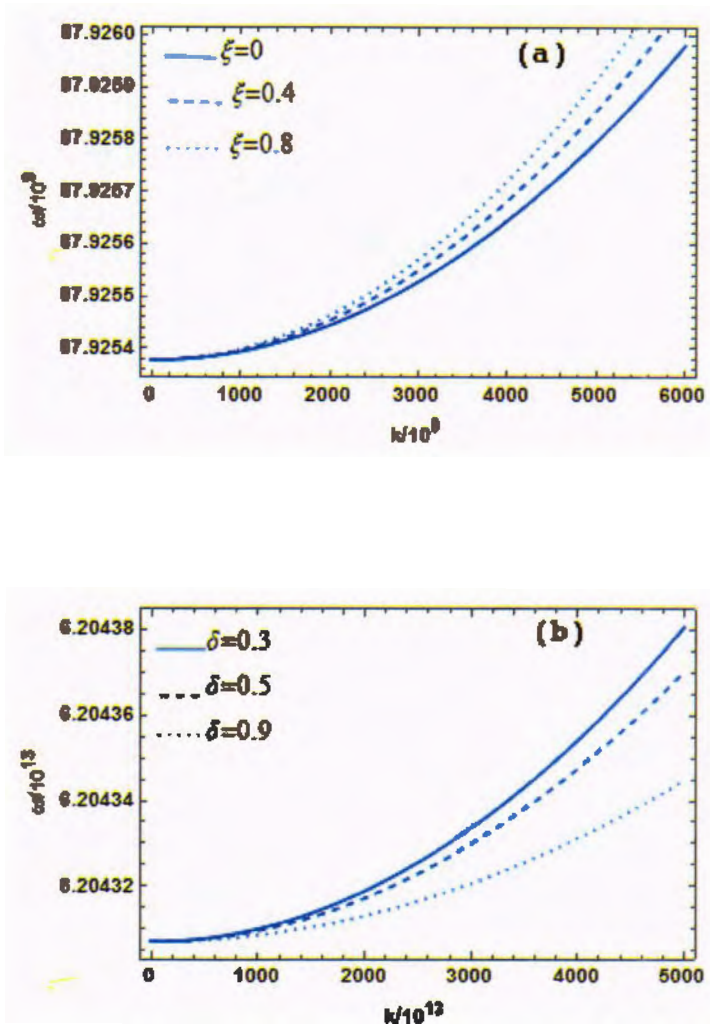
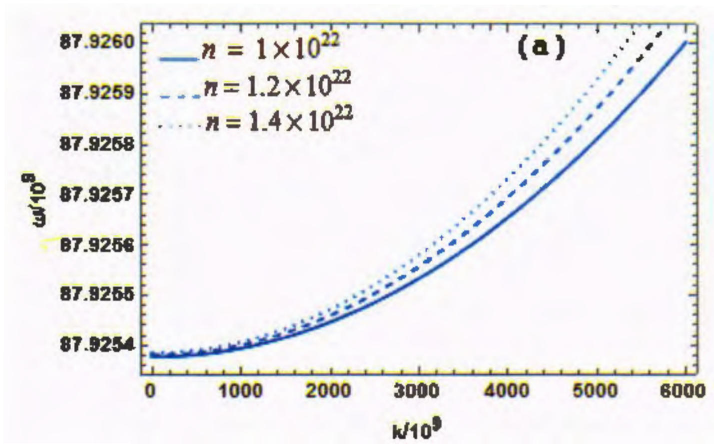


Figure 3.6: Dispersion plots for EM wave modes (Resonance frequency) propagating perpendiculer to the magnetic feild ($\mathbf{k}\perp\mathbf{B}_0$) for both (a) nearly non degenerate and (b) nearly degenerate quantum plasma for various values of (a) $\xi = 0$ (solid line), 0.4 (dashed line) and 0.8 (dotted line) and of (b) $\delta = 0.3$ (solid line), 0.5 (dashed line) and 0.9 (dotted line). The number density and magnetic feild for plot (a) are $n = 10^{22}/m^3$, $\mathbf{B}_0 = 0.5T$ and for (b) are $n = 10^{36}/m^3$, $\mathbf{B}_0 = 10^3T$ For case (a) we have $T_e = 100$ T_{Fe} and for case (b) $T_e \ll T_{Fe}$.



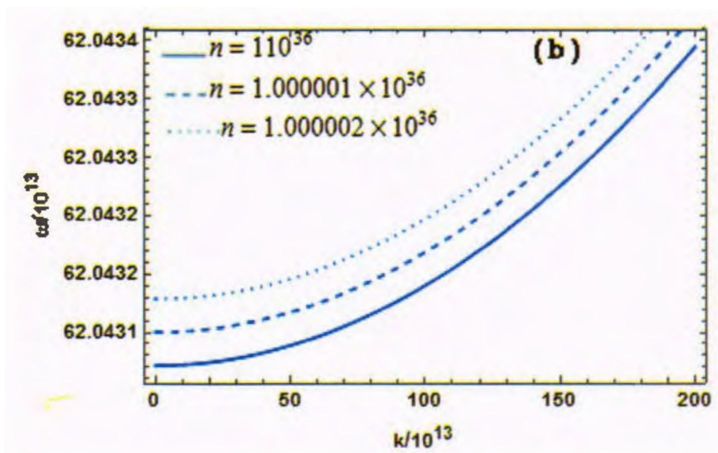
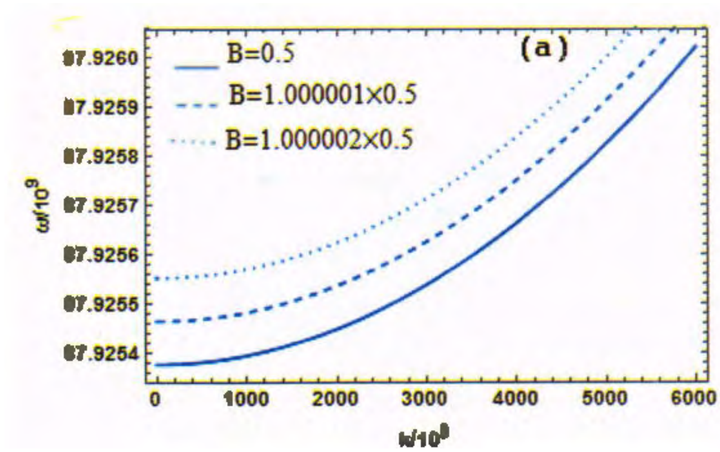
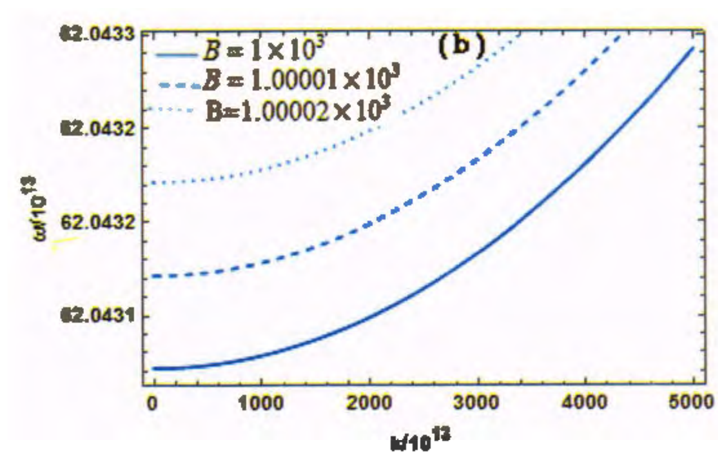


Figure 3.7: Dispersion plots for EM wave modes (Resonance frquency) propagating perpendiculer to the magnatic feild ($\mathbf{k}\perp\mathbf{B}_0$) for both (a) nearly non degenerate and (b) nearly degenerate quantum plasama. For various values of (a) $n = 1 \times 10^{22}/m^3$ (solid line), $1.2 \times 10^{22}/m^3$ (dashed line) and $n = 1.4 \times 10^{22}/m^3$ (dotted line) and of (b) $n = 1 \times 10^{36}/m^3$ (solid line) , $1.000001 \times 10^{36}/m^3$ (dashed line) and $n = 1.000002 \times 10^{36}/m^3$ (dotted line). The value of magnetic feild and ξ for plot (a) are $\mathbf{B}_0 = 0.5T$, $\xi = 0.1$ and values of magnetic feild and δ for plot (b) are $\mathbf{B}_0 = 10^3 T$, $\delta = 0.001$. For case (a) we have $T_e = 100\ T_{Fe}$ and for case (b) $T_e \ll T_{Fe}$.


(a)
B=0.5
B=1.000001×0.5
B=1.000002×0.5
ω/10^9
0
1000
2000
3000
4000
5000
6000


(b)
B = 1×10^3
B = 1.00001×10^3
B=1.00002×10^3
ω/10^13
62.0432
0
1000
2000
3000
4000
5000

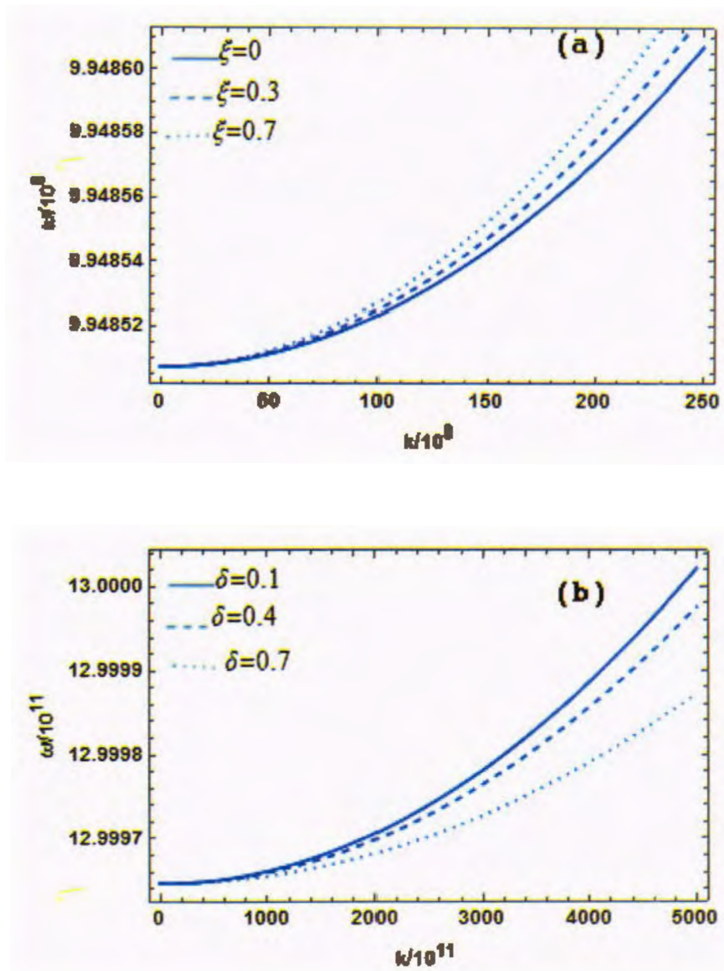
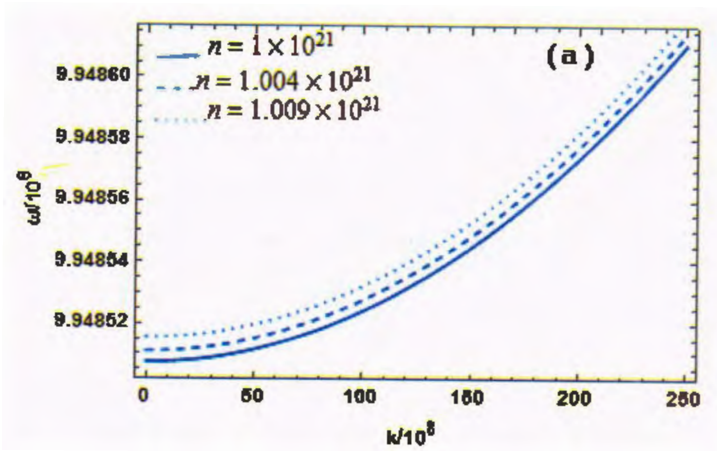


Figure 3.8: Dispersion plots for EM wave modes (cut-off frequency) propagating perpendiculer to the magnetic feild ($\mathbf{k}\perp\mathbf{B}_0$) for both (a) nearly non degenerate and (b) nearly degenerate quantum plasma for various values of (a) $\xi = 0$ (solid line), 0.3 (dashed line) and 0.7 (dotted line) and of (b) $\delta = 0.1$ (solid line), 0.4 (dashed line) and 0.7 (dotted line). The number density and magnetic feild for plot (a) are $n = 10^{21}/m^3$, $\mathbf{B}_0 = 0.008T$ and number density and magnetic feild for (b) are $n = 10^{30}/m^3$, $\mathbf{B}_0 = 10^3T$, For case (a) we have $T_e = 100\ T_{Fe}$ and for case (b) $T_e \ll T_{Fe}$.



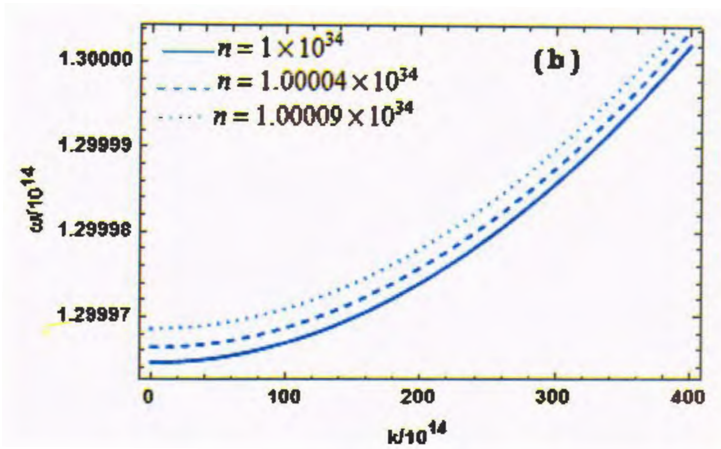
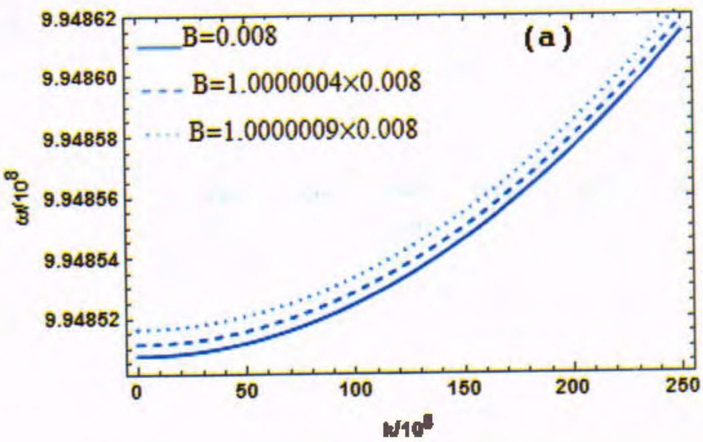
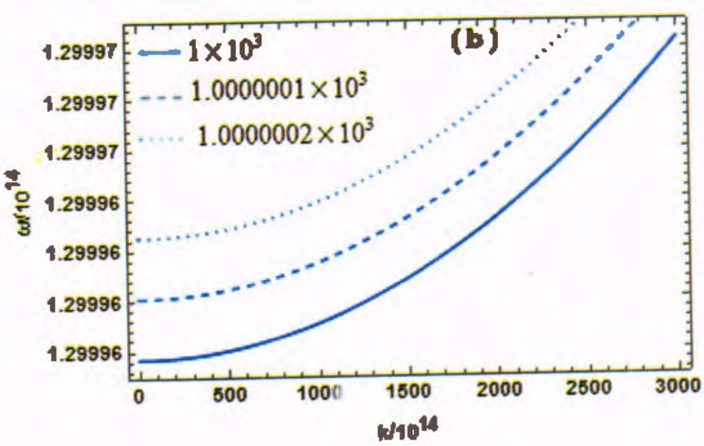


Figure 3.9: Dispersion plots for EM wave modes (cut-off frequency) propagating perpendiculer to the magnetic feild ($\mathbf{k}\perp\mathbf{B}_0$) for both (a) nearly non degenerate and (b) nearly degenerate quantum plasma for different values of (a) $n = 1\times10^{21}/m^3$ (solid line), $1.004\times10^{21}/m^3$ (dashed line) and $1.009\times10^{21}/m^3$ (dotted line) and of (b) $1\times10^{34}/m^3$ (solid line), $1.00004\times10^{34}/m^3$ (dashed line) and $1.00009\times10^{34}/m^3$ (dotted line). The values magnetic feild and ξ for plot (a) are $\mathbf{B}_0 = 0.008T$, $\xi = 0.1$ and values of magnetic and δ for (b) are $\mathbf{B}_0 = 10^3T$, $\delta = 0.0003$. For case (a) we have $T_e = 100\ T_{Fe}$ and for case (b) $T_e \ll T_{Fe}$.



B=0.008
B=1.0000004×0.008
B=1.0000009×0.008
(a)
ω/10^8
k/10^8



1×10^3
1.0000001×10^3
1.0000002×10^3
(b)
ω/10^14
k/10^14