Healthcare Resources Allocation Decision Support Tool: A Forecasting-Simulation-Fuzzy Optimization Approach

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A Dissertation
Submitted in the Partial Fulfillment
of the Requirements for the Degree of
MASTER OF SCIENCE
IN
MATHEMATICS

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Certificate

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We accept this dissertation as confirming to the required standard

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DEDICATION

Thesis is proudly dedicated to my beloved family who always advices me and encouraged me thank you for your endless love, sacrifices and prayers.

My humble and respected Supervisor Dr. Sajida Kousar, thank you for all your efforts and guidance.

Declaration

I, Tania Hussain Satti, solemnly declares that the work presented in this dissertation entitled "Healthcare Resources Allocation Decision Support Tool: A Forecasting-Simulation-Fuzzy Optimization Approach" is original or otherwise acknowledged. This work has not been submitted as a whole or in part for any other degree to any other university in Pakistan or abroad.

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Forwarding Sheet by Research Supervisor

The Thesis entitled "Healthcare Resources Allocation Decision Support Tool: A Forecasting Simulation-Fuzzy Optimization Approach" submitted by Tania Hussain Satti, Reg. No 745- FBAS/MSMA/S21 in partial fulfillment of MS Degree in Mathematics has been completed under my guidance and supervision. I am satisfied with the quality of her research work and allow her to submit this thesis for further process to graduate with Master of Science degree from the Department of Mathematics & Statistics, as per IIUI rules and regulations.

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Abstract

Due to the increasing population, there has been an inefficient allocation of resources in the health care department. In the past years, significant demand and limitation of capacity were faced across every specialty in hospital but mostly this issue has been found in accident and emergency department. However, a hospital bed seems to be crucial resource for all health care system. Therefore to extract the best solutions for the allocation of existing resources in A & E department, various optimization techniques were applied. In emergency department, different sort of uncertainties occurs such that to deal with these uncertainties fuzzy optimization techniques were adopted in the past years. A hybrid approach based on an adaptive neuro-fuzzy inference system (ANFIS) can be adopted to predict about the occurrence of the adverse events. An ANFIS network structure is split into two parts: the premise(IF) is the first component, while the second component is known as the consequence (THEN) and a fuzzy inference system is constructed on fuzzy sets and fuzzy rules and it includes three other components, each of which plays a specific role in reasoning, including fuzzification, inference, defuzzification. By using IF-THEN rules, the fuzzy inference system (FIS) predicts behaviour of the system. These fuzzy inference rules are then applied on the optimization problems and the model is analysed and interpreted for the resource allocation of private and government health care organization. Decision-making tools assist us in the recognition, evaluation, and acceptance of alternatives on the basis of judgements. The essential role of health care decision makers is to determine the most efficient use of limited resources, and distributing the resources, use of technical knowledge provided the information that is available at the moment. One of the most significant area of decision making theory is multi criteria decision making (MCDM). Therefore, an innovative approach known as Best-Worst-Method (BWM) is used to solve MCDM problems. Both best (e.g. most appealing, most essential) and worst (e.g. least appealing, least significant) criteria are evaluated first by the decision maker in accordance with BWM. The scales for criteria are modified to intuitionistic fuzzy form by employing the Intuitionistic fuzzy best worst method (IFBWM). This thesis is classified in to three chapters.

Chapter 1, includes introductory concepts that are essential to understand the subject area. In chapter 2, health care resource allocation planning through fuzzy linear programming model is

presented. The solutions of the problem are computed through ANFIS and it is then compared with the solution of actual problem.

In chapter 3, the concept of fuzzy best worst for decision making are discussed in detail and intuitionistic fuzzy best worst technique is implemented for resource allocation in A & E department. The weights of the various criteria obtained from intuitionistic fuzzy best worst method (IFBWM) are compared with the fuzzy Best worst method.

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Author's Declaration

I hereby state that neither the entirety of this thesis nor any individual parts of it have been transcribed from any other sources. Moreover, I hereby acknowledge that with the cooperation of my mentorDr. Sajida Kousar . i am able to writethis MSthesis entirely . The work presented in this research has not in any manner been produced in support of an application for a degree or other qualification from this or another educational institution.

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Chapter 1

Preliminaries

This chapter is consisting of all the terminologies which further helped to achieve tasks related to later work.

1.1 Fuzzy logic

Fuzzy set theory is a generalization of classical set theory. Fuzzy set theory has been considered to be perfectly natural from the concept of other sciences. A brief study of history may indeed be beneficial to better explain the essence and influencing factors of fuzzy set theory. The researchers goal is to broaden the technical definition of a set and a premise in order to allow the fuzziness as it is relevant to human communication, as well as in human logic, review, and conclusions. This is demonstrated in the first article presented by Zadeh [1] and Goguen [2] on fuzzy set theory. To achieve blurry conceptual phenomena utilizing fuzzy set theory presents a precise mathematical formulation. This concept gained huge acceptance in the last century of with the first significant real-world employment of fuzzy rule-based algorithms for the management of process development, termed as fuzzy logic control. According to Zadeh [3], fuzzy set is a collection of attributes having degree of membership function. A membership function which grants each element to a membership grade between zero and one are distinguished by this kind of set. In the scope of fuzzy sets to incorporate the notions of union, intersection, complement, and various features of these concepts are established. In fact, both fuzzy set theory and probability theory as approachable methods to capture uncertainties occurs in every aspects of the prevailing situations. Fuzzy logics are effectively used in todays organization such as in particularly control system engineering, signal processing, biochemical engineering, factory automation, biotechnology, home appliances, and optimization, in health care etc. In health care department many uncertainties occurs which is unpredictable. And as we know that one of the most important application in the field of health care are medicines in which fuzzy set theory was implemented and presented by Zadeh [4]. The most likely utilisation of fuzzy based theory would be in diagnostic testing and was early forecast by Zadeh theory [5]. Since fuzzy set theory allows to enable interpretation and inferences from inaccurate information, it is proposed to control uncertainties and vagueness in expert systems. Clients information, personal clinical records, as well as other inadequately information are extensively used in the diagnosis of illnesses and in the formulation of medical interventions in the sector of medicines. A rule-based system is a computerized that utilized application of fuzzy inference. Through the use of an open source development environment, it was constructed digitally for the detection of diabetes disease. Web-based Fuzzy Expert System for Diabetes Diagnosis (Web- FESDD)is an intelligent diagnostic system that can be enployed by doctors, diabetic experts, and clients to diagnose. Invalid data that emerges and during disease diagnosis and treatment can also be treated by fuzzy knowledge - based systems [6] The vast majority of research on fuzzy set theory in health care has already been conducted by individuals and is still frequently viewed as unstructured related to his own predictions. For the sake of better understanding and better understanding it is inevitable to learn and investigate the fundamental properties of fuzzy sets and it's generalization.

Definition 1.1.1. Let U be a non-empty set. A **fuzzy set** $\tilde{F}$ of U is a set of ordered pairs represented as.

$$\tilde{F} = \{(u, \hat{\lambda}_{\tilde{F}}(u)) |\ u \in U\}$$

where the first element is from non empty set and second element is it's image under the membership function $\lambda_{\tilde{F}}$ and this membership function is expressed as.

$$\hat{\lambda}_{\tilde{F}} : U \to [0, 1]$$

and $\hat{\lambda}_{\tilde{F}}(u)$ is called the grade of membership of u in the fuzzy set $\tilde{F}$. The value 0 means that u is not a member of the fuzzy set and the value 1 means that u is fully a member of the fuzzy set. A fuzzy set is called normalized when at least one of its elements attains the maximum possible membership grade.

Let U denotes the non-empty set and $\tilde{F}$ and $\tilde{G}$ are the two fuzzy sets where $\hat{\lambda}_{\tilde{F}}$ and $\hat{\lambda}_{\tilde{G}}$ are the corresponding fuzzy membership function. Then their intersection $\tilde{F} \cap \tilde{G}$ and union $\tilde{F} \cup \tilde{G}$ are defined as

$$\hat{\lambda}_{\tilde{F}\cap\tilde{G}}(u) = \min\{\hat{\lambda}_{\tilde{F}}(u), \hat{\lambda}_{\tilde{G}}(u)\} \quad \forall u \in U \tag{1.1.1}$$

$$\hat{\lambda}_{\tilde{F}\cup\tilde{G}}(u) = \max\{\hat{\lambda}_{\tilde{F}}(u), \hat{\lambda}_{\tilde{G}}(u)\} \quad \forall u \in U \tag{1.1.2}$$

Example 1.1.2. Let U $=\{r_1, r_2, r_3, r_4, r_5\}$ and suppose $\tilde{F}$, $\tilde{G}$ are two fuzzy sets

$$\tilde{F} = \{(r_1, 0.2), (r_2, 0.3), (r_3, 0.6), (r_4, 0.7), (r_5, 0.9)\} \quad and$$

$$\tilde{G} = \{(r_1, 0.1), (r_2, 0.4), (r_3, 0.5), (r_4, 0.8), (r_5, 1)\}$$

then

$$\tilde{F} \cap \tilde{G} = \{(u, \lambda_{\tilde{F}\cap\tilde{G}}(u)) \ |\forall u \in U\} \tag{1.1.3}$$

$$\tilde{F} \cup \tilde{G} = \{(u, \lambda_{\tilde{F}\cup\tilde{G}}(u)) \ |\forall u \in U\} \tag{1.1.4}$$

$$\tilde{F} \cap \tilde{G} = \{(r_1, 0.1), (r_2, 0.3), (r_3, 0.5), (r_4, 0.7), (r_5, 0.9)\}$$

$$\tilde{F} \cup \tilde{G} = \{(r_1, 0.2), (r_2, 0.4), (r_3, 0.6), (r_4, 0.8), (r_5, 1)\}$$

The **complement $\tilde{F}^c$ on a fuzzy set $\tilde{F}$** is also a fuzzy set with membership function defined as.

$$\lambda_{\tilde{F}^c}(u) = 1 - \lambda_{\tilde{F}}(u) \quad \forall u \in U$$

For $\alpha \in [0, 1]$,the $\alpha -$ *level* **set** is a subset of U containing all those elements having degree greater than or equal α. to defined as, the set of all those elements belonging to fuzzy set U whose membership function defined on the fuzzy set U is larger than or equal to the degree α. Mathematically, it is denoted as:

$$\tilde{F}_\alpha = \{u \mid \lambda_{\tilde{F}}(u) \geq \alpha \quad \forall u \in U, \quad \alpha \in [0, 1]\}$$

where

$$\acute{\tilde{F}}_\alpha = \{u \mid \lambda_{\tilde{F}}(u) > \alpha \quad \forall u \in U, \quad \alpha \in [0, 1]\}$$

is known as **strong-α level set** or **strong-α cut**.

Definition 1.1.3. Fuzzy numbers are a generalization of crisp numbers to handle vague or uncertain information. A fuzzy number is a mathematical concept that represents a real number with uncertainty or imprecision, and it is defined over a fuzzy set. A triangular fuzzy number is the triplet $(\acute{\xi}, \nu, \psi)$ with the membership function. Graphically, the membership is presented in figure 1.1

$$\lambda_{\tilde{F}}(u; \acute{\xi}, \nu, \psi) = \begin{cases} 0, & \text{if } u \leq \acute{\xi}; \\ \frac{u-\acute{\xi}}{\nu-\acute{\xi}}, & \text{if } \acute{\xi} \leq u \leq \nu; \\ \frac{\psi-u}{\psi-\nu}, & \text{if } \nu \leq u \leq \psi; \\ 0, & \text{if } u \geq \psi; \end{cases} \tag{1.1.5}$$

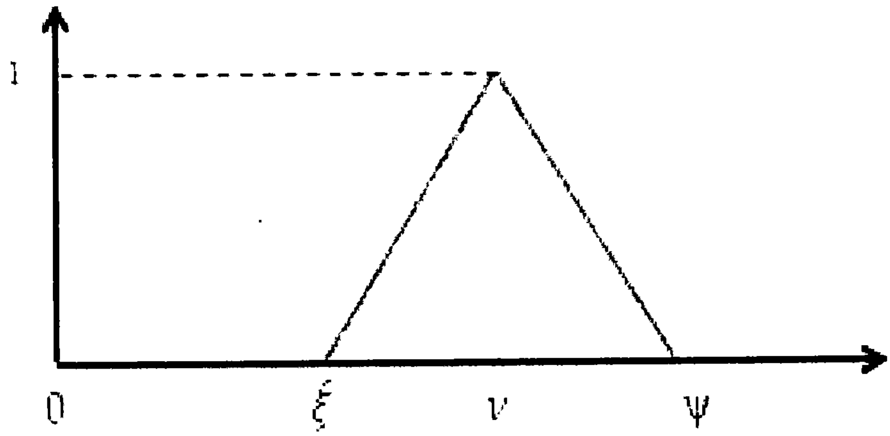


Figure 1.1: **Membership function of triangular fuzzy numbers**

1.2 Intuitionistic fuzzy sets

The concept of intuitionistic fuzzy sets (IFS) was first explained by Atansassov [7] and it is a set of triplet represented as $\{\langle e, (\eta_{\tilde{L}}(e), \varphi_{\tilde{L}}(e))\rangle\}$ where $\eta_{\tilde{L}}(e), \varphi_{\tilde{L}}(e)$ denotes the degree of truth and untruth respectively and $\eta_{\tilde{L}}(e), \varphi_{\tilde{L}}(e) \in [0, 1]$.

Definition 1.2.1. Let $\tilde{S}$ be the non-empty set and mathematically a fuzzy subset $\tilde{L}$ of $\tilde{S}$ is represented as:

$$\tilde{L} = \{\langle e, \eta_{\tilde{L}}(e), \varphi_{\tilde{L}}(e)\rangle \mid e \in \tilde{S}\} \tag{1.2.1}$$

Where $\eta_{\tilde{L}}$ and $\varphi_{\tilde{L}}$ are function from $\tilde{S}$ to the interval $[0, 1]$ commonly called membership and non-membership function with

$$0 \leq \eta_{\tilde{L}}(e) + \varphi_{\tilde{L}}(e) \leq 1 \quad e \in \tilde{S} \tag{1.2.2}$$

1.2.1 Operations defined on intuitionistic fuzzy sets

Let $\tilde{L}_1$ and $\tilde{L}_2$ denotes the two intuitionistic fuzzy sets. then the arithmetic operations and set theory on intuitionistic fuzzy sets are defined as:

$$\tilde{L}_1 + \tilde{L}_2 = \{\langle e, \eta_{\tilde{L}_1}(e) + \eta_{\tilde{L}_2}(e) - \eta_{\tilde{L}_1}(e) \cdot \eta_{\tilde{L}_2}, \varphi_{\tilde{L}_1}(e) \varphi_{\tilde{L}_2}(e)\rangle \mid e \in \tilde{S}\} \tag{1.2.3}$$

$$\tilde{L}_1 \times \tilde{L}_2 = \{\langle e, \eta_{\tilde{L}_1}(e)\eta_{\tilde{L}_2}(e), \varphi_{\tilde{L}_1}(e) + \varphi_{\tilde{L}_2}(e) - \varphi_{\tilde{L}_1}(e)\varphi_{\tilde{L}_2}(e)\rangle \mid e \in \tilde{S}\} \tag{1.2.4}$$

$$\tilde{L}_1 \cup \tilde{L}_2 = \{\langle e, \max(\eta_{\tilde{L}_1}(e), \eta_{\tilde{L}_2}(e)), \min(\varphi_{\tilde{L}_1}(e), \varphi_{\tilde{L}_2}(e))\rangle \mid e \in \tilde{S}\} \tag{1.2.5}$$

$$\tilde{L}_1 \cap \tilde{L}_2 = \{\langle e, \min(\eta_{\tilde{L}_1}(e), \eta_{\tilde{L}_2}(e)), \max(\varphi_{\tilde{L}_1}(e), \varphi_{\tilde{L}_2}(e))\rangle \mid e \in \tilde{S}\} \tag{1.2.6}$$

$$\tilde{L}_1 : \tilde{L}_2 = \{\langle e, \eta_{\tilde{L}_1 : \tilde{L}_2}(e), \varphi_{\tilde{L}_1 : \tilde{L}_2}(e)\rangle \mid e \in \tilde{S}\} \tag{1.2.7}$$

$$\tilde{L}_1 - \tilde{L}_2 = \{\langle e, \eta_{\tilde{L}_1 - \tilde{L}_2}(e), \varphi_{\tilde{L}_1 - \tilde{L}_2}(e)\rangle \mid e \in \tilde{S}\} \tag{1.2.8}$$

$$\eta_{\tilde{L}_1 - \tilde{L}_2}(e) = \begin{cases} \frac{\eta_{\tilde{L}_1}(e) - \eta_{\tilde{L}_2}(e)}{1 - \eta_{\tilde{L}_2}(e)} & \text{if} \quad \eta_{\tilde{L}_1}(e) \geq \eta_{\tilde{L}_2}(e) \\ 0 & otherwise \end{cases} \tag{1.2.9}$$

$$\varphi_{\tilde{L}_1 - \tilde{L}_2}(e) = \begin{cases} \frac{\varphi_{\tilde{L}_1}(e)}{\varphi_{\tilde{L}_2}(e)} & \text{if} \quad \varphi_{\tilde{L}_1}(e) \leq \varphi_{\tilde{L}_2}(e) \\ 1 & otherwise \end{cases} \tag{1.2.10}$$

and:

$$\eta_{\tilde{L}_1 : \tilde{L}_2}(e) = \begin{cases} \frac{\eta_{\tilde{L}_1}(e)}{\eta_{\tilde{L}_2}(e)} & \text{if} \quad \eta_{\tilde{L}_1}(e) \leq \eta_{\tilde{L}_2}(e) \\ & and \quad \eta_{\tilde{L}_2}(e) > 0 \\ & and \quad \eta_{\tilde{L}_1}(e)\varpi_{\tilde{L}_2}(e) \leq \eta_{\tilde{L}_2}(e)\varpi_{\tilde{L}_1}(e) \\ 0 & otherwise \end{cases} \tag{1.2.11}$$

$$\varphi_{\tilde{L}_1 : \tilde{L}_2}(e) = \begin{cases} \frac{\varphi_{\tilde{L}_1}(e) - \varphi_{\tilde{L}_2}(e)}{1 - \varphi_{\tilde{L}_2}(e)} & \text{if} \quad \varphi_{\tilde{L}_1}(e) \leq \varphi_{\tilde{L}_2}(e) \\ & and \quad \varphi_{\tilde{L}_2}(e) > 0 \\ & and \quad \varphi_{\tilde{L}_1}(e)\varpi_{\tilde{L}_2}(e) \leq \varphi_{\tilde{L}_2}(e)\varpi_{\tilde{L}_1}(e) \\ 1 & otherwise \end{cases} \tag{1.2.12}$$

1.3 Fuzzy Inference System

One of the most powerful technique for coping with nonlinear and ill-determined mapping of model parameters to particular output parameters is a fuzzy inference system, which is addressed under fuzzy set theory. A Fuzzy inference knowledge base is constructed on fuzzy sets and fuzzy rules. A fuzzy inference system includes includes fuzzification , inference , defuzzification By using IF-THEN rules, the fuzzy inference system (FIS) predicts behavior of the system. It is the technique of applying probabilistic logic to convert a collection of specific input parameters to an output parameter. Five main operations are performed by fuzzy inference system [8] see figure 1.2

1. A rule base comprising multiple fuzzy IF THEN rules.

2. A collection that outlines the membership criteria for the fuzzy sets used in the fuzzy rules
3. Rules are inferred using a decision-making mechanism.
4. A process of transforming the degree of correspondence with linguistic characteristics from the crisp parameters is fuzzification.
5. A process of converting the fuzzy outcomes into crisp outcomes is known as defuzzification

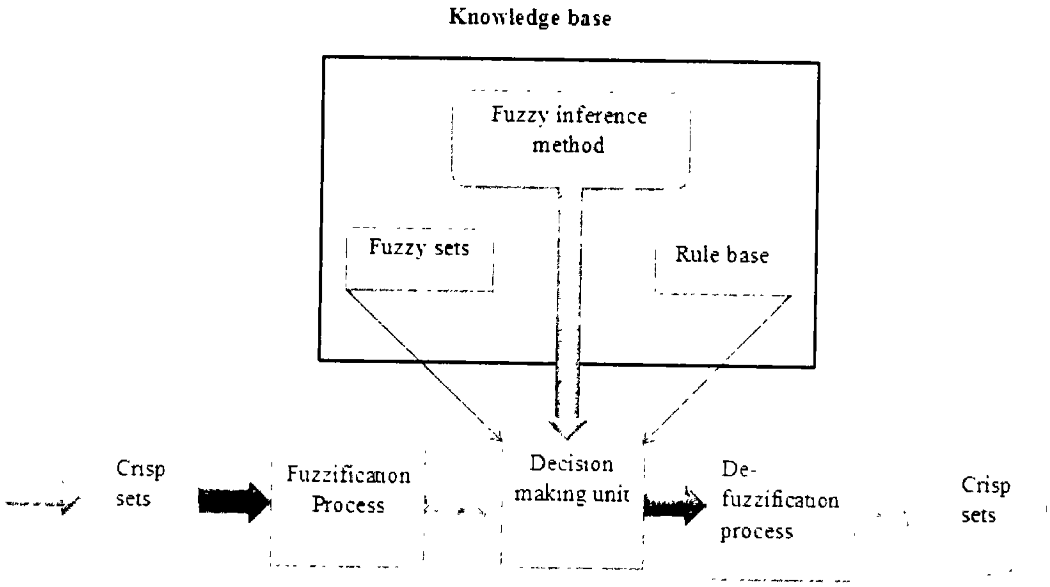


Figure 1.2: **Fuzzy Inference System**

1.3.1 Fuzzy IF-THEN Rules

A fuzzy IF-THEN rule is made up of two components. The first is the IF part and the second is THEN part which are termed as the premises and consequent respectively. Assuming that U is the universal set (the input space) then a fuzzy set $\tilde{F}$ and $\tilde{G}$ in U are described as a collection of ordered pairs.

$$\lambda_{\tilde{F}} = \{(u, \lambda_{\tilde{F}}(u)|u \in U)\}, \lambda_{\tilde{G}} = \{(v, \lambda_{\tilde{G}}(v)|v \in U)\}$$

In the literature, Fuzzy inference system (FIS) have already been devised in a variety of form [9]. It arises from alterations made to the derived component's composition and the defuzzification techniques. One of the most common type identified by Takagi and Sugeno FIS [10] in which the consequential parameter for each rule is expressed as a linear combination of input parameters. The weighted average of the output of each rule is the final outcome. Based on two fuzzy rules a first order Sugeno fuzzy model is developed using two input parameters u, v and one output parameter f [11]. For example

Rule 1: If u is $\tilde{F}_1$ and v is $\tilde{G}_1$, then $f = p_1 u + q_1 v + r_1$ where $\tilde{F}_1, \tilde{F}_2 \in \tilde{F}$ and $\tilde{G}_1, \tilde{G}_2 \in \tilde{G}$.

Rule 2: If u is $\tilde{F}_2$ and v is $\tilde{G}_2$, then $f = p_2 u + q_2 v + r_2$ where $\tilde{F}_1, \tilde{F}_2 \in \tilde{F}$and $\tilde{G}_1, \tilde{G}_2 \in \tilde{G}$,

where p_e, q_e, r_e are the e^{th} rule's succeeding parameters. The $\tilde{F}_e$ and $\tilde{G}_e$ denotes fuzzy sets. Their corresponding membership functions are defined as:

$$w_e = \hat{\lambda}_{\tilde{F}_e}(u) \cap \hat{\lambda}_{\tilde{G}_e}(v) \quad e = 1, 2$$

Where the membership functions of u and v in fuzzy sets $\tilde{F}_e$ and $\tilde{G}_e$ are represented by $\hat{\lambda}_{\tilde{F}_e}$ and $\hat{\lambda}_{\tilde{G}_e}$ respectively.

Fuzzification

It is a mathematical procedure by which crisp input values are transformed in to a membership value of a fuzzy set. Let U denote the universe of discourse for the fuzzy sets $\tilde{F}$ and $\tilde{G}$. The fuzzification process receives the elements u, $v \in U$ and produces their membership degrees.

$$\hat{\lambda}_{\tilde{F}}(u), \quad \lambda_{\tilde{F}}(v), \quad \hat{\lambda}_{\tilde{G}}(u), \quad \hat{\lambda}_{\tilde{G}}(v)$$

Weighted Average The final output of the system is the weighted average of all rules outputs as:

$$\text{Final output} = \frac{\sum_{e=1}^{n} w_e f_e}{\sum_{e=1}^{n} w_e}.$$

1.3.2 Adaptive neuro- fuzzy inference system

Fuzzy and neural networks are combined in the neuro fuzzy inference system which also includes retraining and laboratory tests. The artificial intelligence method which integrates fuzzy logic with artificial neural networks are Neuro-fuzzy. In the literature, when the analysis were conducted various neuro-fuzzy approaches has been presented by Vieira *et al.* [13](see figure 1.3). In the early of 1990 a popular fuzzy inference system (FIS) was launched as *Takagi - Sugeno - Kang*. While optimizing uncertainties to evaluate nonlinear functions both neural networks and fuzzy systems are combined in ANFIS by Jang [12]. To perform instructions from sampling, ANFIS employs a train data set that is the same as that used by artificial neural networks. Therefore the most efficient ANFIS structure for treating the significant issues are determined. To see its influence on samples the generated structure is evaluated. The suitability of ANFIS model reveals the smaller error in the results. It distributes a number of membership functions to each input and presents optimum if-then rules based on optimization methods. An ANFIS network structure is split into two parts: the premise is the first component, while the second component is known as the consequence. ANFIS structure is made up of five layers: [14]

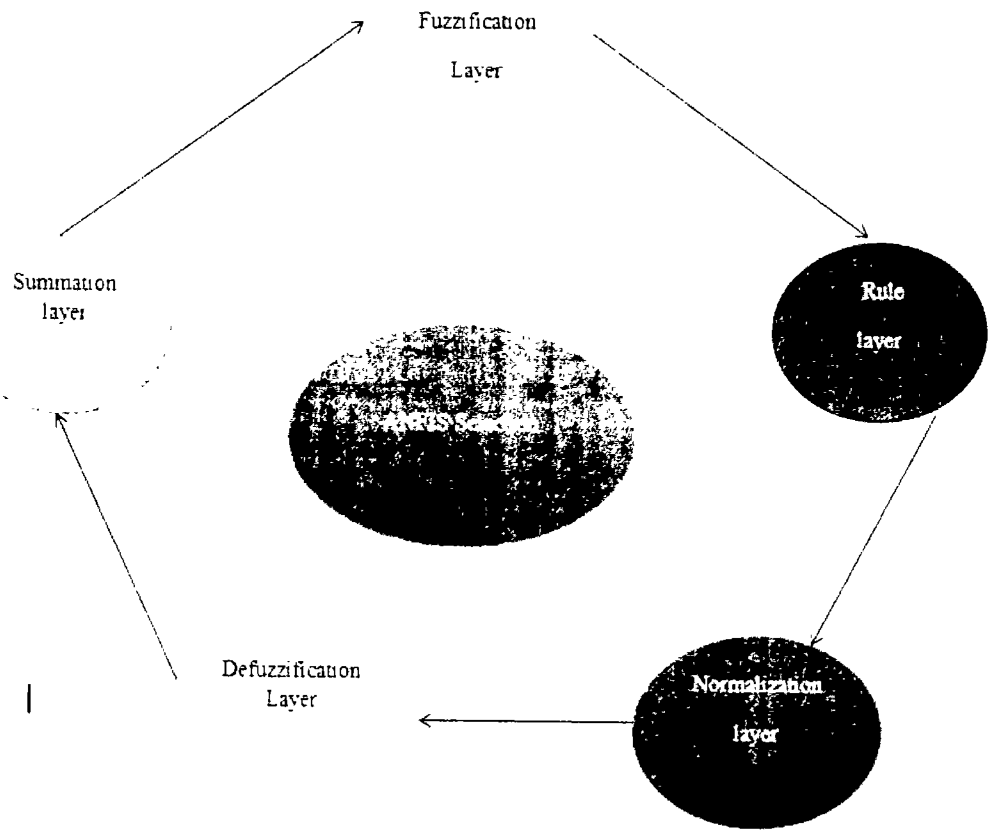


Figure 1.3: **Layer of ANFIS**

Layer 1: Fuzzication layer is the first layer in order to obtain fuzzy clusters from input values by using membership functions. The form of membership function are determined by parameters and these parameters are called premise parameter. The premise parameter set are denoted by $\{\acute{\xi}, \nu, \psi\}$.

$$l_i^1 = \hat{\lambda}_{\tilde{F}}(u, \acute{\xi}, \nu, \psi) = \begin{cases} 0, & \text{if } u \leq \acute{\xi}; \\ \frac{u-\xi}{\nu-\xi}, & \text{if } \acute{\xi} \leq u \leq \nu, \\ \frac{\psi-u}{\psi-\nu}, & \text{if } \nu \leq u \leq \psi, \\ 0, & \text{if } u \succ \psi. \end{cases}$$

Layer 2: This layer is known as rule layer in which the firing strengths (w_ϵ) for the rules are generated by using membership values in fuzzication layer w_ϵ values are computed as:

$$l_e^2 = \bar{w_e} = \lambda_{\tilde{F}}(u) \quad \lambda_{\tilde{G}}(u) \quad e = 1, 2$$

Layer 3: The normalized ring strengths belonging to each rule are calculated by this normalization layer. The ratio of the eth rule's firing strength to the sum of all firing strengths is the normalised value represented as:

$$l_e^3 = \hat{w_e} = \frac{w_e}{\sum_{e=1}^{k} w_e}.$$

Layer 4 . Each node of this layer also known as the defuzzification layer computes the weighted values of the rules. By using rst order polynomial this value is evaluated. (w_e) is the output of the normalization layer and the collection of parameters $p_e, q_e, and r_e$ are known as consequence parameters. In the fourth layer the consequence of every node is represented as:

$$l_e^4 = \hat{w_e} f_e = \hat{w_e}(p_e u + q_e v + r_e).$$

Layer 5 : The summation layer where all the output of ANFIS are determined by adding the results obtained from each rule independently.

$$l_e^5 = \hat{w_e} = \frac{\sum_{e=1}^{k} w_e f_e}{\sum_{e=1}^{n} w_e}$$

Working of ANFIS

The concept of mapping a fuzzy inference system into a neural network structure and then optimizing the membership functions and succeeding component parameters with only a hybrid learning method are built on an ANFIS. The parameters of the membership functions are identified using a neural network learning algorithm in this process

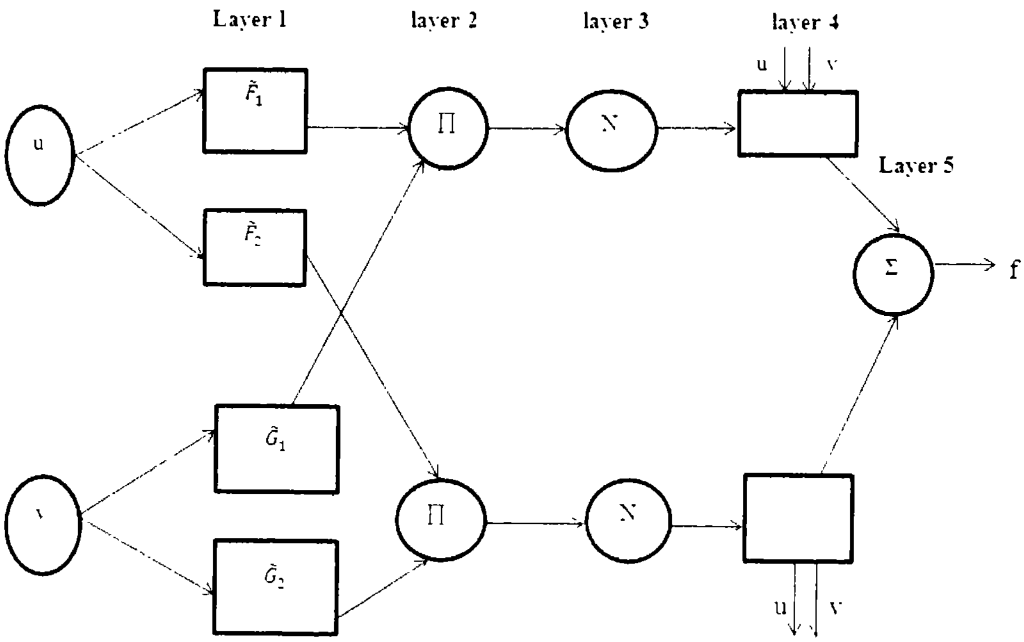


Figure 1.4: **Structure of Neuro-Fuzzy System**

The ANFIS structure contains two input parameters (u, v) and f (objective function) as well as the two rules established in the preceding section. The fuzzifying layer which uses the parameters f_e and g_e is the first phase. The membership functions of these parameters are the result of this layer. The premise parameters are determined in this stage. The firing strength for each rule are determined in the second phase. Algebraic product of the input parameters are the output of this layer. The normalized layer is the third phase in which every node determines the ratio of the firing strength of the i^{th} rule to the average of the firing strength of all the rules are expressed as:

$$\hat{w}_e = \frac{\dot{w}_h}{\dot{w}_1 + \dot{w}_2} \quad h = 1, 2.$$

The output of every node in the fourth layer are represented as.

$$\hat{w}_i f_i = \hat{w}_i (p_i u + q_i v + r_i).$$

The fifth layer calculates the overall capacities as the mean of all input variables, which determines the result

$$\hat{w}_i = \frac{\sum_{i=1}^{n} w_i f_i}{\sum_{i=1}^{n} w_i}.$$

1.3.3 Data set

To extract data from different organization there are many tools used in the research But questionnaire is the best tool used in the research to conduct survey and to collect the accurate information, personal belief, attitudes from respondents. A questionnaire contains a form which is consisting on list of questions and purpose of distributing this survey form in to variety of organization is to gather data and analyze it for future work. This might helps in data collection and for statistical analysis, decision making. In order to design best questionnaire one of the most important attribute in the research form is uniform design and planning. A well structured questionnaire requires proper arrangements needed to design in a proper steps. Following are the main points which should be kept in mind before designing questionnaire:

- Question should be clear, simple, easily understandable.
- Length of questionnaire must be shorter as it also grabs respondent's attention
- Be direct and formal as much as possible according to respondent's perspectives
- Must avoid the words with cryptic meaning and place the questions in a good manner.
- Guidelines must be provided to respondent's in order to obtain accurate data.

A questionnaire has two forms: structured or unstructured. Structured questionnaire are are the questionnaire in which respondents are restricted with fixed scheme like multiple choice question and yes \ no or true \ false. The questions are presented with exactly the same wordings and in the same order for all respondents and respondent's own words are minimized.
For example: where do the people comes from at your hospital?

- rural
- urban

when these attributes are not present in a questionnaire then it is termed as unstructured questionnaire. In this questionnaire, general guidelines are provided to respondent's and then it is the respondent's responsibility to provide the exact formulation of given questions.
For instance: which steps must be taken by the government to make awareness in people to stop taking drugs?

1.4 Fuzzy Linear Programming

There are following steps which are used for developing fuzzy linear programming model.

1. **Developing linear programming model**

   A linear programming model consisting of decision variables, parameters, objective function and constraints is developed to address the real world scenario under consideration

2. **Creating fuzzy numbers**

   Based on the fuzzy information in the problem, design fuzzy coefficients, fuzzy objectives, and fuzzy constraints by converting real values Algebra containing fuzzy numbers: For the easy operation of converted fuzzy numbers, the arithmetic operations are defined and specifically the relations between fuzzy quantities.

3. **Change linear programming to fuzzy linear programming**

   After converting the real values of the linear programming model to fuzzy numbers, a fuzzy linear programming model is formed

4. **Construction of fuzzy membership /non-membership function**

   Fuzzy membership /non-membership functions are created according to the problem which transforms the LP model to FLP

5. **Conversion of Fuzzy linear programming to Linear Programming**

   For solution of fuzzy linear programming model through the existing techniques, new linear programming model is obtained based on aspiration level and membership function through the application of algebraic operations and methods over fuzzy linear programming model

6. **An optimization method as a solution**

   Any suitable optimization method can be used to solve the given crisp model. In this manner, the best optimal solution that satisfies feasible bounds is obtained.

1.5 Best-Worst Multi-Criteria Decision Making Method

Every single day, to make judgements about certain aspects of our lives that are usually associated to our individual problems as well as in career. Decision-making tools assist us in the recognition, evaluation, and acceptance of alternatives on the basis of judgements, beliefs and preferences. Utilizing a variety of criteria, multi-criteria decision making are employed to control and coordinate decision and planning-related challenges. Over the past ten years several multi criteria decision making approaches have been implemented in a variety of supply chain management, health administration, resource management etc. Pairwise comparisons among numerous criteria are essential for all of these techniques One of the most significant area of decision making theory is multi-criteria-decision making (MCDM) Multi criteria problems are typically split into two categories: continuous and discrete with respect to the problem optimal solution. To cope with continuous problems, multi objective Decision-making (MODM) techniques are utilized. Multi-Attribute Decision-Making (MADM) techniques are proposed to deal with discrete problems. While performing pairwise comparisons a vast strategies suffers from inconsistent results and was suggested by Rezaei (2015) [15]. Therefore, an innovative approach known as Best-Worst-Method (BWM) is used to solve MCDM problems and also resolve the inconsistency problem occurs in pairwise comparison . this technique demands for fewer comparisons as compared to previous MCDM strategies. In order to choose the best option in an MCDM problem , a wide range of possibilities are assessed in the context of variety of criteria. Both best (e.g. most appealing, most essential) and worst (e.g least appealing, least significant) criteria are evaluated first by the decision maker in accordance with BWM. By employing pairwise comparison between the best and worst and then compared against the other criteria. By combining the weights from many evaluation criteria and possibilities, the best option is determined, and the outcomes of the acceptable option are computed A consistency ratio is established for the BWM to examine the accuracy of comparisons. Following are the basic steps involved in developing BWM:

Steps To Construct BWM

Step1. To determine a set of criteria for making the decisions

In the first step to assist in decision making relying on the the study of research and individuals point of view, a set of k criteria ($s_1, s_2, s_3, \ldots, s_k$) are determined.

Step2: The best criteria (e.g. the most desirable, the most essential) and the worst criteria(least desirable, least significant) are chosen

Step3. To express your priority for the best criteria over the other criteria, choose a number between 1 and 9 scale. The vector representation of best-to-other vector(BO) can be denoted as:

$$\bar{\alpha}_{\acute{B}} = (e_{\acute{B}1}, e_{\acute{B}2}, e_{\acute{B}3}, \ldots, e_{\acute{B}k})$$

where

e_{Bm} denotes the prefrence of best criteria B over criteria m. Also we have $e_{\acute{B}B}$=1. Every professionals who participated in decision making process may agreed on the obtained final value

Step4: To evaluate the prefrence of almost all of the criteria over the worst criteria using a scale from 1 to 9. The vector representation of others-to-worst(OW) are denoted as:

$$\bar{\alpha}_{\acute{w}} = (e_{1\acute{w}}, e_{2\acute{w}}, e_{3\acute{w}}, \ldots, e_{k\acute{w}})^T$$

where

$e_{W'm}$ denotes the prefrence of all other criteria m over the worst criteria W. Also we have $e_{W'W} = 1$ Every professionals who participated in decision making process may agreed on the obtained final value

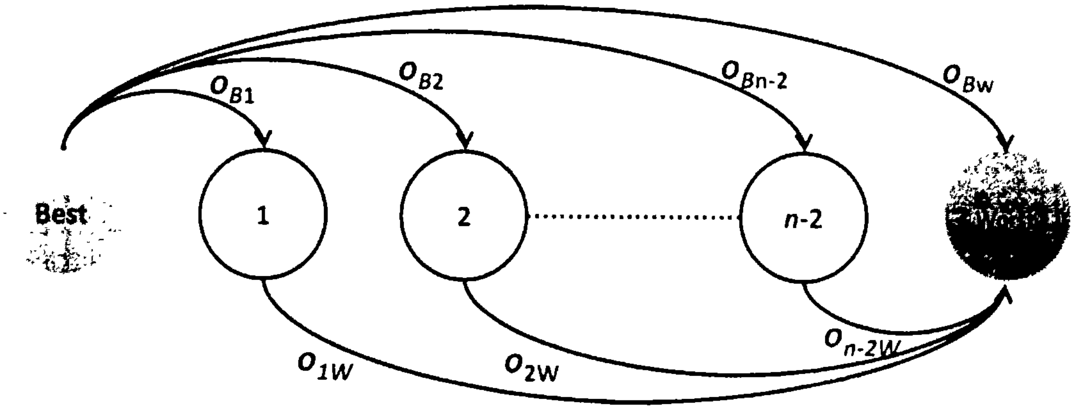


Figure 1.5: **Best to other and other to worst prefrences**

Step5: To determine the optimal weights $(w'_1, w'_2, w'_3, \ldots, w'_k)$

To evaluate the optimal weight for the criteria in case of each pair $\frac{w_B}{w_m}$ and $\frac{w_m}{w_w}$ which then ultimately resulted in the form of weight as: $\frac{w_B}{w_m} = e_{B'm} and \frac{w_m}{w_W} = e_{m'W}$ we must need to figure out the solution which satisfies the above condition for all m and also the greatest possible absolute differences | $\frac{w_B}{w_m}$ - $e_{B'm}$ | and | $\frac{w_m}{w_W}$ - $e_{m'W}$ | are minimized for all m.

Let us consider the sum and non-negativity condition for weights due to which the following problem occurs

$$\tilde{\zeta} = \min / \max_m \left\{ \mid \frac{w_B}{w_m} - e_{B'm} \mid and \mid \frac{w_m}{w_W} - e_{m'W} \mid \right\} \tag{1.5.1}$$

such that $\sum_m w_m = 1$, $w_m \geq 0 \quad \forall m$

Now this programming model is remodeled into the following programming model which are defined as

$$\min \delta$$

such that

$$|\frac{w_B}{w_m} - e_{B'm}| \leq \delta \quad \forall m$$
$$|\frac{w_m}{w_W} - e_{m'W}| \leq \delta \quad \forall m$$

$$\sum_m w_m = 1$$

$$w_m \geq 0 \quad \forall m$$

We will get optimal weights $(\acute{w}_1, \acute{w}_2, \acute{w}_3, \ldots, \acute{w}_k)$ and $\acute{\zeta}$ after solving the above problem (1.5.1) Now,as the BWM is applied on a real world problems so to better understand its implementation example is presented below:

Think about the dress selection problem where the cotton fabric is a criteria. Now, the consistency ratio for the BWM is defined as:

For every m , if the condition $e_{\acute{B}m} \times e_{m\acute{W}} = e_{\acute{B}W}$ is satisfied where $e_{\acute{B}m}$ and $e_{m\acute{W}}$ and $e_{\acute{B}W}$ denotes the corresponding prefrence of best criteria over other criteria m. and the prefrence of other criteria over worst, and the prefrence of best criteria over worst criteria then this comparison is known as fully consistent.

Now to measure the consistency of a comparison as it is more likely for some m not to be fully consistent. so subsequently the concept of consistency ratio is developed. In order to achieve this, by figuring out the minimal consistency of a comparison as illustrated below:

As the maximum possible value of $e_{\acute{B}W}$ is determined by the decision maker but mostly maximum value of $e_{\acute{B}W}$ are 9 so $e_{\acute{i}m} \in \{1, \ldots, e_{\acute{B}W}\}$. Moreover , in case of $e_{\acute{B}m} \times e_{m\acute{W}} < e_{\acute{B}W}$ and $e_{\acute{B}m} \times e_{m\acute{W}} > e_{\acute{B}W}$ or $e_{m\acute{W}} \neq e_{\acute{B}W}$ the inconsistency occurs. If $e_{\acute{B}m}, and e_{m\acute{W}}$ have the maximum value or equal to $e_{\acute{B}W}$ then the inconsistency must attain it's maximum value δ. We know that $\frac{w_B}{w_m} \times \frac{w_m}{w_W} = \frac{w_B}{w_W}$. the highest inequality arise due to allocate the maximum values to $e_{m\acute{W}}$, and $e_{\acute{B}m}$ and Λ is a quantity which must be added in $e_{\acute{B}W}$ although being reduced from $e_{\acute{B}m}$ and $e_{m\acute{W}}$. More accurately, it is represented as:

$(e_{Bm}' - \delta) \times (e_{mW}' - \delta) = (e_{BW}' + \delta)$

Now in case of minimum consistency $e_{\acute{B}m} = e_{mW} = e_{\acute{B}W}$ then we have.

$(e_{BW}' - \delta) \times (e_{BW}' - \delta) = (e_{BW}' + \delta)$

this implies that

$$\delta^2 - (1 + 2e_{\acute{B}W})\delta + (e_{BW}^{2'} - e_{\acute{B}W}) = 0 \tag{1.5.2}$$

Now , the largest possible value of δ can be determined by soling equation (1.3.2) for numerous various values of $e_{\acute{B}W}$ as $e_{\acute{B}W}$ ranges from 1,2,3,. . ,9. This largest value of δ is defined as the consistency index. After determining the consistency index , consistency ratio is calculated with the help of this formula given below

$$consistencyratio = \frac{\acute{\zeta}}{consistencyindex} \tag{1.5.3}$$

Our problem is based on discrete data so to handle the discrete MCDM problems, the discrete data are represented in the form of matrices as:

$$\tilde{H} = \begin{array}{c c} & \begin{array}{ccccc} \sigma_1 & \sigma_2 & \sigma_3 & \cdots & \sigma_n \end{array} \\ \begin{array}{c} \rho_1 \\ \rho_2 \\ \rho_3 \\ \vdots \\ \rho_o \end{array} & \begin{pmatrix} \tau_{11} & \tau_{12} & \tau_{13} & & \tau_{1n} \\ \tau_{21} & \tau_{22} & \tau_{23} & \cdots & \tau_{2n} \\ \tau_{31} & \tau_{32} & \tau_{33} & \cdots & \tau_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tau_{o1} & \tau_{o2} & \tau_{o3} & \cdots & \tau_{on} \end{pmatrix} \end{array}$$

where $\rho_1, \rho_2, \ldots \rho_o$ represents the set of feasible choices and $\sigma_1, \sigma_2, \ldots, \sigma_n$ represents the set of criteria for making that choices , and τ_{ik} represents the grade of choices i in regards to choices k. The target is to determine the choice which is the most appealing or valuable and has the best worth. To evaluate the total worth of choice i , χ_i can be obtained with the help of several approaches. In a generalized form, for the most MCDM techniques to determine χ_i the additive weighted value function are adopted [8] and can be expressed as:

$$\chi_i = \sum_{k=1}^{n} w_k \tau_{ik} \tag{1.5.4}$$

where $w_k \geq 0$. $\sum_k w_k = 1$. Now, the thing which matters in the above equation is the path by which set of vectors or weights of the criteria $w_1, w_2, \ldots w_k$ are determined.

Chapter 2

Health care Resource Allocation Planning through Fuzzy Linear Programming Model

2.1 Introduction

With a population of over 210 million, the Islamic Republic of Pakistan is the thirty-third biggest state in terms of area and the fifth-most populated Pakistan's population is equal to (2.83%) of the worldwide population. In Pakistan, Population ages $0-14$ years (34.63%), $15-64$ years (60.97%) of total population, 65 years and older (4.4%) of total population was reported in 2021 [16]. In the presence of average income per capita, the overall population hierarchy reflects an age structure with a high dependence ratio. Pakistan is one of the world's youngest countries, with more than 60% of the population under 30 and many are unemployed. While a big young generation are now demanding for employment to fulfill their needs due to inappropriate job opportunities, along with poor capital development may change this economic bounty into scourge

It is believed that the health care system of Pakistan is under massive pressure With the passage of time, these days it is quite difficult to make the best use of resources (e.g , bed capacity and average duration of stay, staffing budget, etc.) within the hospitals. So due to the rapid increase in demand and inadequate capacities are experienced across all the specialties (accident and emergency) within hospitals. The increasing ratio of accidents are closely related to worsening prevailing conditions and with the rapid increase in population, that often has multiple complicated situations which occurs in accident and emergency department forms the highest demands for beds Through providing guidelines and make proper awareness in people it may be possible to get overcome on the increasing ratio of accidents It is essential to allocate resources within A & E department due to the cases rising in accident death rates. However, a hospital bed seems to be crucial resources for all health care system. Developments in research and medicines lead notably decreasing length of hospital admissions for inpatient and enhancing the percentage of day cases outpatient, A &E department. A hospital does not have appropriate resources to improve capacity, either in regards of beds or staff, due to inadequate budget. As a result, hospital administrator must explore efficient and effective solutions to use current resources. Therefore to extract the best solutions for the allocation of existing resources in A &E department, various optimization techniques have been devised [17] Optimization is the art to make things more acceptable or to bring things nearer a standard. Many optimization techniques have been invented and applied in engineering, biology, information technology, etc. Mathematicians, professionals used optimization for decision making. The goal of such decision is to increase profit or benefit as much as is practically possible to design a product or to draw something. These are the feedbacks and rough idea of why we need optimization in our daily practice. Optimization tools provide us the best possible means to make things happens in the best possible practical way Through the rigorous use of optimization, we are able to investigate several more hypotheses than a human being is capable of. Optimization methods assures that the solution of given problem is as accurate as possible. Forecasting refers to decision-making strategies that helps to forecast uncertain future events by taking into account past and present events. Forecasting techniques enable in establishing how to distribute resources and in predicting patient demand [18]. When a patient is admitted to a hospital, a physician or nurse must really be informed of the reason for the admittance as well as the patient's condition history Later, he or she will want clinical, radiological, and statistical data, which are some of the most frequently used diagnostic tools. Forecasting techniques enable in establishing how to distribute resources and in predicting patient demand, including inpatient, outpatient, A & E department. It may also be helpful to distribute resources (e.g. bed capacity, staffing cost, average length of stay etc.) in the A& E department

In emergency department, different sort of uncertainties occurs. To deal with the uncertainties fuzzy optimization techniques has been introduced based on the data obtained from private and government health care organizations [19]. These fuzzy techniques then applied on the optimization problems and model is then analyzed and interpreted for the optimal resource allocation of private and government health care organization. Fuzzy sets and fuzzy numbers may be employed to represent uncertain parameters and can be modified using multiple operations on fuzzy sets and fuzzy numbers. The fuzzy set theory established by Zadeh (1965) has a considerable impact on the behavior of a dynamic situation. However, no models have analyzed existing and forecast bed occupancy rate at the hospital within Pakistan. To capture the uncertainties in A& E department by the use of simulation techniques to test at a wider range of activities (average length of stay, treating time, bed occupancy rate) with hospital investments (total revenue, cost) with the aim of testing under different situations. So the optimization is now needed to evaluate the exact bed requirements within hospital under constraints (total targeted beds, nurse to patient ratio, consultation time, consultation hours). The combination of optimization and simulation [20] and forecasting boosts decision-making potential

2.2 Case study:Re-allocating number of beds in emergency department

In order to deal with uncertainties occurs in emergency department and to reallocate number of beds and staffing level in emergency department the concept of fuzzy optimization was introduced. To employ fuzzy optimization technique we adopted the concept of ANFIS by which we introduce fuzzy inference system through inference rules in health care departments. As in current situation in Pakistan the most prevailing diseases are typhoid fever and dengue fever and almost 50% are suffering from this fever and admitted in emergency department due to their critical conditions. Now problem occurs in the emergency is due to the inadequate resources, proper management, inadequate equipments, exceeding ratio of nurse to patient ratio as there are more than thirty patients under the supervision of one nurse at a time, due to the shortage of time period the total consultation time provided by consultant to their patients also affects the patients health improvements. To obtain data from different hospitals we have several inputs from different resources, local data, literature data, financial resources are listed in the model as constraints A quantitative study was conducted thus a questionnaire is used for data collection. The questionnaire is distributed through email to different hospitals In this study, Excel is used for data entry of respondents ethnicity that are collected from hospitals. The questionnaire is consisted of seven section· the first section of the questionnaire is related to the general information of hospitals like hospital name, area of belonging of people who comes to the hospital. The second section of questionnaire is related to patients age group (for instance; age group of patients who are highly preferable for the allocation of beds, age group of patients who have maximum length of stay at hospital during 2021, age group of patients who needs more councelling at the time of discharge, age group of patients who were mostly affected and admitted at hospital due to COVID19, pollen allergy, diarrheal disease etc.) The third section is about hospital stay (for instance about the number of beds currently available, number of admitted patients, number of discharged patients, average duration of stay of patient in emergency department). The fourth section is about resource utilization (proportion of nurses per patient, proportion of consultant to patients , lab technicians in laboratry , ventilators available in ICU) The fifth section is related to hospital management (treating time, daily utilization of operation theatres, duty hours of staff, maximum time of completion of operation and average waiting time required by patients etc). The sixth section is related to staffing cost. average annual earning of nurses and consultants, total revenue from inpatient and outpatient elective specialty. The last section of the questionnaire is related to hospital demand (like we are interested in finding out about the target level of bed occupancy in each specialty).

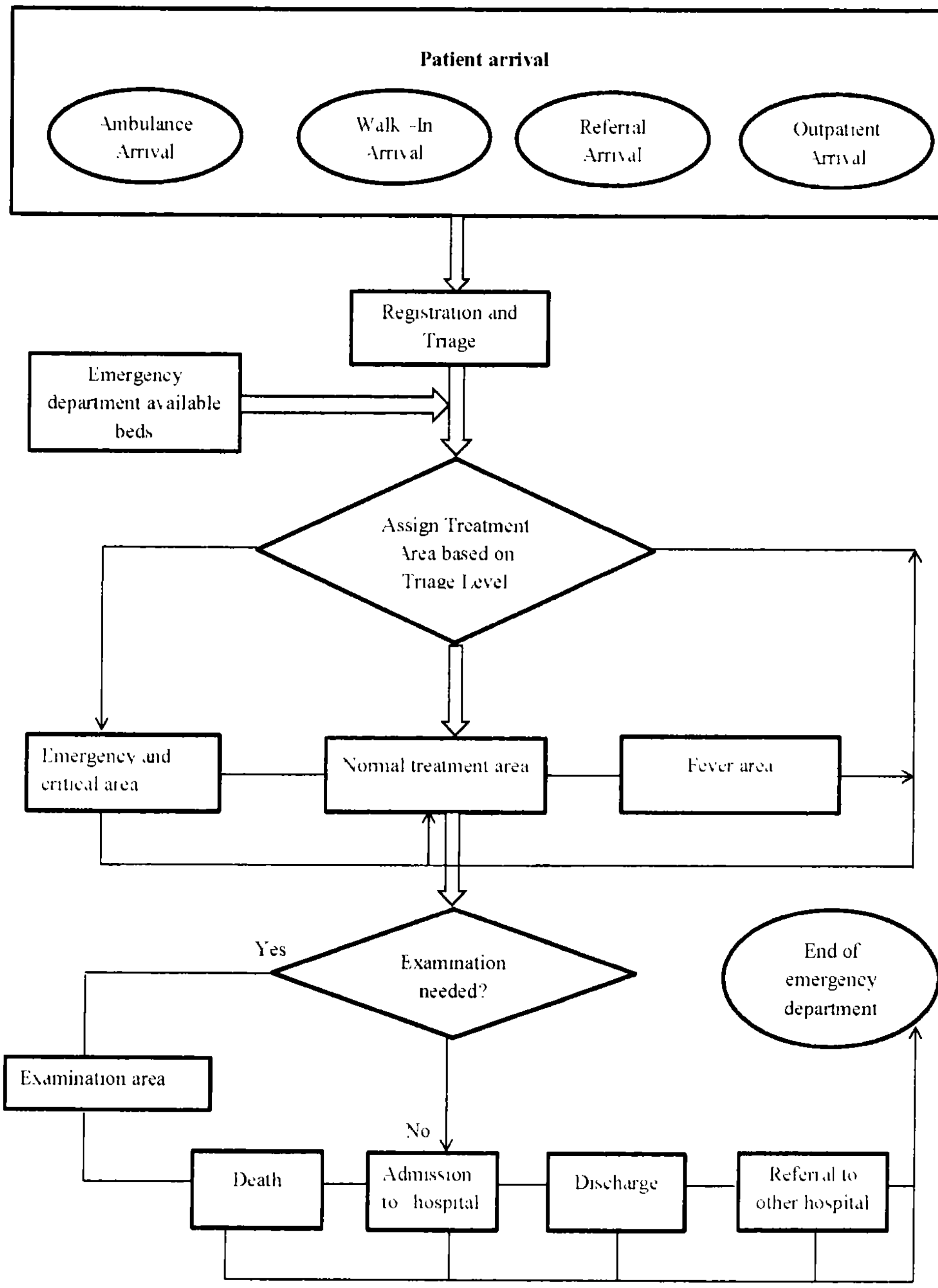


Figure 2.1: **Flow chart for emergency department**

2.2.1 Data analysis

The data is consisted of activities performed at hospital. To gather data from different hospitals a questionnaire was distributed among well known hospitals through which it is easily possible to forecast related to patient's admission and number of beds available in emergency department, nurse to patient ratio, revenue (staffing budget, pay of nurse per shift, pay of consultant per shift, annual average income of consultants and nurses etc. This data is divided in to different parts: **Local data**

- Name of patient
- Gender
- Age group
- Distance (area of belonging of patient)
- Number of patient's admitted and discharged
- Treating time
- Total number of inpatient elective specialty
- Total number of inpatient non elective specialties

Literature data

- Overall ratio of beds occupied as needed
- Average level of bed occupancy rate
- Annual income of specialists
- Average nurse salary per year
- Nurse to patient ratio

Financial resources

- Overtime cost
- Patients related cost (each specialty)
- Staffing budget (inpatient, outpatient)
- Consultant fee per hour(per patient)
- Total income from patient care
- Pay of nurse per shift

- Total revenue from inpatient specialty
- Average revenue at non elective specialty
- Average revenue at elective specialty
- Annual Average revenue earning of consultants
- Average annual earning of nurse

Hospital	Available beds	Admitted patients	Average length of stay	Average revenue	Cost	BOR	Total targeted beds	Nurse to patient ratio	CTS	CHS
Holy Family Hospital	400	28	1-2	175000	11000	75%	1500	18	420	26
Shifa International Hospital	550	25	2-3	200000	79000	85%	2100	3	940	8
Fauji Foundation Hospital	200	150	10-15	2,25000	69750	45%	2100	8	18750	19
Pakistan Institute of Medical Sciences	850	550	3-5	175000	145000	55%	2100	26	22000	3
Rawal Institute of Cardiology	500	107	3-4	2,50000	28000	50%	2300	3	3745	3

Table 2 1: Results obtained from questionnaire

2.3 Mathematical Model

2.3.1 Objective function

To design the model for resource allocation in accident and emergency department we have several objectives:
To maximize

- Number of discharged patients
- Bed occupancy rate
- Financial resources

and to minimize:

- Length of stay
- Overtime cost
- Patient related cost
- Maximum completion time of operation
- Nurse to patient ratio
- Total waiting time of operation

2.3.2 Parameters

The parameters that are used in this linear programming model are defined as
NDP_e =number of patients discharged at speciality e
NB_e =number of beds needed at speciality e
NN_e =number of nurses needed at specialty e
BOR =indicates the ratio of occupied beds at specialty e
Input parameters

- Distance
- Patient's age group
- Beds currently available
- Number of admitted patients
- Number of discharged patients

- Average length of stay
- Proportion of nurses per patient
- Proportion of consultant to patients
- Resource utilization (lab technicians in laboratry, number of ventilators available in ICU)
- Completion time (average maximum completion time of operation, duty hours, average waiting time required by patients)
- Staffing cost (approximate consultant fee per hour, approximate total financial contributions, approximate Pay of nurse per shift, approximate overtime cost of consultants and nurses)
- Revenue (number of patient's with annual average revenue per patient, total revenue from inpatient elective specialty, average annual earnings of consultants, average annual earning of nurses, average annual cost of ventilators in ICU)
- Target level of bed capacity (gynecology, peeds, cardiology, surgery department, accident and emergency department, ICU, others)

General objective function and constraints

The mathematical model for hospitals are as follows:

$$\max \sum_{v=1}^{l} NDP_v \tag{2.3.1}$$

subject to

$$BOR_v < \text{TARGET} \quad \forall v \in V \tag{2.3.2}$$

$$\sum_{v=1}^{l} NB_v \leq \text{BEDS} \tag{2.3.3}$$

$$\sum_{v=1}^{l} \text{Revenue} \geq \sum_{v=1}^{l} \text{Cost} \tag{2.3.4}$$

$$CH_v \geq CT_v \quad \forall v \in V \tag{2.3.5}$$

$$NB_v, DP_v, NN_v, NC_v \in Z^{+} \forall v \in V \tag{2.3.6}$$

The constraint(2.3.1) indicates that the objective function is to maximizes the number of patients discharged to the speciality (v). The constraint (2.3.2) guarantees that the total targeted beds are not exceeded by the bed occupancy levels. Bed capacities are allotted by constraint (2.3.3). The (2.3.4) constraint assures that the total revenue from inpatient care will be greater or equal to the total cost. Constraint (2.3.5) ensures that the overall consultation time provided by all specialists will exceed the total amount of time needed by every patients. The decision variables must be a positive integers guarantees by constraint (2.3.6).

Time constraints

- Maximum completion time of operation
- Total waiting time required by patients of operation
- Daily utilization of operating rooms
- Total surgery time
- Staffing hours
- Theatre utilization by elective inpatient, elective outpatient and non-elective inpatient, outpatient specialty, number of working hours of nurses, patients
- Number of occupied bed days, number of bed days in period

2.3.3 Results

Decision variables are explained below:

$\bar{\varepsilon}_1$: number of bed occupancy rate

$\bar{\varepsilon}_2$: targeted beds

$\bar{\varepsilon}_3$: total revenue

$\bar{\varepsilon}_4$: staffing cost

$\bar{\varepsilon}_5$: nurse to patient ratio

$\bar{\varepsilon}_6$: consultation hours

Holy family hospital

The objective function are defined as

$\bar{f}_1 = 0.75 * \bar{\varepsilon}_1 + \bar{\varepsilon}_2 + 175000 * \bar{\varepsilon}_3 + 11000 * \bar{\varepsilon}_4 + \bar{\varepsilon}_5 + 26 * \bar{\varepsilon}_6$

constraints for linear programming model for hospital are defined as

$\bar{\varepsilon}_1 < 4500$

$\bar{\varepsilon}_2 < 400$

$\bar{\varepsilon}_3 - \bar{\varepsilon}_4 \geq 0$

$\bar{\varepsilon}_5 \leq 18$

$\bar{\varepsilon}_6 \geq 420$

$\bar{\varepsilon}_1, \quad \bar{\varepsilon}_2, \quad \bar{\varepsilon}_3, \quad \bar{\varepsilon}_4, \quad \bar{\varepsilon}_5, \quad \bar{\varepsilon}_6 \quad \geq 0$

the solutions obtained by solving the linear programming model for hospital with the help of Matlab 2018 are:

$\bar{\varepsilon}_1 = 1500$

$\bar{\varepsilon}_2 = 400$

$\bar{\varepsilon}_3 = 100000$

$\bar{\varepsilon}_4 = 100000$

$\varepsilon_5 = 18$

$\varepsilon_6 = 120$

$\bar{Z} = 1.8603e+10$

Objective function	Constraints	Results
Holy Family Hospital		
	$\bar{\varepsilon}_1 \leq 4500$	$\bar{\varepsilon}_1$=4500
$0.75 * \bar{\varepsilon}_1 + \bar{\varepsilon}_2 + 175000 * \bar{\varepsilon}_3$	$\bar{\varepsilon}_2 \leq 400$	$\bar{\varepsilon}_2$=400
$+11000 * \bar{\varepsilon}_4$	$\bar{\varepsilon}_3 - \bar{\varepsilon}_4 \geq 0$	$\bar{\varepsilon}_3$=100000
$+\bar{\varepsilon}_5 + 26 * \bar{\varepsilon}_6$	$\bar{\varepsilon}_5 \leq 18$	$\bar{\varepsilon}_4$=100000
	$\bar{\varepsilon}_6 \geq 420$	$\bar{\varepsilon}_5$=18
	$\bar{\varepsilon}_1, \bar{\varepsilon}_2, \bar{\varepsilon}_3$	$\bar{\varepsilon}_6$=420
	$, \bar{\varepsilon}_4, \bar{\varepsilon}_5, \bar{\varepsilon}_6 \geq 0$	$\bar{Z}$=1.8603e+10
Shifa International Hospital		
	$\bar{\varepsilon}_1 \leq 2100$	$\bar{\varepsilon}_1$ = 2100
$0.85 * \bar{\varepsilon}_1 + \bar{\varepsilon}_2 + 200000 * \bar{\varepsilon}_3$	$\bar{\varepsilon}_2 \leq 550$	$\bar{\varepsilon}_2$=550
$+79000 * \bar{\varepsilon}_4$	$\bar{\varepsilon}_3 - \bar{\varepsilon}_4 \geq 0$	$\bar{\varepsilon}_3$=100000
$+\bar{\varepsilon}_5 + 8 * \bar{\varepsilon}_6$	$\bar{\varepsilon}_5 \leq 3$	$\bar{\varepsilon}_4$=100000
	$\bar{\varepsilon}_6 \geq 940$	$\bar{\varepsilon}_5$=3
	$\bar{\varepsilon}_1, \bar{\varepsilon}_2, \bar{\varepsilon}_3$	$\bar{\varepsilon}_6$=940
	$, \bar{\varepsilon}_4, \bar{\varepsilon}_5, \bar{\varepsilon}_6 \geq 0$	Z=2.7901e+10
Fauji Foundation Hospital		
	$\bar{\varepsilon}_1 \leq 2100$	$\bar{\varepsilon}_1$ = 2100
$0.15 * \bar{\varepsilon}_1 + \bar{\varepsilon}_2 + 2250000 * \bar{\varepsilon}_3$	$\bar{\varepsilon}_2 \leq 200$	$\bar{\varepsilon}_2$=200
$+69750 * \bar{\varepsilon}_4$	$\bar{\varepsilon}_3 - \bar{\varepsilon}_4 \geq 0$	$\bar{\varepsilon}_3$=100000
$+\bar{\varepsilon}_5 + 19 * \bar{\varepsilon}_6$	$\bar{\varepsilon}_5 \leq 8$	$\bar{\varepsilon}_4$=100000
	$\bar{\varepsilon}_6 \geq 18750$	$\bar{\varepsilon}_5$=8
	$\bar{\varepsilon}_1, \bar{\varepsilon}_2, \bar{\varepsilon}_3$	$\bar{\varepsilon}_6$=18750
	$, \bar{\varepsilon}_4, \bar{\varepsilon}_5, \bar{\varepsilon}_6 \geq 0$	$\bar{Z}$=2.9477e+10
Pakistan Institute of Medical Sciences		
	$\bar{\varepsilon}_1 \leq 2100$	$\bar{\varepsilon}_1$ = 2100
$0.55 * \bar{\varepsilon}_1 + \bar{\varepsilon}_2 + 175000 * \bar{\varepsilon}_3$	$\bar{\varepsilon}_2 \leq 850$	$\bar{\varepsilon}_2$=850
$+145000 * \bar{\varepsilon}_4$	$\bar{\varepsilon}_3 - \bar{\varepsilon}_4 \geq 0$	$\bar{\varepsilon}_3$=100000
$+\bar{\varepsilon}_5 + 3 * \bar{\varepsilon}_6$	$\bar{\varepsilon}_5 \leq 26$	$\bar{\varepsilon}_4$=100000
	$\bar{\varepsilon}_6 \geq 22000$	$\bar{\varepsilon}_5$=26
	$\bar{\varepsilon}_1, \bar{\varepsilon}_2, \bar{\varepsilon}_3$	$\bar{\varepsilon}_6$=22000
	$, \bar{\varepsilon}_4, \bar{\varepsilon}_5, \bar{\varepsilon}_6 \geq 0$	$\bar{Z}$=2.7901e+10

Rawal Institute of Cardiology		
	$\bar{\varepsilon}_1 \leq 2300$	$\bar{\varepsilon}_1 = 2300$
$0.5 * \bar{\varepsilon}_1 + \bar{\varepsilon}_2 + 250000 * \bar{\varepsilon}_3$	$\bar{\varepsilon}_2 \leq 500$	$\bar{\varepsilon}_2$=500
$+28000 * \bar{\varepsilon}_4$	$\bar{\varepsilon}_3 - \bar{\varepsilon}_4 \geq 0$	$\bar{\varepsilon}_3$=100000
$+\bar{\varepsilon}_5 + 3 * \bar{\varepsilon}_6$	$\bar{\varepsilon}_5 \leq 3$	$\bar{\varepsilon}_4$=100000
	$\bar{\varepsilon}_6 \geq 3745$	$\bar{\varepsilon}_5$=3
	$\bar{\varepsilon}_1, \quad \bar{\varepsilon}_2, \quad \bar{\varepsilon}_3$	$\bar{\varepsilon}_6$=3745
	$, \bar{\varepsilon}_4, \quad \bar{\varepsilon}_5, \quad \bar{\varepsilon}_6 \geq 0$	$\bar{Z}$=2.7800e+10

2.4 Fuzzy Optimization

Firstly, the linear programming model for hospital is solved by applying optimization techniques and then solution is obtained in case of each hospital. To deal with the uncertainties that occurs in accident and emergency department of hospital it is necessary to employ fuzzy optimization techniques to solve the model. The following steps are kept in mind before designing fuzzy optimization hospital model:

- Firstly, under the several objectives functions it is required to make fuzzy constraints for solving the hospital model.
- Secondly, it is necessary to define the membership function before making inferences.

  $\bar{\varepsilon}_1$: number of bed occupancy rate

  $\bar{\varepsilon}_2$ targeted beds

  $\bar{\varepsilon}_3$: total revenue

  $\bar{\varepsilon}_4$. staffing cost

  $\bar{\varepsilon}_5$. nurse to patient ratio

  $\bar{\varepsilon}_6$ consultation hours

- After deciding the parameters for hospital model fuzzy IF THEN rules are defined over certain criteria which are explained at every stage.
- Set of parameter values and aspiration results are obtained by fuzzy optimization

Fuzzy constraints

The general objective function are defined as:

$max \sum_{k=1}^{6} \acute{\gamma}_k$

subject to

$\hat{\lambda}_1 \geq \acute{\gamma}_1$

$\hat{\lambda}_2 \geq \acute{\gamma}_2$

$\hat{\lambda}_3 \geq \acute{\gamma}_3$

$\hat{\lambda}_4 \geq \acute{\gamma}_4$

$\hat{\lambda}_5 \geq \acute{\gamma}_5$

$\lambda_6 \geq \acute{\gamma}_6$

where$\hat{\lambda}_1, \hat{\lambda}_2, \hat{\lambda}_3, \hat{\lambda}_4, \hat{\lambda}_5, \hat{\lambda}_6$ denotes the membership function of each hospital

$$\hat{\lambda}_{\tilde{F}} \quad U \to [0,1] \quad where \quad \varepsilon \in U$$

and ϑ denotes the membership function defined on $\bar{f}_i$ where i=1, . . . , 6

The objective function and membership function in case of each hospital are defined as

Holy family hospital

$\bar{f}_1 = 0.75 * \bar{\varepsilon_1} + \bar{\varepsilon_2} + 175000 * \bar{\varepsilon_3} + 11000 * \bar{\varepsilon_4} + \bar{\varepsilon_5} + 26 * \bar{\varepsilon_6}$

$$\hat{\lambda}(\bar{\varepsilon_1}) = \begin{cases} 0, & \text{if } \bar{\varepsilon_1} \leq -0.25, \\ \frac{\bar{\varepsilon_1}+0.25}{0.75+0.25}, & \text{if } -0.25 \leq \bar{\varepsilon_1} \leq 0.75, \\ \frac{1.75-\bar{\varepsilon_1}}{1.75-0.75}, & \text{if } 0.75 \leq \bar{\varepsilon_1} \leq 1.75; \\ 0, & \text{if } \bar{\varepsilon_1} \geq 1.75; \end{cases}$$

$$\hat{\lambda}(\bar{\varepsilon_2}) = \begin{cases} 0, & \text{if } \bar{\varepsilon_2} \leq 4499; \\ \frac{\bar{\varepsilon_2}-4499}{4500-4499}, & \text{if } 4499 \leq \bar{\varepsilon_2} \leq 4500, \\ \frac{4501-\bar{\varepsilon_2}}{4501-4500}, & \text{if } 4500 \leq \bar{\varepsilon_2} \leq 4501, \\ 0, & \text{if } \bar{\varepsilon_2} \geq 4501; \end{cases}$$

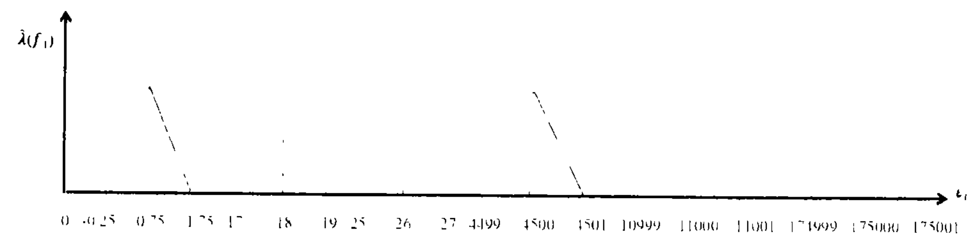
$$\hat{\lambda}(\bar{\varepsilon_3}) = \begin{cases} 0, & \text{if } \bar{\varepsilon_3} \leq 174999, \\ \frac{\bar{\varepsilon_3}-174999}{17500}, & \text{if } 174999 \leq \bar{\varepsilon_3} \leq 175000, \\ \frac{175001-\bar{\varepsilon_3}}{175001-175000}, & \text{if } 175000 \leq \bar{\varepsilon_3} \leq 175001; \\ 0, & \text{if } \bar{\varepsilon_3} \geq 175001; \end{cases}$$

$$\lambda(\bar{\varepsilon_4}) = \begin{cases} 0, & \text{if } \bar{\varepsilon_4} \leq 10999; \\ \frac{\bar{\varepsilon_4}-10999}{11000-10999} & \text{if } 10999 < \bar{\varepsilon_4} < 11000; \\ \frac{11001-\bar{\varepsilon_4}}{11001-11000}, & \text{if } 11000 \leq \bar{\varepsilon_4} \leq 11001, \\ 0, & \text{if } \bar{\varepsilon_4} \geq 11001, \end{cases}$$

$$\hat{\lambda}(\bar{\varepsilon_5}) = \begin{cases} 0, & \text{if } \bar{\varepsilon_5} \leq 17; \\ \frac{\bar{\varepsilon_5}-17}{18-17}, & \text{if } 17 \leq \bar{\varepsilon_5} \leq 18; \\ \frac{19-\bar{\varepsilon_5}}{19-18}, & \text{if } 18 \leq \bar{\varepsilon_5} \leq 19, \\ 0, & \text{if } \bar{\varepsilon_5} \geq 19, \end{cases}$$

$$\hat{\lambda}(\bar{\varepsilon_6}) = \begin{cases} 0, & \text{if } \bar{\varepsilon_6} \leq 25; \\ \frac{\bar{\varepsilon_6}-25}{26-25}, & \text{if } 25 \leq \bar{\varepsilon_6} \leq 26; \\ \frac{27-\bar{\varepsilon_6}}{27-26}, & \text{if } 26 \leq \bar{\varepsilon_6} \leq 27; \\ 0, & \text{if } \bar{\varepsilon_6} \geq 27, \end{cases}$$

Figure 2 2: **Relationship between membership and aspiration**



Shifa international

$\bar{f_2} = 0.85 * \bar{\varepsilon_1} + \bar{\varepsilon_2} + 200000 * \bar{\varepsilon_3} + 79000 * \bar{\varepsilon_4} + \bar{\varepsilon_5} + 8 * \bar{\varepsilon_6}$

$$\hat{\lambda}(\bar{\varepsilon_1}) = \begin{cases} 0, & \text{if } \bar{\varepsilon_1} \leq -0.15; \\ \frac{\bar{\varepsilon_1}+0.15}{0.85+0.15}, & \text{if } -0.15 \leq \bar{\varepsilon_1} \leq 0.85, \\ \frac{1.85-\bar{\varepsilon_1}}{1.85-0.85}, & \text{if } 0.85 \leq \bar{\varepsilon_1} \leq 1.85; \\ 0, & \text{if } \bar{\varepsilon_1} \geq 1.85, \end{cases}$$

$$\hat{\lambda}(\bar{\varepsilon_2}) = \begin{cases} 0, & \text{if } \bar{\varepsilon_2} \leq 2099; \\ \frac{\bar{\varepsilon_2}-2099}{2100-2099}, & \text{if } 2099 \leq \bar{\varepsilon_2} \leq 2100, \\ \frac{2101-\bar{\varepsilon_2}}{2101-2100}, & \text{if } 2100 \leq \bar{\varepsilon_2} \leq 2101, \\ 0, & \text{if } \bar{\varepsilon_2} \geq 2101; \end{cases}$$

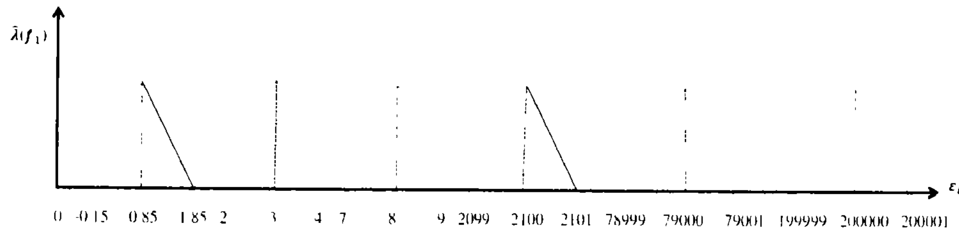
$$\hat{\lambda}(\bar{\varepsilon}_3) = \begin{cases} 0, & \text{if } \bar{\varepsilon}_3 \leq 199999; \\ \frac{\bar{\varepsilon}_3 - 199999}{200000 - 199999}, & \text{if } 199999 \leq \bar{\varepsilon}_3 \leq 200000; \\ \frac{2100001 - \bar{\varepsilon}_3}{2100001 - 200000}, & \text{if } 200000 \leq \bar{\varepsilon}_3 \leq 2100001; \\ 0, & \text{if } \bar{\varepsilon}_3 \geq 2100001; \end{cases}$$

$$\hat{\lambda}(\bar{\varepsilon}_4) = \begin{cases} 0, & \text{if } \bar{\varepsilon}_4 \leq 78999; \\ \frac{\bar{\varepsilon}_4 - 78999}{79000 - 78999}, & \text{if } 78999 \leq \bar{\varepsilon}_4 \leq 79000; \\ \frac{79001 - \bar{\varepsilon}_4}{79001 - 79000}, & \text{if } 79000 \leq \bar{\varepsilon}_4 \leq 79001; \\ 0, & \text{if } \bar{\varepsilon}_4 \geq 79001; \end{cases}$$

$$\hat{\lambda}(\bar{\varepsilon}_5) = \begin{cases} 0, & \text{if } \bar{\varepsilon}_5 \leq 2; \\ \frac{\bar{\varepsilon}_5 - 2}{3 - 2}, & \text{if } 2 \leq \bar{\varepsilon}_5 \leq 3, \\ \frac{4 - \bar{\varepsilon}_5}{4 - 3}, & \text{if } 3 \leq \bar{\varepsilon}_5 \leq 4; \\ 0, & \text{if } \bar{\varepsilon}_5 \geq 4; \end{cases}$$

$$\hat{\lambda}(\bar{\varepsilon}_6) = \begin{cases} 0, & \text{if } \bar{\varepsilon}_6 \leq 7; \\ \frac{\bar{\varepsilon}_6 - 7}{8 - 7}, & \text{if } 7 \leq \bar{\varepsilon}_6 \leq 8, \\ \frac{9 - \bar{\varepsilon}_6}{9 - 8}, & \text{if } 8 \leq \bar{\varepsilon}_6 \leq 9; \\ 0, & \text{if } \bar{\varepsilon}_6 \geq 9; \end{cases}$$

Figure 2.3. **Relationship between membership and aspiration**



Fauji foundation

$\bar{f}_3 = 0.45 * \bar{\varepsilon}_1 + \bar{\varepsilon}_2 + 225000 * \bar{\varepsilon}_3 + 69750 * \bar{\varepsilon}_4 + \bar{\varepsilon}_5 + 19 * \bar{\varepsilon}_6$

$$\lambda(\bar{\varepsilon}_1) = \begin{cases} 0, & \text{if } \bar{\varepsilon}_1 \leq -0.55; \\ \frac{\bar{\varepsilon}_1 + 0.55}{0.45 + 0.55}, & \text{if } -0.55 \leq \bar{\varepsilon}_1 \leq 0.45; \\ \frac{1.45 - \bar{\varepsilon}_1}{1.45 - 0.45}, & \text{if } 0.45 \leq \bar{\varepsilon}_1 \leq 1.45, \\ 0, & \text{if } \bar{\varepsilon}_1 \geq 1.45; \end{cases}$$

$$\lambda(\bar{\varepsilon}_2) = \begin{cases} 0, & \text{if } \bar{\varepsilon}_2 \leq 2099, \\ \frac{\bar{\varepsilon}_2 - 2099}{2100 - 2099}, & \text{if } 2099 \leq \bar{\varepsilon}_2 \leq 2100; \\ \frac{2101 - \bar{\varepsilon}_2}{2101 - 2100}, & \text{if } 2100 \leq \bar{\varepsilon}_2 \leq 2101; \\ 0, & \text{if } \bar{\varepsilon}_2 \geq 2101; \end{cases}$$

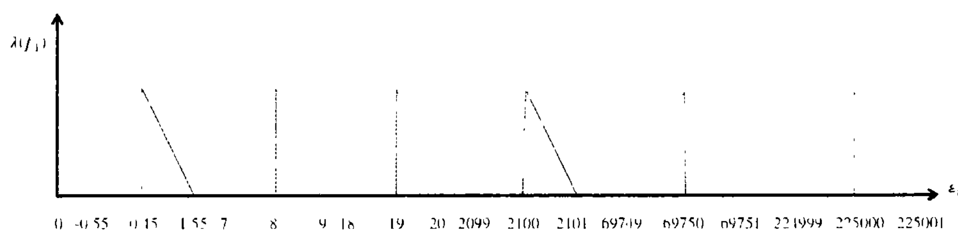
$$\lambda(\bar{\varepsilon}_3) = \begin{cases} 0, & \text{if } \bar{\varepsilon}_3 \leq 224999, \\ \frac{\bar{\varepsilon}_3 - 224999}{225000 - 224999}, & \text{if } 224999 \leq \bar{\varepsilon}_3 \leq 225000; \\ \frac{225001 - \bar{\varepsilon}_3}{225001 - 225000}, & \text{if } 225000 \leq \bar{\varepsilon}_3 \leq 225001; \\ 0 & \text{if } \bar{\varepsilon}_3 \geq 225001, \end{cases}$$

$$\hat{\lambda}(\bar{\varepsilon_4}) = \begin{cases} 0, & \text{if}\, \bar{\varepsilon_4} \leq 69749; \\ \frac{\bar{\varepsilon_4} - 69749}{69750 - 69749}, & \text{if}\, 69749 \leq \bar{\varepsilon_4} \leq 69750; \\ \frac{69751 - \bar{\varepsilon_4}}{69751 - 69750}, & \text{if}\, 69750 \leq \bar{\varepsilon_4} \leq 69751; \\ 0, & \text{if}\, \bar{\varepsilon_4} \geq 69751; \end{cases}$$

$$\hat{\lambda}(\bar{\varepsilon_5}) = \begin{cases} 0, & \text{if}\, \bar{\varepsilon_5} \leq 7; \\ \frac{\bar{\varepsilon_5} - 7}{8 - 7}, & \text{if}\, 7 \leq \bar{\varepsilon_5} \leq 8; \\ \frac{9 - \bar{\varepsilon_5}}{9 - 8}, & \text{if}\, 8 \leq \bar{\varepsilon_5} \leq 9; \\ 0, & \text{if}\, \bar{\varepsilon_5} \geq 9. \end{cases}$$

$$\hat{\lambda}(\bar{\varepsilon_6}) = \begin{cases} 0, & \text{if}\, \bar{\varepsilon_6} \leq 18; \\ \frac{\bar{\varepsilon_6} - 18}{19 - 18}, & \text{if}\, 18 \leq \bar{\varepsilon_6} \leq 19, \\ \frac{20 - \bar{\varepsilon_6}}{20 - 19}, & \text{if}\, 19 \leq \bar{\varepsilon_6} \leq 20; \\ 0, & \text{if}\, \bar{\varepsilon_6} \geq 20; \end{cases}$$

Figure 2.4: **Relationship between membership and aspiration**



Pakistan Institute of Medical Sciences

$\tilde{f}_1 = 0.55 * \tilde{\varepsilon_1} + \tilde{\varepsilon_2} + 175000 * \tilde{\varepsilon_3} + 145000 * \tilde{\varepsilon_4} + \tilde{\varepsilon_5} + 3 * \tilde{\varepsilon_6}$

$$\hat{\lambda}(\tilde{\varepsilon_1}) = \begin{cases} 0, & \text{if } \tilde{\varepsilon_1} \leq -0.45; \\ \frac{\tilde{\varepsilon_1}+0.45}{0.55+0.45}, & \text{if } -0.45 \leq \tilde{\varepsilon_1} \leq 0.55; \\ \frac{1.55-\tilde{\varepsilon_1}}{1.55-0.55}, & \text{if } 0.55 \leq \tilde{\varepsilon_1} \leq 1.55. \\ 0, & \text{if } \tilde{\varepsilon_1} \geq 1.55; \end{cases}$$

$$\hat{\lambda}(\tilde{\varepsilon_2}) = \begin{cases} 0, & \text{if } \tilde{\varepsilon_2} \leq 2099, \\ \frac{\tilde{\varepsilon_2}-2099}{2100-2099}, & \text{if } 2099 \leq \tilde{\varepsilon_2} \leq 2100, \\ \frac{2101-\tilde{\varepsilon_2}}{2101-2100}, & \text{if } 2100 \leq \tilde{\varepsilon_2} \leq 2101, \\ 0, & \text{if } \tilde{\varepsilon_2} \geq 2101, \end{cases}$$

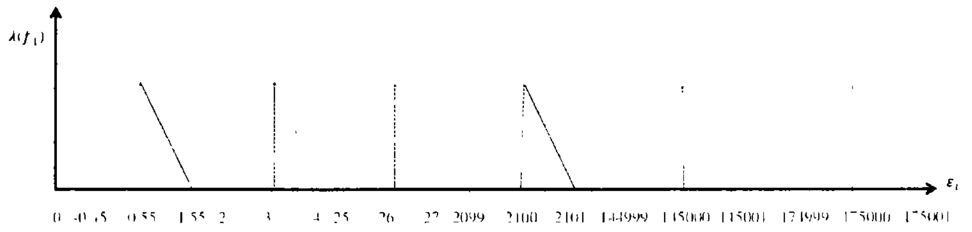
$$\hat{\lambda}(\tilde{\varepsilon_3}) = \begin{cases} 0, & \text{if } \tilde{\varepsilon_3} \leq 174999, \\ \frac{\tilde{\varepsilon_3}-174999}{17500}, & \text{if } 174999 \leq \tilde{\varepsilon_3} \leq 175000; \\ \frac{175001-\tilde{\varepsilon_3}}{175001-175000}, & \text{if } 175000 \leq \tilde{\varepsilon_3} \leq 175001; \\ 0, & \text{if } \tilde{\varepsilon_3} \geq 175001; \end{cases}$$

$$\hat{\lambda}(\tilde{\varepsilon_4}) = \begin{cases} 0, & \text{if } \tilde{\varepsilon_4} \leq 144999, \\ \frac{\tilde{\varepsilon_4}-144999}{145000-144999}, & \text{if } 144999 \leq \tilde{\varepsilon_4} \leq 145000, \\ \frac{145001-\tilde{\varepsilon_4}}{145001-145000}, & \text{if } 145000 \leq \tilde{\varepsilon_4} \leq 145001, \\ 0, & \text{if } \tilde{\varepsilon_4} \geq 145001. \end{cases}$$

$$\hat{\lambda}(\tilde{\varepsilon_5}) = \begin{cases} 0, & \text{if } \tilde{\varepsilon_5} < 25; \\ \frac{\tilde{\varepsilon_5}-25}{26-25}, & \text{if } 25 \leq \tilde{\varepsilon_5} \leq 26; \\ \frac{27-\tilde{\varepsilon_5}}{27-26}, & \text{if } 26 \leq \tilde{\varepsilon_5} \leq 27; \\ 0, & \text{if } \tilde{\varepsilon_5} \geq 27; \end{cases}$$

$$\hat{\lambda}(\tilde{\varepsilon_6}) = \begin{cases} 0, & \text{if } \tilde{\varepsilon_6} \leq 2; \\ \frac{\tilde{\varepsilon_6}-2}{3-2}, & \text{if } 2 \leq \tilde{\varepsilon_6} \leq 3, \\ \frac{4-\tilde{\varepsilon_6}}{4-3}, & \text{if } 3 \leq \tilde{\varepsilon_6} \leq 4, \\ 0, & \text{if } \tilde{\varepsilon_6} \geq 4; \end{cases}$$

Figure 2.5: **Relationship between membership and aspiration**



Rawal hospital

$\bar{f}_7 = 0.5 * \bar{\varepsilon}_1 + \bar{\varepsilon}_2 + 250000 * \bar{\varepsilon}_3 + 28000 * \varepsilon_4 + \bar{\varepsilon}_5 + 3 * \bar{\varepsilon}_6$

$$\hat{\lambda}(\bar{\varepsilon}_1) = \begin{cases} 0, & \text{if } \bar{\varepsilon}_1 \le -0.5, \\ \frac{\bar{\varepsilon}_1 + 0.5}{0.5+0.5}, & \text{if } -0.5 \le \bar{\varepsilon}_1 \le 0.5, \\ \frac{1.5-\bar{\varepsilon}_1}{1.5-0.5}, & \text{if } 0.5 \le \bar{\varepsilon}_1 \le 1.5; \\ 0, & \text{if } \bar{\varepsilon}_1 \ge 1.5; \end{cases}$$

$$\hat{\lambda}(\bar{\varepsilon}_2) = \begin{cases} 0, & \text{if } \bar{\varepsilon}_2 \le 2299; \\ \frac{\bar{\varepsilon}_2 - 2299}{2300-2299}, & \text{if } 2299 \le \bar{\varepsilon}_2 \le 2300, \\ \frac{2301-\bar{\varepsilon}_2}{23012300}, & \text{if } 2300 \le \bar{\varepsilon}_2 \le 2301; \\ 0, & \text{if } \bar{\varepsilon}_2 \ge 2301; \end{cases}$$

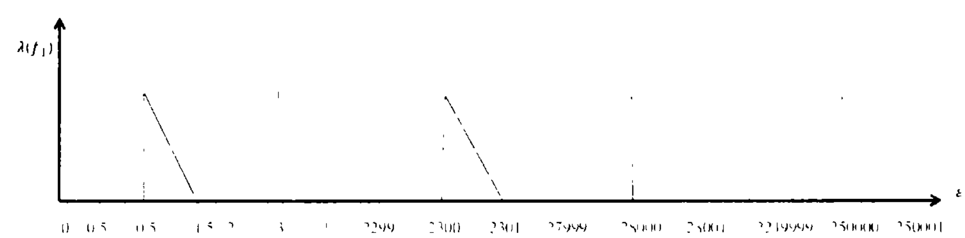
$$\hat{\lambda}(\bar{\varepsilon}_3) = \begin{cases} 0, & \text{if } \bar{\varepsilon}_3 \le 249999; \\ \frac{\bar{\varepsilon}_3 - 249999}{250000-249999}, & \text{if } 249999 \le \bar{\varepsilon}_3 \le 250000; \\ \frac{250001-\bar{\varepsilon}_3}{250001-250000}, & \text{if } 250000 \le \bar{\varepsilon}_3 \le 250001; \\ 0, & \text{if } \bar{\varepsilon}_3 \ge 250001; \end{cases}$$

$$\hat{\lambda}(\bar{\varepsilon}_4) = \begin{cases} 0, & \text{if } \bar{\varepsilon}_4 \leq 27999; \\ \frac{\bar{\varepsilon}_4 - 27999}{28000 - 27999}, & \text{if } 27999 \leq \bar{\varepsilon}_4 \leq 28000; \\ \frac{28001 - \bar{\varepsilon}_4}{28001 - 28000}, & \text{if } 28000 \leq \bar{\varepsilon}_4 \leq 28001; \\ 0, & \text{if } \bar{\varepsilon}_4 \geq 28001; \end{cases}$$

$$\hat{\lambda}(\bar{\varepsilon}_5) = \begin{cases} 0, & \text{if } \bar{\varepsilon}_5 \leq 2; \\ \frac{\bar{\varepsilon}_5 - 2}{3 - 2}, & \text{if } 2 \leq \bar{\varepsilon}_5 \leq 3, \\ \frac{4 - \bar{\varepsilon}_5}{4 - 3}, & \text{if } 3 \leq \bar{\varepsilon}_5 \leq 4; \\ 0, & \text{if } \bar{\varepsilon}_5 \geq 4; \end{cases}$$

$$\hat{\lambda}(\bar{\varepsilon}_6) = \begin{cases} 0, & \text{if } \bar{\varepsilon}_6 \leq 2, \\ \frac{\bar{\varepsilon}_6 - 2}{3 - 2}, & \text{if } 2 \leq \bar{\varepsilon}_6 \leq 3, \\ \frac{4 - \bar{\varepsilon}_6}{4 - 3}, & \text{if } 3 \leq \bar{\varepsilon}_6 \leq 4, \\ 0, & \text{if } \bar{\varepsilon}_6 \geq 4; \end{cases}$$

Figure 2.6: **Relationship between membership and aspiration**



IF part	THEN part	Fuzzy constraints	Aspiration	Decision variable	Objective funtion
Holy Family Hospital					
	$0.75 * \bar{\varepsilon}_1 + \bar{\varepsilon}_2$	$\bar{\varepsilon} + 0.25 \geq \acute{\gamma}_1$	$\acute{\gamma}_1 = 0$	$\bar{\varepsilon}_1 = 1.7500$	
	$+175000 * \bar{\varepsilon}_3$	$1.75 - \bar{\varepsilon} \geq \acute{\gamma}_1$	$\acute{\gamma}_2 = 0$	$\bar{\varepsilon}_2 = 4499$	
	$+11000 * \bar{\varepsilon}_4$	$\bar{\varepsilon} - 4499 \geq \acute{\gamma}_2$	$\acute{\gamma}_3 = 0$	$\bar{\varepsilon}_3 = 174999$	
	$+\bar{\varepsilon}_5 + 26 * \bar{\varepsilon}_6$	$4501 - \bar{\varepsilon} \geq \acute{\gamma}_2$	$\acute{\gamma}_4 = 0$	$\bar{\varepsilon}_4 = 10999$	
		$\bar{\varepsilon} - 174999 \geq \acute{\gamma}_3$	$\acute{\gamma}_5 = 0$	$\bar{\varepsilon}_5 = 17$	
		$175001 - \bar{\varepsilon} \geq \acute{\gamma}_3$	$\acute{\gamma}_6 = 1$	$\bar{\varepsilon}_6 = 25$	
$\mu(\bar{f}_1) = \wedge_{i=1}^{6} \hat{\lambda}(\bar{\varepsilon}_i)$		$\bar{\varepsilon} - 10999 \geq \acute{\gamma}_4$			30745819167.3125
		$11001 - \bar{\varepsilon} \geq \acute{\gamma}_4$			
		$\bar{\varepsilon} - 17 \geq \acute{\gamma}_5$			
		$19 - \bar{\varepsilon} \geq \acute{\gamma}_5$			
		$\bar{\varepsilon} - 25 \geq \acute{\gamma}_6$			
		$27 - \bar{\varepsilon} \geq \acute{\gamma}_6$			
Shifa International Hospital					
	$0.85 * \bar{\varepsilon}_1 + \bar{\varepsilon}_2$	$\bar{\varepsilon} + 0.15 \geq \acute{\gamma}_1$	$\acute{\gamma}_1 = 0$	$\bar{\varepsilon}_1 = 1.8500$	
	$+200000 * \bar{\varepsilon}_3$	$1.85 - \bar{\varepsilon} \geq \acute{\gamma}_1$	$\acute{\gamma}_2 = 0$	$\bar{\varepsilon}_2 = 2099$	
	$+79000 * \bar{\varepsilon}_4 +$	$\bar{\varepsilon} - 2099 \geq \acute{\gamma}_2$	$\acute{\gamma}_3 = 0$	$\bar{\varepsilon}_3 = 199999$	
	$\bar{\varepsilon}_5 + 8 * \bar{\varepsilon}_6$	$2101 - \bar{\varepsilon} \geq \acute{\gamma}_2$	$\acute{\gamma}_4 = 0$	$\bar{\varepsilon}_4 = 78999$	
		$\bar{\varepsilon} - 199999 \geq \acute{\gamma}_3$	$\acute{\gamma}_5 = 0$	$\bar{\varepsilon}_5 = 2$	
		$2100001 - \bar{\varepsilon} \geq \acute{\gamma}_3$	$\acute{\gamma}_6 = 1$	$\bar{\varepsilon}_6 = 7$	
$\mu(\bar{f}_2) = \wedge_{i=1}^{6} \hat{\lambda}(\bar{\varepsilon}_i)$		$\bar{\varepsilon} - 78999 \geq \acute{\gamma}_4$			46240723158.5725
		$79001 - \bar{\varepsilon} \geq \acute{\gamma}_4$			
		$\bar{\varepsilon} - 2 \geq \acute{\gamma}_5$			
		$4 - \bar{\varepsilon} \geq \acute{\gamma}_5$			
		$\bar{\varepsilon} - 7 \geq \acute{\gamma}_6$			
		$9 - \bar{\varepsilon} \geq \acute{\gamma}_6$			

IF part	THEN part	Fuzzy constraints	Aspiration	Decision variable	Objective funtion
		Fauji Foundation Hospital			
$\mu(\tilde{f}_3) = \wedge_{i=1}^{6} \tilde{\lambda}(\bar{\varepsilon}_i)$	$0.45 * \bar{\varepsilon}_1 + \bar{\varepsilon}_2$ $+225000 * \bar{\varepsilon}_3$ $+69750 * \bar{\varepsilon}_4 +$ $\bar{\varepsilon}_5 + 19 * \bar{\varepsilon}_6$	$\bar{\varepsilon} + 0.55 \geq \gamma'_1$ $1.15 - \bar{\varepsilon} \geq \gamma'_1$ $\bar{\varepsilon} - 2099 \geq \gamma'_2$ $2101 - \bar{\varepsilon} \geq \gamma'_2$ $\bar{\varepsilon} - 224999 \geq \gamma'_3$ $225001 - \bar{\varepsilon} \geq \gamma'_3$ $\bar{\varepsilon} - 69749 \geq \gamma'_1$ $69751 - \bar{\varepsilon} \geq \gamma'_1$ $\bar{\varepsilon} - 7 \geq \gamma'_5$ $9 - \bar{\varepsilon} \geq \gamma'_5$ $\bar{\varepsilon} - 18 \geq \gamma'_6$ $20 - \bar{\varepsilon} \geq \gamma'_6$	$\gamma'_1 = 0$ $\gamma'_2 = 0$ $\gamma'_3 = 0$ $\gamma'_4 = 0$ $\gamma'_5 = 0$ $\gamma'_6 = 1$	$\bar{\varepsilon}_1 = 1.4500$ $\bar{\varepsilon}_2 = 2099$ $\bar{\varepsilon}_3 = 224999$ $\bar{\varepsilon}_4 = 69749$ $\bar{\varepsilon}_5 = 7$ $\bar{\varepsilon}_6 = 18$	55489770198.6525
		Pakistan Institute of Medical Sciences			
$\mu(\tilde{f}_4) = \wedge_{i=1}^{6} \tilde{\lambda}(\bar{\varepsilon}_i)$	$0.45 * \bar{\varepsilon}_1 + \bar{\varepsilon}_2$ $+175000 * \bar{\varepsilon}_3$ $+145000 * \bar{\varepsilon}_4 +$ $\bar{\varepsilon}_5 + 3 * \bar{\varepsilon}_6$	$\bar{\varepsilon} + 0.45 \geq \gamma'_1$ $1.55 - \bar{\varepsilon} \geq \gamma'_1$ $\bar{\varepsilon} - 2099 \geq \gamma'_2$ $2101 - \bar{\varepsilon} \geq \gamma'_2$ $\bar{\varepsilon} - 174999 \geq \gamma'_3$ $175001 - \bar{\varepsilon} \geq \gamma'_3$ $\bar{\varepsilon} - 144999 \geq \gamma'_1$ $145001 - \bar{\varepsilon} \geq \gamma'_4$ $\bar{\varepsilon} - 25 \geq \gamma'_5$ $27 - \bar{\varepsilon} \geq \gamma'_5$ $\bar{\varepsilon} - 2 \geq \gamma'_6$ $4 - \bar{\varepsilon} \geq \gamma'_6$	$\gamma'_1 = 0$ $\gamma'_2 = 0$ $\gamma'_3 = 0$ $\gamma'_4 = 0$ $\gamma'_5 = 0$ $\gamma'_6 = 1$	$\bar{\varepsilon}_1 = 1.5500$ $\bar{\varepsilon}_2 = 2099$ $\bar{\varepsilon}_3 = 174999$ $\bar{\varepsilon}_4 = 144999$ $\bar{\varepsilon}_5 = 25$ $\bar{\varepsilon}_6 = 2$	51649682130.8525

IF part	THEN part	Fuzzy constraints	Aspiration	Decision variable	Objective funtion
Rawal Institute of Cardiology					
	$0.5*\bar{\varepsilon}_1+\bar{\varepsilon}_2$	$\bar{\varepsilon}+0.5\geq\gamma'_1$	$\gamma'_1=0$	$\bar{\varepsilon}_1=1.5000$	
	$+250000*\bar{\varepsilon}_3$	$1.5-\bar{\varepsilon}>\gamma'_1$	$\gamma'_2=0$	$\bar{\varepsilon}_2=2299$	
	$+28000*\varepsilon_1+$	$\bar{\varepsilon}-2299>\gamma'_2$	$\gamma'_3=0$	$\bar{\varepsilon}_3=249999$	
	$\bar{\varepsilon}_5+3*\bar{\varepsilon}_6$	$2301-\bar{\varepsilon}\geq\gamma'_2$	$\gamma'_1=0$	$\bar{\varepsilon}_1=27999$	
		$\bar{\varepsilon}-249999\geq\gamma'_3$	$\gamma_5=0$	$\bar{\varepsilon}_5=2$	
$\mu(\tilde{f}_5)=\wedge_{i=1}^{6}\lambda(\bar{\varepsilon}_i)$		$250001-\bar{\varepsilon}\geq\gamma'_3$	$\gamma'_6=1$	$\bar{\varepsilon}_6=2$	0.3283721307 75
		$\bar{\varepsilon}-27999\geq\gamma'_4$			
		$28001-\bar{\varepsilon}\geq\gamma'_4$			
		$\bar{\varepsilon}-2\geq\gamma'_5$			
		$4-\bar{\varepsilon}\geq\gamma'_5$			
		$\bar{\varepsilon}-2\geq\gamma'_6$			
		$4-\bar{\varepsilon}\geq\gamma'_6$			

2.5 Conclusion

With the help of MATLAB 2018, the parameter values for the crisp model and fuzzy model which includesbed occupancy rate, total targeted beds, total revenue, staffing cost, nurse to patient ratio, and consultation hours were computed. When the model's parameter are obtained, graphs are then used to compare the results of the crisp and fuzzy models. Theaim is to maximise bed occupancy rate, total targeted beds, financial resources (staffing cost, total revenue), and to minimise nurse to patient ratio. The graph indicates that the solutions obtained by solving the fuzzy model maximizebed occupancy rate for every hospital more than the crisp model see Figure ?? Resultsproducedfor total targeted beds are improved using a fuzzy model rather than a crisp model see Figure 2.8 The total income and staffing costs that are attained by solving the fuzzy model are greater than those obtained by solving the crisp model see Figure 2.9, Figure 2.10. The nurse to patient ratio is shown in the table issignificantly greater in the crisp model but lower in the fuzzy model see Figure 2.11. The objective function value is more efficientin the fuzzy model than the crisp model see Figure 2.9. The Fuzzy outcomes are more significant than crisp outcomes

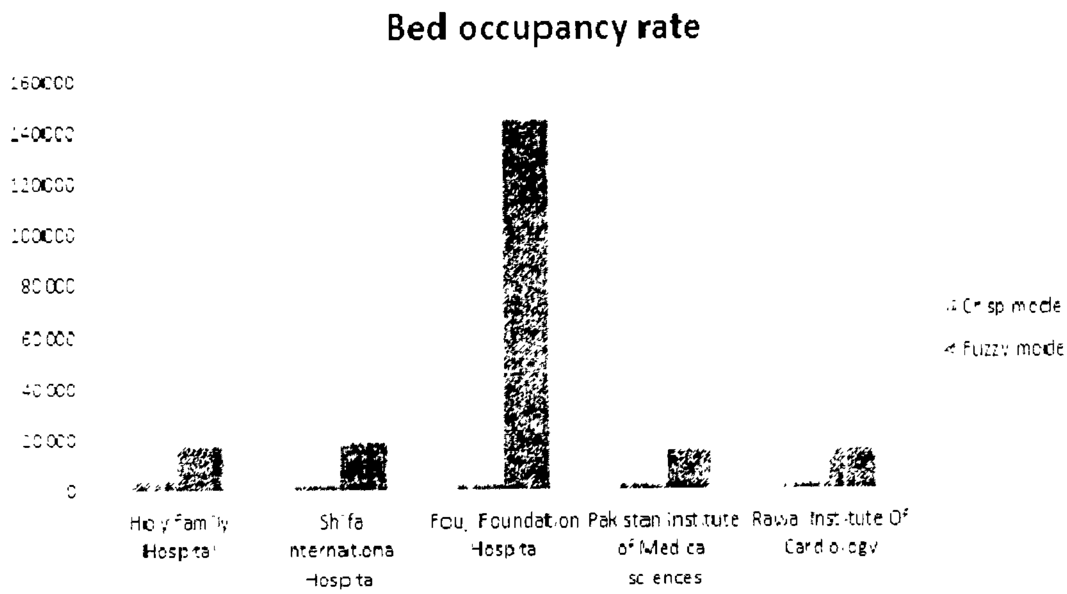


Figure 2.7: **Bed occupancy rate**

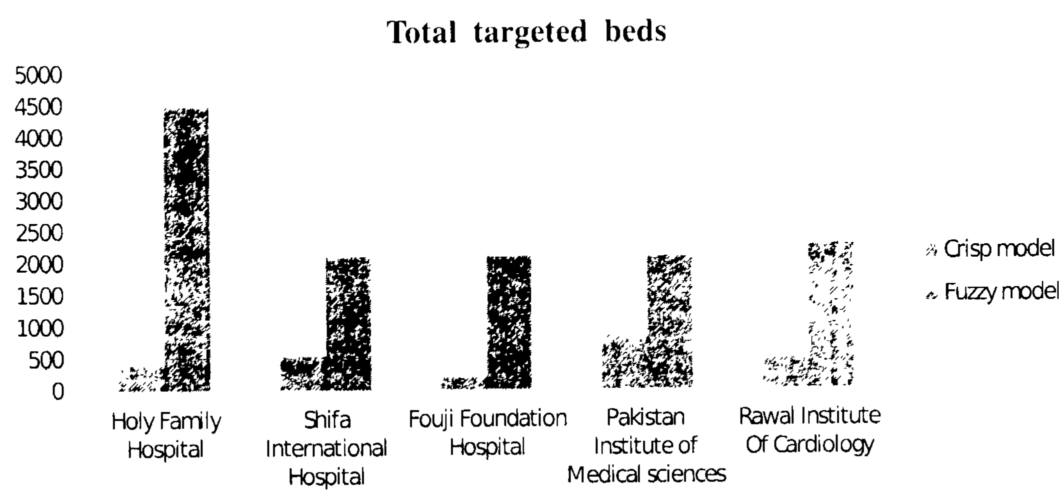


Figure 2.8: **Total targeted beds**

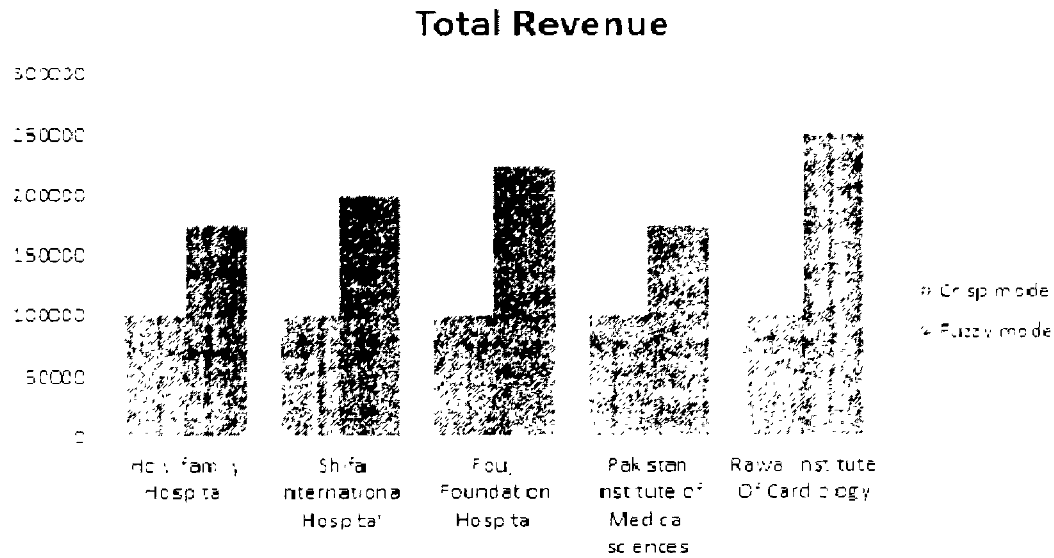


Figure 2.9: Total revenue

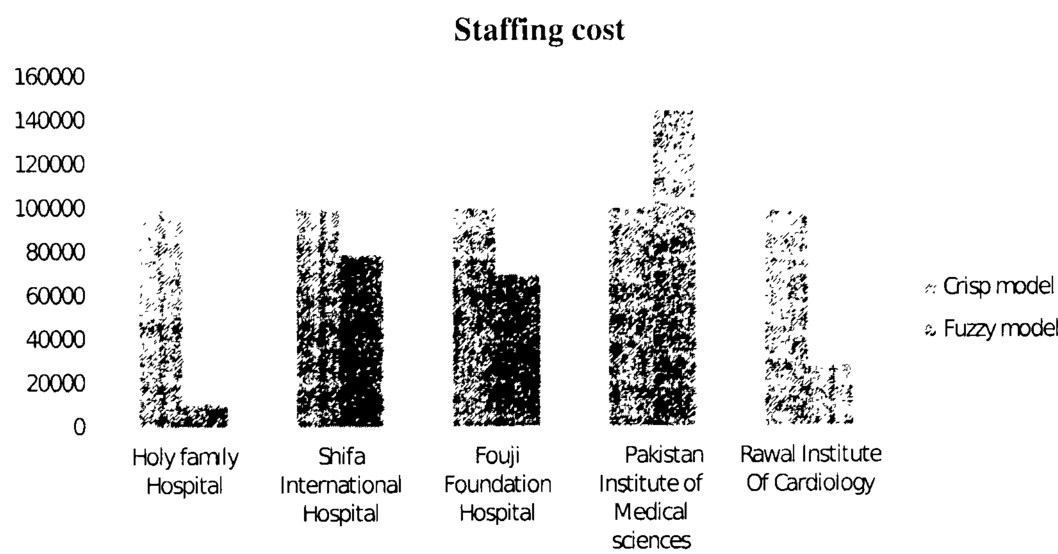


Figure 2.10: **Staffing cost**

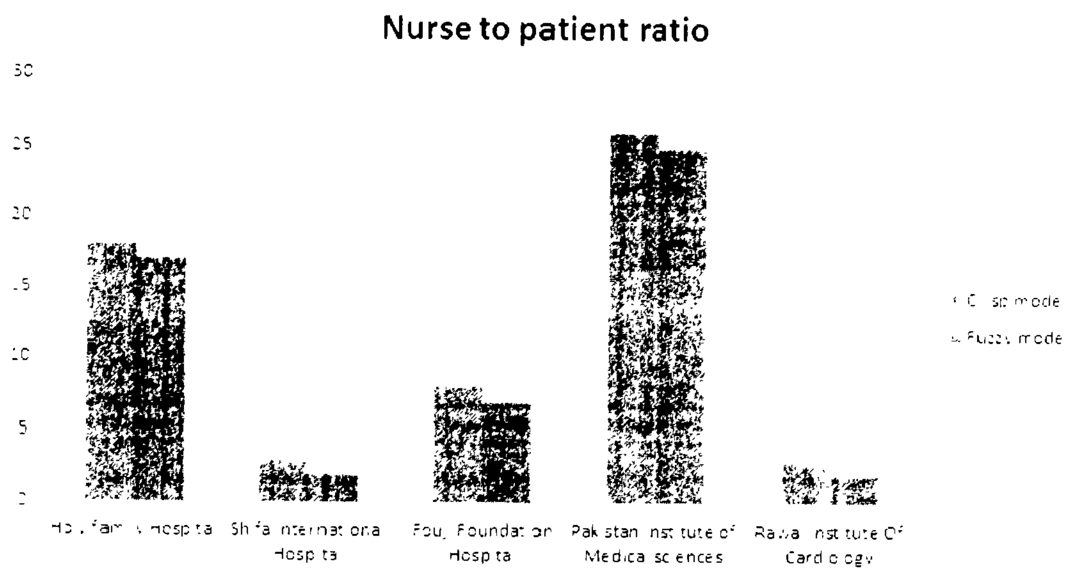


Figure 2.11. **Nurse to patient ratio**

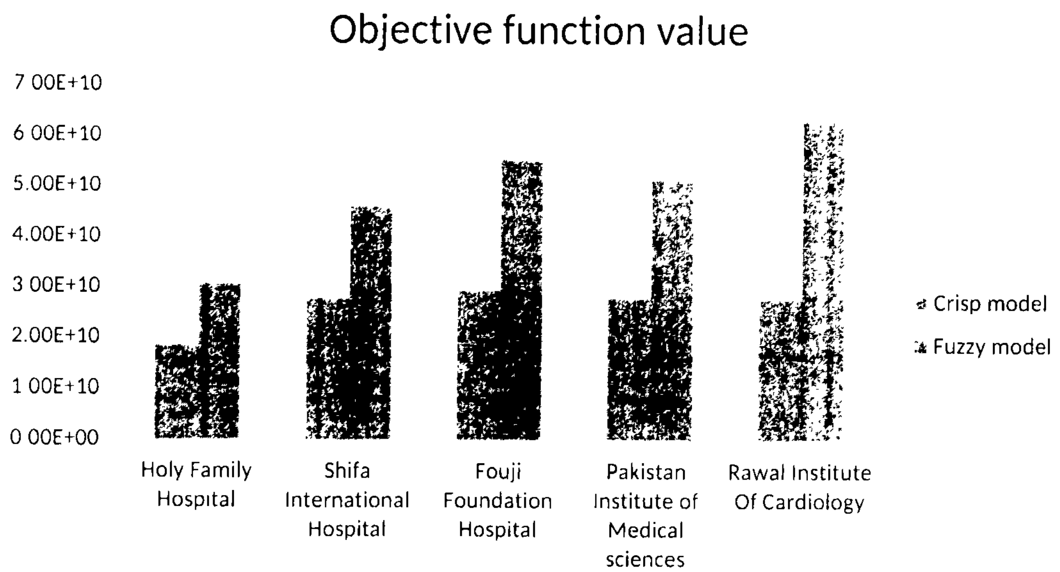


Figure 2 12· **Objective function value**

Chapter 3

Best worst decision making through intuitionistic fuzzy number

It is challenging for an individual decision maker (DM) to take into consideration all the components of a problem , with the increasing complexity of existing socioeconomic changes. In reality recruiting , scheduling , promoting plans , and reorganizing in major industries technical committees make critical decisions. Furthermore , significant judgments are usually constructed with the participation of a dedicated staff and specialists in business areas like health care , judicial frameworks and welfare programs. Based on uncertain data , decision analysis is necessary and acceptable. Over the past years , researchers have put out number of innovative hypothesis testing and strategies. that are being frequently employed to address a wide range of problems and solve problems in the everyday world like banking , administration , economy, production , systemization , transportation etc. The essential role of health care decision-makers is to determine the most efficient use of limited resources , and distributing the resources , use of technical knowledge provided the information that is available at the moment.

Decision making which refers to the reliability and personal satisfaction from set of choices , the term multi criteria decision making (MCDM) describes the process of making decisions using multiple criteria. One of the best MCDM was proposed by Rezaei *et al.* [15] and MCDM is a critical component in program implementation science , system integration , and operational research and have multiple uses in variety of fields, including administration , technology , and finance etc. MCDM is further categorized in to two parts:

- multi-objective decision-making (MODM)
- multi-attribute decision-making (MADM)

Multi-objective decision-making ($MODM$) in which decision variables are continuous in nature and usually are extensive or unbounded number of choices and utilizing vector based optimization approach which is also a mathematical programming approach Although multi-attribute decision-making (MADM) is a form of decision-making in which the decision variables are discrete in nature and are typically known as discrete decision-making ($MCDM$) and usually restricted or bounded the number of choices and it determines the multiple choices at the very start and find the high ranked criteria and the low ranked criteria and then identifies best one

Uncertain decision-making appears in a wide variety of human behaviours , specifically when dealing with sophisticated modelling In management sciences and decision sciences , fuzzy set theory has been extensively used. Over the past years , many approaches that has been presented to handle uncertainties within decision-making challenges becomes a demanding research problem. Concerning to medical and administrative aspects of health care , decision making may have an indirect or direct influence on the patient's welfare According to the perceptions of governments, insurers , and patient's decision-makers are usually remain unsure related to how well the criteria they considered should be valued. In health care service system , when patient's are entered in to the hospital they are not treated as they arrived and alternatively optimized based on the priority and intensity of their requirements to the hospital. To reflect a mechanism during which inpatient beds are granted to elective admittance demands that are placed on hold on a schedule For all waiting patient's with the goal of boosting the accuracy of diagnostic , monitoring , the optimal prioritized allocation strategy must be designed Both qualitative and quantitative aspects in multi-criteria decision making ($MCDM$) are challenging tasks. Now, the assessment of every patient admittance urgency is predicted on the basis of uncertain facts or fuzzy but there might be a significant link in between monitoring and evaluating standards.

Fuzzy judgments are typically used in decision-making to deal with uncertainties that arise in real-world circumstances. It is up to decision makers to determine the most better and worst criteria among a set of criteria while making decisions. A fuzzy planning model in decision making provides more accurate and reliable solution as possible whereas to handle uncertainties or unclear facts decision making is carried out. For the real based data where we have valid knowledge. It is appropriate for the situation where there are no surety related to problem outcomes so in this case decision making becomes too hard and consider to be sophisticated process In the real world application which are under consideration is health care. And in health care there are too many uncertainties happens in accident and emergency (A & E) department On the basis of the intuitionistic fuzzy best-worst technique, a fuzzy multi-criteria decision-making model is developed [21].

3.1 Real World Problem

In the real world , hospital is one of the important area across all over the world Hospital plays a vital role in providing the best services to their patient's in order to improve their repute and to make sure their patient's to better enjoy services In this modern time period , patient s are demanding for quality which increases pressure on hospital administration The case study that is under consideration are emergency department of hospital. Over the past years , across all over the world corona virus was the most prevailing disease and mostly people were affected by this disease. In case of COVID-19 pandemic , a fuzzy multi criteria decision analysis was conducted [22 25] and to evaluate the COVID-19 pandemic performance of insurance company in the health care treatment area an Intuitionistic MACROS fuzzy technique was adopted [26]. In Pakistan , today's most prevailing diseases are typhoid fever , dengue , diarrhea , etc. So in order to deal with these patient's the services provided by hospital includes patient appointments , availability of doctors , treatment time , in case of emergent condition patient's admittance , maintenance of patient's history time required to shift patient's from OPD to ward, allocation of beds nurse care, consultancy time needed to admitted patient's , role of lab technicians , quality of medicines provided to admitted patient's etc. Now the decision making process occurs at every stage for both emergent and non emergent cases It involves evaluation diagnostic, surgical decision making which occurs at every stage In case of emergency department, as most of the time uncertain event occurs so in that time consultants need to take quick decision and select the best treatment area from all of the given opportunities in order to save their patient's life and by using fuzzy best-worst technique, group decision-making was employed in this domain [27]. A case study of maintenance assessment in hospitals using only a fuzzy best-worst multi-attribute decision-making approach utilizing triangular fuzzy numbers was carried in the past [28] Best worst method was further extended to intuitionistic fuzzy best-worst technique by Mou *et al.* [29].

3.1.1 Data collection

The data is gathered from various hospitals in Islamabad and Rawalpindi by means of a questionnaire that inquiries about the allocation of health care resources across accident and emergency department. The overall theme will serve to identify the best criteria and least important, i.e., the worst criterion/theme, in order to devise the health care resource allocation criteria This assessment helps to determine whether there exist sufficient and suitable medical facilities to treat emergency situations. The questionnaire has 7 criteria.

1. Infrastructure

2. Consultancy time

3. Paramedics

4. Hospital stay

5. Impact of health care resource allocation

6. Health professional's satisfaction

7. Improvements in the allocation of health care resource allocation

3.2 Intuitionistic fuzzy best worst method (IFBWM)

Numerous studies have been done in the past with intuitionistic fuzzy sets as a modification of the best-worst technique [30], although to develop an innovative IFBWM such that the generated weight for the criteria in the model will retain in fuzzy intuitionistic format. Steps involved in the IFBWM are constructed as :

Step1: To state the decision making problem as accurately as needed.

Step2: To develop a set of criteria for drawing decisions if there are k criteria $(s_1, s_2, s_3, \ldots, s_k)$ which must be evaluated before drawing conclusions

Step3: Professionals would decide whose criteria have become the best(s_B) and the worst (s_w)

Step4: Evaluation must be performed while evaluating which criteria are preferable over the best ones. The generalized BWM has been modified in this phase to integrate intuitionistic fuzzy sets. It is essential to determine whether all other criteria are preferable over the best criteria in contrast to the traditional BWM where decision-makers indicate their preference for the best criteria over all other criteria. For such an assessment the interval importance degree (IID) scale is utilized. For such a scale, the decision-maker use a number that lies inside the range 0 and 1 to indicate the prefrence of one criteria over the other criteria therefore correspondingly intuitionistic scaling is considered at each defined interval to figuring out the intuitionistic prefrences over the criteria.

Step5: In order to determine the worst citeria prefrence over every other criteria, it relies on the pairwise comparison matrix as defined in the earlier step. So in this IFBWM approach, to calculate the worst citeria prefrence over all other criteria is based on the pairwise comparison matrix of all other criteria over the best criteria. Hence, in this way the corresponding intutionistic scale for the associated prefrences will be evaluated

step6: In the last step, an intuitionistic fuzzy mathematical model is developed and an assessment of the preference of the worst criteria over the other criteria as well as the preference of the other criteria over the best is necessary. As a consequence the model is developed as follows

$$\min \delta$$

such that

$$\left|\frac{w_m}{w_B} - e'_{mB}\right| \leq \delta \quad \forall m$$

$$\left|\frac{w_W}{w_m} - e'_{Wm}\right| \leq \delta \quad \forall m$$

$$\sum_m w_m = 1$$

This model is in crisp environment so to convert this in to intuitionistic fuzzy model we need to replace the decision variables and parameters by intuitionistic fuzzy numbers so the model has the following form

$$\min(\delta_\eta, \delta_\tau)$$

such that:

$$\left|\frac{(w_\eta^m, w_\tau^m)}{(w_\eta^B, w_\tau^B)} - (e'^{mB}_\eta, e'^{mB}_\tau)\right| \leq (\delta_\eta, \delta_\tau) \quad \forall m$$

$$\sum_{m=1} (w_\eta^m, w_\tau^m) \leq 1$$

Moreover, by employing addition operation (1.2.3) and division operation 1.2.7 using (1.2.11, 1.2.12) in the above equation, the final model is

$$\min(\delta_\eta, \delta_\tau) \tag{3.2.1}$$

$$\frac{w_\eta^{m}}{w_\eta^{B}} \leq \delta_\eta + e_\eta'^{\,mB} - \delta_\eta e_\eta'^{\,mB} \quad \forall m \tag{3.2.2}$$

$$\left|\frac{w_\digamma^{m} - w_\digamma^{B}}{1 - w_\digamma^{B}}\right| \geq \delta_\digamma e_\digamma'^{\,mB} \quad \forall m \tag{3.2.3}$$

$$\frac{w_\eta^{W}}{w_\eta^{m}} \leq \delta_\eta + e_\eta'^{\,Wm} - \delta_\eta e_\eta'^{\,Wm} \quad \forall m \tag{3.2.4}$$

$$\left|\frac{w_\digamma^{W} - w_\digamma^{J}}{1 - w_\digamma^{J}}\right| \geq \delta_\digamma e_\digamma'^{\,WJ} \quad \forall m \tag{3.2.5}$$

$$w_\eta^{m} + w_\digamma^{m} \leq 1 \quad \forall m \tag{3.2.6}$$

$$\delta_\eta + \delta_\digamma \leq 1 \tag{3.2.7}$$

$$w_m = \frac{1 + w_\eta^{m} - w_\digamma^{W}}{2} \quad \forall m \tag{3.2.8}$$

$$\sum_{m=1}^{l} w_m = 1 \tag{3.2.9}$$

$$w_\eta^{W}, w_\eta^{B}, w_\digamma^{B}, w_\digamma^{W}, w_\digamma^{m}, w_\eta^{B}, \delta_\digamma, \delta_\eta, w_m > 0 \tag{3.2.10}$$

In the above model equation 3.2.1 denotes the objective function of the model which is always zero. Equation 3.2.2 and 3.2.3 represents the preference of all other criteria over best criteria which will provide the membership and non-membership weight values while equation 3.2.4 and 3.2.5 denotes the preference of worst criteria over the other criteria which will give the value of membership and non-membership weights. The equation 3.2.6 and 3.2.7 ensures the condition for intuitionistic fuzzy sets. Equation 3.2.8 is used to defuzzify the weights. Equation 3.2.9 ensures that sum of weight must be equal to one and equation 3.2.10 describes the non-negativity of decision variables.

Table 3.1: **Fuzzy best worst pairwise comparison of overall criteria**

Criteria	Membership(η)		Non-Membership (φ)		Fuzzy Weights	Defuzzified results
	Best to other (e_{Bm}')	**Other to worst** (e_{mW}')	**Best to other** (e_{Bm}')	**Other to worst** (e_{mW}')		
Infrastructure	8	2	3	9	(0.007352941,0.039215686)	0.4840686275
Consultancy time	9	1	2	1	(0.006535948,0.007169655)	0.4995331165
Paramedics	2	8	9	3	(0.029111765,0.013071895)	0.508169935
Hospital stay	3	7	8	1	(0.019607843,0.014705882)	0.5024509805
Impact of health care resource allocation	7	2	1	9	(0.008403361,0.029111765)	0.189495798
Health professional satisfaction	1	9	1	2	(0.058823529,0.073762838)	0.4925303155
Improvements in health resource allocation	7	2	1	9	(0.016806723,0.033116592)	0.1918300655

Table 3.2: **Fuzzy best worst pairwise comparison of infrastructure**

Criteria	Membership(η)		Non-Membership (φ)		Fuzzy Weights	Defuzzified results
	Best to other ($e_{B\acute{m}}$)	**Other to worst ($e_{\acute{m}W}$)**	**Best to other ($e_{B\acute{m}}$)**	**Other to worst ($e_{\acute{m}W}$)**		
1.1	2	7	9	4	(0.12556251,0.02790278)	0.548829865
1.2	7	2	4	9	(0.035875, 0.06278126)	0.48655
1.3	5	3	6	8	(0.05022501,0.04185417)	0.5042
1.4	7	2	4	9	(0.035875,0.06278126)	0.48655
1.5	8	2	3	9	(0.03139063,0.08360869)	0.4739
1.6	6	8	5	3	(0.04185417,0.04185417)	0.5
1.7	1	9	1	2	(0.17182239,0.10573685)	0.5363
1.8	9	1	2	1	(0.01321711,0.0261311	0.4934
1.9	2	7	9	4	(0.12556251,0.02790278)	0.5525
1.10	7	2	4	9	(0.035875, 0.04626)	0.4918
1.11	4	4	7	7	(0.06278126,0.02643121)	0.51817
1.12	5	3	6	8	(0.05022501,0.03083992)	0.50969
1.13	6	2	5	9	(0.04185417,0.0370079)	0.50242
1.14	8	2	3	9	(0.03139063,0.06167983)	0.4848
1.15	1	4	7	7	(0.06278126,0.02643121)	0.51817
1.16	3	5	8	6	(0.083708, 0.03139063)	0.53029

Table 3.3: **Fuzzy best worst pairwise comparison of consultancy time**

Criteria	Membership(η)		Non-Membership (φ)		Fuzzy Weights	Defuzzified results
	Best to other ($e_{B\acute{m}}$)	Other to worst ($e_{\acute{m}W}$)	Best to other ($e_{B\acute{m}}$)	Other to worst ($e_{\acute{m}W}$)		
2.1	8	2	3	9	(0.05122784,0.1195313)	0.46585
2.2	5	4	6	7	(0.08196455,0.0683039)	0.50683
2.3	7	3	4	8	(0.05851611,0.07220686)	0.49317
2.4	6	4	5	7	(0.06830379,0.06830379)	0.80737
2.5	3	8	8	3	(0.13660758,0.05122784)	0.51269
2.6	8	2	3	9	(0.05122784,0.13660758)	0.45731
2.7	9	1	2	1	(0.04553586,0.01992194)	0.512807
2.8	2	9	9	2	(0.20491137,0.01553586)	0.57969
2.9	1	9	1	2	(0.25041723,0.19921939)	0.52561
2.10	8	3	3	8	(0.051228,0.119532)	0.46584

Table 3.1 **Fuzzy best worst pairwise comparison of paramedics**

Criteria	Membership(η)		Non-Membership (φ)		Fuzzy Weights	Defuzzified results
	Best to other ($e_{\acute{B}m}$)	**Other to worst** ($e_{\acute{m}W}$)	**Best to other** ($e_{\acute{B}m}$)	**Other to worst** ($e_{\acute{m}W}$)		
3.1	6	2	5	9	(0.08352, 0.1136)	0.18497
3.2	5	3	6	8	(0.1002, 0.0916)	0.5028
3.3	6	2	5	9	(0.08352, 0.11356)	0.18497
3.4	5	3	6	8	(0.1002, 0.0418)	0.3162597
3.5	1	9	1	2	(0.2506, 0.3171)	0.1666
3.6	4	1	7	7	(0.12528, 0.0716)	0.5268
3.7	6	2	5	9	(0.081, 0.1002)	0.192
3.8	9	1	2	1	(0.05568, 0.0334)	0.762

Table 3.5: **Fuzzy best worst pairwise comparison of hospital stay**

Criteria	Membership(η)		Non-Membership (φ)		Fuzzy Weights	Defuzzified results
	Best to other ($e_{B\acute{m}}$)	**Other to worst ($e_{\acute{m}W}$)**	**Best to other ($e_{B\acute{m}}$)**	**Other to worst ($e_{\acute{m}W}$)**		
1.1	1	5	1	6	(0.291, 0.1765)	0.5589
1.2	2	5	2	6	(0.1765, 0.02611)	0.5752
1.3	2	1	2	7	(0.1765, 0.0306)	0.5818
1.4	5	1	5	1	(0.01706, 0.0129)	0.51709
1.5	5	2	5	9	(0.0706, 0.0392)	0.5157
1.6	3	3	3	8	(0.1176, 0.02941)	0.54112
1.7	3	3	3	8	(0.11761, 0.0345)	0.5416

Table 3.6 **Fuzzy best worst pairwise comparison of impact of health care resource allocation**

Criteria	Membership(η)		Non-Membership (φ)		Fuzzy Weights	Defuzzified results
	Best to other ($e_{B\acute{m}}$)	Other to worst ($e_{\acute{m}W}$)	Best to other ($e_{B\acute{m}}$)	Other to worst ($e_{\acute{m}W}$)		
5.1	2	2	9	9	(0.0797 0.0351)	0.522
5.2	3	2	8	9	(0.0531 0.0558)	0.199
5.3	3	6	8	5	(0.053 0.0558)	0.199
5.4	3	6	8	5	(0.0531.0.0199)	0.517
5.5	1	1	1	7	(0.2031 0.1591)	0.5219
5.6	3	2	8	9	(0.1062 0.0398)	0.5613
5.7	1	1	7	1	(0.0797 0.0177)	0.5309
5.8	3	6	8	5	(0.106. 0.0558)	0.525
5.9	3	2	8	9	(0.1062 0.0558)	0.525
5.10	2	2	9	9	(0.159 0.035)	0.562

Table 3.7 **Fuzzy best worst pairwise comparison of health professional satisfaction**

Criteria	Membership(η)		Non-Membership (φ)		Fuzzy Weights	Defuzzified results
	Best to other ($e_{B\acute{m}}$)	Other to worst ($e_{\acute{m}W}$)	Best to other ($e_{B\acute{m}}$)	Other to worst ($e_{\acute{m}W}$)		
6.1	1	1	7	1	(0.053 , 0.030)	0.5111
6.2	3	2	8	9	(0.0708 , 0.027)	0.522
6.3	6	3	9	8	(0.1062 , 0.0236)	0.5113
6.4	6	3	9	8	(0.1062 , 0.0236)	0.5113
6.5	6	1	9	7	(0.106 , 0.021)	0.541
6.6	6	3	9	8	(0.1062 , 0.0236)	0.5113
6.7	6	3	9	8	(0.1062 , 0.0236)	0.5113
6.8	6	1	9	7	(0.1062 , 0.0236)	0.5113
6.9	3	2	8	9	(0.0708 , 0.0265)	0.522
6.10	1	1	1	7	(0.133 , 0.1099)	0.511
6.11	6	2	5	9	(0.035 , 0.0379)	0.498

Table 3.8. Fuzzy best worst pairwise comparison of improvements in health resource allocation

Criteria	Membership(η)		Non-Membership (φ)		Fuzzy Weights	Defuzzified results
	Best to other ($e_{B\acute{m}}$)	Other to worst ($e_{\acute{m}W}$)	Best to other ($e_{B\acute{m}}$)	Other to worst ($e_{\acute{m}W}$)		
7.1	3	2	8	9	(0.0401 , 0.0274)	0.5065
7.2	1	1	7	1	(0.0303 , 0.0099)	0.5102
7.3	3	2	8	9	(0.0401 , 0.0274)	0.5065
7.4	2	3	9	8	(0.0606 , 0.024)	0.5181
7.5	3	2	8	9	(0.0401 , 0.0274)	0.5065
7.6	2	2	9	9	(0.0606 , 0.0244)	0.5181
7.7	3	2	8	9	(0.0401 , 0.0274)	0.5065
7.8	3	2	8	9	(0.0401 , 0.0274)	0.5065
7.9	2	2	9	9	(0.0606 , 0.0244)	0.5181
7.10	2	3	9	8	(0.0606 , 0.024)	0.518
7.11	2	3	9	8	(0.0606 , 0.0243)	0.5181
7.12	2	2	9	9	(0.0606 , 0.0243)	0.5181
7.13	1	1	1	7	(0.1398 , 0.1187)	0.4956
7.14	2	2	9	9	(0.1096 , 0.0253)	0.5122
7.15	2	3	9	8	(0.071 , 0.0253)	0.523
7.16	2	2	9	9	(0.081 , 0.0253)	0.5292

Table 3.9: **Consistency index of fuzzy best worst method**

Criteria	Consistency index (CI)	
	Membership	Non-Membership
Infrastructure	0.1153882	0.07259968
Consultancy time	0.15938	0.21060335
Paramedics	0.250555	0.062639
Hospital stay	0.09112	0.071895
Impact of health care resource allocation	0.15938	0.28724
Health professional satisfaction	0.0902655	0.13274
Improvements in health care resource allocation	0.1058942	0.129968
Overall criteria comparisons	0.05882353	0.05882353

Table 3.10. **Intuitionistic fuzzy best worst pairwise comparison of overall criteria**

Criteria	Membership(η)		Non-Membership(φ)		Intuitionistic Weights	Defuzzified results
	Best to other ($e_{B\acute{m}}$)	**Other to worst** (e_{mW})	**Best to other** ($e_{B\acute{m}}$)	**Other to worst** ($e_{\acute{m}W}$)		
Infrastructure	0.125	0.9	0.532	0.19	(0.0588, 0.022098)	0.518
Consultancy time	0.111	1	0.1013	1	(0.03361, 0.0119)	0.5108
Paramedics	0.5000	0.2	0.177	0.57	(0.0196, 0.0112)	0.188
Hospital stay	0.3333	0.3	0.01991	0.508	(0.0235, 0.0663)	0.179
Impact of health care resource allocation	0.1128	0.8	0.3987	0.251	(0.0392, 0.026)	0.506
Health professional satisfaction	1	0.1	1	0.101	(0.1260, 0.082789)	0.5216
Improvements in health resource allocation	0.285	0.1	0.11931	0.225	(0.01622, 0.0283)	0.509

Table 3.11: **Intuitionistic fuzzy best worst pairwise comparison of infrastructure**

Criteria	Membership(η)		Non-Membership(φ)		Intuitionistic Weights	Defuzzified results
	Other to Best ($e_{\acute{m}B}$)	**Worst to Other ($e_{\acute{W}m}$)**	**Other to Best ($e_{\acute{m}B}$)**	**Worst to Other ($e_{\acute{W}m}$)**		
I.1	0.7	0.105	0.3	0.95	(0.1657,0.0793)	0.519
I.2	0.20	0.1	0.6	0.12	(0.0837,0.07930)	0.1865
I.3	0.29	0.26	0.1	0.63	(0.0222,0.07930)	0.50118
I.4	0.20	0.1	0.6	0.12	(0.02221,0.0793)	0.1865
I.5	0.18	0.12	0.6	0.32	(0.02221,0.07930)	0.1739
I.6	0.21	0.1	0.1	0.63	(0.0222,0.0793)	0.5
I.7	1	0.12	1	0.25	(0.10571,0.09903)	0.5362
I.8	0.08	1	0.12	1	(0.0396,0.0088)	0.1931
I.9	0.7	0.64	0.3	0.9	(0.0925,0.0793)	0.5525
I.10	0.20	0.3	0.6	0.57	(0.0617,0.0793)	0.1918
I.11	0.36	0.105	0.59	0.99	(0.01626,0.0261)	0.5182
I.12	0.3	0.13	0.18	0.09	(0.09252,0.0261)	0.5097
I.13	0.21	0.36	0.1	0.07	(0.0617,0.0793)	0.5024
I.14	0.18	0.21	0.6	0.45	(0.06168,0.07930)	0.485
I.15	0.36	0.105	0.59	0.99	(0.0370,0.0261)	0.5182
I.16	0.19	0.158	0.219	0.88	(0.09252,0.019826)	0.530289

Table 3.12 Intuitionistic fuzzy best worst pairwise comparison of consultancy time

Criteria	Membership(η)		Non-Membership(φ)		Intuitionistic Weights	Defuzzified results
	Other to Best ($e_{m\acute{B}}$)	**Worst to Other ($e_{\acute{W}m}$)**	**Other to Best (e_{mB})**	**Worst to Other ($e_{\acute{W}m}$)**		
2.1	0.204	0.89	0.510	0.17	(0.0531, 0.0683)	0.492
2.2	0.327	0.556	0.343	0.29	(0.0398, 0.0888)	0.4755
2.3	0.2	0.7783	0.294	0.276	(0.05312, 0.11099)	0.4711
2.4	0.27	0.67	0.343	0.29	(0.03981, 0.11099)	0.4644
2.5	0.54	0.33	0.26	0.389	(0.03188, 0.1099)	0.4664
2.6	0.204	0.89	0.69	0.15	(0.0531, 0.0683)	0.4948
2.7	0.1817	1	0.100	1	(0.0199, 0.0854)	0.46727
2.8	0.818	0.22	0.97	0.11	(0.0797, 0.0155)	0.51707
2.9	1	0.182	1	0.100	(0.15938, 0.23337)	0.46300
2.10	0.204	0.89	0.600	0.17	(0.1062, 0.0631)	0.5214

Table 3.13. Intuitionistic fuzzy best worst pairwise comparison of paramedics

Criteria	Membership η		Non-Membership φ		Intuitionistic Weights	Defuzzified results
	Other to Best $(e_{mB}^{'})$	Worst to Other $(e_{Wm}^{'})$	Other to Best $(e_{mB}^{'})$	Worst to Other $(e_{Wm}^{'})$		
B1	0.34	0.67	0.36	0.29	(0.0626, 0.0564)	0.50626
B2	0.40	0.56	0.298	0.35	(0.0564, 0.0626)	0.49374
B3	0.33	0.67	0.358	0.29	(0.0626, 0.0564)	0.50626
B4	0.20	0.55	0.13	0.8	(0.1535, 0.1064)	0.5236
B5	1	0.2	1	0.10	(0.2505, 0.2505)	0.5
B6	0.5	0.11	0.22	0.17	(0.2505, 0.167[illegible])	0.5[illegible]
B7	0.33	0.67	0.316	0.33	(0.12528, 0.2505)	0.4371
B8	0.22	0.105	0.105	1	(0.01471, 0.06264)	0.49105

Table 3.11: Intuitionistic fuzzy best worst pairwise comparison of hospital stay

Criteria	Membership η		Non-Membership φ		Intuitionistic Weights	Defuzzified results
	Other to Best (e_{mB})	Worst to Other ($e_{W'm}$)	Other to Best ($e_{m'B}$)	Worst to Other ($e_{W'm}$)		
[illegible]	1	0.16	1	0.07	(0.2, 0.0719)	0.56105
[illegible]	0.[illegible]	0.27	0.15	0.5	(0.1291, 0.0[illegible]36)	0.5129
C3	0.6	0.27	0.15	0.[illegible]2	(0.1291, 0.0636)	0.5129
[illegible]	0.16	1	0.03	1	(0.0353, 0.0196)	0.508
[illegible]	0.24	0.7	0.01	0.33	(0.0863, 0.0653)	0.5104
[illegible]	0.4	0.101	0.07	0.11	(0.1291176, 0.0288)	0.5503
C7	0.[illegible]	0.101	0.07	0.37	(0.1291, 0.02876)	0.55033

Table 3.15: **Intuitionistic fuzzy best worst pairwise comparison of impact of health care resource allocation**

Criteria	Membership(η)		Non-Membership(φ)		Intuitionistic Weights	Defuzzified results
	Other to Best (e'_{mB})	**Worst to Other** (e_{Wm})	**Other to Best** (e_{mB})	**Worst to Other** (e_{Wm})		
5.1	0.39	0.64	0.22	0.500	(0.0638, 0.1489)	0.5071
5.2	0.26	0.67	0.33	0.317	(0.0797, 0.1525)	0.1636
5.3	0.26	0.67	0.33	0.317	(0.0797, 0.1525)	0.1636
5.4	0.26	0.67	0.33	0.089	(0.0597, 0.0300)	0.5148
5.5	1	0.39	1	0.11	(0.15938, 0.3226)	0.1184
5.6	0.52	0.75	0.67	0.11	(0.05312, 0.0667)	0.196
5.7	0.39	1	0.110	1	(0.0265, 0.0177)	0.5055
5.8	0.52	0.75	0.67	0.317	(0.05312, 0.0119)	0.5206
5.9	0.52	0.75	0.67	0.317	(0.05312, 0.0677)	0.193
5.10	0.78	0.110	0.99	0.110	(0.0398, [illegible])	0.5022

Table 3.16: Intuitionistic fuzzy best worst pairwise comparison of health professional's satisfaction

Criteria	Membership(η)		Non-Membership(φ)		Intuitionistic Weights	Defuzzified results
	Other to Best (e_{mB})	Worst to Other (e_{Wm})	Other to Best (e_{mB})	Worst to Other (e_{Wm})		
6.1	0.10	1	0.28	1	(0.0125, 0.053)	0.1947
6.2	0.53	0.75	0.21	0.08	(0.0354, 0.0768)	0.1823
6.3	0.800	0.50	0.21	0.75	(0.0265, 0.0708)	0.177876
6.4	0.800	0.50	0.21	0.7	(0.0265, 0.0708)	0.177876
6.5	0.800	0.50	0.21	0.7	(0.0265, 0.0708)	0.177876
6.6	0.800	0.50	0.21	0.7	(0.0265, 0.0708)	0.177876
6.7	0.800	0.50	0.21	0.7	(0.02655, 0.0708)	0.177876
6.8	0.800	0.50	0.21	0.7	(0.0265, 0.07079)	0.177876
6.9	0.53	0.75	0.21	0.85	(0.03739823, 0.0265)	0.5014
6.10	1	0.39	1	0.36	(0.122, 0.0796)	0.5212
6.11	0.27	0.67	0.31	0.79	(0.0504, 0.0398)	0.50530

Table 3.17. Intuitionistic fuzzy best worst pairwise comparison of improvements in health care resource allocation

Criteria	Membership(η)		Non Membership(φ)		Intuitionistic Weights	Defuzzified results
	Other to Best (c_{mB})	Worst to Other (c_{Wm})	Other to Best (c_{mB})	Worst to Other (c_{Wm})		
7 [illegible]	0.289	0.75	0.18	0.36	(0.01191 0.07599)	0.[illegible]8147
7 2	0.216	1	0.067	1	(0.011329 0.0198)	0.[illegible]9575
7 3	0.288	0.75	0.19	0.36	0.05[illegible]796 [illegible]7599	0.[illegible]89[illegible]
7 [illegible]	0.[illegible]33	0.[illegible]9	0.16	0.[illegible]07	[illegible]	0.[illegible]839
7 5	0.288	0.75	0.18	0.36	0.05[illegible]796 0.075998	0.[illegible]89
7 6	0.133	0.[illegible]9	0.16	0.107	(0.01384 0.076[illegible]	0.18[illegible]
7 7	0.288	0.75	0.18	0.36	0.0[illegible]5.796 0.075998	0.[illegible]89[illegible]
7 8	0.288	0.75	0.18	0.36	[illegible]	0.[illegible]83919
7 9	0.[illegible]33	0.[illegible]9	0.16	0.[illegible]07	0.01[illegible]38 0.0[illegible]759	0.[illegible]83919
7 10	0.288	0.[illegible]9	0.16	0.[illegible]07	0.05[illegible]796 0.07599	0.[illegible]89[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.[illegible]983
7 12	0.[illegible]33	0.[illegible]9	0.16	0.[illegible]07	0.01[illegible]38 0.02[illegible]	0.5096
7 13	1	0.216	1	0.67	(0.11329 0.09803)	0.5076
7 14	0.78	0.28	0.17	0.39	(0.02107 0.[illegible]5066	0.[illegible]867
7 15	0.509	0.[illegible]3	0.17	0.39	[illegible]	0.[illegible]907[illegible]
7 16	0.598	0.36	0.17	0.39	[illegible]	0.[illegible]967

Table 3.18: Consistency index of intuitionistic fuzzy best worst method

Criteria	Consistency index(CI)	
	Membership	Non-Membership
Infrastructure	0.079303	0.11539
Consultancy time	0.15938	0.2[illegible]
Pharmacies	0.25055	0.25055
Hospital stay	[illegible]	0.05882[illegible]
Impact of health care resource allocation	0.15938	0.2872[illegible]
Health care resource allocation	0.07965	[illegible]
Improvements in health resource allocation	0.0793	0.0793
Overall criteria comparisons	0.0588	0.0588

3.3 Results and discussion

[illegible] is drawn between the fuzzy best worst [illegible] best worst method. The best criterion [illegible] table 3.1 [illegible] whereas the worst criterion is consultancy time. [illegible] the preference of best and worst for fuzzy BWM and intuitionistic fuzzy BWM are same. For BWM the scale used to [illegible] the preferences are varies from 1 – 2 [illegible] and it is depending on decision makers while in intuitionistic fuzzy BWM the preferences are [illegible] by using the fuzzy weights. The table 3.1[illegible] shows the pairwise comparison of overall criteria [illegible] fuzzy weights are (0.0073, 0.03922), (0.0294, 0.01307), (0.00654, 0.00747), (0.0196, 0.0147), ([illegible] 0.029[illegible]), (0.0588, 0.0738), (0.01680, 0.0333[illegible]) respectively and also the defuzzified weights [illegible] 0.6862[illegible], 0.4995331465, 0.508169935, 0.5024509805, 0.4894[illegible]9798, 0.4925303455, 0.[illegible]918300655 respectively whereas the weights obtained for intuitionistic fuzzy best worst are (0.05882, 0.022098), (0.03361, 0.01198), (0.019607, 0.0[illegible]195), (0.023529, 0.06629), ([illegible]39, 0.026517), (0.126, 0.08279), (0.016218, 0.0283[illegible]) and the defuzzified weights are [illegible] 518, [illegible] 0.1877, 0.[illegible]7, 0.5064, 0.5216, 0.509 respectively. It is clearly seen by comparison [illegible] 3.1[illegible] the weights obtained by intuitionistic fuzzy BWM gives better results. Table 3.9 describes the consistency index of fuzzy BWM and table 3.18 describes the consistency index of intuitionistic fuzzy BWM. The fuzzy BWM consistency index (CI) of [illegible] pairwise comparison of overall criteria are 0.05882[illegible], 0.05882 and consistency index of intuitionistic fuzzy BWM are ([illegible], 0.1299682[illegible]). The consistency index of intuitionistic fuzzy BWM are higher than fuzzy BWM for both membership and non-membership. Moreover, for the suitability of pairwise comparison consistency level [illegible] need to check that CR [illegible] threshold. If this condition satisfies then pairwise comparison consistency level is acceptable otherwise not acceptable. Table 3.19 – 3.20 indicates the pairwise com-

[illegible] of CR and Threshold of fuzzy BWM and intuitionistic fuzzy BWM respectively. By pairwise comparing the overall criteria using fuzzy BWM, the consistency ratio (CR) for membership function is 0.166666667, and associated threshold is 0.3517 and it is clear that CR [illegible] Threshold so pairwise comparison consistency level is acceptable and in case of non-membership the CR is 0 and associated threshold is 0 so pairwise comparison consistency level is not acceptable. In case of intuitionistic fuzzy BWM for membership and non-membership both CR and Threshold are 0 so pairwise consistency is not acceptable. Table 3.19 indicates the pairwise comparison of CR and Threshold using fuzzy BWM for infrastructure the pairwise comparison consistency level is not acceptable for both membership and non-membership and for consultancy time the pairwise consistency level is acceptable and for paramedics and hospital stay the pairwise consistency level is not acceptable. By the pairwise comparison of impact of health care resource allocation the pairwise consistency level is acceptable for membership but it is not acceptable for non-membership and for health professionals satisfaction the pairwise comparison consistency level is acceptable but for the improvements in health care resource resource allocation the pairwise comparison consistency level is acceptable for membership but for non-membership it is not acceptable. Table 3.20 indicates the pairwise comparison of consistency ratio and threshold using intuitionistic fuzzy BWM for infrastructure the pairwise comparison consistency level is not acceptable for membership but it is acceptable for non-membership. By the pairwise comparison of consultancy time the consistency level is acceptable for both membership and non-membership. The pairwise comparison of consistency ratio and threshold for paramedics, hospital stay, impact of health care resource allocation, health professional's satisfaction the pairwise consistency level is acceptable for both membership and non-membership. For the improvements in health care resource allocation the pairwise consistency level is acceptable for membership and it is not acceptable for non-membership. The comparison between CR and Threshold using fuzzy BWM and intuitionistic fuzzy BWM is presented in the figures 3.1, 3.2 for each criteria respectively.

Table 3.19: Fuzzy best worst pairwise comparison of consistency ratio and threshold

Criteria	Membership η		Non-Membership φ	
	CR	Threshold	CR	Threshold
1	0.8333	0.3662	0.5167	0.3662
2	0.2083	0.3662	0.20833	0.3662
3	0.0972	0.3624	1	0.3254
4	0.5	0.3517	2	0.3254
5	0.208	0.3662	1	0.2716
6	0.2	0.2683	0.2	0.2683
7	0	0.2863	1	0.3662

Fuzzy BWM pairwise comparison of consistency ratio and Threshold

Figure 3.4: Fuzzy best worst pairwise comparison of consistency ratio and threshold

Table 3.20: Intuitionistic fuzzy best worst pairwise comparison of consistency ratio and threshold

Criteria	Membership η		Non Membership φ	
	CR	Threshold	CR	Threshold
1	0.35	0.2683	0.1	0.4667
2	0.65	0.296	0.4667	0.5662
3	1.25	0.28[illegible]	0.75	0.28[illegible]
4	0.5	0.35[illegible]7	0.5	[illegible]
5	0.5714	0.3262	0.5	0.3662
6	0.3514	0.3657	2.333	0.2983
7	0	0.2863	1	0.3662

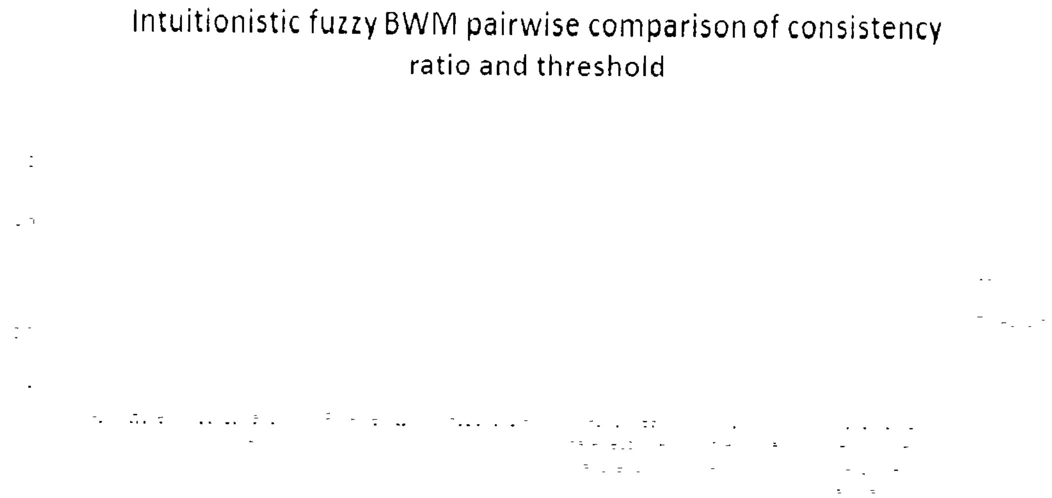


Figure 3.2 Intuitionistic fuzzy best worst pairwise comparison of consistency ratio and threshold

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