# Spreading Codes Analysis under GA-Assisted MUD for Synchronous MC-CDMA System


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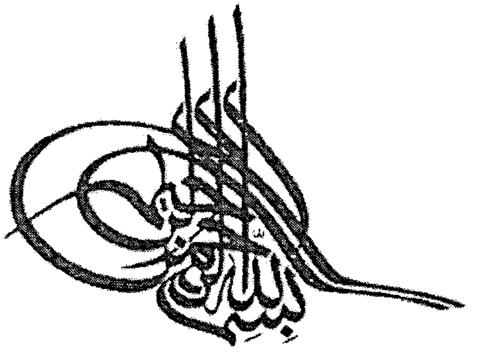
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Dedicated to my parents and teachers

# Abstract

In this dissertation, *receiver optimization techniques* are being investigated into Genetic Algorithm assisted Multiuser Detection scheme for a synchronous, MC-CDMA system.

In multiuser detection, multiple access interference (MAI) is introduced which makes the detection very inefficient. The proposed system is less vulnerable to this hazard in CDMA communication.

In this proposed scheme Orthogonal Frequency Division Multiplexing (OFDM) has been used for attaining frequency diversity gain. Same signal is sent over different carrier frequencies and these carrier frequencies being properly separated in frequency space do not interfere with each other and hence capacity is added up.

In this scheme we also examined the role of Walsh and Gold spreading sequences and demonstrated the results for different number of users. The proposed scheme can perform sufficiently well with very low computational complexity compared to the optimum maximum likelihood scheme with increasing number of users.

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# Contents


Abstract vii

Acknowledgements viii

List of Figures xi

Glossary xii

Chapter 1 Introduction 1

1.1 Introduction to thesis . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1

1.2 Contribution . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2

1.3 Organization . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3

Chapter 2 Multiple Access and Multiuser Detection (Literature Survey) 4

2.1 Introduction to Wireless Technology. . . . . . . . . . . . . . . . . . . . . . 4

2.2 Multicarrier CDMA (MC-CDMA) . . . . . . . . . . . . . . . . . . . . . . . 7

2.3 Basic Multiuser Detector Structure . . . . . . . . . . . . . . . . . . . . . . . 11

2.3.1 Maximum Likelihood Sequence Estimator . . . . . . . . . . . . 12

2.3.2 Linear Decorrelating Detector. . . . . . . . . . . . . . . . . . . . . . 13

2.3.3 Linear MMSE Multiuser Detector . . . . . . . . . . . . . . . . . . 14

2.3.4 Successive Interference Cancellation . . . . . . . . . . . . . . . . 15

2.3.5 Parallel Interference Cancellation Multiuser Detector . . . 16

2.3.6 Genetic Algorithm assisted Multiuser Detector . . . . . . . . 17

Chapter 3 The Genetic Algorithm (Literature Survey) 20

3.1 Introduction. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 20

3.2 Initial Decisions in GA. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 22

3.3 Encoding. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 22

3.4 Initializing a population. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 25

3.5 Defining fitness function . . . . . . . . . . . . . . . . . . . . . . . . . . . . 25

3.6 Population Size. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 26

3.7 Selection Schemes . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 26

3.7.1 Roulette Wheel Selection . . . . . . . . . . . . . . . . . . . . . . . 26

3.7.2 Tournament Selection . . . . . . . . . . . . . . . . . . . . . . . . . 28

3.7.3 Rank Based Selection . . . . . . . . . . . . . . . . . . . . . . . . . 28

3.7.4 Random Number Tournament . . . . . . . . . . . . . . . . . . . . 29

3.8 Crossover Operator . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 29

3.9 Mutation Operator . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 32

3.9.1 Genotype Mutation . . . . . . . . . . . . . . . . . . . . . . . . . . 33

3.9.2 Phenotype Mutation . . . . . . . . . . . . . . . . . . . . . . . . . . 33

3.10 Elitism . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 33

3.11 Population replacement strategies . . . . . . . . . . . . . . . . . . . 34

3.12 GA Termination . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 35

Chapter 4 GA-MUD with orthogonal and non-orthogonal codes 36

4.1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 36

4.2 System Model . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 37

4.2.1 Transmitter . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 39

4.2.2 Channel . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 40

4.2.3 Receiver . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 41

4.2.4 Detection . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 43

4.3 GA assisted Detection . . . . . . . . . . . . . . . . . . . . . . . . . . . 44

4.4 Simulation Results . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 45

Chapter 5 Conclusions 50

References 52

# List of Figures


| | | |
|---|---|---|
| **Figure 2.1** | Wireless access schemes . . . . . . . . . . . . . . . . . . . . . . . | 05 |
| **Figure 2.2** | Classification of CDMA schemes . . . . . . . . . . . . . . . . | 05 |
| **Figure 2.3** | Schematic of decorrelating MUD . . . . . . . . . . . . . . . . . | 13 |
| **Figure 2.4** | Schematic of successive interference canceller . . . . . . | 15 |
| **Figure 2.5** | Schematic of parallel interference canceller . . . . . . . . . | 17 |
| **Figure 3.1** | Basic terminologies of Genetic Algorithms . . . . . . . . . | 24 |
| **Figure 3.2** | Roulette Wheel Selection . . . . . . . . . . . . . . . . . . . . . . | 27 |
| **Figure 4.1** | The transmitter of a MC-CDMA system . . . . . . . . . . . | 38 |
| **Figure 4.2** | Schematic of the GA assisted MUD aided MC-CDMA base station receiver . . . . . . . . . . . . . . . . . . . . . . . . . . . . . | 42 |
| **Figure 4.3** | BER performance of the GA assisted MUD designed for a bit-synchronous MC-CDMA system, using a 32-chip Walsh code. The number of users supported was *K*=10 . . . . . . . . . . | 47 |
| **Figure 4.4** | BER performance of the GA assisted MUD designed for a bit-synchronous MC-CDMA system, using a 124-chip Gold code. The number of users supported was *K*=10 . . . . . . . . . . | 48 |
| **Figure 4.5** | BER performance of the GA assisted MUD designed for a bit-synchronous MC-CDMA system, using a 32-chip Walsh code. The number of users supported was *K*=20 . . . . . . . . . . . | 48 |
| **Figure 4.6** | BER performance of the GA assisted MUD designed for a bit-synchronous MC-CDMA system, using a 124-chip Gold code. The number of users supported was *K*=20 . . . . . . . . . . | 49 |
| **Figure 4.7** | The complexity reduction factor versus number of users | 49 |

# Glossary

| | |
|---|---|
| 2G | *2nd Generation* |
| 3G | *3rd Generation* |
| AMPS | *Advance Mobile Phone System* |
| AWGN | *Additive White Gaussian Noise* |
| BER | *Bit Error Rate* |
| BPSK | *Binary Phase Shift Keying* |
| CDMA | *Code Division Multiple Access* |
| CIR | *Channel Impulse Response* |
| CSI | *Channel State Information* |
| DS | *Direct Sequence* |
| FDMA | *Frequency Division Multiple Access* |
| FFH | *Fast Frequency Hopping* |
| GA | *Genetic Algorithm* |
| GSM | *Global System of Mobile* |
| ISI | *Inter Symbol Interference* |
| ITU | *International Telecommunication Union* |
| MF | *Match Filter* |
| MMSE | *Minimum Mean Square Error* |
| MUD | *Multi User Detection* |
| MRC | *Maximum Ratio Combining* |
| MT | *Multi Tone* |
| OFDM | *Orthogonal Frequency Division Multiplexing* |

| | |
|---|---|
| PIC | *P*arallel *I*nterference *C*anceller |
| SDMA | *S*pace *D*ivision *M*ultiple *A*ccess |
| SFH | *S*low *F*requency *H*opping |
| SIC | *S*uccessive *I*nterference *C*anceller |
| SNR | *S*ignal to *N*oise *R*atio |
| SSS | *S*pread *S*pectrum *S*ignal |
| SUD | *S*ingle *U*ser *D*etection |
| TH | *T*ime *H*opping |
| TDMA | *T*ime *D*ivision *M*ultiple *A*ccess |
| TD-SCDMA | *T*ime *D*uplex-*S*mart antenna aided *CDMA* |
| UMTS | *U*niversal *M*obile *T*elecommunication *S*ystem |
| UTRA | *U*MTS *T*errestrial *R*adio *A*ccess |
| ZF | *Z*ero *F*orcing |

# CHAPTER 1

## Introduction

So far in the literature we have studied many multiple access systems. The one I have selected as a platform for my work is Multi-Carrier Code Division Multiple access (MC-CDMA). In this we merged the concepts of Direct Sequence Code Division Multiple Access (DS-CDMA) and Orthogonal Frequency Division Multiplexing (OFDM). In this way our users are spread in time domain as well as separated in frequency domain because of orthogonal frequencies and since these frequencies are sufficiently separated so the total channel capacity added up and we get maximum frequency diversity gain along with the coding gain.

In scenario when multiple users are communicating over the same channel then at receiver end if we consider only one user as user of interest among all. Then that will be designated as Single User Detection (SUD) and in this way remaining all users' information will be considered as noise. Whilst if we employ the received signal of all the users as a useful information and separate all the users' data, that is known as Multi-User Detection (MUD).

The second approach is very informative and seems very elegant that all the data can be considered as useful information but it costs much more. And we need rather sophisticated techniques to make the things easier as well as robust in nature. So in this way signal processing is the field which is employed.

## 1.2 Contribution

The novel contributions of this dissertation are given below:

i. Genetic Algorithm (GA) [17, 18] is invoked for reducing the complexity of the ML detection based MUD employed in MC-CDMA system. This sub optimum scheme is capable of achieving approximately the same results as ML based MUD, at a significant lower computational complexity.

ii. Investigated the scheme for synchronous DS-CDMA system communication over Rayleigh Fading channel for downlink scenario.

iii. Comparative demonstration using Gold sequences (non-orthogonal) and Walsh codes (orthogonal) as spreading codes for K=10 and 20 users.

iv. Simulation of the complexity reduction factor as compared to optimum detection, versus increasing number of users. This factor is basically the ratio of complexity of optimum detector to proposed scheme's complexity.

## 1.3 Organization

The outline of the dissertation is as follows:

i. **Chapter 2** In this chapter I discussed the wireless access technology along with the vitality and significance of MC-CDMA over single carrier. Further the structure of Multiuser Detectors and their history is discussed in detail. Pros and cons of each scheme are given as well.

ii. **Chapter 3** This chapter is devoted to the Genetic Algorithm, its brief portfolio, detailed terminology and applications.

iii. **Chapter 4** In this chapter a GA-assisted MUD is invoked for Synchronous MC-CDMA system (base station). Simulation results are demonstrated for $K$=10 and 20 number of simultaneous users, using Walsh Codes and Gold Sequences over M=4 orthogonal carriers. Shown the elegance of Walsh Codes (orthogonal) over the Gold Sequences (non-orthogonal). Also complexity reduction factor is simulated versus increasing number of users.

iv. **Chapter 5** In this chapter, the thesis is concluded and future extendable dimensions regarding this scheme are mentioned and references are listed in the end.

# CHAPTER 2

## Multiple Access and Multi-user Detection

### 2.1 Introduction to Multiple Access Technology

Traditional wireless access techniques are consisted of Frequency Division Multiple Access (FDMA) [1], Time Division Multiple Access (TMDA) [1], Code Division Multiple Access (CDMA) [2, 3], and Space Division Multiple Access (SDMA) [4, 5]. The multiple access techniques and their possible types are shown in figure 2.1. In figure 2.1(a) in same frequency and time; signals are separated in space by using different antennas.

The first generation analogue mobile system, which was known as the Advance Mobile Phone System (AMPS), employed the FDMA principle for its access technology. In the TDMA which is shown in Figure 2.1(b) the time-dimension is divided into a number of time slices, where each user is assigned to a dedicated time slice during its communication. A combination of TDMA and frequency-division duplexing (both uplink/downlink) was used in the digital mobile standard, known as Global System of Mobile (GSM) [6], which was the first successful outcome of second generation (2G) mobile communication systems.

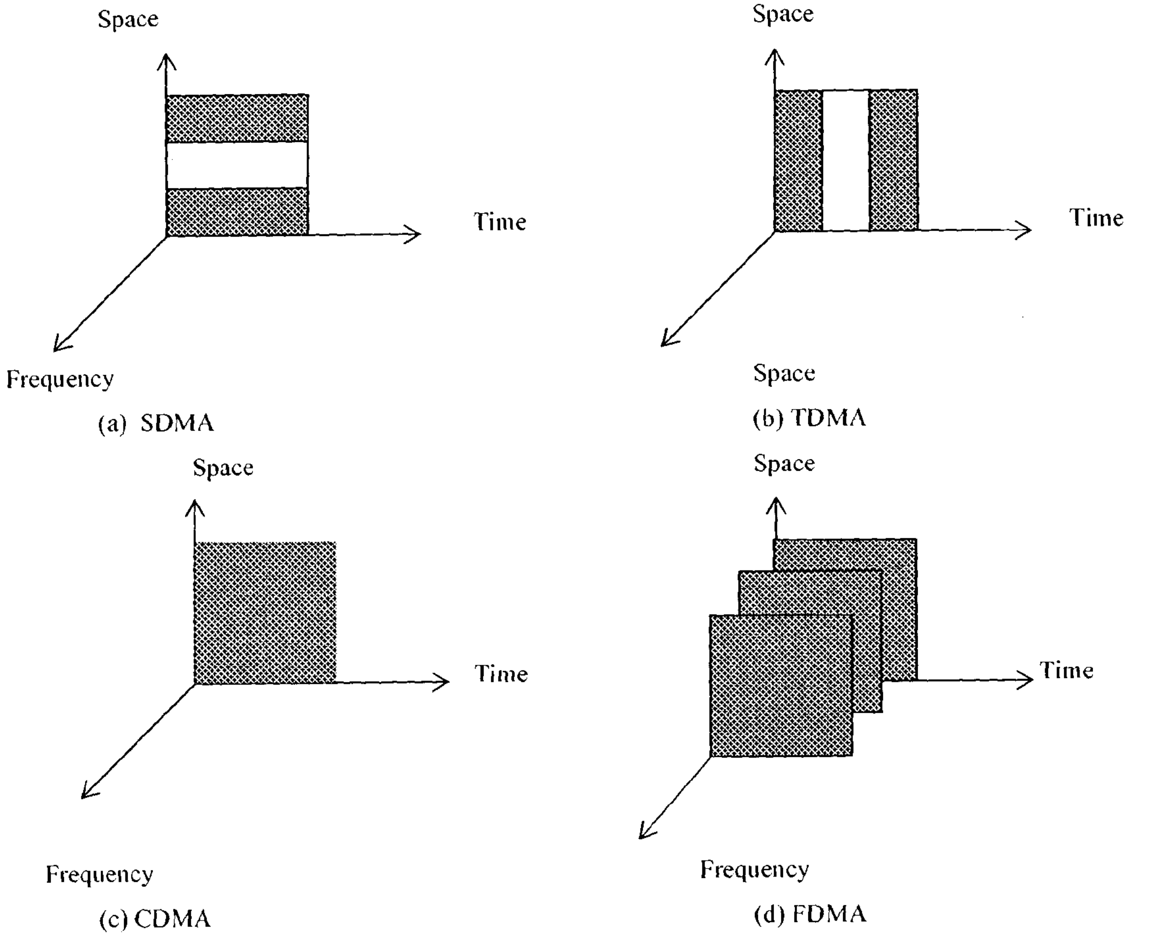



Figure 2.1 Wireless access schemes (Courtesy of University of Southampton)

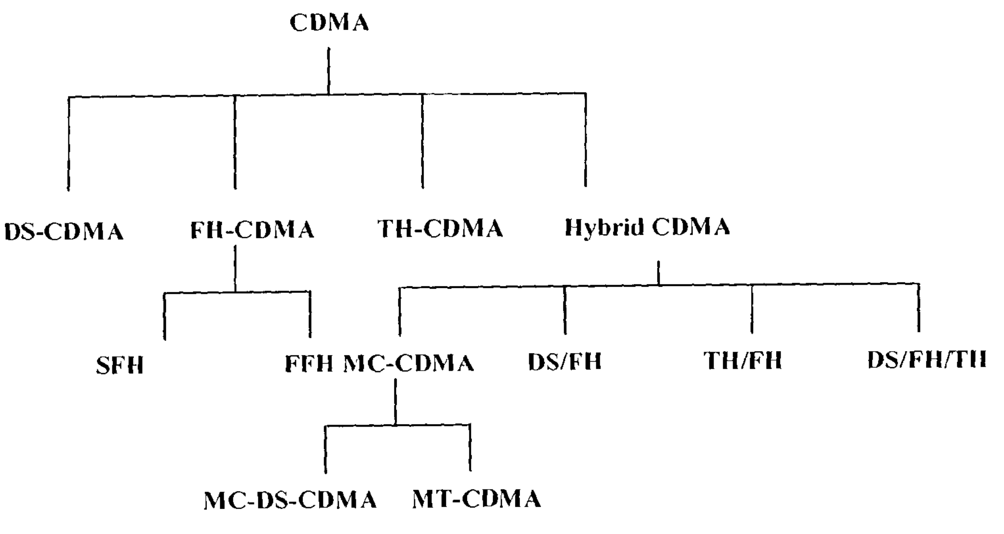



Figure 2.2 Classification of CDMA schemes (Courtesy of University of Southampton)

As shown in Figure 2.1(c) in a CDMA system, all the users share the same frequency bandwidth, all the time, when they transmit, but each user is distinguished by its unique, user specific Pseudo-random Noise (PN) based spreading code. In FDMA, as seen in Figure 2.1(d), we partition the available frequency bandwidth into a number of sub-bands, where each user has a dedicated frequency-band for its communications.

The phenomenon of CDMA originate from military Spread Spectrum (SS) communication principle, where the term "spread" implies that the transmission bandwidth is much higher than that required by the informational signal. There exist several techniques to spread the spectrum of basic information signal (transformation to higher bandwidth). These all are given in the figure 2.2.

The category of spread spectrum communication systems consists of three basic techniques, namely Direct Sequence CDMA (DS-CDMA) [3], Frequency Hopping assisted CDMA (FH-CDMA) [4] and Time Hopping assisted CDMA (TH-CDMA) [7]. Also, as seen in figure 2.2; numerous hybrid CDMA systems can be formed and used by combining the said techniques. In DS-CDMA, a pseudo-random noise sequences is used for directly spreading (modulating) the information bits, which makes a high-bandwidth direct sequence spread signal. In a FH-CDMA system the total transmission bandwidth is divided into a number of sub-bands for the information transfer during successive hops, which are selected at random. So possibly one bit will be sent on different hop and other on different, that is controlled by the frequency hoping pattern; which is no doubt selected at random.

There are further two kinds of FH-CDMA, namely Slow Frequency Hopping (SFH) and Fast Frequency Hopping (FFH). In SFH, the rate of frequency hopping (switching from frequency to frequency) is lower than the information rate (bit rate) means one of more bits can be sent over a single hop. In contrast to SFH, the rate of frequency hopping (frequency switching) in FFH is higher than the rate of information bit stream, which means that a single information bit may be sent over more than one frequency hops. DS-CDMA and FH-CDMA are the most common forms of spread spectrum signaling techniques.

Another spread spectrum assisted signaling technique, which is one similar to FH, is the so-called Time Hopping (TH) technique [7]. In TH, a time that is selected as sufficiently longer than the reciprocal of the information rate ($R = 1/T_b$), is further subdivided into a number of time slices. The information symbols (BPSK, QPSK) are transmitted in pseudo-randomly selected time slices as a frame of one or more symbols.

## 2.2 Multi-carrier CDMA (MC-CDMA)

The terminology of hybrid CDMA comprises a group of techniques that combine two or more of the above-mention spread spectrum techniques. One of these hybrid techniques, known as Multicarrier CDMA (MC-CDMA), is of particular interest in recent many years. Parasad and Hara [1] have given a wonderful overview of MC-CDMA system. Briefly, MC-CDMA scheme may be classified into three major categories

1. Multi Tone CDMA (MT-CDMA) [1].
2. MC-DS-CDMA [3]
3. Frequency domain spreading MC-CDMA [4]

Their common characteristic is that a spreading code is used for spreading user's signal either in time or in frequency domain and that more than one carrier frequency is used for transmission. In this way one is not obtaining the benefits of Spreading Spectrum but also the frequency diversity. In recent years, a number of excellent hybrid CDMA schemes were proposed. For example, SFH assisted MC-CDMA scheme was proposed by Yang and Hanzo [9].

CDMA techniques have been standardized in the regime of several second generations (2G) [12] and third generation (3G) mobile systems [3]. More explicitly, the DS-CDMA technique was first optimized in the context of the Interim Standard-95 (IS-95) [3, 12] in the United States in 1995. In June 1998, the Wideband-CDMA (W-CDMA) [3, 13] and UMTS Terrestrial Radio Access (UTRA) [3, 14] were selected for being used in 3G mobile radio system by the International Telecommunication Union (ITU), both of which are based on DS-CDMA techniques. The cdma2000 standard [3, 15], which is based on MC-CDMA using three subcarriers (frequencies), was also accepted as a prominent candidate for 3G mobile communications by the ITU. Along with all these, the Time-Duplex (TD), smart antenna aided CDMA (TD-SCDMA) [16] system was proposed by China was also ratified by the ITU.

At the end, the least utilized (practically) wireless access method is Space Division Multiple Access (SDMA) [3, 4]. The spatial dimension can be used for users' separation, where according to the terminology of spatial access users are spatially separated from each other. This could be achieved by different sector or smart antennas, which ensures for non overlapping signals and each user is somewhat related to an antenna. So the users' signals are separated in space. As an example, in the TD-SCDMA [16] standard, SDMA techniques are invoked for reducing the Multiple Access

Interference (MAI) which is no doubt a big hazard in detection. The basic ideas of SDMA may be demonstrated with the help of two specific examples.

By using the beamforming techniques [5] spatially selective beams are created for the sake of forming a high-gain antenna beam in the direction of several multipath arriving from the user of interest. This allows the receivers to coherently combine all the useful signal energy arriving by Maximum Ratio Combining (MRC) from the wanted user via a multipath environment. At the same time, the beamformer also creates spatially selective beams for nullifying the multipath components of the interfering users. So the user of interest is enhanced while the interference users are nullified. Whilst this technique potentially reduces the number of resolvable multipath components combined by the beamformer and hence reduces the achievable diversity gain, it reduces the effects of multiuser interference quite significantly and hence has the capability of doubling channel capacity for the users [5].

As a second application of SDMA principles, in [4] multiple antenna elements can be used for supporting multiple users in the uplink MC-CDMA based on OFDM. Since the operating frequency of each antenna is orthogonal so chances of interference are mitigated and hence the capacity is added up. Also the individual users may be identified even within the same bandwidth because of unique user-specific Channel Impulse Response (CIR).

Each of the above access techniques has its own advantages and disadvantages. For example, GSM requires sophisticated frequency / time management for monitoring, which users are supported in which frequency / time slot combination. The benefit of this is that the channels are supposed to be uncorrelated or orthogonal; hence the effects of co-channel interference can be significantly reduced and techniques for minimizing Inter Symbol Interference and fading effects are mostly Channel equalization and frequency hopping. By contrast, an advantage of DS-CDMA is that it has the ability of resolving multipath components with the aid of RAKE receiver as discussed in [10], where each finger of RAKE is associated with a specific user and it takes it as a favored case while rest users as noise.

More specifically, the RAKE-fingers coherently combine the differently delayed, attenuated and phase-rotated signal components that give the diversity gain, which can be increased simply by increasing the number of RAKE-fingers to be combined. However, RAKE combining aided CDMA needs a stringent power control which is also known as near-far problem. In this problem the near users are benefited more than the one located at a distance. So power control could be a best solution. Also CDMA uses power of the orthogonality of the spreading sequences, provided that the orthogonality is not affected by fading

However the orthogonality of codes is really hard to maintain. Especially in the case of uplink scenario where the mobile users are impinging from different sights and co-channel interference really make it impractical. Though downlink scenario implements the same techniques successfully.

In short, CDMA systems suffer from Multiple Access Interference which makes the capacity of CDMA system interference-limited. There are two techniques for detecting the sent signal in multiuser environment.

1. Single User Detection (SUD) where the information for user of interest is considered as of particular interest, while other users' information is considered as noise.

2. Multiuser Detection (MUD) in which other users' signal is considered as useful information as well. So I invoked MUD technique for detection in my proposed system.

## 2.3 Basic Multiuser Detector Structures

Let us have a look at the so-called MUD already been proposed and investigated in literature and their structures. It is convenient to introduce a matrix-vector notation based system model for describing the input/output relationship for MUD. We start with a simple example considering a four-user synchronous system communicating over a non dispersive AWGN channel. The matched filter output related to each user can be written as:

$$\begin{aligned} z_1 &= A_1 b_1 + \rho_{21} A_2 b_2 + \rho_{31} A_3 b_3 + n_1 \\ z_2 &= \rho_{12} A_1 b_1 + A_2 b_2 + \rho_{32} A_3 b_3 + n_2 \\ z_3 &= \rho_{13} A_1 b_1 + \rho_{23} A_2 b_2 + A_3 b_3 + n_3 \end{aligned} \qquad (2.1)$$

Where $A_i$, i=1,2,3 denotes the received signal amplitudes owing to the bits $b_i$, i=1,2,3 transmitted by the three users, $n_i$, i=1,2,3 are the corresponding noise samples and $\rho_{ij}$, i=1,2,3, j=1,2,3 represents the cross-correlation coefficients amongst the user-specific spreading codes. Equation 2.1 may be expressed in a more compact form as follows:

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 1 & \rho_{21} & \rho_{31} \\ \rho_{12} & 1 & \rho_{32} \\ \rho_{13} & \rho_{23} & 1 \end{bmatrix} \begin{bmatrix} A_1 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_3 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} \qquad (2.2)$$

Or as:

$$\mathbf{z} = \mathbf{RAb} + \mathbf{n} \tag{2.3}$$

In the scenario of an $M$-user system, the vector **b, n** and **z** are $M$-dimensional vectors. Where **b** holds the users' transmitted bits, **n** holds the noise and **z** holds the matched-filter output of all $M$ users. The matrix **A** is a diagonal matrix containing the corresponding received signal's amplitudes, while the matrix **R** is an $M \times M$-dimensional correlation matrix, whose entries contain the auto and cross-correlation coefficients of every pair of spreading codes. Note that since the correlation coefficients of the codes satisfy $\rho_{ij} = \rho_{ji}$, the matrix **R** is obviously symmetric. Now we go for the detailed discussion of the structures of MUDs.

### 2.3.1 Maximum Likelihood Sequence Estimator

The Maximum Likelihood Sequence Estimator (MLSE) detects the most likely transmitted $M$-bit vector **b** of $M$-users. The vector **b** is chosen by maximizing the *posteriori probability* of $\mathrm{P}(\mathbf{b} \mid r(t))$, where $r(t)$ is the received signal which is convoluted version of input. It can be read as the probability of receiving particular, given the received signal.

Under the assumption that all $2^M$ possible vectors **b** are equally probable with an equal probability of $2^{-M}$, this detector is referred to as the MLSE. Therefore, this detector does not minimize the bit error probability of any of the $M$-users; it rather minimizes the probability of encountering erroneous $M$-bit vector. The problem associated with the MLSE approach is its enhanced complexity which increases exponentially with number of users $M$. So it is clearly impractical for a real time environment with high number of users. Therefore, numerous reduced-complexity sub-optimum multiuser detectors have been proposed in literature [3, 19].

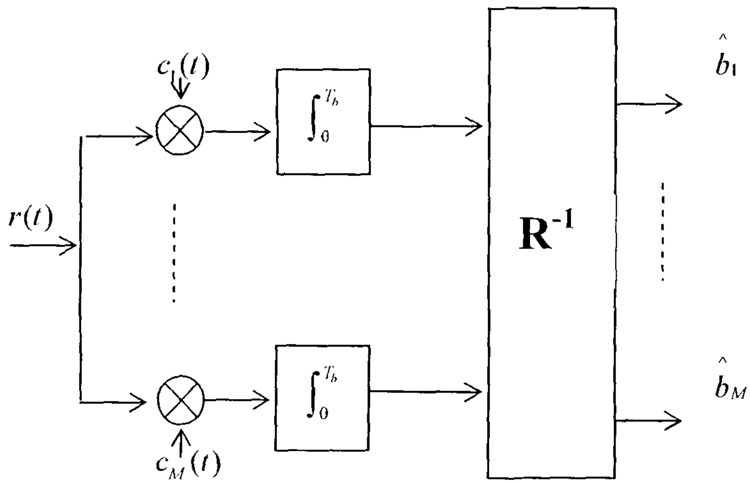



Figure 2.3 Schematic of decorrelating MUD (Courtesy of University of Southampton)

### 2.3.2 Linear Decorrelating Detector

As show in Figure 2.3, the decorrelating MUD applies the inverse of the Cross-Correlation matrix $\mathbf{R}^{-1}$ of the $M$ users' spreading codes at the output of conventional matched filter based detector in order to remove the cross-talk among the $M$ users' spreading codes. Hence, estimated output vector $\hat{\mathbf{b}}$ of the decorrelating detector is given by:

$$\hat{\mathbf{b}} = \mathbf{R}^{-1}\mathbf{z} = \mathbf{A}\mathbf{b} + \mathbf{R}^{-1}\mathbf{n} \tag{2.4}$$

This is consisted of recovered data plus noise term. Here, we can see that the decorrelating detector completely mitigate the MAI terms. The philosophy of this detector is very similar to that of Zero-Forcing (ZF) equalizer [5], which is one of the channel equalization techniques. The advantages of the decorrelating detector are that it:

- Provides significant improved performance compare to conventional SUD
- No near-far problem so does not have to estimate the received signal's amplitude
- It significantly reduces the complexity of optimum detector

However, a disadvantage of this detector is that it amplifies the noise **n** as it can be seen in the equation 2.4. Another disadvantage of this suboptimum detector is that it includes the complexity of matrix inversion **R** that increases with increase in number of users.

### 2.3.3 Linear MMSE MUD

The noise amplification problem of the decorrelating detector is eliminated by the linear MMSE MUD, which minimizes both the effect of the background noise and MAI by using the knowledge of the received signal powers of $M$ users. Moreover, the detector minimizes the expected value (mean) $E[\|\mathbf{Lz}-\mathbf{b}\|^2]$, which is the difference between the sent and received vectors **b** and **z** respectively, and hence the decoupled matrix becomes [19]:

$$\mathbf{L}_{MMSE} = [\mathbf{R} + (\mathbf{N_0}/2)\mathbf{A}^{-2}]^{-1} \qquad (2.5)$$

As seen from Equation 2.5, the MMSE detector uses a modified inverse of the correlation matrix. As a practical measure, the MMSE detector trades off the desire to completely mitigate the MAI with the desire of getting rid off the noise amplification problem. Since the noise is not enhanced in the MMSE detector so it generally provides a better performance over the decorrelating detector. As the background noise reduces, so by putting the term (No/2 ->0) the coefficient of MMSE $\mathbf{L}_{MMSE}$ of converges to the $\mathbf{R}^{-1}$ of decorrelating detector in Equation 2.4.

Drawbacks of this scheme are as under:

- Power estimation
- Complexity even more than Decorrelating detector
- Near-far problem

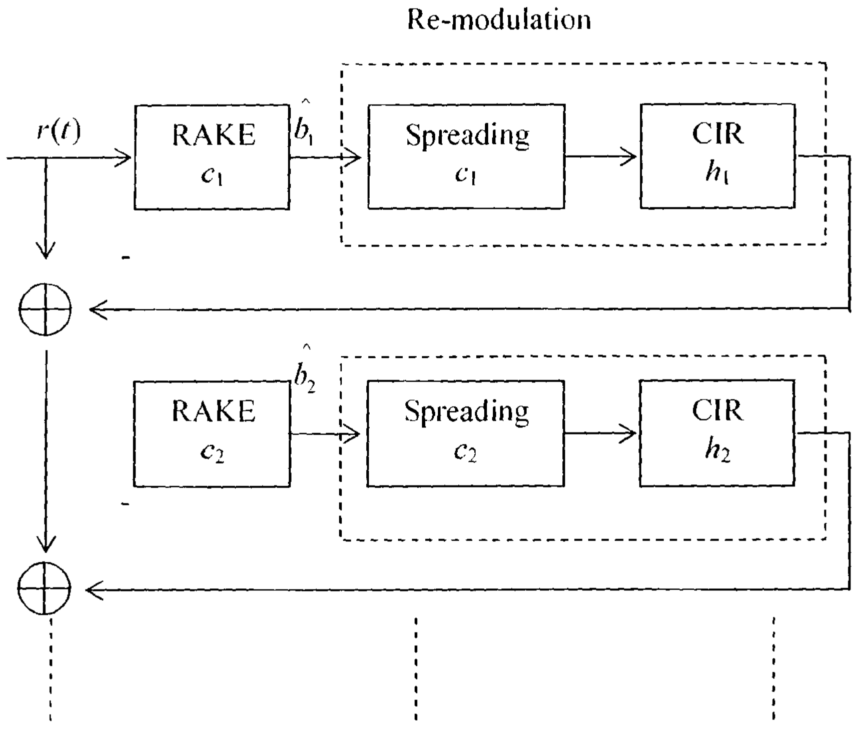



Figure 2.4 Schematic of successive interference canceller (Courtesy of University of Southampton)

### 2.3.4 Successive Interference Cancellation MUD

The schematic of Successive Interference Canceller (SIC) is depicted in Figure 2.4, where all the $M$ users have been ranked with respect to their received signal powers, the user with the highest power is being labeled user-1 and the lowest power as user $M$. After this assignment the signal is processed by the RAKE receiver of user-1 finger. Then this signal is re-spread and reconstructed by taking into account the spreading signature and CIR, so that it becomes compatible with the received signal and then it is subtracted from the received signal.

The remaining signal is then processed by the matched filter or RAKE receiver of user 2 in order to obtain its data estimate. Then again using that estimate we reconstruct the signal and re-spread and again subtract it from received signal. This process repeats and ultimately the last user is processes which actually contains no noise since that is already been subtracted out.

The SIC imposes a significant add-on in performance to the existing MUD with only a little but affordable complexity. The problem associated with this are listed below:

- It offers a delay of one bit after each user is being processed. So the last user M has to wait a lot. So delay must be harmonized with the number of users. Succession could be an overhead.
- All the users must be ranked with respect to their power. So to make sure that ranking is perfect is an overhead.
- Another problem associated with the SIC detector appears; if the first user estimate is not reliable then this will become even mark-able for next users. No matter while rest of the estimates is perfect.

### 2.3.5 Parallel Interference Cancellation Multiuser Detector

This detector works on the same mechanism as that of SIC the only difference is that the estimates are taken, reconstructed and subtracted in parallel so no need to wait for one by one processing. Figure 2.5 shows a single cancellation stage of one user. In each cancellation stage, each user signal is estimated, reconstructed and subtracted from the remaining received composite signal. The benefit of PIC over SIC is that it needs no power estimates to sort the users all the time.

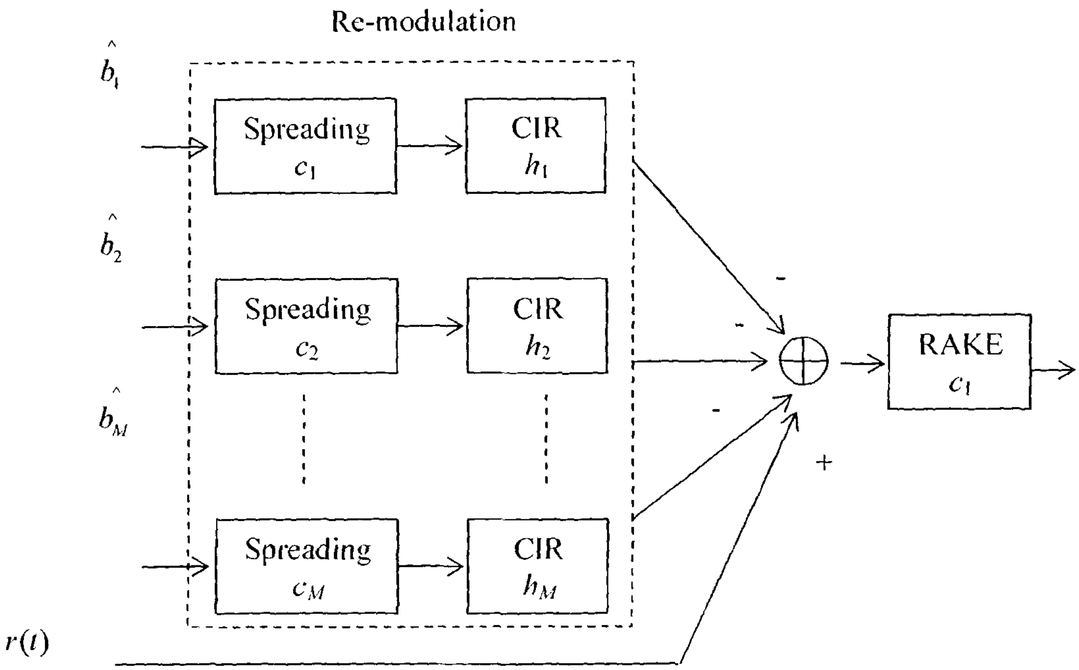



Figure 2.5 Schematic of parallel interference canceller (Courtesy of University of Southampton)

Also being parallel in nature all the users faces same delay. So that is somewhat faster than that of SIC. Also it gives a good performance add-on to conventional MUD with affordable complexity.

### 2.3.6 Genetic Algorithm Assisted Multiuser Detector

Though optimum multiuser detector is capable of achieving a nearly single-user performance by maximizing the following likelihood function [19]:

$$\Omega(\mathbf{b}) = 2\mathbf{bz} - \mathbf{bRb}^T \qquad (2.6)$$

Here the variables used are defined as before. In other words, the multiuser detector will achieve the optimum single-user performance, if it carries out an exhaustive search over the entire search space of the vector **b**. But the related complexity makes this scheme really impractical. So to overcome this problem various optimization techniques are investigated in literature, Genetic Algorithm is one of them [3].

The basic flowchart of GA is shown in Figure 4.3. Let us first assume that the current bit of interest is the $i$th bit of all $M$ synchronous users. GAs starts with the so-called $y$=0$^{th}$ *generation* with an initial population of so-called *individuals*, each consisting of $M$ antipodal bits. The number of $M$-bit individuals in the population is given by *population size P*. Hence, we can express the $p$th individual here as $\tilde{b}_p(y)$.

A *fitness value*, denoted as $f\ [\tilde{b}_p(y)]$ for $p=1,\ldots, P$ is associated with each $M$-bit individual, which is computed by substituting the corresponding individual $\tilde{b}_p(y)$ into a fitness function listed in Equation 2.6. Based upon that, a new population of $P$ individuals is created for the next ($y$+1)th generation through a series of genetic processes, which are referred to in GA terminology as [3, 21]:

- **Selection**

Criteria under which an individual will be selected as best

- **Crossover**

An operation of making new individual from existing ones using some criteria

- **Mutation**

Flipping the bits (genes) of an individual (chromosome) with some probability

- **Elitism**

  Eliminating the weaker individual from mating pool so that it could not participate in next combat

These processes are repeated until the *Y*th generation's individuals are generated. In most cases, the GA is capable of approaching the optimum single-user performance at a fraction of the complexity in comparison to the optimum MUD.

Specially, the case when we are using Walsh Codes, due to their orthogonality they posses less or negligible amount of (Multiple Access Interference) MAI and hence detection is more elegant as compared to Gold Sequences which are non-orthogonal in nature. Also if fading is not there then orthogonality sustains and hence results are really elegant. But unfortunately in real time environments it is seen and practically observed that fading effects distort the orthogonality of codes.

# CHAPTER 3

## The Genetic Algorithm

### 3.1 Introduction

In various scientific fields, optimization plays an important role. Basically there are trade-offs between many factors and to choose the one best is really an art. It is basically need of current era. In many phenomenons it is directly related with the cost of hardware, memory and time. So, best optimization algorithm is the one that gives best performance using least number of resources or complexity etc.

Most of the optimization problems depend on the derivatives of the functions in question or inversion lemmas. The improved estimates of the values that optimize a function are revised from a past estimate and their convergence rate. Even when analytical derivatives are not possible most optimization techniques estimate them by other means. These techniques are useful in many situations but they do not provide the best solution all the time that is why these techniques are known as local optimization techniques. These techniques include RLS algorithm, LMS algorithm, Newton's method etc. In addition to these techniques we have a technique that is based on the Darwin's theory *survival of the fittest*.

This technique has gained recognition and popularity as a powerful optimization technique. It is also accustomed to simulate mostly real and natural phenomenon. GA's have come up as efficient tool to solve optimization problems in a very large multidimensional search space. Like the problem of finding the optimum vector **b**, GA can be used to reduce the search space of Maximum Likelihood Sequence Estimator. GA's can find near optimal solution quickly in a very large search space without getting stuck in local minima and if sometime it is stuck then it can come out with a little more add on to complexity. That is why it is a powerful tool to solve combinatorial problems. Their basic principal is the maintenance of an evolving population (over the generations) of encoded solutions to the problem. The major operations in GA optimization are:

- Selection
- crossover and
- Mutation.

Genetic Algorithms are probabilistic in nature. They are designed to escape from local minima. If they stuck in local minima then some randomness is added so that it seeks for somewhere else. It is also observed that GAs has very fast convergence and approaches to optimal solution quite elegantly. GA's are mostly suggested for the situations where there are a number of parameters to be optimized.

In contrast to differentiable problems GA's have also been found to be a good optimization tool for non-differentiable problems. They have been used successfully in control, signal processing, system identification, Robotics, artificial intelligence etc. GA's have become important optimization tools throughout electromagnetic engineering. They have been applied to the design of devices as diverse as antenna arrays, optical filters, and microwave absorbers. The reason for their popularity is that they are domain independent or general purpose search strategies and are superior in all traditional methods. However it has also been observed that by introducing problem specific knowledge can improve the performance of GA's.

The key features of the GA's are as under:

i. They are very robust in multi-dimensional problems where there is more than one peak in the search space or more coefficients to be optimized
ii. GA's are applied in a situation where rest of the techniques are flop or fail
iii. GA's perform robustly even in a noisy environment in communication system
iv. They are efficient candidates to work on parallel machines (parallel processing)

The only problem associated with GA is their nature of getting trapped in local minima and some time they might miss the global minima.

## 3.2 Initial decisions in GA

Before running a GA, the following decisions should be made:

i. The choice of a genetic representation of the string, candidate solution, i.e. encoding scheme to be used like BPSK or QPSK in communication
ii. The choice of the way to create the initial population of solutions like by applying mutation operator or so on.
iii. Determination of the fitness function that will describe a chromosome fitness
iv. The choice of the crossover operators which are responsible for creating the off springs during evolution
v. Setting of system parameters like, population size, number of generations probabilities of the application of genetic operators like mutation probability.

Following are the discussion of each field.

## 3.3 Encoding

The very first step in using GA's for optimization is to decide the encoding scheme. All those parameters which are needed to be optimized are firstly encoded into a finite length string. Each set of parameters to be optimized is encoded as a string of N characters called genes. Genes are cascaded to form the chromosomes of length L. Chromosomes construct a population. So the population may be seen as a two-dimensional array with N rows and L columns. Fundamental theory of GA's suggests binary encoding like Binary Phase Shift Keying (BPSK) in out dissertation. So in this case the genes are consisted of $\pm 1$.

In proposed scheme all the experiments are carried out for BPSK (Binary Phase Shift Keying) where the string is consisted of binary values.

Other encoding schemes are also available but not used extensively. These schemes include the gray codes, real numbers and combination of binary [5] and real numbers [6], Quadrature PSK (QPSK). The encoding step requires advance knowledge of desired resolution to be used and the lower and upper variable limits. There are two ways to decode the encoded string back to its actual parameter value. Linear transformation of the string in the given range of the parameter. Here the value of the parameter is mapped linearly between some user-determined minimum ($x_{min}$) and maximum ($x_{max}$) values according to the following equation:

$$x = x_{min} + \frac{x_{rep}}{(base-1)}(x_{max} - x_{min}) \qquad (3.1)$$

Where $x_{rep}$ represents the decimal value represented by a l character sub string, base is the base of the number system being used. The values of $x_{max}$ and $x_{min}$ are determined by the user based on the personal knowledge of the problem.

Converting the string back to its decimal value without any scaling process ($x_{rep}$). In this case the first position is devoted to the sign of the parameter and the remaining part is interpreted as the value of the number.

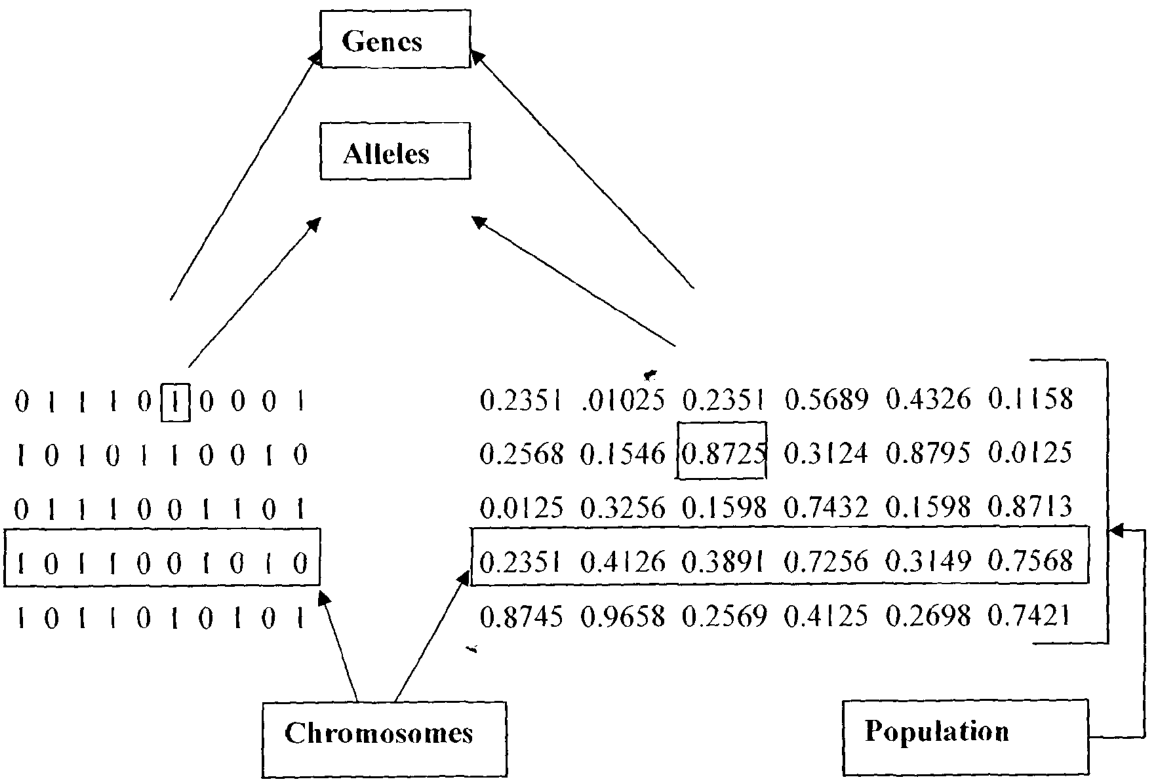



Figure 3.1 Basic terminologies of Genetic Algorithms (Courtesy of [5])

The decimal representation may be real encoding scheme, where a chromosome is a string of real numbers. In Integer representation scheme the chromosome is composed of strings of integer numbers. There are two binary encoding schemes being widely used.

i. **Binary encoding scheme**

Has been widely used in simple genetic algorithms like BPSK

ii. **Gray Coding Scheme**

Gray cods are sometimes a good choice for GA's. Gray codes are essentially the binary numbers but the adjacent numbers differ by one bit. By eliminating Hamming cliff of binary coding Gray codes might improve the performance of GA.

## 3.4 Initializing a population

There are many ways to generate the initial population. Since GA are designed for a global scope so their dependency is not on initial values. But somehow if we want to improve the results then initial conditions may be helpful. As in our example the received maximum ratio combined vector is mutated enough to take first generation.

## 3.5 Defining Fitness function

The fitness function is used to measure the goodness (power or fitness) of the Chromosomes, which is related to the objective function that is what a powerful chromosome should have. Fitness is a absolute quantity. The formation of fitness function is of particular importance on GA. Fitness function must be able to provide a good distinction between good and bad chromosomes. The survival of a good chromosome is dependent on the goodness and accuracy of the objective function. It must be noted that the objective function and the encoding are the only two direct relations between GA and the concrete application.

## 3.6 Population Size

Population size plays an important role in GA, since it is the feature that gives diversity in the chromosomes and make sure to induce the one which is fittest [17]. The population size increases with increase in the chromosome length. Care should also be taken to reduce the number of computations. Usually the number of chromosomes in a population is problem dependent. With a small population the GA might not provide the optimized result and may be stuck in to local minima. Also the number of generations must be sufficient so that the best possible optima may be achieved. Again this is problem specific. Both population size and number of generations are problem specific and complexity is measured by the two.

## 3.7 Selection Schemes

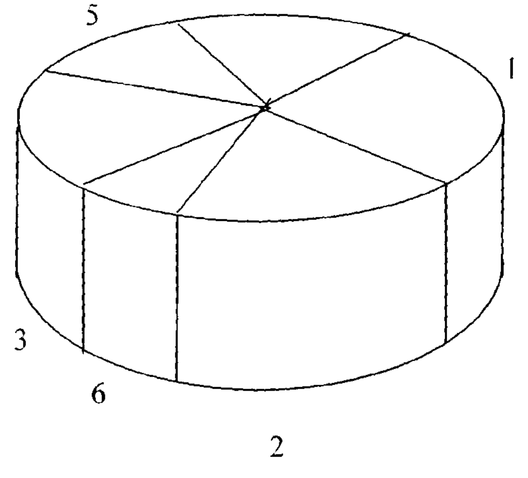
Selective reproduction is one of the main operators of GA [18]. It involves selection of the chromosomes with largest fitness values. These best chromosomes are then used to generate the next population. The probability of selecting a given chromosome to take part in mating is related to its fitness. There are a number of selection schemes. Some of them are listed below.

### 3.7.1 Roulette Wheel Selection

This selection scheme is fitness proportionate selection scheme in which the slots of a roulette wheel are sized according to the fitness of each individual in the population. The wheel is spun. The chromosomes with greater fitness take more space on the wheel and thus have a greater probability of being selected. In this way the parents are selected and copied to a temporary space. The chromosomes thus selected are used to generate the next generation through cross over. The probability of selecting of an individual i, $P_i$ to take part in mating process using Roulette wheel selection scheme is given by,

$$P_i = \frac{f_i}{\sum_{j=1}^{N_p} f_j} \qquad (3.2)$$

Where $f_i$ is the fitness of he individual. The drawback of this scheme is that the best chromosome in the generation may be lost in any generation. This problem can be tackled by using elitism operator, which retains the best chromosome in the population. This selection scheme is implemented as follows. Calculate ft(sum of all fitness values of the string in the population, form ft slots and assign string to the slots according to fitness value of the string. Generate a random number between 1 and ft and use it to index into the slots to find the corresponding string. Copy this string into the mating population. Process is repeated till the mating pool has equal size as the original population.




1: Most fit number

6: Least fit number

Figure 3.2 Roulette Wheel Selection (Courtesy of [5])

### 3.7.2 Tournament Selection

This is another method widely being used in GA. In this method two (or more than two) individuals are selected at random from the current population and the chromosome with large fitness among the two (or the group of chromosomes) is selected. This chromosome is the first parent. The process is repeated to select another parent. These two chromosomes generate two children by mating. Tournament selection has two variants

i. **Tournament selection without replacement**

   Here two individuals are set aside for the next selection operation and they are not replaced into the population until all other individuals have been removed. Since two individual s are removed from the population for every individual selected and the population size remains constant for one generation to next, the original population is restored after the mating population is half filled? The process is repeated for the second time in order to have the mating population of the same size as the original population. In this way, the best individual will be selected twice and the worst individual is not selected at all.

ii. **Tournament selection with replacement**

   In this scheme both individuals are placed back into the original once the better of the two individuals has been selected. This process is repeated until the size of the mating subset is equal to the population size.

### 3.7.3 Rank Based Selection

This selection scheme chooses a specified number of individuals with highest fitness and this subpopulation is used to generate the next generation. This scheme is useful when there are a small number of individuals in a population with very high fitness.

### 3.7.4 Random Number Tournament

This scheme is similar to the previous tournament method but instead of selecting the better individual of the two chromosomes randomly selected, one chromosome is chosen randomly and a random number replaces the other chromosome. In other words, a chromosome with fitness f is chosen from the population at random and a random number r (with $0 < r < 1$) is generated. The randomly selected chromosome is then allowed to take part in the mating if the inequality $f >$ r is satisfied otherwise this chromosome is placed back into the population and another chromosome is randomly selected. This process continues until a second individual is found.

The selection pressure in GA's is the driving force that determines the rate of convergence. A high selection pressure will cause the population to converge quickly, possibly at the expense of sub-optimal result. Roulette wheel selection scheme typically provides the highest selection pressure in the initial generations. Tournament selection provides more pressure in the later generations when fitness values of the individuals are not significantly different.

## 3.8 Crossover Operator

Crossover is one of the basic GA operators. Crossover is partial exchange of the two parent chromosomes to make two off springs [17]. This is accomplished by cutting and the splicing the two parent chromosomes at a specific location and exchanging the bits of the two individuals from the point of crossover onwards. Crossover takes place with specified probability of crossover; $P_c$ [18]. Crossover can be divided in tow broad categories depending on the exchange of the bits among the parents, crossing over.

**i. Genotype (Conventional) Crossover**

Where we define a number of crossover points depending on the type of the cross over used.

**ii. Phenotype (Arithmetic) Crossover**

In this method an appropriate weighted average of the phenotype (the actual parameter values) of the two parents $X_a$, and $X_b$ is found leading to the offspring $X_0$ with

$$x_0 = (1-\lambda)x_a + \lambda x_b \tag{3.3}$$

Where x represents each of the parameter in X. λ is in the range –2 to 2. It may depend on the relative values of fitness of the two parents. The number of children depends on the number of different λ used. For example to generate three children out of two parents λ might be 0.5, -0.5 and 1.5. The conventional crossover fall in the following categories:

**a) Crossover single point**

In this way a single point somewhere in the chromosome is selected at random and two chromosomes interchanges their respective genes to produce two new offspring.

**b) Crossover two points**

In this method two points (either random or fixed) are taken with in two chromosomes (parents). And then they interchange their genes among these points. This is more vital way to get new stronger chromosomes.

c) **Multipoint crossover**

In this method *m* different points are selected along the length of entire chromosome. Then choosing different fragments a lot of children can be produce. This is even better than previous techniques.

d) **Uniform Crossover**

After a parent pair is drawn each chromosome position is crossed with some probability typically one half. Each corresponding pair of genes exchanges their values independently with the same probability of 0.5.

e) **Crossover heuristic**

In this mechanism that area of the chromosome is selected for crossover; where more vital genes are present. After marking that area in two or more chromosomes; offspring are generated.

A random crossing mark is implicitly generated with the probability of 1 at any position typically being set to one half. Characters from the parent strings having ones at the corresponding positions in the crossing mask are swapped in generating the child strings and the remaining characters remain in tact. This scheme has the advantage that it has no position bias. It provides a wise way to alleviate undesirable position bias found in the previous three methods. It should be remembered that the Crossovers could be performed at different levels; hence there are different possibilities.

1. **String Level**

The above-mentioned methods are all string level methods.

2. **Parameter level**

For some applications this type of crossover has proved to be much more useful. In this method genes are swapped among the parents to form new generation. The minimum entity of interest is a gene not a bit.

3. **Exchange of the parameters**

Genotype between the crossovered chromosomes at the same parameter position without any change in the existing parameters combination. The number on sun strings to be exchanged in one crossover is randomly determined. When the number is greater than one then these sub strings are randomly selected. However as crossover operation does not change the value of any parameter in individuals which survive in the population, mutation operation rate should be at least 50% or even higher.

## 3.9 Mutation Operator

This operator ensures diversity in GA's'. Once all the off springs have been generated, the mutation operator operates them upon. Like crossover operator, the application of this operator is also based on probability. But the probability of mutation is small. Mutation protects GA from converging to non-optimal solution. The probability of mutation is set as accordance with the demand and also there is a pseudo randomness which should not repeat. Let us have a vector of 10 bits the possible mutated versions of this vector would not more than 1024. It provides new search space to the GA because for a given population certain regions of parameter space might not be accessible through the crossover procedure alone since it does not introduce new genes into the population. Mutation can be applied either at the parameter level or at the gene level. Mutation can be performed in one of the following types.

### 3.9.1 Genotype Mutation

**i. Random Mutation**

It is applied in the case of non-binary representation scheme. It falls in two categories, static and dynamic.

**ii. Bitwise Complement mutation**

It is applied on bit strings. It selects a bit randomly in a chromosome and inverts it. It occurs with a probability as low as 0.001.

### 3.9.2 Phenotype Mutation

**i. Static Mutation**

Assigning completely new values to one or more randomly selected parameter in the selected parent carries out.

**ii. Dynamic Mutation**

In dynamic mutation the selected parameters are subjected to random displacements form their original values in the parent structure.

## 3.10 Elitism

The individuals with high fitness are no doubt very important in GA optimization. These high fitness individuals may get lost during crossover operation, which may cause loss to the optimization. So there must be some way to preserve high fitness individuals. This is done by using Elitism operator. One elitism strategies try to pass more than one copy of the best member of the population. Another strategy is to maintain a pool of individuals through generations for occasional reintroduction into the population. This tool is quite useful when the GA tend to stagnate. There are two possibilities for applying Elitism.

a) The best individuals of one population are selected in a pool and are carried as a separate pool as newer and newer generations are generated.

b) The second possibility is to add the best member of a population to the next population without any selection or crossover.

## 3.11 Population Replacement Strategies

After generating the offspring population, using the genetic operators on a parent population there are different methods available for choosing the next generation. These are as follows:

i. **Generational Replacement**

In this method, the parent population is totally replaced by the newly generated off springs. This technique is also known as non-overlapping replacement. For this type of replacement the size of two populations are almost equal since the offspring population replaces the parent generation completely, except when elitism is used. However if e elite members are entered into the offspring population, the same number of worst individuals may be deleted form the offspring population beforehand, thus leaving the population constant.

ii. **Overlapping Replacement**

In this method, both the offspring population and some or all of the parent population compete for existence in the next generation. The size of offspring population generated by crossover and mutation might be equal or smaller than the parent population depending on the generation replacement and the level at which elitism is applied (if it is used). There is no restriction on the size of offspring population.

## 3.12 GA Termination

There are different criterions under which GA may be stopped or terminated. Here are a few of them.

i. **On the basis of Time**

The prescribed number of generations has been explored. This is done when we are careful about the computation time. The progress limit is also a stopping criterion. Under this condition GA is terminated when the maximum number of generations that are allowed to be generated successively such that the improvement in the fitness value of the best member of the population is less than some predefined threshold.

ii. **On the basis of precision**

The fitness of the best member of a population exceeds a threshold value. The average fitness of the population reaches a desired value. Any other convergence criterion is fulfilled. Once the GA is terminated, optimal solution can be found either by choosing the best chromosome or weighted average of the entire population.

**Note:** *The major part of material discussed and presented in this chapter is taken from the literature since that is a field of study being utilized for receiver optimization, so its introduction was compulsory in order to get the soul idea behind this technique before going to the main part of dissertation.*

# CHAPTER 4

## GA-MUD with orthogonal and non-orthogonal codes

### 4.1 Introduction

After developing sufficient ground work and related survey of literature, this is the point to start the actual part of dissertation. The scheme is composed of different constituents listed below. The basic theme is to use Multicarrier Code Division Multiple Access, which is a nice merger of DS-CDMA and Orthogonal Frequency Division Multiplexing. Actually this is not the only combination. DS-CDMA can be mixed with even simple FDM (non orthogonal) and that is also known as MC-CDMA.

But in that way proper benefits of the scheme are not utilized since, what we achieve from orthogonality is non comparable. This is the way that total channel capacity is added up. And all the frequencies being adequately separated (orthogonal) does not interfere with each other and hence the signal recognition is easier comparatively. Each user bit is sent over different frequency carrier. So this also exhibits frequency diversity.

So in case of loss of one carrier signal other can equip it and by using Maximum Ratio Combining at the end would help in detection of bit at receiver. Also the benefit of Direct Sequence is blended by spreading each user bit before being modulated, with different signature. Also these signatures differentiate further with other carrier signature. And hence give a dual benefit of spreading and OFDM.

Another thing which is added up further is orthogonality of signatures, which is main theme of this dissertation; and it gives a further enhancement in detection and makes the things easy. And at the end of the day Genetic Algorithm helps in reducing the complexity since things are getting robust but with increase in complexity but GA helps to cut it down. And ultimately a very refine version of idea is built up.

## 4.2 System Model

I have suggested a system with *N* number of users being communicating over the same channel. The channel considered here is Rayleigh fading channel with know Channel State Information (CSI). It is assumed that all these users are simultaneously transmitting the data in a bit synchronized fashion. Each bit of each individual user is being spread using different spreading codes each of length *L* chips.

Now this spread bit is modulated over a set of frequencies *P* which are assuming to be orthogonal. And the spreading code for each frequency is different so the signals are not only orthogonal in time domain but in frequency domain as well. Hence that dual orthogonality helps in separation and demodulation of received composite signal. It also gives frequency diversity gain by using Maximal Ratio Combining (MRC). Hence the total coding gain along each frequency channel is added up and it will be a total of *LP*. So over here we need total of *P* spreading codes with each of length *L*.

The modulating frequencies being used are $\omega_1, \omega_2, \omega_3, ....., \omega_p$. With the fact that all these frequencies are separated by minimum *1/T* interval so no overlap occurs. Hence the capacities will be added up end of the day. Both time and frequency domain orthogonality works as a double edge sword to fight with the impacts of noise and channel distortions. In this work I have utilized both types of codes in time domain namely the Walsh Codes, which are orthogonal in nature and Gold Sequences, which are non-orthogonal but practical ones. Since it is hard to find orthogonal codes for excessive number of users and we needed here a total of $MP$ codes with length $L$.

Following are the components of the system assumed.

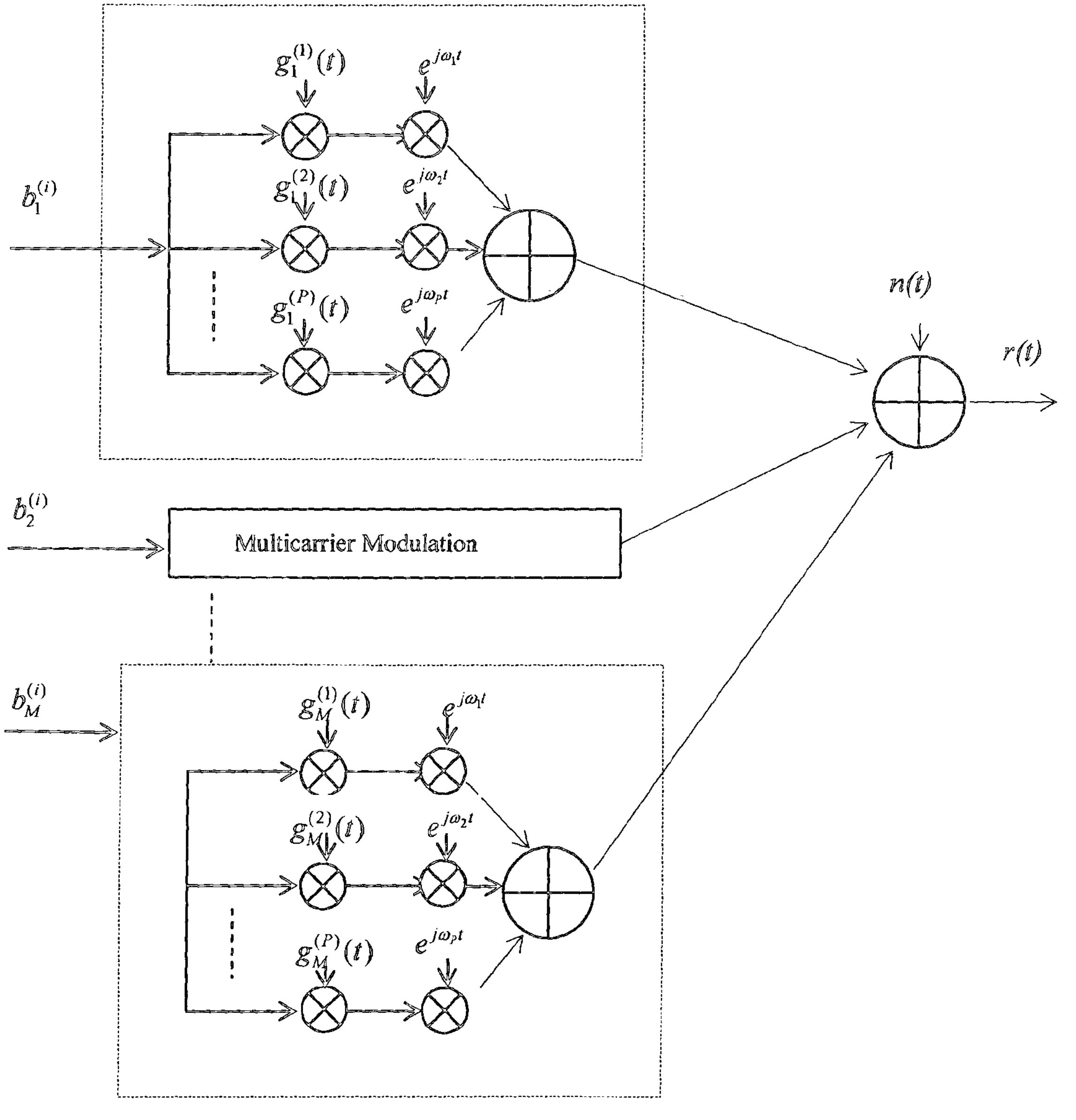



Figure 4.1 The transmitter of a MC-CDMA system

### 4.2.1 Transmitter

We consider a bit-synchronous MC-CDMA system illustrated in Figure 4.1. Here M numbers of simultaneous users are communicating over the same channel. Observe in the figure that the *i*th bit $b_1^{(i)}$ of the *m*th user is spread to *P* parallel subcarriers, each conveying one of the *P* number of *L*-chip spreading signature sequences $g_1^{(p)}(t)$, $p$ = 1, . . ., *P*, each of which spans over (0, $T_b$) interval in time and we have $T_b / T_c = L$, where *Tb* and *Tc* are the bit duration and chip duration, respectively. Each of *P* spreading signatures is mapped onto a different frequency carrier.

In other words, a single-carrier system occupying the same bandwidth as the multicarrier system considered would use a spreading signature having *LP* chips/ bit, and both of these systems have a processing gain of *LP*.

Hence, the transmitted signal of *m*th user associated with the *p*th subcarrier can be expressed in an equivalent low pass representation is given in Equation 4.1; composite signal of *m*th user over all subcarriers can be represented by Equation 4.2; composite signal of all users over *p*th subcarrier can be represented by Equation 4.3; and composite signal of all the users over all the subcarriers can be represented by Equation 4.4.

$$s_m^p(t) = \sqrt{\frac{2\varepsilon_{b,m}}{P}} g_m^{(p)}(t) b_m^{(i)} e^{j\omega_p t} \tag{4.1}$$

$$s_m(t) = \sum_{p=1}^{P} s_m^p(t) = \sum_{p=1}^{P} \sqrt{\frac{2\varepsilon_{b,m}}{P}} g_m^{(p)}(t) b_m^{(i)} e^{j\omega_p t} \tag{4.2}$$

$$s^p(t) = \sum_{m=1}^{M} s_m^p(t) = \sum_{m=1}^{M} \sqrt{\frac{2\varepsilon_{b,m}}{P}} g_m^{(p)}(t) b_m^{(i)} e^{j\omega_p t} \tag{4.3}$$

$$s(t) = \sum_{m=1}^{M} s_m(t) = \sum_{m=1}^{M} \sum_{p=1}^{P} \sqrt{\frac{2\varepsilon_{b,m}}{P}} g_m^{(p)}(t) b_m^{(i)} e^{j\omega_p t} \tag{4.4}$$

where $\varepsilon_{b,m}$ is the $m$th user's signal energy per transmitted bit, $b_m^{(i)}$ belongs to (1, -1) the antipodal signaling symbols, where total number of users are $m = 1,\ldots, M$ and '$i$' denotes the $i$th transmitted bit of $m$th user, while the $m$th user's signature waveform is $g_m^{(p)}$, $p = 1,\ldots, P$, $m = 1, \ldots, M$; on the $p$th subcarrier, which again has a length of $L$ chips, and can be written as:

$$g_m^{(p)}(t) = \sum_{n=1}^{L} g_{m,n}^{(p)}(t) q(t - nT_c), \tag{4.5}$$

Where $T_c$ is the chip duration, $L$ is the number of chips per bit associated with each subcarrier and we have $T_b / T_c = L$ as the coding gain. Again, the total processing gain in $LP$, while $q(t)$ is the rectangular chip waveform employed, can be expressed as:

$$q(t) = \begin{cases} 1, & 0 \le t < T_c \\ 0, & otherwise \end{cases} \tag{4.6}$$

Without loss of generality, we assume that the signature waveform $g_m^{(p)}(t)$ used for spreading the bits to a total of $P$ subcarriers for all the $M$ users has unit energy, which can be written as:

$$\int_0^{T_b} g_m^{(p)2}(t)dt = 1, \quad m = 1, 2 \ldots M, \quad p = 1, 2, \ldots, P \tag{4.7}$$

### 4.2.2 Channel

It is assume that signal of each user $s_m^p(t)$ transmitted on the $p$th subcarrier is propagated over an independent non-dispersive single-path Rayleigh Fading channel and where each user face a different amount of fading independent of each other. Hence, the Channel Impulse Response (CIR) of the $m$th user on the $p$-subcarrier can be expressed as: $\alpha_m^p e^{j\theta_m^p}$, where the amplitude $\alpha_m^p$ is a Rayleigh distributed random variable, while the phase $\theta_m^p$ is uniformly distributed over $[0, 2\pi]$. That means it can take on any value in this range with equal probability.

### 4.2.3 Receiver

Having described the transmitter and the channel, the received signal on the $p$th subcarrier can be expressed as:

$$r_p(t) = \sum_{i=-\infty}^{\infty}\sum_{m=1}^{M}\sqrt{\frac{2\varepsilon_{b,m}}{P}}g_m^{(p)}(t-iT_b)\gamma_m^p b_m^{(i)} e^{(j\omega_p t+\phi_m^p)} + \eta(t) \quad (4.8)$$

Here $M$ is the number of users supported and $\eta(t)$ is the Gaussian noise process with a variance of $N_0/2$. Figure 4.2 depicts the receiver end of the proposed scheme. We shall repeat the same steps but in reverse order to get the things back.

Firstly we shall demodulate the signal which is carried out by Matched Filtering (MF) of each of the $M$ users and the outputs of the $M$ users' match filters are input to the GA-based MUD. It is more convenient to express the associated signal in matrix and vectorial formats, when the sum of the transmitted signals of all users can be expressed in vectorial notation as:

$$r_p(t) = \mathbb{G}_p \mathbb{H}_p \mathbb{A} \mathbb{b} + \mathbb{\eta}, \quad (4.9)$$

Where we have,

$$\mathbb{G}_p = [g_1^p(t), \ldots, g_M^p(t)]$$

$$\mathbb{W}_p = diag[\alpha_1^p e^{j\theta_1^p}, \ldots, \alpha_M^p e^{j\theta_M^p}] \quad (4.10)$$

$$\mathbb{A} = diag[\sqrt{\frac{2\varepsilon_{b,1}}{P}}, \ldots, \sqrt{\frac{2\varepsilon_{b,M}}{P}}]$$

$$\mathbb{b} = [b_1, \ldots, b_M]^T$$

$$\mathbb{\eta} = [\eta_1, \ldots, \eta_M]^T$$

While schematic of the receiver is given on the next page.

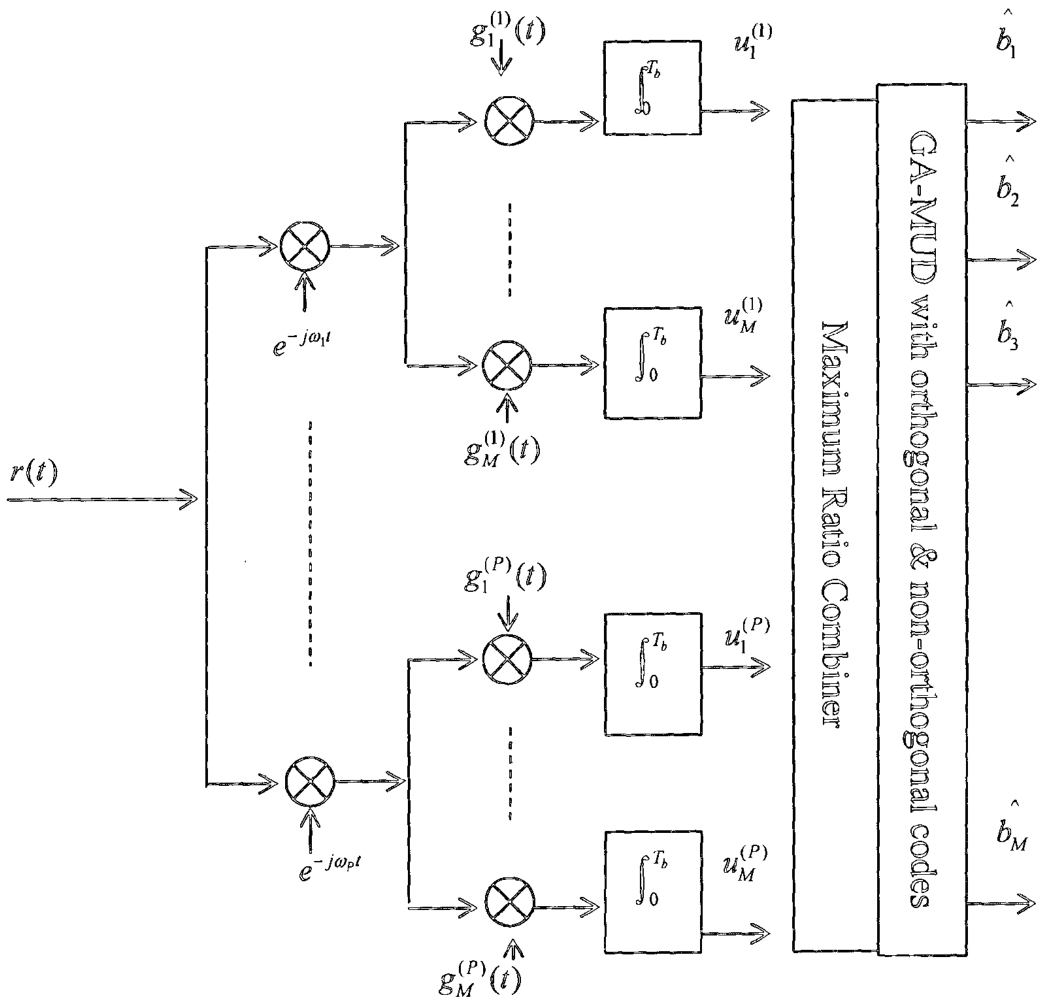



Figure 4.2 Schematic of the GA assisted MUD with MC-CDMA receiver

Based on Equation 4.9, the output vector $\mathbb{U}_p$ of the bank of matched filters displayed in Figure 4.2 can be formulated as:

$$\mathbb{U}_p = \mathbb{G}_p^T(r_p(t)) \tag{4.11}$$

$$= \mathbb{G}_p^T \mathbb{G}_p \mathbb{W}_p \mathbb{A}\mathbb{b} + \mathbb{G}_p^T \mathbb{\eta}$$

$$= \mathbb{R}_p \mathbb{W}_p \mathbb{A}\mathbb{b} + \mathbb{\eta}$$

Further that output of the matched filters is fed to a Maximum Ratio Combiner. Since same data is coming from different multicarriers.

Also in above equation the correlation matrix $\mathbb{R}$ gives the possible correlation between the codes being used. It can be represented as;

$$\mathbb{R}_p = \begin{pmatrix} \rho_{11}^{(p)} & \rho_{12}^{(p)} \cdots & \cdots \rho_{1M}^{(p)} \\ \rho_{21}^{(p)} & \rho_{22}^{(p)} \cdots & \cdots \rho_{2K}^{(p)} \\ \vdots & \vdots & \ddots \vdots \\ \rho_{M1}^{(p)} & \rho_{M2}^{(p)} \cdots & \cdots \rho_{MM}^{(p)} \end{pmatrix} \tag{4.12}$$

Here the elements $\rho_{jk}^{(p)}$ of the matrix $\mathbb{R}_p$ are the auto and cross correlation of the spreading code respectively, which can be expressed as:

$$\rho_{jk}^{(p)} = \int_0^{T_b} g_j^p(t) g_k^p(t) dt \tag{4.13}$$

In case of Walsh codes that matrix is simply turned to Identity, since all cross correlations ends in zero. That can be shown by changing the equation 4.13 to the following.

$$\rho_{jk}^{(p)} = \begin{bmatrix} \int_0^{T_b} g_j^p(t) g_k^p(t) dt = 1; j = k \\ 0; otherwise \end{bmatrix} \tag{4.14}$$

### 4.2.4 Detection

According to [22], the optimum multiuser detector of the *p*th subcarrier will maximize the following objective function:

$$J_p(\mathbb{b}) = 2\,\mathrm{Re}[\mathbb{b}^T \mathbb{A} \mathbb{W}_p^* \mathbb{U}_p] - \mathbb{b}^T \mathbb{A} \mathbb{W}_p \mathbb{R}_p \mathbb{W}_p^* \mathbb{A} \mathbb{b} \tag{4.15}$$

Here the superscript * indicates the conjugate of complex version of matrix. Therefore, combining the contributions of a total of *P* parallel subcarriers, the objective function to be maximized in the context of an optimum multiuser detected MC-CDMA system can be expressed as:

$$J(\mathbb{b}) = \sum_{p=1}^{P} J_p(\mathbb{b})$$

$$= \sum_{p=1}^{P} \{2\,\mathrm{Re}[\mathbb{b}^T \mathbb{A} \mathbb{W}_p^* \mathbb{U}_p] - \mathbb{b}^T \mathbb{A} \mathbb{W}_p \mathbb{R}_p \mathbb{W}_p^* \mathbb{A} \mathbb{b}\} \tag{4.16}$$

Hence the decision rule for Verdu's optimum CDMA multiuser detection scheme based on the maximum likelihood (ML) criterion is to choose the specific M-user bit combination $\mathbb{b}$, which maximizes the metric of Equation 4.14. Hence, we have to find:

$$\hat{\mathbb{b}} = \arg\left\{\max_{\mathbb{b}}[J(\mathbb{b})]\right\} \tag{4.17}$$

## 4.3 GA-assisted Detection

The maximization of Equation 4.14 is a combinational optimization problem, which requires an exhaustive search for each of the $2^M$ combination of $\mathbb{b}$, in order to find the one of that maximizes the metric of Equation 4.14. And in case of non binary symbols this computational complexity is even high. Hence the complexity will increase exponentially with increasing number of users.

Hence, I have utilized GA for finding a solution approximately near to that of optimum ML detector. Also since the orthogonal codes are not very practical so non-orthogonal codes are being utilized and effect of non-orthogonality is measured on the said scenario. Now to start GA we need some initial points and considerations. If we consider M users data as a single vector then that can be designated as the initial chromosome. As $\tilde{\mathbb{b}}_n(y) = [\tilde{b}_{n,1}(y), \ldots, \tilde{b}_{n,M}(y)]$, where $y$, $y = 1,\ldots,Y$ denotes the $y$th generation, and $n$, $n=1,2,\ldots N$ denotes the $n$th individual of the mating pool.

We received the signal from all subcarriers; sum them up using Maximum Ratio Combiner (MRC) and took it as initial chromosome. Then by mutating it in a special manner we get entire generation. The MRC-combined output vector $\hat{b}_{MRC}$ of the matched filter output can be expressed as: $\hat{\mathbb{b}}_{MRC} = [\hat{b}_{1,MRC}, \ldots, \hat{b}_{M,MRC}]$ where we have:

$$\hat{b}_{m,MRC} = \sum_{p=1}^{P} u_m^p \gamma_m^p e^{-j\phi_m^p} \tag{4.18}$$

Having generated $\hat{b}_{MRC}$, I adopted a 'mutated' version of the hard decision vector $\hat{b}_{MRC}$ for creating each individual in the initial population, where each bits of the MRC-vector is toggled according to the mutation probability used, in this case we utilized 0.1; means one of the 10 bits will be toggled. Hence, the first individual of the population namely $\tilde{b}_p(0)$ can be written as:

$$\tilde{b}_p(0) = MUTATION[\hat{b}_{MRC}] \tag{4.19}$$

So one can easily note that MUTATION is an operator, which when applied to a string of (1,-1), will produce toggled versions of initial vector.

## 4.4 Simulation Results

The basic parameters of GA used in our simulations are listed in the following paragraph.

The modulation scheme used in binary phase shift keying (BPSK), spreading codes utilized are both orthogonal (Walsh codes) and non-orthogonal (Gold sequences). Number of subcarriers P is used as 4, the length of subcarrier spreading signature L is (8, 31) for (Walsh, Gold) respectively. So the according coding gain for Walsh is LP=32 and for Gold is LP=124. GA's selection was based upon fitness value returned by the cost function. Mutation was used as standard binary mutation. Crossover methodologies are used as *cross over single point*, *crossover multi-point* etc. Mutation probability was 0.1 while crossover probability is 1.

From Figure 4.3 we can observe that the GA-assisted MUD's performance improves, when the population size $P$ increases. The difference between $P$ = 20 and 30 is more than an order while between 30 and 40 the difference in BER is exactly of an order for Walsh Code in higher SNRs. Similarly, in Gold Sequences population size plays a key role in reducing BER for SNR>13dB.

For example, for Signal to Noise Ratio (SNR) values below 15dB Bit Error Rate (BER) is significantly decreased for $K$=10 users, when evaluating the objective function of Equation 4.14, which imposes a complexity on order of $O$ $(P.Y)$ = $O$ (40.10) = $O$ (400) Furthermore, when the number of users $K$ is increased to 20, the GA assisted MUD has a complexity of $O$ $(P.Y)$ = $O$ (80.20) = $O$ (1600), as seen in Figure 4.5. Further results are obtained for $K$ = 20 number of users using Gold sequences, which are more practical.

These results can be seen in Figure 4.5 and 4.6. We can readily deduce that population size $P$ plays an important role to significantly reduce BER. Here almost same BER is achievable as for $K$ = 10 but at the cost of complexity. Even in this scenario Walsh Codes perform better due to their orthogonality. And for very high SNR like SNR>18 these are capable of achieving BER of $10^{-4}$.

Almost same effect of population size can be seen in Figure 4.7 for Gold sequences. In comparison to Walsh Code, the Gold sequences do not perform well but effect of GA can be seen for both codes. We can also observe that GA-assisted MUD is capable of significantly reducing the complexity of Verdu's optimum MUD. For example, the complexity was reduced by a factor of 1300, when the number of users was $M$=20.

It is very interesting to note that the overall complexity in Figure 4.4 and 4.6 is same for the cases $O$ (40.40) and $O$ (80.20) that is both are equal to $O$ (1600) but graph of $P$ = 80 in both figure shows a better performance.

Hence, for the number of users $M > 14$, population size $P$ dominates the effect of number of generations $Y$. Figure 4.7 demonstrates the complexity reduction factor versus number of users. This is because the increase in population size causes more crossovers and hence more parents are involved so the probability to find the optimum increases. An interesting fact can be seen here that with increase in complexity (though very small compare to optimum) we are able to achieve the same results that of optimum detector. For example in this figure below the complexity of 400 is giving good results and even good can be expected with 500 which is almost half of the optimum case which is 1024 for this case of 10 number of users.

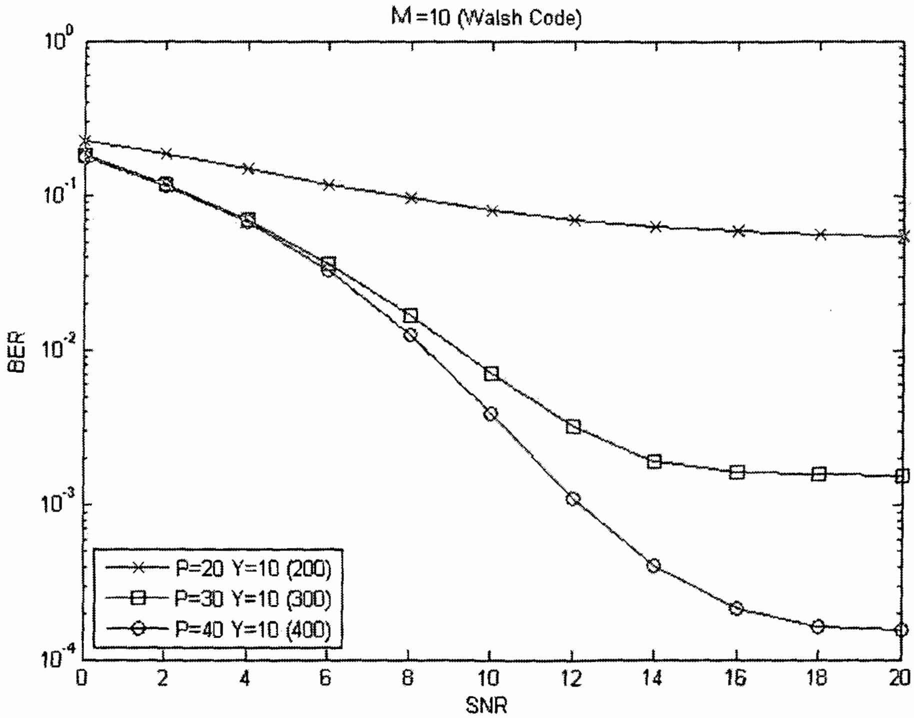



Figure 4.3 BER performance of the GA assisted MUD designed using a 32-chip Walsh code. The number of generations was $Y$=10 and the population size was $P$=20, 30 and 40.

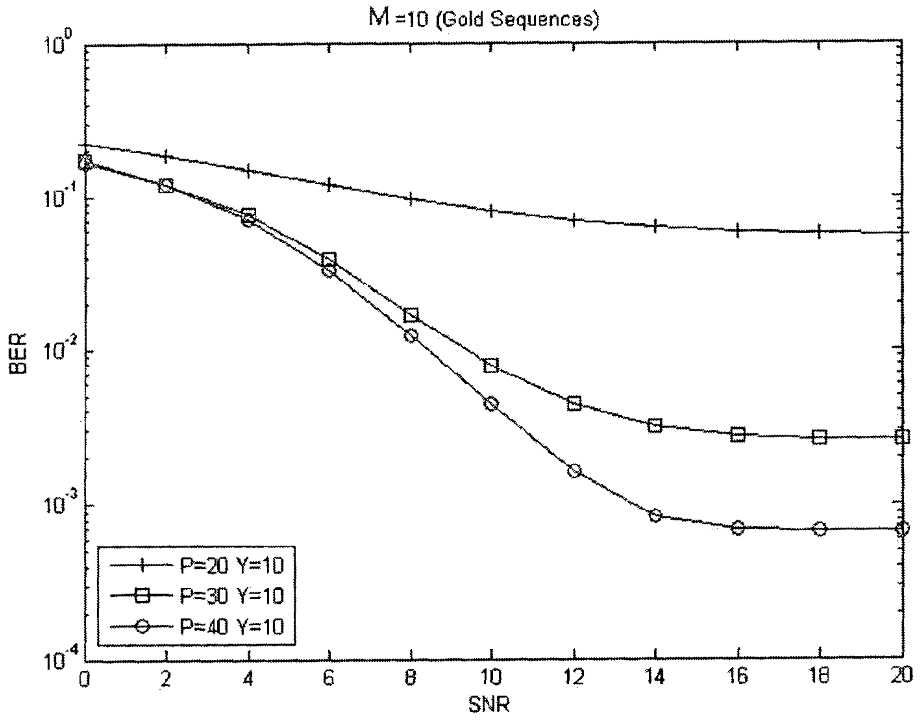



Figure 4.4 BER performance of the GA assisted MUD designed using a 124-chip Gold code with number of generations $Y$=10 and population size $P$= 20, 30 and 40.

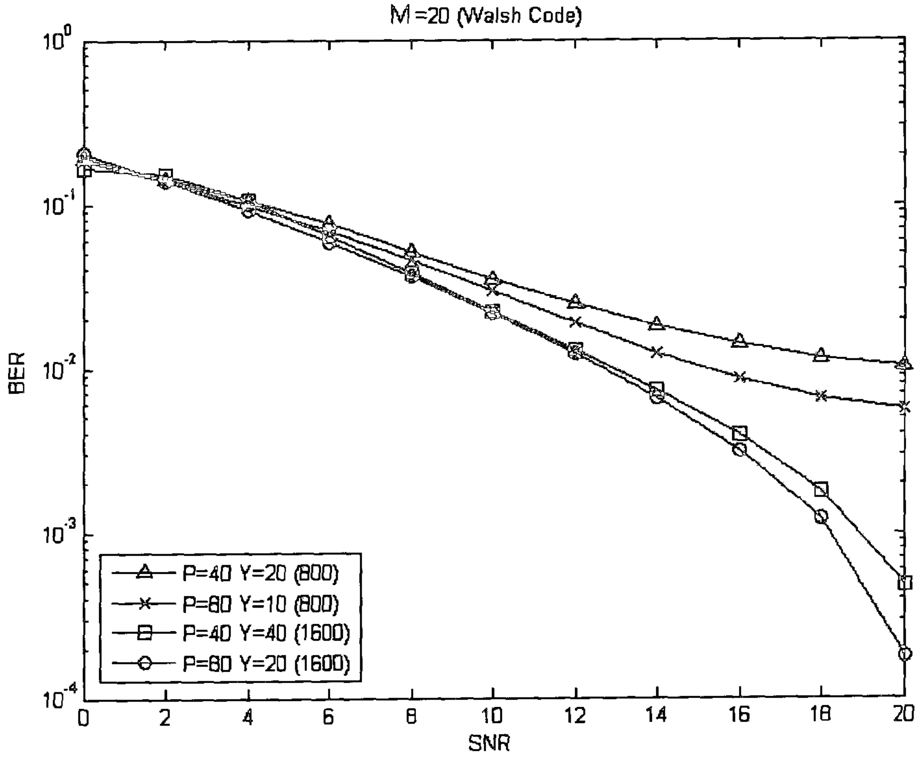



Figure 4.5 BER performance of the GA assisted MUD designed using a 32-chip Walsh code. The number of generations was $Y$=10, 20 and 40 and the population size was $P$= 40 and 80.

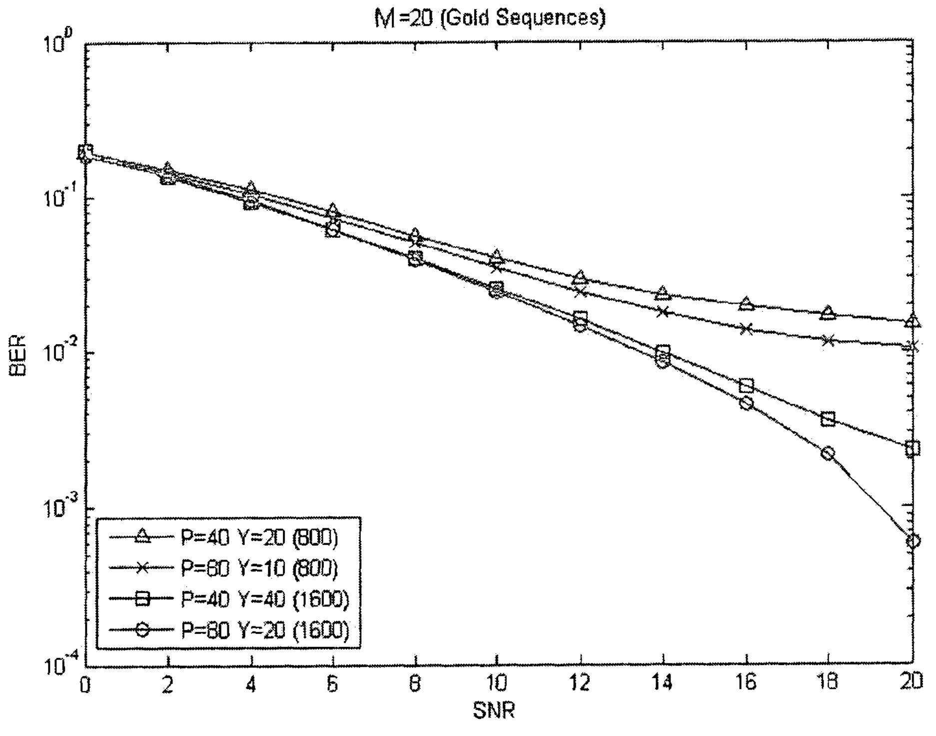



Figure 4.6 BER performance of the GA assisted MUD designed for a bit-synchronous MC-CDMA system, using a 124-chip Gold code.

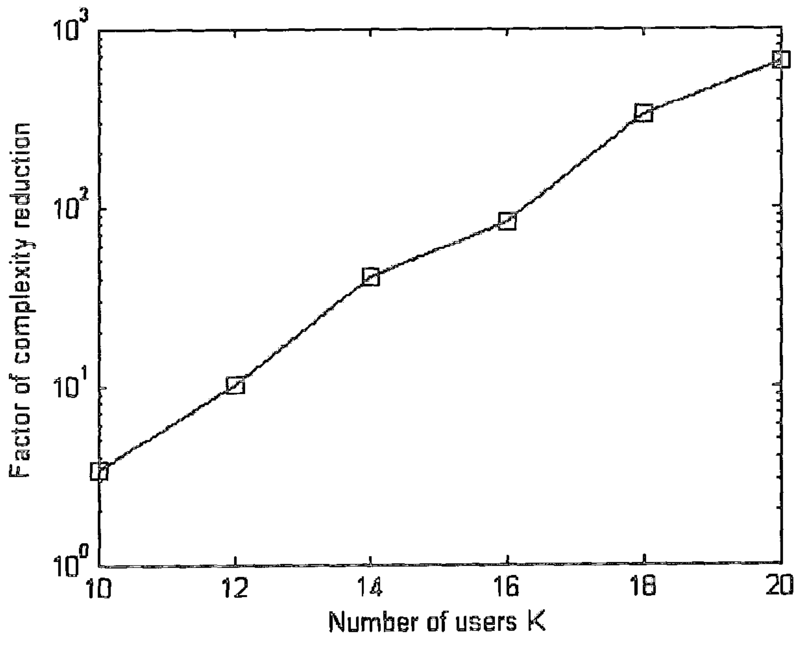



Figure 4.7 The complexity reduction factor of $\frac{2^K}{P \times Y}$ bounded at a BER of $10^{-3}$

# CHAPTER 5

## Conclusions

### Summary

In conclusion, the GA assisted MUD is capable of significantly reducing the detection complexity in comparison with optimum MUD, especially when the number of users supported is higher than $K$=15. Population size plays a key role in decreasing the BER instead of number of generations. Walsh code performs better compared to the Gold code being orthogonal in nature. In practical environment it is hard to find the orthogonal codes, so Gold sequences play an important role. For K=10 number of users no matter what spreading code is being used, the complexity reduction factor is 2.5 and for K=20 users it goes up to 800.

One thing is for sure that orthogonal codes possess good detection capability and less overlapping but hard to find in practice so non orthogonal codes like Gold and Kasami sequences are quite practical. Also with increasing number of users in orthogonal case, number of chips increased significantly so not suitable for systems with low chip-rates. The problem of detection in non-orthogonal codes can be mitigated by using smart detectors at receiver end.

This discussion further can be extended to sufficiently large number of users, change in crossover and mutation techniques as given in chapter 2 may vary the results in a positive direction and with further low complexity. This all was done in an un-coded fashion, further this work can be carried out with channel coding schemes like turbo codes, space time block codes with different transceivers. Also this scheme might be extended to uplink (asynchronous) scenario which is more practical.

For increased users where there is a problem of multi user interference (MUI) is quite high, same receiver can be used with interference cancellers like, parallel interference canceller (PIC), successive interference canceller (SIC)etc.

# References


[1] S.G. Glisic and P. A. Leppannen, *Wireless Communications-TDMA versus CDMA*. Kluwer Academic Publishers, August 1997.

[2] R. Prasad and S. Hara, "Overview of multi-carrier CDMA", in *Proceeding of the IEEE International Symposium on Spread Spectrum Techniques and Applications (ISSSTA),* (Mainz, Germany), pp. 107-114, 22-25 September 1996.

[3] L. Hanzo, L. L. Yang, E. L. Kuan, and K. Yen, *Single and Multi-Carrier DS-CDMA*. John Wiley and IEEE Press, 2003, 1060 pages.

[4] L. Hanzo, M. Munster, B. J. Choi, and T. Keller, *OFDM and MC-CDMA*. John and Wiley and IEEE Press, 2003.

[5] J. Bolgh and L. Hanzo, *Third-Generation Systems and Intelligent Networking,* John and Wiley and IEEE Press, 2002.

[6] R. Steele and L. Hanzo, *Mobile Radio Communications*. IEEE Press-John Wiley, 2 ed., 1999.

[7] J. G. Proakis, *Digital Communications*. Mc-Graw Hill International Editions, 3rd ed., 1999.

[8] R. Prasad and S. Hara, "Overview of Multicarrier CDMA", *IEEE Communications Magazine,* pp. 126-133, December 1997.

[9] L. L. Yang and L. Hanzo, "Software-Defined-Radio-Assisted Adaptive Broadband Frequency Hopping Multicarrier DS-CDMA", *IEEE Communications Magazine,* vol. 4, Mach 2002.

[10] A. Viterbi, *Principles of Spread Spectrum Communications*. Addison-Wesley, August 1995.

[11] R. Prasad, *CDMA for Wireless Personal Communications*. Artech House, Inc., 1996.

[12] L. Miller and J. S. Lee, *CDMA Systems Engineering Handbook*. London, UK: Artech House, 1998.

[13] ARIB/Japan, *Japan's Proposal for Candidate Radio Transmission Technology on IME-2000: W-CDMA*, June 1998.

[14] ETSI/SMG2, *The ETSI UMTS Terrestrial Radio Access (UTRA) ITU-R RTT Candidate Submission,* June 1998.

[15] TIA/US, *The cdma2000 ITU-R RTT Candidate Submission,* June 1998.

[16] CATT/China, *TD-SCDMA Radio Transmission Technology for IMT-2000*, June 1998.

[17] D. E. Goldberg, *Genetic Algorithms in Search, Optimization, and Machine Learning*. ISBN 0201157675, MA USA: Addison-Wesley, August 2001.

[18] M. Mitchell, *An Introduction to Genetic Algorithm*. Cambridge, Massachusetts: MIT Press, 1996.

[19] S. Verdu, *Multiuser Detection*. Cambridge Press, 1998.

[20] L. Hanzo, W. Webb, and T. Keller, *Single and Multi-Carrier Quadrature Amplitude Modulation: Principles and Applications fro Personal Communications, WLANs and Broadcasting*. John Wiley & Sons, Ltd, 2004.

[21] M. Mitchell, *An Introduction to Genetic Algorithms*. Cambridge, MA: MIT Press 1996.

[22] K.Yen and L.Hanzo, "Genetic Algorithm assisted joint multiuser symbol detection and fading channel estimation for synchronous CDMA systems", *IEEE Journel on Selected Areas in Communications,* vol. 19, pp. 985-998, June 2001.

[23] K. Yen and L. Hanzo, "Hybrid Genetic Algorithm Based Multiuser Detection Schemes for Synchronous CDMA Systems", in *Proceeding 51st IEEE Vehicular Technology Conference,* (Tokyo, Japan), pp. 1400-1404, 18 May 2000.

[24] M. Schnell and S. Kaiser, "Diversity Consideration for MC-CDMA Systems in Mobile Communications", in *Proceeding of IEEE ISSSTA 1996,* (Mainz, Germany), pp. 131-135, September 1996.

[25] S. L. Miller and B. J. Rainbolt, "MMSE Detection for Multicarrier CDMA", *IEEE Journal on Selected Areas in Communications,* vol. 18, pp. 2356-2362, November 2000.

[26] P. Zong, K. Wang, and Y. Bar-Ness, "Partial Sampling MMSE Interference Suppression in Asynchronous Multicarrier CDMA System", *IEEE Journal on Selected Areas in Communications*, vol. 19, pp 1605-1613, August 2001.

[27] W. C. Y. Lee, *Mobile Communications Engineering.* New York: McGraw-Hill, 2nd ed., 1998.

[28] R. Prasad and S. Hara, "Overview of multi-carrier CDMA", in *Proceeding of the IEEE International Symposium on Spread Spectrum Techniques and Applications (ISSSTA),* (Mainz, Germany), pp. 107-114, 22-25 September 1996.

[29] J. Bingham, "Multicarrier modulation for data transmission: An idea whose time has come", *IEEE Communications Magazine,* pp. 5-14, May 1990.

[30] X. Gui and T. S. Ng, "Performance of Asynchronous Orthogonal Multicarrier CDMA System in Frequency Selective Fading Channel", *IEEE Transactions on Communications*, vol. 47, pp. 1084-1091, July 1999.