Optimal and Heuristic Approaches for Diversity Optimization in Massive-Scale Social Networks

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2025

DEDICATED TO

My Teachers,

Students

And

Friends

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Title of Thesis: Optimal and Heuristic Approaches for Diversity Optimization in Massive-Scale Social Networks

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ABSTRACT

Social media platforms have taken the lead as main modes of communication while our globe grows more linked. But these platforms are more than just straightforward tools; they are intricate systems that require innovative approaches to work well. In a novel approach, these platforms are compared to electronic circuits. The features of these circuits are reflected in the signal dynamics of the social networks, where the signal represents the flow of information within these networks. Polarization is one of the main issues that arises in such networks and the issue that this research is focused on. Polarization, like noise in circuits, prevents the free exchange of different ideas, creating echo chambers that may support preexisting viewpoints. By devising a method to first identify this polarization and then address it, this research aims to address this problem. Attention is drawn to echo chambers, these solitary concentrations of opinion that greatly contribute to polarization. After locating them, a mechanism is needed to gauge their size and significance. In order to achieve this, it is phrased as an optimization problem, with the objective of reducing the polarization while remaining within a set budget. In the initial phase of this research work, a brand-new metric is proposed to gauge polarization. It is ensured that all the elements that affect polarization are included in the suggested metric after applying Design Thinking with due consideration. This provides a thorough method for quantifying the issue.

Next, strategies are suggested to step in and lessen the polarization, opening up these forums to a wider range of viewpoints. Intelligent recommendation systems that offer the best connections between key nodes are the core components of this approach. These methods give decision-making a level of confidence by different approaches in which one comprises of Genetic Algorithm. These techniques bring about considerable decrease in polarization. This research tests and validates its effectiveness using real-world and synthetic datasets. The outcomes have been positive. These

proposed cutting-edge metric and intervention techniques are quite successful in promoting diversity in these which equates to reducing polarization. Significant progress can be made in eradicating social media networks' echo chambers with this study.

ACKNOWLEDGEMENTS

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This work could not be completed without the help of Adnan Anwar. He methamatical acumen and coding approach were pivotal in showcasing the results.

(Zaka Ul-Mustafa)

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTRODUCTION

The rapid advancement of information technology and the widespread use of smart devices have contributed to a striking increase in web traffic and the popularity of Social Networking Services (SNS) platforms in the age of digital innovation. Due to this upsurge, these platforms are now a veritable wealth of user data and insights. There are two types of the data being discussed: explicit and implicit. Think of "likes" and "ratings" as examples of explicit data. Implicit data, on the other hand, is a little more subtle and captures user interaction patterns and behavioral patterns [1]. The foundation of a recommender system is this data; which are tools that analyze user data to suggest items likely to interest them.

It's interesting that the majority of recommendation systems that rely on cognitive approaches significantly utilize implicit data to identify user preferences [2]. Data mining approaches, however, come into play when it comes to utilizing both explicit and implicit data to fully comprehend the delicate aspects of user preferences, actions, and choices. These are the research areas that are influencing the future of recommendation systems: looking into the user's previous interactions, examining how users communicate with one another, and comprehending their reactions to recommended content [3 - 9]. There have been some quite significant developments in this field over the past ten or so years. To address some of the shortcomings that have persisted, there has been a concerted push to create more sophisticated, intelligent recommendation systems [9 - 13]. All of these systems share a common goal, though, which is to continue to suggest things that previous users have expressed interest in. This overwhelming emphasis on showing similar type of contents is one of major reason to ignite the problem of polarization in Social Networks. However, polarization; lack of diversity, within these networks, promotes secluded filter bubbles and hinders the flow of information. This problem of blockage in information flow has been discussed in various research bodies and is an issue of high importance. The Cambridge Analytica [3] incident serves as a harsh reminder of how social media data may be used to distort public opinion and aggravate polarization. This topic will be tackled head-on, coming up with creative ideas on how to increase diversity in social networks and tear down polarization.

1.1 Problem of Polarization:

Social media has revolutionized global communication, bringing together people, groups, and companies, fostering digital interconnection and influencing political debate, social movements, and public opinion formation [4] . The structure of social media networks can represent that of a signals system in the domain of electronic engineering. In fact, multiple factors in a social media network, like the amount of connections in the graph, amount of polarization and the rate of information spread, can be thought of as the essential components of the signal. If we look at social networks as typical electronic networks then this problem can be addressed and understood better through the lens of signal processing.

The thought process behind this work is to understand and investigate factors that hinder the flow of the signal, information in this case, under consideration in a network because of formation of filter bubbles, due to the lack of diversity also known as polarization. This investigation will help find out the causes of polarization so that we become able to depolarize a network for better information flow. This approach not only theoretically proves that a network is being depolarized rather several real-world examples are also given. Apart from this it is ensured that the methods proposed are computationally efficient by applying greedy approaches that give optimal results. Due to this computational efficiency when the proposed approaches are run on a hardware, they will consume less processing power had the greedy approach not been taken, the same methods could have become heavy on the hardware.

Mapping the above concept of signal and noise to social networks, signal would be the information that has to travel in a network and noise would be polarization which would hinder the flow of information from one group to another. It has been a global problem since centuries; people get trapped in their silos due to similar information they are getting in their lives from different sources. Due to lack of diversity in information spread, many problems like echo chambers [5], demographic likings, tribal thinking and filter bubbles arises. In this research work, the objective is to come up with a wholistic framework of detecting the polarization in a network, measuring it and devising the methods to minimize it

1.2 Effects of Polarization:

It has been observed that, on these Social Networking Services (SNS), after a considerable amount of time, the content shown to users becomes similar to what they usually watch and like. This excessive similarity of content shown to users has led to polarization of users on their ideologies. While the recommendation systems work in the favor of most of the online platforms by recommending the kind of content people usually like however in such cases, recommender system further aggravates the problem by limiting the exposure on the other side of the issue.

Another side effect of social media platforms is that it enables everyone to share their views on a public platform which is good in a way as it helps people to express different viewpoints which may lead to more democratic and diverse society. However, this unsolicited, open and uninhibited access to content often results otherwise and thus, a filter bubble consisting of like-minded opinions, sucks them into it. Experiments have shown that masses accept confirmatory information, bearing false claims [6]. It restricts the open debate on an issue which tantamounts to nurturing extremism and baseless conspiracy theories which aggravates the hostility towards others. It has in a way become a source of spreading misinformation because it allows anyone and everyone to share their opinions. Therefore, because of disinformation, people remain away from the reality and truth. Consequently, a polarized society bears several risks for example, emergence of radicalism or civil war. According to World Economic Forum, this digital divide is a major threat to our policy making [7]. While we get to live in an age where there is no discrimination in information spread but it has its demerits also. Addressing polarization is crucial for maintaining civil dialogue and democratic principles, as unchecked polarization can worsen conflict, divide society, and hinder collective problem-solving [8].

1.3 Solution Space

The detrimental effects of polarization motivate to rally around devising a solution for reducing it. Conventionally, civil societies have devised mechanism to combat this problem by bringing acceptability through diversification. For example, cultural exchange programs and student's outreach programs in universities are designed to bring cultural and talent diversity respectively. Similarly, quota programs are introduced for inter-disciplinary diversity [9]. These steps are taken to tackle this issue on a human level but when we talk about online platform the solution exists in the sub-domains of recommendation system. These sub domains might either be collaborative Filtering or Serendipity. Serendipity promotes diversity and surprise by recommending unexpected items, whereas collaborative filtering predicts user interests based on similar behavior choices [10]. There are more than one definitions of serendipity [11]. A few researches say that it is necessary that contents are relevant and unexpected [12], while others say that contents should be novel as well as unexpected [13]. Yet most of the researches converge both definitions and suggest that all three components namely novelty, relevance and unexpectedness are necessary to qualify a content to be serendipitous [14, 15, 11]. Serendipity can deal with this problem but for that it is imperative to find out the factors that cause this phenomenon of formation of echo-chambers and polarization in a network. Many factors like infobesity, cognitive bias, homophily along with personalized algorithms of social media platforms contribute towards polarization and engulf the user into a filter bubble. Due to these factors, polarization in societies increases which leads to unhealthy controversies that take the societies at the verge of collapse. Polarization reduction affectors typically address these issues by increasing awareness, letting individuals see other sides of an issue with a target to mollify extreme opinions and finding common ground. No doubt, it is a laborious, costly and time taking process.

Proposed Solution:

Main goal in this research project is to develop comprehensive approaches that directly target and reduce the pervasive problem of polarization within networks. The ultimate objective is to apply these approaches to a network that is polarized and watch it change into a more diversified and balanced environment. This research explores and evaluates different approaches to deal with the problem of polarization in a network. It comes up with a complete framework to deal with this issue. This is done by firstly aiming to detect if there is any polarization in the network. If the answer is positive, then it needs to be measured. A novel measuring metric for polarization is devised, called 'Polarization Pointer (β)', which caters to important factors contributing towards polarization in a network. The proposed polarization metric is tested on different datasets and found that it is sensitive to even minor changes in the network. Subsequently, the focus was on increasing the diversity of information exposed to each individual. For this purpose a lot of different interventions were made and finally a few recommended methods of intervention were finalized to reduce the nuisance. Finalized interventions involve selecting a subset of important nodes in a network, the importance is determined by various methods discussed in chapter 4 and chapter 5, and then adding edges between them. The caveat is that while operating within a budget the interventions can't be made to all nodes, so those nodes are selected in each group which, if affected, will reduce polarization in the network. Figure 1.1 shows an example of a polarized and and figure 1.2 demonstrates a non-polarized network.

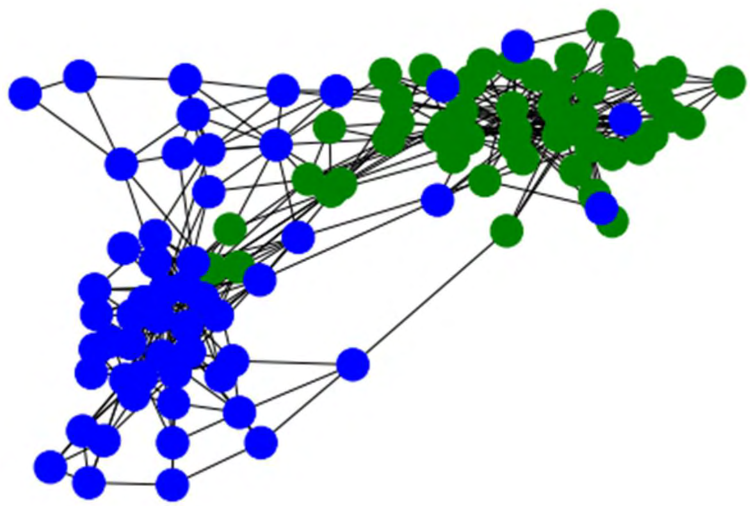


Figure 1.1: A polarized network

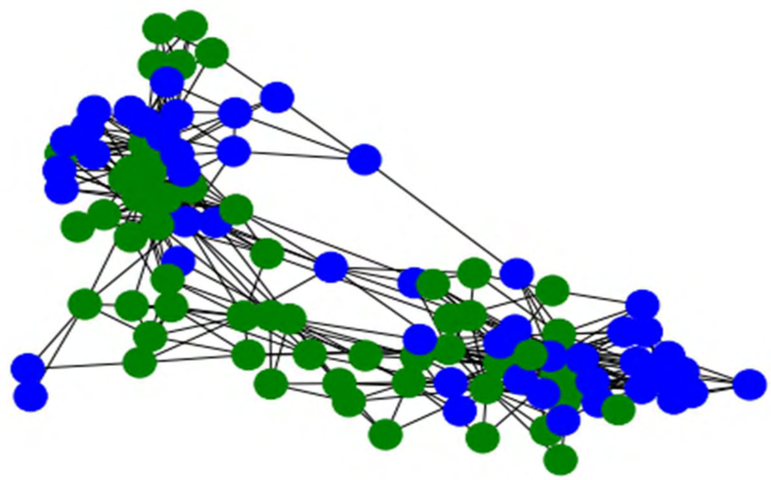


Figure 1.2: An unpolarized network

In figure 1.1, it can be seen that the blue and green nodes are highly polarized and divided into two separate communities whereas, in figure 1.2, the important nodes of the both communities are intervened. This will bring about a decrease in overall polarization. Results show that the recommended interventions reduce the polarization more than any other suggested method.

How This Work is Different From Others:

Substantial work has already been done in this domain as discussed in chapter 2. However, this work significantly differs from any previous work in the sense that it focuses on minimizing polarization after taking into account all factors that are influencing it. Apart from this, the content that an individual is exposed to is also focused and its argued that an individiual's opinion doesn't stay constant over time rather changes on basis of opinion of the connections it has.

Datasets:

In any research work, it is important to verify the research claims. Therefore, it should be supported by results. For this purpose, various types of datasets are used. Manu sources were used to procure and generate specific datasets which will be used for this research. The dataset collection method was explored on various levels and it was attempted to use synthetic data as well as real-world data set for validation of the claims made. This extensive testing on various kinds of data helped in evaluating results in different network situations and configurations to get rid of any potential biases. Limitation and scope of the datasets is stated where required. For example, the number of nodes in the selected dataset versus the practical size of the social media platforms. After working on detection and quantification of the polarization problem, following sources were finalized.

1) Karate:

The dataset represents“a social network of a karate club at a US university in the 1970s [16]. The

social network is partitioned into two distinct equal-sized communities. A graphical representation of the dataset is given in figure 1.3.

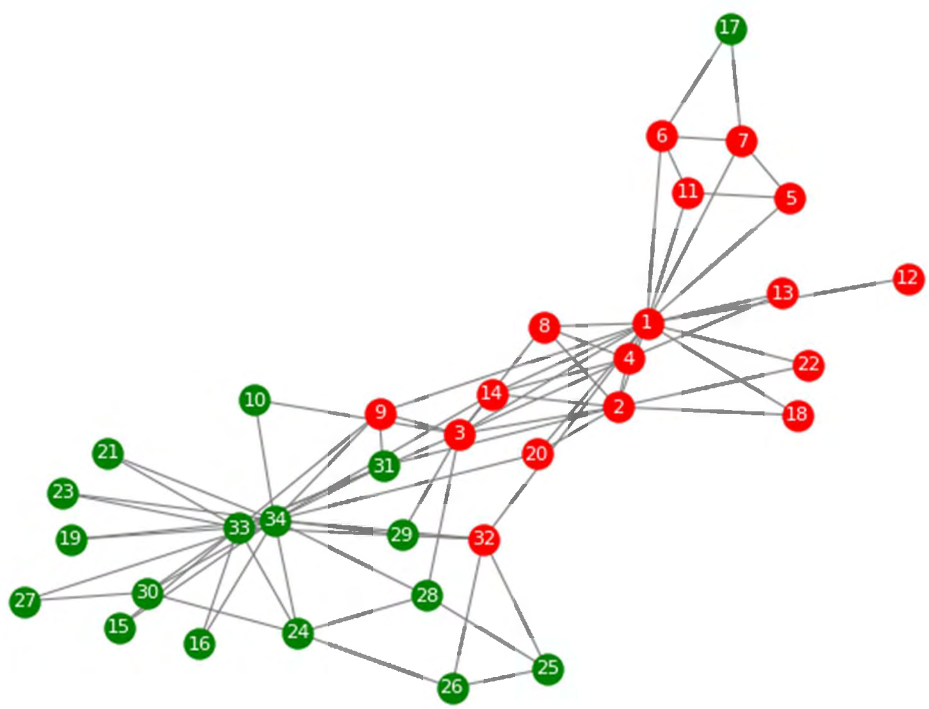


Figure 1.3: A graphical representation of karate dataset

2) Polbooks:

A network of books about US politics, sold by "amazon.com" [17]. Nodes represent the books and the edges represent frequently co-purchased books. Nodes are classified as Liberal (43), Conservative (49), and Neutral (13). Neutral books are randomly assigned to one of the two communities. A plot of the network is shown in figure 1.4.

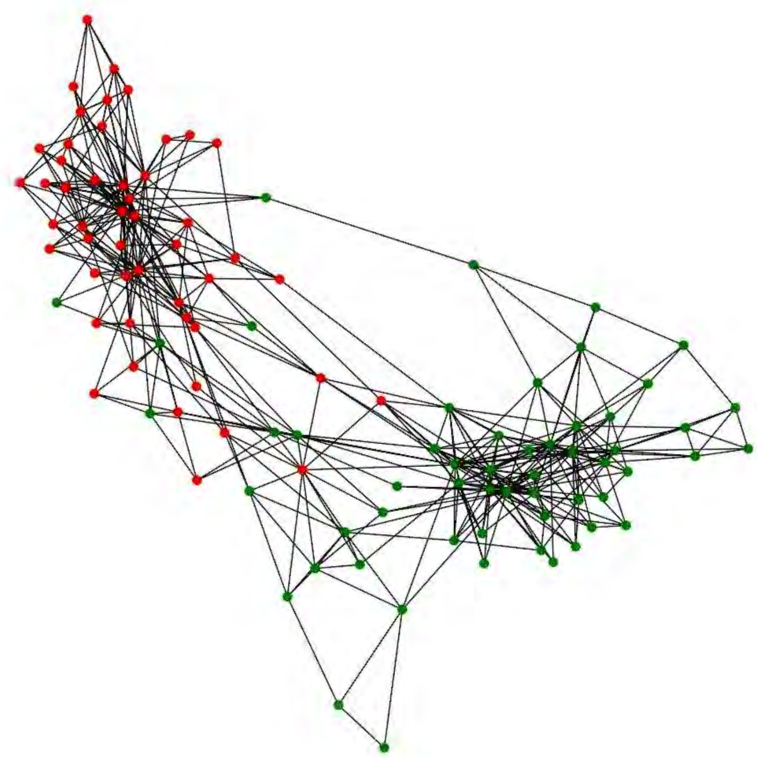


Figure 1.4: A graphical representation of books dataset

2) Polblogs:

A directed network of hyperlinks between weblogs on US politics [18] recorded in 2005. Blogs are classified as either Liberal or Conservative. Edge directions were disregarded and the largest connected component was kept. The resulting dataset contains two communities with 636 and 586 nodes each. A description of the datasets is given in table 1.1. All networks are treated as undirected and all edge weights are set to 1. A plot of the network is shown in figure 1.5.

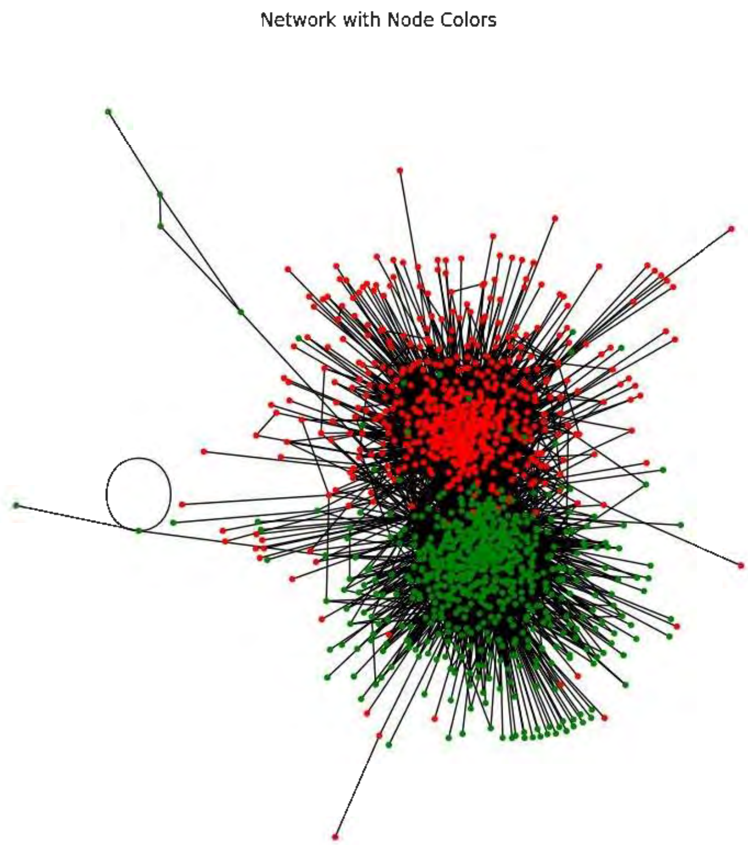


Figure 1.5: A graphical representation of blogs dataset

Table 1.1: Dataset Summary Statistics

Name	Nodes	Edges
Karate	34	78
Polbooks	105	441
Polblogs	1222	16717
Synthetic Data	6000	600000

Work Flow: This Chapter-1 was dedicated for introducing the problem and the issue under discussion. A detailed Literature Review is done in chapter-2 in which the work already done in this domain is discussed. In Chapter-3, an endeavor is made to embark on explaining the development of a model designed to predict variations in user opinions over time. Additionally, a novel 'Polarization Pointer' is proposed. This pointer is applied to various datasets, and the resulting outcomes are thoroughly discussed. Methods to reduce polarization are discussed in Chapter 4 and Chapter 5 in which a set of interventions are recommended. These recommendations are backed by results obtained by applying them on networks of various sizes. The whole work is concluded and recommendations for future work are made in Chapter 6. The Chapter wise road map is given in table 1.2.

Table 1.2: Chapter wise distribution of work

Chapter	**Name**	**Description**
Chapter 1	Introduction	- Role of social media and effects of polarization

		alongwith: - Problem of polarization - Effects of polarization - Solution space
Chapter 2	Literature Review	- Literature review of diversity, polarization, controversy measures and recommendation systems
Chapter 3	Polarization Pointer	- Opinion dynamics and proposed model. - Thought process through which this formula was finalized. - Graphical results which were obtained while reaching to this formula. - Verifying the accuracy and authenticity of the deviced pointer.

Chapter 4	Method and Results	- Come up with an optimal measure to reduce polarization and verify results
Chapter 5	Method and Results	- Use Genetic Algorithm to identify influential nodes in the network and add edges between them.
Chapter 6	Conclusion and Future work	- Concluding thoughts and direction for future work

CHAPTER 2

LITERATURE REVIEW

This research is focused on understanding and addressing the phenomenon of filter bubbles and the resulting polarization on social media platforms. As we keep increasing our dependency on these platforms as primary sources of news and communication, it has become critical to analyze the hazards of content recommendation and the resulting impacts of this algorithmic process on society at large.

This work represents a groundbreaking approach to address to this complex issue. Previous works in this domain were limited to picking on some aspects affecting the polarization and tweaking them whereas this research explores a variety of strategies and makes sure to quantify the polarization level of a network by assessing every potential and impacting factor. After that its explained that how just making a small number of tweakings can result in a considerably less polarized network which can break through the confines of echo chambers and create a more balanced and inclusive society. In this way this work relates to the problem of recommendation systems based on serendipity as this work aims to develop a recommendation system based on aligning polarized users with eachother.

2.1 Diversity Measures:

Substantial amount of work has been done to quantify diversity in social networks, different research works focus on different aspects of the problem to best describe how diverse a gievn network is.

The first work [19] considers social networks in form of a graph such that the graph comprises of (V, E, w) called as G and a vector of opinions denoted as s $\in [-1, 1]^n$ so for all users in V, the diversity index η(G, s) is then defined as:

$$\eta(G, s) = \sum_{i,j\epsilon E} w_{i,j\epsilon E}\left(s_i - s_j\right)^2 \tag{1}$$

They have assumed that the opinions are binary (i.e $s_i \in \{-1, 1\}$) and that all the edge weights and node costs are equal to 1, by employing their technique which will be discussed later, they have simplified their problem to a node selection problem. They are taking difference of opinions of two individuals and multiplying it by the edge weight. Equation (1) will become zero if s_i and s_j are same i.e., either both are -1 or both are +1, and will be non-zero if and only if both s_i and s_j are different. Essentially their diversity index is computing diversity by counting the number of edges between the two communities of people having opposite opinions. However, datasets with only binary opinions do not accurately reflect real world situations where opinions take a range of values from mild to extreme and can be neutral too.

Next another diversity measure proposed in another research work is considered [20] and this work studies the phenomenon of echo chambers on a Twitter dataset from 2017 (French Elections). They assume each user has a newsfeed which contains tweets from their leaders. The tweets have been labeled.

$P(n)s$ = Average proportion of posts supporting party *s* on newsfeed of *n*. This metric is calculated by computing the proportion of time user *n's* news feed contains tweets labeled as *s*. (Tweets are inserted one at a time in the newsfeed, and upon insertion, the previous tweet is deleted).

$p^{(n)}$ = Vector for individual n that describes the distribution of political leanings.

$Echo^{(n)}$= Returns the entry of $p^{(n)}$ corresponding to the user's political affiliation.

To quantify diversity (Between [0-1]):

$$\phi_n = \frac{S}{S-1}\sum_{s=1}^{S} p_s^{(n)}(1 - p_s^{(n)}) \tag{2}$$

Where S is the total number of different types (political parties)

$$Objective\ Function\ =\ argmax_{x,p}\phi$$

Where,

$$\phi \stackrel{\text{def}}{=} \sum_n \frac{\phi_n}{N}$$

denotes the average diversity of the newsfeed over the whole platform and x represents the recommendation policies that describe what type of content should be inserted into the newsfeed and when. It quantifies diversity using the metric n, calculated based on the proportion of posts from different parties in a use The constant in front of the sum ensures that ϕ_n ranges in [0, 1]. A value of 0 indicates that the newsfeed of n only contains posts referring to a single party, describing a perfect echo chamber meaning a polarized/non-diverse network. On the other hand, when ϕ_n = 1 all parties are equally represented on the newsfeed with the same average proportion of 1/S, meaning a maximally balanced/diverse information newsfeed.

2.2 Polarization Measures:

The next paper [21] sets up this problem of quantifying polarization level of a social network as a very interesting boundary problem. They consider a graph divided into two communities G_1 and

G_2, and each community has a boundary. Community boundary is defined for a community G_i, as the subset of nodes $B_{i,j}$ that satisfies two conditions:

1. A node $v \in Gi$ has at least one edge connecting to community G_j;

2. A node $v \in G_i$ has at least one edge connecting to a member of G_i which is not connected to G_j.

B is the set of all boundary nodes. $d_i(v)$is the internal degree of v, i.e the number of edges v has to internal nodes of its own community $d_b(v)$ is the number of edges between v and boundary nodes of other community.

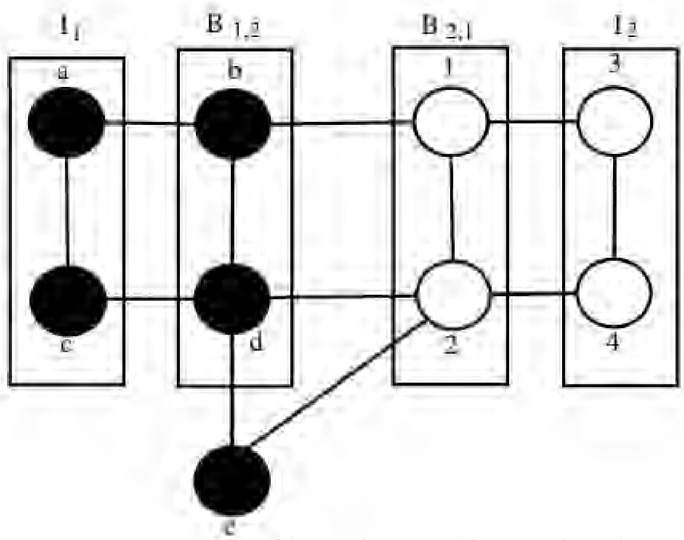


Figure 2.1: Representation of boundary and internal nodes

The polarization P is then defined as:

$$P = \frac{1}{|B|}\sum_{v \epsilon B}\left[\frac{d_i(v)}{d_b(v) + d_i(v)} - 0.5\right] \tag{3}$$

According to Equation (3), Polarization can be measured as the proportion of the edges the boundary nodes have with the members of the internal nodes of its community called E_{int}, if the boundary nodes have more edges with the members of its own community than the members of the other community then the network will be more polarized however if the number of edges between the boundary nodes and the members of its own community are lesser than the edges the boundary nodes have with the nodes of the other community then the network will be less polarized. Here $P \in (-0.5,\ 0.5)$. They argue that the concentration of high-degree nodes in the boundary can correspond to the absence of polarization which is not always true so this method can only be applied to some graphs because it doesn't take into account the structure of the network properly. For example, the case where nodes connected to internal nodes of one community are far greater in number than such nodes of other community cannot be considered through this measure.

Another work [22] taken introduces a term called center of gravity to find out how polarized a network is. They begin by partitioning the nodes into two sets i.e. elite and listeners. The elites have constant opinions of their own, with each opinion $X_s \in [-1, 1]$. The listeners, on the other hand, have an initial opinion of 0 and at each time step they become the average of their neighbors' opinions. Thus, the opinion at time step, t, of a given listener, i, with an in-degree of d_i is given by the following expression:

$$X_i(t) = \frac{\sum_j A_{ij} X_j(t-1)}{d_i}$$

They consider a fraction of people holding a positive (and respectively negative) opinion regarding an issue. They then let these opinions evolve over a course of time by repeatedly averaging the opinions of neighbors of a given node. The overall opinion of a node is the sum of its own opinion and the averaged opinion of its neighbors. They denote the fraction of people holding a positive opinion as A^+ and those holding a negative opinion as A^-. The normalized difference in population sizes, ΔA, is computed as:

$$\Delta A = |A^+ - A^-|$$

Next, the distance between the positive and negative opinions is quantified:

$$gc^+ = \frac{\int_0^1 p(s)sds}{\int_0^1 p(s)ds}$$

$$gc^- = \frac{\int_{-1}^0 p(s)sds}{\int_{-1}^0 p(s)ds}$$

The pole distance, *d*, is defined as the normalized distance between the two gravity centers,

$$d = \frac{|gc^+ - gc^-|}{|s_{max} - s_{min}|} = \frac{|gc^+ - gc^-|}{2}$$

Finally, the *polarization index,* μ, is defined as:

$$\mu = (1 - \Delta A)d \tag{4}$$

In equation (4) μ becomes equal to 1 if the network is highly polarized and becomes equal to 0 incase of a diverse network. The idea of evaluating a network over a course of time to see how its polarization value will change is a good one.

Another paper uses a Generalized Euclidean (GE) distance measure [23], and it estimates how much effort it would take to travel from one opinion to another in the network. Which means quantitatively measuring how different people's beliefs or opinions are on a particular subject. It addresses different components that add to the network polarization:

1. Opinion component: How the people's ideologies diverge.

2. Structural component: How the network is structured (Person X is friends with Person Y). Connections with like-minded individuals. If there is no community structure, each individual is

connected to every other individual and thus exposed to multiple views. If there are clear communities, individuals will only be exposed to ideas within that community.

3. Interplay between Opinion component and Structural Component:

a. The same opinions and the same communities can give rise to different levels of polarization

depending on the meso level (Medium Systems--Tells Size) organization of the system. Communities that can freely interlink regardless of their opinions indicate a lower level of polarization than if communities organize in progressively more extreme echo chambers.

G= (V,E) where V is set of Nodes and E is the set of connections between Node i and j. G must not contain any self-loops and must be connected. The opinion vector is of size $|V|$. One value per node/vertice. These values are bounded between -1 and +1 (Republican vs Democrat). 0 would mean Independent. The polarization measure $\delta_{G,o}$ is modeled as a node vector distance problem. L = estimate of the effective "resistance" between two nodes in a system. This is done using a pseudo-inverse Laplacian to estimate the effective resistance. The result of this operation is then inverted (Moore-Penrose pseudo-inverse) to get L which gives us a good notion of the distance between vectors a and b. To use GE for the purpose of estimating polarization, they split the vector o in two vectors: o+ and o−. o+ contains all positive opinions and zero otherwise; o− contains the absolute value of all negative opinions and zero otherwise. After that, their $\delta_{G,o}$ measure of polarization becomes:

$$\delta_{G,o} = \sqrt{(o^+ - o^-)^T L^\dagger (o^+ - o^-)} \qquad 5$$

one can interpret $\delta_{G,o}$ as the average "distance" between randomly sampled nodes in o+ and o−, weighted by how strongly these nodes hold their opinion (e.g., the distance between two nodes with opinions +1 and −1 is weighted higher than if the nodes had opinions −0.1 and +0.1)

Next another method [24] is discussed that uses a popular opinion formation model called Friedkin and Johnsen (1990). Within this model, opinions are modeled as real numbers between [-1,1]. Each user has an internal opinion Su that is given as an input (and it is fixed). This user also has an expressed opinion Zu. This expressed opinion depends on their internal opinion and opinions in the social network. Within a social network, if a user U takes a random walk, Zu will be the expected opinion the user will reach. High values of Zu means the individual is surrounded by like-minded individuals with extreme opinions and low value of Zu means the U's social media network has moderate and diverse opinions. $|Zu|$ is the degree of polarization of user U. This is measured by looking at the length of the vector z under the $L22$ norm.

How polarization is calculated (Polarization Index):

Given a social network graph G = (V,E) with n nodes and m edges. Each Edge (i,j) is associated with a weight Wij >= 0. (Weight ij). This weight expresses the strength of the connection and the influence they exert on each other.

Friedkin and Johnsen (1990) assumes that every person i has a persistent internal opinion Si and expression opinion Zi, which depends on their internal opinion and the opinions of their neighbors.

$$z_i = \frac{w_{ii}s_i + \sum_{j\epsilon N(i)} w_{ij}z_j}{w_{ii} + \sum_{j\epsilon N(i)} w_{ij}}$$

Where Wii is the weight an individual goes to their own internal opinion. Using this approach, convergence is achieved to Z, which serves as the opinion vector for the entire network. The polarization index of the social network is now defined as follows:

$$\pi(z) = \frac{||\bar{z}||^2}{n} \qquad 6$$

The probability is also affected by the weights wij and wii since they determine the probability that a specific edge is followed. For example, high wii weight means that the user is more likely to be absorbed in her own opinion node than follow a path in the graph to some other node.For a specific node v j , the value |z j | is minimized if node v j has equal probability to reach positive and negative opinions, that is, it has a balanced view of the opinions in the network. On the other hand, if the user is trapped in a filter-bubble of like-minded friends, all with extreme opinions, the value of |z j | will be high. The polarization index becomes high if there are echo chambers in the network, that is, presence of communities in the graph, that are homogeneous with respect to their internal opinions

Another paper [25] attempts to assess and measure how people recommenders contribute to the development of echo-chambers and polarization within social media networks. People recommender algorithms are the ones that show us “People You may Know” or “Who to follow” on different social media platforms. These recommenders consider the network structure (friends of friends) and content (similar interests) when making recommendations. As such, homophilic (similar to one’s self) links are more likely to be suggested. This paper evaluates three people recommender algorithms, Directed Jaccard index, Personalized PageRank and Opinion-biased algorithm. They also incorporate the opinion of nodes, by the use of two opinion dynamic models during their testing. As previously seen, opinion dynamic models show how opinions change in the graph when the users interact with each other.

1. Bounded Confidence Model (BCM):

a. In BCM, interactions modify the opinions of the nodes only when they are within a confidence interval ∈ [0;1] from each other.

b. Opinions in this model are equivalent and interactions happen by people close enough in their opinion. Example: Any political debate (Brexit).

2. Epistemological model.

a. Assumes opinions are not equivalent.

b. One of the opinions is the factual truth and other is its negation.

c. Example: Vaccines cause autism (False) vs Vaccines don't cause autism (Truth).

Metrics:

In order to measure the effect of the recommender systems in terms of echo chambers and polarization phenomena, they employ two global metrics defined over a graph where each node is labeled with an opinion.

1. Neighbors Correlation Index (NCI)

2. Random Walk Controversy Score (RWC)

Neighbors Correlation Index:

This is defined as the Pearson Correlation between the opinion vector O, and the average opinion of each nodes' neighbors.

Average is:

$$o_u^N = \frac{1}{N(u)} \sum_{v \epsilon N(u)} o_v \qquad 7$$

Then the Pearson Correlation is calculated between the nodes opinion and the average calculated above.

A value of -1 means perfect anti-correlation, each node has exactly the opposite opinion of its neighbors. Value of 1 means correlation, each node has exactly the same opinion of the nodes they follow (neighbors).

Random Walk Controversy:

G_X and G_Y

Consider two random walks, one ending in partition X and one ending in partition Y , RWC is the difference of the probabilities of two events:

(i) both random walks started from the partition they ended in

and

(ii) both random walks started in a partition other than the one they ended in.

$$RWC = P_{XX}P_{YY} - P_{XY}P_{YX} \qquad 8$$

Pij= Probability for a random walker that ends in partition j to have started in partition i.

According to this measure, a high value of RWC means low probability of crossing the partitions (with respect to staying in the same partition). Means two sides are very well separated, thus polarized. Similarly, a low value of RWC means high probability of crossing the partitions. Means two sides are not very well separated, thus non-polarized. In this way this measure depicts the behavior of a polarization measure. Values around 0 in this metric reflect an equal probability of crossing sides and staying in the same partition. However, this measure is not influenced by the size of the communities and the total degree of the nodes in the two partitions which means that this measure is ignoring a major factor affecting polarization.

Another study [26] looks at how people behave when they are part of echo chambers; also known as filter bubbles where only like-minded users are present and little to no difference of opinion exists. It also examines how these communities change over time, considering how active the users are and the emotions they express. To understand the changes in these communities, the study uses three growth models: the Gompertz model, the Logistic model, and the Log-logistic model. Both types of communities, whether focused on science or conspiracy content, show similar patterns of growth. At first, they grow rapidly, but then the growth becomes slower until it reaches a certain point where it stabilizes.

Community Evolution:

The authors try to see how a community grows over time using engagement as a proxy. Engagement in this case is defined as user commenting activity. For this, users are divided into:

- U_1 the set of all active users i.e., of all those users that commented at least once
- U_2 the set of all users that commented at least twice, and
- U_3 the set of all users that commented at least five times.

For each set of users, they look at temporal evolution as:

$$S_i(t) = \left\{u\epsilon U_i : \frac{s_u}{n_u} \geq 0.95\right\} \ and \ C_i(t) = \left\{u\epsilon U_i : \frac{c_u}{n_u} \geq 0.95\right\}$$

Where i is {1,2,5}, n_u is the total number of comments made by the user. s_u is the number of comments made by the user on scientific posts and c_u is the number of comments made by the user on conspiracy posts. T is time steps (Here it is days). They use a threshold of 0.95 for membership inside one community. This is basically a method to classify if a person is “Scientific” or “Conspiracy Theorist” using their interaction with labeled data on facebook.

User Sentiment Analysis:

They aim to model the emotional behavior of users as a function of their involvement in the community. For example, if a person engages more within a community, over time does their emotional behavior become more negative?

They use sentiment from the users comments, which they classify using a Machine Learning Sentiment Classifier. A comment is classified into +1 Positive, Neutral (0) and Negative (-1).

1. Mean User Sentiment:

Just a simple average of all the sentiments.

2. Mean negative/positive difference of comments:

$$\delta_{NP(i)} = \frac{1}{T_i}\sum_{j=1}^{T_i}(Neg_j(i) \ - \ Pos_j(i))$$

Where T_i is the number of days user i was active,

$Neg_j\ (i)$ is the number of negative comments in day j and

Pos_j (i) is the number of positive comments in day j.

3. User sentiment polarization

$$p_\sigma(i) = \frac{(N_i - 2k_i - h_i)(N_i - h_i)}{N_i^2} \quad 9$$

Where Ni, ki, hi are respectively the number of all, negative, and neutral comments left by user i, while li = Ni − ki − hi is the number of the positive ones. Note that ρσ(i) ∈ [−1, 1] and that it is equal to 0 if and only if li = ki or hi = Ni, it is equal to 1 if and only if ki = Ni, and it is equal to −1 if and only if li = Ni.

At a Community Level:

1. Community negative/positive difference of comments

$$\delta_{NP}^{C} = \frac{1}{MC}\left(\frac{1}{T}\sum_{j=1}^{T}(Neg_j^C - Pos_j^C\right)$$

where T is the number of days of observations, Neg_j^C the number of negative comments from users belonging to community C during day j, Pos_j^C the number of positive comments from users belonging to community C during day j, MC is the maximum daily activity of community C, and C ∈ {Science, Conspiracy},

2. Mean community sentiment polarization

$$p_\sigma^C = \frac{(N_C - 2k_C - h_C)(N_C - h_C)}{N_C^2} \quad 10$$

where N_C, k_C, h_C are respectively the number of all, negative, and neutral comments left by users of community C, while l_C= $N_C - k_C - h_C$ is the number of positive ones. Where p_σ^C ∈ [-1,1]

It has been observed that echo chambers on Facebook exhibit similar growth patterns in terms of community size, regardless of the difference in content (science or conspiracy). After an initial rapid growth phase, the communities undergo a more gradual growth until reaching a threshold size. They claim that users' emotional behavior within echo chambers is influenced by their level of involvement. Higher user engagement corresponds to a more negative approach, suggesting that as users become more active, they tend to express increasingly negative sentiments. More active users within echo chambers show a faster shift towards negativity compared to less active users. This implies that increased activity and participation intensify the negative sentiments expressed by users. User sentiment polarization, which measures the divergence of sentiments within echo chambers, is generally higher for science users compared to conspiracy users. However, highly active science users tend to decrease their sentiment polarization with increased activity, while conspiracy users tend to increase it.

When talking about opinion dynamins, different studies propose different methods of calculating opinion values and predicting how they will act like in future. Unlike [24], another paper [27] introduces the notion of "steps", which means that opinion change over time in distinct steps can be seen. It is also well known that FJ converges to an equilibrium set of opinions. To calculate Zi for Node Vi:

$$z_i^{(t)} = \frac{s_i + \sum_{j \neq i} w_{ij} z_j{}^{(t-1)}}{d_i + 1}$$

(Internal Opinion + weights sum of external opinions of all neighbors)/(Degree of Vertex)

It can be seen that:

$$z^* = \lim_{t \to \infty} z^{(t)}$$

Z* will contain opinions ranging continuously between -1 and 1.

(This basically means that given enough steps(or time) the opinion of an individual will converge). The authors then define Polarization in this paper as:

Note: Z is a vector of expressed opinions of all the users. For a vector of N opinions $z \in [-1,1]^n$, let mean(z)=$\frac{1}{n}\sum_{j=1}^{n}$ and zj be the mean opinion in Z.

Polarization of Z:

$$P_z \stackrel{\text{def}}{=} \sum_{i=1}^{n}\left(z_i - mean(z)\right)^2 \quad 11$$

Which is technically the Mean Square Error. Here Pz ranges between 0 (when all opinions in Z are equal) to N, when half of the opinions in Z equal 1 and half equal -1.

Disagreement:

(Local Disagreement, DG,z,I). For i∈1, . . . , n, a vector of opinions z∈ [−1,1]n, and social network graph G,

$$D_{G,z,i} \stackrel{\text{def}}{=} \sum_{j\epsilon 1,\ldots,n, j\neq 1} w_{ij}(z_i - z_j)^2$$

(Between two nodes)

An aggregate measure of disagreement is defined as:

(Global Disagreement, DG,z,). For a vector of opinions z∈ [−1,1]n, and social network graph G

$$D_{G,z} \stackrel{\text{def}}{=} \frac{1}{2}\sum_{i=1}^{n} D_{G,z,i}$$

The ½ factor is added so that each edge (i,j) is only counted once.

2.3 Controversy Measures

Some works pose the problem of computing the polarization level of a network as a controversy [28] score problem and instead of computing it as a polarization problem by introducing a concept of computing the controversy score-which is a means to quantify how well-separated the two communities are, using edge betweenness centrality. Edge betweenness centrality is a method of finding the significance of each edge in a given network. It measures the extent to which an edge lies on the shortest paths between different pairs of nodes. If an edge has high value of betweenness centrality then it means that it is a critical link ensuring the flow of information between different parts of the network. This work proposes its unique measure by considering a graph $G = (V, E)$, which is separated into two communities of people having opposite opinions on a topic. Their controversy measure is a way to compute how well-separated the two partitions are. They then define $\sigma_{s,t}$ *(e)* as the number of shortest paths between nodes *s* and *t* that include the edge *e*, and $\sigma_{s,t}$ as the total number of shortest paths between nodes *s* and *t.* They define betweenness centrality of an edge as:

$$bc(e) = \sum_{s \neq v \epsilon V} \frac{\sigma_{s,t(e)}}{\sigma_{s,t}}$$

After that they compute the KL divergence, d_{KL}, of the distribution of edge betweenness centrality of the crossing edges- the edges that are connecting the two partitions, and those that are internal edges- the edges that are part of only one partition. Finally, they define the Betweenness Centrality Controversy of the graph as:

$$BCC = 1 - e^{-d_{KL}} \qquad 12$$

This method can help to see how well separated two partitions or communities are in a graph and can help determine the importance of cut edges that are crucial in bridging structural holes. The value of BCC ranges between 0 – 1. BCC becomes equal to zero when the divergence is small i.e., the betweenness centrality of the edges that connect the two communities doesn't differ much with the edges that are present between the nodes inside the communities meaning which, in this problem setting, means that the graph would be diverse. Similarly, BCC becomes equal to one when the divergence is large i.e., the betweenness centrality of the edges that connect the two communities differs greatly with the edges that are present between the nodes inside the communities meaning which, in this problem setting, means that the graph would be polarized. So the measure written as equation (3) is reliable in order to find out the controversy score, as it gives similar results to a polarization measure. However, the model ignores some important details as it does not take into account the strength of opinions across either side and assume that opinions take one of two possible values instead of a continuum/spectrum.

2.4 Reccomendation Systems:

This problem is related to recommendation systems that work by presenting information based on someone's prior choices or those of other users who share them, recommender systems have completely changed how we interact with content online. According to Mi Zhang and Neil Hurley [29] these recommendations may unintentionally create "filter bubbles" that restrict our access to a variety of content. Because of this, we frequently find ourselves in "echo chambers" where we only hear views that agree with our own, strengthening our preexisting prejudices and impeding our interaction with other viewpoints. Recognizing this situation, efforts have recently been undertaken to diversify recommendation algorithms in order to encourage the investigation of new ideas and themes [30]. The goal is to produce a more interesting and well-rounded online experience. Another body of research [31] says that user satisfaction is greatly increased by topic-diversified recommendation lists. However, as argued in another research [32] finding a balance between customization and diversity continues to be a major difficulty. Although customization adapts content to users' tastes, it can also reinforce preexisting opinions and limit exposure to new viewpoints. The need to maintain this delicate equilibrium has given rise to numerous creative solutions. As an illustration, the idea of "serendipity-based" recommendations, as put forth by Murakami et al. [33] proposes recommendations that diverge from previous user interactions in an effort to introduce novelty and broaden users' horizons. This strategy works particularly well when customers gravitate toward well-known or well-liked products, limiting their exposure to other possibilities.

2.5 Genetic Algorithms:

One of the proposed solutions involves selection of influential nodes in a network and Genetic Algorithm is used to do that. Genetic algorithms are optimization techniques inspired by the process of natural selection. They have been widely used in various domains, including social networks, to solve complex optimization problems. GAs work by evolving a population of candidate solutions over generations through selection, crossover, mutation, and reproduction [34]. Influence maximization is the problem of selecting a set of influential nodes in a social network to maximize the spread of information or minimize polarization. GAs have been applied to this problem in various ways. Doina Bucur and Giovanni Iacca tackled the NP-hard problem of influence maximization in social networks using a Genetic Algorithm. They demonstrated that simple genetic operators could find high-influence solutions comparable to known heuristics without requiring assumptions about the underlying network graph. Their approach also obtained more diverse sets of feasible solutions [35].

A study by Doina Bucur et al. evaluated surrogate-assisted Multi-Objective Evolutionary Algorithms for influence maximization. They used an approximate model of influence propagation instead of Monte Carlo simulations to find the minimum-sized set of most influential nodes. The study emphasized the importance of carefully considering approximate models, as errors induced by these models can significantly impact algorithmic performance [36]. Shiyu Chen et al. proposed a targeted influence maximization solution based on cloud computing. They introduced a tag-aware IC model that considers users' interests, characteristics of the propagated item, and similarity between users and related information. The proposed algorithms achieved speedup and savings in storage compared to state-of-the-art methods [37]. Xiao-tong Qin et al. proposed a topic-aware community independent cascade (IC) model to reduce the complexity of dynamic influence maximization. The model integrates community structural features, community topic features, and time information into an IC model. The proposed algorithm demonstrated better stability, dynamic adaptability, higher computational efficiency, and less space consumption [38].

Canh V. Pham et al. introduced the Influence Maximization with Priority (IMP) problem, focusing on influencing potential users with priority during influence diffusion campaigns. They proposed efficient algorithms, called Integrated Greedy (IG) and Integrated Greedy Sampling (IGS), with provable theoretical guarantees. The proposed algorithm provided better solutions in terms of influence on priority sets while maintaining considerable results in running time, memory usage, and influence spread [39]. While GAs have shown promise in influence maximization, challenges remain. GAs may converge to suboptimal solutions if not properly configured. Handling large-scale social networks may require parallel and distributed GAs. Selecting appropriate population size, mutation rate, and crossover method is essential for efficiency.

This research, in later part, focuses on identifying influential nodes in social networks. This is a critical task for various applications, including viral marketing, political campaigns, and public opinion formation. The concept of influential nodes is closely related to the idea of "hubs" or highly connected nodes in a network that can maximize the spread of influence. Lu Wang et al. proposed a method based on discrete moth-flame optimization to identify influential spreaders in social networks. Their approach considers both the total valuation and variance in valuation of neighbor nodes. The method was found to be effective and robust in tackling the influence maximization problem in five real-world social networks [40]. D. Sivaganesan introduced an algorithm that considers the social behavior of users for influence maximization. The algorithm uses semantic metrics like the interests of the users and their social actions to identify influential nodes. The approach was found to offer improved efficiency in calculation speed on real-world networks [41]. A. Talukder et al. focused on reverse influence maximization to find the seeding cost of target marketing. They proposed a Knapsack-based solution that efficiently resolves the challenges of reverse influence maximization and yields optimized seeding costs [42]. Zahra Aghaee and S. Kianian presented the GIN (Group of Influential Nodes) algorithm that reduces the search space for finding the most influential nodes. The algorithm selects specific nodes from each group and follows the greedy method to select the seed nodes with the highest expected diffusion value [43]. Identifying influential nodes is not straightforward due to various reasons like scalability because as social networks grow, the computational cost for identifying influential nodes increases. Also the influence of a node can change over time, requiring dynamic algorithms.

Genetic Algorithms are used to select influential nodes and they have previously been applied to solve this problem efficiently. Doina Bucur and Giovanni Iacca demonstrated that GAs could find high-influence solutions in feasible runtime, often better than known heuristics [35]. Centrality measures like Degree, Betweenness, and Closeness have been used in conjunction with GAs. S. Pal et al. proposed new centrality measures, Diffusion Degree and Maximum Influence Degree, to find the top influential individuals [44].

Xiaodong Liu et al. proposed an incremental approach, IncInf, for locating top influential nodes in evolving social networks. The method uses GAs to update the influence spread changes efficiently [45]. Xiaodong Chen et al. explored the Influential Node Tracking (INT) problem in dynamic social networks. They proposed the Upper Bound Interchange Greedy (UBI) algorithm, which uses GAs to track influential nodes as the network evolves [46]. GAs have been used to identify influential nodes for viral marketing campaigns. The algorithms aim to maximize the reach of the campaign by selecting nodes that have the highest potential for influence. GAs have also been applied to reduce polarization in social networks by identifying and moderating the opinions of influential nodes. This is particularly relevant in the current socio-political climate. As social networks grow in size, the computational cost of identifying influential nodes increases, posing a challenge for GAs. Genetic Algorithms have shown promise in solving the complex problem of identifying influential nodes in social networks.

2.6 Methods of Polarization Reduction:

After discussing various methods of computing polarization and evaluating how polarized a given network is the next step becomes to find out optimal methods that enable us to reduce this polarization. This has implications in real life as well as it has been shown that social polarization hampers the economic growth of a society [47] so reducing it is imperative to ensure a stable and smoothly running community. For this purpose, a detailed review is done of methods that can be used to tackle this polarization problem and help us reduce it.

In [19] they assume the opinions to be binary such that the opinion vector $s_i \in \{-1, 1\}$ and being given P_{ii} i.e the weighted degree of node i, b_i i.e the cost of changing node i's opinion, and a budget k, they tend to select the nodes with highest values of P_{ii}/b_i, and flip their opinions. Setting all edge weights and node costs to 1, the problem simplifies to selecting k nodes with the highest degrees. They report the results of flipping the opinions of top 0.1n, 0.2n and n nodes respectively in their study. This method seemingly helps to break filter bubbles and increases the diversity of information exposure among connected individuals in social networks. However, just flipping the opinions of such a large number of people in real life is not very practical and realistic.

They [20] study the phenomenon of echo chambers on a Twitter dataset from 2017 and maximize the diversity of content exposed to users using a quadratic program that finds the best recommendations to show to a user. The paper focuses on optimizing personalized content recommendation policies to maximize the average diversity of newsfeeds across the platform.

Next, another method [24] is discussed that talks about reducing the overall polarizability of a given network by convincing people to adopt a more moderate opinion. Given a budget value K, this research focuses on identifying the best set of individuals, where moderating their opinion will reduce the polarization of the whole network the most. They further define two variants of the problem:

1. Moderate Internal:

a. Attempt to moderate the internal opinion Su of individuals and bring it to Su = 0. (Through educational interventions).

2. Moderate External:

a. Attempt to moderate the external opinion Zu of individuals and bring to 0. (This can be done through incentives)

Having discussed various methods to quantify and reduce polarization alongwith their limitations we'll now focus on devising a unique system to measure and reduce polarization. We'll do so while considering all the confines of the previous works in the subsequent chapters.

CHAPTER 3

Polarization Pointer

After discussing the research work done in the domain of detecting, measuring and reducing polarization, this section presents a novel method to know how a user's opinions change over time and proposes a polarization metric. The idea is that a person's opinions about different issues are subject to change. As individuals, we are bound to interact with people around us and this interaction, to some extent, affects our opinion. This human trait is addressed in this section and a mathematical model of opinion propagation is discussed in section 3.1 below.

The metric which is developed to measure polarization in a network is named as "Polarization Pointer". In this section, besides stating the polarization pointer, the thought process of developing it is also explained in section 3.2. As an interesting fact, this Pointer is sensitive to all major factors which contribute in creating the polarization in a social network. Due to this sensitivity, high accuracy is attained in the result. "Design Thinking Technique" [48] is applied in finalizing the parameters which are required to be included in construction of this 'Pointer'. The authenticity and accuracy of this novel metric is attained by testing it on various datasets; both real world as well as synthetic ones. This pointer caters for many factors and is tested for each one of them. Then the results are compared with those of some other metrics mentioned in literature review section. It is also pertinent to mention that while describing the 'polarization pointer' and the factors constituting it. In this report, inter-group interactions are considered instead of intra-group unless it is specified otherwise.

3.1 Opinion Dynamics

Opinion dynamics primarily relates to the evolution and the change in opinion that a particular user goes through after getting affected by his or her neighbors' opinion and after spending some time and dewelling over his or her own opinion. Different factors affect individual opinions such as social influence, media advertising and cognitive biases. To make this model more realistic, different models of opinion dynamics are analysed to propose a unique model that affects an individual's opinions evolution over time. For example, Friedkin-Johnsen Model states that each node i maintains a fixed internal opinion s_i and a publicly expressed opinion z_i which is given by:

$$z_i = \frac{w_{ii} s_i + \sum_{j \in N(i)} w_{ij} z_j}{w_{ii} + \sum_{j \in N(i)} w_{ij}}$$

Proposed Model:

Given that each node i has a fixed innate opinion s_i and an expressed opinion at time t, z_i(t)

$$z_i(t+1) = \alpha . s_i + (1 - \alpha) . \frac{\sum_{j \in V} w_{ij} \, z_j(t)}{\sum_{j \in V} w_{ij}}$$

where $\boldsymbol{\alpha}$ is a parameter and represents the fractional weight that each node gives to its own opinion. Ideally, this can be a function of the strength of the opinion that can vary from person to person. However, this work assumes it to be constant at 0.85. The idea behind taking it 0.85 is that humans, generally give more importance to their own opinion than those of the people they are related to. But the influence of the relatives and a person's social circle still cannot be ignored so that is why it is also assigned some amount of weightage

3.2 Derivation of the proposed Polarization Pointer $\boldsymbol{\beta}$:

In this research a novel measure to quantify polarization is proposed. The methods and a step by step approach to reach to the final proposed measure in equation 4 are listed below.

Exploring Factors Effecting Polarization:

Polarization represents the ideological chasms that form between distinct groups so recognizing the intricate relationship between polarization and various factors within a network is key to understanding and addressing this issue. This section explores the relationships between polarization and several factors that are identified as instrumental in the network structure. By understanding these relationships, effective mathematical models can be devised for measuring and mitigating polarization. As stated in Chapter 1 polarization implies extreme division between different ideological groups. This division is also referred as separation between two or more such groups.

$$\text{Polarization} \;\; \alpha \;\; \text{Group Separation}$$

Similarly, if members of a group or community are well connected with each other then information flow will be high as well as smooth between them. Thus people will have more opportunity to align with each other's opinion. This alignment and jellying together is referred as 'cohesiveness' between them. For example, it is observed that all off-springs of same family or members of the same community follow the same traditions and carry the same opinions which are different from off-springs in any other family or community; it is because of cohesiveness between the members of that family or community. It means that if communities of a network are more cohesive in themselves then polarization will be more in between such communities because such communities will be so densely connected amongst themselves that the flow of information will only occur within those communities whereas information won't flow easily from one community to the other. Similarly, non-cohesiveness means that the two communities of a network are highly connected with eachother as compared to their connections within the communities. Figure 3.1 pictorially represents what is called as a cohesive and non-cohesive network in this research.

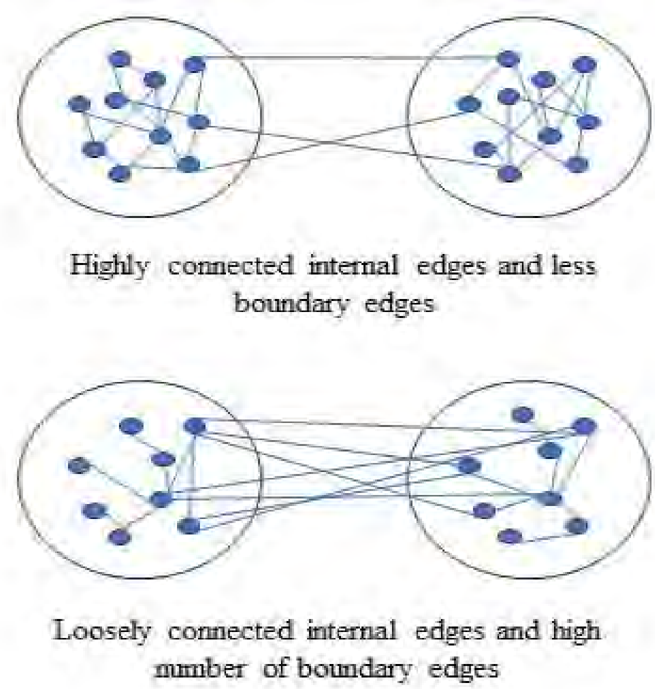


Figure 3.1: Representation of a cohesive and non-cohesive groups in a netwrok

This discussion has taken us to a point where cohesiveness between two groups is quite clear. If same cohesiveness happens to occur between two groups (intergroup) then polarization reduces. As this research focuses on measuring the polarization between two distinct groups, therefore:

$$\text{Polarization} \;\; \alpha \;\; \text{Group Cohesiveness}$$

On the other side, if People between two groups are well connected then they will exchange views with each other and individuals are in a better state to understand each other, hence reducing the polarization. Therefore, increasing connections between two groups results in decreasing the polarization.

$$\text{Polarization } \alpha \ \frac{1}{\text{Group Interaction}}$$

Likewise, if people have an opportunity to get exposed to different contents, different from their own view point, then they would be in a better position to understand other schools of thought. In that case they can exhibit empathy towards other angle of thinking. This exposure to variety of contents will foster diversity in the community; which is inversely proportional to the polarization.

$$\text{Polarization } \alpha \ \frac{1}{Group\ Diversity}$$

It implies that if a network is more diversified than its polarization will be lesser. Diversity can be increased by increasing the exposure to different contents what a certain group is already engaged with, which inturn can be done by increasing interaction between two, or more, groups bearing different opinions.

Above discussion can be summarized as in the form of a rudimentary mathematical relationship:

$$Polarization \equiv \frac{\text{GroupSeparation} \times \text{GroupCohesion}}{\text{GroupInteraction} \times \text{GroupDiversity}} \qquad A$$

If,

S_G = Group separation

D_G = Group diversity

I_G = Group interaction

C_G = Group cohesion

Then above relationship takes the form:

$$Polarization \equiv \frac{S_G \; X \; C_G}{I_G X \; D_G} \qquad \text{B}$$

The next step requires to identify what are the ways to quantify these four factors. Table 3.1 represents the factors to which polarization is sensitive and then some possible ways related to this research are gauged in this table.

Group Separation, S_G , is actually a form of “distance” between opinions of two groups. This distance can be considered as the degree or level of disagreement between the two groups. It implies that if two groups have same opinion, there is no disagreement and thus the ‘distance’ is zero. However, on the contrary, if they have different opinions, this ‘distance’ will keep on increasing with the intensity and level of strife between them. As an extreme case, if the opinions differ by 180 degree then they would be standing on exactly opposite poles. Generally, these extreme opposite poles are mapped by the values -1 and 1. While it is known what group separation is, the ways to compute it needs to be explored.

Group Interaction, I_G , determines the amount of synergy present between the individuals of a group. It can be thought of as the exchange of information, comments, contents as well as their smooth flow between the individuals. If there is substantial exchange of information in a network, it implies that there is more interaction between its nodes (individuals in this case). High level of interaction also ensures more quick, fast and smooth ways of this flow of contents. As an extreme case, this interaction would be at its pinnacle in a mesh topological networks; a network in which every individual is connected with every other in the network.

Group Cohesion, C_G, refers to the influence of one person over the other one. It also includes the factor that how a person can seek alignment with the other one to whom he or she is connected. Moreover, if most of the people are directly connected and there exist more number of shortest path in a network then it means that such a group is more cohesive. In a social network if individuals have considerable amount of confidence on their connections then there will be more cohesion between them. An extreme case will be a cult where all people have blind trust on each other in this case cohesion will be maximum.

Group Diversity, D_G, is the extent of the exposure to a variety of the information. If people in a group are exposed to same type of information then it refers to have Zero diversity. However, high availability of a wide variety of contents exposed to different factions and camps is refered to have more diversity. An extreme case would be when only one type of content exists in a network, in this case group diversity will approximately zero.

Following table, encompasses all major factors which could effect polarization either positively or negatively along with their explanations coupled with various possible methods to measure them.

Table 3.1: Polarizing factors and their methods to measure

Factors	**Explanation**	**Methods to measure**
GroupSeparation	Represents the degree of separation (Distance between groups)	- Community Detection Algorithms - Distance Metrics, for example, distance between the means of two groups. - Using Latent Dirichlet Allocation (LDA) technique to extract dominant topics within each group and compare the differences between them. - Analyze the features which contribute towards the trained classifiers.

GroupInteraction	Represents the level of synergy (the level of communication, exchange of information, or influence between different groups in the network.)	- Analyze the connectivity patterns between nodes belonging to the two groups to assess their level of engagement. - Higher intra-group interactions and lower inter-group interactions indicate a potential polarization. - Running Sentimental analysis to measure whether the interaction like exchange of comments and articles, are predominantly positive or negative. - Network Density (The amount of links in the network as opposed to the total number of possible links)
GroupCohesion	Refers to the degree of unity, agreement, or similarity among individuals within a group,	- Density of intra-group interactions - Questionnaires or Surveys

	indicating the strength of their bonds and shared identity. (As football team, group cohesion refers to how well the players come together, trust each other, and work towards a common goal of winning matches)	- Average path length; short paths implies stronger cohesion. - High Topic consistency of same nature indicates high cohesion.
GroupDiversity	Refers to the variety of characteristics, perspectives, backgrounds, and attributes among individuals within a group	- Measuring type of the contents shared within a group. - Measuring demographic characteristics. - Higher homophily may indicate less diversity. - Jaccard index
Emotional factor	Strong emotional reactions and biases can lead individuals to hold more extreme positions and resist compromising or engaging with alternative perspectives.	- Sentiment Analysis - Emotion Recognition

Frequency of exposure	Increased frequency of exposure to a particular perspective can reinforce existing beliefs and potentially contribute to polarization.	- Content Analysis - Social Media Monitoring
Receptiveness	Receptiveness refers to the openness or willingness of individuals to consider and incorporate new information or perspectives.	- Surveys and Questionnaires - Sentiment Analysis
Correlation	Pearson	- By calculating correlation Coefficients like Pearson Spearman rank correlation. - Topic Co-Occurrence
Echo chamber	A social environment where individuals are exposed to information and ideas that reinforce their existing beliefs, creating a self-reinforcing bubble that	- Echo chamber Metrics - Content Analysis - Network homophily

	excludes dissenting opinions.	

Design Thinking Technique (DTT) is applied and thus all important factors are enumerated in the above table. After listing all possible factors DTT asks to list all factors in descending priority form and then to select only a top few which have major impact on the issue under discussion to achieve maximum ROI (Return on Investment). Hence, a detailed analysis is done to decide upon the best combination of influencing factors and their respective methods after doing extensive testing.

After a lot of deliberation, those methods were picked that could not only be best applied to graph datasets and but also could most accurately map on real world social media platform scenarios. In the context of relationship A, this study will now proceed with aligning the chosen methods with the best suited corresponding polarizing factors.. Lets assume,

CG = center of gravity

N_t = Total nodes in the network

E_t = Total edges in the network

O = individual (Nodes) opinion

W = weight (It may be frequency of received content, influence, trust, timestamp or anything)

So the distance between means (average) of two or more groups is **selected** as method to measure 'Group Separation' between such groups. The *mean of a histogram* has been defined as "Center of gravity" for that particular.

Polarization α $|CG_1 - CG_1|$

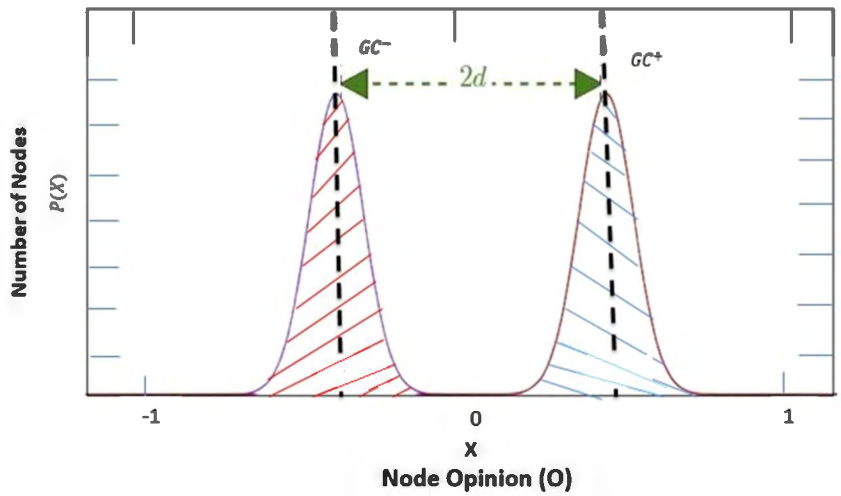


Figure 3.2: Figure showing distance between center of gravities of two distributions

Similarly, Number of Links or edges between two individuals belonging from different groups is **selected** to measure 'Group Interaction'.

$$Polarization\ \alpha\ O_i\ .\ O_j$$

Likewise, Multiplication of opinions of two individuals who are connected with each other is **selected** as metric to measure group diversity.

$$Polarization\ \alpha\ \frac{(N_t\ .\ (N_t\ -\ 1))}{2\ .\ E_t}$$

Whereas, Group Cohesion can be included by further multiplying this diversity with the effect of an individual over the other. 'Weighted factor' is **selected** for incorporating 'Group Cohesion'.

$$Polarization\ \alpha\ W_{i,j}\ .\ O_i\ .\ O_j$$

By incorporating the above-mentioned factors, equation 1 provides a comprehensive way of computing polarization, offering insights into the structure and dynamics of polarized networks which is sensitive to all major factors that are affecting the polarization. Therefore, after replacing the factors mentioned with in above relationship B with the selected methods. Therefor, the relationship B takes the form of the following formula:

$$Polarization = \frac{|CG_i - CG_j|.(N_t . (N_t - 1))}{2 . E_t} \sum (\omega_i . o_i . o_j) \qquad 1$$

In the above equation polarization is quantified by comparing the center of gravity (CG) values between two groups of different or opposite opinions and accounting for their interconnecting edges. The product of weighted sum of opinions (wi * oi * oj) associated with each edge and the difference in center of gravity (|CGi - CGj|) for the two groups yields the polarization value. Subsequently, this value is normalized by the total number of edges in the network (2 * Et).

The above-mentioned equation 1 justifies the relationships of different factors affecting polarization. Though, it correctly shows us how polarized two communities are by comparing their network structures however, during exhaustive testing it was revealed that this formula becomes unable to give us the maximum value which the polarization measure can attain in a given network. The maximum value of polarization is needed to know the extent to which a network can be polarized to so that selected strategies can be applied to check how effective the proposed methods of depolarizing a network are.

To add this feature, several modifications were implemented. Firstly, the distance was normalized, followed by the application of the logarithmic function, which effectively curtails the spread of the polarization range. These adjustments allow for a more controlled and meaningful interpretation of the polarization strength within the network allowing us to stay within a specified range.

$$Polarization = \log\left(\frac{d\ .\ S}{\rho}\right) \qquad 2$$

Where,

S= $\sum(w_i\ .\ o_i\ .\ o_j)$

$\rho = \frac{2.e}{n(n-1)}$

d = $\frac{|CG_+ - CG_-|}{2}$

The above equation 2 for polarization employs a logarithmic transformation on the expression $\frac{d\ .\ S}{\rho}$ to enhance the interpretability and normalization of the polarization value. The added *log* is valuable for addressing scaling issues and facilitating comparisons across diverse networks where polarization values may significantly vary. The logarithmic function helps us with analysing data effectively, but it also poses a challenge of compressing the range of values, which can be advantageous by aiding normalization, but it may also result in information loss. Smaller differences are amplified, while larger ones are suppressed, potentially reducing the distinguishability of subtle polarization variations.

Equation 3 involves dividing by the density measure ρ, derived from the total number of nodes and links connecting them in the same network. In cases of sparse networks with low edge density, ρ can approach zero, leading to potential division by very small numbers. This situation causes numerical instability and sensitivity to changes in edge density.

To address to these observations, a modified approach was adopted in the equation 3. In the modified version, each term is individually normalized. Additionally, utilizing the inverse of ρ by taking (1−ρ) the possibility of ρ reaching its maximum value of 1 is accomodated. As such, the formula 2 becomes:

$$\text{Polarization} = |C_{Gi} - C_{Gj}|. \sum_{i \in n} W_{i,j} . O_i . O_j . (1 - \rho) \qquad 3$$

Where:

$|C_{Gi}$ - $C_{Gj}|$ = Distance between center of gravities of two histogram

O_i, O_j = Opinion of node i, opinion of node j

$W_{i,j}$ = Weight/effect of node i and j on each other

ρ = Density in the network => 1− ρ will become lack of density in the network/sparsity Let the first term $|C_{Gi} - C_{Gj}|$ be denoted by 'd'. And the second term $\sum_{i \in n} W_{i,j} * O_i * O_j$ which is the average connection strength in the network denoted by 's'. Then equation 3 then takes following form:

$$Polarization = \frac{1}{2} (d \times [s + (1 - \rho)]) \qquad 4$$

Where:

s = average connection strength of all the edges of the network

d = distance between center of gravities of opinion groups in the network

Testing:

In the course of developing this proposed polarization metric, it was continuously subjected to rigorous testing and evaluation using diverse datasets. Comparisons were made with multiple state-of-the-art methods put forth by reputable research institutions. This comprehensive assessment was executed to substantiate the reliability and effectiveness of this metric as it was progressively refined.

Case 1

Number of nodes = 1000

Total possible edges = 499500

Table 3.2: Results showing statistics of case1

G1 edges	G2 edges	Boundary edges	Total edges	Equation 1	Equation 2	Equation 4	$Case\ 1$ [23]	$Case\ 2$ [24]	$Case\ 3$ [27]	$Case\ 4$ [49]	$Case\ 5$ [22]
1000	800	500	2300	15319	9.6368	0.5094	12.556	0.3366	336.54	0.4374	0.0013
2000	1500	500	4000	16364	9.7030	0.5359	9.177	0.3366	336.54	0.6267	0.0013
10000	12000	500	22500	28539	10.2590	0.5756	3.780	0.3366	336.54	0.9217	0.0013
50000	90000	500	140500	30566	10.3276	0.5823	1.481	0.3366	336.54	0.9896	0.0013
50000	90000	50000	190000	8599	9.0594	0.4597	1.040	0.3366	336.54	0.9936	0.0013

Table 3.2, shows that polarization increases by increasing internal edges while keeping boundary edges constant. On the contrary, polarization decreases by keeping internal edges constant while increasing the boundary edges. A comparison of this metric is performed with different measures

of polarization given in various papers. Figure 3.3 shows how equation 2 catches the increase in internal edges correctly when the boundary edges are kept constant and starts showing an increase in polarization whereas other methods like that of case 1 shows a decrease in polarization which is counterintuitive. Case 5 and 2 show no change and stay constant whereas equation 4 follows the pattern of equation 2, however their values are too small to be seen visually from the graph.

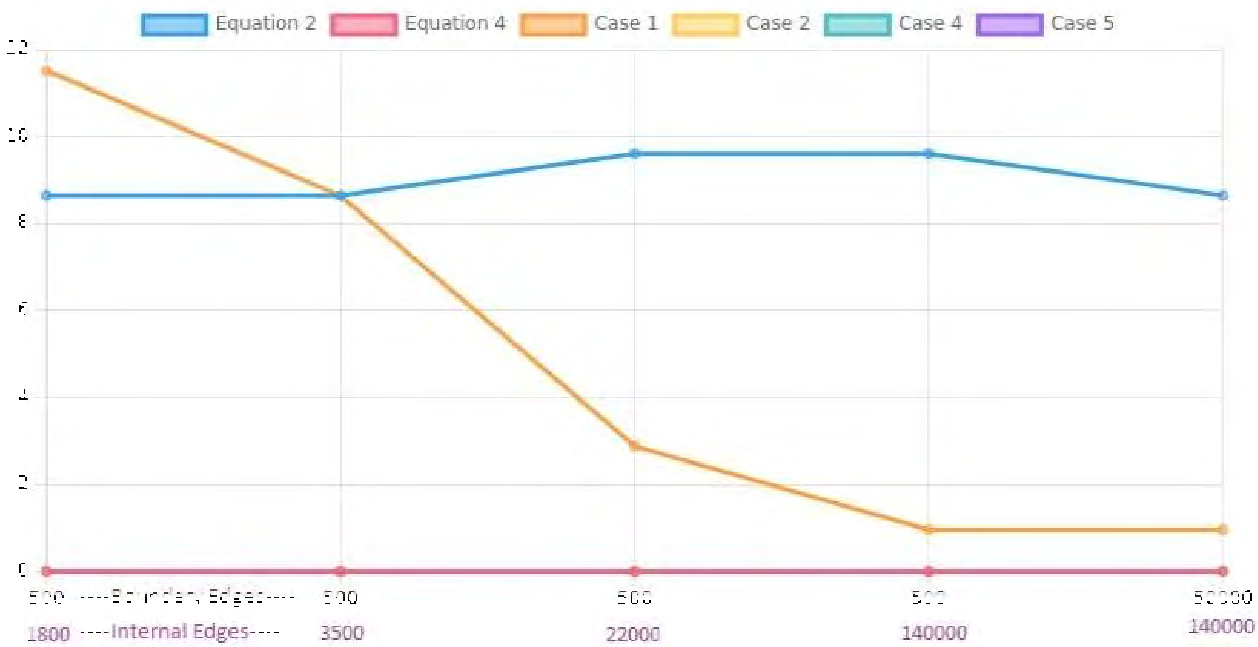


Figure 3.3: Results showing statistics of the proposed equation against different research bodies

Case 2

Number of nodes = 10000

Total possible edges = 49995000

Table 3.3: Results showing statistics of case 2

G1 edges	G2 edges	boundary edges	Total edges	Equation 1	Equation 2	Equation 4	$Case$ 1 [23]
500000	600000	100000	1200000	2533370	14.746	0.5541	0.00224
1000000	800000	100000	1900000	2736759	14.822	0.5602	0.00224
1500000	1000000	100000	2600000	2902547	14.881	0.5631	0.00224
2000000	1500000	100000	3600000	2922395	14.888	0.5672	0.00224
2000000	1500000	1500000	5000000	1238970	14.029	0.4724	0.00224

Table 3.3, shows that increasing number of internal edges by ten times and boundary edges by hundred times shows no change in trend of polarization and it remains same as of mentioned in Table 3.2

Testing on Dataset

A dataset is generated from random values following a uniform distribution between -1 and 1 and the values are then scaled to be centered around 0.8 and -0.9 opinions. The distribution could be understood from the following plot in figure 3.2

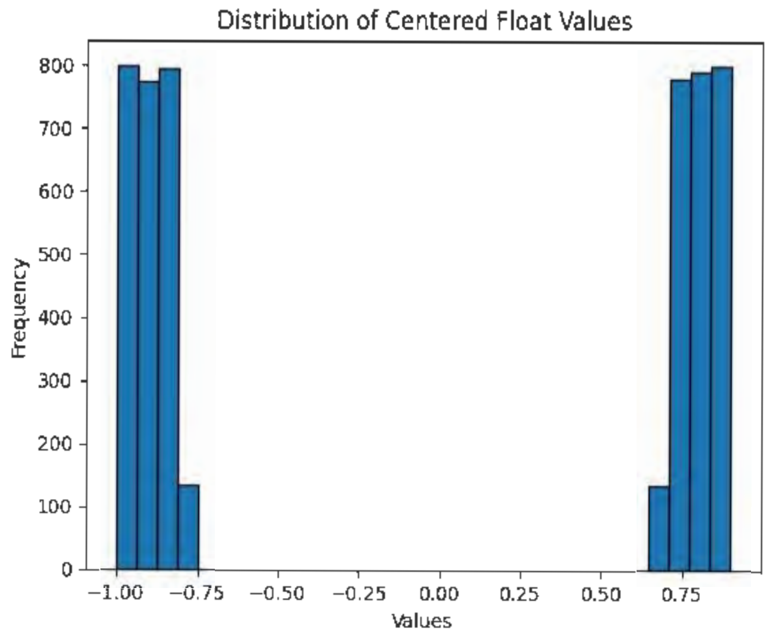


Figure 3.4: Data distribution

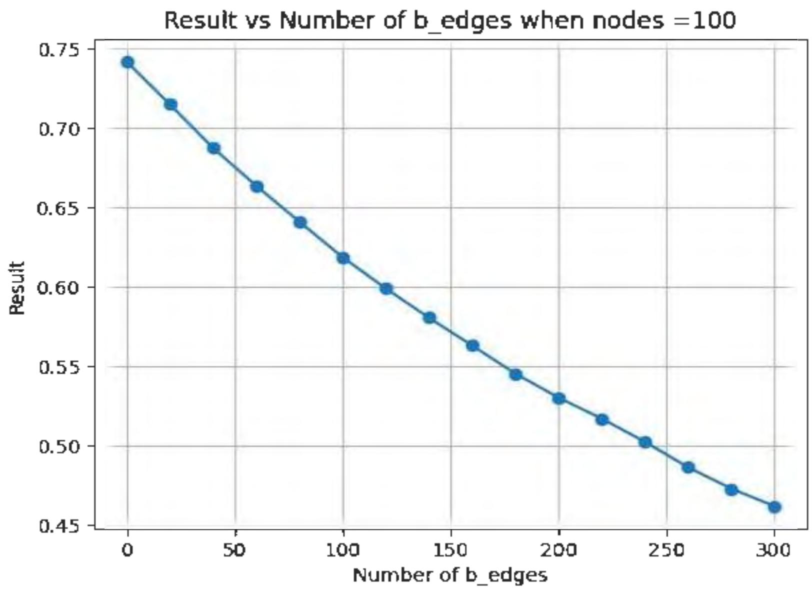


Figure 3.5: An explicit decline in polarization with adding more boundary edges

After showing what the data looks like, several cases are discussed in which different tests are being performed. First, 100 nodes are taken and the number of boundary edges between those nodes is randomly increased. The decreasing trend in the polarization is then observed in Figure 3.6.

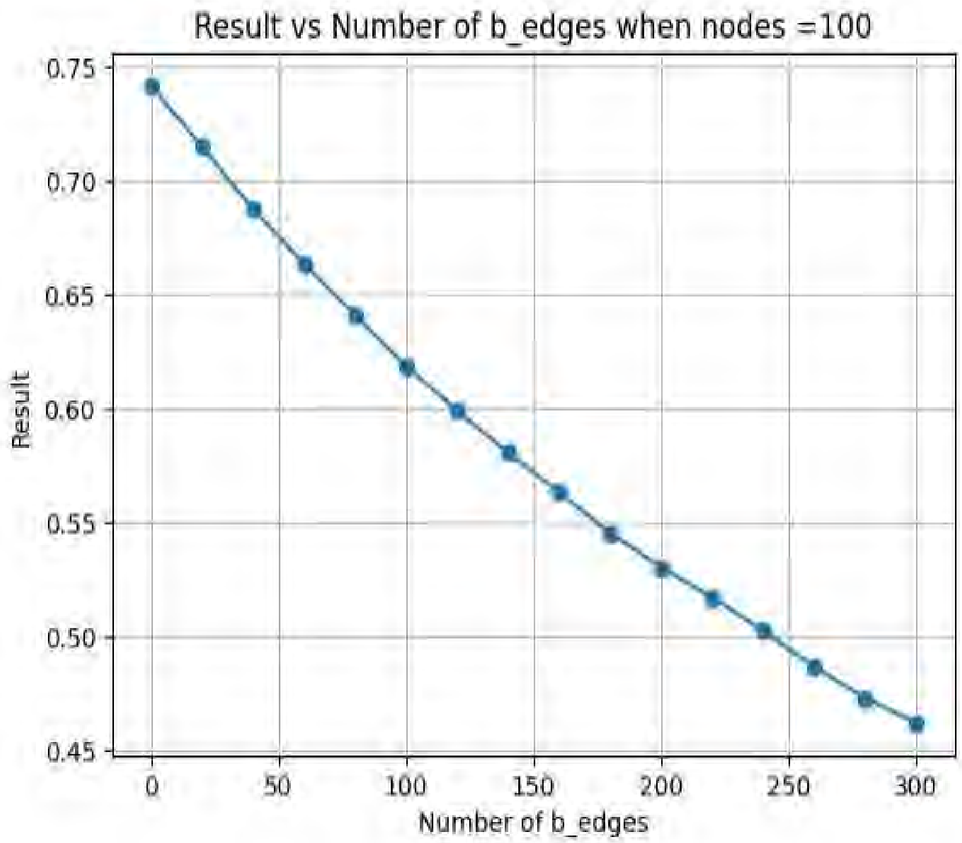


Figure 3.6: An explicit decline in polarization with adding more boundary edges

After that more nodes are added to make it 1000 and the same exercise is repeated of increasing the amount of boundary nodes. The effect of this exercise is observed in the following figures.

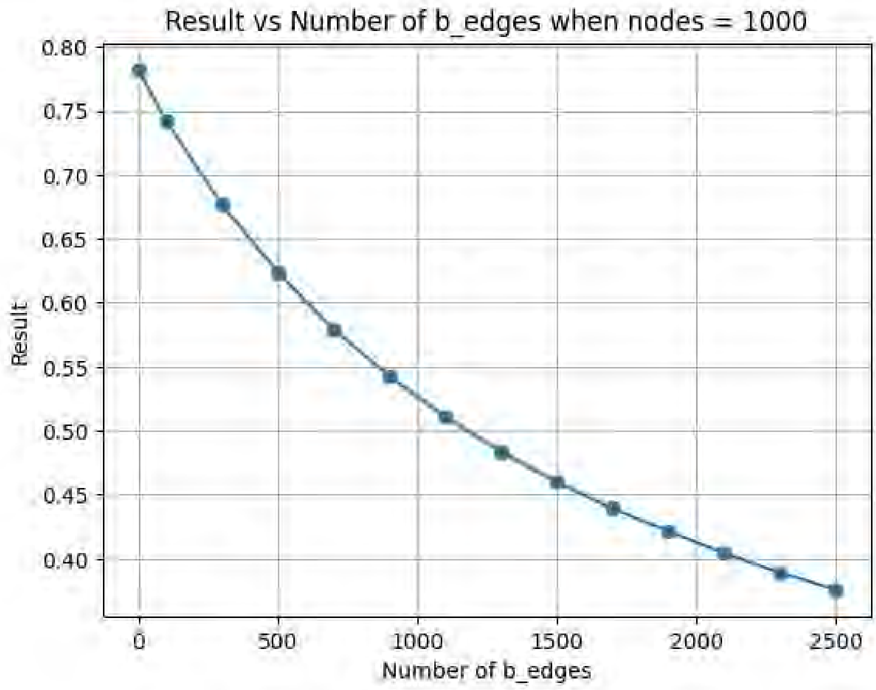


Figure 3.7: An explicit decline in polarization with adding more boundary edges

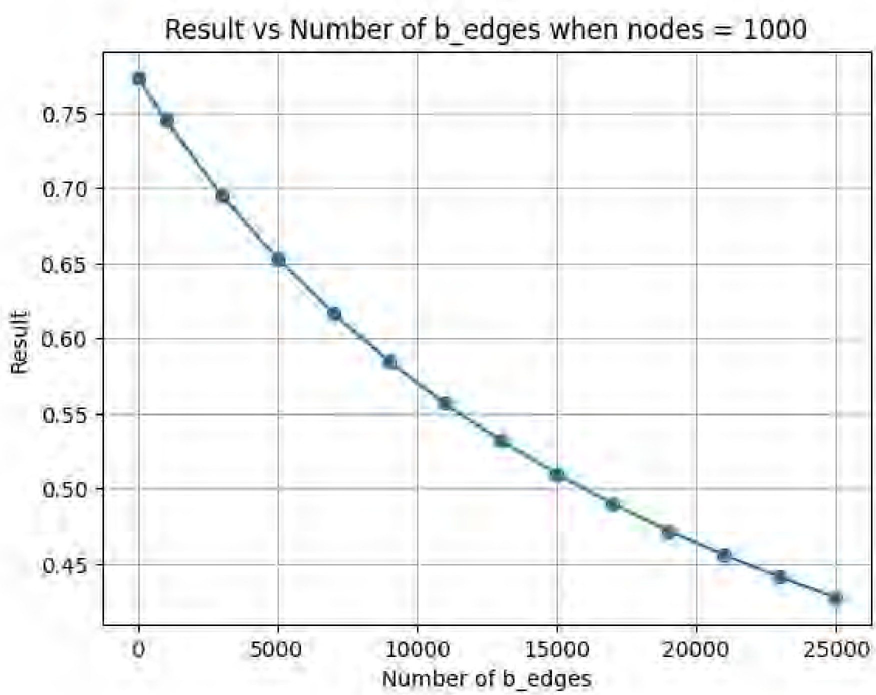


Figure 3.8: An explicit decline in polarization with adding more boundary edges

The number of boundary edges are increased and its effect is seen in Figure 3.7. Figure 3.6, shows that if we increase boundary edges from 0 to 300, polarization decreases from 0.74 to 0.49. Similarly, figure 3.7 and 3.8 shows the same trend when, in figure 3.7, the nodes boundary nodes were increased from 300 to 2500 and in figure 3.8, they are further increased to 25000.

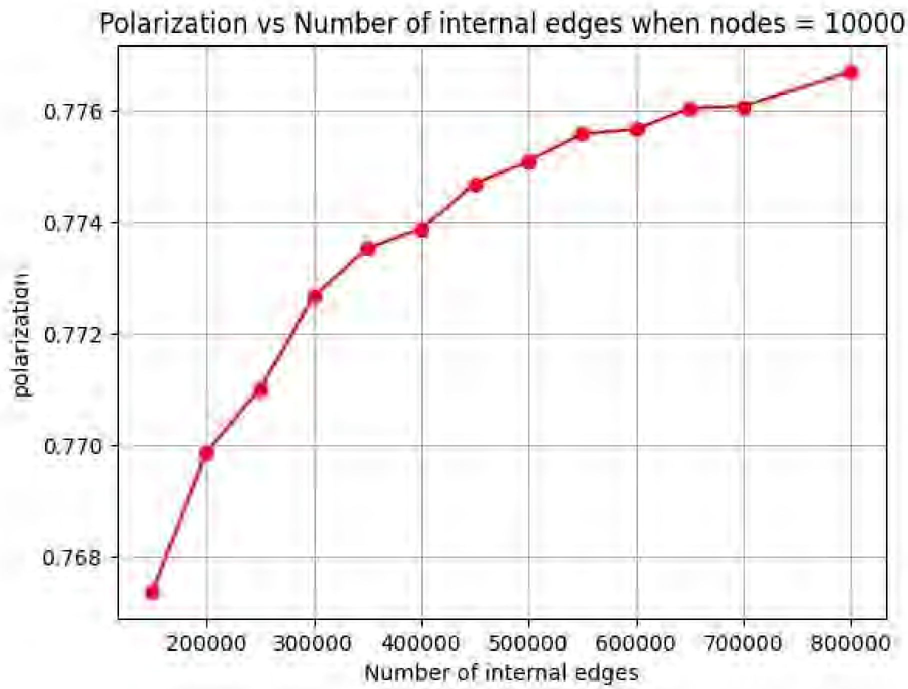


Figure 3.9: Polarization increases with the increase in the internal edges

After observing the effects of adding boundary edges to the overall polarization value, it can be seen how changing the internal edges affects the polarization. Figure 3.6 and 3.7 show that polarization increases when number of internal nodes is increased. This is because of increased Interaction between the members of a same community. This is also exhibiting a factor of group cohesion where when intra group connections become strong they result in increasing the overall polarization value of the network because the people stay in silos and become highly connected in their community without getting exposed to the other side

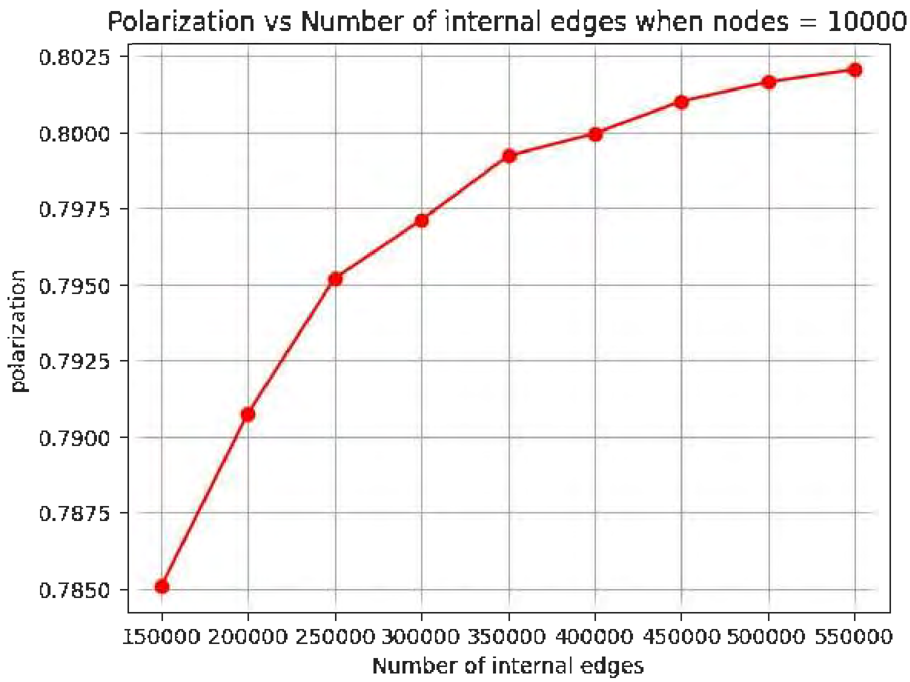


Figure 3.10: Polarization increases with the increase in the internal edges

After this, an interesting phenomenon is observed, shown in Figure 3.11 below which highlights yet another aspect of the polarization measured that needs to be accounted for. It is noticed that upon increasing the number of internal edges the polarization initially increases to a certain level. However, upon further increase in internal edges, an unexpected decrease in polarization is noticed, which contradicts the expected behavior of an increase. This decrease indicates the presence of another aspect of the formula used for polarization calculation that still needs to be evaluated. After a thorough review and analysis of the equation 4, it was upgraded to cater for ground realities. It was observed that when the number of internal edges was being increased, it led us to increased number of boundary edges as well to a point that polarization started decreasing. To cater this exclusive focus was made on boundary edges, introducing an inverse relationship to polarization as updated in equation 5.

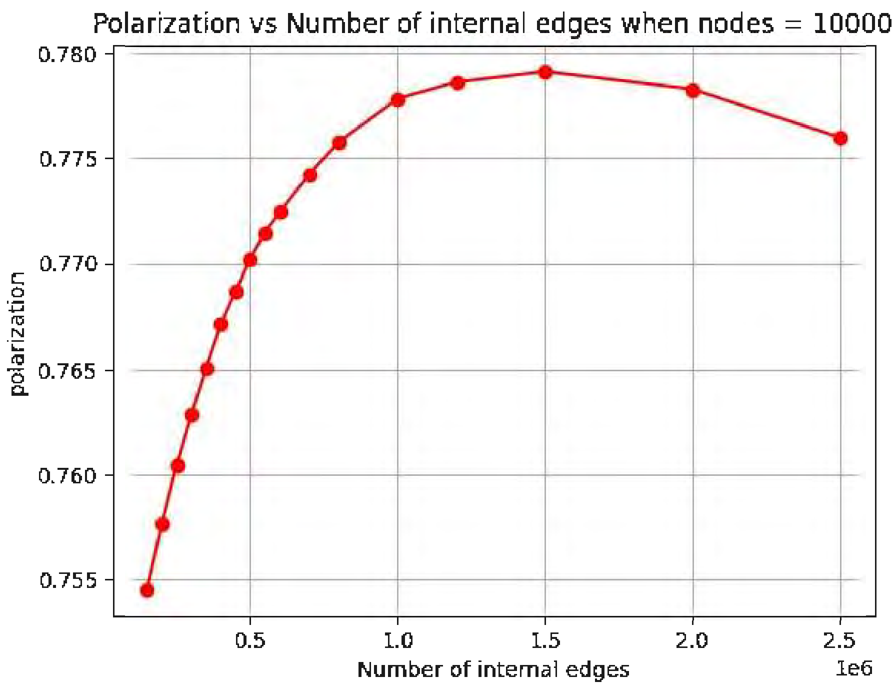


Figure 3.11: Sudden decrease after increase in polarization due to increasing internal edges

The formula is then updated as follows:

$$Polarization = \frac{1}{2}(d.[s + (1 - \rho)]) \quad 5$$

Where,

$s = \frac{\Sigma(w_i \,.\, o_i \,.\, o_j)}{\Sigma i}$

$\rho = \frac{2.e_b}{n(n-1)-n_1(n_1-1)\, n_2(n_2-1)}$

$d = \frac{|CG_+ - CG_-|}{2}$

s = average connection strength in the network

$1-\rho$ = lack of density between the groups/sparsity

d = distance between the center of gravities of opinion groups in the network

After making only the boundary edges count the plot of polarization vs the number of boundary edges added was made again and the trend is shown in Figure 3.12

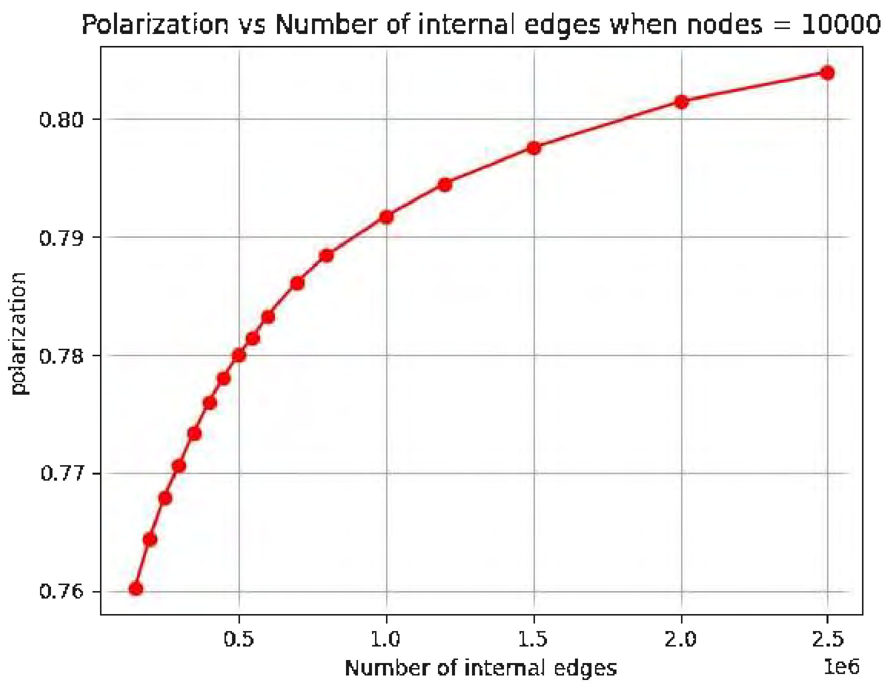
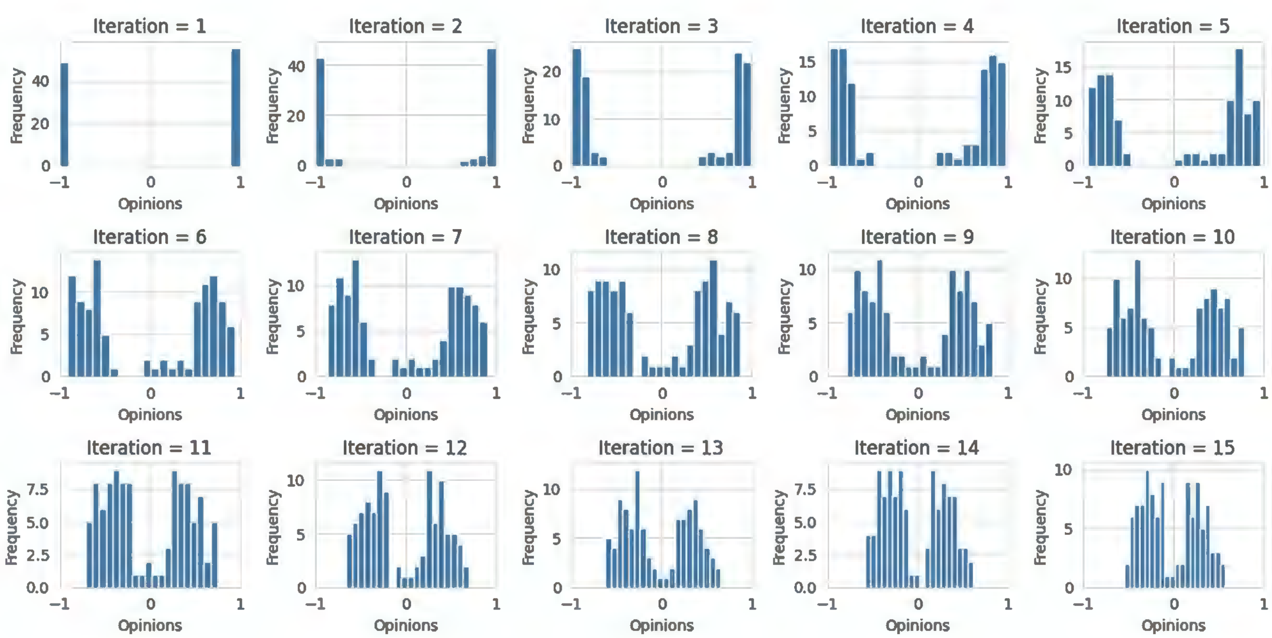


Figure 3.12: Adjustment of sudden decrease in polarization

Figure 3.12 shows that the latent fallacy of equation 4 has been removed and thus with substantial increase in internal edges to 2.5Million the trend remains the same as of figure 3.6 and 3.7. This is also in accordance of the equation 5.

3.3 Testing on Real Dataset

The above proposed equation 5 is then tested on a real-world dataset known as polbooks [13] described in Chapter 1, introduction. Before applying the formula, another case needs to be addressed first. It was proposed in this research that instead of assuming binary opinions, the values of opinions should be taking continuous values between -1 and 1 so that it replicates a real-world setting where everyone is not highly opinionated rather people of opinions with varying intensities coexist. To accomplish this the measure for opinion evolution proposed in this research was applied to identify how people's opinions change over time. It starts with the initially assigned binary opinions and after that it goes on until quite a few number of steps. Results are reported for the first 15 time steps. It can be seen how opinions change from initially assigned binary ones to proper form where they can be evaluated. It can be observed in figure 3.13 that the final plot follows a gaussian distribution showing that opinions follow the pattern of a normal



distribution.

Figure 3.13: Opinions evolution using the proposed model

For the sake of this research, the opinions at time-step number 3 are finalized and the values generated at time-step 3 are used to feed to this model and to analyse the behavior of the proposed polarization measure. Evolved opinions of polblogs data step frozen at time-step 3 can be seen in figure 3.14 below. After collecting the opinions at time-step 3 they were fed then to the proposed measure given in equation 5 and the results are shown below in figure 3.12 a,b. It can be seen in figure 3.12 (a) that as boundary edges are added the polarization keeps going down and the average opinion of the network also becomes less extreme as seen in figure 3.12 (b) to show that consensus is being established in the network

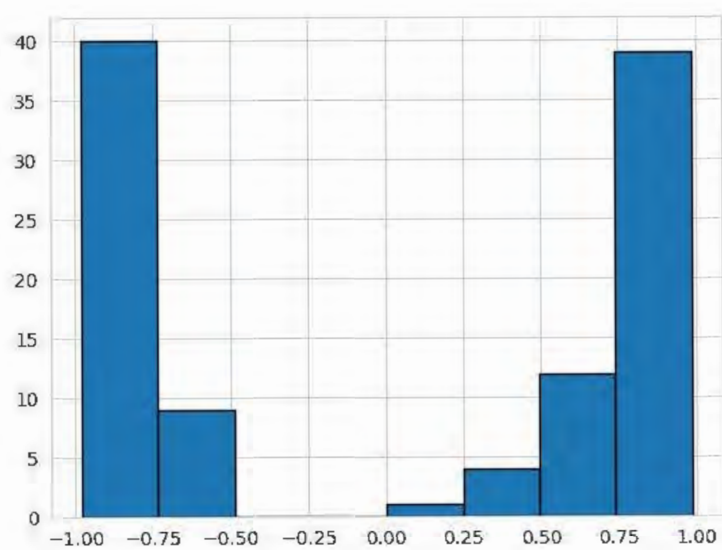


Figure 3.14: Opinion spread of polblog dataset

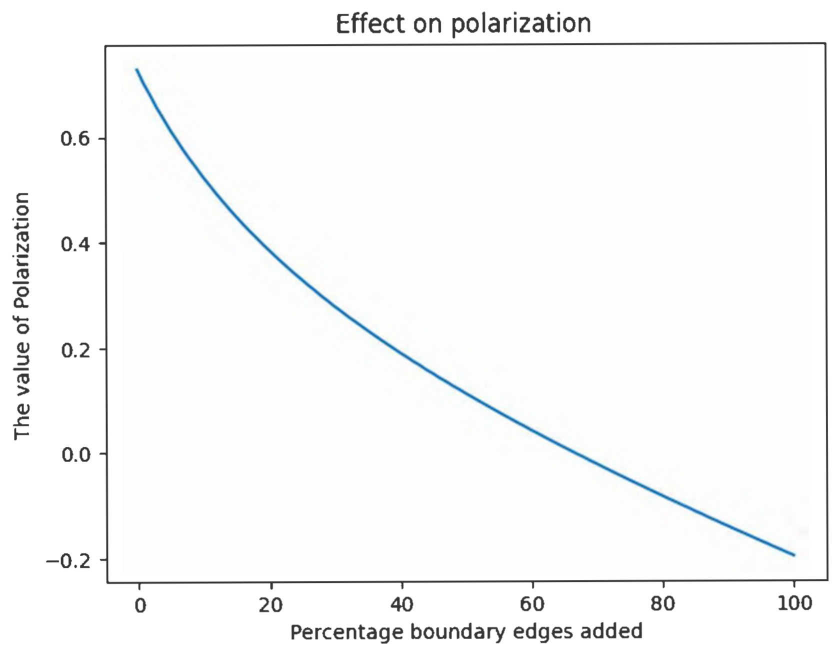


Figure 3.15 (a): Polarization is decreasing with the increase in boundary edges

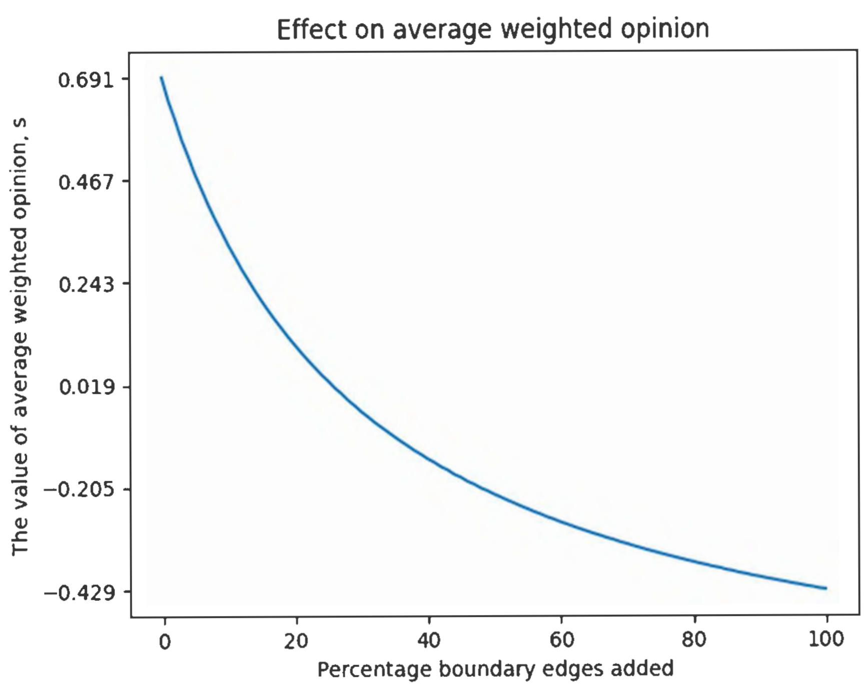


Figure 3.15 (b) Average weighted opinions decrease as more boundary edges are added

Another noticeable point to observe in the above figure is where both the polarization and the value of "s" become negative as the number of boundary edges increases. This phenomenon is mathematically accurate, leading to a situation where the network is no longer polarized. However, from a practical standpoint, negative polarization lacks a meaningful interpretation. Hence, it became evident that further adjustments to the formula are necessary.

To address this concern, a subtle alteration to the formula by introducing the "max" function in the polarization calculation. Specifically, the "max" function was applied to the expression (s + (1 - ρ)) as follows:

$$\max(s + (1 - \rho), 0).$$

By doing so, the result is constrained to zero whenever the factor (s + (1 - ρ)) becomes negative. This modification enables us to obtain more sensible and meaningful results, thereby improving the overall accuracy and relevance of the formula. The modified final formula is then termed as the polarization point and is denoted by β.

$$\boldsymbol{\beta = \frac{1}{2}(d \,.\, max(s + (1 - \rho), 0)} \quad \mathbf{6}$$

Where,

$$s = \frac{\sum(w_i . o_i . o_j)}{\sum i}$$

$$\rho = \frac{2. e_b}{n(n-1) - n_1(n_1 - 1) - n_2(n_2 - 1)}$$

$$d = \frac{|CG_+ - CG_-|}{2}$$

Explanation s = average connection strength in the network $1 - \rho$ = lack of density between the groups/sparsity

d = distance between the center of gravities of opinion groups in the network

The effect of adding boundary edges on ρ can be observed in the figure # below.

Now the proposed polarization pointer consists of three primary components: s, ρ, and d. Each component captures a distinct aspect of the network's dynamics.[10] The average connection strength, denoted by s, reflects the extent to which individuals within the

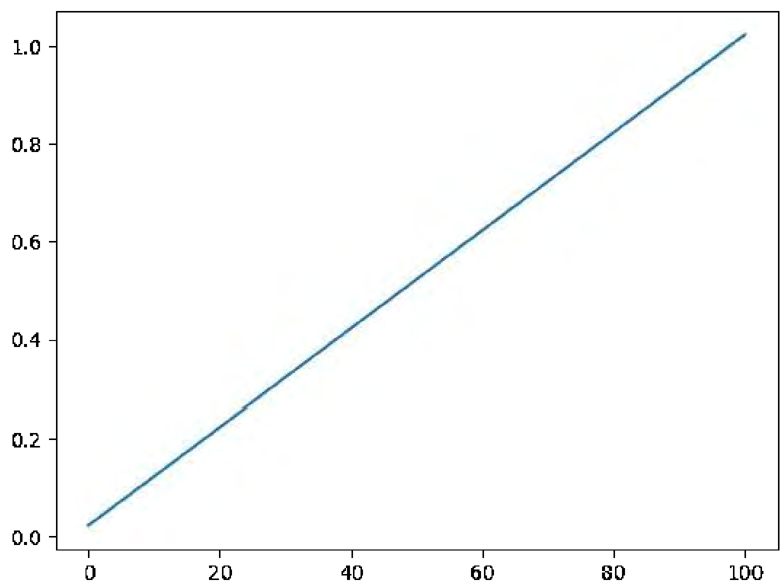


Figure 3.16: Effect on ρ

network are connected and influencing eachother based on their opinions. It is computed as the weighted sum of opinion interactions between individuals, normalized by the total number of interactions. This measure enables us to gauge the intensity and prevalence of opinion-based connections within the network. The sparsity measure, denoted by 1-ρ, quantifies the lack of density between opinion groups. It takes into account the number of edges among different groups as well as the total possible edges between the groups. By considering the relative abundance or scarcity of connections between opinion groups, ρ provides insights into the level of separation and isolation between polarized factions. The distance between center of gravities, represented by d, captures the separation between opinion groups in the network. It is computed as the absolute difference between center of gravities of the positive and negative opinion groups, divided by two. This distance metric allows us to analyze the physical distribution and spatial dynamics of polarized groups within the network.

Special case: where no boundary edges exist between the opinion groups, resulting in a network with no density, the proposed measure can be written as:

$$Polarization = \frac{1}{2}(d.[s+1])$$

In such cases, the absence of connections between opinion groups contributes to a high sparsity measure, emphasizing the stark division between factions. The proposed formula provides a comprehensive approach to quantify polarization in networks, encompassing measures of connection strength, sparsity, and distance between opinion groups. By considering these factors, researchers can gain a deeper understanding of the underlying [2] dynamics shaping polarized networks. The analysis of the special case, where no boundary edges exist, sheds light on the distinctive characteristics and implications of polarization in networks with no density. This formula serves as a valuable tool for researchers investigating polarization and its impact on various domains, enabling a more nuanced understanding of social dynamics in polarized environments.

CHAPTER 4

Method and Results - I

As discussed previously that polarization in social networks can lead to increased divisions and hinder constructive discussions among different groups. To address this issue, an optimization framework is proposed that is aimed at minimizing polarization within the network. In equation 6 the key factors influencing polarization are s and ρ so the objective is to minimize s while maximizing ρ to achieve a reduction in polarization. Additionally, this works aims to find the optimal configuration of added edges between opinion-based separated groups to maximize the impact on polarization reduction while adhering to a constraint on the maximum number of edges allowed to be added.

Variables and Definitions

Before proceeding, it is important to define key variables and explain what each variable signifies.

Table 4.1: Key variable explanations

Variable	Explanation
s	The segregation measure, calculated as the ratio of the sum of the product of weights wi and opinion values oi and oj to the sum of indices i. This variable captures the level of separation between different groups within the social network
ρ	The ρ, computed as the ratio of twice the number of boundary edges eb to the total possible edges n(n − 1) − n1(n1 − 1) − n2(n2 − 1), where n1 and n2 represent the sizes of two separate opinion-based groups. This variable indicates the degree of similarity and affinity between connected nodes

d	The distance measure, representing the difference between the sizes of the two separated groups, divided by 2. This measure provides insights into the distance between the groups based on their opinion values

Polarization Formula

The overall polarization measure is defined as:

$$\beta = \frac{1}{2}(d \,. max(s + (1 - \rho),0)$$

Minimization Objective

The goal is to minimize the polarization measure β by strategically adding edges between opinion-based separated groups. To achieve this, focus is made on minimizing the segregation measure s and maximizing the measure ρ. However, the constraint is that there's a limit to the number of edges that can be added between the groups.

Optimization Problem Formulation

Let E be the set of all possible edges between opinion-based separated groups, and E_{added} be the set of edges to be added. Thus, the optimization problem can be formulated as follows:

$$Minimize: \beta = \frac{1}{2}(d \,. max(s + (1 - \rho),0) \quad 7$$

$$Subject\ to: |E_{added}| \leq MaxEdgesToAdd \quad 8$$

where MaxEdgesToAdd is the maximum number of edges that can be added between the groups. The edges added must be selected optimally, since this work considers this as a constraint, to maximize the reduction in polarization.

Solving the Optimization Problem

The minimization problem at hand as represented by relationship 7, presents a complex optimization task involving both discrete and continuous variables due to the presence of MaxEdgesToAdd as a discrete variable, and s, ρ, and d as continuous variables. To efficiently determine the optimal configuration of added edges, a systematic technique is proposed as outlined in the following sections.

Node Grouping and Selection

Initially, the nodes are categorized into groups based on their positive and negative opinions. Subsequently, these groups are sorted in descending order based on the magnitude of their opinion values. From each group, selectively the top 5% of nodes are chosen. The purpose of this selection process is to identify optimal nodes for potential connections.

Identifying Optimal Node Pairs

To minimize the segregation measure, denoted by s, the focus is on connecting nodes from the selected groups with opposing opinions. These nodes are the top 5% nodes of their respective groups, descendingly sorted on magnitude of their opinions. For each possible combination of nodes between the selected groups, the product of their opinions is calculated. By sorting these products in descending order, the node pairs that yield the maximum effect on reducing s when connected with an edge can be identified.

Edge Addition Strategy

Based on the computed product values, adding edges is prioritized between the node pairs that contribute the most to minizing s. This strategic approach allows to maximize the impact of the added edges on polarization reduction. By following this technique, the aim is to effectively reduce the value of the segregation measure s within the social network. The reduction in s is achieved by connecting nodes from opposing opinion groups, where the product of their opinions becomes negative, thus contributing to the desired decrease in polarization. In addition to the proposed technique, leverage advanced optimization algorithms are leveraged that can efficiently explore the solution space while adhering to the constraints imposed on the number of edges that can be added. The ultimate objective is to identify a configuration that leads to a substantial reduction in polarization, fostering a more cohesive and inclusive social network.

Algorithm 1: Pseudocode for adding boundary edges

Data: Graph G, nodes group: g_1, g_2

$g_1 \leftarrow Sort\_W.R.T\_Opinion(g_1)$;

$g_1 \leftarrow Sort\_W.R.T\_Opinion(g_2)$;

for n_i in Top_percent (g_1) **do**

    for n_j in Top_percent (g_2) **do**

        $op_{ij} \leftarrow$ opinion_value ($n_i \times n_j$);

    end

end

$edges_1 \leftarrow \{n_i n_j | n_i, n_j \epsilon\ OP\}$;

G_with_edges ← Copy(G);

for edge in edges **do**

    Add_edge(G_with_edges, edge);

end

Segregation

The segregation measure (s) is a metric used to assess the degree of separation or clustering between different groups of nodes within the social network. It quantifies how much nodes with similar opinions tend to be connected to each other, potentially forming isolated clusters or echo chambers. The formula for calculating the segregation measure (s) is as follows:

$$s = \frac{\sum(w_i \, . \, o_i \, . \, o_j)}{\sum i}$$

Where:

• w_i represents the weight of the edge between nodes i and j.

• o_i and o_j are the opinion values of nodes i and j, respectively.

The segregation measure s yields a real-valued result. Positive values of s suggest that nodes with similar opinions are more likely to be connected, leading to the formation of groups or clusters with similar beliefs. Conversely, negative values of s indicate that nodes with opposing opinions are more likely to be connected, promoting a more diverse and interconnected network.

4.1 Testing on datasets

In figure 4.1 a, b, c, d, the results of given data are visualized by plotting a histogram of nodes with their evolved opinions on the said datasets. Karate [16], Polbooks [17] and Polblogs [18] are real word dataset whereas the results are also shown on synthetic dataset. Opinions are evolved for 3 time steps and observed how the binary values of opinions changed to continuous values ranging between -1 and 1.

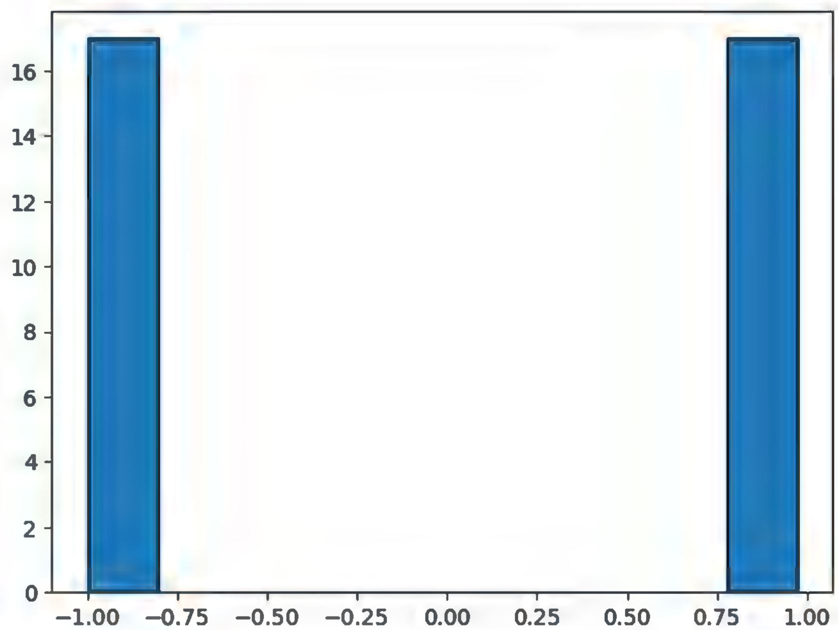


Figure 4.1a: Evolutions of opinions in 3 time steps for karate dataset

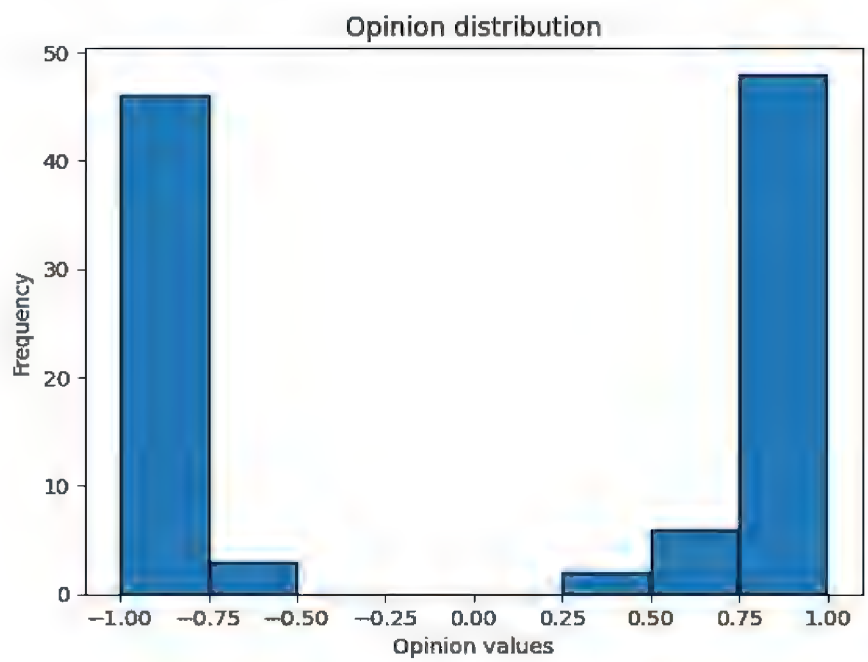


Figure 4.1b: Evolutions of opinions in 3 time steps for Polbooks dataset

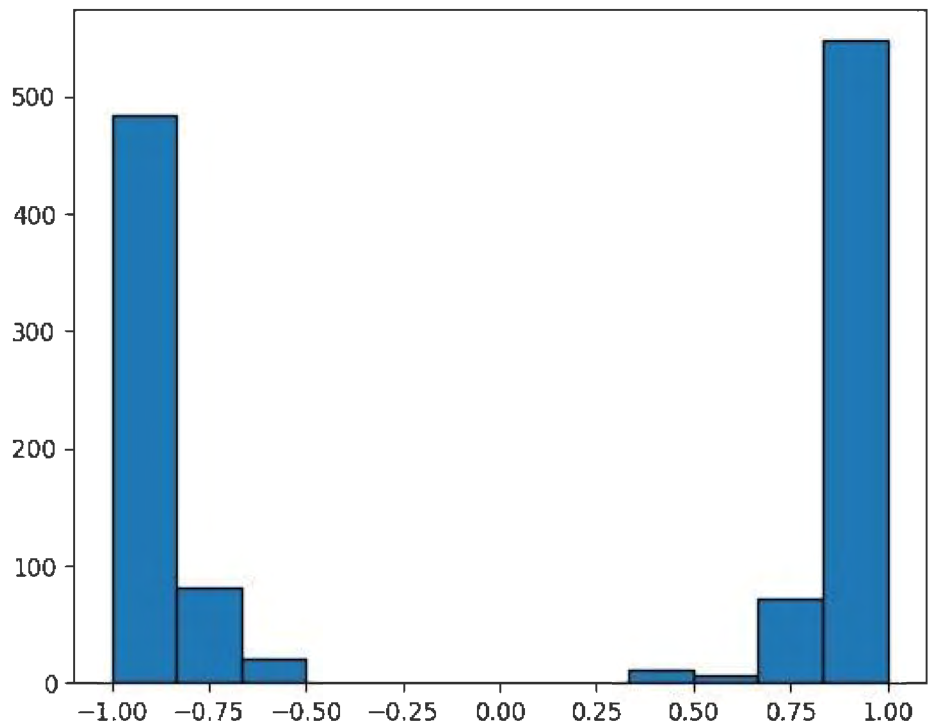


Figure 4.1c: Evolutions of opinions in 3 time steps for Polblogs dataset

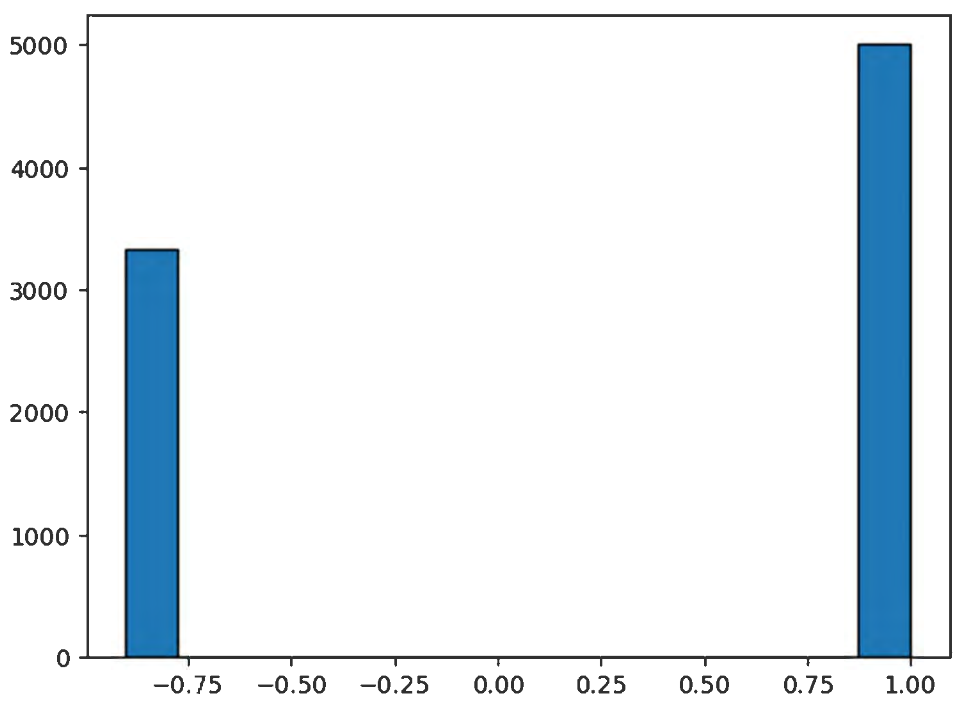


Figure 4.1d: Evolutions of opinions in 3 time steps for synthetic dataset

Next, top nodes from both the communities are picked, according to the proposed method and edges are added between them. The reason for adding edges between the boundary nodes is that these nodes are essentially acting as bridges between the two communities and increasing their number should be causing a decrease in the overall polarization value measured by β. Figure 4.2a, b, c and d# is showing the behavior of polarization as top n% of boundary edges are added.

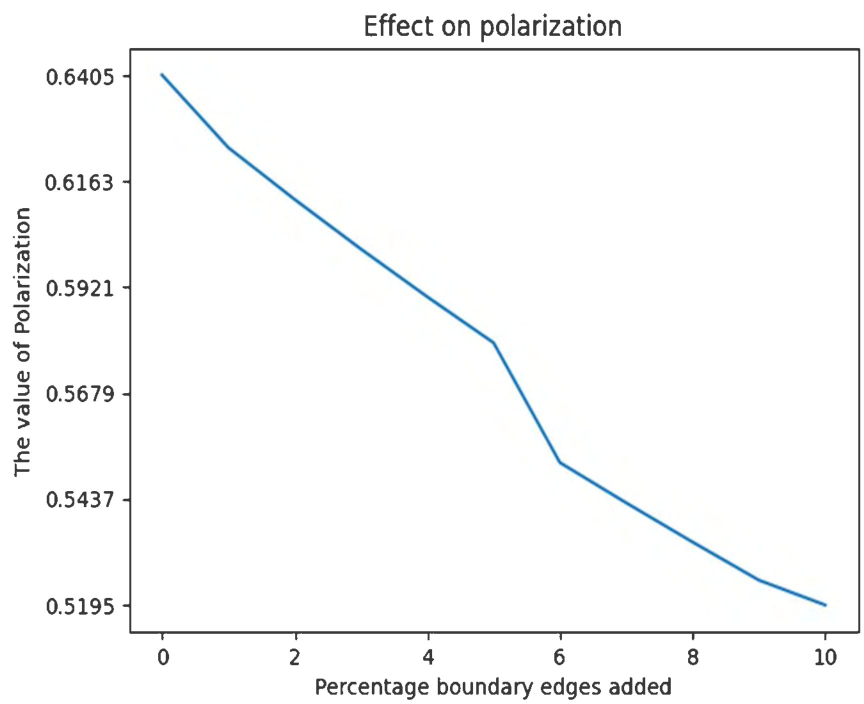


Figure 4.2a: Decrease in polarization with adding more boundary edges in Karate dataset

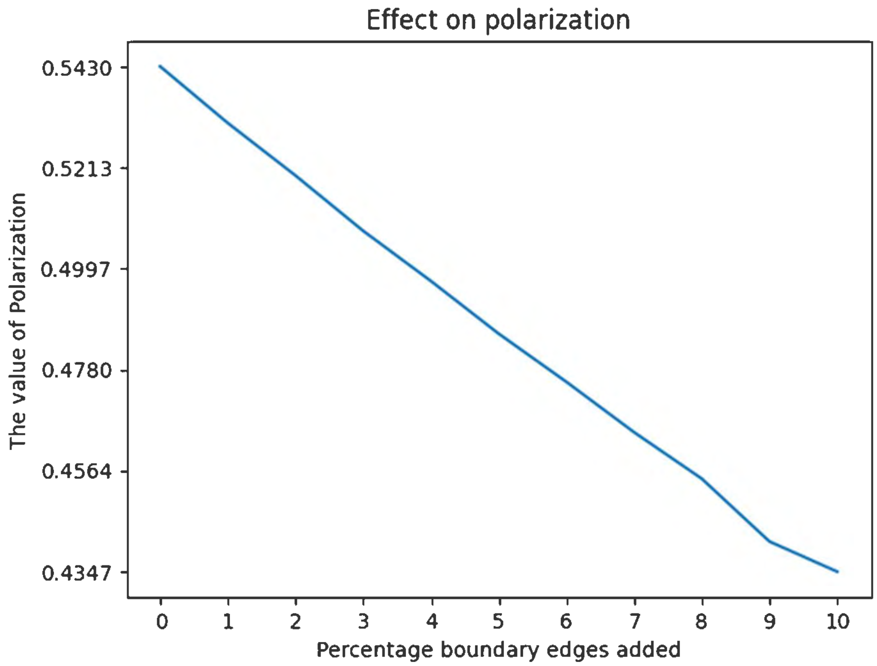


Figure 4.2b: Decrease in polarization with adding more boundary edges in Polbooks dataset

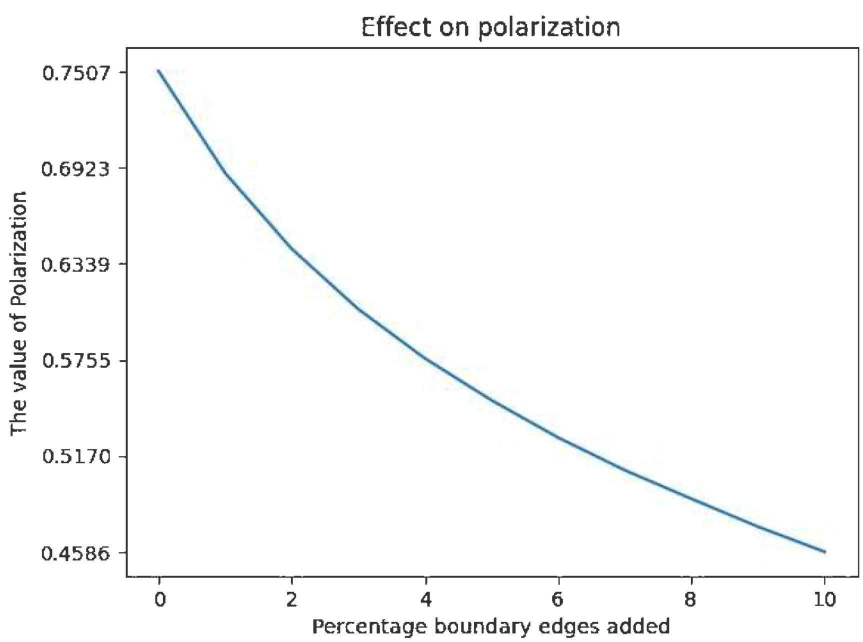


Figure 4.2c: Decrease in polarization with adding more boundary edges in Polblogs dataset

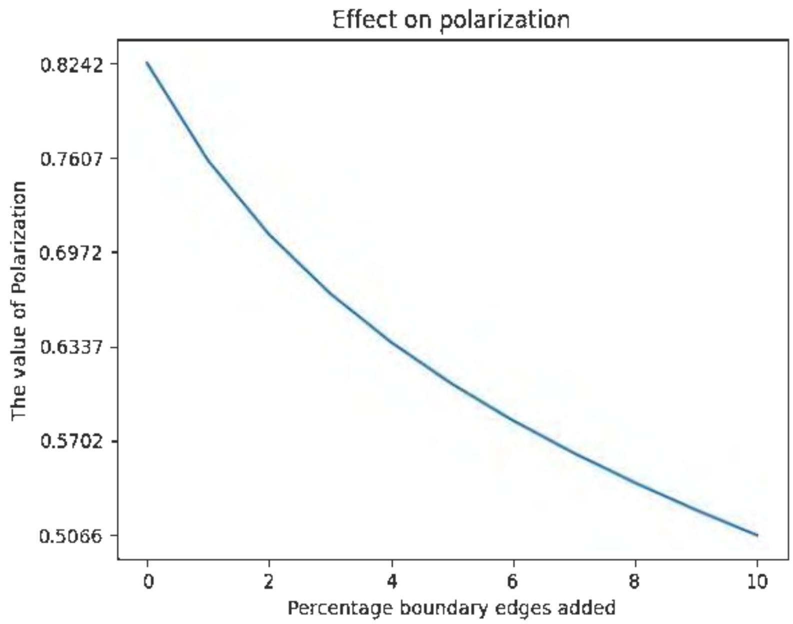


Figure 4.2d: Decrease in polarization with adding more boundary edges in synthetic dataset

Next, the results are reported of how the average weighted opinion of the community changes as the boundary edges are added in Figure 4.3 a, b, c and d . It is obvious that if the number of boundary edges added keeps increasing, the value of average weighted opinion of the network will keep decreasing until it reaches a point after which it becomes constant.

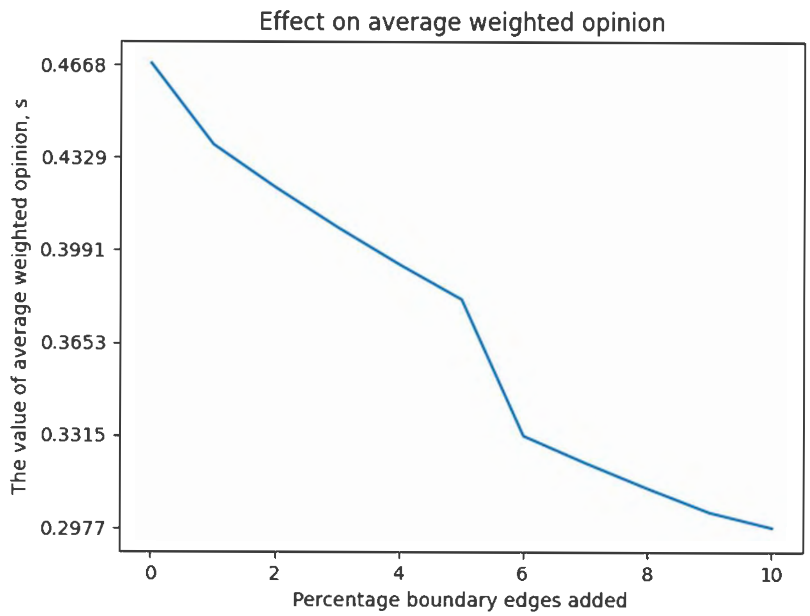


Figure 4.3a: Decrease in the value of average weighted opinions and boundary edges are added in karate

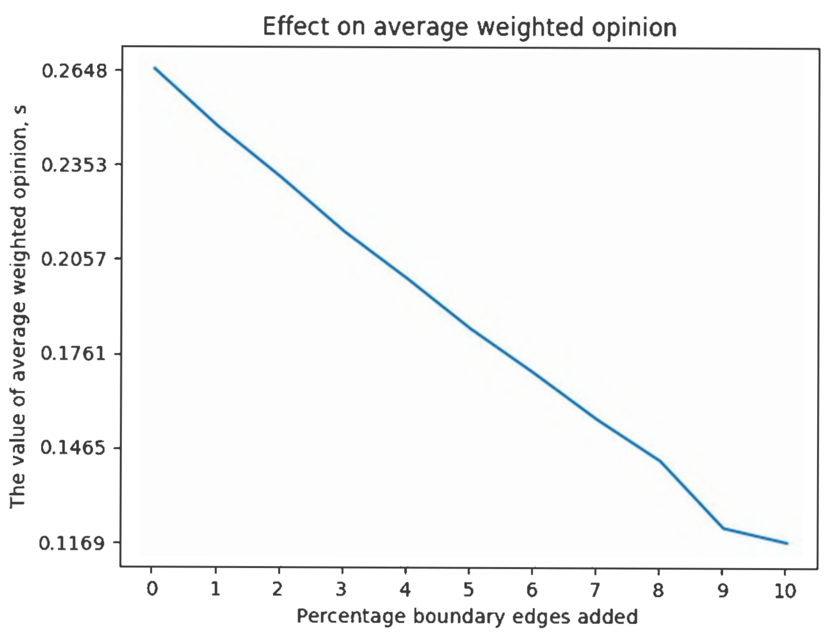


Figure 4.3b: Decrease in the value of average weighted opinions and boundary edges are added in Polbooks

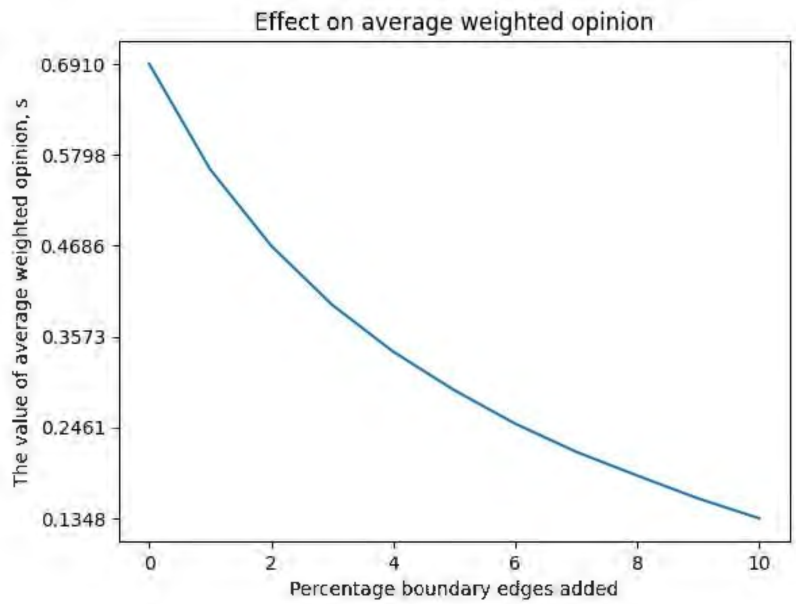


Figure 4.3c: Decrease in the value of average weighted opinions and boundary edges are added

Polblogs

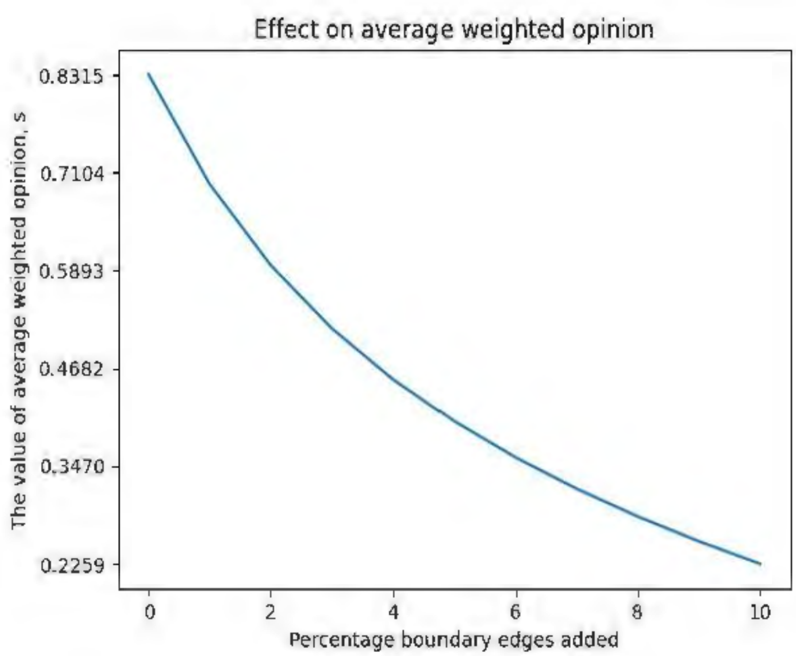


Figure 4.3d: Decrease in the value of average weighted opinions and boundary edges are added

in synthetic dataset

The decrease in the value of average weighted opinions shows that as more and more boundary edges are added, the average opinion of the groups becomes less extreme relatively. This shows that the network is moving towards a state of consensus. In this way it is showing cohesiveness within a group and exhibiting how it is increasing with the increase in the number of edges connecting the boundary nodes.

After that in Figure 4.4 a, b, c and d it is observed the density which basically portrays how many edges between the two communities are added. In this way it is measuring the density of boundary edges. Density is increasing as more and more boundary edges are added.

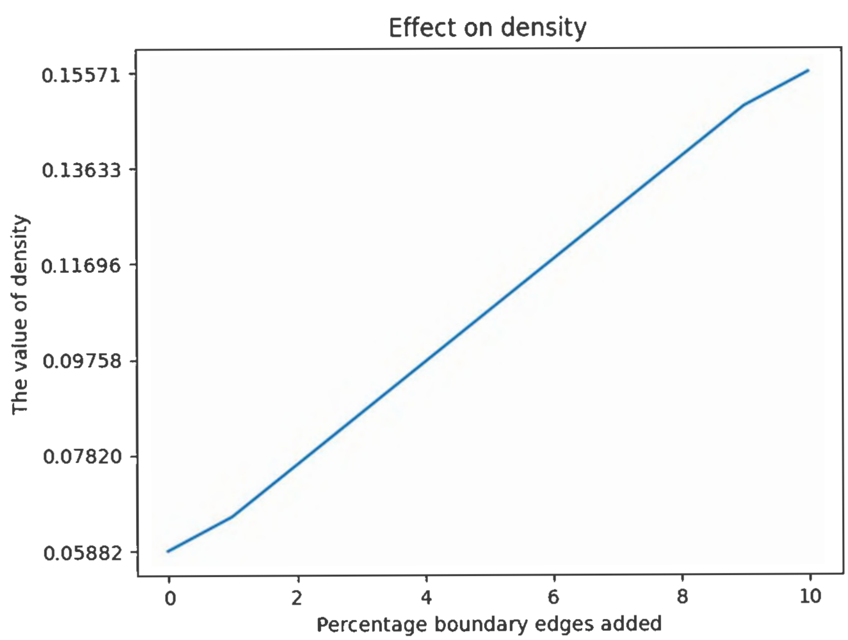


Figure 4.4a: Effect on density with adding more boundary edges in karate dataset

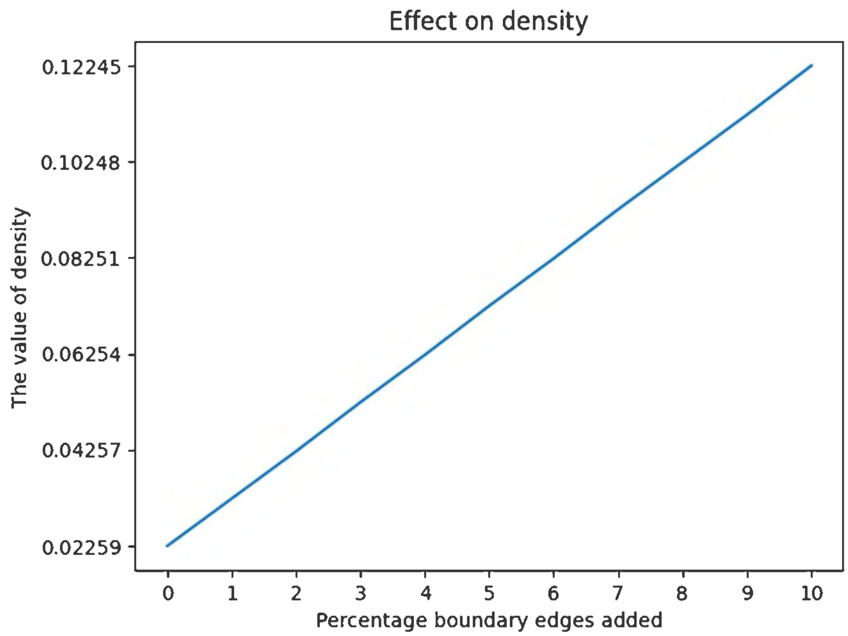


Figure 4.4b: Effect on density with adding more boundary edges in books dataset

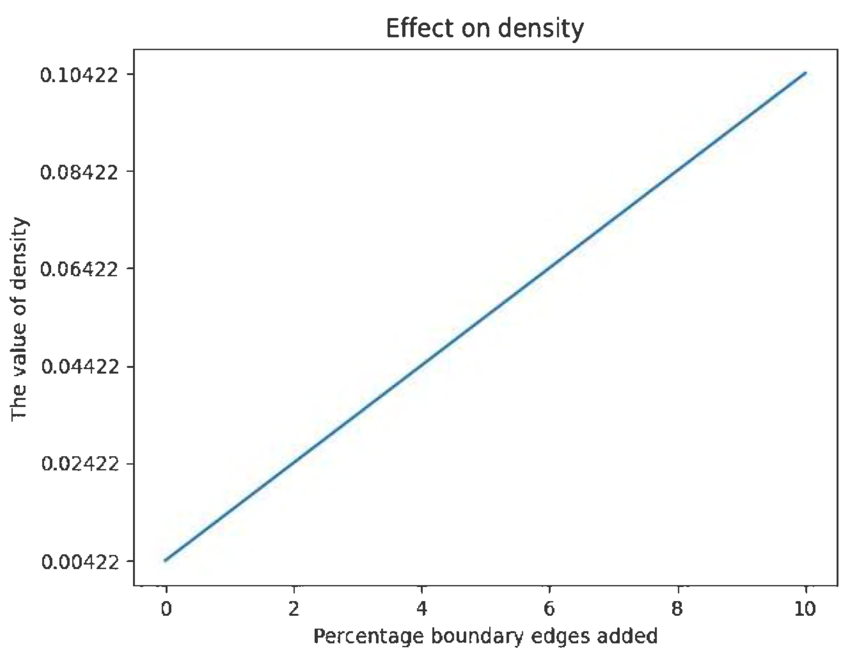


Figure 4.4c: Effect on density with adding more boundary edges in Polblogs dataset

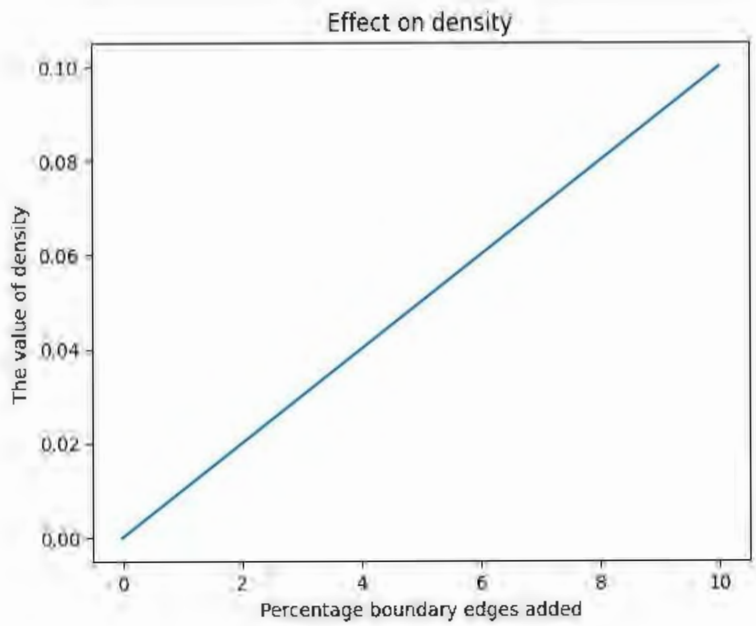


Figure 4.4d: Effect on density with with adding more boundary edges in synthetic dataset

4.2 Testing on Different Methods

After viewing the results on different datasets it was decided to test proposed method against different state-of-the-art approaches to see how well this measure performs. Results are shown by selecting edges using various methods, including the proposed method. Among other well-known measures chosen for comparison are betweenness centrality and degree centrality. Edges are also added using a random manner, and the outcomes are plotted. In Figure 4.5 a, b, c, and d, it is evident that the proposed method outperforms all other methods in reducing polarization. Similar trend can be seen on average weighted opinion in Figure 4.6 a, b, c and d, its clear that the proposed approach is outperforming different established methods in every aspect.

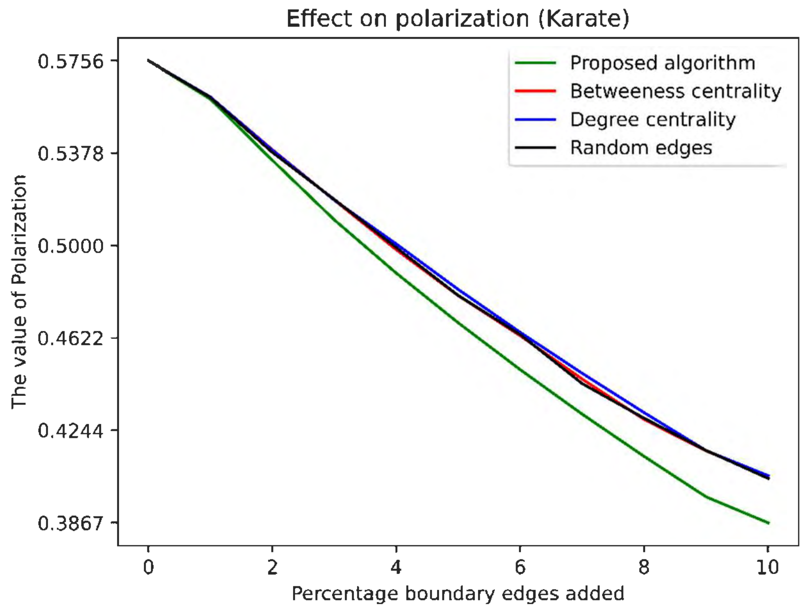


Figure 4.5a: Effect on polarization with adding more boundary edges in Karate dataset using the proposed algorithm and other methods

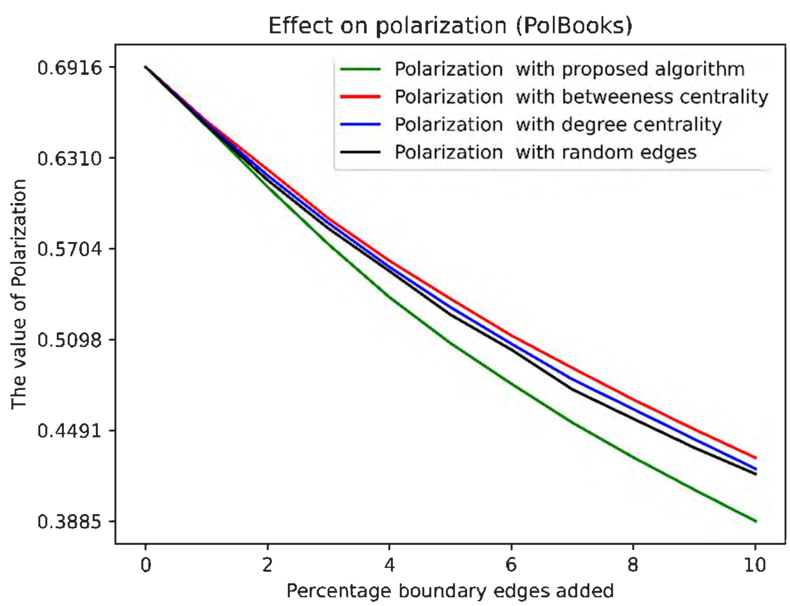


Figure 4.5b: Effect on polarization with adding more boundary edges in Polbooks dataset using the proposed algorithm and other methods

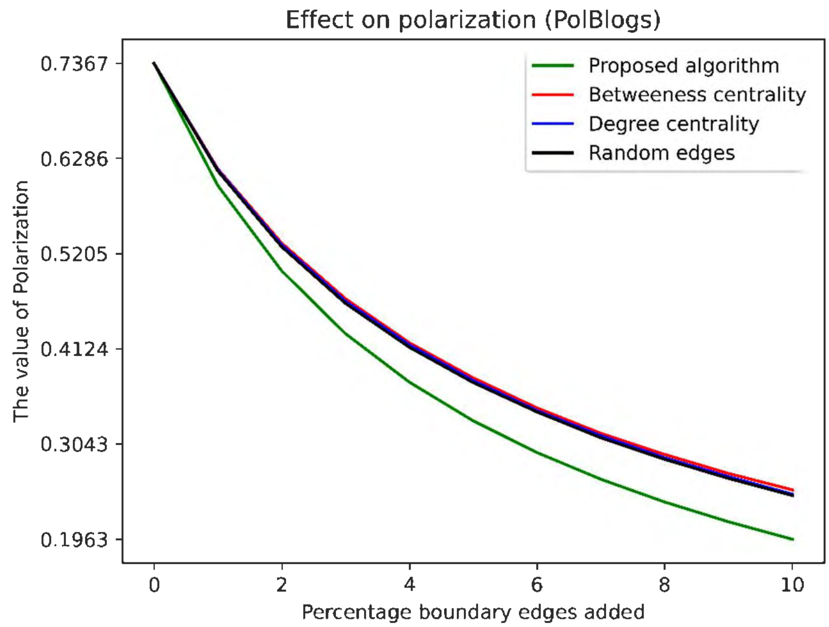


Figure 4.5c: Effect on polarization with adding more boundary edges in Polblogs dataset using the proposed algorithm and other methods

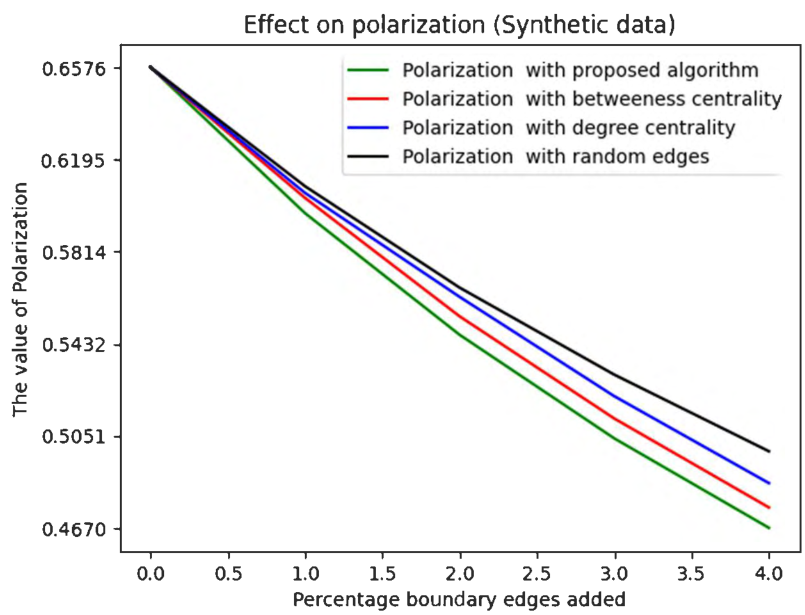


Figure 4.5d: Effect on polarization with adding more boundary edges in synthetic dataset using the proposed algorithm and other methods

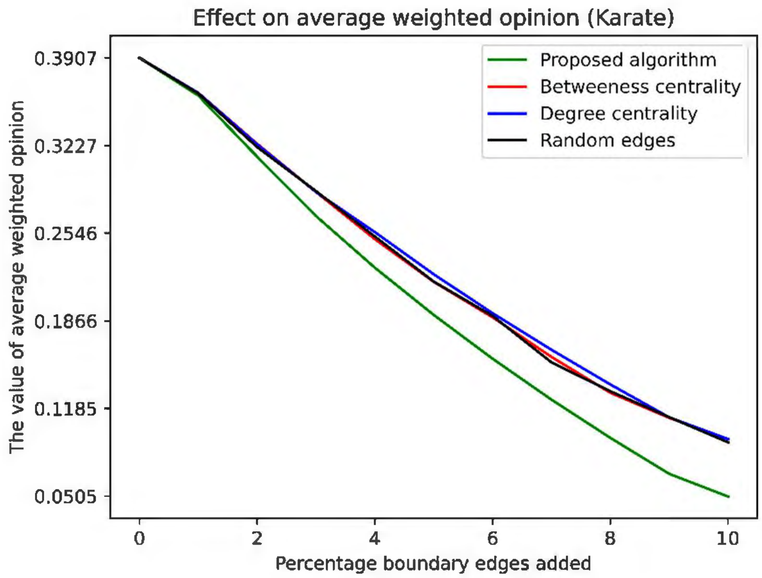


Figure 4.6a: Decrease in the value of average weighted opinions and boundary edges are added in Karate dataset

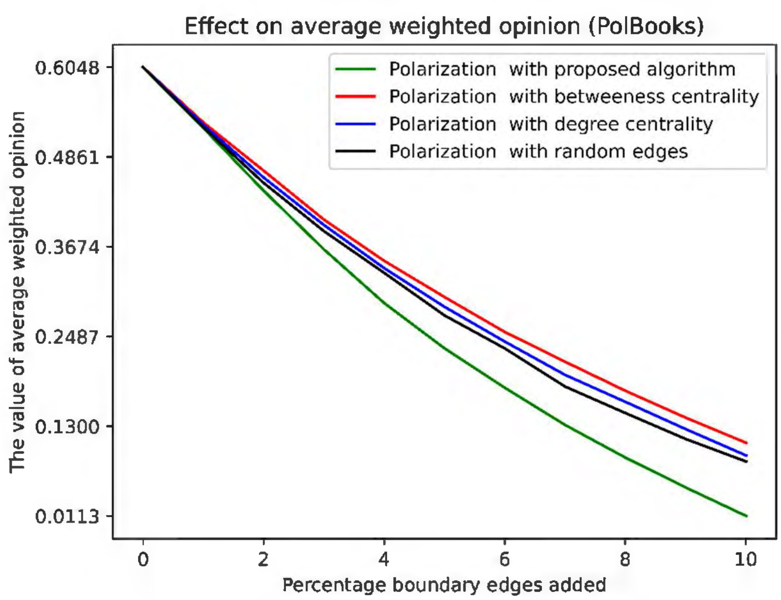


Figure 4.6b: Decrease in the value of average weighted opinions and boundary edges are added in Polbooks dataset

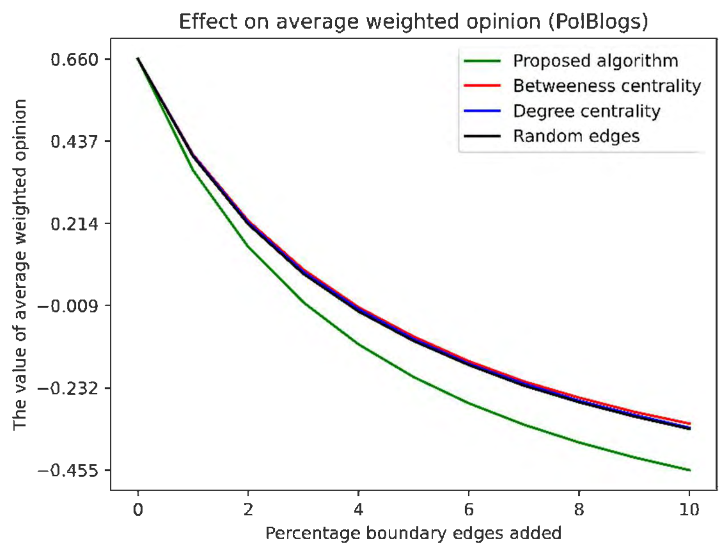


Figure 4.6c: Decrease in the value of average weighted opinions and boundary edges are added in Polblogs dataset

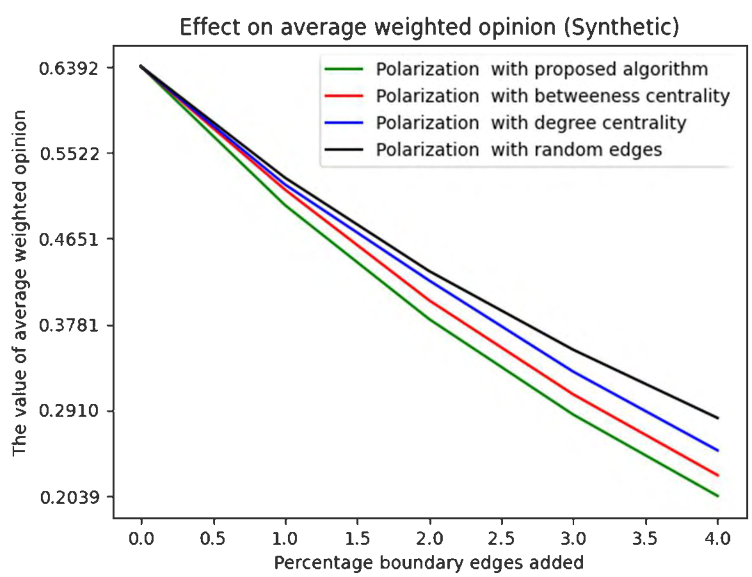


Figure 4.6d: Decrease in the value of average weighted opinions and boundary edges are added in synthetic dataset

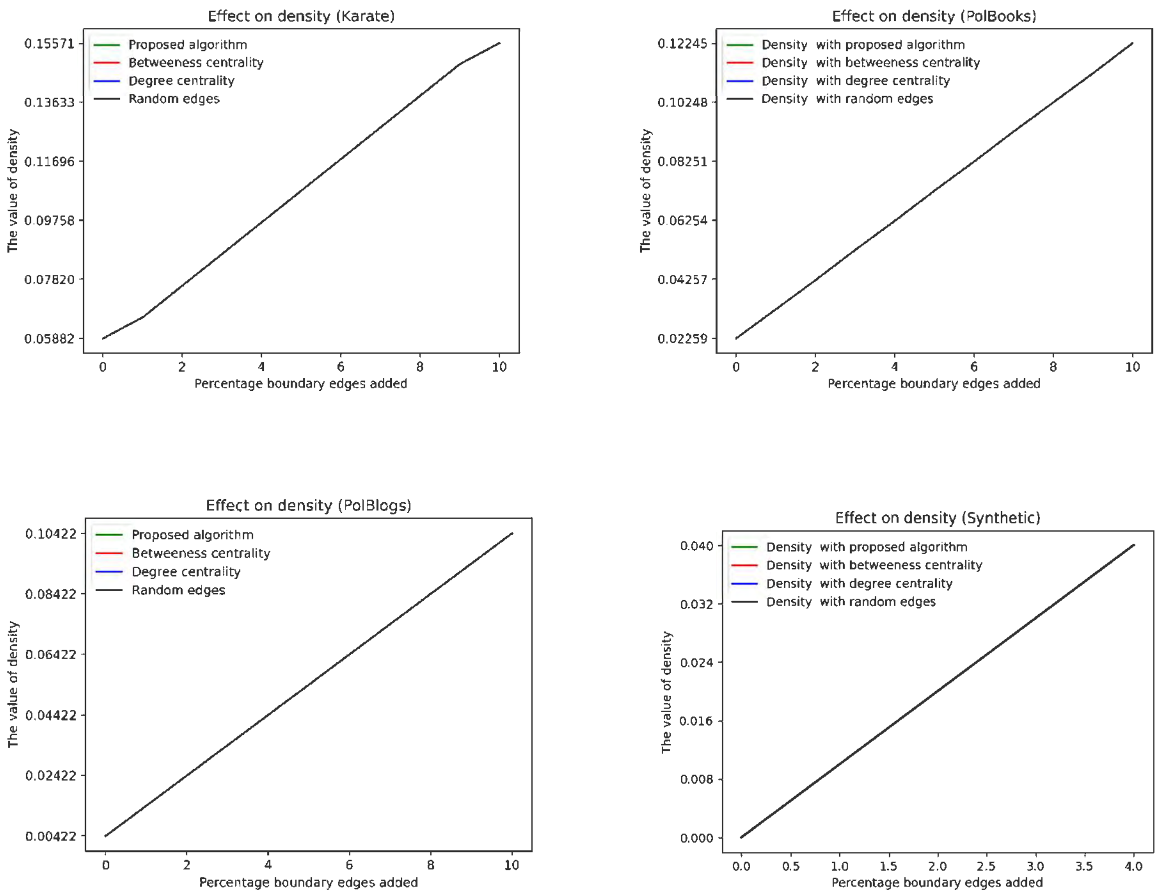


Figure 4.7: Density of given datasets increases in the same fashion for every method

An interesting insight that obtained from Figure 4.7 is that the effect on density remains same for each method applied, this shows that while the same number of edges were added in all cases, the proposed method still outperformed in every aspect in terms of reducing polarization and reducing the value of average weighted opinions.

CHAPTER 5

Method and Results - II

As mentioned in Section 2.5, Genetic Algorithms have been effectively utilized to solve a variety of problems in different domains. In this chapter, the aim is to mitigate the pervasive problem of polarization that characterizes present-day social networks. Utilizing Genetic Algorithms, this section identifies nodes of high influence within a given network. Following this identification, strategic interventions are proposed to foster greater diversity and equilibrium within the network. This approach commences by establishing the social network as an undirected graph G = (V, E), where V signifies the set of individual nodes and E denotes the set of edges characterizing relationships. Each node i within V is attributed with an opinion value o_i, denoting the individual's stance on a given subject. Positive opinion values symbolize affiliation with one group, while negative values signify alignment with another.

Node Selection Using Genetic Algorithm:

To identify influential nodes that could mitigate polarization, a Genetic Algorithm (GA) for optimization is employed.

1. Genetic Algorithm Parameters: The GA is configured with essential parameters, including the size of the population (population_size), the number of generations (num_generations), and the probability of mutation (mutation_rate).

2. Initial Population Generation: The initial population of candidate solutions is generated by sorting nodes based on their degree centrality and selecting the top nodes. These nodes possess a higher likelihood of exerting influence within the network.

3. Fitness Calculation: The fitness of each solution within the population is computed. This involves evaluating the summation of various centrality metrics, including degree centrality, closeness centrality, eigenvector centrality, PageRank, and Katz centrality, weighted by their corresponding opinion values. The fitness function, which quantifies the fitness of a solution (set of nodes) in the genetic algorithm, can be represented mathematically as follows:

Let S be a solution containing a subset of nodes from the network, and let n be the number of nodes in S. The fitness function F (S) can be defined as:

$$F(S) = \frac{1}{\sum_{i=1}^{n} o(i) \,.\, (\sum_{j=1}^{n} (DC(i) + CC(i) + EC(i) + PR(i) + KC(i)))}$$

Where:

$o(i)$ represents the opinion value of node i.

DC(i) is the degree centrality of node i.

CC(i) is the closeness centrality of node i.

EC(i) is the eigenvector centrality of node i.

PR(i) is the PageRank score of node i.

KC(i) is the Katz centrality of node i.

Lower fitness values indicate solutions with greater potential for reducing polarization.

4. Parent Selection: A probabilistic approach is adopted to select parents from the population. The probability of selection is proportional to the fitness score of each solution. Parents are selected for crossover based on their fitness scores. The selection probability of a solution S is determined by its fitness value relative to the total fitness of the population. Mathematically, the selection probability Pselect(S) can be defined as:

$$P_{select}\,(S) = \frac{F(S)}{\sum_{S' \in population} F(S')}$$

Where:

F (S) is the fitness value of solution S.

F (S′) is the fitness value of solution S′.

The summation is performed over all solutions in the population.

These equations provide a mathematical foundation for understanding the mutation and parent selection processes within the genetic algorithm framework.

5. Crossover and Mutation: Crossover is performed between pairs of parents to generate offspring. Subsequently, a mutation operation is applied to offspring with a probability determined by the mutation rate. Mutation involves adding a node to the solution, enhancing the diversity of the population.

The mutation operation introduces diversity into the population by randomly adding a node to a solution with a certain probability. Mathematically, the mutation can be represented as follows:

Let S be a solution (set of nodes), and Sm be the mutated solution obtained from S. The mutation operation is applied with a probability of pmutate. If mutation occurs, a random node i is added to the solution:

$$S_m = \begin{cases} S \cup \{i\}, & with\ probability\ p_{mutate} \\ S, & otherwise \end{cases}$$

Where:

- p_{mutate} is the mutation probability.

- S is the original solution.

- S_m is the mutated solution.

6. Replacement Strategy: Offspring replace the least fit solutions in the population. This process ensures that the population evolves towards more optimal solutions.

7. Influential Node Selection: Ultimately, the most influential nodes are identified by selecting the solution with the highest fitness score. These nodes are anticipated to have a significant impact on reducing polarization.

Opinion Evolution and Polarization Assessment:

Opinions are evolved of the given dataset using the proposed model of opinion evolution as discussed in section 3.1 and then the polarization value is computed using the polarization pointer **β**.

This pointer integrates opinion values and network structure.

1. Polarization Metric: The metric characterizes polarization under varying conditions by iteratively introducing edges between nodes holding opposing opinions. For a range of parameter values k, edges are formed between positive and negative opinion nodes. This encourages interaction among influential nodes.

2. Group Opinion Means and Polarization Parameter: The computed opinions are employed to

ascertain the group means, denoted as gc_plus and gc_minus, for the positively and negatively opinionated nodes, respectively. The difference between these group means is halved to yield the polarization parameter d. This parameter serves as an indicator of polarization intensity.

3. Edge Opinion Sum and Group Interaction: The interaction between groups is quantified

by summing the product of opinions for edges linking nodes across different groups. This calculation is integral to the determination of the polarization metric.

4. Polarization Calculation and Visualization: The polarization metric, incorporating the polarization parameter d, the sum of the product of opinions, and an attenuation factor ρ, is computed. The relationship between network structure, opinion dynamics, and polarization is unveiled through visualization, with polarization values plotted against parameter values k.

5.1 Result and Discussion

This part talks about what was found when a plan was looked at to make disagreements in social media less intense. The discoveries are explained in detail, especially how certain important nodes in the network affect how disagreements spread. The investigation was meticulously structured, encompassing distinct phases of analysis and experimentation. The initial phase of this study entailed the selection of influential nodes, a pivotal task executed through the careful application of the genetic algorithm, the intricacies of which have been thoroughly detailed earlier. The algorithm successfully identified nodes that possessed a pronounced potential for mitigating polarization. These identified nodes were subsequently harnessed as critical components in the ensuing stages of the analysis.

Building upon the identification of influential nodes, a strategy was formulated to produce meaningful inter-group connections within the network. These inter-group connections were established among the most influential nodes that held divergent opinions, orchestrating a deliberate interplay between contrasting stances. The number of connections to make between different groups of opinions, based on all the possible ways they could be connected, was decided. This procedural configuration was methodically tested across datasets, encompassing both synthetic constructs and real-world instances, thus assuring a comprehensive and rigorous assessment. This meticulous evaluation was designed to establish the method's generalizability and effectiveness across a spectrum of scenarios, reinforcing the credibility of the findings made in this work. The discussion revolves around the observations that were made and then details are explained of bringing together pivotal points and connecting opposite groups. Through this study, insights were gained into the mechanisms by which disagreements become less intense in complex social networks.

Polblog dataset

Figure 5.1 shows the effect of adding boundary edges to a social network on the polarization of the network. The x-axis of the graph represents the percentage of boundary edges added, and the y-axis represents the value of polarization. The line graph shows that as the percentage of boundary edges added increases, the value of polarization decreases.

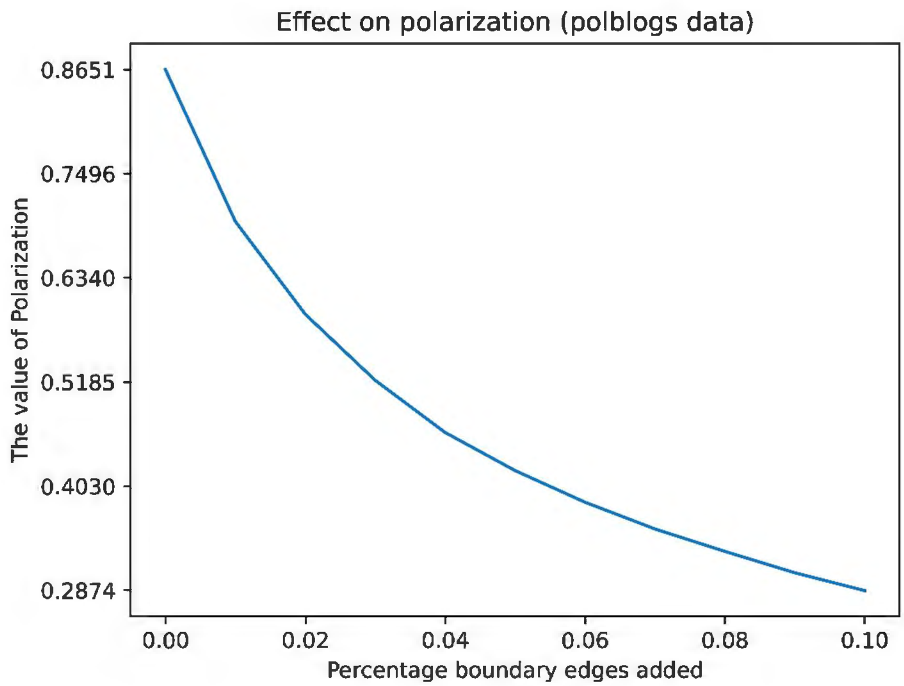


Figure 5.1: Effect of adding boundary edges between most influential nodes in polblogs

In this case, the boundary edges represent connections between nodes that are in different groups.

When these connections are added, it makes it easier for the nodes in the two groups to communicate with each other. This can lead to a decrease in polarization, as the two groups become more aware of each other's viewpoints. In this case, the percentage decrease in polarization is approximately 67%.

Figure 5.1 shows that the rate of decrease in polarization slows down as the percentage of

boundary edges increases. This is because, as more and more boundary edges are added, the network becomes more connected, and it is more difficult for the two groups to remain isolated from each other. Overall, the image provides evidence that adding boundary edges to a social network can be an effective way to reduce polarization. However, it is important to note that the effect is not linear, and the rate of decrease in polarization slows down as more boundary edges are added.

Polbooks dataset

Figure 5.1 shows the effect of adding boundary edges to a social network on the polarization of the network. The x-axis of the graph represents the percentage of boundary edges added, and the y-axis represents the value of polarization. The line graph shows that as the percentage of boundary edges added increases, the value of polarization decreases.

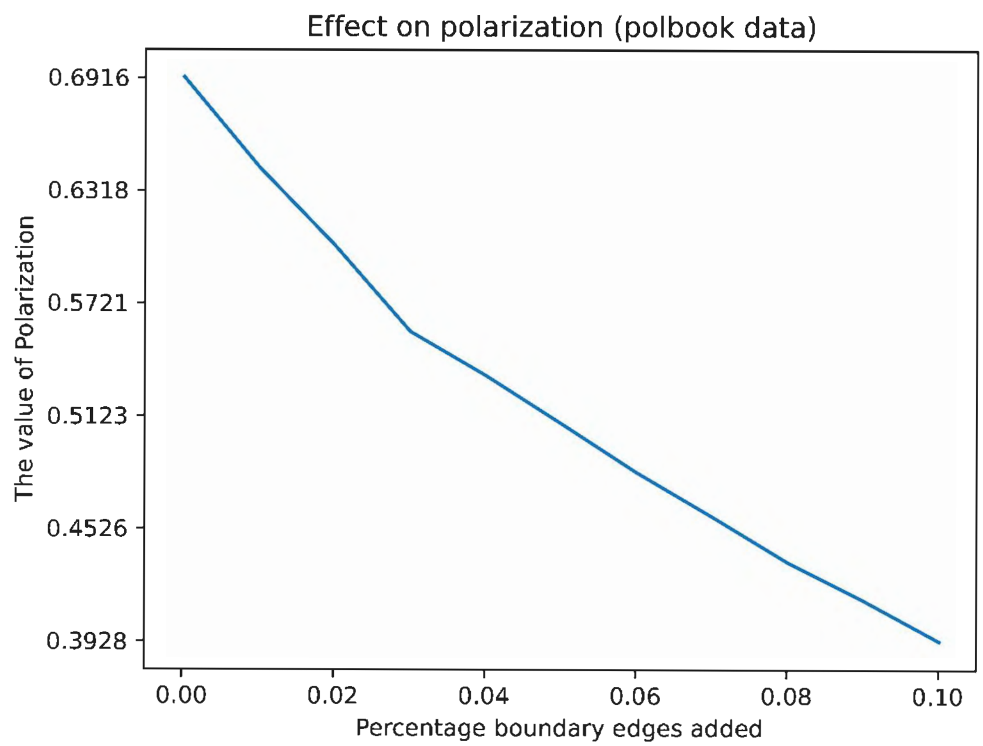


Figure 5.2: Effect of adding boundary edges between most influential nodes in polbooks

The percentage difference in polarization reduction can be calculated by finding the difference between the value of polarization before the boundary edges were added and the value of polarization after they were added and then dividing that difference by the initial value of polarization. In this case, the percentage difference in polarization reduction is approximately 43%.

Figure 5.2 also shows that the rate of decrease in polarization slows down as the percentage of boundary edges increases. This is because, as more and more boundary edges are added, the network becomes more connected and it becomes more difficult for the two groups to remain isolated from each other. Overall, the image provides evidence that adding boundary edges to a social network can be an effective way to reduce polarization. However, it is important to note that the effect is not linear, and the rate of decrease in polarization slows down as more boundary edges are added.

Table 5.1: Percentage reduction and difference for each dataset

	Polblogs		Polbooks	
% edges added	Reduction %	Difference %	Reduction %	Difference %
0	0.00	-	0.00	-
1	19.41	19.41	6.92	6.92
2	11.93	11.93	12.79	5.87
3	7.92	7.92	19.49	6.70
4	5.92	5.92	22.49	3.00
5	4.05	4.05	26.50	4.01
6	3.16	3.16	30.36	3.86
7	3.49	3.49	34.02	3.66
8	3.98	3.98	37.18	3.16
9	3.19	3.19	40.16	2.98
10	3.40	3.40	43.26	3.10

The aim of this research was to enhance network diversity through the implementation of a minimal set of targeted interventions designed to yield optimal outcomes. Results presented in Table 5.1 validate the efficacy of the proposed method, as evidenced by a consistent trend of reduced polarization across both datasets under study. To quantify the impact of the interventions, a comparative analysis was conducted on the state of the network before and after the implementation of the proposed changes. This comparison revealed that the method not only reduces polarization but does so with minimal alterations to the existing network structure. This finding suggests that achieving a more diverse and less polarized network does not necessitate sweeping or disruptive changes. Instead, targeted, minimal interventions can effectively shift the network towards a more balanced state.

Moreover, the speed at which the method moves the network towards this balanced state is noteworthy. This rapid transition is particularly beneficial in scenarios where timely decision-making is crucial, such as during political campaigns or public health crises. In summary, the research successfully meets its overarching goal: to develop an efficient, minimally invasive method for reducing polarization in social networks. The results indicate that the proposed approach can serve as a viable strategy for fostering diversity and balance in various types of social systems.

Chapter 6

Conclusion and Future Work

6.1 Conclusion

This work engages with polarization problem in social networks. While conducting a detailed literature review, to understand the dynamics of this area, major gaps were identified where contributions could be made. It was observed that all available work was addressing only a specific factor of the polarization problem for example, some research bodies were not taking edges into the account while most were ignoring the influence a person can have over the other. Even the solutions proposed were taking substantial assumptions due to which there was a gap between the results obtained and the actual situation. Therefore, people have to compromise on the accuracy of the measurements. This research work, tried to include all those factors which influence the polarization of a network. A systematic approach was adapted to identify those factors. Work was also done on opinion evolution to see how a person changes his or her opinions according to the sorroundings, with time.

These factors were captured in a mathematical model. A gradual working and through process evolved, which ultimately culminated in a state-of-the-art metric, was also shared. A protocol is devised in this research using which two things can be effectively done. 1) Predicting a person's opinion as it changes with time 2) Drafting minimizing polarization as an optimization problem subject to constraint 3) Wholistically measuring polarization in a network by encompassing all factors contributing to it. Results on various data sets supports this claim. The goal was to come up with a comprehensive framework to address polarization. Besides accurately measuring the polarization, optimal solutions were also proposed through which the issue of polarization can be mitigated. The optimal solutions comprised of selecting key nodes through different methods. One method involved the use of Genetic Algorithm for this selection to be optimal. The other method selected nodes based on the intensity of their opinion values. After selecting the key nodes, edges were added between them which was bound to cause a significant decrease in polarization. The effectiveness of this strategy was evaluated by comparing the levels of polarization before and after suggested interventions. The results validated the claims made in this research. A holistic framework is developed for guaging social network polarization as well as a comprehensive toolkit to tackle this problem head-on.

6.2 Future Work

The exploration of the problem of social media polarization has been interesting and significant, yielding many ideas that could help us use technology to promote a more inclusive discussion. Since it's not just about doing more, but better, this work make sure to keep the efforts resource-wise efficient. From here, there are a number of fascinating directions to take:

- More intervention strategies can be devised to further lower the polarization in social networks. Some people are most resistant towards taking in other's point of view so how to keep this in account while reducing polarization as the aim is to spend the resources on people who can be easily convinced. This will help us to keep on evaluating the most useful factors having maximum Return on Investment (ROI).
- We're thrilled about the prospect of tailoring this approach to fit particular circumstances. In some environments, depending on context, certain interventions might be more effective. This adaptability in this strategy can increase the effectiveness of the polarization-busting strategies.

- This work has an impact beyond the realm of social media. It can be investigated how this work could spur innovation in the electronics sector. For example, consider etching this algorithm onto a silicon device. That might greatly increase processing rates and responsiveness, opening up a completely new world.

- AI is a potent ally in the struggle against polarization when it comes to forecasting. Network polarization flare-ups can be predicted by using advanced AI techniques. Then, this Polarization Pointer could intervene to evaluate the precision of these AI forecasts. It's like having a yardstick and a crystal ball in one.
- A consideration of intervention methods would be incomplete without considering their ethical implications. This could shed light on potential benefits and drawbacks, assisting us in using technology responsibly. It provides an explanation for disturbing events like the "Cambridge Analytica" [50] case and suggests ways to avoid them in the future.

- Collaborating with social media platforms would enable us to test our initiatives in a real-world environment. The aim would be to observe our findings come to life on the web and in real-world social media settings.

So there is still plenty to learn about and do to solve polarization in social networks. We're prepared to continue looking further, seeking a more skilled strategy and richer insights.

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